

Essays in Macroeconomics and Political Economy

by

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Submitted to the Department of Economics
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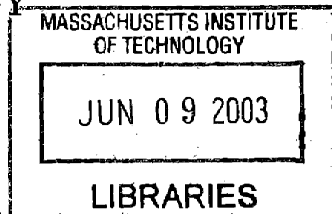
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Abstract

Chapter 1: This chapter provides an answer to the Bulow and Rogoff (1989) sovereign debt paradox based on a political economy model of debt. It shows that the presence of political uncertainty reduces the ability of a country to save, and hence to replicate the debt contract after default. In a model where different parties alternate in power, an incumbent party with a low probability of remaining in power has a high short-term discount rate and is therefore unwilling to save. The current incumbent party realizes that in the future whoever achieves power will be impatient as well, making the accumulation of assets unsustainable. Because of their inability to save, politicians demand debt ex-post and the desire to borrow again in the future enforces repayment today.

Chapter 2: In this chapter, I present a political economy model of government savings. Two political parties alternate in power every period. The party in power controls the government and decides how to allocate spending this period and how much to save for the future. No party has the ability to commit and at any point in time a party can spend all the income of the government in her own consumption and save nothing for the future. If both parties behave as previously described, then these strategies are the worst subgame perfect equilibria. However, parties are long run players in this political game, and they might be expected to coordinate and play more efficiently. I characterize the set of efficient subgame perfect equilibria.

Chapter 3: This chapter studies the implementation of new ideas by managers who choose between contracts offered by an existing firm and a competitive venture capitalist. Relying on existing assets makes implementation cheaper. But it also reduces contractual flexibility which is valuable in the presence of behavioral or informational frictions. To implement an idea, the incumbent firm has to pay the manager an amount that depends on the venture capitalist offer. Venture capital affects the innovation policy of incumbents by changing both the threat of new ideas and their price. The value of an incumbent firm

is endogenous and negatively related to the intensity of venture capital pressure. More innovative projects tend to be implemented in new ventures because of the importance of contractual flexibility.

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A mis viejos

Chapter 1

A Political Model of Sovereign Debt Repayment

The history of sovereign lending is characterized by three broad facts: governments have at times been able to borrow substantial amounts of funds from foreign entities; much of what they borrowed they eventually repaid; and repayment was often complicated, involving delay, renegotiation, public intervention and default¹.

Sovereign debt is fundamentally different from private debt because a government cannot use many things as collateral for a loan and the ability to take a government to court is extremely limited. This gives rise to the question: Why do sovereign debtors pay back their debts?

The oldest explanation of why countries repay is that they must maintain a good reputation in foreign financial markets to be able to borrow more in the future. Eaton and Gertsovitz (1981) formalize this idea in the context of a small country subject to income shocks². A defaulting government loses access to international credit markets. Default is costly because the country will not be able to smooth consumption later on. The desire to borrow in the future therefore induces the country to pay back its debts today.

This explanation is revisited by Bulow and Rogoff (1989). They show that if countries are able to save in rich asset markets, then reputation considerations alone cannot enforce repayment and countries will eventually default on any debt contract. The idea behind

¹Eaton and Fernandez (1995).

²Several authors have extended the reputation approach to sovereign lending. See for example Atkeson (1991), Grossman and van Huyck (1988), Worrall (1990).

their argument is simple and illuminating: in any debt contract there is a point in time where the value of the debt of a borrower country reaches (or is very close to) a maximum. At that point the country would default and start a sequence of savings in a way that perfectly replicates the original debt contract but generates extra income (the interest that is not repaid). This sequence of savings is possible as long as the international markets offer a menu of assets indexed on the same contingencies as the original debt contract. So, if international asset markets are rich enough, countries will always default on their debts. Bulow and Rogoff conclude: “loans to LDCs are possible only if the creditors have either political rights, which enable them to threaten the debtor’s interests outside its borrowing relationships, or legal rights”.

Several other explanations of why countries repay their debts have been proposed. Researchers have studied the possibility that reputation spillovers to other valuable relationships might be costly enough to enforce repayment (Cole and Kehoe (1995), (1996)). Another approach looks at the assets available to the country after default. Technological restrictions (Kletzer and Wright (2000)) or collusion among banks (Wright (2002)) might reduce the range of savings mechanisms available to the country after default. Another branch of the literature studies the punishments available to creditors, from military intervention to trade embargoes³.

This paper takes another look at reputation models of lending. I will argue that even when international financial markets are quite complete, political considerations restrain a country from implementing the saving sequence that the Bulow and Rogoff argument requires.

In order to arrive at this result the paper builds on the simple insight that politicians are not continuously in power. Because the nature of the political process does not assure the incumbent politician that he will be in power again tomorrow the politician is impatient. This impatience has already been used to explain politicians reluctance to save (Alesina and Tabellini (1990)). Incumbent politicians have a bias towards the present but this bias does not affect their discount rates between dates in the distant future. They are more patient in the distant future than they are today because the uncertainty over who will be in power in the future has yet to be revealed. They know however, that when tomorrow arrives, whoever is in power will be impatient in the short-

³In a very interesting paper, Rose (2002) has shown that after a country defaults, its international trade is significantly reduced, identifying a channel through which external creditors might be punishing the defaulting country.

run as well. This time-inconsistency can generate strong inefficiencies in the savings done by governments. I argue that it can also explain two seemingly contradictory facts:

- Politicians don't save and they spend too much.
- Most of the time, they pay back their debts even in the absence of punishments or clear political costs.

This paper shows that political uncertainty generates inefficient savings and makes the replication strategies of Bulow and Rogoff (1989) not possible. A political economy model with political turnover is presented and the equilibrium behaviour of the parties is characterized. It is shown that when political turnover is positive, parties tend to consume too much out of the stock of assets of the country.

In the Bulow and Rogoff (1989) argument the country is always better off by defaulting on any debt contract. The country can save in the asset markets and generate the same consumption allocation that the debt contract was generating (without having to pay back the interest rate). However, the presence of political uncertainty reduces the ability of the country to keep the assets around for long. The parties in power realize that if they were to default, future governments will inefficiently overspend and the country will run out of its assets too fast. Because they might be in power again in the future, this inefficient overspending lowers their utility today.

Debt reduces this inefficiency. The reason is that the parties in power can borrow from foreigners when the asset stock is low enough. This in turn implies that by repaying their debts, they do not have to keep a high stock of assets around for smoothing purposes (they can borrow when times are bad), and this reduces the temptation of future governments to inefficiently overspend from accumulated savings. This improvement in the allocation of resources can be valuable enough for the parties today to enforce repayment of previous debts.

The paper relates to the political economy literature on fiscal deficits (Alesina and Tabellini (1990), Persson and Svensson (1989)). However, these papers do not consider the possibility of default. Tabellini (1991), and Dixit and Londregan (2000) present models of sustainability of domestic debt⁴. In these models, the lenders are citizens and

⁴For other papers in intergenerational redistribution see Rotemberg (1990), Grossman and Helpman (1998) and Mulligan and Sala-i-Martin (1999).

thus have political rights (they can vote). I analyze a model of *sovereign* debt, where lenders reside outside the country and have no political rights.

The paper builds on the techniques developed by Harris and Laibson (2001) in their characterization of the hyperbolic consumer problem. These techniques proved very useful in the analysis of the political game.

Finally, the paper is related to recent work by Gul and Pesendorfer (2002). In their work, the authors develop a theory of preferences for commitment, providing a modeling alternative to the standard hyperbolic discounting framework. They study a consumer with these preferences and show how Bulow and Rogoff (1989)'s result might be overturned. Their model does not connect to political economy, as this paper does; and hence does not have clear empirical predictions for sovereign debt. The main motivation behind their work is the desire to eliminate the multiplicity of "selves" that appears in the hyperbolic discounting literature. While this is a desirable characteristic for an intra-personal game; in a political game the multiplicity of decision makers seems to be a much better approximation of reality.

The sequence of the paper is as follows. First, I setup the model without debt. The model consists of a small economy with different political parties subject to endowment and political shocks. I define the equilibrium and characterize some of its main properties. This is done in Section 1.2. Section 1.3 analyzes the model at the limit when the political shocks are very likely. For this case I have closed form solutions for the equilibrium and a uniqueness result. Section 1.4 introduces the possibility of borrowing from outsiders and characterizes the equilibrium behaviour of the parties when they have access to international lending. Section 1.5 presents the argument of debt sustainability. I characterize under which conditions the country repays its debts and under which it will default. I show that the argument of Bulow-Rogoff does not in general hold except in the particular case when there is no political turnover. Section 1.6 shows the differences in debt sustainability that should be expected across political systems. Section 1.7 concludes. Before entering into the model, I quickly review some of the relevant empirical literature.

1.1 Empirical Evidence

There were several historical instances where the accumulation of state surpluses was politically impossible. Cole, Dow and English (1994) report the interesting case of the

United States in the mid-1830s. At that time, the accumulation of a large federal surplus was controversial and at the end, the surplus was distributed to the states. The states did not hold the money for long, and spend or distributed it. Years later, Benjamin R. Curtis (a supreme court judge) specifically argued that a state's reputation in credit markets was important because U.S. states could not accumulate surpluses, and in an emergency they might need more resources than they could tax in a single year. Alexander Hamilton made a similar point in the case for repayment of the U.S. Revolutionary War debt.

In their analysis of international reserve-holding behavior of developing countries, Aizenman and Marion (2002) provide evidence that countries with higher political uncertainty (measured as the probability of a leadership change) tend to accumulate lower levels of reserves. Their argument is that higher political uncertainty reduces the optimal size of buffer stocks held by a government because it increases the opportunistic behavior of the policy maker.

Political uncertainty has been associated to other fiscal problems. For example, Cukierman, Edwards and Tabellini (1992) document the fact that higher political uncertainty is positively associated with seignorage. They argued that seignorage reflects the high costs of administering and enforcing the collection of regular taxes, but that the evolution of the tax structure of a country depends on the political system. When there is high political turnover, incumbent politicians might choose to maintain an inefficient tax system so as to constrain the behavior of future governments, which current incumbents might disagree with.

Political uncertainty tends to be associated with inefficient fiscal behavior in general. Governments seem to have a lower ability to save, a more inefficient tax system and more problems controlling spending, the higher the political uncertainty faced by incumbents is. In this paper I argue that these inefficiencies⁵, in particular the savings one, might be the reason why governments repay their foreign debts even in the absence of punishments or direct political costs.

⁵Lane (2000) does a detailed analysis of international lending to LDCs and finds evidence that better government anti-diversion policies (policies that reduce rent-seeking activities by politicians) are associated with lower amounts of sovereign borrowing. This negative relationship weakens as other controls are included, but surprisingly enough it never becomes positive.

1.2 The Model

In this section the political model is set up. I first analyze the equilibrium behavior of the political parties without debt. In later sections the possibility of sovereign lending is introduced.

Consider a small economy which has m political parties indexed by i . Each party has the following utility defined over the continuous flow of consumption provided by the government at every instant t .

$$U_0^i = E_0 \left[\int_0^\infty e^{-rt} u(c_t^i) dt \right]$$

where c_t^i is the consumption provided to party i at time t by the government.

Assume the CRRA utility representation

$$u(c) = c^{1-\rho}$$

with⁶ $0 < \rho < 1$.

For notation purposes, all stock variables are uppercase while flow variables are lowercase.

Every instant the party in power decides how much to provide to different parties and how much to save in an asset market.

How does the government finance its spending flows?

- With Poisson probability λ there is an endowment shock.
- Immediately after the shock, the country receives a stock Y of income.
- The rest of the time, the country does not receive any endowment and spends out of previous savings.

At any point in time a given party control the government. I proceed now to characterize how power is allocated among the different parties. The following simple political structure is assumed⁷:

- With Poisson probability γ there is a political shock.

⁶Under this condition ($0 < \rho < 1$) we have that $u(0) = 0$. This is important because parties do not always consume in the equilibrium, and utility comparisons cannot be made unless $u(0) > -\infty$.

⁷In a previous version of the paper we had a more general political structure. Ruling coalitions were selected out of the political parties, and these coalitions decided on spending and savings. A ruling coalition was constrained to provide the same consumption to the parties in the coalition. This structure generalized the set up but did not add anything to the results and made the notation cumbersome.

- Immediately after the shock, a party is randomly chosen to govern.
- The probability that a party is selected⁸ is $\alpha \in (0, 1)$.

The political shock and the endowment shock are assumed to be independent events.

Let p_t be the party in power at time t .

Notice that the parties consume only through government provision. This is the case if the government is the only entity that can provide the public goods that the parties desire (like roads, schools, dams, etc.).

Where do these political shocks come from? They could be the outcome of elections, strikes that force a change in government, variability in the bargaining power of the different parties in the coalition, the possibility of impeachments, or just the political breakdown of the ruling coalition. The current incumbents always face this risk and this uncertainty make them impatient about the future. The incumbents do not know for sure that they will be governing again tomorrow.

Assumption 1.1 *There is **political turnover**: $\gamma(1 - \alpha) > 0$ and $\alpha > 0$.*

From standard concave utility arguments the parties have a desire to smooth through time the consumption flow, and the ruling parties would like to save part of the government's income for the future. However, there is a chance that current incumbents won't be in power tomorrow to consume out of the savings they made today. This reduces the amount they save. Savings are shown to be inefficient from the perspective of all parties. Politicians consume too much out of their stock of assets.

The options available to the government for savings are specified in the next section.

1.2.1 (Cash-in-Advance) Asset Market

There is a foreign spot asset market populated by foreign investors that are risk neutral and share the same discount rate r . The government can save in the foreign spot asset market.

There is a riskless bond that returns a constant flow of r (note that parties and the outsiders share the **same discount factor**). Let B_t denote the holdings on the bond a country has at any time t .

⁸We will think of α as continuous from $(0,1)$. One can reinterpret this political set up as a coalitional set up where α could take any value. See previous footnote.

The other relevant asset is a Lucas tree. The return on this tree is assumed to be contingent on the realization of the country's endowment shock. The reason why this asset is introduced is the following⁹. The Bulow-Rogoff argument says that if a country has access to an asset market that provides contracts (assets) indexed by the same contingencies as a debt contract, then the country will at some point in time default on this debt contract and will start saving in the asset market. The main point of this paper is to show how the ability of the country to sustain debt changes when political risk is introduced into a model where the Bulow-Rogoff result would otherwise hold (something that is expected to happen with the introduction of this contingent asset).

Let A_t be the holdings of this tree at time t and A_{t+dt} be the holdings of the tree at time $t + dt$. Then

$$A_{t+dt} = \begin{cases} 0 & ; \text{ if the endowment shock happens} \\ (1 + (r + \lambda) dt) A_t & ; \text{ if the endowment shock doesn't happen} \end{cases}$$

The return from holding the tree is $r + \lambda$ when there is no endowment shock¹⁰.

This contingent Lucas tree could be thought of as the result of the ability of foreign investors to make instantaneous commitments contingent on the aggregate endowment shock. In any interval Δt , the foreign investors can write a short term contract (that lasts only one Δt period) contingent on the realization of the endowment shock. Competition on the investors' side will tie the return on the asset to the zero profit condition. As Δt goes to zero the instantaneous Lucas tree is obtained.

The assumption underlying this contingent Lucas tree is that foreigners have a commitment technology that allows them to credibly offer these instantaneous cash-in-advance contracts to the country. These are contracts where the country pays upfront and receives non-negative payments ex-post. The case of interest is when the country *cannot* commit to repay its past obligations with the foreigners and can default on a contract that requires it to make a payment ex-post. I first study the case when the government cannot borrow from foreign investors and I analyze the ability to repay in later sections.

⁹To the reader familiar with the Bulow and Rogoff (1989) argument, this Lucas tree represents the cash in advance contracts available to a country upon default.

¹⁰Why is the return $r + \lambda$? Let W be the value of holding T units of the contingent tree for a risk neutral investor. W is given by the value equation $rW = \hat{r}T + \lambda(-T)$; where \hat{r} is the return on the tree. Given that the asset is unit priced and a zero profit condition holds we have that $W = T$ (the value of holding the T units of the tree for a foreign investor is equal to its price). From this we obtain that $\hat{r} = r + \lambda$.

Let W_t denote the total asset income of the country. The following is assumed

Assumption 1.2 *The country cannot short the riskless bond : $A_t \leq W_t$*

$B_t = W_t - A_t$ represents the investments done in the riskfree bond at time t . Then, the amount that is invested in the riskless bond, B_t , cannot be negative. This assumption will be relaxed in section 1.4 once the possibility of borrowing from abroad is introduced.

Summing up, the country is subject to two shocks: endowment and political shocks. The endowment shocks create a desire for smoothing the government provision flow. In the next section I characterize how the political forces interact with the smoothing needs of the parties.

1.2.2 The Political Equilibrium

Let a provision profile \hat{c}_t be the vector of spending allocations to every party at time t , $\hat{c}_t = (c_t^1, c_t^2, \dots, c_t^m)$. Any point in time is characterized then by the vector $h_t = \{p_t, \hat{c}_t, A_t\}$ detailing the ruling party, the spending allocation done and the savings (portfolio) decision A_t . A history is then a correspondence from the $[0, T)$ to possible vectors h_t .

A strategy for party i at time t is a mapping from all possible histories up to time t and states at time t where party i is in power to instantaneous spending allocations and savings decisions.

The nature of this dynamic game allows for multiple subgame perfect equilibria. These equilibria are difficult to characterize, and in a related paper (Amador (2002a)) I study the asset management when the parties are constrained to play only equilibria in the pareto frontier of the perfect payoff set. In this paper, however, I will use a different characterization.

Definition 1.1 *A **Stationary Markov strategy** for party i is a profile of consumption $\hat{c}_i(W) = (c_i^1(W), c_i^2(W), \dots, c_i^m(W))$ and an investment function $A_i(W)$.*

The correspondence $\hat{c}_i(W)$ determines the consumption allocation to all parties that party i will choose if she is in power at some time with an asset level of W . A consumption allocation $c_i^j(W)$ is the consumption party i will provide to party j if she were in power with W assets. The function $A_i(W)$ details how much of the savings are done in the contingent Lucas tree if party i is in power with W assets.

The following is assumed

Assumption 1.3 *Parties play only stationary Markov strategies.*

So, politicians only play strategies that are a function of the payoff relevant variables at a given point in time. There is no reputation building under this assumption. Whatever happened in the past that does not affect the income of today and the future does not matter for the politicians' behaviour. The paper focuses on Markov Perfect Equilibria, i.e. subgame perfect equilibria that use Markov strategies.

Suppose that party i is the ruling party. Let t_1 be the time when the first political shock arrives after $t = 0$. Then, the utility of party i today is:

$$V(W_0) = E_{t_0} \left[\int_0^{t_1} e^{-rt} u(c_i^i(W(t))) dt + e^{-rt_1} [\alpha V(W(t_1)) + (1 - \alpha) V_o(W(t_1))] \right]$$

where V is the expected utility of the incumbent at time 0 with asset level W_0 and V_o is the expected utility of the party once out of power. This equation tells us that the party in power consumes up to the time when the political shock arrives (t_1). At that time, with probability α the party in power remains as the incumbent and receives a value of V ; and with probability $(1 - \alpha)$ she is out of power and receives a value V_o . Below, I will be more specific about the latter value function.

The party i faces (in the absence of an endowment shock) the following instantaneous budget constraint

$$dW = \left((r(W - A_i) + (r + \lambda) A_i) - c_i^i - \sum_{j \neq i} c_i^j \right) dt$$

and if $W = 0$, then $c_i^j = 0$ for all j .

To understand this equation, notice that $r(W - A_i) + (r + \lambda) A_i$ is the return on the asset holdings if A_i is the amount of wealth invested in the contingent Lucas tree: the return on the riskless bond is $r(W - A)$ and the return on the contingent tree is $(r + \lambda)A$. And $c_i^i(W) + \sum_{j \neq i} c_i^j(W)$ is the total spending flow done by the government. The change in wealth (dW) is then the return from holding the assets minus the spending done in that instant. When there is an endowment shock, the asset level W_t will jump to $(Y + W - A)$.

Once a political shock arrives, only total wealth at that time matters. Sharing of the spending in the past is irrelevant for future play. So, the party in power today maximizes $V(W)$ and for a given total amount of spending spends everything on its own

consumption. There is no reason to share with the outsiders if tomorrow's play will not be affected by it. The following proposition states this.

Proposition 1.1 *In a Markov Equilibrium, for any $i \neq j$, $c_i^j = 0$ for almost all W .*

This reduces the level of complexity of the problem considerably. The party in power always gives zero provision to the outsiders. The only decision left to be made at time t is the amount of total spending c_t , subject to the budget constraint. The symmetry of the game implies that any party in power at time t faces exactly the same problem. I concentrate then on symmetric equilibria.

Definition 1.2 *A control $x = (c, A)$ is **feasible** if c and A are such*

- $c : [0, \infty) \rightarrow [0, \infty)$ with $c(0) = 0$
- $A : [0, \infty) \rightarrow (-\infty, \infty)$ with $A(W) \leq W$

This definition tell us that a control is feasible if it satisfies the short-sale constraint ($A(W) \leq W$) and the parties cannot consume when there is no wealth ($c(0) = 0$).

Given a control x the evolution of wealth is defined as follows.

Definition 1.3 *For a given control $x = (c, A)$, **wealth evolves according to x from W_0** if when the endowment shock does not occur the asset level of the country follows*

$$dW = (rW + \lambda A(W) - c(W)) dt \quad (1.1)$$

where $W(t)$ jumps to $(Y + W(t) - A(W(t)))$ when the endowment shock occurs; and $W(0) = W_0$.

For a given control (c, A) , every instant the country receives a flow of income equal to $rW + \lambda A$; where rW is the return from all the asset holdings, and λA is the extra return from the holdings of the Lucas tree. This flow of income minus the consumption flow is equal to the instantaneous change in wealth.

Let W_y^x denote the case when W is evolving according to x from y .

For two feasible controls $x = (c, A)$ and $x_1 = (c_1, A_1)$ the value $V(W|x_1, x)$ is defined to be the expected value for an incumbent party if she follows (c_1, A_1) and everybody

else follows (c, A)

$$V(W_0|x_1, x) = E \left[\int_0^{t_1} e^{-rt} u(c_1(W_{W_0}^{x_1}(t))) dt + e^{-rt_1} (\alpha V(W_{W_0}^{x_1}(t_1)|x_1, x) + (1 - \alpha) V_o(W_{W_0}^{x_1}(t_1)|x_1, x)) \right]$$

where

$$V_o(W_0|x_1, x) = E [e^{-rt_1} [\alpha V(W_{W_0}^x(t_1)|x_1, x) + (1 - \alpha) V_o(W_{W_0}^x(t_1)|x_1, x)]]$$

and where t_1 is the first time after $t = 0$ when a political shock arrives.

The value function $V(\cdot|x_1, x)$ captures the value for an incumbent of following the strategy x_1 when everybody else follows x . The incumbent consumes c_1 as long as she is in power and wealth is evolving according to her strategy (x_1) . When a political shock arrives (at t_1), with probability α the current incumbent remains in power, and with probability $(1 - \alpha)$, she is out of office. Once out of office, she receives a utility level captured by the value function $V_o(\cdot|x_1, x)$. This value function tells us that the party out of power does not consume while out of government and wealth is evolving according to the strategies of the other parties (x) . Once a political shock arrives, with probability α the party becomes the incumbent, and with probability $1 - \alpha$, she remains an outsider.

A Symmetric Markov Equilibrium is defined as follows:

Definition 1.4 *A Symmetric Markov Equilibrium is a feasible control $x^* = (c^*, A^*)$ such that for any other feasible control $x = (c, A)$,*

$$V(W|x^*, x^*) \geq V(W|x, x^*) \quad ; \text{for } W \in [0, \infty) \quad (1.2)$$

for all $W \in [0, \infty)$

The definition tells us that a symmetric Markov equilibrium is characterized by two functions c^* and A^* such that for any other feasible functions c and A the value generated by following the first strategies is at least as high as the value generated by following c and A while the given party is in power and when everybody else follows c^* and A^* .

Notice that a symmetric Markov equilibrium as defined above is subgame perfect. It is not difficult to show that a best response to a Markov strategy is also Markov. So, equation (1.2) is enough for perfection.

In general there may be many solutions to (1.2). The method used to select among equilibria is the technique developed by Harris and Laibson (2001) in their study of a hyperbolic consumer problem. The technique is described in detail in the Appendix I. The reason why this technique can be used is that a solution to (1.2) is equivalent to a Markov solution of a well-chosen hyperbolic program.

The Hyperbolic Equivalence

To show this result, notice that $\exists W \in [0, \infty)$ such that $V(W|x_1, x) > V(W|x, x)$ if and only if

$$E \left[\int_0^{t_1} e^{-rt} u(c_1(W_{W_0}^{x_1}(t))) dt + e^{-rt_1} (\alpha V(W_{W_0}^{x_1}(t_1)|x, x) + (1 - \alpha) V_0(W_{W_0}^{x_1}(t_1)|x, x)) \right] > V(W_0|x, x) \quad (1.3)$$

for some $W_0 \in [0, \infty)$, where the main difference between (1.3) and (1.2) is that (1.3) considers only deviations by incumbents during the period before the *first* political shock arrives.

Now, let

$$J(W|x) = \frac{1}{\alpha} [\alpha V(W|x, x) + (1 - \alpha) V_0(W|x, x)]$$

Using the value functions and solving out,

$$J(W_0|x) = E \left[\int_0^{\infty} e^{-rt} u(c(W_{W_0}^x(t))) \right]$$

So, $J(W|x)$ is the utility of a party that uses a control x and is continuously in power.

Let the value function \tilde{V} be defined as

$$\tilde{V}(W_0|x_1, x) = E \left[\int_0^{t_1} e^{-rt} u(c_1(W_{W_0}^{x_1}(t))) dt + e^{-rt_1} (\alpha J(W_{W_0}^{x_1}(t_1)|x)) \right]$$

Notice that \tilde{V} corresponds to the left hand side of (1.3). Then, x^* is a Symmetric Markov Equilibrium if for any feasible control x ,

$$V(W|x^*, x^*) \geq \tilde{V}(W|x, x^*)$$

for $W \in [0, \infty)$

The value function¹¹ \tilde{V} is equivalent to the value function of a hyperbolic consumer which faces a vanishing “present” (from 0 to t_1), and discounts all the “future” (from t_1 onwards) by $\alpha < 1$. The techniques of Harris and Laibson (2001) can then be used to characterize the political equilibrium. For expositional purposes, the main body of the paper presents only the results and refrains from developing the techniques in detail (this is done in the appendix).

The Bellman System

The following proposition states the associated Bellman equation for a Markov equilibrium¹².

Proposition 1.2 *A Symmetric Markov Equilibrium is a feasible control $x^* = (c^*, A^*)$ such that $\exists V, V_o$ where (c^*, A^*) solve*

$$rV(W) = \max_{c,A} u(c) + \lambda [V(Y + W - A) - V(W)] + V'(W)(rW + \lambda A - c) + \gamma(1 - \alpha)(V_o(W) - V(W)) \quad (1.4)$$

where V_o is given by

$$rV_o(W) = \lambda [V_o(Y + W - A) - V_o(W)] + V'_o(W)(rW + \lambda A - c) + \gamma\alpha(V(W) - V_o(W)) \quad (1.5)$$

The two value functions V and V_o capture the expected utility for a party in or out of power respectively. The Bellman equations have a very intuitive expected utility interpretation:

The first equation tell us that the current utility flow for a party in power is equal to the consumption flow she receives plus the probability that the endowment shock arrives (λ) times the corresponding change in the value function ($V(Y + W - A) - V(W)$), plus the change in value due to accumulation or decumulation of the asset stock ($V'(W)(rW + \lambda A - c)$) and plus the probability that a political shock arrives and the

¹¹Notice that $\tilde{V}(W|x, x) = V(W|x, x)$

¹²See Appendix for a formal derivation.

current incumbent is not in power anymore ($\gamma(1 - \alpha)$) times the corresponding change in value ($V_o(W) - V(W)$).

The second equation tell us that the current utility flow of being out of power is equal to the probability that an endowment shock arrives (λ) and the asset level moves from W to $Y + W - A$ times the corresponding change in value [$V_o(Y + W - A) - V_o(W)$], plus the change in value due to accumulation or decumulation of the asset stock ($V'_o(W)(rW + \lambda A - c)$) and plus the probability that a political shock arrives and the party is in power ($\gamma\alpha$) times the corresponding change in value ($V(W) - V_o(W)$).

The main difference between being in power and not is two fold :

- The parties not in power do not receive a government provision.
- The parties not in power have no decision to make, while the party in power selects the spending flow for the instant.

Taking the first order condition of the system with respect to c , when $W > 0$, the following equation is obtained:

$$u'(c^*) = V'(W)$$

The current flow of spending is only constrained when $W = 0$. For any $W > 0$, consumption is unconstrained and the first order condition will hold with equality. This first order condition is also sufficient for optimality, because W is fixed at any instant and u is concave by assumption. The condition says that the marginal utility of consumption is equal to the marginal value of wealth. Differentiating this equation with respect to the state variable

$$u''(c^*(W))c^{*'}(W) = V''(W)$$

By concavity of u , we have that $u'' < 0$. As long as spending is monotonically increasing in W , $c^{*'}(W) > 0$, the value function is also concave in W .

Taking the first order condition with respect to A (for $W > 0$):

$$V'(Y + W - A) \leq V'(W)$$

with equality for $A < W$.

Suppose now that $W \leq Y$. In this case for any $A < W$, $Y + W - A > W$. If the value function is strictly concave (if $c^{*'}(W) > 0$) then $V'(Y + W - A) < V'(W)$. This does not satisfy the first order condition (which holds with equality for $A < W$), so A has to be W : when $W \leq Y$, then $A^* = W$, i.e. all the savings are done in the contingent tree.

When $W > Y$, then from the first order condition $A^* = Y$. The following result obtains

Result 1.1 *If $c'(W) > 0$ for all $W > 0$, then the following holds in equilibrium*

$$A^*(W) = \begin{cases} W & \text{for all } W \leq Y \\ Y & \text{for all } W > Y \end{cases} \quad (1.6)$$

Notice that A^* does not depend on α or γ . The instantaneous portfolio decision is not affected by the political distortions. The political distortions affect the aggregate level of consumption and eventually whether or not W is less than Y , but not the way assets are allocated at any instant for a given W . This means that there are inefficiencies not because the incumbent is not doing the savings *right* in the sense that it is not using the asset market efficiently but rather that it will be consuming too much. This will become clear when I analyze the efficiency of the political equilibrium, which is done in the next section.

1.2.3 Constrained Efficiency

In this section I study the constrained efficient solution to the savings problem.

If there were no political turnover, ($\gamma(1-\alpha) = 0$), the incumbent party maximizes a standard exponential problem. However, when there is political turnover ($\gamma > \gamma(1-\alpha) > 0$) a Markov equilibrium is clearly inefficient. Parties that are not in power do not receive any provision allocation from the government. I call this inefficiency the “sharing” inefficiency. There is however another inefficiency. The perceived return on the savings by the party in power today has been reduced because of political risk. Incumbents save too little. I call this the “savings” inefficiency.

Why does the savings inefficiency happen? Suppose that the country has just received a political shock but the uncertainty about the ruling party has not yet been realized. From the perspective of all parties, they all have the same probability (α) of being in power, so they are identical. I ask the question: assuming that only the political winners receive a government provision, if the parties could commit to a provision rate (without knowing who among them will be in power) what rate would they pick? The parties maximize their ex-ante value (before knowing who will be in power).

Recall that $J = \frac{1}{\alpha} [\alpha V + (1-\alpha) V_o]$. The ex-ante value (under commitment) is then given by $\alpha J^C = \alpha V^C + (1-\alpha) V_o^C$. The optimal commitment solution (c^C, A^C) is such

that

$$rJ^C(W) = \max_{c,A} u(c) + \lambda [J^C(Y + W - A) - J^C(W)] + J^{C'}(W)(rW + \lambda A - c) \quad (1.7)$$

This program is a standard exponential program. The consumption rate that all parties would like to commit to before the uncertainty about the political shock is realized is exactly the same as the one that a party continuously in power would pick. This is a constrained efficient result: it is the rate that a central planner constrained to provide consumption flows only to parties in power would pick. The following holds

$$c^C(W) = \begin{cases} (r + \lambda)W & ; \text{for } W \in (0, Y] \\ rW + \lambda Y & ; \text{for } W > Y \end{cases} \quad (1.8)$$

The intuition for this result is very simple. Because the interest rate of the assets and the discount rate of the parties are the same, the optimal spending is to maintain the level of wealth constant across time and states of nature. Given that there is a borrowing constraint, this desire implies that the consumption flow is $(r + \lambda)W$ whenever wealth is below Y (where $(r + \lambda)W$ is the return from holding all the assets in the contingent tree). And the consumption flow is $rW + \lambda Y$ when wealth is above Y , where this is the return from holding the assets in this case (Y is now the amount invested in the contingent tree). The constrained efficient solution is then to maintain the asset level. Notice that this will also be the aggregate spending that an unconstrained social planner will pick (a social planner that provides consumption to all parties, irrespective of whether they are in power or not).

Political uncertainty distorts the savings decision. Once the uncertainty about the political shock is realized, the incumbents choose (as it is shown in the next sections) a provision flow equal to $c^*(W) > c^C(W)$. There is too much spending from the ex-ante perspective of all parties. Notice that the distortion of savings (savings inefficiency) is the outcome of the inability of the parties to share the intra-instant provision flows (sharing inefficiency), but it is different in nature. For example, this savings inefficiency can clearly be reduced if the government has access to an illiquid savings technology. Even if the illiquid technology could do nothing to improve the sharing within a given instant it would constrain the parties to an aggregate consumption flow that is smaller than the one they would otherwise choose. However, in the current set-up, all assets available to the government are assumed to be completely liquid.

1.2.4 A Description of the Equilibrium

Suppose for now that the value function of being in power is concave. For the particular case of $W \leq Y$, the following results hold

Proposition 1.3 *For any $\alpha < 1$,*

$$c^*(W) > c^C(W) = (r + \lambda)W$$

For $\alpha < 1$, politicians are consuming faster out of the asset stock than a central planner would. This means that starting from $W \leq Y$, the wealth process never leaves the interval $[0, Y]$.

Politicians also consume faster the higher is the political uncertainty.

Proposition 1.4 *For any $\alpha < 1$,*

$$\begin{aligned} \frac{dc^*(W)}{d\gamma} &> 0 \\ \frac{dc^*(W)}{d\alpha} &< 0 \end{aligned}$$

The higher is the probability of a political shock (γ) the faster the incumbent will run out of asset holdings.

The higher is the political risk (lower α), the faster the incumbent will run out of asset holdings.

This description of the equilibrium will be valid as long as the value function V is concave for all the domain of W ($W \in [0, \infty)$). However, in general there might be cases where $c'(W) < 0$ and hence V will be convex for certain values of W . This implies that the optimal portfolio decision A isn't in general continuous or monotonic in W . To be able to generalize the intuitions of this section, the techniques developed by Harris and Laibson (2001) in their study of the hyperbolic consumer when the "present" vanishes away are used. In my case, this implies the study of the economy as the political shock becomes very likely ($\gamma \rightarrow \infty$).

1.3 The Limit Economy

In this section the equilibrium as $\gamma \rightarrow \infty$, or when there is a political shock every instant is studied. See Appendix I for a full derivation of the setup. I call the limit policy functions the policy functions of the limit economy.

This rest of the paper exploits the continuous time setup. It is possible to obtain closed form solutions for the equilibrium and we can compute comparative statics.

There are two cases to consider.

1.3.1 Case 1: $\alpha + \rho - 1 > 0$

Suppose that $\alpha + \rho - 1 > 0$. This assumption is satisfied when the political uncertainty and the elasticity of substitution are sufficiently small.

The first fundamental result is

Proposition 1.5 *In the limit economy, the value function of being in power is concave.*

This proposition tell us that equation (1.6) characterizes the optimal investment strategy.

The following proposition is obtained.

Proposition 1.6 *In the limit economy, the value function of being in power is, for $W \leq Y$:*

$$rV_{\infty}(W) = \frac{r}{r + \lambda} \psi^{\rho} [(r + \lambda) W]^{1-\rho} + \frac{\lambda}{r + \lambda} \psi^{\rho} [(r + \lambda) Y]^{1-\rho}$$

for $W > Y$:

$$rV_{\infty}(W) = \psi^{\rho} (rW + \lambda Y)^{1-\rho}$$

and the consumption flow is

$$c^*(W) = \begin{cases} \psi^{-1} (r + \lambda) W & ; \text{for } W \in (0, Y] \\ \psi^{-1} (rW + \lambda Y) & ; \text{for } W > Y \end{cases}$$

where $\psi = \frac{\alpha + \rho - 1}{\rho} < 1$.

We know from section 1.2.3 that once there is a political shock, the efficient level of spending is $c^C(W) = (r + \lambda) W$ for $W \leq Y$ and $c^C(W) = rW + \lambda Y$ for $W > Y$. In the limit economy, political shocks happen at every instant, and it is not surprising that

the optimal consumption level is $c^C(W)$. However, the lack of commitment generates inefficiencies in the limit economy in the way the assets are managed. There is too much current provision.

Corollary 1.1 *In limit economy, because parties cannot commit to a given $c(W)$, the government spends too much : $c^*(W) > c^C(W)$.*

A number of comparative statics of the limit economy are worth noting. First,

$$\frac{\partial c^*(W)}{\partial \alpha} < 0$$

As the probability of being elected to power diminishes, the parties in power spend more and save less.

Notice also that

$$\frac{\partial c^*(W)}{\partial \rho} < 0$$

As the elasticity of substitution increases $\left(\sigma = \frac{1}{\rho}\right)$, the parties consume faster out of the stock of assets. Notice that the second-best policy calls for a constant spending flow which is independent of ρ and α . These results confirm the intuition that the inefficiency that is created from the political risk is amplified when the probability of being elected is reduced and when the intertemporal elasticity of substitution is increased. In the first case, as the probability of being in power in the future decreases, the incumbent today cares less about the future and consumes at a faster rate. And, in the second case as the elasticity of substitution increases, the parties have less incentive to smooth their consumption flow, and hence will consume more today.

The parties will eventually deplete the asset stock for any $\alpha < 1$. The following proposition states this.

Proposition 1.7 *For any given $w > 0$ and $W_t > 0$,*

$$\lim_{T \rightarrow \infty} \Pr \left(\inf_{\tau \in [t, T]} \{W_\tau\} < w \right) = 1$$

The country will find itself in equilibrium with practically no assets.

I now consider the other case.

1.3.2 Case 2: $\alpha + \rho - 1 < 0$

The previous results on the savings equilibrium were obtained under the assumption that $\alpha + \rho - 1 > 0$. Now the case when $\alpha + \rho - 1 < 0$ is analyzed.

In this case, the limit economy as previously shown is not well defined. In particular, the previous results were based on the fact that the consumption rate converges to a finite value as γ tends to infinite. However, in the case when $\alpha + \rho - 1 < 0$ this can be shown not to be true.

Proposition 1.8 *If $\alpha + \rho - 1 < 0$ the consumption function $c^*(W)$ is such that*

$$\lim_{\gamma \rightarrow \infty} c^*(W) = \infty$$

As $\gamma \rightarrow \infty$ the ruling party spends faster and faster out of the government's stock of assets. The value functions converge to zero. In the limit, everything is spent in the instant a party takes power. The political risk has a dramatic impact on savings.

Proposition 1.9 *If $\alpha + \rho - 1 < 0$ the associated value function $V(W)$ satisfies that*

$$\lim_{\gamma \rightarrow \infty} rV(W) = 0$$

The reason why this is so is that the increase in the consumption rate lasts only for an instant dt . The assets are depleted during that instant and the provision is zero thereafter until an endowment shock arrives¹³.

This proposition can be link to proposition (1.7). It is an extreme version of that result. In case 1, I showed that the parties will deplete the asset stock in the absence of endowment shocks. Here a similar result holds, except that it is happening at a much faster rate. The parties deplete the asset stock in a single instant.

Remark: *Even when there are no savings done in equilibrium, the parties in power **do not** put a zero discount factor on the future. They still care about the future, because there is a significant probability ($\alpha > 0$) that they might come to power again. However,*

¹³The party in power is consuming a stock W in a very short time. As an approximation, the utility she gets is $u\left(\frac{W}{\Delta t}\right) \Delta t$, as $\Delta t \rightarrow 0$. If the utility were bounded then it is clear that $u\left(\frac{W}{\Delta t}\right) \Delta t \rightarrow 0$. If the utility is not bounded, then by L'Hopital, we have that $\lim_{\Delta t \rightarrow 0} u\left(\frac{W}{\Delta t}\right) \Delta t = \lim_{x \rightarrow \infty} \frac{u(Wx)}{x} = \lim_{x \rightarrow \infty} u'(Wx)W$. If the Inada conditions are satisfied (which is true in our CRRA setup), we know then that $\lim_{x \rightarrow \infty} u'(x) = 0$. So, the utility over the consumption of all the wealth in an instant is zero.

in equilibrium, incumbents overspend because they expect the next instant incumbent to overspend as well, and so on. The return to savings is reduced not only because of the extra-impatience of the party in power today, but mainly because of the equilibrium behavior of future incumbents.

1.4 Sustaining Stationary Promises of Repayment

We know by proposition (1.7) that for positive political turnover, in equilibrium the government will eventually find itself with practically no asset holdings. The parties in power would like then to borrow against the future endowment shocks. They would like to short-sell the riskless bond.

Suppose that every instant the country can issue *promises to repay* in case that the endowment shock hits. Let $X_t \in (0, Y]$ be the amount of these promises issue at time t . How much is a foreigner willing to pay for such a promise? Given the risk neutrality of the foreign investors, they are willing to pay λX_t . Under the promises, if the endowment shock happens, wealth (W_t) jumps to $Y + W_t^- - (A_t^- + X_t^-)$. If the endowment shock does not happens, then wealth under the promises evolves according to

$$\dot{W}_t = rW_t + \lambda A_t + \lambda X_t - c_t$$

A **Stationary Borrowing Contract** is a triplet $\Upsilon(W) = (A(W), c(W), X(W))$ with associate value function $V(W; \Upsilon)$.

After a default, a country can still save in the asset market, but it cannot issue promises for repayment. A Stationary Borrowing contract is sustainable if the party in power at any time prefers to maintain repay its promises, and remain in the contract, rather than default. The value for the party in power in case of default is the value characterized in the previous section, $V(W)$. The following definition follows,

Definition 1.5 *A Stationary Borrowing Contract is **sustainable** if for all W*

$$\begin{aligned} V(W; \Upsilon) &> V(W) \\ V(Y + W - A(W) - X(W); \Upsilon) &> V(Y + W - A(W)) \end{aligned} \tag{1.9}$$

This definition says that the party in power prefers to be in the contract all the time rather than out of it. And whenever is called to repay its promises, it prefers to do it

rather than defaulting. It is assumed that the country can keep its assets after defaulting. As it will be shown, this is not going to change the results to come because the party in power has the highest temptation to default when the country has no assets.

There are many $A(W), c(W), X(W)$ that make this inequality true. For example, contracts that use the default as a trigger mechanism to sustain reputation between the political parties. I will focus on cases without this implicit reputation. For this reason, I will produce an allocation which is an equilibrium when default is not an option and later on check that under this allocation, parties have no incentive to default on their promises.

Definition 1.6 *A Feasible Equilibrium under $X(W)$ is characterized by*

- An **asset** function $A : [0, \infty) \rightarrow (-\infty, \infty)$ with $A(W) \leq W$
- A **consumption** function $c : [0, \infty) \rightarrow [0, \infty)$ with $c(0) \in [0, \lambda X(0)]$;
Such that $\exists V, V_0$ where:
 $c(W), A(W)$ solve

$$rV(W; X) = \max_{c, A \leq W} \{u(c) + \lambda [V(Y + W - A - X(W); X) - V(W; X)] \\ + V'(W; X) (rW + \lambda A + \lambda X(W) - c) + \gamma(1 - \alpha) (V_0(W; X) - V(W; X))\}$$

and V_0 is

$$rV_0(W; X) = \lambda [V_0(Y + W - A(W) - X(W); X) - V_0(W; X)] \\ + V'_0(W; X) (rW + \lambda A(W) + \lambda X(W) - c(W)) + \gamma\alpha (V(W; X) - V_0(W; X))$$

This corresponds to the previous definition of Symmetric Markov Equilibrium if $X_t = 0$. But now the possibility that the country issues promises is allowed.

The objective in this paper is to show that a country can issue promises for repayment, even in the absence of direct punishments. So, it suffices to show the existence of one contract that is sustainable. For simplicity and to be able to solve the model, I assume the following promises function.

The promises function $X(W)$ has a debt-limit D and is

$$X(W) = X(W|D) \equiv \begin{cases} D & ; \text{ for } W \in [0, Y - D) \\ Y - W & ; \text{ for } W \in [Y - D, Y] \\ 0 & ; \text{ for } W \in (Y, \infty) \end{cases}$$

Remark: *This promises function has a simple interpretation. It is the amount of borrowing a country will get if it were facing a short-sale constraint of D and the value function were concave. However, notice that in the model, the country is not free to pick the amount it borrows. It is monitored by the foreigners who lend exactly $X(W)$. The foreigners observe the state of the country every instant and decide how many promises for repayment to buy.*

Rewrite the value function $V(W, X(\cdot|D))$ as $V(W|D)$.

The first order condition for consumption is $u'(c) = V'(W|D)$. And A is such that

$$\max_{A \leq W} \{ \lambda V(Y + W - A - X(W|D)|D) + \lambda V'(W|D) A \}$$

If the value function is concave, the optimal portfolio decision is

$$A = \begin{cases} W & ; \text{ for all } W \leq Y \\ Y & ; \text{ for all } W > Y \end{cases}$$

I will analyze the problem as $\gamma \rightarrow \infty$ (see Appendix I). Again the analysis is separated into two cases.

1.4.1 Case 1

This is the case when $\alpha + \rho - 1 > 0$. The following theorem holds (see Appendix I):

Theorem 1.1 (Representation Theorem) *For any debt-limit D , the value function*

of being in power, V_∞ , in the limit economy is given by

$$V_\infty(W|D) = \begin{cases} \frac{\alpha}{r+\lambda} \left[(1-\rho) u((r+\lambda)W + \lambda D) v \left(\ln \frac{(r+\lambda)W + \lambda D}{\lambda D} \right) \right. \\ \left. + \lambda V_\infty(Y - D|D) \right] & ; \text{ for } W \in [0, Y - D] \\ \frac{\alpha}{r} (1-\rho) u(rW + \lambda Y) v \left(\ln \frac{rW + \lambda Y}{\lambda D} \right) & ; \text{ for } W > Y - D \end{cases}$$

where v is a function such that

1. $v(0) = \frac{1}{1-\rho}$
2. $v' < 0$ on $(0, \infty)$
3. v asymptotes to $v(\infty) = \frac{\psi^\rho}{\alpha} \frac{1}{1-\rho}$.
4. for a given l , $v(l)$ is independent of λ, D , and r .
5. and $(1-\rho)v + v'$ is positive for any finite l and is increasing in l .

The dynamics under a stationary debt-limit D can be analyzed. From before it is obtained that $V'_\infty(W|D) = u'(c^*(W))$ for $W > 0$. Taking the first derivative of $V_\infty(W|D)$ with respect to W :

$$u'(c^*) = V'_\infty = \begin{cases} \alpha ((1-\rho)v + v') u'((r+\lambda)W + \lambda D) & ; \text{ for } W \in (0, Y - D] \\ \alpha ((1-\rho)v + v') u'(rW + \lambda Y) & ; \text{ for } W > Y - D \end{cases}$$

The sum $(1-\rho)v + v'$ is increasing in W and hence, it converges to $(1-\rho)v(\infty) = \frac{\psi^\rho}{\alpha} < 1$. This implies that $(1-\rho)v + v' < 1$, for all W . So $u'(c^*) = V'_\infty < u'((r+\lambda)W + \lambda D)$ for $W \in (0, Y - D]$ and $u'(c) < u'(rW + \lambda Y)$ for $W > Y - D$. Then, consumption is always higher than $(r+\lambda)W + \lambda D$ for $W \in (0, Y - D]$ and higher than $rW + \lambda Y$ for $W > Y - D$. But $(r+\lambda)W + \lambda D$ and $rW + \lambda Y$ are the respective income flows the country gets from holding the assets, borrowing and receiving endowment shocks. The country consumes more than the income flow it receives every instant. The following then holds.

Proposition 1.10 *Under debt-limit D , the country consumes more than its income flow and wealth monotonically decreases towards zero in the absence of endowment shocks.*

The parties eventually consume their wealth down to zero. But if the country can borrow, once the wealth disappears the parties still have the ability to borrow against the endowment shock.

1.4.2 Case 2

In this case under any debt-limit D , the value function of being in power, $V_\infty(W|D)$, in the limit economy is given by

$$\lim_{\gamma \rightarrow \infty} V(W|D) = \frac{\alpha u(\lambda D)}{r} \quad (1.10)$$

The parties are unable to save, and consume all of their income in a single instant. However, they can borrow and receive a constant flow of λD . This result is again similar to Proposition 1.10, the striking difference is that the consumption of all the wealth is taking place now in an instant of time.

1.5 Sustaining Debt

In this section, I study the sustainability of debt, if the government can default on its previous promises.

1.5.1 Sustaining Debt, Case 1

Are the parties in power going to repay their debts? For that it is necessary to check that the parties have no incentive to default. From (1.9), the following has to hold

$$V_\infty(Y - D|D) > V_\infty(Y); \text{ for all } W \in [0, Y - D]$$

and

$$V_\infty(W|D) > V_\infty(Y); \text{ for all } W \in (Y - D, Y]$$

Where the first inequality clearly implies the second (given that V_∞ is increasing).

Using the representation theorem,

$$V_{\infty}(Y - D|D) = \frac{\alpha(1 - \rho)}{r} u((r + \lambda)Y - rD) v\left(\ln\left(\frac{(r + \lambda)Y - rD}{\lambda D}\right)\right) \quad (1.11)$$

And in the case with no debt,

$$V_{\infty}(Y) = \frac{\alpha(1 - \rho)}{r} u((r + \lambda)Y) v(\infty) \quad (1.12)$$

Dividing (1.11) by (1.12), the equilibrium under short-sale constraint D is sustainable if for all $W \in [0, Y]$

$$\left[\frac{u((r + \lambda)Y - rD)}{u((r + \lambda)Y)} \right] \left[\frac{v\left(\ln\left(\frac{(r + \lambda)Y - rD}{\lambda D}\right)\right)}{v(\infty)} \right] > 1 \quad (1.13)$$

The first term in square brackets in (1.13) is always less than one (for any $r > 0$) and the second is always strictly greater than one. However, as the interest rate goes down, the first term approaches one, and the second remains bounded above one for any $W \in [0, Y]$. Their product approaches a value strictly greater than one for all $W \in [0, Y]$. The following proposition follows.

Proposition 1.11 *For any $D \in (0, Y]$, there exists an $\bar{r} > 0$, such that for any $0 < r \leq \bar{r}$, the feasible equilibrium under debt-limit D is sustainable.*

The Bulow-Rogoff argument is not holding in this economy. Political parties repay the debt even when the credit market is as complete as the asset market and the only punishment available to the foreign investors is denial of future lending in case of default. The reason lies in the inability of the parties to save enough. Even when the parties would all like to save more, once in power they rationally choose not to. They tend to consume too much out of their asset holdings. The country eventually has very little wealth and the parties desire to borrow from the foreign creditors. If they had defaulted in the past, they won't be able to borrow again. This could be a strong enough punishment to enforce repayment of the debt. When the interest rate is low enough, parties are more patient and hence care more about the future, and the benefits of default are reduced because the return on savings is small.

How does this ability to repay relate to the political risk? The following proposition answers this question.

Proposition 1.12 *Let $\bar{r}(D)$ be the highest interest rate at which the feasible equilibrium under debt-limit D is sustainable; then $\bar{r}(D)$ is decreasing with α .*

As the political risk increases (α goes down) savings are more distorted. This proposition tells us that as the political risk increases, parties will repay the debts more easily (they can sustain debt contracts at a higher interest rate). However, as the political risk vanishes (as α goes to one) the following holds.

Proposition 1.13 (Bulow-Rogoff) *For any debt-limit D ,*

$$\lim_{\alpha \rightarrow 1} \bar{r}(D) = 0$$

As α increases, the incumbent is more likely to remain in power in the future. The distortions on the savings margin are reduced and the incumbent party will find the default option more attractive. As α goes to one, a Bulow-Rogoff type of result obtains: the parties will only repay their debts if the interest rate is zero. For any positive interest rate, debt is not sustainable as an equilibrium. This proposition makes clear that the reason why parties repay the debt lies in the inefficiencies in savings that appear when the political uncertainty is high. Once the political risk vanishes and the parties are able to save more efficiently they do not need the credit market anymore and would default for any positive interest rate.

The next proposition analyzes the other extreme, when political uncertainty is high. In this case the following applies.

Proposition 1.14 *For any $D \in (0, Y]$, there exists an $\bar{\alpha} \in (1 - \rho, 1)$, such that for any $\alpha \in (1 - \rho, \bar{\alpha}]$ the feasible equilibrium under debt-limit D is sustainable.*

When the political risk is high enough (α low enough), any debt-limit D can be sustained.

Remark: (A Comparison with Harris and Laibson) *Harris and Laibson (2001) has shown that the instantaneous hyperbolic program under case 1 is equivalent (in value functions) to the program of a time-consistent consumer with a wealth-contingent utility function. However, the results on debt sustainability rely on the fact that the political parties are time-inconsistent. Both results can be reconciled once it is noticed that in the*

Harris and Laibson (2001) equivalence result, the wealth-contingent utility function depends on the income available in the states where $W = 0$, so as the amount D is changed, the equivalent consumer's utility function is changing. Under default, the equivalent consumer has a different utility function than under the positive short-sale constraint; and clearly the Bulow-Rogoff argument does not hold with a consumer that has a different utility function once he has defaulted.

In this subsection the sustainability of debt under case one has been studied. I will now show the dramatic results that occur when $\alpha + \rho - 1 < 0$.

1.5.2 Sustaining Debt, Case 2

In the case with a debt-limit D (see equation (1.10) the following holds

$$\lim_{\gamma \rightarrow \infty} V(W|D) = \frac{\alpha u(\lambda D)}{r}$$

Recall from before that without debt,

$$\lim_{\gamma \rightarrow \infty} V(W) = 0$$

The following proposition is then immediate

Proposition 1.15 *If $\alpha + \rho - 1 < 0$ then for any $\alpha > 0$ and $D > 0$ the feasible equilibrium under debt-limit D is sustainable.*

The inefficiencies in savings created by the political risk are so large that doesn't matter what the interest rate or the elasticity of substitution are, debt would always be sustainable. Notice why: As $\gamma \rightarrow \infty$ the spending rate converges to infinity. This is the dramatic outcome of the logic : if tomorrow they (whoever are in power) are going to eat a lot, I will today eat much more. The reason why this happens is that once the savings are made, the total stock of assets belongs to the next party in power. This new incumbent will consume as much as it desires out of the total stock, making the present incumbent very reluctant to save. Debt eliminates this because once tomorrow arrives the parties hit their borrowing constraint and the dramatic logic previously exposed does not apply.

1.6 Autocracies versus Democracies

The previous section analyzed the sustainability of debt in a model with political turnover. One key ingredient of that model was the stability of the political parties: the parties remain in the political game forever (the value function of being out of power is not zero). I showed that as the political uncertainty increased, the ability of the parties to sustain sovereign lending increased. This was due to the inefficiencies in savings created by the political structure. I think of this political structure as representing a modern **democracy**, with several long-lived parties.

Suppose now that there is no political resurrection. Once a party is out of government, it is out forever. I call this case an **autocracy**. In an autocracy an incumbent rules continuously, but once a political shock comes and the incumbent is removed from power, she cannot return to the political game¹⁴. In this situation the value function of an incumbent is

$$V^A(W) = E \left[\int_0^{\infty} e^{-(r+\gamma(1-\alpha))t} u(c_t) dt \right] \quad (1.14)$$

where $\gamma(1-\alpha)$ is the probability that the incumbent is removed from power. The political instability makes the incumbent impatient (she has an effective discount rate higher than r) but it does not make her time-inconsistent. The value function (1.14) is a standard exponential value function. In this case the Bulow-Rogoff result holds. The incumbent will always default on any debt contract. The model thus predicts that

- In a democracy, political turnover is positively related to debt sustainability
- In an autocracy, political turnover is not related to debt sustainability.

1.7 Conclusion

In this paper, I proposed a theory of sovereign debt repayment based on political economy considerations. Bulow and Rogoff (1989) show that a country that has access to a sufficiently rich asset market cannot commit to repay its debts and therefore should be unable to borrow. I show that the presence of political uncertainty reduces the ability of a

¹⁴The case I have in mind is a dictator (and his associates) who faces exile or death once he loses power. The autocracy case is related to a system where individuals instead of parties govern, and individuals are clearly less likely to return to power than parties are.

country to save, and hence to replicate the original debt contract after default. In a model where different parties alternate in power, an incumbent party with a low probability of remaining in power has a high short-term discount rate and is therefore unwilling to save. The current incumbent party realizes that in the future whoever achieves power will be impatient as well, making the accumulation of assets unsustainable. Because of their inability to save, politicians demand debt ex-post and the desire to borrow again in the future enforces repayment today.

1.8 Appendix I : The Full Model under Case 1

In this section I derive the instantaneous system and its properties. I follow closely Harris and Laibson (2001).

The equilibrium selection technique developed by the previous authors involves three main steps. First, noise is added to the asset holdings, this guarantees that the consumption function is continuous. It is possible to show existence in this case of a viscosity solution to the Bellman system (the reader is referred to Harris and Laibson (2001)). Second, I analyze the system as $\gamma \rightarrow \infty$, where a uniqueness result holds. And finally, I study the equilibrium as the noise vanishes.

The problem without ability to borrow is a subcase of the more general case with debt-limit D , so I will study the general problem for any value of D .

Let us add noise to the asset holdings. Both assets now evolve according to

$$\begin{aligned} dA_t &= (r + \lambda) A_t dt + \sigma A_t dw_t \\ dB_t &= r B_t dt + \sigma B_t dw_t \end{aligned}$$

when there is no endowment shock and where w_t is a standard Brownian motion process. Notice that the process w_t is the same process for both assets. Total wealth is given by

$$W_t = A_t + B_t$$

When no endowment shock happens the wealth process evolves according to

$$dW = (rW + \lambda(A + X) - c) dt + \sigma W dw_t$$

Let us redefine the value functions. Let $J = \frac{1}{\alpha} [\alpha V + (1 - \alpha) V_0]$. Then I can write system in proposition 2 in the following way

For $W \geq 0$:

$$\begin{aligned} rV &= u(c) + \lambda[V(Y + W - (A + X)) - V] + V'(rW + \lambda(A + X) - c) + \\ &\quad + \gamma(\alpha J - V) + V'' \frac{\sigma^2 W^2}{2} \end{aligned}$$

$$rJ = u(c) + \lambda[J(Y + W - (A + X)) - J] + J'(rW + \lambda(A + X) - c) + J'' \frac{\sigma^2 W^2}{2}$$

For $W = 0$:

$$\begin{aligned} rV &= u(c) + \lambda[V(Y - (A + X)) - V] + V'(\lambda(A + X) - c) + \gamma(\alpha J - V) \\ rJ &= u(c) + \lambda[J(Y - (A + X)) - J] + J'(\lambda(A + X) - c) \end{aligned}$$

With associated FOC:

$$u'(c) = V' ; \text{ for } W > 0 \quad (1.15)$$

$$c = \arg \max_{c \in [0, \lambda D]} \{u(c) - V'c\} ; \text{ for } W = 0 \quad (1.16)$$

And

$$A = \arg \max_{A \in (-\infty, W+D]} \{V(Y + W - (A + X)) - AV'(W)\}$$

Now, if the value functions were concave (I will check this later on), the optimal A is given by

$$A = \begin{cases} W & ; \text{ for all } W \leq Y \\ Y & ; \text{ for all } W > Y \end{cases}$$

Taking the limits of the Bellman system as $\gamma \rightarrow \infty$, it converges to
For $W > Y - D$:

$$-rJ + u(c) + J'(rW + \lambda Y - c) + \sigma^2 W^2 J'' = 0 \quad (1.17)$$

For $0 < W \leq Y - D$:

$$-rJ + u(c) + J'((r + \lambda)W + \lambda D - c) + \lambda J(Y - D) - \lambda J + \sigma^2 W^2 J'' = 0 \quad (1.18)$$

For $W = 0$:

$$-rJ + u(c) + J'(\lambda D - c) + \lambda J(Y - D) - \lambda J = 0 \quad (1.19)$$

with $\alpha J = V$. And $c \in [0, \lambda D]$ for $W = 0$.

From (1.15) and using the fact that $u(c) = c^{1-\rho}$; then for $W > 0$,

$$c = \left[\frac{V'}{1-\rho} \right]^{-\frac{1}{\rho}}$$

So, $u(c) - J'c = \left[\frac{V'}{1-\rho} \right]^{\frac{\rho-1}{\rho}} - J' \left[\frac{V'}{1-\rho} \right]^{-\frac{1}{\rho}}$. Using the fact that $V' = \alpha J'$:

$$u(c) - J'c = \frac{\rho\psi}{\alpha} \left(\frac{1-\rho}{\alpha} \right)^{\frac{1-\rho}{\rho}} J'^{\frac{\rho-1}{\rho}} \equiv h(J')$$

for $W > 0$.

For the case when $W = 0$, using (1.16):

$$u(c) - J'c = \begin{cases} \frac{\rho\psi}{\alpha} \left(\frac{1-\rho}{\alpha} \right)^{\frac{1-\rho}{\rho}} J'^{\frac{\rho-1}{\rho}}; & \text{if } J' > \frac{u'(\lambda D)}{\alpha} \\ (\lambda D)^{1-\rho} - J'(\lambda D); & \text{if } J' \leq \frac{u'(\lambda D)}{\alpha} \end{cases} \equiv h_0(J')$$

It is possible to rewrite (1.17), (1.18) and (1.19) as

For $W > Y - D$:

$$-rJ + J'(rW + \lambda Y) + h(J') + \sigma^2 W^2 J'' = 0 \quad (1.20)$$

For $0 < W \leq Y - D$:

$$-rJ + J'((r + \lambda)W + \lambda D) + \lambda J(Y - D) - \lambda J + h(J') + \sigma^2 W^2 J'' = 0 \quad (1.21)$$

For $W = 0$:

$$-rJ + J'\lambda D + \lambda J(Y - D) - \lambda J + h_0(J') = 0 \quad (1.22)$$

Harris and Laibson (2001) show that a system like (1.20), (1.21) and (1.22) has a unique viscosity solution J . This is the solution that will be characterized.

Proving Concavity

Let us check now that the viscosity solution J that solves (1.20), (1.21) and (1.22) is concave.

Suppose now that $J'' = 0$. Taking first derivatives (from both equations (1.17) and (1.18)): $J''' \sigma^2 W^2 = [J' - u'(c)] c'$. But when $J'' = 0$, from the FOC of consumption we have that $\alpha J' = V' = u'(c) \Rightarrow \alpha J'' = u''(c) c' \Rightarrow c' = 0$. So $J''' = 0$ whenever $J'' = 0$. If there is any W_1 such that $J''(W_1) \geq 0$, this implies that $J''(W_t) \geq 0$, for all $W_t > W_1$. Then, J grows at least linearly for any $W > W_1$, which contradicts the CRRA (boundness) assumption.

So $J''(W)$ cannot be non-negative, for any $W > 0$. The value function in the limit is concave and A is optimal. It is also possible to show uniqueness of this solution as

the instantaneous hyperbolic program (1.17), (1.18) and (1.19) can be rewritten as the program of a time-consistent consumer with a wealth-dependent utility function. See Harris and Laibson (2001).

The Viscosity Solution When The Noise Vanishes

I now let the noise vanish. In particular as $\sigma^2 \rightarrow 0$, the value functions $J(\sigma^2)$ uniformly converge on compact subsets of $[0, \infty)$ to the unique viscosity solution of the following system :

For $W > Y - D$:

$$-rJ + J'(rW + \lambda Y) + h(J') = 0 \quad (1.23)$$

For $0 < W \leq Y - D$:

$$-rJ + J'((r + \lambda)W + \lambda D) + \lambda J(Y - D) - \lambda J + h(J') = 0 \quad (1.24)$$

For $W = 0$:

$$-rJ + J'(\lambda D) + \lambda J(Y - D) - \lambda J + h_0(J') = 0 \quad (1.25)$$

Let v be define as the following

$$v(l) = \begin{cases} (r + \lambda) \frac{J\left(\frac{\exp(l + \ln(\lambda D)) - \lambda D}{r + \lambda}\right) - \frac{\lambda}{r + \lambda} J(Y - D)}{(1 - \rho) u(\exp(l + \ln(\lambda D)))} & ; \text{ for } l \in [0, \ln((r + \lambda)Y - rD) - \ln(\lambda D)] \\ r \frac{J\left(\frac{\exp(l + \ln(\lambda D)) - \lambda Y}{r}\right)}{(1 - \rho) u(\exp(l + \ln(\lambda D)))} & ; \text{ for } l > \ln((r + \lambda)Y - rD) - \ln(\lambda D) \end{cases} \quad (1.26)$$

Substituting v into the system, we have that J satisfies (1.23), (1.24) and (1.25) iff v satisfies

For $l > 0$:

$$((1 - \rho)v + v') - v + (1 - \rho)^{-\frac{1}{\rho}} h((1 - \rho)v + v') = 0 \quad (1.27)$$

For $l = 0$:

$$((1 - \rho)v + v') - v + (1 - \rho)^{-\frac{1}{\rho}} h_0 ((1 - \rho)v + v') = 0 \quad (1.28)$$

The advantage of this result is that this new system, (1.27) and (1.28), is independent of wealth W . I can then show that there exists a smooth function $H(\cdot)$ such that $v = H(v')$.

It is possible to show the following

- $H'' > 0$
- $H'(0) = 0$
- $\min H = H(0) = \frac{\psi^\rho}{(1-\rho)\alpha} < \frac{1}{1-\rho}$

Let $v_0 = \frac{1}{1-\rho}$; (here v_0 corresponds to the value when a country has no wealth, endowment shocks never happen, and the parties consume always λD). The unique viscosity solution $v(l)$ to the system (1.27) and (1.28) is such that

- $v(0) = v_0$
- $v' < 0$ on $(0, \infty)$
- v asymptotes to $H(0) = v(\infty)$.
- for a given l , $v(l)$ is independent of λ , D , and r .
- and $(1 - \rho)v + v'$ is positive for any finite l and is increasing in l .

The following figure plots the function H in the (v', v) space.

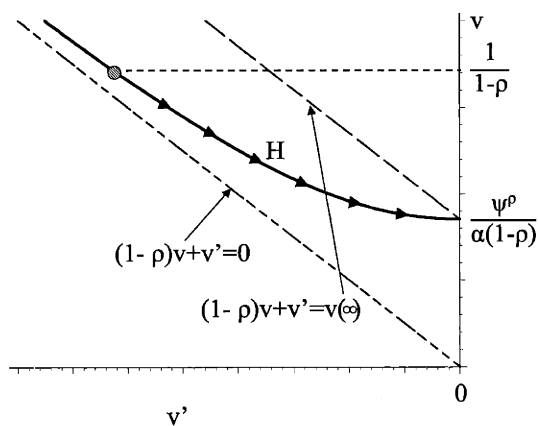


Figure 1-1: The function H

Figure 1.2 shows the graph of v as a function of l .

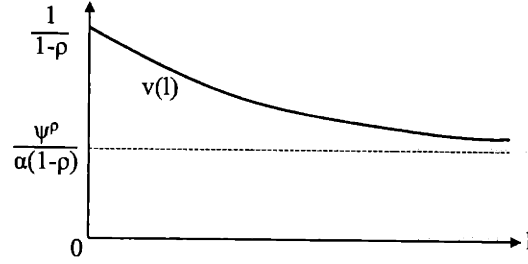


Figure 1-2: The function $v(l)$

The Representation Theorem

Going back to the original system and using (1.26), I have that J can be represented as

$$J(W) = \begin{cases} \frac{1}{r+\lambda} [(1-\rho) u((r+\lambda)W + \lambda D) v(\ln((r+\lambda)W + \lambda D) - \ln(\lambda D)) \\ + \lambda J(Y-D)] & ; \text{ for } W \leq Y-D \\ \frac{1}{r} (1-\rho) u(rW + \lambda Y) v(\ln(rW + \lambda Y) - \ln(\lambda D)) & ; \text{ for } W > Y-D \end{cases}$$

This is the representation theorem.

I can also compute the value when there is no debt. When $D \rightarrow 0$, the solution converges to $H(0)$. The value function is given by

$$J(W) = \begin{cases} \frac{1}{r+\lambda} [(1-\rho) u((r+\lambda)W) v(\infty) + \lambda J(Y)] & ; \text{ for } W \leq Y \\ \frac{1}{r} (1-\rho) u(rW + \lambda Y) v(\infty) & ; \text{ for } W > Y \end{cases}$$

For any D , the associated (limit) value function J is concave. This is just because J is obtained as the limit of concave functions (as noise vanishes).

1.9 Appendix II: Proofs

Proof Proposition (1.3): For $W \leq Y$, let $V(W) = \phi W^{1-\rho} + \beta$ and $V_o(W) = \phi' W^{1-\rho} + \beta'$. Substituting in the assumed value functions, letting $\mu = c/W$, and using the FOC for consumption, it is obtained that $\mu^{-\rho} = \phi$. Solving for the value function

$$\phi = \frac{\mu^{1-\rho}}{(r+\lambda)\rho + (1-\rho)\mu} \left[\frac{(r+\lambda)\rho + (1-\rho)\mu + \gamma\alpha}{(r+\lambda)\rho + (1-\rho)\mu + \gamma} \right]$$

$$r\beta = \left(\frac{(r+\lambda)\rho + (1-\rho)\mu + \gamma\alpha}{(r+\lambda)\rho + (1-\rho)\mu} - \frac{\gamma(1-\alpha)}{r+\gamma} \right) \frac{\lambda\mu^{1-\rho}Y^{1-\rho}}{(r+\lambda)\rho + (1-\rho)\mu + \gamma}$$

Let μ^* be such that $F(\mu^*) = 0$ where $F(\cdot)$ is obtained from the FOC:

$$F(\mu) = \frac{1-\rho}{r+\lambda}\mu^2 + \left(2\rho - 1 + \frac{\gamma}{r+\lambda} \frac{\alpha + \rho - 1}{\rho} \right) \mu - ((r+\lambda)\rho + \gamma) \quad (1.29)$$

It is easy to see that $F(0) < 0$ and it is a parabola that opens up, so it has a unique positive root. Taking the derivative of the implicit function $F'(\mu) = 2\frac{(1-\rho)}{r+\lambda}\mu + 2\rho - 1 + \frac{\gamma}{r+\lambda} \frac{(\alpha - (1-\rho))}{\rho}$; which evaluated at $r + \lambda$ yields $F'(r + \lambda) = 1 + \frac{\gamma}{r+\lambda} \frac{(\alpha - (1-\rho))}{\rho} > 0$. Evaluating the implicit function at $r + \lambda$ I obtain that $F(\mu^{SB}) = \gamma \frac{\alpha - (1-\rho)}{\rho} - \gamma$ which implies that $F(\mu^{SB}) < 0$. Given that $F(r + \lambda) < 0$ and $F'(r + \lambda) > 0$, then μ^* that solves $F(\mu^*) = 0$ is such that $\mu^* > (r + \lambda)$ ■

Proof Proposition (1.4): For $\frac{d\mu^*}{d\gamma} > 0$: Differentiating the implicit function

$$\frac{d\mu^*}{d\gamma} = \frac{1 - \mu^* \frac{(\alpha + \rho - 1)}{(r + \lambda)\rho}}{\frac{2(1-\rho)}{r+\lambda}\mu^* + 2\rho - 1 + \gamma \frac{(\alpha + \rho - 1)}{(r + \lambda)\rho}}$$

Given that $\mu^* > r + \lambda$, the denominator is always positive. We know that $\mu^* \rightarrow r + \lambda$ as $\gamma \rightarrow 0$ and $\mu^* \rightarrow \left(\frac{(\alpha + \rho - 1)}{(r + \lambda)\rho} \right)^{-1}$ as $\gamma \rightarrow \infty$. That means that if there exists $\gamma' < \infty$ such that $\mu^*(\gamma') > \left(\frac{(\alpha + \rho - 1)}{(r + \lambda)\rho} \right)^{-1}$, there has to exist a $\gamma'' < \infty$ such that $\mu^*(\gamma'') = \left(\frac{(\alpha + \rho - 1)}{(r + \lambda)\rho} \right)^{-1}$. Now, $F\left(\left(\frac{(\alpha + \rho - 1)}{(r + \lambda)\rho}\right)^{-1}\right) = 0$ implies that $\frac{1-\rho}{r+\lambda} \frac{(r+\lambda)^2 \rho^2}{(\alpha + \rho - 1)^2} + 2\rho \frac{(r+\lambda)\rho}{(\alpha + \rho - 1)} - (r + \lambda)\rho = 0$. This equation is not a function of γ and for any $\alpha < 1$, this equation does not hold. There is no $\gamma'' < \infty$ such that $\mu^*(\gamma'') = \left(\frac{(\alpha + \rho - 1)}{(r + \lambda)\rho} \right)^{-1}$ and hence $\mu^* < \left(\frac{(\alpha + \rho - 1)}{(r + \lambda)\rho} \right)^{-1}$ for any γ . This implies that the numerator is always positive.

For $\frac{d\mu^*}{d\alpha} < 0$: Differentiating the implicit function

$$d\mu = -\frac{\frac{\gamma}{r+\lambda} \frac{1}{\rho} \mu}{2 \frac{1-\rho}{r+\lambda} \mu + \left(2\rho - 1 + \frac{\gamma}{r+\lambda} \frac{(\alpha - (1-\rho))}{\rho}\right)} d\alpha$$

The numerator is positive and the denominator has been previously shown to be positive. This completes the proof. ■

Proof of proposition (1.5): See previous Appendix I. ■

Proof of proposition (1.6): From previous Appendix I, the representation theorem implies that

$$J(W) = \begin{cases} \frac{1}{r+\lambda} [(1-\rho) u((r+\lambda)W) v(\infty) + \lambda J(Y)] & ; \text{ for } W \leq Y \\ \frac{1}{r} (1-\rho) u(rW + \lambda Y) v(\infty) & ; \text{ for } W > Y \end{cases}$$

Substituting into for $v(\infty) = \frac{\psi^\rho}{\alpha(1-\rho)}$ and $u(c) = c^{1-\rho}$ we can obtain the value function $V_\infty = \alpha J$. Taking the first derivative of V_∞ with respect to W we have that $u'(c) = V'(W)$. We can then solve for $c^*(W) = \frac{(V')^{-\rho}}{1-\rho}$. And this gives us the equilibrium consumption rule. ■

Proof of corollary (1.1): It is clear as $\psi < 1$. ■

Proof of proposition (1.7): Given that in the $c(W)$ is higher than $(r+\lambda)W$ when $W \leq Y$ and higher than $rW + rY$ when $W > Y$, wealth is monotonically decreasing to zero in the absence of endowment shocks. This implies that for any initial W_0 , the wealth process eventually converges to $[0, Y]$. For any $W_t \in (k, Y]$, there exists a $T < \infty$ such that if no endowment shock happens, $W_{t+T} \leq k$. So I need the endowment shock not to happen in an interval of size T . This is a positive probability event. Given that time is infinite, it will happen with probability one. ■

Proof of proposition (1.8): See Harris and Laibson (2001) ■

Proof of proposition (1.9): See Harris and Laibson (2001) ■

Proof of theorem (1): See previous Appendix I. ■

Proof of proposition (1.10): The first part is proven in the text. The second part follows immediately. ■

Proof of proposition (1.11): In the text. ■

Proof of proposition (1.12): Suppose that a contract D is just sustainable for some

$r > 0$. This implies that $\left[\frac{v(\ln(\frac{(r+\lambda)Y-rD}{\lambda D}))}{v(\infty)} \right] \left[\frac{u((r+\lambda)Y-rD)}{u((r+\lambda)Y)} \right] = 1$. Now, as α increases, $v(\infty)$ increases, and $v(l)$ moves down to $v(\infty)$. This implies that $\frac{v(l)}{v(\infty)} (> 1)$ decreases with α . And the previous equality breaks. So r can not sustain the contract anymore. This implies then that $\bar{r}(D)$ has been reduced with the increase in α . ■

Proof of proposition (1.13): As $\alpha \rightarrow 1$, $v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)$ converges to $\frac{1}{1-\rho}$. This implies that $\frac{v(\ln(\frac{(r+\lambda)Y-rD}{\lambda D}))}{v(\infty)}$ converges to 1 as $\alpha \rightarrow 1$. Given that for any $r > 0$, $\frac{u((r+\lambda)Y-rD)}{u((r+\lambda)Y)} < 1$, then for some α close to 1, $\left[\frac{v(\ln(\frac{(r+\lambda)Y-rD}{\lambda D}))}{v(\infty)} \right] \left[\frac{u((r+\lambda)Y-rD)}{u((r+\lambda)Y)} \right] < 1$. I can do that for all $r > 0$, so as $\alpha \rightarrow 1$, only $r = 0$ is sustainable. ■

Proof of proposition (1.14): It is easy to see that as $\alpha \rightarrow 1 - \rho$, $v(\infty) \rightarrow 0$ and $v\left(\ln\left(\frac{(r+\lambda)Y-rD}{\lambda D}\right)\right)$ remains bounded below by a strictly positive value. This implies that $\left[\frac{v(\ln(\frac{(r+\lambda)Y-rD}{\lambda D}))}{v(\infty)} \right]$ goes to infinity as $\alpha \rightarrow (1 - \rho)$, so for any r , I can find an $\bar{\alpha}$ such that for any $\alpha \in (1 - \rho, \bar{\alpha}]$, $\left[\frac{v(\ln(\frac{(r+\lambda)Y-rD}{\lambda D}))}{v(\infty)} \right] \frac{u((r+\lambda)Y-rD)}{u((r+\lambda)Y)} > 1$; which proves the proposition. ■

Proof of proposition (1.15): Immediate because $\lim V = 0 < \lim V^D$ for any $D > 0$ as long as $\alpha > 0$. ■

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Chapter 2

Political Compromise and Savings

How does a government controlled by different parties allocate spending intra period and across time? The main characteristic of such a political game is the lack of commitment, the inability to tie the hands of future governments to follow today's incumbent wishes. However, cooperation is efficient. Parties are not continuously in power and would like to smooth their share of government spending.

In this chapter, I present a political economy model of government savings. Two political parties alternate in power every period. The party in power at any period controls the government and decides how to allocate spending this period and how much to save for the future. No party has the ability to commit and at any point in time a party can spend all the income of the government in her own consumption and save nothing for the future. If both parties behave as described, then these strategies are a subgame perfect equilibria. However, parties are long run players in this political game, and they might be expected to coordinate and play more efficient equilibria. I study, then, the set of efficient subgame perfect equilibria.

Two main results are obtained. First, as expected, cooperation is sustainable for low discount rates and low elasticities of intertemporal substitution. They both reduce the incentive to cheat relative to to cooperate. However, differently from previous results, there is no cooperation in cases where the marginal utilities of consumption of the parties in the future are equal to infinity. I show that the set of efficient equilibria can be very sensitive to small parameter changes. Small decreases in the discount rate can discontinuously make cooperation disappear.

In the technical side, the paper shows how to bypass the non-convexity that appears

when incentive constraints are dependent on previous play¹. In the case of power utility and no endowment I propose a technique to compute the equilibrium set numerically and characterize the intra period spending and asset holdings.

These results also highlight the role that illiquid assets might play in generating more efficient allocations. Illiquidity is a useful characteristic because it reduces the temptation of current governments from overconsuming, and allows political parties to smooth spending accross time more easily. So, as in the case of hyperbolic consumers (see Laibson (1994)) there is a demand for illiquidity. This calls into question the efficiency of the “privatization” programs that have been applied in Latin America in the last decade. A government might hold illiquid assets even when their rate of return is lower than alternatives, because these might be the only type of assets that they are not tempted to consume once in power. If liquidating such assets becomes suddenly cheaper, previous allocations that supported cooperation across parties might become unsustainable.

Several papers have analyzed the impact of lack of commitment in political economy models (see for example, Acemoglu and Robinson (2001), Alesina and Tabellini (1990), etc.). This paper extends this literature by showing how the asset holdings of a government are influenced by the political constraints when parties cannot commit to future plays. The set up of the model is close to Dixit, Grossman and Gul (2000). These authors characterize the set of efficient equilibria payoffs in a model where parties alternate in government probabilistically every period. Every period the government receives an endowment which cannot be pledge nor saved, but can be consumed by the parties. In this paper however, the focus is in the role that assets play in a political equilibrium, and the parties are allowed to save or disave. However the savings technology is common to all parties, and the party in power at any period controls all the income received from previous government savings.

The next section explicitly sets up the model.

2.1 Political Compromise and Savings

There are two parties, A and B. They generate utility from the consumption of two public goods (g_t^A and g_t^B) provided every period by the government. Given a sequence of government spending $g = \{(g_0^A, g_0^B), (g_1^A, g_1^B), \dots\}$, the utilities of both parties are given

¹A similar point in a different model have been made by Wright (2001)

by

$$v_0^A(g) = \sum_{t=0}^{\infty} \beta^t u(g_t^A)$$

$$v_0^B(g) = \sum_{t=0}^{\infty} \beta^t u(g_t^B)$$

where $u(x) = x^\theta$ with² $\theta \in (0, 1)$.

The government can save in a one period bond with a rate of return R . Every period, before deciding on spending and savings, the government receives an endowment y plus the return of the previous period bond holdings. Let a_t be the savings done by the government in period t .

The government at period t faces the following budget constraint

$$y + Ra_{t-1} = a_t + g_t^A + g_t^B$$

I assume that the government cannot borrow, so $a_t \geq 0 \forall t$. It is also assume that

Assumption 2.1 *Let $\beta R = 1$.*

With this assumption, in a first best allocation it is always optimal to maintain the asset level constant.

The political structure is described in the next assumption

Assumption 2.2 *In odd periods, party A controls the government. In even periods, party B controls the government. The party that controls the government at some time t chooses the spending allocation at time t and the savings done by the government for next period.*

The game is then as follows. At any time, the party that controls the government has discretion about choosing the savings and spending done by the government in that period.

This game might obviously have multiple equilibria. As we will see, there exist equilibria where parties cooperate and share from government spending every period. There

² θ is restricted to be bigger than zero, because otherwise the punishment of no cooperation is too strong, and first best allocations are always incentive compatible.

are also equilibria where parties spend only on their own goods while in power which is clearly inefficient. We would like to characterize the set of efficient equilibria of this dynamic game. Following Abreu, Pearce and Stachetti (1990), all efficient equilibria can be sustained by the threat of moving to the worst possible subgame perfect equilibrium. We need first to characterize this worst possible equilibrium, and this is what follows.

2.2 Worst Equilibrium

The worst possible equilibrium is supported by the following strategies. The party in power consumes all of the government income and saves nothing. The other party behaves just like that. Given that both parties follow the previous strategy, it is easy to see that it constitutes a subgame perfect equilibrium.

What are the payoffs to the parties in power? Suppose that the party in power receives an asset income of Ra . The party in power spends on her good all of the current government income, that is $y + Ra$, and thereafter consumes 0 every time she is not in power and y every time she is. The utility for the party is then

$$\bar{v}(Ra) = u(y + Ra) + \frac{\beta}{1 - \beta^2} (u(0) + \beta u(y)) = u(y + Ra) + \frac{\beta^2}{1 - \beta^2} u(y)$$

Note that this is the minimum payoff that any party with assets holdings of Ra can get in any equilibrium.

What are the dynamics of savings in the worst equilibrium? Parties every period save nothing for the next, $a_t = 0$ for all $t > 0$. The political economy problem does not allow a party today to appropriate the return on the savings she has made. The party in power in the future will use the savings in the consumption of her own goods, from which the party today derives no utility. This inability to tie the hands of the party in power tomorrow, makes the party in power today unwilling to save. This is a result well known and studied in the political economy literature (Alesina and Tabellini (1991), Persson and Svensson (1991)), politicians are impatient and tend to consume too much out of their assets once in power. The objective of this paper is to expand these results for the case when parties are able to sustain reputation, and analyze the role that assets play in a reputational equilibrium. Next section begins to characterize the set of efficient subgame perfect equilibria, equilibria that are sustained by the threat of reverting to the worst equilibrium described above.

2.3 Efficient Subgame Perfect Equilibria

In this section we proceed to characterize the set of efficient equilibria. A couple of definitions follow:

Definition 2.1 We say that g is feasible if $g_t^A \geq 0$ and $g_t^B \geq 0 \quad \forall t$; and

$$\sum_{i=1}^t \left(\frac{1}{R}\right)^{i-1} (y - (g_i^A + g_i^B)) + Ra_0 \geq 0; \quad \forall t$$

Definition 2.2 For any g , we say a_t to be generated by g if

$$a_t = y + Ra_{t-1} - (g_t^A + g_t^B)$$

Note that feasibility is equivalent to $a_t \geq 0$.

Definition 2.3 We say that a feasible g is incentive compatible if

$$v_t^A(g) \geq u^A(y + Ra_{t-1}) + \frac{\beta^2}{1 - \beta^2} u(y) \quad \text{for all odd } t$$

$$v_t^B(g) \geq u^B(y + Ra_{t-1}) + \frac{\beta^2}{1 - \beta^2} u(y) \quad \text{for all even } t$$

with a_t being generated by g .

Call $F(a_0)$ the set of all g that are feasible with initial assets a_0 .

Call $I(a_0)$ the set of all g that are feasible and incentive compatible with initial assets a_0 . Clearly $I(a_0) \subset F(a_0)$.

The set of efficient SPE is characterized by the solutions to the following program

$$V(a_0, \psi) : \max_{g \in I(a_0)} v_0^B(g) \quad \text{subject to } v_0^A(g) \geq \psi \quad (2.1)$$

Where $V(a_0, \psi)$ represents the maximum payoff possible for party B subject to providing a minimum payoff of ψ to party A . Note that in the first period party A is in power.

Before analyzing the program in (2.1), I proceed to characterize the first best allocations of the game (the allocations under commitment). This is done in the next section.

2.4 First Best Allocations

A first best allocation solves the following relaxed program

$$V^{FB}(a_0, \psi) : \max_{g \in F(a_0)} v_0^B(g) \quad \text{subject to } v_0^A(g) \geq \psi$$

The difference is that the program is now restricted only to allocations that are feasible, and incentive compatibility is not taken into account.

Under Assumption 2.1, in a first best, allocation total spending by the government is constant, and both parties receive a constant share of it every period.

Result 2.1 *A first best allocation is characterized by a constant savings decision $a_t = a_0$, and $g_t^A + g_t^B = (R - 1)a_0 + y = g^T$ and $g_t^A = \alpha g^T$; for some $\alpha \in [0, 1]$.*

It is possible then to index the set of first best allocations by the fraction α of total spending allocated to party A's consumption. The respective payoffs under a first best allocation for the parties are

$$v^A = \frac{u(\alpha g^T)}{1 - \beta} \quad \text{and} \quad v^B = \frac{u((1 - \alpha) g^T)}{1 - \beta}$$

When is a first best allocation incentive compatible? It is now known that all first best allocations are index by α , and each party receive a constant payoff throughout. To check incentive compatibility, the payoffs under the first best allocation should be compared to the payoffs (which is also constant) under the worst equilibrium.

2.4.1 Incentive Compatibility of First Best Allocations

Note that in a first best allocation, the payoff to the parties are stationary. For a first best α to be incentive compatible the following inequalities have to hold

$$\begin{aligned} \frac{u(\alpha g^T)}{1 - \beta} &\geq u(y + Ra_0) + \frac{\beta^2}{1 - \beta^2} u(y) \\ \frac{u((1 - \alpha) g^T)}{1 - \beta} &\geq u(y + Ra_0) + \frac{\beta^2}{1 - \beta^2} u(y) \end{aligned}$$

At any time a party comes to power, she has to receive under the first best allocation more than what she can get by deviating towards the worst equilibrium. Given the

symmetry of the model, a first best allocation will be incentive compatible if the first best allocation with $\alpha = \frac{1}{2}$ is.

Lemma 2.1 *If for some asset level a_0 , the first best allocation indexed by $\alpha = 1/2$ is not incentive compatible, then no first best allocation for the same asset level will be subgame perfect .*

For the first best allocation with $\alpha = 1/2$ to be a subgame perfect, the following condition has to be satisfied,

$$\frac{u\left(\frac{y+(R-1)a_0}{2}\right)}{1-\beta} \geq u(y+Ra_0) + \frac{\beta^2}{1-\beta^2}u(y)$$

Or equivalently,

$$H(a_0) = u\left(\frac{y+(R-1)a_0}{2}\right) - (1-\beta)u(y+Ra_0) - \frac{\beta^2}{1+\beta}u(y) \geq 0 \quad (2.2)$$

The function $H(a)$ is positive whenever there is enough surplus in a first best allocation

to satisfy the incentive constraints of both parties whenever they come to power.

As $\beta \rightarrow 0$, $H(a_0)$ converges to $u\left(\frac{y+(R-1)a_0}{2}\right) - u(y+Ra_0) < 0$. As $\beta \rightarrow 1$, $H(a_0)$ converges to $u(y/2) - \frac{u(y)}{2} > 0$ (where the inequality follows by concavity of u). The next result then follows.

Result 2.2 *For any initial asset level a_0 there exists $\underline{\beta}(a_0)$ with $0 < \underline{\beta}(a_0) < 1$, such that for any $\beta \geq \underline{\beta}(a_0)$, there exists a first best allocation that is subgame perfect; and for any $\beta < \underline{\beta}(a_0)$ no first best allocation is subgame perfect.*

How does $H(a_0)$ change with a_0 ? When $a_0 = 0$, $H(a_0 = 0) = 2^{-\theta} - \frac{1}{1+\beta}$. The function $H(a_0)$ starts positive then if $2^\theta(1+\beta)^{-1} < 1$. What happens when a_0 go to infinity? We can write

$$H(a_0) = (a_0)^\theta \left[\left(\frac{y}{2a_0} + \frac{(R-1)}{2} \right)^\theta - (1-\beta) \left(\frac{y}{a_0} + R \right)^\theta - \frac{\beta^2}{1+\beta} u\left(\frac{y}{a_0}\right) \right]$$

As a_0 goes to infinity, the cooperation value is growing at a rate of $\left(\frac{R-1}{2}\right)^\theta$ and the temptation at a rate $(1-\beta)R^\theta$. So if $\left(\frac{R-1}{2}\right)^\theta > (1-\beta)R^\theta \Rightarrow$

$$2^\theta (1-\beta)^{1-\theta} < 1, \quad (2.3)$$

eventually the cooperation gains from having more assets will overcome the increase in the value of the worst equilibrium, and first allocations will be incentive compatible for high enough level of assets. If the condition (2.3) holds with opposite sign, then as a_0 increases, the value from non-cooperation will overtake the benefits, and no first best allocation will be incentive compatible.

Figure 1 plots $H(a_0)$ for different parameter values.

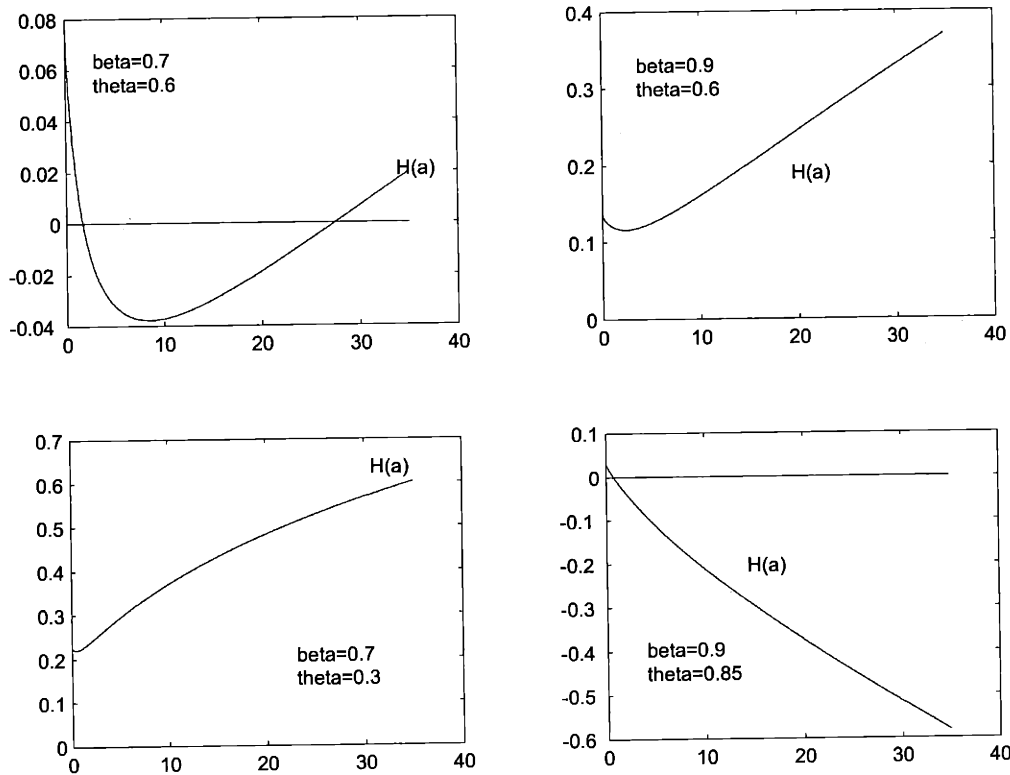


Figure 2-1: Is First Best Incentive Compatible?

As it is possible to see from the graph, the relation is not monotonic. In the top left panel, for example, there is an intermediate level of assets for which no first best

allocation is incentive compatible, even when there are incentive compatible first best allocations for low and high asset levels.

2.5 Efficient Subgame Perfect Equilibria

In this section, I study the set of incentive compatible and feasible allocations that are efficient.

Let a_0 be the initial amount of asset holdings. Let ψ be the utility level promised to party A (in power in period 1). Then, the set of efficient SPE is parameterized by a_0 and ψ and is the set of solutions to the following program :

$P(a_0, \psi) :$

$\max_{\{g^A, g^B, a\}} E_0 \sum_{i=0}^{\infty} \beta^i u^B(g_{i+1}^B)$ subject to

$$\sum_{i=0}^{\infty} \beta^i u^A(g_{i+1}^A) \geq \psi$$

$$a_t \geq 0; g_t^A \geq 0; g_t^B \geq 0; \forall t$$

$$y + Ra_{t-1} = g_t^A + g_t^B + a_t; \forall t$$

and subject to the incentive constraints,

$$\sum_{i=0}^{\infty} \beta^i u(g_{t+i}^A) \geq u(y + Ra_{t-1}) + \frac{\beta^2}{1 - \beta^2} u(y); \text{ for all even } t \geq 2 \quad (2.4)$$

$$\sum_{i=0}^{\infty} \beta^i u(g_{t+1}^B) \geq u(y + Ra_{t-1}) + \frac{\beta^2}{1 - \beta^2} u(y); \text{ for all odd } t \geq 2 \quad (2.5)$$

The first problem is that the constraint set is not convex. Because previous choices of asset holdings affect the incentive constraint in the next period, choice variables of the past will appear as a convex function on the left hand side of the incentive constraint. The constraint set is, then, not convex. Hence, even if the value function were differentiable first order conditions won't be sufficient for optimality.

Let $V(a, \psi)$ be the maximum value attainable to the party **not** in power today when the party in power has been promised a utility level of ψ and the savings done yesterday were equal to a . Define the domain constraint of $V(a, \psi)$ as a correspondence $D(a)$ such

that $D(a)$ is an interval of the real line :

$$D(a) = [\underline{\psi}(a), \bar{\psi}(a)]$$

where $\underline{\psi}(a) = u(y + Ra) + \frac{\beta^2}{1-\beta^2}u(y)$. Let the operator T^V be defined in the following way:

$$T^V \{V(a, \psi), D(a)\} = \max_{g_1, g_2, a', \psi'} \{u(g^1) + \beta\psi'\} \quad (2.6)$$

subject to:

$$u(g_2) + \beta V(a', \psi') \geq \psi \quad (2.7)$$

$$y + Ra - g_1 - g_2 - a' = 0 \quad (2.8)$$

$$a' \geq 0 \quad (2.9)$$

$$\psi' \in D(a') \quad (2.10)$$

where condition (2.7) is the promise keeping constraint: the utility delivered to the party in power today cannot be smaller than what was promised to her yesterday. Condition (2.8) is the budget constraint of the government today. Condition (2.9) is the non-borrowing constraint of the government. And condition (2.10) is the incentive constraint for the party in power tomorrow and a feasibility constraint on the promise utility (whatever is promised for tomorrow cannot be too high, otherwise it won't be feasible). A prime on top denotes next period choices.

And let the operator T^D be

$$T^D [V(a, \psi), D(a)] = [\underline{\psi}(a), T\bar{\psi}(a)]$$

where

$$T\bar{\psi}(a) = \max_{\substack{0 \leq g_2 \leq Ra+y \\ \bar{\psi}' \in D(y+Ra-g_2)}} \left\{ u(g_2) + \beta V(y + Ra - g_2, \bar{\psi}') \right\} \quad (2.11)$$

Remark: The operator T^V maps a function V (with a domain D) into a new function $T^V V$ that is the maximum value attainable to the party not in power subject to all the relevant constraints. The operator T^D maps a domain of V into a new domain $T^D D$ which basically reads that whatever is promised tomorrow to the party in power has to be higher than her autarky value tomorrow and smaller than the maximum promise that

can be made, (2.11). The solution to our original program is the biggest fixed point of (T^V, T^D) .

The recursive formulation can be interpreted as follows. The value offered to the party not in power today -given a promised utility value to the party in power today and an asset level- is obtained by maximizing over a new non-negative asset level, consumption levels and a new promised utility such that the party in power today obtains at least a utility level equal to her promised utility (2.7), the budget constraint is satisfied (2.8), and the party in power tomorrow cannot be promised a utility level below the worst equilibrium payoff (2.10), given the asset level selected for tomorrow.

It's not known whether the value function is concave or differentiable, but it is still possible to derive a few interesting results.

Proposition 2.1 *If the incentive constraint at time $t + 1$ is not binding and $a_t > 0$, then $g_{t+1}^A = g_t^A$ and $g_{t+1}^B = g_t^B$*

Proof. Suppose w.l.o.g. that party A was in power in period $t + 1$ and that $g_{t+1}^A \neq g_t^A$. Then you can bring g_t^A closer to g_{t+1}^A without affecting a_{t+1} , g_t^B and g_{t+1}^B by changing the amount saved at time t (a_t). This change has no effect in the utility of party B at time t and $t + 1$, and increases the utility of party A by $|u'(g_{t+1}^A) - u'(g_t^A)|$, without affecting her incentive constraint tomorrow (it was slack). The incentive constraint of party B was not affected by this change (her utility remained the same, and the assets held at time t (a_{t-1}) did not change). If the incentive constraint at time $t + 1$ is not binding, it has to be the case then that in an efficient allocation $g_t^A = g_{t+1}^A$. If $g_t^B \neq g_{t+1}^B$ a similar argument applies, a marginal movement of the assets at time t will increase the utility of party B at time t (making her incentive constraint slack) and will not tight the incentive constraint of party A at time $t + 1$ (which was slack). ■

This characterization says that if the incentive constraint is not binding at some period, then it is efficient to maintain the spending allocation between this period and the previous one. If incentive constraints never bind, then the following is clear,

Corollary 2.1 *If in an efficient incentive compatible allocation g no incentive constraint ever binds, then g is a first best allocation.*

What about if the incentive constraints are binding?

Proposition 2.2 *If party i is in power at time $t + 1$ and her incentive constraint is binding, then $g_{t+1}^i \geq g_t^i$*

Proof. Let w.l.o.g. $i = A$. The proof proceeds by contradiction. Suppose that $g_{t+1}^A < g_t^A$. Then it is possible to increase the savings done at time t (a_t), by one unit, and assign all the returns (R) to party A's consumption at time $t + 1$. This does not affect party B's payoff at time t , nor her incentive constraint at time t (a_{t-1} hasn't change). However, party A's payoff for deviating at time $t + 1$ increased by $Ru'(y + Ra_t)$ while her utility from cooperating at time $t + 1$ increased by $Ru'(g_{t+1}^A)$. Given that $y + Ra_t > g_{t+1}^A$ the benefits from cooperation increased more than the costs at time $t + 1$, so the incentive constraint of party A at time $t + 1$ is holding. Note also that the utility of party A at time t is now higher by an amount $u'(g_{t+1}^A) - u'(g_t^A) > 0$. This is then a more efficient allocation than the initial one, a contradiction. ■

A similar proposition can be proved for the consumption behavior of the party not in power at the time an incentive constraint binds,

Proposition 2.3 *Let $a_t > 0$. If party i is in power at $t + 1$ and her incentive constraint is binding, then $g_{t+1}^{-i} \leq g_t^{-i}$.*

Proof. Let w.l.o.g. $i = A$. The proof proceeds by contradiction. Suppose now that $g_{t+1}^B > g_t^B$. Then you can reduce the savings done at time t , a_t , without affecting a_{t+1} , g_{t+1}^A and g_t^A , this reduces the assets holdings at time $t + 1$, reducing the benefit from deviating of party A while the benefit from cooperation at time $t + 1$ has not changed. The incentive constraint for party A at time $t + 1$ is now slack. However, the decrease in the savings increases g_t^B and decreases g_{t+1}^B , at a rate R . The utility of party B at time t is then increased. This also makes her incentive constraint slack, the benefits from cooperation increased, while the value of deviating remained the same (a_{t-1} hasn't changed). So, this movement is incentive compatible and has increased the utility of party B while keeping party A's payoff constant. Then, an allocation with $g_{t+1}^B > g_t^B$ cannot be efficient. ■

The previous propositions characterize the behavior of government spending across parties with the changes of political power and whether the incentive constraint are binding or not. Whenever an incentive constraint binds, the consumption allocated to the party in power at that time will increase and the consumption allocated to the party out

of power will decrease. The proofs have shown how assets are playing a double role in an efficient allocation. Assets help to smooth consumption across time, but at the same time, they change the incentive constraints of the parties in power by directly affecting the value from moving to the worst equilibrium at any time. It has proven difficult to analyze the behavior of the asset holdings for the general case. Next section characterizes the efficient set when there is no endowment ($y = 0$). The power utility structure is exploited very heavily throughout that section. As will be clear, with no endowment and power utility, the program becomes homogenous, which allow me to bypass the non-convexity of the constraint set and characterize the efficient equilibria.

2.6 The Savings Game with No Endowment

In this section the case with no endowment is analyzed.

2.6.1 First Best Allocations and Incentive Compatibility

Result 2.3 *If there is no endowment ($y = 0$), there exists a first best allocation that is subgame perfect if and only if*

$$1 \geq (1 - \beta)^{1-\theta} 2^\theta \tag{2.12}$$

Remark: *Note that this condition is equivalent to condition (2.3), that apply in the limit as a_0 approaches infinity. In this case the endowment has no relevance, and it is not surprising that the condition for sustainability of the first best is the same as in the no-endowment case.*

Note also that the condition for the existence of an incentive compatible first best allocation with no endowment is independent of the level of initial assets a_0 . The reason is that given power utility, the utility under the worst equilibrium and the utility under a first best allocation is proportional to $(a_0)^\theta$. So, if for some asset level there exists an incentive compatible first best allocation, then it exists for any asset level. Note also that as θ goes to zero, the inequality is more likely to hold; and the opposite occurs as θ goes to 1. The intuition for that result is that as $\theta \rightarrow 1$, the elasticity of intertemporal substitution of the parties increases and the desire to smooth spending across time is reduced. This makes cooperation harder to sustain. In the limit when $\theta = 1$, the parties are linear, and there is no gain from intertemporal smoothing.

2.6.2 The Efficient Subgame Perfect Frontier

The first thing to notice is the following.

Proposition 2.4 *In the game with no endowment, if in an incentive compatible allocation for some t , $a_t = 0$, then this allocation is the worst equilibrium.*

If at some point in time, savings are zero, then the party at that time in power at time will spend all the assets on her own good (by incentive compatibility). This implies then that the party in power a period before will not save either and will consume all the assets on her own good, and so on. The allocation is then the autarky equilibrium.

Let us redefine the value function as $V(a, \psi) = \hat{V}(a, \tilde{a})$, where $u(R\tilde{a}) = \psi$. Using the fact that there is no endowment ($y = 0$), the operator T (2.6) is an operator in the value function, T^V

$$T^V \left[\hat{V}(a, \tilde{a}), \hat{D}(a) \right] = \max_{\{a', \tilde{a}', g_1, g_2\}} \{u(g_1) + \beta u(R\tilde{a}')\}$$

subject to:

$$u(g_2) + \beta \hat{V}(a', \tilde{a}') \geq u(R\tilde{a}) \quad (2.13)$$

$$Ra - g_1 - g_2 = a' \quad (2.14)$$

$$a' \geq 0 \quad (2.15)$$

$$\tilde{a}' \in \hat{D}(a') \quad (2.16)$$

and an operator T^D in the domain constraint,

$$T^D \left[\hat{V}(a, \tilde{a}), \hat{D}(a) \right] = [a, \bar{\chi}(a)]$$

where

$$\bar{\chi}(a) = \frac{1}{R} u^{-1} \left(\max_{\substack{0 \leq g_2 \leq Ra \\ \tilde{a}' \in \hat{D}(Ra - g_2)}} \{u(g_2) + \beta \hat{V}(Ra - g_2, \tilde{a}')\} \right) \geq a$$

We are interested in the biggest fixed point of T , $V^*(a, \tilde{a})$, $D^*(a)$ such that $V^* = T^V V^*$ and $D^* = T^D D^*$.

We say that a correspondence $D(a)$ is homogenous if $\lambda \hat{D}(a) = \hat{D}(\lambda a)$ for λ positive.

Lemma 2.2 *The operator T is such that for all $\hat{V}(a, \tilde{a})$ continuous and homogeneous of degree θ in the associated domain \hat{D} homogenous and compact valued we have that*

- $T^V \hat{V}$ is homogenous of degree θ
- $T^D \hat{D}$ is homogenous.

Proof. The domain operator is homogenous clearly if \hat{V} is homogenous of degree θ . Let $g_1(a, \tilde{a}), g_2(a, \tilde{a}), a'(a, \tilde{a})$ and $\tilde{a}'(a, \tilde{a})$ be the policy functions that solve $T^V V(a, \tilde{a})$, then it is easy to see that $\lambda g_1(a, \tilde{a}), \lambda g_2(a, \tilde{a}), \lambda a'(a, \tilde{a})$ and $\lambda \tilde{a}'(a, \tilde{a})$ satisfy the constraint set for $T^V V(\lambda a, \lambda \tilde{a})$ with λ positive. This implies then that $\lambda^\theta T^V V(a, \tilde{a}) \leq T^V V(\lambda a, \lambda \tilde{a})$. Given that the choice of λ is arbitrary (we could have chosen $1/\lambda$) then $\lambda^\theta T^V V(a, \tilde{a}) = T^V V(\lambda a, \lambda \tilde{a})$. ■

Lemma 2.3 *The operator T is monotone. For any two value functions V_1 and V_2 with respective domains D_1 and D_2 then if*

$$V_1 \leq V_2 \text{ and } D_1 \subset D_2 \Rightarrow T^V V_1 \leq T^V V_2 \text{ and } T^D D_1 \subset T^D D_2$$

Theorem 2.1 *The value function $V^*(a, \tilde{a})$ is homogenous of degree θ .*

Proof. Start with $V_0(a, \tilde{a})$ of the first best allocations with $D_0(a) = \left[a, a \left(\frac{R}{R-1} \right)^{\frac{1-\theta}{\theta}} \right]$. In the first iteration, clearly we have that $T^V V_0 \leq V_0$ and $D_0 \subseteq T^D D_0$. Now, because $V_0(a, \tilde{a})$ is homogenous of degree θ , then $T^V V_0$ is homogenous as well (the same applies for the domain operator). The value of V^* is such that $V^* \leq V_0$, so monotonicity implies that $(T^V)^n V^* = V^* \leq (T^V)^n V_0$, (the same for D_0). We have sequences of $(T^V)^n V_0$ and $(T^D)^n D_0$ that are monotonically decreasing, and are always bigger than V^* and D^* so, the sequences have to converge. Given that they converge to a fixed point, it has to be the case that $\lim_{n \rightarrow \infty} (T^V)^n V_0 = V^*$ ($[V^*, D^*]$ is the biggest fixed point). Because $(T^V)^n V_0$ are homogenous for all n , we have that V^* is homogenous. ■

The interesting cases to study are when the incentive constraints are binding. From now on, the following assumption is made

Assumption 2.3 *There is no incentive compatible first best allocation:*

$$1 < (1 - \beta)^{1-\theta} 2^\theta$$

The incentive constraints will be binding.

Theorem 2.2 *If no first best allocation is incentive compatible, then the incentive constraints are always binding.*

To proof Theorem 2.2, the following lemma is used.

Lemma 2.4 *If no first best allocation is incentive compatible, then an incentive compatible allocation cannot provide both parties at the same time with a utility level higher than their autarky value.*

Proof. By Assumption (2.3), there is no first best allocation that can give each party at the same time a utility level higher than their autarky value. Now, first best allocations give the maximum amount of utility to one party constrained to providing certain value to the other. Given a utility value for one party, any other allocation will provide a utility level to the other party smaller than the corresponding first best allocation. Now, given that no first best allocation was incentive compatible, that implied that at any time it was not possible in a first best allocation to provide both parties with utility levels higher than their autarky values, then no other allocation will do it. ■

This lemma says that if no first best allocation is incentive compatible, then at any time t if one party receives a utility level higher or equal to $u(Ra_t)$, then other party has to receive a value strictly smaller than $u(Ra_t)$ (Notice that the party who receives less than $u(Ra_t)$ is the party out of power at time t , otherwise her incentive constraint won't be holding).

Now, it is possible to proof Theorem 2.2.

Proof of Theorem 2.2. First, it is shown that if the incentive constraint is not binding at time $t+1$, it cannot be binding at time $t+2$. Suppose not, that the incentive constraint is binding at time $t+2$ and it was not binding at $t+1$. Let w.l.o.g. party A be in power at time t . Then by Proposition 2.1, $g_{t+1}^A = g_t^A = g_A$ and $g_{t+1}^B = g_t^B = g_B$. At time $t+2$, party A is receiving a utility level equal to her autarky value (the incentive constraint

is binding), so $V_{t+2}^A = u(Ra_{t+2})$. At time $t + 1$, by Lemma 2.4 the utility to party A ($u(g_A) + \beta V_{t+2}^A$) is smaller than her autarky value and the utility to party B is higher than her autarky value (her incentive constraint is not binding at $t + 1$), so

$$u(g_A) + \beta u(Ra_{t+2}) < u(Ra_{t+1}) < u(g_B) + \beta V_{t+2}^B \quad (2.17)$$

Where V_{t+2}^B is the value party B receives in period $t + 2$. In period $t + 2$, party B is out of power, and receives by Lemma 2.4 a value smaller than her autarky value, $V_{t+2}^B < u(Ra_{t+2})$. Plugging into (2.17),

$$\begin{aligned} u(g_A) + \beta u(Ra_{t+2}) &< u(g_B) + \beta V_{t+2}^B < u(g_B) + \beta u(Ra_{t+2}) \\ u(g_A) - u(g_B) &< 0 \Rightarrow \end{aligned}$$

$$g_A < g_B \quad (2.18)$$

The incentive constraint of party A is binding at time $t + 2$, so the incentive constraint at time t for party A implies that

$$V_t^A = u(g_A) + \beta u(g_A) + \beta^2 u(Ra_{t+2}) \geq u(Ra_t) \quad (2.19)$$

By Lemma 2.4, party B in period t , receives a utility value that has to be smaller than her autarky value. So $u(g_B) + \beta V_{t+1}^B \leq u(Ra_t)$. But, $V_{t+1}^B > u(Ra_{t+1})$ (her incentive constraint is not binding at time $t + 1$), so

$$u(g_B) + \beta u(Ra_{t+1}) < u(Ra_t) \quad (2.20)$$

Then by (2.18), (2.19) and (2.20) it is obtained

$$\frac{u(Ra_t) - \beta^2 u(Ra_{t+2})}{1 + \beta} \leq u(g_A) < u(g_B) < u(Ra_t) - \beta u(Ra_{t+1})$$

or equivalently,

$$\beta(1 + \beta)u(Ra_{t+1}) - \beta u(Ra_t) - \beta^2 u(Ra_{t+2}) < 0 \quad (2.21)$$

Let $g^T = g_A + g_B$. Then, the following relation holds

$$\begin{aligned} R^2 a_t - Rg^T &= Ra_{t+1} \\ R^3 a_t - R^2 g^T - Rg^T &= Ra_{t+2} \end{aligned}$$

Let $J(g^T) = (1 + \beta)\beta u(R^2 a_t - Rg^T) - \beta u(Ra_t) - \beta^2 u(R^3 a_t - R^2 g^T - Rg^T)$. Minimizing J

$$g^{T*} = \{\arg \min J(g^T)\} = [R - 1] a_t$$

Plugging back into the value of $J(g^T)$, $J(g^{T*}) = 0$. So, $\min_{g^T} [J(g^T)] = 0 \Rightarrow J(g^T) \geq 0$ for all feasible g^T , and (2.21) can not hold. So, if an incentive constraint is not binding at time $t + 1$, then is not binding at time $t + 2$ and by induction, is never binding for any $\tau > t + 1$. The allocation starting from time $t + 1$ is then a first best allocation, but this is a contradiction, because no first best allocation is incentive compatible by Assumption (2.3). ■

From Theorem 2.2 it follows then that $u(R\tilde{a}') = u(Ra') \Leftrightarrow \tilde{a}' = a'$. Using the homogeneity of the value function, the promised utility constraint can be rewritten as $u(g_2) + \beta(a')^\theta \hat{V}(1, 1) \geq u(R\tilde{a})$ and the program under a binding incentive constraint is now

$$\hat{V}(a, \tilde{a}) = \max \{u(g_1) + \beta u(Ra')\} \quad (2.22)$$

subject to :

$$\begin{aligned} u(g_2) + \phi \beta (a')^\theta &\geq u(R\tilde{a}) \\ Ra - g_1 - g_2 &= a' \\ a' &\geq 0 \end{aligned}$$

where $\phi = \hat{V}(1, 1)$.

It is possible to see that in program (2.22) (when incentive constraints are binding) the policy functions are proportional to the initial asset level a . Because from the second period forward, the value function is always evaluated at $V(a', a')$, two different programs will have optimal allocations with the same ratios g_1/a and g_2/a from period 2 onwards. Knowing $V(a, a) = \phi a^\theta$ will be enough to characterize the Pareto frontier of the subgame perfect equilibria. The problem has been reduced from two dimensions, to just one. It is necessary now to compute ϕ .

What is the value of ϕ ? Suppose there is one unit of the asset and party A is in power. Let $g_t^A = (1 - \beta)^{\frac{1}{\theta}} R$. This constant path of consumption delivers a utility to party A equal to $u(R)$. From the budget constraint it is possible to compute the associated consumption allocation for party B subject to a constant savings of one unit, $g_t^B = (R - 1) \left[1 - \left(\frac{R-1}{R} \right)^{\frac{1-\theta}{\theta}} \right] > 0$. The utility level generated by g_t^B , $\frac{u(g_t^B)}{1-\beta}$, is the maximum utility that could be delivered to party B constrained to providing a utility level of $u(R)$ to party A. However, this allocation is not incentive compatible (no first best allocation ever is). So the value to party B, $\hat{V}(1, 1) \equiv \phi$, cannot be as high as $\frac{u(g_t^B)}{1-\beta}$. The next lemma follows,

Lemma 2.5 *When no first best allocation is incentive compatible, then ϕ is strictly smaller than $\bar{\phi} \equiv R^\theta \left[\left(\frac{R}{R-1} \right)^{\frac{1-\theta}{\theta}} - 1 \right]^\theta < R^\theta$.*

The behavior of the asset level can also be characterized.

Notice that as $\theta \rightarrow 1$, $\bar{\phi} \rightarrow 0$, and hence $\phi \rightarrow 0$, $g_2(a) \rightarrow Ra$ and $a'/a \rightarrow 0$. In the limit as θ converges to one, the unique equilibrium is autarky. This is not surprising, as the elasticity of intertemporal substitution goes to infinity, there are no gains from trade, and cooperation is not incentive compatible.

For the second period forward, $\bar{a} = a$. The problem is

$$\max_{\substack{Ra = a' + g_1 + g_2 \\ a' \geq 0}} \left\{ u(g_1) + \beta R^\theta u(a') + \kappa \left(u(g_2) + \phi \beta (a')^\theta \right) \right\} \quad (2.23)$$

where κ is the lagrange multiplier of the promise keeping constraint, with $\kappa > 0^3$. This is a convex program, and first order conditions will be sufficient for optimality. I will first study the problem for any given $\kappa > 0$, and compute equilibrium characteristics that will have to hold for all κ .

Taking the first order conditions (let μ be the lagrange multiplier on the budget constraint) $u'(g_1) = \mu = \kappa u'(g_2) = (\beta R^\theta + \kappa \phi \beta) u'(a')$. Solving out for the spending

³Under Assumption 3, the promise keeping constraint is binding in any fixed point of program (2.22) for $\bar{a} = a$. The proof follows by contradiction. Suppose not, then if in a fixed point the promise keeping constraint is not binding, $g_2 = 0$. Solving for $\phi a'^\theta = \max_{0 \leq g_1 \leq (Ra - a')} [u(g_1) + \beta u(Ra')]$, it is obtained $\phi = \frac{R^{2\theta}}{(1+R)^{\theta-1}}$. But, given that the promise keeping is not binding, $\phi > R(1+R)^\theta$. Using the computed value of ϕ , it has to be the case that $1 > \left(\frac{1+R}{R} \right)^{(2\theta-1)} R$. But $\left(\frac{1+R}{R} \right)^{(2\theta-1)} R \geq 2^\theta \left(\frac{1+R}{R} \right)^{(\theta-1)} R \geq 2^\theta (1-\beta)^{(1-\theta)} > 1$ where the last step follows from assumption 3. This is a contradiction then and the promise keeping constraint is binding in a fixed point for $\bar{a} = a$.

amounts as a function of a'

$$g_1 = [\beta R^\theta + \kappa \phi \beta]^{\frac{1}{\theta-1}} a', g_2 = \left[\frac{\beta R^\theta + \kappa \phi \beta}{\kappa} \right]^{\frac{1}{\theta-1}} a' \quad (2.24)$$

Using the budget to solve for a' ,

$$Ra = a' + g_1 + g_2 = a' \left(1 + [\beta R^\theta + \kappa \phi \beta]^{\frac{1}{\theta-1}} + \left[\frac{\beta R^\theta + \kappa \phi \beta}{\kappa} \right]^{\frac{1}{\theta-1}} \right)$$

Putting $\beta R = 1$, and solving out for the ratio of assets

$$\frac{a'}{a} = \frac{R}{1 + \frac{1 + \kappa^{\frac{1}{1-\theta}}}{(1 + \kappa \phi R^{-\theta})^{\frac{1}{1-\theta}}} R} \quad (2.25)$$

Let $h(\kappa) \equiv \left(\frac{1 + \kappa^{\frac{1}{1-\theta}}}{(1 + \kappa \phi R^{-\theta})^{\frac{1}{1-\theta}}} \right)$. I will compute the minimum value of $h(\kappa)$ as a function of κ . This can then be used as an upper bound for a'/a .

Taking derivatives of $h(\kappa)$ with respect to κ ,

$$\frac{\partial h(\kappa)}{\partial \kappa} = \frac{\frac{1}{\theta-1} \kappa^{\frac{\theta}{1-\theta}} (1 + \kappa \phi R^{-\theta})^{\frac{1}{1-\theta}} - \frac{1}{\theta-1} (1 + \kappa \phi R^{-\theta})^{\frac{\theta}{1-\theta}} \phi R^{-\theta} (1 + \kappa^{\frac{1}{\theta-1}})}{\left((1 + \kappa \phi R^{-\theta})^{\frac{1}{1-\theta}} \right)^2}$$

The sign of $\frac{\partial h(\kappa)}{\partial \kappa}$ is equal to the sign of $\kappa - [\phi R^{-\theta}]^{\frac{1-\theta}{\theta}}$. When $\kappa < [\phi R^{-\theta}]^{\frac{1-\theta}{\theta}}$, $h(\kappa)$ is decreasing, and when $\kappa > [\phi R^{-\theta}]^{\frac{1-\theta}{\theta}}$, $h(\kappa)$ is increasing. Then, $h(\kappa)$ is minimized at $[\phi R^{-\theta}]^{\frac{1-\theta}{\theta}}$. An upper bound on $\frac{a'}{a}$ is

$$\frac{a'}{a} < \frac{R}{1 + h([\phi R^{-\theta}]^{\frac{1-\theta}{\theta}}) R} = \frac{R}{1 + \frac{1 + [\phi R^{-\theta}]^{\frac{1}{\theta}}}{(1 + [\phi R^{-\theta}]^{\frac{1}{\theta}})^{\frac{1}{1-\theta}}} R} < 1$$

where the last inequality holds because $\phi < \bar{\phi} \equiv R^\theta \left[\frac{R^{\frac{1-\theta}{\theta}}}{(R-1)^{\frac{1-\theta}{\theta}}} - 1 \right]^\theta$. For all κ , we have that $a' < a$. The following theorem is thus proved,

Theorem 2.3 *In the case of no endowment when no first best allocation is incentive*

compatible, if in an efficient allocation $a_t > 0$ for some $t \geq 1$, then the level of assets holdings decreases at a constant rate from period 2 onwards.

If there are positive savings done in an efficient allocation, assets are decreasing continuously towards zero. The intuition is the following. If in an incentive compatible allocation, a constant level of assets is maintained, then the ratio of g_1/g_2 is going to be low. An incentive compatible allocation would have to provide too much consumption to the party in power to keep her from consuming all the asset holdings, given that the asset holdings are going to be high in the future, the same applies for the party in power tomorrow. Given that there is party turnover, the consumption allocated to a given party will vary widely, according to whether she is in power or not. Now, by reducing the asset level, total consumption is higher, and is possible to achieve a higher g_1/g_2 ratio. Now, when parties alternate in power, the changes in their consumption paths are less drastic than before, but have a decreasing slope (less is saved for the future). This trade-off between asset efficiency and the sharing of consumption in the power utility case with no endowment is stronger in the need for consumption sharing, and asset holdings are reduced in an efficient allocation.

I can also characterize the rest of the efficient frontier. Let $a'(a, \tilde{a})$, $g_1(a, \tilde{a})$ and $g_2(a, \tilde{a})$ be the optimal policies with initial asset level a and promised utility $u(R\tilde{a})$. Then, the following holds,

Proposition 2.5 *For any a , as the utility level promised to the party in power (\tilde{a}) increases (in the domain), then*

$$\begin{aligned} a'(a, \tilde{a}) & \text{ decreases} \\ \frac{g_1(a, \tilde{a})}{g_2(a, \tilde{a})} & \text{ decreases} \end{aligned}$$

Proof. To proof this proposition, it is first shown that $\kappa > 1$. To see this, note that Proposition 2.2 implies that $g_1(a, a) \leq g_2(a', a') = \frac{a'(a, a)}{a} g_2(a, a)$. Given that $a'/a < 1$, then $g_2 > g_1$. Using (2.24), it is obtained that $\kappa^{\frac{1}{1-\theta}} > 1$, which implies that $\kappa > 1$. Increasing \tilde{a} is equivalent to increase κ in problem (2.23) -a tightening of the promised keeping constraint- and given that $\kappa > 1 > (\phi R^{-\theta})^{\frac{1-\theta}{\theta}}$, it implies by (2.25) that a' is decreasing in \tilde{a} for a given a . The fact that g_1/g_2 decreases is clear by (2.24). ■

It is possible to characterize the rest of the model by solving for ϕ and κ . From the promise keeping constraint,

$$(g_2)^\theta + \phi\beta (a'/a)^\theta = R^\theta$$

And from the value function definition

$$\hat{V}(1, 1) \equiv \phi = (g_1/a)^\theta + \beta (Ra'/a)^\theta$$

Setting $a = 1$ in (2.25) and solving out for g_1 and g_2 using (2.25), a system of three equations with three variables $(\phi, a'/a, \kappa)$ is obtained. Call Γ the set of all such trios with κ in the extended reals.

Let $\Gamma^* = \{(\phi, a', \kappa) \in \Gamma \text{ such that } \phi \leq \bar{\phi}\}$. The solution to our program is the maximum $\phi \in \Gamma^*$. Note that $(0, 0, \infty) \in \Gamma^*$, so Γ^* is non-empty. Obtaining comparative statics on ϕ has proved hard. Next section presents a simple numerical algorithm to compute the values of ϕ , a'/a , and g_1/g_2 .

2.6.3 The Value of ϕ and Sensitivity to Parameters

This section presents a general and simple numerical algorithm to compute the values of ϕ and the ratio of savings a'/a for any set of parameter values (β and θ).

Define the following operator, $T(\hat{\phi})$:

$$T(\hat{\phi}) = \max_{g_1, g_2, a'} \left\{ g_1^\theta + \beta (Ra')^\theta \right\}$$

subject to :

$$\begin{aligned} g_2^\theta + \hat{\phi}\beta (a')^\theta &\geq R^\theta \\ R - g_1 - g_2 &= a' \\ a' &\geq 0 \end{aligned}$$

The operator T is monotone. The parameter ϕ is the maximum fixed point of T such that $\hat{\phi} \in [0, \bar{\phi}]$.

Remark: *The computational approach is as follows : start from some $\phi_0 \geq \bar{\phi}$ such that $T(\phi_0) < \phi_0$. By monotonicity of T , the sequence of positives values $\phi_{t+1} = T(\phi_t)$ is monotonically decreasing. It converges to some $\phi_\infty = \lim_{n \rightarrow \infty} T^n(\phi_0)$. If $\phi_\infty \in [0, \bar{\phi}]$*

then $\phi = \phi_\infty$.

Notice that $0 = T(0)$. When $\hat{\phi} = 0$, then $g_2 = R$, $g_1 = 0$ and $TV(0) = 0$. This is the worst equilibrium allocation.

Table 2.1 shows the computed biggest fixed point ϕ for different values of β , with $\theta = 0.4$ and $2^\theta (1 - \beta)^{1-\theta} > 1$. As β goes down, ϕ decreases. Also, as predicted from the previous section, all ratios a'/a are smaller than one. As β decreases, the government is saving less (a'/a) and the ability to share (as measured by g_1/g_2) decreases.

Values of β	0.200	0.230	0.260	0.290	0.320	0.350
ϕ	1.013	1.144	1.253	1.341	1.405	1.460
a'/a	0.546	0.643	0.726	0.792	0.836	0.879
g_1/g_2	0.107	0.135	0.162	0.186	0.203	0.222

Table 2.1: Values of ϕ , $\frac{a'}{a}$ and $\frac{g_1}{g_2}$ for $\theta = 0.4$

Figure 2-2 plots the operator T for the values in the table. We can see the existence of two fixed points (zero and the one showed in table 1).

The reading of figure 2-2 might suggest that ϕ is continuous in β . However, this is not true. Table 2.2 presents similar calculations for $\theta = 0.55$. In this case, ϕ (and the policy functions) appear to be discontinuous. For β low enough, cooperation is not sustainable, and the only possible payoff is the autarky value.

Values of β	0.400	0.440	0.483	0.484	0.500	0.520	0.540	0.560
ϕ	0	0	0	0.920	1.121	1.230	1.292	1.341
a'/a	0	0	0	0.628	0.736	0.845	0.901	0.947
g_1/g_2	0	0	0	0.122	0.131	0.175	0.205	0.233

Table 2.2: Values of ϕ , $\frac{a'}{a}$ and $\frac{g_1}{g_2}$ for $\theta = 0.55$ and different values of β

To explore the discontinuity, figure 2-3 plots the operator $T(\cdot)$ for the corresponding different parameter values. The graph of T is clearly not concave, and *kisses* the forty five degree line around $\hat{\beta} \cong 0.484$ (for $\theta = 0.55$). For values of β lower than that, the operator T never again touches the forty five degree line for values greater than zero. This non concavity of T generates discrete jumps in ϕ for small changes in parameter values.

This is a surprising result. There are some positive β values for which there is a unique equilibrium, the autarky equilibrium, where parties never cooperate, spend everything

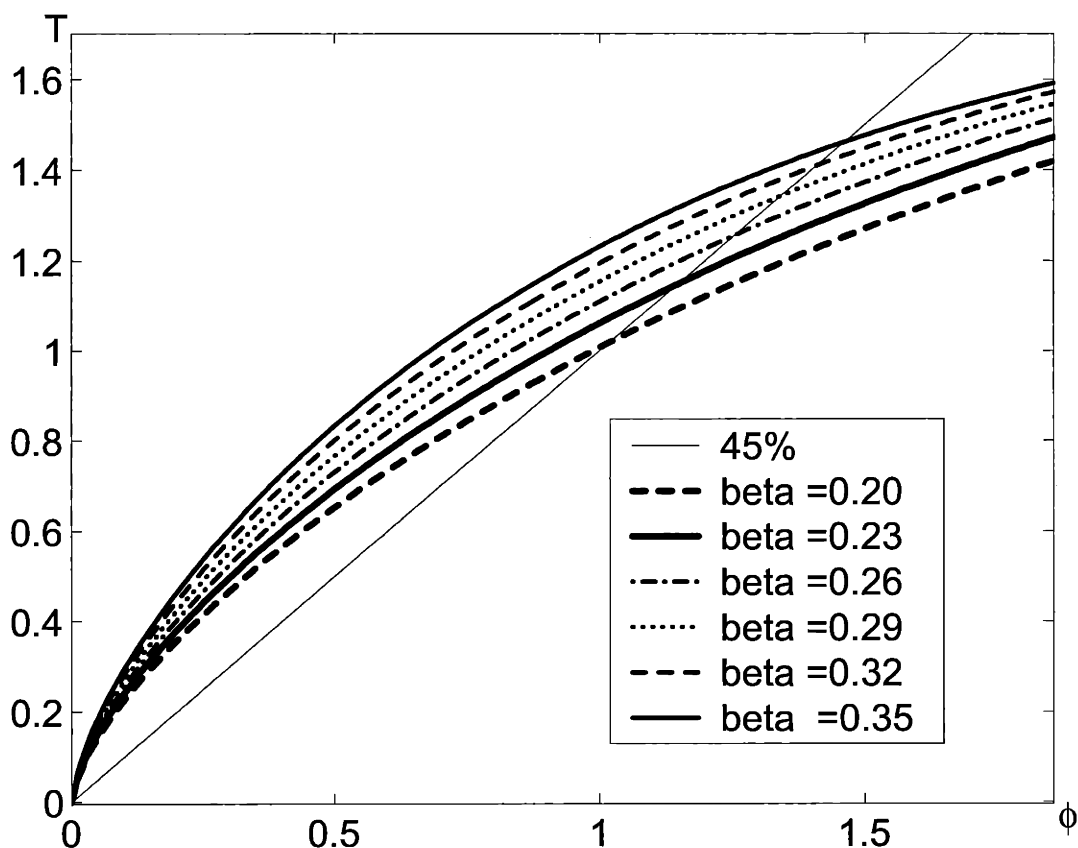


Figure 2-2: The T operator for $\theta = 0.4$ and different values of β

on their own goods, and never save for the future. *Note that this is so even when the parties' future marginal utilities of consumption are equal to infinity.*

Kocherlakota (1993) characterizes the efficient frontier of a game with two players without commitment. He shows that if the marginal utility of consumption in the future under autarky is sufficiently high, there exist always equilibria with more consumption sharing than autarky. The intuition is the following. Every period there is an amount y of income that the parties could share. The parties cannot borrow nor save, and assume for simplicity a political structure identical to the one described in Section 1. Suppose that today the parties are in autarky. Suppose now that they promise each other small amounts of consumption (the party in power gives a small amount of consumption to the party not in power every period). Given that the marginal utility of consumption next period for the party in power under autarky is infinity (she is not receiving any

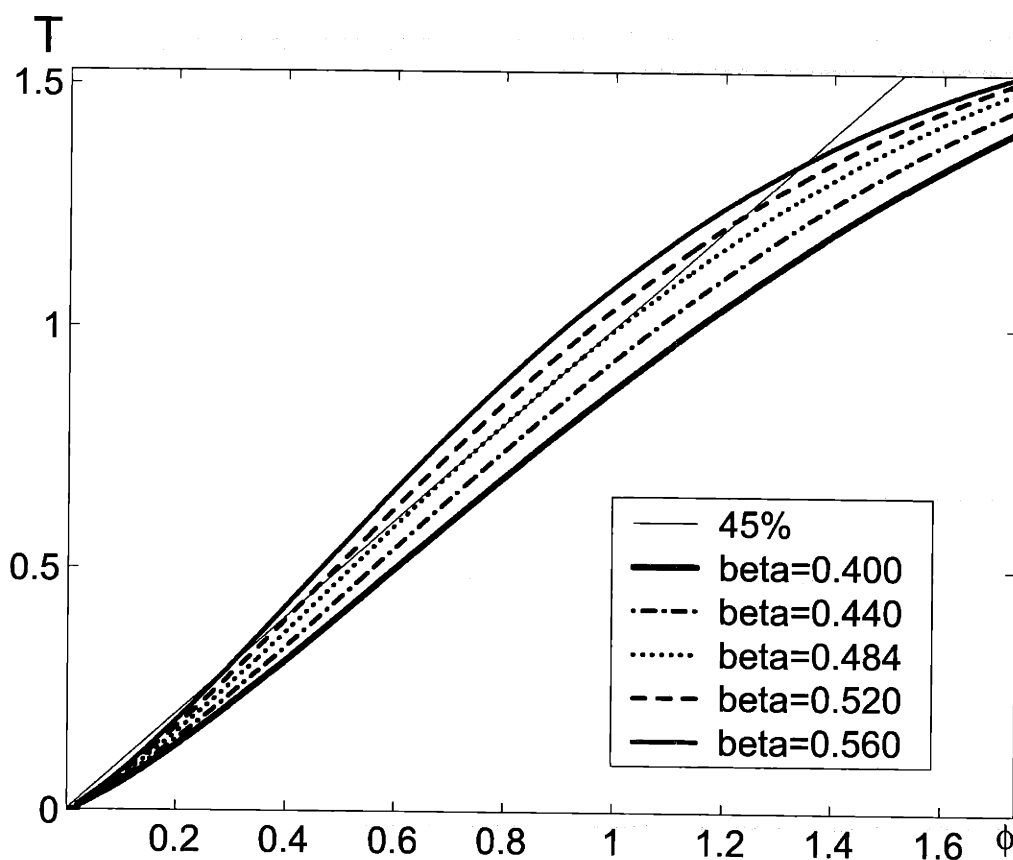


Figure 2-3: The T operator for $\theta = 0.55$ and different values of β

consumption when she is out of power) this strictly increases the utility of both parties above autarky, and hence is incentive compatible.

In the model with savings and no endowment this intuition does not apply. In particular, in the autarky equilibrium, the party tomorrow has no income (no assets have been passed on to the future) and hence nothing to share. The only way to consume in the future is through savings done today by the party in power. In a sharing allocation, the party in power has to provide consumption to the other party and save for the future. She will not receive all the return on those savings however, because the party in power tomorrow will have to be “convinced” to share as well, and hence will have to be provided with consumption. This reduces the incentive to cooperate today, and for low betas and a sufficiently high elasticity of intertemporal substitution, there is no equilibrium but autarky.

Next section explores how the equilibria set might be improved with the use of illiquid assets.

2.6.4 The Role of Illiquid Assets and Efficiency

Suppose now that the government had access to a savings technology that is illiquid⁴. The budget constraint of the government is now

$$\begin{aligned} R(k + a) &= k' + a' + g_1 + g_2 \\ g_1 + g_2 &\leq Ra \\ a', k' &\geq 0 \end{aligned}$$

Where k denotes the amount saved in the previous period into this illiquid asset. The important thing to notice is that the government can not liquidate s in the current period, so total government spending is constrained to be smaller than the amount of liquid funds, Ra . The government can transform illiquid funds into liquid funds that could be consumed next period.

Because the illiquid funds are not available this period, the current deviation by the party in power can only attain a maximum consumption of Ra . After a deviation has occurred the worst subgame perfect equilibrium is just the consumption of all the income available to the government every period. Given that the use of the illiquid assets is not going to benefit the current party (because she expects the party in the future to spend all the income in her own good), in the worst equilibrium, she won't liquidate any of it⁵. The worst equilibrium payoff to a party in power is then $u(Ra)$, or the consumption of liquid funds.

When are first best allocations incentive compatible with illiquid assets?

A first best allocation is as before characterized by a fraction α allocated to a party out of a constant total spending. For a given total assets $k + a$, there is an incentive compatible first best allocation if for $\alpha = 1/2$ the following holds

⁴See Laibson(1997).

⁵This could be relaxed without affecting the results.

$$\frac{u\left(\frac{(R-1)(a+k)}{2}\right)}{1-\beta} \geq u(Ra) \quad (2.26)$$

Imposing the condition that total consumption $(R-1)(a+k)$ is done out of the liquid savings:

$$\begin{aligned} (R-1)(a+k) &\leq Ra \\ (R-1)k &\leq a \end{aligned}$$

To minimize the right hand side of (2.26), a is set to its smallest possible value $(R-1)k$. Plugging this back into (2.26), the following is obtained,

Result 2.4 *In the no endowment case with illiquid asset holdings, a first best allocation is incentive compatible if*

$$1 \geq 2^\theta (1-\beta)$$

This is a weaker condition than (2.12). In particular,

Result 2.5 *If $\beta > \frac{1}{2}$, an incentive compatible first best allocation always exists.*

The ability to save in illiquid assets, increases the possibility of sustaining a first best allocation. Hence there is a demand for illiquidity as in the case of hyperbolic consumers studied by Laibson (1997). The intuition for this result is very similar, illiquid assets reduce the temptation of the parties to refuse to cooperate, because they can not liquidate easily the wealth of the government. This lowers the incentive constraint for the party in power and more efficient allocations are now sustainable.

2.7 Conclusion

In this paper I studied the behavior of savings under political compromise. Parties in power face a trade-off between consuming today all of the income available or maintaining a reputation for cooperation. I proposed a numerical implementation to calculate the set of efficient subgame perfect equilibria following the work of Abreu, Pearce and Stachetti. It is shown that the savings are inefficiently done in equilibrium, and that parties might demand illiquid assets to improve the equilibrium allocation. It is also shown that the

efficient set of subgame perfect equilibria is discontinuous in the parameters, and that there are cases when the marginal utilities of consumption in the future are equal to infinity for all players, and there is still no cooperation.

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Chapter 3

Entrepreneurial Pressure, Innovation, and Rent Cannibalization

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3.1 Introduction

Would ideas that are incorporated as new ventures be incorporated in existing organizations, were the venture capital market less competitive? Does the venture capital market force or discourage incumbents to innovate?

In this paper, we propose a theory of innovation and entrepreneurship which determines:

1. How venture capital markets indirectly affect the innovation policy of established companies.
2. Under what circumstances new ideas are incorporated in existing or in new organizations.

New ideas occur to managers in a given sector. These ideas can be implemented in a new firm financed by venture capital or inside an existing firm. The willingness of an incumbent firm to adopt a new idea depends on the rents it has to give to the manager,

on the incremental value of this idea, and on the threat that this idea constitutes for the firm if implemented outside.

Implementing an idea inside an existing organization allows the sharing of assets and might therefore be cheaper. But the advantage of implementing the new project on the existing organization also comes at a cost: in the presence of contractual imperfections, it is harder to reward the manager with the cash-flows generated by his project in the existing firm than in a new firm –where any positive cash-flow comes from the entrepreneur’s project. If contingent contracting is valuable ex-ante –because of asymmetric information, moral hazard or differences in beliefs, this gives to the venture capitalist an advantage over the existing firm at financing new projects.

Whether a project is done inside or outside depends therefore on the balance of two related comparative advantages: established firms, by relying on existing assets, might face a lower cost of implementing the idea, but venture-capitalists can write contracts that depend exclusively on cash-flows generated by the project. The more innovative a project is, the more attractive is venture capital financing. For the most innovative projects, the incumbent firm cannot compete with venture capital. Such projects are implemented outside. In general, we determine whether a project is done in the incumbent firm, in a new firm or nowhere, depending on its characteristics. We describe in this perspective the life-cycle of products and firms.

In our model, venture capital does not only affect innovation through the creation of new ventures, it also affects the willingness of incumbent firms to innovate. This effect is ambiguous. If entrepreneurs can easily finance projects outside, incumbent firms are forced to innovate (if they don’t do it, it will get done in new firms). On the other hand, better venture capital markets imply higher rents to managers and reduce the value of innovation.

The value of a firm comes from the combination of the cash-flows generated by the current technology and the option to exploit new ones. Both are negatively affected by the efficiency of the venture capital market which changes the appropriability of new ideas and the life-cycle of technologies. Our model endogenizes the value of the firm and provides a framework for the valuation of innovative companies.

Our paper is related to three strands of literature. The first one is the literature on internal vs. external markets. Getner, Scharfstein and Stein (1994) develop a model of internal versus external capital markets, focussing on moral-hazard problems and asset redeployability. More recently, Gromb and Scharfstein (2001) study the organizational

choice between internal or external venturing for new projects. The interaction between the incentives and redeployability trade-off on the one hand and the equilibrium of the labor market on the other hand determines the optimal form of organization.

Second, our paper is related to the standard endogenous growth literature and the literature on innovation and incomplete contracts which endogenizes the organizational form of R&D. The main difference of our model with this two literatures is that we focus on the implementation rule for new ideas (e.g. where are new ideas implemented?) rather than on the production of new ideas.

Third, our paper is also related to recent contributions on the appropriability of firms' assets by their employees. Rajan and Zingales (2001) study how firms organize in reaction to the threats by employees of appropriating its rents. They relate the intensity of this threat to the efficiency of financial markets and mention that internal competition (the firm versus its employees) might be more effective at forcing firms to innovate than external competition. Hellmann (2002) studies when employees choose to become entrepreneurs, depending on the property right environment and the nature of projects. Cassiman and Ueda (2002) study what projects are done inside an established firm and which ones outside. They find that this depends on the project's complementarity with the existing firm's assets and on the capacity constraint of the firm. The mechanism they explore is the option value that an incumbent has to wait for a better use of its innovative capacity. The optimal management of this real option leads to a decision rule that favors projects that have high value or high complementarity. By contrast, we focus on contractual frictions, and we find that high value projects tend to be done outside the incumbent firm.

The paper delivers a set of empirical predictions. In particular, the model predicts that big ideas (or high valued projects) will be implemented outside the existing firms. Sectors with more contractual frictions, like for example human capital intensive sectors, are sectors where more innovations will be implemented in outside ventures (Section 3.5). The model also shows that more efficient venture capital markets shortens the life-cycle of firms in a sector. However, more efficient venture capital markets has ambiguous effects in the life cycle of products (the innovation rate can be either increased or reduced).

Next section introduces a static version of the model, where the main intuitions and the contractual environment are explained. Section 3.3 presents the full dynamic model and endogenizes the value of the firm. Section 3.4 characterizes the efficiency properties of the equilibrium and the effect of venture capital in the innovation level. Section 3.5

generalizes the contractual friction. Section 3.6 concludes.

3.2 The Static Set Up

There are three risk-neutral actors in an economy: an incumbent firm, a manager and a competitive venture capitalist.

The objective of the firm is to maximize shareholders' value. Let q be an index of the current technology's productivity. The incumbent firm holds the technology and let $V > 0$ be the value to shareholders at the beginning of the period. The manager has an idea (or project) that if successful will replace the incumbent's technology with a new one of value τV (with $\tau > 1$). For now, we take τ as given. This idea can be implemented at a cost C inside the incumbent firm and at a cost C' within a new organization. Once implemented, the idea succeeds with probability p , in which case the old technology becomes obsolete. If the project is not implemented by the firm, we assume that it is not possible to prevent the manager from trying it outside. So, if the firm buys a project it will implement it.

The manager decides the organizational form (new or old) for his project; and this decision changes the implementation cost, but it also affects the contractual environment. Even if both types of organizations have access to the same contractual instruments, new organizations have a contractual edge. As will be shown, subject to the same constraints, the contracts offered by a new organization are more contingent on the project's payoffs. This advantage increases the managerial rents of creating a new firm in the presence of moral hazard, asymmetric information or managerial optimism.

The game takes place as follows. A manager in the firm receives the idea. After this, the firm makes a take-it or leave-it offer to the manager. The manager thus chooses the most rewarding of the two following options:

- he can implement the project outside, if the venture capital market is willing to finance it.
- he can accept the offer of the firm, if there is one.

We now specify the contractual environment for these offers.

Assumption 3.1 *In any organizational form, wealth can only be transferred to the manager with cash and stock of the company.*

Let $V^{out}(\tau)$ be the value the manager gets by implementing a project of size τq through the venture-capitalist. If the firm wants to do the project inside, it has to offer contractual package that the manager values slightly more than his outside option as an entrepreneur. Let $T(\tau)$ denote the cost of this package to the current shareholders. When no venture capitalist is willing to finance the project, $V^{out}(\tau) = 0$ and therefore $T(\tau) = 0$. Let \mathcal{O} be the set of projects that a venture capitalist is willing to finance.

The firm maximizes shareholder value,

$$\max_{\mathcal{I}} \left\{ 1_{\{\tau \in \mathcal{I}\}} [(1-p)V + p\tau V - T(\tau)] + 1_{\{\tau \notin \mathcal{I}, \tau \in \mathcal{O}\}} [(1-p)V] + 1_{\{\tau \notin \mathcal{O} \cup \mathcal{I}\}} [V] \right\} \quad (3.1)$$

where \mathcal{I} is the set of projects that are implemented inside the firm. The objective function (3.1) is interpreted as follows. If the project is implemented inside ($\tau \in \mathcal{I}$), then the firm has to pay a transfer $T(\tau)$ to the manager and with probability p the project is successful, generating a value of τV to the current shareholders. If the firm does not implement the project and the project is implemented outside ($\tau \notin \mathcal{I}, \tau \in \mathcal{O}$) then with probability p the shareholders lose their firm, so the shareholders receive an expected value of $(1-p)V$. If the firm does not implement the project, and the project cannot be implemented outside ($\tau \notin \mathcal{O} \cup \mathcal{I}$), then the shareholders retain the value of the firm V .

We now explain how the ex-ante surplus of the project is affected by the way the manager is rewarded. We assume the following,

Assumption 3.2 *The manager is optimistic relative to the incumbent and the venture capitalist with respect to his own idea. Let \tilde{p} denote the optimistic prior belief of the manager, and p the rational prior belief. Then*

$$1 \geq \tilde{p} > p \geq 0$$

An investor's ability to offer contingent contracts to the manager now matters. The reason is that contingent contracts allow to pay the manager with "dreams", which is not possible with non-contingent claims. The cheapest way for a rational investor to pay the manager is to give him the money in the state of nature that he overvalues (i.e. in case that his project succeeds).

The competitive venture capitalist offers to finance the project as long as $p\tau V > C'$,

i.e. for any $\tau > \tau^{out}$ where

$$\tau^{out} = \frac{C'}{pV}$$

The most attractive offer that he can make to the manager is an equity claim on the project of objective value $p\tau V - C'$ so that he just breaks even. Given the structure of the project, an equity claim is contingent on the success of the project, which the manager overvalues. The venture capitalist then, will offer the manager the highest claim possible in the state of the world where the firm is a success such that he just breaks even. The venture capitalist gets $\frac{C'}{p}$ from the cash flows of a succesful project and the subjective value of the remaining equity for the manager is $\tilde{p} \left[\tau V - \frac{C'}{p} \right]$.

Proposition 3.1 *The subjective return of the external market offer for the manager with idea τ is $V^{out}(\tau)$ where*

$$V^{out}(\tau) = \begin{cases} \frac{\tilde{p}}{p} [p\tau V - C'] & ; \text{ when } p\tau V > C' \\ 0 & ; \text{ otherwise} \end{cases}$$

Due to their optimistic beliefs, managers overestimate the value of their outside option as entrepreneurs by a factor $\frac{\tilde{p}}{p}$.

If she wants the project to be done inside, the incumbent has to make an offer to the manager of subjective value V^{out} .

We first establish that the firm's offer, if any, consists exclusively of stock.

Proposition 3.2 *The offer of the firm consists exclusively of stock. To transfer of a subjective value $V^{out}(\tau)$ to the manager the firms has pay the manager an objective cost of $T(\tau)$ where*

$$T(\tau) = \begin{cases} \frac{p\tau + (1-p)}{\tilde{p}\tau + (1-\tilde{p})} V^{out}(\tau) > 0 & ; \text{ when } \tau > \tau^{out} \\ 0 & ; \text{ otherwise} \end{cases}$$

Notice that the transfer the firm is making to the manager is smaller than the V^{out} , $\left(\frac{p\tau + (1-p)}{\tilde{p}\tau + (1-\tilde{p})} < 1 \right)$. This is so because the firm is paying the manager with "dreams". The intuition for this proposition is as follows. If the firm had to pay with cash the cost to do so would be $V^{out} = \frac{\tilde{p}}{p} [p\tau V - C']$. Stocks are a cheaper way to pay since the manager overestimates the potential impact of his innovation on the value of the firm. The shareholders' cost of giving away a fraction x of the firm is $[p\tau + (1-p)]xV$; but the manager's subjective valuation is higher, $[\tilde{p}\tau + (1-\tilde{p})]xV$. Therefore, the cheapest

way to pay the manager is to pay him exclusively with stock at a shareholders' cost of $\frac{p\tau+(1-p)}{\tilde{p}\tau+(1-\tilde{p})}V^{out}$. The firm can bridge part of the gap in beliefs with the manager, but not as well as the venture capitalist. To see this, notice that to transfer a unit of subjective value to the manager, the venture-capitalist only has to give a claim of value $\frac{p}{\tilde{p}} < 1$ but the firm has to give a claim of value $\frac{p\tau+(1-p)}{\tilde{p}\tau+(1-\tilde{p})} > \frac{p}{\tilde{p}}$.

When does the incumbent firm implement the project? Two cases have to be distinguished depending on whether the manager has the opportunity to implement the project outside or not.

For a given τ , if $V^{out}(\tau) = 0$ ($\tau \notin \mathcal{O}$), then $T(\tau) = 0$ and from (3.1) the firm implements the project when $p(\tau - 1)V - C > 0$. In this case, the firm cares about the incremental gains of the project and compares these to the implementation cost C . Let τ^{in} be

$$\tau^{in} = 1 + \frac{C}{pV}$$

The interpretation of this threshold is the following: for any $\tau > \tau^{in}$ the firm is willing to finance the project in the absence of venture capital pressure.

When V^{out} is positive ($\tau \in \mathcal{O}$), the firm faces the threat of the project being done outside. If the project is implemented outside the shareholders lose V with probability p . That implies that the firm does not only consider the incremental gains of the project: it also has to take into account the loss incurred if the project succeeds outside and the transfer required to keep the manager inside. The value of doing the project is $p(\tau - 1)V - C - T(\tau)$. The project will be done inside then, when $p(\tau - 1)V - C - T(\tau) > -pV$,

$$p\tau V - C - \frac{\tau + (1-p)/p}{\tau + (1-\tilde{p})/\tilde{p}} [p\tau V - C'] > 0$$

Let $\tilde{\alpha} = (1 - \tilde{p})/\tilde{p}$ and $\alpha = (1 - p)/p$ (where $\tilde{\alpha} < \alpha$). The previous equation can be expressed as

$$\frac{\alpha - \tilde{\alpha}}{\tau + \tilde{\alpha}} [p\tau V - C'] < C' - C \quad (3.2)$$

where $\frac{\alpha - \tilde{\alpha}}{\tau + \tilde{\alpha}} [p\tau V - C']$ is the mispricing of the outside option of the manager due to the optimism bias. It increases with τ and converges to $(\alpha - \tilde{\alpha})pV$ when τ goes to $+\infty$. This is the result of two competing effects: on the one hand, the valuation gap is increasing with τ ; on the other hand, for high τ 's, the potential contribution of the manager is less

diluted in the existing technology and therefore the firm can bridge the gap in beliefs more easily.

If $1 - (C' - C)/pV(\alpha - \tilde{\alpha}) < 0$, then (3.2) always holds for any τ ; and if $1 - (C' - C)/pV(\alpha - \tilde{\alpha}) > 0$, we can rewrite it as

$$\tau < \frac{\tau^{out} + \tilde{\alpha}(C' - C)/pV(\alpha - \tilde{\alpha})}{1 - (C' - C)/pV(\alpha - \tilde{\alpha})}$$

Let τ^{dis} be defined as

$$\tau^{dis} = \begin{cases} \frac{\tau^{out} + \tilde{\alpha}(C' - C)/pV(\alpha - \tilde{\alpha})}{1 - (C' - C)/pV(\alpha - \tilde{\alpha})} & ; \text{ when } C' - C < pV(\alpha - \tilde{\alpha}) \\ +\infty & ; \text{ otherwise} \end{cases}$$

For $\tau < \tau^{dis}$ the inequality (3.2) holds and the firm is willing to do the project inside if $V^{out} > 0$.

Proposition 3.3 (The Venture Capital Advantage) *For any $\tau > \tau^{dis}$, if $V^{out} > 0$, the project will be done outside.*

Venture capital is financing projects, even when they have a higher cost of implementation. Why is this happening? At the threshold τ^{dis} , the gap in beliefs becomes too costly for the firm to afford doing the project inside. This happens for high quality projects (high τ). The reason is that the mispricing of the entrepreneurial outside option V^{out} is increasing with τ : this mispricing is $(\alpha - \tilde{\alpha})pV$ when τ is very large. For high levels of τ , the price that is required to convince a manager to do the project inside becomes too high for the firm to afford it. This mispricing is not an issue for the venture-capitalist who can effectively offer a totally contingent contract.

In this section we have characterized the equilibrium behavior of innovation for a given value of the firm V . We have seen that the venture capitalists have an advantage because they can offer “de-facto” more contingent contracts. This allows them to attract managers with ideas, even when they have implementation costs that are higher than the firm’s. We show that the advantage of the venture capitalist appears even when both the firm and the venture capitalist have access to the *same* type of contracts. The difference in the payoff structure that they face, generates the venture capital advantage, because they can give the manager payoff that are more contingent on the success of their projects than the firms. The manager values this, because as in our case, he overvalues

the probability of success. In the next section we proceed to endogenize the value of the firm in a dynamic set up.

3.3 Dynamic Set-up: Endogenizing the Value of the Firm

The value of an idea for a manager, $V^{out}(\tau)$, depends on the value of becoming the incumbent τV . The net present value of cash-flows received by the incumbent's shareholder, V , depends on the life-cycle of technologies and the share of the revenues from new technologies that the incumbent is able to appropriate. Both terms are endogenous. In this section, we obtain V, T and V^{out} as the solutions to a dynamic problem faced by the incumbent.

Time is continuous. Technologies are indexed by their quality, q , as in a vertical quality ladder model. We consider a linear specification of the model where the incumbent receives an instantaneous profit flow of¹:

$$\pi(q) = \pi q$$

and where the implementation costs are $C_q = Cq$ and $C'_q = C'q$.

With poisson arrival rate λ , a manager gets an idea about a way to switch to a higher technology, one of quality τq ($\tau > 1$). Where τ is drawn from a time invariant and quality independent distribution $F(\cdot)$.

The value of the current incumbent firm is denoted by $V(q)$. The transfer offered by the firm to the manager with an idea τ is denoted by $T(\tau q; q)$ and the outside subjective valuation for the manager is denoted by $V^{out}(\tau q; q)$.

In this context the transfer and the outside subjective valuation are homogenous of degree one. So we can write $T(\tau q; q) = qT(\tau)$ and $V^{out}(\tau q; q) = qV^{out}(\tau)$. The threshold τ^{in} , τ^{out} and τ^{dis} are independent of q .

Let $\mathcal{O} = [\tau^{out}, \infty)$ be the set of projects that can be implemented outside.

The value of the firm is linear, $V(q) = qV$, and is the outcome of the following

¹We provide in appendix 1 the microfoundations for this specification.

maximization:

$$rV = \max_{\mathcal{I}} \left\{ \pi + \lambda \int_{\mathcal{I}} \{p[\tau - 1]V - C - T(\tau)\} dF(\tau) - \lambda \int_{\mathcal{O} - \mathcal{I}} pV dF(\tau) \right\} \quad (3.3)$$

where \mathcal{I} represents the set of projects that are implemented inside and Let \mathcal{I}^* denote the optimal policy rule.

Note that q is irrelevant for the characterization (it plays just a multiplicative role) so we can omit it from the analysis.

The value equation (3.3) says that the incumbent firm enjoys a flow of profits equal to π every instant it remains the monopolist. With Poisson probability λ , an idea arrives to a manager. If the idea is implemented inside ($\tau \in \mathcal{I}^*$), then the total cost of implementing the idea for the firm is the direct cost of implementation C , plus the transfer that is made to the manager $T(\tau)$. With probability p , the idea is a success, and the firm enjoys an increase in value equal to $\tau V - V$ (the difference between being the τ -incumbent versus being the current incumbent). If the idea is implemented outside ($\tau \in (\mathcal{O} - \mathcal{I})$), the current incumbent only loses if the idea is succesful (an event of probability p). In that case, the current incumbent is replaced by a new one and the current shareholders loose the total value of the firm V . Equation (3.3) is the dynamic version of (3.1).

To characterize the optimal decisions depending on C' we analyze three distinct regions given the values of τ^{dis} , τ^{out} , and τ^{in} defined in the previous section.

Proposition 3.4 *For any $C' > 0$, the values of τ^{in} , τ^{out} and τ^{dis} fall in one of the following three regions:*

Region I:

$$\tau^{dis} < \tau^{out} < \tau^{in}$$

Region II:

$$\tau^{out} < \min \{ \tau^{dis}, \tau^{in} \}$$

Region III:

$$\tau^{in} < \tau^{out} < \tau^{dis}$$

For any $C' < C$, the equilibrium lies in region I. There exists $\bar{C} > C$, such that for $C' > \bar{C}$, the equilibrium lies in region I; for $C' \in (\bar{C}, C)$ the equilibrium lies in region III.

In Region I, all projects are done outside.

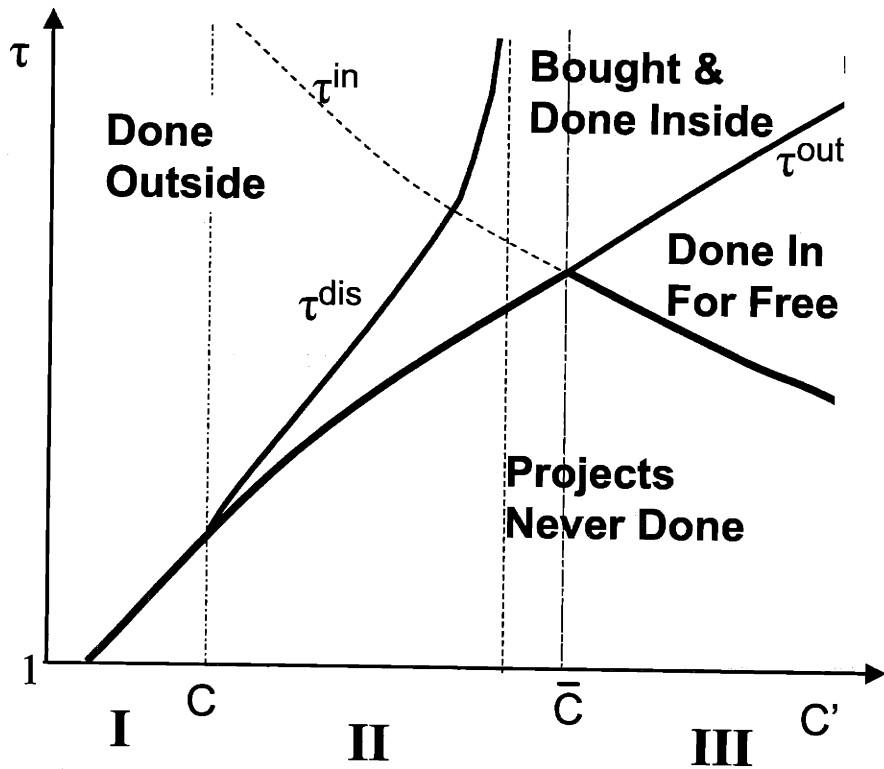


Figure 3-1: The route of innovation

In Region II, the projects with a $\tau < \tau^{out}$ will never be done. Projects with $\tau \in (\tau^{out}, \tau^{dis})$ will be done inside, with a positive transfer to the manager. And projects with $\tau > \tau^{dis}$ will be incorporated outside the firm.

In Region III, all the projects with $\tau \leq \tau^{in}$ will never be done. Projects with $\tau \in (\tau^{in}, \tau^{out})$ will be done inside the firm with no payment to the manager because her outside option is zero. For values of τ between τ^{out} and τ^{dis} , the firm will do the project inside, but makes a positive transfer to the manager. When τ is larger than τ^{dis} , the firm cannot compensate the manager, and the project is incorporated outside the firm.

We can obtain a comparative statics of $V, \tau^{dis}, \tau^{out}$ and τ^{in} with respect to the efficiency of the venture capital market (C'). The difficulty is that V is endogenous and depends itself of C' . It is therefore required to use the value equation to find these results. The proof is given in appendix 2.

Proposition 3.5 *The equilibrium values of τ^{dis}, τ^{out} and τ^{in} are monotonic in C' and*

have the following derivatives

$$\begin{aligned}\frac{\partial \tau^{dis}}{\partial C'} &> 0 \\ \frac{\partial \tau^{out}}{\partial C'} &> 0 \\ \frac{\partial \tau^{in}}{\partial C'} &< 0\end{aligned}$$

and $\frac{\partial V}{\partial C'} > 0$ if $C' > C$ and $\frac{\partial V}{\partial C'} = 0$ if $C' < C$.

Figure 1 shows the functions τ^{dis} , τ^{out} and τ^{in} in a (C', τ) space. Notice that we solve the equilibrium for a given value of C' , and then compute the different implementation rules as functions of τ , given this C' .

We can also consider the limiting case where C' is very big, so that the incumbent behaves as a monopoly. This asymptotic behavior is described by:

$$\left\{ \begin{array}{l} \lim_{C' \rightarrow +\infty} \tau^{in} = \tau^\infty < \infty \\ \lim_{C' \rightarrow +\infty} \tau^{out} = \infty \\ \lim_{C' \rightarrow +\infty} (\tau^{dis} - \tau^{out}) = \infty \\ \lim_{C' \rightarrow +\infty} V = V^\infty < \infty \end{array} \right.$$

where τ^∞, V^∞ are the solutions of

$$\left\{ \begin{array}{l} \tau^\infty = 1 + \frac{C}{pV^\infty} \\ rV^\infty = \pi_0 + \lambda \int_{\tau^\infty}^{\infty} (p(\tau - 1)V - C)dF(\tau) \end{array} \right.$$

3.4 Innovation and Efficiency

3.4.1 Innovation Rate

In our model all the projects above a certain threshold $\tilde{\tau}$ are implemented. When this threshold goes down, the rate of innovation $\lambda(1 - F(\tilde{\tau}))$ increases. This means that the life-cycle of products becomes shorter but also that the average “size” of innovation (the average implemented τ) becomes smaller.

We first want to describe how the innovation rate changes with the cost of external venturing C' .

Proposition 3.6 *The innovation threshold $\tilde{\tau}$*

- *decreases with C' if $C' > \bar{C}$*
- *increases with C' if $C' < \bar{C}$*

The interpretation is the following: as long as $C' > \bar{C}$, the margin of innovation $\tilde{\tau}$ corresponds to the zone where $V^{out} = 0$, so that C' does not affect the decision of the monopolist through the transfer to the manager but only through its impact on the value of being an incumbent: the role of C' in this region has purely a rent sharing effect between the shareholders and the managers. A lower C' means a loss of monopoly rents and therefore decreases the willingness of the incumbent to innovate. On the contrary, when $C' < \bar{C}$, the margin of innovation $\tilde{\tau}$ corresponds to zone where $V^{out} > 0$. Decreasing C' increases the range of projects that are feasible externally and therefore increases innovation. We proceed now to determine how the welfare of shareholders and managers is affected by changes in C' .

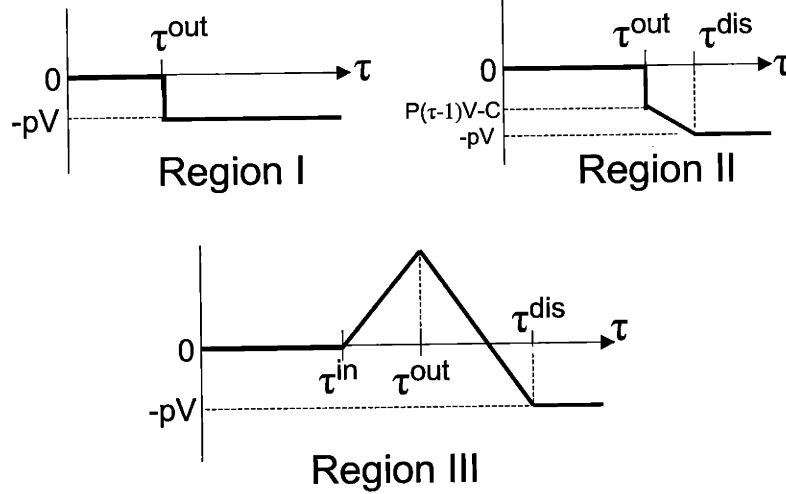
3.4.2 Monopoly Rents

We ask now the following question : How does the value of an incumbent firm react to the occurrence of an idea τ ?

The impact on value of an innovation differs according to the three regions previously described as it is shown in figure 2. We proceed now to describe in detail how this graph was obtained.

Consider region I. On this region, all the projects are done outside. Once a manager gets an idea that can be implemented, the incumbent firm is destroyed with probability p . So, the incumbent's return to an idea in this region is 0 when $\tau < \tau^{out}$ and $-pV$ when $\tau \geq \tau^{out}$.

Consider region II. Projects with a τ between τ^{out} to τ^{dis} , are bought by the monopolist and done inside. And for $\tau > \tau^{dis}$, the manager implements the project outside. The monopolist return to an idea is then 0 for $\tau < \tau^{out}$. For τ between τ^{out} and τ^{dis} , the return is $(p - \tilde{p})\tau V - pV + \frac{\tilde{p}}{p}C' - C$ which is negative for $\tau = \tau^{out}$ and decreases with τ , up to τ^{dis} where it becomes equal to $-pV$. For any $\tau > \tau^{out}$, the project is done outside and the return to the firm is $-pV$.



Incumbent's Return to Innovation

Consider now region III, projects with $\tau < \tau^{in}$ are never done. For $\tau \in (\tau^{in}, \tau^{out})$ the projects are done inside for free ($T(\tau) = 0$) and the private return to innovation is $p(\tau - 1)V - C$ which is increasing in τ . For $\tau \in (\tau^{out}, \tau^{dis})$ the project has to be bought and the return is now $p(\tau - 1)V - C - T(\tau) = (p - \tilde{p})\tau V - pV + \frac{\tilde{p}}{p}C' - C$ which is decreasing in τ and becomes $-pV$ for $\tau = \tau^{dis}$. Notice that the return is continuous at τ^{out} . Last, for $\tau > \tau^{dis}$, the project is done outside with an ex-ante loss to the firm of $-pV$.

3.4.3 Aggregate Efficiency

In this section we analyze the efficiency of the equilibrium.

The social planner values every unit of production at $\frac{\pi_0}{\beta}$, where β is the profit share of output accrued to the monopolist. We can define the social value function of an existing firm as

$$rU = \frac{\pi_0}{\beta} + \lambda \int_{\tau \in In} (p(1 - \tau)U - C) d\tau + \lambda \int_{\tau \in Out} (p(1 - \tau)U - C') d\tau$$

Where In is the set of innovations done inside the firm and Out is the set of the ones done outside.

If $C < C'$, then it is always efficient to do the project inside the incumbent (if it is to be done). A project is socially profitable inside whenever the value generated by it is

bigger than the cost of implementation :

$$p(1 - \tau)U > 0$$

Define

$$\tau^{fb} = 1 + \frac{C}{pU}$$

The social value U is given by the following set of equations (for $C < C'$):

$$\begin{aligned} rU &= \frac{\pi_0}{\beta} + \lambda \int_{\tau^{fb}}^{\infty} (p(1 - \tau)U - C) d\tau \\ \tau^{fb} &= 1 + \frac{C}{pU} \end{aligned}$$

From which we can infer that:

Proposition 3.7 *The incumbent, when not exposed to entrepreneurial pressure ($C' = +\infty$), tends to underinnovate:*

$$\tau^{fb} < \tau^{\infty}$$

The reason why this is so is very simple. If there is no entrepreneurial pressure, then the monopolist will innovate whenever $p(\tau - 1)V > C$. Given that $V < U$, the monopolist does not value an innovation as much as the social planner does (it only values a share β of the production), so the monopolist underinnovates.

Corollary 3.1 *The following holds*

- *If $C' > \bar{C}$, there is underinnovation and therefore a motive for public start-up subsidies.*
- *Depending on the parameters, $C' = C$ might lead to under or over-innovation.*

The central planner can implement the first best level of innovation by using various public policy tools. If C' needs to be decreased, start-up subsidies can be used. If C' needs to be increased, taxes on firm-creation can be used. It is also possible to reinforce laws on non-compete or non-disclosure agreements or to make bankruptcy more costly.

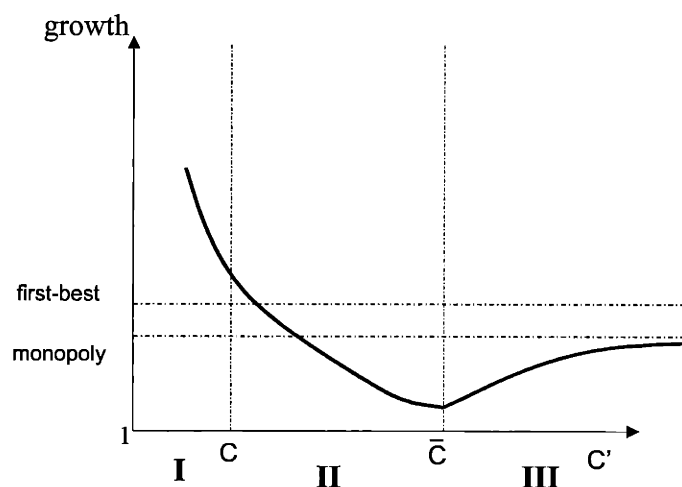


Figure 3-2: Efficient Innovation

3.5 The Value of Contracting Outside

A crucial feature of our model is that financing a project as a new venture allows for more contingent contracts. We have shown that this result holds when both the incumbent and the new firms are subject to the same constraint: paying with stock or cash. Restricting the analysis to these two instruments has the advantage of keeping the dynamic part of our model simple. To illustrate the generality of this point we show in the static framework that the comparative advantage of the venture-capitalist subsists for large class of contracts. The space of possible contracts for both parties (the incumbent and the venture-capitalist) can be arbitrarily close to perfect contracting.

The structure of the game is as in section 1. The only change is that we relax assumption 2 in the following way. It is also possible to write contracts contingent on the new project's payoff, but such contracts are enforceable only with probability θ and void with probability $1 - \theta$. Stock and cash are still available.

Assumption 3.3 *In any organizational form, wealth can only be transferred to the manager with the three following instruments:*

1. Cash,
2. Stock,

3. *Contracts contingent on the new project's payoff. Such contracts are enforceable only with probability θ and void with probability $1 - \theta$.*

This change in the contractual environment has no impact from the point of view of a venture capitalist, for whom stock is purely contingent on the project's payoff –the third instrument is therefore redundant with the second. But it enhances the contractual flexibility of the firm. In particular, for $\theta = 1$, the firm has the same contractual edge as the venture-capitalist.

The assumption captures the fact that large organizations might try to commit to reward "internal entrepreneurs" through contingent contracts (the most sophisticated rely on the use of tracking stocks), but in many cases such contracts are mere promises. They are hard to enforce, because they are contingent on accounting variables that the incumbent's management can manipulate.

In that context, the cheapest way for the incumbent to transfer an amount of subjective value V^{out} to the manager is the following:

If $\theta\tau V > \frac{V^{out}}{\tilde{p}}$, the incumbent promises to the manager a transfer $\frac{1}{\tilde{p}} \frac{V^{out}}{\theta}$ if the project generates positive cash-flows. Both parties know ex-ante that this contract will be enforceable with probability θ and void otherwise.

If $\theta\tau V < \frac{V^{out}}{\tilde{p}}$ the incumbent offers to the entrepreneur a package that includes the integrality of the cash-flows generated by his project plus a fraction α of the firm. From the point of view of the entrepreneur, the value of this offer is $\theta\tilde{p}\tau V + \alpha[\tilde{p}(1 - \theta)\tau V + (1 - \tilde{p})V]$. Therefore,

$$\alpha = \frac{V^{out}/V - \theta\tilde{p}\tau}{\tilde{p}(1 - \theta)\tau + (1 - \tilde{p})}$$

Considering the value of the transfer to the entrepreneur from the point of view of the incumbent's shareholder, there are again two cases to consider:

- If $\theta > \theta^*(\tau) = \frac{V^{out}}{\tilde{p}\tau V}$, the value to the shareholder of the transfer is $\frac{\tilde{p}}{\theta} V^{out}$, which is the same as the objective value transferred by the VC to the manager. In this case, the incumbent has the same transformation rate than the venture-capitalist. \$1 of subjective value is transferred to the entrepreneur at cost $\frac{\tilde{p}}{\theta}$.
- If $\theta < \theta^*(\tau)$, the value the incumbent's shareholder has to transfer to the manager for matching the outside offer is:

$$T(\tau) = \theta\tilde{p}\tau V + \frac{\tilde{p}(1-\theta)\tau + (1-\tilde{p})}{\tilde{p}(1-\theta)\tau + (1-\tilde{p})} (V^{out} - \theta\tilde{p}\tau V) > \frac{\tilde{p}}{\theta} V^{out}.$$

This means that as θ becomes small, the "transformation rate" of the venture-capitalist deteriorates, implying a larger cost of financing new projects. In particular, when $\theta = 0$, we are back to the framework of the first section with a transformation rate of $\frac{p\tau + (1-p)}{\tilde{p}\tau + (1-\tilde{p})}$.

The class of contracts we consider is large enough to allow situations where the incumbent firm is not at a contracting disadvantage compared to the venture capitalist. This happens when $\theta > \theta^*(\tau)$. However, for projects that are sufficiently innovative, the venture-capitalist always has an advantage. Since $V^{out}(\tau) = \frac{\tilde{p}}{p}(p\tau V - C')$, $\theta^*(\tau) = (1 - C'/p\tau V)$ which is arbitrarily close to 1 for τ large enough. Therefore, for τ large enough, $T(\theta, \tau) > \frac{p}{\tilde{p}}V^{out}$.

Proposition 3.8 *For any θ , there exist τ^* such that for any $\tau > \tau^*$, the incumbent has a contracting disadvantage relative to the venture-capitalist, i.e.*

$$T(\theta, \tau) > \frac{p}{\tilde{p}}V^{out}(\tau)$$

The analysis in this part has been done in the static framework, i.e. taking V as exogenous. It is possible to carry it to the dynamic framework without affecting the results

What is θ related to? The parameter θ captures the inability of a firm to make promises contingent in a project payoff. This inability to contract is going to be more severe the more intangible the project is (when the ability to value the project through accounting procedures becomes harder). Also, a higher θ might be associated with the strength of the legal system. Weak legal system will be unable to commit to enforce contracts, and hence θ will be low.

3.6 Conclusion

By increasing the number of projects that can be financed externally, better venture capital markets exert pressure on incumbent companies to innovate more than they would otherwise. There is an opposite effect however: Because they force shareholders to give-up more rents to managers, better capital markets decrease the value of being the incumbent firm and therefore the incentive to innovate. This leads to a non-monotonic relationship between capital market efficiency and growth.

Which projects are done inside vs. outside depends on the balance between two comparative advantages: the incumbent firm can use existing assets while the venture capitalist can write contracts contingent on the project's outcome. In the presence of contractual frictions, the most innovative projects are implemented in new ventures. The established firm cannot match the attractiveness of the entrepreneurial outside option for such projects.

If the marginal innovation is done under pressure from outside, a better VC market increases the innovation rate. If the marginal innovation would have been implemented without outside pressure, a better VC market, by decreasing the rents of being the incumbent firm, decreases the rate of innovation. Therefore, in equilibrium, the relation between innovation and the efficiency of external capital markets has an inverse-U shape.

3.7 Appendix I. The Static Aggregate

Consumers are linear,

$$U_0 = E_0 \int_0^{\infty} c_t e^{-rt} dt$$

There is a competitive final good that uses a continuum of intermediates, indexed from 0 to 1, and labor (L) to produce units of consumption according to the following production function :

$$Y = \left[\int_0^1 q(i) k(i)^{1-\alpha} di \right] L^\alpha$$

$q(i)$ represents the quality of the leading machine in sector i . $k(i)$ is the numbers of machines from sector i that are used in the production of the final good. The final good will be our numeraire.

Call $\psi(i)$ the price of one machine in sector i . The demand for machines for a given sector i is :

$$k(i) = [(1 - \alpha) q(i) / \psi(i)]^{\frac{1}{\alpha}} L$$

There is a monopolist in every sector that holds the patent for the leading quality machine. The monopolist can create machines at a constant marginal cost of $\phi q(i)$. We assume that innovations are drastic (the old technology becomes obsolete). Given the isoelastic demand functions, the monopolists solve the following profit maximizing problem.

$$\max \pi_i(q) = k(i) [\psi(i) - \phi q(i)]$$

The FOC conditions deliver :

$$\psi(i) = \frac{\phi q(i)}{1 - \alpha}$$

$$k(i) = \left[\frac{(1 - \alpha)^2}{\phi} \right]^{\frac{1}{\alpha}} L$$

so profits for any particular sector with machine q are

$$\pi_i(q) = \pi_0 q(i)$$

where $\pi_0 = \left[\frac{(1-\alpha)^2}{\phi} \right]^{\frac{1}{\alpha}} \frac{\alpha\phi}{1-\alpha} L$. This is the profit function used in section 2, once we introduced the dynamic version of the model.

3.8 Appendix II. Comparative Statics

Assumption 3.4 *The rate of growth of the economy can never be higher than the discount rate.*

$$r > \lambda p \int_1^{\infty} \tau dF(\tau)$$

This assumption guarantees that all value functions are bounded. We then move on to prove the following proposition,

Proposition 3.9

- *The value function is given by:*

- *If $C' > C$:*

$$rV = \pi_0 + \lambda \int_{\min(\tau_{in}, \tau_{out})}^{\infty} \{ [p\tau V - C - T(\tau)]^+ - pV \} dF(\tau)$$

- *where $T(\tau) = \frac{\tau+(1-p)/p}{\tau+(1-p)/p} [p\tau V - C']^+$*

- *If $C' < C$:*

$$rV = \pi_0 - \lambda p(1 - F(\tau_{out}))V$$

- *A smaller cost of doing the project outside C'*

- *reduces the value of being an incumbent:*

$$\frac{\partial V}{\partial C'} > 0$$

- *reduces the threshold at which external projects become viable:*

$$\frac{\partial \tau_{out}}{\partial C'} > 0$$

- *reduces the threshold τ_{in} at which the incumbent becomes willing to do a project if there is no transfer to the manager:*

$$\frac{\partial \tau_{in}}{\partial C'} < 0$$

First, consider the case where $C' < C$ so that all innovation occurs outside:

$$rV = \pi_0 - \lambda pV(1 - F(\tau_{out}))$$

Differentiating this equation, we obtained

$$[r + \lambda p(1 - F(\tau_{out}))]dV = \lambda pV f(\tau_{out})d\tau_{out}$$

Using the fact that $pV\tau_{out} = C' \Rightarrow pVd\tau_{out} = dC' - p\tau_{out}dV$. Substituting back we get

$$\frac{d\tau_{out}}{dC'} = \frac{r + \lambda p(1 - F(\tau_{out}))}{\lambda pV f(\tau_{out}) + [r + \lambda p(1 - F(\tau_{out}))]pV} > 0$$

So, $\frac{\partial \tau_{out}}{\partial C'} > 0$ and

$$\frac{dV}{dC'} = \frac{\lambda f}{r + \lambda p(1 - F(\tau_{out})) + \lambda p f}$$

So, in this region:

$$\frac{dV}{dC'} > 0$$

Which implies :

$$\frac{d\tau_{in}}{dC'} < 0$$

We now need to consider the case where $\tau_{in} > \tau_{out}$ (i.e. $C' - C < pV$). The incumbent does not innovate when the threat is not credible because an innovation always have a negative incremental value compared to the statu-quo. In this case,

$$rV = \pi_0 + \lambda \int_{\tau_{out}}^{\infty} [[p\tau V - C - T(\tau)]^+ - pV] dF(\tau)$$

For $\tau \in [\tau_{out}, \tau_{dis}]$ we have that

$$p\tau V - C - T(\tau) = p\tau V - C - \frac{\tau + (1-p)/p}{\tau + (1-\tilde{p})/\tilde{p}} [p\tau V - C'] > 0$$

Plugging back into the value function

$$rV = \pi_0 + \lambda \int_{\tau_{out}}^{\tau_{dis}} \left[-pV - C - \frac{\alpha - \tilde{\alpha}}{\tau + \tilde{\alpha}} p\tau V + \frac{\tau + \alpha}{\tau + \tilde{\alpha}} C' \right] dF(\tau) - \lambda pV(1 - F(\tau_{out}))$$

Computing the comparative statics:

$$\left[(r + \lambda p(1 - F(\tau_{out})) + \lambda p \int_{\tau_{out}}^{\tau_{dis}} \frac{\alpha - \tilde{\alpha}}{\tau + \tilde{\alpha}} \tau dF \right] \frac{dV}{\lambda} = (pV - (C' - C)) f d\tau_{out} + \left(\int_{\tau_{out}}^{\tau_{dis}} \frac{\tau + \alpha}{\tau + \tilde{\alpha}} d\tau \right) dC'$$

Using the fact $pV\tau_{out} = C' \Rightarrow pVd\tau_{out} = dC' - p\tau_{out}dV$ and replacing $d\tau_{out}$ by $(dC' - p\tau_{out}dV)/pV$, we see that in this region:

$$\frac{dV}{dC'} > 0$$

This implies:

$$\frac{d\tau_{in}}{dC'} < 0$$

Using the fact that $r > \lambda \int_{\tau_{out}}^{\tau_{dis}} \tau dF$, replacing this time dV by $(dC' - pVd\tau_{out})/(p\tau_{out})$, we have that:

$$\frac{d\tau_{out}}{dC'} > 0$$

Third, consider the case where $\tau_{in} < \tau_{out}$ (i.e. $C' - C > pV$) so that the incumbent does innovate at the margin where the outside value of the project is zero. Then, the value is given by

$$rV = \pi_0 + \lambda \int_{\tau_{in}}^{\infty} \{ [p\tau V - C - T(\tau)]^+ - pV \} dF(\tau)$$

At τ_{in} , the function we are integrating equals zero (the firm is indifferent between doing or not). Using the fact that $r > \lambda p \int_{\tau_{out}}^{\tau_{dis}} \tau dF$, we conclude as before.

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