On Proprietary Disclosures of Investment Institutions

by

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ABSTRACT

We analyze issues related to proprietary disclosures by specialized investment institutions such as hedge funds to their trading counterparties and creditors. In this paper, disclosure can be costly because of the potential for exploitation through competitive trade or "front-running".

In chapter 1, we consider a model with a direct revelation mechanism between leveraged investors and their lenders. In this model, the investors need to borrow from lenders with heterogeneous risk-exposures in order to trade. Investors may obtain advantageous terms of borrowing by disclosing their investment strategy, thereby revealing its correlation to the lender's existing risk-exposure. Investors risk being "front-run" by their lender if they disclose, however. We show that in the presence of front-running, the "unraveling" result of full disclosure may not hold. Mandating disclosure has ambiguous welfare effects since it can not only lead to the matching of uncorrelated risks, but also to concentrations of risk. These results have implications for regulations on leveraged investors in financial markets.

In chapter 2, we consider an indirect revelation made by an arbitrageur (e.g., hedge fund) to trading counterparties through traded securities. In this model, the arbitrageur has private information about the relative value of two or more securities. We conjecture that in a segmented dealer market, the arbitrageur trades each security with a different dealer so that each dealer sees only one piece of the total position. We show that this "break-up" strategy can be optimal and unique even if given an array of redundant strategies and securities, including "swaps" of the various securities. The analysis of this equilibrium has implications for the kinds of claims held by an arbitrageur's counterparties in a leveraged scenario and their resulting stability.

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There is no doubt to whom I owe the most, however. I dedicate this thesis to my loving and devoted parents, to whom I owe nothing less than everything.
The near-collapse of Long-Term Capital Management in 1998 sparked a furious interest in the level of transparency between leveraged market participants and their lenders. LTCM conducted its activities in a highly non-transparent way, disclosing very little information to its creditors, counterparties, and investors. The Basle Committee’s policy report on the crisis, for example, documents that “counterparties did not generally receive meaningful information about leverage or the concentration of exposure in certain types of positions, risk factors, trading strategies ... to enable them to form a more comprehensive picture of the true risk profile of LTCM.”

The conventional wisdom that emerged was that better information sharing could have mitigated the severity of the crisis by fostering more prudent risk-management. In addition to providing monitoring and control over the size and leverage of LTCM’s portfolio, greater transparency could have potentially reduced the concentrations of risk that many institutions were exposed to. Specifically, many institutions were vulnerable because they held highly correlated assets in their proprietary and loan portfolios that simultaneously experienced losses. The crisis opened the question of whether regulation might be needed to foster transparency between leveraged investment institutions and their creditors.

The “unraveling” argument of Milgrom (1981) and others says that such regulation is unnecessary because there ought to be full voluntary disclosure in the absence of regulation. The argument is that better types will always want to disclose to differentiate themselves from worse types so that all types will become known in equilibrium. The argument applies when there are no costs associated with disclosure.

This chapter posits that the main barrier or cost to information sharing by leveraged investors is that disclosed information can often be exploited through competitive trade in the markets. If the disclosure reveals information regarding an investor’s proprietary trading strategies, for example, the creditor (typically a bank or securities firm) can often duplicate this strategy and extract profits. We refer to this duplication as front-running. Anecdotally, LTCM and other hedge funds clearly saw the threat of front-running as a primary motive to withhold information.1 As its first recommendation for enhancing information-sharing between counterparties, an industry-formed policy group on counterparty risk-management has proposed improving “information barriers between

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1Lowenstein (2000) documents that “[John Meriwether] felt that investment banks were rife with leaks and couldn’t be trusted not to swipe his trades for themselves. Indeed, most of them were plying similar strategies. Thus, as a precaution, Long-Term would place orders for each leg of a trade with a different broker. Morgan would see one leg, Merrill Lynch another, and Goldman yet another, but nobody would see them all.”
the firm's traders... and the credit risk managers who determine counterparty credit" because of "the intensely competitive nature of the relationship between credit providers and credit users in aspects of their respective market businesses." Such "Chinese walls" have existed in the past but have clearly not been entirely effective at containing information leakage.

In this chapter, we develop a theory of disclosure in the presence of front-running costs. We seek to understand first, the conditions under which a leveraged investor will voluntarily disclose information and second, the conditions under which a regulator ought to alter free-market disclosure practices to improve social welfare.

We study these issues in a stylized model where an investor's market exposure is unknown and can be disclosed to heterogeneous lenders with different preferences for this exposure. In our model, an investor (i.e., a hedge fund manager) has information about a profitable but risky investment opportunity in one of two possible markets. The investor has limited capital and needs a loan from the banking sector to make the investment. Banks have existing risky assets with heterogeneous exposures to the possible markets of the investment. In our stylized setup, banks' assets are either correlated or uncorrelated to the investment. These banks also have concave objectives and want to avoid new correlated risks as a result of their diversification motive.

The investor may obtain favorable terms of lending by disclosing her investment strategy, thereby revealing its correlation to the lender's existing assets. The cost of disclosure is that the lender can front-run and attempt to duplicate the strategy. We find that, in equilibrium, the investor may choose disclosure or non-disclosure depending on parameters in the economy. These parameters affect the relative cost of front-running versus the interest-rate on the loan. Specifically, we find that there is non-disclosure for high NPV investments, high investor capitalization, and low bank risk-aversion.

We next analyze the welfare effects of disclosure regulations. Such regulations should be designed to prevent risk concentrations and other conditions that may lead to systemic market and institutional failures. We find that when the investor chooses non-disclosure, there may be a social gain from mandating disclosure. The social gain comes from the fact that uncorrelated investors and banks can match when there is transparency. When the investor's type is not disclosed, banks can not discern between a correlated or an uncorrelated investor and must match randomly. Although mandating disclosure generates front-running and a loss for the investor, this loss is not necessarily a social loss because it simply transfers wealth from the investor to other agents.

Mandating disclosure has ambiguous welfare effects, however, since it can not only result in the matching of uncorrelated risks, but also in concentrations of risk. If compelled to disclose, the
investor may actually prefer to disclose to a correlated lender over an uncorrelated lender. Though an uncorrelated bank will offer a superior interest-rate, it will also front-run the investor more aggressively because it views the risk more favorably. The investor can prefer a correlated bank if this front-running cost is sufficiently high.

We also consider various extensions of the model. We study the effect of capital regulations on welfare and the voluntary disclosure policy when raising capital is costly and the investor chooses an amount of capital ex-ante. We also consider a natural extension where the investor has unknown default risk that can only be disclosed by “opening the books”, i.e., by simultaneously revealing the investor’s trading strategy and market exposure. In this setting, it is now true that decreasing the level of transparency in the economy can be socially beneficial.

In this model, correlated banks front-run less than uncorrelated banks because of aversion to concentrated risks. One can imagine feasible alternatives, however. For example, a correlated bank could conceivably front-run more because the correlated bank is in the same market and may have a greater ability to process information disclosed by the investor. To address such critiques, we appeal to a more general interpretation of the model. The general feature of the model is the presence of heterogeneous lenders with different preferences for the investor’s risk. Different characteristics may drive this preference, including differing financial technology, capitalization, number of lenders in a syndicate, etc. With heterogeneity in these characteristics, mandating disclosure can be harmful because the investor may choose a lender for whom the risk is costly in order to minimize front-running. Hence, our welfare conclusions can maintain their validity even outside the specific form and assumptions of the model presented here.

The existing literature on disclosure is extensive. We should first mention the papers of Jovanovic (1982), Verrecchia (1983), and Dye (1986), which analyze disclosure with costs. In these papers, disclosures have an exogenous and “deadweight” cost whereas in ours, the cost of front-running is an endogenous transfer from one party to another. This chapter is most related to the papers of Campbell (1979), Bhattacharya and Ritter (1983), Ueda (2002), and Yoshia (1995), which study optimal disclosure policies when there are exploitation costs associated with disclosure. These papers deal with disclosures by firms about their real investments that competitors can exploit in the product market. This chapter is specific to disclosures between financial institutions concerning financial market information. As such, we attempt to capture features relevant to financial institutions and can consequently analyze the effects of capital and risk-management policies on our

\[\text{We present an alternative model in a previous draft (Ko (2001)) where the heterogeneity arises from differing financial technology in the market. The results of this model are similar to the ones presented here.}\]
equilibrium. Exploitation costs are particularly relevant in financial markets because of the ease of duplication and the inefficacy of legal protections for proprietary information.

This chapter proceeds as follows. Section 1.2 describes the setup of the model. Section 1.3 describes the equilibrium in a benchmark case where there is no front-running. In this case, disclosure is costless, and there is full disclosure in accord with the Milgrom argument. In section 1.4, we consider the equilibrium with front-running but where disclosure is mandatory. This case is intended to isolate the investor’s choice of correlated versus uncorrelated lender. Section 1.5 discusses the voluntary equilibrium and examines the investor’s choice of disclosure versus non-disclosure. In section 1.6, we examine the effects of regulation on disclosure policies and welfare. In section 1.7, we add heterogeneous credit qualities to our model and study the optimal voluntary and regulated policies in this setting. Section 1.8 concludes.

1.1 Model Setup

The model has three dates. At date-0, lending contracts are established and disclosures are made. At date-1, investments are made. At date-2, assets pay off, and all agents consume.

1.1.1 Assets

There are three assets in the economy. The first is a riskless asset with a zero rate of interest. We refer to this asset as “cash”. The other two are risky assets with payoffs specified below.

- Asset $x$ has payoff: $2\tilde{V}_x \cdot \tilde{X}$.
- Asset $y$ has payoff: $2\tilde{V}_y \cdot \tilde{Y}$.
- $\tilde{V}_x$, $\tilde{V}_y$, $\tilde{X}$, and $\tilde{Y}$ are binary and independent random variables. $\tilde{X}$ and $\tilde{Y}$ have outcomes of 0 or 1 with equal probability. $\tilde{V}_x$ and $\tilde{V}_y$ have outcomes of 0 or $V$ with equal probability. The factors and assets are shown in figure 1.1.1.

1.1.2 Agents

There are three types of agents.

- **Investor:**

  There is one investor who is of either type $x$ or type $y$ with equal probability. The investor’s type is private information. An investor of type $x$ observes $\tilde{V}_x$, and an investor of type $y$ observes $\tilde{V}_y$. 

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Figure 1: These four random variables are independent, and all outcomes above are equally likely.

The investor is endowed with a small amount of cash: Δ. We also refer to Δ as the investor’s capital.

The investor is risk-neutral.

- **Banks:**

  Banks can also be of either type x or type y, and there is a large number of banks of each type. Banks do not observe $V_x$ or $V_y$. A bank’s type, unlike an investor’s, is common knowledge.

  Banks have a risky endowment, which consists of a large amount of cash, $W$, plus a risky payoff of $q \cdot X$ for type x banks and $q \cdot Y$ for type y banks.

  Banks are risk-averse and have a thrice continuously differentiable, monotone and concave utility function, $u$. This concave objective is meant to capture the bank’s risk-management motives in a reduced-form.\(^3\) The relevant measure of risk-aversion for our purposes is the fractional difference in marginal utility between the high and low wealth outcomes of a bank’s risky endowment, given by the parameter $k$:

  \[
  k = \frac{u'(W) - u'(W + q)}{u'(W + q)} \tag{1}
  \]

  We also impose some technical conditions on the derivatives of $u$, which are given in the appendix.

- **Counterparty:**

  There are two counterparties.

\(^3\)This risk-aversion is a stand-in for the severity of the institution’s risk-management policy, which may be related to its cost of accessing external capital markets as in Froot, Scharfstein, and Stein (1993). The other agents in this model are meant to represent institutions as well. However, their risk-management is not crucial for our results.
Counterparty x is endowed with one unit of asset x, and counterparty y is endowed with one unit of asset y.

Counterparties have a private value for assets x and y. Specifically, they achieve a payoff of \( V' \) from assets x and y in all states where \( V' \) satisfies the following inequalities:

\[
V > V' > \max \left\{ \frac{V}{2}, \Delta \right\}
\]  

(2)

The first inequality implies that there is a gain from trade when \( \tilde{V}_j = V \ (j \in \{x, y\}) \). In this state, the counterparty should sell the asset to the other agents since they have a higher expected value for the asset than the counterparty. When \( \tilde{V}_j = 0 \), however, there is no gain from trade since the counterparty values the asset more. There is also no gain when the agent has no information since \( V' > \frac{V}{2} \). One motivation for this private valuation comes from tax considerations. The counterparty’s value can come from a higher tax rate that decreases both capital gains and capital losses, consistent with the fact that \( V > V' > 0 \). Anecdotally, it seems that it is not uncommon for institutions to face restrictions that prevent them from realizing the full value from financial assets in certain states of the world and that these restrictions can generate substantial profits for arbitrageurs.\(^4\)

The second inequality also ensures that the investor’s capital alone is insufficient to make an investment in the asset and that external finance is needed, therefore.

Finally, we assume that the assets represent gambles that are small relative to the bank’s existing risky endowment or that:

\[
q \gg V
\]  

(3)

In this chapter, we use a first-order approximation to compute the bank’s utility from payoffs from the asset. In particular, suppose we add a small deviation of \( \delta \ll q \) to existing wealth of \( w \). The resulting utility is, to first-order: \( u(w + \delta) \approx u(w) + u'(w) \cdot \delta \).

We end this section by discussing motivations for two assumptions of our model. First, we assumed that a bank’s type was common knowledge while an investor’s was not. This assumption is made mainly for simplicity. The results of our model are intuitively consistent with one where the bank’s type is private information and can be disclosed to the investor as well as vice versa. Such

\(^4\)For example, Edwards (1999) states, “Even staunch believers in efficient markets will readily admit that price inefficiencies may exist when regulations restrict the flow of capital into particular financial sectors or into particular investment strategies. For example, the short-sale portfolio restrictions imposed on institutional money managers may create an opportunity that hedge funds can exploit.”
a model would be quite elaborate, unfortunately. Alternatively, we mentioned in the introduction that one can think of the bank’s risk-aversion as being driven by characteristics such as differing financial technology in the market, which are more likely to be commonly known (as in Ko (2001)).

The other key assumption concerns the endowments of banks. Why should these endowments be undiversified when type x and type y banks clearly have a motive to share their risks? One simple answer is that adverse selection can hinder the market for risk-sharing. Consider a variation of our model where banks have private information on the payoff of their endowments. Suppose that banks can now engage in state-contingent “swaps” of $X$ and $Y$. A type x bank may want to short a swap in $X$ for risk-sharing reasons. A type y bank may need compensation to take the other side of this trade, however, because of the possibility that the type x bank may be trading for informational reasons when the payoff of $X$ is low. The cost imposed by adverse-selection can be high enough that the original bank will choose to bear its risks rather than swapping them unprofitably. Again, we choose not to explicitly model this feature for simplicity.\footnote{In fact, we have analyzed such a model, and the results are identical. This alternative model has some analytically cumbersome details, unfortunately.}

1.1.3 Timing and Actions

Loan commitments:

The investor in this model requires financing to invest in either asset x or y. We assume that this financing is provided by banks only through loan commitments.\footnote{We can endogenize banks as the sole lenders in the economy by assuming the existence of investors with high-risk, negative-NPV investments that only banks can distinguish.} We exogenously take the size of the loan to be $1 - \Delta$, which, when added to the investor’s capital of $\Delta$, gives the investor total assets of $\$1$. Giving borrowers and lenders choice over the size of the loan changes no results of the model.

The contract specifies the face value of the loan, i.e., the maximum claim that the lender has on the borrower’s assets at date-2: $R$ (we abuse terminology slightly and also refer to $R$ as the loan’s interest-rate). All claims are limited liability.\footnote{It can be shown that parties will endogenously choose debt without having to make an exogenous restriction since debt claims are optimal from risk-sharing considerations. It is likely that in other reasonable models, banks will have an incentive to offer equity contracts to curtail their own incentive to front-run. However, it may still be optimal for the investor to retain a substantial potion of the equity in order to provide ongoing incentives to monitor and maintain profitable positions.} The contract is a one-sided loan commitment since the borrower has the option to take the loan or not at date-1.
Figure 2: At date-0, banks offer interest-rates for non-disclosure and the two types of disclosure. The investor then chooses an offered interest-rate and hence, to disclose or not to a particular bank (which has correlation, ρ). At date-1, all agents can submit bids in both auctions.

**Date 0:**

Prior to date 0, all agents observe their type. At date 0, the following game is played to determine the terms of the loan.

1. Each bank declares a menu of interest-rates for loans: \([R_x, R_y, R_N]\). The declaration is a commitment to providing loans with the following interest-rates: \(R_x\) if the investor discloses that she is of type \(x\), \(R_y\) if the investor discloses that she is of type \(y\), and \(R_N\) if the investor makes no disclosure.

2. The investor chooses at most one bank for the loan. She also selects whether or not to disclose her information to this bank and locks in the corresponding interest-rate.

The investor (of type \(j\)) then observes \(\tilde{V}_j\). If the investor has agreed to disclose, the lending bank observes the investor’s type and signal.

The timeline is shown in figure 2.

**Date 1:**

Trade at date-1 occurs in first-price sealed-bid auctions for assets \(x\) and \(y\).

1. The investor and banks may simultaneously submit bids for both auctions: \(p_x\) and \(p_y\).

2. Counterparty \(x\) can choose to sell asset \(x\) to the highest bidder for this asset at the highest price and similarly for counterparty \(y\). For simplicity, we assume that ties go to the investor. The counterparty will, of course, accept the bid only if it is no less than his private value, \(V'\).
1.1.4 Claims and Objective

Suppose the investor takes a loan with face value, $R$, and buys asset $je \{x,y\}$ for an amount, $p$, when $\tilde{V}_j = V$. The investor’s total assets consist of cash of $1 - p$ plus the risky asset. These provide a gross payoff at date-2 of: $1 + 2V \cdot \tilde{J} - p$. The investor takes an equity claim on this payoff, and the bank takes a debt claim.

The bank’s net payoff is as follows:

$$\min \{1 + 2V \cdot \tilde{J} - p, R\} - 1 + \Delta$$

(4) since the bank lends $1 - \Delta$ at date-1 and takes a debt claim on the date-2 payoff. This debt is risky and exposed to factor $\tilde{J}$ since the investor may default if $\tilde{J} = 0$.

The investor’s net payoff on her equity claim is given as follows:

$$\max \{1 + 2V \cdot \tilde{J} - p - R, 0\} - \Delta$$

(5)

The investor will, therefore, attempt to minimize the objective: $p + R$. This objective is the price paid in the auction plus the face value of debt or, roughly speaking, the investor’s front-running plus interest-rate costs. In our discussion of equilibrium, our focus will be on the investor’s choice of disclosure policy and lender type in equilibrium. The investor can choose to either disclose or not and can choose either a correlated lender (e.g. type $x$ bank to a type $x$ investor) or an uncorrelated lender (e.g. type $y$ bank to a type $x$ investor). In these choices, the investor trades off interest-rate versus front-running costs.

We now analyze the equilibria of different versions of this game.

1.2 Benchmark: Equilibrium with no Front-Running

In this section, we consider the simplest benchmark case where banks can not participate in the auction at date-1 and consequently, can not front-run the investor. Disclosures are costless, therefore, and investors ought to voluntarily disclose their information in accord with the Milgrom unraveling result. In addition, investors should match with uncorrelated lenders since these lenders are tolerant to the investor’s risk and can offer favorable interest-rates.

We start by considering the investor’s trade at date-1. The investor has already procured terms for a loan at date-0. The investor will attempt to trade as long as she can earn positive profit from the loan. If $E[\max \{1 + 2V \cdot \tilde{J} - V' - R, 0\}|s_J = 1] \geq \Delta$ and $\tilde{V}_j = V$, the investor will buy the asset
by bidding the minimum price in the auction, \( V' \). If \( \bar{V}_j = 0 \), the investor will not buy the asset because it pays off zero with certainty.

We now consider the interest-rates offered by banks at date-0. In this chapter, we only consider interest-rate equilibria of the following symmetric form. First, all type x banks offer one menu and all type y banks offer another menu. Type x banks offer rates: \([R_{yx}^*, R_{yx}^*, R_{Ny}^*]\); type y banks offer rates: \([R_{ty}^*, R_{ty}^*, R_{Ny}^*]\). Second, correlated risks are offered one interest-rate, uncorrelated risks are offered another, and non-disclosed risks are offered yet another. In this equilibrium:

\[
\begin{align*}
R_0^* &= R_{xy}^* = R_{yx}^* \\
R_1^* &= R_{xx}^* = R_{yy}^* \\
R_N^* &= R_{Nz}^* = R_{Ny}^*
\end{align*}
\]

(6)

where \( R_0^* \) is the interest-rate for uncorrelated risks (e.g., that a type y bank offers a type x investor), \( R_1^* \) is the interest-rate for correlated risks, and \( R_N^* \) is the interest-rate for non-disclosed risks.

The following proposition summarizes the equilibrium actions of the game:

**Proposition 1** The equilibrium interest-rates offered at date-0 are given by:

\[
\begin{align*}
R_0^* &= 1 - V' + 2(V' - \Delta) \\
R_1^* &= 1 - V' + (2 + k)(V' - \Delta) \\
R_N^* &= 1 - V' + (2 + \frac{k}{2})(V' - \Delta)
\end{align*}
\]

In equilibrium, the investor chooses \( R_0^* \) and consequently discloses to an uncorrelated bank.

**Proof:** See Appendix.

The competitive interest-rates in equation 7 can be understood from a decomposition of the investment assets. The total assets can be decomposed into two parts: \( 1 - V' \) of the assets are kept as riskless cash and the remainder is invested in the risky asset. The investor has borrowed an amount, \( 1 - \Delta \), which needs to be repaid. \( R^* \), therefore, consists of two parts. The first is the riskless repayment of \( 1 - V' \), and the second is the risky repayment of \( V' - \Delta \). This second term is multiplied by 2 because it is received with probability \( \frac{1}{2} \) (uncorrelated banks have risk-neutral valuations because we are using a locally-linear approximation for the utility function). A correlated bank applies an additional discount of \( k \) to this term because of risk-aversion. The non-disclosure interest-rate has half this discount because the bank does not know whether the correlation of the bank is 0 or 1.\(^8\) These interest-rates simply reflect the risk-tolerance of each type of lender. An uncorrelated lender will offer the lowest interest-rate because of tolerance to the risk of the loan.

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\(^8\)We restrict to "symmetric" pure-strategy trembling-hand perfect equilibria where we can apply the definition on
The investor will choose the minimum interest-rate, \( R_0 \) (since there are no front-running costs to consider), and, therefore, disclose to an uncorrelated lender in equilibrium. This confirms our intuition that there is full disclosure in a benchmark where disclosure is costless. \(^9\)

1.3 Mandatory Disclosure Equilibrium

We now add the feature that banks can front-run and start by considering the case where the investor’s disclosure is mandatory. The investor can choose only to which type of lender (correlated or uncorrelated) she will disclose her information. As before, an uncorrelated bank will offer a lower interest-rate than a correlated bank because it views the risk more favorably. An uncorrelated bank will also front-run the investor more aggressively for the same reason, however. This tradeoff makes the choice between correlated and uncorrelated lender a priori unclear.

We denote the equilibrium interest-rate offered by a bank with correlation of \( \rho \) by \( R_\rho^* \). The price paid in the auction, which we denote \( p_\rho^* \), depends on the amount of front-running done by this bank.

The following proposition summarizes the equilibrium values of \( R_\rho^* \) and \( p_\rho^* \).

**Proposition 2** The equilibrium prices in the auction are:

\[
\begin{align*}
p_0^* &= V \\
p_\rho^* &= \max \left\{ V', \left(1 + \frac{k}{2}\right)^{-1}V \right\}
\end{align*}
\]

The equilibrium interest-rates offered by banks are:

\[
\begin{align*}
R_0^* &= 1 - p_0^* + 2(p_0^* - \Delta) \\
R_\rho^* &= 1 - p_\rho^* + (2 + k)(p_\rho^* - \Delta)
\end{align*}
\]

**Proof:** See Appendix.

The investor then chooses the lender with the minimum \( p_\rho^* + R_\rho^* \), and this choice is summarized in the following corollary:

---

page 351 of Fudenberg-Tirole (1996) by restricting bids to a finite number of values. The equilibrium is symmetric in the sense that both type x and y investors play the pure same strategy (either disclosure or non-disclosure). The interest-rates, \( R_\rho^* \) and \( R_X \), are not unique because they are not chosen in equilibrium. We choose to specify these rates competitively as above for simplicity, however.

\(^9\)There is an alternate equilibrium where investors will choose non-disclosure and match with uncorrelated banks. We can eliminate such equilibria by assuming that the investor has small heterogeneous costs from taking loans from different banks and that these costs are independent of the bank’s correlation.
Corollary 1 There exists a $V^*_c$, which we call the “concentration threshold”, such that:

- If $V' < V^*_c$, the investor will choose a correlated bank.
- If $V' > V^*_c$, the investor will choose an uncorrelated bank.

Similarly, there exists a $V_c$ such that for $V > V_c$, a correlated bank is preferred, and for $V < V_c$, an uncorrelated bank is preferred.

Corollary 1 says that the choice of lender depends on the NPV of the investment. The NPV increases with $V$ and decreases with the minimum trade price, $V'$. If the investment has high NPV (i.e., $V'$ is low), front-running is a dominant concern, and the investor will choose a correlated lender. If the investment has low NPV (i.e., $V'$ is high), interest-rate concerns dominate, and the investor chooses an uncorrelated lender. We now offer some intuition for this result and for the trade prices and interest-rates in equations 8 and 9.

As before, the investor will only bid in the auction if $\hat{V}_j = V$. At date-1, banks that do not observe the investor’s information will not bid in the auction because of inequality 2.

If an uncorrelated bank observes that $\hat{V}_j = V$, its reserve price for the asset is: \(^{10}\)

$$\bar{p}_0 = V$$  \hspace{1cm} (10)

If the correlated bank observes that $\hat{V}_j = V$, its reserve price is:

$$\bar{p}_1 = (1 + \frac{k}{2})^{-1} V$$  \hspace{1cm} (11)

To win the asset, the investor’s bid must be greater than or equal to the reserve prices of the competing banks. The investor must also bid at least the reserve price of the counterparty, $V'$. Hence, if the investor discloses to a correlated bank, she must pay $p_1^* = \max \{V', \bar{p}_1\}$ as in equation 8. If the investor discloses to an uncorrelated bank, she must pay $p_0^* = V$ (since $V > V'$ always).

The interest-rates in equation 9 can be understood using the same decomposition as in the previous section. The $1 - p^*$ term represents a riskless repayment and the $p^* - \Delta$ term represents a risky repayment, which the correlated bank discounts by a factor that depends on $k$.

\(^{10}\)The discussion in this section is not precise because the terms of the loan affect how much each party values the asset, which affects the prices offered. Here, we have each party offering prices as if there were no loan and they were making investments entirely from their own capital. The proof in the appendix shows that this heuristic argument gives the correct results.
Corollary 1 can be understood in the following way. The uncorrelated bank front-runs “all the way” and pushes the price to the expected value of the asset: $V$. There is (approximately) zero profit left in the investment at this point. The correlated bank, on the other hand, does not front-run “all the way” and some profit potentially remains for the investor. The investor does not realize the entire profit of $V - p_1^*$, however. Part of this profit is paid to the correlated bank as compensation for the undesirable risk of the investment so that the investor’s profit is: $V - p_1^* - \frac{k}{2}(p_1^* - \Delta)$. Correlated borrowing dominates uncorrelated until $V - p_1^*$ becomes so small (or alternatively, when $V'$ becomes large) that the compensation for risk drives the investor’s profit below zero. Hence, for high NPV investments, a correlated lender is preferred whereas for low NPV investments, an uncorrelated lender is preferred.

The parameters, $\Delta$ and $k$, which represent the institutional characteristics of capitalization and risk-management policy, have the following comparative statics:

**Corollary 2** There exist $\Delta_c$ and $k_c$ such that for $\Delta > \Delta_c$ and $k < k_c$, a correlated bank is preferred, and for $\Delta < \Delta_c$ and $k > k_c$, an uncorrelated bank is preferred.

Increasing $\Delta$ decreases the credit-risk borne by the lender. Since a correlated bank is more averse to this credit-risk than an uncorrelated bank, the correlated bank’s interest-rate decreases by more than the uncorrelated’s interest-rate as a result. Hence, increasing $\Delta$ increases the relative attractiveness of correlated disclosure.

Increasing $k$, the correlated bank’s risk-aversion, increases the correlated bank’s interest-rate. It can also decrease the correlated bank’s front-running when $V' < \tilde{p}_1$ ($k$ is low). In this region, the front-running benefit dominates the interest-rate cost, and the investor’s profit actually increases with $k$. The investor’s profit from correlated disclosure in this region is $\frac{k}{2}\Delta$ and is always greater than that from uncorrelated disclosure. When $V' > \tilde{p}_1$ ($k$ is high), the investor’s profit decreases with $k$ since changing $k$ no longer affects the auction price and hence, front-running costs. It is in this region that the profit from correlated disclosure becomes less than the profit from uncorrelated disclosure (zero) for $k$ great enough.

### 1.4 Voluntary Equilibrium

We now consider the equilibrium in a free market where investors have the choice of disclosure versus non-disclosure.

We denote by $p_N^*$ and $R_N^*$, the price paid in the auction at date-1 in the case of non-disclosure and the interest-rate for non-disclosure, respectively. The investor may also disclose to either a
correlated or an uncorrelated lender as before.

The following proposition summarizes the equilibrium auction prices and interest-rates.

**Proposition 3** The equilibrium prices for trade in the case of disclosure are given by \( p_0^* \) and \( p_1^* \) in equation 8. The price for non-disclosure is:

\[
p_N^* = V'
\]  

The equilibrium interest-rates for disclosure are given by \( R_0^* \) and \( R_1^* \) in equation 9. The interest-rate for non-disclosure is:

\[
R_N^* = 1 - p_N^* + \left(2 + \frac{k}{2}\right)(p_N^* - \Delta)
\]  

**Proof:** See Appendix.

There is no front-running in the case of non-disclosure since the leading bank now does not observe the investor’s type or signal. Hence, the investor can pay \( V' \) for asset as in equation 12. The interest-rate for non-disclosure is consequently the same as in the case of no front-running as given in equation 13. The following corollary obtains for the investor’s choice of disclosure policy:

**Corollary 3** There exists a \( V_d' \), which we call the “disclosure threshold”, such that:

- If \( V' < V_d' \), the investor will choose not to disclose.
- If \( V' > V_d' \), the investor will choose to disclose.

where \( V_d' > V_d' \). Similarly, there exists a \( V_d \) such that for \( V > V_d \), the investor does not disclose, and for \( V < V_d \), the investor discloses.
The fact that $V'_d > V'_e$ means that when the investor voluntarily discloses, she will match with an uncorrelated lender. One can see this in figure 3. The investor will never voluntarily disclose to a correlated lender and, therefore, will never create concentrations of risk with probability one. The reason is that the investor will always prefer non-disclosure over correlated disclosure since both its front-running and interest-rate costs are (weakly) lower. It is clear that non-disclosure does not have a higher front-running cost than either type of disclosure. Non-disclosure has a lower interest-rate cost than correlated disclosure because a correlated risk is the most costly type of risk to the bank and with non-disclosure, there is some chance that the investor’s risk is uncorrelated.

The argument for why non-disclosure is preferred for high NPV (low $V'$) and disclosure is preferred for low NPV (high $V'$) is similar to the previous one. Disclosure to an uncorrelated bank results in front-running “all the way” to the expected value of the asset, pushing profits to zero. With non-disclosure, some profit potentially remains but the investor must compensate the bank for the undesirable non-disclosed risk through the interest-rate. Hence, non-disclosure is preferred until the NPV of the investment is so low that the interest-rate drives the investor’s profit below zero.

Finally, we have the disclosure decision as function of $\Delta$ and $k$.

**Corollary 4** There exist $\Delta_d$ and $k_d$ such that for $\Delta > \Delta_d$ and $k < k_d$, the investor does not disclose, and for $\Delta < \Delta_d$ and $k > k_d$, the investor discloses.

The reasoning for $\Delta$ is similar to that of the previous section. The investor’s profit from non-disclosure is decreasing in $k$ since an increase in $k$ increases the interest-rate cost but not the front-running cost of non-disclosure. The investor, therefore, does not disclose for $k$ low enough.

1.5 Regulations and Welfare

Until now we have considered the investor’s voluntary choice of lender and disclosure policy. We now address the question of whether this voluntary choice is optimal from the standpoint of social welfare, considering whether regulations such as mandated disclosure can be potentially beneficial. In this model, regulations on the investor can not be pareto-improving. The reason is that regulations impose constraints on the investor’s profit maximization, which make the investor worse off. Our main goal here is to show that various forms of regulation can, in fact, be pareto-suboptimal. We also show that certain regulations can be welfare-improving if the regulator has specific objectives in mind. We pursue our analysis by considering a welfare function of the following form:
\[ \omega \Pi_i + (1 - \omega) \Pi_{cp} \] (14)

where \( \Pi_i \) is the profit of the investor, \( \Pi_{cp} \) is the profit of the counterparty, and \( \omega \in [0, 1] \) is the investor's weight in this function. We omit the bank's utility because the bank acts competitively and realizes only its reserve utility in any equilibrium.\(^{11}\)

We assume that the regulator can set overall guidelines pertaining to disclosure policy, capitalization, and risk-management but can not control the specific lending activities and assets of institutions in the economy. The assumption here is that monitoring of specific activities is infeasible either of high direct costs or indirect costs such as information leakage.

We should mention that our analysis here is incomplete since many factors are left out that deserve consideration: particularly, the investor's incentive to gather information. Our arguments for the pareto suboptimality of regulation are only strengthened to the extent that they affect this incentive adversely. In general, however, we view our results as indicative and a starting point for further analysis.

1.5.1 Disclosure Regulation

We mentioned that mandating disclosure can not be pareto-improving since the investor's profit can only decrease as a result of regulations. However, mandating disclosure can improve welfare if the regulator has specific objectives in mind. We saw in our previous analysis that mandating disclosure can change the matching between investor and bank risk-types.

In the absence of regulation, there is non-disclosure when \( V' < V'_d \) and consequently, random matching between correlated and uncorrelated types. When \( V' > V'_c \), mandating disclosure can induce the investor to match exclusively with uncorrelated types. When \( V' < V'_d \), however, investors will match only with correlated banks and create concentrations of risk. The regulator cares about this risk-matching when \( \omega = \frac{1}{2} \) and the objective is to maximize the total surplus (or profit) from trade. With this objective, matching between correlated types results in a loss of welfare because the full surplus of the trade, \( V - V' \), is not realized when some of the risk is held by a correlated lender averse to this risk. Although mandating disclosure generates front-running and a loss for the investor, this loss in not necessarily a social loss because it simply transfers profit from the investor to other agents. Hence, mandating disclosure is potentially beneficial when it eliminates

\(^{11}\)Because of the market mechanism and the indivisibility of the asset in this model, the bank does not actually profit from front-running and all benefits accrue to the counterparty. In a more general model, however, the bank could potentially make positive profits from front-running.
concentrations of risk \((V' > V_c')\) and potentially hazardous when it exacerbates them \((V' < V_c')\).

These concentrations of risk can, in fact, be sufficiently detrimental that they result in a pareto loss of welfare.

The following proposition concerns the welfare effects of mandating disclosure.

**Proposition 4** If \(\omega = \frac{1}{2}\), mandating disclosure is suboptimal if and only if the investor prefers disclosing to a correlated bank over an uncorrelated bank (i.e., \(V' > V_c'\)).

Mandating disclosure can be pareto suboptimal if the investor prefers disclosing to a correlated bank over an uncorrelated bank (i.e., \(V' < V_c'\)).

Although mandating disclosure hurts the investor, it can potentially benefit the counterparty because disclosure generally results in more front-running and higher prices for the asset. Mandating disclosure may be pareto suboptimal, however, when it results in the investor choosing a correlated lender. In particular, front-running does not increase when the correlated bank’s reserve price is less than the counterparty’s reserve value, \(V'\) (i.e., in the region, \(\tilde{p}_1 < V' < V_c'\)). There is a pareto loss in this case because the investor’s profit decreases while the counterparty’s stays the same. Correlated risk-matching does not always result in a pareto loss, however, since the correlated bank’s reserve price can exceed \(V'\).

The following proposition pertains to disclosure regulation in the opposite direction, i.e., mandating non-disclosure.

**Proposition 5** Mandating non-disclosure is always pareto suboptimal.

In this particular version of the model, mandating non-disclosure or decreasing transparency is never beneficial for social welfare. The reasons are first, that altering the investor’s voluntary policy can only decrease the investor’s profits and second, that decreasing transparency decreases front-running and the profit of the counterparty. Hence, mandating non-disclosure can only result in a pareto loss. In section 1.6, we consider an extension of the model where this property no longer holds.

It may be difficult for a regulator to determine the optimal disclosure policy based on \(V'\) since the NPV of an institution’s investments may not be readily observable. Instead, the regulator may rely on corollary 2 to determine the optimal policy. Mandating disclosure may result in concentrations of risk and a loss of welfare when the investor’s capital, \(\Delta\), is high and the bank’s risk-aversion, \(k\), is low. Hence, the regulator might rely on a “rule-of-thumb” that says that enhancing information
sharing through regulations ought to be considered when institutions are poorly capitalized and financially weak, i.e., when $\Delta$ is low and $k$ is high.

1.5.2 Capital Regulation

In this section, we consider the welfare effects of capital regulations on our model. To this end, we consider a variation of our model where the investor has a convex cost of capital, $\Delta + C(\Delta)$, where $C(\Delta)$ represents any net costs of raising capital $\Delta$. For example, these costs can represent the net costs of external finance. The investor optimally chooses an amount of capital, $\Delta^*$, prior to date-0. In addition, we assume that the investor's capital is observable to the banks (e.g., through a disclosure of NAV). With this added feature, the following statement can be made concerning capital requirements:

**Proposition 6** Implying a minimum capital requirement on the investor is pareto suboptimal.

**Proof:** See Appendix.

When capital requirements compel the investor to hold additional capital above $\Delta^*$, there is a loss of welfare that comes from two sources. First, according to corollary 4, additional capital can cause the investor to switch from disclosure to non-disclosure.\(^{12}\) Hence, the counterparty can lose rents from a reduced ability to front-run. Second, the investor also loses profits because the voluntary amount of capital, $\Delta^*$, results in optimal profits whereas any regulation pushes profits away from this optimum.

Of course, additional capital results in a reduction in the investor's default risk.\(^{13}\) There is no social gain from this reduction, however, because the cost of the additional capital outweighs the gain from the reduced default risk. Specifically, the benefit of reduced default risk is fully internalized by the investor since this benefit accrues to the investor through a lower interest-rate from the bank. The investor, therefore, chooses $\Delta^*$ in the absence of regulation such that the marginal social benefit of capital is equal to its marginal cost:

\[ \Pi'/(\Delta^*) = C'(\Delta^*) \]

\(^{12}\) Obviously, if $C(\cdot) > 0$ and the investor participates only for positive profits, there are cases when the investor will opt not to raise any capital nor participate in markets because profits will be negative. In these cases, capital requirements are still pareto suboptimal because they can push an investor from participation to non-participation in markets.

\(^{13}\) To be exact, by default risk, we do not mean the probability of default but the firm's liquidation value in the event of default.
where $\Pi_t(\Delta)$ is the investor’s profit and the social benefit of capital from reduced default risk. Hence, regulation can only move capital to a social suboptimum.

1.6 Disclosure with Heterogeneous Credit Risk

We now analyze a variation of our basic model where investors can have different capitalizations and hence, different credit risk, as well as having different market exposures as before. In addition, capitalization is no longer observable in isolation through aggregated accounting figures such as NAV. Disclosures now reveal both capitalization and market exposure simultaneously and consequently expose the investor to front-running. The motivation for such a model comes from cases when a leveraged investment institution can only verifiably disclose its credit-risk profile by “opening the books,” i.e., disclosing detailed information on its investments.\textsuperscript{14} Hence, investment information and credit risk are revealed simultaneously.

In particular, there are two types of credit risk for investors; good types have high capitalization, $\Delta_g$, and bad types have low capitalization, $\Delta_b < \Delta_g$. This type is privately observed by the investor, and the prior probability for a good type is $\gamma$ and for a bad type is $1 - \gamma$. The average capitalization is given by: $\bar{\Delta} = \Delta_b + \gamma(\Delta_g - \Delta_b)$. Again, the investor can be of type $x$ or type $y$, and this type is independent of her capitalization.

In this version of the model, an investor’s disclosure simultaneously reveals her investment’s correlation ($x$ or $y$) and credit risk (good or bad). This model has the unfortunate feature that types may signal their capitalizations through the size of the loan taken because the investment is of fixed scale. A more general model with a continuous choice of scale would not have this uneconomic feature, and we choose to eliminate it by simply assuming all loans have a fixed size, i.e., both the good and bad types take loans of size, $L$.

There are now four interest-rates for disclosure in equilibrium: $R_{t5}^x$, $R_{t9}^x$, $R_{t5}^b$, and $R_{t9}^b$, one for each type of investment (correlated and uncorrelated) and one for each type of credit risk. We denote by $R_0^*(\Delta)$, $R_1^*(\Delta)$, and $R_N^*(\Delta)$, the two interest-rates for disclosure and one for non-disclosure as a function of $\Delta$. Specifically:

\textsuperscript{14}There are cases when NAV and other aggregated accounting figures are subject to enough manipulation to make them noisy measures of capitalization and associated default risk. One such case is when the institution has discretion to mark the value of its own illiquid assets. Detailed portfolio disclosures would provide a more accurate measure of capitalization in this case.
\[ R_0^*(\Delta) = L + \Delta - p_0^* + 2(p_0^* - \Delta) \]
\[ R_1^*(\Delta) = L + \Delta - p_1^* + (2 + k)(p_1^* - \Delta) \]
\[ R_N^*(\Delta) = L + \Delta - p_N^* + (2 + \frac{k}{2})(p_N^* - \Delta) \]

(16)

As is shown in the appendix, the equilibrium auction prices do not depend on the credit risk of the investor. We can, therefore, write the equilibrium prices as before with no reference to \( g \) or \( b \): \( p_0^* \). The following proposition and corollary summarize the equilibrium:

**Proposition 7** The equilibrium auction prices are given by \( p_0^* \), \( p_1^* \), and \( p_N^* \) in equations 8 and 12.

The equilibrium interest-rates for disclosure are given by:

\[
\begin{align*}
R_{0g}^* &= R_0^*(\Delta_g) & R_{0b}^* &= R_0^*(\Delta_b) \\
R_{1g}^* &= R_1^*(\Delta_g) & R_{1b}^* &= R_1^*(\Delta_b)
\end{align*}
\]

(17)

The equilibrium interest-rates for non-disclosure are such that:

\[
\begin{align*}
R_N^* &= R_N^*(\bar{\Delta}) & \text{if } & \Gamma_g^* > p_N^* + R_N^*(\bar{\Delta}) \\
R_N^* &= R_N^*(\Delta_b) & \text{if } & \Gamma_g^* < p_N^* + R_N^*(\bar{\Delta})
\end{align*}
\]

(18)

where \( \Gamma^* = \min\{p_0^* + R_0^*(\Delta), p_1^* + R_1^*(\Delta)\} \).

**Proof:** See Appendix.

**Corollary 5** The following statement summarizes the investor’s disclosures in equilibrium.

- If \( \Gamma_g^* > p_N^* + R_N^*(\bar{\Delta}) \), the good type does not disclose. The bad type will also not disclose in this equilibrium.

- If \( \Gamma_g^* < p_N^* + R_N^*(\bar{\Delta}) \), the good type discloses. The bad type will then disclose to an uncorrelated type if \( \Gamma_b^* < p_N^* + R_N^*(\Delta_b) \) and otherwise not disclose.

\( \Gamma_g^* \) and \( \Gamma_b^* \) are the disclosure costs (front-running plus interest-rate) for the good and bad types, respectively. We note first that the bad type will always withhold information if the good type withholds. Since \( \Gamma_g^* < \Gamma_b^* \), if the good type finds non-disclosure preferable over disclosure, the bad type will as well.

There are now two possible interest-rates for non-disclosure. If \( \Gamma_g^* > p_N^* + R_N^*(\bar{\Delta}) \), the good type will not disclose. Banks will, therefore, offer a non-disclosure interest-rate of \( R_N^*(\bar{\Delta}) \) since the expected amount of capital held by non-disclosing investors is the unconditional average. If
\( \Gamma^*_g < p^*_N + R^*_N(\bar{\Delta}) \), the good type will disclose, and banks will offer a non-disclosure interest-rate of \( R^*_N(\Delta_b) \) since now it is only possible for bad types to choose non-disclosure.

The presence of the bad type increases the good type's interest-rate for non-disclosure. In other words, \( R^*_N(\Delta_b) > R^*_N(\bar{\Delta}) > R^*_N(\Delta_g) \) where \( R^*_N(\Delta_g) \) is the non-disclosure interest-rate when there are only good types in the economy. Hence, the good type discloses for more parameter values than if she were the only type, in an attempt to distinguish herself from the bad type. The lower the average type, \( \bar{\Delta} \), the more the good type discloses. By similar reasoning, the bad type discloses less than if she were the only type. The bad type now has an additional benefit to withholding information: that of pooling with a better credit risk.

We wish to emphasize that the good type's incentive to disclose and differentiate herself from the bad type may be strong enough that the good type will disclose even when she prefers a correlated bank. This is in contrast to the previous voluntary equilibrium, in which the investor never disclosed to a correlated lender. The bad type, however, still always prefers non-disclosure over correlated disclosure since non-disclosure is now even more attractive to the bad type. It is still true, therefore, that there are no risk concentrations with probability one in a free market.

### 1.6.1 Disclosure Regulation

We proceed with our welfare analysis using the following welfare function: \(^{15}\)

\[
\omega [\eta \Pi_g + (1 - \eta) \Pi_b] + (1 - \omega) \Pi_{cp}
\]

where \( \Pi_g \) is the expected profit of the good-type investor, \( \Pi_b \) is the profit of the bad-type investor, \( \Pi_{cp} \) is the profit of the counterparty, and \( \omega, \eta \in [0, 1] \) are social weights.

**Proposition 8** Mandating disclosure can be pareto suboptimal if both good and bad type investors prefer disclosing to a correlated bank over disclosing to an uncorrelated bank.

This result is identical to the one before. Mandating disclosure is never pareto-optimal because it can only hurt both types of investors. Since investors always have the option of disclosure, compelling them to disclose can not increase their profits. As before, however, disclosure can benefit the counterparty by increasing front-running. It may not benefit the counterparty when both types choose correlated lenders since front-running does not increase when the correlated

---

\(^{15}\)We limit our welfare discussion in this section to the analysis of disclosure regulations since the analysis of capital regulations is now problematic. Specifically, there is a hold-up problem associated with the ex-ante choice of capital when capital has a non-negative cost and is not observed by the lender.
bank's reserve price is less than the counterparty's. Hence, mandating disclosure can result in a pareto loss when it induces risk concentrations. In all other cases, mandating disclosure is either redundant or has positive benefits for the counterparty.

The following proposition concerning mandating non-disclosure distinguishes this model from the previous:

**Proposition 9** Mandating non-disclosure can increase social welfare for some welfare weights and parameter values.

Unlike before, mandating non-disclosure can now increase welfare in some cases. In particular, mandating non-disclosure can create a subsidy from the good type to the bad type. This happens when the good type is unwillingly compelled to withhold information, which lowers the non-disclosure interest-rate for the bad type since the average non-disclosing type has improved. As before, mandating non-disclosure hurts the good type of investor and the counterparty. Hence, mandating non-disclosure can only be preferable when the social weight of the bad type \((\omega \cdot (1 - \eta))\) is high enough.

It is now true that *decreasing* the amount of voluntary information sharing in the economy can be beneficial whereas before it was not. Hence, the addition of heterogeneous credit risks complicates our welfare analysis and causes us to lose some of the sharpness of our previous results.

### 1.7 Conclusion

In this chapter, we have studied disclosure by leveraged investors to their creditors when there are front-running costs. We have analyzed both voluntary and regulated disclosure in the context of a model of risk-matching between institutions with heterogeneous risk exposures.

In our model, voluntary disclosure occurs for low NPV investments, low investor capitalization, and high lender risk-aversion. In addition, mandatory disclosure is potentially beneficial when it results in the investor matching with an uncorrelated lender. When a regulator wants to maximize the total surplus from trade, the investor's loss from the front-running induced by transparency is outweighed by the gains from better risk-matching in the economy. Mandating disclosure, however, is suboptimal when it results in the investor matching with a correlated lender or in *concentrations of risk*. Mandating disclosure can, in fact, be pareto-suboptimal when the losses from these concentrations of risk are sufficiently high.

Mandating non-disclosure or decreasing the level of transparency in the economy is pareto suboptimal in our initial model where investors have heterogeneous market exposures only. With
heterogeneous credit risks, on the other hand, decreasing transparency can have potential benefits. Good credit risks now have an additional incentive to disclose and sometimes voluntarily disclose to correlated lenders. The adverse effects of these risk concentrations can be great enough to justify mandating non-disclosure. We also studied the effects of imposing minimum capital requirements on the risk-matching and disclosure properties of our economy. We found that the effects were unambiguously bad and decreased the likelihood of voluntary disclosure.

This model has some straightforward empirical implications. Though the proprietary nature of the data makes actual testing impractical in most cases, the model can be tested, in principle. A simple implication of the model is that the return correlation between a leveraged investment institution and its creditors ought to be positively related to the investor’s capitalization and negatively related to the creditor’s risk-aversion. Another implication is that front-running can imply a *non-decreasing relationship between the investor’s profit and the lender’s risk-aversion* as mentioned in section 1.3. In the absence of front-running, this relationship ought to be *strictly* decreasing. To demonstrate a non-decreasing relationship empirically could be evidence of non-negligible front-running on the part of the creditor.

In this model, welfare losses are the result of one lender’s utility loss from suboptimal risk-matching. Our model does not capture welfare losses from the “systemic” risks that were the prime concern of regulators in the LTCM crisis. In particular, we do not capture the possibility that one institution’s distress could spill over into other institutions and markets, even ones not directly related to the original institution. In this respect, concentrations of risk were problematic in the LTCM situation because of the potential for “knock-on” effects whereby a liquidation of LTCM’s assets could have led to substantial losses for other institutions, possibly resulting in further defaults and liquidations. It would be interesting to extend the model to capture such effects. For example, one could incorporate these issues by introducing intermediate shocks in asset prices in an economy with multiple investors and lenders. To study the interaction of disclosure policy and lender choice with properties of asset prices including liquidity and volatility in crisis situations would be an interesting agenda for future research.

\[\text{\textsuperscript{16}}\text{According to our analysis, such a non-decreasing relationship may only be observed when correlated disclosure is observed, i.e., in regulated environments or when credit-risk revelation is an important component of costly disclosure.}\]
The leveraged trading of hedge funds almost caused a meltdown of global financial markets in the fall of 1998. The funds, the largest of which was Long-Term Capital Management, were mainly involved in the relative-value trading of fixed-income instruments. Though the activity of these relative-value funds has dropped since the crisis, they are still active players in many markets. JWM Partners, the newly-named fund of LTCM's founders, has a reported leverage of 10 to 1 on an equity base of around $400 million. In other markets, leveraged relative-value trading has actually increased since the crisis. According to informal conversations with bank risk-managers, activity in equities has gone up since 1998. There is substantial interest, therefore, in understanding the trading behaviors of these players.

Relative-value trading is defined in this chapter as any trading strategy using two or more securities which are mispriced relative to each other. The classic example involves two similar bonds. LTCM would often trade one bond long and the other short and make a profit when the yield spread "converged" to some anticipated level.

This chapter deals with a feature of trading that is unique to relative-value trade: namely, that a relative-value trade can be "broken-up" into its component securities in a multiple dealer market. Each leg of the trade can be done with a different dealer. The alternative would be to "consolidate" the trade, i.e., to trade all legs through a single dealer. This behavior is important because it affects how many counterparties are connected to a hedge fund through leveraged positions. In addition, it affects the types of claims that each counterparty has on the fund. Some of LTCM's problems in the 1998 crisis may have come from the fact that its trades were broken-up between counterparties. Consequently, LTCM had a huge number of different counterparties with diverse claims. An orderly renegotiation was effectively impossible without a regulatory intervention.

The practice of breaking up trades was typical at LTCM, as is documented in the recent popular book, "When Genius Failed: The Rise and Fall of Long-Term Capital Management":

[John Meriwether] felt that investment banks were rife with leaks and couldn't be trusted not to swipe his trades for themselves. Indeed, most of them were plying similar strategies. Thus, as a precaution, Long-Term would place orders for each leg of a trade with a different broker. Morgan would see one leg, Merrill Lynch another, and Goldman yet another, but nobody would see them all. Even Long-Term's lawyer was kept in the dark; he would hear the partners speak about "trading strategy three," as though

\footnote{Figures reported in the Wall Street Journal as of July 2000}
Long-Term were developing a nuclear arsenal (pg. 48).

The intuitive rationale for breaking up trades is clear: each leg of the trade alone contains an incomplete piece of the hedge fund’s information. The fund can add noise to each dealer’s inference by breaking up the trade. Noise may be desirable if the dealer has the ability to exploit this information at the fund’s expense.

To fix ideas, we start with a simple example. Suppose the interest-rate spread between French and German government bonds of a certain maturity is too large. An arbitrageur knows this and wants to take a short position in this spread. He can take this position by buying French bonds and shortselling German bonds. The breakup strategy here is to trade the French bond with dealer 1 and the German bond with dealer 2. The consolidation strategy is to trade a swap of French and German interest-rates with a single dealer.

Why are dealer inferences noisy in the breakup case? Dealer 1 sees a long French bond position and may not know which bond was shorted with dealer 2. The bond on the other side may be a German bond or an Italian bond, for example. Seeing the French-German swap, on the other hand, would resolve this uncertainty.

We have made a number of implicit assumptions. First, there is private order flow in this market so that dealer 1 can not see the trades of dealer 2. Second, there is a specific structure on the uncertainty. The dealers know that the arb is trading the spread between two bonds but does not know which two bonds in particular.18

This chapter presents a model where information is “broken-up” through trade on different securities. There is an informed arbitrageur with information about a particular market factor. One important feature of the model is that this factor is unknown to dealers and other market participants. For example, the dealer in our example did not know whether the fund had information about the French-German spread or the French-Italian spread. The other feature is that some securities - the “bonds” - are perceived as noisy and others - the “swaps” - are fully revealing. The bonds are noisy because every “type” of arbitrageur trades only in bonds where a type corresponds to a possible factor that is privately observed by the arb. Different types mix their signals together by trading in the same set of bonds. If a dealer sees a trade in a particular bond, that dealer can not perfectly distinguish which type made the trade. This chapter, therefore, introduces a concept of “endogenous” noise on different securities. The noise is endogenous because it can only be sustained if every type of arbitrageur finds it optimal to trade exclusively in noisy bonds.

18The arb is not, for example, trading a spread between two swaps. If that were the case, the French-German swap may be perceived as only one leg of a larger trade and would not be fully revealing.
We use our model to study the existence and uniqueness of this "breakup" equilibrium. These issues are not trivial. Consider the fact that the security structure is redundant and the arbitrageur can accumulate a position in infinitely many ways. For instance, an arb can take a position in the French-German spread by going long a French-Italian swap with dealer 1, long an Italian bond with dealer 2, and short a German bond with dealer 3. We prove that such alternate strategies are undesirable. We also show that our equilibrium strategy is unique in a certain class so that there is only one possible way that the arbitrageur can trade to create noisy inferences.

Breakup occurs in our model because dealers, in effect, "front-run" on an arbitrageur's information, resulting in a loss. Breakup adds noise to the dealer's inference which decreases the amount of front-running. A number of empirical predictions come out of the model. One implication is that an arb's profit from breakup is increasing in the risk-aversion of its counterparty. This happens because breakup creates noise, which has a greater impact on more risk-averse agents. Another implication is that breakup can induce correlations between factors that are fundamentally uncorrelated. Breakup gives dealers noisy information about which factor is mispriced. If these dealers trade aggressively on this noisy information, they often submit net demands for unrelated factors, moving their prices.

Although we deal here with the informational issues of trade alone, we believe that our framework is useful for thinking about issues related to systemic risk. For example, we stated that a broken-up trade is most likely with a risk-averse counterparty. If the observed risk-aversion of an institution is positively related to its leverage ratio, these trading arrangements could be highly unstable. In particular, an arrangement of non-netted trades with a multitude of counterparties who are highly leveraged could be quite vulnerable to negative shocks.

This chapter presents two stylized representations of relative-value trade. The first is a two-factor structure which we discuss in sections 1.2 and 1.3. We start with this structure because it is easy to see why breakup occurs in this setup. We develop our ideas concerning the existence and uniqueness of the equilibrium here. Though transparent, the two-factor structure is a special case with non-generic empirical implications. In section 2.4, we consider a different structure with three factors and derive the general empirical results mentioned before. Section 2.5 concludes.
2.1 The Model

2.1.1 Two-Factor Structure

We first describe the information and security structure of our two-factor representation of relative-value trade. We start with this simple case because it makes the motive for breakup quite clear. We develop our initial thinking with this two-factor structure and derive results concerning the existence and uniqueness of the equilibrium. This structure, unfortunately, is a special case and gives non-generic results in its empirical predictions. This gives us the motive to consider a different security structure later and derive general empirical predictions.

In our model, there is one arbitrageur with private information and multiple dealers.

- There are two underlying binary risky factors: $X$ and $Y$. $X$ and $Y$ are independently and identically distributed. $X$ can have a high outcome ($+\sigma$) or a low outcome ($-\sigma$), each with equal probability (same for $Y$).\(^{19}\)

- The arbitrageur has private information on either $X$ or $Y$ but not both. With probability $\frac{1}{2}$, the arb sees $X$ perfectly, and with probability $\frac{1}{2}$, the arb sees $Y$ perfectly. Dealers do not know a priori which factor the arb has information in.

There are, therefore, four possible types of arb: $\omega \in \{x_H, x_L, y_H, y_L\}$.

These types correspond to the two possible factors and the two possible outcomes (high or low) that the arb could observe.

- Dealers offer trade in pure securities, $X$ and $Y$. In addition, they offer trade in two hybrid securities, $A$ and $B$, which are linear combinations of $X$ and $Y$: $A = \frac{1}{2}(X + Y)$ and $B = \frac{1}{2}(X - Y)$.

If the arb wants to buy one unit of $X$, he can buy either "pure" $X$ or two hybrids: $A + B$. Similarly, the arb can buy one unit of $Y$ by buying $A - B$.

This information structure is similar to standard asymmetric information models of trade except that there is one additional dimension of uncertainty: the factor of speculation. In our initial

\(^{19}\)The results of the two-factor model would not change if $X$ and $Y$ were correlated or had non-zero mean. They would change, however, if $X$ and $Y$ were not identical because the binary setup is fragile. If $X$ and $Y$ were not identical, dealers would be able to infer the arb's type from quantities traded. We could eliminate this problem by considering continuous distributions at the cost of analytic complication.
example, dealers did not know a priori which yield spread the arb was speculating in (e.g., French-German spread or French-Italian spread). Similarly, arbs here can be speculating in either X or Y.

Trade using A and B is the breakup strategy, and trade in X and Y is the consolidation strategy. The reason is that A and B can act like the bonds of our initial example. Recall that a French bond dealer could not discern which of two possible yield spreads was mispriced. If a dealer sees an arb buying A, he may not be able to discern whether a type $x_H$ is buying $A + B$ or a type $y_H$ is buying $A - B$. Similarly, X and Y can function like swaps. A trade in X can resolve this uncertainty in the same way a trade in a French-German swap can.

The structure represents a situation where an arb could be betting on the common factor between two related securities ($A + B$) or the differential factor between them ($A - B$). An example of this kind of uncertainty is if A and B are the stocks of two competitors in an industry. $A - B$ would be a long-short position betting on the success of one competitor versus the other. $A + B$ would be a position in the common factor: the risk of the industry.

2.1.2 Sequence of Events

There are two main stages of trade in our model. First, arbs and dealers trade. Second, dealers trade with noisy external agents. The timing goes as follows:

- One arb with private information appears in the market prior to trade. There are multiple (greater than four) competitive dealers.

- In stage-1, the arb asks dealers for bid-ask prices in two securities (the specific rules are given next). Dealers quote competitive bid-ask prices to the arb. The arb then trades with two dealers at these prices. Finally, dealers make inferences based on this trade.

- In stage-2, dealers submit market orders to a centralized market with an exogenous supply curve. The exogenous supply curve represents the demands of uninformed external traders. Security payoffs are realized after period 2.

In stage-1, dealers act competitively, and in stage 2, dealers act non-competitively. The two dealers who trade at stage-1 gain information not possessed by the rest of the market, which they use at stage-2. It is a critical assumption of the model that the arb not be able to commit to trading with only one dealer at stage-1. We will discuss this point in the next section.
This two-stage model of trade is standard in the microstructure literature. For example, the models of Madhavan [1995], Roell [1992], and Naik, et al [1996] have this structure: competitive bidding for trade with an informed trader, followed by the imperfectly competitive use of this information in subsequent trade with uninformed traders. This paradigm captures a salient aspect of most multiple-dealer markets. Specifically, each dealer has a private order flow which can sometimes provide an informational advantage over the rest of the market.

Formally, the game that we are describing is defined by the following players, information sets, and sequence of actions.

There are five players in the game. The first is an arbitrageur with privately observed type: \( \omega \in \{x_H, x_L, y_H, y_L\} \). The other four players are dealers who we label: 1, 2, 3, and 4.\(^{20}\)

The actions proceed as follows:

1. The arb declares two securities: \( J \) and \( J' \) where \( J, J' \in \{X, Y, A, B\} \).

   He also declares corresponding quantities (absolute size) for each security: \( |\tilde{\theta}| \) and \( |\tilde{\theta}'| \) where \( |\tilde{\theta}|, |\tilde{\theta}'| \in [\frac{1}{2}, \infty) \).\(^{21}\)

   Dealers 1 and 3 observe \( |J, |\tilde{\theta}| \). Dealers 2 and 4 observe \( |J', |\tilde{\theta}'| \).

2. Dealers 1 and 3 quote bid and ask prices for security \( J \) at size \( |\tilde{\theta}| \).

   Dealers 2 and 4 quote bid and ask prices for security \( J' \) at size \( |\tilde{\theta}'| \).

   In equilibrium, these prices are given by \( \bar{p}_J(|\tilde{\theta}|), \bar{p}_J(-|\tilde{\theta}|) \) (ask and bid for \( J \)) and \( \tilde{p}_{J'}(|\tilde{\theta}'|), \tilde{p}_{J'}(-|\tilde{\theta}'|) \) (ask and bid for \( J' \)). These prices are per unit of \( J \) and \( J' \).

3. The arb declares a buy or sell order for security \( J \) (\( J' \)). He chooses to execute this trade with either dealer 1 or 3 (2 or 4). Without loss of generality, the arb chooses dealer 1 for \( J \) (2 for \( J' \)).

   The executed trade \( [J, \tilde{\theta}] \) (\( [J', \tilde{\theta}'] \)) is the private information of the dealer 1 (2).

4. Dealers 1 and 2 submit market orders for \( X \) and \( Y \) on the centralized market.

   In equilibrium, dealer 1 submits demands: \( [\theta_x(J, \tilde{\theta}), \theta_y(J, \tilde{\theta})] \).

   Dealer 2 submits demands: \( [\theta_x(J', \tilde{\theta}'), \theta_y(J', \tilde{\theta}')] \).

\(^{20}\)The game functions the same with more than four dealers. As long as there are least two dealers per security proposed by the arb, the offered prices at stage-1 are perfectly competitive. This is the only effect we are trying to achieve here.

\(^{21}\)A minimum quantity of trade is needed for somewhat subtle reasons that are discussed in sections 3.7 and 3.8 of the appendix.
Finally, we mention the specific parameters of the model’s preferences and supply curves.

- Preferences:
  Arbs are infinitely risk-averse.
  This assumption is made to simplify analysis and match the real-world practices of “arbitrage” hedge funds. In the model, an arb of type $x_H$ and $x_L$ will not want to bear any unhedged exposure to Y and vice versa. Hence, we can limit our attention to simple equilibria where arbs of type $x_H$ go long X only, $y_H$ go long Y only, and so on.
  Arbitrage funds attempt to hedge away all exposures except the one in the realm of their expertise. When betting on a bond spread, for instance, all foreign-exchange risk, term-premium risk, etc. is hedged away until a pure exposure to the desired factor remains.
  Explicitly, the arb’s utility over random wealth, $\widetilde{W}$, is given by:

  \[ U(\widetilde{W}) = \inf_{\{\text{states}\}} (\widetilde{W}) \]  

  (20)

  Dealers in the model have general concave monotone utility given by: $u(\cdot)$.

- Exogenous supply curves:
  The centralized market has exogenous linear supply curves in X and Y. Prices are given by:
  $p_x = b\Theta_x$ and $p_y = b\Theta_y$
  where $\Theta_x$ is the sum of all orders for X (also for Y). Hence, there is a market impact with elasticity, $b$.

  Some of the exogenous assumptions of the model and their rationales are discussed in the appendix.

2.1.3 First-Best

We mentioned before that a critical assumption of the model is that the arb can not commit to trading with only dealer. Suppose instead that the arb can make this commitment. Arbs and dealers would then be able to function as a monopoly at stage-2. In this case, arbs would choose to fully reveal their information to one dealer. Breakup would not occur. The arb could achieve highest profits by sharing information with only one dealer who would make monopoly profits against the noisy supply curve at stage-2. This profit would be completely transferred through the dealer’s competitive price at stage-1.
The arb must not be able to commit himself to one particular dealer. Without a binding commitment to one dealer, the arb would choose to trade again with other dealers and extract further profits. When multiple dealers have the information, they “overtrade” at stage-2 and compete away first-best profits. This is the sense in which dealers “front-run”. Strictly speaking, dealers front-run each other and not the arb directly. However, because of rational anticipation, this front-running results in a loss for the arb relative to the first-best. Breakup adds noise to inferences, decreases front-running, and restores lost profits.

We start our analysis by considering the hypothetical case where the arb has access to the centralized market. We call this benchmark case the “first-best”. The arb will act as a monopoly and maximize profits against a linear supply curve.

Without loss of generality, we take the case when the arb is of type $x_H$ from now on.

The arb’s objective is given by:

$$\max_\theta [\sigma - b \theta] \theta$$  \hspace{1cm} (21)

The optimal demand is given by:

$$\theta_{FB} = \frac{\sigma}{2b}$$  \hspace{1cm} (22)

This is simply the familiar result that a monopolist produces (trades) halfway up the demand (supply) curve. First-best profit is given by:

$$\Pi_{FB} = \frac{\sigma^2}{4b}$$  \hspace{1cm} (23)

### 2.1.4 Second-Best

In reality, the arb does not have direct access to this supply curve and may not be able to achieve the first-best profit when trading through dealers. We now consider our model when the arb can not make a commitment to the monopoly outcome. Also, only pure securities are offered: X and Y.

At stage-1, an arb of type $x_H$ will buy X through two dealers.\(^{22}\) These dealers infer the full information and submit demands for X in stage-2. They maximize their profit with demands of $\theta^1$ and $\theta^2$ against the linear supply: $b(\theta^1 + \theta^2)$.

\(^{22}\)In equilibrium, type $x_H$ will buy X, type $x_L$ will sell X, and similarly for Y. The proof that these actions are a unique equilibrium is available upon request.
In equilibrium, both dealers submit equal demands given by:

$$\theta_{SB} = \frac{\sigma}{3b}$$

(24)

The profit to each dealer is given by:

$$\pi_{SB} = \frac{\sigma^2}{9b}$$

(25)

The prices that dealers offer to the arb at stage-1 are competitive. This implies that all profits dealers anticipate making on the centralized market are immediately transferred to the arb. Since they rationally anticipate Cournot competition at stage-2, they each transfer $$\pi_{SB}$$ to the arb. The arb makes total profit of:

$$\Pi_{SB} = \frac{2\sigma^2}{9b}$$

(26)

Because of the duopoly competition at stage-2, this profit is less than first-best. The arb wants everyone ex-ante to expect that he will only trade with one dealer, but he can not commit to do this. Ex-post, it is always in the arb’s interest to trade with both dealers because the arb extracts anticipated profits immediately through each dealer he trades with.

### 2.2 Breakup Equilibrium

We now consider the full equilibrium where the arb has access to both pure (X and Y) and hybrid (A and B) securities. In the previous section, the arb had access to X and Y only. Only second-best profit could be achieved because the two dealers at stage-2 “over-traded” relative to the first-best.

Recall that introducing A and B can create noise in dealers’ inferences. This noise can restore the first-best outcome by dampening dealers’ demands at stage-2. This mitigates “over-trading” in the Cournot game.

In fact, the arb can achieve first-best profits by using A and B to breakup information. This result holds for every level of dealer risk-aversion for this particular security structure. We show this for the extreme cases of zero and infinite risk-aversion. We put off discussing the formal equilibrium for a moment in order to develop the intuition here.

Suppose that all arb types find it optimal to breakup trade using A and B. Arbs of the four types will execute the following strategies:
\[ x_H \rightarrow A + B \quad y_H \rightarrow A - B \]
\[ x_L \rightarrow -A - B \quad y_L \rightarrow -A + B \]  

(27)

A positive A dealer (who executed a buy order for A) will infer that the arb is of type \( x_H \) or \( y_H \) with equal probability. Similarly, a positive B dealer will infer that the arb is of type \( x_H \) or \( y_L \). Suppose that the arb trades a quantity of \( \tilde{\theta} \). The +A dealer will bear an inventory of \((-\tilde{\theta}/2, -\tilde{\theta}/2)\) after trade with the arb (the left entry is the inventory in factor X and the right in Y).

We consider the equilibrium in the stage-2 Cournot game with uncertainty. Each dealer submits an optimal demand to the centralized market. Since the +A dealer’s inferences and inventories are symmetric in X and Y, we conjecture equilibrium demands of: \((\theta_1, \theta_1)\). By similar reasoning, we conjecture the following demands for all dealer types at stage-2.

\[ +A \rightarrow (\theta_1, \theta_1) \quad +B \rightarrow (\theta_1, -\theta_1) \]
\[ -A \rightarrow (-\theta_1, -\theta_1) \quad -B \rightarrow (-\theta_1, \theta_1) \]  

(28)

We now solve for these demands in the case of +A dealer with similar arguments for all other dealers. Our goal is to show that the aggregate demand at stage-2 is first-best and therefore, first-best profit accrues to the arb at stage-1.

The +A dealer infers the state to be either \( x_H \) or \( y_H \). The arb did a complementary trade of +B in the former case and -B in the latter. The +A dealer chooses stage-2 demands \((\theta_x, \theta_y)\) to maximize the following utility:

\[
\max_{(\theta_x, \theta_y)} \frac{1}{2} E \left[ u \left( \sigma \cdot (\theta_x - \frac{\theta}{2}) - b(\theta_1 + \theta_x) \cdot \theta_x + \tilde{Y} \cdot (\theta_y - \frac{\theta}{2}) - b(-\theta_1 + \theta_y) \cdot \theta_y \right) \right]_{x_H} + \\
\frac{1}{2} E \left[ u \left( \tilde{X} \cdot (\theta_x - \frac{\theta}{2}) - b(-\theta_1 + \theta_x) \cdot \theta_x + \sigma \cdot (\theta_y - \frac{\theta}{2}) - b(\theta_1 + \theta_y) \cdot \theta_y \right) \right]_{y_H}
\]  

(29)

where \( \tilde{X} \) and \( \tilde{Y} \) are the risky outcomes of X and Y in states where the arb can not discern their outcome.\(^{24}\)

We now solve for \( \theta_x \) and \( \theta_y \), first in the risk-neutral case and then in the infinite risk-aversion case.

\(^{23}\)The quantities traded for all dealer types is the same because the optimization problems are similar. A dealer who infers a state of \( x_H \) will buy as much X as a dealer who infers a state of \( x_L \) will sell.

\(^{24}\)We are not including cash from the stage-1 transaction with the arb in this equation. Throughout, we use preferences with "no wealth effects" so the omission does not change the optimization. Our analysis could be extended to more general preferences at the expense of complication. Also, we are assuming that inventories prior to stage-1 are zero.
Risk-neutral case:

In the risk-neutral case, the dealer puts a value on risky wealth equal to its expectation. He maximizes the objective:

$$
\max_{(\theta_x, \theta_y)} \left[ \frac{1}{2} \left( \sigma \cdot (\theta_x - \frac{\sigma}{2}) - b(\theta_I + \theta_x) \cdot \theta_x - b(-\theta_I + \theta_y) \cdot \theta_y \right) + \right. \\
\left. + \frac{1}{2} \left( -b(-\theta_I + \theta_x) \cdot \theta_x + \sigma \cdot (\theta_y - \frac{\sigma}{2}) - b(\theta_I + \theta_y) \cdot \theta_y \right) \right]
$$

(30)

First, we note that the $\theta_I$ terms cancel in this equation so that there are no strategic interactions between dealers. Optimizing ($\theta_x, \theta_y$) yields:

$$
\theta_x = \theta_y = \frac{\sigma}{4b}
$$

(31)

The dealer submits half of the first-best demand in each factor. The noise creates uncertainty about which factor is mispriced, and effectively, the amount of information in each factor is cut in half. His expected profit in stage-2 is given by:

$$
\pi_I = \frac{\sigma^2}{8b}
$$

(32)

The dealer offers a competitive price at stage-1, which fully transfers the profit from this trade to the arb. The arb collects half of the first-best profit from both the A dealer and the B dealer and hence, earns first-best profit. Another simple way to see this is from the fact that the aggregate demand in any state is first-best. For example, when the arb is of type $x_H$, he buys $A + B$. The $+A$ dealer submits demands of ($\frac{\sigma}{4b}, \frac{\sigma}{4b}$), and the $+B$ dealer submits demands of ($\frac{\sigma}{4b}, -\frac{\sigma}{4b}$). The net demand for $Y$ cancels and the aggregate demand is first-best: ($\frac{\sigma}{2b}, 0$).

Risk-averse case:

In the infinitely risk-averse case, the dealer’s utility over his risky wealth is given by:

$$
\min_{(\tilde{X}, \tilde{Y})} \left[ \sigma \cdot (\theta_x - \frac{\sigma}{2}) - b(\theta_I + \theta_x) \cdot \theta_x + \tilde{Y} \cdot (\theta_y - \frac{\sigma}{2}) - b(-\theta_I + \theta_y) \cdot \theta_y, \\
\tilde{X} \cdot (\theta_x - \frac{\sigma}{2}) - b(-\theta_I + \theta_x) \cdot \theta_x + \sigma \cdot (\theta_y - \frac{\sigma}{2}) - b(\theta_I + \theta_y) \cdot \theta_y \right]
$$

(33)

since he values a risky distribution of wealth as the minimum possible outcome. The solution to dealer 1’s demand has two regions:

$$
\begin{align*}
\theta_x &= \theta_y = \frac{\tilde{\theta}}{2} \text{ if } \tilde{\theta} < \frac{\sigma}{b} \\
\theta_x &= \frac{\sigma}{2b} \text{ if } \tilde{\theta} \geq \frac{\sigma}{b}
\end{align*}
$$

(34)
Intuitively, if the dealer’s inventory from stage-1 is small, infinite risk-aversion drives the dealer to trade his risky inventory to zero in stage-2. If the dealer’s inventory from stage-1 is large enough, however, the dealer takes the market impact of his trades in stage-2 into account and is only willing to trade up to a certain amount.

In the region $\tilde{\theta} < \frac{\bar{\theta}}{6}$, the dealer passes off all trade from the arb to the centralized market. At stage 1, the dealer will charge a price to the arb for security A that is exactly equal to the centralized market price: a “pass-through” price. Hence, the arb effectively has direct access to the centralized market for quantities low enough. Since this region contains the first-best demand (i.e., $\theta_{FB} < \frac{\bar{\theta}}{6}$), the arb can achieve first-best profits.

We have thus seen how the noise generated from breakup dampens dealers’ demands and improves profits relative to Cournot competition with perfect information. In the risk-neutral case, noise decreases the amount of information perceived in each factor and consequently decreases trading aggressiveness. In the infinitely risk-averse case, noise causes dealers of A and B to function as perfect pass-throughs to the centralized market. The arb can achieve first-best profit in both cases. We shall see in section 4 that the result for infinite risk-aversion is general but for risk-neutrality is specific to this security structure.

### 2.2.1 Formal Equilibrium Concept

We now define our equilibrium concept formally in order to address the questions of existence and uniqueness for the “breakup” equilibrium we have proposed. We focus here on the stage-1 equilibrium between arbs and dealers. The analysis of the stage-2 equilibrium is straightforward. In this section, we simply take the outcome of the stage-2 game as given. The Bayesian-Nash Equilibrium at stage-1 is defined by the arb’s trading strategy and the dealers’ inferences and pricing strategy.

The arb’s equilibrium trading strategy is given by:

$$\Psi(\omega) = \begin{pmatrix} \langle J, J' \rangle \\ \langle \tilde{\theta}, \tilde{\theta}' \rangle \end{pmatrix}$$ (35)

The left component of the $\langle \rangle$ bracket is the trade with dealer 1, and the right is with dealer 2. The top component of the vector is the security type and the bottom is the quantity. In general, the arb can mix several different trading strategy vectors in equilibrium. The arb chooses this trade to maximize his utility given the dealers’ pricing strategy: $\tilde{p}_J(\tilde{\theta})$. 

38
Dealer 1 observes the arb's trade, \([J, \tilde{\theta}]\), and forms an inference about the arb's information, \(\omega\), and the information set of dealer 2, \([J', \tilde{\theta}]\). We denote this conditional probability by:

\[
\tau_{[J, \tilde{\theta}]} \left( \omega, \begin{bmatrix} J' \\ \tilde{\theta} \end{bmatrix} \right)
\]  

(36)

These inferences satisfy Bayes' rule if the trade, \([J, \tilde{\theta}]\), is played by any arb type in equilibrium. We use a version of the Cho-Kreps intuitive criterion for off-equilibrium inferences. We discuss this issue further in a moment.

The dealer's pricing strategy, \(\tilde{p}_J(\tilde{\theta})\), maximizes his utility given his inferences: \(\tau_{[J, \tilde{\theta}]}\). Because of Bertrand competition at stage-1, these prices are set so that the dealer achieves a utility of 0 by trading with the arb (where 0 is the utility of not trading).

We conjecture the following arb trading strategies and dealer inferences as a BNE:

\[
\Psi(x_H) = \begin{pmatrix} \langle A, B \rangle \\ (\theta_{FB}, \theta_{FB}) \end{pmatrix} \quad \Psi(x_L) = \begin{pmatrix} \langle A, B \rangle \\ (-\theta_{FB}, -\theta_{FB}) \end{pmatrix}
\]

\[
\Psi(y_H) = \begin{pmatrix} \langle A, B \rangle \\ (\theta_{FB}, -\theta_{FB}) \end{pmatrix} \quad \Psi(y_L) = \begin{pmatrix} \langle A, B \rangle \\ (-\theta_{FB}, \theta_{FB}) \end{pmatrix}
\]

(37)

This equilibrium has the arb breaking up his information by trading in the A and B hybrid securities only. We will use a shorthand of \(\langle A, -B \rangle\) where the A indicates a buy order for A and the \(-B\) indicates a sell order for B.

Dealer inferences over hybrid securities, A and B, are given by:

\[
\tilde{\theta} > 0: \quad \tau_{[A, \tilde{\theta}]} \left( x_H, \begin{bmatrix} B \\ \tilde{\theta} \end{bmatrix} \right) = \tau_{[A, \tilde{\theta}]} \left( y_H, \begin{bmatrix} B \\ -\tilde{\theta} \end{bmatrix} \right) = \frac{1}{2}
\]

\[
\tau_{[B, \tilde{\theta}]} \left( x_H, \begin{bmatrix} A \\ \tilde{\theta} \end{bmatrix} \right) = \tau_{[B, \tilde{\theta}]} \left( y_L, \begin{bmatrix} A \\ -\tilde{\theta} \end{bmatrix} \right) = \frac{1}{2}
\]

\[
\tilde{\theta} < 0: \quad \tau_{[A, \tilde{\theta}]} \left( x_L, \begin{bmatrix} B \\ \tilde{\theta} \end{bmatrix} \right) = \tau_{[A, \tilde{\theta}]} \left( y_L, \begin{bmatrix} B \\ -\tilde{\theta} \end{bmatrix} \right) = \frac{1}{2}
\]

\[
\tau_{[B, \tilde{\theta}]} \left( x_L, \begin{bmatrix} A \\ \tilde{\theta} \end{bmatrix} \right) = \tau_{[B, \tilde{\theta}]} \left( y_H, \begin{bmatrix} A \\ -\tilde{\theta} \end{bmatrix} \right) = \frac{1}{2}
\]

(38)

For \(\tilde{\theta} = \theta_{FB}\), these inferences are rational according to Bayes' rule and the arb's strategy in equation 37. For example, when the arb buys A, the dealer thinks with probability \(\frac{1}{2}\), \(\omega = x_H\) and
with probability $\frac{1}{2}$, $\omega = y_H$ exactly as we discussed in the previous section. Dealer inferences over pure securities, $X$ and $Y$, are given by:

$$
\tilde{\theta} > 0 : \quad \tau_{[X, \tilde{\theta}]} \left( x_H, \begin{bmatrix} X \\ \tilde{\theta} \end{bmatrix} \right) = \tau_{[Y, \tilde{\theta}]} \left( y_H, \begin{bmatrix} Y \\ \tilde{\theta} \end{bmatrix} \right) = 1 \quad (39)
$$

$$
\tilde{\theta} < 0 : \quad \tau_{[X, \tilde{\theta}]} \left( x_L, \begin{bmatrix} X \\ \tilde{\theta} \end{bmatrix} \right) = \tau_{[Y, \tilde{\theta}]} \left( y_L, \begin{bmatrix} Y \\ \tilde{\theta} \end{bmatrix} \right) = 1
$$

Of course, the question remains as to whether these inferences are reasonable for off-equilibrium trades. Intuitively, dealers see the arb as playing two possible strategies. A type $x_H$ almost always plays the equilibrium strategy, $\langle A, B \rangle$, but he also infrequently plays an off-equilibrium strategy, $\langle X, X \rangle$.

We use a version of the Cho-Kreps criterion to show that it is reasonable for $x_H$ to play $\langle X, X \rangle$ as an off-equilibrium strategy. Cho-Kreps says that a type $\omega$ may have positive probability conditional on a trade of $J$ only if it is conceivable for $\omega$ to play $J$ given all possible inferences by the dealer. In this case, there are peculiar inferences that would make the dealer price $X$ low enough so that $x_H$ would prefer $\langle X, X \rangle$ over $\langle A, B \rangle$.

The added complication in our setup is the fact that the dealer infers not only the type, $\omega$, but also the complementary trade, $J'$. If a type $x_H$ buys $X$ from dealer 1, which trades with dealer 2 are reasonable? The answer depends on how much rationality there was in the choice of that trade. The choice of $X$ with dealer 1 represents a mistake on the part of $x_H$. Was the trade with dealer 2 another mistake or was it chosen rationally given the mistake in the first trade? We discuss these issues in the appendix and present a version of Cho-Kreps in which our off-equilibrium inferences are reasonable.

The final step in proving that the breakup equilibrium exists is checking that the arb’s trading strategy in equation 37 is optimal. This procedure is not trivial because there are many possible alternative trading strategies. Not only should breakup, $\langle A, B \rangle$, dominate consolidation, $\langle X, X \rangle$, but it should also dominate $\langle A, -Y \rangle$, $\langle B, Y \rangle$, and all combinations of securities and quantities. It turns out that $\langle A, B \rangle$ does, in fact, dominate every conceivable alternative. The results on the existence and uniqueness of the equilibrium are given in the next section.
2.2.2 Results

The following theorem summarizes the results on arb's trading profits and the existence of the equilibrium.

**Theorem 1** For the two-factor structure and any monotone, concave dealer preferences:

1. The breakup equilibrium exists. All arb types achieve highest utility from trading in hybrid securities, A and B.

2. The breakup strategy is the unique noisy strategy in a symmetric equilibrium.

3. The breakup profit is always first-best: \( \Pi_I = \Pi_{FB} \)

The proof of 1 and 3 are given in the appendix.

The proof of 2 is also in the appendix. However, we should say a word here about the definition of a symmetric equilibrium. These are equilibria where types play strategies that are symmetric in two ways. First, \( x_H \) and \( x_L \) play the same strategies but with opposite sign (also for \( y_H \) and \( y_L \)). Second, \( x_H \) and \( y_H \) play mirror-image strategies (also for \( x_L \) and \( y_L \)). For example, if \( x_H \) plays \( (X, X) \), \( y_H \) will play \( (Y, Y) \). The precise definition of this symmetry is given in the appendix.

There is one additional restriction which is also discussed in the appendix.

There is one other feasible equilibrium strategy that \( x_H \) can play: \( (X, X) \), consolidation. Depending on off-equilibrium inferences, this strategy can be sustained in equilibrium. However, there is only one possible noisy strategy: \( (A, B) \), breakup. The implication is that given a particular security structure, there is only one way for arbitrageurs to break up their information. One might think that it is possible to have many alternate noisy equilibria where arbitrageurs accumulate their positions in convoluted ways. For example, the German-French arb in our initial example could use swaps instead of bonds to make inferences noisy. He could break up his information in the French-German position by taking a long position in a French-Italian swap with one dealer and a short position in an Italian-German swap with another. It turns out that these strategies do not work as an equilibrium.\(^{25}\)

\(^{25}\)We can generalize our uniqueness result to structures such as the one with the French, German, and Italian bonds.
2.3 Three-Factor Structure

In this section, we consider a different (three-factor) information and security structure. As we mentioned before, the previous structure can be seen as a special case because some of its properties do not hold for general security structures. In particular, breakup profit is not usually constant as a function of risk-aversion. It is usually strictly highest for high risk-aversion as we shall see in this section.

Consider an information and security structure similar to the one before except that there are three factors of speculation: X, Y, and Z. Arbs can have information about one of these factors. There are three pure securities in these factors. There are also three hybrid securities: \( A = \frac{1}{2}(X + Y - Z) \), \( B = \frac{1}{2}(X - Y + Z) \), and \( C = \frac{1}{2}(-X + Y + Z) \). An arb can construct a trade in a pure factor with any two hybrids: \( X = A + B \), \( Y = A + C \), and \( Z = B + C \). This structure represents a case where an arbitrageur’s strategy involves two of three securities, but it is unknown which two. For example, the arb in the introduction could have been exploiting the relative-value between any two of three bonds: a German bond, a French bond, and an Italian bond.\(^{26}\)

The model is almost the same as before. There are now six types of arbs: \( \omega \in \{x_H, x_L, y_H, y_L, z_H, z_L\} \). The arb trades with two dealers at stage-1. There are three identical supply curves at stage-2: \( p_x = b\theta_x \), \( p_y = b\theta_y \), and \( p_z = b\theta_z \). The one essential difference between this model and the previous one is that dealer preferences are explicitly parameterized. Unlike in the previous section, the breakup profit varies with risk-aversion. So we explicitly parameterize risk-aversion and derive profit as a function of the parameter.

For analytic convenience, we use a proxy for risk-aversion. Specifically, there is a quadratic cost of bearing inventories in any factor with a noisy outcome. For example, suppose the trade \([A, \tilde{\theta}]\) gives noise about the arb’s type. After this trade, the dealer maximizes the following objective at stage-2:

\[
\max_{\tilde{\theta}} E \left[ \pi(\tilde{\theta}) | A, \tilde{\theta} \right] - \alpha \left[ \left( \theta_x - \frac{1}{2} \tilde{\theta} \right)^2 + \left( \theta_y - \frac{1}{2} \tilde{\theta} \right)^2 + \left( \theta_z + \frac{1}{2} \tilde{\theta} \right)^2 \right]
\]

(40)

where the \( \tilde{\theta} = (\theta_x, \theta_y, \theta_z) \) are demands submitted at stage-2, and \( \pi(\tilde{\theta}) \) is the profit of the arb’s position after all trade. The amount of risk-aversion is captured by the parameter \( \alpha \).\(^{27}\) If a trade

---

\(^{26}\) This analogy is not precise because we have \( X = A + B \) and not \( X = A - B \), for example. In the latter case, the speculative factors \( X, Y, \) and \( Z \) would be redundant (e.g., \( X = Y + Z \)), whereas they are not in the case above. However, we can extend our analysis to this case as we do in section ??.

\(^{27}\) The dealer is penalized for holding every factor exposure equally regardless of the "amount" of noise in its outcome.
fully reveals the arb’s type, there is no cost of inventory in the corresponding factor.

The breakup equilibrium has the same form as before: arbs of all types trade only in hybrid securities. A type of $\omega = x_H$ would trade $\langle A, B \rangle$, $y_H$ would trade $\langle A, C \rangle$, and $z_H$ would trade $\langle B, C \rangle$. The “low” types would execute strategies of the opposite sign.

Inferences are rational given this trading strategy and Bayes’ rule. For example, a dealer observes the arb buying $A$, would infer the state to be $\omega = x_H$ or $y_H$, each with probability $\frac{1}{2}$. In other words, for security $A$ and $\bar{\theta} > 0$:

$$
\tau_{[A, \bar{\theta}]} \left( x_H, \begin{bmatrix} B \\ \bar{\theta} \end{bmatrix} \right) = \tau_{[A, \bar{\theta}]} \left( y_H, \begin{bmatrix} C \\ \bar{\theta} \end{bmatrix} \right) = \frac{1}{2}
$$

(41)

Off-equilibrium inferences are the same as before except that we now have security $Z$. For $\bar{\theta} > 0$, for example:

$$
\tau_{[Z, \bar{\theta}]} \left( z_H, \begin{bmatrix} Z \\ \bar{\theta} \end{bmatrix} \right) = 1
$$

(42)

Off-equilibrium inferences are once again reasonable according to the Cho-Kreps criterion. Our results are given in the next section.

2.3.1 Results

The following theorem summarizes results for the three-factor structure.

**Theorem 2** For the three-factor structure and quadratic cost-of-inventory preferences for dealers:

1. The breakup equilibrium exists.

2. The breakup strategy is the unique noisy strategy in a symmetric equilibrium.

3. The breakup profit is strictly increasing in risk-aversion: $\frac{\partial \Pi_I(\alpha)}{\partial \alpha} > 0$. It is first-best in the limit of infinite risk-aversion: $\Pi_I(\infty) = \Pi_{FB}$.

The only difference between this and the previous theorem is result 3. Here, the breakup profit starts below first-best for zero risk-aversion and increases, approaching the first-best level as risk-aversion approaches infinity. The graph of breakup profit as a function of risk-aversion is given in figure 4.

We want to show why profits in the risk-neutral case are less than first-best. Again, without loss of generality, we take the case of $\omega = x_H$ where the arb will trade $\langle A, B \rangle$. It can be shown
that with these inferences, the equilibrium at stage-2 has the A dealer submitting a demand of 
\((\frac{2\gamma}{95}, \frac{2\gamma}{95}, -\frac{\gamma}{95})\) (in the XYZ basis) to the centralized market.\(^{28}\) Similarly, the B dealer will submit: 
\((\frac{2\gamma}{95}, -\frac{\gamma}{95}, \frac{2\gamma}{95})\).

When we sum the three demands, we get: \((\frac{4\gamma}{95}, \frac{1\gamma}{95}, \frac{1\gamma}{95})\). We notice that there are net positive demands for Y and Z, which have expected value, 0. The dealers take a loss in their trades in Y and Z. Since their expected profit is fully transferred to the arb through competition, the arb earns less than first-best profit.

We contrast this with the two-factor structure. In the risk-neutral case, when the arb has information \(\omega = \omega_H\), the dealer's demands sum to: \((\frac{5\gamma}{95}, 0)\), the first-best outcome. There is zero demand for factor Y. The two-factor structure has the special property that there are no net demands in irrelevant factors. The arb's cumulative demand (the "input") for factor X is unchanged when it goes into the centralized market (the "output"). The special property of the two-factor structure is the orthogonality of A and B when written in the basis of X and Y factors. Like the three-factor structure, most structures do not have this property. For low levels of risk-aversion, dealers submit net demands for irrelevant factors, causing a loss.

As risk-aversion approaches infinity, the noise in A, B, and C causes the dealer act like a pass-through. Dealers no longer submit net demands for irrelevant factors because they simply pass through the arb's order. As in the two-factor case, the arb earns first-best profits in this regime.

With a generic structure, there are net demands for irrelevant factors. Profits are less than first-best for low risk-aversion and approach first-best for high risk-aversion. Hence, the profit from breakup is, in general, strictly highest for high risk-aversion (the pass-through region).

There is one additional implication of this analysis. When X has a high expected value, both Y and Z are temporarily pushed up in price at stage-1. Fundamentally, X, Y, and Z are completely uncorrelated. However, because breakup creates net demands in irrelevant factors, it induces endogenous or excess correlations in securities prices. In our initial example, one should expect to see excess comovement in the spreads between different European-sovereign bonds. When an arbitrageur takes a position in the French-German spread, dealers will attempt to accumulate positions in this spread as well as the other sovereign spreads because of imperfect information. One should expect to see excess correlations when dealers have low-risk aversion. In this regime,

\(^{28}\)The A dealer submits a net demand for factor Z even though he knows that the arb has no information about Z. The reason is that he knows that in any state of the world, the complementary dealer will submit a demand of \(\frac{2\gamma}{95}\) for Z (by considerations of symmetry). Hence, Z is overpriced relative to its expected value, 0.
dealers trade aggressively in unrelated securities in spite of noisy inferences.

2.4 Conclusion

In this chapter, we developed a model for how information can be broken up through trade on different securities. In our framework, certain securities were perceived as noisy and others were not. Securities were noisy because types with different information mimicked one another by trading exclusively in the set of noisy securities. Hence, this chapter introduces a concept of “endogenous” noise on different securities.

We illustrated this kind of equilibrium with two simple security structures. We showed that the proposed equilibrium does, in fact, exist and has certain uniqueness properties as well. Uniqueness implies that given an exogenous security structure, it is well-determined exactly how arbitrageurs will trade to hide their information. This result is somewhat surprising given the huge number of ways that an arb could trade to confound their counterparties’ inferences.

In section 2.3.1, we presented the main empirical predictions of the model. First, the profit from breakup is generally highest when counterparties have high risk-aversion. If we believe that institutions fear financial distress, we can draw a rough correspondence between high risk-aversion and high leverage-ratio. An intermediary with high leverage and tight risk-management policy would, therefore, be the most likely counterparty in a broken-up trade. Since the lack of netting in such a trade can be unstable, these arrangements with highly leveraged counterparties may be highly unstable.

A corollary of this result is that there may be correlations between factors in excess of their fundamental correlations. These factors represent the most likely positions that a hedge fund could be holding. For example, suppose that a hedge fund is actually holding a position in a certain sovereign bond spread and that dealers individually can not discern exactly which spread. The true spread will then have excess comovements with other sovereign spreads because of this imperfect inference.

The implications for systemic risk and stability have yet to be explored and are left for future research. For example, whereas consolidation results in all counterparties having identical claims, breakup results in diverse claims which may be difficult to renegotiate in a distress scenario. The symmetry in the information and security structures may make it easier or harder for one counterparty or another to make concessions in this process. Finally, one could explore the incentives for financial innovation in this market. Typically markets become less noisy when new securities are introduced. In this market, new securities actually make the world more noisy because these
securities are traded in segmented markets. Even when there is a complete tableau of securities, there are incentives to introduce new securities because arbitrageurs desire this noise. It may be interesting to study innovation in this context.
3 Appendix

Utility Conditions:

We adopt the following notation:

\[
\begin{align*}
    u_+(x) &= u(W + q + x) \\
    u_-(x) &= u(W + x) \\
    u_\pm(x) &= u_+(x) + u_-(x)
\end{align*}
\] (43)

The following technical conditions must hold for the local derivatives of \( u \):

\[
1 + k = \frac{\frac{u''_-(0)}{u'_-(0)}}{\frac{u''_+(0)}{u'_+(0)}} < \frac{\frac{u''_-(0)}{u'_-(0)}}{\frac{u''_+(0)}{u'_+(0)}} < \frac{\frac{u''_+(0)}{u'_+(0)}}{\frac{u''_-(0)}{u'_-(0)}} \] (44)

One can show that these conditions are true for power utility.

3.1 Benchmark Equilibrium: No Front-running

We define \( R_0^e, R_1^e, \) and \( R_N^e \) as the competitive interest-rates for the corresponding type. Explicitly:

\[
\begin{align*}
    U_x(\min\{1 - V' + 2V \cdot \bar{Y}, R_0^e\} - (1 - \Delta)) &= U_x(0) \\
    U_x(\min\{1 - V' + 2V \cdot \bar{X}, R_1^e\} - (1 - \Delta)) &= U_x(0) \\
    \frac{1}{2}U_x(\min\{1 - V' + 2V \cdot \bar{X}, R_N^e\} - (1 - \Delta)) &= U_x(0) \\
    \frac{1}{2}U_x(\min\{1 - V' + 2V \cdot \bar{Y}, R_N^e\} - (1 - \Delta)) &= U_x(0)
\end{align*}
\] (45)

where \( U_x(\bar{w}) = E[u(W + q\bar{X} + \bar{w})] \).

Proof of Proposition 1:

To first-order: \( U_x(a + b\bar{X}) = U_x(0) + \frac{1}{2}u'_+(0) \cdot (a + b) + \frac{1}{2}u'_+(0) \cdot (1 + k)a \) and \( U_x(a + b\bar{Y}) = U_x(0) + \frac{1}{2}u'_+(0) \cdot (a + b) + \frac{1}{2}u'_+(0) \cdot a \). Hence, \( R_0^e, R_1^e, \) and \( R_N^e \) are given as in equation 7 to first-order.

Since \( R_0^s < R_N^s < R_1^s \), it is clear that no bank can improve its utility by offering an interest-rate for correlated disclosure or non-disclosure that is less than or equal to \( R_0^s \). Also, no bank can improve its utility by offering less than \( R_0^s \) for uncorrelated disclosure. Hence, our equilibrium exists.

Our interest-rate equilibrium is unique in the sense that the interest-rate for uncorrelated disclosure is unique and that this rate is always the minimum rate (i.e., the investor will always choose it). Suppose there is a different equilibrium where the interest-rates are given by \( R_s \) for \( s \in \{0, 1, N\} \).
We first show that $\min_s R_s \leq R_0$. Suppose instead that $\min_s R_s > R_0$. A bank could then offer an uncorrelated interest-rate of $R$ such that $R_0 < R < \min_s R_s$. It is obviously incentive-compatible for the investor to take this offer and individually rational is $R - R_0$ is small enough. The bank would profit since $U_x(\min\{1 - V' + 2V \cdot \bar{Y}, R\} - (1 - \Delta)) > U_x(0)$ for $R > R_0$. Hence, $\min_s R_s \leq R_0$.\footnote{If the number of dealers is large enough, the bank earns a profit that is at most infinitesimally small from offering the same interest-rate as all other banks of the same type. Hence, this deviation improves utility.}

The investor does not choose correlated disclosure in equilibrium because the bank would suffer a loss. The investor would only choose correlated disclosure if $R_1 = \min_s R_s \leq R_0$. But since $R_1 > R_0$, $U_x(\min\{1 - V' + 2V \cdot \bar{X}, R_1\} - (1 - \Delta)) < U_x(0)$. Hence, this is not individually rational.

Finally, we can show that the investor does not choose non-disclosure in equilibrium. Assume the contrary. If the investor chooses non-disclosure in equilibrium, the bank must infer that the investor is correlated with probability $\frac{1}{2}$ and uncorrelated with probability $\frac{1}{2}$. The reason is that we restrict to symmetric equilibria where $x$ and $y$ types play the same strategy. Also, we assumed that the investor faces a vanishingly small i.i.d. cost for contracting with each bank and that this cost is unrelated to the bank’s correlation. Hence, $R_N = \min_s R_s \leq R_0$ is not individually rational for any bank.

### 3.2 Mandatory Disclosure Equilibrium

We first discuss the auction equilibrium in a hypothetical case where the investor has sufficient capital to buy the asset and does not take a loan. Suppose that a competing bank of correlation $\rho$ observes that $\bar{Y}_2 = V$ (otherwise, the bank does not bid). This bank’s “no-loan” reserve price is denoted by $\bar{p}_\rho$. $\bar{p}_1$ is defined by:

$$U_x(2V \cdot \bar{X} - \bar{p}_1) = U_x(0)$$

(46)

$\bar{p}_0$ is defined similarly with $\bar{Y}$ replacing $\bar{X}$. In this auction, the investor wins by bidding $p_\rho^* = \max\{\bar{p}_\rho, V'\}$ (since the investor is risk-neutral and has the highest reserve price). Now suppose there is a loan of face value, $R$, between the bank and the investor. This loan changes how much each party values the asset. The investor’s reserve price, $\bar{p}_i(R)$ is given by:

$$E[\max\{1 + 2V \cdot \bar{X} - \bar{p}_i(R) - R, 0\}] = \Delta$$

(47)

The correlated bank’s reserve price $\bar{p}_1(R)$ is given by:
\[ U_x(\min\{1 + 2V \cdot \tilde{X} - \tilde{p}_1(R), R\} - 1 + \Delta) = U_x(2V \cdot \tilde{X} - \tilde{p}_1(R)) \] (48)

since the bank compares winning the auction and owning the asset directly to losing and allowing the investor to take the loan. Again, \( \bar{p}_0(R) \) is defined similarly with \( \tilde{Y} \) replacing \( \tilde{X} \). It is easy to show that \( \bar{p}_p(R) \) is non-increasing in \( R \) since increasing \( R \) increases the value of the loan, which increases the needed return on the asset.

If \( \bar{p}_i(R) \geq \max\{\bar{p}_p(R), V\} \), the investor will win by bidding: \( p^*_\rho(R) = \max\{\bar{p}_p(R), V\} \).

Also our conditions in equation 44 assure that \( -\bar{p}'_\rho(R) < 1 \) for \( V \) small enough as we prove at the end of this section.

**Proof of Proposition 2:**

We define \( R^*_p \) and \( R^*_0 \) as:

\[
\begin{align*}
U_x(\min\{1 + 2V \cdot \tilde{X} - p^*_1, R^*_1\} - 1 + \Delta) &= U_x(0) \\
U_x(\min\{1 + 2V \cdot \tilde{Y} - p^*_0, R^*_0\} - 1 + \Delta) &= U_x(0)
\end{align*}
\] (49)

where \( p^*_1 \) and \( p^*_0 \) are the equilibrium prices in the no-loan auctions. It is immediate that if \( \bar{p}_i(R^*_p) \geq p^*_p \), the equilibrium auction prices are \( p^*_\rho(R^*_p) = p^*_p \) for our conjectured interest-rates as in equation 8. If \( \bar{p}_i(R^*_p) < p^*_p \), the investor simply does not take this offer.

First, we prove the existence of this equilibrium. Suppose that a bank deviates by offering \( R > R^*_p \) for \( \rho \) disclosure. \( p^*_\rho(R) + R > p^*_\rho(R^*_p) + R^*_p \) since \( -\bar{p}'_\rho(R) < 1 \). Hence, the investor will not take this offered interest-rate so that this deviation is not profitable.

If a bank instead offers \( R < R^*_p \), this increases \( p^*_\rho(R) \), which drives the bank's utility below zero. For example, with correlated disclosure and \( R < R^*_1 \):

\[
U_x(\min\{1 + 2V \cdot \tilde{X} - p^*_1(R), R\} - 1 + \Delta) < U_x(\min\{1 + 2V \cdot \tilde{X} - p^*_1, R^*_1\} - 1 + \Delta) = U_x(0)
\] (50)

Hence, this deviation is unprofitable, and our equilibrium exists.

We now prove uniqueness. We denote by \( \rho^* \), the preferred bank correlation in our proposed equilibrium, i.e., \( \rho^* = \arg\min_{\rho\in[0,1]} [p^*_\rho + R^*_\rho] \). Suppose we can conjecture an alternate equilibrium where the interest-rates are \( R_p = [R_0, R_1] \). We want to prove that \( R_{\rho^*} = R^*_\rho \) and that the investor chooses \( \rho^* \) in this alternate equilibrium.

We first show that \( \min_\rho[p^*_\rho(R_p) + R_p] \leq p^*_\rho + R^*_\rho \). Suppose this is not true and that \( \min_\rho[p^*_\rho(R_p) + R_p] > p^*_\rho + R^*_\rho \). A bank can then offer a rate, \( R > R^*_\rho \), for \( \rho^* \) disclosure such that \( p^*_\rho(R) + \)
\( R < \min_{\rho} [p_{\rho}^*(R_\rho) + R_\rho] \). Again, it is incentive-compatible for the investor to take this offer and individually-rational for \( R - R_{\rho}^* \) small enough. Since \( R > R_{\rho}^* \) and \( p_{\rho}^*(R) \leq p_{\rho}^* \), the bank obviously profits from this deviation. For example, suppose \( \rho^* = 1 \):

\[
U_x(\min\{1 + 2V \cdot \tilde{X} - p_{\rho}^*(R), R\} - 1 + \Delta) > U_x(\min\{1 + 2V \cdot \tilde{X} - p_{\rho}^*(R_1), R_1\} - 1 + \Delta) = U_x(0)
\]

for \( R > R_1^* \). Hence, this deviation is profitable, and consequently, \( \min_{\rho} [p_{\rho}^*(R_\rho) + R_\rho] \leq p_{\rho}^* + R_{\rho}^* \) in any equilibrium.

Finally, \( R_\rho \geq R_{\rho}^* \) in any trembling-hand perfect equilibrium. The reason is that any lower interest-rate results in negative utility for the bank if the investor takes the offer (and zero utility in any other case). Hence, if \( \rho \) is an inferior choice in our equilibrium in proposition 2 (i.e., \( p_{\rho}^* + R_{\rho}^* > p_{\rho}^* + R_{\rho}^* \)), it is an inferior choice in any equilibrium since \( p_{\rho}^*(R_\rho) + R_\rho \geq p_{\rho}^* + R_{\rho}^* \geq R_{\rho}^* \geq \min_{\rho} [p_{\rho}^*(R_\rho) + R_\rho] \). QED.

**Proof that \(-\tilde{\rho}_0(R) < 1\):**

In this proof, we assume that \( 1 + 2V - \tilde{\rho}_0(R) \geq R \geq 1 - \tilde{\rho}_0(R) \). It is not individually rational for an investor to take an offer of \( R > 1 + 2V - \tilde{\rho}_0(R) \geq 1 + 2V - \tilde{\rho}_0^*(R) \) so that the first condition must hold. Also, \( R < 1 - \tilde{\rho}_0(R) \) implies that \( R < R_0^* \) since \( R_0^* \geq 1 - p_0^* = \tilde{\rho}_0(R_0^*) \). Since an offer of \( R < R_0^* \) is not individually rational for a bank, the second inequality must hold. \( \tilde{\rho}_0(R) \) is, therefore, defined by:

\[
[u_\pm (R - 1 + \Delta) - u_\pm (2V - \tilde{\rho}_0)] + [u_\pm (-\tilde{\rho}_0 + \Delta) - u_\pm (-\tilde{\rho}_0)] = 0
\]

We want to prove that the second inequality in equation 44 implies that \(-\tilde{\rho}_0^*(R) < 1\).

We first expand equation 52 to first-order in \( V \):

\[
R - 1 + \Delta - 2V + \tilde{\rho}_0 + \frac{u_\pm'(-\tilde{\rho}_0)}{u_\pm'(2V - \tilde{\rho}_0)} \Delta + O(V^2) = R - 1 + \Delta - 2V + \tilde{\rho}_0 + \Delta + O(V^2) = 0
\]

since \( \frac{u_\pm'(-\tilde{\rho}_0)}{u_\pm'(2V - \tilde{\rho}_0)} = 1 + O(V) \). Expanding to second-order gives:

\[
R - 1 + \Delta - 2V + \tilde{\rho}_0 + \frac{u_\pm'(2V - \tilde{\rho}_0)}{u_\pm'(2V - \tilde{\rho}_0)} (R - 1 + \Delta - 2V + \tilde{\rho}_0)^2 + \frac{u_\pm'(-\tilde{\rho}_0)}{u_\pm'(2V - \tilde{\rho}_0)} \Delta + \frac{1}{2} \frac{u_\pm'(-\tilde{\rho}_0)}{u_\pm'(2V - \tilde{\rho}_0)} \Delta^2 + O(V^3) = 0
\]

Differentiating equation 52 with respect to \( R \) gives the following expression for \(-\tilde{\rho}_0^*(R)\):

\[
50
\]
\[ -\bar{p}_0'(R) = \frac{u'_\pm(R - 1 + \Delta)}{u_\pm(2V - \bar{p}_0) - [u'_\pm(-\bar{p}_0 + \Delta) - u'_\pm(-\bar{p}_0)]} \] (55)

Hence, a sufficient condition for \(-\bar{p}_0'(R) < 1\) is:

\[ [u'_\pm(R - 1 + \Delta) - u'_\pm(2V - \bar{p}_0)] + [u'_\pm(-\bar{p}_0 + \Delta) - u'_\pm(-\bar{p}_0)] < 0 \] (56)

We take a second-order expansion of this condition:

\[ R - 1 + \Delta - 2V + \bar{p}_0 + \frac{1}{2} \left[ u''_{\pm}(2V - \bar{p}_0) \right] (R - 1 + \Delta - 2V + \bar{p}_0)^2 + \frac{1}{2} \left[ u''_{\pm}(-\bar{p}_0) \right] \Delta + \frac{1}{2} \left[ u''_{\pm}(2V - \bar{p}_0) \right] \Delta^2 + \mathcal{O}(V^3) > 0 \] (57)

Substituting the expansions for \(R - 1 + \Delta - 2V + \bar{p}_0\) in equations 54 and 53 gives:

\[
\begin{align*}
\frac{1}{2} \left[ u''_{\pm}(2V - \bar{p}_0) - u''_{\pm}(2V - \bar{p}_0) \right] (R - 1 + \Delta - 2V + \bar{p}_0)^2 + \\
\left[ \frac{u'_{\pm}(-\bar{p}_0)}{u'_{\pm}(2V - \bar{p}_0)} - \frac{u'_{\pm}(-\bar{p}_0)}{u'_{\pm}(2V - \bar{p}_0)} \right] \Delta + \frac{1}{2} \left[ \frac{u''_{\pm}(-\bar{p}_0)}{u'_{\pm}(2V - \bar{p}_0)} - \frac{u''_{\pm}(-\bar{p}_0)}{u'_{\pm}(2V - \bar{p}_0)} \right] \Delta^2 + \mathcal{O}(V^3)
\end{align*}
\] (58)

where \(\Gamma_0 = \frac{u'_{\pm}(0)}{u'_{\pm}(0)} - \frac{u'_{\pm}(0)}{u'_{\pm}(0)}\). Expanding \(\left[ \frac{u'_{\pm}(-\bar{p}_0)}{u'_{\pm}(2V - \bar{p}_0)} - \frac{u'_{\pm}(-\bar{p}_0)}{u'_{\pm}(2V - \bar{p}_0)} \right] \) gives:

\[
\begin{align*}
\frac{u'_{\pm}(-\bar{p}_0)}{u'_{\pm}(2V - \bar{p}_0)} - \frac{u'_{\pm}(-\bar{p}_0)}{u'_{\pm}(2V - \bar{p}_0)} &= \frac{1 - \frac{u''_{\pm}(0)}{u'_{\pm}(0)} \bar{p}_0}{1 + \frac{u''_{\pm}(0)}{u'_{\pm}(0)} (2V - \bar{p}_0)} - \frac{1 - \frac{u''_{\pm}(0)}{u'_{\pm}(0)} \bar{p}_0}{1 + \frac{u''_{\pm}(0)}{u'_{\pm}(0)} (2V - \bar{p}_0)} + \mathcal{O}(V^2) = \\
&= -2\Gamma_0 V + \mathcal{O}(V^2)
\end{align*}
\] (59)

Substituting into equation 58 gives the sufficient condition:

\[ \Gamma_0 \Delta - 2V + \mathcal{O}(V^3) > 0 \] (60)

Since \(\Delta - 2V < 0\), a sufficient condition for \(-\bar{p}_0'(R) < 1\) is that \(\Gamma_0 < 0\) for \(V\) small enough.

QED.

**Proof that \(-\bar{p}_1'(R) < 1\):**

Again, we assume that \(1 + 2V - \bar{p}_1(R) \geq R \geq 1 - \bar{p}_1(R)\). The first inequality holds by our previous argument. If the second does not hold, one can show that \(-\bar{p}_1'(R) = (1 + \frac{1}{2}k)^{-1} < 1\). \(\bar{p}_1(R)\) is, therefore, defined by:

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\[ [u_+(R - 1 + \Delta) - u_+(2V - \tilde{p}_1)] + [u_-(\tilde{p}_1 + \Delta) - u_-(\tilde{p}_1)] = 0 \] (61)

We want to prove that the first inequality in equation 44 implies that \(-\tilde{p}_1'(R) < 1\).

We expand equation 61 to first-order in \(V\):

\[
R - 1 + \Delta - 2V + \tilde{p}_1 + \frac{u_1'(-\tilde{p}_1)}{u_1'(2V - \tilde{p}_1)} \Delta + O(V^2) = 0
\]
\[ R - 1 + \Delta - 2V + \tilde{p}_1 + \frac{u_1'(-\tilde{p}_1)}{u_1'(0)} \Delta + O(V^2) = 0 \] (62)

since \(\frac{u_1'(-\tilde{p}_1)}{u_1'(2V - \tilde{p}_1)} = \frac{u_1'(0)}{u_1'(0)} + O(V)\).

Differentiating equation 61 with respect to \(R\) gives the following expression for \(-\tilde{p}_1'(R)\):

\[
-\tilde{p}_1'(R) = \frac{u_1'(R - 1 + \Delta)}{u_1'(2V - \tilde{p}_1) - [u_1'(-\tilde{p}_1 + \Delta) - u_1'(-\tilde{p}_1)]}
\] (63)

Hence, a sufficient condition for \(-\tilde{p}_1'(R) < 1\) is:

\[
[u_1'(R - 1 + \Delta) - u_1'(2V - \tilde{p}_1)] + [u_1'(-\tilde{p}_1 + \Delta) - u_1'(-\tilde{p}_1)] < 0
\] (64)

Expanding this condition to first-order gives:

\[
R - 1 + \Delta - 2V + \tilde{p}_1 + \frac{u_1'(-\tilde{p}_1)}{u_1'(2V - \tilde{p}_1)} \Delta + O(V^2) -
\[ R - 1 + \Delta - 2V + \tilde{p}_1 + \frac{u_1'(0)}{u_1'(0)} \Delta + O(V^2) > 0 \] (65)

Substituting the expansion for \(R - 1 + \Delta - 2V + \tilde{p}_1\) in equation 62 gives:

\[
\Gamma_1 \cdot \Delta + O(V^2) > 0
\] (66)

where \(\Gamma_1 = \frac{u_1''(0)}{u_1'(0)} - \frac{u_1'(0)}{u_1'(0)}\). Hence, \(\Gamma_1 > 0\) is a sufficient condition for \(-\tilde{p}_1'(R) < 1\) for \(V\) small enough. QED.

### 3.3 Voluntary Equilibrium

**Proof of Proposition 3:**

We have argued that uninformed banks (i.e., banks not observing \(\bar{V}_j\)) do not bid in the auction. We prove formally at the end of this section that the lending bank never bids in the auction when the investor does not disclose since any such equilibrium is not individually rational. Hence, \(p^*_N(R_N) = V'\) when the investor chooses non-disclosure.
We first prove the existence of our interest-rate equilibrium. There is no profitable deviation from the interest-rates for disclosure, \( R_0^* \) and \( R_1^* \) (in equation 9) by the arguments in the previous section. There is also obviously no incentive for a bank to offer a non-disclosure interest-rate of \( R > R_N^* \) since the investor will not take this offer. A bank will also not offer \( R < R_N^* \) since this can not result in positive utility. If the investor takes this offer, the bank must infer that the investor is of type \( x \) and type \( y \) with probability \( \frac{1}{2} \) each by arguments from the previous section. Hence, the bank can only realize non-positive utility with such an offer.

We again prove uniqueness. We denote by \( s^* \), the investor’s choice in our proposed equilibrium where \( s^* = \arg \min_{s \in \{0,1,N\}} [p_s^*(R_0^*) + R_s^*] \). Suppose we conjecture an alternate equilibrium where the interest-rates are \( R_s = [R_0, R_1, R_N] \). We again want to prove that \( R_{s^*} = R_{s^*}^* \) and that the investor chooses \( s^* \) in this alternate equilibrium.

We again show that \( \min_s [p_s^*(R_s) + R_s^*] \leq p_{s^*}^* + R_{s^*}^* \). Assume the contrary: \( \min_s [p_s^*(R_s) + R_s^*] > p_{s^*}^* + R_{s^*}^* \). A bank can offer a rate, \( R > R_{s^*}^* \), for choice \( s^* \) such that \( p_{s^*}^*(R) + R < \min_s [p_s^*(R_s) + R_s] \). This offer is incentive-compatible for the investor and profitable for the bank by our arguments from the previous section. Hence, \( \min_s [p_s^*(R_s) + R_s] \leq p_{s^*}^* + R_{s^*}^* \).

Again, \( R_0 \geq R_0^* \) and \( R_1 \geq R_1^* \) in any trembling-hand equilibrium. Also, if \( N \) is chosen in this alternate equilibrium (i.e., \( N = \arg \min_s [p_s^*(R_s) + R_s] \)), it is easy to see that \( R_N \geq R_N^* \) since \( R_N^* \) is the competitive non-disclosure interest-rate when the probability of type \( x \) and type \( y \) is \( \frac{1}{2} \) each.

If \( \rho \) disclosure is an inferior choice in our equilibrium in proposition 3, it as an inferior choice in any equilibrium since: \( p_\rho^*(R_\rho) + R_\rho \geq p_s^* + R_s^* \geq \min_s [p_s^*(R_s) + R_s] \). If \( N \) is an inferior choice in our equilibrium in proposition 3, it also can not be chosen any equilibrium. Assume the contrary. This implies that \( V' + R_N = \min_s [p_s^*(R_s) + R_s] \leq p_{s^*}^* + R_{s^*}^* < V' + R_N^* \) which implies that \( R_N \leq R_N^* \) which is a contradiction.

**Proof that** \( p_N^*(R_N) = V' \):

In this proof, we will use the fact that if \( E[\alpha + \beta_x \tilde{X} + \beta_y \tilde{Y}] \leq 0 \) and \( \beta_x \geq 0, U_x(\alpha + \beta_x \tilde{X} + \beta_y \tilde{Y}) < U_x(0) \). In other words, negative profit investments result in negative utility for banks, a straightforward consequence of risk-aversion.

We want to show that it is not individually rational for an uninformed bank to bid more than \( V' \) in either auction. Assume the contrary, i.e., the bank bids \( p_b > V' \).

If \( \tilde{V}_j = 0 \), the bank wins the auction since no informed agents bid in this state.

If \( \tilde{V}_j = V \), the bank either wins the auction or takes a claim on the asset through the investor.

The total profit from these trades is \( E[2\tilde{V}_j \tilde{J} - p_b] = \frac{V}{2} - p_b < \frac{V}{2} - V' < 0 \). The investor’s profit
is $\Pi_i \geq 0$ from individual rationality. Hence, the bank's profit is $E[2\tilde{V}_j \tilde{J} - p_b] - \Pi_i < 0$. It is clear that the bank's claim has $\beta_j \geq 0$ in all states.

The bank's utility from this trade is less than zero, therefore, from the fact above. It is clear then that when $R_N$ is individually rational so that $\frac{1}{2}U(\min\{1 - V' + 2V \cdot \tilde{X}, R_N\} - (1 - \Delta)) + \frac{1}{2}U(\min\{1 - V' + 2V \cdot \tilde{Y}, R_N\} - (1 - \Delta)) \geq U(0)$, that the bank does not bid more that $V'$ in any auction since doing otherwise gives negative utility.

3.4 Regulations and Welfare

3.4.1 Disclosure Regulation

Proof of Proposition 4:

The investor and counterparty's profits are given by:

$$
\Pi_i = \frac{1}{2}[1 + 2V - p^* - R^*] - \Delta = V - p^* - \frac{\kappa}{2}(p^* - \Delta) \\
\Pi_{cp} = p^* - V'
$$

(67)

when $\tilde{V}_j = V$ (profits are zero when $\tilde{V}_j = 0$) where $\kappa$ corresponds to the costliness of the risk to the lender where $\kappa = k$ for correlated disclosure, 0 for uncorrelated disclosure, and $\frac{k}{2}$ for non-disclosure. Hence, $\Pi_{soc}(\omega = \frac{1}{2}) = \frac{1}{2}\Pi_i + \frac{1}{2}\Pi_{cp} = V - V' - \frac{\kappa}{2}(p^* - \Delta)$

For non-disclosure:

$$
\Pi_{soc}(\omega = \frac{1}{2}) = V - V' - \frac{k}{4}(V' - \Delta) \\
$$

(68)

For uncorrelated disclosure:

$$
\Pi_{soc}(\omega = \frac{1}{2}) = V - V' > V - V' - \frac{k}{4}(V' - \Delta) \\
$$

(69)

For correlated disclosure:

$$
\Pi_{soc}(\omega = \frac{1}{2}) = V - V' - \frac{k}{2}(p^*_1 - \Delta) < V - V' - \frac{k}{4}(V' - \Delta) \\
$$

(70)

since $p^*_1 \geq V'$.

Hence, if $V' < V'_c$ mandating disclosure causes the investor to switch from non-disclosure to correlated disclosure which is strictly suboptimal (for $\omega = \frac{1}{2}$). If $V' > V'_c$ mandating disclosure is weakly optimal because it either causes the investor to switch from non-disclosure to uncorrelated disclosure (when $V'_d > V' > V'_c$) or has no effect (when $V' > V'_d$).
We now want to show that mandating disclosure is pareto-suboptimal only when the investor prefers correlated disclosure to uncorrelated disclosure. Again, mandating disclosure obviously has no effect when \( V' > V'_d \). Mandating disclosure is not pareto-suboptimal when \( V'_d > V' > V'_c \) since \( \Pi_{cp} = p^*_N - V' = 0 \) in the voluntary equilibrium and \( \Pi_{cp} = p^*_0 - V' > 0 \) in the mandatory disclosure equilibrium. Mandating disclosure is pareto-suboptimal when \( V'_c > V' > \bar{p}_1 \) since \( \Pi_i = V - V' - \frac{k}{4}(V' - \Delta) \) in the voluntary equilibrium and \( \Pi_i = V - V' - \frac{k}{2}(V' - \Delta) \) in the mandatory disclosure equilibrium. Also, \( \Pi_{cp} = 0 \) in both equilibria so that the investor suffers from mandatory disclosure while the counterparty is unaffected.

**Proof of Proposition 5:**

When \( V' < V'_d \), mandating non-disclosure obviously has no effect. When \( V' > V'_d \), there is a pareto loss since \( \Pi_{cp} = 0 \) for non-disclosure and \( \Pi_{cp} = p^*_0 - V' = V - V' > 0 \) in the voluntary equilibrium. Also, the investor’s profit decreases because her profit from uncorrelated disclosure is greater than that from non-disclosure.

#### 3.4.2 Capital Regulation

The investor’s profit (gross of \( C(\Delta) \)) in the voluntary equilibrium is, therefore, given by:

\[
\Pi(\Delta) = \max\{0, V - V' - \frac{k}{4}(V' - \Delta)\}
\]  \hspace{1cm} (71)

The counterparty’s profit is given by:

\[
\Pi_{cp} = 1(\Delta < \Delta_d)(V - V')
\]  \hspace{1cm} (72)

where \( 1(\cdot) \) is an indicator function and \( \Delta_d = V' - \frac{k}{2}(V - V') \) is the disclosure threshold for \( \Delta \), below which the investor discloses and above which the investor withholds.

Suppose a regulator imposes a minimum capital requirement of \( \Delta_{min} \). The investor will then choose an optimal \( \Delta^*(\Delta_{min}) \) to maximize the following objective:

\[
\max_{\Delta} [\Pi(\Delta) - C(\Delta)] \text{ s.t. } \Delta > \Delta_{min}
\]  \hspace{1cm} (73)

It is now clear that imposing \( \Delta_{min} > 0 \) will always decrease both the investor’s and the counterparty’s profit. Imposing \( \Delta_{min} > 0 \) can only increase \( \Delta^*(\Delta_{min}) \) over \( \Delta^*(0) \) by inspection so that the profit of the counterparty in equation 72 decreases. Imposing \( \Delta_{min} > 0 \) also decreases the investor’s profit because the investor must now choose the optimal \( \Delta \) over a smaller region. Hence,
imposing or raising minimum capital requirements results in a pareto loss of welfare. Incorporating an individual rationality constraint for the investor is straightforward and does not change our result as noted previously.

3.5 Heterogeneous Credit Risk

$R_N^*(\bar{\Delta})$ is defined by:

\[ \frac{1}{2} \gamma U_x(\min\{1 - V' + 2V \cdot \bar{X}, R_N^*(\bar{\Delta})\} - (1 - \Delta_g)) + \frac{1}{2} \gamma U_y(\min\{1 - V' + 2V \cdot \bar{Y}, R_N^*(\bar{\Delta})\} - (1 - \Delta_b)) \]

\[ + \frac{1}{2} (1 - \gamma) U_x(\min\{1 - V' + 2V \cdot \bar{X}, R_N^*(\bar{\Delta})\} - (1 - \Delta_b)) + \frac{1}{2} (1 - \gamma) U_y(\min\{1 - V' + 2V \cdot \bar{Y}, R_N^*(\bar{\Delta})\} - (1 - \Delta_b)) = U_x(0) \]  

(74)

and $R_N^*(\Delta_b)$ is defined by:

\[ \frac{1}{2} U_x(\min\{1 - V' + 2V \cdot \bar{X}, R_N^*(\Delta_b)\} - (1 - \Delta_b)) + \frac{1}{2} U_y(\min\{1 - V' + 2V \cdot \bar{Y}, R_N^*(\Delta_b)\} - (1 - \Delta_b)) = U_x(0) \]  

(75)

We label the equilibria as 1 and 2 where equilibrium 1 has $p_N^* + R_N^*(\bar{\Delta}) < \Gamma_g^*$ (the good type does not disclose) and equilibrium 2 has $p_N^* + R_N^*(\bar{\Delta}) > \Gamma_g^*$ (the good type discloses).

Proof of Proposition 7:

With our proposed interest-rate equilibrium, auction prices for disclosure are the same as before: $p_{p,\tau}(R_{p,\tau}) = p_p^*$ where $\tau \in \{g, b\}$. Also, $p_N^* = V'$ because our arguments from section 3.3 apply.

We first prove the existence of the interest-rate equilibrium. There are no profitable deviations from the interest-rates for disclosure ($R_g^*$ and $R_b^*$) by our previous arguments. Again, a bank will obviously not offer a non-disclosure interest-rate of $R > R_N^*$ since the investor will not take this offer. A bank will also not offer $R < R_N^*$. First consider equilibrium 1 where $R_N^* = R_N^*(\bar{\Delta})$. Both the good and bad types would take such an offer. But since $R < R_N^*(\bar{\Delta})$, the bank would realize negative utility. Next consider equilibrium 2 where $R_N^* = R_N^*(\Delta_b)$. Suppose a bank offers a non-disclosure rate of $R < R_N^*$ such that $p_N^* + R > \Gamma_g^*$. In this case, only the bad type will take the offer and the bank will realize negative utility because $R < R_N^*(\Delta_b)$. Suppose instead that the bank offers a rate such that $p_N^* + R \leq \Gamma_g^*$. The good type can now take this offer, but the bank again will realize negative utility. The reason is that $p_N^* + R \leq \Gamma_g^* < p_N^* + R_N^*(\bar{\Delta})$ implies that $R < R_N^*(\bar{\Delta})$. 

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We now proceed to uniqueness. $s^*(\tau)$ is the equilibrium choice of a type $\tau$ investor in our proposed equilibrium: $s^*(\tau) = \arg \min_{s \in \{0,1,N\}} [p_s^* + R_{s,\tau}^*]$ (of course, $R_{N,\theta}^* = R_{N,(b)}^*$). We again conjecture an alternate equilibrium of $R_{s,\tau}$. We want to prove that $R_{s^*(\tau),\tau}^* = R_{s^*(\tau),\tau}^*$ and that an investor of type $\tau$ chooses $s^*(\tau)$ in this alternate equilibrium.

Again, the disclosure interest-rates are bounded below by $R_{p,\tau}^*$ by trembling-hand perfection, i.e., $R_{p,\tau}^* \geq R_{p,\tau}^*$ for $\rho \in \{0,1\}$, which implies that $\min_{p} [p_{p,\tau}^*(R_{p,\tau}) + R_{p,\tau}] \geq \Gamma^*_\tau$.

We first consider the case when our proposed equilibrium is 1 where both types do not disclose, and we show that $R_N = R_{N}^* = \mathcal{R}_N^*(\Delta)$ so that both types do not disclose in our conjectured equilibrium (since $p_N^* + R_N = p_N^* + R_N < \Gamma^*_N \leq \min_{\rho} [p_{\rho,\tau}^*(R_{\rho,\tau}) + R_{\rho,\tau}]$).

If $R_N < R_{N}^*$, both types will choose non-disclosure. Since $R_N < \mathcal{R}_N^*(\Delta)$, this interest-rate is not individually rational for the bank. Suppose instead that $R_N > R_{N}^*$. A bank can then offer a non-disclosure interest-rate of $R$ such that $R_N < R < R_{N}^*$ and:

$$p_N^* + R < \Gamma^*_N \leq \min_{\rho} [p_{\rho,\tau}^*(R_{\rho,\tau}) + R_{\rho,\tau}] \quad (76)$$

for both $\tau \in \{g,b\}$. Both the good and bad type will take this offer, and this bank earns positive utility since $R > R_{N}^* = \mathcal{R}_N^*(\Delta)$. Hence, this deviation is profitable.

Now consider the case where our proposed equilibrium is equilibrium 2 where the good type discloses. We first show that:

$$\min_{s} p_{s,\tau}^*(R_{s,\tau}) + R_{s,\tau} \leq p_{s^*(\tau)}^* + R_{s^*(\tau),\tau} \quad (77)$$

Suppose to the contrary that $\min_{s} p_{s,\tau}^*(R_{s,\tau}) + R_{s,\tau} > p_{s^*(\tau)}^* + R_{s^*(\tau),\tau}$ (where $s^*(g)$ is a type of disclosure). As before, a bank can offer an interest-rate of $R > R_{s^*(g),g}^*$ for choice $s^*(g)$ such that:

$$p_{s^*(g),g}^*(R) + R < \min_{s} p_{s,\tau}^*(R_{s,\tau}) + R_{s,\tau} \quad (78)$$

so that the good type investor takes this offer. Since $R > R_{s^*(g),g}^*$, this bank can achieve positive utility.

Now suppose that $\min_{s} p_{s,\tau}^*(R_{s,\tau}) + R_{s,\tau} > p_{s^*(b),b}^* + R_{s^*(b),b}^*$, $s^*(b) = N$. The same argument applies here. A bank can offer $R > R_{s^*(b),b}^*$ such that $p_{s^*(b),b}^*(R) + R < \min_{s} p_{s,\tau}^*(R_{s,\tau}) + R_{s,\tau}$. If $s^*(b) = N$, the bank achieves positive utility whether the good type takes this offer or not since: $R > \mathcal{R}_N^*(\Delta_b) = \mathcal{R}_N^*(\Delta)$. We now show that non-disclosure is never chosen by the good type in equilibrium. Assume the contrary. The bad type will also choose non-disclosure since $\min_{\rho} p_{\rho,b}^*(R_{\rho,b}) + R_{\rho,b} \geq \Gamma^*_b \geq \Gamma^*_g \geq \min_{s} [p_{s,g}^*(R_{s,g}) + R_{s,g}] = p_N^* + R_N$. But $\Gamma^*_g < p_N^* + \mathcal{R}_N^*(\Delta)$ implies that $R_N < \mathcal{R}_N^*(\Delta)$ so that this equilibrium is not individually-rational for banks. By our previous arguments invoking trembling-hand perfection, therefore, the good type chooses $s^*(g)$ in any equilibrium.
We finally show that if the bad type does not choose non-disclosure in our proposed equilibrium \((R^*_{e,r})\), she does not choose non-disclosure in the alternate equilibrium \((R^*_{e,r})\). Assume the contrary. \(p^*_N + R_N = \min_b[p^*_s(R^*_s,b) + R^*_s,b] \leq \Gamma^*_b < p^*_N + \mathcal{R}^*_N(\Delta_b)\) implies that \(R_N < \mathcal{R}^*_N(\Delta_b)\) which means that this equilibrium is not individually-rational for banks. Again, by our previous arguments, the bad type chooses \(s^*(b)\) in any equilibrium. QED.

3.5.1 Disclosure Regulation

Proof of Proposition 8:

The counterparty's profit is now given by:

\[
\Pi_{cp} = \gamma p^*_s(g) + (1 - \gamma)p^*_s(b) - V'
\]  

(79)

We first want to prove that mandating disclosure is not pareto-suboptimal when at least one investor type prefers uncorrelated disclosure over correlated disclosure. Suppose first that both types prefer uncorrelated disclosure. Mandating disclosure obviously has no effect if both types disclose voluntarily. If one or both types do not disclose voluntarily in equilibrium, mandating disclosure obviously strictly increases the counterparty's profit since \(p^*_0 > V'\).

Now suppose that that only the bad type prefers uncorrelated disclosure (if the good type prefers uncorrelated disclosure over correlated disclosure, so does the bad type by corollary 2). Consider first the case where the bad type does not disclose voluntarily. Mandating disclosure strictly improves counterparty utility since \(p^*_0 = p^*_N = V'\) without regulation and \(p^*_0 = p^*_0 > V'\) with regulation.

Next consider the case where the bad type discloses voluntarily. The good type also discloses voluntarily in this equilibrium by corollary 5. In this case, mandating disclosure is redundant and not pareto-suboptimal.

Mandating disclosure is pareto-suboptimal when \(\bar{p}_1 < V' < V'_c(\Delta_b)\) where \(V'_c(\Delta_b)\) is the concentration threshold for the bad type. The bad type prefers correlated over uncorrelated disclosure and so does the good type by corollary 2. The bad type will choose non-disclosure voluntarily by corollary 5. Mandating disclosure strictly decreases the profit of the bad investor and weakly decreases the profit of the good investor. It also does not increase the profit of the counterparty since both investor types choose correlated disclosure and \(p^*_0 = V'\) in this parameter region.

Proof of Proposition 9:
We want to show that mandating non-disclosure can improve the profit of some agent. If so, mandating non-disclosure is beneficial for a social welfare function with a high enough weight on this agent. We will show that the bad investor benefits with the following parameters:

\[
V' = (1 + \frac{k}{4})^{-1} \left[ V + \frac{k}{4} \Delta_b - \epsilon \right] \\
\Delta_g > \Delta_b + \left[ 1 - \gamma(1 + \frac{k}{4}) \right]^{-1} \epsilon
\]

(80)

We will show that in this equilibrium, the bad investor does not disclose and the good investor discloses. The bad investor’s profit without regulation is, therefore: \( \Pi_b = \frac{1}{2} [1 + 2V - p_N \gamma - R_N^*(\Delta)] - \Delta \). This profit increases with mandatory non-disclosure to: \( \Pi_b = \frac{1}{2} [1 + 2V - p_N \gamma - R_N^*(\Delta)] - \Delta \).

We first show that the good type discloses voluntarily. A sufficient condition is that: \( p_N^* + R_N^*(\bar{\Delta}) > p_b^* + R_b^*(\Delta) \). Manipulating this inequality gives:

\[
(1 + \frac{k}{4})(V' - \bar{\Delta}) > V - \Delta_g \\
(1 + \frac{k}{4})(V' - \Delta_b - \gamma(\Delta_g - \Delta_b)) > V - \Delta_g \\
V - \Delta_b - \epsilon - \gamma(1 + \frac{k}{4})(\Delta_g - \Delta_b) > V - \Delta_g \\
\Delta_g > \Delta_b + \left[ 1 - \gamma(1 + \frac{k}{4}) \right]^{-1} \epsilon
\]

(81)

where the second inequality implies the third by the first equation in equation 80. Hence, the second inequality in equation 80 gives the desired condition.

We next show that the bad type chooses non-disclosure voluntarily. Since the good type does not disclose in equilibrium, a sufficient condition is that: \( p_N^* + R_N^*(\Delta_b) > p_b^* + R_b^*(\Delta_b) \). Manipulating this inequality gives:

\[
(1 + \frac{k}{4})(V' - \Delta_b) < V - \Delta_b \\
V' < (1 + \frac{k}{4})^{-1}(V + \frac{k}{4} \Delta_b)
\]

(82)

Equation 80 gives this relation. QED.

3.6 Exogenous Assumptions of the Model in Chapter 2

The model’s specific frictions and simplifications are listed below:

1. Only dealers have access to the supply curve on the centralized market. Arbs can only trade with dealers.

There is a simple reason why arbs do not have direct access to this supply curve. This supply curve comes from noisy clients who do not know whether the dealer is trading for informational or risk-sharing reasons. If these clients were to trade with the arb, their demands would change
because the arb has only informational motives. In this sense, dealers are providing the arb with anonymity and hence, profits.

2. Incomplete contracts:

There is a contracting imperfection in the model that prevents commitment to a monopoly between arb and dealer. In particular, no contracts can be written on the specific quantities traded by various parties.

If such contracts can be written, then the arb can commit to trading only $\eta$ amount with one dealer. The arb can not break this commitment if a contract stipulates that he will trade no more than $\eta$. If however, these contracts can not be written, parties will try to make extra profits by doing extra trades. In effect, they will "front-run" each other.

We believe in real dealer markets, hedge funds have an imperfect ability to form long-term commitments to particular dealers. Commitments are at best short-term because these markets are highly competitive and the cost of switching dealers is near zero.

3. Only market orders in centralized market:

When the arb breaks up trade, he splits his information into two pieces between two different dealers. In aggregate, however, the market has the full information. The market prices of X and Y at stage-2 will reveal it if the market is not too noisy.

Breakup only matters when it creates some persistent noise in the dealer's inference. This noise should not vanish immediately by observing market variables. Hence, the dealers must not be able to extract the arb's information perfectly at stage-2. Restricting the dealer to market orders is a proxy for an informationally incomplete market; at stage-2. If the dealer can not condition his demand on the outcome of market prices, the noise from stage-1 will affect his trading behavior.

3.7 Off-Equilibrium Inferences

It would be useful to review the standard Cho-Kreps criterion for signaling games before we discuss our modification. The notation here is taken from Fudenberg-Tirole [1996].

A signaling game has the following form. Player 1 has unobserved type, $\theta$, and moves first with action $a_1$. Player 2 observes $a_1$ and then chooses action $a_2$. Cho-Kreps (the non-iterated version) says that it is feasible to have a positive probability for type, $\theta$, conditional on $a_1$, if $a_1$ gives greater utility than the equilibrium action for some conceivable response, $a_2$.  

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In other words, suppose \( u(a_1, a_2, \theta) > u^*(\theta) \) for some \( a_2 \in BR(a_1, \Theta) \) where \( u^*(\theta) \) is the equilibrium utility of \( \theta \) and \( BR(a_1, \Theta) \) is the class of best-responses to \( a_1 \) for all possible inferences about \( \theta \) (as defined in Fudenberg-Tirole). If this is true, then it is feasible for the probability of \( \theta \) conditional on \( a_1 \) to be positive. We denote this as: \( \tau_{a_1}(\theta) > 0 \).

Now we consider a game that resembles our model with three players and two actions by the informed player. Player 1 is informed, and players 2 and 3 are uninformed. Player 1 moves first and takes action \( a_1 \), observed by player 2, and \( a'_1 \), observed by player 3. Player 2 chooses an action \( a_2 \), and player 3 chooses an action, \( a'_2 \). Player 2 makes an inference about \( \theta \) and \( a'_1 \) (similarly for player 3).\(^{30}\) We offer the following two alternatives as criteria for determining reasonable inferences about \( \theta \) and \( a'_1 \) after observing \( a_1 \).

**Alternative 1:** The off-equilibrium action, \( a_1 \), is a mistake on the part of player 1. The Cho-Kreps criterion says that type \( \theta \) may play \( a_1 \) if it is within a reasonable class of mistakes. Alternative 1 treats \( a'_1 \) as the same kind of mistake as \( a_1 \).

Explicitly, suppose there is an \( a_2 \in BR(a_1, \Theta \otimes A) \) and an \( a'_2 \in BR(\hat{a}'_1, \Theta \otimes A) \) such that \( u(a_1, a_2, a'_1, a'_2, \theta) > u^*(\theta) \) where \( BR(a_1, \Theta \otimes A) \) is the class of best-responses to \( a_1 \) for all possible joint inferences about \( \theta \) and the complementary action, \( a'_1 \). This is the minimal condition for \( \tau_{a_1}(\theta, a'_1) > 0 \) to be feasible \((\tau_{a'_1}(\theta, a_1) > 0 \) is, of course, also feasible).

**Alternative 2:** This alternative treats the choice of \( a'_1 \) more rationally than alternative 1. \( \theta \) and \( a'_1 \) are feasible conditional on \( a_1 \), if \( a'_1 \) is the best choice of action for type \( \theta \) given that \( a_1 \) had already been chosen.

Explicitly, suppose \( a'_1 = \arg\max_{a'_1} u(a_1, a_2, \hat{a}_1, a'_2(\hat{a}_1), \theta) \) and \( u(a_1, a_2, a'_1, a'_2(a'_1), \theta) > u^*(\theta) \) for some \( a_2 \in BR(a_1, \Theta \otimes A) \). \( a'_2(a'_1) \) is the equilibrium best-response of player 2. This is the minimal condition for \( \tau_{a_1}(\theta, a'_1) > 0 \).

We call this condition Cho-Kreps with conditional dominance. The choice \( a'_1 \) must be dominant for type \( \theta \) conditional on an exogenous choice of \( a_1 \). With \( a'_1 \), player 1 conjectures the true equilibrium response; whereas with \( a_1 \), he mistakenly conjectures a response: \( a_2 \in BR(a_1, \Theta \otimes A) \). Hence, the mistake about responses affects only \( a_1 \) and not \( a'_1 \). \( a'_1 \) is chosen rationally. In alternative 1, the mistake about responses affects both \( a_1 \) and \( a'_1 \).

Alternative 2 is stronger than 1 in the sense that there are fewer feasible inferences. Here, we show that our off-equilibrium inferences from equation 39 in section 2.2.1 satisfy alternative 2 and therefore, also satisfy alternative 1.

---

\(^{30}\)Player 2 may also make an inference about \( a'_2 \), but we assume that \( a'_1 \) summarizes any information about \( a'_2 \) as in our model.
Conditional on $X$ ($\tilde{\theta} > 0$), the dealer believes a type $x_H$ played $(X, X)$. We want to justify this inference. The type $x_H$ corresponds to $\theta$ above, $(X, X)$ corresponds to $a_1$ and $a'_1$, and dealer prices, $\bar{p}(\tilde{\theta})$, correspond to $a_2$ and $a'_2$.

A type $x_H$ will prefer $(X, X)$ over the equilibrium action, $(A, B)$ if a dealer plays action: $p_2(\tilde{\theta}) = -\frac{1}{\phi}\frac{\phi^2}{\Psi}$. We can check that this price is a best-response if: $\tau_X(y_H, [B, -\theta_{FB}]) = 1$.

The only thing left to check is that if $x_H$ buys $X$ from dealer 1, it is optimal to buy $X$ from dealer 2, our condition of conditional dominance. This is quite easy. Because the arb must buy any security in minimum quantity, $\frac{\phi}{\phi_0}$ (see section 2.1.2), a type $x_H$ must bear some net inventory in $Y$ with any other security besides $X$. It is easy to show that the disutility from this inventory is sufficient to make $X$ conditionally dominant.

3.8 Proof of Theorem 1

1. Existence:

A general proof is given in section 3.13 of the appendix.

2. Uniqueness:

We mentioned previously that a symmetric equilibrium is one where different types play strategies that are symmetric in two ways. First, $x_H$ and $x_L$ play the same strategies but with opposite sign (same for $y_H$ and $y_L$). The second symmetry is defined by the following linear transformation:

\[
\begin{align*}
f(A) &= -B & f(X) &= Y \\
f(B) &= A & f(Y) &= -X
\end{align*}
\]  

(83)

where $f$ transforms the strategy of $x_H$ into the strategy of $y_H$ (also for $x_L$ and $y_L$). This transformation is a symmetry because it preserves all relationships between $A$, $B$, $X$, and $Y$. For example, $A = \frac{1}{2}X + \frac{1}{2}Y$. $f$ preserves this relationship: $f(A) = -B = \frac{1}{2}Y - \frac{1}{2}X = \frac{1}{2}f(X) + \frac{1}{2}f(Y)$.\textsuperscript{31}

There is one additional restriction on the strategies played in equilibrium. These strategies must be fully-hedged. In other words, a type $x_H$ must hold a cumulative long position only in $X$, and $y_H$ a cumulative long position in $Y$, etc. We call these strategies FH strategies and the corresponding equilibria FH equilibria.

There are several possible symmetric and FH equilibria besides the breakup equilibrium:

\textsuperscript{31}There is one other important symmetry in addition to this one. It is defined by: $f(A) = A$, $f(B) = -B$, $f(X) = Y$, $f(Y) = X$. We prefer the symmetry above because it transforms $A$ and $B$ the same as it transforms $X$ and $Y$.  

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1. \( x_H \rightarrow (2B, Y) + \) other mixed strategies
\[ y_H \rightarrow (2A, -X) + \text{other mixed strategies} \]

2. \( x_H \rightarrow (X, X) \)
\[ y_H \rightarrow (Y, Y) \]

3. \( x_H \rightarrow (X, X) \)
\[ \langle A, B \rangle \]
\[ y_H \rightarrow (Y, Y) \]
\[ \langle A, -B \rangle \]

We write \((2B, Y)\) to mean a trade where the quantity of \(B\) is twice the quantity of \(Y\). Equilibrium 2 and 3 are consistent with our theorem. The breakup strategy here is the only noisy strategy. We must, therefore, only show that equilibrium 1 does not exist. 32

1. We can rule out candidate equilibrium 1 by showing that no set of prices supports this equilibrium. We show that if prices make \((2B, Y)\) incentive-compatible for \(x_H\) and \((2A, -X)\) incentive-compatible for \(y_H\), there is a contradiction. We denote the equilibrium quantity at which \(A\) and \(B\) are traded as \(\theta^*\), and the quantity for \(X\) and \(Y\) as \(\theta^*_X\). In this proof, we denote \(\pi_B = -\theta^* \cdot \bar{p}_D(\theta^*)\) (same for \(A\)) and \(\pi_X = -\theta^*_X \cdot \bar{p}_Y(\theta^*_X)\) (same for \(Y\)).

In this equilibrium, \(x_H\) must prefer \((2B, Y)\) over \((A, B)\) and \((X, X)\) where these trades are at the equilibrium quantities. This implies the following inequalities:

\[
\sigma^{\theta^*}_2 + \pi_B + \pi_Y \geq \sigma^{\theta^*} + \pi_A + \pi_B,
\]
\[
\sigma^{\theta^*} + 2\pi_X
\]

Similarly, \(y_H\) must prefer \((2A, -X)\) over \((A, -B)\) and \((Y, Y)\) which implies:

\[
\sigma^{\theta^*_X}_2 + \pi_A + \pi_X \geq \sigma^{\theta^*} + \pi_A + \pi_B,
\]
\[
\sigma^{\theta^*} + 2\pi_Y
\]

This system of inequalities is inconsistent. Summing the top inequalities and the bottom inequalities gives:

\[
\pi_X + \pi_Y \geq \sigma^{\theta^*} + \pi_A + \pi_B
\]
\[
\pi_A + \pi_B \geq \sigma^{\theta^*} + \pi_X + \pi_Y
\]

32There is yet another symmetric equilibrium similar to 1 except that \(x_H\) plays \((2A, -Y)\). We can rule out this alternative by a proof very similar to the one above.
There is an obvious contradiction when $\theta^* > 0$. QED.

3. Profit is First-Best

We want to prove the following identity for the ideal model:

$$\Pi_I = \Pi_{FB}$$ (87)

for all dealer preferences with monotone and (weakly) concave utility function. In other words, the arb will always achieve FB profits by breakup regardless of the dealer's level of risk-aversion.

The proof goes in two steps. First, we prove that the arb's profit is always at least first-best since the dealers can always pass-through the arb's demand to the centralized market. If the dealer chooses not to pass-through, it must be an improvement and will show up as a discount in the price. The arb gets a price that is no worse than the centralized market price. Second, we prove that the arb can do no better than first-best if the dealer is risk-averse. The dealer will transfer no more than the profit he makes on the centralized market.

As usual, the arb is of type $\omega = x_H$. The arb buys $\bar{\theta}$ units of A from dealer 1 and $\bar{\theta}$ units of B from dealer 2.

Dealer 1 then submits a market order of $(\theta, \bar{\theta})$ to the centralized market. Dealer 1's risky wealth after trade is $\bar{W}(\theta) + \bar{p}_A(\bar{\theta}) \cdot \bar{\theta}$ where $\bar{W}(\theta)$ is:

$$\bar{W}(\theta) = \sigma \cdot (\theta - \frac{\bar{\theta}}{2}) - p_x(\theta) \cdot \theta + Y \cdot (\theta - \frac{\bar{\theta}}{2}) - p_y(\theta) \cdot \theta$$ (88)

Dealer 1 chooses $\theta$ to maximize his utility. If a weakly suboptimal choice of $(\frac{\bar{\theta}}{2}, \frac{\bar{\theta}}{2})$ is chosen, his wealth is such that:

$$\bar{W}(\theta = \frac{\bar{\theta}}{2}) = -p_x(\frac{\bar{\theta}}{2}) \cdot \frac{\bar{\theta}}{2} - p_y(\frac{\bar{\theta}}{2}) \cdot \frac{\bar{\theta}}{2}$$ (89)

By competition:

$$u(\bar{W}(\theta = \frac{\bar{\theta}}{2}) + \bar{p}_A(\bar{\theta}) \cdot \bar{\theta}) \leq u(0) = E[u(\bar{W}(\theta) + \bar{p}_A(\bar{\theta}) \cdot \bar{\theta})]$$ (90)

Hence:

$$\bar{p}_A(\bar{\theta}) \cdot \bar{\theta} \leq p_x(\frac{\bar{\theta}}{2}) \cdot \frac{\bar{\theta}}{2} + p_y(\frac{\bar{\theta}}{2}) \cdot \frac{\bar{\theta}}{2}$$ (91)
By a similar argument for dealer 2:

\[
\tilde{p}_B(\tilde{\theta}) \cdot \tilde{\theta} \leq p_x(\frac{\tilde{\theta}}{2}) \cdot \frac{\tilde{\theta}}{2} - p_y(\frac{\tilde{\theta}}{2}) \cdot \frac{\tilde{\theta}}{2}
\]  

(92)

We now compute the total amount the arb pays for his position:

\[
(\tilde{p}_A(\tilde{\theta}) + \tilde{p}_B(\tilde{\theta})) \cdot \tilde{\theta} \leq p_x(\tilde{\theta}) \cdot \tilde{\theta}
\]  

(93)

Hence, the arb’s price always beats or matches the centralized market price irrespective of the dealer’s preferences. If we consider a demand of \( \tilde{\theta} = \theta_{FB} \), then the arb’s profit will be bounded below by his first-best profit:

\[
\Pi_I \geq \Pi_{FB}
\]  

(94)

We now prove the opposite inequality for all (weakly) risk-averse dealer preferences:

\[
\Pi_I \leq \Pi_{FB}
\]  

(95)

We let \( \Pi_i(\omega) \) be the profit dealer i makes in the centralized market in state \( \omega \). We let \( \bar{\Pi}_i(\omega) \) be the profit the arb makes from dealer i in state \( \omega \). Explicitly:

\[
\Pi_i(x_H) = \sigma \cdot \theta - p_x(\theta) \cdot \theta + \bar{Y} \cdot \theta - p_y(\theta) \cdot \theta
\]  

(96)

\[
\bar{\Pi}_i(x_H) = \sigma \cdot \frac{\theta}{2} + \bar{Y} \cdot \frac{\theta}{2} - \tilde{p}_A(\theta) \cdot \tilde{\theta}
\]

with similar expressions for other states and dealers.

Dealer i’s total wealth after trade is \( \Pi_i(\omega) - \bar{\Pi}_i(\omega) \). Again, by competition and Jensen’s inequality:

\[
E \left[ u(\Pi_i(\omega) - \bar{\Pi}_i(\omega)) \right] = u(0)
\]  

(97)

\[\Rightarrow E \left[ \Pi_i(\omega) - \bar{\Pi}_i(\omega) \right] \geq 0
\]  

(98)

Summing over i:

\[
E \left[ \Pi_1(\omega) + \Pi_2(\omega) - \bar{\Pi}_I(\omega) \right] \geq 0
\]  

(99)

By symmetry, \( \Pi_I(\omega) \equiv \Pi_I \) across all states. Finally, since \( \Pi_1(\omega) + \Pi_2(\omega) \leq \Pi_{FB} \), we have proved the result.
3.9 Proof of Theorem 2

Profit is increasing in $\alpha$. Explicit calculation.

3.10 General Security Structures

We now develop some notation which we will use later. We write a security structure as a matrix, $\Gamma$, where each column is a different hybrid security, and the entries are pure factor exposures. For example, in the three-factor example above:

$$\Gamma = \frac{1}{2} \begin{bmatrix} 1 & 1 & -1 \\ 1 & -1 & 1 \\ -1 & 1 & 1 \end{bmatrix}$$ (100)

And in the ideal model:

$$\Gamma = \frac{1}{2} \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$$ (101)

We now develop some results about general security structures. There are different kinds of generalizations that we can make based on the previous setups. We could consider different correlations between speculative factors or different dealer prors concerning the mispricing. We could also consider cases where it is not clear whether a given security is pure and hybrid. In many real-life cases, a dealer does not know when a particular trade represents the arb's cumulative position or simply one leg of a larger trade.

For the sake of ease, we generalize along one particular direction. We simply change the number of factors and the hybrid security structure and consider the properties of the breakup equilibrium (i.e., the equilibrium where all types trade exclusively in noisy, hybrid securities). We develop some results on structures for which the equilibrium has nice properties. Finally, we use our reasoning to make conclusions about real-world relative-value trading with general information and security structures.

Our general setup simply modifies our previous setups to include more factors. In general, there are $n$ factors that the arb could be speculating in: $X_1, X_2, \ldots, X_n$. The arb in the market has an equal prior probability of having information about each of the $n$ factors. The $n$ factors also have independent and identical binary outcomes ($\sigma$ and $-\sigma$, each with probability $\frac{1}{2}$).

A general security structure has $n$ pure securities in factors, $X_1, X_2, \ldots, X_n$, and $n$ hybrid securities, $A_1, A_2, \ldots, A_n$. The hybrid securities are linear combinations of $X_1, X_2, \ldots, X_n$. Each
dealer offers prices in all $2n$ pure plus hybrid securities. Again, the security structure will be written as a matrix, $\Gamma$, with columns that correspond to hybrid securities and rows that represent pure factor exposures. The security $A_i$ corresponds to the $i$'th column of $\Gamma$: $\Gamma_i$.

In the breakup equilibrium, the arb trades only in hybrid securities which are noisy. The structure needs to be invertible, therefore, in order for there to be a unique hybrid strategy for every arb type. An arb of type $X_i$ will accumulate one unit of $X_i$ by trading the quantities $\begin{pmatrix} \Gamma_{1i}^{-1} & \Gamma_{2i}^{-1} & \ldots & \Gamma_{ni}^{-1} \end{pmatrix}$ of the hybrids $\begin{pmatrix} A_1, A_2, \ldots, A_n \end{pmatrix}$. We call this strategy the unique hybrid strategy (without reference to the overall quantity of the position in $X_i$).

Finally, we call the matrix $\Gamma^{-1}$ the inference matrix. The columns of $\Gamma^{-1}$ correspond to different pure types for arbs and the rows to the quantities of hybrids they use to accumulate a position. A dealer of a certain hybrid security looks across the corresponding row to infer which possible arbs could be trading this hybrid and in which states.

We now define: 1) a simple security structure and 2) a strongly symmetric security structure. A simple security structure has a $\Gamma^{-1}$ inference matrix with only $\{-1, 0, 1\}$ entries. We restrict ourselves to simple structures to avoid a problem which arises because of the discrete outcomes of $X_i$. If different types of arbs were to use different quantities of a given security to construct a position of a given size, a dealer would be able to infer the arb's type from the quantity traded. A simple security structure does not have this problem.

A strongly symmetric structure is one with symmetric $\Gamma^{-1}$. Also, every column of $\Gamma^{-1}$ has the same number of non-zero entries. We define $m$ as this number of non-zero entries. We can also think of $m$ as the number of hybrid securities needed to construct a pure position. Our previous two examples were simple and strongly symmetric with $m = 2$. It turns out that simple and strongly symmetric structures have certain desirable properties for relative-value trading. We will offer economic interpretations in section ??.

Our proxy for risk-aversion is to assume a cost of holding inventories whenever there is noise about the outcome. To be specific, whenever the arb trades a quantity $\tilde{\theta}$ of hybrid security $A_i$, there is noise about the outcome. The dealer makes inferences from the trade and submits demands of $\tilde{\theta} = (\theta_1, \theta_2, \ldots, \theta_n)$ into the centralized market. He anticipates making an uncertain profit of $\pi(A_i, \tilde{\theta})$ from this trade. \footnote{\pi(A_i, \tilde{\theta}) is only profit from trades executed on the centralized market, not from any trades done with the arb} The dealer maximizes the objective:

$$
\pi_f(A_i, \tilde{\theta}) = \max_{\tilde{\theta}} E \left[ \pi(A_i, \tilde{\theta}) \bar{A}_i, \tilde{\theta} \right] - \alpha \left( \tilde{\theta} - \tilde{\theta}_\Gamma \right)^T \left( \tilde{\theta} - \tilde{\theta}_\Gamma \right)
$$

(102)
The dealer is penalized for holding every pure factor exposure equally.

A pure trade with the arb is fully revealing according to the inferences from the breakup equilibrium. A dealer will simply maximize profits after a pure trade, and there is no cost of bearing inventory in the pure factor that has been traded.

3.11 Exogenous Assumptions

Assumptions 1, 2, 4, and 6 are the same as in section 2.1.1. Assumptions 3 and 5 vary slightly.

- Exogenous supply curves:
  
  There are \( n \) linear supply curves for \( X_1, X_2, \ldots, X_n \) on the centralized market. Factor \( X_i \) has the supply curve: \( p_{X_i} = b\Theta_{X_i} \)

  where \( \Theta_{X_i} \) is the sum of all orders for \( X_i \).

- Trade with at most \( m \) dealers:
  
  The arb is allowed to trade with at most \( m \) dealers in period 1.

  It is a bit arbitrary that we allow different arbs to trade through different numbers of dealers purely because their trading strategy is different. If the constraint reflects some physical limit or some etiquette rule imposed through relationships, it probably should not vary with security structure.

  We do this mainly for ease. Using \( m \) dealers allows us to maintain the unique hybrid strategy as a possible strategy. We can justify the constraint if we think of one time period not as a fixed amount of time, but the time it takes to accumulate a position in the desired factor. An arb with a more complicated strategy will take a longer time.

  The arb chooses to trade through at most \( m \) dealers in that window of time because there is some fixed cost for every dealer that sees a part of the position. The cost is there because the arb wants to preserve the option to return to the market later to either buy more of the position or liquidate it.

3.12 Breakup Equilibrium

We restrict our attention to simple and strongly symmetric structures because their existence properties are nice. Because of the symmetry, each hybrid dealer expects the same profit in the centralized market. Therefore, \( \pi_f(A_i, \tilde{\theta}) \) from equation does not depend on \( A_i \): \( \pi_f(A_i, \tilde{\theta}) \equiv \pi_f(\tilde{\theta}) \).

The arb captures \( \pi_f(\tilde{\theta}) \) from each dealer because of competition.
We conjecture a breakup equilibrium that looks similar to the ones considered previously. The arb trades in hybrid securities to create noise in the dealer’s inference.

The arb’s breakup strategy is:

$$\Psi(X_i) = \begin{bmatrix} <A_1, A_2, \ldots, A_n> \\ \bar{\theta}^*(\alpha) <\Gamma_{11}^{-1}, \Gamma_{21}^{-1}, \ldots, \Gamma_{ni}^{-1}> \end{bmatrix}$$ (103)

where $\bar{\theta}^*(\alpha)$ is the solution to the following maximization:

$$\bar{\theta}^*(\alpha) = \arg\max_{\tilde{\theta}} \pi_I(\tilde{\theta})$$ (104)

The dealer makes inferences about outcomes and complementary trades with other dealers given this equilibrium trading strategy. These inferences are noisy for hybrid trades, in general. The dealer infers that pure trades are fully revealing and that the arb executed identical pure trades with all other dealers.

The dealer’s pricing strategy is:

$$\bar{p}_{X_i}(\tilde{\theta}) = \text{sgn}(\tilde{\theta}) \alpha - \frac{1}{\bar{\theta}} P_{II}$$

$$\bar{p}_{A_i}(\tilde{\theta}) = \text{sgn}(\tilde{\theta}) \frac{\sigma^2}{m} - \frac{1}{\bar{\theta}} \pi_I(\tilde{\theta})$$ (105)

These prices are written in terms of their information and profit components as before.

The term $\Pi_I(\alpha) = \max_{\tilde{\theta}}[m\pi_I(\tilde{\theta})]$ is the total profit made by the dealer by using the breakup strategy. It is the certainty equivalent of the dealer’s profit collected from n dealers. $\Pi_{II}$ is the profit from consolidation, which is equal to the total profit in a m-player Cournot game: $\Pi_{II} = \frac{m}{(m+1)^2} \sigma^2$.

We now think about whether the breakup equilibrium exists. In other words, is it incentive-compatible for the arb to play the breakup strategy of equation 103? We decompose price into information and profit components as before. With the profit piece we ask simply if $\Pi_I(\alpha) \geq \Pi_{II}$. With the information piece, we ask whether the arb can pay less for a position of a given size by using an alternative strategy. If both properties are true, the breakup equilibrium exists.

### 3.13 Proof of Existence for General Structures

Consider a simple, strongly symmetric security structure, $\Gamma$, with dimension n. There are n pure factors that the arb could have information about, which we name: $X_1, X_2, \ldots, X_n$. There are n hybrid securities: $A_1, A_2, \ldots, A_n$. The hybrid securities have pure factor exposures given by the matrix, $\Gamma$: $A_i = \Gamma_{i1}X_1 + \Gamma_{i2}X_2 + \ldots + \Gamma_{ni}X_n$. Or in vector form: $A_i = [\Gamma_{i}]$, where $\Gamma_{i}$ is the i’th column of $\Gamma$. 

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We have already derived the dealer's pricing strategies in the breakup equilibrium. For pure security, \( X_i \), and hybrid security, \( A_i \):

\[
\tilde{p}_{X_i}(\bar{\theta}) = \tilde{p}_{X_i}(\bar{\theta}) - \frac{1}{\bar{\theta}} \frac{\Pi_i}{m}
\]

\[
\tilde{p}_{A_i}(\bar{\theta}) = \tilde{p}_{A_i}(\bar{\theta}) - \frac{1}{\bar{\theta}} \frac{\Pi_i(\alpha)}{m}
\]

where:

\[
\tilde{p}_{X_i}(\bar{\theta}) = \text{sgn}(\bar{\theta}) \sigma
\]

\[
\tilde{p}_{A_i}(\bar{\theta}) = \text{sgn}(\bar{\theta}) \frac{\sigma}{m}
\]

\( \tilde{p}_{X_i} \) and \( \tilde{p}_{A_i} \) are the "informational" components of the price. Simply, they are the expected value of these securities given the arb's conjectured trading strategy.

To show that the breakup equilibrium exists, we must simply show incentive compatibility for the arb, i.e., the breakup strategy dominates all other trading strategies. A general trading strategy with \( m \) dealers is given by:

\[
\Psi = \begin{bmatrix}
< J_1, J_2, \ldots, J_m > \\
< \tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_m >
\end{bmatrix}
\]

The strategy sums to the following pure factor exposures for the arb:

\[
\tilde{\theta}_1 J_1 + \tilde{\theta}_2 J_2 + \ldots + \tilde{\theta}_m J_m = \Gamma(\Psi) = 
\begin{bmatrix}
\Gamma_1(\Psi) \\
\Gamma_2(\Psi) \\
\vdots \\
\Gamma_n(\Psi)
\end{bmatrix}
\]

Without loss of generality, we take the arb's type to be \( X_1 \). The breakup strategy for this type is given by:

\[
\Psi^* = \begin{bmatrix}
< A_1, A_2, \ldots, A_n > \\
\tilde{\theta}^*(\alpha) < \Gamma_{11}^{-1}, \Gamma_{21}^{-1}, \ldots, \Gamma_{n1}^{-1} >
\end{bmatrix}
\]

where the above expression has \( m \) columns in reality (dealing only through \( m \) dealers) because we don't count entries equal to zero. We want to show that following expression for the arb's utility is maximized by the breakup strategy:
\[ U(\Psi) = \sigma \Gamma_1(\Psi) - [\tilde{p}_{j_1}(\tilde{\theta}_1)\tilde{\theta}_1 + \tilde{p}_{j_2}(\tilde{\theta}_2)\tilde{\theta}_2 + \ldots + \tilde{p}_{j_m}(\tilde{\theta}_m)\tilde{\theta}_m] \\
+ \inf[X_2\Gamma_2(\Psi) + X_3\Gamma_3(\Psi) + \ldots + X_n\Gamma_n(\Psi)] + \Pi_\Psi \]  

(111)

\[ = \tilde{U}(\Psi) + \Pi_\Psi \]

We attack this problem in pieces. First, we remove the profit piece, \( \Pi_\Psi \), and consider it later. We are left with the informational piece: \( \tilde{u}(\Psi) \). This piece is linear in the scale of the position. We prove a kind of “constrained” incentive-compatibility for this piece when the scale is fixed.

Specifically, we will show that when the arb wants to accumulate a position in \( X_1 \) of some fixed size (i.e., \( \Gamma_1(\Psi) = 1 \)) the best way is to use the unique hybrid strategy.

Explicitly:

**Lemma 1** For every \( \Psi \) s.t. \( \Gamma_1(\Psi) = 1 \):

\[ \sigma - [\tilde{p}_{A_1}(\Gamma^{-1}_{11})\Gamma^{-1}_{11} + \tilde{p}_{A_2}(\Gamma^{-1}_{21})\Gamma^{-1}_{21} + \ldots + \tilde{p}_{A_n}(\Gamma^{-1}_{n1})\Gamma^{-1}_{n1}] \geq \tilde{U}(\Psi) \]

(112)

This lemma is sufficient to prove that the breakup strategy is optimal whenever \( \Pi_\Psi \) is incentive-compatible. The proof that lemma 1 is sufficient is available upon request.

### 3.14 Proof of Lemma 1

To prove lemma 1, we need one additional fact: we only need consider strategies over hybrid securities: \( A_1, \ldots, A_n \). Any additional trades in pure securities can not improve \( \tilde{U}(\Psi) \).

The reason is simple. We consider trading strategies with a fixed strategy in hybrids. Without loss of generality, suppose a type \( X_1 \) arb has some positive inventory of factor \( X_2 \): \( \Gamma_2(\Psi) > 0 \). The arb can either try to hedge away this exposure by trading in pure securities or simply bear the non-zero inventory. In the first case, the arb pays a “spread” of \( \sigma \). He sells the pure exposure for: \(-\sigma\Gamma_2(\Psi)\). In the second case, he bears the exposure and values the position as much as its minimum payout: \(-\sigma\Gamma_2(\Psi)\).

Hence, full or partial hedging by pure securities does not improve utility. We restrict ourselves to strategies over hybrid securities only and prove the following lemma:

**Lemma 2** For every pure strategy in hybrid securities, \( \Psi \), s.t. \( \Gamma_1(\Psi) = 1 \) where:
\[ \Psi = \begin{bmatrix} < A_1, A_2, \ldots, A_n > \\ < \tilde{\theta}_1, \tilde{\theta}_2, \ldots, \tilde{\theta}_n > \end{bmatrix} \] (113)

the following relation holds:

\[ \begin{align*}
-\left[ \tilde{\rho} A_1 (\Gamma_{11}^{-1}) \Gamma_{11}^{-1} + \tilde{\rho} A_2 (\Gamma_{21}^{-1}) \Gamma_{21}^{-1} + \ldots + \tilde{\rho} A_n (\Gamma_{n1}^{-1}) \Gamma_{n1}^{-1} \right] & \geq \\
-\left[ \tilde{\rho} A_1 (\tilde{\theta}_1) + \tilde{\rho} A_2 (\tilde{\theta}_2) + \ldots + \tilde{\rho} A_n (\tilde{\theta}_n) \right] & + \inf \left[ X_2 \Gamma_2(\Psi) + X_3 \Gamma_3(\Psi) + \ldots + X_n \Gamma_n(\Psi) \right]
\end{align*} \] (114)

Note that we are no longer considering the m-dealer constraint so that this lemma is stronger than we need.

The RHS of equation 114 can be written as:

\[ -\left[ \sum_i |\tilde{\theta}_i| \frac{\sigma^2}{m} \right] + \inf \left[ X_2 \Gamma_2(\Psi) + X_3 \Gamma_3(\Psi) + \ldots + X_n \Gamma_n(\Psi) \right] \]

\[ = -\left[ \sum_i |\tilde{\theta}_i| + \sigma \| \Gamma(\Psi) \| \right] + \sigma \]

where \( \| \Gamma(\Psi) \| \) is the absolute-value norm of the vector: \( \| \Gamma(\Psi) \| = |\Gamma_1(\Psi)| + |\Gamma_2(\Psi)| + \ldots + |\Gamma_n(\Psi)| \).

We can discard the constants in our consideration of equation 115. Our constrained objective becomes:

\[ \begin{align*}
\{ |\tilde{\theta}_1| + \ldots + |\tilde{\theta}_n| \} \frac{1}{m} + |\tilde{\theta}_1 \Gamma_{11} + \tilde{\theta}_2 \Gamma_{12} + \ldots + \tilde{\theta}_n \Gamma_{1n}| \\
+ |\tilde{\theta}_1 \Gamma_{21} + \tilde{\theta}_2 \Gamma_{22} + \ldots + \tilde{\theta}_n \Gamma_{2n}| \\
\vdots \\
+ |\tilde{\theta}_1 \Gamma_{n1} + \tilde{\theta}_2 \Gamma_{n2} + \ldots + \tilde{\theta}_n \Gamma_{nn}| \\
\end{align*} \] (116)

s.t. \( \tilde{\theta}_1 \Gamma_{11} + \tilde{\theta}_2 \Gamma_{12} + \ldots + \tilde{\theta}_n \Gamma_{1n} = 1 \)

We want to show that the constrained maximization is solved by \( \begin{bmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_n \end{bmatrix} = \Gamma_1^{-1}. \)

We can eliminate the constraint by making a change of basis:

\[ \begin{bmatrix} \tilde{\theta}_1 \\ \vdots \\ \tilde{\theta}_n \end{bmatrix} = \Gamma_1^{-1} + \sum_i \lambda_i \Gamma_i^{-1} \] (117)
which is equivalent to the linear constraint.

Finally, we rewrite our objective in terms of the $\lambda_i$'s:

\[
\begin{align*}
|\Gamma_1^{-1} + \lambda_2 \Gamma_{12}^{-1} + \ldots + \lambda_n \Gamma_{1n}^{-1}| \\
+|\Gamma_2^{-1} + \lambda_2 \Gamma_{22}^{-1} + \ldots + \lambda_n \Gamma_{2n}^{-1}| \\
\vdots \\
+|\Gamma_n^{-1} + \lambda_2 \Gamma_{n2}^{-1} + \ldots + \lambda_n \Gamma_{nn}^{-1}| \frac{1}{m} + |\lambda_2| + \ldots + |\lambda_n|
\end{align*}
\]

(118)

Now all we must do is to show that the objective is minimized when $\lambda_i = 0$. This is quite easy by inspection. Suppose $\lambda_i \neq 0$ and so $|\lambda_i| = k > 0$. Suppose we then change $\lambda_i$ to 0. The $|\lambda_i|$ term decreases from $k$ to 0 and the rest of the objective increases by at most $k$. Hence, we have achieved a weak improvement. QED.

References


