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THE EFFECT OF ACCELERATED DEPRECIATION ON INVESTMENT

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ABSTRACT

The Effect of Accelerated Depreciation on Investment
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Submitted to the Department of Economics on July 1, 1966, in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics.

This thesis represents an attempt to determine the effects of accelerated depreciation on investment. Motivation is provided by recent tax law changes which have resulted in liberalized depreciation provisions. In 1954 the sum of the year's digits and double declining balance methods were permitted in place of the straight line method, in 1958 a limited 20% initial allowance was introduced, and in 1962 a reduction in asset life for tax purposes and an investment credit of 7% were authorized.

In theory investment behaviour will be influenced by the changes in present discounted value and liquidity which result from an acceleration of depreciation. That is, an acceleration of deductions not only increases an asset's discounted revenue stream and hence its profitability, but also provides a permanently higher level of cash flow for a growing firm, and to the extent that there is an advantage to financing from internal sources the profitability of investment projects is increased. Although the elasticity of investment expenditures with respect to discounted value and liquidity changes is unknown, it is nevertheless interesting to compare such changes for different methods of acceleration as well as for relevant parameters such as the asset life, discount rate and growth rate of investment.

In practice the effectiveness of the two factors will depend on the nature of the investment decision-making process used. Interview evidence and a study of the extent of reliance of firms on internal financing suggest that although discounting techniques are rarely considered explicitly by firms, the level of cash flow has a strong influence on investment decisions. For this and other reasons the liquidity effect forms the basis of the empirical analysis. A general model of investment, dividend, and external finance behaviour is estimated which, as well as being of interest in itself, is used to obtain estimates of the increase in investment in the two-digit manufacturing industries attributable to the 1954 and 1962 accelerated depreciation provisions. The 1958 allowance is quantitatively unimportant because of the annual limitation to $2,000.

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Chapter 1

INTRODUCTION

By accelerated depreciation is meant any change in the timing of depreciation deductions over the life of an asset which results in an increase in the present discounted value (to be denoted pdv) of these deductions. This increase of course depends on the discount rate used in calculating the present value, and therefore a specific rate or range of rates is required in order to determine what constitutes accelerated depreciation. That is, depreciation deductions could be altered in such a manner as to yield an increase in pdv at some discount rates and a decrease at others. For the methods of acceleration which have occurred in practice and to be considered below, however, this is not possible since they all involve increased deductions in early year(s) with corresponding lower deductions in later years, and the pdv of such a series is greater than zero for all positive discount rates.

Since accelerated depreciation is defined in relative terms, that is, as a change from an existing to a new system, the existing system is itself of importance. The straight line depreciation method (to be denoted SL) is generally considered as the existing system or norm compared to which the methods of double declining balance (DDB) and sum of the year's digits (SYD) are said to be accelerated.[1] On the other hand, any of these methods may be taken as the existing system compared to which the introduction of an initial allowance is said to be accelerated. An initial allowance of b percent of cost results in an increased deduction in the first year of b times cost, together with a corresponding decrease in the depreciable base of the asset over the remaining n-1 years.

Two methods of stimulating investment which have been used in practice but which do not satisfy the above definition of accelerated depreciation are the investment credit and reduction in asset life for tax purposes. The former results in a decrease in taxes by the amount of the credit in the first year of an asset's life, but leaves depreciation deductions unchanged. The latter is essentially different because it changes the period over which deductions are taken. However, it may be thought of in terms of the above definition by considering the new deductions over the longer life, that is, as deductions of $1/n_1$ for n_1 years and 0 for

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[1]Under the SL method deductions of 1/n are permitted in each year, for an asset with a life of n years. Under the DDB method the allowable deduction in the k^{th} year of the asset's life is 2/n times the undepreciated value of the asset. Since the latter is given by $(1-2/n)^{k-1}$, the DDB deduction is $(2/n)(1-2/n)^{k-1}$. Under the SYD method allowable depreciation in the k^{th} year is n-(k-1) divided by the sum of the first n digits (n(n+1)/2), and hence equals 2(n-k+1)/(n(n+1)). It should be noted that these expressions are given for an asset with unit cost. In the analysis to follow all examples will have this property unless otherwise stated.

$n_2 - n_1$ years, where n_1 is the shorter, n_2 the longer life. For convenience in the work to follow both the credit and reduction in life are classified as methods of accelerated depreciation.

The four major methods of acceleration which are therefore to be studied in detail are a switch to SYD, an initial allowance, an investment credit, and a reduction in asset life for tax purposes. This choice of methods is motivated by recent tax law changes which have resulted in an acceleration of depreciation in practice. The relevant tax provisions are reviewed briefly before consideration is given to the theoretical effects of accelerated depreciation.

Although specific methods of computing depreciation deductions were not specified by the Treasury prior to 1954, methods used were required to be reasonable, to conform with a recognized trade practice, and to be adopted by the taxpayer in his own account. Useful lives for tax purposes were intended to correspond to the length of time assets were retained in use, the life of each asset therefore depending on the particular circumstances of its employment. Estimated lives contained in Treasury mortality tables, such as Bulletin F, were averages and were not meant to apply to all assets or taxpayers.

The Internal Revenue Code of 1954 specifically authorized the following three methods of computing depreciation: the straight line method, the declining balance method at not more than twice the straight line rate, and the sum-of-the-years digits method. Any other consistent method was allowed, provided the deductions at the end of each year during the first two-thirds of the useful life of the property did not result in a greater cumulative deduction than under double declining balance. The option of switching at any time from double declining balance to straight line was permitted in order to recover the total cost of the asset. These methods were applicable to all new assets with a useful life of three or more years acquired or constructed after December 31, 1953. The 1954 Revenue Code did not include any changes with respect to determination of useful lives for tax purposes, although Revenue Ruling 90 issued by the Internal Revenue Service at the time instructed agents not to adjust lives used by taxpayers unless there was a clear basis for change.

New or used property with a useful life of over 5 years acquired after December 31, 1957, was eligible for a 20% initial allowance. The allowance could be claimed on property with a value of not more than $10,000 in any taxable year.

The 1962 Revenue Act required that entrepreneurs claim a tax credit equal to 7% of qualified investment in new or used machinery bought after December 31, 1961. Qualified investment was defined as: zero for assets with useful lives of less than 4 years, one-third of cost for assets with lives greater than 3 and less than 6 years, two-thirds of cost for assets with lives greater than 5 and less than 8 years, and full cost for assets with lives of 8 or more years. The depreciable base of qualified investment had to be reduced by the amount of the credit taken. In any one year the credit was limited to the first $25,000 of tax liability plus one-fourth of any remaining tax liability. Any unused credit could be carried back 3 years and then forward 5 years until exhausted.

In July, 1962, the I.R.S. published Revenue Procedure 62-61 to replace Bulletin F for the purpose of determining useful lives. Use of the procedure was optional. Useful lives were suggested in general by industry groupings, and by certain Guideline classes that crossed industries such as office furniture and transportation equipment. The Guidelines were applicable to existing as well as to new facilities. The new lives could be used for three years after which they were required to conform with actual lives as demonstrated by retirement practice. The reserve ratio test was intended to provide an objective basis for determining if this conformity was met.[2] Table 1.1 contains a comparison

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[2]Basically the reserve ratio for each Guideline class is computed by dividing the total depreciation reserves for all assets in that class by their corresponding basis (including any assets which have been removed from the accounts but which are still in use). The reserve ratio calculated in this manner must lie in the acceptable range prescribed by the Treasury, where the acceptable range depends on the method of depreciation, the Guideline life (n) and the rate of growth of investment over the preceding n years.

of Guideline lives and estimates of actual average tax lives in use at the time of introduction of the Guidelines.[3,4]

The Revenue Act of 1964 repealed the provision introduced in 1962 which required the depreciable base of assets to be reduced by the amount of tax credit taken. The depreciable base of assets purchased and subjected to such a reduction in 1962 and 1963 could be increased by a corresponding amount beginning in 1964.

The two major effects of accelerated depreciation, which will be analysed in detail in Chapters 2 and 3, are the present discounted value and liquidity effects. The former refers to the change in the pdv of a single asset's net revenue stream resulting from an accelerated depreciation provision. That is, when depreciation deductions are increased in early years, taxes are reduced and hence net revenues increased by the amount of the deductions times the corporate tax rate. Of course there is an equivalent decrease in net revenues in later years, but with a positive discount rate the pdv of these changes is positive. By the pdv effect then is meant the pdv of the change in depreciation deductions times the corporate tax rate, but since the latter

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[3]All numbered tables appear at the end of their respective chapters.

[4]Table 1.1 also contains for each manufacturing industry its Standard Industrial Classification number which will be used for reference purposes in the analysis to follow.

is assumed throughout to be constant (at 50%), it is sufficient to analyse the former.

There are a number of reasons for analysing changes in pdv. Although the mechanism through which such changes might be expected to result in changes in investment is unknown, a number of hypotheses are possible. First, investment decisions may rest on pdv calculations themselves. Second, investment decisions may be a function of rate of return measures which are affected by pdv changes. Third, the investment process may be formulated in terms of an adjustment process involving a desired capital stock the magnitude of which depends on the pdv of depreciation deductions (assuming of course a positive corporate tax rate).

There exists in the literature on accelerated depreciation, a number of studies in which pdv changes are analysed.[5] There does not exist, however, a comprehensive analysis of pdv effects such as the one presented in Chapter 2, which allows a comparison to be made of the effects of the major methods of accelerated depreciation over a wide range

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[5] See for example, E. C. Brown, "The New Depreciation Policy Under the Income Tax: An Economic Analysis", National Tax Journal, March, 1955, in which the effect on pdv of a switch from SL to SYD is studied; and M. M. Dryden, "Capital Budgeting and the Investment Credit", Working Paper 24-63, School of Industrial Management, M.I.T., June, 1963, in which the pdv change resulting from the 1962 credit is analysed. Other relevant works include Richard Goode, "Accelerated Depreciation Allowances as a Stimulus to Investment", Q.J.E., Vol. LXIX (May, 1955), pp. 191-220 and George Terborgh, Realistic Depreciation Policy, M.A.P.I., 1954.

of asset lives and discount rates. This is important because it is very difficult in the absence of such a comparison to determine the relative incentive to investment provided by the different methods. The effect on pdv changes of variations in the discount rate and asset life can also be analysed, and is of interest in discovering the relative incentive provided to assets with different lives and of different degrees of riskiness. The latter is possible to the extent that the discount rate may be interpreted as a measure inclusive of risk. Finally a table is presented which allows a direct comparison to be made of the effects of the 1954 and 1962 provisions, for asset lives which are intended to approximate the average lives used in the two-digit manufacturing industries.

The second major effect of accelerated depreciation is the liquidity effect. The liquidity measure to be considered is the ratio of total depreciation deductions to total investment.[6] This ratio gives the fraction of investment in any period which can be financed internally from depreciation allowances. Such a concept is of interest if there exists an advantage to financing investment internally. The nature of this advantage will be discussed in Chapter 4.

Of course total cash flow consists of net profits as well as depreciation allowances, and an increase in the

[6]This terminology differs from standard usage in that the liquidity measure defined here is a flow rather than a stock concept.

latter due to accelerated depreciation will reduce taxable and hence net profits. It is not hard to show, however, that cash flow will increase by the tax rate times the change in depreciation deductions. Consider the following simplified identity in which D is depreciation, P^n is net profit, P^g is taxable profits less all deductions except depreciation, and T is the corporate tax rate, then $P^n = (P^g - D)(1-T)$. Cash flow (CF), which equals $P^n + D$, is therefore given by $CF = (P^g - D)(1-T) + D$ or $CF = P^g(1-T) + DT$, which shows that an increase in depreciation deductions, *ceteris paribus*, increases cash flow by the tax rate times the change in deductions. This increase in cash flow is an upper bound to the amount (depending on the fraction of profits retained) by which the internal financing of investment can increase as a result of accelerated depreciation. Since this increase is given by a constant (the tax rate) times the change in depreciation deductions, it is sufficient to concentrate on the latter in order to determine the effect on internal financing.

In analysing the effect of accelerated depreciation on liquidity it is necessary to distinguish between the case of a single asset and that of a stream of assets. This distinction is not necessary for pdv analysis, but liquidity analysis is relevant only in the context of a stream of assets. That is, the variable of interest in any period is the ratio of total depreciation to current investment where the former includes depreciation on assets purchased at different times in the past. For a single asset of course the behaviour of the depreciation-investment ratio over time is simply given by the depreciation rate itself, and any increase in deductions in early years by definition equals the decrease in later years. But for a stream of assets the total depreciation deduction in any period is a function of investment expenditures over the preceding n years, where n is the average asset life. In order to determine the total depreciation deduction then it is necessary to make an assumption about the past growth of investment. For a positive growth rate, the increase in deductions on new or recent assets due to accelerated depreciation will not equal the decrease on older assets, because the stock of newer assets is permanently larger.

There exists in the literature a number of studies in which the advantages to be gained from accelerated depreciation under conditions of growth are recognized. Probably the first authors to explicitly analyse the time path of deductions for different methods for a stream of growing assets were R. Eisner and E. D. Domar.[7] The former showed that with a positive growth rate of investment, the aggregate

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[7]Robert Eisner, "Accelerated Amortization, Growth, and Net Profits", Q.J.E., Vol. LXVI (November, 1952), pp. 533-544; and Evsey D. Domar, "The Case for Accelerated Depreciation", Q.J.E., Vol. LXVII (November, 1953) pp. 493-519.

depreciation-investment ratio would increase as the length of asset life decreased, under the SL method. To illustrate the effect, hypothetical depreciation values were calculated for the U. S. economy using actual investment figures but assuming different SL amortization periods.

E. D. Domar studied the behaviour of the ratio of accelerated to normal depreciation under conditions of an exponential growth rate of investment and no initial capital stock. The methods of accelerated depreciation studied were the combinations of DDB and an initial allowance, SL and an allowance, and SL with a shorter life. Advantages accruing to new and growing firms were emphasized. But Domar's conclusions (which have essentially become the commonly held views in the literature) rest entirely on the assumption that the most appropriate measure of advantage from accelerated depreciation is the ratio of accelerated to normal deductions. One important implication of such an assumption, for any of the methods of acceleration studied here or in Domar's work, is that the gain from acceleration will decrease (or remain constant) during transition to steady state conditions. It will be argued below that the difference of depreciation deductions rather than the ratio of such deductions is a more suitable measure of the advantage from acceleration, in which case some of Domar's conclusions, and in particular the one just noted, must be modified.

A detailed account of the literature on the subject of depreciation deductions under conditions of growth will not be presented. A partial list of the contributing authors, however, would include E. C. Brown, E. D. Domar, R. Eisner, and R. Goode.[8] In spite of the substantial number of articles relating to the behaviour of depreciation deductions under conditions of growth, there exists neither a comprehensive study nor one in which the relation to liquidity factors is clearly stated. The analysis to be presented in Chapter 3 may be considered comprehensive for the following reasons. It allows a comparison to be made of the effects of the various major methods of accelerated depreciation, as well as of different asset lives, growth rates, and types of growth. The transitional effects for growing firms are studied carefully since their relevance for n years (the average asset life) after introduction of the new methods makes them important. Steady state depreciation to investment ratios are analysed and shown to be equivalent to pdv expressions when the growth rate is interpreted as a discount rate. Finally, the relevance of the change in the depreciation-investment ratio resulting from accelerated depreciation is studied in terms of the advantages to be gained from the increased capability to finance investment internally.

The analysis of the liquidity and pdv effects outlined and presented in detail in Chapters 2 and 3 is straightforward in that it simply involves computing changes in the

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[8]See Brown, op. cit., Domar, op. cit., Eisner, op. cit., and Goode, op. cit.

relevant parameters. A much more difficult problem is to determine the manner in which and the extent to which these changes affect investment decisions. Such a step is of course necessary for an empirical determination of the importance of the 1954 and 1962 Revenue Act provisions. If perfect rationality could be attributed to all entrepreneurs and if the exact manner of reaching investment decisions were known, then the pdv and liquidity factors, which theoretically should affect investment, could be translated into actual changes in investment. However, neither of these assumptions is acceptable. First, entrepreneurs do not always follow objectively rational practices when making investment decisions, whether because subjective preferences are considered more important, or because of ignorance of appropriate methods. Second, there exists in the literature a wide variety of determinants which are hypothesized to affect investment (to varying degrees) while attempts to describe investment behaviour econometrically have not resulted in general acceptance of any particular subset of these.

An assumption must be made therefore about the factors which determine investment decisions in order to investigate the effect of accelerated depreciation on them and hence on investment. If these factors are influenced by pdv and liquidity considerations then their incorporation (if possible) into an empirical model provides a means for determining empirically the effects of depreciation changes. Two points should be mentioned. First, even if pdv and liquidity changes do affect investment decisions it may be possible to argue, in view of the orders of magnitude of such changes, and in view of the probably rough predictions of future revenues and costs required in making investment decisions, that one or both of the effects is essentially negligible. Second, since investment decisions may be based on not entirely rational grounds, some mechanism may exist through which accelerated depreciation affects investment other than the two mentioned above.

In order to gain further insight into the factors which are considered important by entrepreneurs in reaching investment decisions, a brief report on two recent interview studies of corporation executives is given in Chapter 4. Attention centers on rate of return measures used by entrepreneurs in analysing investment projects, and particular emphasis is placed on determining whether such measures are affected in general by accelerated depreciation. (The extent to which such measures are affected is analysed in Chapter 5.)

In view of the fact that accelerated depreciation results in a permanent increase in liquidity (for growing firms), Chapter 4 also contains an analysis of the advantages of financing expenditures from internal sources, and a description of the extent to which this practice is followed. Aside from rational reasons for preferring internal funds, probably the major one of which is due to differences in tax rates on dividends and capital gains, entrepreneurs exhibit a strong subjective preference for them which in some cases may not be entirely rational. Whether for rational reasons or not, however, the existence of such a preference suggests that the liquidity effects resulting from accelerated depreciation may well be important.

Chapter 5 contains a comprehensive investigation of the orders of magnitude involved in rate of return changes resulting from specific accelerated depreciation provisions. In spite of the fact that the elasticity of investment with respect to such changes will in general be unknown, there are at least two reasons for analysing them. First, they are of interest in comparing the different methods of accelerated depreciation, and in comparing variations in asset lives and initial rates of return. Second, the orders of magnitude involved are of interest. In particular, if acceleration results in very small changes in rate of return measures, one might be justified in assuming their influence on investment negligible in view of the roughness with which such measures are likely to be constructed, being based on revenue predictions over the asset's entire life.

There exists in the literature a scattered discussion of changes in rate of return measures resulting from accelerated depreciation. In particular analysis has centered on the internal rate of return. G. Terborgh has calculated the effect of various measures of accelerated depreciation assuming an initial internal rate of return of 10% and a linearly declining revenue stream.[9] M. Dryden has tabulated the effect of the 1962 investment credit for various initial internal rates and with a linearly declining revenue stream, and has experimented slightly with the revenue stream assumption.[10] There exists, however, no comprehensive analysis of rate of return changes such as the one presented in Chapter 5. The effects on the internal rate of return for various initial rates and asset lives, and under the assumptions of constant and linearly declining revenue streams are given for the four major methods of accelerated depreciation mentioned above. The effects on a modified internal rate of return, which avoids the assumption of reinvestment at the internal rate, are also given. Finally in view of the reportedly widespread use of such a rate of return measure, the change in an asset's payout period due to accelerated depreciation is analysed.

It is important to recognize the relation between the pdv and liquidity effects discussed in Chapters 2 and 3, and

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[9]George Terborgh, Incentive Value of the Investment Credit, the Guideline Depreciation System, and the Corporate Rate Reduction, M.A.P.I., Washington, D. C., 1964; and New Investment Incentives, M.A.P.I., Washington, D. C., 1962.

[10]Miles M. Dryden, "How do Recent Changes in Tax Laws Affect Investment Decisions?" Working Paper 25-63, School of Industrial Management, M.I.T., June, 1963.

the rate of return analysis in Chapter 5. The changes in the internal rate of return and the modified rate of return considered in Chapter 5 are essentially pdv changes translated into rate of return terms. That is, such changes arise because of variations in the timing of an asset's (discounted) depreciation deductions, although the total amount of deductions remains the same. For the internal rate of return, depreciation deductions are discounted at the internal rate (whatever it may be), while for the modified internal rate deductions are discounted at the firm's cost of capital.

Variations in an asset's payout period due to accelerated depreciation do not depend on the pdv or the liquidity effects as defined above. That is, since discounting is ignored the effect is not one of present values, not is it concerned with the effect on the aggregate depreciation-investment ratio of a stream of assets. Rather accelerated depreciation alters an asset's payout period simply by increasing net revenues in early years thereby reducing the period of time taken for revenues to accumulate to investment cost. The payout period is strictly speaking not a rational profitability measure, and hence is not included in the analysis in Chapters 2 and 3 of the two major effects of accelerated depreciation. It is included in the discussion on rate of return measures because of its reportedly widespread use in practice.

No account is taken in the rate of return calculations in Chapter 5 of the liquidity effect considered in Chapter 3.

The rate of return calculations are concerned with changes in a single asset's revenue stream resulting from accelerated depreciation, while the liquidity factor as defined above is relevant only for a stream of assets. These two concepts, may be related however, in the following manner. Since the liquidity effect in any period after introduction of acceleration reduces the cost of financing investment by allowing more to be financed internally, then it results effectively in an increase in the rate of return of each asset purchased in that period. That is, the cost of financing each asset may be considered reduced and hence its rate of return increased. The reduction in cost will depend not only on the extent of the increase in internal financing made possible by the acceleration, but also on the importance of this increase to the firm. The former is exactly what is analysed in Chapter 3 under the heading of the liquidity effect and depends therefore on the average asset's life and the growth rate of investment. The importance of this increase to the firm, however, is not readily determinable because it depends on the subjective preference of the firm for internal funds, as well as on the relative costs of internal and external funds. For this reason no attempt is made to translate liquidity changes into rate of return changes, and in the rate of return analysis presented in Chapter 5 financing costs are assumed constant.

In contrast to the many discussions which exist in the literature on the theoretical effects of accelerated depreciation on investment, empirical analyses are almost nonexistent. The author is aware of only two (as yet unpublished) papers in which an attempt is made to determine empirically the effects of accelerated depreciation. The first is a paper by R. E. Hall and D. W. Jorgenson. The second is a Doctoral Dissertation by R. M. Coen.[11] The analyses are very similar in that they both assume that investment expenditures depend on the difference between the existing capital stock and a desired capital stock. The latter is made a function of the "user cost" of capital, and this cost in turn depends on the pdv of depreciation deductions earned by the assets involved (assuming a positive corporate tax rate of course). A change in the pattern of such deductions therefore changes their present discounted value, and hence the desired and actual capital stocks. (This alleged direct dependence of investment expenditures on the discounted value of depreciation deductions provides additional motivation for the pdv analysis of the next chapter.)

Probably the major reason for the lack of empirical work in this field is the fact that any such analysis will of necessity depend crucially on the nature of the investment

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[11] R. E. Hall and D. W. Jorgenson, "Tax Policy and Investment Behaviour", 1966, (to be published in the A.E.R.), and R. M. Coen, Accelerated Depreciation, The Investment Tax Credit, and Investment Decisions, (Preliminary), Unpublished Manuscript, December, 1965.

function assumed. The papers mentioned above, for example, rely entirely on the assumption that investment expenditures respond (in a specified manner) to changes in the present discounted value of depreciation deductions earned on fixed assets. This means that entrepreneurs must employ precise discounting procedures, or act as if they did, when making investment decisions. If entrepreneurs do not in general use discounting methods, another formulation of investment behaviour might be more appropriate. The real problem then lies in the fact that, as mentioned above, a wide variety of determinants are hypothesized to affect investment while attempts to describe investment behaviour econometrically have not resulted in the general acceptance of any particular investment function.

The model of investment behaviour hypothesized in this paper and studied in detail in Chapter 7 is oriented more towards the profit models in the literature than towards the Jorgenson capital model as outlined above. The assumption is made that the firm's cash flow (depreciation plus net profits) plays a major role in influencing investment decisions. The motivation for making such an assumption is the preference (some reasons for which are discussed in Chapter 4) of entrepreneurs for internal funds. Pressure on capacity, the availability of external funds, and the current liquid position of the firm are also assumed to affect investment.

The investment equation is postulated to be one of a system of equations that involves a simultaneous determination of investment, dividend, and external finance behaviour. Dividend and investment expenditures both rely heavily on cash flow, which in turn depends on investment due to its depreciation component. The level of external finance is affected by investment opportunities relative to the supply of internal funds, while investment itself is influenced by the availability of external finance. The budget constraint of the firm requires that these decisions be consistent.

Dividend behaviour in general is assumed to follow the basic Lintner model in which the change in dividends in any period represents partial adjustment towards a desired level of dividends, with the latter being a constant fraction of cash flow.[12] Variations from this pattern may result due to differences in the liquid position of the firm. The cash flow variable is used rather than net profits in view of recent findings by several authors which suggest that cash flow is the superior income variable.[13] By far the most

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[12]John K. Lintner, "Distribution of Incomes of Corporations Among Dividends, Retained Earnings, and Taxes", Proceedings of the American Economic Review, Vol. 46, No. 2, (May, 1956), pp. 97-113.

[13]See in particular John A Brittain, Corporate Dividend Policy, The Impact of the Tax Structure and Other Factors, (Preliminary Manuscript), March, 1965; R. Sutch, "Some Comments on Corporate Dividend Behaviour", Unpublished Manuscript, January, 1966; R. Gordon, "Explaining Corporate Payout Behaviour", Unpublished Manuscript, July, 1965, and E. Kuh, "Income Distribution over the Business Cycle", Chapter 8 of The Brookings Quarterly Econometric Model of the United States, Chicago, 1965, pp. 275-278.

comprehensive treatment of the subject is provided by J. A. Brittain, whose basic behavioural hypothesis is that firms are aware of the depressing effect of changing depreciation provisions on their ability to pay dividends, and will take this into account when making such payments. Three arguments are offered in support of the proposition. First, firms may think of depreciation as a purely accounting charge in which case cash flow will be viewed as one source of funds to be distributed between dividends and investment. Second, firms may regard stability of dividends more important than investment expenditures, and consequently finance dividends directly from cash flow. Finally, in a period of changing depreciation regulations firms will desire to utilize consistent depreciation rules for determing dividend payments, and for simplicity may use cash flow as an approximation.

The external finance behaviour of firms is analysed in considerable detail. Such behaviour is assumed to depend not only on current investment expenditure and the supply of internal funds but also on the cost of financing externally, the firm's current liquid position, and the relation of long term debt to equity. The latter assumes that borrowing decisions are influenced by the difference between an optimal and the actual debt-equity ratio. An attempt is made to determine whether the resort to outside funds is best represented by past, current, or future expectations of investment expenditure, and to determine if it depends in a nonlinear fashion on financing needs. The major components of external finance, long term bank borrowing and corporate bond issues, are analysed separately in an attempt to determine the extent to which their determinants differ.

In a recent book by W. H. Locke Anderson an attempt is made to explain investment in fixed assets, short and long term borrowing, and the accumulation of cash and government securities.[14] The analysis is based on quarterly time series data for the two-digit manufacturing industries. A major drawback of the study is its failure to allow for simultaneity in the estimation procedure while stressing the interdependence of financial decisions in the theoretical discussion.

The author is aware of only one other study in the literature in which a simultaneous model of investment, dividend, and external finance behaviour is statistically estimated. It is a recent paper by P. J. Dhrymes and M. Kurz involving a cross section analysis similar in some respects to the time series analysis presented here.[15]

The reduced form of such a system of equations may be used to determine the effect of any method of accelerated

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[14] W. H. Locke Anderson, Corporate Finance and Fixed Investment, Boston, 1964.

[15] Phoebus J. Dhrymes and Mordecai Kurz, Investment, Dividend and External Finance Behaviour of Firms, (Preliminary), presented at the Conference on Investment Behaviour, sponsored by Universities-National Bureau Committee for Economic Research, June 10-12, 1965.

depreciation. Of particular interest are the changes in the 1954 and 1962 Revenue Acts. The mechanism through which accelerated depreciation affects the endogenous variables is of course by changing the pattern of depreciation deductions, and therefore cash flow, over time. Variations in cash flow result in variations in dividend payments, the level of external finance, and investment expenditures, with the latter feeding back onto cash flow through a further change in depreciation deductions. Using an initial set of lagged endogenous variables and the actual values of exogenous variables, the reduced form can be used to generate values of endogenous variables which are functions of any desired accelerated depreciation parameter.

The following identity (in simplified form) contains the depreciation parameters which can be altered in order to analyse the different methods.

$$D_t = D_{t-1} + v_t I_t + C_t + R_t$$

D_t is depreciation, v_t is the depreciation rate applied against current investment, C_t is a correction term which is required if the depreciation method results in unequal deductions over time, (and is therefore required for all methods but SL), and R_t is current retirements of fixed assets. By appropriately adjusting C_t and v_t and then using the reduced form to generate values of endogenous variables, any method of accelerated depreciation may be analysed. For example, if SYD were used instead of SL, then basically v_t would be

$2(n-k+1)/n(n+1)$ rather than $1/n$, and C_t would have to be adjusted to take into account the fact that under SYD deductions on any asset decrease by an amount equal to $2/n(n+1)$ each year. A reduction in asset life for tax purposes from n_2 to n_1 would require that n_1 be used instead of n_2 in computing v_t and C_t. This general method of analysis is used in Chapter 8 in an attempt to determine the effects of the liberalized depreciation provisions introduced in 1954 and 1962.

A major problem in determining the effect of the introduction of accelerated methods in 1954 arises in connection with the fact that entrepreneurs did not immediately accept such methods but adopted them only slowly over the years. A problem arises because there is no direct information available on the extent of use of the accelerated methods by two-digit industry, nor is the author aware of any estimates of their use. Clearly information on the rate of adoption is required as a part of the parameter v_t in the depreciation identity given above. That is, in analysing a switch from SL to SYD, v_t will be a weighted average of the two depreciation rates, with weights equal to the amounts of investment written off under the two methods.

For this reason an attempt is made in Chapter 6 to estimate an adoption rate or learning function for accelerated depreciation. Satisfactory results are obtained for all industries but textiles and petroleum. Although there appears to be no good reason for the poor results in the textile industry, there is in the case of the petroleum industry, in that depletion provisions may make accelerated depreciation less advantageous than straight line. The statistical techniques derived and used in obtaining learning function estimates have not, to the author's knowledge, appeared in the literature.

Before turning to the analysis of pdv effects in the next chapter, a few details will be given concerning the different methods of accelerated depreciation.

As mentioned above, DDB results in a deduction in year k of an asset's life of an amount equal to $(2/n)(1-2/n)^{k-1}$. Since $\sum_{k=1}^{n} (2/n)(1-2/n)^{k-1} = 1 - (1-2/n)^n$ it is clear that at the end of n years the asset will not be completely depreciated. For this reason the law permits a switch from DDB to SL at any time during the asset's life. Profit maximization requires a switch when the annual deductions under the two methods are equal, and for an asset with life n this occurs in year n/2+1, calculated as follows. The percent of cost written off after t years is given by $\sum_{k=1}^{t} (2/n)(1-2/n)^{k-1} = 1-(1-2/n)^t$, leaving $(1-2/n)^t$ for later years. The switch occurs in year k+1 determined by equating deductions: $(1-2/n)^k/(n-k) = (2/n)(1-2/n)^k$, from which $k+1 = n/2+1$.

If complete rationality is not assumed and switching does not occur, it can be shown that for certain values of the discount rate (r) and asset life (n), the SL method results in a higher present discounted value of deductions than the DDB method. That is, by solving for the value of r which equates deductions under the two methods for a given n, one obtains the discount rate below which the present value of the deductions using SL exceeds that of DDB. For asset lives of 5, 10, 15, 20 and 25 years respectively the critical discount rate is approximately given by 9, 7, 5, 4, and 3%. Such calculations illustrate the crucial role of the discount rate in the definition of accelerated depreciation.

The pdv and liquidity computations under DDB are complicated if switching is assumed, and the problem mentioned above is encountered if it is not. For this reason, the SYD method of depreciation is used in the analysis in Chapters 2, 3, and 4 to represent both the accelerated methods (DDB and SYD) introduced in 1954. The error involved in using SYD in place of DDB is small, since the two methods (assuming switching) result in essentially the same pattern of deductions.

An initial allowance, which results in a larger deduction in the first year with an equal reduction in later years, is more beneficial under SL than SYD or DDB. This follows (for $n > 2$) because, although the gain is always taken in the first year, the write-down of the base occurs closer to the present using an accelerated method. For $n = 2$ there is no difference since the remainder of the asset is completely depreciated in the second year in any case. Of course the combination of SYD or DDB and an allowance remains preferable to SL and an allowance.

The continuous formulations of the three methods of depreciation are used to some extent in the analysis in order to simplify the mathematics. The SYD rate is the only one in which a change is evident, since the SL and DDB rates remain as $1/n$ and $2/n(1-2/n)^k$ respectively. The continuous SYD rate applicable at time k of an asset's life is given by $2(n-k)/n^2$ and since $\int_0^n 2(n-k)/n^2 dk = 1$, the asset is completely depreciated as required.

In practice the depreciable base of an asset must be reduced by its estimated salvage value before applying the SL or SYD methods, but not the DDB method. Since there is little to be gained in the theoretical discussions from assuming varying amounts of salvage (in relation to cost), and since no relevant data exist for the empirical work, salvage considerations are ignored in the analysis to follow.

Table 1.1

COMPARISON OF 1962 GUIDELINE LIVES AND AVERAGE LIVES USED IN PRACTICE (1962)

Industry Description	Industry Number	Current Lives	Guideline Lives
Food and Beverage	20	15	13
Textile-mill Products	22	16	13
Paper and Allied Products	26	19	15
Chemicals and Allied Products	28	13	11
Petroleum and Coal Products	29	18	15
Rubber Products	30	14	13
Stone, Clay, and Glass Products	32	18	16
Primary Metal Industries	33	21	17
Machinery except Transportation and Electrical	35	14	12
Electrical Machinery and Equipment	36	14	11
Motor Vehicles and Equipment	371	14	12
Transportation Equipment Except Motor Vehicles	372	12	9

Source: Based on asset lives in the Treasury Depreciation Survey, Treasury Department, Office of Tax Analysis, November, 1961, Table 1, (Unpublished), and Depreciation Guidelines and Rules, (Revenue Procedure 62-61), U.S. Treasury Department, I.R.S., Publication No. 456, Revised, August, 1964, pp. 6-13. For any industry in which more than one Guideline life appears in Revenue Procedure 62-61 the entry in Table 1.1 is a weighted average (using 1962 investment values) of these lives. All asset lives have been rounded to the nearest integer.

Chapter 2

THE EFFECT OF ACCELERATED DEPRECIATION ON PRESENT DISCOUNTED VALUES

As mentioned in Chapter 1 the two major effects of accelerated depreciation are the pdv and liquidity effects. The purpose of this chapter is to analyse the former. The four basic methods of accelerated depreciation to be considered are: a switch from SL to SYD, the introduction of an initial allowance, the introduction of an investment credit, and the adoption of a shorter asset life for tax purposes. As mentioned above reasons for making such calculations rest on the assumption that investment is affected by pdv changes. Although the precise elasticity of investment with respect to such changes may be unknown, the calculations are of interest in that when combined with order of magnitude elasticity estimates, they provide some idea of the orders of magnitude involved. A comparison of incentives across methods as well as for different asset lives and interest rates is also of interest.

It should be recalled that the pdv analysis in this chapter is concerned with a single asset while the liquidity analysis in the next chapter involves a (constant or growing) stream of assets.

The Effect on PDV of a Switch from SL to SYD

Let n be the tax life of an asset and r the rate at which deductions are discounted. The change in net revenue in any period from using SYD instead of SL equals the change in depreciation deduction for that period times the corporate tax rate. Discounting these changes by the rate r and summing gives the change in present discounted value. Assuming SL and continuous discounting, the present value of depreciation deductions is given by:

$$PDV(SL) = \int_0^n e^{-rt}/n \; dt$$

and under SYD the corresponding expression is:

$$PDV(SYD) = \int_0^n (2(n-t)/n^2)e^{-rt}dt.$$

Let y_s = PDV(SYD)-PDV(SL), then y_s times the corporate tax rate is the gain in discounted value from using SYD. Table 2.1 gives values of y_s for selected r and n.[1]

From the table it can be seen that y_s is neither a monotonic function of r for fixed n, nor of n for fixed r. Considering y_s first as a function of r only, the introduction of SYD increases the present value of early deductions and decreases the value of later ones. A higher discount rate reduces both early and late deductions. The discount rate for which the gain in deductions is a maximum is therefore the one for which a higher rate reduces near deductions more than it reduces future ones. For each n this value of r can be calculated by setting the partial derivative of y_s with respect to r equal to zero, and solving to obtain r. Table 2.2 contains such values of r for n less than 40 years, although

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[1]Unless otherwise stated all such tabulations of changes resulting from an acceleration of depreciation are based on annual rather than continuous discounting.

values of r greater than 25% are not recorded.

From the fact that y_s declines for large values of r the conclusion is often drawn that the benefits from switching to SYD decrease for risky assets. That is, the discount rate in the preceding calculations may be thought of as playing a dual role -- that of discounting for time per se and for risk. A time discount rate is applied because revenues are received in the future. If uncertainty is involved in the outcome a risk discount factor may be applied as well. Generally the latter will be an increasing function of time since more risk is associated with distant revenues, either because of greater probability of not receiving them or they are predicted with less certainty. One plausible manner in which to discount for risk is to discount revenues in year t by $(1+r)^t$ thus resulting in a discounted value calculation of the usual sort.[2] But since there are an infinite number of ways to discount, each depending on predictions about the future, different conclusions from those based on Tables 2.1 and 2.2 might be reached.

Even if the particular assumption that revenues in period t are discounted for risk by $(1+r)^t$ is accepted, care must be taken in interpreting Tables 2.1 and 2.2 since the discount rate appearing in the tables combines both the time and risk factors. Let r_1 be the time, and r_2 the risk discount

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[2]For simplicity it is assumed (although perhaps unrealistically) that the same risk discount rate is applied to gross revenues as to depreciation deductions.

rate, then r in Tables 2.1 and 2.2 is related to these two rates by the following equation:

$$(2.1) \quad (1+r) = (1+r_1)(1+r_2)$$

This means that the risk discount rates for which the benefits from accelerated depreciation are a maximum are considerably less than the values given in Table 2.2. For example since y_s for n = 20 reaches a maximum at r = .13, then for a time discount rate of .05, the risk discount rate (r_2) which maximizes pdv may be calculated as:

$$(1+r_2)(1.05) = 1.13 \text{ or } r_2 = .076$$

In conclusion, if risk is associated with discounting revenues in period t by $(1+r_2)^t$, then for any given asset life (n) and time discount rate (r_1) there exists a value of the risk discount rate r_2^*, above which the gain from accelerated depreciation decreases. Using Table 2.2 and equation (2.1) it can be seen that for large n and a high time discount rate, r_2^* may well equal zero. If this is the case, any discount for risk decreases the benefit derived from accelerated depreciation, and the maximum incentive is for investment in riskless assets.

If y_s is considered as a function of n only, an analysis of the same form as above would reveal for any r, the n which maximizes the gain. This has not been done but an idea of the orders of magnitude involved can be obtained from Table 2.1. For example, discount rates of .16 and .24 yield the maximum advantage for assets with lives of approximately 20 and 16 years respectively.

Present value calculations for a change from SL to DDB (assuming the switching provision is used) are not presented but it is clear that the results will be similar to those given in Table 2.1. On the other hand, as stated in the preceding chapter, if DDB is used ignoring the switching provision then for low discount rates the SL method will have a higher pdv than the DDB method.

<u>The Effect on PDV of an Initial Allowance</u>

As mentioned in Chapter 1, the introduction of an initial allowance is more beneficial if SL rather than an accelerated method such as SYD is in use. An initial allowance of b% of cost results in a gain in deductions in the first year of b, with a corresponding loss of b over the remaining n-1 years. The discounted value of the net gain from introducing the allowance, assuming SL is in use (and before multiplying by the tax rate) is therefore given by:

$$y_a(SL) = b\int_0^1 e^{-rt}dt - b\int_1^n e^{-rt}/(n-1)\,dt$$

The corresponding expression assuming SYD is:

$$y_a(SYD) = b\int_0^1 e^{-rt}dt - b\int_1^n (2(n-t)/(n-1)^2)e^{-rt}dt$$

Note that the loss of b in deductions is spread over n-1 years in proportion to a depreciation rate applicable to an asset of n-1 years. For this reason 1/(n-1) is the SL rate in the second part of $y_a(SL)$ and $((n-1)-(t-1))/(n-1)^2$ is the corresponding SYD rate. Values of $y_a(SL)$ and $y_a(SYD)$ appear in Table 2.3 for b = 100% and selected r and n. In order to compare the effects resulting from an allowance with other accelerated methods the entries in this table must be multiplied by the value of the allowance.

A priori one would expect the gain from using the allowance to follow the same pattern with respect to the discount rate as the gain from using SYD in place of SL. That is, for a given asset life the gain should increase with r at first and then decrease. Table 2.3 shows this to be the case, although the effect is not very pronounced because the only year with increased deductions is the first.

With respect to asset lives, however, there is a basic difference between the gain resulting from an initial allowance and that from a switch to SYD. In the case of an allowance, the net gain increases monotonically with n for any acceptable depreciation allowance (defined below). This proposition is not difficult to prove.

Let y_a = the increase in pdv resulting from an initial allowance

b = initial allowance as a percent of cost

r = discount rate

n = asset life

T = corporate tax rate

h(t,n) = depreciation deduction on an asset of age t with life n. h(t,n) must satisfy the following conditions for all $n > 0$.

(a) $h(t,n) > 0$ for $0 \leq t \leq n$

$= 0$ for $t > n$

(b) $\int_0^n h(t,n)\, dt = 1$

(c) $d/dn(h(t,n)) \leq 0$ for $0 \leq t \leq n$

Condition (a) states that all deductions over the asset's life must be positive, (b) requires that the total deduction be equal to cost, and (c) requires that the deduction in any year be smaller (or the same) for a longer lived asset.

y_a is therefore given by:

$$(2.2)\quad y_a = bT\int_0^1 e^{-rt}dt - bT\int_1^n h(t-1,n-1)e^{-rt}dt$$

and differentiating y_a with respect to n gives:

$$(2.3)\quad dy_a/dn = -bT(h(n-1,n-1)e^{-rn} + \int_1^n d/dn\ (h(t-1,n-1))e^{-rt}dt$$

From condition (b) above, $\int_1^n h(t-1,n-1)dt = 1$ and differentiating with respect to n gives:

$$h(n-1,n-1) + \int_1^n d/dn(h(t-1,n-1))\ dt = 0$$

Substituting for h(n-1,n-1) in (2.3) yields:

$$(2.4)\quad dy/dn = -bT(\int_1^n (d/dn(h(t-1,n-1)))(e^{-rt} - e^{-rn})dt)$$

But $e^{-rt} - e^{-rn} \geqslant 0$ for t = 1,n and $d/dn(h(t-1,n-1)) \leqslant 0$ from condition (c). Therefore the integral in (2.4) is negative since it consists of all negative terms, and hence dy_a/dn itself is positive.

This shows that the gain from an initial allowance is an increasing function of n, which is a plausible result if the allowance is thought of as an interest free loan in the first year, to be paid back over the life of the asset. The longer the life the more benefit is obtained. A switch from SL to SYD can not be thought of in these terms because the period during which the loan occurs is not restricted to the first year, but varies with the asset life.

Using y_a as defined above in (2.2) it can be shown that the gain from an allowance, as a function of r, increases at first and then decreases. Differentiating y_a with respect to r gives: $dy_a/dr = -(bT/r)\int_0^1 e^{-rt}tdt + (bT/r)\int_1^n h(t,n)e^{-rt}tdt$ Since the second term is positive, dy_a/dr would be positive if it were not for the fact that the gain is taken over the first period and must be discounted. For small r the first period discounting will be unimportant and dy_a/dr will be positive, but for large r the first term will dominate. This shows that for all depreciation functions h(t,n) the gain from an initial allowance increases at first, but decreases for r greater than some r^*, which depends on h(t,n).

The Effect on PDV of an Investment Credit

The change in discounted value resulting from an investment credit is simply the amount of credit k discounted by r over the first period, that is, $k\int_0^1 e^{-rt}dt$. This value decreases with r and is independent of n and the corporate tax rate.

The investment credit introduced in the 1962 Revenue Act consists of a 7% tax credit in the first period together with a write-down of the base over the asset's life. The required write-down of the base means that the pdv of the credit will depend on the asset's life and the corporate tax rate. The gain in pdv resulting from such a credit is given by: $y_k = .07\int_0^1 e^{-rt}dt - .07T\int_0^n h(t,n)e^{-rt}dt$

As with an initial allowance $dy_k/dn > 0$ for all n but $dy_k/dr > 0$ only for r less than some r^*.

Table 2.4 gives values of the change in pdv resulting from the 1962 credit for various asset lives and discount rates. Since the credit is applicable only to machinery and equipment, asset lives greater than 24 years are not presented. The required reduction in credit for lives of less than 8 years is taken into account in the calculations. The table indicates that credit is more beneficial for assets with long lives and if SL rather than SYD is in use. The absolute gain does not appear to vary much with the discount rate, but the pattern of increase followed by decrease, as r increases is discernible.

<u>The Effect on PDV of a Change in Asset Life</u>

Let n_1 be the new shorter asset life for tax purposes, and n_2 the old life. Then the increase in pdv due to using the shorter tax life is the difference between the depreciation deductions under the two lives. This increase in pdv may also be thought of as resulting from a change in deductions of $h(t,n_1)-h(t,n_2)$ in the first n_1 years and of $-h(t,n_2)$ in the remaining n_2-n_1 years, where $h(t,n)$ is the depreciation deduction on an asset of age t with life n. The increase in pdv is then:

$$y_c = \int_0^{n_1} (h(t,n_1)-h(t,n_2))e^{-rt}dt - \int_{n_1}^{n_2} h(t,n_2)e^{-rt}dt$$

$$= \int_0^{n_1} h(t,n_1)e^{-rt}dt - \int_0^{n_2} h(t,n_2)e^{-rt}dt$$

which shows that y_c equals the difference between depreciation deductions under the two lives.

Since it is inconvenient to tabulate y_c extensively, values are presented in Table 2.5 only for changes in asset lives which approximate the 1962 revisions for the two-digit manufacturing industries. Values are tabulated under the assumption of both SL and SYD methods in use, and for various discount rates. The fact that the positive changes in pdv precede the negative changes means that the benefit from a reduction in asset life increases with r at first and then decreases, as indicated in the table.

Comparison of Different Methods of Accelerated Depreciation

Before comparing the benefits obtained from the various methods, a summary is given of the general behaviour of such benefits with respect to asset life and discount rate changes. Considering the former it appears that the maximum incentive resulting from a switch to SYD ranges from assets of over 40 years (with a discount rate of .04) to 12 years (with a rate of .28). For an initial allowance and investment credit the gain in pdv increases monotonically with the asset's life. Therefore if pdv calculations affect investment and if substitutibility exists among assets with different lives, the latter two methods provide an incentive towards investment in longer lived assets.

With respect to the discount rate the gain from switching (although remaining substantial) decreases for large r. For an initial allowance or investment credit this diminishing effect is not nearly as pronounced since the only gain in depreciation deductions occurs in the first year. Tables 2.3

and 2.4 indicate that except for very large n and r, the gain from these forms of accelerated depreciation is an increasing function of r. The benefit from a reduction in asset life follows the same pattern as that from a switch to SYD, and falls rapidly for large r because the gain in depreciation deductions is spread over the original life of the asset (under SL).

The dependence of pdv changes on the discount rate is of interest primarily because the latter generally includes an element of risk. In this respect care must be taken in interpreting Tables 2.1 - 2.6. However, if it is assumed that revenues in period t are discounted for risk by $(1+r_2)^t$, then for a wide range of time and risk discount rates, the gain from accelerated depreciation decreases with the riskiness of the asset. This suggests that for a switch to SYD and for a change in asset life, the main incentive will occur in less risky or riskless assets. On the other hand the opposite incentive occurs, except for very large r and n, following the introduction of an initial allowance or investment credit.

The increase in the pdv of depreciation deductions times the corporate tax rate forms the basis for comparing a switch to SYD, an initial allowance, and a reduction in asset life. Since the first year gain from an investment credit is independent of the tax rate only the later pdv changes must be multiplied by this rate. Table 2.4 is calculated in such a manner assuming a tax rate of 50%. The entries in Tables 2.1 (switch from SL to SYD) and 2.5 (reduction in tax life) must be multiplied by the corporate tax rate in order to give comparable values. Table 2.3 entries must be multiplied by the tax rate and an initial allowance rate. Computations may be carried out for various initial allowance and tax rates in order to compare the different methods (but remembering that the credit calculations assume a 50% tax rate).

One such comparison is presented in Table 2.6, which contains the relevant pdv changes for the four methods (as well as a true 7% credit to be discussed below) assuming a tax rate of 50%, an initial allowance of 20%, and asset lives approximating actual average lives used in the two-digit manufacturing industries. It is assumed that SYD is in use and that the shorter asset lives are relevant for the introduction of the investment credit and initial allowance. The longer asset lives are assumed to be relevant for the switch from SL to SYD, and the reduction in asset lives is assumed to take place under SYD. These assumptions are intended to approximate conditions existing at the time of introduction of the 1954, 1958, and 1962 liberalized depreciation provisions. For the case of the 1958 allowance longer asset lives would probably be more appropriate, and since SYD had not been completely adopted by entrepreneurs at that time SL depreciation would have some relevance. On the other hand the assumptions made above permit a direct comparison to be made of the allowance and the 1962 credit.

The final entry in Table 2.6 contains the changes in pdv resulting from a true 7% investment credit. Since the credit involves no write-down of the asset's base these values depend only on the discount rate, decreasing slightly with the latter since it is assumed that the benefit accrues during the first year. It is clear that the gain from a true 7% credit is greater than the gain from any other method of accelerated depreciation, for the asset lives and discount rates recorded here.

For discount rates of 8% or more the switch from SL to SYD results in a larger gain in pdv for all industries than any of the other methods of accelerated depreciation, except the true 7% credit. Industry 33, with the longest asset life ($n_2 = 21$), obtains the maximum benefit from the switch for discount rates of 16% or less. For higher rates no generalizations are possible and the gain obtained by almost all industries is roughly the same; although it should be noted that for a discount rate of 28% the maximum gain is obtained by the industry (28) with the shortest asset life.

The benefit from the 1962 credit, which essentially includes an element of fixed subsidy, does not vary much with the discount rate or with the asset life, but does of course increase with the latter. In this respect the maximum benefit is obtained by Industry 33, for which $n_1 = 17$, although such an advantage, relative to changes resulting from other methods of accelerated depreciation, appears to be very small.

The benefit from the initial allowance follows the same pattern with respect to the discount rate and asset life as does the credit, but of course shows more variability. Industry 33 again obtains the maximum benefit. It is interesting to note that for discount rates in the range 16-24% the gain from a 20% initial allowance is approximately the same as that from the 1962 credit for most industries.

The gain from the reduction in lives varies considerably with Industry 30 obtaining the least benefit, due to a change from 14 to 13 years, and Industry 372 the most, due to a reduction from 12 to 9 years. The gain resulting from a large reduction in asset life is of course partially offset by the reduction in gain due to taking the 1962 credit on a short life. Table 2.6 indicates that the former considerably outweighs the latter. Industries 35 and 36 provide a good example. The original asset life is 14 years in each case, with a reduction to 12 years in the former and 11 years in the latter industry. For a discount rate of 16% the difference between the industries in the pdv increase due to the asset life change is .011, while the investment credit difference is negligible.

Table 2.1

CHANGE IN PDV DUE TO A SWITCH FROM SL TO SYD

n \ r	4	8	12	16	20	24	28
4	.018	.032	.043	.052	.058	.064	.068
8	.038	.064	.081	.092	.099	.102	.104
12	.056	.087	.104	.112	.115	.114	.112
16	.071	.104	.117	.121	.119	.115	.109
20	.084	.115	.124	.122	.117	.110	.103
24	.094	.123	.126	.120	.112	.103	.095
28	.103	.127	.126	.117	.106	.096	.087
32	.110	.130	.124	.112	.100	.090	.081
36	.116	.130	.120	.107	.094	.083	.075
40	.121	.130	.117	.102	.089	.078	.069

n = asset life in years
r = discount rate in percent

Table 2.2

VALUES OF THE DISCOUNT RATE WHICH MAXIMIZE THE GAIN FROM SWITCHING TO SYD FROM SL

n	r	n	r	n	r
10	25	22	12	32	8
12	21	24	11	34	8
14	18	26	10	36	7
16	16	28	9	38	7
18	14	30	9	40	7
20	13				

n = asset life in years
r = discount rate in percent

Table 2.3

CHANGE IN PDV DUE TO AN INITIAL ALLOWANCE OF 100%
ASSUMING SL IN USE

n \ r	4	8	12	16	20	24	28
4	.072	.131	.178	.217	.248	.274	.295
8	.137	.237	.311	.365	.404	.433	.453
12	.196	.325	.411	.468	.506	.530	.544
16	.249	.398	.487	.542	.574	.591	.600
20	.297	.458	.547	.595	.621	.633	.636
24	.340	.508	.593	.636	.655	.661	.660
28	.380	.551	.630	.666	.680	.682	.678
32	.416	.587	.660	.690	.699	.698	.691
36	.449	.618	.684	.709	.714	.710	.702
40	.479	.644	.704	.724	.727	.720	.710

CHANGE IN PDV DUE TO AN INITIAL ALLOWANCE OF 100%
ASSUMING SYD IN USE

n \ r	4	8	12	16	20	24	28
4	.060	.110	.151	.185	.213	.236	.255
8	.105	.185	.245	.292	.328	.356	.376
12	.146	.249	.322	.375	.412	.439	.457
16	.184	.305	.385	.439	.475	.498	.513
20	.219	.353	.437	.490	.523	.543	.554
24	.252	.396	.481	.531	.560	.577	.585
28	.283	.434	.518	.565	.590	.603	.608
32	.312	.467	.549	.593	.615	.625	.627
36	.338	.497	.576	.616	.635	.642	.642
40	.363	.523	.599	.636	.652	.656	.655

n = asset life in years
r = discount rate in percent

Table 2.4

CHANGE IN PDV DUE TO THE 1962 INVESTMENT CREDIT
ASSUMING SL IN USE

n \ r	4	8	12	16	20	24	28
4	.012	.012	.012	.012	.012	.012	.012
5	.012	.012	.012	.012	.012	.012	.012
6	.024	.025	.026	.026	.026	.026	.026
7	.025	.026	.026	.027	.027	.027	.027
8	.038	.040	.041	.041	.042	.041	.041
10	.039	.041	.043	.043	.044	.044	.043
12	.040	.043	.044	.045	.045	.045	.045
14	.041	.044	.046	.047	.047	.047	.046
16	.042	.045	.047	.048	.048	.048	.047
18	.043	.047	.048	.049	.049	.049	.048
20	.044	.048	.049	.050	.050	.049	.048
22	.044	.049	.050	.051	.051	.050	.049
24	.045	.049	.051	.051	.051	.050	.049

CHANGE IN PDV DUE TO THE 1962 INVESTMENT CREDIT
ASSUMING SYD IN USE

n \ r	4	8	12	16	20	24	28
4	.012	.012	.011	.011	.011	.011	.011
5	.012	.012	.012	.012	.012	.012	.011
6	.024	.024	.024	.024	.024	.024	.024
7	.024	.025	.025	.025	.025	.025	.024
8	.037	.037	.038	.038	.038	.038	.038
10	.037	.039	.039	.040	.040	.040	.039
12	.038	.040	.041	.041	.041	.041	.041
14	.039	.041	.042	.043	.043	.043	.042
16	.039	.042	.043	.044	.044	.044	.043
18	.040	.043	.044	.045	.045	.045	.044
20	.041	.044	.045	.046	.046	.045	.045
22	.041	.044	.046	.047	.047	.046	.046
24	.042	.045	.047	.047	.047	.047	.046

n = asset life in years
r = discount rate in percent
A corporate tax rate of 50% is assumed.

Table 2.5

CHANGE IN PDV DUE TO REDUCTION IN ASSET LIFE

			r						
Ind	n_1	n_2	4	8	12	16	20	24	28
20	13	15	.027	.037	.040	.039	.037	.034	.031
			.019	.029	.034	.035	.035	.034	.033
22	13	16	.040	.055	.058	.057	.053	.049	.045
			.029	.043	.049	.051	.051	.049	.048
26	15	19	.050	.065	.066	.062	.057	.051	.046
			.037	.053	.058	.059	.058	.055	.052
28	11	13	.028	.041	.046	.046	.045	.042	.040
			.020	.031	.037	.040	.040	.040	.039
29	15	18	.038	.050	.051	.048	.044	.040	.036
			.028	.040	.045	.046	.046	.043	.041
30	13	14	.014	.019	.021	.020	.019	.018	.017
			.010	.015	.017	.018	.018	.018	.017
32	16	18	.025	.033	.033	.031	.028	.025	.022
			.018	.026	.029	.030	.029	.027	.026
33	17	21	.048	.060	.059	.054	.048	.043	.039
			.036	.049	.053	.053	.051	.048	.045
35	12	14	.028	.039	.043	.043	.041	.038	.035
			.020	.030	.035	.037	.038	.037	.036
36	11	14	.042	.060	.067	.067	.064	.062	.056
			.030	.046	.054	.058	.058	.058	.056
371	12	14	.028	.039	.043	.043	.041	.038	.035
			.020	.030	.035	.037	.038	.037	.036
372	9	12	.044	.066	.076	.079	.078	.075	.072
			.031	.049	.060	.065	.067	.068	.067

Ind = industry
n_1 = average asset life after 1962 Guideline change
n_2 = average asset life before 1962 Guideline change
r = discount rate
The first line of the table for each industry is based on SL in use and the second line on SYD.

Table 2.6

COMPARISON OF PDV EFFECTS FOR DIFFERENT METHODS

			r						
Ind	n_1	n_2	4	8	12	16	20	24	28
20	13	15	.038	.040	.041	.042	.042	.042	.042
			.016	.026	.034	.039	.043	.046	.047
			.033	.050	.057	.059	.058	.057	.055
			.010	.015	.017	.017	.017	.017	.016
22	13	16	.038	.040	.041	.042	.042	.042	.042
			.016	.026	.034	.039	.043	.046	.047
			.035	.051	.058	.060	.059	.057	.054
			.014	.021	.024	.025	.025	.024	.024
26	15	19	.039	.041	.043	.043	.043	.043	.043
			.017	.029	.037	.042	.046	.049	.050
			.041	.056	.061	.061	.059	.055	.052
			.018	.026	.029	.029	.029	.027	.026
28	11	13	.038	.039	.040	.041	.041	.040	.040
			.014	.023	.030	.036	.039	.042	.044
			.030	.046	.054	.057	.058	.057	.056
			.010	.015	.018	.020	.020	.020	.020
29	15	18	.039	.041	.043	.043	.043	.043	.043
			.017	.029	.037	.042	.046	.049	.050
			.033	.050	.057	.059	.059	.057	.055
			.014	.020	.022	.023	.023	.022	.020
30	13	14	.038	.040	.041	.042	.042	.042	.042
			.016	.026	.034	.039	.043	.046	.047
			.031	.048	.055	.058	.058	.057	.055
			.005	.017	.018	.019	.019	.019	.018
32	16	18	.039	.042	.043	.044	.044	.044	.043
			.018	.030	.039	.044	.047	.050	.051
			.033	.050	.057	.059	.059	.057	.055
			.009	.013	.014	.015	.014	.013	.013
33	17	21	.040	.042	.044	.044	.044	.044	.044
			.019	.032	.040	.045	.049	.051	.052
			.043	.058	.062	.061	.057	.054	.050
			.018	.024	.026	.026	.025	.024	.022

Table 2.6 (Continued)

Ind	n_1	n_2	r = 4	8	12	16	20	24	28
35	12	14	.038	.040	.041	.041	.041	.041	.041
			.015	.025	.032	.037	.041	.044	.046
			.033	.048	.055	.058	.058	.058	.055
			.010	.015	.017	.018	.019	.018	.018
36	11	14	.038	.039	.040	.041	.041	.040	.040
			.014	.023	.030	.036	.039	.042	.044
			.033	.048	.055	.058	.058	.058	.055
			.015	.023	.027	.029	.029	.029	.028
371	12	14	.038	.040	.041	.041	.041	.041	.041
			.015	.025	.032	.037	.041	.044	.046
			.033	.048	.055	.058	.058	.058	.055
			.010	.015	.017	.018	.019	.018	.018
372	9	12	.037	.038	.039	.039	.039	.039	.039
			.011	.020	.027	.031	.035	.038	.040
			.028	.043	.052	.056	.057	.057	.056
			.015	.024	.030	.032	.033	.034	.033
	Credit		.069	.067	.060	.065	.063	.062	.061

Ind = industry
n_1 = average asset life after 1962 Guideline change
n_2 = average asset life before 1962 Guideline change
r = discount rate
For each industry:
Line 1 = 1962 investment credit (assuming SYD and n_1 in use)
Line 2 = 20% initial allowance (assuming SYD and n_1 in use)
Line 3 = switch from SL to SYD (assuming n_2 in use)
Line 4 = 1962 asset life reduction (assuming SYD in use)

Credit = true 7% investment credit
A corporate tax rate of 50% is assumed.

Chapter 3

THE EFFECT OF ACCELERATED DEPRECIATION ON LIQUIDITY

The two major effects of accelerated depreciation are the pdv and the liquidity effects. The preceding section contained an analysis of the changes in the pdv of an asset's revenue stream resulting from the various types of accelerated depreciation. The purpose of this section is to analyse the changes in liquidity.

For reasons explained in Chapter 1 the liquidity variable of interest is the depreciation-investment ratio for a stream of assets, and the analysis involves a comparison of this ratio before and after the introduction of accelerated depreciation. Clearly either the difference of this ratio, or the ratio of its values calculated before and after the change, can be used to measure the gain from accelerated depreciation. The former seems more relevant since it gives the actual increase in the fraction of current investment which can be financed internally, and the larger is this increase the more benefit is obtained. On the other hand the ratio measure essentially gives the percent increase in the amount of investment financeable internally, and although this is certainly a well-defined concept, it does not seem appropriate to make comparative statements about the gain from new methods in terms of such a measure. Reliance on the latter for example, would mean that an increase in the depreciation-investment ratio from 50 to 100% would be considered (much) less advantageous than an increase from 2 to 5%. This is clearly unacceptable. The decision to analyse the absolute rather than the percent change in the depreciation-investment ratio is not inconsequential in that it leads to some general quantitative conclusions (particularly those relating to transition effects) which differ from commonly held views in the literature.

The analysis in this section involves introducing accelerated depreciation into a system in which investment is growing at a constant exponential rate g. This growth rate is assumed to remain constant when comparing the new depreciation-investment ratios with values that would have existed without acceleration. The assumption that g prevails after introduction of the new method is unrealistic, at least for the period of transition, since the purpose of such a measure is to stimulate investment expenditures. Changes in the depreciation-investment ratio given below will therefore be approximations to actual changes. The steady state growth rate on the other hand may well be the same after introduction of accelerated depreciation as before.

The analysis to follow centers on tracing changes in the depreciation-investment ratio resulting from the introduction of SYD, an initial allowance, an investment credit, and a reduction in asset life. The behaviour of the change as a function of the asset life and growth rate, as well as a comparison among the different methods is of interest.

The results given for the investment credit must be interpreted carefully. Under a true credit depreciation deductions remain the same while profits and cash flow increase by the amount of the credit. For this reason changes in the ratio of cash flow to investment (CF/I) rather than depreciation to investment are tabulated. Changes in the depreciation-investment ratio resulting from other methods of accelerated depreciation may be compared with these cash flow ratios only after being multiplied by the tax rate.

General Method of Analysing the Liquidity Effect

Consider an investment stream (I_t) of assets with life n, and the introduction of an accelerated method of depreciation at time 0. Then the distinction $t < n$ and $t \geqslant n$ is important since the latter represents return to the steady state with respect to the introduction of the new method.

Assume first that I_t is constant over time. Then for $t < n$ the total annual deduction will be larger after introduction of accelerated depreciation, since the latter results for each asset in larger deductions in early years. The gain will increase as long as all assets are experiencing larger deductions under the new method, and will begin to decrease when a lower deduction must be taken on any asset. For $t \geqslant n$, the total annual deductions will be the same with the new method as the old since the gains on relatively new assets will cancel exactly with the losses on older assets. This means that with a constant investment stream there is no permanent liquidity effect from accelerated depreciation, although there is a transition benefit for n years. Of course for a true investment credit there are no losses in later years, and annual deductions will remain permanently higher by the amount of the credit.

The case of a constant exponential growth rate (g) of investment expenditures is more interesting. Let $I_t = e^{gt}$, and let D_t be the total depreciation deduction from all assets at time t. Assume steady state conditions prior to t = 0, (i.e. the growth rate g has prevailed for at least n years) at which time accelerated depreciation is introduced. In order to compare D/I after the change with values that would have prevailed if the change had not occurred, it is convenient to carry out the calculations assuming no investment prior to t = 0. This is permissible since the contribution of investment before time 0 to each D/I ratio cancels when taking the difference between ratios for the two methods. In general, ignoring I_t for $t < 0$ and assuming an exponential growth rate (g) of investment, D/I ratios may be calculated as follows.

Aggregate depreciation is the sum of all deductions since time 0 and is therefore given by: $D_t = \int_0^t h(t-v,n)I_v dv$ for $t < n$. Substituting $I_v = e^{gv}$ and dividing by I_t gives: $D_t/I_t = \int_0^t h(t-v,n)e^{-g(t-v)}dv$, and letting $w = t-v$, D_t/I_t becomes:

$$(3.1) \quad D_t/I_t = \int_0^t h(w,n)e^{-gw}dw \quad \text{for } t < n.$$

For $t \geqslant n$, $D_t = \int_{t-n}^t h(t-v,n)e^{gv}dv$ and therefore

(3.2) $D_t/I_t = \int_0^n h(w,n)e^{-gw}dw$

Equation (3.2) gives the steady state value of D/I, which is of course independent of t. It is interesting to note that if g is interpreted as a discount rate, then the steady state expression for the aggregate D/I ratio is the same as that for the pdv of depreciation deductions for a single asset. Discounting by the growth rate is reasonable in the sense that it determines the importance of recent acquisitions to the stock of assets and weights their contribution to total depreciation accordingly.

This interpretation is particularly helpful in analysing the effect on D/I of asset life and growth rate changes since the pdv analysis of the preceding chapter is directly applicable. Values of the growth rate which are of practical interest, however, will in general be smaller than the relevant values of the discount rate r. Therefore to the extent that conclusions in the preceding section are dependent on the magnitude of r, they may not be of interest here.

Before analysing specific accelerated depreciation provisions it is interesting to determine the effect on steady state D/I ratios of variations in growth rates and asset lives. D/I will be a decreasing function of g by analogy with pdv considerations. Differentiating (3.2) with respect to n shows that D/I is also a decreasing function of n since the expression for the derivative (for $t \gg n$):

$$d(D_t/I_t)/dn = h(n,n)e^{-gn} + \int_0^n (d(h(w,n))/dn)e^{-gw}dw$$

was shown to be negative in the preceding chapter. Therefore the faster the growth of investment and the longer the average life of the asset, the lower will be the (steady state) fraction of investment which can be financed through depreciation deductions.

The general analysis described above is useful in determining the effect on D/I of the introduction of SYD, an initial allowance, an investment credit and a change in asset life.

The Effect on D/I of a Switch from SL to SYD

In order to determine the effect on D/I of a switch from SL to SYD, (3.1) must be evaluated for each method and the difference (denoted $y_s(t)$) obtained. Table 3.1 contains values of $y_s(t)$ for selected n, g, and t. The value in the table for any g, n and t=n, represents the permanent increase in D/I from using SYD in place of SL.

The table indicates that for a given n and g (with $t < n$) the gain increases at first with t as more and more assets are subject to larger deductions under the accelerated method, then decreases as these same assets are subject to smaller deductions in later years. The maximum gain occurs when deductions under the two methods are equal because after this point $y_s(t)$ includes negative terms. The maximum gain therefore occurs in year $(n+1)/2$ obtained from: $1/n = 2(n-t+1)/n(n+1)$. Values of $t = n/2+1$ are included in the table to indicate the magnitude of this gain. (Since tabulated asset lives are even, either $n/2+1$ or $n/2$ may be used to approximate $(n+1)/2$.) This suggests that if the percent of current investment which can be financed internally is a relevant factor in investment decisions, then the maximum incentive from using SYD will occur $(n+1)/2$ years after its introduction. For the two-digit manufacturing industries considering machinery only and assuming immediate adoption of the accelerated methods, the maximum incentive would occur roughly during the period 1959-65.

The behaviour of the gain as a function of the growth rate for fixed t and n with $t < n$ depends on t. For low t the gain decreases monotonically with g, while for large t the opposite is true (at least for the range of g given in the table). In particular for $t < (n+1)/2$ then as noted above all terms in $y_s(t)$ will be positive, and since these terms are effectively discounted by the growth rate, an increase in the latter will diminish $y_s(t)$. For $t \geqslant (n+1)/2$, $y_s(t)$ also includes negative terms and the effect of an increase in g is ambiguous. The conclusion that the slowest growing firms obtain the most benefit from accelerated depreciation (in terms of the change in D/I) for a substantial number of years after the introduction of the new method, is in contrast to the steady state conclusion that the fastest growing firms experience the most benefit.

Considering steady state situations it can be seen that for the range of g and n in Table 3.1, $y_s(n)$ is a monotonically increasing function of both g and n. For larger growth rates or asset lives this does not hold since values given by (3.2) are equivalent to pdv changes and therefore are not monotonic functions of g or n. The values of g for which $y_s(n)$ reaches a maximum for fixed n may be obtained from Table 2.2. These values of g range from over 30% for n=10 to 7% for n=40, which suggests that for all practical purposes the advantage due to SYD increases with the growth rate. Similarly some idea of the values of n for which $y_s(n)$ reaches a maximum for fixed g may be obtained from Table 2.1. It appears that for growth rates of 8% or less the maximum occurs for asset lives of over 40 years.

It may be thought that the results in Table 3.1 depend crucially on the assumption of an exponential growth rate of investment. That this is not true is indicated by the results in Table 3.2 which are based on a linear growth pattern. Instead of assuming that $I_t = e^{gt}$ it is assumed that $I_t = I_o + rt$, where I_o is the initial period investment. If r is expressed as a percent of I_o and called g, i.e. $I_t = I_o(1+gt)$, then the gain from using accelerated depreciation as a function of g, n, and t may be tabulated in a manner comparable to Table 3.1. Table 3.2 contains such calculations and a comparison with Table 3.1 indicates that the growth assumption makes virtually no difference for values of $t \leq n$. However, for $t > n$ the gain is not constant in the linear case but approaches zero over time, and for this reason Table 3.2 contains a value for $t > n$. From the table (and from calculations not presented) this gain appears to approach zero very slowly.

The Effect on D/I of an Initial Allowance

It is assumed that an initial allowance of b% of cost is applied on assets acquired during the period t-1 to t. Total depreciation for $t < n$ may then be written as:

$$D_t = \int_0^{t-1} (h(t-v,n)-b(h(t-v-1,n-1))e^{gv}dv + \int_{t-1}^{t} (h(t-v,n)+b)e^{gv}dv$$

$$= \int_0^{t} h(t-v,n)e^{gv}dv + b\int_{t-1}^{t} e^{gv}dv - b\int_0^{t-1} h(t-v-1,n-1)e^{gv}dv$$

That is, total depreciation is calculated on investment over the preceding t years, with the current year's depreciation increased by b% of the current year's investment, and with depreciation from time 0 to t-1 reduced by the appropriate depreciation rate times b% of the investment undertaken during that period. The first term on the right hand side of the equation (after regrouping) is the value which total depreciation would take in the absence of an allowance. After moving this term to the left hand side, dividing by $I_t = e^{gt}$ and substituting $w = t-v$, the change in D/I due to an initial allowance (denoted $y_a(t)$) becomes:

$$(3.3) \quad y_a(t) = b\int_0^{1} e^{-gw}dw - b\int_1^{t} h(w-1,n-1)e^{-gw}dw$$

for $t \leq n$. The steady state value for the change in D/I is derived in a similar manner and is given by:

$$(3.4) \quad y_a(t) = y_a(n) = b\int_0^{1} e^{-gw}dw - b\int_1^{n} h(w-1,n-1)e^{-gw}dw$$

for $t \geq n$.

Values of $y_a(t)$ for an initial allowance of 100% and for various asset lives and growth rates are tabulated in Table 3.3. The table is based on the assumption that the method of depreciation in use is SYD and although analogous values for SL are not presented previous considerations suggest that such values will follow the same general pattern but will be slightly larger.

For fixed n and g the maximum gain occurs in the first year because this is the only year in which there are no reductions in depreciation on any assets. The gain decreases monotonically with t until year n as more and more assets are subject to a reduction in base. For fixed n and t with $t < n$, the gain as a function of g depends on t. It decreases monotonically for t=1 since $y_a(1)$ contains no negative terms, but for $t > 1$ the direction of change is ambiguous. Although it is impossible to determine even the general pattern of this change from Table 3.3 because of the small number of t values appearing, untabulated values indicate that $y_a(t)$ is a decreasing function of g only for a few years after introduction of the allowance.

The steady state gain is a monotonically increasing function of g for the range of g given in the table, and is of course an increasing function of n by analogy with pdv considerations.

The Effect of an Investment Credit on the Ratio of Cash Flow to Investment

The effect of a true investment credit (k) is to increase net profits (P) by the amount of the credit, and to leave depreciation deductions unchanged. Letting CF_t be cash flow in period t, then the credit increases CF_t/I_t (or P_t/I_t) by an amount equal to $(k \int_{t-1}^{t} e^{gv}dv)/e^{gt} = k \int_{0}^{1} e^{-gw}dw$, which is a function only of k and g. The value of this expression for a 7% credit and a 5% growth rate appears in Table 3.6, which contains a comparison of the different methods of accelerated depreciation, to be discussed shortly. It should be recalled that changes in CF/I can be compared with changes in D/I only after the latter have been multiplied by the tax rate, since as shown in Chapter 1 an increase in D (because it reduces P) increases cash flow by an amount equal to the change in D times the tax rate.

An investment credit such as the one introduced in 1962 may be analysed in essentially the same manner as an initial allowance. The only major difference is that the calculations must include the tax rate explicitly since the credit involves an element of subsidy (in the first year), together with a change in depreciation deductions due to the write-down of the base in later years. A minor difference is that the latter occurs over n rather than n-1 years. In view of these remarks and equation (3.3) above, the increase in CF/I due to an investment credit of k% is given by:

$$(3.5) \quad y_k(t) = k \int_0^1 e^{-gw}dw - kT \int_0^t h(w,n)e^{-gw}dw \text{ for } t \leq n,$$

where T is the corporate tax rate. The steady state increase is obtained by substituting n for t in this expression.

Values of $y_k(t)$ are presented in Table 3.4 for T =.5 and k =.07, and for selected values of n, t, and g. The general quantitative behaviour of $y_k(t)$ will not be described in detail since it is essentially the same as that of $y_a(t)$ given above. The only noticeable difference is the much smaller variability of the credit with respect to changes in n and g. This is due to the element of subsidy in the credit. That is, ignoring discounting, the reduction in the depreciable base with an investment credit is only .5 (the tax rate) times the first year gain, whereas with an initial allowance it is equal to the entire first year gain. Since it is the reduction in base which is substantially affected by differences in n and g, this reduces the variability of the benefit from the credit considerably. (The first year gain is of course affected slightly by g, but not by n.)

The Effect on D/I of a Change in Asset Life

In order to determine the effect on D/I of a reduction in asset life (3.1) must be evaluated for each life and the difference obtained. Table 3.5 contains changes in D/I for various growth rates assuming SYD is in use, and for asset life reductions approximating the 1962 revisions for the two-digit manufacturing industries.

The year in which the maximum increase in D/I occurs for fixed g may be determined as follows. Under SL it is n_1 (the shorter asset life) years after introduction of the change because all assets earn larger deductions for n_1 years. Under SYD larger deductions are not obtained for n_1 years, and the critical year must be determined by equating deductions under the two lives. That is, assuming the continuous formulation of SYD for convenience of computations, the year of maximum increase in D/I is t^* obtained by solving: $2(n_1-t^*)/n_1^2 = 2(n_2-t^*)/n_2^2$, from which $t^* = n_1n_2/(n_1+n_2)$.

(If annual instead of continuous discounting is assumed the corresponding t^* is $(n_2+1)(n_1+1)(n_2-n_1)/(n_2(n_2+1)-n_1(n_1+1))$.) The value of D/I corresponding to t^* is presented in Table 3.5 in order to provide an idea of the maximum change in D/I resulting from the reduction in asset lives.

Considering the change in D/I as a function of g for fixed $t < n$, similar reasoning shows that D/I decreases monotonically with g at least for $t < t^*$. This means that the slowest growing firms will obtain the most benefit from a reduction in asset life for a substantial number of years after the change (at least $n_1 n_2/(n_1+n_2)$ under SYD and n_1 under SL).

By analogy with pdv calculations the change in the steady state D/I value will not be an increasing function of the growth rate for all g, although the table indicates that it is for all practical purposes.

Summary and Comparison of Methods

The analysis of the effect of accelerated depreciation on liquidity involves studying the behaviour over time of the depreciation-investment ratio (D/I) for a stream of assets. D/I is studied because an increase in depreciation deductions increases cash flow (CF) by the tax rate times this amount. With a constant tax rate, therefore, it suffices to study changes in D/I in order to determine the effect on the availability of internal funds. For an investment credit CF/I rather than D/I is studied because the depreciation deductions remain constant, with the change in cash flow being equal to the credit. For reasons given above the absolute rather than the percent change in D/I is used to measure the benefit obtained from accelerated depreciation in comparing different methods, growth rates, and asset lives.

The distinction $t < n$ and $t \geq n$ is important in the analysis since the former period involves transition effects only while the latter represents return to steady state conditions. Steady state changes in D/I are shown to be equivalent to pdv changes, interpreting the growth rate as a discount rate. The general conclusions to be drawn about the effect of accelerated depreciation on such changes are therefore the same as those given in the preceding chapter. In general, however, relevant values of the growth rate are lower than corresponding values of the discount rate, and for this reason steady state changes in D/I are for all practical purposes increasing functions of the growth rate (while pdv changes are not uniformly increasing functions of the discount rate). For this reason also, the gain from switching to SYD is an increasing function of asset life, at least for lives of up to 30 years and growth rates of up to 9%.

Transition effects are important for three reasons. First, they are the incentive effects to investment which are immediately available. Second, they are operative for a substantial period after introduction, since for the two-digit manufacturing industries under study average machinery lives currently range from 9 to 17 years. Third, they are in all cases much larger than steady state changes.[1]

The transition behaviour of D/I over time (with a fixed growth rate) is of interest. As shown above the year of maximum benefit (largest increase in D/I) depends on the method of accelerated depreciation. For an initial allowance or credit (such as the 1962 credit) the maximum gain occurs in the first year because in all later years a write-down in the base is required. For a switch from SL to SYD, D/I is a maximum in year (n+1)/2 since before this time more assets are obtaining higher deductions and none lower deductions. For a change in asset life from n_2 to n_1 years, D/I is a maximum in year n_1 if SL is assumed, and in year $n_1 n_2/(n_1+n_2)$ if SYD (and continuous discounting) is assumed.

The transitional behaviour of D/I as a function of the growth rate is also of interest, and depends on t and the particular method of accelerated depreciation. In general, slower growing firms experience a greater benefit from acceleration than faster growing ones for a substantial number of years after introduction of the new method, although for an initial allowance this appears to hold for only a few years. After n years of course, steady state conditions prevail and the fastest growing firms obtain the

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[1]It is true of course that in absolute terms the additional steady state investment financeable from acceleration will at some point exceed the maximum transitional amount, simply because of the growth in investment.

most benefit, at least for growth rates tabulated in this study. Basically the reason slow growing firms experience a larger gain is because the (positive) changes in depreciation deductions which occur in the first few years remain a relatively more important determinant of the total D/I ratio for slow growing firms than for fast growing firms. That is, past annual increases in depreciation deductions are discounted by the growth rate of investment in calculating the change in the current D/I ratio, and the higher this growth rate, the smaller the contribution of annual deduction increases to the change in D/I.

The effect of the different methods of accelerated depreciation on liquidity may be compared using Tables 3.1, 3.3, 3.4, and 3.5. The following adjustments are required in order to make all entries comparable to those in Table 3.4 which are based on a 50% tax rate. Table 3.1 and 3.5 entries must be multiplied by the corporate tax rate (.5) and Table 3.3 entries by the latter and an initial allowance rate. Table 3.6 contains changes in CF/I (that is, D/I changes which have been multiplied by the tax rate) for the special case of a 50% tax rate, a 20% initial allowance, and asset lives which are intended to approximate actual lives in the two-digit manufacturing industries. A 5% exponential growth rate of investment is assumed. For the same reasons as in the pdv comparison of the preceding chapter it is assumed that SYD is in use and that the shorter asset lives

(n_1) are relevant for the introduction of the investment credit and initial allowance, while the longer lives (n_2) are assumed to be relevant for the switch from SL to SYD. The reduction in asset lives is assumed to take place under the SYD method of depreciation.

Table 3.6 also contains the change in CF/I resulting from a true 7% investment credit assuming a growth rate of 5%. It should be noted that since the credit does not depend on t there are no transition effects, and hence the tabulated value represents the permanent change in the cash flow-investment ratio which would result from introduction of the credit.

There is no need to comment extensively on the steady state values recorded in Table 3.6 since they follow the same pattern as the pdv computations of the previous chapter. It suffices to say that the true investment credit results in the greatest permanent benefit by far while the other methods, ranked in a general (since they depend on n) order of importance, are the switch from SL to SYD, the 1962 credit, the 20% initial allowance, and the asset life changes.

As noted above transition benefits are much larger than steady state benefits. For all methods of accelerated depreciation tabulated in Table 3.6 except the 1962 credit and small asset life reductions, transition changes in some period are larger than the (steady state) changes obtained from a true credit. The particular transition values of t presented in the table correspond to the maximum increase in D/I for the different methods. For example, in Industry 26 the greatest benefit from asset life reduction occurs in the eighth year, while for a switch to SYD it occurs in the tenth year. The credit and allowance of course provide the greatest benefit for all industries in the year of introduction.

Table 3.6 indicates that in general the transition effect is greater in all years for a switch from SL to SYD than for any other method, except for an allowance and credit in the first few years, and for a true credit in the last few years of transition. It is interesting to note that although the steady state gain from switching to SYD increases with the asset life for the range given in the table, the maximum transition effect is approximately the same for all industries although it decreases slightly with the asset life. As noted above the maximum benefit from switching to SYD occurs in year (n+1)/2, and since the method was introduced in 1954 this corresponds to the period 1959 to 1965 for the two-digit manufacturing industries.

The maximum increase in D/I due to a reduction in asset life varies considerably across industries. Industry 372 experiences the most benefit, in the fifth year after introduction, while Industry 30 shows almost no gain at all. The maximum benefit from asset life reduction under SYD occurs in year $n_1n_2/(n_1+n_2)$ (with continuous discounting), and since the reduction was instituted in 1962, this corresponds to the period 1968-1972 for the two-digit manufacturing industries.

The 20% initial allowance provides more benefit than other methods in the first few years after introduction but its effect diminishes rapidly, and when steady state conditions are reached it provides less benefit than all other methods except the asset life reductions.

Table 3.1

CHANGE IN D/I DUE TO A SWITCH FROM SL TO SYD
EXPONENTIAL GROWTH OF INVESTMENT

		g				
n	t	1	3	5	7	9
6	2	.188	.183	.178	.174	.169
	4	.188	.184	.179	.175	.171
	6	.008	.022	.034	.045	.053
10	2	.143	.139	.136	.132	.129
	4	.214	.205	.197	.190	.183
	6	.214	.205	.198	.190	.184
	10	.014	.038	.056	.071	.082
14	2	.113	.109	.106	.104	.101
	4	.186	.178	.171	.164	.158
	8	.222	.211	.200	.191	.182
	14	.020	.052	.074	.090	.101
18	2	.092	.090	.087	.085	.083
	4	.160	.153	.146	.140	.135
	10	.226	.212	.199	.187	.176
	18	.026	.064	.089	.105	.114
22	2	.078	.076	.074	.072	.070
	4	.139	.133	.127	.122	.117
	12	.228	.211	.195	.182	.170
	22	.031	.074	.100	.115	.122
26	2	.067	.065	.064	.062	.060
	4	.122	.117	.112	.107	.103
	14	.228	.209	.191	.176	.163
	26	.036	.084	.109	.122	.127
30	2	.059	.058	.056	.055	.053
	4	.109	.104	.100	.095	.091
	16	.228	.206	.187	.171	.157
	30	.041	.092	.117	.127	.129

n = asset life in years
t = number of years after introduction of SYD
g = exponential growth rate of investment in percent

Table 3.2

CHANGE IN D/I DUE TO A SWITCH FROM SL TO SYD
LINEAR GROWTH OF INVESTMENT

		g				
n	t	1	3	5	7	9
6	2	.188	.183	.178	.174	.170
	4	.188	.184	.180	.176	.172
	6	.008	.021	.031	.038	.045
	8	.008	.020	.028	.035	.040
10	2	.143	.140	.136	.132	.129
	4	.214	.206	.198	.192	.186
	6	.214	.206	.199	.193	.188
	10	.014	.034	.048	.058	.065
	12	.013	.032	.045	.053	.060
14	2	.113	.110	.107	.104	.101
	4	.186	.179	.172	.166	.160
	8	.223	.213	.204	.197	.190
	14	.019	.044	.061	.072	.079
	16	.018	.043	.057	.067	.073
18	2	.092	.090	.087	.085	.083
	4	.160	.153	.147	.142	.137
	10	.226	.214	.205	.197	.189
	18	.024	.054	.071	.082	.089
	20	.023	.052	.067	.077	.084
22	2	.078	.076	.074	.072	.070
	4	.139	.133	.128	.123	.119
	12	.228	.214	.204	.195	.188
	22	.028	.061	.079	.090	.097
	24	.028	.060	.076	.085	.091
26	2	.067	.065	.064	.062	.060
	4	.122	.117	.112	.108	.104
	14	.230	.214	.203	.194	.186
	26	.033	.068	.086	.097	.103
	28	.032	.066	.083	.092	.098
30	2	.059	.058	.056	.055	.053
	4	.109	.104	.100	.096	.093
	16	.229	.213	.201	.192	.185
	30	.037	.074	.092	.102	.108
	32	.036	.072	.089	.098	.103

n = asset life in years
t = number of years after introduction of SYD
g = linear growth rate of investment in percent

Table 3.3

CHANGE IN D/I DUE TO AN INITIAL ALLOWANCE OF 100%

		g				
n	t	1	3	5	7	9
6	2	.663	.657	.650	.643	.637
	4	.212	.235	.255	.273	.289
	6	.022	.064	.101	.134	.163
10	2	.794	.782	.771	.760	.749
	4	.472	.481	.489	.496	.502
	8	.096	.148	.194	.233	.267
	10	.035	.098	.151	.197	.237
14	2	.850	.836	.823	.810	.797
	4	.606	.608	.609	.610	.610
	12	.076	.152	.215	.267	.310
	14	.048	.130	.198	.253	.300
18	2	.881	.866	.852	.838	.824
	4	.685	.683	.681	.677	.674
	16	.076	.172	.248	.309	.358
	18	.060	.160	.240	.303	.354
22	2	.901	.885	.870	.855	.841
	4	.738	.733	.727	.722	.716
	20	.082	.195	.283	.350	.403
	22	.072	.188	.278	.347	.401
26	2	.915	.898	.883	.867	.853
	4	.775	.768	.761	.753	.746
	24	.090	.220	.316	.388	.442
	26	.083	.215	.313	.386	.441
30	2	.925	.908	.892	.876	.861
	4	.803	.794	.785	.776	.768
	28	.100	.244	.347	.422	.477
	30	.095	.241	.346	.421	.477

n = asset life in years
t = number of years after introduction of initial allowance
g = exponential growth rate of investment in percent
SYD is assumed

Table 3.4

CHANGE IN CF/I DUE TO 1962 INVESTMENT CREDIT

		g				
n	t	1	3	5	7	9
4	2	.015	.015	.015	.014	.014
	3	.013	.013	.013	.012	.012
	4	.012	.012	.012	.012	.012
6	2	.034	.034	.033	.033	.032
	4	.027	.027	.026	.026	.026
	6	.023	.024	.024	.024	.024
8	2	.055	.054	.053	.052	.051
	4	.045	.044	.044	.044	.043
	6	.038	.039	.039	.039	.039
	8	.035	.036	.037	.037	.038
10	2	.057	.056	.055	.054	.054
	4	.048	.048	.047	.047	.046
	8	.037	.038	.039	.039	.040
	10	.036	.037	.038	.038	.039
14	2	.064	.059	.058	.057	.056
	4	.053	.052	.052	.051	.051
	12	.037	.039	.040	.041	.042
	14	.036	.038	.039	.040	.041
18	2	.062	.061	.060	.059	.058
	4	.056	.055	.055	.054	.053
	16	.037	.039	.041	.042	.043
	18	.037	.039	.041	.042	.043
22	2	.063	.062	.061	.060	.059
	4	.058	.057	.057	.056	.055
	20	.037	.040	.042	.044	.045
	22	.037	.040	.042	.044	.045

n = asset life in years
t = number of years after introduction of investment credit
g = exponential growth rate of investment in percent
SYD is assumed.
A corporate tax rate of 50% is assumed.

Table 3.5

CHANGE IN D/I DUE TO REDUCTION IN ASSET LIFE

						g		
Ind	n_1	n_2	t	1	3	5	7	9
20	13	15	2	.033	.032	.031	.030	.029
			4	.054	.052	.050	.048	.046
			7	.067	.064	.060	.057	.054
			15	.006	.016	.023	.027	.031
22	13	16	2	.046	.045	.044	.042	.041
			4	.077	.074	.071	.068	.065
			7	.097	.092	.087	.082	.078
			16	.009	.023	.033	.040	.045
26	15	19	2	.046	.045	.044	.043	.041
			4	.080	.076	.073	.070	.067
			9	.111	.104	.097	.092	.087
			19	.012	.030	.042	.050	.055
28	11	13	2	.043	.042	.040	.039	.038
			4	.069	.066	.063	.061	.058
			6	.078	.075	.071	.068	.065
			13	.006	.016	.024	.029	.033
29	15	18	2	.036	.035	.034	.034	.033
			4	.063	.060	.057	.055	.053
			8	.085	.080	.075	.071	.067
			18	.009	.023	.032	.038	.042
30	13	14	2	.017	.017	.016	.016	.016
			4	.029	.027	.026	.025	.024
			7	.035	.033	.031	.030	.028
			14	.003	.008	.011	.014	.016
32	16	18	2	.023	.022	.022	.021	.021
			4	040	.038	.036	.035	.033
			9	.055	.052	.049	.046	.043
			18	.006	.015	.021	.025	.027
33	17	21	2	.038	.037	.036	.035	.034
			4	.066	.063	.060	.058	.055
			10	.099	.092	.086	.081	.076
			21	.012	.029	.040	.047	.051

Table 3.5 (Continued)

Ind	n_1	n_2	t	g = 1	3	5	7	9
35	12	14	2	.037	.036	.035	.034	.033
			4	.061	.058	.056	.054	.052
			7	.072	.069	.065	.062	.059
			14	.006	.016	.023	.028	.032
36	11	14	2	.060	.058	.057	.055	.054
			4	.097	.093	.089	.086	.083
			6	.113	.011	.102	.097	.093
			14	.009	.024	.035	.043	.049
371	12	14	2	.037	.036	.035	.034	.033
			4	.061	.058	.056	.054	.052
			7	.072	.069	.060	.062	.059
			14	.006	.016	.023	.028	.032
372	9	12	2	.082	.079	.077	.075	.073
			4	.126	.120	.116	.111	.107
			5	.134	.128	.122	.117	.113
			12	.009	.025	.037	.046	.053

Ind = Industry
n_1 = average asset life after 1962 Guideline change
n_2 = average asset life before 1962 Guideline change
t = number of years after introduction of the 1962 Guideline change
g = exponential growth rate of investment in percent
SYD is assumed

Table 3.6

COMPARISON OF CHANGES IN CF/I FOR VARIOUS METHODS
EXPONENTIAL GROWTH RATE = 5%

Ind	n_1	n_2						
			t = 2	4	8	13	15	
20	13	15	.058	.051	.042	.039	-	
			.081	.059	.030	.019	-	
			.050	.082	.101	.065	.039	
			.015	.025	.060	.017	.011	
			t = 2	4	7	8	13	16
22	13	16	.058	.051	.044	.042	.039	-
			.081	.059	.035	.030	.019	-
			.047	.079	.100	.101	.075	.040
			.022	.035	.043	.043	.027	.016
			t = 2	4	8	10	15	19
26	15	19	.059	.053	.045	.042	.040	-
			.083	.063	.036	.029	.021	-
			.041	.070	.097	.100	.079	.046
			.022	.036	.049	.047	.032	.021
			t = 2	4	7	11	13	
28	11	13	.056	.049	.042	.038	-	
			.079	.053	.028	.016	-	
			.056	.088	.102	.067	.035	
			.020	.032	.035	.020	.012	
			t = 2	4	8	9	15	18
29	15	18	.059	.053	.045	.043	.040	-
			.083	.063	.036	.032	.021	-
			.043	.073	.099	.100	.072	.044
			.017	.027	.038	.037	.023	.015
			t = 2	4	7	13	14	
30	13	14	.058	.051	.044	.039	-	
			.081	.059	.035	.019	-	
			.053	.085	.101	.052	.037	
			.008	.013	.015	.008	.005	
			t = 2	4	9	16	18	
32	16	18	.059	.053	.044	.040	-	
			.084	.065	.035	.022	-	
			.043	.073	.100	.064	.044	
			.011	.018	.024	.014	.010	

Table 3.6 (Continued)

Ind	n_1	n_2						
			t = 2	4	10	11	17	21
33	17	21	.060	.054	.044	.043	.040	-
			.085	.067	.034	.031	.023	-
			.038	.065	.098	.098	.077	.048
			.018	.030	.043	.042	.028	.020
			t = 2	4	7	12	14	
35	12	14	.057	.050	.043	.039	-	
			.080	.056	.032	.018	-	
			.053	.085	.102	.066	.037	
			.017	.028	.030	.019	.012	
			t = 2	4	6	7	11	14
36	11	14	.056	.049	.043	.042	.038	-
			.079	.053	.035	.028	.016	-
			.053	.085	.100	.102	.066	.037
			.028	.044	.051	.050	.032	.017
			t = 2	4	7	12	14	
371	12	14	.057	.050	.043	.039	-	
			.080	.056	.032	.018	-	
			.053	.085	.102	.066	.037	
			.017	.028	.030	.019	.012	
			t = 2	4	5	6	9	12
372	9	12	.054	.046	.043	.040	.037	-
			.075	.045	.034	.026	.014	-
			.059	.092	.099	.102	.083	.032
			.038	.058	.061	.060	.041	.018

Credit .068

Ind = industry
n_1 = average asset life after 1962 Guideline change
n_2 = average asset life before 1962 Guideline change
t = number of years after introduction of accelerated method

For each industry:
Line 1 = 1962 investment credit (assuming SYD and n_1 in use)
Line 2 = 20% initial allowance (assuming SYD and n_1 in use)
Line 3 = switch from SL to SYD (assuming n_2 in use)
Line 4 = 1962 asset life reduction (assuming SYD in use)

Credit = true 7% investment credit
A corporate tax rate of 50% is assumed.
An exponential growth rate of investment of 5 % is assumed.

Chapter 4

INTERVIEW EVIDENCE

The purposes of this chapter are first to report on two recent interview studies of corporation executives describing capital budgeting procedures, and second to outline the advantages obtainable from, and the extent of use of, internal financing.

The aspect of the capital budgeting procedure of primary concern is the manner in which entrepreneurs make rate of return calculations. Interest centers on determining whether or not such measures are explicitly affected by liberalized depreciation provisions, and in particular if it is possible for the pdv effect of acceleration to be operative. Clearly, if entrepreneurs rarely use discounting techniques in their rate of return calculations, then the pdv effect (in as much as it is taken into account explicitly) will be unimportant. However, such a finding would not necessarily mean that the pdv effect was irrelevant, since entrepreneurs could be implicitly applying discount procedures. That is, the combination of subjective judgment and various rules of thumb by entrepreneurs might result in investment decisions consistent with those that would be reached if discounting were considered explicitly. But such a procedure, because of its very subjective nature, would be only approximate and in fact might not be affected at all by small accelerated depreciation changes.

Donald F. Istvan has recently attempted to determine the nature of the capital expenditure decision-making process in large corporations by interviewing executives and by studying the forms and manuals used by the firms in dealing with investment problems.[1] The interviews covered 48 firms in 10 industries, with the former being among the 10 largest in their respective industries. In 1959 these firms accounted for more than $8 billion of the $33 billion of plant and equipment expenditure reported by the Department of Commerce.

One aspect of Istvan's work was a study of the measure of acceptability used by the corporations in ranking projects if funds were limited, or in providing a minimum level of acceptability if funds were not limited. The following table summarizes the results of this investigation.

Summary of Employment of Various Measures of Acceptability[2]

Measure of Acceptability	Number of Firms Using as the Primary Measure	Number of Firms Using in a Supplementary Manner
Time adjusted rate-of return	5	9
MAPI formula	2	0
Simple rate-of-return	24	8
Payback	13	21
Subjective judgment	4	44
Total	48	

[1]Donald F. Istvan, Capital-Expenditure Decisions, Indiana Business Report No. 33, Indiana University, 1961.

[2]*Ibid.*, p. 96.

Use of the time adjusted rate-of-return means either that the internal rate of return was used to rank projects or that present discounted value calculations (using a minimum acceptable rate of return for discounting) were used for the accept-reject decision. The reason for not using discounting techniques, according to one-half of the firms, was either that present techniques were satisfactory, or the belief that operating personnel would never be able to understand or apply such techniques.

The MAPI (Machinery and Allied Products) formula, used by only 2 companies, is a shortcut discounted value method based on certain assumptions about the capital mix, interest cost and return on equity. It is applicable only to replacement investment.

The simple rate of return, used by 32 companies, does not employ the present value principle. There are 2 different simple rate of return measures, the initial and average. The initial simple rate of return, used by 16 of the 32 firms, is calculated by averaging simple rates (over the asset's life) or by dividing the average net revenue over the project's life by average investment. It is interesting to note that 25 of the 32 firms used an after-tax version of the simple rate of return while of these 25, 11 considered taxes correctly, 8 incorrectly, and for 6 the procedure was not ascertained.

The payback period is the number of years required to recover the cost of the investment. Among the 13 firms using the payback as a primary measure, the acceptable number of years for recovering an investment ranged from 1 to 5 years. None of the executives interviewed was able to provide objective reasons for the set levels of acceptability. Of the 34 firms using the measure, 16 used it correctly after taxes, 13 failed to account for taxes and the procedure was not ascertained for the other 5.

Subjective judgment was used mainly for urgent proposals, in particular when it was necessary to maintain a competitive position.

From the interview evidence it appears unlikely that the pdv effect of accelerated depreciation will be very important, since only 7 of the 48 firms interviewed used discounting techniques (explicitly) as a primary measure in their investment calculations. Use of the payback period as a primary measure by 13 firms with probably about one-half taking taxes into account correctly, suggests some effect on investment through this channel. The most extensively used measure was the simple rate of return, with the average and initial simple rates being of about equal importance. The average measure is unaffected by an acceleration of depreciation since it is calculated in terms of revenues earned over the life of the asset and does not involve discounting. The initial rate, on the other hand, takes only first year revenues into account and is therefore affected by a change in the pattern of depreciation deductions. An initial allowance in particular will be extremely important since the (first year) gain is taken into account while the write-down in the base in later years is not. An allowance of b% of cost increases the initial simple rate by bT, and hence a 20% allowance together with a 50% tax rate (T) provide a (very substantial) rate of return increase of .1. Other methods of acceleration result in much smaller gains in the first year, and hence are less effective. The potential incentive provided by all methods is diminished of course by the fact that at least 15 of the 32 firms using the simple rate do not consider taxes correctly.

Thomas M. Stanback, Jr. has recently attempted to determine the effect of accelerated depreciation allowances on modernization expenditures in the textile industry.[3] The investigation relied on interviews with executives of 25 textile firms, and analysis of published financial reports. The study revealed that in general a rate of return calculation was the single most important criterion applied in determining the acceptability of investment proposals. The following table, drawn from a preliminary summary of Stanback's work, presents the distribution of firms according to the types of investment formulas used for modernization projects.[4]

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[3] Thomas M. Stanback, Jr., An Evaluation of the Influence of Liberalized Depreciation and the Investment Credit on Modernization Expenditures in the Textile Industry, (Unpublished Preliminary Summary), N.B.E.R., December, 1965.

[4] Ibid., p. 18.

	Number of Firms
Pre-tax pay-back period only	16
After-tax pay-back period only	4
Combination after-tax pay-back period and rate of return during pay-back period	1
Combination after-tax rate of return on investment and after-tax pay-back period	1
Combination after-tax rate of return and pre-tax pay-back period	1
Combination discounted cash flow and selected additional after-tax formulas	2

Probably the most surprising result is that 16 of the 25 firms interviewed did not take taxes into account at all in their investment formula computations. In addition another firm was making calculations in such a way that tax effects would be imperfect if existent at all, thus leaving only 8 of 25 firms in a position to be explicitly influenced by acceleration through rate of return calculations. However, these 8 firms were in general larger than the others, and hence accounted for a relatively larger share of total capital outlays (than given by 8/25). The table indicates that only two firms used discounted cash flow techniques, thus suggesting that the pdv effect of accelerated depreciation (at least to the extent that it is explicitly considered) will be unimportant.

In addition to analysing investment formulas Stanback attempted to analyse the cash flow effect of accelerated depreciation explicitly. The procedure relied heavily on interview material involving the following three questions:

"1. Does the firm have a policy of limiting itself primarily to internally generated funds?

2. Does the firm see itself as having faced financial restrictions?

3. Is there direct evidence that the increased availability of funds arising out of liberalized depreciation has resulted in increased modernization expenditures?"[5]

The resulting information was summarized and firms were classified in terms of probable cash flow influence, as recorded in the following table.[6]

Class	Number of Firms
A. Maximum cash flow effect	5
B. Strong cash flow effect	7
C. Partial cash flow effect	6
D. Virtually no cash flow effect	7

The table indicates that over one-half of the firms were significantly influenced by a cash flow effect. But when further classified by size, 62% of the larger firms were in groups C and D, and 67% of smaller firms in A and B, thus reducing the importance of such an effect. The fact remains, however, that 18 of 25 of the firms were influenced at least partially by cash flow considerations whereas in considering rate of return effects only 8 used measures defined in after-tax terms, and only 2 of these used pdv calculations.

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[5] Ibid., p. 25.

[6] Ibid., p. 25.

The evidence cited here can of course not be considered comprehensive since it is based on only two interview studies, but it does provide some idea of the type of explicit rate of return calculation used by entrepreneurs. The most common criteria appear to be a simple rate of return and a payback period, neither of which involves discounting. However, to the extent that the "initial" version of the simple rate is used, and taxes are taken into account correctly, both the simple rate and the payback will be affected by accelerated depreciation. The studies suggest, though, that in many cases either pre-tax measures are used, or taxes are not considered correctly, thus diminishing the effectiveness of acceleration. Discounting techniques appear to be used sparingly in rate of return calculations, which suggests that the (explicit) pdv effect of acceleration will be unimportant.

On the other hand, Stanback's evidence indicates that the liquidity effect of accelerated depreciation may play a much more significant role in influencing investment decisions. The liquidity effect results in a permanent increase in the level of cash flow (for a growing firm), and its importance therefore depends on the extent to which internal financing is considered advantageous. Theoretically, additional investment will be undertaken in projects unprofitable before acceleration, but profitable after due to the reduction in cost made possible by the increased availability of internal funds. If the latter are considered much less expensive than external funds, then the increase in cash flow from an acceleration of depreciation will provide a large stimulus to investment. In addition it is possible, that internal funds will be strongly preferred to external funds for reasons which are not strictly rational, in which case investment expenditures may be effectively restricted to the amount of funds generated internally. To the extent that this occurs in practice the liquidity effect of accelerated depreciation will be perhaps even more important. It should be realized that this does not mean rates of return or profitability measures do not set (ultimate) bounds to investment, but rather that the extent of investment in projects (which may be profitable even before accelerated depreciation) is strongly influenced by the level of cash flow.

The advantages of internal financing are outlined briefly below in terms of the disadvantages to external financing (where the latter includes both debt and equity issues), and a table describing the sources and uses of corporate funds in recent years is presented in order to give some idea of the extent of use of internal funds.

There are at least four disadvantages to debt financing. First, debt involves a fixed interest burden which must be met even in times of depression. Second, an increase in debt, _ceteris paribus_, increases the riskiness of the firm (that is, it increases the danger of default on debt and raises the risks of common stock ownership), which may have undesirable repercussions to the extent that the firm's capital structure is taken into account by investors. Third, debt financing may reduce managerial flexibility due, for example, to restrictions on the use of money or the imposition of minimum balance sheet requirements. Finally, the separation of ownerwhip and control in the modern firm makes debt financing asymetrically risky for management. That is, from management's point of view there is little to be gained from debt financing a risky project, simply because management's profit as compared with salary income is generally small, while there is much to lose if the project fails and bankruptcy ensues. As Meyer and Kuh point out, the converse of this proposition is that internal financing will be advantageous: "if debt financing is asymmetrically risky for professional management, expansion out of retained earnings is beneficial for exactly the same reasons."[7]

There are two major disadvantages to stock financing. First, it often results in dilution of earnings or control. Second, equity issues are generally a relatively expensive method of raising capital except for the largest firms, and in fact such an option may not even exist for small firms. Costs involved in equity financing include the necessity to underprice the issue, and the actual commission to underwriters

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[7]John R. Meyer and Edwin Kuh, The Investment Decision: an Empirical Study, Cambridge, 1957, pp. 19-20.

for services in marketing the securities. Also, the fact that dividend payments are taxable while interest payments are not, makes equity financing much more expensive in relation to debt.

Internal financing is advantageous because it involves none of the disadvantages listed above attributable to external sources of funds. In addition, the differential taxation of dividends and capital gains provides an incentive to firms to finance from internal sources. That is, earnings paid out as dividends are subject to tax at the marginal personal income tax rate (of the recipient), while earnings retained and invested are subject to tax at the capital gains rate (due to the resulting increase in value of the stock). To the extent that the capital gains rate is less than the personal rate there is an advantage to financing from retentions, because under such a procedure a given value of current earnings yields (eventually) a larger return to the existing stockholders (disregarding the timing disadvantage of postponed payment). Of course the fraction of earnings retained in any one period is limited by the preferences of stockholders (as reflected in changes in the value of the stock) for present as compared with future income.

There is no need to present detailed documentation in support of the proposition that internal financing plays an important role in the investment decision process. Meyer and Kuh cite an extensive list of studies containing empirical evidence relating to the preference for internal funds; and themselves state in summarizing the empirical investigations that: "By far the most outstanding aspect of the direct inquiries is their virtual unanimity in finding that internal liquidity considerations and a strong preference for internal financing are prime factors in determining the volume of investment."[8]

An idea of the extent to which internal and external funds have been used in the manufacturing industries in recent years may be obtained from Table 4.1.[9] A summary of relevant information from the latter is presented in the table below, which contains the ratios of external long term to total sources of funds, and of internal to total sources of funds. The table also contains the ratios of internal sources to total long term sources and ratios of the former to plant and equipment. Internal sources consist of retained profits plus depreciation, while total long term sources consist of all sources less short run sources.

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[8]*Ibid.*, p. 17.

[9]Table 4.1 gives values for manufacturing and mining since no such (consistent) table is available for the former alone. Although an approximate sources and uses table could be constructed for the manufacturing industries by using data from various sources, this was not done since accurate data are not available on external long term financing, one of the variables of primary interest. Further, even if the mining industry results differ, their effect on the totals is likely to be insignificant due to size considerations.

Summary of Sources and Uses Table

	year					
1957	1958	1959	1960	1961	1962	1963
.22	.17	.06	.08	.11	.09	.09
.90	.86	.71	.86	.71	.76	.73
.80	.83	.93	.91	.87	.89	.89
.98	1.12	1.40	1.03	1.16	1.26	1.27

Line 1 gives the ratio of external long term to total sources.
Line 2 gives the ratio of internal to total sources.
Line 3 gives the ratio of internal to total long term sources.
Line 4 gives the ratio of internal sources to plant and equipment.

The fraction of long term funds in any year from internal sources ranges from .80 to .93, while the ratio of total internal funds to total long term funds summed over the period is .87 (although not shown explicitly), thus indicating a predominant reliance on internal financing. Further, in every year but one the amount of internal funds exceeds investment expenditures, and in some years by a considerable amount, indicating again the importance of retentions. (Since the latter comparison involves both sources and uses, the substantial size of the discrepancy term should be noted.)

The purposes of this chapter were to study briefly the type of rate of return calculations employed by entrepreneurs in the investment decision process, and to investigate the advantages from internal financing and the extent to which such financing is used in practice. The purpose of the next chapter is to provide some idea of the orders of magnitude involved in rate of return changes resulting from the different methods of accelerated depreciation.

Table 4.1

SOURCES AND USES OF CORPORATE FUNDS[10]

(Billions of Dollars)

	Year						
	1957	1958	1959	1960	1961	1962	1963
Sources, total	18.6	17.0	25.4	19.6	23.8	25.8	28.5
Retained Profits[1]	7.1	4.4	7.5	5.7	5.1	6.1	6.8
Depreciation	9.6	10.2	10.6	11.1	11.7	13.5	14.1
External long term sources[2]	4.1	2.9	1.4	1.6	2.6	2.4	2.6
Stocks	1.4	(3)	.5	.4	.2	-.6	-1.0
Bonds	1.8	2.2	.3	.5	2.0	1.5	1.8
Short term sources[4]	-2.3	-.4	5.8	1.2	4.5	3.8	5.1
Uses, total	17.4	14.0	22.5	16.3	22.2	22.9	25.7
Plant and Equipment	17.0	12.2	12.9	15.3	14.5	15.5	16.5
Inventories (book value)	.8	-2.3	4.4	1.0	1.3	2.6	2.2
Receivables and miscellaneous assets	.1	2.6	3.9	2.3	5.1	3.1	4.8
Cash and U.S. Gov't. securities	-.6	1.4	1.3	-2.3	1.3	1.7	2.2
Discrepancy (uses less sources)	-1.2	-3.0	-2.9	-3.3	-1.6	-2.9	-2.8

[1]Includes depletion
[2]Also includes long term bank loans, mortgages, and other long term debt
[3]Less than $50 Million
[4]Includes short term bank loans, trade payables, federal income tax liabilities, and miscellanous liabilities.

Source: U. S. Dept. of Commerce, O.B.E., based on S.E.C. and other financial data.

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[10]U. S. Dept. of Commerce, O.B.E., Survey of Current Business, Vol. 44, No. 11 (November, 1964), p. 9.

Chapter 5

THE EFFECT OF ACCELERATED DEPRECIATION ON RATE OF RETURN MEASURES

The purpose of this chapter is to investigate the effect on the internal rate of return (to be denoted iror), a modified internal rate (to be defined below), and the payback period, of the various methods of accelerated depreciation. These effects are compared between methods, and for different asset lives and initial rates of return. Interest centers also on the orders of magnitude involved in the changes, since very small changes may indicate that effects on investment are essentially negligible. As mentioned in Chapter 1 only the pdv effect is considered when analysing the internal rate and modified rate, with financing costs being assumed constant.

Although the analysis of the internal rate centers on discounted value changes, a basic difference is introduced when considering changes in rate of return measures as opposed to pdv changes. Rate of return measures involve a compounding over time, which means basically that for a fixed change in the pdv of the revenue stream, the longer the asset's life the less effect this change will have on the rate of return. This is important when analysing the effects of accelerated depreciation as between assets of different lives (by comparing rate of return measures) since short lived assets derive a relatively greater advantage compared with long lived assets from accelerated depreciation in terms of rate of return changes than in terms of pdv changes.

The following general method of analysis permits a determination of the effects of accelerated depreciation on the internal rate of return.

General Method of Analysis

Consider an asset which costs C in the first period and yields positive net cash flows of R(t) over n years. Then the internal rate r_1, assuming continuous discounting, is defined by:

$$(5.1) \quad C = \int_0^n R(t)e^{-r_1 t}dt$$

If accelerated depreciation is adopted thus changing the allowable depreciation deductions and therefore net revenues in each period, a new internal rate r_2 will result. Let w(t,n)C be the increase in revenue t years after introduction of a particular method of accelerated depreciation. Then r_2 is defined by:

$$(5.2) \quad C = \int_0^n R(t)e^{-r_2 t}dt + \int_0^n w(t,n)Ce^{-r_2 t}dt$$

Rewriting the latter and substituting for C from (5.1) gives:

$$(5.3) \quad 1 = (\int_0^n e^{-r_2 t}R(t)dt)/(\int_0^n e^{-r_1 t}R(t)dt) + \int_0^n w(t,n)e^{-r_2 t}dt$$

which is an implicit relation involving r_2 and r_1 and R(t). The relation between r_1 and r_2 can not be determined without making an assumption about the shape of R(t). Two convenient assumptions are constancy and linear decline. For the former (5.3) becomes:

$$(5.4) \quad 1 = (\int_0^n e^{-r_2 t}dt)/(\int_0^n e^{-r_1 t}dt) + \int_0^n w(t,n)e^{-r_2 t}dt$$

If R(t) is assumed to decline linearly to 0 after n years, that is $R(t) = g(n-t)$ then (5.3) becomes:

$$(5.5)\quad 1 = \left(\int_0^n e^{-r_2 t}(n-t)dt\right)/\left(\int_0^n e^{-r_1 t}(n-t)dt\right) + \int_0^n w(t,n)e^{-r_2 t}dt$$

Equations (5.4) and (5.5) form the basis for determining orders of magnitude involved in rate of return increases due to various types of accelerated depreciation. Before presenting such results, however, it is interesting to observe some general conclusions about the relation between the gain from accelerated depreciation and the length of asset life assumed.

As mentioned above short lived assets generally will experience larger rate of return increases from accelerated methods than long lived assets. For an investment credit in particular the following argument shows that the increase in the internal rate is a monotonically decreasing function of n. In the case of a credit of k percent of cost, (5.2) reduces to:

$$(5.2)'\quad C = \int_0^n R(t)e^{-r_2 t}dt + kC\int_0^1 e^{-r_2 t}dt$$

Differentiating (5.1) with respect to n gives:

$$dC/dn = 0 = e^{-r_1 n}R(n) + \int_0^n e^{-r_1 t}R(t)(-t(dr_1/dn))dt$$

which may be solved for dr_1/dn:

$$dr_1/dn = e^{-r_1 n}R(n)/\left(\int_0^n e^{-r_1 t}R(t)dt\right)$$

Differentiating (5.2)' with respect to n and solving in a similar manner for dr_2/dn gives:

$$dr_2/dn = R(n)e^{-r_2 n}/\left(\int_0^n R(t)^{-r_2 t}tdt + k\int_0^1 e^{-r_2 t}tdt\right)$$

Therefore $dr_2/dn < dr_1/dn$ if and only if:

$$R(n)e^{-r_2 n}/\left(\int_0^n R(t)e^{-r_2 t}dt + k\int_0^1 e^{-r_2 t}dt\right) <$$

$$R(n)e^{-r_1 n}/\left(\int_0^n e^{-r_1 t}R(t)dt\right)$$

which after slight simplification becomes:

$$\int_0^n e^{r_1(n-t)} tR(t)dt < \int_0^n e^{r_2(n-t)} tR(t)dt + k\int_0^1 e^{r_2(n-t)} tdt$$

But $r_2 > r_1$, and $R(t) \geqslant 0$ for all t by assumption, and therefore $d(r_2-r_1)/dn < 0$, which means the maximum increase in the internal rate due to an investment credit, regardless of the shape of the revenue stream, occurs for the shortest lived assets to which the credit is applicable.

It is clear that this is true for a credit, only because the resulting increase in the discounted value of the revenue stream ($kC\int_0^1 e^{-r_2t}dt$) does not depend on n. Consequently for any accelerated method for which the change in discounted value is an increasing function of n, the behaviour of the rate of return increase can not be determined _a priori_. If the increase in $\int_0^n w(t,n)e^{-r_2t}dt$ is outweighed by the compounding factor as n increases, the shortest lived assets will experience the most gain. Calculations presented below show this to be the case for a switch from SL to SYD, but not for the 1962 credit or for an initial allowance.

As mentioned above equations (5.4) and (5.5) provide the basis for determining the extent of increase in the internal rate of return due to a switch from SL to SYD, the introduction of an initial allowance, investment credit, and a reduction in asset life.

Change in the Internal Rate due to a Switch from SL to SYD

The final term required for a solution to (5.4) and (5.5) is the tax rate times the discounted value of the difference in depreciation deductions under the two methods, that is:

$$\int_0^n w(t,n)e^{-r_2 t}dt = T\int_0^n (2(n-t)/n^2-1/n)e^{-r_2 t}dt$$

Table 5.1 gives values of r_2 for various initial rates (r_1), asset lives (n), and a tax rate of 50%, assuming both a constant and linearly declining revenue stream. The calculations indicate that the absolute increase in the rate of return is a monotonically decreasing function of n. This occurs because the compounding factor consistently outweighs the increase in $\int_0^n w(t,n)e^{-r_2 t}dt$. The table indicates that the gain is larger under a linearly declining revenue stream than under a constant one, and that the absolute change in the internal rate is larger, the larger the initial rate. The percentage change, although not shown explicitly, decreases slowly.

Change in the Internal Rate Due to An Initial Allowance

The term required in (5.4) and (5.5) to obtain the change in the internal rate resulting from an initial allowance of b% of cost is given by:

$$\int_0^n w(t,n)e^{-r_2 t}dt = bT\int_0^1 e^{-r_2 t}dt - bT\int_1^n h(t-1,n-1)e^{-r_2 t}dt$$

The choice of SL or SYD, and of a constant or linearly declining revenue stream gives four implicit relations connecting the initial and new internal rates. Table 5.2 contains values of r_2 for various initial rates, asset lives, a tax rate of 50% and an initial allowance of 20%. In contrast to the pdv and liquidity calculations in preceding chapters a specific initial allowance rate is required here to obtain a solution to the problem and consequently tabulations are given for the 20% rate only.

Table 5.2 indicates that the behaviour of the gain from an allowance as a function of n depends on the revenue stream assumption and on the initial rate of return. Only for an initial rate of 4% is the gain monotonically decreasing in all cases for the range of n given in the table, while for higher initial rates the gain appears to first decrease and then increase. The gain is larger of course under SL than SYD and assuming a linearly declining revenue stream. As mentioned in Chapter 1 the reason for the former is that the write-down in the base occurs closer to the present under SYD.

Change in the Internal Rate Due to the 1962 Investment Credit

Let k be the credit as a percent of cost then the term required in equations (5.4) and (5.5) is given by:

$$\int_0^n w(t,n)e^{-r_2t}dt = k\int_0^1 e^{-r_2t}dt - kT\int_0^n h(t,n)e^{-r_2t}dt$$

Values of r_2 for various initial rates and asset lives are presented in Table 5.3. Since the credit is applicable only to machinery and equipment, the maximum asset life given is 24 years.

The strong incentive to short lived assets provided by a true investment credit is diminished in the 1962 case by the reduction in credit for short lives and by the writedown in the asset's base. When these provisions are taken into account the maximum increase in the internal rate of return, as indicated in the table, occurs for assets of 8 years. That the gain for lives of over 8 years is in almost all cases a decreasing function of n is due to the fact that the compounding factor outweighs the relative advantage to long lived assets from the base reduction.

The investment credit is more beneficial under a linearly declining revenue stream than under a constant one and if SL is in use rather than SYD. Since most firms by 1962 were using accelerated methods, the SYD calculations probably give a better idea of the gain from the credit than do the SL calculations. The table indicates that for a constant revenue stream, assuming SYD, the internal rate increases approximately from 4 to 5, 8 to 9, 12 to 13.2, 16 to 17.3 and 20 to 21.4% for assets with lives of 8 years (the maximum gain). Only for low initial rates therefore does the gain appear to be significant, since for higher initial rates, although absolute increases are larger, the corresponding percent increases are very small. Analogous results assuming a linearly declining revenue stream are somewhat more favourable.

<u>Change in the Internal Rate Due to a Reduction in Asset Life</u>

For purposes of calculating the new internal rate, it is assumed that the useful life of the asset remains the same, with only the tax life and hence depreciation deductions changing. That is, the new internal rate is calculated over the longer life, as was the initial rate. The term required in (5.4) and (5.5) is given by:

$$\int_0^{n_2} w(t,n_2)e^{-r_2t}dt = \int_0^{n_1} h(t,n_1)e^{-r_2t}dt - \int_0^{n_2} h(t,n_2)e^{-r_2t}dt$$

where n_1 is the shorter and n_2 the longer life. The SL and SYD methods of depreciation together with the two revenue assumptions provide, once again, four implicit relations involving the old and new initial rates. Values of r_2 are given in Table 5.4 for various initial rates and for asset life changes which are intended to approximate actual changes in 1962 for the two-digit manufacturing industries.

The table indicates that absolute changes in the internal rate increase very slowly with the initial rate, and that percentage changes decrease. It appears that with a constant revenue stream and assuming SYD in use, the change in the internal rate is seldom greater than one percentage point, except of course for very high initial rates. The linearly declining revenue assumption results in slightly higher values but in the majority of industries the increases are still very small.

Comparison of Effects

One of the reasons for presenting the tables in this section is to permit a comparison of rate of return changes resulting from different methods of accelerated depreciation. Although a detailed table such as the ones provided in connection with the liquidity and pdv analyses has not been drawn up, Tables 5.1-5.4 provide a good indication of the different effects. A ranking of methods according to the general benefit provided indicates that the switch to SYD, the 1962 credit, and the 20% allowance are all approximately comparable, while most asset life reductions yield much less benefit.

An idea of the overall order of magnitude involved is provided by noticing that for a constant revenue stream the rate of return increase is less than 2 percentage points in all cases except for short lived assets involved in a switch from SL to SYD, and for assets with high initial rates subject to a 20% initial allowance. With a linearly declining revenue stream this holds for initial rates of 4, 8, and 12% with the former exception. The maximum advantage under the credit is for an asset of 8 years, and under the switch to SYD for the shortest-lived asset. No such generalization is possible for the initial allowance changes. Both the allowance and the switch to SYD result in substantial rate of return increases for certain combinations of asset lives and initial rates, which are considerably greater than the maximum increases provided by the credit.

The gain in the internal rate is always greater with declining than constant revenues, and for the credit, allowance, and asset life change if SL rather than SYD is in use. The absolute gain increases in all cases with the initial rate, while the percentage gain decreases for all methods but the initial allowance, which shows a slight increase.

<u>The Modified Internal Rate of Return</u>

There has been much discussion in the rate of return literature concerning the advantages and disadvantages of internal rate of return calculations as a criterion for investment decisions.[1] It is generally accepted that present value calculations, using the firm's cost of capital (to be denoted i) as the discount rate lead to the correct solution of investment expenditure problems. Internal rate of return calculations, depending on the circumstances, do not always provide the same results.

Assume first that net cash outlays (to be denoted C) occur only in the first period and there is no ceiling to expenditures in this period. Under the pdv criterion all projects are undertaken for which pdv > C, and under the iror criterion if iror > i. These clearly yield identical results since iror > i if and only if pdv > C. Assume now a ceiling to expenditures in the current period, then under the pdv considerations pdv/C is maximized, that is, the projects are ranked according to pdv/C and investment continues until the ceiling is reached. Under iror considerations projects are

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[1]See in particular the articles by A. A. Alchian, E. Renshaw, J. H. Lorie and L. J. Savage, and Ezra Solomon in Chapter II, and the article by J. Hirshleifer in Chapter IV of <u>The Management of Corporate Capital</u>, edited by Ezra Solomon, University of Chicago, 1963. See also H. M. Weingartner, <u>Mathematical Programming and the Analysis of Capital Budgeting Problems</u>, Englewood Cliffs, N. J., 1963, and "The Excess Present Value Index--A Theoretical Basis and Critique", <u>Journal of Accounting Research</u>, Vol. 1, No. 2, Autumn, 1963, pp. 213-224.

selected according to their iror values, and this may lead to a different set of projects. The case of mutually exclusive projects is handled correctly by the iror method by using the Fisher rate of return.[2]

Assuming net cash outlays in periods other than the first it can be shown that the usual internal rate of return calculations may give ambiguous results in that more than one iror may result. If net cash outlays in more than one period are combined with expenditure constraints over time the investment decision problem becomes very complex and one must resort to linear programming techniques such as those developed by H. M. Weingartner.[3] In general then the pdv technique is superior to the iror technique although in some cases the latter does give correct results. For example the iror measure will determine correctly the cut-off point for a group of investments, if there is no rationing, and net outlays occur in the first period only. For more complicated problems, or in order just to compare the profitability of different projects, the iror is not appropriate.

Several authors have pointed out that it is the implicit reinvestment assumption contained in the iror formulation which leads in certain cases to incorrect results.[4]

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[2]See for example A. A. Alchian, op. cit., pp. 67-71.

[3]H. M. Weingartner, op. cit.

[4]See for example Ezra Solomon, op. cit.

This can be seen by rewriting the iror equation (5.1) as

$$(5.6) \quad Ce^{rn} = \int_0^n R(t)e^{(n-t)r}dt$$

That is, r is the rate of return on cost which compounds the cost to equal the revenues earned over the asset's life. This is an acceptable definition of a rate of return measure except for the fact that the intermediate cash flows are assumed to be reinvested at this particular rate of return, which will vary for each project. Ideally the reinvestment rates to use are those which will be prevailing in years 1 to n. Assuming that accurate prediction of these rates is not possible it seems more reasonable to allow the cash flow for all projects to be reinvested at the firm's (current) cost of capital (i) than to allow the flows from each project to be reinvested at that project's internal rate. The modified internal rate of return (r*) defined by (5.7), incorporates this proposition.

$$(5.7) \quad Ce^{r^*n} = \int_0^n R(t)e^{(n-t)i}dt$$

It should be noted that (5.6) and (5.7) agree except for the reinvestment assumption.[5]

It is not hard to show that r* provides the correct cut-off for investment if there is no rationing, according to the rule: invest if $r^* > i$. Rewriting (5.7) gives:

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[5]Introduction of the cost of capital into the calculations means of course that the resulting rate of return can in no sense be considered "internal". The terminology is used solely to emphasize the similarity to the internal rate of return.

$$(5.8) \quad e^{r^*n}/e^{in} = \int_0^n R(t)e^{-it}dt/C$$

which shows that since $\int_0^n R(t)e^{-it}dt$ is the asset's pdv, then pdv $\gtrless$ C as $r^* \gtrless i$. Mutually exclusive projects are ranked correctly by r* in the following manner. Suppose there are two projects with costs C^1 and C^2 and revenues $R^1(t)$ and $R^2(t)$. Let $C^1 > C^2$, and calculate r* for the project with cost $C^1 - C^2$ and revenues $R^1(t) - R^2(t)$, then invest in project 1 if $r^* > i$ and if not, invest in 2. That this rule ranks as does net pdv can be seen as follows. By the definition of r*:

$$C^1 - C^2 = (\int_0^n (R^1(t) - R^2(t))e^{(n-t)i}dt)/e^{r^*n}$$

Assuming $r^* > i$ then:

$$C^1 - C^2 < (\int_0^n (R^1(t) - R^2(t))e^{(n-t)i}dt)/e^{ni} =$$

$$\int_0^n (R^1(t) - R^2(t))e^{-ti}dt$$

which may be written as $\int_0^n R^1(t)e^{-ti}dt - C^1 > \int_0^n R^2(t)e^{-ti}dt - C^2$. That is, the net pdv of the first project is greater than the second, which is as desired. For the case of rationing in the current period it is clear that r* does not necessarily rank correctly. From (5.8) above pdv/C = e^{r^*n}/e^{in} which shows that ranking by pdv/C is not equivalent to ranking by r*, and the latter will rank projects correctly only if they have approximately the same lifetime. The internal rate of return, however, does not rank correctly even in this case.

In summary, r* is an appropriate rate of return measure in the sense that:

1. It leads to the correct "invest or not" decision if there

is no rationing, according to the rule: invest if $r^* > i$.

2. It leads to the correct choice between mutually exclusive projects.

3. It can be interpreted as a "rate of return" on cost.

4. It assumes reinvestment of intermediate cash flows at a rate equal for all assets to the firm's cost of capital.

5. When rationing exists in 1 period, it ranks projects of approximately the same life correctly, that is according to pdv/C.

Although r^* fails when entrepreneurs take account of rationing in many periods or when investment projects are not independent, so do all other simple rate of return measures and more sophisticated techniques are required.

The effects on the modified internal rate of the various methods are now considered.

Effect of Accelerated Depreciation on the Modified Internal Rate of Return

The following analysis provides a bound to the change in the modified internal rate resulting from any method of accelerated depreciation. The bound is interesting because it results in orders of magnitude suggestive of a very small effect on investment. As above let $w(t,n)C$ be the increase in revenue t years after the introduction of a method of accelerated depreciation. Let i be the firm's cost of capital and r_1 and r_2 the modified internal rates before and after accelerated depreciation. The two relations analogous to (5.1) and (5.2) are

$$(5.9)\quad Ce^{r_1 n} = \int_0^n R(t)e^{i(n-t)}dt$$

$$(5.10)\quad Ce^{r_2 n} = \int_0^n R(t)e^{i(n-t)}dt + e^{in}\int_0^n w(t,n)Ce^{-it}dt$$

Substituting (5.9) into (5.10) and dividing by C gives:

$$(5.11)\quad e^{r_2 n} = e^{r_1 n} + e^{in}\int_0^n w(t,n)e^{-it}dt$$

This implicit relation allows determination of the change in the modified internal rate due to any method of accelerated depreciation. It should be noted that the change is independent of the shape of the revenue stream, and that the expression $\int_0^n w(t,n)e^{-it}dt$ is simply the pdv of the change in depreciation deductions (times the tax rate) and hence is exactly what appears in Chapter 2. An interesting bound to the change in the modified internal rate may be determined in the following manner. Let $h = r_2 - r_1$ then

$$e^{r_2 n} = e^{r_1 n + hn} = e^{r_1 n}(1 + hn + h^2n^2/2! + \ldots\ldots)$$

Substituting this expression for $e^{r_2 n}$ in (5.11) above gives

$$e^{r_1 n}(1 + hn + h^2n^2/2! + \ldots\ldots) = e^{r_1 n} + e^{in}\int_0^n w(t,n)e^{-it}dt$$

which, after dividing by $e^{r_1 n}$ becomes:

$$hn + h^2n^2/2! + \ldots\ldots = (e^{in}\int_0^n w(t,n)e^{-it}dt)/e^{r_1 n}$$

or $hn < (e^{in}\int_0^n w(t,n)e^{-it}dt)/e^{r_1 n}$. But $r_1 > i$ is required of the project to be feasible, which gives:

$$(5.12)\quad h = r_2 - r_1 < (\int_0^n w(t,n)e^{-it}dt)/n$$

The latter clearly shows that the relative gain accruing to short lived assets is larger in rate of return terms than pdv terms, since the numerator of the expression is precisely the gain in pdv resulting from accelerated depreciation.

(5.12) can be used to obtain an idea of the orders of magnitude involved in rate of return changes resulting from the major methods of accelerated depreciation. The gain from a true credit is easily calculated and provides an upper bound to changes for all methods. A credit of 7% of cost increases pdv by slightly less than .07 and hence increases the modified internal rate by less than .07/n. Rate of return changes for assets with lives of 5 and 10 years are therefore .015 and .007 respectively, suggesting that only short lived assets obtain any recognizable advantage. For the credit currently in effect of course the rate is scaled down for assets with lives of less than 8 years. In general it appears that the effects of accelerated depreciation on the modified internal rate of return are negligible.

Effect of Accelerated Depreciation on the Payback Period

The third measure to be analysed is the payback (or payout) period of an asset, defined as the number of years required for revenues (gross of depreciation) to accumulate to investment cost. Although the payback period is essentially a liquidity and not a profitability measure, its importance lies, as suggested in the preceding chapter, in the fact that it is widely used by entrepreneurs in the investment decision process. Accelerated depreciation affects an asset's payback period by increasing net revenues in early years thereby reducing the period of time taken for revenues to accumulate to cost. This effect is a peculiar one in that the ignoring of discounting means that the effect is not one of present values, nor is it concerned with a stream of assets. If the payout period is relevant it simply means that, for a single asset, even if the time factor is ignored, accelerated depreciation will affect the asset's rate of return. This particular mechanism is not analysed in Chapters 2 or 3, which include only the "rational" pdv and liquidity effects.

In analysing the effect of accelerated depreciation on the payback period it is necessary to make an assumption about the shape of the revenue stream. Both constancy and linear decline are studied. Under the former the payback period (n^*) is defined by the equation: $\int_0^{n^*} R dt = 1$, where R is the constant revenue per period. Under the latter revenues are assumed to decline linearly to zero in n years (the asset life for tax purposes), that is, $R(t) = g(n-t)$. The payback period is therefore defined by $\int_0^{n^*} g(n-t)dt = 1$. An alternative linear decline assumption is that revenues reach zero in n^* years. But this is unrealistic (and hence is not analysed below) because it implies no revenue is earned after n^* years, and therefore the asset is not earning a positive rate of return. The order in which the various methods of accelerated depreciation are studied differs from previous sections because the credit and allowance effects are simpler to analyse and hence are considered first.

Effect on the Payback Period of an Investment Credit

Assume first a true investment credit of k% of cost, an asset life for tax purposes of n years, and an original payback period of n* years. Then with a constant revenue stream, revenue per period is $(1/n^*)C$, and since the only change due to the introduction of the credit is a first year gain of kC, the new payback period n' is given by the relation:[6]

$$k + n'/n^* = 1$$

where n'/n^* represents revenues of $1/n^*$ for n' years. Solving for n' yields: $n' = n^*(1-k)$. Alternatively this result may be derived by recognizing that both the credit and revenue per period are expressed as a percent of cost. A revenue increase of k in the first period therefore reduces the payback period by $k/(1/n^*) = kn^*$, resulting in a new period of $n^*(1-k)$. In percentage terms the reduction is of course simply k. No tabulations are given of payback period changes because of the simplicity of calculations, with the change being neither a function of the asset life or tax rate. A 7% credit for example, results in a change of only $.07n^*$, and hence if $n^* = 5$ this is .35 years and if $n^* = 10$, .7 years. The reduction as a percent of the original payback period is of course just 7%.

The analysis is complicated by the introduction of a declining revenue stream. The new payback n' is defined by:

$$k + \int_0^{n'} g(n-t)dt = 1$$

which may be solved for n' using the definition of n* given

[6]It is convenient to normalize on C by considering revenues each period as a fraction of cost.

above: $\int_0^{n^*} g(n-t)dt = 1$. Table 5.5, which contains values of n' for selected n and n* and a 7% credit, indicates that the payback changes are considerably larger under a declining than under a constant revenue stream. The following considerations are important in this regard.

In analysing the dependence of payback period changes on the shape of the revenue stream or length of asset life assumed, the relevant factors are the magnitude of, and rate of decline of, the revenue stream in year n*. For a given gain in the first year (due for example to a true credit), the payback period change will be greater the faster are revenues falling, and the smaller their value, in year n*. But for a given initial payback period revenues are smaller and are falling faster at n* under a declining than under a constant revenue stream. This proposition concerning revenues holds also for short as compared with long asset lives, assuming declining revenues only. For these reasons, the values in Table 5.5 exceed those for constant revenues (in which the gain is always .07n*), and the gain for a given payback period decreases with n.

The 1962 investment credit requires in addition a write-down in the base of the asset. Assuming a constant revenue stream and SL depreciation, the payback period change due to such a credit (of k%) may be obtained by solving for n' in the following expression:

$$k + n'/n^* - n'(Tk/n) = 1$$

This differs from a true credit by the term $n'(Tk/n)$, which represents the write-down in the asset's base over the payback period. The corresponding expression for a declining revenue stream is:

$$k + \int_0^{n'} g(t-n)dt - n'(Tk/n) = 1$$

which may be solved for n' in terms of n^* and n (by using the definition of n^* given above for declining revenues). Table 5.6 contains values of n' for various n^* and n, assuming SL in use, and for constant and linearly declining revenue streams. The reduction in credit for short-lived assets is taken into account. Although not tabulated, corresponding changes under SYD will be slightly smaller.

With a constant revenue stream all values are of course less than the limit set by a true credit, that is, $.07n^*$. The gain for a given payback increases with the asset life because the larger the value of n, the lower the reduction per year due to the write-down in the base, and hence the greater the gain. This means that the larger the value of n the less will the entries in Table 5.6 differ from those for a true investment credit.

Payback period changes under declining revenues are larger than under constant revenues, but smaller than the true credit values given in Table 5.5. For a given initial payback the behaviour of the change as a function of n can no longer be determined, *a priori*, since the change, as n increases will tend to decrease due to the declining revenue assumption, and to increase due to the base reduction factor. It appears that for initial payback periods of over 5 years the former effect outweighs the latter.

Effect on the Payback Period of an Initial Allowance

Assume an initial allowance of b% of cost, with a constant revenue stream and SL in use. Then the allowance results in increased revenue of Tb in the first year, with a reduction in all later years of Tb/(n-1) and hence the new payback period (n') is defined by the relation:

$$Tb + n'/n^* - (n'-1)(Tb/(n-1)) = 1$$

Table 5.7 contains values of n' for selected n*, n, and an initial allowance rate of 20%. The table indicates that the gain is an increasing function of n, which by analogy with 1962 credit considerations, is due to the fact that the base of the asset must be written down. The limit to the gain derivable from the allowance may be determined by considering n indefinitely large, thus resulting in no effective write-down in the base. The relation given above becomes in the limit: $Tb + n'/n^* = 1$, which is equivalent to the expression for an investment credit with Tb replaced by k. Since the value of Tb is .10 (that is, .2 x .5), a bound to the gain obtainable from an allowance of 20% is 10% of the original payback period. The table indicates, however, that except for large n together with small n*, actual changes will be far below this value.

With declining revenues and SL, n' is defined by:

$$Tb + \int_0^{n'} g(t-n)dt - (n'-1)(Tb/(n-1)) = 1$$

Table 5.7 contains values of n' for an initial allowance of 20%, and selected n and n*. The table indicates that the payback changes are substantially larger with a declining than with a constant revenue stream particularly for short lived assets with large initial paybacks, although such changes are still in all cases only fractions of a year even for large initial payback periods. The gain is an increasing function of asset life, which means that the relative advantage to long lived assets (in terms of a small base write-down over the payout period) outweighs the advantage to short lived assets arising from the declining revenue assumption.

<u>Effect on the Payback Period of a Switch from SL to SYD</u>

A switch to SYD results in a change in revenue of $T(2(n-t)/n^2)$ in year t, and hence the new payback n', assuming SL and a constant revenue stream, is defined by:

$$n'/n^* + \int_0^{n'} T(2(n-t)/n^2 - 1/n)dt = 1$$

With a linearly declining revenue stream n' is defined by:

$$\int_0^{n'} (g(n-t) + T(2(n-t)/n^2 - 1/n))dt = 1$$

Table 5.8 gives values of the change in the payback period resulting from a switch to SYD for both revenue assumptions. The gain as a function of n increases and then decreases under a constant revenue stream, and decreases monotonically under a declining revenue stream. The reason for the former is that with a given initial payback only the gains from switching to SYD are relevant at first in the payback calculations, but as n increases the corresponding later year losses also become relevant. Under declining revenues the initial advantage as n increases is outweighed by the advantage to the shortest lived assets arising from the declining revenue assumption.

It is noteworthy that almost all values given in Table 5.7 are larger than the corresponding values for any of the other methods of accelerated depreciation, including a true 7% credit.

Effect on the Payback Period of a Change in Asset Life

Let n_1 be the shorter and n_2 the longer tax life. Since the change in revenue in any period under SL is $T(1/n_1 - 1/n_2)$, then with a constant revenue stream n' is given by:

$$n'/n^* + n'T(1/n_1 - 1/n_2) = 1$$

and with a linearly declining revenue stream by:

$$\int_0^{n'} g(t-n_2)dt + n'T(1/n_1 - 1/n_2) = 1$$

Table 5.9 contains values of n' for selected n^* assuming both a constant and linearly declining revenue stream, and for asset life changes which are intended to approximate actual changes in 1962. The effects of the asset life reductions appear in general to be small, except perhaps in industries such as 36 and 372 which experienced large percent reductions in lives. Excluding these two industries, all changes are less than one-half of a year for original payback periods of under 8 years.

Summary

Three rate of return measures are analysed: the internal rate of return, the modified internal rate, and the payback period. For the former two the advantage to short lived assets, as measured in rate of return terms compared with pdv or liquidity terms, is emphasized. Determination of the effects of accelerated depreciation in all cases depends on the shape of the net revenue stream. The iror and payback period are analysed under the assumptions of constancy and linear decline, while an upper bound to changes in the modified internal rate is determined which is independent of the shape of the revenue stream.

It is of course impossible to determine the extent to which rate of return changes resulting from the different methods of accelerated depreciation will affect investment decisions. However, it seems likely *a priori* that the effects of changes at least in the modified internal rate will be negligible. Internal rate of return changes are somewhat larger, but as mentioned above are less than 2 percentage points in all cases except for short lived assets involved in a switch to SYD, and for assets with high initial rates subject to a 20% initial allowance. This result, in conjunction with the finding in the preceding chapter that discounting techniques are not generally used in practice, and in view of the fact that such rate of return calculations require estimates of future revenues, which are likely to be only approximate, suggests that the pdv effect of accelerated depreciation (to the extent that it is explicitly taken into account) will be unimportant. Further, the fact that such rate of return changes are small may mean that their effect on any subjective implicit discounting procedures, used in the investment decision process, will also be unimportant.

Payback period changes are much more dependent than internal rate of return changes on the revenue stream assumption. Such changes under a constant revenue stream do not in general seem large, being small fractions of a year for low initial paybacks and in almost all cases smaller than one year. But under a declining revenue stream the payback period changes are always larger and indeed are significantly larger in some cases. Therefore to the extent that investment decisions are influenced by a payback criterion, and revenues are assumed to decline to zero over the asset's life, it is possible that accelerated depreciation will affect investment through this channel.

In the empirical work to follow the mechanism through which accelerated depreciation is assumed to affect investment is through a change in the level of cash flow. The only other plausible empirical formulation, and as mentioned in the first chapter the one used by Hall and Jorgenson, and Coen, is to postulate a discounted value mechanism; that is, that accelerated depreciation affects investment through changes in the pdv of an asset's revenue stream. However, in view of the survey evidence presented in Chapter 4, and in view of the orders of magnitude involved in rate of return changes calculated in this chapter, the fact that a discounting mechanism is not included in the simulations can not be considered a serious omission. The simulations of course do not take payback period changes into account explicitly, nor in fact does there appear to be an empirical formulation suitable for such a purpose.

Table 5.1*

EFFECT ON THE INTERNAL RATE OF RETURN OF A SWITCH FROM SL TO SYD

Assuming Constant Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.75	9.40	13.97	18.49	22.94	27.36
8	4.70	9.25	13.68	18.03	22.33	26.59
12	4.66	9.12	13.46	17.72	21.94	26.12
16	4.62	9.02	13.29	17.50	21.67	25.81
20	4.59	8.93	13.16	17.34	21.48	25.59
24	4.56	8.86	13.06	17.21	21.33	25.42
28	4.53	8.80	12.97	17.11	21.21	25.29
32	4.51	8.75	12.90	17.02	21.11	25.17
36	4.49	8.71	12.85	16.95	21.02	25.08
40	4.47	8.67	12.79	16.88	20.95	25.00

Assuming Linearly Declining Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	5.22	10.26	15.15	19.92	24.59	29.18
8	5.13	9.96	14.59	19.08	23.46	27.78
12	5.05	9.73	14.19	18.52	22.76	26.95
16	4.98	9.54	13.89	18.12	22.28	26.40
20	4.92	9.39	13.65	17.83	21.94	26.03
24	4.86	9.26	13.47	17.60	21.69	25.75
28	4.81	9.15	13.32	17.42	21.49	25.53
32	4.77	9.06	13.20	17.28	21.33	25.37
36	4.73	8.98	13.10	17.16	21.20	25.23
40	4.69	8.92	13.01	17.06	21.10	25.12

n = asset life in years
r_1 = initial internal rate of return in percent
A corporate tax rate of 50% is assumed.
Table gives values of the internal rate of return after switching to SYD.

*Tables 5.1-5.4 are based on continuous discounting.

Table 5.2

EFFECT ON THE INTERNAL RATE OF RETURN DUE TO A 20% INITIAL ALLOWANCE

Assuming SL and Constant Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.42	8.81	13.17	17.50	21.81	26.10
8	4.41	8.78	13.10	17.40	21.69	25.96
12	4.40	8.75	13.06	17.35	21.63	25.91
16	4.40	8.73	13.03	17.32	21.62	25.91
20	4.39	8.71	13.01	17.32	21.63	25.94
24	4.38	8.70	13.01	17.32	21.65	25.98
28	4.38	8.69	13.01	17.34	21.67	26.01
32	4.37	8.69	13.01	17.35	21.70	26.05
36	4.37	8.69	13.02	17.37	21.73	26.07
40	4.36	8.69	13.03	17.39	21.75	26.10

Assuming SL and Linearly Declining Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.67	9.28	13.82	18.33	22.79	27.21
8	4.65	9.21	13.69	18.13	22.52	26.88
12	4.63	9.15	13.59	17.99	22.35	26.68
16	4.62	9.11	13.52	17.89	22.24	26.57
20	4.60	9.07	13.46	17.83	22.17	26.49
24	4.59	9.04	13.42	17.78	22.12	26.44
28	4.57	9.01	13.38	17.74	22.08	26.41
32	4.56	8.99	13.36	17.72	22.06	26.39
36	4.55	8.97	13.34	17.70	22.04	26.38
40	4.55	8.96	13.33	17.69	22.04	26.37

Table 5.2 (Continued)

Assuming SYD and Constant Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.31	8.61	12.89	17.15	21.41	25.65
8	4.29	8.57	12.82	17.07	21.31	25.55
12	4.28	8.55	12.80	17.05	21.30	25.56
16	4.28	8.54	12.79	17.05	21.32	25.60
20	4.28	8.54	12.79	17.07	21.36	25.67
24	4.27	8.53	12.80	17.10	21.41	25.73
28	4.27	8.54	12.82	17.13	21.46	25.79
32	4.27	8.54	12.83	17.16	21.50	25.84
36	4.27	8.55	12.85	17.19	21.54	25.89
40	4.27	8.55	12.87	17.22	21.57	25.92

Assuming SYD and Linearly Declining Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.49	8.95	13.37	17.78	22.16	26.52
8	4.45	8.87	13.25	17.61	21.96	26.28
12	4.44	8.84	13.20	17.55	21.88	26.19
16	4.43	8.81	13.17	17.51	21.84	26.16
20	4.42	8.80	13.15	17.50	21.82	26.14
24	4.42	8.79	13.14	17.48	21.82	26.14
28	4.41	8.78	13.13	17.47	21.82	26.15
32	4.41	8.77	13.12	17.47	21.82	26.16
36	4.40	8.77	13.12	17.48	21.83	26.17
40	4.40	8.77	13.12	17.48	21.83	26.17

n = asset life in years
r_1 = initial internal rate of return in percent
A corporate tax rate of 50% is assumed.
Table gives values of the internal rate of return after introduction of a 20% initial allowance.

Table 5.3

EFFECT ON THE INTERNAL RATE OF RETURN DUE TO THE 1962 INVESTMENT CREDIT

Assuming SL and Constant Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.63	8.67	12.69	16.73	20.76	24.79
5	4.52	8.56	12.59	16.63	20.66	24.70
6	4.89	8.98	13.06	17.14	21.20	25.29
7	4.78	8.88	12.96	17.04	21.20	25.21
8	5.07	9.21	13.34	17.48	21.61	25.74
10	4.90	9.04	13.18	17.32	21.47	25.61
12	4.77	8.93	13.08	17.23	21.38	25.54
14	4.69	8.85	13.00	17.16	21.32	25.49
16	4.63	8.79	12.94	17.11	21.28	25.46
18	4.58	8.74	12.90	17.08	21.26	25.45
20	4.54	8.71	12.87	17.05	21.25	25.44
22	4.51	8.68	12.85	17.04	21.24	25.44
24	4.48	8.65	12.83	17.02	21.23	25.44

Assuming SL and Linearly Declining Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.95	9.00	13.04	17.09	21.14	25.18
5	4.78	8.84	12.89	16.94	20.99	25.03
6	5.35	9.48	13.59	17.70	21.81	25.91
7	5.19	9.32	13.44	17.55	21.66	25.76
8	5.63	9.83	14.02	18.20	22.37	26.53
10	5.36	9.57	13.76	17.95	22.13	26.30
12	5.18	9.39	13.59	17.78	21.97	26.14
14	5.05	9.27	13.47	17.66	21.85	26.03
16	4.95	9.18	13.38	17.57	21.77	25.95
18	4.88	9.10	13.30	17.51	21.70	25.90
20	4.82	9.04	13.25	17.45	21.65	25.85
22	4.77	8.99	13.20	17.41	21.61	25.81
24	4.72	8.95	13.16	17.37	21.58	25.78

Table 5.3 (Continued)

Assuming SYD and Constant Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.61	8.63	12.65	16.67	20.70	24.72
5	4.50	8.53	12.55	16.58	20.60	24.63
6	4.86	8.92	12.98	17.04	21.10	25.16
7	4.75	8.82	12.88	16.94	21.01	25.08
8	5.02	9.12	13.23	17.33	21.44	25.56
10	4.84	8.96	13.07	17.19	21.32	25.44
12	4.73	8.85	12.97	17.10	21.24	25.38
14	4.65	8.77	12.90	17.04	21.19	25.35
16	4.58	8.72	12.85	17.00	21.16	25.33
18	4.54	8.67	12.81	16.97	21.15	25.33
20	4.50	8.64	12.79	16.96	21.14	25.33
22	4.47	8.61	12.77	16.94	21.13	25.33
24	4.44	8.59	12.75	16.94	21.13	25.34

Assuming SYD and Linearly Declining Revenue Stream

	r_1					
n	4	8	12	16	20	24
4	4.92	8.95	12.98	17.01	21.04	25.07
5	4.75	8.79	12.82	16.86	20.90	24.93
6	5.29	9.38	13.46	17.55	21.63	25.71
7	5.13	9.23	13.32	17.41	21.49	25.58
8	5.54	9.69	13.83	17.98	22.12	26.26
10	5.28	9.44	13.60	17.75	21.91	26.06
12	5.10	9.27	13.44	17.60	21.77	25.93
14	4.98	9.15	13.32	17.50	21.67	25.84
16	4.88	9.06	13.24	17.42	21.60	25.78
18	4.81	9.00	13.18	17.36	21.55	25.73
20	4.75	8.94	13.12	17.32	21.51	25.70
22	4.70	8.89	13.08	17.28	21.48	25.67
24	4.66	8.86	13.05	17.25	21.45	25.65

n = asset life in years
r_1 = initial internal rate of return in percent
A corporate tax rate of 50% is assumed.
Table gives values of the internal rate of return after introduction of the 1962 Investment Credit.

Table 5.4

EFFECT ON THE INTERNAL RATE OF RETURN DUE TO REDUCTION IN ASSET LIFE

Assuming SL and Constant Revenue Stream

			r_1					
Ind	n_1	n_2	4	8	12	16	20	24
20	13	15	4.21	8.34	12.40	16.44	20.47	24.48
22	13	16	4.31	8.47	12.56	16.62	20.65	24.68
26	15	19	4.33	8.49	12.58	16.62	20.65	24.67
28	11	13	4.26	8.42	12.51	16.57	20.61	24.64
29	15	18	4.26	8.39	12.46	16.49	20.52	24.53
30	13	14	4.11	8.18	12.21	16.23	20.25	24.26
32	16	18	7.17	8.25	12.29	16.31	20.33	24.33
33	17	21	4.29	8.42	12.48	16.51	20.53	24.54
35	12	14	4.24	8.37	12.45	16.50	20.53	24.55
36	11	14	4.37	8.58	12.71	16.79	20.85	24.89
371	12	14	4.24	8.37	12.45	16.50	20.53	24.55
372	9	12	4.45	8.74	12.93	17.05	21.14	25.20

Assuming SL and Declining Revenue Stream

			r_1					
Ind	n_1	n_2	4	8	12	16	20	24
20	13	15	4.33	8.50	12.58	16.63	20.64	24.65
22	13	16	4.47	8.70	12.82	16.87	20.89	24.90
26	15	19	4.50	8.73	12.82	16.86	20.87	24.86
28	11	13	4.40	8.62	12.75	16.82	20.86	24.88
29	15	18	4.39	8.58	12.65	16.69	20.69	24.69
30	13	14	4.17	8.27	12.31	16.34	20.35	24.35
32	16	18	4.25	8.37	12.42	16.44	20.44	24.43
33	17	21	4.43	8.61	12.67	16.69	20.69	24.68
35	12	14	4.36	8.56	12.66	16.71	20.74	24.75
36	11	14	4.57	8.88	13.04	17.14	21.18	25.20
371	12	14	4.36	8.56	12.66	16.71	20.74	24.75
372	9	12	4.71	9.13	13.37	17.53	21.62	25.67

Table 5.4 (Continued)

Assuming SYD and Constant Revenue Stream

			r_1					
Ind	n_1	n_2	4	8	12	16	20	24
20	13	15	4.16	8.27	12.35	16.42	20.48	24.53
22	13	16	4.22	8.38	12.50	16.60	20.68	24.75
26	15	19	4.24	8.41	12.53	16.63	20.72	24.75
28	11	13	4.19	8.32	12.43	16.52	20.59	24.67
29	15	18	4.19	8.32	12.42	16.50	20.56	24.62
30	13	14	4.08	8.14	12.18	16.22	20.25	24.28
32	16	18	4.12	8.21	12.27	16.32	20.36	24.39
33	17	21	4.21	8.36	12.46	16.54	20.61	24.67
35	12	14	4.17	8.29	12.39	16.46	20.53	24.59
36	11	14	4.26	8.46	12.60	16.73	20.84	24.93
371	12	14	4.17	8.29	12.39	16.46	20.53	24.57
372	9	12	4.32	8.56	12.76	16.92	21.07	25.19

Assuming SYD and Declining Revenue Stream

			r_1					
Ind	n_1	n_2	4	8	12	16	20	24
20	13	15	4.24	8.40	12.51	16.60	20.66	24.71
22	13	16	4.34	8.57	12.72	16.84	20.93	25.00
26	15	19	4.37	8.61	12.76	16.88	20.96	25.02
28	11	13	4.28	8.49	12.63	16.75	20.84	24.91
29	15	18	4.29	8.48	12.60	16.69	20.76	24.81
30	13	14	4.12	8.21	12.27	16.32	20.35	24.38
32	16	18	4.19	8.31	12.38	16.44	20.48	24.51
33	17	21	4.33	8.52	12.65	16.74	20.80	24.85
35	12	14	4.26	8.44	12.57	16.67	20.74	24.80
36	11	14	4.40	8.69	12.89	17.05	21.17	25.27
371	12	14	4.26	8.44	12.57	16.67	20.74	24.80
372	9	12	4.49	8.86	13.13	17.35	21.52	25.66

Ind = industry
n_1 = average asset life after 1962 Guideline change
n_2 = average asset life before 1962 Guideline change
r_1 = initial internal rate of return in percent
A corporate tax rate of 50% is assumed.
Table gives values of the internal rate of return after 1962 Guideline change.

Table 5.5

CHANGE IN PAYBACK PERIOD DUE TO A TRUE 7% INVESTMENT CREDIT

Assuming Linearly Declining Revenue Stream

	n*							
n	3	4	5	6	7	8	9	10
4	.43							
6	.30	.50	.86					
8	.27	.40	.58	.86	1.33			
10	.25	.36	.50	.68	.92	1.27	1.82	
12	.24	.34	.46	.60	.77	1.00	1.30	1.71
14	.24	.33	.44	.56	.70	.87	1.08	1.35
16	.23	.32	.42	.53	.66	.80	.97	1.17
18	.23	.32	.41	.51	.63	.76	.90	1.07
20	.23	.31	.40	.50	.61	.72	.85	1.00

n* = original payback period in years
n = average asset life in years

Table gives values of the payback period change due to a true 7% credit.

Table 5.6

CHANGE IN THE PAYBACK PERIOD DUE TO THE 1962 INVESTMENT CREDIT

Assuming SL and Constant Revenue Stream

	n*							
n	3	4	5	6	7	8	9	10
4	.05							
6	.11	.13	.14					
8	.18	.22	.26	.28	.29			
10	.19	.24	.28	.31	.34	.35		
12	.19	.24	.29	.33	.36	.39	.36	.43
14	.20	.25	.30	.34	.38	.42	.41	.47
16	.20	.25	.31	.35	.40	.44	.45	.50
18	.20	.26	.31	.36	.41	.45	.47	.53
20	.20	.26	.32	.37	.42	.46	.49	.54

Assuming SL and Linearly Declining Revenue Stream

	n*							
n	3	4	5	6	7	8	9	10
4	.11							
6	.16	.24	.42					
8	.22	.31	.43	.60	.93			
10	.22	.30	.39	.50	.65	.89		
12	.21	.29	.37	.47	.58	.72	.91	1.19
14	.21	.29	.37	.45	.54	.65	.78	.95
16	.21	.29	.36	.44	.53	.62	.73	.86
18	.21	.28	.36	.44	.52	.60	.70	.81
20	.21	.28	.36	.43	.51	.59	.68	.78

n* = original payback period in years
n = average asset life in years
A corporate tax rate of 50% is assumed.
Table gives values of the payback period change due to the 1962 Investment Credit.

Table 5.7

CHANGE IN THE PAYBACK PERIOD DUE TO A 20% INITIAL ALLOWANCE

Assuming SL and Constant Revenue Stream

	n*							
n	3	4	5	6	7	8	9	10
4	.11							
6	.19	.17	.11					
8	.22	.24	.23	.19	.11			
10	.24	.28	.29	.29	.25	.20	.11	
12	.25	.30	.33	.35	.34	.31	.27	.20
14	.26	.32	.36	.39	.40	.40	.37	.33
16	.27	.33	.38	.42	.44	.45	.45	.43
18	.27	.34	.39	.44	.47	.49	.50	.50
20	.27	.34	.41	.46	.50	.53	.55	.56

Assuming SL and Declining Revenue Stream

	n*							
n	3	4	5	6	7	8	9	10
4	.28							
6	.28	.35	.41					
8	.29	.36	.42	.47	.55			
10	.29	.37	.43	.49	.55	.60	.67	
12	.29	.37	.45	.51	.57	.62	.68	.74
14	.29	.38	.45	.52	.59	.65	.70	.75
16	.29	.38	.46	.53	.60	.67	.72	.78
18	.29	.38	.46	.54	.61	.68	.74	.80
20	.29	.38	.47	.55	.62	.69	.76	.82

n* = original payback period in years
n = average asset life in years
A corporate tax rate of 50% is assumed.
Table gives values of the payback period change due to a 20% initial allowance.

Table 5.8

CHANGE IN THE PAYBACK PERIOD DUE TO A SWITCH FROM SL TO SYD

Assuming SL and Constant Revenue Stream

	n*							
n	3	4	5	6	7	8	9	10
4	.24							
6	.31	.40	.36					
8	.29	.43	.54	.57	.45			
10	.27	.41	.56	.67	.74	.71	.52	
12	.24	.39	.54	.68	.80	.89	.91	.83
14	.22	.36	.51	.66	.80	.93	1.03	1.08
16	.20	.33	.47	.63	.78	.93	1.06	1.16
18	.19	.31	.45	.60	.75	.90	1.05	1.18
20	.17	.29	.42	.56	.72	.87	1.03	1.17

Assuming SL and Linearly Declining Revenue Stream

	n*							
n	3	4	5	6	7	8	9	10
4	.68							
6	.50	.83	1.26					
8	.41	.67	.98	1.36	1.87			
10	.35	.57	.84	1.15	1.50	1.93	2.50	
12	.30	.50	.74	1.01	1.31	1.65	2.04	2.52
14	.26	.44	.66	.90	1.18	1.48	1.81	2.18
16	.24	.40	.60	.82	1.07	1.34	1.64	1.96
18	.21	.36	.54	.75	.98	1.24	1.51	1.81
20	.20	.33	.50	.69	.91	1.14	1.40	1.68

n* = original payback period in years
n = average asset life in years
A corporate tax rate of 50% is assumed.
Table gives values of the payback period change due to a switch from SL to SYD.

Table 5.9

CHANGE IN THE PAYBACK PERIOD DUE TO REDUCTION IN ASSET LIFE

Assuming SL and Constant Revenue Stream

			n*							
Ind	n_1	n_2	3	4	5	6	7	8	9	10
20	13	15	.05	.08	.12	.18	.24	.32	.40	.49
22	13	16	.06	.11	.17	.24	.34	.44	.55	.67
26	15	19	.06	.11	.17	.24	.33	.43	.53	.66
28	11	13	.06	.11	.17	.24	.33	.42	.53	.65
29	15	18	.05	.09	.14	.19	.26	.34	.43	.53
30	13	14	.02	.04	.06	.10	.13	.17	.22	.27
32	16	18	.03	.05	.09	.12	.17	.22	.27	.34
33	17	21	.05	.09	.14	.20	.26	.34	.43	.53
35	12	14	.05	.09	.14	.21	.28	.36	.46	.56
36	11	14	.09	.15	.23	.33	.45	.58	.73	.89
371	12	14	.05	.09	.14	.21	.28	.36	.46	.56
372	9	12	.12	.21	.32	.46	.62	.80	1.00	-

Assuming SL and Linearly Declining Revenue Stream

			n*							
Ind	n_1	n_2	3	4	5	6	7	8	9	10
20	13	15	.05	.09	.15	.23	.34	.47	.64	.86
22	13	16	.07	.13	.21	.31	.45	.62	.82	1.08
26	15	19	.07	.12	.20	.29	.41	.56	.73	.94
28	11	13	.07	.13	.22	.33	.49	.69	.96	1.33
29	15	18	.05	.10	.16	.24	.34	.46	.61	.79
30	13	14	.03	.05	.09	.13	.19	.28	.39	.54
32	16	18	.03	.06	.10	.15	.22	.30	.39	.52
33	17	21	.05	.10	.16	.23	.32	.44	.57	.73
35	12	14	.06	.11	.18	.28	.41	.56	.78	1.06
36	11	14	.10	.18	.29	.44	.62	.86	1.17	1.55
371	12	14	.06	.11	.18	.28	.41	.56	.78	1.06
372	9	12	.14	.26	.42	.64	.92	1.29	1.75	-

Ind = industry
n_1 = average asset life after 1962 Guideline change
n_2 = average asset life before 1962 Guideline change
n* = original payback period in years
A corporate tax rate of 50% is assumed.
Table gives values of the payback period change due to the 1962 Guidelines.

Chapter 6

ESTIMATION OF AN ACCELERATED DEPRECIATION LEARNING FUNCTION

As mentioned in Chapter 1 a major problem involved in determining the effect of the accelerated depreciation methods introduced in 1954 is that there is no information available for the two-digit manufacturing industries on the rate of adoption of such methods. In this chapter an attempt is made simultaneously to estimate the fraction of investment in each year written off by accelerated methods and to fit a learning curve to these values.

The estimation procedure is based on the following recursive relation between the total amount of accelerated depreciation in two consecutive years.

$$(6.1)\quad z_t = z_{t-1} + y_t I_t h(1,n) - \sum_{p=1}^{t-N} y_{t-p} I_{t-p} B(p,n) - R(t-1,N)$$

where: z_t = total accelerated depreciation in year t

I_t = investment in year t

y_t = the (unknown) percent of I_t written off by accelerated methods in year t

$h(t,n)$ = accelerated depreciation rate on assets of age t with tax life of n years. Under DDB $h(t,n) = 2/n(1-2/n)^{t-1}$.

$B(p,n)$ = $h(p,n) - h(p+1,n)$; representing the change in the annual depreciation rate on assets of age p as compared to p+1. Under DDB $B(p,n) = 2/n(1-2/n)^{p} - 2/n(1-2/n)^{p+1}$.

$R(t,N)$ = depreciation in t on assets bought since year N and retired in year t. This factor is ignored in further calculations since concern centers on years immediately following the adoption of new methods.

N = the year of introduction of accelerated methods

Basically equation (6.1) says that the total amount of accelerated depreciation in t is equal to accelerated depreciation in t-1, plus the depreciation on the fraction of current investment subject to the new method, minus a correction factor due to the fact that the accelerated method does not provide for equal annual deductions, minus depreciation on (accelerated) assets retired during t-1.

The y values are unknown, but if total accelerated depreciation (z) is known in each year since N and an average life (n) of assets is assumed, then the y's can be obtained by solving (6.1) recursively. If some values of z are unknown, however, the y's can be determined only for years prior to the first unknown z. For later years (6.1) provides a set of linear relations in the y's. In particular, if the first z value is unknown then no value of y can be determined. This is the case for the current problem. The total accelerated depreciation figures (z) are available only for fiscal years 1954, 1955, 1957, and 1960.[1] Since the Revenue Act of 1954 permitted the new methods to begin in accounting periods ending after December 1953 and since the 1954 fiscal year began with accounting periods ending July 1954, the initial

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[1]The exact nature and source of all data used in the estimations are given in the appendix.

accelerated depreciation value is unknown. The set (6.1) therefore consists of 3 linear relations in the unknown y's connecting years 1954-55, 1955-57, and 1957-60.

These relations may be written in matrix notation as $Ay = c$, where A is a 3 x 8 matrix, y is an 8 x 1 vector and c is a 3 x 1 vector. The matrix A is a function of investment values (I_t), the accelerated depreciation rates (h(t,n)), and the correction terms (B(p,n)). The y vector represents the unknown percent of annual investment depreciated under accelerated methods from 1953 to 1960. The 3 elements of c are: $z_{55}-z_{54}$, $z_{57}-z_{55}$, and $z_{60}-z_{57}$, where for example z_{55} is total accelerated depreciation in 1955. The problem is to simultaneously estimate the y values and to fit a learning curve to them. Before proceeding to the method of estimation some consideration must be given to details of the data.

A timing problem exists in matching the depreciation with the investment figures. There are three issues involved. a) The Internal Revenue Code states that depreciation in the first year of an asset's life should be proportional to the length of time the asset is available, which means that ideally all investment should be multiplied by an appropriate time factor. Some averaging alternatives are allowed, however, and in particular the "half-year convention" is used by some firms, although there appears to be no good evidence on the extent of its use. Under this convention one-half of a year's depreciation is taken in the first year of an asset's life. The approximation used in the current analysis, which is essentially a compromise between the two methods, is to assume that investment is centered in each quarter.

b) Depreciation can not be taken on an asset until it is ready for use, therefore investment figures may not represent the true base for depreciation in the year. For machinery this may not be serious since the discrepancy between investment figures and installed equipment is probably small. A lag certainly exists for plant, but there seems to be no information on its magnitude. The fact that the weight given to plant in the calculations is small helps to make this assumption on timing less crucial. (Plant life is 40 years, and the fraction of investment which is in plant is approximately .3.)

c) The annual depreciation data must be related to the quarterly investment data. The accelerated depreciation value for year t includes corporations with accounting periods ending July t to June t + 1. The assumption is made that investment expenditures are distributed among these firms in the same proportion as total assets. The latter are used since neither the depreciation nor the depreciable asset data are classified by accounting periods. This means that if w_2 is the fraction of total assets attributable to firms whose accounting periods end in the second quarter of each year then it is assumed that the fraction of total investment in any quarter accounted for by these firms is also w_2.

In the numerical calculations involving the set (6.1), the DDB and SYD depreciation figures are combined and referred to as accelerated depreciation, with the DDB rate being used as the accelerated rate to be applied on investment. Investment in each year is divided into plant and equipment expenditure. The tax life for the latter is taken from a U. S. Treasury Department Release (1961) estimating actual life in practice, while a life of 40 years is assumed for plant.

Estimation Procedure

As mentioned above, (6.1) may be written in matrix form as $Ay = c$, where the y's represent the (unknown) percent of annual investment depreciated under the accelerated methods. The problem is to simultaneously estimate these y's and to fit a learning curve to them. However, before considering the special case represented by equation (6.1), a general solution to the problem will be given. Stated generally, the problem is to fit a curve to $y = f(x) + e$ subject to the restriction that $Ay = c$, where:

- y is an unknown T x 1 vector
- f is a function of x whose parameters (p_f) are to be estimated
- x is a known matrix, with dimensions T x n where n is the number of parameters to be estimated in f
- c is a known k x 1 vector
- A is a known k x T matrix
- e is a T x 1 vector of disturbances, the elements of which are assumed to be normally distributed independent random variables with $E(e) = 0$ and $E(ee') = \sigma_e^2 I$.

Two methods of solution are considered.[2]

Method I

A straightforward maximization of the likelihood function requires a minimization of $e'e$ or $(y-f(x))'(y-f(x))$ subject to $Ay = c$. Letting m be a k x 1 column vector of Lagrange multipliers, then the problem is to minimize $g = (y-f(x))'(y-f(x)) + 2m'(Ay-c)$ over m, p_f, and y. Equating first partial derivatives of g with respect to m, p_f, and y to zero gives

(6.2) $Ay = c$

(6.3) $df(x)/dp_f\ (y-f(x)) = 0$

(6.4) $y - f(x) + A'm = 0$

If f is not linear in its parameters, then $df(x)/dp_f$ will be a function of p_f and these $k + n + T$ equations (in m, p_f, and y) will not generally be solvable in a straightforward manner. A nonlinear minimization procedure is then needed to determine the estimates of y and p_f.

Linear Case

If f is linear in its parameters, that is, $y = xb + e$ with b an n x 1 vector of unknown parameters, then (6.2), (6.3), and (6.4) can be solved as follows. Setting $f(x) = xb$ and $p_f = b$, (6.3) and (6.4) become respectively

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[2]To the author's knowledge this problem has not been dealt with in the literature on restricted regressions. The latter deals with restrictions of one form or another on the parameters of f, with the y values assumed known, while the current problem involves restrictions on the (unknown) y values themselves.

(6.5) $b = (x'x)^{-1}x'y$

(6.6) $y - xb + A'm = 0$

Premultiplying (6.6) by A one can solve for $m = (AA')^{-1}(Axb - Ay)$ and substituting back into (6.6) gives

(6.7) $y = xb + A'(AA')^{-1}(Axb - Ay) = 0$

Since from (6.5) and (6.2) $x'y = x'xb$ and $Ay = c$, then premultiplying (6.7) by x' yields $\hat{b} = (x'A'(AA')^{-1}Ax)^{-1}x'A'(AA')^{-1}c$. $\hat{b}$ is now in terms of observables and $\hat{y}$ is obtained by substituting $\hat{b}$ into (6.7) to give $\hat{y} = x\hat{b} - A'(AA')^{-1}(Ax\hat{b} - c)$.

<u>Method II</u>

An alternative method of solution, which is essentially a generalized least-squares procedure, is to suppress the unobservable y values first. In the general case of $y = f(x) + e$ subject to $Ay = c$, this requires premultiplication of y by A to give $c = Ay = Af(x) + Ae$ which is in terms of observables. Maximization of the likelihood function requires a minimization over p_f of the generalized sum of squares $(c-Af(x))'(AA')^{-1}(c-Af(x))$, which provides estimates of p_f but not of y. The following considerations show that these estimates of p_f are the same as those that would be obtained under method I.

Method I involved a minimization of $g = (y-f(x))'(y-f(x)) + 2m'(Ay-c)$ over m, p_f, and y; yielding the first order conditions (6.2), (6.3) and (6.4). Premultiplying (6.4) by A gives $m = (AA')^{-1}A(f(x)-y)$, which when substituted back into (6.4) gives $y-f(x) = A'(AA')^{-1}A(y-f(x))$.

Premultiplying by (y-f(x))' yields

(6.8) $(y-f(x))'(y-f(x)) = (c-Af(x))'(AA')^{-1}(c-Af(x))$

since Ay = c. The values of y which minimize g must satisfy (6.8) and hence minimization of g over m, p_f, and y is equivalent to minimization of $(c-Af(x))'(AA')^{-1}(c-Af(x))$ over p_f.

Linear Case

In the linear case the estimation of b reduces to a more familiar generalized least-squares problem. Given $y = xb + e$, $Ay = c$, $E(e) = 0$ and $E(ee') = \sigma_e^2 I$, premultiplying by A gives $Ay = Axb + Ae$ or $c = zb + u$, where $z = Ax$ and $E(uu') = \sigma_e^2 AA'$. The generalized least-squares estimate of b is therefore

$$\hat{b} = (z'(AA')^{-1}z)^{-1}z'(AA')^{-1}c$$
$$= (x'A'(AA')^{-1}Ax)^{-1}x'A'(AA')^{-1}c$$

as before. The $\hat{y}$'s may now be estimated by minimizing $(y-y_b)'(y-y_b) + m'(Ay-c)$ where $y_b = x\hat{b}$. The estimates of y so obtained are the same as those obtained under method I.

Estimation of Specific Functions

The first relation to be considered is $y_t = 1 - v^t + e_t$, where the parameters to be estimated are v and the y's, e_t is the disturbance term, and t takes on values 1 through 8, starting in 1953. In the notation of the preceding section, v corresponds to p_f, the t values to x, and $1 - v^t$ to f(x). The estimates of y, denoted $\hat{y}$, are restricted to satisfy the set of 3 linear relations described previously. There are a number of reasons for considering the above curve. It is monotonic, increases rapidly at first, passes through the origin, is asymptotic to 1, and contains only one parameter to estimate. Description by one parameter permits easy comparison between industries of the extent of adoption of accelerated methods. A nonlinear minimization algorithm recently developed by Goldfeld, Quandt, and Trotter, has been used to obtain the estimates.[3]

A linear relation of the form $y_t = b_1 + b_2t + e_t$ is also estimated. In the notation of the preceding section b_1 and b_2 correspond to p_f, the t values and the constant term correspond to x, and $b_1 + b_2t$ corresponds to f(x). This curve should prove superior to the nonlinear form for industries in which the percent of investment subject to accelerated methods has remained essentially constant since introduction, at some level other than 100%. The linear estimators derived in the previous section are used to obtain the values of b_1, b_2 and the $\hat{y}$'s.

Discussion of Results

Table 6.1 contains estimates of linear and nonlinear learning functions for 12 two-digit industries. The estimated y values ($\hat{y}$), generated y values (y_v and y_b), learning function parameters (v, b_1, and b_2) and the standard errors of the latter are presented. Standard errors of the $\hat{y}$'s, although not tabulated, are respectable for all industries but textiles

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[3] S. M. Goldfeld, R. E. Quandt and H. F. Trotter, "Maximization by Quadratic Hill-Climbing", Princeton University, Econometric Research Program R. M. #72, January, 1965 (to be published in Econometrica).

and petroleum, with values in most cases being less than one-sixth of their respective coefficients.

Since the $\hat{y}$'s are restricted to satisfy a set of relations such as (6.1) there is no way to ensure that they lie between 0 and 1. However, they will if there are no errors of observation in the data and if the learning functions are correctly specified. In the nonlinear case the values of $\hat{y}$ are less than 1 in all but 2 industries, and in the linear case in all but 3 industries. On the other hand the estimates generated by the nonlinear learning functions (y_v) will lie between 0 and 1 if the estimates of v are themselves in this range. The estimates of v in Table 6.1 range from 0 to .94. In the linear case the values of y_b need not lie between 0 and 1, and indeed for large t will probably lie above 1 if the true learning function is nonlinear.

Measures of Goodness of Fit

Two measures of goodness of fit appear in the table. $\bar{R}_y^2$ is defined as:

$$\bar{R}_y^2 = 1 - \frac{\sum_1^T (\hat{y}-y_q)^2/(T-(n+T-k))}{\sum_1^T (\hat{y}-\bar{\hat{y}})^2 \;/(T-(T-k)-1)}$$

where T = number of observations

n = number of estimated parameters in the learning function

k = number of linear relations (number of rows of A)

q = b in the linear, and v in the nonlinear estimations

n + T - k degrees of freedom are lost in calculating the sum of squared residuals since n learning function parameters are used together with T - k independently determined y values. In the denominator the latter plus one degree of freedom for estimation of $\bar{\hat{y}}$ are required.

The major drawback of $\bar{R}_y^2$ is that it does not take into account that part of the problem concerned with estimation of the y values themselves. This is well illustrated by the nonlinear estimation in the electrical machinery industry. The negative value of $\bar{R}_y^2$ results because the values of y_v are constant and hence the sum of squared residuals is larger than the sum of squared deviations of y about its mean. In spite of the negative value of $\bar{R}_y^2$ the estimates are of interest since the problem is to determine the y's as well as to approximate them by a curve, and in this case the y's appear to be constant at 1, suggesting that accelerated depreciation was immediately adopted.

A second measure of the goodness of fit which avoids this problem and which is wholly in terms of fits to observables is defined as $\bar{R}_c^2$.

$$\bar{R}_c^2 = 1 - \frac{\sum_1^K (c-c_q)^2/(k-n)}{\sum_1^K (c-\bar{c})^2\ /(k-1)}$$

where $c_q = Ay_q$ and n, T, k, and q are as before. $\bar{R}_c^2$ indicates how well the estimation is doing in explaining variation about the known vector c. n degrees of freedom are lost in calculating the sum of squared residuals since the n estimated parameters of the learning function are required to calculate y_q, and hence Ay_q or c_q. In the denominator one degree of freedom is lost in calculating $\bar{c}$, the sample mean.

As can be seen from Table 6.1 the results in terms of $\bar{R}_c^2$ are respectable for all industries except textiles and petroleum. The fact that for these two industries the estimates of the y's are very low suggests that even after seven years firms may not have adopted the accelerated methods. There is a good reason for expecting a low adoption rate in the petroleum industry. Depletion allowances are important, and under the percentage method, allowable depletion is computed as 27½ percent of annual gross income from the property, but cannot exceed 50 percent of net income. Since accelerated depreciation reduces net income, SL might be advantageous under certain circumstances. This line of reasoning is supported by statistics in the Treasury Depreciation Survey[4] of 1961. According to the latter, accelerated methods have been applied to approximately 37 percent of total investment since 1953 in the "oil and natural gas production and refining" category while for all other manufacturing the figure is 74 percent. The fact that circumstances may exist under which accelerated depreciation is not advantageous makes estimation of a learning function for this industry less meaningful.

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[4]Treasury Department, Office of Tax Analysis, Treasury Depreciation Survey, November 1961, Table 14, (unpublished). The survey does not provide information by two-digit industry or by year, but presents the total amount of accelerated depreciation claimed by broad industrial classes between 1954 and 1959.

A possible hypothesis concerning the poor results in the textile industry is that they are due in part to the inability of firms to absorb the larger allowances resulting from accelerated depreciation. That is, a basic assumption throughout the analysis is that firms earn sufficient profits to warrant using the accelerated methods. If this assumption is violated in any given year SL will be more advantageous than SYD or DDB since it results in a larger total deduction in later years. It is difficult to determine the importance of this hypothesis but the following calculations suggest that if it is relevant at all then of the 12 industries under study, the textile industry is the one most likely to be affected.

Data are available both for corporations with net income and for those without net income. The fraction of an industry's gross sales or total assets accounted for by firms with net income is a rough measure of the profitability of the industry. Although a great deal of significance cannot be attached to the values of such ratios, a comparison across industries is of interest. For fiscal years 1954 to 1960 the percent of each industry's gross sales and total assets accounted for by firms with net income has been calculated. For 1954 to 1958 inclusive both ratios were lower in the textile industry than in any other industry. The average value of the sales ratio for these years was .844 for textiles and .917 for all manufacturing. The analagous total asset values were .840 and .923. According to these measures of general industry profitability the performance of the textile industry was much inferior to that of the other manufacturing industries from 1954 to 1958.

Asset Life Assumptions

The above estimation procedure requires the assumption of a fixed asset life for machinery from 1953 to 1960, and to the extent that this assumption is violated the results will be biased. Unfortunately there is very little evidence available on the behaviour of asset lives during the period. However, according to the preliminary report of the Treasury Depreciation Survey of 1961 (covering over one-half of total depreciable property accounts of corporations) there has been essentially no change in tax lives since 1953. The survey states "Questionnaire responses by both large and small firms indicated relatively few material changes in service lives . . . . . during the period since 1953."[5] Only one-sixth of large firms reported a material change, and only one-third of these reported shorter lives. The average values since 1953 reported by the survey are the ones used in the above estimations.

It is important to determine the sensitivity of the parameter estimates to the (constant) asset life assumed. If the estimating procedure is such that a small change in the asset life produces markedly different results, then little

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[5]Treasury Department, Office of Tax Analysis, Preliminary Report on Treasury Depreciation Survey, January 1961, page 3, (unpublished).

faith can be put in the learning functions presented above. For this reason the nonlinear equations were rerun with all asset lives arbitrarily increased by two years. The results appear in Table 6.2. The fits as measured by $\bar{R}_c^2$, and the values of v are very similar for most industries.

The Treasury Survey notwithstanding, a shortening of asset lives may have occurred in the period, in which case the results will be biased. No attempt is made in the paper to test explicitly for this bias, mainly because there exists no good evidence on the extent or rate of decline of lives. The following reasoning, however, suggests that such a bias, if it exists at all, will probably not be serious.

Assume for the moment a constant asset life over the period. From 6.1 it can be seen that the adoption rates (y) are determined essentially by dividing accelerated depreciation changes by the product of investment values and depreciation rates. A reduction in the latter due to an increase in asset life (n) will therefore basically result in larger y values. For an increase in n the behaviour of the correction term, involving B(p,n) and past investment and adoption rates, is not determinable. It will tend to increase due to the higher adoption rates and decrease due to B(p,n), but since it only involves terms in $1/n^2$ its effect will be of second order in any case. In the estimation procedure, of course, the y values are also affected by the form of the learning function assumed, but for a small increase in asset life the effect should be to increase the estimated rates of adoption. The re-estimation of the learning functions with all asset lives increased two years supports this line of reasoning. For every industry the estimate of v is lower, implying a faster adoption of accelerated methods.

If asset lives are allowed to decline during the period from n+2 to n, where n is the original life, then the same intuitive reasoning as above may be applied. The values near n+2 at the beginning of the period will tend to drive the adoption rate estimates above those given in Table 6.1, while the values near n at the end of the period will tend to drive the estimates below those given in Table 6.2. Without re-estimating it is imposible to determine whether the results will actually lie between those given in the two tables, but intuitively it appears plausible. In any case it appears unlikely that the results will differ substantially from those given in the tables.

No account has been taken in the analysis of the dispersion of asset lives about the average life. Clearly if information on the distribution of capital expenditures by asset life were available for each quarter it would be used. The lack of such information and the necessity to use an average life results in errors in the estimated parameters. The following analysis provides some idea of the error in effective depreciation rates which results from assuming an average life, under steady state conditions and a uniform distribution of assets. The steady state assumption is of course unrealistic but the analysis nevertheless provides an insight into the problems involved. Further, although this too may be unrealistic, it can be shown that exactly the same results hold if the steady state assumption is replaced by the assumption of constant net investment.

Assume that steady state conditions prevail and that the stock of assets is distributed uniformly with respect to asset lives, with n_1 the minimum and n_2 the maximum life. Let k(n) be the stock of, and I(n) investment in, assets of life n. Then in order to maintain the steady state I(n) must equal k(n)/n. Total investment I_T is therefore $\int_{n_1}^{n_2} I(n)dn$, which reduces to $k\log(n_2/n_1)$ since k(n)=k, a constant for all n. Depreciation (assuming DDB) on assets of age t is then given by:

$$\text{Dep}(t) = \int_{n_1}^{n_2} (2/n)I(n)(1-2/n)^{t-1}dn$$
$$= ((1-2/n_2)^t - (1-2/n_1)^t)/(t\log(n_2/n_1))I_T = r(t)I_T,$$

where r(t) is the effective depreciation rate on total investment for assets of age t. Alternatively if depreciation is calculated by applying the average asset life ($\bar{n}$) to total investment, then since $\bar{n} = \int_{n_1}^{n_2} n(I(n)/I_T)dn = (n_2-n_1)/\log(n_2/n_1)$, total depreciation becomes:

$$\text{Dep}'(t)=(2\log(n_2/n_1)/(n_2-n_1))(1 - 2\log(n_2/n_1)/(n_2-n_1))^t$$

$= r'(t)I_T$ where r'(t) is the effective depreciation rate on total investment for assets of age t. Table 6.3 contains values of r(t)-r'(t) for capital stocks which are uniformly distributed with respect to asset lives, and which have distribution means of 12, 15 and 18 years.

Since 1960 is the 7th year after introduction of DDB (ignoring the last half of fiscal 1953), Table 6.3 values are given for $t \leq 7$ only. The table indicates a rapid reduction in error after the first year with even the latter error appearing small. Considering a set of equations such as (6.1) it can be seen that the only error of any consequence is in this first year rate. The other term in (6.1) involving depreciation rates is given by $\sum_{p=1}^{t-n} y_{t-p} I_{t-p} B(p,n)$, where B(p,n) is the difference in depreciation rates for two consecutive years. The error in B(p,n) will therefore be given not by the values in Table 6.3, but by first differences of such values. These first differences are in general negligible, and even the sum of such differences (up to a maximum of 7 terms) appears negligible. It should be noted that although r(t) as given above is not valid for $t > n_1$, this does not seriously affect the current problem since values of t greater than 7 are not of interest.

Possibility of Switching to SL

Under the DDB method of depreciation the option exists of switching to SL. Profit maximization requires a switch when the annual deductions under the two methods are equal, and for an asset with life n this occurs in year n/2+1, as was shown in Chapter 1. To the extent that switching has been used in practice the adoption rate estimates given above will be biased downward. This results from the fact that the 1960 accelerated depreciation figures will not include depreciation on assets subject to switching between 1957 and 1960. Since these figures are used in the estimation procedure, together with the assumption of no switching, the adoption rates will be underestimated.

A revision of the procedure to account for switching is not possible since it would require knowledge of the age distribution of the investment series. However, the following considerations suggest that the error from switching may not be serious. First, switching is not permitted in group accounts, which apparently are in common use, although there is no good evidence as to what extent. Second, switching is not possible under SYD, and although the DDB rate represents both methods in the estimations, to the extent that SYD is used in practice, the bias will be reduced. Third, the amount of investment subject to switching will be small compared to total investment since only relatively short lived assets purchased in the first few years after introduction of the new methods will be eligible.

Additional First-Year Depreciation

As mentioned in Chapter 1, a 20 percent additional first-year depreciation allowance of up to $2000, applicable on assets with a life greater than five years, is permitted on property acquired after December 31, 1957. Since the base of such assets must be written down by the amount of the allowance, and since the DDB and SYD depreciation figures exclude the additional first-year depreciation, then the investment figures used in the above calculations and considered eligible for the accelerated methods should be reduced by an amount equal to the write-down in the base of such assets. This correction has not been made because the data are unavailable. Data do exist, however, on the amount of first-year depreciation taken in 1960. The ratio of the latter to total investment gives the fraction of investment in 1960 which, in the analysis, is wrongly considered eligible for the accelerated methods. The maximum value of this fraction is .0037 (for the food industry) indicating that the error involved in ignoring the allowance is negligible.

5-Year Amortization Provisions

No consideration has been given in the above analysis to the extent of use of the 5-year amortization provisions, applicable mainly to grain storage and emergency facilities. According to the 1954 Revenue Code, amortization includes deductions taken in lieu of depreciation for emergency facilities and grain storage facilities, (erected after December 31, 1952) deductions taken for experimental expenditure capitalized but not subject to depreciation, and certain organizational expenditures, and mine exploration and development expenditures. The investment figures used in the current study include capital outlays subject to these amortization provisions, and a slow adoption rate according to the learning function may just reflect extensive use of such provisions.

In order to compare the rate of adoption of accelerated methods the learning functions should be re-estimated excluding from investment that portion eligible for the amortization allowances. Unfortunately no direct data exist on the amount of investment subject to amortization. Estimates of the latter, however, may be obtained by using the following recursive relation.

$$(6.9) \quad Am_t = Am_{t-1} + (I_t^{Am} - I_{t-d}^{Am})/d$$

where: Am_t = the total amortization deduction in year t

I_t^{Am} = investment subject to amortization at rate 1/d

d = amortization period, in this case 5 years

In order to construct the investment series using (6.9) Am_t must be known for all years since the introduction of the amortization allowances. Since Am_{1954} is unavailable and can be estimated only roughly, the learning functions were not re-estimated. The following calculations, however, provide some idea of the orders of magnitude involved.

Amortization taken on emergency facilities only, is available in the Quarterly Financial Reports.[6] This series is used to estimate Am_{1954} by interpolating linearly the ratio of amortization on emergency facilities to total amortization between 1953 and 1955. Equation (6.9) then provides an investment series (I_t^{Am}) representing expenditures subject to amortization. The percent of total investment not subject to amortization ($I_t - I_t^{Am}$), written off by accelerated methods, is then given by $y_v I_t/(I_t - I_t^{Am})$, where y_v is the original learning function estimate of the percent of total investment written off by accelerated methods.

Using this expression learning function values changed

[6]Federal Trade Commission - Securities and Exchange Commission, Quarterly Financial Report for Manufacturing Corporations, 1953 - 1955.

by less than .05 in all industries but paper, chemicals, primary metals, and other transportation, and only in the latter two did the values change substantially. These rough revisions, of course, are different from those that would be obtained by re-estimating the learning functions. The latter procedure was felt to be not worthwhile because of the possibility that the constructed I_t^{Am} series are not accurate.

As mentioned in Chapter 1, an empirical analysis of the 1954 accelerated depreciation provisions requires for each industry not only the estimation of a learning function for the accelerated methods, but also the estimation of a model of investment, dividend, and external finance behaviour. The following chapter is devoted to a study of the latter.

Table 6.1

Estimated Depreciation Learning Functions

for U. S. Manufacturing Industries

Percent of Investment Written off by Accelerated Methods

Fiscal Year		1954	1955	1956	1957	1958	1959	1960
Industry		Nonlinear Form		$y_t = 1 - v^t + e_t$				
Food and Beverage	$\hat{y}$	=.53	.66	.70	.79	.86	.90	.93
	y_v	=.49	.63	.74	.81	.87	.90	.93
		v = .72 (.015)		$\bar{R}^2_y$ = .96		$\bar{R}^2_c$ = 1.00		
Textile-mill Products	$\hat{y}$	=.28	.52	.75	.54	.36	.37	.46
	y_v	=.22	.31	.39	.46	.53	.58	.63
		v = .88 (.053)		$\bar{R}^2_y$ = -.94		$\bar{R}^2_c$ = -.19		
Paper and Allied Products	$\hat{y}$	=.41	.56	.66	.69	.72	.76	.82
	y_v	=.38	.51	.62	.70	.76	.81	.85
		v = .79 (.017)		$\bar{R}^2_y$ = .89		$\bar{R}^2_c$ = .97		
Chemicals and Allied Products	$\hat{y}$	=.23	.33	.43	.50	.56	.61	.66
	y_v	=.24	.33	.42	.49	.56	.61	.66
		v = .87 (.002)		$\bar{R}^2_y$ = 1.00		$\bar{R}^2_c$ = 1.00		
Petroleum and Coal Products	$\hat{y}$	=.15	.30	.41	.33	.22	.25	.34
	y_v	=.12	.17	.22	.27	.32	.36	.40
		v = .94 (.020)		$\bar{R}^2_y$ = -.86		$\bar{R}^2_c$ = -.14		
Rubber Products	$\hat{y}$	=.33	.52	.81	.77	.72	.73	.81
	y_v	=.39	.53	.63	.71	.78	.83	.86
		v = .78 (.039)		$\bar{R}^2_y$ = .71		$\bar{R}^2_c$ = .86		

Table 6.1 -- Continued

Percent of Investment Written off by Accelerated Methods

Fiscal Year		1954	1955	1956	1957	1958	1959	1960
Industry		Nonlinear Form			$y_t = 1 - v^t + e_t$			
Stone, Clay, and Glass Products	$\hat{y}$ =	.59	.68	.64	.84	.98	1.05	1.03
	y_v =	.55	.70	.80	.86	.91	.94	.96
		v = .67 (057)		$\bar{R}_y^2$ = .78		$\bar{R}_c^2$ = .92		
Primary Metal Industries	$\hat{y}$ =	.34	.47	.59	.71	.80	.85	.88
	y_v =	.38	.51	.62	.70	.76	.81	.85
		v = .79 (.014)		$\bar{R}_y^2$ = .97		$\bar{R}_c^2$ = .99		
Machinery except Transportation and Electrical	$\hat{y}$ =	.49	.68	.96	.96	.93	.95	.97
	y_v =	.59	.74	.83	.89	.93	.96	.97
		v = .64 (.052)		$\bar{R}_y^2$ = .85		$\bar{R}_c^2$ = .97		
Electrical Machinery and Equipment	$\hat{y}$ =	1.09	1.01	.85	.87	.91	.88	.92
	y_v =	1.00	1.00	1.00	1.00	1.00	1.00	1.00
		v = .00 (.456)		$\bar{R}_y^2$ = -.68		$\bar{R}_c^2$ = .56		
Motor Vehicles and Equipment	$\hat{y}$ =	.76	.82	.84	.72	.71	.68	.78
	y_v =	.53	.68	.78	.85	.90	.93	.95
		v = .69 (.151)		$\bar{R}_y^2$ = -9.19		$\bar{R}_c^2$ = .81		
Transportation Equipment, except Motor Vehicles	$\hat{y}$ =	.38	.48	.43	.58	.74	.81	.83
	y_v =	.33	.45	.55	.63	.70	.76	.80
		v = .82 (.022)		$\bar{R}_y^2$ = .87		$\bar{R}_c^2$ = .96		

Table 6.1 Continued

Percent of Investment Written off by Accelerated Methods

Fiscal Year		1954	1955	1956	1957	1958	1959	1960
Industry		Linear Form		$y_t = b_1 + b_2t + e_t$				
Food and Beverage	$\hat{y}$ =	.56	.63	.72	.78	.84	.90	.97
	y_b =	.57	.63	.70	.77	.84	.91	.98
	b_1 =	.429 (.020)	b_2 =	.068 (.004)	$\bar{R}_y^2$ =	1.00	$\bar{R}_c^2$ =	1.00
Textile-mill Products	$\hat{y}$ =	.37	.44	.76	.58	.42	.39	.39
	y_b =	.52	.51	.49	.48	.46	.45	.44
	b_1 =	.548 (.382)	b_2 =	-.014 (.070)	$\bar{R}_y^2$ =	-.91	$\bar{R}_c^2$ =	.33
Paper and Allied Products	$\hat{y}$ =	.45	.53	.68	.69	.71	.76	.83
	y_b =	.49	.55	.60	.66	.72	.78	.84
	b_1 =	.372 (.102)	b_2 =	.058 (.019)	$\bar{R}_y^2$ =	.84	$\bar{R}_c^2$ =	.97
Chemicals and Allied Products	$\hat{y}$ =	.23	.32	.44	.49	.54	.61	.69
	y_b =	.25	.33	.40	.48	.55	.62	.70
	b_1 =	.107 (.052)	b_2 =	.074 (.009)	$\bar{R}_y^2$ =	.97	$\bar{R}_c^2$ =	.99
Petroleum and Coal Products	$\hat{y}$ =	.19	.26	.42	.35	.26	.25	.27
	y_b =	.28	.28	.28	.28	.29	.29	.29
	b_1 =	.275 (.201)	b_2 =	.002 (.038)	$\bar{R}_y^2$ =	-.99	$\bar{R}_c^2$ =	.42
Rubber Products	$\hat{y}$ =	.36	.49	.83	.76	.70	.74	.82
	y_b =	.48	.55	.61	.67	.74	.80	.86
	b_1 =	.358 (.314)	b_2 =	.063 (.058)	$\bar{R}_y^2$ =	.15	$\bar{R}_c^2$ =	.71

Table 6.1 -- Continued

Percent of Investment Written off by Accelerated Methods

Fiscal Year		1954	1955	1956	1957	1958	1959	1960
Industry		Linear Form		$y_t = b_1 + b_2t + e_t$				
Stone, Clay and Glass Products	$\hat{y}$ =	.61	.67	.66	.81	.94	1.04	1.12
	y_b =	.56	.65	.74	.84	.93	1.01	1.11
	b_1 =	.377 (.115)	b_2 =	.092 (.021)	$\bar{R}_y^2$ =	.91	$\bar{R}_c^2$ =	.98
Primary Metal Industries	$\hat{y}$ =	.35	.46	.60	.69	.76	.86	.96
	y_b =	.37	.47	.57	.67	.77	.87	.97
	b_1 =	.172 (.050)	b_2 =	.099 (.009)	$\bar{R}_y^2$ =	.98	$\bar{R}_c^2$ =	.99
Machinery except Transportation and Electrical	$\hat{y}$ =	.52	.66	.98	.94	.89	.95	1.05
	y_b =	.63	.71	.78	.86	.93	1.00	1.08
	b_1 =	.482 (.288)	b_2 =	.074 (.054)	$\bar{R}_y^2$ =	.41	$\bar{R}_c^2$ =	.86
Electrical Machinery and Equipment	$\hat{y}$ =	1.09	1.01	.84	.89	.92	.90	.86
	y_b =	1.02	.99	.96	.93	.90	.87	.84
	b_1 =	1.08 (.169)	b_2 =	-.030 (.032)	$\bar{R}_y^2$ =	.02	$\bar{R}_c^2$ =	.74
Motor Vehicles and Equipment	$\hat{y}$ =	.79	.79	.83	.77	.73	.70	.69
	y_b =	.82	.80	.78	.76	.74	.72	.70
	b_1 =	.859 (.075)	b_2 =	-.020 (.014)	$\bar{R}_y^2$ =	.41	$\bar{R}_c^2$ =	.98
Transportation Equipment, except Motor Vehicles	$\hat{y}$ =	.40	.47	.44	.57	.71	.81	.88
	y_b =	.35	.44	.53	.61	.70	.78	.87
	b_1 =	.182 (.126)	b_2 =	.086 (.023)	$\bar{R}_y^2$ =	.89	$\bar{R}_c^2$ =	.97

Table 6.2

Comparison of Nonlinear Estimates Using Original Asset Lives and Original Lives Increased by Two Years

Industry	Original Asset Lives R_c^2	Original Asset Lives v	Original Asset Lives plus 2 years R_c^2	Original Asset Lives plus 2 years v
Food and Beverage	1.00	.72	.99	.66
Textile-mill Products	- .19	.88	- .20	.87
Paper and Allied Products	.97	.79	.98	.76
Chemicals and Allied Products	1.00	.87	1.00	.86
Petroleum and Coal Products	- .14	.94	- .03	.93
Rubber Products	.86	.78	.88	.74
Stone, Clay, and Glass Products	.92	.67	.90	.62
Primary Metal Industries	.99	.79	.99	.77
Machinery except Transportation and Electrical	.97	.64	.96	.57
Electrical Machinery and Equipment	.56	.00	.79	.00
Motor Vehicles and Equipment	.81	.69	.84	.53
Transportation Equipment except Motor Vehicles	.96	.82	.96	.79

Table 6.3

Error in Effective Depreciation Rates

due to Assuming an Average Asset Life

$\tilde{n} = 12$

t	$n_1 = 10$ $n_2 = 14$	$n_1 = 8$ $n_2 = 16$	$n_1 = 6$ $n_2 = 18$
1	.0015	.0070	.0191
2	.0007	.0032	.0077
3	.0002	.0008	.0010
4	.0000	-.0006	-.0027
5	-.0002	-.0014	-.0044
6	-.0003	-.0017	-.0050
7	-.0004	-.0018	

$\tilde{n} = 15$

t	$n_1 = 13$ $n_2 = 17$	$n_1 = 11$ $n_2 = 19$	$n_1 = 9$ $n_2 = 21$
1	.0008	.0034	.0086
2	.0004	.0019	.0048
3	.0002	.0009	.0021
4	.0000	.0002	.0002
5	.0000	-.0002	-.0008
6	-.0001	-.0005	-.0016
7	-.0001	-.0007	-.0020

$\tilde{n} = 18$

t	$n_1 = 16$ $n_2 = 20$	$n_1 = 14$ $n_2 = 22$	$n_1 = 12$ $n_2 = 24$
1	.0004	.0019	.0047
2	.0003	.0012	.0030
3	.0001	.0007	.0017
4	.0000	.0003	.0007
5	.0000	.0000	.0000
6	.0000	-.0001	-.0004
7	.0000	-.0002	-.0007

where: $\tilde{n}$ is the mean of the uniform distribution describing the stock of assets

n_1 and n_2 are the minimum and maximum asset lives respectively

t is the asset's age

DDB is assumed

Chapter 7

INVESTMENT, DIVIDEND AND EXTERNAL FINANCE BEHAVIOUR

The purpose of this chapter is to investigate the investment, dividend and external finance behaviour of firms. As mentioned in Chapter 1 a simultaneous model of behaviour is hypothesized which, as well as being of interest in itself, provides the means for determining empirically the effects of various accelerated depreciation provisions.

The data are quarterly with all regression equations based on 50 observations, running from the second quarter of 1952 through the fourth quarter of 1964.[1] The level of aggregation follows the two-digit industry classification.

The model contains the following variables, equations, and identities.

Endogenous Variables

I^1 = investment in fixed assets[2]

Dep = depreciation of fixed assets[2]

Div = dividends

EF^1 = external finance in the form of long term debt

EF^2 = external finance in the form of new stock issues

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[1]The sources of these data are given in the appendix.

[2]The notation used in this chapter for investment and depreciation differs from that of preceding chapters. Investment in fixed assets is here denoted I^1 instead of I and depreciation is Dep instead of D. The latter symbol is used to represent the amount of long term debt outstanding.

CF = cash flow

I^2 = change in the current position of the firm

$\widetilde{EF}^1$ = EF^1 subject to a lag distribution (defined by identity 4)

$\widetilde{EF}^2$ = EF^2 subject to a lag distribution (defined analogously to $\widetilde{EF}^1$)

$\widetilde{CF}$ = CF subject to a lag distribution (defined analogously to $\widetilde{EF}^1$)

<u>Exogenous Variables</u>

P^g = gross profits (before deduction of depreciation or taxes)

MI = Moody's industrial bond rate

LTBR = the rate on term loans from banks

s_i = seasonal dummies

DC = a measure of debt capacity

WCI = Wharton School capacity utilization index

$\widetilde{WCI}$ = WCI subject to a lag distribution (defined analogously to $\widetilde{EF}^1$)

C+GS = stock of cash and government securities at the beginning of the quarter

$\widetilde{C+GS}$ = C+GS subject to a lag distribution (defined analogously to $\widetilde{EF}^1$)

T = effective tax rate on corporate profits

v = depreciation rate on current fixed investment

R = retirement of fixed assets

c = constant term

All variables are current unless otherwise stated.

<u>Structural Equations</u>

1. $I_t^1 = f_1(\widetilde{EF}_t^1, \widetilde{EF}_t^2, \widetilde{CF}_t, \widetilde{WCI}_t, \widetilde{C+GS}_t, s_1, s_2, s_3, c)$

2. $Div_t = f_2(CF_t, Div_{t-1}, C{+}GS_t, s_1, s_2, s_3, c)$

3. $EF^1_t = f_3(I^1_t, CF_t, EF^2_t, C{+}GS_t, MI_t, LTBR_t, DC_t, c)$

4. $EF^2_t = f_4(I^1_t, CF_t, EF^1_t, C{+}GS_t, DC_t, c)$

Identities

1. $CF = Dep + (1-T)(P^g - Dep)$

2. $Dep_t = Dep_{t-1} + I_t v_t + I_{t-1} v_{t-1} - R_t$

3. $Div + I^1 + I^2 = EF^1 + EF^2 + CF$

4. $\widetilde{EF}^1_t = \sum_{i=0}^{s} EF^1_{t-i} w_i$

Values for the weights w_i are assumed a priori and are not estimated. It should be noted that $P^n = (1-T)(P^g-Dep)$ is after tax profits but since it is not used explicitly it is combined with Dep to form CF.

Equation 4 is not estimated because observations on EF^2 are available only by subtraction using identity 3. Such a procedure, however, has the undesirable property of attributing to EF^2 errors of observation from all the other variables in the identity. Another drawback is that the data are not all from the same source, and as a result any variable obtained as a residual will contain an additional "statistical discrepancy" error. For this reason also, it is not appropriate to use EF^2 (obtained by subtraction) in the structural equations.

Two alternatives are to drop EF^2 entirely from the equations or to replace it by a proxy, namely, all the other variables in identity 3. The problem with the latter procedure is that estimation of equations 1 and 3 is essentially reduced to estimation of an identity, thus obscuring the structural parameters. That is each equation will contain all the variables in identity 3 except for EF^2, and if the latter is of minor significance or of small variability then the structural estimates obtained will reflect in part the relation described by identity 3. In view of the fact that post-war values of EF^2 for the manufacturing industries have been small compared to values of EF^1, structural equations 1 and 3 are estimated omitting EF^2 entirely.[3]

The question arises as to whether I^2, which represents all short term changes, can be assumed exogenous without serious specification error resulting. That is, a basic assumption throughout is that the fixed investment, external finance, dividend decision-making process, may be thought of in long run terms, leaving inventory fluctuations and the meeting of short term obligations to be financed by short term means such as bank loans, commercial paper, etc. On the other hand to the extent that short term factors interact

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[3]Since this results in inconsistent estimates of the structural coefficients, an attempt is made at the end of the chapter to determine the direction of inconsistency to be expected for each coefficient. The effect of estimating the equations with EF^2 replaced by the proxy suggested above is also discussed.

with and are affected by decisions to invest, finance externally, or pay dividends, this represents a misspecification. For example, if short and long term borrowing are substitutes in financing investment, or if current investment or dividend payments result in a cut back in (intended) inventory accumulation, then I^2 will not really be exogenous and a specification error will exist.

There is, however, a more compelling reason for considering I^2 as endogenous. Among other things it consists of changes in holdings of cash and government securities. To the extent that long term borrowing results in temporary increases in such holdings, I^2 will be directly affected by borrowing and hence must be considered endogenous.

The model to be estimated then consists of 6 endogenous variables (ignoring EF^2 and those variables defined by lag assumptions), 3 identities, and 3 structural equations (1-3). The latter will now be considered in detail.

<u>Investment Equation</u>

Investment expenditures are hypothesized to depend on the level of cash flow (CF), the amount of external finance (EF), a measure of quick liquidity (C+GS), and the rate of capacity utilization (WCI). *A priori* one would expect these factors to have a positive effect on investment.

Little need be said concerning inclusion of the cash flow variable in the equation in view of the discussion on internal financing presented in Chapter 4. The overwhelming preference of business for financing from internal funds suggests that the level of cash flow will strongly influence investment expenditures. But resort to external funds will of course be necessary in some instances, and for this reason the aggregate of long term bank borrowing and corporate bond issues is included in order to reflect the availability or ease of obtaining such funds.

The rate of capacity utilization, as measured by the Wharton School Index, is included for obvious reasons. Other things being equal increased pressure on existing facilities will result in attempts to expand capacity. The C+GS variable is intended to measure the current liquid position of the firm at the end of the previous quarter, and hence should affect investment positively.

Dividends are excluded from the investment equation on the grounds that their only effect on investment decisions arises from their role as a competing use for funds. That is, the cash flow constraint of the firm implies than an increase in dividend payments, _ceteris paribus_, will result in a decrease in investment expenditures. But this is not a sufficient reason for including dividends in the structural equation for investment behaviour since any "competing use" considerations are taken care of by the cash flow identity itself. Inclusion of dividends in the investment equation is permissible only if it is hypothesized that investment expenditures are affected by the level of dividend payments through some mechanism other than the cash flow constraint. Since no such mechanism appears to exist, dividends are excluded.

The problem of a lag between investment determinants and expenditures is met by assuming the existence of an average relationship between capital expenditures and capital appropriations. The latter represent the appropriation of funds for investment projects, and hence may be thought of as the essential investment decision variable. Capital appropriations are assumed to depend on current values of determining variables, with the fixed relation between appropriations and investment linking the latter to its determinants.

The assumption that appropriations depend on current determinants is of course an arbitrary one. The following remarks suggest, however, that it is probably the most feasible in that the choice of an alternative assumption is made difficult by the necessity to specify the direction as well as the time span of the dependence. That is, to the extent that investment decisions lag changes in the determining variables, some lag pattern will be relevant. This may occur, for example, because it takes a continued pressure on capacity or build up of liquidity through large cash flows before the decision to invest is made. On the other hand to the extent that the pattern of future expenditures resulting from current appropriations is recognized by businessmen, expected future values of cash flow and external funds will be relevant, thus suggesting leads rather than lags.

Although the fixed relation between appropriations and investment is assumed to hold on the average, it is clear that the distribution of investment expenditures resulting from capital appropriations will vary from one project to another due to such factors as the start-up time involved, the possibility that waiting lines for construction materials may be encountered, and the construction period of the project itself. Use of the average is clearly an approximation to these conditions, but as long as there is no systematic bunching of (for example short run) projects, then the error involved in making the assumption will be small. Perhaps a more serious problem is the possibility that the pattern of investment payments may itself be a function of the level of cash flow and availability of external finance. To the extent that this results in large changes in the relation between appropriations and investment over time, the results given below will be in error.

The above considerations lead to the assumption of a relation of the form:

(7.1) $A_t = X_t b + u_t$

where: A_t = capital appropriations in period t

I_t = capital expenditures in period t

X_t = the determinants of A_t

w_i = the fraction of A_t resulting in investment expenditure in period t+i

u_t = disturbance term in period t

Further, assuming that current appropriations result in investment expenditures over the next s periods, (including the current period) according to the weights w_i gives the relation:

$$(7.2) \quad I_t = \sum_{i=0}^{s} A_{t-i} w_i$$

where $\sum_{i=0}^{s} w_i = 1$. Substituting (7.1) into (7.2) yields:

$I_t = (\sum_{i=0}^{s} X_{t-i} w_i) b + \sum_{i=0}^{s} u_{t-i} w_i$ which may be written as

$$(7.3) \quad I_t = \tilde{X}_t b + \tilde{u}_t$$

using the notation that $\tilde{X}_t = \sum_{i=0}^{s} X_{t-i} w_i$ and $\tilde{u}_t = \sum_{i=0}^{s} u_{t-i} w_i$.

The weights w_i are assumed to follow an inverted-V distribution over 8 quarters. This choice of weights is motivated by consideration of recent empirical findings by Shirley Almon.[4] The latter has estimated a relation between appropriations and investment for each two-digit manufacturing industry (for the 1000 largest firms). Her estimated weights involve lags ranging from 5 to 10 quarters, and the number of industries with weights running for 5 to 10 quarters respectively are 1, 1, 1, 3, 4, and 2. A goodness of fit measure and the existence of no negative weights are the essential criteria used for determining the length of lag. In this respect the author reports that after a lag of

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[4] Shirley Almon, "The Distributed Lag Between Capital Appropriations and Expenditures," Econometrica, Vol. 33, No. 1 (January, 1965).

sufficient length is reached there is little change in the goodness of fit for distributions with longer lags, and "usually by the time 8 or 9 quarters are included.....a smooth curve takes shape."[5] Further, although the Almon procedure does not restrict the distributions to be symmetric, they are approximately symmetric in most industries. These considerations suggest that there is little to be gained by using a specific distribution for each industry rather than a simplified symmetric distribution ranging over 8 or 9 quarters for all industries. For this reason the inverted-V distribution of 8 quarters length is used, resulting in weights of .05, .10, .15, .20, .20, .15, .10, and .05 respectively over the preceding 8 quarters (starting with the current quarter).[6]

It should be noted that Almon's estimations are for the 1000 largest manufacturing firms, while the analysis here involves all firms. The possibility that the appropriation-investment process may differ for small firms makes the decision to use a standard distribution for all industries (rather than the specific one chosed by Shirley Almon for each industry) even more acceptable.

The petroleum industry exhibits the shortest Almon lag (of 5 quarters). In order to test the appropriateness of an

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[5] Ibid., p. 184.

[6] Frank de Leeuw, "The Demand for Capital Goods by Manufacturers: a Study of Quarterly Time Series," Econometrica, Vol. 30, No. 3 (July, 1962), pp. 407-423.

8 quarter lag in this industry, the investment equation is also estimated using an inverted-V distribution over 4 quarters.

Specification of the model in terms of an average relation between investment and appropriations suggests that serial correlation will be a serious problem. That is, if it is assumed that the basic error term u_t satisfies the usual Markov assumptions, then the composite error term $\tilde{u}_t = \sum_{i=0}^{s} w_i u_{t-i}$ will not, since its successive values contain many of the same u values (s-1 to be exact). Since the weights w_i are known, estimation of the investment equation by generalized least squares is possible, thereby taking into account the nonspherical distribution of $\tilde{u}_t$. The problems involved in such an estimation procedure are now considered.

Assume for the moment that the investment equation is not part of a simultaneous system. Then considering relations (7.1) and (7.2) above it is clear that the following relations must hold for $\tilde{u}_t$, if it is assumed that for u_t itself $E(u) = 0$ and $E(uu') = \sigma_u^2 I$.

$$E(\tilde{u}_t) = 0$$

$$E(\tilde{u}_t \tilde{u}_t') = \sum_{k=0}^{s-(j-i)} w_k w_{k+j-i} \quad \text{for all i, and } j = i, \ldots\ldots i+s-1$$

$$= 0 \quad \text{for all i, and } j = i+s, \ldots\ldots T$$

In short, $E(\tilde{u}_t \tilde{u}_t') = \sigma_u^2 \Omega$, where Ω is not diagonal, but is a known function of the weights w_i. Under these conditions, the best linear unbiased estimate of b in relation (7.3) above is given by: $\hat{b} = (\tilde{X}\Omega^{-1}\tilde{X})^{-1}\tilde{X}'\Omega^{-1}I$.

Assuming now that the investment equation is part of a simultaneous system, it is not hard to show that the (second stage) estimation is of the same generalized least squares form as above, that is, the appropriate weighting factor is Ω. Let the structural equations be given by:

(7.4) $Y\Gamma + XB + U = 0$

where the Y's are endogenous and the X's exogenous variables and the disturbances U satisfy the usual assumptions. Assume that in the m^{th} equation the b^{th} endogenous variable is appropriations (A). The coefficient of the latter is normalized to give: $A = Y_m\gamma_m + X_mB_m + U^m$, where Y_m represents all the endogenous variables in the m^{th} equation (except the b^{th}), X_m are the exogenous variables, and U^m is the m^{th} column of U.[7] Replacing appropriations by investment (denoted y now for convenience) gives:

(7.5) $\sum_{i=0}^{s} A_{t-i}w_i = y = \tilde{Y}_m\gamma_m + \tilde{X}_mB_m + \tilde{U}^m$

where the variables with a $\sim$ are as before (subject to the lag distribution), and $E(\tilde{U}^m\tilde{U}^{m'}) = \Omega\,\sigma^2_{u^m}$. The $\tilde{Y}_m$ are endogenous and should be regressed on the exogenous variables X. But for each variable in $\tilde{Y}_m$, say the first, we have $\tilde{y}_{t,1} = \sum_{i=0}^{s} y_{t-i,1}w_i$ and therefore it is appropriate to regress $\tilde{Y}_m$ on X defined analogously, which gives:

(7.6) $\tilde{Y}_m = \tilde{X}P + \tilde{V}_m$

7In the analysis to follow subscripts denote a group of parameters or vectors while superscripts denote a single parameter or vector.

The estimation of equation (7.6) using ordinary least squares (OLS) is inefficient compared to using the unweighted X and Y values for two reasons. First s observations are lost, and second the disturbance term is not spherical. The latter problem can be overcome by using generalized least squares, but such a procedure is not necessary since a (more efficient) estimate of P may be obtained by OLS using the unweighted X and Y values. That is, equation (7.6) implies the following reduced form between X_m and Y_m:

$$(7.7) \quad Y_m = XP + V_m$$

and under the usual assumptions concerning error terms in a structural model (since the reduced form disturbances V are linear combinations of the structural disturbances U), we have $E(V^i) = 0$, and $E(V^i V^{i\prime}) = k_i I$ for i = 1 to m, where V^i is the vector of disturbances from the regression of the i^{th} endogenous variable on all the exogenous variables. P can therefore be estimated by OLS from (7.7) and then used to give $\hat{Y}_m = X\hat{P}$, and $\tilde{\hat{Y}}_{m,t} = \sum_{i=0}^{s} \hat{Y}_{m,t-i} w_i$. Substituting $\tilde{\hat{Y}}_m$ into (7.5) gives the equation to be estimated as:

$$(7.8) \quad y = \tilde{\hat{Y}}_m \gamma_m + \tilde{X}_m B_m + (\tilde{U}^m + \tilde{V}_m \gamma_m)$$

It remains to be shown that $E(\tilde{U}^m + \tilde{V}_m \gamma_m)(\tilde{U}^m + \tilde{V}_m \gamma_m)' = \Omega s'$ where s' is a scalar and Ω is as above.

The reduced form of (7.4) is given by Y = XP + V, where $P = -B\Gamma^{-1}$ and $V = -U\Gamma^{-1}$. Rewriting the latter after postmultiplying by Γ gives $V\Gamma = -U$, which shows that in general the disturbance from any structural equation is a linear combination of all the reduced form disturbances, with the weights being the coefficients of the endogenous variables in that structural equation. This holds for $\tilde{V}$ and $\tilde{U}$ as well thus giving $\tilde{V}\Gamma = -\tilde{U}$, and for the m^{th} structural equation in particular $\tilde{U}^m = -\tilde{V}_{m*}\gamma_{m*}$, where m* represents all the endogenous variables in the equation. Now the term $\tilde{V}_m\gamma_m$ in (7.8) includes all but the b^{th} variable and hence $\tilde{V}_{m*}\gamma_{m*} = \tilde{V}_m\gamma_m + \tilde{V}^b\gamma^b$, where $\tilde{V}^b$ is the vector of disturbances from the reduced form equation with y_b the dependent variable. Using this expression and recalling that $\gamma^b = 1$ from the mormalization rule, the error term in (7.8) may be simplified as follows: $\tilde{U}^m + \tilde{V}_m\gamma_m = -\tilde{V}_m\gamma_m - \tilde{V}^b\gamma^b + \tilde{V}_m\gamma_m = -\tilde{V}^b$. But $E(V^b) = 0$ and $E(V^bV^{b'}) = s'I$ because V^b is a vector of reduced form disturbances. Therefore $E(\tilde{V}^b\tilde{V}^{b'}) = \Omega s'$ since $\tilde{V}_t^b = \sum_{i=0}^{s} V_{t-i}^b w_i$ (which defines Ω). This shows that the disturbance from equation (7.8) is such that $E(\tilde{U}^m + \tilde{V}_m\gamma_m)(\tilde{U}^m + \tilde{V}_m\gamma_m)' = E(\tilde{V}^b\tilde{V}^{b'}) = \Omega s'$ and the equation should be estimated at the second stage by a generalized least squares procedure using Ω.

Dividend Equation

Dividend behaviour is determined (basically) according to a Lintner-type model. That is, an optimal payout ratio r* is assumed, together with the usual partial adjustment process, thus giving:

$$(7.9)\quad \Delta Div_t = a_0 + a_1(Div_t^* - Div_{t-1}) + a_2(C+GS)_t$$

where $Div_t^* = r^*CF_t$. The equation is actually estimated in the following form, which may be obtained by moving Div_{t-1} to the right hand side.[8]

$$Div_t = a_0 + a_1 r^*CF_t + (1-a_1)Div_{t-1} + a_2(C+GS)_t$$

The term involving C+GS permits variations from (partial adjustment towards) the long run desired level of dividend payments, depending on the current liquid position of the firm, and a_2 is therefore expected to be positive. Since a_1 represents a reaction coefficient it is expected to be positive and less than one, and hence the coefficient of Div_{t-1} should be in the same range. The coefficient of CF ($a_1 r^*$) will of course be positive, and an estimate of the desired payout ratio r* may be obtained by dividing it by a_1.

As mentioned in Chapter 1, CF is used instead of net profits because in similar studies other authors have found it to be the superior variable in terms of reasonableness of parameter estimates.

An alternative to including C+GS as a separate term is to assume that it affects the desired payout ratio r*. Such a formulation is plausible only if r* is interpreted more in short run than long run terms. That is, if r* is intended to represent the long run desired payout ratio, then it is not appropriate to assume that it varies in each quarter with the liquid position of the firm. On the other hand if it is interpreted as the payout ratio which is desired in any particular quarter, then quite possibly it will

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[8]The parameter estimates are the same of course for both forms.

vary with C+GS. This leads to estimation of the following equation.

$$\Delta \, Div_t = a_0 + a_1(r_t^* CF_t - Div_{t-1})$$

$$= a_0 + a_1((a_2 + a_3(C+GS_t))CF_t - Div_{t-1}),$$ if it is assumed that $r_t^* = a_2 + a_3(C+GS_t)$.

E. Kuh has recently noted that inclusion of the constant term in the first dividend equation given above, but without C+GS included, is inconsistent with the underlying theory.[9] That is, if it is hypothesized that $\Delta Div_t = a_0 + a_1(Div_t^* - Div_{t-1})$, then the change in dividends equals a_0 even when actual dividends in the preceding period are at the desired level, which is not appropriate. Theoretically this problem does not arise if C+GS is included as a separate variable because then even if $Div_t^* = Div_{t-1}$, short run fluctuations in dividend payments (about the long run desired level) can occur as a result of variations in liquidity conditions.

The dividend equation is estimated both with and without the constant term, with C+GS excluded, in order to determine the effect of the latter on the parameter estimates and to study the explanatory power of these alternative specifications.

A further test of the validity of the dividend model and one which arises because of possible serial correlation

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[9]Edwin Kuh, Capital Stock Growth: A Micro-Econometric Approach, Amsterdam, 1963, p. 17.

of the disturbances is the following. Suppose that lagged dividends do not affect current dividend payments, and that the dividend equation is subject to serial correlation, thus implying the relations:

$$(7.10)\quad Div_t = a_0 + a_1 CF_t + a_2 (C+GS)_t + e_t$$

and $e_t = \lambda e_{t-1} + e^*_t$ where e^*_t may or may not be serially correlated. Then it is clear that if Div_{t-1} is included in the equation it will be correlated with e_t, and since Div_{t-1} is considered predetermined in the estimating procedure it is possible that it will take on a spurious significance.

Z. Griliches has recently suggested the following test to distinguish between models such as (7.9) and (7.10).[10] Substituting $\lambda e_{t-1} + e^*_t$ for e_t in (7.10), and then replacing e_{t-1} by the expression for e_{t-1} obtained from equation (7.10) lagged once, gives:

$$(7.11)\quad Div_t = a_0(1-\lambda) + a_1 CF_t - a_1\lambda CF_{t-1} + a_2(C+GS)_t - a_2\lambda(C+GS)_{t-1} + \lambda Div_{t-1} + e^*_t$$

If (7.10) is the correct specification then the coefficients of the variables will be related as suggested by this equation. For example, the coefficient of CF_{t-1} will equal minus the product of the coefficients of CF_t and Div_{t-1}, with a similar relation holding for C+GS. On the other hand if the original model is more appropriate these relations will not hold, and insignificant coefficients on CF_{t-1} and $C+GS_{t-1}$ are to

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[10] Z. Griliches, "Distributed Lags: A Survey", <u>Unpublished Manuscript</u>, 1965.

be expected. The dividend equation is estimated in form (7.11) in order to check the possibility that the alternative specification (7.10) is appropriate, and that lagged dividends appear significant simply because of the serial correlation of the disturbances.

External Finance Equation

External finance (EF^1) is defined as the first difference of long term debt outstanding. The latter consists of long term bank loans and "other long term debt", mainly corporate bond issues. External finance is hypothesized to depend on the demand for funds as represented by fixed investment expenditures, the supply of internally generated funds (CF), the liquid position of the firm (C+GS), the cost of borrowing as represented by an interest rate series, and a measure of the debt capacity of the firm (DC). The variable which appears to be the most consistent determinant of borrowing behaviour is the latter, and is defined in the following manner.

It is assumed that an optimum debt-equity ratio d* exists, and that the discrepancy between d* and the actual debt-equity ratio affects long term borrowing behaviour. Let D be debt outstanding and E total stockholders equity, then the desired debt level at the beginning of period t, which corresponds to the existing equity level E_{t-1}, is given by d^*E_{t-1}. It is further assumed that the change in debt in the period arising from considerations about the debt-equity ratio is a fraction a_1 of the difference between this desired debt level and the actual debt level D_{t-1}. That is, a partial adjustment process similar to the one given above for dividends is hypothesized.

The following basic difference, however, exists between the dividend and external finance models. In the former it is possible to hypothesize that the change in dividends is a function of one term only, namely the difference between desired and actual dividends. In fact as noted above, if no other determinants are assumed to affect dividends, then even the inclusion of a constant term in the equation is at odds with the underlying theory. In the external finance model on the other hand it is unreasonable to hypothesize that the difference between the desired and actual debt levels is the only determinant, since this would rule out further borrowing once the desired debt-equity ratio were reached. Inclusion of other terms such as investment expenditures, cost of borrowing, and availability of internal funds is therefore required, and consequently there can be no "constant term" problem as in the dividend case.

The above considerations lead to the estimation of an equation of the following form:

$$\Delta D_t = EF_t^1 = a_0 + a_1(d^*E_{t-1} - D_{t-1}) + \text{other terms}$$

$$= a_0 + a_1 d^*E_{t-1} - a_1 D_{t-1} + \text{other terms}$$

In principle such an equation can be estimated directly to provide estimates of a_1, $a_1 d^*$, and therefore d^*, but in practice D_t and E_t are so highly collinear that it is difficult to determine their separate effects. Simple correlation coefficients between D_t and E_t are greater than .95 in 8 of the 12 two-digit industries. If an extraneous estimate of d* were available then the composite variable $(d^*E_{t-1} - D_{t-1})$ could be formed and used to represent debt capacity. A natural candidate for d*, barring any good a priori information on its magnitude, is the average debt-equity ratio over the period. Use of the average as an estimate of d* requires basically that the short run fluctuations about the desired ratio cancel in the averaging process. An examination of the behaviour of D/E over time, however, indicates that in most industries the ratio has been steadily increasing over time. This suggests a changing desired debt-equity ratio, in which case the trend value of D/E would be more appropriate than the average value. The debt-equity ratio for each industry is regressed on time and a constant term, and the estimated values of D/E are used to represent d* in calculating (d*E-D).[11]

The other variables appearing in the EF^1 equation require little explanation. The cost of obtaining external funds is represented by an interest rate series. Since the EF^1 variable includes long term borrowing from banks as well

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[11]This is of course a very superficial treatment in the sense that no attempt is made to explain changes in the desired debt-equity ratio in terms of economic factors such as the relative costs of debt and equity financing.

as proceeds from corporate bond issues, both the interest rate on term loans from banks and the rate on Moody's Industrials are analysed. The availability of internal funds is represented by the cash flow variable, while general liquidity conditions are given by the stock of cash and government securities at the end of the preceding quarter. The demand for funds, of course, is hypothesized to depend on investment expenditures.

In the above formulation, the explanatory variables enter in current terms. It is possible, however, that the true relationship is more of a long run affair with increased financing resulting from a continued build-up of investment opportunities, a prolonged low interest rate, or a continued large debt capacity. To test this hypothesis the equations are also estimated with the explanatory variables averaged over the preceding three quarters together with the current quarter.

Alternatively it is possible to hypothesize the existence of leads rather than lags in one or more of the explanatory variables. This seems particularly relevant for the case of investment expenditures. Under the assumption of an average lag between appropriations and investment the firm knows at any point of time what past appropriations will be spent in the coming year, and may also have a good idea of the amount of funds to be appropriated within the year. This estimate of expenditures together with profit predictions for the year ahead may well be an important determinant of external financing. Actual values of I^1 and CF totalled over the current and succeeding three quarters are used to represent current expectations of the two variables. For the reason just indicated investment expenditures will probably be predicted with more accuracy than profits and hence use of actual values as proxies for expectations will be more justifiable in this case.

The possibility that EF^1 is responsive to investment relative to cash flow is explicitly tested by using various combinations of I^1, CF, and I^1/CF in lead form. The square of the latter is also included in an attempt to test the hypothesis that EF^1 depends in a nonlinear fashion on financing needs (I^1/CF). That is, if it is assumed that external financing occurs only when expected investment becomes large in relation to expected internal funds, or if it is assumed that the effect of I/CF increases in importance as I/CF itself increases, then inclusion of a nonlinear term will be more appropriate than the straight linear formulation.

As mentioned above the external finance variable includes both long term bank loans and other long term debt, the latter consisting mainly of corporate bond issues. Since data are available on both these series and since there are reasons for believing that they may behave differently, the series are also analysed separately. The belief that the two components may vary independently rests on the following considerations.

Banks in general supply long term funds to business in essentially two ways: through intermediate term loans and through interim credits of one or two years maturity. The former are attractive to small firms that do not have ready access to the securities market, and to all firms in as much as they represent a quicker and perhaps cheaper source of funds than bond or equity issues. Such loans may be of an initial maturity of anywhere from 4 to 8 years, being repaid generally from internal cash flows. Interim credits on the other hand, of one or two years maturity, may be used by business when embarking on a heavy capital investment project requiring funds at various stages of construction. Firms may be reluctant to borrow the entire sum at the outset, preferring to use interim funds to finance the project during construction. These funds are then repaid from the proceeds of new bond issues obtained at or shortly after completion of the project, or at a time when the cost of such issues is relatively low.

In view of these remarks it seems reasonable to hypothesize that the bank loan component of external finance will be affected mainly by investment pressures in relation to the supply of internal funds, debt capacity considerations, and perhaps the cost of obtaining funds as measured by the rate on term loans from banks. The timing of corporate bond issues, on the other hand, may well be affected more by cost considerations and less by current pressure on funds. There are at least two reasons for the latter. First, it will result to the extent that firms can rely on interim financing until a time when the relative cost of borrowing in the bond market is low. In this connection, it has been estimated that from 1959-1962 in the manufacturing industries interim financing by banks covered about one-quarter of the capital expenditures eventually financed in the securities market.[12] Second, even if interim financing from banks is not used, firms may tend to float larger bond issues when costs are low, then are immediately required for investment projects, simply because of cost considerations. Of course in the case of an unpredicted or continuing boom, it is possible that investment opportunities will make direct resort to the bond market appropriate even at a time of high interest rates. In general though the above reasoning suggests that the cost of bond issues, as measured by an average industrial bond yield, may well be the most important factor in determining the extent and timing of their use. It also suggests that long term bank loans of the preceding few years may be a relevant determinant of bond financing.

When bank loans and bond issues are combined and analysed as one external finance variable, it is clear that

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[12] George Budzeika, "Commercial Banks as Suppliers of Capital Funds to Business," Federal Reserve Bank of New York, Monthly Review, Vol. 45, No. 12 (Dec., 1963), P. 186.

their movements will in some cases be offsetting. That is, to the extent that corporate bond issues are used to refund interim loans, the net effect on the aggregate will be zero. This may tend to reduce the importance of the industrial bond rate in explaining the behaviour of the aggregate external finance variable.

<u>Method of Estimation</u>

An instrumental variable approach is used to estimate the three structural equations. For the dividend and borrowing equations, the set of instruments is chosen to be all the exogenous variables together with endogenous variables of 1 lag or more if they appear in the model. The variable $\widetilde{CF}_t$ (and similarly $\widetilde{EF}^1_t$) is broken into $CF_t + \sum_{i=1}^{s} CF_{t-i}$, where the latter term consists entirely of predetermined variables and hence can be considered as one predetermined variable for estimation purposes , call it $\widetilde{CF}_{-1}$. In this way no instruments are omitted and the procedure is two-stage least squares. The set of instruments used in the estimation of the dividend and borrowing equations is therefore:

$\widetilde{WCI}$	S_3	Dep_{-1}
$\widetilde{C+GS}$	Div_{-1}	$\widetilde{CF}_{-1}$
C+GS	MI	$\widetilde{EF}^1_{-1}$
S_1	LTBR	I^1_{-1}
S_2	P^g	c
	DC	

Certain variables listed as exogenous in the model as outlined at the beginning of the chapter are not used as instruments for the following reasons. The tax rate variable T is not used since it represents the effective tax rate and is therefore not truly exogenous. R_t is omitted because no independent observations exist on it, and v_t is not included since it is available only annually.

A different set of instruments is required in estimating the investment equation because of the problems involved in using lagged endogenous variables as instruments. Since the disturbance in the investment equation ($\tilde{u}$) consists of a weighted average of disturbances from the appropriations equation (u) dating back s periods, it is clear that any endogenous variable lagged s or fewer periods will be correlated with the current disturbance from the investment equation.[13] For this reason only exogenous variables are included as instruments in the estimation. The procedure used involves replacing the two endogenous variables in the equation ($\widetilde{CF}$ and $\widetilde{EF}$) by $\hat{\widetilde{CF}}$ and $\hat{\widetilde{EF}}$, obtained as follows. CF (and similarly EF) is regressed on all the exogenous variables in the system (Z) using OLS to give $\hat{CF} = Z\hat{P}$. These estimated $\hat{CF}$ values are then weighted using the w_i to give $\hat{\widetilde{CF}} = \sum_{i=0}^{s} \hat{CF}_{t-i} w_i$. As mentioned above an alternative estimation of P, by regressing $\widetilde{CF}$ on $\tilde{Z}$ (where $\tilde{Z}_t = \sum_{i=0}^{s} Z_{t-i} w_i$) is

[13] The exact relation between the disturbance and a lagged endogenous variable is given in a short note at the end of the chapter.

inefficient for two reasons. First the regression involves s fewer observations than the first method, and second the (reduced form) disturbance term is not in the correct form for least squares estimation.

The results of estimating the three structural equations by the instrumental variable method are given in the tables at the end of the chapter. For each equation estimates of the coefficients and their standard errors, the value of R^2 and the Durbin-Watson statistic appear. R^2 for the simultaneous estimations is obtained by calculating the variance of the residuals after substituting the structural estimates back into the original equation, and hence must necessarily be less than the R^2 which would be obtained by using ordinary least squares on the original equation. It should be realized that R^2 calculated in this manner is not generally appropriate for measuring the goodness of fit of a structural equation. For this reason it is used in the analysis to follow only in comparing the original estimates of the investment equation with the generalized least squares estimates. In this case the fact that the generalized least squares estimates yield negative R^2 values in one-half of the industries suggests that they are inferior to the original estimates.

Discussion of Results

Investment Equation

The results of estimating the investment equation by the instrumental variable method described above (but not the generalized least squares method) are given in Table 7.1. Out of 12 industries signs of coefficients are as expected for the $\widetilde{WCI}$, $\widetilde{C+GS}$, $\widetilde{CF}$ and $\widetilde{EF}$ variables in 9, 9, 10, and 7 industries respectively. Since the sampling distribution of the parameter estimates obtained by using instrumental variable techniques is unknown for small samples, no significance tests are available. However, the values in brackets below the estimated coefficients are the asymptotic standard errors, and to the extent that these reflect significance of the estimates, the cash flow variable appears to perform much better than the others.

The fact that the $\widetilde{EF}$ variable does not perform as well as the others in terms of the number of correct signs or as measured by the ratio of the coefficient to its standard error may rest on considerations mentioned above in discussing the EF equation. That is, to the extent that the timing of long term borrowing in the form of bond issues is governed strictly by cost considerations, the relation between I and $\widetilde{EF}$ will be a loose one, and could conceivably be inverse, with borrowing occurring predominantly in slack periods when interest rates are low. On the other hand to the extent that this is true, it is difficult to interpret recent results of other investigators in which the interest rate on industrial bonds (or Moody's aaa rate) is found to be a significant determinant of investment.[14] Supposedly such formulations are condensed versions (and incorrectly specified if behaviour is simultaneous) of the model spelled out explicitly here. That is, changes in the cost of external financing result in changes in the rate of borrowing, which in turn are reflected in increased investment expenditures. This problem is further analysed below in connection with the results of estimating the EF equation.

As mentioned above the investment equation for the petroleum industry is also estimated using an inverted-V distribution over 4 quarters. According to Shirley Almon's calculations this industry has the shortest lag between appropriations and investment (5 quarters), and appropriates the majority of funds in the first quarter of each year. Although the results of re-estimating the investment equation are not recorded here, it appears that there is nothing to be gained from the shorter lag. CF enters positively, but C+GS and EF are negative, while coefficient to standard error ratios remain approximately the same.

The very low Durbin-Watson statistics (to be denoted DW statistics) in Table 7.1 indicate the existence of positive serial correlation, which as mentioned above is to be

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[14] See in particular R. W. Resek, "Investment by Manufacturing Firms: A Quarterly Time Series Analysis of Industry Data," Unpuplished Manuscript, Tables I and II; Frank de Leeuw op. cit., p. 419; and E. Kuh and J. R. Meyer, "Investment, Liquidity, and Monetary Policy," Research Study Three in Impacts of Monetary Policy, C. M. C., Englewood Cliffs, N. J., 1963, p. 381.

expected in view of the formulation of the model. It is not difficult to determine the approximate value of the DW statistic to be expected under the assumption that the u's from equation (7.1) satisfy E(u) = 0 and E(uu') = $\sigma_u^2 I$, and hence the $\tilde{u}$'s of the equations to be estimated satisfy the relations given on page 168 above. It is well known that the Durbin-Watson statistic is approximately equal to $2(1-\hat{r})$ where $\hat{r}$ is the sample correlation coefficient between $\tilde{u}_t$ and $\tilde{u}_{t-1}$. The true correlation between $\tilde{u}_t$ and $\tilde{u}_{t-1}$ assuming stationarity in the u's is given by $r = E(\tilde{u}_t\tilde{u}_{t-1})/E(\tilde{u}_t^2)$, which becomes $(\sum_{k=0}^{5} w_k w_{k+1})/(\sum_{k=0}^{5} w_k^2)$ using the expression for $E(\tilde{u}\tilde{u}')$ cited above. Under the assumption that the w's follow an inverted-V distribution over 8 quarters, this expression equals 14/15 and hence the DW value should be close to 2/15 or .13. The DW values in the table are larger than this but are based on the fitted residuals from an equation, and also the equation takes no account of the form of the residuals. That is, a more efficient estimate of the correlation of the residuals will be obtained in conjunction with the generalized least squares estimation of the equations given below.

The results of applying a generalized least squares procedure to the investment equations appear in Table 7.2. The purpose, as outlined above, is to obtain more efficient estimates of the parameters. The estimates, however, do not (on _a priori_ grounds at least) appear superior. The values of R^2 are much lower in all, and negative in one-half of the industries, and the coefficient to standard error ratios of almost all the variables are much lower, and in fact close to zero in many cases. It is true that one would expect higher standard errors and lower values of R^2 after removal of serial correlation, but the results given above seem to be extreme. Further, considering signs of coefficients, although $\widetilde{EF}^1$ is now positive in 9 rather than 7 industries as before, $\widetilde{WCI}$ is positive in only 3 instead of 9 industries. Estimates of the coefficient of $\widetilde{CF}$ are now greater than one in 5 industries, which does not seem reasonable. On the other hand the Durbin-Watson statistics are closer to the value of .13 mentioned above. Although not recorded in Table 7.2, the only variables for which the coefficient to standard error ratio increases significantly are the seasonal dummies.

The question arises as to whether the generalized least squares procedure is removing the serial correlation as intended. A reasonable test is to calculate the DW statistic for the investment equation after multiplying the entire equation by P, where P is the matrix such that $P\Omega P' = I$. The following table contains the DW values calculated in this manner, and the fact that most of them are near 2 suggests that the procedure is appropriate.

Durbin-Watson Statistics for Transformed Investment Equation

Industry	DW	Industry	DW	Industry	DW
20	2.0	29	1.7	333	1.7
22	2.0	30	2.1	36	2.4
26	2.4	32	2.1	371	1.8
28	2.4	331	1.4	372	2.6

The generalized least squares procedure is of course intended to remove not only the first-order (serial) correlation between the $\tilde{u}$'s but also the correlation of other orders. The DW statistic tests only for first-order correlation, and to the extent that the procedure is not removing other correlations the parameter estimates can not be considered more efficient. It is not practical (or possible) to test for the correlation between the disturbances of all orders, but the following procedure is intended to provide an estimate of the fourth-order correlations. The latter is chosen for study because of the possibility that annual factors are important thus resulting in a positive relation between disturbances four quarters apart.

The first line of Table 7.3 for each industry contains values of the DW statistic obtained by estimating the investment equation for each quarter of the year separately. The results suggest that the generalized least squares procedure is not removing the fourth-order correlation as intended. For the fourth quarter in particular the DW values are low for all industries but two. The second line of the table for each industry gives the DW values for the original investment equations, that is, before the attempted removal of the correlation from the disturbances. The approximate DW value to be expected in this case may be calculated in a manner similar to that given above for first-order correlation. Since the correlation between $\tilde{u}_t$ and $\tilde{u}_{t-4}$ is given by

$r = E(\tilde{u}_t \tilde{u}_{t-4})/E(\tilde{u}_t^2)$, which becomes $\sum_{k=0}^{s} w_k w_{k+4} / \sum_{k=0}^{s} w_k^2 = 1/3$, the DW to be expected is approximately 1.33. Tabled values are in general higher.

An admittedly very rough measure of the extent to which the generalized least squares procedure is removing the correlation is to calculate the number of industries in which the DW value is closer to 2 after the method is applied. Considering the 12 industries, this occurs for the first to fourth quarters respectively in 4, 6, 4, and 4 industries, or in 18 of 48 cases. These results together with parameter estimates which are less acceptable on a priori grounds, cast doubt on the intended increased efficiency of the generalized least squares procedure. The ultimate test of the investment model in the present context, however, is the ability of the corresponding reduced form to generate sensible values of endogenous variables over time. For this reason both investment models are used in the attempt to analyse empirically the effects of accelerated depreciation in the following chapter.

Dividend Equation

The results of estimating the first model of dividend behaviour formulated above (7.9) are given in Table 7.4. Out of 12 industries signs of coefficients are as expected for the CF, Div_{-1}, and C+GS variables in 12, 9 and 9 industries respectively. The cash flow variable outperforms the others in terms of the number of correct signs and the ratio of coefficient value to standard error. Table 7.5 contains estimates (in columns 1-3 respectively) of the reaction coefficients (a_1), and of the desired long run payout ratio (r*) obtained from the equations, as well as the ratio of total dividends to total cash flow which prevailed during the period. The latter is included for comparison with the estimated r* values, and may be considered an approximation to the desired payout ratio if it is assumed that over a long period of time actual dividend payments equal desired payments. This may not be an appropriate assumption, however, if the cash flow variable is essentially a steadily increasing series over the period, since then the partial adjustment process will result in permanently lower actual payments than desired, and the average payout ratio will underestimate the desired ratio.

Estimation of the equation with the desired payout ratio a function of C+GS provides very similar results in terms of reaction coefficients and payout ratios and for this reason the results are not tabulated here.

From Table 7.5 it can be seen that in three industries the reaction coefficients are greater than 1, indicating an overadjustment to desired dividend levels. Although this behaviour can not be ruled out as irrational it is not what one would expect. The fact that, as indicated in Table 7.4, these industries are also the ones with the lowest coefficient-standard error ratios for lagged dividends, gives one cause to view the results skeptically. The values of r* obtained by estimating the equations for these industries omitting lagged dividends (and thus arbitrarily setting the reaction coefficient at 1) are almost identical with those given in Table 7.4. A reaction coefficient of 1 of course implies that short and long run adjustments to changes in desired dividends are identical. This seems unrealistic for a quarterly model even in view of the fact that the depreciation component of cash flow makes the latter a sluggish series, thus increasing the probability of a high reaction coefficient.

The desired payout ratio is less than the average over the period in all but one industry. As mentioned above this is contrary to expectations in view of the nature of the cash flow variable and the partial adjustment process assumed. Explanations for this discrepancy are not obvious. One possibility is that systematic overestimation of the reaction coefficient is resulting in underestimation of the desired payout ratio (since the latter is obtained by dividing the coefficient of CF by the reaction coefficient). If any biases are present in the estimation of the reaction coefficient, however, one expects them to be in the opposite direction in view of the possibility that serial correlation is a problem. Assuming it is, then to the extent that part of the effect of the serial correlation is being attributed to lagged dividends, the coefficient of the latter $(1-a_1)$ will be overestimated, the reaction coefficient (a_1)

underestimated, and hence the desired payout ratio overestimated.[15] This is clearly not the case. Further, in Table 7.5 there is no obvious relation between high reaction coefficients and the discrepancy of actual from desired payout ratios as would be expected under the hypothesis.

The possibility that the C+GS variable is influencing the estimate of the long run payout ratio is tested by omitting the former from the equations. As mentioned above this results in a problem of interpreting the constant term, and therefore the equation is estimated both with and without the intercept.[16] The values of r* obtained from these equations appear in columns 4 and 5 of Table 7.5. The exclusion of C+GS makes very little difference to the calculation of r* (as is expected under our hypothesis about C+GS) and in only two industries (331 and 371) does it substantially increase.

Suppression of the constant term, on the other hand, provides markedly different results, and in fact the estimates of r* are now extremely close to the dividend cash flow ratio in all industries, with the maximum discrepancy

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[15] Z. Griliches, "A Note on Serial Correlation Bias in Estimates of Distributed Lags," Econometrica, Vol. 29, No. 1 (January, 1961), pp. 65-73.

[16] It should be realized that suppression of the constant term while retaining the seasonal dummies involves estimation of an equation in which the latter sum to zero. This is accomplished computationally by using 3 dummies (S_i) and no constant term, where $S_i = 1$ in the i^{th} quarter and -1 in the fourth quarter, for i = 1, 2 and 3.

being 3 percentage points. The corresponding reaction coefficients, given in column 6 of Table 7.5, are lower in every industry than in the original model, and in only one instance is the coefficient greater than 1. Clearly this form of the model is a better representation of the hypothesized desired-payout partial-adjustment process of dividend behaviour than the others. Table 7.6 contains the complete set of estimated parameters and statistics for the equation. Signs of coefficients are as expected in all industries except 371, in which the coefficient of lagged dividends is negative. Because of the marked superiority of this equation in terms of the model of dividend behaviour hypothesized, it is used in the accelerated depreciation analysis in the following chapter.

As mentioned above the dividend equation is estimated including CF and C+GS lagged one period in order to test the model against the alternative specification that lagged dividends are irrelevant. Values of the coefficients are not presented because they in no way support the alternative formulation of the model. In fact the lagged cash flow variable enters positively in all but 1 industry and the lagged liquidity variable in all but 4 industries. If the alternative formulation were valid both variables would have negative coefficients. In the analogous test involving the form of the equation in which the constant term and C+GS are excluded, the lagged cash flow variable enters positively in all cases, thus refuting the alternative specification.

External Finance Equation

Before analysing the results of estimating the EF equation it is appropriate to consider the debt capacity measure (DC) in more detail. As mentioned above DC is calculated by regressing the debt-equity ratio for each industry on time and a constant term, and the estimated values of D/E are used to represent d* in calculating (d*E-D). The regression results appear in Table 7.7. $\hat{d}^*_{64}$ is the value of the desired debt-equity ratio at the end of 1964 obtained from the equations, while initial values are of course given by the constant term. The coefficient of time in the equations is over twice its standard error in every industry but rubber (30).

The rubber industry is the only one in which there appears to be no trend in the debt-equity ratio. For this industry the average debt-equity ratio over the period is used rather than the trend value in calculating the debt capacity measure, although the small magnitude of a_1(-.0002) indicates that these two methods would yield very similar results. In 10 of the other 11 industries the debt-equity ratio is an increasing function of time, with only the petroleum industry ratio showing a slight decline. Values of the desired debt-equity ratio vary considerably across industries with the average being .224 in 1964. The automobile industry has the lowest ratio (.114), and the "transportation except automobile" industry the highest ratio (.306).

The results of estimating the external finance equation are given in Table 7.8. Only one interest rate (Moody's Industrial rate) is included because of very high collinearity with the long term bank rate. Out of 12 industries, signs of coefficients are as expected for the MI, DC, C+GS, I^1 and CF variables in 7, 12, 10, 9 and 8 industries respectively. Both in terms of the number of correct signs and the ratio of coefficient value to standard error, the measure of debt capacity outperforms the other variables.

Estimates of the reaction coefficients for the debt relation, which appear as the coefficients of the DC variable, are fairly uniform across industries. The reaction coefficient represents the fraction of the gap between the existing debt level, and the one desired in terms of the amount of outstanding equity, which is closed by resort to external financing in the period. Ignoring the automobile industry (371), values of this coefficient range from .15 to .58 with an average value of .30. For the automobile industry the value is .03 but this carries little weight in view of the fact that the debt-equity ratio in the industry is much lower than in all other industries, suggesting that debt considerations are unimportant.

Although the signs of coefficients are generally as expected, the coefficient to standard error ratios of the variables (excluding DC) are not particularly impressive.

Efforts to obtain more significant representations of external finance behaviour center, for reasons mentioned above, on testing the appropriateness of the lag assumptions involved, as well as analysing the bank loan and other long term debt series separately.

The results of estimating the EF equation with the right hand variables averaged over the preceding 4 quarters are not tabulated since they in no way represent an improvement over the original model. Out of 12 industries signs of coefficients are as expected for the MI, DC, C+GS, I^1 and CF variables in 5, 12, 7, 10 and 8 industries respectively, while coefficient to standard error ratios appear in general to be lower.

The results of estimating the equation with the cash flow and investment variables averaged over the succeeding 4 quarters do not differ much from the original model. In terms of signs of coefficients the only change is one more industry with a negative CF variable, while coefficient to standard error ratios are generally about the same.

Since there appears to be no advantage in either the lead or lag version of the EF model, the analysis to follow will be restricted to the "current" version. It is possible of course that more sophisticated computational techniques would result in the establishment of a lead or lag in the equation. However, such experimentation is not considered worthwhile in view of the fact that there are really no good *a priori* reasons for deciding on even the direction of the dependence, and in view of the fact that the preliminary results given here are in no way encouraging.

Attempts to determine nonlinearities in the external finance equation by considering the investment to cash flow ratio and its square explicitly lend no support to the hypothesis, and for this reason the results are not tabulated here.

As mentioned above there are reasons for believing that the bank loan and corporate bond components of the external finance series might react differently to the determinants which are hypothesized to affect the aggregate. In particular it is hypothesized that bank loans will tend to be relatively important when investment opportunities are large relative to existing internal funds, whereas corporate bond issues may well tend to be influenced more by cost considerations (as represented by an interest rate series) than by pressure on funds.

The results of the regressions of the two different series (to be referred to as bank loans and bond issues) are not given in detail but the following discussion presents the points of interest. In general it appears that, although the results certainly do not run contrary to the hypotheses mentioned above, neither do they clearly support them. Considering signs of coefficients, the investment variable is positive in 10 industries for bank loans and 8 for bond issues, cash flow is negative in 6 and 5 industries respectively for these categories, and the interest rate is negative in 8 industries for both. Although these slight differences in sign for I^1 and CF are in accord with our hypotheses, the generally low coefficient to standard error ratios involved suggest that a great deal of weight should not be attached to them.

C+GS is the only variable whose sign clearly indicates a difference in determinants for the two series, being negative in 11 industries for bond issues but negative in only 6 for bank loans. The basic rationale for the hypothesized negative coefficient on C+GS is of course that a temporary depletion of liquid balances can substitute for external funds. _A priori_, there appears to be no reason for such a substitution to be more relevant for bond issues than bank loans, and in fact just the opposite might be expected. That is, the running down of liquid balances as a substitute for interim financing from banks seems more reasonable than as a substitute for bond issues, especially if the latter are determined primarily by cost considerations and are planned in essentially long run terms.

A comparison of goodness of fit measures indicates that R^2 is higher in exactly one-half of the industries for the bank loan series, thus implying that the relatively low R^2 values for the aggregate series are not a result of one component being essentially random (in the sense that it can not be explained by the hypothesized determinants). In view of the fact that there is no compelling evidence to suggest that the separate series are affected differently by the explanatory variables, the aggregate EF series is used in the empirical analysis of the affects of accelerated depreciation, in the following chapter. Before turning to the latter, however, three points mentioned earlier in this chapter are discussed. First an attempt is made to determine the direction of inconsistency to be expected in the estimated coefficients of the external finance (EF^1) and investment equations due to omitting EF^2. Second the effect of including I^2 and Div in the structural equations is analysed, and finally the procedure of using lagged endogenous variables as instruments in the investment equation is considered.

Omission of EF^2 from the investment and external finance (EF^1) equations results in inconsistent estimates of the structural parameters. An expression for the inconsistency may be obtained by considering the auxiliary regressions of EF^2 on the variables included in the structural equations.[17] Let the coefficient of EF^2 in the correctly specified investment equation (for any particular industry) be b*, and the regression of EF^2 on the other variables in the investment equation be as follows:

$$EF^2 = b_0 + b_1 CF + b_2 EF^1 + b_3(C+GS) + b_4 WCI,$$

[17] See for example H. Theil, Economic Forecasts and Policy, Amsterdam, 1961, pp. 212-215.

then the inconsistency in the estimated CF coefficient, for example is given by b^*b_1.[18]

It is of course impossible to obtain values for b* and the b's, and even conjectures about their orders of magnitude will be very approximate (although b* can probably be assumed to be less than 1). On the other hand predictions about the signs of b* and the b's are likely to be more accurate, thus revealing the direction of inconsistency in the estimated structural coefficients. Assuming that b* is positive, only the signs of the b's need be studied. By analogy with the EF^2 equation b_1 and b_3 will probably be negative, while b_2 will be positive to the extent that equity and debt issues are substitutes. The sign of b_4 may be positive but in any case the relation between WCI and EF^2 is likely to be a weak one. This reasoning suggests that the coefficients of CF and C+GS will be underestimated, and those of EF^1 and WCI overestimated in the structural equations for investment.

One implication of the underestimation of the CF coefficients is that the simulated changes in investment

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[18]Since the structural equations are part of a simultaneous system the appropriate form of CF and EF^1 to use in the auxiliary regressions is the one obtained after regressing these variables on all the instruments. Further, although the equation is written above without time subscripts, the auxiliary regression for the EF^1 equation will be in current terms, while all variables in the auxiliary regression for the investment equation will be lagged according to the inverted-V distribution.

attributable to accelerated depreciation (which appear in the following chapter) will be too small. On the other hand, the very limited use of equity funds by most industries in recent years suggests that the relation between EF^2 and the right hand variables may be a weak one with the level of EF^2 being determined more by subjective factors, in which case the inconsistencies will not be serious. Similar reasoning applied to the structural equations for EF^1 suggests that the coefficients of CF, C+GS, and DC may be underestimated and those of MI and I^1 overestimated.

As mentioned above the alternative to omitting EF^2 from the structural equations is to replace it by a proxy, namely all other variables in identity 3. This procedure involves including I^2 and Div in the investment and external finance (EF^1) equations, and although the detailed results of such estimations are not presented here, in general the investment equations improve only slightly while the EF^1 equations improve considerably. For the latter the R^2 values more than double in 8 industries, the coefficient to standard error ratios of I^1 and CF increase while DC loses significance, and I^2 enters positively in 9 industries with a coefficient to standard error ratio greater than 3.

However appealing these results may be at first sight, a little consideration reveals that they are not the desired structural estimates. The result of using I^2 and Div in the EF^1 equation (and the investment equation too, of course) is to essentially reduce the problem to estimating an identity. The equation to be estimated is:

$$EF^1 = f(I^2, \text{Div}, I^1, CF, C{+}GS, MI, LTBR, DC, c)$$

and identity 3 may be written as:

$$EF^1 = I^1 + \text{Div} + I^2 - CF - EF^2$$

The equation to be estimated contains all the terms in the identity except for EF^2, and if the latter is of minor importance or of small variability compared with the other variables, then what is being estimated is essentially identity 3. It is not surprising then that the R^2 values double in 8 industries, and that the variables which also appear in identity 3 become more significant. There are a number of reasons why the R^2 values, although much larger, are not near one. The estimation procedure is not ordinary least squares since a structural equation is involved, the EF^2 variable is omitted, and as mentioned above, the data are from different sources, thus resulting in a "statistical discrepancy" term in the identity. Probably the most reasonable interpretation to be placed on the highly significant I^2 variable is that borrowing results in a temporary increase in the stock of cash or government securities (included in I^2), and hence the causation is not from I^2 to EF^1, but the reverse.

As mentioned above estimation of the investment equation requires strictly speaking that no endogenous variables of s or fewer lags be used as instruments. The following analysis shows that the error committed in using such lags as instruments decreases as the lag increases. What is of interest is the correlation between the error term in (7.8) which was shown to be $\tilde{V}_t^b$, and a lagged endogenous variable $\tilde{y}_{t-j}$ where $\tilde{y}_{t-j} = \sum_{i=0}^{s} y_{t-j-i} w_i$, and y is any endogenous variable. We have therefore $\text{cov}(\tilde{V}_t^b, \tilde{y}_{t-j}) = \text{cov}(\sum_{i=0}^{s} V_{t-i}^b w_i, \sum_{k=0}^{s} y_{t-j-k} w_k)$. But V_t^b is a reduced form disturbance and hence all terms are zero unless $t-i = t-j-k$ or $k = i-j$, which gives:

$$\begin{aligned}\text{cov}(\tilde{V}_t^b, \tilde{y}_{t-j}) &= \sum_{i=0}^{s} \text{cov}(V_{t-i}^b, y_{t-i}) w_i w_{i-j} \\ &= (\sum_{i=j}^{s} w_i w_{i-j}) \text{cov}(V^b, y)\end{aligned}$$

since $\text{cov}(V_t^b, y_t) = \text{cov}(V^b, y)$ independent of t. Also $\text{var}(\tilde{y}_{t-j}) = (\sum_{i=0}^{s} w_i^2)\sigma_y^2$ and $\text{var}(V_t^b) = (\sum_{i=0}^{s} w_i^2)\sigma_{V^b}^2$, and therefore

$$r(\tilde{V}_t^b, \tilde{y}_{t-j}) = (\sum_{i=j}^{s} w_i w_{i-j} / \sum_{i=0}^{s} w_i^2) r(V^b, y)$$

where $r(\tilde{V}_t^b, \tilde{y}_{t-j})$ is the correlation between $\tilde{V}_t^b$ and $\tilde{y}_{t-j}$.

This shows that the correlation between the error term in (7.8) and a lagged endogenous variable used (incorrectly) as an instrument depends not only on the correlations between all the structural disturbances and the endogenous variable y (since V^b is a linear combination of all the U's) but also on the w's. The correlation decreases as the lag increases since there are fewer terms in the expression containing the w's, and if the lag is s periods the correlation is zero and the instrument is legitimate. No lagged endogenous variables are used as instruments in estimating the investment equation because it is assumed that the explanatory power of endogenous variables lagged 8 or more quarters is essentially negligible. To the extent that this is not true, of course, the estimates of the coefficients will be less efficient.

Table 7.1

REGRESSION RESULTS--INVESTMENT EQUATION

$$I_t^1 = a_0 + a_1\widetilde{WCI}_t + a_2(\widetilde{C+GS})_t + a_3\widetilde{CF}_t + a_4\widetilde{EF}_t$$

Industry	a_1	a_2	a_3	a_4	R^2	DW
20	285.40 (366.26)	-.0346 (.0585)	.2555 (.0652)	-.4179 (.1997)	.70	.70
22	- 53.71 (203.93)	-.0089 (.0369)	.8781 (.2262)	1.8490 (.4410)	.58	.30
26	339.12 (179.58)	.0867 (.0264)	.8222 (.1913)	-.0637 (.0732)	.91	.79
28	1241.80 (335.86)	-.0299 (.0714)	.2385 (.0568)	.9438 (.3010)	.68	.43
29	5038.70 (1194.40)	.1259 (.1225)	-.0626 (.0821)	-.3363 (.5336)	.71	.63
30	49.65 (24.45)	.0488 (.0195)	.4292 (.0528)	.0679 (.1285)	.85	.97
32	-166.92 (124.73)	.3111 (.0739)	.1984 (.0842)	-.9209 (.2831)	.74	.65
331	143.26 (143.60)	.1618 (.0474)	.3188 (.2119)	.6575 (.8543)	.44	.30
333	20.17 (65.42)	.1131 (.0335)	.6560 (.0794)	.5579 (.1189)	.78	.55
36	175.37 (78.93)	.0146 (.0133)	.1686 (.0261)	.2597 (.1086)	.85	.54
371	1056.40 (208.86)	.0383 (.0411)	-.1824 (.0899)	.8292 (1.4159)	.69	.46
372	-85.20 (70.13)	.0787 (.0414)	.6616 (.0911)	-.1221 (.2768)	.72	.39

For each industry:
Line 1 gives coefficient estimates.
Line 2 gives asymptotic standard errors of coefficients.
DW is the Durbin-Watson statistic.
Quarterly seasonal dummies, although not shown, are included in the equations.
Variables appearing in the regressions are defined on pages 158-159.
Based on 50 observations, 1952:3--1964:4.

Table 7.2

REGRESSION RESULTS--INVESTMENT EQUATION
(Estimated by the Method of Generalized Least Squares)

$$I_t^1 = a_0 + a_1\widetilde{WCI}_t + a_2(\widetilde{C+GS})_t + a_3\widetilde{CF}_t + a_4\widetilde{EF}_t$$

Industry	a_1	a_2	a_3	a_4	R^2	DW
20	-4957.6 (5881.8)	.8345 (.8486)	1.1772 (1.1320)	1.4318 (2.4499)	-2.22	.15
22	- 167.9 (1284.2)	-.0760 (.3036)	.9632 (1.6568)	- .0791 (3.4789)	.63	.26
26	- 398.5 (2056.8)	.1499 (.3810)	2.0365 (2.2781)	.3747 (1.0179)	.65	.25
28	-1613.2 (3906.4)	.7570 (.5361)	.6244 (.5943)	.9108 (4.0582)	-.14	.17
29	3899.4 (14230.0)	2.1757 (1.8545)	-1.4753 (1.8196)	.7053 (9.6566)	-1.80	.20
30	- 242.8 (530.1)	.2336 (.4200)	1.1445 (1.0859)	- .8032 (2.2896)	-.53	.26
32	522.2 (1434.2)	-.7635 (.8908)	1.2850 (1.3672)	.2052 (4.1986)	-.68	.14
331	- 294.1 (1389.5)	.1142 (.4709)	.9227 (1.5537)	-.7261 (4.8294)	.08	.22
333	- 239.1 (645.7)	.2918 (.4672)	.3367 (1.4267)	.4296 (1.7053)	.44	.25
36	- 627.0 (1245.5)	-.0843 (.2081)	.4148 (.5465)	.8712 (1.9494)	.00	.21
371	74.0 (1830.6)	.1482 (.3066)	-.1623 (1.0484)	3.3189 (15.237)	.01	.27
372	- 119.7 (521.8)	.1285 (.6558)	1.5458 (1.2574)	.2252 (4.4789)	-.30	.10

For each industry:
Line 1 gives coefficient estimates.
Line 2 gives asymptotic standard errors of coefficients.
DW is the Durbin-Watson statistic.
Quarterly seasonal dummies, although not shown, are included in the equations.
Variables appearing in the regressions are defined on pages 158-159.
Based on 50 observations, 1952:3--1964:4.

Table 7.3

DURBIN-WATSON STATISTICS FOR THE ORIGINAL AND TRANSFORMED INVESTMENT EQUATIONS--BY QUARTER

Industry	Q_1	Q_2	Q_3	Q_4
20	2.4	1.5	1.4	1.0
	1.6	1.3	1.3	1.4
22	1.7	1.7	3.4	1.2
	1.3	1.2	1.1	1.0
26	1.4	1.9	2.2	1.3
	2.9	3.1	1.6	1.8
28	1.0	1.7	2.5	1.6
	1.7	1.5	1.7	1.6
29	1.6	1.2	1.5	1.4
	.9	1.4	1.2	1.2
30	1.1	1.9	.5	1.6
	1.4	2.3	3.0	2.2
32	2.7	1.3	.8	1.8
	2.5	2.6	2.3	2.7
331	.9	1.6	1.2	1.3
	1.6	2.1	2.7	1.8
333	.7	2.3	1.8	1.2
	2.2	2.9	2.6	2.6
36	1.4	1.4	2.5	2.2
	1.6	1.6	1.8	1.4
371	1.6	1.2	2.4	.8
	2.7	2.8	2.4	2.6
372	1.9	1.4	1.9	1.3
	2.5	2.4	2.1	2.1

For each industry:

Line 1 gives the Durbin-Watson statistic for the transformed investment equation, by quarter.

Line 2 gives the Durbin-Watson statistic for the original investment equation, by quarter.

Q_i is the i^{th} quarter for $i = 1$-4.

Table 7.4

REGRESSION RESULTS--DIVIDEND EQUATION

$$Div_t = a_0 + a_1 CF_t + a_2 Div_{t-1} + a_3 (C+GS)_t$$

Industry	a_1	a_2	a_3	R^2	DW
20	.1236 (.0371)	.3951 (.1730)	.0039 (.0146)	.93	2.3
22	.0374 (.0107)	.4399 (.1309)	.0115 (.0045)	.87	2.6
26	.0552 (.0288)	.7081 (.1395)	-.0036 (.0041)	.95	2.9
28	.2428 (.0435)	.2402 (.1310)	-.0248 (.0138)	.92	2.1
29	.1654 (.0409)	.4738 (.1211)	.0215 (.0192)	.95	2.6
30	.2051 (.0386)	-.1183 (.1726)	-.0145 (.0118)	.74	2.2
32	.0720 (.0458)	.4206 (.1784)	.0324 (.0166)	.82	2.5
331	.0270 (.0130)	.8501 (.0565)	.0032 (.0041)	.90	2.6
333	.1860 (.0474)	-.1482 (.1479)	.0279 (.0124)	.64	2.1
36	.0478 (.0317)	.7927 (.1254)	.0026 (.0049)	.93	3.0
371	.3744 (.0683)	-.2093 (.1534)	.0439 (.0216)	.69	1.7
372	.0902 (.0260)	.3195 (.1305)	.0256 (.0099)	.71	2.3

Line 1 gives coefficient estimates.
Line 2 gives asymptotic standard errors of coefficients.
DW is the Durbin Watson statistic.
Quarterly seasonal dummies, although not shown, are in luded in the equations.
Variables appearing in the regressions are defined on pages 158-159.
Based on 50 observations, 1952:3--1964:4.

Table 7.5

ESTIMATED REACTION COEFFICIENTS AND DESIRED PAYOUT RATIOS FOR DIVIDEND MODELS

Industry	(1)	(2)	(3)	(4)	(5)	(6)
20	.60	.20	.29	.17	.29	.19
22	.56	.05	.24	.06	.23	.11
26	.29	.19	.28	.20	.30	.13
28	.76	.31	.38	.33	.38	.61
29	.53	.30	.28	.30	.29	.45
30	1.12	.19	.23	.20	.23	.97
32	.58	.13	.29	.16	.28	.32
331	.15	.12	.29	.20	.30	.12
333	1.15	.16	.37	.17	.35	.78
36	.21	.23	.34	.24	.37	.09
371	1.21	.30	.43	.39	.40	1.07
372	.68	.13	.29	.14	.27	.31

Column	Description
(1)	Estimated reaction coefficient with C+GS and intercept included
(2)	Estimated desired payout ratio with C+GS and intercept included
(3)	Actual ratio of total dividends to cash flow (1952:3-1964:4)
(4)	Estimated desired payout ratio with C+GS excluded and intercept included
(5)	Estimated desired payout ratio with both C+GS and intercept excluded
(6)	Estimated reaction coefficient with both C+GS and intercept excluded

Table 7.6

REGRESSION RESULTS--DIVIDEND EQUATION
(Intercept and C+GS Excluded)

$$Div_t = a_1 CF_t + a_2 Div_{t-1}$$

Industry	a_1	a_2	R^2	DW
20	.0569 (.0317)	.8119 (.1093)	.91	2.6
22	.0262 (.0086)	.8858 (.0376)	.83	2.9
26	.0390 (.0264)	.8704 (.0946)	.94	3.0
28	.2297 (.0447)	.3930 (.1211)	.91	2.2
29	.1314 (.0335)	.5464 (.1206)	.95	2.6
30	.2267 (.0380)	.0263 (.1647)	.70	2.3
32	.0916 (.0494)	.6781 (.1754)	.76	2.9
331	.0340 (.0112)	.8840 (.0361)	.89	2.4
333	.2701 (.0479)	.2162 (.1380)	.48	2.2
36	.0346 (.0288)	.9059 (.0920)	.93	3.1
371	.4273 (.0550)	-.0713 (.1392)	.66	1.8
372	.0829 (.0291)	.6938 (.1098)	.59	2.5

For each industry:
Line 1 gives coefficient estimates.
Line 2 gives asymptotic standard errors of coefficients.
DW is the Durbin-Watson statistic.
Quarterly seasonal dummies, although not shown, are indluded in the equations.
Variables appearing in the regressions are defined on pages 158-159.
Based on 50 observations, 1952:3--1964:4.

Table 7.7

REGRESSION RESULTS--DEBT-EQUITY RATIO EQUATION

$$(D/E)_t = a_0 + a_1 t$$

Industry	a_0	a_1	R^2	$\hat{d}^*_{64}$
20	.1583	.0010	.94	.217
22	.0864	.0019	.89	.195
26	.1653	.0016	.51	.258
28	.1629	.0012	.42	.235
29	.2048	-.0009	.82	.151
30	.2774	-.0002	.01	.263
32	.0982	.0015	.86	.187
331	.1476	.0015	.59	.234
333	.1477	.0025	.65	.290
36	.1325	.0018	.57	.237
371	.0535	.0010	.45	.114
372	.0665	.0041	.93	.306

t = time in quarters, beginning in 1950:3
$\hat{d}^*_{64}$ = estimated value of D/E in the fourth quarter of 1964 ($a_0 + a_1 \times 58$)

Based on 58 observations (1950:3-1964:4).

Table 7.8

REGRESSION RESULTS--EXTERNAL FINANCE EQUATION

$$EF_t^1 = a_0 + a_1 MI_t + a_2 DC_t + a_3 (C+GS)_t + a_4 I_t^1 + a_5 CF_t$$

Ind	a_1	a_2	a_3	a_4	a_5	R^2	DW
20	-.1412 (.2085)	.4974 (.1378)	.0542 (.0659)	.1798 (.5483)	.1854 (.1327)	.34	2.2
22	-.4168 (.1661)	.1534 (.0847)	-.0673 (.0529)	.0395 (.3013)	.5166 (.3415)	.22	2.3
26	-.2346 (.4884)	.5825 (.1283)	-.0520 (.1379)	.6613 (.6035)	-1.3853 (.6418)	.31	1.9
28	-1.2588 (.4878)	.2197 (.0986)	.0003 (.0480)	.5620 (.2203)	.1253 (.1292)	.33	.19
29	.0086 (.4263)	.2576 (.1134)	-.0849 (.0653)	.0491 (.1650)	-.0247 (.1218)	.15	1.4
30	-.5761 (.1565)	.4004 (.0929)	-.0366 (.0516)	1.2302 (.6945)	.0702 (.2489)	.33	1.7
32	.0920 (.1019)	.2333 (.0922)	-.0910 (.0577)	.1753 (.2518)	-.0612 (.1269)	.19	1.5
331	-.0658 (.2544)	.3430 (.0919)	-.1694 (.0416)	-.0673 (.2406)	-.0329 (.1167)	.36	2.1
333	-.0853 (.2633)	.1844 (.0959)	-.0089 (.1005)	.3825 (.3588)	-.3814 (.3182)	.22	2.5
36	.0896 (.4113)	.2374 (.1019)	-.1311 (.0749)	-.0164 (.5905)	-.1660 (.2777)	.10	1.6
371	.1390 (.2073)	.0284 (.0549)	-.0092 (.0180)	.1753 (.1312)	-.0926 (.0586)	.14	2.6
372	.2908 (.1564)	.1683 (.0939)	-.0186 (.0793)	-.3704 (.4595)	-.0510 (.2904)	.08	1.6

Ind = industry
For each industry:
Line 1 gives coefficient estimates.
Line 2 gives asymptotic standard errors of coefficients.
DW is the Durbin-Watson statistic.
Variables appearing in the regressions are defined on pages 158-159.
Based on 50 observations, 1952:3--1964:4.

Chapter 8

SIMULATION RESULTS

The model of investment, dividend and external finance behaviour estimated in the preceding chapter forms the basis for determining empirically the effects of accelerated depreciation. The procedure involves using the reduced form of the structural system to generate values of the endogenous variables under different assumptions about the depreciation parameters. That is, actual values of exogenous variables are used together with initial values of endogenous variables (corresponding to the time period immediately preceding the introduction of the accelerated methods) to generate over time a new set of endogenous variables. This procedure is followed both for depreciation parameters which are intended to represent existing conditions, that is, with depreciation accelerated, and for parameters which represent no acceleration. The difference in these sets of values represents the effect of accelerated depreciation. Both the 1954 and 1962 changes in depreciation provisions are studied, but the 1958 initial allowance is not, because as mentioned in Chapter 6 the annual limitation to $2,000 makes its effect negligible.

Values generated under conditions which are assumed to actually exist represent a stringent test of the model since such values are anchored to actual values only through the exogenous variables, and the initial set of endogenous variables. To the extent that the model is not a good representation of behaviour these generated values will diverge from actual values. For this reason it is appropriate to measure the effect of accelerated depreciation by comparing the two sets of generated values rather than the actual. That is, a divergence from actual values reflects both the introduction of accelerated depreciation and the inability of the model to predict exactly, and since the purpose of the simulation is to isolate the former, the effect of the latter must be suppressed. Of course if values generated under the assumption of actual conditions differ greatly from actual values then little faith can be put in such a procedure, and the effects of accelerated depreciation will probably not be determined accurately.

The set of structural equations estimated in the preceding chapter, together with the identities of the model used in calculating the reduced form, are presented again for convenience.

<u>Structural Equations</u>

1. $I_t^1 = a_{11} + a_{12}\widetilde{CF}_t + a_{13}\widetilde{EF}_t^1 + a_{14}\widetilde{WCI}_t + a_{15}(\widetilde{C+GS})_t +$ seasonal dummies

2. $Div_t = a_{21}CF_t + a_{22}Div_{t-1} +$ seasonal dummies

3. $EF_t^1 = a_{31} + a_{32}I_t^1 + a_{33}CF_t + a_{34}(C+GS)_t + a_{35}MI_t + a_{36}DC_t$

Identities

1. $CF = Dep + (1-T)(P^{g}-Dep)$

2. $Dep_t = Dep_{t-1} + I_t v_t + I_{t-1} v_{t-1} - C_t - R_t$

3. $Div + I^1 + I^2 = EF^1 + EF^2 + CF$

4. $\widetilde{EF}^1 = \sum_{i=0}^{s} EF^1_{t-i} w_i$

5. $\widetilde{CF}_t = \sum_{i=0}^{s} CF_{t-i} w_i$

This set of equations and identities in matrix form, omitting the disturbance terms, may be written as: $y_t = By_t + \Gamma z_t + \sum_{i=1}^{t-N} \Gamma_{t-i} y_{t-i}$, where the y's are the endogenous and the z's the exogenous variables. The expression $\sum_{i=1}^{t-N} \Gamma_{t-i} y_{t-i}$ contains cash flow and external finance values with lags of up to 7 quarters, due to the inverted-V weighting assumption. The expression also involves investment values, resulting from the C_t term in identity 2 (to be explained below), which run from t-4 back to the time of introduction of the accelerated method (year N). Solving this set of equations for y gives the reduced form which is used to generate the endogenous variables:

$$(8.1) \quad y_t = (I-B)^{-1}(\Gamma_0 z_t + \sum_{i=1}^{t-N} \Gamma_{t-i} y_{t-i})$$

It should be noted that the nonlinearities in identities 1 and 2 do not present a problem in computing the reduced form since they involve products of endogenous with exogenous variables, and not products of more than one endogenous variable. The fact that they occur in current terms, however, means that the matrices B and $(I-B)^{-1}$ will not be constant over time but will be functions of v_t and the T (the exogenous variables involved in the nonlinearities), and therefore different for all periods.

A problem arises in deciding on the set of structural equations and identities to use in the simulations. If the model is used exactly as outlined above, then the cash flow constraint (identity 3) is essentially inoperative, in that since I^2 appears in no other equation it takes up all the slack in the system. The reduction of I^2 to residual status is not in accord with the dividend-investment competing use behaviour outlined above.

One solution to the problem is to consider I^2 as exogenously determined, in which case identity 3 may be used to determine one of the other endogenous variables, thus making its structural equation redundant. A natural candidate for this variable, in view of the problems encountered in the previous chapter of obtaining a reasonable representation of external finance behaviour, is EF^1. Such a procedure makes the cash flow constraint operative, in a statistical sense because EF^1 appears also in the investment equation, and economically in that dividend and investment expenditures cannot be financed at will from liquid balances. The problem with the procedure, however, is that I^2 is considered as exogenously determined while the model itself (as evidenced by the I^1 and EF^1 equations) postulates that investment expenditures can be financed through depletion of liquid balances, and that the latter can substitute, temporarily at least, for external funds.

A more reasonable solution, and one in accord with the basic tenets of the model, is to recognize explicitly the feedback mechanism from I^2 to the other endogenous variables. That is, to the extent that I^2 includes changes in the stock of cash and government securities, future values of both I^1 and EF^1 are affected by changes in I^2. This proposition is incorporated into the model by breaking I^2 into a component which equals the change in cash and government securities, and one which is to be considered exogenous ($\bar{I}^2$) in the simulations. Identity 6 expresses this relation: $I^2_t = \bar{I}^2_t + (C+GS)_{t+1} - (C+GS)_t$. It is clear that I^2 is no longer simply a residual since it appears both in identities 3 and 6. On the other hand $(C+GS)_{t+1}$ takes on a residual status because it appears only in identity 6, but the dependence of I^1 and EF^1 on lagged values of C+GS means that a cash flow constraint mechanism is in effect operative. For example, abnormally high current dividends may be financed from cash and government securities in the current period, but this results in increased EF^1 and decreased I^1 in future periods due to the dependence on C+GS lagged. This appears to be a reasonable formulation of behaviour, and in particular is more in accord with the implications of the structural equations and the concept of a cash flow constraint, than the alternative formulations given above.

The specification of C+GS as endogenous for simulation purposes requires an identity (7) linking $\widetilde{C+GS}$ to C+GS, of the same form as identities 4 and 5. The 3 structural equations and 7 identities are used in an attempt to determine the effects of the different methods of accelerated depreciation. Identity 2 forms the basis for analysing a switch from SL to DDB or SYD and a change in asset life, while identity 1 forms the basis for analysing a true investment credit.

Before studying the simulation results it is appropriate to consider whether any general statements can be made about the behaviour of the model over time, without actually simulating. For example, it would be interesting to determine the behaviour of the percent change in investment resulting from the introduction of SYD or an investment credit. The analysis given below shows that even for a simplified version of the model it is not possible to determine the behaviour of the percent change in investment for the former, although weak statements can be made about the latter.

Consider a system involving an investment equation and a profits identity, with the latter incorporating explicitly the effects of an investment credit. That is, let $I_t = A_1 + bP_t$ and $P_t = A_2 + kI_t$, where I_t is investment,

P_t is net profits, k is the credit rate, and A_1 and A_2 represent variables assumed exogenous in the simulations. With no credit the investment series (I_t) is given by: $I_t = A_1 + bA_2$, and with a credit introduced at time 0, the resulting investment series (I_t') is: $I_t' = A_1 + (A_2 + kI_t')$. The difference between the series t years after introduction (ΔI_t) is then simply bkI_t', while the percent difference is bk/(1-bk). That is, the percent increase in investment is constant, and depends only on the coefficient of profits in the investment equation, and on the magnitude of the credit.

A serious drawback of this formulation, however, is that it ignores the additional depreciation resulting from an increase in investment, which in turn further affects investment expenditures through changing the level of cash flow. Clearly the inclusion of such a feedback in the model will result in a larger percent change in investment. The following depreciation identity incorporates this proposition: $Dep_t = Dep_{t-1} + vI_t - R_t$, where R_t is retirements and v is the effective depreciation rate on current investment. If it is assumed for simplicity that SL depreciation is in use, then no "correction term" is needed in the identity, and the value of v is 1/n, where n is the average asset life. Since the credit is introduced at time 0 it is convenient to express current depreciation as a sum of t terms, and then substitute it into the investment equation to obtain, for $t < n$:

$$I_t = A_1 + b(P_t + v \sum_{j=0}^{t} I_j + Dep_0 - \sum_{j=0}^{t} R_j)$$

The corresponding investment series (I'_t) generated with the credit in effect is given by:

$$I'_t = A_1 + b(P_t + kI'_t + v\sum_{j=0}^{t} I_j + Dep_0 - \sum_{j=0}^{t} R_j) \quad \text{for } t < n,$$

from which $\Delta I_t = bv \sum_{j=0}^{t} \Delta I_j + bkI'_t$ and

$$\Delta I_t/I_t = bk/(1-bk-bv) + (bv\sum_{j=0}^{t-1} \Delta I_j)/((1-bk-bv)I_t) \quad \text{for } t < n.[1]$$

Therefore the percent change, although not constant, is always greater than bk/(1-bk-bv), which is slightly larger than the previous model value of bk/(1-bk). It is impossible to determine the behaviour of $\Delta I_t/I_t$ over time without simulating, because it depends on I_t and all previous values of ΔI_t. The fact that I_t appears in the denominator on the right hand side of the equation, however, suggests that the percent change will decrease if I_t is growing sufficiently rapidly.

Of course the simplified model presented here differs considerably from the one used in the simulations, and for this reason nothing definite can be said about the latter. The simplified model, does, however, provide an idea of the general behaviour to be expected from the type of accelerated depreciation mechanism under study. Similar reasoning suggests that no general statements of interest can be made concerning the long run behaviour of the percent change in

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[1] For $t \geqslant n$ the lower limit of the summation becomes t-n+1.

investment following the introduction of SYD or DDB.[2]

Before considering the simulation estimates of the effects of accelerated depreciation on investment, it is interesting to study the predictive ability of the model itself. As mentioned earlier, a simulation over 44 quarters (1954-1964) provides a stringent test of the equations since the endogenous variables are anchored to actual values only through the exogenous variables (and the initial set of endogenous variables). Two formulations of the model are tested, both involving the 3 structural equations, but in one the C+GS variable is assumed endogenous in the simulations, (to be referred to as the constrained case) while in the other it is exogenous (the unconstrained case). As discussed above the former is thought to be a more realistic interpretation of entrepreneurial behaviour.

Simulation results are available for only 10 of the two-digit industries under study.[3] Industries 371 and 29 are not analysed because, contrary to expectations, the coefficient of cash flow in their respective investment equations is negative. Since the principle mechanism through which an

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[2]Consequently the only manner in which to determine the steady state effects of accelerated depreciation is to simulate for a large number of periods using extrapolated values of the exogenous variables. This remains as an interesting possibility for future work.

[3]Further, industry 33 data on asset lives, the learning function, and the investment credit are used in the simulations for industries 331 and 333 since appropriate data are not available for the latter separately.

acceleration of depreciation is hypothesized to affect investment behaviour is by changing the level of cash flow, and since the effect is hypothesized to be positive, a simulation involving a negative relation between cash flow and investment is not of interest. This represents an unfortunate failing of the model in the case of the motor vehicle industry since there is no apparent reason for the negative cash flow coefficient or for assuming that depreciation allowances are not important. For the petroleum industry on the other hand extensive use of depletion allowances renders depreciation considerations less important and determination of the effects of accelerated depreciation on investment less meaningful. Of course the predictive ability of the equations in both of these industries could still be tested, but this has not been done since the primary purpose of the model is to determine the effects of accelerated depreciation.

A summary of the predictive ability of the model appears in Table 8.1. For the cash flow, investment and dividend variables, the annual percent deviation of actual from predicted values for 1954-1964 has been calculated. Although these annual deviations are not presented, the average (over 11 years) of their absolute values appears in the table for both the constrained and unconstrained simulations. Prediction of cash flow is of course not difficult since it depends only on the generated depreciation values. The fairly accurate dividend predictions are not unexpected for the following reasons. First, dividends are a smooth series in general. Second, most of the estimated dividend equation parameters imply reasonable payout ratios and reaction coefficients, and all but one of the estimated coefficients has the correct sign. Finally asymptotic standard errors are generally low in relation to coefficient values in these equations.

Investment predictions on the other hand, particularly for industries 22, 331, and 333, are not impressive. A study of the annual percent changes indicates that in 7 of the 10 industries the model overpredicts investment expenditures in 1954 and 1958, and in many cases the maximum annual deviations occur in these years. Further although expenditures are not generally overpredicted in 1961, they are underpredicted in 8 industries in 1962. A comparison of the simulated and actual investment series reveals that the model generally predicts turning points correctly, but underestimates the magnitude of the swings, and for this reason large deviations occur in years such as 1954 and 1958. It should be mentioned that for the steel industry (331) overpredictions in the strike years of 1956 and 1959 contribute to the high average deviation, with the discrepancy in 1959 along being 40%.

In spite of the fact that the model does not predict investment accurately for certain industries, it is noteworthy that there is no tendency for the predictions to drift over time. That is, deviations in later years are not generally larger than those in early years.

Table 8.1 indicates that in industries 20 and 22 the constrained estimates are greatly inferior to the unconstrained estimates. The basic reason is that the coefficient of C+GS in the investment equation for these two industries is negative (and hence contrary to expectations). The introduction of a cash flow constraint under such circumstances yields perverse results in that an initial increase in the stock of cash and government securities causes a reduction rather than an increase in investment expenditures, and this in turn results in a further increase in liquid balances and reduction in investment. The omission of the cash flow constraint in the simulations for these two industries, and for industry 28 in which the C+GS coefficient is also negative, seems preferable to assuming such unrealistic behaviour.

Analysis of the 1954 Depreciation Provisions

The effect of a switch in depreciation methods, say from $h(w,n)$ to $h^*(w,n)$ may be determined in the following manner. Assume that b_t is the fraction of investment in period t which is depreciated under the new method (h^*) and that n is the average asset life, then the coefficient of current investment in identity 2 may be written as : $v_t = b_t h^*(1,n) + (1-b_t)h(1,n)$. If the investment series is also classified by plant and equipment, then the coefficient of total investment is the weighted average of two such terms, one involving the average life of plant and the other of equipment. The weights of course are the fractions of investment in each category. The coefficient of lagged investment (v_{t-1}) in identity 2 is calculated analogously, the term itself appearing because of the assumption that investment is centered in each quarter and that depreciation is taken on one-half of the investment occurring in the quarter. The correction term for accelerated depreciation (C_t) in identity 2 is basically (as given in Chapter 6 but with different notation): $C_t = \sum_{p=1}^{t-N} b_{t-p} I_{t-p} B(p,n)$ where N is the year of introduction of the new method and $B(p,n) = h^*(p,n) - h^*(p+1,n)$. The term is required because accelerated depreciation does not involve equal annual deductions.

In the 1954 case of a switch from SL to DDB or SYD, (DDB will be used to represent both methods) $h^*(w,n) = 2/n(1-2/n)^{w-1}$ and $h(w,n) = 1/n$. Estimates of b_t, the fraction of investment written off by DDB, are the ones derived in Chapter 6. Values of the endogenous variables are generated from 1954 to 1964 first with b_t taking actual values and then with $b_t = 0$. The difference between these sets of values represents the effect of the introduction of accelerated methods in 1954.

The simulation results are summarized in Table 8.2 which contains the estimated annual percent increases in investment from 1954 to 1964 due to the introduction of accelerated methods in 1954. The first line of the table for each industry is based on the assumption that the C+GS variable is exogenous in the simulations (the unconstrained case) while the second line assumes that it is endogenous (the constrained case). The former assumption is thought to be a more realisitc interpretation of behaviour in general, although for reasons suggested above the unconstrained estimates are more appropriate for industries 20, 22, and 28. For industry 26 also, the unconstrained estimates may be considered more appropriate in view of the unreasonably large coefficient of cash flow in the external finance equation (a value of -1.39). Since this coefficient is greater than minus one in absolute value an increase in cash flow will result in a correspondingly larger decrease in external financing, and <u>ceteris paribus</u> will result in a reduction in the stock of liquid balances. Such behaviour is certainly a factor contributing to the samll change in investment in the constrained simulation, and for this reason the unconstrained estimate may be more representative of actual behaviour.

Similar reasoning suggests that even the constrained results in industry 22 will be overestimates of the actual effects of acceleration. For this industry the value of the coefficient of external finance in the investment equation is greater than one (a value of 1.85), which seems unreasonably high. In addition the cash flow coefficient in the external finance equation is positive, which means that an initial increase in cash flow will result in an increase (of almost the same magnitude) in investment due simply to the fact that external financing has risen. The result of this inappropriate feedback mechanism will probably be to attribute larger investment changes to acceleration than actually occurred.

In general the simulations involving the cash flow constraint will provide larger estimates of investment changes than the unconstrained simulations if in the latter case the simulated values of I^2 (which are generated using identity 3) are greater after the introduction of accelerated depreciation. Imposition of the cash flow constraint under these circumstances results in a further increase in investment due to the fact that the larger values of I^2 (and hence correspondingly higher levels of C+GS) are allowed to affect investment.

Considering the unconstrained estimates for industries 20, 22, 26, and 28, and the constrained estimates for the other industries, Table 8.2 indicates that, except for the first years after introduction of accelerated depreciation, there is no general pattern to the percent changes in investment. This is not surprising since as mentioned above no conclusions could be drawn concerning such changes even for a very simple model of investment behaviour. For industries 20, 22, 26, 30 and 372 the percent changes appear to be slowly increasing, while for the other industries there is no clear trend. Of course the fact that investment expenditures display a high variability means that percent changes will fluctuate even if absolute gains from acceleration increase steadily over time. In addition to the investment base itself, however, the percent changes depend on many factors, including the rate of adoption of the accelerated method, the industry's average asset life, the fraction of investment in machinery, and the coefficients of the variables considered endogenous in the simulations. The latter include the coefficient of cash flow in the 3 structural equations, the coefficient of investment in the external finance equation and of external finance in the investment equation, and the coefficient of the stock of cash and government securities in the investment and external finance equations.

On the other hand, there are several reasons for expecting the percent change in investment to increase for a number of years following introduction of the accelerated methods. These include the fact that the methods were not immediately adopted, the use of an inverted-V lag distribution for investment expenditures covering 8 quarters, and the fact that in the first few years after introduction all assets are subject to higher depreciation rates under SYD or DDB than under SL.

Alternative estimates of the investment equation were obtained in Chapter 7 using a generalized least squares procedure, and although they appeared inferior on several grounds, their ability to explain investment behaviour (in a reduced form context) was suggested as a further test of their appropriateness. Constrained simulations analogous to those in Table 8.1 have been carried out with the original investment equation replaced by its corresponding generalized least squares estimate. The two investment equations are compared using a statistic based on the sum of squared deviations of the actual investment series from the series generated by the reduced form in each case. More sophisticated and comprehensive measures of the ability of the equations to explain all the endogenous variables could be considered, but since the investment equation is the only one that differs between simulations and since interest centers on investment behaviour in particular, the simple measure suggested above seems adequate. Table 8.3 contains values of R_I^2 calculated for the original investment estimates, and the generalized least squares estimates. R_I^2 is defined as 1-var(e)/var(I) where e is, as above, the deviation of the actual investment value from its estimated value, and var(I) is the variance of the actual investment series. In all industries but textiles the original investment equation is superior to the generalized least squares equation.

<u>Analysis of the 1962 Asset Life Reduction</u>

The 1962 tax life reduction was applicable to both old and new assets. The effect of the reduction applied to new assets may be determined for simulation purposes by using the shorter lives in calculating v and C in identity 2.

The difference between the values of the endogenous variables generated with the two lives represents the effect of this method of accelerated depreciation. The increase in depreciation resulting from applying the shorter lives to old assets is much more difficult to determine since it depends on the magnitude and age distribution of the existing capital stock. An estimate of such an increase in depreciation may be obtained by applying the change in average life to investment incurred during the preceding n_2 years (where n_2 is the longer, and n_1 the shorter life). Using this procedure the gain in depreciation for assets subject to SL is given by:

$$G(SL) = (1/n_1 - 1/n_2) \sum_{j=1}^{n_2} I_{t-j}$$

and the corresponding gain for assets subject to DDB is:

$$G(DDB) = \sum_{j=1}^{n_2} (2/n_1 - 2/n_2)(1-2/n_2)^j I_{t-j}$$

The term $(1-2/n_2)^j$ is required because under DDB the reduction in lives is applicable to the undepreciated base of the asset, which is $(1-2/n_2)^j$ after j years.

For the two-digit manufacturing industries values of G(SL) and G(DDB) have been calculated using investment series dating back the minimum of n_2 or 17 years. The latter is chosen since 1946 is the first year of reliable investment data. The results appear in Table 8.4. The total for each industry is an estimate of the potential increase in depreciation available in 1962 as a result of applying the new tax lives to the existing stock of assets. The effect on investment of the increase may be obtained by comparing values of endogenous variables generated first with the depreciation gain included in identity 2 and then with it excluded.

Such a procedure for estimating the effects of the Guideline revisions assumes that the shorter lives were completely adopted. Although it is difficult to determine the validity of this assumption, there is evidence to suggest that it may not be appropriate. In particular, data appearing in the Statistics of Income for 1962 reveal that in only 4 of the two-digit industries did the amount of depreciation taken under the Guideline lives exceed one-half of total 1962 depreciation.[4] Although these data suggest that the Guidelines were not readily accepted, such a conclusion is unwarranted in view of the fact that an average (pre-Guideline) life of n_2 years is consistent with use by some entrepreneurs of lives as short as, or perhaps even shorter than, the Guideline lives. For this group there is no advantage to using the latter, and hence an adoption rate of less than 100% is to be expected. For purposes of the simulation model it is necessary to know the extent to which nonguideline users were influenced by reasons other than that there was no advantage in terms of a reduction in lives. Since no such evidence exists, the simulations presented below are based on the assumption of complete adoption of the Guideline lives.

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[4] U. S. Treasury Department, I.R.S., Statistics of Income 1962, Corporation Income Tax Returns, Table 33, p. 314.

It should be mentioned that a questionnaire survey conducted by the Office of Business Economics in April and May of 1963 presents data on the extent of use of, and additional depreciation due to using the Guidelines, as well as data on reasons for not adopting them.[5] Since the estimates of the extent of use of the shorter lives differ widely from those given in the 1962 Statistics of Income, and since the latter is generally considered a reliable and comprehensive source of data, little faith can be put in the results of the survey. A comparison of the percent of 1962 depreciation taken under the Guidelines for the two sources is presented in Table 8.5. The latter indicates that not only are the estimates of Guideline use larger for all industries in the 1963 questionnaire survey, but in many cases are half as large again as the Statistics of Income estimates.

The tax law incorporating the Guideline revisions was passed in July 1962 with the shorter lives being applicable to depreciation claimed in all accounting periods ending after that date. For simplicity the simulations are based on the assumption that all assets purchased in 1962 were eligible for the shorter lives at the time of purchase. This is an understatement to the extent that certain 1961 investment expenditures may have been eligible for the credit (but this is likely to be a very small amount), and

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[5]Lawrence Bridge, "New Depreciation Guidelines and the Investment Tax Credit", Survey of Current Business, July, 1963, Table 1, p. 4.

is an overstatement in that it permits the influence of shorter lives to begin in January rather than July of 1962 (although the total depreciation change is of course approximately the same). The simulation results given below provide estimates of the (hypothetical) effects on the endogenous variables of the Guideline lives applied to new and old assets separately, as well as estimates of the (actual) combined effect.

Analysis of the 1962 Investment Credit

Identity 1 forms the basis for analysing the 1962 investment credit. A true credit of k% of cost increases after tax profits by kI^m where I^m is investment in machinery, and its effect may therefore be taken into account by simulating with kI^m added to the right hand side of identity 1. Since the 1962 credit when first introduced involved a write-down in the asset's base, it is also necessary to adjust C in identity 2. As mentioned in Chapter 1, however, the base reduction stipulation was repealed in the 1964 Revenue Act, and depreciation not claimed in 1962 and 1963 because of the requirement, could be claimed in 1964. For simplicity the simulations given below are based on the assumption that a true credit was introduced in 1962, and although this attributes too much additional depreciation to 1962 and 1963 and correspondingly too little to 1964, the amounts involved are likely to be negligible.

The effect of the 1962 credit is diminished by the lower rates applicable to short lives, and by the limit on the amount of credit which can be taken in any one year. It is possible to estimate roughly the importance of these two factors from data appearing in Statistics of Income for 1962. Data are available on the total cost of property on which the credit would be applied if there were no limitations (C^T), and on the cost of property actually used in caluclating the credit (C^U), where the latter differs from the former to the extent that short lived assets are taken into account.[6] Unfortunately data on the total cost of property exclude investment in assets with lives of less than 4 years, and investment in used assets exceeding $50,000 (neither of which qualifies for credit). Consequently the rate given by $.07(C^U/C^T)$, when applied to all investment in machinery, overestimates the actual effective credit rate somewhat, but is certainly a closer approximation than .07.

Data are available on the tentative (Cr^t) and actual (Cr^a) credit for 1962. The former is 7% of C^U while the latter is less than this amount to the extent that firms are prevented from claiming the credit by the absence of net income or by the upper limit to the credit. The latter is $25,000 plus one-quarter of any tax liability above $25,000. The effective investment credit rate used in the simulations, which takes both the short life and the income limitation factors into account, is therefore given by $.07(C^U/C^T)(Cr^t/Cr^a)$.

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[6]U. S. Treasury Department, op. cit., Table 1, pp. 50-52 and Table 14, p. 192.

Table 8.6 gives values of C^U/C^T, Cr^t/Cr^a and products of such values for 1962.

Since data are not yet available on these ratios for 1963 and 1964, the 1962 ratios are used for all years. *A priori* this is a satisfactory assumption for C^U/C^T as there is no reason to expect it to vary. The behaviour of Cr^t/Cr^a on the other hand is more difficult to predict since unused credit carried forward from 1962 must be taken into account. Preliminary data for 1963, available only for the category of "all industrial divisions", suggest that 1962 values may be good approximations at least to 1963 values. The 1963 ratio of actual to tentative credit, where the former includes the amount carried forward from 1962, is .78, while the corresponding 1962 ratio is .74.[7] A possible explanation for the approximate equality of the ratios is that the credit carried forward could not be used in 1963 for precisely the same reasons as in 1962. This hypothesis is supported by the preliminary statistics for 1963 which show that (for all industrial divisions) although the cost of property used for the investment credit increased by 23% over 1962, the unused credit increased by 106%.

The investment credit was introduced in October 1962 with all assets purchased after December 31, 1961 being eligible for the credit. For simplicity the simulations

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[7]U. S. Treasury Department, I.R.S., *Statistics of Income 1963, Corporation Income Tax Returns, Preliminary*, Table C, p. 3.

given below are based on the assumption that all assets obtained in 1962 were eligible for the credit at the time of purchase. This involves an error in timing in that the credit becomes effective in January rather than October, but of course the amount of credit taken will be approximately the same. Further the error will be less serious to the extent that entrepreneurs anticipated the credit, and the possibility of such an anticipation was (ostensibly) one of the main reasons for making the credit retroactive. Simulation results given below are based on the assumption that the credit was taken on all investment in machinery.

Table 8.7 contains estimates of the increase in 1964 investment attributable to the 1962 accelerated depreciation provisions. The (hypothetical) effects of the separate components of the 1962 change as well as the total (actual) effects are presented. A true 7% credit is also studied.[8] Due to the simultaneity involved, addition of the separate effects of the 1962 revision will provide slightly different results from those obtained by considering the total (actual) depreciation change. This is evident in industries such as 22 and 26 experiencing large percent increases, although for

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[8]It should be noted that in the empirical analysis the difference between the true and actual credit is due to the lower rates on short-lived assets and the annual limitation to the amount of credit. On the other hand, in the discussion of pdv, liquidity, and rate of return effects in previous chapters the difference between the true credit and the "1962 credit" is due to the lower rates on short-lived assets and the reduction in the asset's base.

small increases the results appear to be almost identical.

Both the constrained and unconstrained simulation results are presented, and are of course much more in accord than for the 1954 change since the time period involved is 3 rather than 11 years. For the same reasons as above, however, the unconstrained estimates for industries 20, 22, 26 and 28 may be considered more appropriate than the constrained estimates, and the summary statistics to follow are based on this assumption. Further, although reasons are given above which suggest that the effects of accelerated depreciation will be overestimated in the textile industry (22), the extremely high percent changes recorded in Table 8.7, reflect as well an unreasonably large effect resulting from the application of the asset life reduction to the existing capital stock. Consequently the results for this industry should be viewed skeptically.

The table indicates that in all but one industry (30) the effect of the asset life reduction when applied on both existing and new assets is greater than the effect of the actual credit, and that this holds for all but three industries if a true credit is considered. Moreover the reduction in lives appears to be much more important when applied to the existing capital stock than when applied on new assets, with the latter in general resulting in percent increases in investment of approximately one-fourth the magnitude of the former. In contrast to the gain on new assets, however, which increases in importance as more assets are subject to the shorter lives, the gain in depreciation on old assets is a transitory effect which can only exist (and in diminishing importance) for fewer than n_1 years. It should be recalled that the simulations are based on the assumption of complete adoption of the Guideline lives, that is, the change in average life (from n_2 to n_1) is applied to all investment in machinery, and to the extent that this overestimates use of the Guidelines the simulation results will overestimate their effectiveness.

In view of the transitory nature of the application of Guidelines to the existing capital stock, and in view of the possibility that this effect is overestimated, it is interesting to determine changes due to the more permanent aspects of the 1962 provisions, that is, those resulting from the credit and application of Guidelines to new assets. This may be accomplished (approximately) by adding columns 1 and 3 of Table 8.7. It appears that in most industries the combined effect of these two measures is comparable to that of a true 7 % credit, with the average increase in 1964 investment being 2.6%.[9]

In most industries the incentive provided by the investment credit is substantially reduced due to the lower

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[9]The average is calculated by weighting industry changes, excluding textiles, with 1964 investment values obtained from simulations under the assumption of no acceleration. All aggregate percent changes in investment reported below are calculated in this manner.

rates on short-lived assets, the income limitation, and the restriction on the amount of credit in any one year. The extent to which the incentive is reduced of course depends on the ratios given in Column 3 of Table 8.6, and (very close) approximations to the actual credit effects may be obtained by applying these ratios to the results for the true credit which appear in Column 2 of Table 8.7. Since the (unweighted) average of these fractions for the industries under study is .73, the actual credit is, roughly speaking, 73% as effective as a true 7% credit. Estimates of the effects of a credit involving only one of the limitations listed above may be obtained in a similar manner using the appropriate ratios in Columns 1 and 2 of Table 8.6.

The combined effects of the 1962 liberalized depreciation provisions appear to have resulted in fairly substantial increases in investment. For the industries under study percent increases in 1964 investment range from approximately 1 to 11% with the average being 5.1%. In all cases the application of Guideline lives to new assets provides the least benefit, with the application to old assets and the credit providing, in general, approximately equal incentives. Guideline application to old assets is of course only transitory, and excluding this effect the average increase in investment is 2.6%. The actual credit is approximately three-quarters as effective as a true 7% credit due to the lower rate applicable to short lives, the possibility of no net income and the limit to the amount of credit taken in any year.

Summary

In summary the various mechanisms through which an acceleration of depreciation might be expected to affect investment, both in theory and in practice, are reviewed and the relative merits of the four major methods of acceleration under study are compared.

In theory investment behaviour will be influenced by both the pdv and liquidity effects of accelerated depreciation. Under the former a change in the pattern of depreciation deductions increases the asset's discounted revenue stream and hence its profitability. Under the latter the change in depreciation deductions results in a permanently higher level of cash flow for a growing firm, and to the extent that there is an advantage to financing from internal sources the profitability of investment projects is increased. It is a straightforward matter to calculate the pdv and liquidity changes (the latter in the form of depreciation to investment ratios) resulting from the introduction of different methods of acceleration. The magnitude of these changes may be compared for the different methods as well as for relevant parameters such as the asset life, discount rate and growth rate of investment. The response of investment expenditures to such changes, however, is not determinable since it depends also on the relative costs of financing from internal and external sources, and on the volume of investment projects which becomes profitable as a result of the depreciation change.

In practice therefore the effectiveness of the pdv and liquidity factors will depend on the nature of the investment decision-making process used. In this respect the interview evidence summarized in Chapter 4 and a study of the reliance of firms on internal financing suggest that although discounting techniques are rarely considered explicitly by firms, the level of cash flow has a strong influence on investment decisions. The interview evidence also indicates that a payback period criterion is in common use, suggesting that investment may be affected through a mechanism other than changes in liquidity or pdv.

The liquidity effect forms the basis of the attempt to determine empirically the influence of accelerated depreciation on investment. The decision to rely on a cash flow rather than a pdv mechanism rests not only on evidence just cited concerning the relative importance of these factors in practice, but also on orders of magnitude involved in internal rate of return changes resulting from an acceleration of depreciation. That is, even if discounting techniques are relevant, to the extent that they are employed in the form of internal rate of return calculations, their effect is likely to be small. Computations suggest that internal rate of return changes due to most methods of accelerated depreciation will be negligible in view of the fact that such rate of return calculations must rely on revenue and cost predictions over the entire asset's life. Therefore the fact that a discounting mechanism is not included in the simulations cannot be considered a serious omission. Although payback period changes may influence investment decisions independently of cash flow considerations (in the sense that cost reduction due to the increase in cash flow is not the determining factor) no account is taken of this effect in the simulations since there does not appear to be an empirical formulation suitable for such a purpose.

The four methods of accelerated depreciation studied are an investment credit (both with and without a write-down in base), an initial allowance, a switch from SL to SYD, and a reduction in asset life for tax purposes. The relative merits of these methods are compared below in terms of their effect on pdv, liquidity and rate of return measures, and in view of the empirical results just presented. The relative incentives provided by the different methods to short and long lived assets, and to slow and fast growing firms are reviewed.

The true investment credit (of 7%) is in many respects the most effective measure studied. It differs from other types of acceleration in that it involves a direct subsidy and not just a change in the timing of depreciation deductions, and consequently increases in the level of cash flow resulting from a credit are due to an increase in net profits and not depreciation. Comparison of the credit with other methods of accelerated depreciation rests on the assumption that changes in cash flow regardless of their origin are treated comparably as far as investment decisions are concerned.

The pdv and steady state liquidity changes resulting from a true credit are in most cases greater than those for any other method. Both effects are independent of the asset life, although the former decreases slightly with the discount rate and the latter with the growth rate of investment. Assuming a constant revenue stream and SL in use a credit of k% decreases an asset's payback period by k% of the original payback, while under a linearly declining revenue stream the change is somewhat larger. The simulated effect of a true 7% credit introduced in 1962 is to increase 1964 investment in the manufacturing industries by approximately 2.8%. Although the future behaviour of the change in investment cannot be predicted it should be noted that at least in the simplified model studied above the percent change remains permanently positive.

The effectiveness of a true investment credit is considerably diminished by a write-down in the asset's base, and by a reduction in the credit rate on short-lived assets such as the one required of the 1962 credit when first introduced. For such a credit the pdv and liquidity changes are smaller than those provided by a switch to SYD for a large range of discount rates, asset lives and growth rates; and are smaller than those attributable to a 20% initial allowance in a few cases. In contrast to a true credit, both the pdv and liquidity effects vary with the asset's life, being monotonically increasing functions of the latter. Steady state liquidity changes increase uniformly with the growth rate (for plausible rates), while pdv changes decrease slightly for high discount rates. Internal rate of return changes are of approximately the same order of magnitude as those due to a 20% initial allowance and a switch to SYD. The maximum internal rate of return increase occurs for asset lives of 8 years, and in only a few cases are such changes greater that 2 percentage points. Payback period changes are of course less than those arising from a true credit, (k% of the original payback for a constant revenue stream).

For reasons given above simulations involving the 1962 credit do not assume a reduction in base, although the lower rates for short-lived assets are taken into account, as are the restrictions arising from the annual limit on the amount of credit and the possibility of no net income. According to the simulation results the actual credit is approximately three-quarters as effective as the true credit, providing an increase in 1964 of 2.1%.

The initial allowance is similar in many respects to the 1962 credit (as first introduced) and although the empirical effect of the 1958 allowance is essentially negligible due to the annual limitation to $2,000, it is interesting to compare pdv, liquidity and rate of return changes with those of other methods, particularly the credit. As mentioned in Chapter 2, for discount rates in the range of 16-20% the change in pdv from a 20% allowance is approximately the same as that from the 1962 credit for asset lives prevailing in the manufacturing industries. The gain increases monotonically with the asset life, and in fact for large values of the latter (and high discount rates) exceeds the benefit from all other methods including a true credit. Liquidity changes resulting from an allowance are large in the first few years after introduction, but when steady state conditions are reached, are smaller than for all other methods except certain asset life reductions. Internal rate of return changes are approximately equal to those from the 1962 credit and switch to SYD, although they are larger for high initial rates and long asset lives. Payback period changes are larger than those from the 1962 credit for low, but not for high initial paybacks.

The switch from SL to SYD is studied primarily because of the introduction of such an accelerated method in 1954. A comparison of effects with other methods is of interest, however, in view of the possibility that a future policy measure might incorporate a change in methods approximately comparable to the change from SL to SYD. In general pdv and steady state liquidity changes resulting from a switch to SYD are second only to those of a true 7% credit, while transition liquidity effects far exceed the latter. Both effects increase at first and then decrease with the asset life, although for a plausible range of growth rates the liquidity effect is a monotonically increasing function of the asset life. Steady state liquidity changes increase uniformly with the growth rate, but pdv changes do not with the discount rate. Internal rate of return changes are comparable to those due to the 1962 credit or 20% allowance, except that short-lived assets obtain a significantly larger benefit, which is in fact far greater than that provided by other methods. Payback period changes are larger in almost all circumstances than for any other method of acceleration, including the true credit.

A simulation of the introduction of accelerated methods in 1954 yields an increase in 1964 investment of 3.7%. Although it is not possible to predict future changes, a study of Table 8.2 indicates that in most industries the percent increases have been either rising or have remained approximately constant from 1960-1964, and in only one or two industries have they been declining.

Asset life reductions approximating those introduced in 1962 provide in general less benefit than any other method of accelerated depreciation studied. In fact, barring the few industries with the largest percent reductions (such as 36 and 372), pdv, liquidity and rate of return changes are uniformly lower than for any other method. In view of this fact it is not surprising that the simulated effect of the reductions is to increase 1964 investment by only .6%.

A reduction in asset life applied on existing assets differs from other methods studied in that it results in a transitory effect which can only prevail, and in diminishing importance, for fewer than n_1 years (the shorter life). Determination of the effectiveness of such a reduction is difficult since it depends on the magnitude and age distribution of the capital stock. Analysis of the 1962 provision is further hampered by the fact that no accurate data are available on the extent of adoption of the new lives. However, a simulation assuming complete acceptance of the Guidelines yields an increase in 1964 investment of 2.4%. This value can be expected to decline rapidly in view of the transitory nature of the acceleration.

Table 8.1

MODEL TEST SUMMARY

Percent Deviation of Simulated from Actual Values

Industry	Investment	Dividends	Cash Flow
20	4.1	1.3	.3
	19.1	1.4	1.1
22	18.7	6.0	2.0
	69.6	6.9	8.6
26	6.3	2.7	.4
	7.6	2.8	.6
28	9.0	4.3	.7
	11.2	4.3	.9
30	7.6	4.5	.4
	6.3	4.5	.3
32	7.2	3.0	.6
	15.6	2.9	.5
331	19.4	8.5	1.4
	19.1	8.5	1.3
333	16.3	6.5	1.1
	22.4	5.8	.6
36	7.8	4.5	.5
	7.7	4.6	.5
372	14.7	8.6	1.6
	11.6	9.1	.9

Table gives the average (1954-1964) annual percent deviation of simulated from actual values.

For each industry:
Line 1 is based on simulations assuming C+GS exogenous.
Line 2 is based on simulations assuming C+GS endogenous.

Table 8.2

PERCENT CHANGE IN INVESTMENT DUE TO
1954 ACCELERATED DEPRECIATION PROVISIONS

Ind	1954	1955	1956	1957	1958	1959	1960	1961	1962	1963	1964
20	-	.2	.4	.6	.8	1.0	1.0	1.1	1.2	1.2	1.3
	-	.1	-.1	-.1	-1.9	-4.0	-6.5	-10.7	-17.0	-24.7	-37.2
22	.2	.8	1.2	3.2	12.1	8.9	5.8	8.9	9.4	9.8	11.4
	.2	.8	1.1	3.2	28.2	10.3	4.7	38.9	-14.0	6.8	10.7
26	.1	.5	.9	1.6	3.7	4.8	4.3	5.5	6.0	6.6	7.1
	.1	.4	.7	1.2	2.4	2.4	1.7	1.6	1.4	1.5	1.5
28	-	.3	.5	.8	1.5	2.7	3.1	3.5	3.4	3.2	3.2
	-	.2	.4	.6	1.0	1.5	1.3	1.0	.4	-	-.4
30	.1	.4	.8	1.5	2.5	2.8	2.2	2.9	3.0	2.9	2.9
	.1	.5	1.0	2.1	4.0	5.1	4.9	6.7	7.7	7.8	8.6
32	-	.2	.4	.7	1.2	1.3	1.2	1.6	1.6	1.7	1.9
	-	.6	1.5	2.6	4.2	5.5	4.0	4.1	7.6	9.4	7.1
331	.1	.2	.3	.5	.9	1.4	1.9	2.5	3.7	4.2	3.0
	-	.2	.4	.7	1.1	1.8	2.5	3.2	4.7	5.2	3.6
333	-	.2	.3	.5	1.5	4.0	4.0	4.0	4.5	4.1	3.6
	-	.2	.4	.7	1.9	6.0	6.8	4.8	3.9	2.8	2.1
36	-	.3	.5	.8	1.0	1.0	.8	.8	.7	.7	.8
	-	.3	.4	.5	.6	.5	.4	.4	.4	.4	.5
372	.1	.4	.9	1.9	2.9	3.6	4.3	4.6	4.4	4.2	5.0
	.1	.4	1.1	2.5	3.0	3.1	3.6	4.0	4.2	4.4	5.3

Ind = industry

For each industry:
Line 1 values are based on simulations assuming C+GS is exogenous.
Line 2 values are based on simulations assuming C+GS is endogenous.

Table 8.3

COMPARISON OF THE ORIGINAL INVESTMENT EQUATIONS WITH THOSE ESTIMATED BY GENERALIZED LEAST SQUARES

Industry	(1)	(2)
20	-959.31	-1.53
22	.68	-2.11
26	.29	.86
28	-9.59	.62
30	.02	.85
32	*	.22
331	.71	.79
333	.38	.67
36	-.01	.84
372	.03	.41

Table gives values of $R_I^2 = 1 - var(e)/var(I)$, where I is investment and the e's are the deviations of actual from simulated investment values.

Column 1 values are based on the generalized least squares estimates of the investment equation.
Column 2 values are based on the original estimates of the investment equation.

* Less than -1,000

Table 8.4

ESTIMATED POTENTIAL INCREASE IN DEPRECIATION ON EXISTING CAPITAL STOCK RESULTING FROM 1962 ASSET LIFE REDUCTION

Industry	n_1	n_2	G(SL)	G(DDB)	Total
			($ Millions)		
20	13	15	38.7	37.5	76.2
22	13	16	49.2	39.6	88.8
26	15	19	42.3	49.1	91.4
28	11	13	98.5	62.3	160.8
29	15	18	183.3	60.0	243.3
30	13	14	5.4	6.5	11.9
32	16	18	15.3	24.9	40.2
331	17	21	70.1	72.1	142.2
333	17	21	20.7	21.1	41.8
36	11	14	26.6	69.0	95.6
371	12	14	50.9	60.6	111.5
372	9	12	22.1	30.1	52.1

n_1 = average asset life after Guideline change

n_2 = average asset life before Guideline change

$$G(SL) = (1/n_1 - 1/n_2)\sum_{j=1}^{n_2} I_{t-j}$$

$$G(DDB) = (2/n_1 - 2/n_2)\sum_{j=1}^{n_2} (1-2/n_2)^j I_{t-j}$$

Total = G(SL) + G(DDB)

I_t = investment

Table 8.5

COMPARISON OF 1962 GUIDELINE DEPRECIATION

Percent of 1962 Depreciation taken under the Guidelines

Industry	(1)	(2)
20	32.3	60.0
22	23.0	57.6
26	59.6	87.1
28	52.6	88.4
29	41.0	na
30	47.3	59.3
32	42.5	64.4
33	68.6	na
36	47.7	72.8
371	73.4	96.7
372	29.6	36.8

Column 1 Source: Statistics of Income, 1962, Table 33, p. 314.
Column 2 Source: S.C.B., July 1963, Table 1, p. 4.
na: not available

Table 8.6

1962 INVESTMENT CREDIT STATISTICS

Industry	C^U/C^T	Cr^t/Cr^a	$(C^U/C^T)(Cr^t/Cr^a)$
20	.84	.82	.69
22	.93	.86	.80
26	.89	.81	.72
28	.93	.88	.82
29	.91	.44	.40
30	.89	.89	.79
32	.87	.75	.65
33	.95	.88	.83
36	.78	.89	.70
371	.89	.94	.83
372	.68	.81	.55

C^T total cost of property qualified for the credit
C^U cost of property used in calculating the credit
Cr^t tentative credit
Cr^a actual credit taken

Source: Statistics of Income, 1962, Table 1, pp. 50-52 and Table 14, p. 192.

Table 8.7

ESTIMATED EFFECT ON INVESTMENT OF THE
1962 ACCELERATED DEPRECIATION PROVISIONS

Percent Increase in 1964 Investment

Ind	n_1	n_2	(1)	(2)	(3)	(4)	(5)	(6)
20	13	15	.6	.8	.2	.6	.8	1.4
			-.7	-1.1	-	-.9	-.9	-1.7
22	13	16	8.9	11.2	2.3	14.3	16.8	27.3
			6.6	8.4	1.9	11.0	13.1	20.8
26	15	19	3.8	5.4	1.5	5.2	6.7	10.9
			2.7	3.7	1.3	3.5	4.8	7.7
28	11	13	2.2	2.7	.5	2.2	2.8	5.0
			1.6	2.0	.5	1.6	2.0	3.7
30	13	14	2.0	2.6	.3	1.0	1.3	3.3
			3.0	3.8	.3	1.5	1.8	4.9
32	16	18	.7	1.1	.2	.7	.9	1.6
			3.7	5.7	.7	3.7	4.4	8.2
331	17	21	1.4	1.7	.4	1.9	2.3	3.7
			1.8	2.1	.5	2.4	2.9	4.7
333	17	21	2.2	2.7	.4	2.0	2.3	4.7
			2.5	3.0	.4	2.1	2.5	5.1
36	11	14	.4	.7	.3	.8	1.1	1.6
			.3	.4	.2	.5	.7	1.0
372	9	12	1.8	3.3	1.7	3.6	5.4	7.3
			2.0	3.8	1.9	4.0	6.0	8.2

Column	Description
1	Actual Credit
2	True Credit
3	Asset Life Reduction on new assets only
4	Asset Life Reduction on old assets only
5	Asset Life Reduction on old and new assets
6	Asset Life Reduction on old and new assets plus Actual Credit

Ind = industry
n_1 = average asset life after 1962 Guideline change
n_2 = average asset life before 1962 Guideline change
Line 1 values are based on simulations assuming C+GS exogenous.
Line 2 values are based on simulations assuming C+GS endogenous.

APPENDIX

SOURCES OF DATA

The sources of all data used in the analysis (except those for which sources appear in the text) are classified below under the chapter in which the data are first discussed.

Chapter 6

1. The investment data are from various issues of the Survey of Current Business, U. S. Department of Commerce, O.B.E.

2. The accelerated depreciation data are from the following sources:

1954 - Supplementary Depreciation Data from Corporation Income Tax Returns (Statistics of Income 1959), U. S. Treasury Department, I.R.S., June 1965, Appendix, Table 11, p. 118.

1955 - ________, Appendix, Table 6, pp. 103-105.

1957 - Statistics of Income, Corporation Income Tax Returns, U. S. Treasury Department, I.R.S., 1957-58, Table 23, p. 115.

1960 - Supplementary Depreciation Data from Corporation Income Tax Returns, Appendix, Table 4, pp. 90-96.

It is assumed that "returns showing depreciation methods" account for the main part of accelerated depreciation and these values are used as estimates of total accelerated depreciation in all years. The 1954 accelerated depreciation figures (available only for firms with accounting periods ending December 1954 through June 1955) were blown up by the ratio of total investment to investment attributable to firms with accounting periods ending in these months. The distribution of investment over firms according to accounting periods is as described above.

3. The fraction of investment in machinery and equipment for 1946-1962 is from an unpublished study prepared for the Department of Commerce, O.B.E., by Michael Gort. Data for 1963 and 1964 are from the 16th and 17th annual McGraw-Hill Survey of Business' Plans for New Plants and Equipment.
4. The distribution of assets by accounting period is an average of such data from Statistics of Income, Corporation Income Tax Returns, 1954-55, Table 14, pp. 88-94, and from the same publication for 1958-59, Table 12, pp. 110-116.
5. Amortization data are from various issues of Statistics of Income, Corporation Income Tax Returns, and the F.T.C.-S.E.C. Quarterly Financial Report for Manufacturing Corporations.

Chapter 7

1. The Quarterly Financial Reports for Manufacturing Corporations (F.T.C.-S.E.C.) are the basic sources for data used in the regression analysis. Variables obtained from these reports are: depreciation, dividends, profits

before and after tax, long term debt, total stockholders' equity, cash, and government securities. All data are spliced in 1951 and 1956 using the method employed by W. H. L. Anderson, and in 1958 by multiplying pre-1958 values by a correction factor.[1] The latter for each series is the ratio of revised to unrevised 1958 values.

2. Moody's industrial bond rate is from various issues of the Survey of Current Business.

3. The interest rate on term loans from banks is from an unpublished memo: "Rates Charged Customers on Long Term Commercial Loans," provided by Mr. James Eckert of the Board of Governors of the Federal Reserve System, Washington, D. C.

4. The capacity utilization index is an unpublished series provided by Mr. F. Gerard Adams of the Wharton School of Finance and Commerce.

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[1]W. H. Locke Anderson, Corporate Finance and Fixed Investment, Boston, 1964, p. 29.

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