A DIAGRAMMATIC REPRESENTATION OF CERTAIN PROBLEMS IN GENERAL EQUILIBRIUM THEORIES.

by

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ABSTRACT

A Diagrammatic Representation of Certain Problems in General Equilibrium Theories

by John Ching-Han Fei

Submitted for the Degree of Doctor of Philosophy in the Department of Economics and Social Science on May 9, 1952.

This is a study of a collection of problems which belong to the field of static general equilibrium theories in economic literature. The problems are: the equalization of factor prices, the analysis of "specialization status", and the study of the "world" productive efficiency in the international trade theories. Furthermore, certain problems in the history of economic doctrines, in regard to the value and distribution theories of the Austrians, Ricardo and Marshall, are examined from the viewpoint of general equilibrium analysis.

The systematic use of diagrammatic methods provides the unifying scheme of the otherwise unrelated problems. The thesis, then, is integrated from the viewpoint of method of analysis. The ease with which the unrelated problems can be similarly treated testifies the belief that there exists a group of explanatory principles which are applicable to all the problems selected.

The economic problems which are studies in this thesis must be amenable to the two dimensional limitation inherent in the diagrammatic methods. Simplified assumptions will have to be made. This means that any conclusions to be drawn from the use of these methods will only be approximations of reality, and the thesis is, therefore, highly abstract. On the other hand, the writer believes that most of the problems studied are of such a nature that they cannot be satisfactorily treated by a literary exposition.

In spite of the clumsiness of diagrammatic methods, as compared, for example, with the algebraic methods, the arguments in this thesis are developed in a rigorous and logical order. The individual economic problems, instead of being treated exhaustively at once, are introduced in to the development of the arguments at convenient stages to allow a more systematic exploitation of the "methods of analysis".

No mathematical background beyond high school algebra is required of the readers. The merit of the thesis, to a large extent, is pedagogical.

24 Agassiz Street
Cambridge, Massachusetts
May 9, 1952

Professor Joseph S. Newell
Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge 39, Massachusetts

Dear Sir:

In accordance with the requirements for the degree of Doctor of Philosophy at the Massachusetts Institute of Technology, I herewith submit a thesis entitled: "A Diagrammatic Representation of Certain Problems in General Equilibrium Theories."

Respectfully submitted

John Ching-Han Fei

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The painstaking supervision and the kind suggestions of Professor R. L. Bishop, under whose guidance this thesis was written, have contributed much toward the quality of the contents and the exposition of this thesis. Mr. J. W. Hanson, a fellow student of the writer at M.I.T., offered his generous criticism in the embryonic stage of this thesis. For the help and assistance of the aforementioned the writer is extremely grateful. However, all errors and shortcomings of this thesis are entirely the responsibility of the writer. Final thanks must be extended to Inez J. Crandall for her expert typographical assistance.

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Chapter I.

Production Functions

Section one: Production Functions - General Considerations

Production functions describe the quantitative relationships between inputs (or factors of production) and outputs (or commodities). From the viewpoint of the economic analysts, they are engineering knowledge assumed (or taken for granted) for an analytical problem at hand. The empirical verification of the validity of such assumptions constitutes the work of engineering research and does not concern the economist as such.

The expression "engineering knowledge" suggests that a production function is technical "know-how" possessed by a "conscious mind", which is another entity that the economist assumes to exist (e.g. economic man, "entrepreneur","factor owner"). Since it is also assumed, as a general practice of the economic analyst, that the conscious mind has the purpose of realizing an optimum result, the production function describes, in one sense or another, the maximum output obtainable with various combinations of the inputs. For simplicity, we shall speak of a production function as if there were only one output and two factors of production. Throughout this thesis, we shall not discuss the more complicated cases.

Rigorously, the properties of any entity in an analytical system is describable and definable only in terms of the operational relationships that are assumed to exist between the entity and the other entities belonging to the same system. A production function, then, defines the factors of production and the outputs involved - since it describes the operational relationships among them. In other words, there may be other interesting properties of, for example, a factor of production (physical, chemical, ethical or philosophical etc.), but they do not concern the analytical economist if these properties are not defined in terms of the operational relationships between a factor of production and the other entities. On the other hand, the operation relationships, when fully given, sufficiently describe an entity such as a factor of production.

A production function, however, defines only one aspect of the factors of production and the outputs. The other aspect of these entities are defined by the operational relationships as related in the preference system of a conscious mind. The former may be called the productive aspect of the factors of production and the commodities and the latter, the psychological aspect. The former aspect is "engineering" in nature and the latter is "psychological" in nature. Both of them are data assumed by the economist. In the present chapter, we are concerned with the former aspect, or the definitions of the factors of productions and the commodities. That is to say, we are concerned with the production function.

Section two: Factors and outputs

Both a factor of production and an output, related in a production function, are, rigorously speaking, the "services" yielded (or yieldable) by some durable (or non-durable) agents. The conceptual distinction of the "services" from the "agent" which generates them, represents a most significant advance in thinking in the history of economic thought.[1/] However, in this thesis, we are concerned only with the static theory; so this distinction can be neglected. We can speak, indiscriminately, of the "service of a factor of production (or product)" or the "factor of production (or the product)" themselves.

However, we must not infer from this practice that the time dimension is completely suppressed for a production function --if we can claim any relevancy of our analysis to the facts of the realistic world at all. In other words, we have to imagine that a production function relates the output and the factors of production as applied during a certain interval of time; only, for the sake of simplicity, the time interval is assumed to be uniform for all the problems considered. The production function itself, then, has a time dimension of one unit period.

Another simplification that we want to make with respect to the properties of the factors of production and the output is that they are finely divisible (as related in a production function). The purpose of such an assumption grew out of the requirement of <u>marginal</u> analysis, that is, we want to know, for example, the effect on output of an addition

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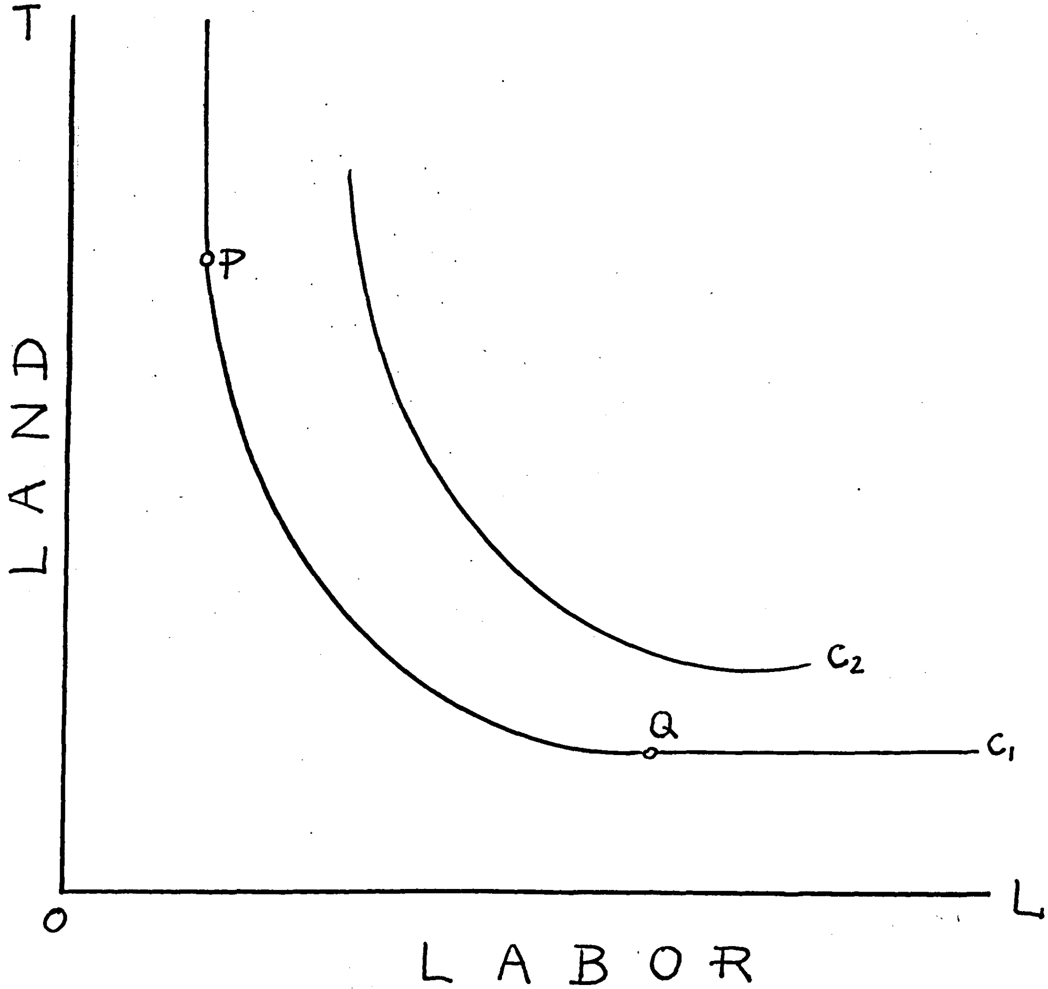
1/It was a contribution of the great French economist, L. Walras. See, for instance, G. J. Stigler, <u>Production and Distribution Theories</u>, Macmillan, 1941, p. 246 ff.



Diag. 1

(or subtraction) of a small quantity of a factor of production.[2/]

Section three: Properties of Production Functions

There are certain properties of the production function which will be assumed throughout this thesis. Some of these properties are derivable from the nature of a production function so far assumed; others are justifiable only by empirical research hence constituting new assumptions. In any case, our discussions will be brief, since these properties are usually assumed by economists and discussions on them are easily found.[3/]

The properties of a production function which we want to assume can be best described for our purpose with the aid of a diagram. For the case of two factors of production and one output, a system of production contours (diagram one) may be drawn which shows the maximum output obtainable at various combinations of the two inputs. The two factors are called land (T) and labor (L) and the output is called clothing (C). Since the output is cardinally defined (i.e. we can add or subtract two outputs and obtain a sum or difference), there is an "index" for every iso-product curve - i.e. the combinations of the factors which yield the same output as defined by the index. We shall call these iso-product curves the production contours. (e.g. c_1, c_2, in diagram one).

2/It is tempting to justify the assumption of "perfect divisability" by referring to the time dimension of the service (and the output). This justification, however, raises serious problems for a static theory which cannot be easily handled -- for then the factor (and product) takes on a two-dimensional character which nullifies the effort to standardize the time period of the production function. The problem is non-static. (The first serious attempt to deal with a problem of this kind is probably made by Jevons. See e.g. Stigler, Production and Distribution Theories, p. 26.)

3/See, e.g., Stigler Theory of Prices, p. 69 on indifference curves

(1) <u>The production contours cannot have a positive slope</u>.

This property is derivable from the assumed "maximum" property of a production function. If a production contour has a positive slope, the same output can be produced by several combinations of the two factors with some combinations representing more units of input of <u>both</u> factors (i.e. represented by the lower points on a contour with positive slope). These inefficient ways of production, from the engineer's viewpoint, must be ruled out provided there is no "disposal" problem. (And the "disposal" problem may be neglected for all practical purposes).

(2) <u>Production contours cannot cross each other</u>.

This is another property derivable from the assumed properties of the production function given so far. If two production contour cross each other, the output represented by the two contours - i.e. the indices of the two contours - must be exactly the same. With two negatively sloped production contours crossing each other and with the same index of output, the argument leading to the justification of property one, applies in this case too.

(3) <u>The production contours should not be concave to the origin</u>.

(They may be convex to the origin, horizontal or vertical (straight) lines - Diagram 1). The slopes of the production contours may be called the marginal rates of substitution. It indicates the units of one factor that have to be added (or given up) when one unit of the other factor is withdrawn (or added) if the output is to remain unchanged. This property of a production function states, then, that the marginal rate of substitution must not be increasing; or, it should be more difficult to substitute one factor by another (and in the limiting cases, it becomes impossible to substitute any more) after a substitution in the same direction had taken place.4/

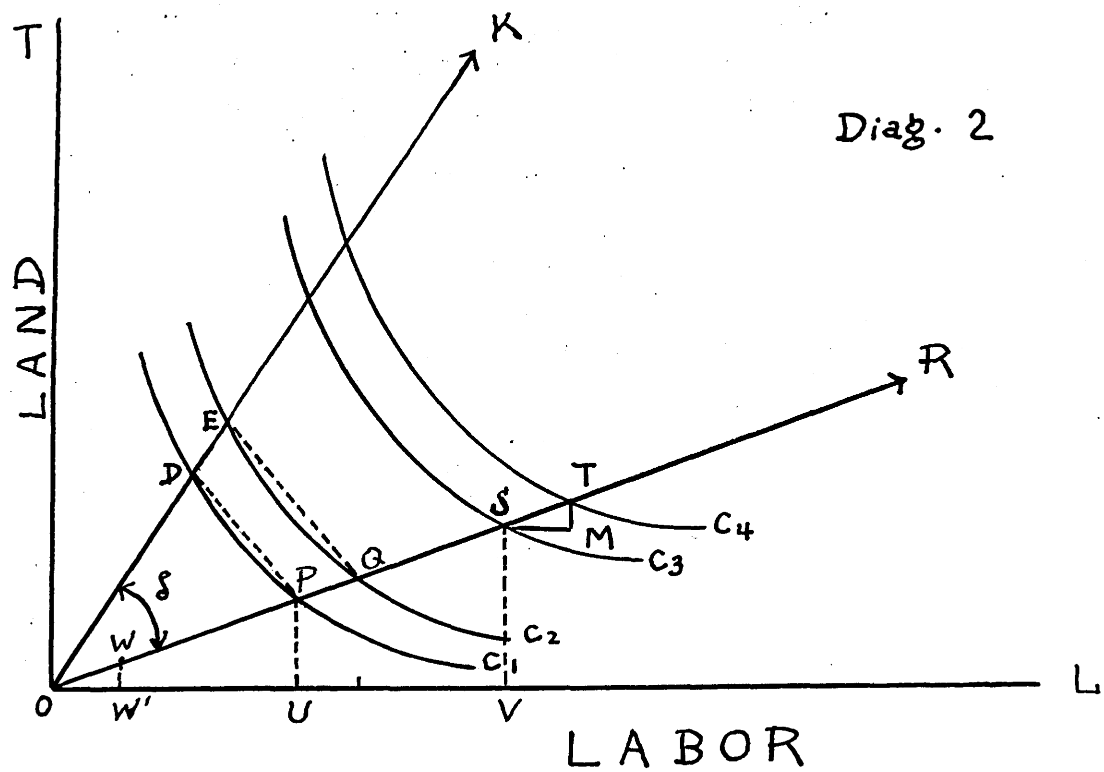
Property (3), then, may be called the "imperfect substitutability assumption of the two factors", in contrast to the case of "perfect substitutability" under which a production contour will be represented by a (negatively sloped) straight line. This latter case seems to suggest that the two factors are exactly the same as far as their relationships with the particular level of output is concerned. The two factors differ from each other only in that one factor can be looked upon as a constant multiple of the other. When the factors are finely divisible, this distinction can be eliminated by a redefinition of the unit of one of the factors. Hence, our assumption of diminishing marginal rate of substitution seems to be justifiable by the assumption of two different factors of production, which is apparently what our interest dictates when we postulate two factors of production instead of one.

However, the rule of the diminishing marginal rate of substitution is justifiable only by empirical observation. That is to say, there is no logical necessity that we can derive this property from the other assumptions of a production function made so far. On this account, the findings of engineering studies probably do not contradict the assumptions made by the economists.

4/The two limiting points (P and Q in diagram 1) mark the places where the marginal rate of substitution becomes zero or infinite. They may be so close to each other that the middle part of the curve is eliminated, in which case we have "complete non-substitutability" for that output. This case will not be considered in this thesis. On the other hand, the two limiting points may be so far apart that the horizontal and vertical portions of the contours may be neglected. For simplicity we will often consider this special case.

Finally, we shall assume throughout this thesis the property of "constant returns to scale" for all the production functions. This means that the total output of a commodity will be proportional to the quantities of the inputs applied. A more rigorous formulation of this property, in terms of the map of production contours, will be given in the following chapter --where a number of properties of the contour maps, deductible from this assumption, will be discussed.[5]

5/Economists seem to hold different opinions as to whether the property of constant returns to scale is deductible from other properties of the factors of production and outputs --especially the assumption of fine divisibility. (See e.g. Professor Chamberlin "The Theory of Monopolistic Competition" Sixth Edition, Appendix B, "The cost curves of the individual producer.") It seems to the writer that the controversy of "proportion" vs. "size" is "engineering" in nature. It should be settled by the engineers rather than the economists. The writer frankly makes this assumption as an unverified hypothesis. As will be evident in our later analysis, this is a drastic simplification from the viewpoint of geometrical presentation.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Diag. 2
LAND
R
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M
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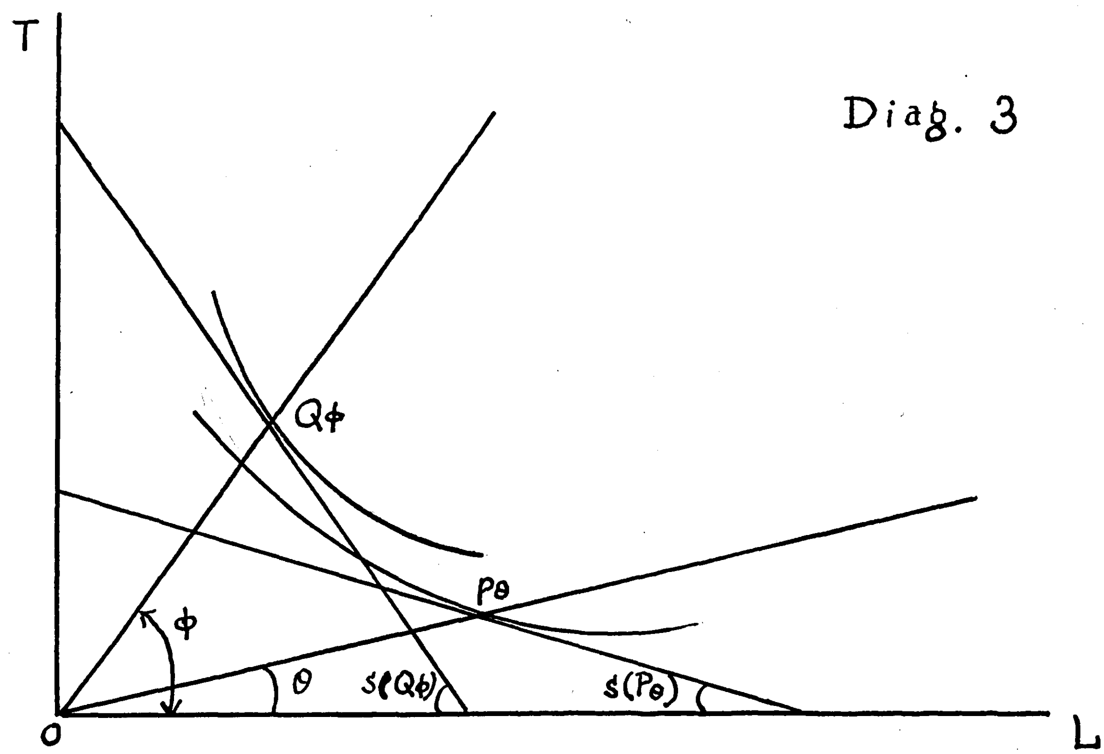
T
Diag. 3
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Pθ
φ
θ
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s(Pθ)
O
L

Chapter II.

Deduced Properties of Production Functions With Constant Returns to Scale

In Chapter I we have assumed, or deduced, four basic properties for the production functions which will be adhered to throughout this thesis. The production contours are: 1) cardinally defined iso-output curves with negative slope; 2) non-crossing; 3) non-concave (to the origin); and 4) satisfy the condition of constant returns to scale. In this chapter we shall deduce a number of properties - which will be called "rules" - of such a production function which will be used in our analysis in the later chapters.

All the "proofs" in this chapter will be geometrical, in line with the spirit of this thesis. A number is attached to each property (i.e. rule), for more convenient reference in our later chapters.

Rule one: On any diagonal line, the outputs at any two points are proportional to the radial distances.

In diagram two, let P and Q be any two points on OR which is any radial line. Let c_1 and c_2 be the indices of the production contours passing through P and Q respectively. Rule one states that:

$$OP/OQ = c_1/c_2.$$

This merely states, in a more precise way, the meaning of constant returns to scale, no proof is required. (Obviously, OP/OQ measures the ratio of inputs (for either factor) at these two points.)

Rule two: On any radial line, equal distances measure equal increment (or decrement) of output.

In diagram two, let the distance between S and T equal the distance P and Q. Let the indices of the production contours passing through P, Q, S and T be c_1, c_2, c_3, and c_4 respectively.

Prove: $c_2 - c_1 = c_4 - c_3$

Proof: by rule one, $OQ/OP = c_2/c_1$; $OT/OS = c_4/c_3$ and $OP/OS = c_1/c_3$

we have, $(c_2 - c_1)/(c_4 - c_3)$

$$= (c_1/c_3) \cdot \frac{(c_2 - c_1)/c_1}{(c_4 - c_3)/c_3}$$

$$= (c_1/c_3) \cdot \frac{(c_2/c_1) - 1}{(c_4/c_3) - 1}$$

$$= (OP/OS) \cdot \frac{(OQ/OP) - 1}{(OT/OS) - 1}$$

$$= (OP/OS) \cdot \frac{(OQ - OP)/OP}{(OT - OS)/OS}$$

$$= \frac{(OQ - OP)}{(OT - OS)} = \frac{PQ}{ST} = 1$$

So, $c_2 - c_1 = c_4 - c_3$ QED.

Rule three: The whole system of production contours can be deduced from any one production contour.

Proof: In diagram 2, let the production contour representing c_1 units of output be given. Let c_2 be any quantity of output for which we want to find the production contour.

Let OR be any radial line intersecting c_1 at P. On OR, mark the radial distance OQ such that OQ equals to OP x (c_2/c_1), a known quantity.

So we have: $OQ/OP = c_2/c_1$

Hence, by rule one, point Q is a point lying on the production contour with c_2 units of output. (We implicitly assume that for any point in the map there is one and only one output). Similarly we can find all other points on the production contour with c_2 units of output by taking all other radial lines; and we can also build up any production contour in this way. This property will simplify our exposition in the later sections --since we can, then, concentrate on one contour instead of drawing out the whole system.

Rule four: When two radial lines determine a series of pairs of points on the same production contours, straight lines joining each pair of points are parallel.

In diagram 2, let OR and OK be any two radial lines intersecting the production contours c_1 and c_2 at points P, Q, D and E. Join the straight lines PD and EQ.

Prove: $DP // EQ$

Proof. $OQ/OP = OE/OD = c_2/c_1$..........by rule one.

Hence triangles OPD and OQE are similar.

We have, $DP // EQ$

QED

Rule five: The slopes of the production contours at points intersected by the same (any) radial line will be the same.

Proof: In the proof of Rule (4) diagram (2), let OK approach OR (i.e. let the angle δ approach to zero). The slopes of DP and EQ approach the slopes of the production contours at points P and Q respectively. Since DP always parallels EQ (rule four), the slopes of the production contours at P and Q must be the same. QED.

The economic significance of this property is quite obvious. It states that the marginal rate of substitution of the two factors will be the same for any given input-ratio of the two factors.

There are certain advantages, as will be evident in our later analysis, if we write this rule in another system of notations. Let points P_θ and Q_ϕ be any two points in a contour map. Let the subscripts in P and Q represent the input-ratios corresponding to the two points - see diagram 3. Let $S(P_\theta)$ and $S(Q_\phi)$ represent the slopes of the production contours passing through these points. (In diagram 3, these values are represented by the slopes of the tangent lines at P and Q). With this notation, we can write out rule five more neatly as follows:

$$S(P_\theta) = S(Q_\phi) \text{ if } \theta = \phi$$

We may also take this opportunity to adopt a convention which will be adhered to throughout this thesis. This convention involves an agreement as to the way we speak of the <u>ratios</u> and the ways we shall represent these ratios in our diagrams. We may formally list our conventions as follows:

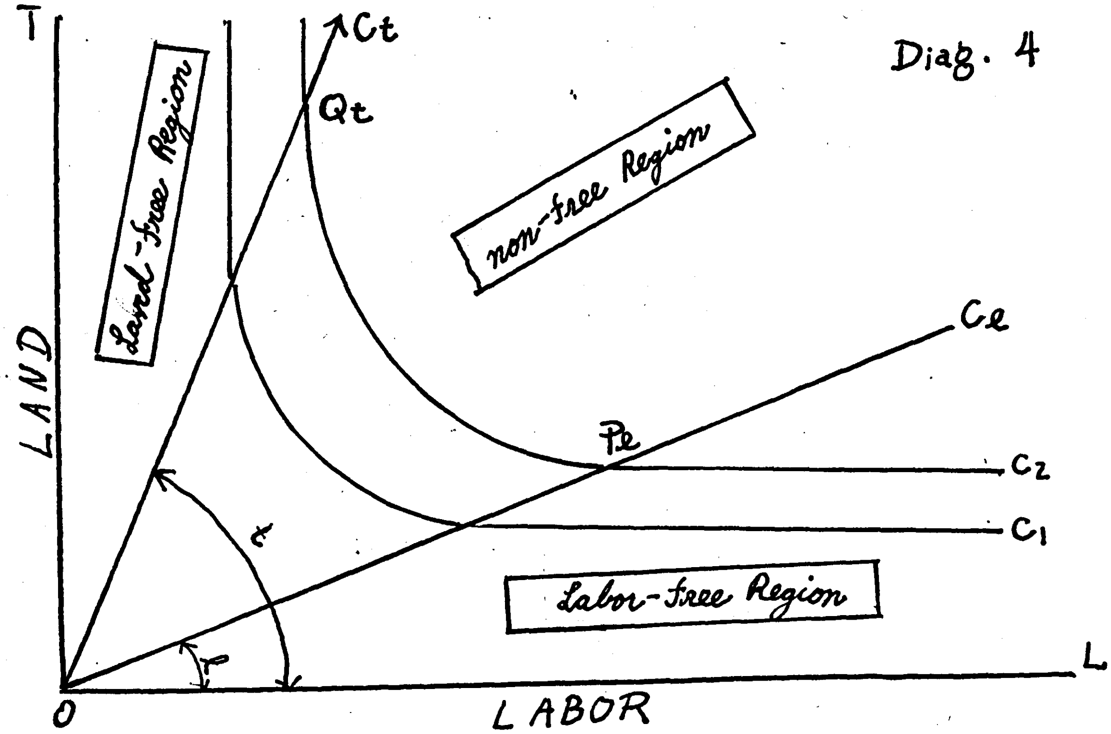
1) When we speak of the input ratio, we take land as the numerator (and labor as the denominator).
2) When we speak of the marginal rate of substitution, we take the <u>units</u> of land that are required to substitute <u>one</u> unit of labor. (Land is again the numerator). In view of our discussion in Chapter one, the marginal rate of substitution, in our convention, equals the "wage-rent ratio", this means that, in our convention, when a ratio is <u>high</u>, <u>labor</u> is,

or tends to be better off (and land worse off). This "humanitarian" association of a "<u>high</u>" with "<u>labor better off</u>" (rather than land) may be a useful mnemonic device.

3) In our diagrams, we shall always take the vertical axis (Y-axis) for land and the horizontal axis (X-axis) for labor.
4) The input ratio is then represented by the natural slopes of the radial lines --which are always positive. (These values are represented by the subscripts of the points in the contour map). In this way, as convention number one (above) dictates, a <u>higher</u> input ratio is represented by a "<u>larger</u>" radial angle, and vice versa.
5) For the marginal rate of substitution, we will take the absolute value - i.e. a positive value. This value is then represented by the tangent of the smaller angle made by the horizontal axis and the tangent to any production contour at points for which the marginal rate of substitution is considered. Our conventions (No. 2 above and the present one) implies that a <u>higher</u> marginal rate of substitution will be represented by a "<u>larger</u>" angle.

Referring to diagram 3, we see, for example, that when the angle ϕ is <u>greater</u> than θ, the angle $S(Q\phi)$ is <u>greater</u> than $S(P\theta)$.

In conclusion, in the conventions we have adopted: "highs" in <u>words</u> are reflected by "large" in geometrical expression.



T
Ct
Qt
Diag. 4
Land-free Region
non-free Region
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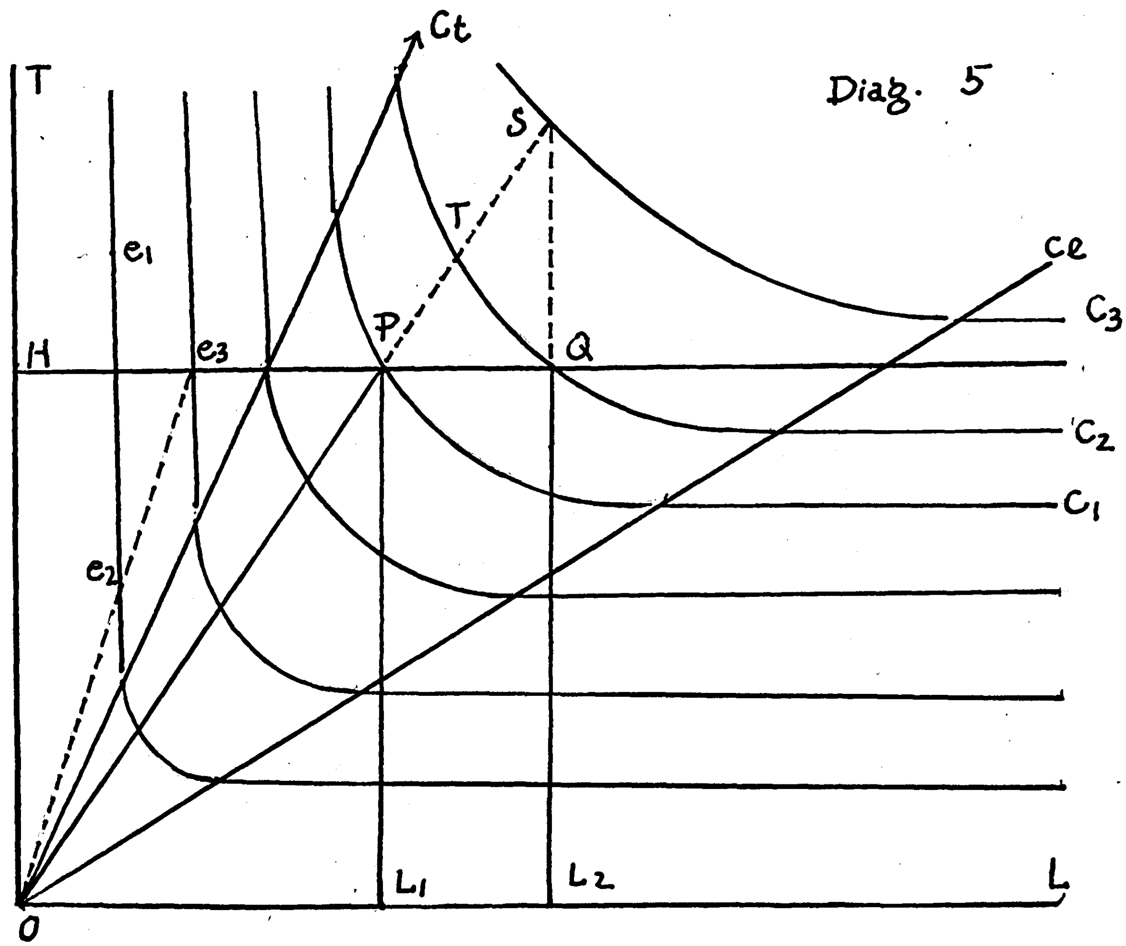
LAND
Pe
C2
t
C1
Labor-free Region
L
0
LABOR



Ct
T
Diag. 5
S
T
e1
ce
C3
H
e3
P
Q
C2
C1
e2
L1
L2
L
0

<u>Rule six</u>: <u>The labor-free ridge line (O_l) and land-free ridge line (O_t) which are radial lines, divide the whole contour map into three regions</u>: <u>the land-free region, the labor-free region and the non-free region</u>. <u>The slopes of the production contours in the three regions are, respectively, infinite, finite, and zero.</u> (Also, any region may be empty). (See Diagram 4)

We may call the lines joining the points of the production contours having the same slope "iso-slope lines". By rule five above, we know that the "iso-slope-lines" are radial lines.

There are two iso-slope-lines which are of particular interest to our later analysis, namely, the two iso-slope lines (or radial lines) which intersect the production contours at points where the contours become horizontal, or vertical, respectively. We will call the former the <u>labor-free ridge line</u> and denote it by O_l; and the latter the <u>land-free ridge line</u> and denote it by O_t. (See Diagram 4).

Since the production contours cannot be concave to the origin, the shape of any one production contour, in the general case, must be as depicted in Diagram 3. There are three portions for any contour: a vertical portion, a horizontal portion and a portion with finite slope, demarked by two points - e.g.(P_l, Q_t for contour c2). The two ridge-lines (O_l and O_t), then, pass through the two demarkation points, respectively.

But, as we know, the whole systems of contours may be generated by any one contour (Rule three), and the slopes of the contours at the same input ratio must be the same (Rule five), hence the ridge-lines trisected the whole map of contours into three regions, in which the slopes of the contours are infinite, finite and zero respectively. (See diagram 4).

We may call the three regions: "land free" (slope infinite), "non-free" (slope finite) and "labor-free" (slope zero). If an equilibrium position is established in any region, under perfect competition, factor rewards will be as the name of the region would indicate. This is true because the factor price ratio, in equilibrium, equals the marginal rate of substitution.

The ridge lines, then, indicate the input ratios at which the two factors are on the margin (or "ridge") of becoming free. The subscripts in C_t and C_1 (i.e. "t" and "1") have double significances: they represent the factor that is becoming free ("t" for land and "1" for labor) at <u>what input ratio</u>. The capital letter in C_t and C_1 (i.e. "C") will be used to represent the commodity for which the contour map is drawn (i.e. "C" stands for clothing). We shall use other capital letters for other commodities. However, in our later usage, we will take "C_t" or "C_1" as representing <u>either</u> the ridge lines themselves <u>or</u> the input ratios at which the ridge lines occur, depending upon the connotation of the text.

It is evident that, with our convention, R_t is necessarily greater than R_1. The economic interpretation is that more lands have to be combined with one unit of labor in order to render land "redundant" than to render labor "redundant."

It is further evident that, in conforming with the spirit of "proportionality" and "constant returns to scale", there is a symmetrical relationship with respect to the ratios of input of the two factors.

Hence, in our following "proofs", we shall confine our attention to <u>one</u> of the two "free regions"; the other half of the proof will be taken as self evident.

The existence of all three regions is probably generally true as an engineering fact. However, in our following analysis, we shall often neglect the existence of the two "free-regions" for the purpose of simplication. (There is one particular case where the free regions cannot be neglected - i.e. the case of the Ricardian rent theory which occupies our attention in a later chapter).

<u>Rule seven:</u> <u>In the non-free region, the marginal rate of substitution is greater the greater is the input ratio; in the land-free and labor-free regions, the marginal rates of substitutions are infinite and zero respectively</u>.

With our notations developed earlier, rule seven may be written as:

7.1) $S(Q\phi) > S(P\theta)$ if $\phi > \theta$ and if $C_1 < \phi < C_t$ or $C_1 < \theta < C_t$ or both.

7.2) $S(P\theta) = 0$ if $\theta < C_1$

7.3) $S(P\theta) =$ infinity if $\theta > C_t$

These properties are merely precise statements of what was implied in our discussion of Rule (6) and the non-concave (to the origin) property of the production contours. No proof is required.

Rule eight: In the non-free region, we have $S(Q\phi) \gtreqless S(P\theta)$ if and only if $\phi \gtreqless \theta$, where the equality and inequality signs correspond.

This property follows directly from Rule (5) and Rule (7) above. It merely states the necessary and sufficient conditions for the equality (and inequality) of the slopes of the production contours in terms of the slopes of the radial lines.

Rule nine: The average physical productivity of any factor of production remains unchanged when the input ratio is fixed, i.e. regardless of the size of the inputs.

The average physical productivity for any factor of production is defined as the total output divided by the units of input of that factor. With reference to diagram two, the average physical productivities of labor at points P and S (on OR radial line) are, respectively:

c_1/OU at P
and c_3/OV at S, where PU and SV are perpendicular to the horizontal axis.

Prove: $c_1/OU = c_3/OV$

Proof: Triangles OUP and OVS are similar, we have

$$OS/OP = OV/OU$$

but $OS/OP = c_3/c_1$.......by rule one.

We have $c_3/s_1 = OV/OU$

or $c_1/OU = c_3/OV$ QED.

(The proof applies to all three regions since we have made no reference to the slopes of the production contours). The economic interpretation of this property follows directly from the property of constant returns to scales and need no further elaboration.

Rule Ten: <u>In the non-free region, the average physical productivity of labor (land) is higher the higher (lower) is the input ratio; in the free-regions, the average physical productivity of the non-free factor remains constant.</u>

Let the subscript in APP_{e1} - (i.e. "el") represent the point for which the average physical productivity is considered. The second half of Rule (10) is readily proved as follows (Diag. 5). In the land-free region, consider APP_{e1} and APP_{e3} of labor (i.e. the non-free factor). Let the radial line O_{e3} intersect the production contour, on which el lies, at e2, we have:

$APP_{e2} = APP_{e3}$.........by rule (9)

and $APP_{e1} = APP_{e2}$.........by the fact that the production contours in this region are straight lines; both output and input at el and e2 are the same.

We have: $APP_{e1} = APP_{e3}$ QED.

(Similarly we can prove that in the labor-free region the APP of land remains constant).

Let us next prove the first half of Rule (10). In Diagram 5, let points P and Q be two points, in the non-free region, with the same amount of the input of land (i.e. OH). Let the input of labor at P and Q be OL_1 and OL_2 respectively; and let the outputs be c_1 and c_2 respectively. (For labor, $APP.P = c_1/OL_1$ and $APP.Q = c_2/OL_2$). Let L_2 be greater than L_1 so that the input:ratio at P is greater than the input ratio at Q. So for the case of labor, all we have to do is to prove that:

$$APP.P > APP.Q \text{ or equivalently } c_1/OL_1 > c_2/OL_2$$

Proof: Join the OP radial line, which, when extended, intersects the vertical line (passing through Q) QL_2 at point S. Let the output at S be c_3. The production contour c_2 necessarily intersects the OS radial line at a point lower than S - e.g. at T, (by the assumption that in the non-free region the contours are negatively sloped.) We have:

$$OT < OS \text{ or } c_2 < c_3$$

By rule nine, APP.P = APP.S

$$APP.Q = c_2/OL_2 < c_3/OL_2 = APP.S$$

$$\text{So } APP.Q < APP.S = APP/P$$

$$\text{hence } APP.Q < APP.P \qquad QED$$

What was proved above can be stated in a form familiar to partial equilibrium analysis. In the non-free region, if we hold the quantity of land constant (at H) and successively add more and more labor, the average physical productivity of labor declines continuously. That this is a necessary condition for competitive equilibrium had been frequently pointed out in economic literature.[1/]

By Rule (9) above, the <u>general</u> statement of the first half of Rule (10) is proved, namely, the average physical productivity of labor declines when the input <u>ratio</u> is smaller. (e.g. the APP of any point on the OS radial line (since they are all equal) will be greater than the APP at Q --and greater than any APP at points with the same input ratio as point Q).

Similarly, the symmetrical case, for the average physical productivity of <u>land</u>, can be proved.

1/See e.g. Hicks, <u>Value and Capital</u>, page 81

Rule eleven: a) In the non-free region, the marginal physical productivity of labor increases when the input ratio is higher.

b) In the land-free region the marginal physical productivity of labor is constant and equal to average physical productivity of labor.

c) In the labor-free region, the marginal physical productivity of labor is zero. (The symmetrical cases for the marginal physical productivity of land can be similarly stated)

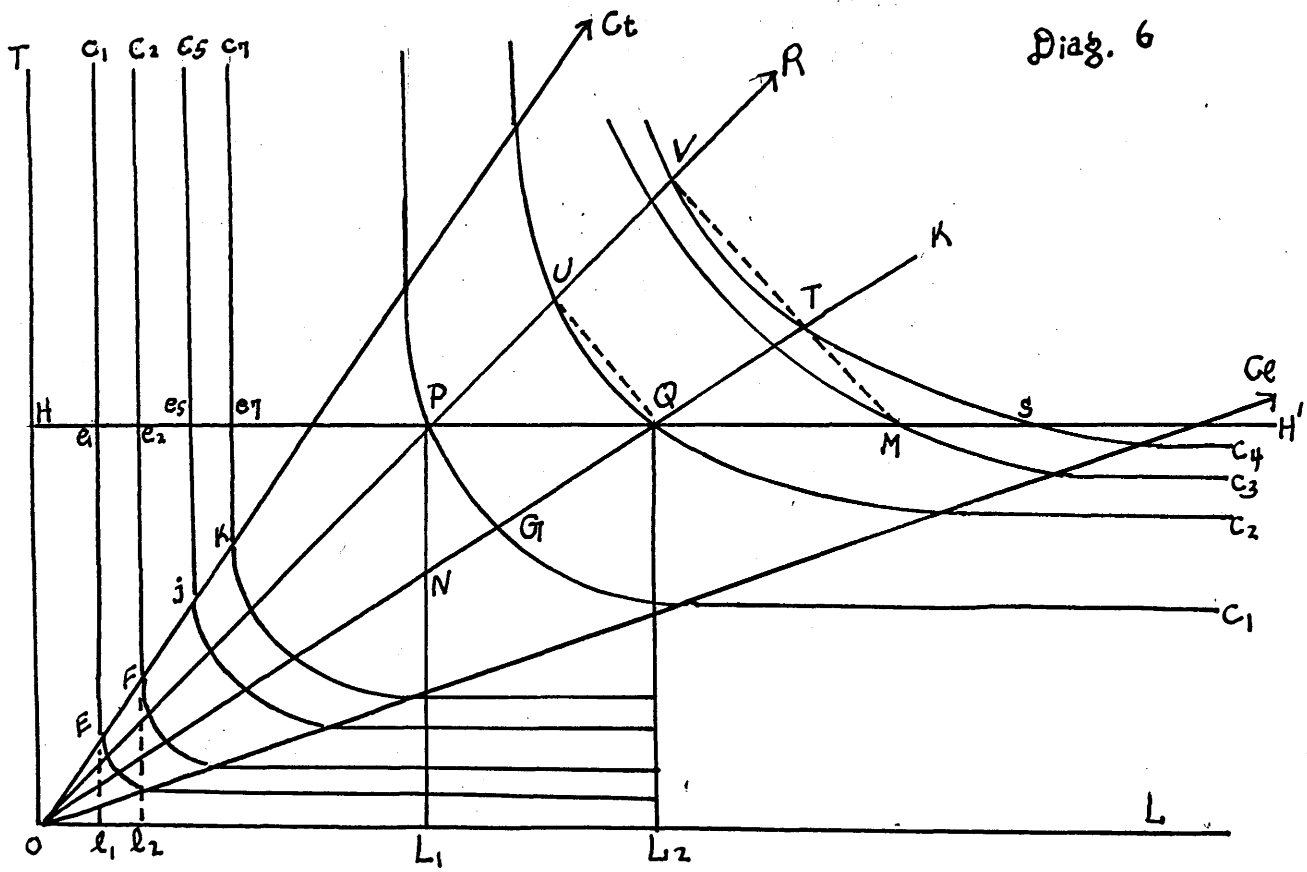
Marginal productivity is defined as the increment (or decrement) of total output, per unit of labor, when the increase (or decrease) of labor is small, holding the other factor constant. Part (c) of Rule (11) can be seen directly from, e.g. Diagram 4. In the labor-free region, since the production contours are horizontal lines, total output will not be affected when labor input alone is increased.

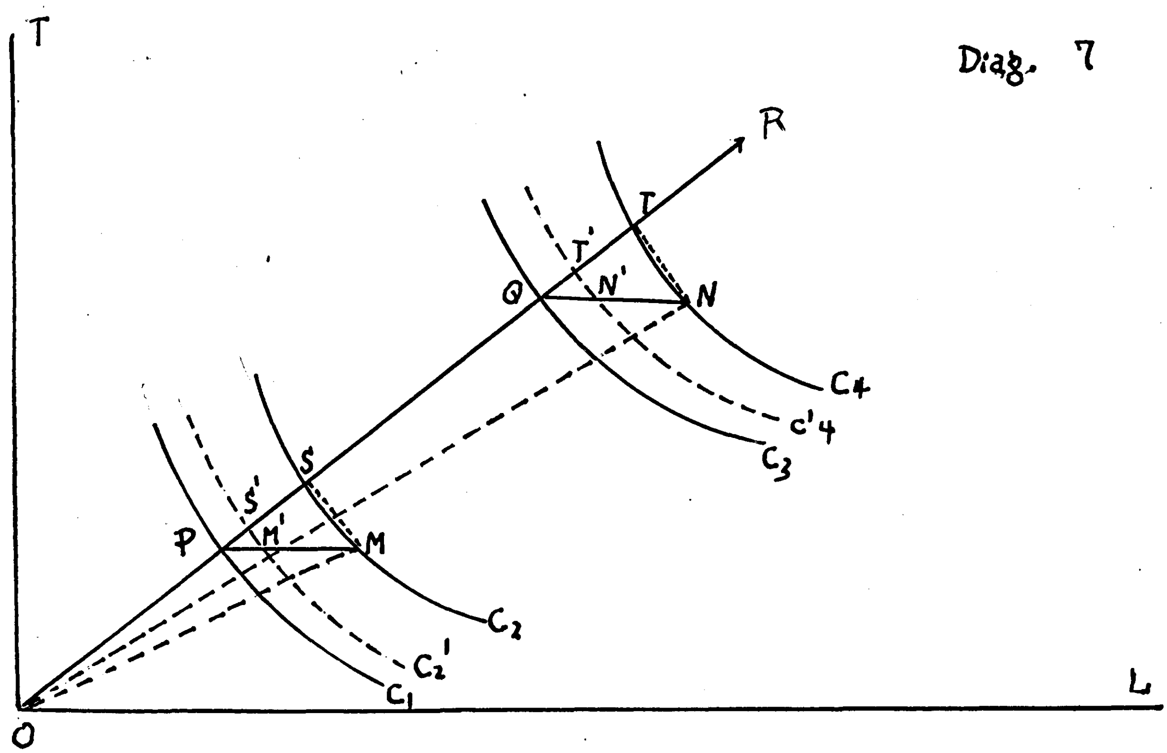
Part (b) can be proved as follows. In Diagram 6, let HH' be any horizontal line - i.e. the input of land is being held constant on this line. In the land-free region, take four points on HH', e1, e2, e5, and e7, such that the increment of labor from e1 to e2 equals to that from e5 to 47. Furthermore, define these increments as one unit of labor. Let the outputs at the four points be c_1, c_2, c_5 and c_7 respectively.

We have, $MPP_{e1} = c_2 - c_1$ and $MPP45 = c_7 - c_5$

Prove: $c_2 - c_1 = c_7 - c_5$

Proof: Let the four contours intersect Ct (the land-free ridge line) at E, F, J, and K respectively. Obviously, the distances EF and JK are equal - since the distances e1e2 and e547 are equal. So we have: $c_2 - c_1 = c_7 - c_5$........by rule (2) QED.


Diag. 6
T
C1
C2
C5
C7
Ct
R
K
V
U
T
Q
P
H
e1
e2
e5
e7
S
M
Cl
H'
C4
C3
C2
C1
G
N
K
j
F
F
E
O
l1
l2
L1
L2
L


Diag. 7
T
R
T
T'
N'
N
Q
C4
c'4
C3
S
S'
P
M'
M
C2
C2'
C1
O
L

It is our purpose to show next that in this region the <u>MPP</u> of labor equals the <u>APP</u> of labor, i.e. we want to

Prove: $APP._{e2} = c_2 - c_1$ (the righthand side of the equality is seen to be the MPP at e2)

Proof: Since the distance between l_1 and l_2 has been defined as one unit of labor, the input of labor at e2 is Ol_2/l_1l_2

So we have:

$$APP._{e2} = c_2/(Ol_2/l_1l_2)$$
$$= c_2 \times l_1l_2/Ol_2$$

$$\text{But } l_1l_2/Ol_2 = EF/OF = (OF-OE)/OF = 1 - OE/OF$$
$$= 1 - (c_1/c_2) \text{.......by rule one}$$
$$= (c_2 - c_1)/c_2$$

$$\text{So } APP._{e2} = c_2.(c_2 - c_1)/c_2$$
$$= c_2 - c_1$$
$$= MPP._{e2}$$

QED.

It is our purpose to prove, next, part (a) of Rule (11), namely, the marginal physical productivity of labor decreases as ratio of input decreases, in the non-free region. For this purpose, let us first prove a special case, namely, the case under which the quantity of land is held constant - HH' in Diagram 6.

Let P be any point in the non-free region, through which a horizontal line HH' and the radial line OR are drawn. From point P, mark off, successively, equal distances on OR, i.e. PU and UV. Let the output at P, U and V be c_1, c_2 and c_4 respectively. We have:

1) $c_2 - c_1 = c_4 - c_2$..........by Rule (1).

Let the production contours of c_2 and c_4 intersect HH' at points Q and S respectively. Through point Q draw the radial line OK intersecting the production contour c_4 at T. Point T is necessarily higher than point S.

Join the straight lines UQ and VT, we know:

UQ // VT........by rule (4)

Let VT intersect HH' at point M. Draw the production contour through point M, i.e. c_3. By the convexity property of the production contour (c_4) point M lies to the left of S. This given:

(2) $c_3 < c_4$

We have, PQ = QM......by the facts that PU = UV and UQ // VM

Let us define these distances as representing one unit of labor, so that MPP of labor at point P and point Q are $(c_2 - c_1)$ and $(c_3 - c_2)$ respectively. We want to

Prove: MPP.P > MPP.Q, or equivalently, $c_2 - c_1 > c_3 - c_2$

Prove: We have $c_2 - c_1 = c_4 - c_2$.......by (1) above.

$c_4 - c_2 > c_3 - c_2$.......by (2) above.

Hence $(c_2 - c_1) > (c_3 - c_2)$

QED.

What we have proved above is the familiar assertion in the partial equilibrium analysis that marginal physical productivity of labor decreases if successively more labor is applied on the same amount of land - or the so-called "law of diminishing returns". (The symmetrical case for the "diminishing returns" of land applied to a fixed quantity of labor can be similarly proved).[2]

The completion of the proof for part (a) of rule (11) depends upon the proof that, for any given input ratio, the marginal physical productivity of any factor of production is fixed - i.e. regardless of the size of inputs.[3] (If this is true, together with the partial proof given above, the marginal physical productivity of labor would be higher the higher the input ratio, which is what part (c) of rule (11) asserts.)

2/It is obvious that the "law of diminishing marginal return" (and "average return" as proved in Rule (1)) above) is derivable from the assumption of "imperfect substitutability" of the two factors of production and the assumption of constant returns to scale. (For instance, in the "free-regions" the law of diminishing returns does not hold for marginal and average product, because, in the free-regions the production contours are straight lines. The general case of "negatively sloped" "straight" production contours (i.e. the general case of perfect substitutability) under which the laws of diminishing returns do not hold if the assumption of constant returns to scale is assumed, can be similarly proved). Mrs. Joan Robinson stated (Economics of Imperfect Competition, Macmillan, 1948 page 330) "What the Law of Diminishing Returns really states is that here is a limit to the extent to which one factor of production can be substituted for another.....The Law of Diminishing Returns then follows from the definition of a factor of production and requires no further proof." It must be obvious that Mrs. Robinson was taking the assumption of "constant returns to scale" for granted - or else the statements are not true. (Prof. Hicks has commented on this point in Value and Capital, page 95, footnote 2).

3/This proof is given as rule (13) below.

Rule twelve: <u>In the non-free region, the marginal physical productivity of any factor is smaller than the average physical productivity of the same factor</u>.

In Diagram 6, define the distance between P and Q (i.e. L_1L_2) as one unit of labor (as we did in the proof of Rule (11)). Marginal physical productivity of labor at point P is $(c_2 - c_1)$. It is our purpose to

Prove: $APP.P > c_2 - c_1$.

Proof: The inputs of labor at point P are OL_1/L_1L_2 units. The average physical productivity of labor at P is then:

$$APP.P = c_1/(OL_1/L_1L_2)$$

$$= c_1 \cdot (L_1L_2/OL_1)$$

But $L_1L_2/OL_2 = NQ/ON > GQ/OG$

$$= \frac{c_2 - c_1}{c_1} \text{ ......by rules (1) and (2)}$$

We have $APP.P = c_1\ (L_1L_2/OL_1) > c_1\ (\frac{c_2 - c_1}{c_1}) = c_2 - c_1$

i.e. $APP.P > (c_2 - c_1)$

QED

This property is evident enough in the partial equilibrium analysis - i.e. when average physical productivity is falling the marginal physical productivity must be lower than the average physical productivity. This rule, together with the rules proved above (especially rule 10 and rule 11) enable us to plot the traditional "partial equilibrium diagram" (e.g. where total output is plotted against one variable input with the other input being held constant.[4/] We will have an occasion to use these "partial equilibrium diagrams" in our later analysis.

<u>Rule thirteen:</u> <u>When the input ratio is fixed, the marginal physical productivity of any factor of production is determined</u> - i.e. regardless of the size of the input.

The proofs for the two non-free regions have been conveniently demonstrated earlier.[5/] All we have to prove is for the non-free region.

In diagram 7, let OR be any radial line in the non-free region. Let P and Q be any two points on OR with outputs c_1 and c_3 respectively. Mark off, horizontally, from P and Q two equal distances PM and QN. Let the production contours passing through points M and N represent c_2 and c_4 units of output respectively. Let c_2 and c_4 intersect OR at points S and T respectively. Join the straight lines, SM, TN, ON and OM.

If we define the distances PM and QN as representing one unit of labor, $(c_2 - c_1)$ and $(c_4 - c_3)$ will be the marginal productivities of labor at points P and Q respectively. It is our purpose to prove that

$$MPP.P = MPP.Q \text{ or, equivalently, } (c_2 - c_1) = (c_4 - c_2)$$

4/See for example the article "On the Law of Variable Proportions" by Professor J. M. Cassels, reprinted in "Readings in the Theory of Income Distribution" page 103. The production contours there produced (page 110) emphasized the "disposal problem" which was neglected by the present writer, - i.e. some portions of the production contours are positively sloped.

5/See above part (b) and (c) of rule (11) on page 22

Proof: _If_ we know that PS = QT, then, by rule (2) above, we definitely know that $(c_2 - c_1)$ equals $(c_4 - c_3)$ and the theory is proved. (But PS does not equal to QT when the unit of input is large).

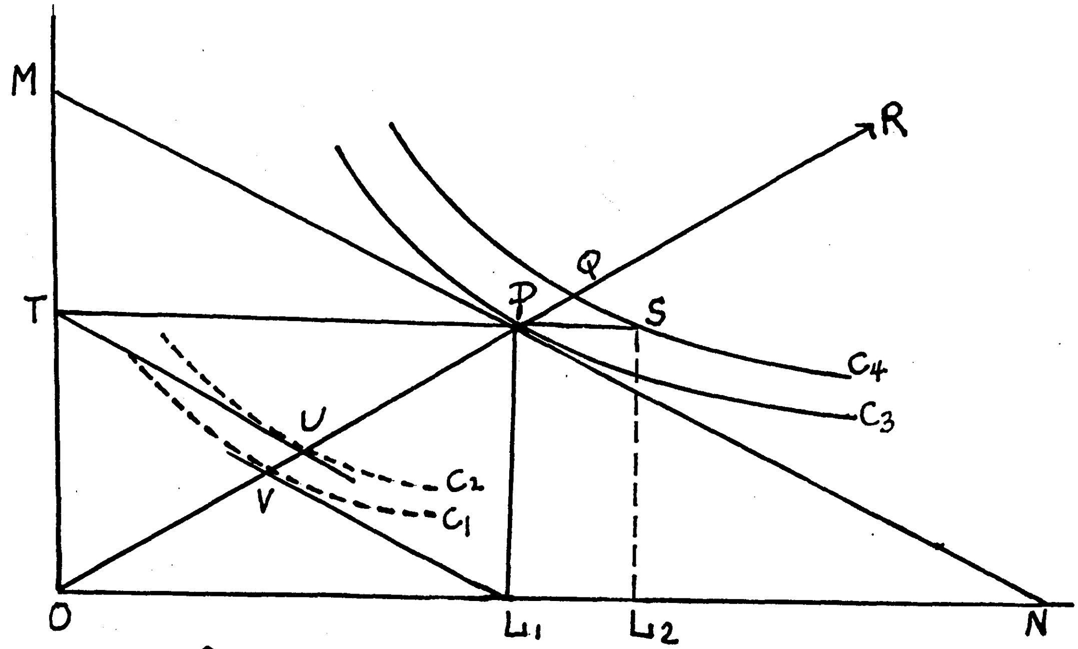
Consider the triangles PMS and QNT. These two triangles would be equal if SM and TN are parallel. (Since the angles SPM and TQM are equal; PM equals QN by construction).

Thus, when the factors of production are finely divisible - which is our assumption - we can let the increment of labor be small by redefining the unit (e.g. PM' and QN' are equal). Should this be done, the straight lines joining, e.g. S'M' and T'N' are approximately parallel by rule (4) above. (These straight lines are so close to the production contours that we cannot even show them separately in our diagram, although we can still show the increments of inputs and outputs quite "comfortably"). Hence, when the increments of the inputs are small, triangles PM'S' and QN'T' are approximately equal. PS' approximately equals QT' and the theory is approximately proved.

Similarly, the case for the equalization of the marginal physical productivity of land can be proved.

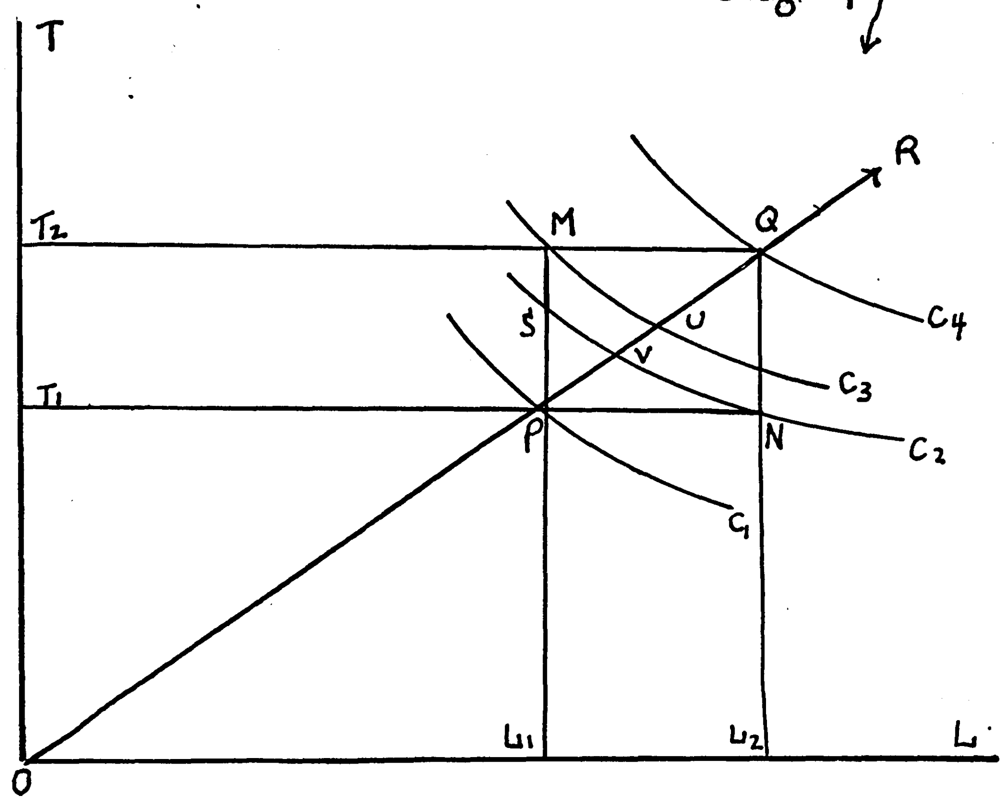
Rule fourteen: _The marginal physical productivity of labor times the number of units of labor plus the marginal physical productivity of land times the units of land equals total output_. (The Euler Theorem)

The proof for the non-free regions is again implied in our proofs earlier. What we need to prove is for the non-free region.



Diag. 8

Diag. 9



In diagram 8, let point P be any point in the map of production contours, let the production contour passing through point P be c_3. Let the quantities of inputs at P be OL units of labor and OT units of land.

Through point P draw the tangential line MN. Through points L and T draw LV and TU, respectively, parallel to MN, intersecting the radial line OP at points V and U. (By rule (5) above, we know that LV and TU are tangential to the production contours at points V and U.) Let the production contours passing through V and U be c_1 and c_2 respectively.

We know, first of all, that the triangles OVL and TUP are equal (TP equals OL; $\angle VOL = \angle UPT$; $\angle VLO = \angle UTP$). We have:

$$UP = OV$$

$$\text{and } OV + OU = OP$$

By rule (2) we know:

$$c_1 + c_2 = c_3$$

Hence, it is only necessary for us to prove that; at point P

(OL units of labor) x MPP_l = c_1.....(1)

(OT units of land) x MPP_t = c_2.....(2)

Let us prove (1). Let the distance LL' on the horizontal axis represent the increment of one unit of labor. Let the vertical line SL' intersect the horizontal line TP (extended) at S. Let the production contour passing through point S be c_4. It is then obvious that at point P:

$$MPP_l = c_4 - c_3 \text{.....(3)}$$

If LL' represents one unit of labor, we further know that the input of labor at point T is

$$OL/LL' \text{ units.}$$

So we have: OL units of labor x MPP_1 = $OL/LL' \times (c_4 - c_3)$.....(4)

Let c_4 intersect OR at point Q. Since we know that the

distance OP/c_3 marks off one unit of output

along the OR radial line, (by rule (2)).....(5)

so we have, by rule (2), c_4 units of output equal to $OQ/(OP/c_3)$ units of output. Hence, we know:

$$\begin{aligned} c_4 - c_3 &= (OQ/(OP/c_3) - c_3) \\ &= (OQ/OP - 1)c_3 \\ &= (OQ - OP)(c_3/OP) \\ &= (PQ/OP)c_3 \quad \text{substitute into (4)} \end{aligned}$$

we have: $(OL/LL')(PQ/OP)c_3$

$$= (OL/OP)(PQ/LL')c_3 \text{.....(6)}$$

But if the increment of labor is small, the portion of production contour SQ approaches a straight line parallel to MN (and LV), hence we know: (from the similar triangles PSQ and OLV)

$$PQ/LL' = OV/OL \quad \text{substitute in (6),}$$

we have: $(OL/OP)(OV/OL)c_3$

$$= OV/(OP/c_3)$$

By (5) above, we know that this last expression equals C_p units of output. Since this expression is derived from (4), we know (1) is proved. Similarly, (2) can be proved.

QED.

Rule (14) states that if all factors receive payment <u>in products</u> equal to the amount of their respective marginal physical products, the total output will be exactly exhausted.

<u>Rule fifteen</u>: <u>The marginal rate of substitution equals the ratio of marginal physical productivities of the two factors of production</u>.

In diagram 9, let c_2 intersect OR and PM at points V and S respectively; let c_3 intersect OR at U. The marginal rate of substitution at point V, along the c_2 contour is the ratio of PS units of land to PN units of labor. But if PN and PM are representing one unit of labor and land respectively, PS units of land is PS/PM units of land ("PS" refers to the geometric distance and PS/PM refers to the number of units when PM is defined as a unit). So the marginal rate of substitution is:

$$MRS. = \frac{PS/PM}{PN/PN} = \frac{PS/PM}{1} = PS/PM$$

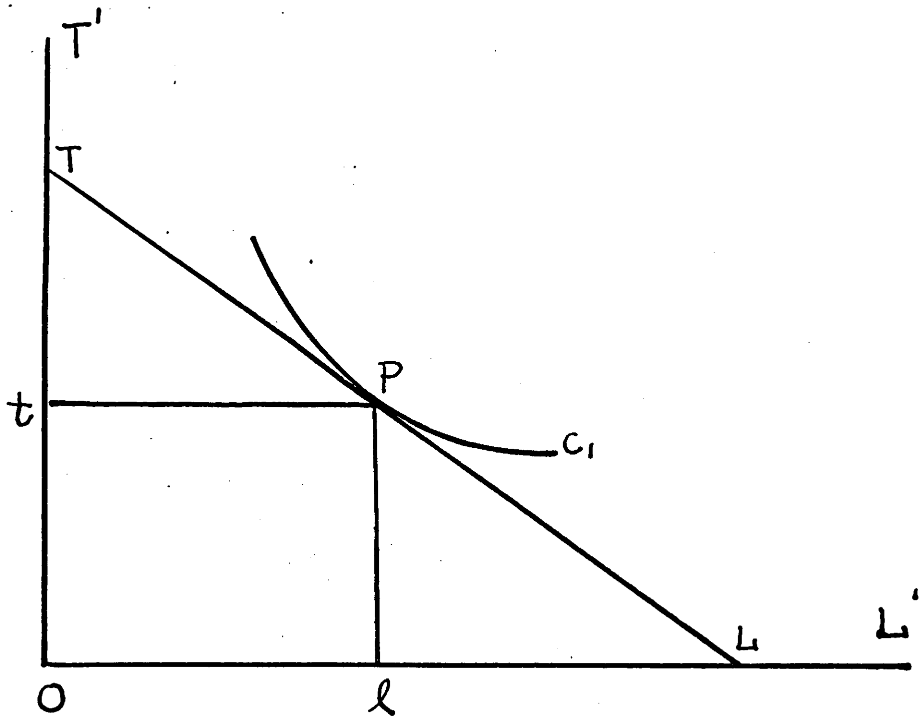
When the changes are small, the increments of output are proportional to the increments of inputs:

$$MRS = PS/PM \text{.......by rule (5) which states that the production contours are approximately parallel on OR.}$$
$$= PV/PU$$

$$= \frac{c_2 - c_1}{c_3 - c_1} \text{ ....by rule (1) and rule (2)}$$

$$= \frac{MPP.l}{MPP.t}$$

QED.



Diag. 9.5

This property of a production function enables us to represent the ratio of factor reward by the slope of the production contours; - which represents the marginal rate of substitution - _if_ the factors are paid respectively their marginal physical product.

Rule sixteen: _If all the factors are paid their respective marginal physical products, the exchange value of the total output, at any point in the map of production contour, in terms of the "wage unit" and "rent unit" respectively, can be represented, respectively by the X-intersect and Y-intersect of the straight line tangent to the production contour at this point._

In diagram 9.5 the exchange value of the total output equals the market value of Ol units labor plus Ot units of land - by rule (14) which states the "product exhaustion" property of the production function of constant returns to scale. Since Ot units of land equal in exchange value lL units of labor (by rule(15) which states that the ratio of factor rewards equals the slope of the production contours), the exchange value of total output (c_1) equals to Ol plus lL, or OL units of labor. (Similarly, we can prove that it equals OT units of land in exchange value). The distance OL (OT), then represents the total value of output, or the total income of factors, and the distances Ol and lL(tT, and Ot) represent the size of wage bill and rent bill, respectively, in terms of wage (rent) unit. This geometrical property of a production contour map will be very useful for our later analysis. Needless to say, this property is generally true only under the assumption of "constant returns to scale" and the assumption that factor reward, in terms of "product unit", equals the marginal physical product.

Chapter III.

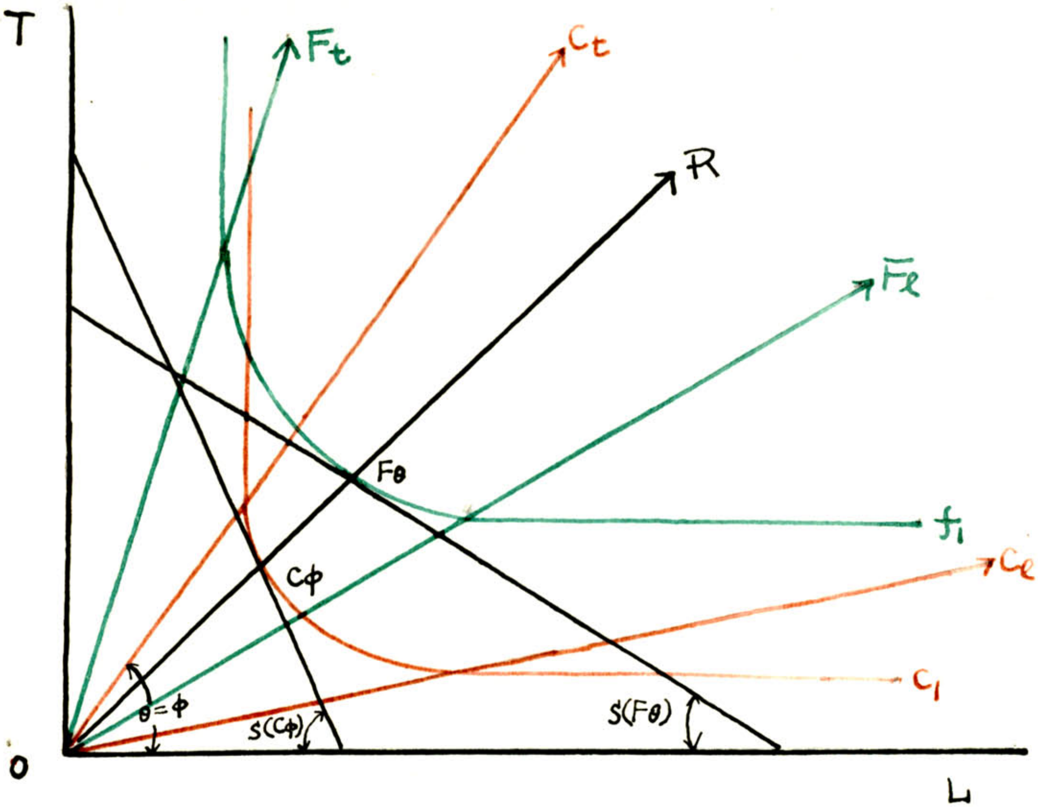
The Box Diagrams

Section one: Two Products

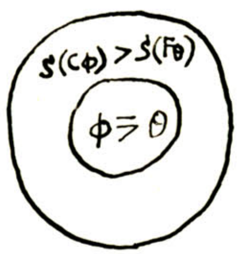
For an analysis of the problems of "exchange", naturally, we have to introduce into our analytical framework at least two commodities, for which two production functions (with constant returns to scale) must be postulated.

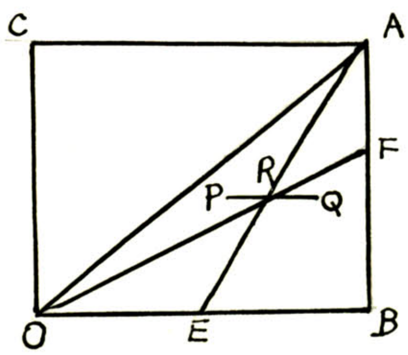
As we have mentioned above, the production functions define the productive aspect of the products and factors involved; when there are two or more products (and factors of production), the production functions, postulated for each of the commodities, jointly define the operational relationships that are assumed to exist among all the commodities and the factors of production, namely, as a group-relationships. The quantitative relationships between them necessarily become more complicated, so that in order to reduce the problem to a manageable extent, relative to our purpose of "diagrammatic representation", certain assumptions have to be made to simplify our problems.

Throughout this thesis, we will assume that there are two products, and for the production of each commodity the same two factors of production are required. Despite this drastic simplification, we shall discover in our later analysis that even under this simplified assumption the analysis of the problems of productive equilibrium of a static economy is a problem which cannot be satisfactorily handled by our method - i.e. diagrammatic method.



Diag. 10





<u>Section two</u>: <u>Relative factor intensities of the two commodities</u>.

We want to define the two commodities, which will be analyzed, in such a way that they are <u>different</u> from each other, as far as their productive aspects are concerned. For this purpose, the concept of relative factor intensities of the two commodities must be introduced. This can be most conveniently done with the aid of the maps of production contours of the two commodities.

Let us call the two commodities clothing (C) and food (F). The maps of production contours may be drawn. However, by rule (3) of the previous chapter, we can take <u>one</u> production contour from each map as representative of the whole map - since any one production contour may generate the whole map under the assumption of constant returns to scale. The two representative production contours, one from each map, are shown in diagram 10, where c_1 is the production contour for clothing and f_1 is the production contour for food.

The relative factor intensities of the two commodities may be operationally defined as follows:

> "Food is a relatively land intensive commodity if, at any input ratio (except in a factor-free region for <u>both</u> commodities), the marginal rate of substitution of the two factors is lower for the production of food than for the production of clothing.

Referring to diagram 10,[1/] and with the notations developed earlier

---

1/According to our verbal definition given above, the definition should be written:

$$S(F_\theta) < S(C_\phi) \text{ if } \theta \leqq \phi$$

The "inequality sign" can be placed in the "if-clause" by the convexity property of the production contours - i.e. by rule 8.

this definition may be stated more neatly in the following form:

Rule (16) $S(F_\theta) < S(C_\phi)$ if $\phi \gtreqless \theta$ and if $F_1 < \theta < F_t$

or if $C_1 < \phi < C_t$

where F_1, F_t, C_1 and C_t represent the input ratios at the labor-free and land-free ridge lines for the two commodities.

It is further obvious that, by the definition of the relative factor intensities of the two commodities, we have:

Rule (18) $Ft > Ct$

and $Fl > Cl$

This means, for example, that the input ratio corresponding to the land free ridge line for food (Ft) must be greater than that corresponding to the land-free ridge line for clothing (Ct). Otherwise, the definition of relative factor intensities is contradicted - see diagram 10.

There are several intuitive explanations of this definition of relative factor intensities. Food is land intensive if, at the same input ratio, it takes a <u>smaller</u> quantity of land to substitute the same amount of labor, as compared with the production of clothing. In other words land is more important for the production of food than for clothing - vice versa for labor. Another way to realize the significance of this definition (intuitively) is to draw a number of production contours for each product in the same diagram. Then, it will be seen that the production contours for food have lower slope at any given point than those for clothing. This makes it likely that an addition of labor alone will affect the total output of clothing more readily than that of food (and vice versa for a net increase of land). Still another way of looking at the definition is in terms of rule (18) above, which states that land becomes a redundant (free) agent at a higher input ratio (more land per unit of labor) for the production of food than for clothing.

Several miscellaneous observations may be made with respect to this definition:

a) Our definition is unambiguous by rule (5) of production functions of constant returns to scale, which states that the marginal rate of substitution is fixed for any input ratio. (In other words, this definition is "possible" because of the assumption of constant returns to scale).

b) By the concavity properties of the production contours, this definition may be restated in the following form: Rule (16.5) $\theta > \phi$ if $S(F_\theta) \gtrless S(C_\phi)$.[2/] This form will be relevant to our analysis in certain cases below.

c) This definition always holds as defined. Specifically we do not allow the case under which the marginal rates of substitution, at certain input ratios, are higher for food than for clothing. In other words, we assume that food is <u>always</u> relatively land intensive.

2/Subject to the same qualifying conditions (if $F_l < \theta < F_t$ or if $C_l < \phi < C_t$) as in rule (16) on page 39. That rule (16.5) is valid can be readily seen from the logical diagram of rule (16) accompanying diagram 10.

d) We can easily and comfortably give such a definition of relative factor intensities for the case of two factors and two products. When there are more factors and products, the formulation of any such qualitative conditions of the production functions is not readily treated by diagrammatical method. Consequently we want to realize and emphasize the severe limitations that have been imposed upon the scope of content of this thesis which is nothing more than "illustrative" of certain theoretical problems in static theory.

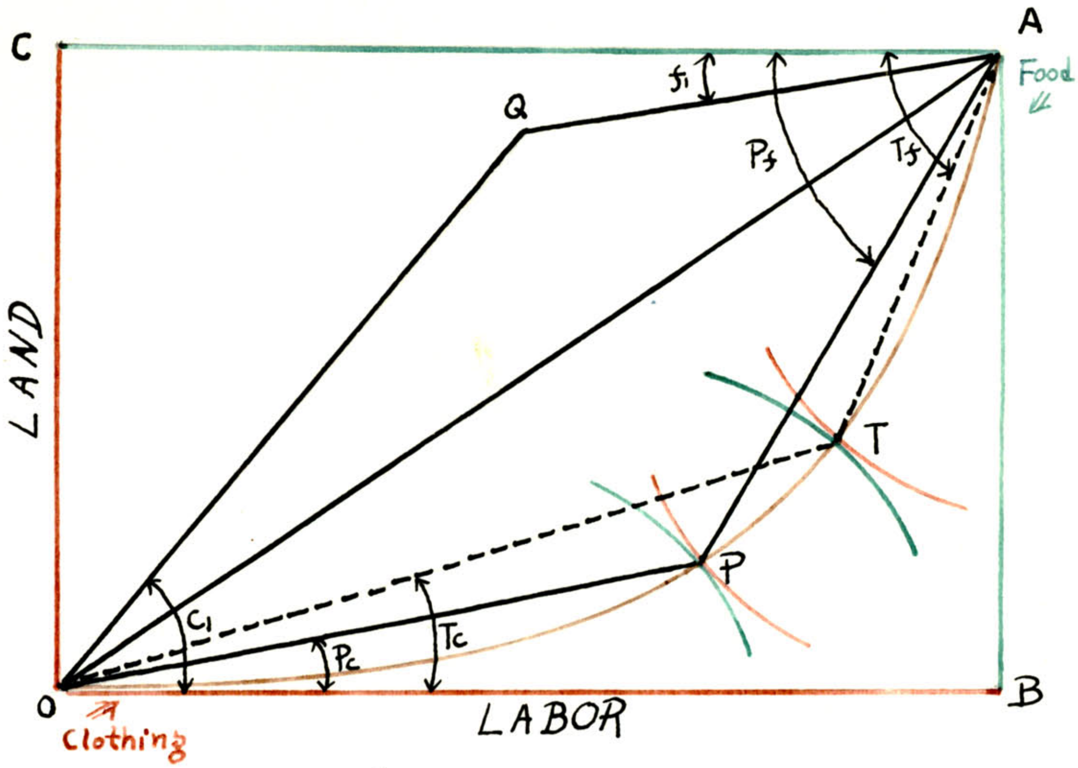
Section three: The box diagram

When the endowments of the two factors of production for a particular "economy" are known, a box diagram may be constructed to show the optimum patterns of allocation of resources for the production of the two commodities. Let us first construct such a box diagram and then briefly explain the relevance of the box diagram to the analysis of the operation of the "economy" at the end of the present section. Throughout the present chapter we shall neglect the ridge lines of the two maps of production contours.[3/]

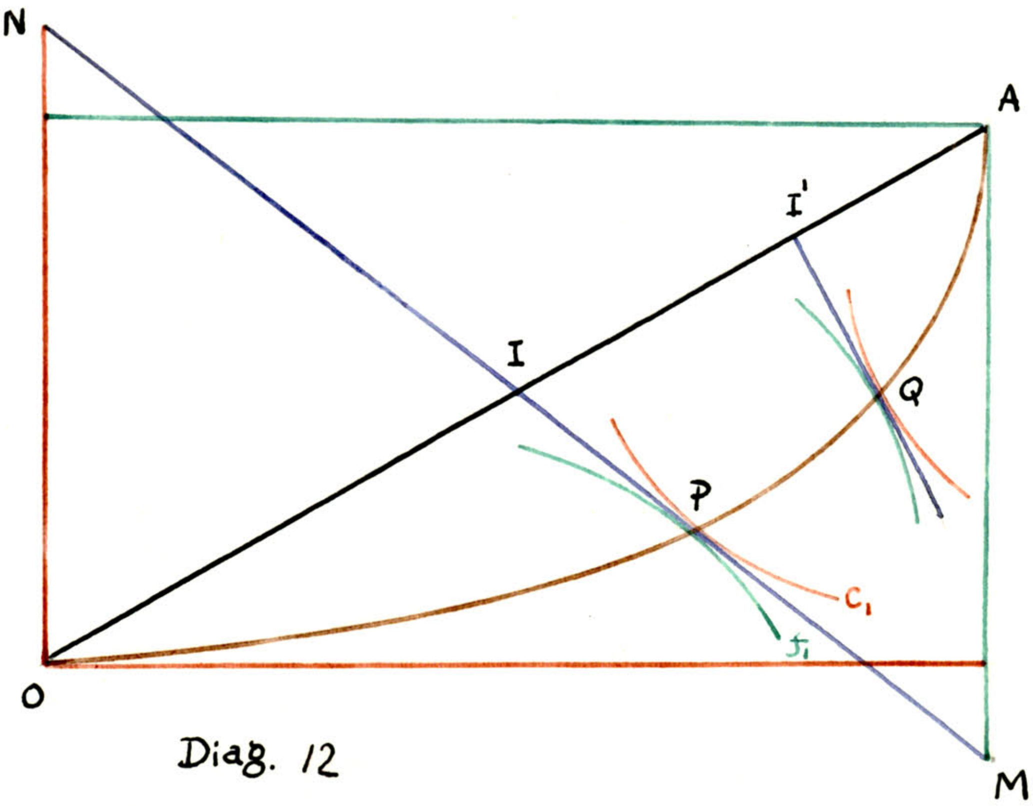
In diagram 11, let the contour map for clothing first be drawn, taking the lower-left corner as the origin.[4/] The horizontal and vertical

3/That is to say, we assume that the non-free region coincides with the entire map. The general case, under which this is not true, will be more conveniently discussed in a later chapter.

4/This convention will be adopted throughout the thesis, i.e. the lower-left corner of a box will always be taken as the origin for the clothing-map, the upper-right corner for the origin of food-map.



Diag. 11



Diag. 12

distances of point A to point O, can be taken to represent the endowments of land and labor respectively for a given economy A (which will be called a "country A"). Point "A" can then be taken as the origin of the food map - plotted up side down. A box, OBAC is obtained.

The locus of the points of tangencies of the contours - belonging to the different maps, e.g. points P and T, will be called the curve of optimum allocation of resources, or, more simply, the "optimum-allocation curve" for country A, (i.e. curve OPA in diagram 11). This name is proper, since, away from this curve, it is always possible to increase the production of both commodities by re-allocating factors of production in such a way as to move toward the curve. Expressed differently, any point on the OA curve represents an optimum-allocation pattern in the sense that when the output of one commodity is predetermined, the output of the other commodity is the "maximum" at any point on the curve, relative to the given endowments of resources.

When nothing is known of the conditions of operation of the economy - except the given endowments of resources which must be used and the technology of production (represented by the production functions) - the OA curve, as its name implies, represents the ideal pattern of allocation of the resources (from the viewpoint of "production efficiency"). It obviously represents the ideal ways of production (of the two commodities) under any economic institution in which the economizing of the productive resources is a social goal. In other words, the OA curve furnishes a criterion on which the productive efficiency of an economy may be judged (or the productive efficiencies of different economic systems may be compared) - e.g. by investigating whether the "actual" performance of an economy will likely be established at a point on the OA curve given the "institutional set-up". The optimum allocation of resources curve, then, is something belonging to the sphere of study of "welfare economics".

A competitive capitalistic system can claim its superiority on this account - i.e. the "rule of operation" of a competitive system, when fully adhered to, will likely bring about the ideal production efficiency pictured above. The assertion, however, constitutes the analysis of the operation of the competitive system. Under the assumption of static competition, full employment of resources will always be established. Equilibrium will then be established at some point in the box —representing full employment. If the "equilibrium position" is not established at a point on the optimum allocation-curve OA, the ratios of rewards of the two factors of production (represented by the slope of the production contours) will be different for the two industries. Some factors of production are apparently not satisfied - reconcentrating will occur to bring the equilibrium position to a point on the optimum allocation curve.

Section four: The geometric properties of the Optimum-Allocation Curves

There are certain geometrical properties of an optimum allocation curve which may be pointed out. The following properties are stated in terms of the conventions already laid down - on the choosing of the axes (for the factors of production) and the origins (for the outputs). (It must also be remembered that the assumption of constant returns to scale and the assumption of the relative factor intensities are retained throughout the thesis.)

Rule nineteen: The optimum allocation curve (OA) can only lie below the diagonal line (Straight line OA)

In diagram 11, this property can be formally proved as follows: Let point Q be a point which lies on or above the diagonal OA. Join OQ and AQ making the angles QOB and QAC (which are designated by c_1 and f_1 respectively in diagram 11). It is obvious:

$$c_1 \geqslant f_1$$

So by rule (16), we have:

$$S(Qc_1) > S(Qf_1)$$

Hence, point Q cannot be a point on the optimum allocation curve if it lies on or above the OA diagonal - for the two production contours do not have the same slope, as shown.

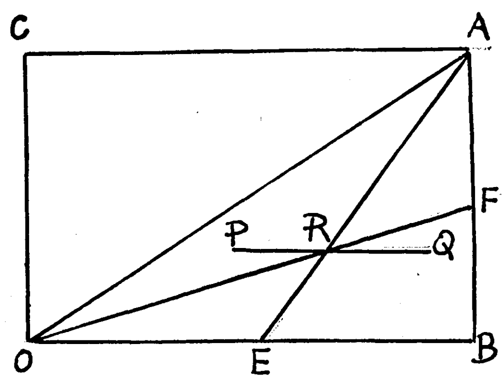
QED

The economic interpretation of Rule (19) is: when equilibrium is established, the input ratio for the production of clothing is necessarily lower than the input ratio for the production of food. This is ensured by the definition of the relative factor-intensities of the two commodities as was implied in the proof.

Rule twenty: The straight line joining any point on the optimum-allocation curve with either of the two origins (A or O) cannot intersect the optimum allocation curve - (i.e. a straight line passing through either origin can intersect the OA curve at most once - not counting the origin.)

This property makes it so that the slope of the optimum-allocation curve must be positive - i.e. OA runs from the lower left origin upward to the upper-right origin.[5/] Since the slopes of the production contours are non-negative, this implies that a movement along the optimum allocation curve upward indicates an increase of the production of clothing and decrease of food, vice versa.

The proof for this property can be more conveniently carried out in the course of the proof for the following property.



5/This can be readily proved as follows: Let PQ be a horizontal, vertical, or a negatively sloped portion of the optimum allocation curve. Take a point on PQ, namely point R. Join OR and AR, extended to F and E. Point Q necessarily lies in the enclosed boundary of the rectangular REBF. Since the optimum allocation curve necessarily passes through the origin, so, no matter to which origin (A or O) the optimum allocation is drawn from point Q, the curve will have to intersect either RF or RE. This contradicts rule (20), hence the optimum allocation curve cannot be horizontal or negatively sloped.

This property of the optimum allocation curve - i.e. positive slope - can be used to prove Rule (21) below. Hence, it is seen, rule (20) and rule (21) mutually imply each other.

<u>Rule twenty-one:</u> <u>A movement upward along the optimum-allocation curve toward point "A" indicates increased input ratios for both commodities</u>. (i.e. higher points on OA represent higher land-labor ratio according to our convention). <u>And hence the factor-price ratio will be higher too</u>.

In diagram 11, let P and T represent two points on the optimum allocation curve OA, such that T is higher than P. Join the straight lines OP, AP, OT and AT. Let the input ratios at P and T be P_c, P_f, T_c and T_f (as indicated by the subscripts). Let the production contours passing through point P and T be drawn.

We have: $S(P_c) = S(P_f)$

and $S(T_c) = S(T_f)$....by the property of production contour.

By Rule (8) we have

$$S(P_c) \lesseqgtr S(T_c) \text{ if and only if } P_c \lesseqgtr T_c$$

$$S(T_f) \lesseqgtr S(P_f) \text{ if and only if } T_f \lesseqgtr P_f$$

Combining these equalities and inequalities we have:

$$S(T_f) = S(T_c) \gtreqless S(P_c) = S(P_f) \gtreqless S(T_f) \text{ if, and only if,}$$

$$T_c \gtreqless P_c \text{ and } P_f \gtreqless T_f$$

(the equality and inequalities signs follow order).

We have: if $T_c = P_c$ then $P_f = T_f$ (otherwise there is a contradiction.)

This proves rule (20) above - i.e. "any straight line passing through one of the origins (O or A) and a point on the OA curve cannot intersect OA curve again".

From the same conditional equality, and inequality, we derive:

when $T_c > P_c$, then $T_f > P_f$

This proves rule (21) namely, the input ratios for the two commodities always change in the same direction as equilibrium position changes from one point on the optimum allocation curve to another.

As will be evident in our later analysis, this property of the optimum allocation curve proves to be very important.

Since we know, by rule (20) above, that the slope of the optimum allocation curve is positive, and that a movement upward along the optimum allocation curve indicates more output of clothing, rule (21) implies the fact that as more outputs of clothing are produced the input ratios are higher in the production of both commodities, and the factor price ratio also becomes higher (by rule 8).

Section five: The exchange value of total outputs and the product price ratio.

As will be evident in our later analysis, it is highly desirable if we can find a geometrical expression for the product-price ratios in the box diagram. For this purpose let us first establish the following rules for a box diagram:

Rule twenty-two: The ratio of the exchange values of the total outputs of the two commodities (i.e. value of clothing divided by value of food) at any point of equilibrium on the optimum allocation curve can be represented by the ratios of the distances along the main diagonal of the box, respectively from the two origins, to the point on the main diagonal intersected by a straight line tangential to the production contours passing through that point of equilibrium.

In diagram 12, let P be a point on OA curve. Through point P draw a straight line MN tangential to the production contours c_1 and f_1 passing through (and tangential at) point P. Let MN intersect the main diagonal AO at point I. Let the exchange value of c_1 units of clothing to f_1 units of food be denoted by Ex. It is our purpose to prove that

$$Ex = OI/IA$$

Proof: Let MN intersect the vertical sides (extended) of the box at points M and N. The exchange value of c_1 units of clothing, in terms of "rent unit" is ON (by rule 16). Similarly, the exchange value of f_1 units of food, in terms of rent unit, is AM. Hence,

$$Ex = ON/AM$$

By the similar triangles, OIN and AIN we have:

$$Ex = ON/AM$$

$$= OI/IA$$

QED

We are only one step removed from the derivation of a geometrical expression of the product-price ratios in the box diagram. If c_1 and f_1 are defined respectively as one unit of clothing and food, then OI/IA represents the product price ratio, at point Pl.

We know, in any case (i.e. regardless of the definition of units), the geometrical expression OI/IA represents the ratio of values of output by "industrial sectors", this understanding will be helpful to our later analysis of the Ricardian rent theory. It may be called the "industrialization ratio".

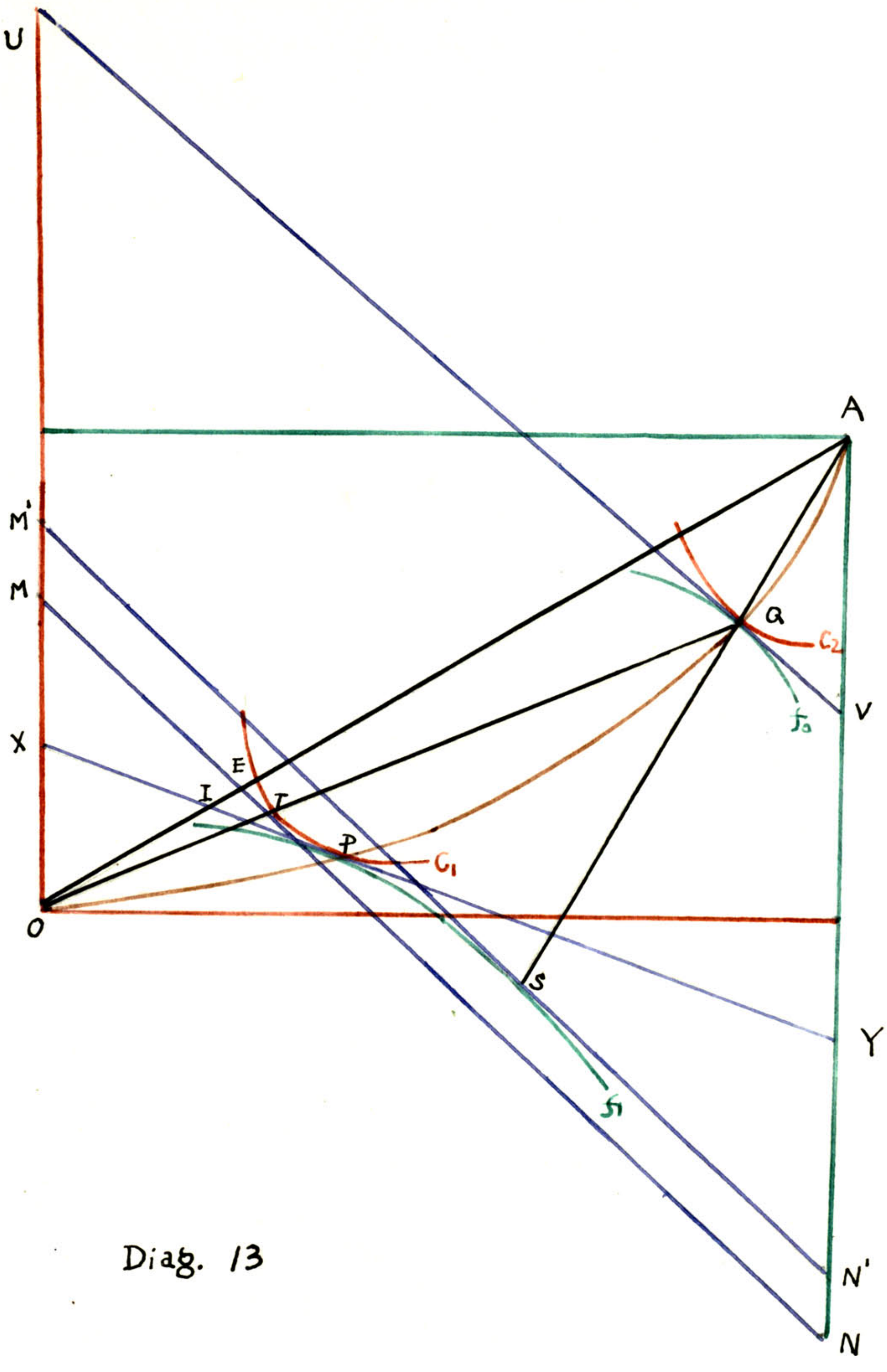
From diagram 12 it is easily seen that the industrialization ratio is higher at a higher point on the optimum allocation curve - e.g. point Q is higher than P and $OI'/I'A > OI/IA$. This is because of the fact that point Q is higher than P <u>and</u> QI' is steeper than PI (by rule 21) - the economic interpretation of these two effects (causing higher industrialization ratio) are obvious: not only more clothing is produced but the price of clothing will be higher, as will be proved immediately.

<u>Rule twenty-three</u>: <u>A movement along the optimum-allocation curve upward toward point A indicates an increase of product price ratio</u> (i.e. higher ratio of "price of clothing" to "price of food" by our convention).

This proposition is intuitively quite obvious - for we know that, e.g. a higher price ratio must be established, in favor of clothing, to <u>call forth</u> an increasing supply (i.e. production) of clothing, vice versa. It is our purpose to prove this intuitively obvious fact by the diagrammatic method.

In diagram 13, let a box with optimum-allocation curve OA be constructed. Let the outputs at point P, on OA curve, be c_1 units of food and f_1 units of clothing. Let point Q be another point on OA with outputs c_2 and f_o. By rule (20), on page 45, we know that the output (ratio) of clothing is higher at point Q than at point P.

Join the radial lines OQ and AQ. Let OQ intersect the production contour c_1 at point T; and let AQ (extended) intersect the production f_1 at point S. Through points Q, S, and T draw the tangential lines UV, M'N' and MN respectively, intersecting the vertical axes at the points



Diag. 13

indicated. Let MN intersect OA diagonal at point E. Through point P draw the tangential line XY, intersecting the OA diagonal at point I. It is immediately obvious, by rule (8) that:

UV // MN // M'N' and all these straight lines are steeper than XY.......(1)

Let us prove the following (two) propositions:

a) OI smaller than OE

b) AN' smaller than AN.

Proposition (a) is true because of the facts that point T lies between IY and IA [6/] and that MN is steeper than XY (by (1) above). Proposition (b) is true because of the facts that point P lies above MN and below M'N' (by the convexities of the production contours).

If we define c_1 and f_1 as one unit of clothing and food respectively, we know, by rule (22), that

product price ratio at P is OI/IA.......(2)

We want to derive a geometrical expression for the product price ratio at point Q. First, we know that the units of outputs at Q are c_2/c_1 for clothing and f_0/f_1 for food, if c_1 and f_1 are defined one unit of the two commodities respectively. By rule (1) and by (1) above, we can derive readily the geometrical expressions for the units of outputs:

Units of output of clothing at Q: c_2/c_1 = OQ/OT = OU/OM...(3)

Units of output of food at Q: f_0/f_1 = AQ/AS = AV/AN'...(4)

Since the total values of output, in terms of rent unit, at point Q, are OU for clothing and AV for food (rule 16), we derive, immediately the following equalities for point Q:

6/This is true because OQ is less steep than OA diagonal by rule (19) and the production contour c_1 lies above XY by construction.

From (3) price of clothing in rent unit:

$$OU/(OU/OM) = OM$$

From (4) price of food in rent unit:

$$AV/(AV/AN') = AN'$$

This gives:

Product-price ratio at point Q: OM/AN'

$> OM/AN$....by (a)

$= OE/EA$....by (1)

OI/IA....by (b)

The last expression is seen to be the product price ratio at point P (by (2) above). Hence rule (23) is proved.

QED.

<u>Section six</u>: <u>The Social Distribution Ratio</u>

In anticipation of our later analysis - especially on the Ricardian rent theory and the related issue - it is desirable if we can find a geometrical expression for the "social distribution ratio" in the box diagram.

The "social distribution ratio" may be defined as the ratio of the income of the owners of "labor factor" to the income of the owners of the "land factor". In other words, it is the total <u>wage bill</u> divided by the total <u>rent bill</u>. For this purpose, let us establish the following rule:

Rule twenty-four: The social distribution ratio (Sr) corresponding to any point on the optimum-allocation curve can be diagrammatically represented by the ratio of the "vertical distance" between the two points on the "vertical axes" intersected by a straight line tangential to the (two) production contours passing through the point of equilibrium to the distance of the "vertical" side of the box.

(The inverse of the social distribution ratio - i.e. 1/Sr - can be represented diagrammatically by the ratio of the distances as described above except for "vertical" the expression "horizontal" may be substituted)

In diagram 14, let OBAO be the box. Let P be any point on the optimum-allocation curveOA. Let MN be the straight line passing through point P and tangential to the production contours passing through point P. Let MN intersect the two vertical axes at points N and M (and let it intersect the two horizontal axes at points U and V).

Through point N draw a horizontal line NH intersecting the vertical axis on the opposite side at point H. (Through point U draw a vertical line UI intersect the horizontal axis on the opposite side at point I.)

The vertical distance between the two points (M and N) intersected by the tangential line MN (or UV) on the vertical axes is HM. (The horizontal distance between the two points (U and V) intersected by the tangential line UV (or MN) on the horizontal axes is IV.)

It is obvious that: HM/AO = OO/IV [7/]

7/The triangles NHM and UIV are similar. This gives: $\frac{HM}{HN} = \frac{UI}{IV}$. But HN = AO and OO = UI. Substitute these values in the equality and the result is shown.

This gives: HM/OC = AO/IV

It is our purpose to prove that these expressions may be used to represent the social distribution ratio (Sr). (See the statement of rule (24) above)

Prove: Sr = HM/OC

Proof: Through point P draw the horizontal line JK intersecting the vertical axes at points J and K.

Through P draw the vertical line ST intersecting the horizontal axes at points S and T.

By rule (16) the total value of the output of clothing, in terms of "rent unit" is ON, which is divided to the share of wage bill (JN) and rent bill (OJ). Similarly, the value of the total output of food, in terms of the rent unit, is AM - divided into the wage bill of KM and rent bill of AK. Hence, we have:

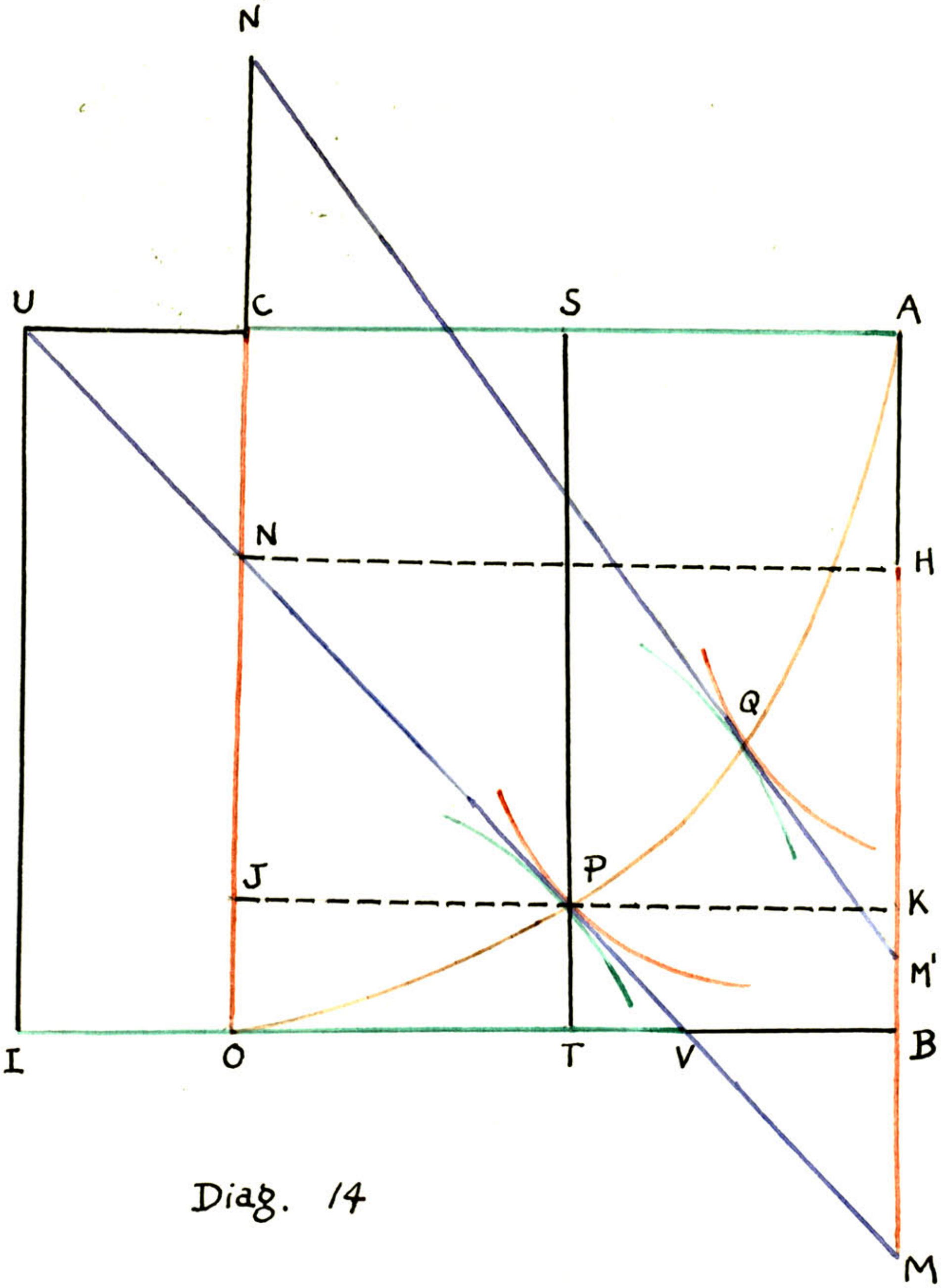
$$\text{Total wage bill} = JN + KM$$
$$= HK + KM$$
$$= HM$$

$$\text{Total rent bill} = OJ + AK$$
$$= OJ + JC$$
$$= OC$$

So we have: $$Sr = \frac{\text{total wage bill}}{\text{total rent bill}} = \frac{HM}{OC}$$

QED.

(The direct proof of Sr = AO/IV makes use of the "wage units" into which all values are converted).



Diag. 14

We can make certain applications of this rule (24). First of all it can be equally proved that the social distribution ratio is higher - i.e. "labor as whole better off relative to land" - when more clothing (the labor-intensive commodity) is produced. This intuitively obvious fact can be unambiguously proved as follows:

In diagram 14, let point Q be a higher point on OA curve than P. By rule (20) above, point Q represents more output of clothing than P. The tangential line passing through Q (i.e. M'N') is steeper than MN, the vertical distance of the two points (M' and N') intersected by M'N' on the vertical axes must be longer (i.e. H'M' > HM). The fact alone is sufficient to prove that the social distribution ratio at point Q is greater than that at point P, by rule (24), since the distance OC is unchanged. As a corollary, we can generalize our findings in this and the previous sections into the following convenient rule:

Rule twenty-five: The "output-ratio", "input ratios in the production of both commodities", the "marginal rate of substitution (the factor-price ratio)", the "product price ratio", the "industrialization ratio" and the "social distribution ratio" increase and decrease together - represented, diagrammatically - by moving along the OA curve upward and downward, respectively.

Chapter IV.

Factor Price Equalization and Specialization Status

In the previous chapters we have derived a number of "rules" for production functions with constant returns to scales, for the definition of the relative factor intensities of the two commodities (food and clothing), and for the optimum allocation curves in the box diagrams. In the present chapter we shall make use of these results for the analysis of certain problems in international trade theory.

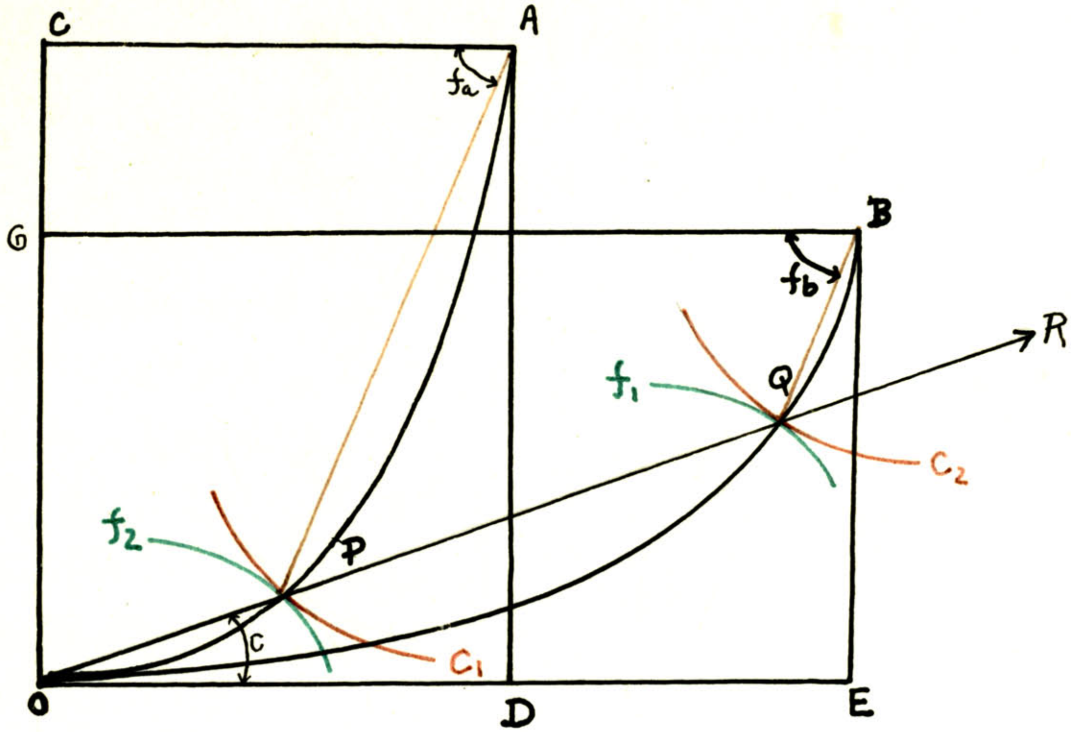
Section one: Equalization of factor prices in International trade equilibrium

The necessity of a complete equalization of factor prices between trading countries under certain equilibrium conditions (which will be more precisely stated later) has been proved by Professor Samuelson.[1/] In this section we prove the same theory by diagrammatic method.

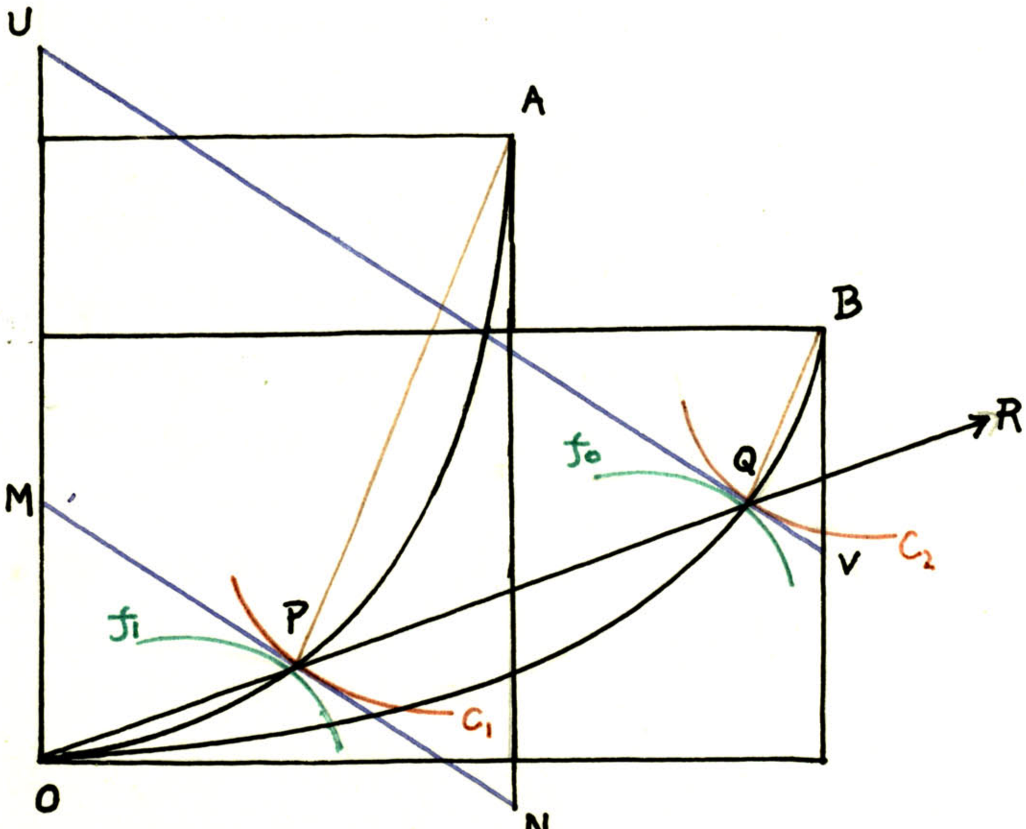
Professor Samuelson made the following assumptions in his proof:

1. two countries with given factor endowments
2. two factors of production - land (T) and labor (L)
3. two outputs with different factor intensities - food (F) and clothing (C).
4. production functions with constant returns to scale.
5. identical techniques of production of both commodities for the two countries.

1/P. A. Samuelson, "Factor Price Equilization Once Again". Economic Journal, June, 1949.



Diag. 15



Diag 16

6. free trade
7. perfect competition in a static world
8. no transportation cost or other trade barriers
9. complete factor mobility within each country; complete factor immobility between countries.

All these assumptions will be maintained for the time being. It is immediately seen that our preparations in the previous chapters are directly applicable to a problem limited by this list of assumptions.

When the quantities of factor endowments for the two countries are given, we can construct two box diagrams, one for each country, and obtain two optimum allocation curves. However, in diagram 15, the two boxes are superimposed on each other in such a way that the contour systems, for the production of clothing, of the two countries coincide. There are three contour-maps in this diagram: a map for the production of clothing for both countries (which takes the lower left corner as the "common origin") and two maps of production contours for food, one for each country, which take, respectively, the upper-right corners of the two boxes as the origins. The two countries may be labeled country A and country B; the quantities of factor endowments for the two countries are seen to be OD and OE units of labor and OC and OG units of land for countries A and B respectively. The fact the contour systems for the production of clothing coincide is ensured by the assumption that the techniques of production are the same for the two countries.

Two optimum-allocation curves can be derived - i.e. curves OA and OB. Through point O draw a radial line OR intersecting the two optimum allocation curves at point P (on OA) and Q (on OB). Join the straight lines PA and QB. It is our purpose to prove, first of all, that PA and QB are parallel.

Let the production contours passing through point P and Q be c1, c2, f1 and f2 respectively for the outputs of clothing and food for countries A and B. (See diagram 15) Let the angle ROE be denoted by "c" and let the angles PAC and QBG be denoted by "fa" and "fb" respectively. Then, with the notations we have developed earlier, we have:

$S(P_c) = S(P_{fa})$

$S(Q_c) = S(Q_{fb})$.....by construction, point P and Q are points of tangency.

but $S(P_c) = S(Q_c)$.....by rule (8) and by construction.

hence $S(P_{fa}) = S(Q_{fb})$

We have fa = fb........by rule (8) above.

hence PA $\parallel$ QB

QED.

This property of a "box diagram" involving two countries is so important for our development that we want to call it rule (25) which reads:

Rule twenty-five: In a box diagram of two countries, the straight lines joining the two origins of food maps, respectively, with the two points on the optimum allocation curves intersected by the same radial line from the common origin of clothing map are parallel.

Whenever a pair of points, one on each optimum allocation curve, is obtained as the points of intersection by any OR radial line from the origin for clothing, we will say, by rule (25) that the pair of points satisfy "the parallel relationship".

If a pair of points satisfies the parallel relationship, we know that the factor price ratios of the two countries are completely equalized —if the pair represents the equilibrium position of the two countries. That this is true is implied in our derivation of Rule (25) above —the input ratios for the production of the same commodity by different countries are the same (for food and for clothing) by Rule (8), the marginal rates of substitution (or factor price ratios) must be equalized between the two countries. This observation may be generalized as rule (26) which reads:

Rule twenty-six: If a pair of points satisfies "the parallel relationship" the input ratios for the production of the same commodity by different countries are the same; the marginal rates of substitution at the two points are equalized, and consequently the factor price ratios are equalized between the two countries if the pair of point represents equilibrium positions.

It is our purpose to prove, next, the following property of a box diagram involving two countries:

Rule twenty-seven: If the equilibrium positions of the two countries are established at a pair of points satisfying the "parallel relationship" then the product-price ratios are completely equalized between the two countries.

Diagram 16 is a reproduction of diagram 15. Let the radial line OR intersect the optimum allocation curves at points P and Q respectively. Let the production contours passing through these points be c_1, c_2, f_1 and f_2. Through points P and Q draw the tangential lines MN and UV, intersecting the vertical axes at points M, N and U, V respectively for the two countries.

If c_1 and f_1 are defined as <u>one unit</u> of clothing and food respectively, the product price ratio for country A equals OM/AN by rule (22). Let the product price ratio, for country B, at point Q be represented by P_b, it is our purpose to

Prove: P_b = OM/AN

Proof: The outputs of the two commodities, for country B, at point Q are c_2/c_1 units of clothing and f_o/f_1 units of food - since c_1 and f_1 are defined as one unit of clothing and food respectively. By rule (1) we have:

Units output of clothing at Q = c_2/c_1 = OQ/OP

Units output of food at Q = f_o/f_1 = BQ/AP

The exchange values of total outputs, in terms of rent units, are OU (for OQ/OP units of clothing) and BV (for BQ/AP units of food) by rule (16). Hence we have:

$$Pb = \frac{OU/(OQ/OP)}{BV/(BQ/AP)}$$ the numerator and denominator being the prices of clothing and food, in terms of rent units, respectively.

$$= OU/BV \times OP/OQ \times BQ/AP \ldots\ldots.(1)$$

Since MN and UV are parallel (by rule 26 above), we know that the triangles OPM and OQU are similar; also, the triangles AFN and BQV are similar (all three sides are parallel by the parallel relationship). We derive the following equalities:

$$OP/OQ = OM/OU$$

$$\text{and} \quad BQ/AP = BV/AN$$

Substitute these equalities in (1) above, we have

$$Pb = OU/BV \times OM/OU \times BV/AN$$

$$= OM/AN$$

QED.

Rules (27) and (26) together state that _if_ the equilibrium positions of the two countries can be represented by a pair of points satisfying "the parallel relationship" both _factor-price ratios_ and _product-price ratios_ are completely equalized between the two countries.

Section two: The proof

Having equipped ourselves with this knowledge it is a simple matter to prove the theory of "equalization of factor prices". However, we want to state very clearly, first, what we want to prove. The theory actually reads as follows:

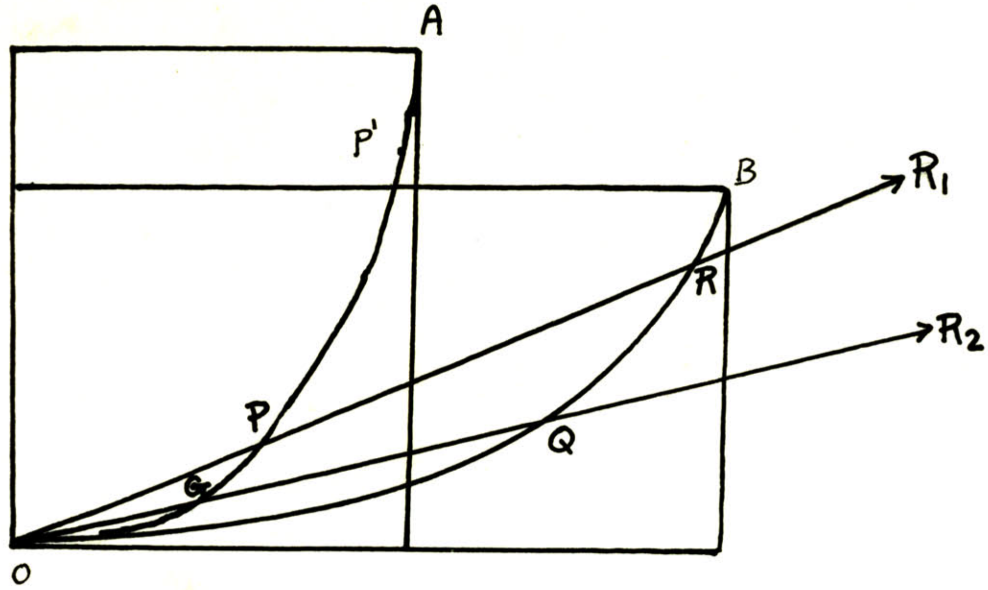
"If the equilibrium is established in such a way that _neither country is completely specialized_, then factor-price ratios will be completely equalized between the two countries"

First of all, it must be emphatically pointed out,[2] that the theory concerns only "_incomplete specialization for both countries_". The theory does _not_ claim to assert that "incomplete specialization for one or both countries" _will be_ the _actual_ equilibrium positions; all it asserts is that _if_ both countries are incompletely specialized _then_ we have factor price equalization.[3]

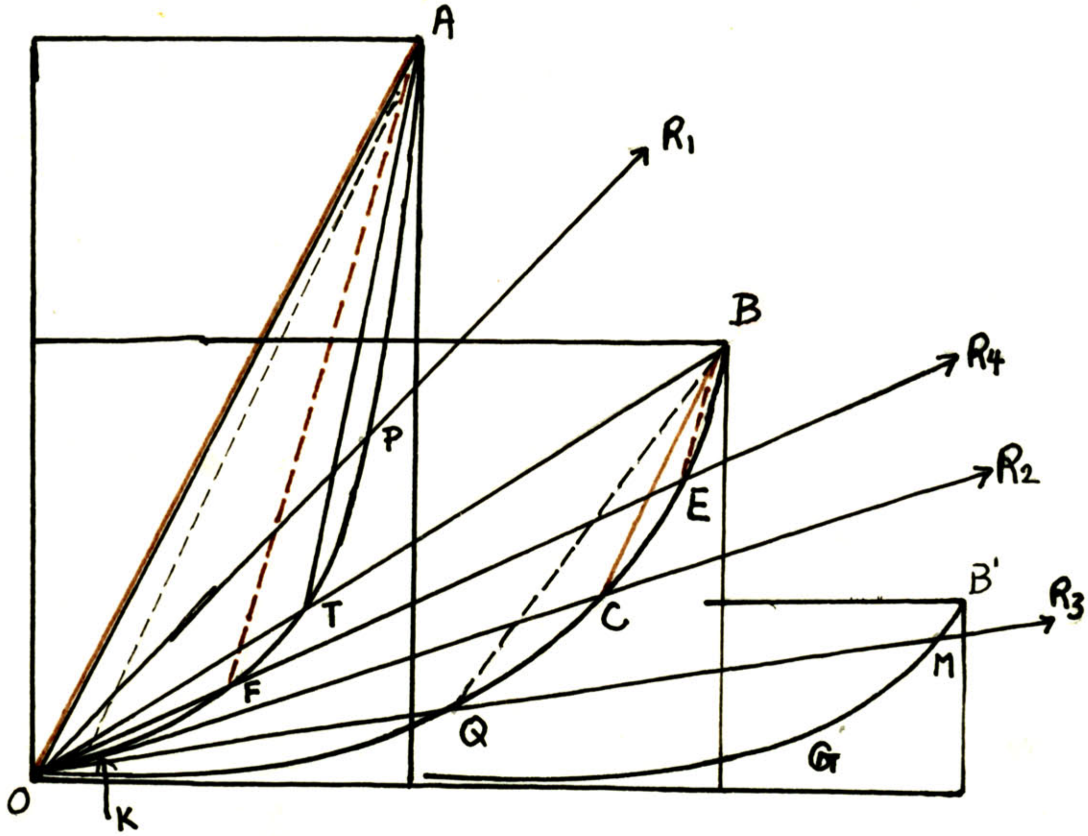
We can arbitrarily pick a pair of points - e.g. (P and Q) in diagram 17 - one on each optimum allocation curve (OA and OB). The pair should be picked in such a way that neither country is completely specialized. (Hence we are forbidden to pick point O, A and B because they represent complete specialization.) We now ask: under what conditions will the arbitrary pair of points picked in this way represent a conceivable equilibrium position?

2/See P. A. Samuelson, op.cit. Economic Journal June 1949, page 182

3/In other words, when one or both bountries are _actually_ completely specialized, the theory becomes irrelevant.



Diag. 17



Diag. 18

This pair of points may or may not satisfy the parallel relationship - which can be easily tested by drawing a radial line from point O, through either P or Q and see if it intersects the other point or not; if it does, the pair does satisfy the relationship, otherwise the pair doesn't. If the pair does satisfy the relationship, we know by rule (26) and rule (27) that both factor price ratios and product price ratios are completely equalized between the two countries. (The pair is compatible with the requirement that "product-price ratios" must be equalized in the equilibrium position --since, by assumption, there is only one product "market" in the whole "world".)

If this arbitrarily constructed pair of points does not satisfy the parallel relationship - as shown in diagram 17 - we can draw a radial line through either point, e.g. OR1 through point P. The radial line OR1 then must intersect the optimum allocation curve OB at a point other than point Q,[4/] e.g. point R on OB. Point R then serves as a "reference point".

The product price ratios between the "reference point R" and point P, by rule (27) above, must be the same. The product price ratios between R and Q must be different - by rule (25) which states that for different points on the same optimum allocation curve, the product price ratio must be different. (Actually, in the case depicted in diagram 17, we can make a stronger statement that the product ratio is higher at R than at point Q). Hence, the product price ratio at point P is different from the product price ratio at point Q. This proves: if a pair of points does not satisfy the relationship it cannot represent an equilibrium position -

4/Namely, if it intersects OB curve at all. Let us assume for the moment that it does intersect OB curve.

because it is incompatible with the assumption of equalization of product price ratios when there is only "one" product "market" in the "world". Hence the theory is proved --since if all the possible equilibrium positions must satisfy the parallel relationship, we know by rule (26) that factor-price ratios are completely equalized.

Section three: Further elaborations on the Proof

It is evident that the completion of our pfoof hinges upon possibility of finding a reference point - e.g. point R in diagram 17. If the reference point cannot be found our proof fails. In this section we will analyze this situation and the methods which we will have to employ to overcome this difficulty.

In diagram 18, let diagram 17 be reproduced in which the two diagonals OA and OB are shown. It is immediately obvious that if the input ratio for the production of clothing at point P is higher than the endowment ratio of country B - represented by the slope of the diagonal OB the radial line passing through point P (i.e. OR1) cannot intersect the other optimum allocation curve OQB.

Through point B draw a straight line BC parallel to AO diagonal. Let point C be the point of intersection of BC and the optimum allocation curve OB. Draw the radial line OC (i.e. OR2). Let point Q be another point lying on the OCB curve below point C. We want to show that a radial line drawn through point Q (i.e. OR3) cannot intersect the optimum allocation curve OA at a point other than the origin "O".

Now suppose OR3 does intersect the OPA curve at a point K. Join AK and BQ, which must be parallel by the parallel relationship (rule 25).

But we know AK is steeper than OA diagonal - by rule (19); and BQ is less steep than BC - by rule (25). Since BC parallels AO by construction, we know BQ must be less steep than Ak. This contradicts rule (25), namely, the parallel relationship. Hence we know: a radial line drawn through a point on OB curve lower than point C cannot intersect the optimum-allocation curve OA (e.g. point F).5/

Let the point of intersection of the OB diagonal with the optimum-allocation curve OA be point T. By our discussion above, we know: only, and all, radial lines lying between points T and C intersect both optimum-allocation curves - OA and OB. In other words, a radial line, intersecting the OA curve at a point higher than point T (e.g. OR1), cannot intersect the OB curve; a radial line, intersecting the OB curve at a point lower than C, cannot intersect the OA curve.

If an arbitrary pair of points, one on each optimum allocation curve is picked in such a way that, for instance, P is above T and Q is below C, it is obvious that the radial line passing through either point cannot intersect the "other" optimum allocation curve. In other words, we cannot obtain a single "reference point" in the way described in the previous section, and our proof is not completed. But in this case, the situation can be easily salvaged by drawing a "reference radial line" (e.g. OR4) intersecting both optimum-allocation curves at two reference points (F and E). We can then argue that the price ratio at point Q

5/If E(onOCB) is higher than C, OE necessarily intersetc OA(eg. F)
5/This can be easily proved as follows: Join BE which is steeper than BC and AO. Through point A draw a straight line, parallel to BE, i.e. AF which must necessarily intersect OA optimum-allocation curve at a point (F). The fact that radial line passing through point F on OA curve necessarily intersects OB at point E can be easily seen by the parallel relationship - for BE parallels AF.

must be lower than at point P - with reference to the two reference points - and hence we can arrive at the same conclusion required for the completion of our proof, namely, as long as a pair of points does not satisfy the parallel relationship it cannot represent the equilibrium position -- since it is incompatible with product-price equalization under conditions of incomplete specialization.

Difficulty arises when the factor endowment ratios of the two countries are so different from each other --the method of "reference-radial-line" described above fails us. Let the endowment ratio of country B'be lowered to a value lower than the "input ratio in the production of clothing" corresponding to point C. (e.g. point B' in diagram 18 is the upper-right corner of the new box for country B', for which an optimum-allocation curve OGB' is drawn.) In this case, it is obvious that no radial line drawn from the origin "O" will ever intersect both optimum-allocation curves (OA and OB'). In other words, the "parallel relationship" cannot be satisfied by any pair of points, one on each optimum allocation curve (e.g. P and G), no matter how the pair is picked. Obviously we cannot find a single "reference radial line" intersecting both optimum allocation curve.

The difficulty caused by this situation can be overcome by interposing a "reference country" which "bridges the gap" between the two countries under investigation. For instance, country B can now be taken as a "reference country" interposed between country A and B'. Let the pair of points be (P and G), The radial lines R4 and R3 are drawn in such a way that R4 intersects both OA and OB curves and R3 intersects both OB and OB' curve. (We have one reference country, (B), two reference radial lines (OR4 and OR3) and four reference points (F,E,Q,M).) We can then argue in a zig-zag way that the product-price ratio at point G must be lower than that at point P - taking successively the inequalities and equalities through points G,M,Q,E,F and P. The same conclusion required for the completion of the proof is obtainable, i.e. the incompatibility of a pair of points not satisfying the parallel relationship with product price equalization.

However, the method of interposing a "single" reference country fails us again if the endowment ratios of the two countries under investigation are "too different". We have to interpose more than one reference country to "bridge the gap". Exactly how many reference countries do we have to interpose before the gap can be bridged will be analyzed in the following sections. It turns out that we have to investigate a problem, which is not the concern of the theory of equalization of factor prices, before the question can be conveniently answered, namely the problem of "the compatibility of incomplete specialization for both countries with various endowment ratios of the two countries". This will be the subject of the following section - which is really a digression from the process of the proof of theory of "equalization of factor prices" and may be considered as "other applications of the box diagrams".

Section four: Specialization status

In this section we will consider the "specialization statuses" of the two countries under various assumptions of the endowment ratios of the two countries. By specialization status we mean either "completely specialized" or "incompletely specialized" in production of a country - the two "events" are mutually exclusive.

In diagram 18, returning to the two original countries A and B with optimum-allocation curves OTA and OCB, we have proved that if point Q lies below point C (a critical point) then product-price ratios between the two countries cannot be equalized no matter where point P is located on the optimum-allocation curve OA. What, then, is the specialization status of country A if point Q is the actual equilibrium position of country B? It is fairly obvious that, as long as country A is incompletely specialized, product price ratios cannot be equalized between the two countries, and hence it cannot represent the equilibrium position of country A. In this case, a stronger statement can be made, namely, country A becomes completely specialized in the production of food, the equilibrium position of country A must be represented by the origin point O.[6/] So we derive the rule: when the equilibrium position of country B is lower than C, country A is completely specialized in the production of food.

But we have pointed out above, that a radial line higher than the radial line OR2 (e.g. OR4 intersects OB curve at E which is higher than C) necessarily intersects the OA curve; and that a radial line lies below OR2 (e.g. OR3) cannot intersect the OA curve. Hence we know that the slope

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6/From a purely logical standpoint, we have to argue, first, that the equilibrium position of country A must be represented by either point A or point O (i.e. if country A must be completely specialized, it must either completely specialize in the production of clothing, at "A", or of "food" at "O"), the former point must be ruled out by a certain "dynamic" consideration or by the fact that given the product price ratio, as determined at point "Q" in country B, the exchange value of total output at A is lower than that at O, and hence A must be ruled out as the equilibrium position of country A (a "static" reasoning"). The point is definitely trivial - we will briefly discuss the point in Appendix A.

of the radial line OR2 represents the slope of the optimum-allocation curve OA near the end point O. In other words, the slope of the OA curve at the end point O defines the input ratio for the production of clothing in country A when a little bit of clothing is produced in this country. In view of our discussion above we derive the following rule: (taking country A as the "home" country.)

Rule twenty-eight: If the equlibrium position is established in such a way that the input-ratio in the production of clothing in the "other country" is equal to, or lower than, the slope of the optimum-allocation curve of the "home" country at the lower end, the home country becomes completely specialized in the production of food.

In a similar way, we can first argue that the slope of the production contour at the upper end (B) for country B can be represented by the slope of the straight line AT. If the equilibrium position of country A (the "other" country) is such that the input ratio in the production of food is higher than the slope of AT (e.g. at point P on OA curve), then country B must be completely specialized in the production of clothing.[7/] Hence, taking country B now as the "home" country, we derive a rule parallel to rule (28):

7/All these propositions can be proved in an exactly similar way (as we did in the parallel case) if, instead of point "O", we choose the origin of the food map as the common origin for the boxes of the two countries (i.e. let the production contours for food, for both countries, coincide.)

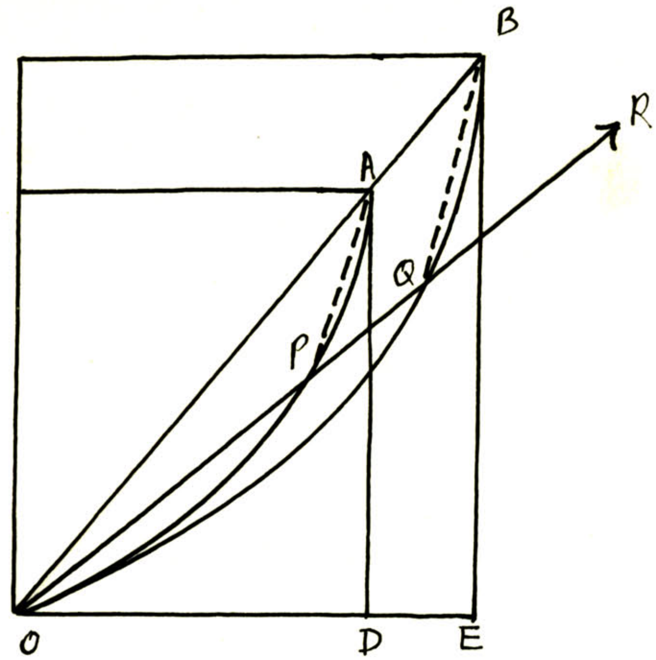
Rule twenty-nine: If the equilibrium position is established in such a way that the input-ratio in the production of food in the "other country" is equal to, or higher than, the slope of the optimum-allocation curve of the "home" country at the upper end, the home country becomes completely specialized in the production of clothing.

Hence we see the slopes of the optimum allocation curves at the upper and lower ends define a pair of critical input ratios - representing, respectively, the input-ratio when "a little bit" of food and clothing are produced.[8/] We may call them the "upper critical ratio" and the "lower critical ratio" --respectively for the upper-end slope and the lower-end slope for any given optimum-allocation curve. We can restate rules (28) and (29) in the following comprehensive form:

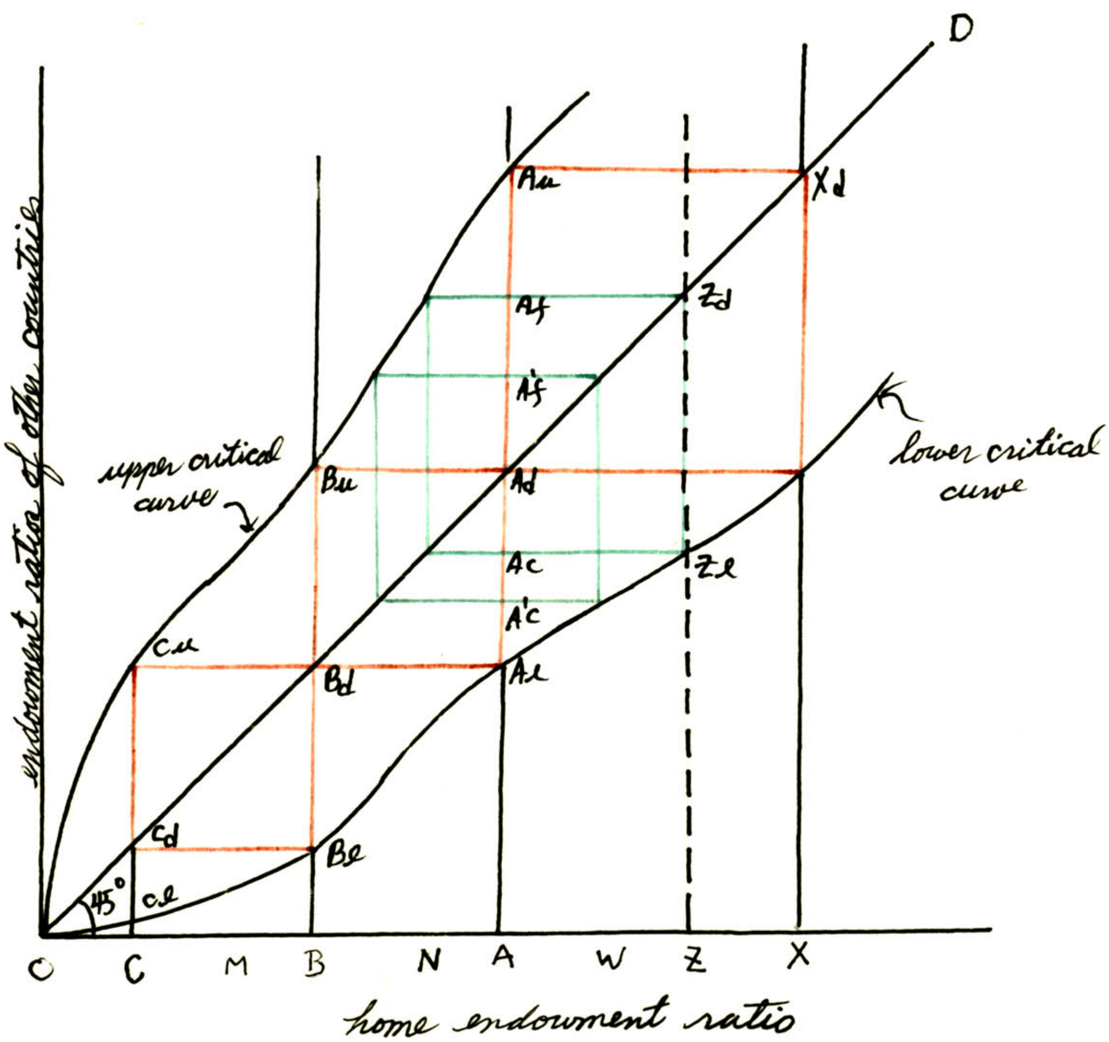
Rule thirty: The upper and lower end slopes of any optimum-allocation curve define the upper and lower critical ratios of a given country such that if the input-ratio for the production of clothing (food) of another country is equal to, or higher (lower) than, the upper (lower) critical ratio, the home country is completely specialized in the production of clothing (food).

The significance of this rule will be more evident if we can prove the following proposition:

8/The writer cannot see any economic significance of the slope of the optimum-allocation of resource curves except at the end points - which, as we will see, are full of meanings.



Diag. 19



Diag. 20

Rule thirty-one: The upper and lower critical ratios are determined by the endowment ratio of a given country — the size of the country is immaterial.

This proposition can be proved very easily. In diagram 19 let the endowment ratios of two countries A and B be the same (i.e. represented by the slope of the "common" diagonal OAB). The optimum allocation curves are OPA and OQB. Let OR be any radial line intersection OA and OB curves at points P and Q respectively. Join PA and QB, which are parallel (by the parallel relationship). Now let the OR radial line gradually approach the diagonal OAB; PA and BQ approach the slopes of the optimum-allocation curves near the end-points (point A and B). The proposition is proved for the upper critical ratio. The lower critical ratio can be similarly proved.[9/]

By rule (31), we can plot the upper and lower critical ratios against the endowment ratio of the home country. In diagram 20, let the endowment ratio of country A be represented, on the horizontal axis (which is marked as the "home endowment ratio") by the distance OA. The vertical distances AA_u and AA_d represent the upper and lower critical ratio of country A. Let the 45° line OD be drawn, intersecting the vertical line AA_u at point A_d. The vertical distance AA_l then represents the endowment ratio of country A. If we take all endowment ratios of the "home" country and plot the upper and lower critical ratios, we obtain the "upper critical curve" and "lower critical curve". The diagram may be called the "critical diagram".

9/If the origin of the food-map is chosen as the common origin, the proof will be exactly the same.

Section five: The critical diagram

There are certain geometrical properties of the critical curves which may be explicitly pointed out.

a) the upper and lower critical curves straddle the 45° line OD

This property can be easily proved. Referring to the box diagram of any country (e.g. diagram 18, take the "box" for country A), we know that the upper and lower critical ratios are really the input ratios for the production of food and clothing respectively (when the outputs are small). Since the optimum-allocation curve necessarily lies in the lower half of the "box" bisected by the main diagonal (OA), (by rule 19), it is immediately obvious that the upper critical ratio is higher than the lower critical ratio, and the two straddle the endowment ratio — which lies on the 45° line OD.

b) the two critical curves pass through the origin

In diagram 18, it is seen that the optimum allocation curve for country B', (with very low endowment ratio) is flattened. Thus, when the endowment ratio approaches zero, the optimum-allocation curve approached the main diagonal, both the upper and the lower critical ratio approach zero.

c) the two critical curves approach infinity as the endowment-ratio approaches infinity

When the endowment ratio of a country approaches infinity, the optimum-allocation curve again approaches the main diagonal - which has an "infinite" slope. The two critical ratios approach infinite too.

d) <u>the slopes of the two critical curves are positive</u>

This property can be more conveniently proved at a later point (see below, page 80 , footnote 13). For the time being we will take it as an unproved hypothesis. The economic interpretation of this property is: when the endowment ratio is higher both the upper and the lower critical ratios become higher.

e) <u>the upper and lower critical curves are symmetrical to OD (the 45°) line in their geometrical shapes</u>

In diagram 20, through the point A1 draw a horizontal line A_1B_d, intersecting OD at Bd. Through B_d draw a vertical line BB_u intersecting the horizontal axis and the upper critical curve at points B and B_u respectively. (The distances OB or BB_d represents the endowment ratio of country B, which is lower than that of country A). It is our purpose to prove that the distances B_dB_u and B_dA_1 are the same.

What we want to prove can be stated in an alternative way. If, after the point B_u is obtained in the way described above, a horizontal line is again drawn through B_u, we can alternatively prove that this horizontal line necessarily passes through the point A_d - which marks the endowment ratio for which the lower critical ratio is marked by A_1. In other words, what we want to prove is that we can inscribe <u>perfect squares</u> between the two critical lines if we take any point on the OD line as the lower-left corner of a "square".

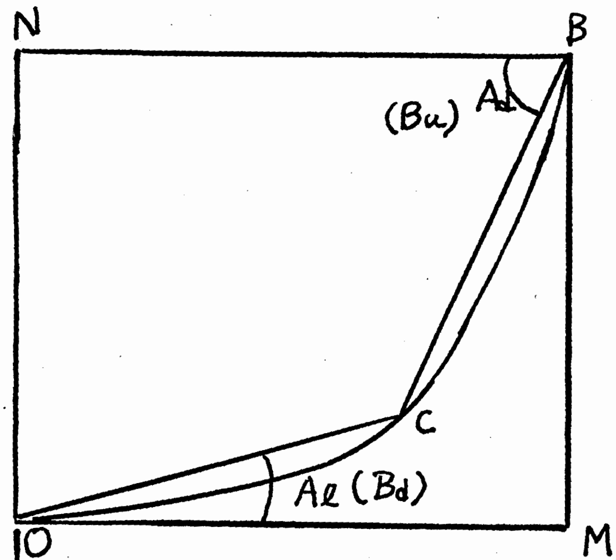
We can accomplish what we want to prove more readily if we can state it in terms of economic language. We want to prove that <u>if</u> the endowment ratio (B_d)[10/] of country B equals the lower critical ratio (A_l) of another country A, the upper critical ratio of country B (B_u) equals the endowment ratio of country A (A_d), vice versa. As we will see below, that if this is not true, there is an apparent absurdity from the viewpoint of economic reasonings.

The proof formally begins as follows: In diagram 18, we recall that point C is obtained on the optimum-allocation curve of country B by drawing a straight line through point B, the slope of which equals the endowment ratio of country A. If a radial line is drawn through point O passing through point C, (OC or OR2) we know that the slope of this radial line represents the lower critical ratio of country A. It is then obvious that we can take the box of any suitable size for "country B" and do the same thing - i.e. locating the lower critical ratio of country A.

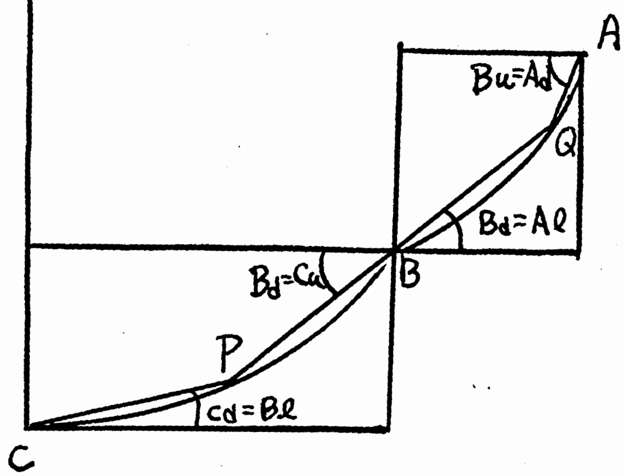
Thus, in diagram 21, let the box of any suitable size (i.e. suitable endowment ratio) represent the box of a country - i.e. the box OMBN. Through point B draw the radial line with a value of slope equal to the endowment ratio of country A (i.e. A_d), and obtain point C. The slope of the straight OC then equals the lower critical value of country A.[11/]

10/We will use the notations Au, A_d, A_l, B_u, B_d, B_l, C_u, C_d, C_l.....to represent the upper and lower critical ratios, the endowment ratios of the various countries indicated by the capital letters (the subscripts: "u" "l" and "d" stands for the upper critical ratio, lower criticalratio and endowment ratio respectively.)

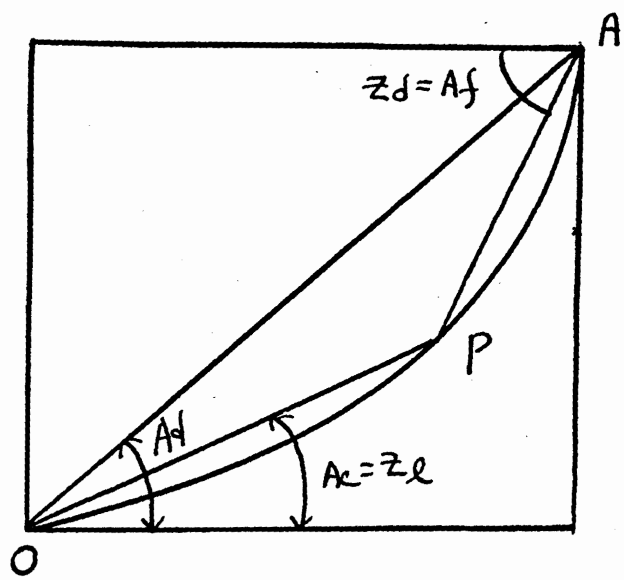
11/The fact that we will always obtain the same lower critical value, given the endowment ratio, by taking the box of any suitable size (endowment ratio) is ensured by the parallel relationship.



Diag. 21



Diag. 22



Diag. 23

The box of country B, in diagram 21, is, then, a geometrical device with the aid of which we can derive the lower critical ratio given the endowment ratio of a given country. (This geometrical device is of some value since we know that it is usually not very easy to draw, accurately, the tangential line to a curve at the end point of the curve.) By exactly parallel argument, we know that if the endowment ratio of a country (B_d in diagram 21) is represented by the slope of a radial line drawn from the lower-left origin, the slope of the radial line passing through the point of intersection (C) from the upper-right origin represents the upper critical ratio corresponding to the given endowment ratio (B_u)[12/]

If this is true then the fact that the two critical curves (diagram 21) are symmetrical to the OD line is already proved. For what is stated in diagram 21 is: if the endowment ratio of country B (B_d) equals the lower critical ratio of country A (A_l), then the upper critical ratio of country B (B_u) equals the endowment ratio of country A (A_d). Thus property (e) (page 77) of the two critical curves are proved.[13/]

12/Refer back to diagram 18, the slope of OT represents the endowment ratio of country B and the slope of T'A represents the upper critical ratio of country B.

13/In view of our discussion above, we can derive the "critical map" (i.e. diagram 21) more accurately when the optimum-allocation curves can be more or less accurately drawn. The geometrical device is as follows; in diagram 22, let a number of boxes be placed as shown. The optimum-allocation curves CB, BA can be drawn. First, through point B draw a straight line PQ with a value of slope equal to the endowment ratio of a given country B (i.e. B_d). Draw the straight lines from the origins A and C to the points of intersection P and Q - i.e. CP and AQ. The slope of CP and AQ represent the lower and upper critical ratios of a country with the endowment ratio equal to that of country B. If we vary the slope of the straight line PBQ (shifting the points of intersection P and Q correspondingly) we can find the upper and lower critical ratios corresponding to endowment ratios at the neighborhood of country B. It is then obvious that when the endowment ratio is higher, both the upper and the lower critical ratios will be higher - by rule (25). This means that the slopes of the upper and lower critical curves must be positive.

Section six: The Equilibrium Position (of trade) as seen in the Critical Map

Referring to diagram 18, we know that the input ratios for the production of clothing of a given country are straddled by the endowment ratio of the country (as the upper limit) and the lower critical ratio of the country (as the lower limit). (Take country A for instance, the input ratio for the production of clothing lies between the slopes of the diagonal OA and the radial line OC (or OR2). This is ensured by the fact that the optimum allocation curve can only lie below the diagonal, and the fact that the lower critical ratio defines the lower limit of the input-ratio for the production of clothing.)

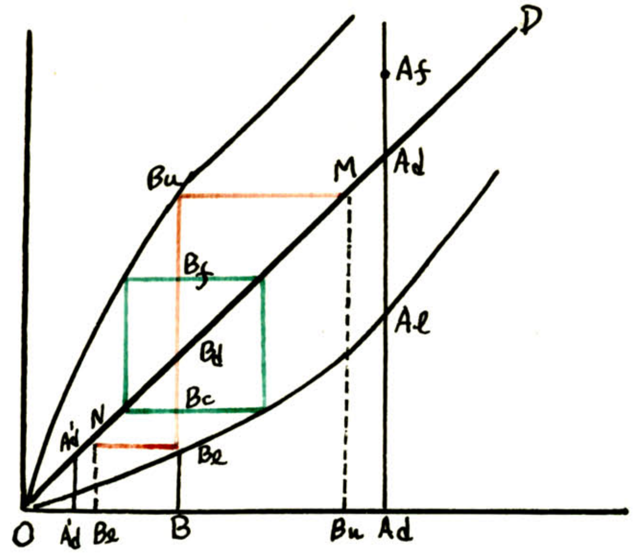
Similarly, we know that for any given country, the input-ratios in the production of food are straddled by the endowment ratio (as the lower limit) and the upper critical ratio (as the upper limit) of the given country.

Referring now to diagram 20, we know that the input ratios for the production of food and clothing, respectively, for country A (for instance) must be represented by a pair of points A_f and A_c, respectively, in such a way that A_f lies in the range A_dA_u and Ac lies in the range A_lA_d.

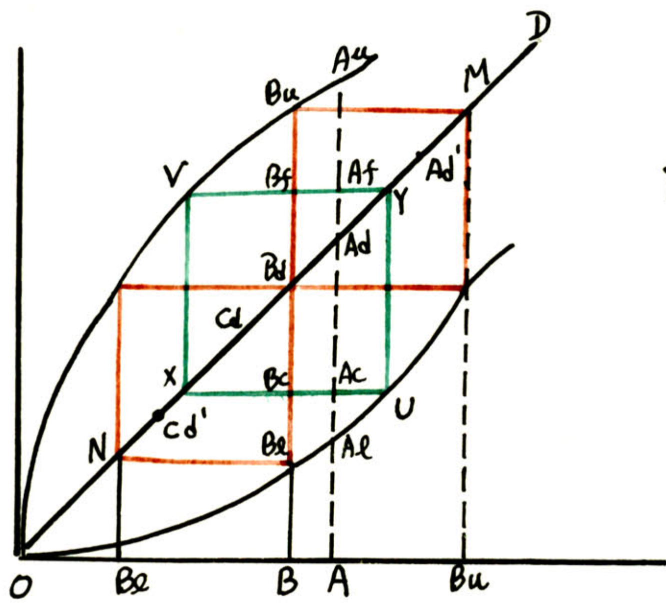
Referring to diagrams 18 and 20, if the equilibrium position of country A (represented by the point P in diagram 18) moves upward, as caused e.g. by a strangthening of the demand for clothing, both A_f and A_c (in diagram 20) shift upward. The limiting position is reached when country A becomes completely specialized in the production of clothing - in which case point P (diagram 18) reaches point A, and A_f and A_c (in diagram 20) approaches A_u and A_d simultaneously. (Similarly we know that as the relative strength of demand for clothing decreases A_f and A_c (diagram 20) shift downward, reaching A_d and A_l simultaneously as country A becomes completely specialized in the production of food.)

In diagram 20, when one of the two equilibrium input ratios is known (e.g. A_f of the pair A_f and A_c) the other input ratio can be easily obtained diagrammatically by "inscribing a perfect square" (green, as shown) with the upper-horizontal side passing through A_f; the other equilibrium input ratio (A_c) is obtained as the point of intersection of the "lower horizontal side" of the perfect square with the vertical line AA_u (corresponding to the endowment ratio of the country). Two such perfect squares, (green) with (A_f, A_c) and (A'_f, A'_c) as the two sets of equilibrium input ratios, are shown in diagram 20.

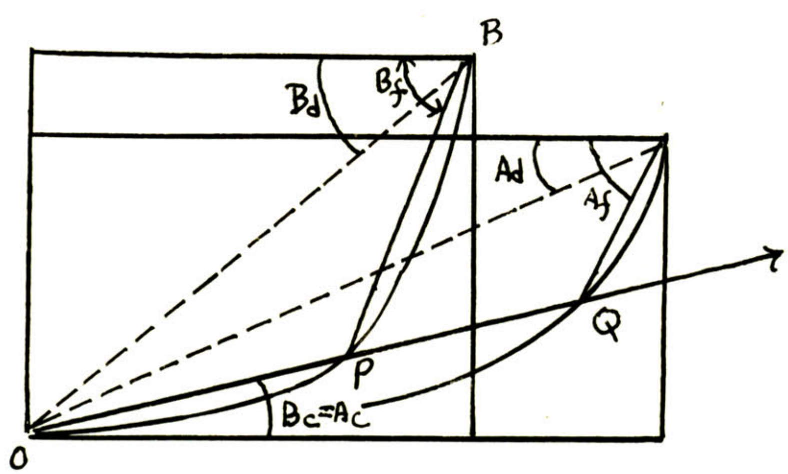
This can be easily proved as follows: in diagram 20, let the horizontal sides of the perfect square corresponding to A_f and A_c intersect the 45° line OD and the lower critical curve at Z_d and Z_l respectively. Let the country with the endowment ratio of Zd (and the lower critical ratio of Z_l) be country Z. In diagram 23, let the box for country A be shown. The equilibrium position of country A is shown to be point P with input ratios of A_f and A_c respectively for food and clothing. Now if the endowment ratio of country Z equals the input ratio for the production of food in country A (i.e. Z_d equals A_f), it is immediately obvious that the lower critical ratio of country Z (i.e. Z_l) equals the input ratio for the production of clothing in country A (i.e. A_c). Hence the method described above, for the location of the other input ratio (A_c) given one input ratio (A_f) on the vertical line AAu (diagram 20) is proved to be correct.



Diag. 24



Diag. 25



Diag. 26

Section seven: Applications of the Critical Map

With the aid of the critical maps we can analyze the problems relating to specialization status, mentioned earlier (in Section 4), in a more systematic way. Categorically we will present these applications in order.

1. Let us first demonstrate rules (28), (29) and (30) of section 4 above in the critical map..

In diagram 24, let the critical map be redrawn. If the endowment ratio of a country B is given, (Bd), we can locate the points B_u and B_1 representing the upper and lower critical ratios for country B. Through points B_u and B_1 draw the horizontal lines B_uM and B_1N intersecting 45° line OD at points M and N respectively. The lines dropped from points M and N vertically (i.e. MBu and NB1) intersect the horizontal axis at B_u and B_1 which again mark the upper and lower critical ratios of country B on the horizontal axis. (Since OD is the 45° line).

Let the endowment ratio of another country A be A_d such that A_d is greater than B_u. It is our purpose to show that incomplete specialization in production for both countries is not obtainable for these endowment ratios (B_d and A_d) of the two countries. We can even make a stronger statement: in this case, either country A will be completely specialized in the production of food or country B will be completely specialized in the production or clothing if the other country is incompletely specialized. That this statement is true is exactly what has been said in rules(28) and (29) - which can be seen in the following way.

Referring to diagram 24 we know that if country B is incompletely specialized, the equilibrium input ratios of country B (i.e. B_f and B_c)

can be represented by the pair of points (B_f and B_c on the vertical line BB_u intersected by (the upper and lower horizontal sides of) a (green) perfect square inscribed between the two critical curves.

(1) $Bc < Bd$ .....

But if Ad is greater than Bu by construction we have

(2) $Al > Bd$ .....by the fact that the slopes of the critical curves are positive.

From (1) and (2), we have:

$\underline{Bc < Al}$

This condition states that the input ratio for the production of <u>clothing</u> (B_c) for one country (B) which is incompletely specialized, is lower than the lower critical ratio of country A. Hence the result of rule (28) is directly applicable. So we conclude that <u>if country B is incompletely specialized country A must be completely specialized in the production of food</u>.

On the other hand, if country A is incompletely specialized, we can derive the inequality $A_f > B_u$ by exactly similar argument (i.e. $A_f > A_l > B_u$) and the result of rule (29) is directly applicable. We conclude, then, <u>if country A is incompletely specialized country B is completely specialized in the production of clothing</u>.

By exactly parallel argument, we can show that if the endowment ratio of another country (A'd) is such that it is lower than B_1, either country B will become completely specialized in the production of food or country A' will become completely specialized in the production of clothing, if the other country is incompletely specialized.

Hence we can restate rule (30), with an application of the critical map, in the following form:

> "When the endowment ratio of a country is known, the upper and lower critical ratios of the country are completely determined (by technological considerations). If the endowment ratio of another country is greater (smaller) than the upper (lower) critical ratio of this country, then, _if_ the home country is incompletely specialized, the other country will become completely specialized in the production of food (clothing); and _if_ the other country is incompletely specialized, the home country is completely specialized in the production of clothing (food)."

2. We can now analyze the cases in which the endowment ratios of the other countries _are_ straddled by the upper and lower critical ratios of the home country - which is taken as country B.

In diagram 25 let the endowment ratio of country B be given as B_d. We construct the upper and lower critical ratio for country B on the horizontal axis as before (B_u and B_l). Let us assume that the equilibrium position of country B is _known_ and we know the equilibrium input ratios of country B (i.e. B_f and B_c) fall on the horizontal sides of a (green) perfect square (diagram 25).

Let the endowment ratio of the other country be variously represented by A_d, A'_d, C_d and C'_d. These points are chosen such that _all of them are straddled_ by the upper and lower critical ratios of country B (B_u and B_l). (Apparently these points, lying on the 45° line OD, must be included in one or the other perfect square (red) constructed from (B_u,B_d) and (B_d,B_l) respectively - see diagram 25.

Let the point A_d be included in the green perfect square. This point, then, represents the endowment ratio of a country straddled by the _input ratios_ for the production of the two commodities (B_f and B_c)

of country B which, we assume, is an incompletely specialized country. We want to prove that country Ad is incompletely specialized too.

That this is true is seen from the direct application of the parallel relationship (i.e. rule 25) which states that: when the input ratios for the production of clothing are equalized between the two countries (i.e. $B_c = A_c$) the input ratios for the production of food must be equalized too. That the product-price ratios are equalized between the two equilibrium positions (i.e. represented by (B_f,B_c) for country B and (A_f,A_c) for country A) is further ensured by rule (27).14/

Similarly we can prove that countries with the endowment ratios like C_d in diagram 25 - i.e. straddled by B_d and B_c - must also be incompletely specialized countries.15/

14/From a rigorous logical viewpoint, there is a missing link in this argument, namely, (referring to diagram 26 below) if equilibrium of country B is established at point P, how do we know that the equilibrium position of country A must be established at point Q even though P and Q satisfy the parallel relationship? This seems to be a trivial point from the viewpoint of the economists. It will be investigates in Appendix I of this chapter.

15/Hence we see that there are at least two significances of the perfect squares inscribed between the two critical curves. Referring to diagram 25 the two red perfect squares, constructed from the endowment ratio (Bd), the upper critical ratio (Bu) and the lower critical ratio (B) of a country (B) tell us whether incomplete specialization for both countries will be obtainable - depending upon whether or not the endowment ratio of "the other" country (marked on OD) is contained in one or the other (red) perfect square. The green perfect square in diagram 25 (or rather the upper and lower horizontal sides of the green perfect square) indicates the equilibrium input ratios (for food and clothing) of all the incompletely specialized countries. These "colors" will be adhered to throughout this chapter. A third economic interpretation of the perfect squares will be assigned still another color (grey).

A second application of the critical map may be stated in the following form:

> "If the endowment ratio of a country (Ad or Cd in diagram 26) lies between the input ratios (e.g. B_f and B_c) of a non-specializing country (e.g. country B), then, the country is incompletely specialized too."

3. On the other hand, if the endowment ratio of "another" country (e.g. country A'_d), while <u>lower</u> than the <u>upper critical</u> ratio (B_u) than the incompletely specializing country (B) is actually <u>higher</u> than the <u>input ratio</u> in the production of food (B_f) of the incompletely specializing country, it is evident that the "other" country becomes completely specialized in the production of food, by rule (28). Similarly, we know that a country with endowment ratio C'_d satisfying the condition $B_l < C'_d < B_c$ becomes completely specialized in the production of clothing.

Hence, we can state our conclusion in the following form:

> "When the endowment ratios of the two countries are not so different that incomplete specialization of production for both countries is <u>obtainable</u>, this state of affairs may or may not be <u>realized</u>. When the endowment ratio of a country is straddled by the input ratios for the production of food and clothing of an incompletely specializing country, the country is also incompletely specialized; if the endowment ratio is greater (lower) than the input ratio, for the production of food (clothing) of an incompletely specialized country, the country becomes completely specialized for the production of food (clothing)." 16/

16/Another remark may be added for the application of this result: when the endowment ratios of two countries are given, the actual <u>determination</u> of the specialization status of the countries involved is a <u>completely unanswered question</u>. For an analysis of this question, we have to introduce into our analytical framework the relative strength of demand for the two products of the two countries. Until then the question must be postponed. (See below, Chapter V)

4. Another application of our critical map, which is immediately derived from our discussions above may be explicitly pointed out. The proposition may be stated in the following form:

> "When any <u>one</u> input ratio of an incompletely specialized country is known, a range of endowment ratios is completely determined such that if the endowment ratio of a country falls within, above, or below this range, the country is, respectively, incompletely specialized, completely specialized in the production of food, or completely specialized in the production of clothing."

This proposition is diagrammatically represented in diagram 25. Let the critical curves be drawn. If we know the input ratio for the production of one commodity at the equilibrium position (e.g. B_c), we can readily derive the other input ratio (e.g. Bf). The meaning of our statement above can be easily verified.

Appendix I.

The propositions derived in the present chapter are valid from the standpoint of non-rigorous economic reasonings. However, from the viewpoint of a rigorous geometrical (i.e. logical) proof, there are a number of unproved propositions which have been pointed out in the footnotes in the relevant sections. These loopholes will receive our explicit attention in this appendix.

(a) We have stated 1/ that if the equilibrium position of an incompletely specializing country is given (represented by a point on the optimum-allocation curve), and if there exists a point on the optimum-allocation curve of another country such that the pair of points satisfies the parallel relationship, then the equilibrium position must be represented by the pair of points - in other words, the equilibrium position of the "other" country must be represented by the point mentioned.

Referring to diagram 26, our (unproved) assertion is that: if the equilibrium position of country B is represented by point P, then equilibrium position of country A must be represented by point Q. (P,Q satisfy the parallel relationship.)

For the completion of the proof of this proposition we can, first, rule out all points on the optimum curve OA (i.e. excepting points O, A and Q) as incompatible with equalization of product-price ratios of the two countries. 2/ So what we want to rule out now are the two end points

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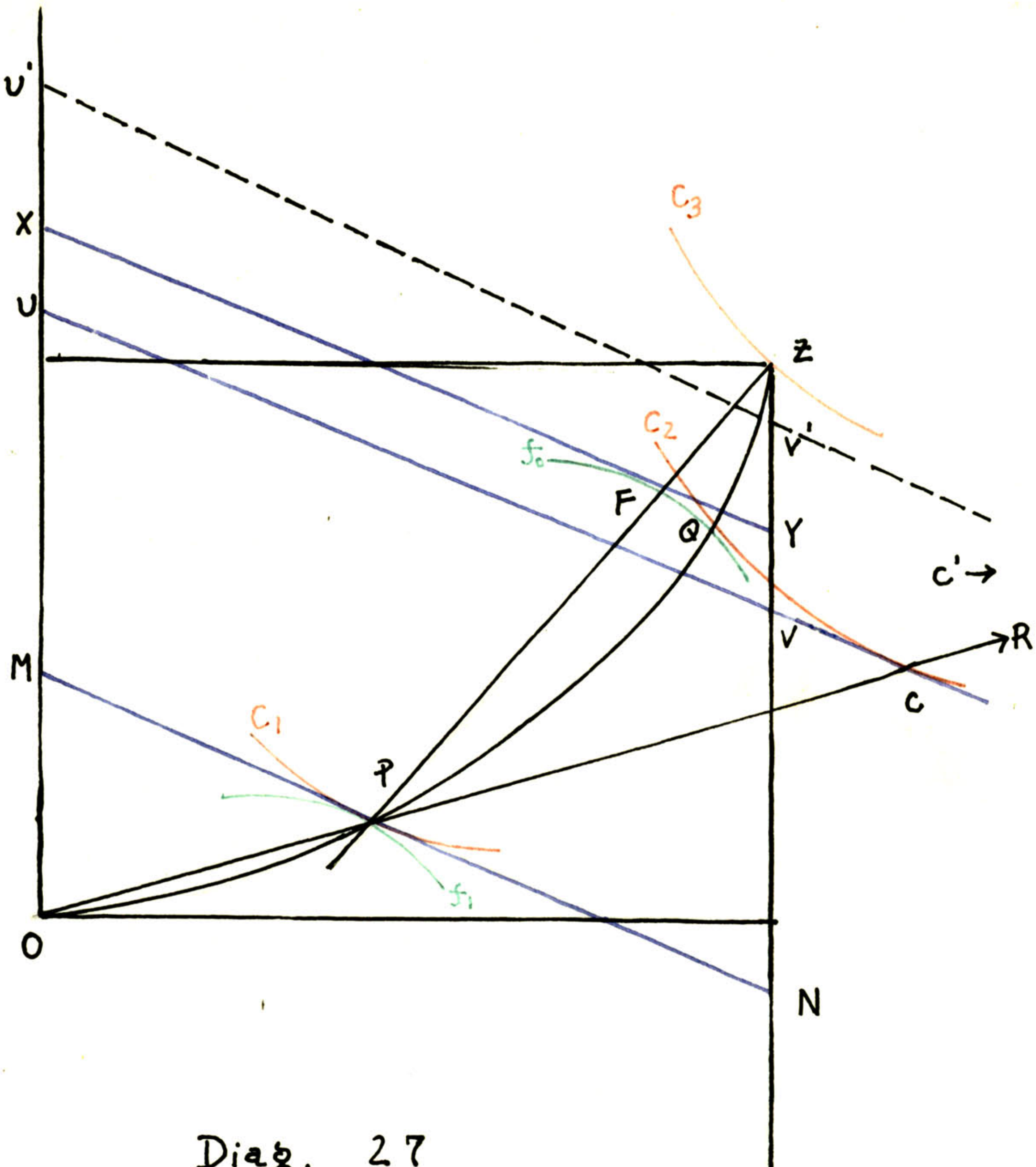
1/See footnote 14 on page 87 above.

2/In this case point Q can be taken as a reference point for the purpose of "ruling out" all other points.



Diag. 27

O and A - representing the positions where country A becomes completely specialized in food or clothing.[3/]

That this seemingly trivial problem, from the viewpoint of economic reasonings, actually does constitute a problem from the viewpoint of a rigorous geometrical proof is evident if we realize the fact that when we ruled out the other points on OA curve (as incompatible with the equilibrium position) we make use of the condition of "non-equalization of product-price ratios"; but if country A is completely specialized (in the production of either commodity) we cannot obtain a meaningful measure of the "domestically determined product-price ratio" which can be compared with the product-price ratio determined by the productive force in the foreign country.[4/]

For the completion of this proof, let us first prove the following general proposition for a "domestic economy": (i.e. a domestic problem)

> "the total value of output (of the two commodities) is a maximum if production is carried out at the equilibrium position rather than on any other points on the optimum-allocation curve under the product-price ratio prevailing at the equilibrium position."

In diagram 27, let the equilibrium position of country Z be represented by point P on the optimum-allocation curve OP - with outputs C_1 and f_1 units of clothing and food respectively. Join ZP and OP (i.e. OR). Let point Q be a point, lying on the optimum curve above point P,

---

3/That point "Q" is compatible with all the requirements of the equilibrium position is ensured by rule (27). We want to prove that it is the necessary equilibrium position of country A.

4/If we allow a certain "dynamic" consideration, the problem becomes trivial. However, a "dynamic consideration" is not the concern of the present thesis - by agreement.

with outputs C_2 and f_o. Let OR and ZP intersect C_2 and F_o at point C and F respectively. Through points P, C and F draw the tangential lines MN, UV and XY respectively intersecting the vertical axis at points indicated. These tangential lines are parallel by rule (8). By rule (16) above, we know that the total value of output, in terms of rent units for instance, can be represented by the sum of the distances:

<u>OM plus ZN</u>

Under the same product-price ratio, <u>if</u> production is actually carried out at point Q, then the total value of C_2 units of clothing and f_o units of food can be represented, respectively, by the distances OU and ZY. Total value of output would be:

<u>OU plus ZY</u>

It is our purpose to

Prove: OU plus ZY $<$ OM plus ZN

Proof: This inequality can be restated in the form:

$$OU - OM < ZN - ZY$$

which is equivalent to

$UM < NY$ in view of our diagram.

Since the tangential lines are parallel, we derive:

$$UM = VN \quad .....(1)$$

By the convexities of the production contours, and by the fact that XY is parallel to UV, we derive:

$$NY > VN \quad .....(2)$$

From (1) and (2) we derive:

$UM = VN < NY$ which gives

<u>OU plus ZY $<$ OM plus ZN</u>

QED.

It may be pointed out that this theorem applies regardless of the location of the point Q. When point Q is located at point Z, for instance, we know that the tangential line U'V' (C' is the point of intersection of OR with c3) must be lower than point Z, we know immediately:

$$\underline{\text{OU' plus zero} < \text{OM plus zn}}$$

When point Q lies below point P (or even at point O) the proof is similar.

Now referring back to diagram 26: if the equilibrium position of country B __is__ established at P, we know that the product-price ratio corresponding to this equilibrium position will equally hold in country A - by the assumption that there is only one output market. If the equilibrium position of country A is __not__ established at point Q, which is a point having the same product-price ratio, then the total value of outputs in country A is __not maximized__ regardless of the actual location of the "equilibrium point" of country A - by the theory we just proved. If we assume perfect knowledge in a static world, this will be incompatible with the equilibrium position.[5/]

(b) Another loophole which can now be investigated in the validity of rule (28).[6/] Referring to diagram 18, the rule says that if equilibrium position of country B is represented by point Q, with input-ratio

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5/We can __imagine__ that in this case, country A is producing for a __world__ market and has relatively little influence on the __world__ product-prices established therein - because we have assumed that the equilibrium position of country B __is__ at point P.

6/Rules (28)(29) and (30) are similar in nature, so only rule (28) will be investigated. See footnote 6 on page 71. It may be pointed out that our analysis of specialization status in section 7 is largely dependent upon the assumption of these rules.

for the production of clothing lower than the lower critical ratio of country A, then country A is completely specialized in the production of food.

In the present case, if a "reference radial line" (e.g. OR4) with two "reference points" (e.g. E and F) can be found, we can immediately rule out all points on the optimum allocation OA, except the two complete-specialization points A and O, as incompatible with the equilibrium price ratios established at point Q in country B - which, we assume, is the equilibrium position. So we need to rule out only point A — which means country A can only be completely specialized in the production of food.

First, let us state a proposition which is immediately obvious:

> "If a country is completely specialized in the production of food, then, when the price (ratio) of food increases, the country will remain completely specialized in the production of food". 7/

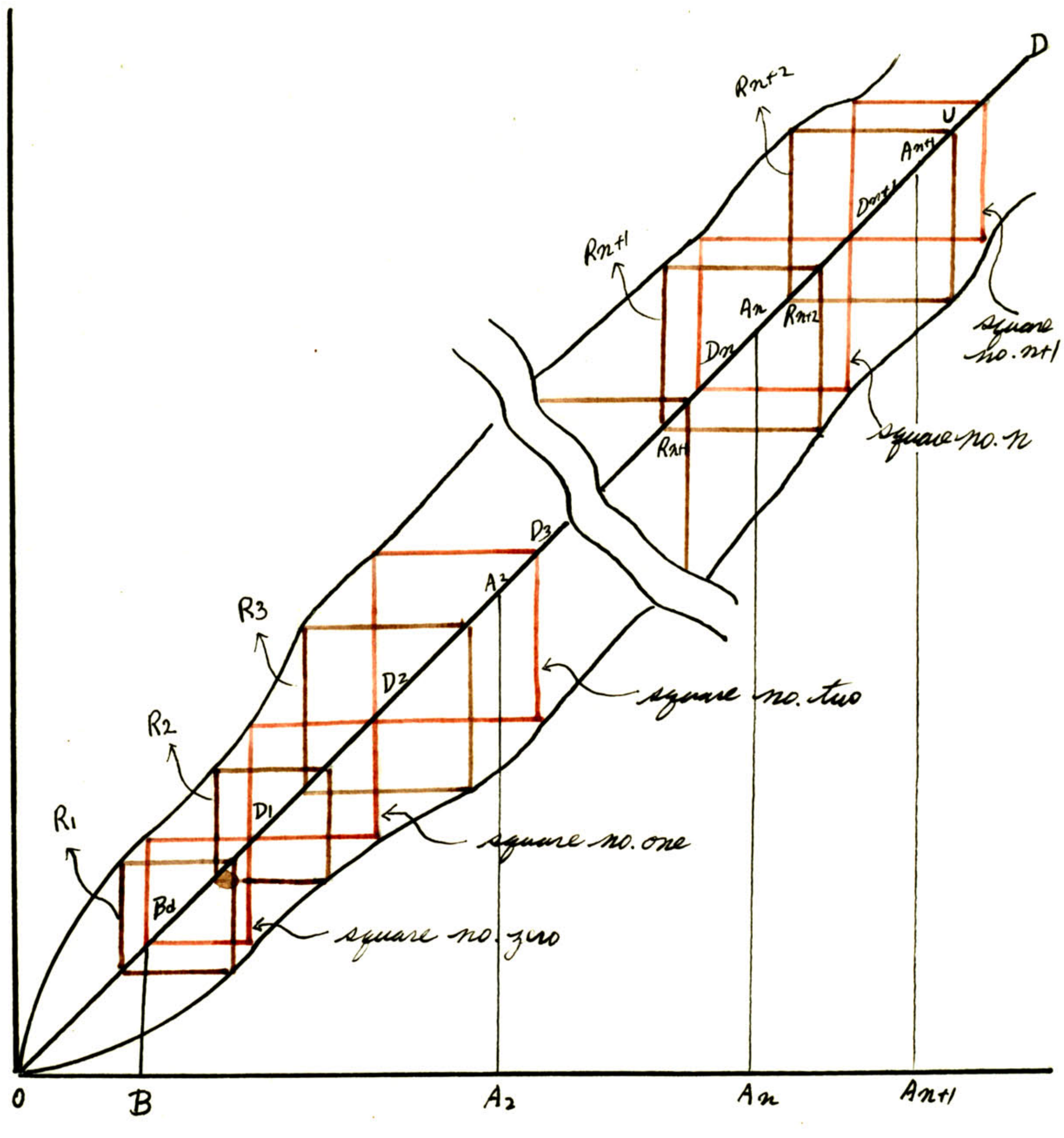
Now if we take the radial line OR2 as the reference radial line (OR2 intersects OA curve at point O) we know that had the equilibrium position of country B been established at point C, (i.e. we take point C as the reference point) the product-price ratio at this point would have caused country A to become completely specialized in the production of food - because the total value of output would have been maximum only at

---

7/Even this self evident statement is stronger than what we need relatively to our purpose. What will be sufficient for our purpose is the weaker statement: if a country is already completely specialized in the production of food, then when the price (ratio) of food increases, the country cannot become completely specialized in the production of clothing (!) --since we have ruled out all the incomplete-specialization-points on the optimum-allocation curve.

this point (o). But point Q of country B actually represents a lower product-price ratio (i.e. higher price for food) than that of point O (by rule). Hence we know country A would be even more assuredly completely specialized in the production of food, by our statement on the previous page. Hence rule (28) is ensured to be valid. (Also rule 29)

(c) In our analysis of the specialization status of the various countries, we made little mention of the sizes of the various countries -- except that the critical ratios are completely determined by the endowment ratios. We can complete our analysis by the remark that: the specialization status of two countries will always precisely be the same if their endowment ratios are the same. That this is true can be immediately seen from diagram 19 — in view of our analysis undertaken above.


D
U
square no. zero
square no. one
square no. two
square no. n
square no. n+1
O
B
A_2
A_n
A_{n+1}

Diag. 28

Appendix II

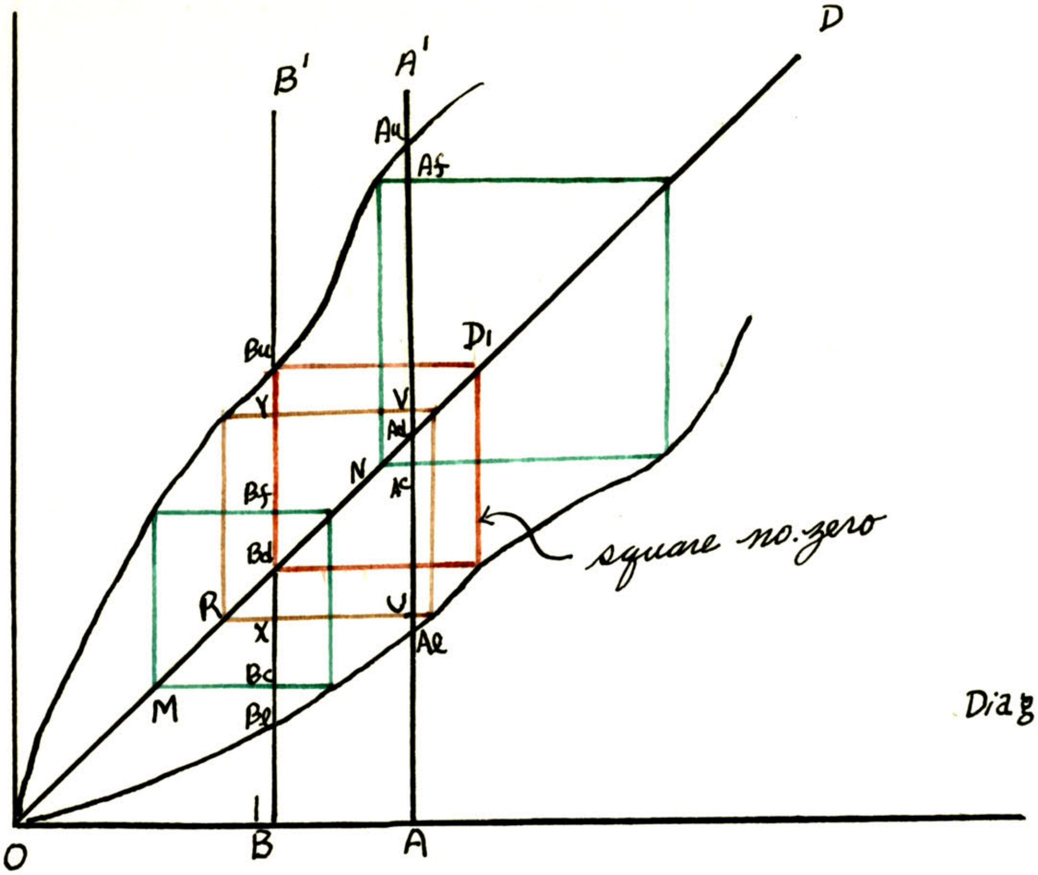
The completion of the proof of the theory of equalization of factor prices.

We have pointed out, at the end of section 3, that the completion of the proof of equalization of factor prices hinges upon the possibility of finding the reference points and the reference radial lines in such a way that the product-price ratios between any two arbitrarily picked points can be compared.

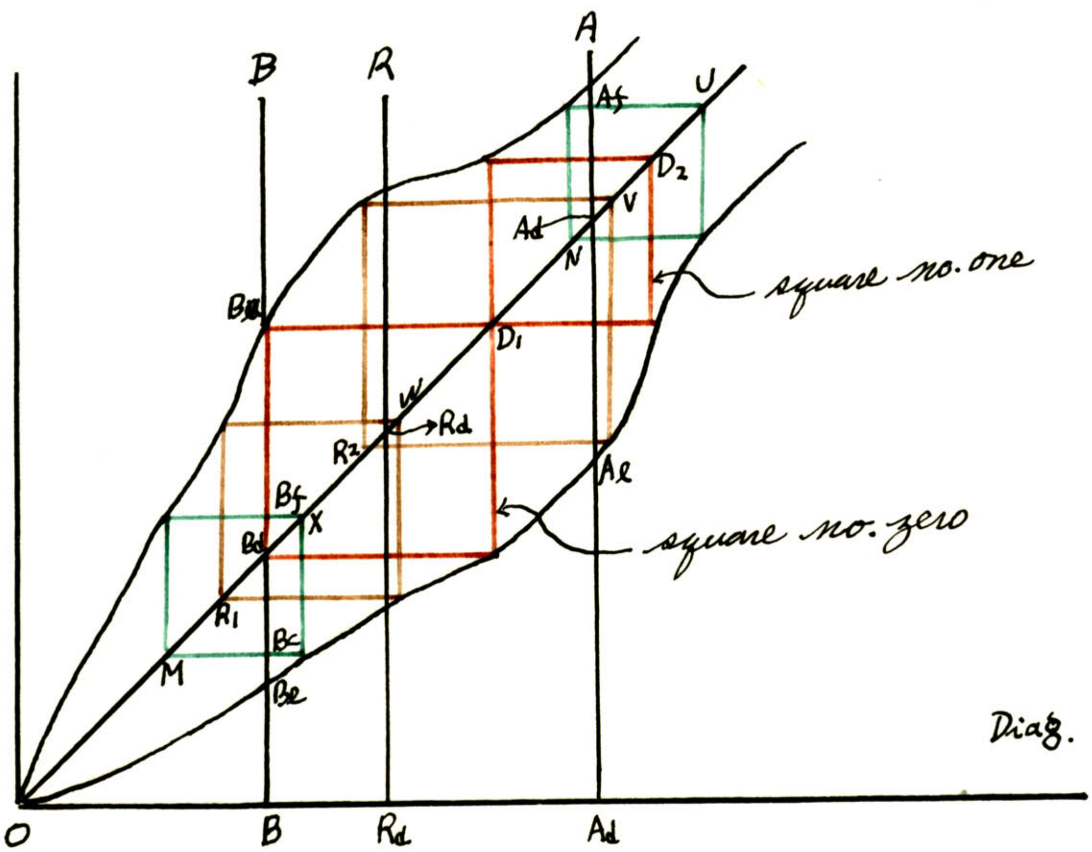
We have left the proof uncompleted — since we have not investigated the possibility of finding such reference points and reference radial lines. Still more, when the endowment ratios of the two countries are so different such that the parallel relationship cannot be satisfied at any pair of arbitrarily chosen points, we have to use other methods to bridge the gap. This possibility has not been investigated at all.

With the aid of the critical map we can now analyze these problems in a more systematical way.

In diagram 28 (neglect the grey square) let the endowment ratio of country B be given as B_d. Taking B_d, on OD, as the lower-left corner, we can inscribe a red perfect square with the upper-right corner located at point D_1. From point D_1, taken as the lower-left corner, another perfect square can be inscribed, with the upper right origin at the point D_2. The same process can be repeated. In this way, given the endowment ratio of a country B_d, we can inscribe a series of red perfect squares, taking successively the points, B_d, D_1, D_2, D_3.....D_n, D_{n+1}..... as the lower left corners. If we identify the perfect squares by their



Diag. 29



Diag. 30

respective lower left corner, we can call them: "Square" number 0, 1, 2, 3.....n, n+1,....., (as marked in the diagram, starting from the endowment ratio of country B - i.e. Bd -).

If the endowment ratio of country A is higher than B_d, A_d must be located on OD at a point higher than B_d. Let us define the "degree of difference of endowment ratios between countries A and B," denoted by D_{ab} in the following way:

> "If Ad is included in, or lying on the lower-left corner of the nth perfect square, constructed from the endowment ratio ratio of B_d, the degree of difference of endowment ratios between Ad and Bd will be called nth degree, namely: $D_{ab} = n$."

As shown in the diagram, when A_2 (i.e. the endowment ratio of country A) is included in the second square or located at the point D_2, the endowment ratios of countries A and B differ by two degrees - i.e. Dab = 2. Similarly the degree of difference between Bd and An in the nth perfect square would be equal to n-degree, and so on.

We can first show, that when Dab equals zero, we can always complete our proof. In diagram 29, let Ad fall inside the perfect square number zero or on B_d. We can arbitrarily designate two "incomplete specialization points", one for each country; namely, the lower green square M and the upper green square N. The lower (green) square M intersects BB_u at B_f and B_c which represents the equilibrium input ratios for the production of food and clothing respectively for country B. The upper (green) square N intersects AA_u at A_f and A_c representing the input ratios for country A at this arbitrarily chosen equilibrium position. It is obvious that the lower green square M contains point B_d, and the upper green square N contains the point A_d — if the condition of incomplete specialization for both countries is satisfied. If Dab = o, we can always construct a (grey) square (R) such that it contains both A_d and B_d because of the fact that the slopes of the critical curves are positive.

The grey square then necessarily intersects BB' and AA' at four points, X,Y,U and V, such that all the four points are included between the two critical curves. This is sufficient for the completion of the proof of the theory — for the grey square corresponds to a suitably chosen reference radial line and the input ratios at (X,Y) and (U,V) correspond to that indicated at the two reference points, one on each optimum-allocation curve of a country, intersected by the same radial line.[1] In other words, for Dab = D, no reference country needs to be postulated.

If the endowment ratio differs by one or more degree (i.e. $D_{ab} \geqslant 1$) this device fails. Referring to diagram 30, when A_d is located in the (red) square No. 1, we apparently cannot find a grey box containing both points, B_d and A_d - even when Ad is as low as D_1.[2] In this case we have to postulate a reference country, which can be done in the following way.

1/The correspondance between this case in the critical diagram and e.g. diagram 18, is as follows: (descriptions in the parentheses refer to diagram 29)

point Q (corresponds to B_f, B_c)
point P (corresponds to A_f, A_c)
radial line OR4 (corresponds to the grey square)
point E (corresponds to X,Y)
point F (corresponds to U,V)

2/This means that there can be no reference radial line intersection the optimum-allocation curves of both countries at (two) non-end points.

First, construct two grey squares, R_2 and R_1, such that R_1-square contains B_d and R_2-square contains A_d and such that the lower left corner of R_2-square (i.e. point R1) is included in R_1-square. (In other words, the two grey squares are interlocked). This is possible even when Ad is almost (but not quite) as high as D_2, - as long as D_{ab} equals one degree). There must then be at least one vertical line, e.g. RR_d intersecting the two grey squares at four points. The country with endowment ratio R_d can be taken as the reference country and the two grey squares can be taken as the two reference radial lines; the four pairs of points, intersected by the upper and lower horizontal sides of the two grey squares on the vertical lines BB_d, (one pair), RR_d (two pairs) and AA_d (one pair) are the four reference points.3/ The equilibrium positions of the two countries can then be postulated as represented by the two green squares M and N, with the aid of the reference points and the reference radial lines, the proof can be completed.

In view of our discussions so far made, we can derive the following conclusion, which will be referred to as rule A_1:

> "When the endowment ratios of the two countries differ by zero degree, the proof of equalization of factor prices can be completed without postulating a reference country; when the endowment ratios differ by one degree, the proof can be, and can only be, completed by postulating one, and at least one reference country with two reference radial lines, and four reference points."

3/The correspondance between this case and diagram 18 is as follows: let the countries be A and B' (diagram 19). Country B becomes the "reference country", two reference radial lines are OR4 and OR3, and the four reference points are F,E,Q,M. These reference country,radial lines, points can then be used to bridge the gap between the two arbitrarily chosen points P (on OA) and G (on oB). The inequality of the product price ratios at points P and G can be obtained by arguing in a "zigzag" way.

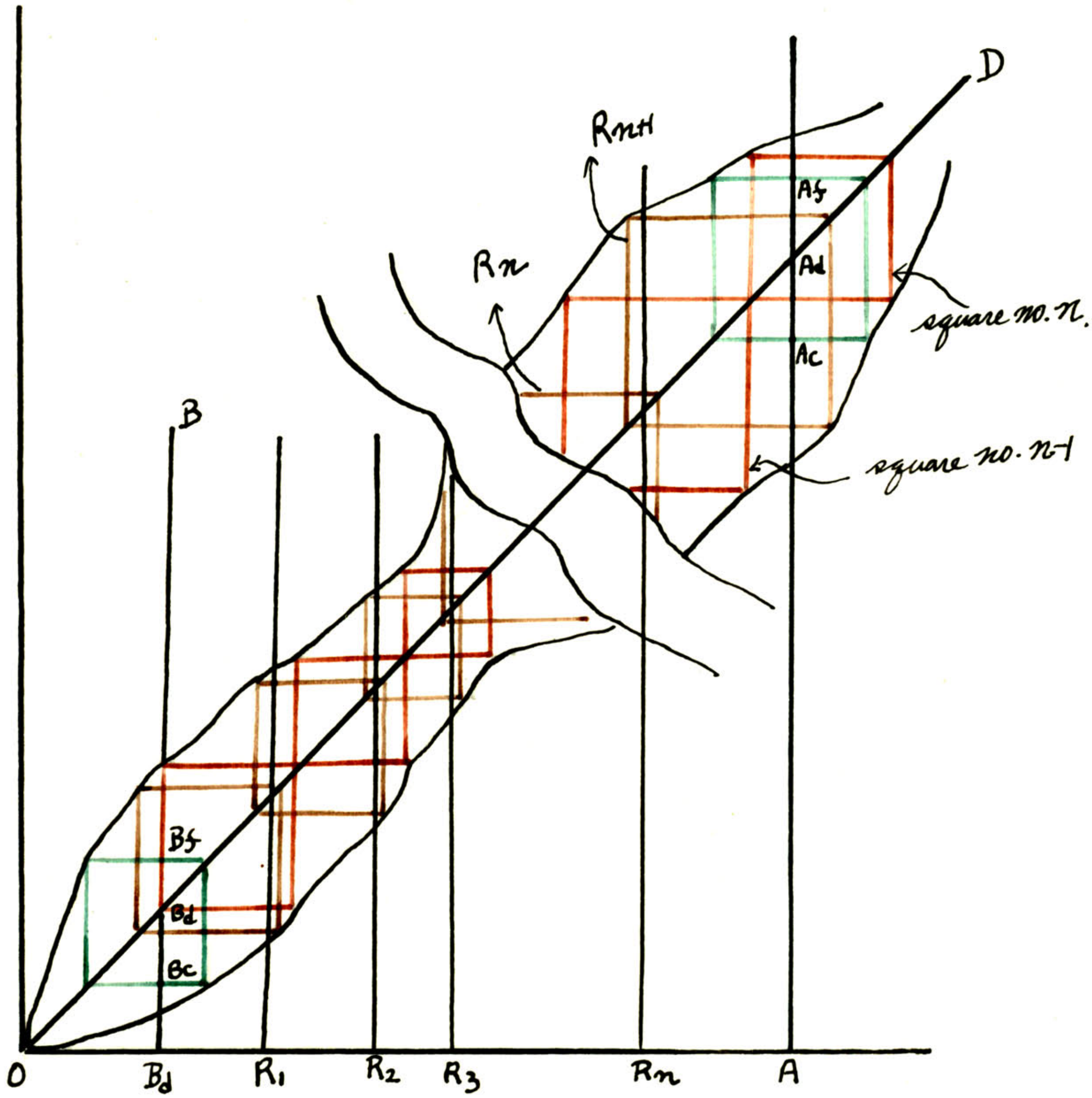
In order to generalize our result to the n-degree case (i.e. Dab equals n) we have to make use of the principle of induction.[4/] For this reason let us first investigate the relationship between the number of interlocked (grey) squares and the degree of difference of the two endowment ratios (Dab).

We can first prove the following propositions: when the endowment ratios Ad and Bd differ by n-degree, we can construct a series of exactly n+1 "interlocked squares" (grey) such that the first (grey) square includes the point Bd and the nth square includes the point Ad.[5/]

Referring to diagram 28, let An+1 be the endowment ratio of country A which is included in the perfect square number N+1, or lies on the point Dn+1 - i.e. Dab equals n+1. We can always construct a grey square R_{n+2} containing the point A_{n+1} such that the point R_{n+2} is contained in the (red) square number n — this is true even the point An+1 is almost as high as D_{n+2}. If P(n) is true, R_{n+2} must now be included in the grey square Rn+1 which is the "last" square of a series of interlocked (grey) squares --with the "first" (grey) square containing the point Bd. The squares Rn+2 and Rn+1 are then interlocked. Hence when P(n) is true P(n+1) must necessarily be true. So P(n) is true for all n.

4/The induction principle which we will make use of is as follows: if proposition one P(1) is true and if "P(n) is true implies that P(n plus one) is true", then P(n) is true for all n. (We make use of the critical map for the completion of the proof of our theory mainly because of the fact that the induction principle can be more conveniently applied).

5/Applying the induction principle, we know first that P(1) is true - i.e. when Bab equals one, we need two interlocked (grey) squares to "interlock" points Bd and Ad (by rule A1, see diagram 30). What we need to prove is: "if P(n) is true then P(n plus 1) is true.



D
Rn+1
Rn
Af
Ad
Ac
square no. n.
square no. n-1
B
Bf
Bd
Bc
O
Bd
R1
R2
R3
Rn
A

Diag. 31

For any two conjacent interlocked (grey) squares, we can postulate a reference country with endowment ratio represented by a point (on OD) contained in <u>both</u> comjacent interlocked square - e.g. in diagram 30, R_d (on OD) is included in the squares R_1 and R_2. For n+1 interlocked (grey) squares, there must be exactly n such reference countries which we can postulate.

For the case B_{ab} equal to n, the completion of our proof is demonstrated in diagram 31 - where the two green squares represent the arbitrarily chosen equilibrium positions of the two countries. We conclude:

> "<u>When the resources endowments of countries A and B differ by n-degree ($D_{ab} = n$), the proof of the theory of equalization of factor prices can be completed by the postulating of not more than n reference countries, with (n+1) radial lines and Z(n+1) reference points</u>."

Chapter V.

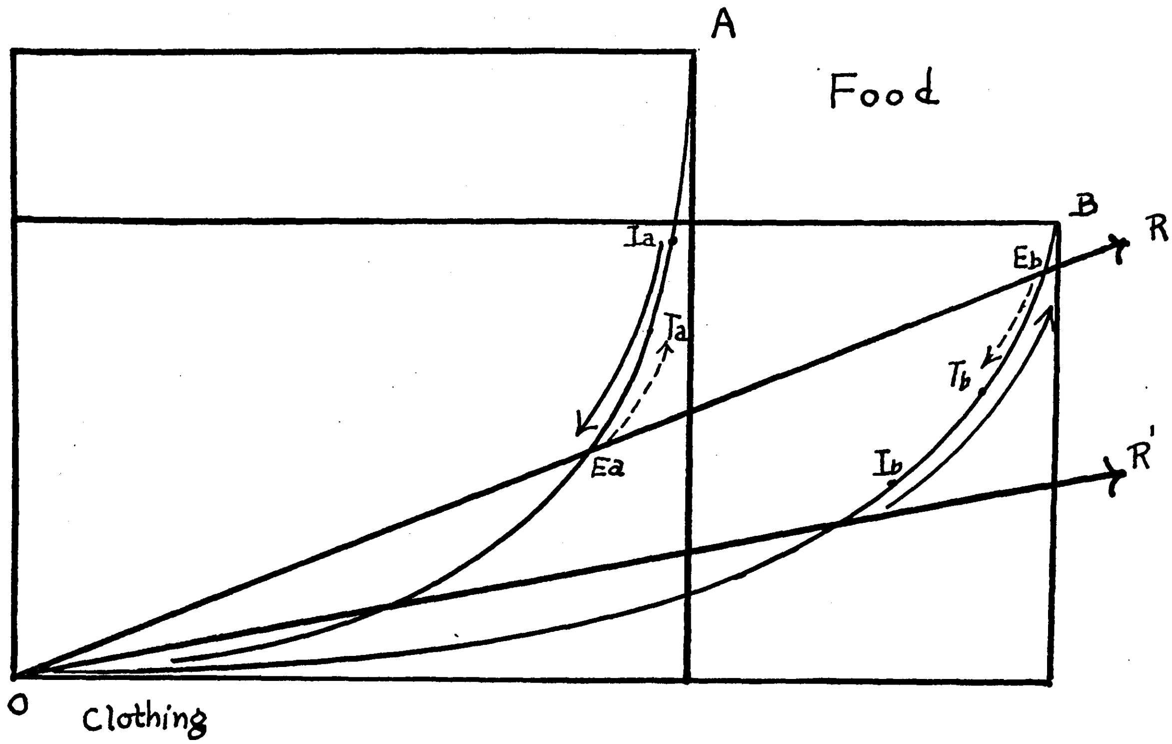
The Determination of the International Equilibrium Positions and the Analysis of the "World" Production Efficiencies

In our previous chapter we have made certain applications of the box diagrams for the analysis of problems in the field of international trade theory. We shall consider certain other applications, in the same field, in this chapter. The topics which will be analyzed are as indicated in the title of the chapter, which will be more fully explained.

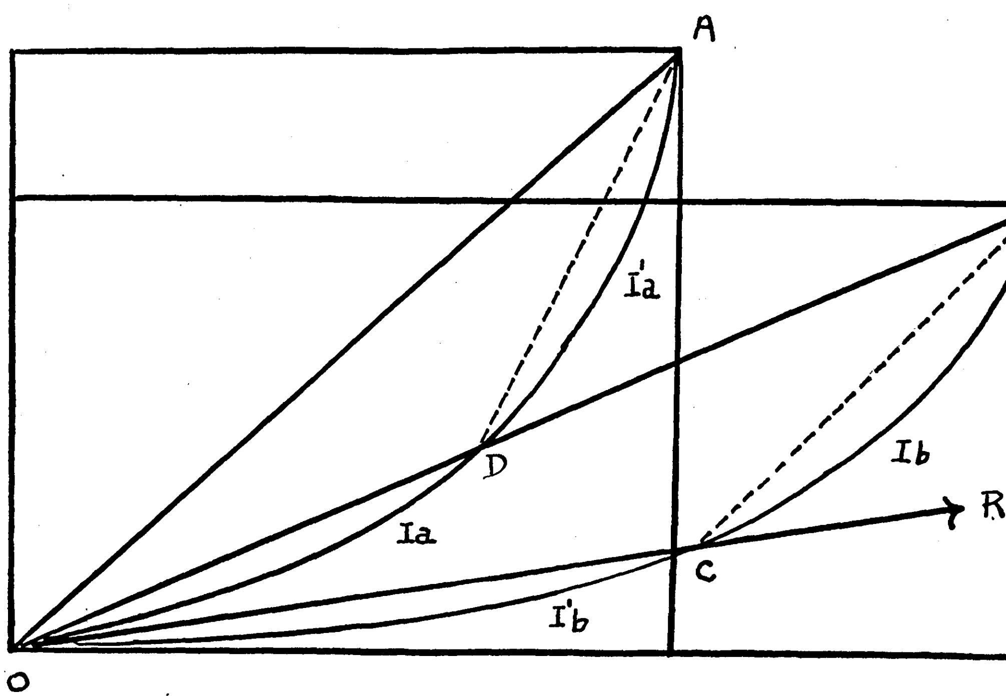
Section one: The Relative Strength of Demand and the Determination of the General Equilibrium System, with Applications.

In our analysis in the previous chapter, we have taken the equilibrium position, and the changes of the equilibrium positions for granted. Specifically, we have neglected the relative strengths of demand for the two commodities, by the two countries, so that we could not have meaningfully talked about the determination of the general equilibrium system. Here we shall try to show at least vaguely, the relative strengths of demand "at work" — and hence the determination of the international equilibrium position, all in one diagram.

We can do this, most simply, by taking one point for each country on its optimum allocation curve to represent the "isolation equilibrium position" of this country - i.e., the equilibrium position which would have been established had the country been completely isolated. The isolation point (or rather the output ratio corresponding to the isolation point) can then be taken as a measure of the relative strength of demand for the commodities in this country.



Diag 32



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In diagram 32 let Ia and Ib represent the isolation equilibrium position of countries A and B respectively. If all trade barriers disappear and if the relative strengths of demand in the two countries are normal,[1] equilibrium position, after trade, will be represented by a pair of points - e.g., Ea and Eb - satisfying the parallel relationship[2] and located in such positions that Ia lies above, and Ib lies below that radial line (OR) passing through the pair of points (Ea and Eb)[3].

The production equilibrium position of country A shifts downward, after trade, from Ia to Ea; and that of country B shifts upward from Ib to Eb, (indicated by the solid arrows). The exact equilibrium position is not determinable by this "diagramatic method". We only know that OR must run between Ia and Ib. This is true because, as depicted in the diagram for instance, the product price ratio must have been higher in country A than in country B in isolation. The equilibrium product price ratio becomes lower for country A (and higher for country B) so that country A becomes an exporter of food and importer of clothing; for country B the opposite is true.

1/We can define "normal" (or "stable" in the Hicksian Sense in Value and Capital) in the intuitively obvious way: when the price ratio is higher (i.e. price of clothing becomes relatively higher), the demand ratio becomes lower (i.e. more food and less clothing will be demanded at the equilibrium position).

2/Otherwise, product price ratios cannot be equalized by rule (27).

3/Namely, the "equilibrium radial line" OR must be located between the two isolation points such that one isolation point lies above, and the other lies below, the equilibrium radial line. If this is not true, Needless to say, the introduction of the relative strength of demand into our analytical framework in this way (i.e. the diagramatic solution of the final equilibrium position, given the initial isolation equilibrium positions) is highly unsatisfactory. On the one hand, the determination of the "isolation equilibrium position", itself, is left unexplained. 4/ On the other hand, given the initial isolation equilibrium positions, the determination of the final equilibrium position is again quite vague. 5/ However, if the relative strengths of demand are normal, in the sense understandable in common sense, we can still say that our diagramatic solution of the problem does indicate the general tendencies (or the general direction) of the (correct) final equilibrium positions.

Fn. 3/cont'd.

the relative strengths of demand for the two commodities by both countries cannot be normal. For instance, if the final equilibrium position is established at OR^1, it is easily seen that, relative to the product price ratios established at Ia and Ib, the price of food becomes relatively higher for both countries, and hence, if the relative strengths of demand for both countries are normal (by assumption), the consumptions of food will decline, absolutely, for both countries. But, the equilibrium position OR^1, indicates that the output of food by both countries has increased -- this contradicts the assumption of the normality of demand for both countries, and hence, OR^1 cannot represent the final equilibrium position. (See discussion in text and footnote 4/)

4/As indeed, it is probably impossible to demonstrate the solution of a complete determination of a general equilibrium system, all in one diagram, on the level of abstraction involving two commodities and two factors of production. We have to postulate, in addition, at least two trading individuals, with two preference systems indicating the supply (of factor) and demand (of product) conditions. There will then be more dimensions than a two-dimensional diagram can handle.

5/There seem to be (at least) two distinct difficulties which have been assumed away. Firstly, we know that there are two equilibrium conditions which have to be satisfied in the equilibrium position of international

If the results of our diagramatic solutions are generally acceptable, we can derive, immediately, the following propositions - which can be considered as the direct applications of the box diagram (with the forces of the relative strengths of demand introduced):

1. The equilibrium product-price ratio lies between the initial isolation-equilibrium price ratios of the two countries.
2. The equilibrium factor-price ratio lies between the initial isolation-equilibrium factor-price ratios of the two countries.
3. When a protective tariff (or other trade barriers) is imposed, the new production equilibrium positions of the two countries (represented by Ta and Tb in diagram 32 shift toward the isolation equilibrium positions of the two countries respectively, (indicated by the dotted arrows); and when the protective tariff is "prohibitive", the limiting positions (Ia, Ib) will be reached.

Fn. 5/ cont'd.

trade: (a) complete equalization of product price ratios; (b) complete equalization of total values of import and export for any country. The second condition fails to be representable in the box diagram. (The first condition is being represented by the parallel relationship). For a more satisfactory diagrammatic analysis of this problem, making use of the "national preference system" and the (so-called) "map of production frontier", see Dr. W. W. Leontieff "The use of Indifference Curves in the Analysis of Foreign Trade," reprinted in the Readings in the Theory of International Trade. The second fifficulty which has been assumed away is the dynamic process of approaching the equilibrium position, from the initial equilibrium positions - i.e., the converging process. Apparently our diagrams are far from adequate to handle this problem.

These observations are immediately seen from diagram 32, and hardly need any further explanation.[6/]

Especially worth noticing is proposition (2). For, by this proposition, if we know the isolation equilibrium positions of the two countries, we readily know the impact of foreign trade on functional distribution of one and/or both countries; and we also know the impact of foreign trade on the "social distribution ratio" of one and/or both countries. Coupled with proposition (3), we then know what should be the proper, e.g. "protection policy" to bring about certain desirable results.[7/] This problem seems to be a realistic one and has received the treatment of the economists on the theoretical level.[8/]

6/For propositions (1) and (2) see rule (25) Proposition (3) can be diagrammatically proved by Dr. Leontieff's method. See footnote /5 .

7/Take the case depicted in diagram 32 for instance, where the functional distribution ratio becomes unfavorable for labor in country A after trade. By rule (25), we know that the social distribution ratio is also adversely affected (for labor). If the desired goal of policy is to improve the relative position of labor, the proper policy for country A to adopt is to impose trade restrictions - to bring Ta higher. We can say that if the relative strengths of demand for the two commodities are approximately the same for the two countries - as indicated, for instance, by the fact that the output ratios at Ia and Ib are approximately the same - the final equilibrium will be established in such a way that the land-abundant country (as measured by the relative endowment ratios of the two countries, e.g. country A in diagram 32) becomes the exporter of food, which is the situation depicted in diagram 32 This means, of course, that ordinarily the imposition of trade barriers by a land-abundant country tends to improve the distribution ratio (functional and social) in favor of labor, which was the argument used in the "Australian Case" cited in footnote (8) below. (In other words, ordinarily, a labor-abundant country cannot use this argument).

8/The case of "Australian Protective Tariff" is an example. See e.g. M. C. Samuelson "The Australian Case for Protection Re-examined." Q.J.E. LIV (1939-49) 143-151.

Another application of our diagrammatic methods may be pointed out. We know that if the initial equilibrium positions of one or both countries are higher, the final equilibrium positions will be higher.[9/] Our diagrammatic method, then, not only solves a static general equilibrium problem but also will be of some use for the analysis of a "comparative static" problem - or, at least, for an indication of the general directions of the solution of a comparative static problem.

Section two: The Determination of the Specialization Status of the Trading Countries.

In our analysis of the specialization status of the trading countries in the previous chapter, we observed that we were incapable to solve the problem then.[10/] (This is obviously true because we could not have hoped to analyze the determination of the equilibrium position without knowing the relative strengths of demand.) We can now briefly analyze this problem.

In diagram 33, let the optimum allocation curves OA, and OB be shown - for countries A and B. Let the critical point C be located - by drawing BC parallel to the OA diagonal.[11/] We know that the slope of OC represents the lower critical ratio of country A and the slope of AD represents the upper critical ratio of country B.

9/This statement is not always true, but it would be true if, for example, the initial equilibrium point, or points, are raised sufficiently high. This is true because the equilibrium radial line OR always runs between the two initial equilibrium points.

10/See footnote 16 on page 88

11/See page 67 above. (BC11AD)

When the initial equilibrium positions of the two countries are given, we can readily derive the following conclusions, with the aid of diagram , (and with the understanding that the equilibrium radial line necessarily runs between the initial equilibrium points of the two countries):

a) If Ia lies in the portion (on the OA curve) between points O and D and if Ib lies in the portion (on the OB curve) between points C and B, both countries will be incompletely specialized.

b) If I'a lies between points D and A and if Ib lies between C and B, country B may become completely specialized for the production of clothing but country A cannot become completely specialized.

c) If Ia lies between O and D and if I'b lies between O and C, country A may become completely specialized in the production of food but country B cannot become completely specialized.

d) If I'a lies between A and D and I'b lies between O and C all the results in a) b) and c), above, are possible.[12/]

12/Apparently the four propositions exhaust all the possible significant combinations of the locations of the initial equilibrium positions of the two countries —if neither country is initially completely specialized.

The geometrical validity of these statements is immediately seen from the diagram and no further elaboration is required. The economic interpretations of these results are also quite obvious. Take proposition (b), for instance. It states that if the forces of demand for clothing are strong in both countries, the labor-abundant country (B) may be "forced" into the status of complete specialization in production for clothing (before the land-abundant country (A) will be forced to do so.)

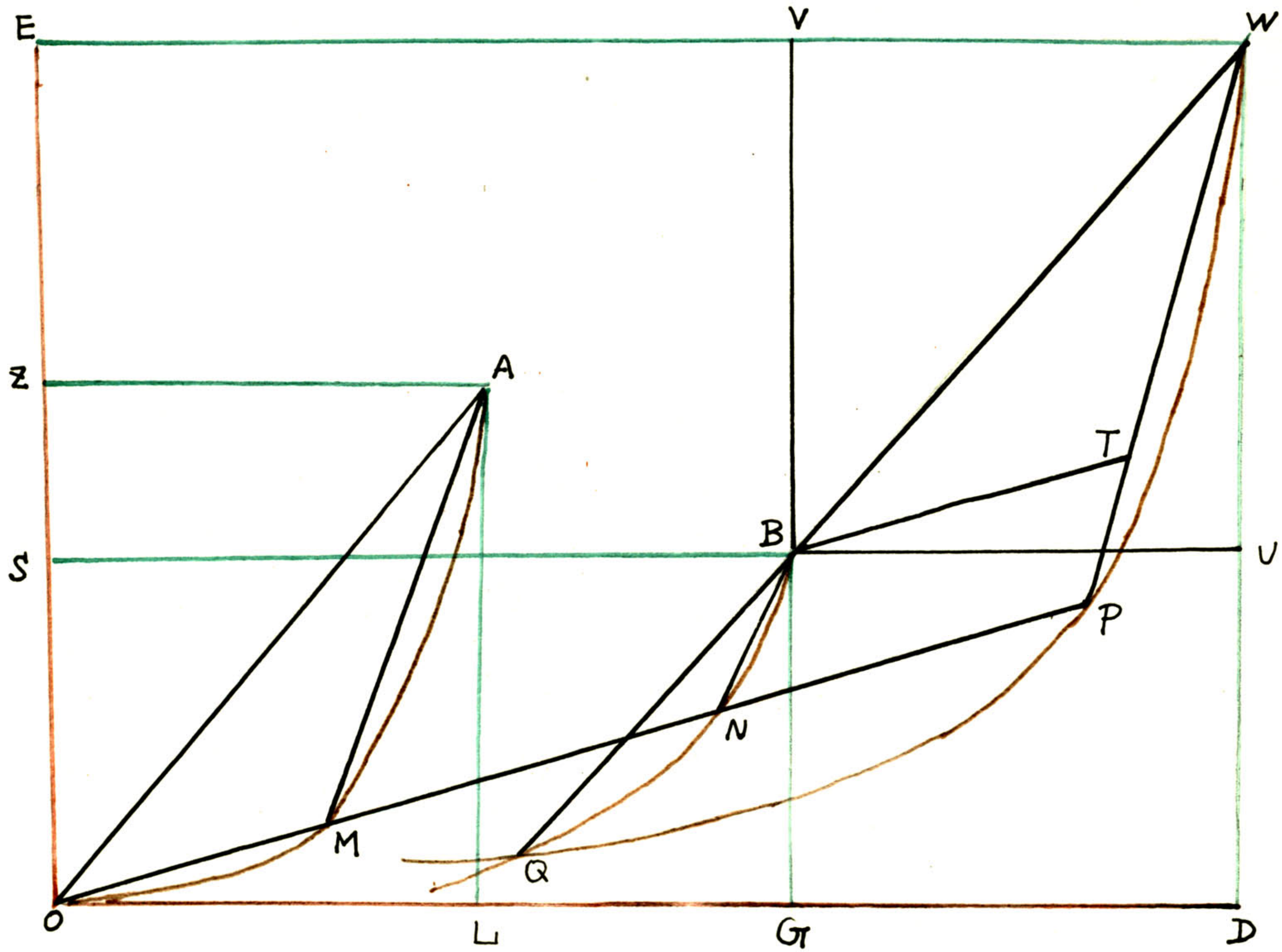
This diagrammatical method can be easily extended to the analysis of the other interesting cases. Take the case in which the endowment ratio of country B is so different[13/] from that of country A that point B, in diagram 33, actually lies below OR. In this case, no matter where Ia and Ib are located, incomplete specialization for both countries is not obtainable. The conclusion supports our previous assertion in Chapter IV.[14/]

The strength of our conclusions on the analysis of specialization status can be further improved if we take into consideration the "size" of the countries, in addition to the endowment "ratios" of the various countries. Intuitively we know that it is more likely for a small country to be completely specialized than for a big country. This intuitively obvious conclusion finds support in the analysis by our diagrammatic method. This will be done more conveniently in a later chapter when the problems of multiple equilibrium, i.e. equilibrium involving more than two trading countries, are discussed.[15/]

13/In other words the endowment ratios of the two countries must differ by more than "zero" degree. See page 100 above.

14/See page 86 above. (Notice, apparently we can make the stronger statement there cited).

15/See below page 148



Diag. 34

Section three: Factor Immobility and World Production Efficiency.

In this section we apply the technique of box diagram to study "the effect of international immobility of factors of production on the production efficiency from the world-viewpoint under the assumption of complete and incomplete specialization."

In diagram (34) let factor endowments of country A and B be represented by boxes OLAZ and OGBS respectively. The "world" endowment will be ODWE -- the box BUWV equals to OLAZ. The optimum allocation curves for countries A, B and the "world" can be drawn -- the curves O A, O B and O W. (We have three systems of contour maps with origins at A, B, and W for food). The curve OW represents the world production efficiency with perfect factor mobility between countries.

We define the situation where the combined output of the two countries can be represented by a point on OW (i.e., the "world" optimum allocation curve) as cases where the "production efficiency from the world viewpoint" is not impaired. Otherwise, we considered the world production efficiency as being impaired, by definition. The OW curve defines, in a schedule sense, the optimum output of the entire world, from the viewpoint of production efficiency. It actually becomes the optimum allocation curve of the entire world if the two countries are completely integrated -- or, in the language of our assumptions, if the factor immobility between countries is relaxed. When the assumption of factor immobility is <u>not</u> relaxed, the OW curve properly belongs to the sphere of study of "international welfare economics" and should be distinguished from the OA curve and the OB curve which belong to the study of the operation of a competitive system.

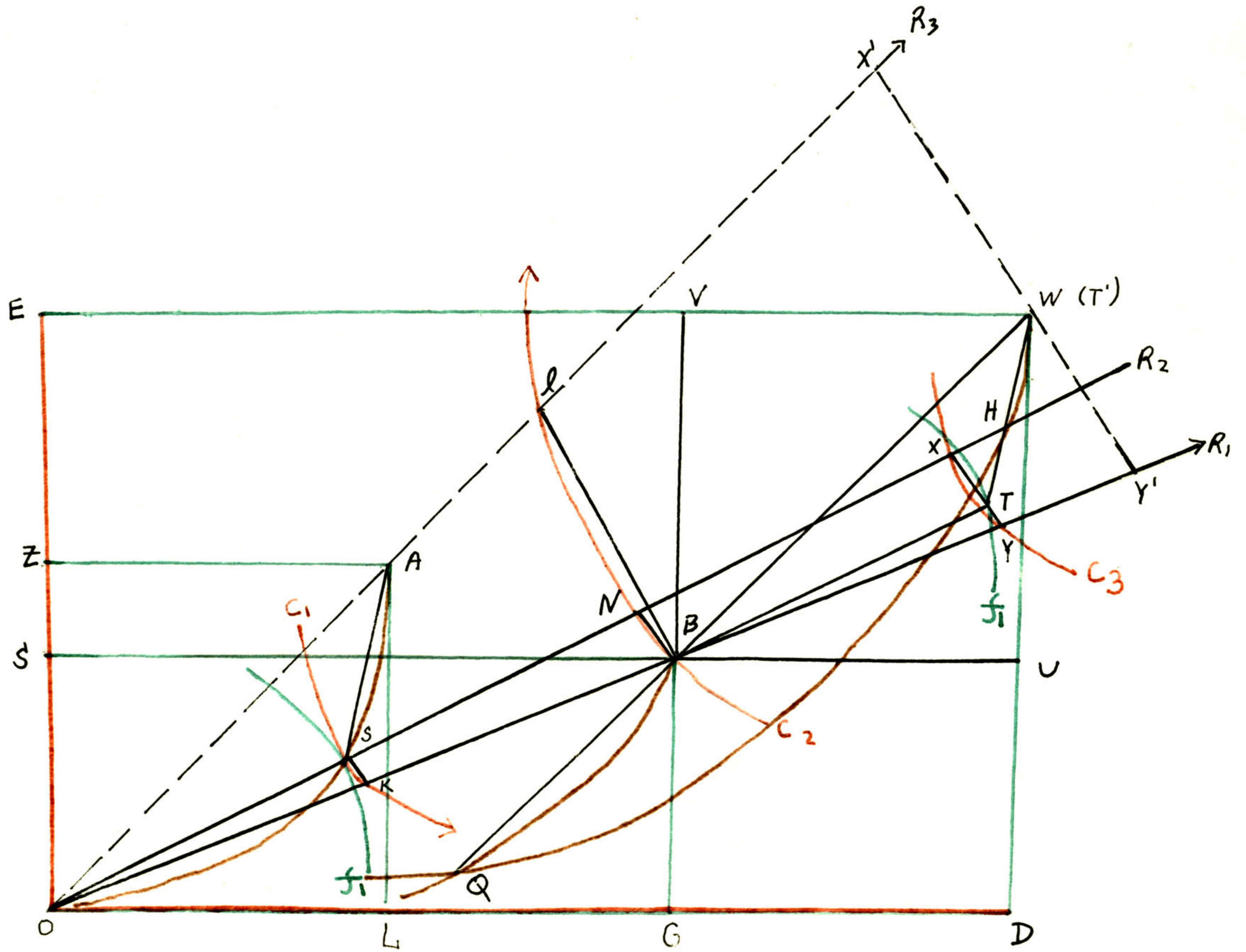
We shall study, first of all, the cases where neither country is completely specialized in trade equilibrium. Let the straight line OR represent any equilibrium position where neither country is completely specialized. OR intersects the three optimum-allocation curves at M, N and P. Join the parallel lines MA, NB and PW. Through B draw BT parallel to OR intersecting WP at T. NBTP is a parallelogram. Draw the diagonal lines OA and PW, which are parallel. The triangles OAM and BWT are equal. Thus we have:

OM equal to BT equal to NP;

BN equal to TP. AM equal to WT. So we have OM plus ON equals OP and AM plus BN equals WP.

Under the assumption of constant returns to scale, the last two equalities mean that the combined output of the two countries can be represented by a point on the world optimum allocation curve (OW). The conclusion must be: under any equilibrium position, the world production efficiency cannot be improved by an integration of the two countries (i.e., by an elimination of the barriers to international factor mobility, e.g. "Immigration laws").

When one country is completely specialized the conclusion is different. In diagram (35) OR1 represents the equilibrium position where country B is on the margin of complete specialization (for clothing). Let OR2 represent the actual equilibrium position, which intersects OA (curve) at S and OW at H. Let the production contours passing through the points S and B be C1 and C2, respectively, for clothing. Let C1 intersect OR1 at K; and C2 intersect OR2′ at N. By the assumption of constant returns to scale the straight lines SK and NB are parallel.



Diag. 35

Join the straight lines AS and WH. Through B draw a straight line BT parallel to OR2 intersection WH (extended) at T. Through T draw a straight line parallel to NB (and SK) intersecting OR2 and OR1 at X and Y respectively. NBTX is a parallelogram. The triangles OKS and BYT are equal. (This can be readily seen if the relative positions of the curves in the boxes OZAL and BYWU are compared). So we know:

OK equals BY and OS equals BT equals NX

and OK plus OB equals OY and OS plus ON equals OX

Since the combined output of clothing by countries A and B is represented by C2 plus C1, the combined output must be represented by a contour passing through point X and point Y by the assumption of constant returns to scale. Let this contour be C3.

The output of food -- which is now produced by country A alone -- is represented by a contour (belonging to the contour map of the box OLAZ) passing through point S. Since AS and WT are equal and parallel, the output of food in the "world" box must be represented by a contour passing through point T, i.e. F1. The contours C3 and F1, which represent the actual combined output in the "world" box, obviously cannot be tangent to each other -- for C3 cannot pass through point T if it passes through point X and point Y -- by the concavaty of the contours. Thus, actual combined output cannot be represented by a point on the "world" optimum allocation. The conclusion must be: "in an equilibrium position, when one or both countries are completely specialized, the production efficiency of the world is impaired.[16/] This is a consequence of international immobility

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16/Three remarks may be added:

(a) The case in which both countries become completely specialized in the production of the same commodity is apparently the limiting case. In diagram 13, draw OA diagonal and call this line OR3. Points X', Y' and T', of factor of production." The common sense interpretation is that the best way to use the world resources cannot be achieved. It is possible to increase the output of both commodities by a reallocation of resources across the national frontier in order to realize a better result obtainable under the given state of technology of production.

Fn. 16/ cont'd.

which are connected by a straight line, must fall on OR3, OR1 and point W respectively. The straight line connecting them is parallel to B1. Contour O4 (passing through X and Y) represents world output of clothing with no output of food.

(b) In diagram (34) or (35), point Q is the point of intersection of OB curve with OW curve. The general rule for the intersection of two or more optimum allocation curves is as follows: if the straight line connecting the two upper right vertices of two boxes intersects the optimum allocation curve of one box, it intersects the optimum allocation curve of the other box at the same point (e.g. OB curve, OW curve and WB line (extended) meet at Q).

In our case, because BQ is parallel to AO (since WB is parallel to), point Q becomes the point where "the parallel relationship" is on the margin of being unable to be satisfied between country A and B. (See footnote on page 67; point Q corresponds to point C in diagram 18).

The economic interpretation of point Q is as follows. When OR intersects OB curve below point Q, country A becomes completely specialized. It can be similarly shown that this impairs world production efficiency. This is symmetrical to the case proved in the text.

(c) Our conclusion does not apply to the case where the endowment ratios of the two countries are the same. When both countries are completely specialized in clothing, for instance, X', Y' fall on W -- world production efficiency is not impaired.

Section four: Product Imperfect Mobility and World Production Efficiency

We can study the effect of a tariff on the production efficiency from the world point of view.[17/] Let us make the simplifying assumption that when the tariff equilibrium is established, no country is completely specialized. The technique used in this study closely follows the technique developed in the last section -- therefore the boxes and optimum allocation curves in Diagram 36 need no explanation.

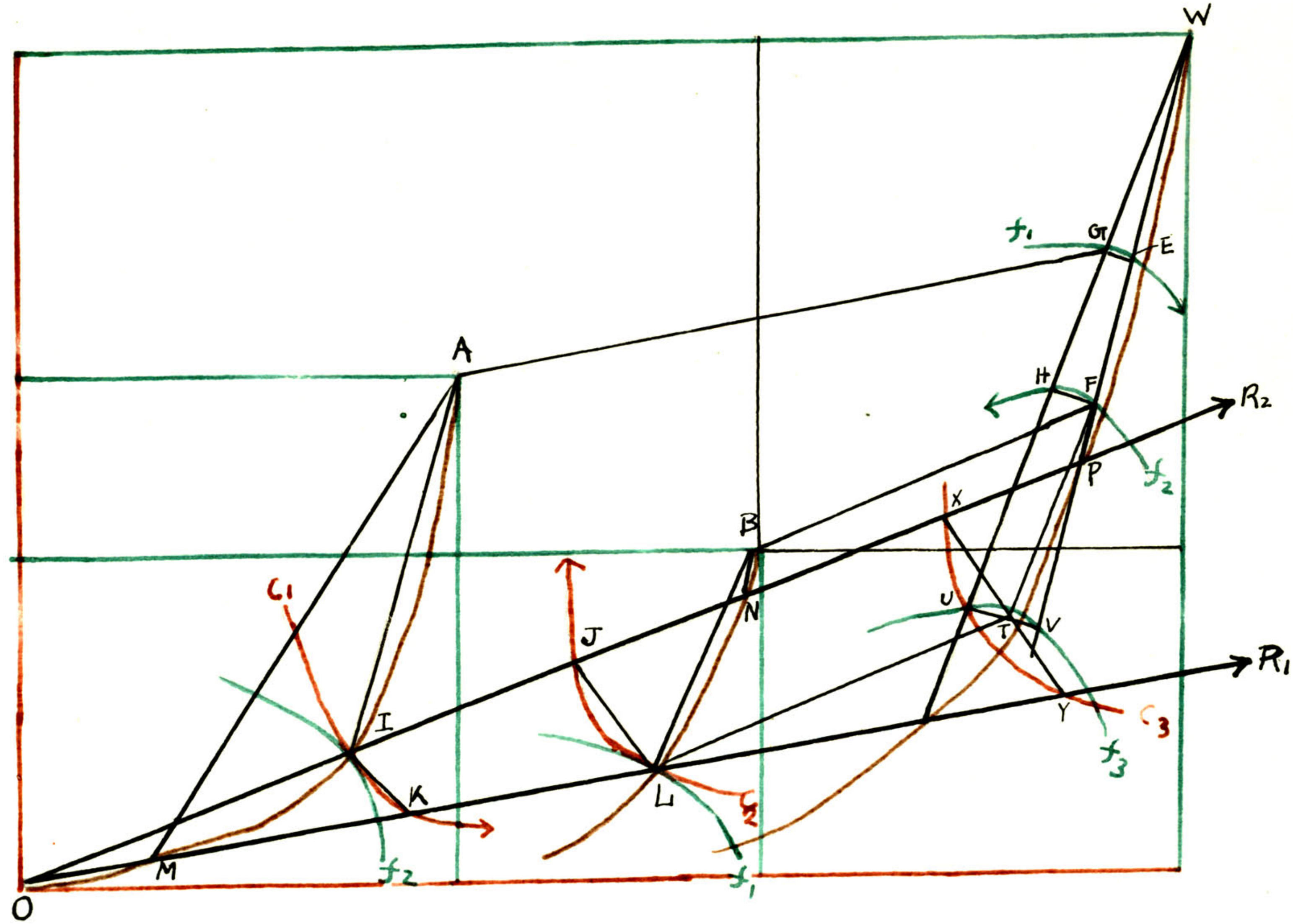
Let the tariff equilibrium positions of the two countries be represented by points I and L on the two optimum allocation curves. Points I and L are picked in such a way that they do not satisfy "the parallel relationship" -- otherwise product price ratios will be equalized between the two countries which is generally incompatible with the existence of a tariff. Through I and L, draw the radial lines OR2 and OR1 intersecting the three optimum allocation curves at (I, N, P) and (M, L, Q) respectively. Join the parallel lines (AM, BL, WQ) and (AI,BN,WP).

From B draw BF parallel to OR2, intersecting WP at F. BNPF is a parallelogram. From A draw AG parallel to OR1, intersecting WQ at G. AMQG is a parallelogram.

BN equals FP. AM equals GQ.

Since AI plus BN equals WP, so AI equals WF. Since AM plus BL equals WQ, so BL equals WG. Thus we know, in the food-contour-map of the "world", the output of food of country A is represented by the contour passing through point F (i.e. F_2); and that of country B is represented by the contour passing through point G (i.e. F_1). Let F1 intersect WP at E; let F2 intersect WQ at H. Join the parallel lines (GE, FH).

17/The technique of the box diagram suggested in this section applies equally well to the case of a "subsidy".



Diag. 36

In the clothing contour map, let C1 pass through point I; and let C2 pass through point L -- C1 and C2 represent the output of clothing by country A and B respectively. Let C1 intersect OR1 at K; and let C2 intersect OR2 at J. Join the parallel lines IK, LJ.

From point L, draw LT parallel OR2. From point F, draw FT parallel WQ. LT and FT intersect at T. Through T draw a straight line parallel to JL (and IK) intersecting OR1 and OR2 at Y and X respectively. Through T draw another straight line parallel to HF (and GE) intersecting WP (extended) and WQ at V and U respectively.

Triangles FTV and WGE are equal;[18/] FTUH is a parallelogram. From these relationships, we readily derive:

WG equals HU. FV equals WE.

and hence WG plus WH equals WU
and WE plus WF equals WV.

Triangles LTY and OIK are equal[2];[19/] LTXJ is a parallelogram. From these relationships, we readily derive:

LY equals OK. OI equals JX.

and hence OI plus OJ equals OX

and OK plus OL equals OY.

By the assumption of constant returns to scale, we know that in the "world" contour systems, combined output of food is represented by a contour passing through points U and V; combined output of clothing is

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18/All three sides are parallel, and FT equals WG (equals BL), since FBLT is a parallelogram.

19/All three sides are parallel and LT equals OI (equals BF). TLBF is a parallelogram. It is seen that OI equals BF because "AI equals and parallels WF."

represented by a contour passing through points X and Y. Let F3 and G3 be the two contours.

By the property of concavity of the contours, C3 and F3 necessarily straddle point T -- hence, they cannot be tangent to each other. Total output cannot be represented by a point on the world optimum allocation curve (OW). So we conclude:

"Tariff impairs production efficiency from the world viewpoint."

So we see (in view of the last section) that both imperfect mobility of factor and imperfect mobility of product tend to impair the production efficiency of the world as a whole.

These are cases where "the rule" of Dr. Lerner is not satisfied. [20/] However, the tariff is probably more irksome not only because it is more artificial, but also, as we saw, because it always impairs world production efficiency, while factor immobility only does that sometimes. [21/]

20/A. P. Lerner "The economics of Control" The Macmillan Company, 1944.

21/This remark is subject to the qualification that under certain conditions factor immobility impairs world efficiency too. This will be the case where the difference of the endowment ratios between the two countries is so great that "the parallel relationship" can never be satisfied between them.

Chapter VI.

The Box Diagram and The Map of Production Frontiers - Applications to Multi-equilibrium Problems in International Trade.

Section one: Introduction

Our diagrammatic analysis of the problems of international trade, undertaken in the previous chapters, can now be generalized in two ways. First, we can relax the assumption of "two countries" and analyze the multi-equilibrium problems of international trade - i.e. equilibrium involving any number of countries. Secondly, we can analyze the cases under which one factor of production may become a free agent when equilibrium is established. The present chapter is devoted for the analyses of these problems.

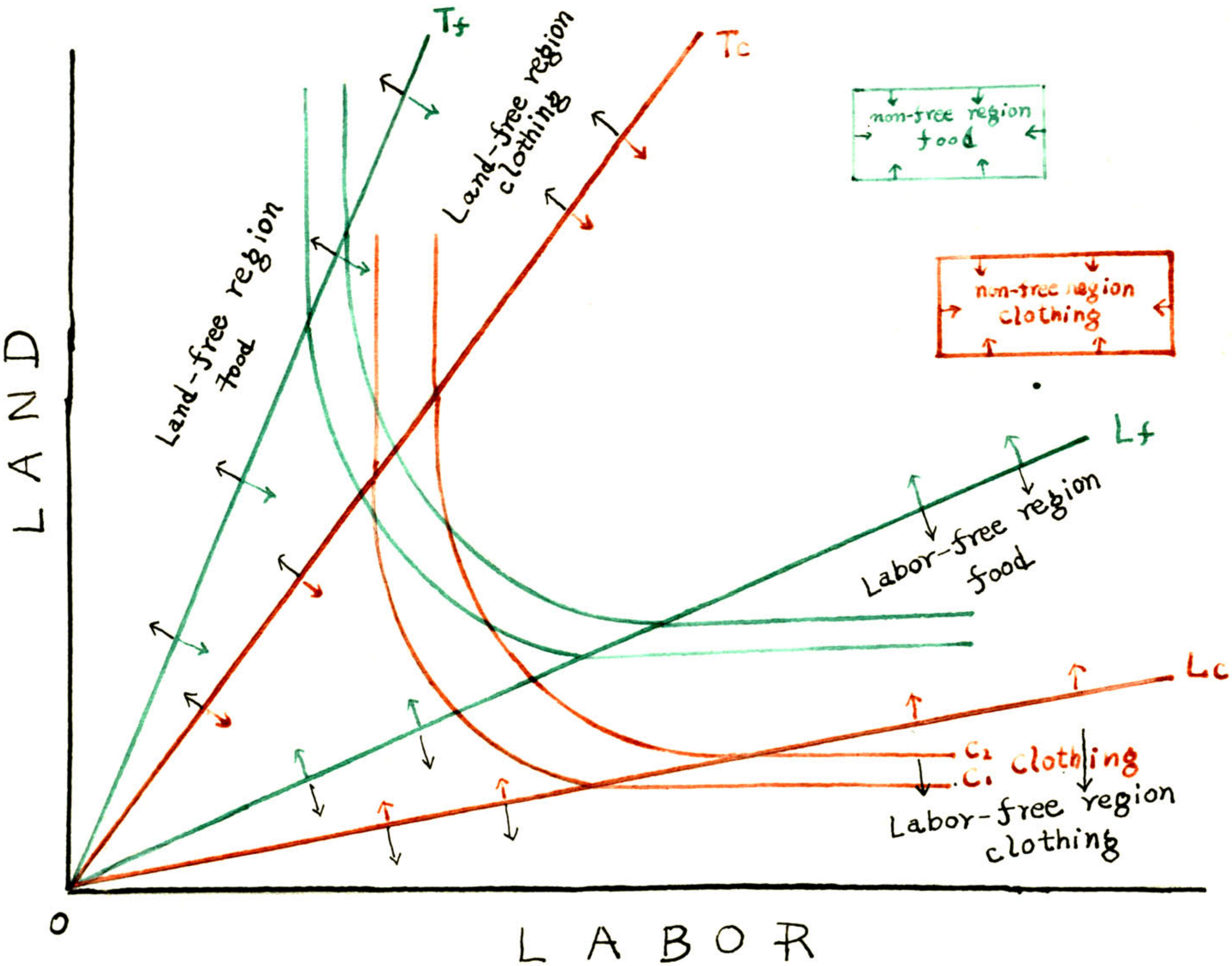
The undertakings of this chapter also pave the way for our analysis in the following chapter --on certain problems in the history of economic doctrines. For this reason, we will develop, in this chapter, the so-called "map of production frontiers" from the box diagram, which facilitates our later exposition.

Finally, we include an appendix at the end of this chapter dealing with the construction of the "critical map"1/ under the generalized conditions as indicated in the first paragraph above.

Section two: The Ridge Lines

The analysis in the previous chapters ruled out the possibility that one of the factors may become a free agent. We want to generalize our diagrammatic representation in order to take care of this possibility.

1/The "critical map" as developed in Chapter IV above, See page 76


Tf
Tc
Land-free region clothing
Land-free region food
non-free region food
non-free region clothing
Lf
Labor-free region food
Lc
C2
C1
clothing
Labor-free region clothing
LAND
LABOR
O

Diag. 37

That one factor of production may become a free agent is the consequence of the fact that the marginal rates of substitution of the two factors (as defined by the production contour maps) may become infinite or zero. Geometrically, this stage of affairs is represented by the vertical and the horizontal portions of the productions contours. The economic significances of the "Ridge Lines" of the contour maps, discussed earlier,[2] must now be exploited.

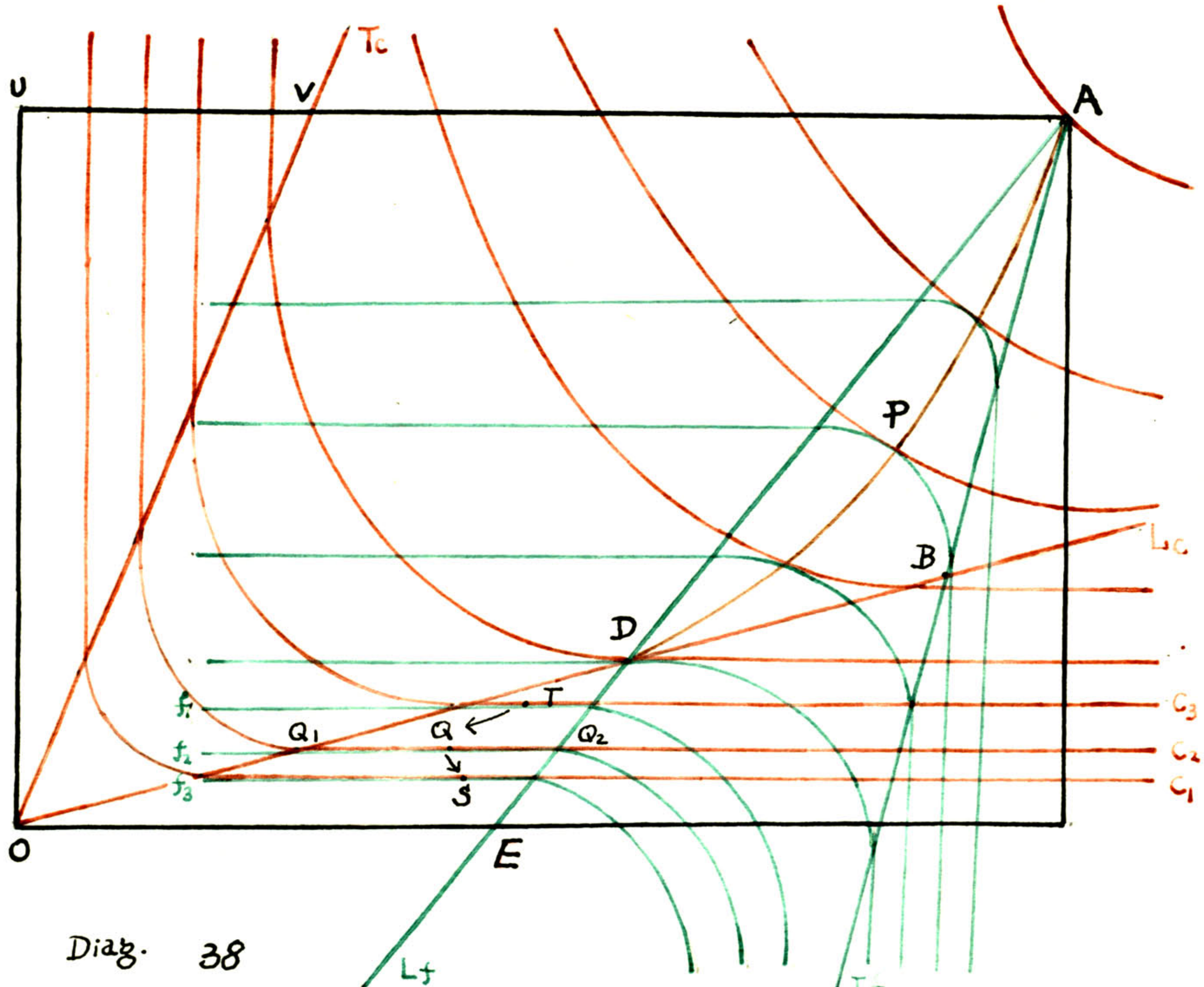
In diagram 37 the two maps of production contours - red for clothing and green for food - are superimposed upon each other, right side up. The four ridge lines - Tf, Tc, Lf, and Lc - are shown.[3] We recall that they are straight lines, by the assumption of production functions with constant returns to scale[4] and that Tf > Tc and Lf > Lc, by the definition of the relative factor intensities of the two commodities.[5]

2/See above page 16 , Chapter I

3/See page above. In footnote _/ on page , we have assigned a double meaning to these expressions, e.g. Tf represents the land (Capital T) free ridge line for the production of food (subscript f) and also represents the input ratio corresponding to the (same) ridge line.

4/See above page 16

5/See above page 39 . Rigorously, the only limitations on the input ratios for the "set" of ridge lines are as stated in the text. However, in our exposition throughout this chapter, we have added another explicit assumption, namely, Tc   Lf (see diagram 37). The implication of this simplication will be discussed in the appendix.



Diag. 38

We also recall that the region of a map of production contour, straddled by the two ridge-lines of the (same) map is called the non-free region of the map.[6/] The non-free regions of both maps, and the labor-free and land-free regions of both maps, are indicated in diagram

In diagram 38, a box diagram is constructed with the four ridge lines shown - Tc, Lc, Tf and Lf. It is seen that the common ground covered by the non-free regions of both maps (for food and clothing) is included in the triangle ABD. The area enclosed by the triangle ABD, then, may be called the non-free region of the box diagram. From diagram , it is clearly seen, that the optimum allocation curves - e.g. APD - lie, and only lie, in the non-free region of a box diagram. (This is true because of the fact that, e.g. the optimum allocation curve passes through point D, which is clearly a point of tangency of the production contours.)

In the triangle DEO (diagram 38), the (horizontal) contours of the two maps coincide. Labor is the redundant factor and land is the scarce factor. The marginal rates of substitution are always zero in both industries. It is evident that, in this region, a commodity has constant opportunity cost in terms of the other commodity. The product price ratio is completely governed by the ratio of the quantities of the scarce factor (in the present case, land) "embodied" in each unit of the two commodities.[7/]

6/See above page 16

7/This is true by the assumption of production functions with constant returns to scale. Let the vertical distances between (S, Q) and (Q, T) - all in triangle DEO - be the same. (Diag. 38) Then (c3-c2) = (c2-c1) and (f2-f1) = (f3-f2) (See page 11 above.) Starting from point T, When equilibrium is established in the non-free region of a box diagram - i.e. in the case represented in diagram 38, when the demand for clothing is sufficiently strong so that equilibrium position is established at a point on the (grey) curve APD - neither factor will be a free agent. This is true because of the fact that the slopes of the production contours at the points of equilibrium are neither zero nor infinite.[8/] Otherwise, one factor of production - i.e. in this case labor - will become a free agent, (e.g. when equilibrium position is established at point Q).[9/]

Fn. 7/ cont'd.

when more food is successively produced, e.g. f1, f2, f3, the successive increments of (f2-f1) and (f3-f2) units of food are obtained at the opportunity costs, in terms of clothing, of (c3-c2) and (c2-c1) units successively. The product price ratio always equals to:

$$\frac{\text{price of clothing}}{\text{price of food}} = \frac{f2-f1}{c3-c2} = \frac{f3-f2}{c2-c1}$$

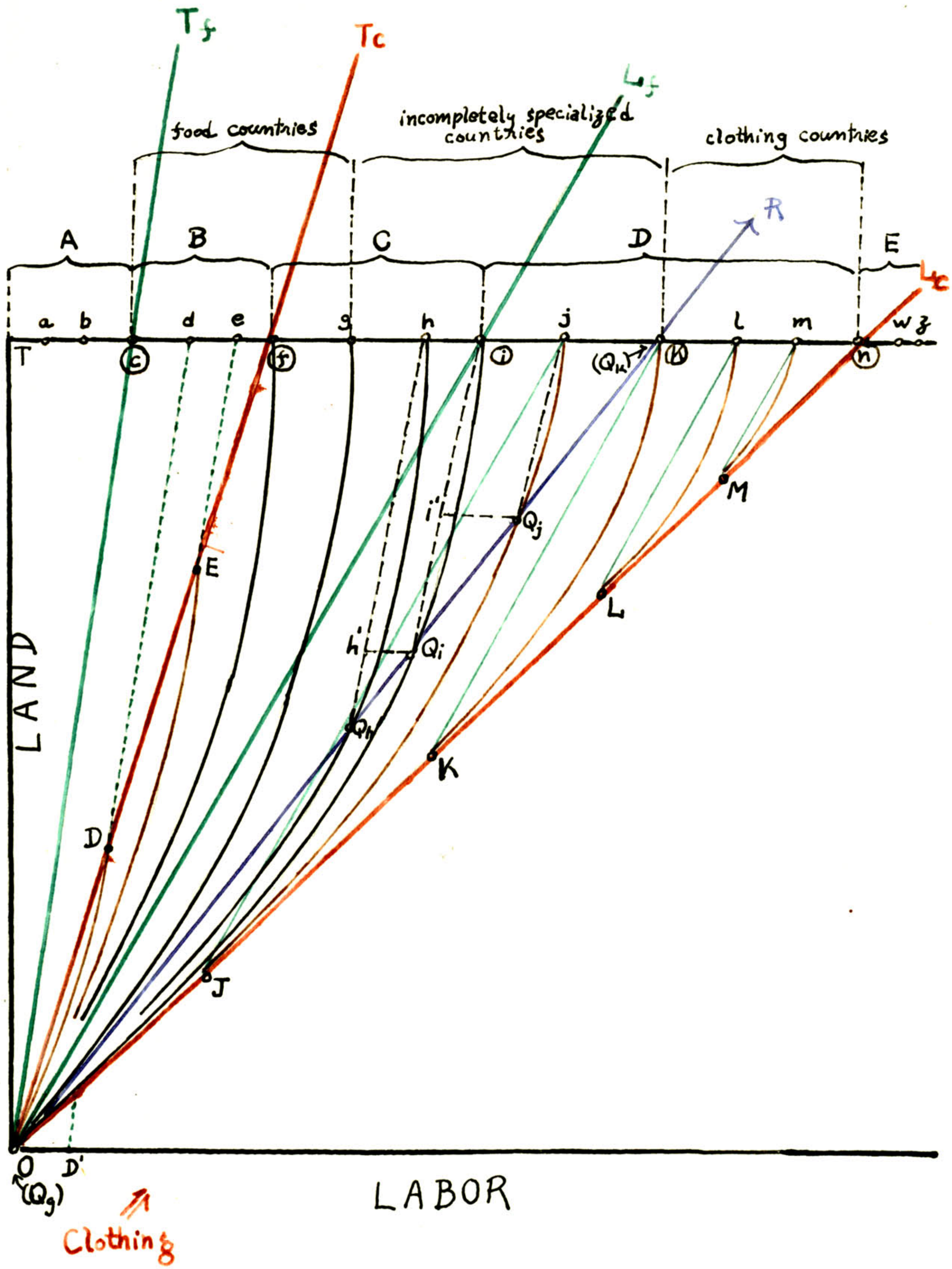
Let the vertical distances between T, Q and Q,S be k-units of land. The product price ratio can be alternatively represented by:

$$\frac{\text{price of clothing}}{\text{price of food}} = \frac{k/(c3-c2)}{k/(f_2-f_1)} = \frac{k/(c2-c1)}{k/(f_3-f_2)}$$

The numerators and denominators represent the amounts of the scarce factor (land) embodied in one unit of clothing and food, respectively.

8/This state of affairs in the present case included the upper end point "A" but excluded the lower end point D of the optimum allocation curve APD. For, at point D, obviously labor is free. At point A, on the other hand, the production contours for the production of food are irrelevant; it is the slope of the production contour for clothing, at point A, which indicates the equilibrium factor price ratio at point A.

9/It is obvious that, in this case, it is immaterial at which point the equilibrium position should be represented in DOE as long as the vertical position is "correct" --i.e. corresponding to the relative strength of demand for the two commodities. (For example, the equilibrium position point Q can be alternatively represented by any point on the horizontal line between Q1 and Q2, diag. 38)


Tf
Tc
Lf
Lc
R
food countries
incompletely specialized countries
clothing countries
A
B
C
D
E
T
a
b
c
d
e
f
g
h
i
j
(Qk)
k
l
m
n
w
z
D
E
h'
i'
Qh
Qi
Qj
J
K
L
M
LAND
LABOR
O
(Qg)
D'
Clothing

Diag 39

<u>Section three:</u> <u>The "Complete Box Diagram"</u>

In diagram 39, let the quantity of the land-endowments for all countries a, b, c, d, ...be the same, namely, OT; and let the labor-endowments of these countries be represented by the horizontal distances Ta, Tb, Tc, Td......etc. The box diagrams of these countries are superimposed upon each other. (The right-vertical sides of all the boxes are not shown).

Draw all the four ridge lines, Tf, Tc, Lf and Lc from the lower-left (common) origin of the boxes of the various countries (as in diagram 37). Let the points of intersection of the ridge-lines with the upper-horizontal side of the boxes be the points c, f, i, and n respectively. (See diagram 39). These four points demarked five regions: A, B, C, D and E, in an ascending order of relative labor abundancy. The special case described in diagram 38 above, is a country in region D -- e.g. country m, where the straight lines (green) mM and Lf are parallel.

For other countries, the optimum allocation curves can be readily found when the regions to which they belong are known. For instance, country d, in region B has an optimum allocation curve running from point O to point D which is the point of intersection of Tc and a straight line (dotted green) parallel to Tf, passing through point d. This is true because of the fact that the common ground covered by the two non-free regions of the maps of production contours for food and clothings (namely the non-free region of the box diagram), is enclosed by the triangle ODD'.[10/]

10/A straight line (dotted green) should be completed between D and D'.

Hence, it is seen, all the countries in region D, i.e. all the countries with endowment ratios between Lf and Lc, have optimum allocation curves which terminate, at the lower ends, on the ridge line Lc. This is true because of the fact that the straight lines (solid green) - e.g. jJ, kK, lL, mM...... - through the upper-right corners of the boxes of these countries, - e.g. j, k, l, m...... - necessarily intersect the ridge line Lc if they are parallel to Lf (solid green) e.g. at points J, K, L, M...... Referring back to diagram 38, it is evident that the optimum allocation curves of these countries must be terminated at these points - e.g. J, K, L, M....on Lc. 11/ 12/

Similarly, all countries, in region B - i.e. the countries with endowment ratios between Tf and Tc, such as countries d and e - have optimum allocation curves which terminate, at the upper ends, on Tc. 13/

Hence, countries in regions B and D have "incomplete" optimum allocation curves. (They are represented by the grey curves in diagram 39)

For countries in regions A and E no optimum allocation curve can be drawn - as indeed, there is no technical problem of allocation of resources for these countries, since one of the factors is always redundant.

11/These points are, respectively, the lowest points in the non-free regions. They correspond to point D in diagram 38.

12/This is true because of the fact that Lc is less steep than Lf.

13/That is, the straight lines (dotted green) dD, eE, which are parallel to Tf (solid green), necessarily intersect the ridge line Tc - e.g. at points D and E which are the highest points, respectively, in the non-free regions of the box diagrams.

The only countries which have optimum allocation curves of "full" length - i.e. running from point O to the upper-horizontal sides of the boxes - are all the countries in region C. (These countries have endowment ratios between Lf and Tc; their optimum allocation curves are represented by black curves in diagram 39). These observations can be easily verified, in each case, by drawing in the ridge lines (parallel to Tf and Lf) and by observing the location of the "non-free regions" of the maps of the production contours in the box diagrams.[14]

Section four: The Multi-equilibrium of International Trade

In diagram 39, an equilibrium position of international trade can be represented by a radial line, e.g. OR, intersecting the optimum allocation curves of the various countries at a series of points, e.g. Qg, Qh, Qi, Qj......which are the equilibrium positions of countries g, h, i, j,...... The series of points of intersection indicate the patterns of resource allocation of the various countries. This is true because of the fact that the "parallel relationship" still holds for any pair of countries (i.e. the dotted black lines hQh, iQi, jQj....are parallel). From this it can be readily proved that the product price ratios, between any pair of countries, are equalized[15] and hence, by induction, the product price ratios of all countries must be equalized at the series of points.

14/Hence, it is seen, that our discussions in the previous chapters are only special cases. In diagram 39, let Tc and Tf approach the vertical axis OT and let Lf and Lc approach the horizontal axis. All the countries, regardless of the endowment ratios, will be in region C.

15/See above page 62.

If we may assume, as an expository device, that every point on the ceiling of diagram   represents a "country" - i.e. if we may assume that there are an infinite number of countries, each with endowment of land OT, representing all possible endowment ratios - then, for any equilibrium position, two countries stand out on the margin of complete specialization, one for each commodity. For instance, at equilibrium position OR, country g and country k are on the margin of complete specialization for food and clothing respectively. Let us call them the "marginal food country" (country g) and the "marginal clothing country" respectively.

It is obvious, by the "parallel relationship", that the input ratio for the production of clothing (food) for any incompletely specialized country, equals the endowment ratio of the marginal clothing (food) country. Hence, when the equilibrium position of any incompletely specialized country is given, it is a simple matter to locate the two marginal countries in e.g. diagram 39 (i.e. to determine the endowment ratios of the two marginal countries.)

We will make certain applications of our diagram (39) in the three following sections (V, VI and VII). Our analysis will be brief; and, in most cases, we will only state the conclusions of the analysis. This is due to the fact that the validity of the assertions which will be made are either implicit in the construction of diagram 39, or can be readily deduced therefrom.

Section five: The Prospect of Factor Reward

First of all, an "internal problem" can be easily analyzed with the aid of our diagram - namely, the "prospects" of reward for the two factors of production, relative to the endowment ratios of the countries. For this problem, the significant classifications of the countries, according to the ratios of factor-endowment, are the five regions: A, B, C, D, and E - in an ascending order of relative labor abundance (diagram 39):

(a) Region A includes all countries with endowment ratios greater than Tf. For these countries, land is always redundant and is always a free agent regardless of the relative strength of demand for the two commodities. Labor - the scarce factor - always received all the product - i.e. the full distributive share is accruable to labor. For this reason, Region A may be called the land-absolutely-abundant region, (or, the countries included in this region, e.g. a, and b, may be called the land-absolutely-abundant countries.)

(b) Region B includes all countries with endowment ratios between Tf and Tc where land may be free, depending upon the relative strength of demand for the two commodities. (If the demand for the land-intensive commodity (food)is strong enough, land will not be free, otherwise it will be a free agent). Labor, on the other hand, is never a free agent. For this reason, the countries in this region (B) may be called the land-relatively-abundant countries.

(c) Region C includes all countries with the endowment ratios between Tc and Lf. Neither factor will ever be a free agent regardless of the relative strength of demand. For this reason, the countries in this region may be called the <u>non-free-countries.</u>

(d) Region D, which includes the countries with the endowment ratios between Lf and Lc, is symmetrical to region B. Labor may become free while land is never a free agent. For this reason, Region D may be called a labor-relatively-abundant region.

(e) Region E includes all boundaries with endowment ratios higher than Lc. It is symmetrical to Region A and may be called the labor-absolutely-abundant region where labor is always free while land is never free.

As is implied in the construction of diagram 39, the classification of the five regions is completely governed by the technological considerations, i.e. they are determined by the input ratios corresponding to the four ridge lines.

The distinction of the five regions is somewhat interesting from the viewpoint of the development of value theory in the history of economic doctrine. This problem will receive our detailed analysis in the following chapter. 16/

Section six: The Specialization Status and the Theory of Equalization of Factor prices.

In a previous chapter, we have considered the theory of international equalization of factor prices, of Professor Samuelson, for the case of two countries. 17/ In this section, we extend the analysis to include "many countries", and we also make the more generalized assumption that one of the two factors may become free. 18/

16/See below Chapter VII

17/See above page 58.

18/See section one of this chapter.

We assume, first of all, that the equilibrium position is already determined - e.g. as represented by the radial line OR in diagram For the analysis of the equalization of factor prices, at this particular equilibrium, the significant classifications of the countries, according to the endowment ratios, are as follows:[19/] (See diagram 39)

I. Region A - the land-absolutely-abundant countries defined in the previous section.

II. Region E - the labor-absolutely-abundant-region defined in the previous section.

III. The incompletely-specialized countries - the countries with endowment ratios falling between the endowment ratios of the two marginal countries (i.e. countries g-k inclusive for OR in diagram).

IV. The infra-marginal-food-countries - the countries with endowment ratios lying between Tf and the endowment ratio of the marginal food countries (i.e. countries c-g inclusive).[20/]

V. The infra-marginal-clothing-countries - countries with endowment ratios lying between Lc and the endowment ratio of the marginal clothing country (i.e. countries k-n inclusive).

The determination of the five regions depends upon technological considerations and the equilibrium product-price ratio (which can be

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19/Not to be confused with the classification (of five regions) undertaken in the previous section which dealt, entirely, with an internal problem. The classifications, here undertaken, are marked at the top of diagram

20/It may be observed that Region A (i.e. the land-absolutely-abundant region) is also included in the infra-marginal region for complete specialization in food. However, relative to the purpose of this section, it should be separated.

traced back to the forces of the relative strength of demand). As will be pointed out later,[21] this classification of the countries (into five groups) is possible (for any given equilibrium position) only under particular assumptions.

The five groups of countries, as identified above, are indicated at the top of diagram 39. With the aid of this classification (and diagram 39), we may readily state the following conclusions with respect to the theory of equalization of factor prices, (with only moderate elaborations):

(a) For incompletely specialized countries (i.e. countries g-k inclusive), factor prices are equalized, both absolutely and relatively, between the countries.

The fact that factor price ratios of these countries are equalized (i.e. factor prices are equalized relatively) is a straightforward extension of our earlier analysis. We have been able to prove that for any pair of countries in this group, the factor price ratios must be equalized.[22] Hence, by induction, the factor price ratios of all countries in this group must be completely equalized.

What we mean by the assertion that factor prices are equalized absolutely can be illustrated by the reward of labor in different countries. The marginal physical productivity of labor, for the production of the same commodity (food or clothing) must be completely equalized

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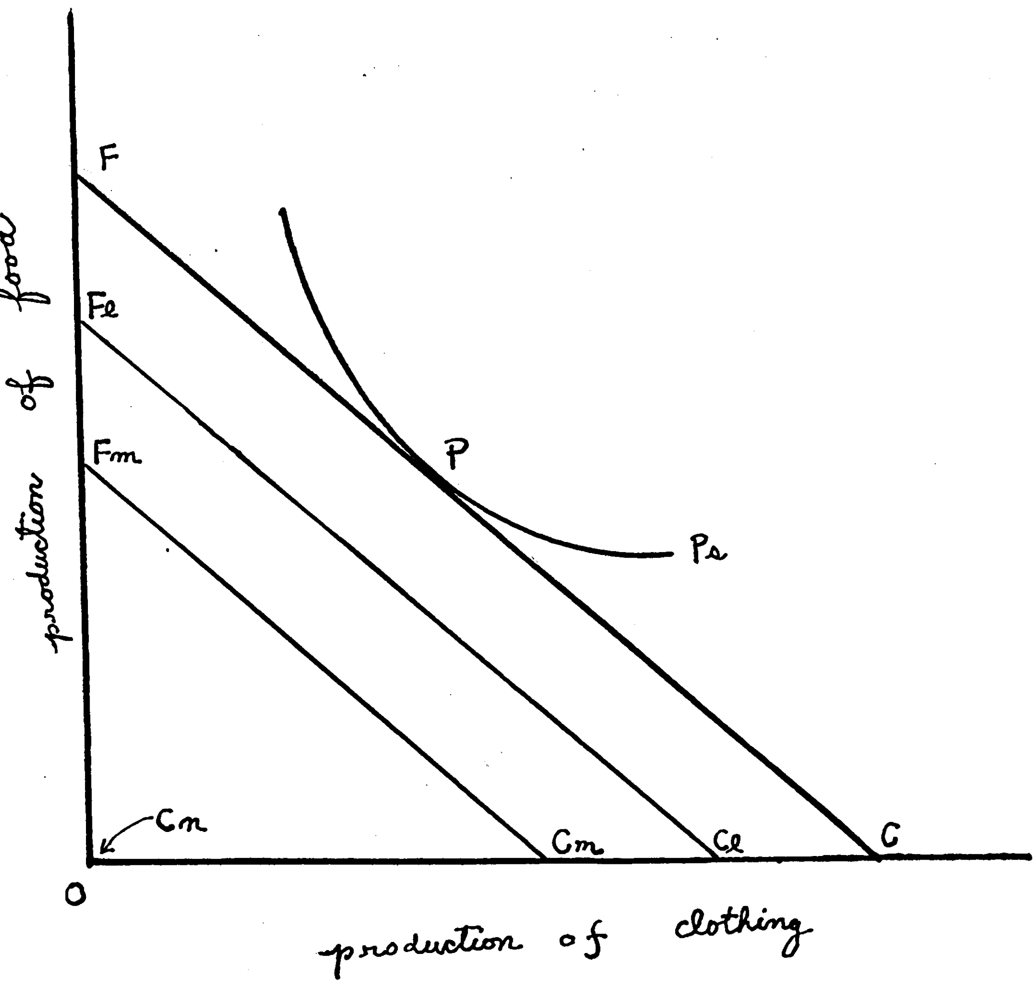
21/See below page 176.

22/See section II on page 64 above.



Diag. 40 Marginal Physical Productivity of Labor

between the countries in this group. This is true by Rule (13) on page 28 , which states that the marginal physical productivities are completely determined by the ratio of inputs (for the production of either commodity).[23/]

The "real wage" of labor, at this particular equilibrium position, can be represented as in diagram 40. The marginal physical productivity of labor, for the production of clothing, is plotted on the x-axis (e.g. OC), and the marginal physical productivity of labor for the production of food is plotted on the Y-axis (e.g. OF). The slope of the straight line FC, then represents the equilibrium product price ratio. This is true due to the fact that OC and OF represent the reward of labor, (in terms of clothing-unit and food-unit), employed in the different industries of the same country. The exchange value of OF units of food, necessarily equals OC units of clothing --by the assumption of perfect mobility of labor between the two industries.[24/] Hence, OF/OC represents the product price ratio (price of food/price of clothing).

The straight line FC in diagram 40 then, can be taken to represent the real wage of labor, in the sense that the actual consumption of labor can be represented by a point on it.[25/] (The problems of index numbers, which is the basis on which an "ambiguous measure" of real wage can be obtained, are avoided in diagram 40)

23/Apparently the ratios of input, for the production of the same commodities, by the different countries in this group are equalized - by the parallel relationship.

24/Boiling down to its logical content, a trivial version of labor theory of value was born out of this fact. See chapter VII below, page 218.

25/This is true by the fact that the slope of FC represents product price ratio. The actual consumption point - e.g. point P in diagram 40 - is obtained as the point of tangency of FC with the psychological preference systems of the individual labors.

Since the marginal physical productivities of labor in all countries are equalized (and the product price ratios are, of course, also equalized), diagram 40 (i.e. the curve FC) can be taken to represent the real wage of all countries at this particular equilibrium level. Hence, the real wages are equalized absolutely between the various countries in this group.

Similarly, the real rents of the various countries, at this particular equilibrium level, must also be completely equalized, absolutely.

(b) For infra-marginal-food (clothing) countries, both factor price ratios, and absolute level of factor rewards, are different for all countries. The higher the endowmant ratio (as compared with the other countries) the higher will be the factor-price ratio and the higher (the lower) the real wage (real rent). (The limiting position is reached when endowment ratio is as high (low) as Lc(Tf) where Region E (A) is reached and labor (land) becomes a free agent.)

This conclusion is easily supported -- with the aid of diagram 39 Imagine that the production contours for clothing are there. The increasing endowment of labor is represented by moving from point k to point n along the horizontal line kn (i.e. for the infra-marginal country for the production of clothing). The slopes of the production contours must be decreasing as the point moves toward point n - by rule (11) on page (18). This proves that the factor price ratio is decreasing as the endowment ratio is decreased.26/

26/The contour maps for the production of food are irrelevant for the determination of the distribution --because all the factors of production are being allocated for the production of clothing, for this group of countries.

Since the input ratios for the production of clothing are higher when the endowment ratios of these countries are higher, rule (11) on page 22 states that the marginal physical productivity of labor will be higher when the input ratio is higher. Referring to diagram 40, the marginal physical productivities of labor in countries k, l, m, n,.... can be represented by the horizontal distance of the points C, Cl, Cm, Cn. The (dotted) straight lines, (which are parallel to the straight line FC) represent the real wage of these countries --since the slope of FC represents the equilibrium product price ratios for all countries. It is seen that e.g. the real wage of country m is smaller than that of country l (i.e. the straight line FmCm is everywhere lower than the straight line FlCl.[27/]

In diagram 40 , when the endowment ratio is as low as Lc - i.e. country n - the production contour at point n will be horizontal. This means that the factor price ratio and the real wage will be zero.

Similarly, for the infra-marginal-food countries - i.e. countries c-g inclusive - it can be easily shown that both the factor price ratio and the absolute level of factor reward are different from country to country. When the input ratio is higher (e.g. country d rather than country e), the factor price ratio is higher and the level of real wage (rent) is higher (lower), until point c (i.e. the endowment ratio Tf) is reached where the factor-price ratio is infinite, the level of real rent is zero (and the level of real wage is the highest).

27/See footnote 25/ above on page 141.

(c) For countries in Region A (E) where land (labor) is always free, the absolute wages (rent) (and trivially, the factor price ratios) are equalized between themselves, and are higher than the countries in the other regions.

This is easily seen from diagram 39. Take countries w and z - in region E - for instance. The marginal and average physical productivities of land at points w and z are apparently the same (it is the production contours for the production of clothing that are relevant.) Similarly, it can be easily proved that the marginal and average physical productivities of labor, at point a and b (in Region A), for the production of food, are completely equalized. Furthermore, it can be easily seen that the configuration of factor reward for countries in region E (A) is similar to that of country n (c), in which, the reward of land (labor) is higher than all countries not in region E (A).

In view of the observations made above, the theory of Professor Samuelson may be modified in the following form:

(1) Incomplete specialization is a sufficient (but not necessary) condition for the complete equalization of factor price, both absolutely and relatively.

(2) For countries with a factor of production which is always redundant (i.e. countries in region A or E), factor rewards are completely equalized between countries with the same redundant factor, regardless of the equilibrium position.

Section seven: Generalization

Two assumptions which we have made in the analysis of the last two sections may be relaxed: (1) that the endowment of "land" for all countries is the same (e.g. OT in diagram 39). (2) that there are an infinite number of countries.

(1) With the aid of the box diagrams, it can be easily shown that the configurations of factor rewards (i.e. the factor price-ratios and the absolute level of factor reward) will be equalized for all countries with the same endowment ratio when equilibrium is established between them. The fact that equilibrium factor rewards are independent of the sizes of the countries are ensured by the assumption of production functions with constant returns to scale.28/

Since our analysis in the previous section (Section six and diagram 39) included all endowment ratios, it automatically takes care of countries with any pattern of factor endowment - ratio and size.

With respect to the internal problem analyzed in section five - i.e. on the prospect of factor rewards - it can also be easily shown that the implications of the classification of the various countries, according to the endowment ratios, into the five regions (A, B, C, D and E as determined by Tf, Tc, Lf and Lc -- diagram 39) will be equally valid when countries of all sizes are considered.29/ In other words, the prospect of factor rewards is completely determined by the endowment ratios -- and is independent of the sizes of the countries.

Hence it is seen that our analysis in the two previous sections are general and exhaustive of all possible patterns of factor endowment.30/

28/Compare with the analysis undertaken on page 75 earlier.

29/For the implications of the classifications (i.e. for the meaning of "prospect of factor rewards) see section five above.

30/See, however, page 113, below

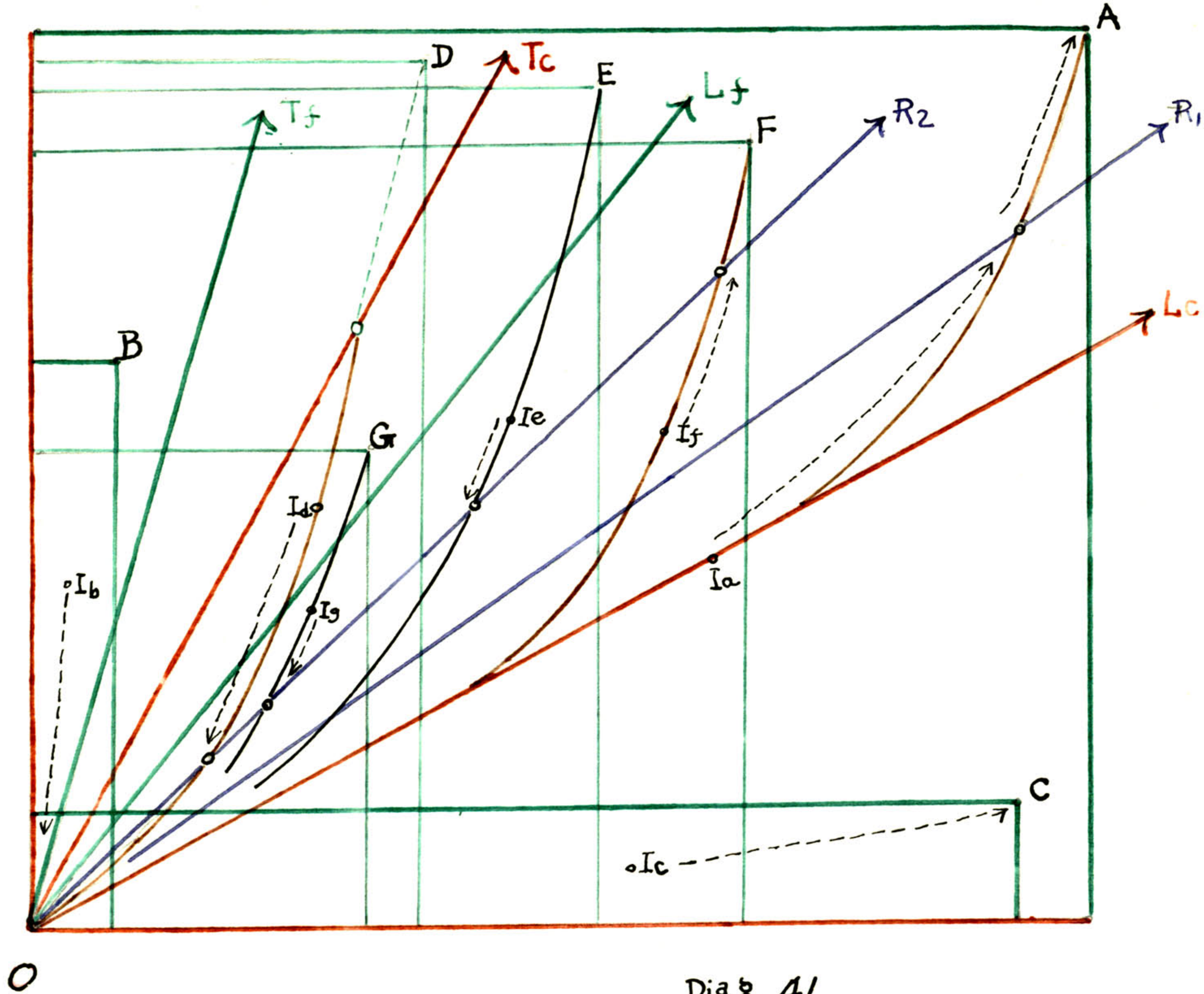
This generalization (from "endowment ratios" to "sizes of factor endowment) can be easily made because of the assumption of production functions with constant returns to scale.

(2) The assumption of an infinite number of countries is clearly an expository device. The technique of analysis developed in the last two sections should apply equally well to any number of countries. All the countries represented in e.g. diagram 39 (or even in the more general case, where all countries with any size of the factor endowments, may be represented in a diagram -- e.g. something like diagram 41 below) can be treated as reference countries, with the aid of which a designated group of countries can be analyzed. 31/

We can even say that: it is the technique of diagrammatic analysis which should be emphasized rather than the concrete conclusions which have been reached in the previous sections. 32/

31/More precisely, when a finite number of countries are under investigation, the positions of the critical countries isolated in the previous sections (i.e. countries with the endowment ratios Tf, Tc, Lf and Lc and the marginal food and clothing countries - see diagram 39) are often more interesting and crucial. They (alone) can be taken as the reference countries (and their optimum allocation curves plotted as auxiliary curves) which are often sufficient to throw much light on the analysis of a designated group of countries.

32/The significance of this assertion will be fully realized in the appendix of chapter VI (see page 176), where it will be shown that our analysis in the previous sections is not exhaustive of all the possibilities at all.



Diag. 41

Section eight: The Determination of Specialization Status for Many Trading Countries.

In chapter IV we have analyzed the "determination" of the specialization status of the trading countries by the diagrammatic methods, for the case of two countries.[33/] This analysis will be generalized in the "many countries" case in this section. As before, our analysis in this section is, to a large extent, intuitive -- the validity of the conclusions appeals to "common sense" rather than to precise quantatitive reasonings; and the conclusions must be accepted with reservations.

In diagram 41, let there be three countries (A, B and C) initially in the "world" for which the "isolation equilibrium positions" are shown -- i.e. Ia, Ib and Ic.[34/] These points are chosen so as to indicate the fact that the relative strengths of demand for the two commodities are approximately the same in the individual countries throughout the world. (The relative strengths of demand will be "isolated" (methodologically) in this way throughout this section).

The equilibrium position, after trade, between the three countries, can be represented by the radial line OR1. It is seen that countries A and C are the exporter of clothing and country B is the exporter of food. Country A is incompletely specialized while country C (B) is completely specialized in the production of clothing (food).

33/See above, page 112

34/For the meaning of "isolation equilibrium position", see page 108 above. The isolation equilibrium positions of countries C and B (for which no optimum allocation curves can be drawn) can be represented by any point in the "box" with the "correct" vertical and horizontal positions, respectively. (See footnote 9/ on page 130 above.)

Is it probable that these specialization statuses (of the three countries) likely hold under all "normal" circumstances --under the assumption that the relative strengths of demand for the two commodities are everywhere alike? It is very improbable, in this case, that country A becomes completely specialized after trade. This is true (intuitively) because of the fact that if country A is completely specialized (for the production of food, for instance) the structure of production of this country must have changed drastically, relative to the volume of trade, as compared with the isolation equilibrium position. (i.e. the production equilibrium position of country A has changed from Ia to point O). We expect such a thing to happen only when there exists an important world market for food (i.e. importing countries for food), to which country A can export (food). Since country C, then, will be the only importing country (for food, in this case when country A is completely specialized), and because of the fact that country C is a small country, it cannot be the country which absorbs the hugh export of country A (plus that of country B) and be the only supplier of clothing - to all the three countries.[35/] Hence country A could not have been completely specialized.

This seems to be suggestive of the fact that the completely specialized countries are likely to be the small countries - e.g. countries B and C in the present case. However, a more comprehensive conclusion

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35/This is most likely true under the assumption that the relative strengths of demand are everywhere the same, but is most certainly true when the relative strength of demand is also price-inelastic. (If country A is completely specialized in the production food, there would be too much food and too little clothing for the whole world (supplied by country C alone) as compared with the isolation outputs of the world).

seems to be that: the completely specialized countries are likely to be those countries with the endowment ratios very different from the world average. (In the present case, the mere fact that country A is large ensures that the endowment ratio of country A cannot differ drastically from the world average. When there are relatively few countries the size of country A as "heavily weighted" in the computation of the world average. Hence, the underlined conclusion above still applies in the present case.)

The likelihood of this conclusion can be seen when more countries are added in the way shown in diagram 41 - i.e. countries D, E, F and G are newly added countries. The equilibrium position is represented by the radial OR2. (Countries A, F and C are the exporters of clothing and countries B, D, E, G are the exporters of food). It is seen that country A is now completely specialized in the production of clothing - to supply the world market which is now composed of the importing countries (for clothing) of E and G (in addition to B) which are more suitable for the production of food than A.[36]

From diagram 41, it is seen that the endowment ratio of country A is "more different" from the world average (than before) because of the fact that the newly added countries (D,E,F and G with endowment ratios higher than that of country A) have raised the world average. If the

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36/The newly added countries are countries D,E,F and G. Relative to country A, all these countries are relative land-abundant countries, more suitable for the production of food than country A.

relative strengths of demand of the newly added countries (as represented by the isolation equilibrium positions Id, Ie, If and Ig) are similar to that of the other countries, the world price of clothing must now become higher because of the fact that the newly added countries are relatively land-abundant countries. The increased price of clothing has "forced" country A into the status of complete specialization for the production of clothing. Hence, our formula holds.[37/]

Our formal analysis on the problem of specialization status - i.e. our formal exercise of the technique of box-diagram - is now completed. We will say a few words, in the remainder of this section, on the significance (or the value) of our analysis of this problem - from the utilitarian viewpoint.

It is needless to say that we cannot very easily find examples in the realistic world to test the validity (and still less, the predictive power) of our analysis because of the fact that the assumptions of our analysis are so simple. Yet, broadly speaking, it is probably true that our analysis is not entirely irrelevant to the facts of the realistic world in the sense that, in international and inter-regional trade, we often find highly specialized regions of production which can be explained by the drastic difference in resources endowments under the operation of (loose) market force.[38/]

37/That is when there are many countries, the weight of the sizes of a single country is relatively not as influential (on the world average). It is the deviation from world average which determines, approximately, the specialization status of the individual countries - when there are either many or few countries.

38/e.g. the one-product-colonial economy, or the "mining area" within an economy etc.

As it is well known, there are many reasons causing the completely specialized countries (especially agricultural countries) to refuse the dictation of the market force and to become incompletely specialized.[39/] A usual way to achieve this goal is through industrialization - which represents, essentially, an effort to change the endowment ratios (e.g. capital to labor)which may be called on economic adjustmant) - testifying the correctness of the general spirit of our analysis.

Another way to achieve the status of incomplete specialization (which is more effective) is through political adjustment. The countries concerned may become (artifically) "isolated" - i.e. insulated from the dictation of the market force outside the political boundary by the adoption of the various kinds of trade restriction). These facts can be easily demonstrated by our diagrammatic methods.

The writer believe that, barring transitional and dynamic consideration, the long run desirability of free trade throughout the world, under the favorable conditions of perfect competition, should be recognizes as an acceptable international ethics — guiding the enactments of trading policies.[40/] If this belief is acceptable, the question of specialization status takes on a significance roughly comparable to the "infant industry argument" — namely, if incomplete specialization, as such, is being considered as "undesirable" from the viewpoint of the

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39/See, for example, N. S. Buchanan and F. A. Lutz, "Rebuilding the World Economy", The Twenty Century Fund 1947, page 181..

40/See Professor C. P. Kindleberger "The Dollar Shortage", The Technology Press of M.I.T., 1950, pages 222-224.

individual countries, the question is then; how long the political adjustment (e.g. tariff) must last before the economic adjustment (e.g. industrialization) can be accomplished to such a degree that will enable the individual countries to obtain a degree of industrialization (considered desirable) and to participate in the family of world trade with "ethical" trading policies - i.e. free trade.

Faced with a problem of this kind, the classification of the countries, into two groups, namely, completely specialized countries and incompletely specialized countries, as had been done in an earlier section, is somewhat significant.

This is true because of the existence of the infra-marginal (food and clothing) countries as far as specialization status is concerned.[41/] If we only know that a country is incompletely specialized, we know reasonably well that "the day is not far off" for the country to improve her endowment ratio (through, e.g. real capital accumulation in the industrialization process) to the desirable direction and extent so that the country will be able to participate in world trade as an ethical member - even though the country has to rely on political adjustment to effect the transition temporarily.

On the other hand, if a country is completely specialized, we do not know "how infra" is the position of the country - the endowment ratio of the country may be "way off" from the "desirable" ratio of factor

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41/See Diagram 39

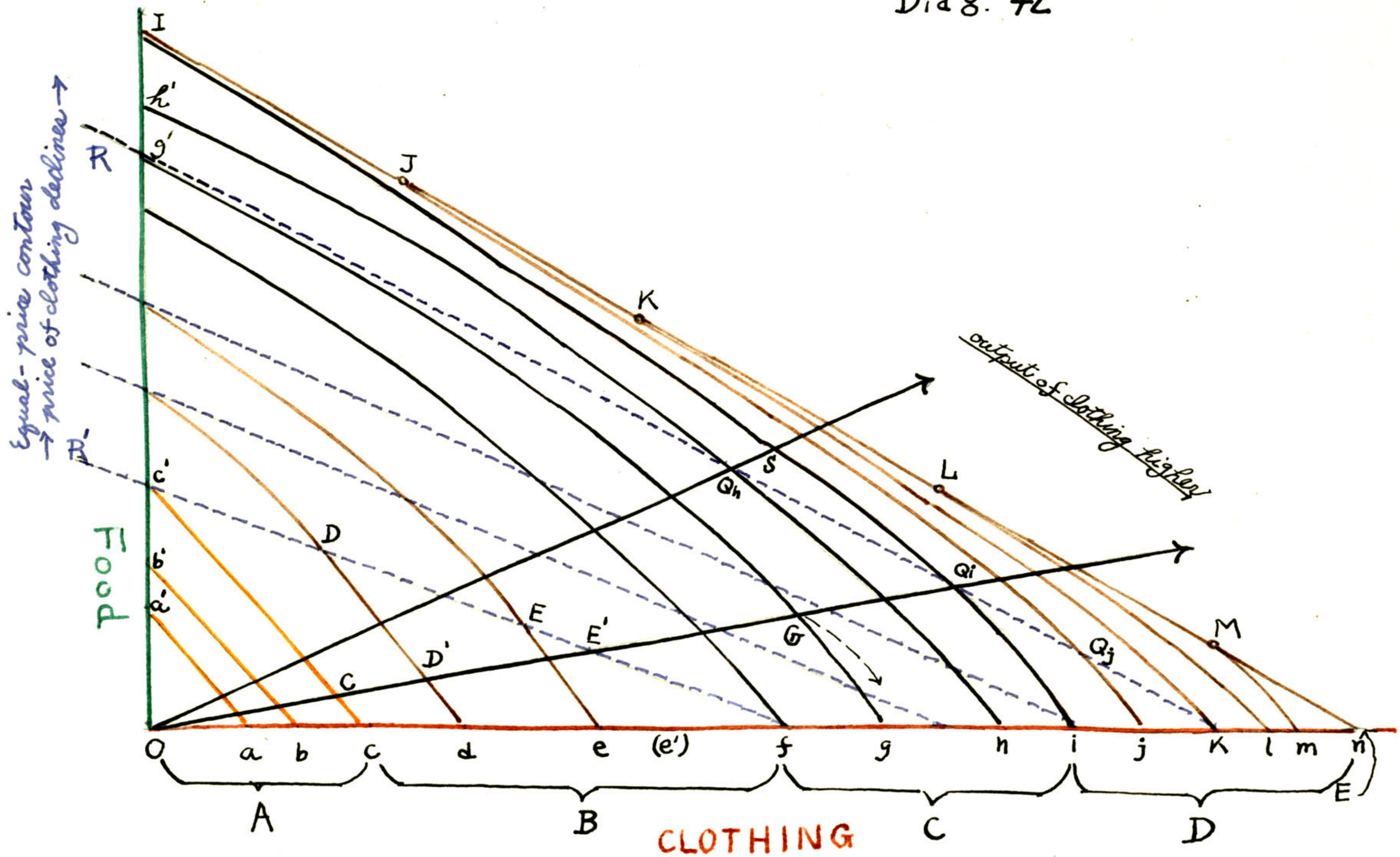
endowment (namely, that ratio which is compatable with free trade and the acceptable degree of industrialization).[42/]

It is safe to conclude, however, that: what must be emphasized is, again, the method of analysis: given the equilibrium position of the international trade, and the endowment ratio of a country, we know, approximately, to what extent a country has to modify her endowment ratio in order to achieve a certain (desirable) degree of industrialization.[43/] It is fairly easy to demonstrate the logic of the analysis of a problem of this kind, with the aid of the diagrammatic methods developed in this chapter.

42/On a closer examination, however, it seems advisable to say that our argument in this connection (in the text) is more (or less) meaningful depending upon the difference between the endowment ratios of the marginal food country and the marginal clothing country. Referring to diagram39 , if point g and point k are close together, our argument in the text is more meaningful. This is true because of the fact that, in such case, it will be relatively easy for a country which is already incompletely specialized, to change her degree of industrialization, through a slight modification of her endowment ratio and produce a higher ratio of industrial output. When the distance between point g and point k is great, our argument loses force. Hence, it is seen that the solution of our problem hinges upon the character of the production functions, in addition to the other equilibrium conditions.

43/This is more easily true, if the country involved is a small country --otherwise it is no longer reasonable to assume that the equilibrium position will remain the same after the program of industrialization of the country is completed.

Diag. 42



I
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g'
R
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a'
Food
J
K
L
M
S
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Qi
Qj
D
E
E'
D'
C
G
output of clothing higher
Equal-price contour
→ price of clothing declines →
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CLOTHING

Section nine: The Map of Production Frontiers

In this section we will show the relationships between the box diagram and the so-called map of production frontiers --or the map of optimum output curves. In diagram 42a map of production frontiers is constructed. It is derived from diagram 39. The vertical axis and the horizontal axis are taken to represent the outputs of food and clothing, respectively.[44/]

For every optimum a location curve in diagram 39, there is a corresponding production frontier in diagram 42 (solid curves). The five regions: A, B, C, D and E, identified in section five from the box diagram (39) are indicated in the frontier map below the horizontal axis. The correspondence between the two maps can be developed in the following ways:

1) Countries in Region C - From diagram 39, it is seen that this group of countries have optimum allocation curves with full length. By rule (25) on page 57, we know that the product price ratio is monotonically increasing, between the two points representing complete specialization for clothing and food, respectively --as the equilibrium position changes from a lower point to a higher point on the optimum allocation curves (i.e. when more clothing - and less food - are produced).

44/It is unfortunate that when this system of axes is chosen, the output ratio, which is taken throughout this thesis as the units of clothing per unit of food, must be represented by the "inverse slope (rather than the slope) of the radial line passing through any point for which the output ratio is considered. The reason that this (inconvenient) system of axes is adopted will be evident in our analysis in the following chapter.

This is reflected in diagram 42 , where it is seen that the production frontiers are entirely "curved" for countries in region C — on any (one) production frontiers in this region, the absolute value of the slope of the frontier is higher at a lower point between the two points representing complete specialization (e.g. between point h and point h' for the production frontier of country h).

(The production frontiers of countries in region C are represented by solid black curves which are the type of curves used for the optimum allocation curves for these countries. The two countries lying at the margins, at each end, of this region, are country f and country i, respectively, as can be seen from both maps - diagram 39 and diagram 42 .)

2) Countries in Region D - From diagram 39, it is seen that the optimum allocation curves for this group of countries are "incomplete" at the lower ends --indicating the fact that when the output of food has increased to a certain point, the product price ratio, instead of declining, becomes constant with every further increase of the output of food 45/ (until complete specialization for the production of food is reached).

This state of affairs is reflected in diagram 42 by the fact that the production frontiers in this region (D) become straight lines (rather than curved) at the upper portions. (The solid grey lines are used for the production frontiers and the optimum allocation curves for this group of countries.)

45/See above, footnote 7/.

The fact that, in diagram 39, the optimum allocation curves of all countries in region D terminate on Lc, (e.g. at points J, K, L. and M...) is reflected in diagram 42 by the fact that the production frontiers of these countries "become" straight lines at the points J, K, L, and M. The fact that these "turning points" fall on the same straight line - i.e. the straight line In - will be proved later.[46/]

We may observe, first of all, that the lower limit of the product price ratios for all the countries in this group are identical. This can be easily proved with the aid of such diagrams as 38, where it is seen that the product price ratio becomes constant when the equilibrium position is established at a lower point than D.[47/]

It can also be easily proved that, when the equilibrium position of a country in this region (D) is established in the constant cost region, labor becomes redundant so that any further addition of labor (with the quantity of land being held fixed) will not affect total outputs (specifically, will not increase total outputs).[48/] In the map of production

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46/From diagram 39, it is obvious that as the radial line OR approaches Lc, the product price ratios established at points J, K, L and M are completely equalized. This fact alone ensures that J, K, L and M fall on one straight line - which is the limiting case of a proposition which will be proved later. (See page 162 below). (Footnote 56/)

47/Namely, this "constant opportunity cost" will be the same for all countries in region D.

48/In diagram 38, (which is a box diagram of a country in region D), when the equilibrium position is established at point Q, any further addition of labor only makes labor more redundant -- because land is the bottleneck factor. Any addition of labor will shift the upper-right corner of the box (i.e. point A), further to the right, horizontally --with the whole map of production contours for food shifts horizontally. The fact that the "new" country (with a lower endowment ratio) can only produce the same amount of, e.g. food, (i.e. f2 at point Q) if she produces the same amount of clothing (i.e. c2 at point Q) as compared with country A is clearly indicated by the fact that the production contour f2, always coincides with the production contour c2, over the horizontal portions, when f2 is shifted horizontally.

frontiers, this is reflected by the fact that the "straight-line portions" of the production frontiers of this group of countries necessarily coincide. In diagram 42, the straight line In is, then, composed of the straight line portions of the production frontiers of all the countries in region D.

3) Countries in Region E - The countries in Region E, for which no optimum allocation curve can be drawn (in diagram 39), have the straight line In as the common production frontier in diagram42 . This is supported by a simple extension of the arguments used above.49/

4) Countries in Region B - The optimum allocation curves for the countries in this region are incomplete at the higher ends (diagram 39). This is reflected in diagram42 , by the fact that the production frontiers of this group of countries become straight lines at the lower ends, indicating the fact of constant opportunity cost. (The solid grey curves are used for the optimum allocation curves and the production frontiers of these countries.)

It can be easily proved that the straight-line portions of the production frontiers of these countries (e.g. Ee and Dd) are parallel. Furthermore, the point (e.g. D and E) where the production frontiers 'become' straight line fall on one straight line (fc' dotted blue).50/

49/The production frontier for country n is the straight line In in diagram42 . Further addition of labor-endowment to that of country n will no longer produce any effect on outputs - because country n always produces under the condition of constant opportunity cost. (See footnote 48)

50/In diagram 39, when OR approaches Tc, the equilibrium price ratios established at points E and D (of countries e and d) are equalized. If the point D and E, (which are the "turning points" of the production frontiers for countries d and e in diagram 39) represent product price equalization points, the assertion in the text can be easily proved with the observation which will be made on page162 below. footnote 56/.

The positions of the production frontier of a country in this region is always higher than that of another country if the labor endowment is higher (than the other country: e.g. the production frontier of country e is higher than the production contour of country d). Furthermore, the distances between the parallel (straight-line) portions of the production frontiers (e.g. dD and Ee) are proportional to labor endowments of the various countries.[51/] It is seen that there is asymmetry of the production frontiers of the countries in region B and region D.)

5) Countries in Region A - For these countries, (for which the optimum allocation curves cannot be shown in diagram 39, the production frontiers are parallel, and the horizontal distances between them are proportional to the endowment of labor (i.e. the scarce factor). These assertions can be readily proved.[52/]

51/This observation can be easily proved with the aid of diagrams like : postulating two countries (in diagram 38 with the upper-right corners of the boxes lying between points U and V. It is easily seen that, e.g. when the output of one commodity is fixed, the output of the other commodity is higher for the country with more labor endowed. This asymmetry between the frontiers in regions B and D is due to the fact that the quantity of land (rather than labor) is being held fixed for all countries considered. When the constant opportunity cost range is reached, in this case, land is the redundant agent and labor is the scarce (bottleneck) agent. When more labor is added, the redundant land will be gradually "absorbed", and the optimum outputs of the two commodities will be proportional to the endowment of the scarce factor (labor) as long as there are still redundant factors (land).

52/These production frontiers - e.g. aa', bb'...etc. - are, of course, parallel to dD and eD of countries in region B. In other words, the triangle Ofc' in diagram42 enclosed the constant price-region, in which land is always redundant and free.

Equal-Price Contours:

The dotted (blue) lines in diagram 42 are the equal-price-contours - i.e. they connect points on the production frontiers of the various countries with equal slope (representing equal product price ratios).

The equal price contours are straight lines. This can be readily proved with the aid of diagram 39. Take any radial line OR, for instance. The points of the optimum allocation curves which it intersects - i.e. Qh, Qi, Qj... - represent equilibrium positions with completely equalized product price ratios (by rule 27 on page 62). At these points, the increments of the output from country h to country i and from country i to country j are in the ratios of QhQi/QiQj for clothing and Qhh'/Qii' for food,[53/] by rule (1) on page 10. But, by the similar triangles QhQih' and QiQji' (all three sides are parallel), we have:

Qhh'/QhQi = Qii'/QiQj

This condition states that: the ratio of increments of the two outputs from country h to country i equals that from country i to country j. Referring to diagram 42 this condition ensures that Qh, Qi and Qj fall on the same straight line if the product price ratios are equalized between them.[54/]

53/Qih' and Qji' are horizontal lines by construction.

54/In diagram 42 the ratio of increment of the two commodities from Qh to Qi is represented by the slope of the straight line connecting the points Qh and Qi. The same ratio is represented by the slope of the straight line connection Qi and Qj. In other words, the straight line connecting any pair of points with product price equalization must have the same slope. The equal-product-price lines (dotted blue) must be a straight line.

By the theory of equalization of factor prices, investigated earlier,[55] all the points (on the product frontiers, e.g. Qh, Qi, Qj) on the same equal-product-price contour (e.g. the straight line kg') also represents complete equalization of factor prices, absolutely and relatively.[56] [57] This fact is somewhat interesting relative to our analysis in the following chapter.

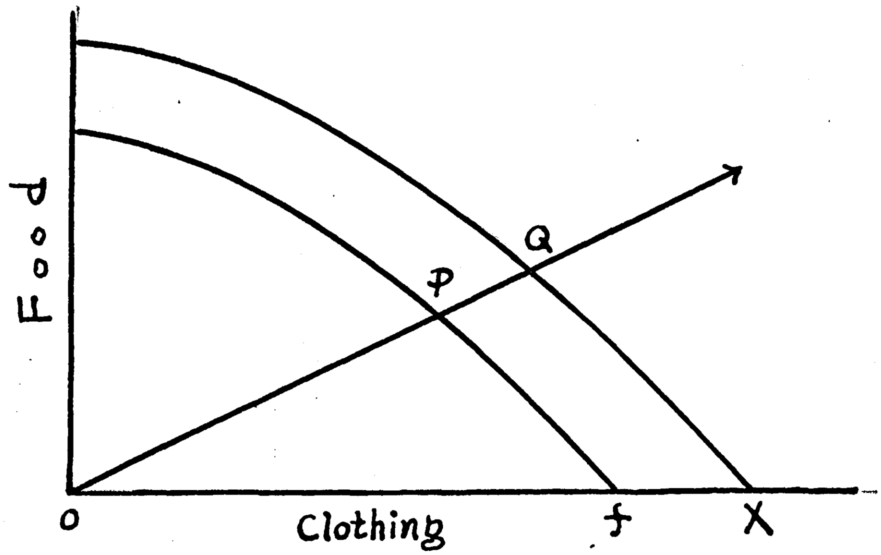
55/See above section six on page 137

56/See above page 139. These points obviously represent the equilibrium positions of the incompletely specialized countries. The production frontiers of the marginal food (clothing) countries - i.e. countries g and k respectively - intersect the equal-price contour - i.e. g'k (dotted blue) - at the upper and lower ends, (of g'k), respectively. Incidentally, it is then obvious that the points J, K, L and M (which are the "turning points" of the production frontiers in region D) fall on one straight line In, in diagram 42 (also, points E and D, etc. fall on the same straight line c'f) because of the fact that the product price ratios are equalized at these points. (See above foot-note 46/, 50/)

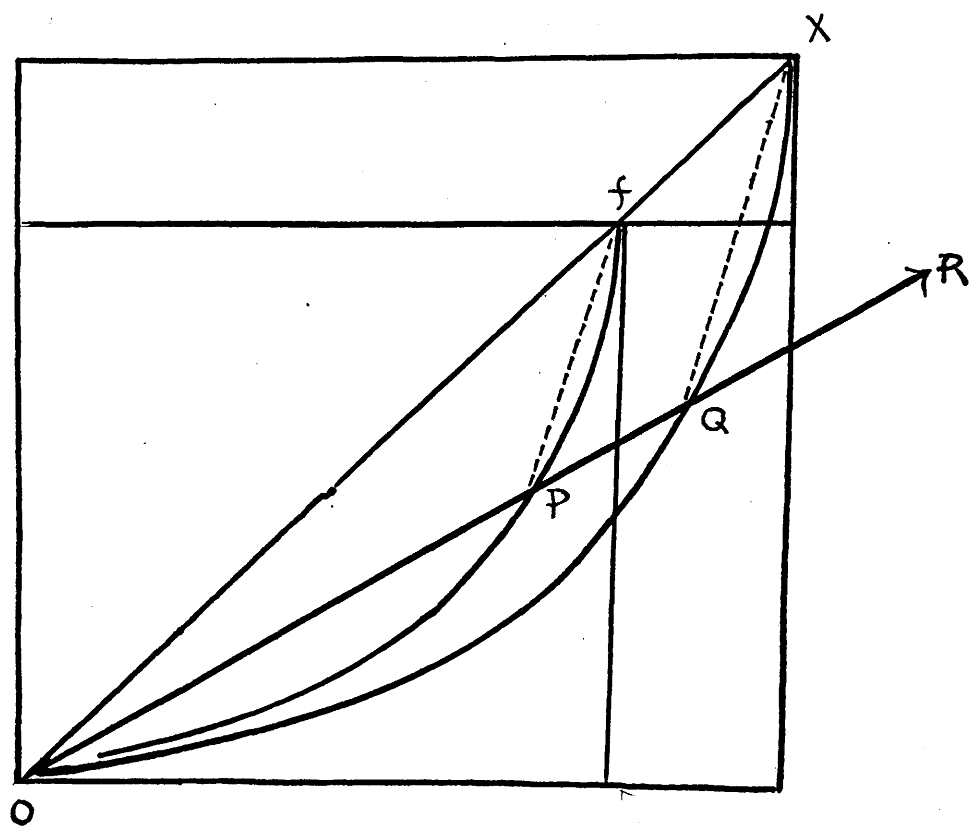
57/It can also be shown that the ratios of increments of (both) outputs, from point Qh to Qi and from Qi to Qj, equals to the ratio of the increment of labor-endowments from country h to country i and from country i to country j. Referring to diagram 39, it is easily seen that:

$$\frac{Qhh'}{Qii'} = \frac{QhQi}{QhQj} = \frac{hi}{ij}$$

(The last ratio in thie equality is the ratio of the successive increments of labors).



Diag. 43



Diag. 44

Generalized Production Frontier Map:

So far, for the construction of diagram 39, we have held the quantity of land endowment constant for all countries (i.e. OT in diagram 39. When the quantity of land is variable, the production frontiers of any country can be easily derived, geometrically, from diagram 44, in the following way: when the endowment ratio, of a country ~~is~~ given i.e. country X 58/ - the size of factor endowments of country X must be a multiple of one country represented in diagram 39 (Or 42). 59/ Let this country be country f.

The production frontier of country X can be geometrically derived from the production frontier of country f, by projecting the latter production frontiers (which is known), in the radial direction, in such a way that for any output ratio, the radial distance OP' and OQ' (see diagram 43) bear the same proportion as the ratio of factor endowments of the two countries. This can be easily proved by the method of box diagram. The equal-price contour between the two countries, are "connected" by the radial lines, e.g. OR in diagram 43. 60/

58/Country X has endowment of land other than OT in diagram 39; we want to construct a production frontier for this country.

59/This is true because of the fact that all the endowment ratios are represented in these diagrams.

60/In diagram 44, for any equalibrium position OR, it is readily seen, by the parallel relationship that,

$$\frac{OA}{OB} = \frac{fP}{XQ} = \frac{Of}{OX}$$

These equalities state that when the product price ratios are equalized, the ratios of output of clothing (OA/OB) and food (jA/XB) equal the ratio of factor endowments (Oj/OX).

When equilibrium is established between these two countries, it is readily seen that the factor prices are equalized, absolutely and relatively.[61]

61/See footnote 60/above.

Appendix

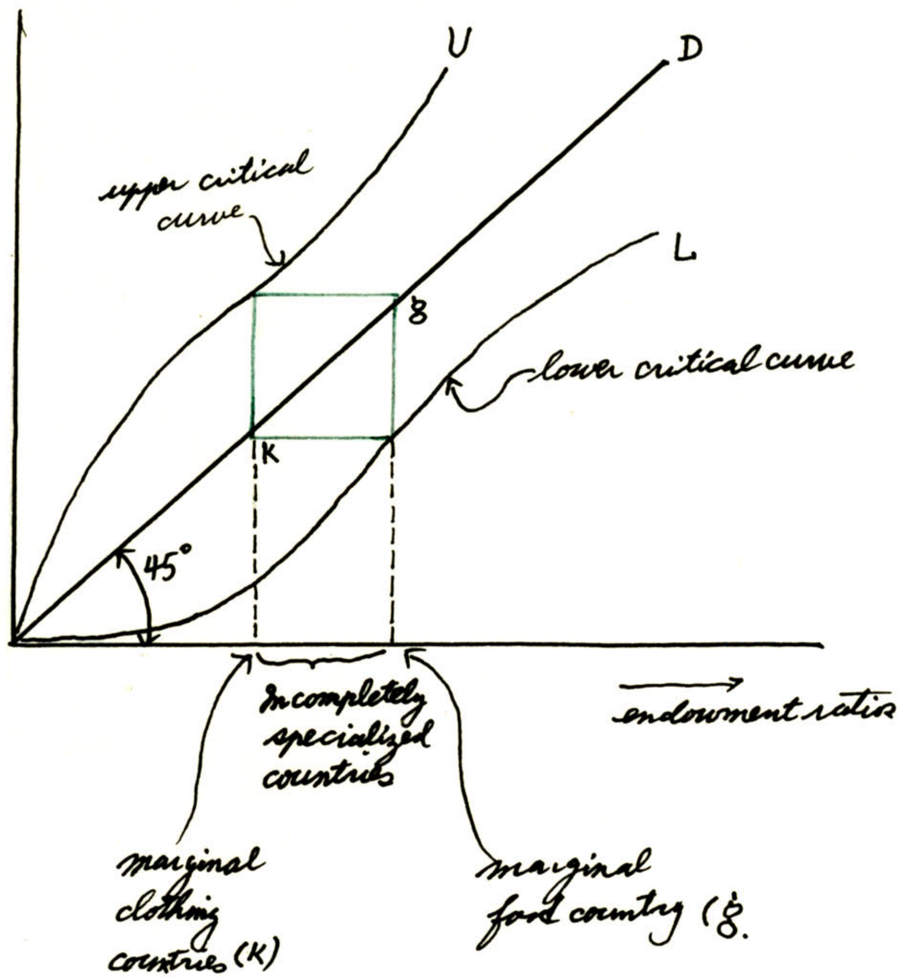
The Generalized Critical Map

In chapter IV above, we have developed a critical map from the box diagram. Such a map is reproduced in diagram 45. We recall that the upper and lower critical curves indicate the upper and lower critical ratios, for any endowment ratio(of a country) as indicated on the horizontal axis. The upper and lower critical ratios are the upper and lower enc-slopes of the optimum allocation curve (with "full" length) in the box diagram, of a country with "that" endowment ratio. The upper and lower critical curves are positively sloped and are symmetrical with respect to the 45-degree line - OD.[1/]

For any equilibrium position, the equilibrium input ratios of the two industries, of the various incompletely specialized countries, can be represented by a (green) perfect square inscribed between the two critical curves - e.g. perfect square k-g in diagram 45. This reflects the equilibrium condition that the input ratios for the production of the same commodity, of all the incompletely specialized countries, are the same. Furthermore, the endowment ratio of the marginal food (clothing) country, equals the input ratio for the production of food (clothing) of any (and all) incompletely specialized countries.[2/] Hence, in diagram 45 countries k and g are the marginal clothing and food countries, respectively for the equilibrium position depicted in diagram 39 (i.e. equilibrium position represented by OR).

1/See Chapter IV, sections 5.

2/See above page 135.


U
D
L
upper critical curve
lower critical curve
45°
K
g
Incompletely specialized countries
endowment ratio
marginal clothing countries (K)
marginal food country (g.

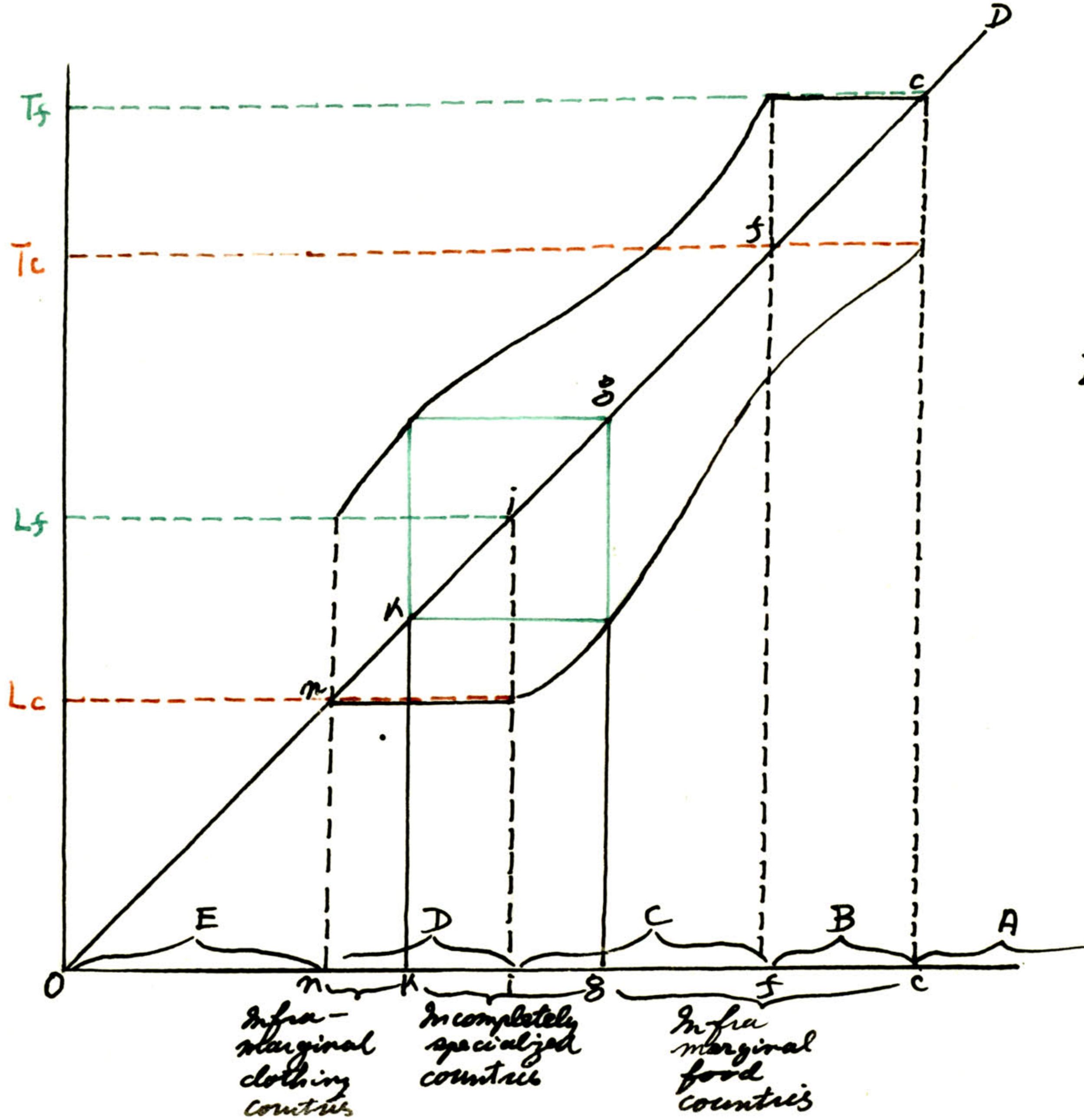
Diag. 45

This critical map can now be generalized to include the possibility that one of the factors may become a free agent at the equilibrium positions. The critical map for this more general case is represented in diagram 46 which is derived from diagram 39 in the following way. (First, mark off the four input ratios of the ridge lines Tf, Tc, Lf and Lc on the vertical axis. Draw the horizontal lines through these points intersecting the 45-degree line OD at points c,f,i, and n which are projected directly on the horizontal axis. The five regions A, B, C, D, and E - identified in section V - for the study of the "internal problem" - are marked above the horizontal axis.[3/]

Consider country i in diagram 39. It is the country with the endowment ratio Lf and hence lies on the margin between regions C and D.[4/] Of all the countries which have "full-length" optimum-allocation curves (black) i.e. countries in region C, it is the country with the lowest endowment ratio (Lf). The <u>lower-critical ratio</u> - i.e. the lower end slope of the optimum-allocation curve - <u>of country i equals Lc.</u> This can be proved as follows:

3/See above section V

4/Country i is the country in which labor is on the margin of "may be free" (see page 136 above). It should not be confused with country k which is the <u>marginal clothing country</u>. (While the critical position of country i is completely determined by technology considerations, the critical position of country k is partially determined by the particular equilibrium position - which is governed by the relative strength of demand for the two commodities.)


D
c
Tf
Tc
f
Diag. 46
g
Lf
i
k
Lc
n
E
D
C
B
A
O
n
k
i
g
f
c
Infra-
marginal
clothing
countries
Incompletely
specialized
countries
Infra
marginal
food
countries

Diag. 46

In diagram 39 the straight lins (dotted black) jQ_j and iQ_i are parallel by the parallel relationship - since the points Qj and Qi are intersected by the radial line OR by construction. Now let the point Q_j approach point J which lies on Lc, the radial line passing through Qj (e.g. OR) then approaches the radial line Lc, and the point Qi, on the optimum-allocation curve of country i (i.e. iQiO) approaches point O.[5/] This proves that the lower end slope of the curve iQ_i equals the slope of the radial line OR, which approaches the radial line Lc as the limiting position.

Hence we know: when the endowment ratio of a country (e.g. country i) equals Lf, the lower critical ratio equals Lc. This is indicated in diagram 46. Similarly, when the <u>endowment ratio of a</u> country - e.g. country f - <u>equals Tc</u>, the <u>upper critical equals Tf</u>, as indicated in diagram 46.

It is our purpose to show next that the country with the <u>endowment ratio Lc</u> (e.g. country n in diagram 39) has an <u>upper critical ratio</u> Lf. This can be proved as follows:

Consider the optimum allocation curve of country i, namely iQ_iO, in diagram 39. If we let Qi approach point O as before, the radial line passing through the point Q_i approaches the ridge line Lc - as shown immediately above - and the (dotted black) line iQ_i approaches Lf. This fact is sufficient to prove that when the input ratio equals Lc (i.e. OR approaches Lc), the upper critical ratio approaches Lf (i.e. iQi approaches Lf).

5/This is true because of the fact that the jQ_j and iQ_i always parallel (dotted black); and they approach, respectively, the parallel lines (solid green) jJ and iO which are parallel by construction.

This fact is indicated in diagram 46 - i.e. when the endowment ratio equals L_c, the upper critical ratio equals L_f - i.e. for country n. Similarly, when the endowment ratio equals T_f (e.g. that of country c) the lower critical ratio equals T_c as indicated in diagram 46.

In diagram 39 it is clearly seen that the lower critical ratios of all countries in Region D - i.e. countries i-n inclusive - have the same lower critical ratio (i.e. L_c). The lower critical ratios in the present case signify that: when these countries begin to produce clothing in such a way that labor is no longer free, the input ratios for the production of clothing of all countries in region D equal Lc.[6]

Hence in diagram 46 we see that the lower critical curves become the horizontal line - with height L_c - for all countries in region D. Similarly, we see that the upper critical ratios of all countries in region B equals T_f - i.e. the horizontal portion with height T_f. The critical diagram "begins" and "ends" with "perfect squares".

The equilibrium position OR, in diagram 39, is represented again by the (green) perfect square in diagram 46. The specialization status of the various countries is indicated in diagram 46 below the horizontal axis.

When the world demand for food is stronger, the equilibrium position will be represented by a lower perfect square in diagram 46 (and by

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6/Whether the lower end slopes of the (grey) optimum-allocation curves of these countries (e.g. the slope at points J, K, L, M....) equal Lc or not is economically irrelevant, because of the fact that the lower end slopes at these points no longer represent the input ratios, then the optimum-allocation curves are "incomplete" at the lower (and higher) ends.

a lower retail line (than OR) in diagram 39. Corresponding to this weakening of the world demand for clothing, the endowment ratios of the marginal food country and the marginal clothing country become lower, and the input ratios for the production of both commodities by all the (new group of) non-specializing countries will be lower (than the input ratios of the old groups of non-specializing countries before the change of the strength of demand).7/ The product price ratio becomes lower too.

With a continuous weakening of the world demand for clothing, the limiting position will be reached. In diagram 46 this limiting position is represented by the perfect square n-i, and in the diagram 39 by the radial line which is coinciding with the ridge line Lc. The marginal food country approaches country i and the marginal clothing country approaches country n. When this limiting position is reached, the world product price ratio reaches the lowest limit. A further lowering of product price ratio is not only impossible but is also inconsistant with the existance of international trade because the range of constant opportunity cost has been reached.

Similarly, the product price ratio established in the position under which country f (c) is the marginal clothing (food) oountry (i.e.

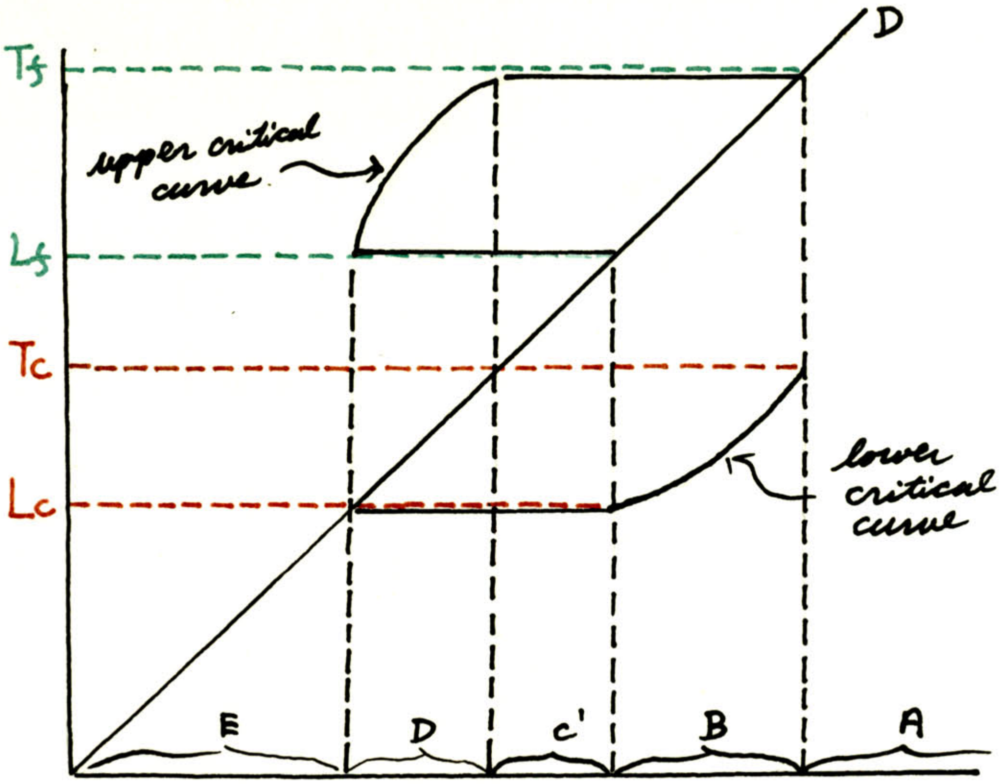
7/It can be easily seen that with the change of the relative strength of demand - a "comparative static problem" - the factor price ratios of some countries will not be affected. In the case discussed in the text, the factor price ratio of the (old) infra marginal food countries (and countries in Region A) will not be effected. Other cases can be similarly analyzed. In short, functional distribution ratio will only be effected when the pattern of allocation of resources (of a particular country) is effected.

in diagram 39 OR approaches Tc, and in diagram 46 the green square approaches the perfect square f-c) the product price ratio there established will be the upper limit of all possible price ratios. The product price ratios of the world will always be straddled by the higher and lower limits established at the limiting positions at the two extremes.

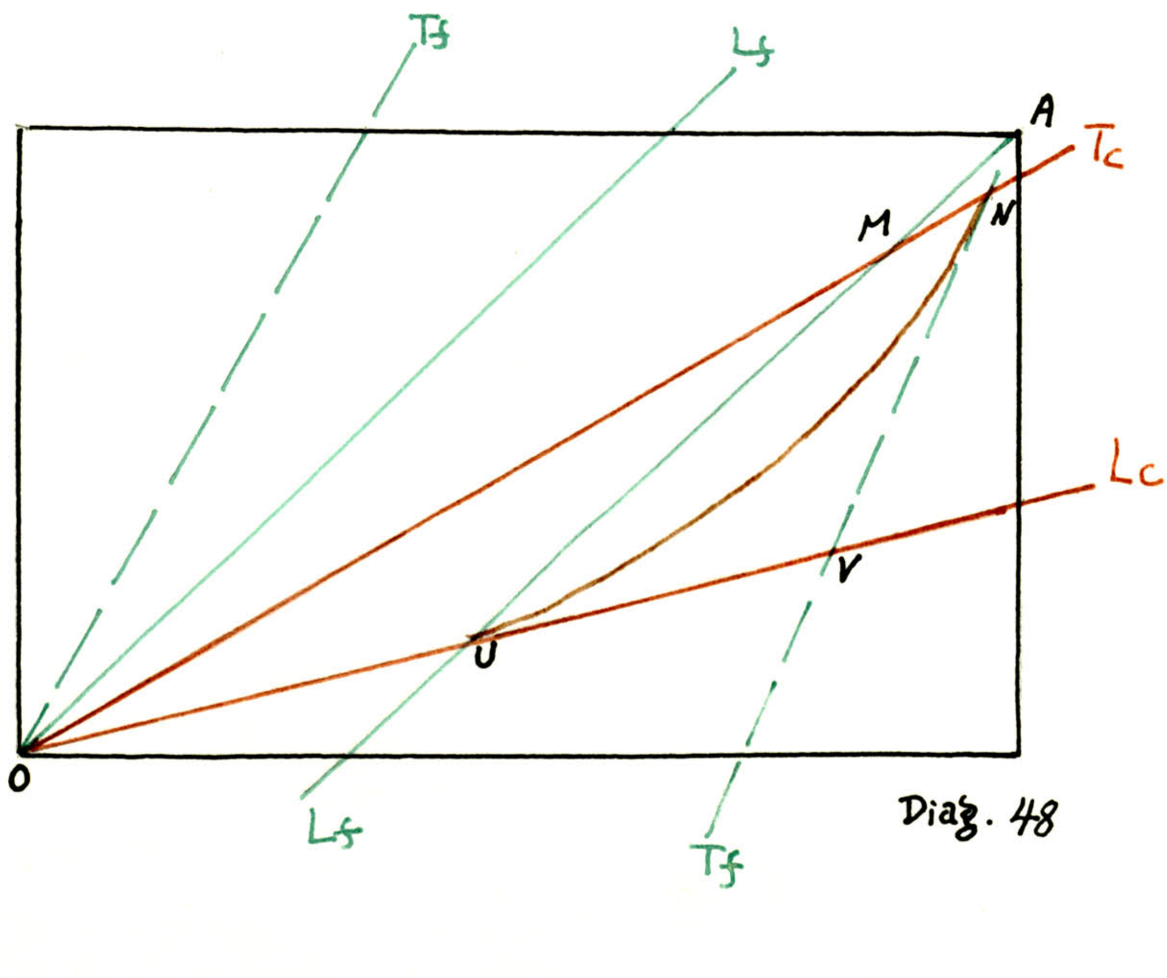
The positions of the ridge lines (or rather the set of values of the input ratios corresponding to the ridge lines Tf, Tc, Lf and Lc) are taken as assumed values. When Lf and Lc approach zero and Tf and Tc approach infinity, all the countries necessarily fall in region C, namely, the non-free region in which neither factor will ever be free. This can also be clearly seen from diagram 46: if we let Lf and Lc approach zero, and Tf and Tc approach infinity, diagram 46 becomes diagram 45 which is clearly a special case.

Through this chapter we have added another restriction of the set of values assigned to Tf, Tc, Lf and Lc, namely, in addition to Tf $>$ Tc and Lf $>$ Lc (which are true by the definition of the relative factor intensities of the two commodities) we have made the additional assumption that Tc $>$ Lf.[8/] This additional assumption was made to facilitate our exposition, and the implication of this restriction must now be investigated.

8/See above footnote 5/on page 127. From diagram 38 it is seen that this added assumption amounts to the assertion that there should be at least one common input ratio which lies in the non-free regions of the maps of production contours for the production of both commodities.



Diag. 47



Diag. 48

When the assumption Tc > Lf is relaxed the critical map is represented in diagram 47 where it is seen that Lf > Tc.[9/] Consequently the horizontal portions of the upper critical curve and the lower critical curve overlap over the range marked C' on the horizontal axis. The endowment ratios of all countries in this Region (C') lie between Lf and Tc.

The box diagram of a country in Region C' is constructed as in diagram 48. In this diagram it is seen that the condition Lf > Tc is satisfied (i.e. Lf is steeper than Tc, drawn from point O). That country A is a country in the Region C' is indicated by the fact that point A (the upper-right corner of the box) is straddled by Lf and Tc (drawn from point O). If Lf and Tc are drawn from point A (paralleling those drawn from point O, respectively,) the common ground covered by the non-free regions of the two maps is enclosed by the rectangle MNVU - and the optimum-allocation curve of this country must lie in this region, running from point N to point U. The important thing to notice is that the optimum-allocation curve is "incomplete" at both ends.[10/]

Country A in (Region C') as indicated in diagram 48 is, then, a country in which both factors may become free agents depending upon the

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9/The conditions Tf > Tc, Lf > Lc must of course be retained. The case where Lf = Tc is a limiting case and need not be discussed.

10/This is always true if point A lies between Lf and Tc.

relative strength of demand for the two commodities. In diagram 47 the countries in Region D (i.e. countries with the endowment ratios between Lc and Tc) are the labor relatively abundant countries, and the countries in region B (i.e. countries with the endowment ratios between Lf and Tf) are the land relative abundant countries.[11] That the optimum-allocation curves for these two groups of countries are broken, respectively, at the lower and the upper ends can be easily checked by the method of box diagram.[12]

In spite of the fact that countries in Region C' have not received our explicit attention throughout this chapter, they probably represent the most general cases — in the sense that the optimum-allocation curves of the countries in the other regions can be readily derived from the optimum-allocation curves of this group of countries (and appear to be special cases). For example, in diagram 48 let Tc be higher than point A, point N, then, reached point A and we have a country in Region D. Let both point Tc and point Lc become higher than point A, the optimum-allocation curve disappears, and we have a country in region E. Other cases can easily be generated from this optimum-allocation curve by similar methods.

Hence it is seen that the classification of the countries undertaken in both section V and section VI are only special cases, after all.

11/See section V above.

12/The interpretations (and the naming of) of regions A and E in diagram 47 are identical to that in e.g. diagram 46. This can also be easily checked.

(Region C disappears in diagram 47 - i.e. under the assumption Lf > Tc - and region C' disappears in diagram 46 - i.e. under the assumption Tc > Lf.) Fortunately, we have exhausted all the possibilities - of the set of values which could be assigned to Tf, Tc, Lf and Lc - by the cases investigated under map 46 and map 47 if the other assumptions which we made are to be satisfied.

However, it is safe for us to draw the concluding remark that what should be emphasized are the diagrammatic methods of analysis which are being employed throughout this thesis, rather than the concrete conclusions which have been deduced throughout the previous sections. It is probably true that from the viewpoint of intellectual progress, the training in analytical ability is more important than the derivation of concrete conclusions — which are the major concerns and interests of the "applied economics."

Chapter VII.

Diagrammatic Representations of the Value and Distribution Theories of Ricardo Marshall and the Austrians.

Section one: Introduction

In this chapter our attention turns to certain issues in the history of economic doctrine related to the value and distribution theories. We shall try to represent, by diagrammatic methods, the value and distribution theories of D. Ricardo, A. Marshall and the "Austrians" who have contributed so much to the development of this branch of economic theories.

In the short space allotted to a treatment of the development (nearly 100 years) of such a broad subject as value and distribution theories, it is necessary for us to be selective and concentrate on certain aspects of the theories of these economists. In this chapter the writer hopes to demonstrate what he believes to be the most significant features of the theories of these economists, in contrasting to each other, and from the viewpoint of the evolution of economic doctrines, by diagrammatic methods.

Another word of caution may be explicitly registered with respect to the undertaking of this chapter. In our diagrammatic representations of the various theories under consideration, it is obvious that we cannot tolerate any ambiguity as to the quantitative analytical assumptions - which is an advantage (or disadvantage?) of our method. In our undertakings, then, we encounter the difficulty that the theorists considered might not have learned (or did not care) to state, explicitly, their assumptions in the forms which are desirable from the modern analytical viewpoint. In that event we must attribute to them certain analytical assumptions in our exposition, for the purpose of building up models which are considered representative, perhaps, from the viewpoint of "maximum likelihood" of the theories of these economists.

We can say that the names of the three economics are chosen for their representative values and are "impressionistic". The models which will be labeled as "Austrians", "Ricardian" and "Marshallian" are merely representative of the theories associated with these economists, in the senses which will be explained in due course.

In the following section, let us first demonstrate the value and distribution theories of the Austrians.

Section two: The Austrians.

In chronological sequence, W. S. Jevons was the forerunner of the "utility" or "subjective" theorists of value — the "new" theories being gradually popularized by C. Menger and his followers as the Austrian School.1/

According to the subjective theorists, the causal relationship between cost and price was somewhat inverted from the position taken by the "old" Classical School — the subjective (or psychological) element was taken as the "causal factor" and the cost of production was obtainable through a process of "imputation". The "real cost" theory of value, of e.g. Ricardo, was no longer accepted.2/

1/See e.g. Prof. Knight "Capital and Interest", reprinted in "Readings in the Theory of Income Distribution" page 386.

2/Knight, op. cit. page 386.

The recognition of the subjective elements brought about many revolutionary changes in economic theories. Production, in the "new" theory, consists of using factors of production of all kinds in a relationship of symmetrical cooperation for the purpose of creating (final) consumers' service,[3/] and distribution simply became the valuation of the productive services in the imputation process - i.e. the determination of the <u>functional distribution</u> to the factors of production.[4/] "The radical transformation of the classical system may be dated from the promulgation of 'utility' or 'subject' theory of value...".[5/]

The transformation from the "old" to the "new", especially with respect to the integration of the value and distribution theories under the "new" spirit, was not accomplished by Jevons.[6/] Professor Stigler has pointed out[7/] "Jevons' theory of distribution contributed <u>little</u> to the solution of the problem of <u>distribution</u>, although they contain the germs of some important later development".

The model which will be built shortly is more properly called "Austrian" for the (additional) technical reason that the quantities of the resources are assumed to be fixed. As Professor Stigler has pointed

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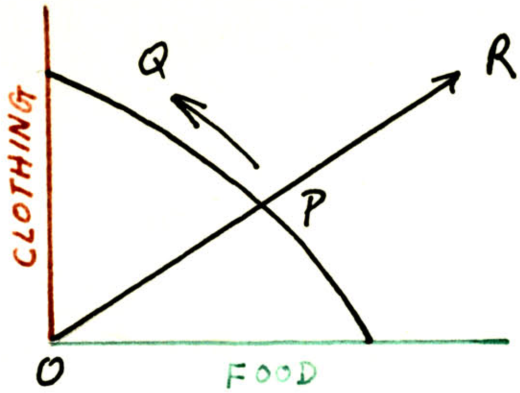
3/In contrast to the old classical conception (e.g. Smithian and Ricardian) of production as creation of <u>tangible wealth</u>. <u>Op.cit</u>. Knight 386-387.

4/For a discussion of the significance of the impact of the subjective theory on <u>distribution</u>, see Professor Stigler, "Production and Distribution Theories", Chapter I.

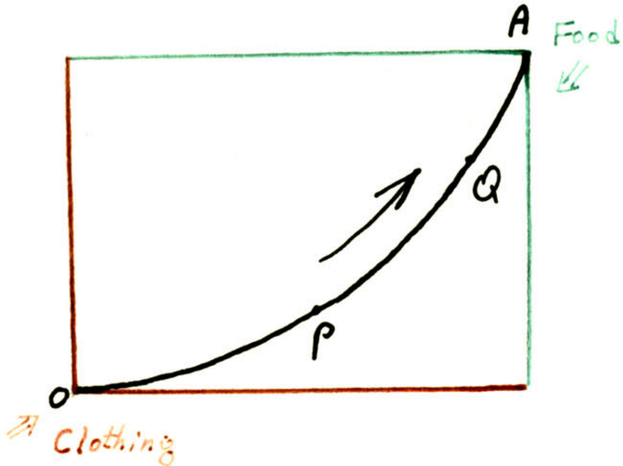
5/Knight, <u>op.cit</u>. page 386.

6/As has been pointed out by Prof. Stigler, the process of integration of value and distribution theories was a slow one. <u>op.cit</u>. page 3

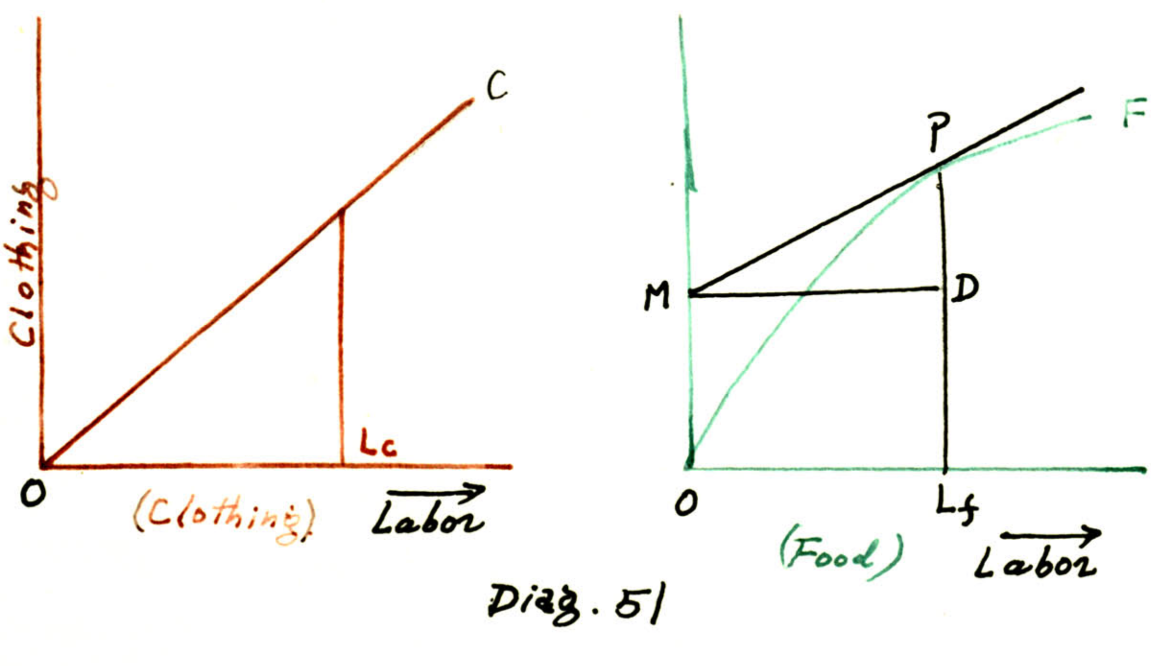
7/Stigler <u>op.cit</u>. page 35



Diag. 49



Diag. 50



Diag. 51

"...Bohm-Bawerk's detailed exposition differs in one important respect. The usual Austrian assumption of complete fixity of the quantity of productive resources (or, more properly, of the flow of productive services) is an explicit part of his analysis".8/

The diagrammatic representation of the Austrian Theory of value and distribution has already been undertaken in our analysis in the previous chapters. In diagrams 49 and 50 a box diagram (of country A) and the corresponding map of production frontier are shown. In the frontier map, take the radial line OR - or rather the slope of the radial line OR - as representing the relative strength of demand for the two commodities. A relatively stronger demand for clothing, for instance, will be represented by a steeper OR line (i.e. OR moves toward the direction indirected by the arrow) representing a higher demand ratio (or output ratio) in favor of clothing. In the box diagram, the equilibrium position will then be represented by a higher point (Q rather than P). By rule 25, we know that the product price ratio and the functional distribution ratio (i.e. factor price ratio) become higher. (The geometrical expressions for these measurements in the frontier map and/or the box diagram have been discussed in the previous chapter.9/)

The analytical assumptions underlying this model are quite obvious.10/ For our later purposes we may explicitly point out here that we have assumed for this model that for both labor and land the supply is completely inelastic with respect to factor rewards and that both factors

8/Stigler op.cit. page 183

9/See above page 57.

10/See e.g. the assumptions listed on page 58 above.

are useful for the production of both commodities.[11/] In other words, we did not differentiate the two factors of production, either from the viewpoints of the productive properties nor from the viewpoint of the supply conditions. This is representative of the spirit of the subjective theorists (referring to "rent"),

> "It will be seen that exactly the same principle applies to wage. A man who can earn six shillings a day in one employment will not turn to another kind of work unless he expects to get six shillings a day or more from it."
>
> "The parallelism between the theories of rent and ages is seen to be perfect in theory."
>
> "Rates of wages are governed by the same formal laws as rents." [12/]

In the following section, we shall try to construct a, what may be called, static Ricardian Model.

Section three: A Static Ricardian Model

The old Classical economists, in their discussion of the "distribution" problems, made only the differentiation of agricultural products (or "raw produce") on the one hand, and industrial products on the other — i.e. they neglected the individual products within the two broad classifications. This has been clearly pointed out by Professor Buchanan, who wrote:[13/]

11/As shown in diagrams 49 and 50, we did not postulate a "free-region" for any factor of production.

12/Quotations from W.S. Jevons, "Theory of Political Economy."

13/Prof. D. H. Buchanan "The Historical approach to Rent and Price Theories" reprinted in Readings in the Theory of Income Distribution.

> "Until the last paragraph but one of the introductory section he (i.e. Adam Smith) speaks of rent in general terms...giving no hint that he is dealing with the rent paid for producting a "particular commodity, taken separately" (609)

> "They (i.e. Ricardo and Malthus) did not place one kind of raw produce over against another, but placed "raw produce" over against "manufactures". At no place in West's Application of Capital to Land, Malthus's Nature and Progress of Rent, or in Ricardo's chapter on Rent in the Principles is there any discussion of the supply of a particular product..." (page 619, italic original).

If this is the case, it is reasonable to attribute to the Old Classical economists, the assumption of the production functions with constant returns to scale. 14/ In what follows, we will take "food" as the representative agricultura (raw) product and "clothing" as the representative industrial (manufactured) product, for the Old Classical economists, and for the production of these products, we assume production functions with constant returns to scale.

What will be called a "static Ricardian Model" in this section, is then representative ("impressionistic") of the positions taken by the Old Classical economists, in general, in this respect.

Another characteristic of the analytical assumption of the Old Classical economists is that: labor was assumed to be the only factor useful for the production of the manufactured product (clothing) -- or

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14/See footnote 30/on page 194 below where it will be pointed out that this assumption was attributed to the classical economists by e.g. Wicksteed.

land is only useful for the production of raw produce (food).[15/] This is again explicitly pointed out by Professor Buchanan:

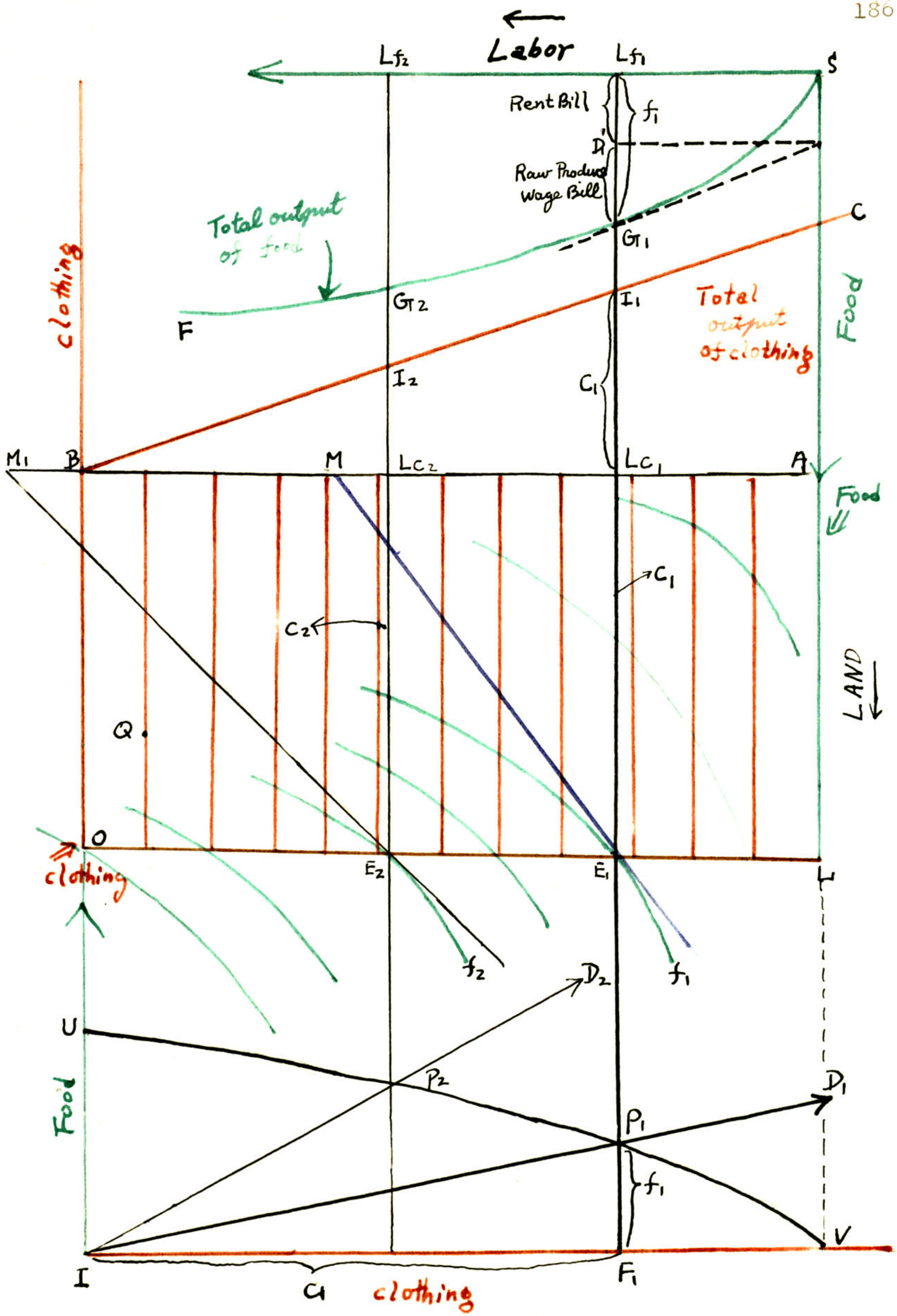
> "This land (i.e. referring to the distribution theories of Adam Smith) was in a very different situation from the fields which had sharply competing uses...It must be used for generalised raw produce or return to nature" (607, underline supplied)

> "Ricardo's treatment makes much of the shifting of labor (and capital) between raw produce and manufactures, but never comes to the shifting of land, because by his hypothesis land had only one use and did not shift to manufactures" (620) (underline supplied)

The production functions, for the production of food and clothing, then, can be represented, by the diagrammatic methods usually found suitable for "partial equilibrium" analysis, as in diagram 51. The total outputs of clothing, when various quantities of labor (only) are applied, can be represented by the radial (straight) line OC, reflecting the condition of constant returns to scale. The total output of food, when various quantities of labor are applied on a fixed amount of land, can be represented by the curve OF. (When equilibrium is established at point P, the distances L_fD and DP represent the size of the total rent bill and the partial wage bill originated in the farming sector).[16/] [17/]

15/In other words, while labor is useful for the production of both food and clothing, land is useful for the production of food only. What is meant by "useful" or "useless" will be translated into "diagrammatical language" shortly. It is true that, for Ricardo, capital is another factor of production. We neglect the capital for a reason which will be discussed below. See page 205.

16/Given point P, (or the distance of OL_f), point D is obtained in such a way that it lies on the same horizontal line passing through point M, which is the point of intersection of the tangential line at P with the vertical axis. The fact that PD equals to wage bill (in terms of food) can be easily proved by the fact that the slope of the tangential line represents the marginal physical productivity of labor. The fact that DL_f represents the Rent Bill - i.e. marginal physical productivity of land times the units of land - is assured by the Euler Theorem - i.e. rule 14.


Labor
Lf2
Lf1
S
Rent Bill
f1
D'
Raw Produce Wage Bill
Total output of food
G1
C
G2
I1
Total output of clothing
F
I2
C1
clothing
Food
M1
B
M
Lc2
Lc1
A
Food
C1
C2
LAND
Q
O
clothing
E2
E1
L
f2
f1
D2
U
P2
D1
Food
P1
f1
V
I
F1
C1
clothing

Diag. 52

These production functions can be represented, alternatively by the method of box diagram. In diagram 52, the box diagram is constructed (middle) when the endowments of resources are given - i.e. OL units of labor and AL units of land. The only "peculiarity" of this box is that the production contours for the production of the industrial product (clothing) are vertical (straight) lines (with "indices" proportional to the horizontal distances from the lower left origin O) reflecting the assumption that land is useless for the production of clothing. We see, then, the assumption that land has no alternative use from the viewpoint of the "farming sector" is only a special case of the more general assumption of the productive properties of land. 18/ 19/

Fn. 16 cont'd.
The proof of the Euler Theorem from diagram II (for food) was first accomplished by S. J. Chapman (Stigler op.cit. page 337. "He presents a most elegant diagrammatic proof that the residual share is equal to the marginal product of the factor receiving the residual." The proof was reporduced by Prof. Stigler on page 337-339) The writer is inclined to think that this diagrammatic proof is more complicated than the one given as rule (14) in this thesis - making use of the production contours-and also, by the very nature of the theorem, it is more desirable (from the pedogogical viewpoint) to prove it from the contour map.

17/In drawing the total output curve for the production of food, we made the assumption inplicitly that the marginal physical productivity of labor is constantly diminishing and does not approach zero in the relevant range. This amounts to the assumption that the ridge lines, for the production of food, coincide with the two axes, respectively, so that the whole map of production contour is in the non-free region. (See below page 186) The more general case will be discussed in a later section. (See below Section VII)
Since the marginal physical productivities of the factors are determined by the ratio of input (rule 13), the curve marginal to the total product curve (OF) is fixed if we take the horizontal axis measuring input ratios.

18/Since the production contours are vertical, the withdrawal or addition of land has no effect on the output of clothing. We can say that this is the special case where the land-free ridge line (and consequently the labor-free ridge line) for the contour map of clothing, coincides with the horizontal side of the box.

The optimum allocation curve for this box is the (grey) lower-horizontal side of the box, (i. e. OL) reflecting the fact that all the land will always be allotted to the agricultural product, as should be expected.[20/]

The production frontier map corresponding to this optimum allocation curve is plotted in the diagram 52 immediately below - i.e. curve UV - in such a way that the vertical axis (food-axis) is lined up with the lower-left origin of the box diagram. (The unit of clothing is defined in such a way that when OL units (or all the) labors are allocated for the production of clothing, the total output (of clothing) is IV units in the frontier map such that IV equals OL in geometrical length.) The production frontier UV is concave to the origin everywhere.

19/As has been pointed out in footnote 17 on page 187, the contour map for the production of food, as shown in the diagram, is also a special case. The two ridge-lines for the production of this commodity are not shown because they coincide, respectively, with the two axes AL and AB.

20/From the viewpoint of welfare economics, the lower-horizontal side is obviously the optimum allocation curve - e.g. if equilibrium is not established at a point on OL (e.g. at point Q) the production of food can be increased, without effecting the output of clothing, by reallocating more land to the agricultural sector.
That OL also represents the possible equilibrium position as a result of the operation of the competitive system is evident - by the fact that if equilibrium is established at point Q, (and if full-employment is ensured by perfect competition, equilibrium position must be represented by a point in (or on the sides of) the box), land owners receive nothing in the industrial sector and receive something from the agricultural sector. This is impossible under the assumption of "perfect mobility" of land between the two sectors. (Incidentally we see that the "equality" at the margin is not a necessary description for the equilibrium position. What is relevant for economic analysis is the "incompatability with the equilibrium conditions at points other than those satisfying the equilibrium conditions" - a meaningful tautalogy.

The diagrams, suitable for partial equilibrium analysis (introduced earlier) are plotted in the upper region of diagram 52. The upper-horizontal side of the box (BA) is used as the horizontal axis, (with origin at point B) for the total output curve of clothing (i.e. BC radial line in this diagram is OC in diagram 51). The total output curve for the production of food (i.e. OF in diagram 51) is placed up-side-down in this diagram with the origin (S) lined up with upper-right corner of the "box" - i.e. point A.

In this model, if the relative strength of demand is given, as representative by the slope of the radial line OD_1 in the frontier map, equilibrium will be established at point P_1. The equilibrium positions in the box diagram, the total output curve for clothing, and the total output curve for food will be represented respectively, by the points E_1, I_1, G_1.

The straight line (blue) ME_1, tangential to the production contour of food, passing through point E_1 can be drawn. Let ME1 intersect the AB axis at point M. From point G_1 (on the total food-output curve SF) we can locate point D'_1 marking the two distributive shares in the farming sector of the economy - i.e. D'_1Lf_1 and D'_1G_1 for rent bill and wage bill respectively.[21/]

In this set of diagrams, many significant measurements at the equilibrium positions can find geometrical expressions. For example; outputs of clothing (food) (c_1f_1) contour in the box diagram or the horizontal (vertical) distance IF_1 (F_1P_1) in the map of production frontier

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21/See footnote 16 on page 185 above.

or the vertical distance L_cI_1 (LfG_1) in the total output map), product price ratio (slope of ME1 in the box diagram) social distribution ratio (OL divided by MLc in the box diagram)[22/] [23/] etc.

With the aid of our diagrams, we can say quite a few things, on certain issues in the history of economic doctrine with respect to the value and distribution theories. Let us present them in the following (random) order:

(1) For the Old Classical economists, distribution means the "distribution" of annual output to the various social classes (in our model, the land-owning class and the laboring class). In the words of Professor Buchanan:

> "But rent as a share in the distribution of the annual produce of the nation was something different, it was the total income of a "class"of society (607 quotation mark original, discussion referring to Adam Smith)

> "The corn law discussion centered about the question as to what determined the price of raw produce to the urban population. But it was not a question of the value of particular commodities...It was a question of the value of the gross produce furnished by one class, the rural class. It was dominated by the class point of view." (617) and "Their discussions were dominated by the point of view of distribution between social classes." (618, discussion referring to Ricardo and Malthus, italic original)

22/By rule 24.

23/We also see readily the expression of wage bills and rent bills, in terms of products, directly in the total output maps as shown. We can find similar geometrical expressions of the wage bill and rent bill in the box diagram by rule 24. Also OE1 divided by AM gives us the ratio of the value of the industrial products to the farming products.

The fact that the theoretical interest of the Old Classical economists was centered about the social (or Class) distribution problems can be explained by historical incidences and the then prevailing industrial and social background. (Adam Smith more or less inherited the problem from the French Physiocrats.[24/] Ricardo and Malthus were agitating the practical corn-law issue at the turn of the century.)[25/] With the passing of the historical incidents, the theoretical interest of the younger generation, symbolized by the person of Jevons, whose interest was similar to the Austrians, has changed, as has been pointed out by Professor Buchanan:

> "Jevons represents the further development of those influences which were apparent in Mill. He lived under different conditions and was interested in a different aspect of economic study. After a thirty-year campaign the corn-law question had been solved and almost forgotten before Jevons reached mature age..... Jevons was not concerned with the problem which chiefly concerned men in Ricardo's time, the practical distribution of the annual produce among the different "classes" of the community. He was interested in.... a theory of exchange." (628)

In other words, in the "static Ricardian model" constructed above, the critical measurements for the Old Classical economists are such ratios as OL/MLc rather than the slope of ME1. It was the social distribution ratio rather than the functional distribution ratio which was the major concern of the Classical economists.[26/]

24/See Buchanan op.cit. Sections II and III

25/op.cit. Section IV

26/Under a simplified (but probably realistic) assumption as to the distribution of the ownership of resources (i.e. labors only own "labor", and landlords only own "land"), the old Classical economists "equated" the two problems.

(2) It is evident from our diagram (52) that in order to solve the functional distribution and the social distribution problem, "the other blade of the scissors" is completely necessary in our model. In other words, the Old Classical economists could not have solved the functional distribution nor the social distribution problem — since they have neglected the relative strength of demand. The absurdity of the "labor theory of value" of the Old Classical economists will receive our detailed criticism at a later section of this chapter.[27]

(3) Our static Ricardian model of this section is probably very misleading since we have neglected the supply conditions of the two factors (labor and land) postulated by the Old Classical economists. This question will be analyzed in the following sections where we will construct other models more representative of the Ricardian Rent Theory.

(4) The two total output curves which have been placed on top of the box diagram (in diagram (52) are clearly redundant as far as the solving of the general equilibrium problem is concerned — there is nothing which we can read from the two curves that we cannot read in the box diagram and the frontier map. They are placed there (diagram 52) in order to show the relationship between the "partial equilibrium analysis" and the "general equilibrium analysis".

Especially worth mentioning is the Marshallian "strategy" of exposition which "built up" the analysis of the general equilibrium problem from partial equilibrium analysis.

27/See below Section VIII page 214.

The Marshallian rules of game, as is well known, differentiated between short run and long run, which is essentially an adjustment "lag" type of analysis. With respect to the land, Marshall imagined that not only the total quantity of land is fixed in the "old" country, but also that the land, in parcels, used by any individual firm, is fixed in the short run.28/ The land is, then, a fixed factor in the short run for a firm to which various amounts of the other factors (e.g. labor) must be added. In this way, the payment of "rent", constitutes a "short run surplus", from the view point of the firm.29/

28/For the arguments in this problem, see Professor D. Worcester, Reconsideration of Rent Theory, A.E.R. Vol. XXXVI, No. 3, June 1946. The fact that Marshall's treating of the factor "land" on a different basis than the other factors of production was mainly caused by the desire of Marshall to facilitate exposition for the purpose of general discussion, as was pointed out by Professor Worcester (page 261 footnote 6) "Jevons, Wicksteed, Davenport and many others have argued against the Marshallian concept, holding that rent should not be measured as a surplus since it is unnecessary to do so and it adds nothing to exposition. Moreover, it makes the theory unnecessarily complex by putting rent on a basis different than other expenses... Furthermore, they think that it ought so to be regarded because of the smooth way in which it would then fit in the larger framework of economic theory. Marshall acknowledges the final point and condones it for this purpose, but not for general discussion. The dispute, then, was almost exclusively about the implications of the method and not the definition of the result." Again on page 275, Professor Worcester quoted Marshall in his defence of the "fixed factor concept". "Thus he wrote, '...in discussions written specially for mathematical readers it is no doubt right to be very bold in the search of wide generalizations...but it is not in the treatise such as the present in which mathematics is used only to express in terse and more precise language those methods of analysis and reasoning which ordinary people adopt, more or less consciously in the affairs of everyday life!" The meaning of this rather long footnote is to support the assertion of the writer in the text, that the partial equilibrium of the Marshallian type is merely a strategy for the exploitation of the general equilibrium problem.

29/The "short-run-surplus", from the viewpoint of the firm, owing to adjustment lag, is drastically different from the Old Classical conception of "rent as a surplus" which, the writer believes, should be explained by the differentiated long-run supply conditions of land and labor.

The exposition is based upon a partial equilibrium psychology and the total output curves (and the related marginal and average output curves and cost curves) are especially suitable for this kind of strategy.

This strategy, in spite of its questionable relevancy to the facts of the realistic world, is too "Classical" in flavor — in that rent was treated as a surplus. Under the assumption of production functions with constant returns to scale, the equivalent of the "surplus" approach and the "generalized positive" approach, was first accomplished by Professor P. H. Wicksteed's "Co-ordination", which have received the systematic analysis of Professor Stigler.30/

Section four: The Classical "Land" and "Labor".

In the last section we have examined the different "measurements" that interested Ricardo and the Austrians. (We have been able to find geometrical expressions for these measurements, i.e. functional distribution vs class distribution, in our diagram) But we came to the conclusion that our Ricardian model constructed in the last section cannot truly represent the major interest of Ricardo — so that, in order to do

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Fn. 29 cont'd.

(See Appendix A below). We may quote Professor Stigler's observation on the personality of Professor Marshall (op.cit. 63) "The other important characteristic (i.e. the Marshallian works in general), from our viewpoint, is Marshall's veneration for the classical economists...he had a pronounced tendency so to phrase his own doctrines as to minimize the change from the classical tradition."

30/See Stigler op.cit. Chapter XII, especially section on Wicksteed (page 323 ff). On page 327 of this treatment, we found that the assumption of production functions with constant returns to scale was attributed to Ricardo by Professor Wicksteed. This attribution of Prof. Wicksteed has received at least a tacit sanction of Professor Stigler who wrote, as an introductory remark to his exposition of Wicksteed (326) "Because he (Wicksteed) says perhaps as many judicious things about the Ricardian theory as one man has ever said, this portion of his analysis deserves detailed presentation."

justice to Ricardo, we have to construct new Ricardian models. This inevitably involves the analysis of the additional assumptions made by Ricardo.

It facilitates our exposition if we consider, from the rigorous analytical viewpoint, the various legitimate assumptions which we are entitled to make with respect to such an entity as "factor of production" — and then investigate the Ricardian assumptions.

Take "land" for instance, it was conceived by Ricardo as "the original, indestructible powers of the soil" net of any "improvement" (or the "capital element") artificially added.[31/] This "power", as variously interpreted by the Classical economists,[32/] consists of the "geometrical relationship", the "rain fall", "the weather", a "bounty gift of nature" (for the Physiocrats) a "niggardly given gift of nature" (for Ricardis) etc., etc.

To the modern analytical economists, it is clear, however, that a description of the physical, chemical, ethical or theological properties of a factor of production is neither important nor interesting. What is "relevant" and what we definitely want to know are the operational relationship between "factors" and the other entities in the analytical system. As we have pointed out earlier,[33/] the significant descriptions

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31/Ricardo, "Principles of Political Economy" Everyman's Library Edition by Ernest Rhys, J. M. Dent & Son, Ltd., page 33.

32/The land considered here is the "land" in the "old" country of Marshall.

33/See Chapter I, page 2

of a factor of production, from the analytical viewpoint, consists of two, and only two, categories:

1) the productive aspect of a factor of production which describes the "efficiency of production" of a factor relative to, and in conjunction with, the other factors of productions, for the production of various commodities.

2) the supply conditions of a factor which describe the conditions (definable in terms of other analytical entities in the same system) under which the factor, or factors, will be supplied to various uses.[34]

These two aspects of information on a factor, when fully given as "data", are sufficient descriptions of a factor — other informations are redundant. Since we have already defined the productive aspect of the factors - i.e. by the production contours - what remains to be considered are the Ricardian assumptions with respect to the supply conditions of land and of labor.

In spite of their occasional digressions on "irrelevant descriptions", the Classical economists, on the whole, in their capacities as analytical economists, made rather clear cut assumptions as to the supply conditions of land — namely, the supply of land is completely inelastic with respect to the variation of the reward payable to the owners.[35]

34/As we have pointed out earlier, (page 2), these aspects are, respectively, engineering knowledge and psychological knowledge in nature.

35/Ricardo, page 110.

For the supply conditions surrounding "labor", the Classical economists conceived the idea that, in the long run, it is infinitely elastic at a (minimum) real wage level — namely, the so-called iron law of wages. For Ricardo stated:

> "The natural price of labor is the price which is necessary to enable the laborers, one with another, to subsist and to perpetuate their race, without either increase or diminution" (52)

This supply condition is subsequently more clearly (and operationally) defined:

> "When the market price of labor exceeds its natural price...labor rears a numerous family...(or)...by the encouragement which high wages give to the increase of population, the number of laborers is increased...(and conversely)...when the market price of labor is below its natural price...it is only after their privations have reduced their number that the market price of labor will rise to its natural price".

Hence, if we take the "owner" as a family unit, the supply of labor (as a factor of production) is infinitely elastic at the minimum wage level in the long run. The only ambiguity is the "minimum wage" or the "natural price". However, Ricardo's subsequent discussions suggested that by natural price he meant a stock of wage goods with specific composition, i.e. the natural price is the "real wage" of labors.[36/]

For the short run supply of labor, the assumption is less clear cut. However, it is obvious that Ricardo admitted the possibility of

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36/See Ricardo, pages 58,59 where the spending pattern is computed, by numerical examples, of the income of a worker — where he considered a worker "worse off" when he cannot purchase the same quantity of "industrial product" as before. (See also footnote 37 on page 198).

temporary divergent of actual wage established in the market and the minimum wage — for indeed, this divergence is crucial for the "dynamic mechanism" of the Ricardian distribution theory which will be considered in the following section.37/ There we will interpret (or attribute to him) his assumption as to the short run supply condition of labor.38/

We may add a remark that, with respect to the supply conditions of labor; Ricardo made no allowance for the "sectoral preference" - i.e. labors (or owners) are quite indifferent as to the two sectors (farming or industrial) to which they will sell their services as long as the rewards are the same.39/ This rather trivial remark proves to be of some interest for the value theory of Ricardo.

These fairly clear cut assumptions as to the supply conditions of labor and land of the Classical school have been, unfortunately, entangled with other unnecessary philosophical or sociological properties of the two factors, which, as we have observed, are completely irrelevant for economic analysis.

Section five: Ricardo's Dynamic Distribution Theories

Once the supply conditions of the two factors of production are introduced into the analytical framework, the analysis of the "dynamic distribution theory of Ricardo" begins. Let us point out, first of all, certain features of this theory. (The actual construction of a "Ricardian model" will be undertaken in the following section which will be based upon the simplifications deduced from the analysis of this section.)

37/See Gaide and Rist "History of Economic Doctrine"

38/See below, page 201.

39/See Buchanan, op.cit. page 620.

That a large part of Ricardo's analysis was dealing with a dynamic distribution theory had been clearly pointed out by Professor Harrod (in "Toward Dynamic Economics").[40] A brief outline of Ricardo's dynamic theory ("dynamic" in the "Harrod" sense) was given in the same book.[41] Let us first outline this "outline" in the following form:

1. The motivating force of a growing economy is capital accumulation, which is governed by profit expectation.

2. Capital is the wage fund, the function of which is to motivate (or "advance to") "productive" labor.[42]

3. In the short run when wage fund increases, and with constant population, real wage increases.

4. In the long run population increases by the (assumed) operation of the Iron law of wages.

5. As population and the accumulated capital increase, land is more intensively cultivated. Hence, rent increases by the assumption of law of diminishing returns.

6. Consequently, the product-less-rent share that goes to labor and capital (i.e. wage plus profit) decreases percentagewise.

7. In the long run, real wage will always be maintained at the minimum wage level; so profit declines as accumulation proceeds.

40/Prof. Harrod pointed out (page 15) that "dynamic theory --- occupied at least half of the attention of the Old Classical School".

41/Toward Dynamic Economics, page 15-20. For another diagrammatic representation of this theory, see Baumol: Economic Dynamics, Chapter 2.

42/Let us neglect the "equipment" category of capital and concentrate on the "wage fund" category. This diverges from Prof. Harrod's interest but is perhaps more representative of the dynamic distribution theory of Ricardo.

8. In conclusion, the long run tendency is for the rent share to increase, wage rate maintain constant, and profit rate diminishes - to the point where capital accumulation ceases and the long run (pessimistic) stationary state is reached.[43]

From the modern viewpoint, it is not difficult to detect at least the following ambiguity of this dynamic distribution theory. (And we know: our "diagrammatic" analysis cannot stand any ambiguity).

1. With regard to the "supply condition of labor", we noticed that it must behave in such a way as to allow a short run lag of adjustment so that actual wage, established in the "market" place, can be different from the "minimum" wage — otherwise the dynamic mechanism is spoiled. Since this "short run lag" is crucial, the supply conditions of labor, in the short run, must be more precisely defined. To simplify matters we can imagine that the dynamic process of growth is composed of successive (shorter) periods and that during each period the supply of labor is completely inelastic with respect to "real wage", like land. The size of population, in the next period will increase (or decrease) if difference between actual wage and minimum wage, in the current period, is positive (or negative). (We could have postulated a functional relationship between the rate of population increase and the "rate of change" of this "difference" (or the "duration" of a certain level of difference) so as to have a quantatively determined growth-path through time.

43/We neglect such elements of Ricardian theory as the technical invention, the possibility of the change of minimum wage through time, etc.

But for the sake of simplicity, let us neglect this refinement and satisfy ourselves with a postulation merely on the direction of population change.)

The assumption of the short run inelastic supply of labor seems to be the logical interpretation of the wage fund theory of capital in the rigorous form.[44/] It serves to determine the actual wage in every period if the stock of wage fund (capital) is known in each period.

2. Unfortunately, the size of the wage fund coming to the market in each period was left unexplained in the dynamic theory outlined above. This fundamental defect of the wage fund theory has received the most severe criticism from Professor Knight.[45/] In his words the question becomes:

> "...what actually determined the division of the produce-less-rent between capitalist and the laborers... or what determines the amount which the capitalist must pay as wages, before he gets his own share, fixed by subtraction (?)" (189)

In other words, what determined the size of wage fund, which, together with the (given) inelastic supply of labor in any given period, will serve to determine the actual wage in that period?

44/Speaking on the "old" Classical dynamic models, Professor Baumol remarked, op.cit. page 16, "McCulloch went so far as to adopt a perfectly rigid wage fund theory which held that wages were given by the total capital in existence divided by the number of workers, i.e. by the quantity of capital per wage earner". This implies of course, the assumption that the short run supply of labor is completely inelastic. Just how a less rigid wage fund theory will fit into the analysis was an unanswered question. As indeed, the diagrammatic representation of the dynamic theory of the old classical school, by Prof. Baumal, did not face squarely the difficulties considered in the following paragraphs of this section - and was very vague. (This is a criticism on Ricardo rather than on Prof. Baumol).

45/F. Knight "The Ricardian theory of Production and Distribution" - The Canadian Journal of Economics and Political Science, I, 1935, page 185 ff.

The answer clearly cannot be, e.g. what is required for the support of the labor, for then, in any period, real wage always exactly equals the minimum wage and the dynamic mechanism is spoiled — which is a highly unfavorable interpretation of Ricardo. As Professor Knight has said:

> "since the theory purports to explain, and must explain, the division of the joint labor-capital share which actually takes place, must be short-run monetary theory, and not one which merely states or explains a long-run tendency." (189)

The long-run tendency is what will eventually happen in the long-run pessimistic state (of Ricardo); and the short-run determination of the real wage - through the determination of the size of wage fund - is what actually happens in (each of) the successive short periods. This, according to Professor Knight, was left unexplained by Ricardo. The answer, according to Professor Knight, is:

> "Thus the subsistence theory of wages rests on the deeper assumption that the employer-capitalist makes the division arbitrarily and this is the clear import of the text" (190) and again

> "The deeper aspect of this theory that capitalists make the division between themselves and their laborers by arbitrary fiat, is strongly confirmed by the tone of the discussion in Smith and Mill" (190)

Hence we know that the size of wage fund is determined by arbitrary decisions of the capitalists in aggregate. However, the difficulty is still there. This is true because: if it is within the power of the capitalists (the "masters") to stipulate the necessary payment to the workers (the "servants") it is natural, to the self-interest of the "masters", to fix the actual wage at the minimum wage level; and if this is true, there can be no deviation of the actual wage from the minimum wage level and the dynamic mechanism of Ricardo is again spoiled.

There are at least two ways to escape this dilemma - i.e. to reconcile the co-existence of the wage fund theory and the (desired) result that there can be a deviation of actual wage from the minimum wage in the short period. The first one is to postulate a bargaining power on the part of the labor class (the servants) — labors would not be so helplessly subjecting their fate to the mercy of the capitalists. The actual short run wage is then determined by "class-struggle". 46/ The second way out is to postulate another determinant of the size of short run wage fund - i.e. the savings of the capitalists; and the modern economists would have no difficulty identifying this "other determinant" as, e.g. the "consumption function" - in which the level of income is the determinant of savings. 47/

With respect to the "first way out", we know that this line of thought leads to the "exploitation school". In the absence of an impartial market force, a "personal force" must be substituted in its place so that the economists could find a solution of an apparently realistic social problem. If the validity of this line of thought is accepted, there is little need for the modern distribution theory. 48/ Of course, we cannot

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46/In modern terminology, this would become a bilateral monopoly problem — and, generally, no definite solution is obtainable. Mrs. J. Robinson, writing on the long-period theory of employment of Marx, (Essay on Marxian Economics) wrote "In these circumstances, the level of real wages is determined by the bargaining power of capitalists as a class and workers as a class. So long as the workers do not combine, they are helpless, and must take what they can get. Wages therefore tend to be depressed to the lower limit set by subsistence level."

47/See Keynes, "The General Theory"

48/The modern distribution theory, under which functional distribution is determined by market force was a contribution of the subjective theorists (1870-1895). Contrasting the "new" with the "old", Prof. Stigler stated reject the exploitation theories only on the ground that they render useless the modern (elegant) distribution theory. The main reason for the rejection of this school of thought is that we question the relevancy of the hypothesis of this school to the facts of the realistic world.[49/] Consequently, we cannot take this escape to build our dynamic Ricardial model.

The second escape seems to be more promising. However, on a closer examination, it is highly doubtful that this would be a faithful interpretation of the "dynamic distribution theory" of Ricardo.[50/] Consequently,

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Fn. 48 cont'd.
(Production and Distribution Theory, page 1) "It was in this quarter-century that economic theory was transformed from an art, in many respects literary, to a science of growing rigour".

49/In one way or another, the exploitation school assumes the existance of "classes" which is more a "belief" than a testable hypothesis. They may be criticized in the words of Professor Knight, who wrote ("Profit", reprinted in "Readings in the Theory of Income Distribution") "The labor theories of value and of production rest in the first place on a confusion between ethical and economic or scientific explanatory principles" (536) which is criticism of the exploitation school in general, and again "...wages were supposed to be determined independently, the final share of the capitalist being left as a residuum. The most important commentary on this classical scheme of distribution is the negative statement that it failed completely to "implement" the process of distribution through any discussion of the actual workings of competitive (or monopolistic) principle of price fixing. Fruitful treatment of the distribution problem...came about gradually as a result of the new treatment of value introduced by the utility theorists" (534) which can be taken as a criticism of the distribution theory of the exploitation school in particular.

50/The writer believes that no modern economic student, in his rightfull mind, will regard the (income-determined) savings as a wage fund.

the second escape can not be adopted on which our Ricardian Model will be erected.

What we will do, for the construction of a dynamic Ricardian model, is to neglect the "capital" element completely.[51/] For what it is worth, we can construct a dynamic model for the study of the "production and distribution" theory without the capital element — although much of the spirit of the Ricardian theory is lost.[52/]

However, in all fairness to that great name, we can still call our model - which will be constructed presently - "Ricardian". For what we will do is to retain most of the "salvagable" elements of his theory from the viewpoint of the modern distribution theories. With respect to his distribution theory, we know labor and capital are "dosed" together for production, and a dose is applied on "land". The determination of the two distributive shares: rent and "reward for capital and labor" was rightfully regarded by Ricardo as determined by a competitive mechanism, and the solution of the problem foreshadowed the later "marginal analysis". This was the most significant contribution of Ricardo as far as distribution theories are concerned.[53/]

51/We neglect both the "wage fund" and the "equipment" elements of Capital of Ricardo.

52/For we know that capital accumulation is really the motivating force of the Ricardian theory. (See above page 199)

53/As Professor Knight has stated, op.cit. page 178 "...It is in connection with rent that we find the nearest approach in the classical writings to a real theory of distribution meaning a process of imputation on the basis of final increment".

Another element which we definitely want to preserve (as a salvageable element) is the "dynamic" nature of his theories. The problem of "growth" is a much neglected problem in the New-classical Tradition.[54/] In this respect, the wisdom of this "old" classical economist deserves the highest praise.

The Ricardian Model that will be constructed below should be read with the understanding that it is not a "faithful" representation of the Ricardo's dynamic distribution theory, rather, it is a very favorable interpretation of his theories.

Section six: Dynamized Ricardian Model

Within the limitation of our interpretation of the "dynamic Ricardian model", the actual construction of the model (diagrammatically) is a relatively easy task — since we have acquired all the geometrical background. As indeed, diagram (42) (on page 159 - i.e. the map of production frontier - which we have constructed for the study of the "multiple equilibrium problems" (in the field of international trade theories) can be used without modification.[55/] All we need to point out is that diagram 42 is derived from diagram 39,[56/] in which the quantity of

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54/See Harrod, op.cit. Chapter I

55/This offers an example of the similarity of the problems in the whole field of economic study - i.e. it seems to suggest that there exists a system of unified principles which explain a wide range of economic problems; and precisely owing to this fact, the related (group of) economic problems constitute a separated field of study, namely, a distinct scientific subject.

56/See discussions on pages 156-162 for the derivation of the map of production frontiers (diag.42) from the box diagram (diagram 39).

land is being held constant for all "countries". Instead of "all countries", the different boxes - or, the different optimum-allocation curves and the different production frontiers - now represent the different sizes of population for a "given country". The quantity of land of this country is being held fixed as population increases. This is probably not a misrepresentation of Ricardo and the whole Classical school.[57/] The (five) regions now represent the five stages of growth - in the order of the sizes of the population. The distinction of the five states of growth will be relevant to our analysis in the following section.

There is one feature of this dynamized Ricardian Model which we want to point out; namely, in our present model, land is assumed to be useful for the production of both commodities —whereas, in our Ricardian model constructed in Section III above, we assume that land has no use in the production of clothing.

The analysis of the dynamic distribution theory of Ricardo can be formally represented as follows: historically a country is in stage one - i.e. point C in diagram 42, - where land is a free factor.[58/] If the "growth path" is known (e.g. the straight line OQ_i) intersecting higher and higher production frontiers corresponding to successively larger sizes of population (higher solid curves) at a series of points e.g. D', E', G, Q_i'...(with sizes of population successively, d, e', g, i,...)

57/At least Marshall is included in this group. See footnote 32/on page 195

58/The area included in OfC' is the land free region the series of points is the successive equilibrium position in each period.

The growth path itself is determined, among other things, by the relative strengths of demand for the two commodities in each of the successive period. Had the relative strength of demand been more favorable for food, (i.e. the ratio of output of food to clothing is larger), the growth path would have been represented by a steeper radial line - e.g. OQh. The growth path, of course, need not be represented by a straight line; but for simplicity, let us assume for the moment that the growth path can be represented by a straight line in order to isolate the "demand" side of the problem. The dynamic mechanism can be briefly indicated. We first postulate a level of "minimum" wage — which is assumed to be fixed throughout. When equilibrium is established at point C, labors received the whole national product - (since land was a free agent). If the actual wage established at point C is higher than the minimum wage, the size of population in the next period becomes greater, and equilibrium position will be established at a higher point - e.g. point D'. Again, the actual wage established at point D' is compared with the "fixed" minimum wage; if actual wage still exceeds minimum wage, the size of population increases again. In this way, a sequence of equilibrium points (i.e. C,D',E',O,Q_i) is generated by this dynamic mechanism.

If the growth path is a straight line (economically the relative strength of demand for the two commodities remains price inelastic at a fixed output ratio), the radial line OQ_i intersects higher and higher price line (dotted blue). This means, of course, that the (actual) real wage gradually declines through time. A point Q_i is finally reached where actual wage equals "minimum wage". The population, then, ceases to grow and the long run (pessimistic) stationary state, (for Ricardo) is reached. The size of population is assumed to be equal to i, when the minimum wage is established.59/

If, after the stationary state is reached, the relative strength of demand for food increases - represented, e.g. by a more steep radial line OQ_h - the equilibrium position will be established at point Q_h which is the point of intersection of the radial line OQ_h and the equal-price contour (g'K dotted blue) passing through point Q_i. This is true because with a reduction of the size of the population, the minimum wage can be maintained at this "demand ratio". In other words, the long run equilibrium will always be established at a point lying on the equal - price contour g'K, depending upon the relative strength of demand for the two commodities. (This is true because we know that the level of real wage remains unchanged - at the "minimum wage" level - if equilibrium is established anywhere on g'K.60/

It is then obvious in the long run stationary state (in our model) the adjustment to change of relative strength of demand requires population change. If, however, the "fluctuation" of the relative strength of demand is rather sudden then the population adjustment fails to respond

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59/The final equilibrium point Q_i must stop short of stage five, (but may reach certain point in stage four depending upon the relative strength of demand for the two commodities) for when stage five is reached, labor becomes a free agent and is clearly "impossible".

60/See above, page 137.

instantaneously (in the "short run")[61/], the product price ratio will have to change. For instance, if the size of population has been adjusted to the point Q_i (with population size i), the short run equilibrium position, when demand ratio shifted to the radial line OQ_h, will be established at point S which represents a point with higher food price than that established at point Q_i with the same size of population. (Eventually, equilibrium will shift to point Q_h in order to maintain both the minimum wage and the demand ratio).

We readily identify the adjustment along the production frontier Ii (or along any production frontier corresponding to a fixed size of population) the Austrian adjustment.[62/] The movement along the Radial line OQ_i (or along any growth path not necessarily a straight line) is the Ricardian adjustment, and the movement along g'K (or along any equal-price line representing minimum wage) the Marshallian Problem, for a reason which will be suggested below.[63/]

In the following sections we will make certain comparisons of the theoretical conclusion of the three models so far constructed.

61/Although both the "short run" and the "short period" (in our exposition of the dynamic Ricardo) are phenomena caused by the lag in population adjustment, they should be distinguished. The "short period" is relevant to a "process analysis" and the "short run" is more properly a "comparative static analysis". Economically, the short period refers to certain points of time during a growth process and the short run refers to a lag of adjustment when the stationary state is already reached.

62/See above section II, page 179.

63/See below page 223.

Section seven: The Equilibrium Configurations of the Three Models

We can easily derive certain conclusions, respectively for the "three" models, with the aid of our diagram (diagram 42). Let us point them out orderly.

(1) Dynamic Ricardo Model

In the dynamic Ricardian Model, if the growth path is a straight line - i.e. if the relative strength of demand for the two commodities remains approximately the same - we know:

a) The price of food increases gradually (relative to the price of clothing).64/

b) The price of labor (or rather the price of "capital and labor" since they are dosed together) decreases absolutely and relatively to the price of land service.65/

c) The effect of population growth on social distribution ratio (i.e. wage bill divided by rent bill) is indeterminant.66/

64/Referring to diagram 42, if the growth path is a straight line, the ratios of output for all sizes of population, will be equalized. Referring to diagram 42, it is seen that if the equilibrium position before population increase is Qi (i.e. population size i) when population increases to size j, the new equilibrium position must be represented by a point lower than Q_j if the same (old) output ratio is to be maintained. (Since $\frac{OQ_i}{OQ_j} < \frac{iQ_i}{jQ_j}$ which gives $\frac{OQ_i}{iQ_i} < \frac{OQ_i}{jQ_i}$, the ratio of output of clothing to food is higher at Q_j than at Q_i). The new product price ratio must be lower than the old product price ratio (established at Q_i before population increase) if the output ratio is to be maintained.

65/See Fn. 64/ above.

66/In other words, the labor's share (relative to the land-owner's share) of the total national product is effected by two opposing forces, a) the declining functional distribution ratio adversely effected the social distribution ratio, from the viewpoint of the labor class; and b) the increasing size of labor force tends to effect social distribution ratio more "favorably" to labor. When the growth path is a straight line, the net result is generally unknown. This can be proved by geometric methods.

Hence, in spite of the fact that social distribution ratio was what interested Ricardo most we cannot support the thesis that social distribution ratio necessarily declines as the population-growth (and capital accumulation) proceeds (when we "isolated" the forces of the relative strength of demand for the two commodities[67/] by assuming a radial-line-growth-path).

If the growth path is not along a radial line, these conclusions apparently do not hold. For instance, if the growth path (in diagram 42) after point G, concaves downward sufficiently (along the dotted arrow) so that the growth path intersects successive "lower" equal-price contour and "higher" production frontiers (i.e. larger size of population) the price of clothing becomes higher and higher and the functional distribution becomes more and more favorable to labor. The social distribution ratio then necessarily becomes more favorable to labor class.[68/] This state of affairs is brought about by the fact that

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67/Hence, strictly speaking, the "pessimism" of Ricardo was unfounded, if the "social distribution" ratio is emphasized. (If the functional distribution ratio is emphasized, then the "pessimist is justified, as has been pointed out above in footnote 64 on page 211)

68/In other words, both factors, identified in footnote 66 on page 211, will be favorable to labor class.

as population increases the relative strength of demand becomes more and more favorable to the "labor-intensive commodity" —the clothing. This is, of course, what should be the expected intuitively.

If we take the function distribution radio as a measure of relative welfare positions of the labor (and landowners) we have just come to the conclusion that the laborers will likely become worse off as population increases and when the relative strength of demand is isolated. This probably will be the case if we can make the additional assumption that food is a "necessity" and clothing is a "luxury" (relative to each other) - which is probably not a very misleading assumption. If this is true then, as populatinn increases, the society as a whole becomes poorer and poorer (since we assume that the quantity of land is unaugmentable which serves as a bottle-neck for the aggregate income of the nation) and a larger share of the resources must be devoted for the production of the "necessity"— food - to feed the increasing population. 69/ The growth path will then more likely be along a radial line if not actually concaved upward - representing an ever increasing output ratio in favor of food. And if the growth path concaves upward sufficiently, not only the functional distribution ratio, but also the "social distribution ratio" will be deteriorated from the labor (and capital) standpoint. In other words, the qualitative conclusion of Ricardo is largely supported if this assumption - on the nature of the demand of the two commodities - is made.

69/This assertion is not rigorous. Since we do not know the personal distribution of resources in particular and "wealth" in general we do not know the actual relative strength of demand — and we do not know the exact location of the growth path. Our assertion is essentially a deduction of the "non-rigorous type" — which are not lacking in the history of economic thought.

(2) The Austrian Model

Qualitative conclusions for the Austrian Model - which has been identified as a "short run population lag model"[70/] are more easy to make. We simply repeat what we have said earlier.[71/] The social distribution ratio, functional distribution ratio, output ratio, product price ratio, input ratios all rise and fall together, as governed by the relative strength of demand of the two commodities, and they change in the same direction.

(3) Marshallian Model

In the long run Marshallian Model, in which the size of population is adjusted to the optimum size for the maintainance of minimum wage and a particular demand ratio (represented by a movement along the e.g. equal price contour g'K) the product price ratio is always the same. The functional distribution ratio is also maintained at a constant level. The social distribution becomes more favorable for labor class when the relative strength of demand becomes more favorable to the labor intensive commodity - clothing.[72/]

Section eight: The Role of the Relative Strength of Demand

Let us now briefly examine the role played by the relative strength of demand for the two commodities in the three models considered

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70/See above pages 210.

71/See rule (25)

72/This is obvious: when the wage-rent ratio is fixed, the social distribution ratio (way bill - rent bill ratio) is higher if labor-rent ratio is higher → as will be brought about by a strengthening of the demand for clothing.

in the last section and the "static Ricardial Model" constructed in Section III above. Our purpose is to examine the special features of the value and distribution theories of the various models involved.

1. The Austrian Model

The central feature of the Austrian Model constructed above is the strategic importance assigned to the force of relative strength of demand — almost everything else seems to be the "effect" of "demand". The subjective (psychological) valuation of the consumers seems to be the sole "cause".73/ Our model, then, represents the spirit of the enthusiasm of the early proponents of the subjective theories of value.

We can even say, with Marshall, that e.g. Jevons errored on the other extreme - i.e. by over emphasizing the demand factor. From the viewpoint of static general equilibrium theories, "it takes two blades of scissors to cut" — and Marshall was no doubt right.74/

2. The Static Ricardian Model

In Section III we have constructed a static Ricardian model which differs from the Jevonian model only in that land was assumed to have no use for the industrial sector — for the production of clothing. It is now apparent to us that in this Ricardian model the relative strength of demand for the two commodities is as indispensable, for the determination of the

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73/The concept of "cause and effect relationship" (i.e. "causality") as a scientific principle is obsolete. What we mean by "cause and effect" in this connection, is strictly from the viewpoint of "method of analysis" — i.e. owing to the peculiar construction of our model (in which the supplies of the factors are assumed to be fixed) the only thing that can vary significantly and independently is the relative strength of demand for the commodities.

74/See Marshall, Principle Eight Edition Appendix I. From a philosophical standpoint, one perhaps can make an argument out of the assertion that "Psychological valuation" rather than "cost" is more fundamental for value. However, philosophical observations are not relevant to an analytical problem.

equilibrium position, as in the Jevonian model. (See diagram 52 on page 186) However, it is well known that Ricardo had arrived at the conclusion that the relative prices are determined by the "marginal labor cost" - i.e. the "labor theory of value". We can launch a belated attack on the obsolete and erroneous labor theory of value, with the aid of our diagrams.

Starting from the nation that "labor is the real price" of everything, Ricardo apparently conceived the idea that the exchange value (which Ricardo strenuously distinguished from "wealth")[75/] of a stock of commodities is strictly proportional to the quantity of labor pain "embodied" in them.[76/] (This "humanitarian" way of thinking, if represented in our diagram 52 becomes something as follows: at point P1, the exchange value of the total output of food to clothing will be OE1/ E1 which is the ratio of the quantities of labor embodied in the two stocks of commodities produced at point P1).

Realizing that this is apparently <u>not</u> what was actually found in the market place, Ricardo skillfully pointed his finger <u>at the margin</u> and argued that it is the labor embodied in the product produced <u>under the most difficult circumstance</u>,[77/] - meaning by this the cultivation at the extensive margin (i.e. the worst land currently being used) which, Ricardo knew, is equal to the intensive margin of cultivation - that <u>counts</u>. In this way, he reasoned, the labor theory of value is inpregnable. But to a modern reader, his "no rent land" and his emphasis on "differential rent" (his slightening of the "scarcity rent") are all devices for the preservation of his <u>marginal labor theory of value</u>.[78/]

75/Ricardo, Chapter XX

76/See Ricardo <u>op.cit</u>. Chapter I. After quoting the famous "beaver and deer" parable of Smith, Ricardo stated, dogmatically: "That this is really the foundation of exchange valve of all things, excepting those which cannot be increased by human industry, is a doctrine of the utmost importance...If the quantity of labor realized in commodities regulates

It is to be noted, however, that Ricardo was perfectly right in the sense that the exchange ratio does equal the quantities of labor embodied at the margin. The big question is: how and where do we locate the margin of cultivation? (In diagram 52, if we let OD (changes e.g. to OD_2 the margins will change). The modern answer is, of course "by the other blade of the scissors" which Ricardo slighted.79/

If we take a very sympathetic explanation of Ricardo, as Marshall would have us do,80/ by interpreting him as "taking the demand for granted" this means, in our diagram (diagram 52) for instance, we have to hold OD constant. In this way Ricardo is absolved from the responsibility of "determining the margin". But the difficulty does not end here.

Footnote 76 cont'd.
their exchange value, every increase of the quantity of labor must augment the value of that commodity on which it is exercised, as every diminution must lower it."

77/For the unconditional labor pain cost theory only holds in the stage of growth before the land is appropriated. See Ricardo page 37 of Chapter II.

78/See Gide and Rist "History of Economic Doctrine" page 152.

79/If we assume a one product economy the question, of course, does not arise — for then, given the assumption of full employment (under the assumption of static competition, full employment is always realized), the margin of cultivation is determined when the endowments of the two factors are given (if the supply of factors is perfectly inelastic). If there are two (or more) products, this becomes a problem! Ricardo could have neglected the problem of "value" - and the labor theory of value - and concentrate on the analysis of "distribution" which, we know, was his major interest, by assuming a one product economy.

80/See Marshall, Principle Appendix I page 813 where Marshall states "If then we seek to understand him rightly, we must interpret him generously", and on page 814 Marshall stated, at the margin of the page, "He took utility for granted, because its influence is relatively simple."

In the first place, the "pain" (labor) cost theory of value is still completely wrong if any ethical inference is implied. The marginal unit of wheat is exchanged with the marginal unit of clothing strictly in the proportion of labor embodied (in one unit each of the two commodities) - perhaps, in harmony with social justice 81/ - but all the other (infra-marginal) units of wheat are exchanged for clothing not proportional to embodied labor pain → and strictly involves social unjustice. 82/

In the second place, if we take the static Austrian model 83/ then all what has been said of "labor and pain" above, can be substituted by "land and non-pain" and all the statements still hold - namely, products are exchanged strictly in proportion to the service of "land" embodied in one unit each of the two commodities. 84/

We we know, no matter how favorable we interpret Ricardo, labor theory of value is completely trivial. For in both the Austrian and in the static Ricardian models, the marginal labor costs (labor embodied) only indicated (or reflected) the exchange ratio. Analytically, this is determined by the assumption that there is free mobility of labor between the two sectors and that there is no sectorial preference in the supply

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81/For instance, something like "money cost", as determined by competitive force, equals "real cost" which was the position taken by Marshall (for the Stationary State, at least. See principle page 810)

82/We may wittedly add, the total units of social unjustice exceed the total units of social justice by total units of output of "wheat" minus two! (Since one unit of social unjustice is cancelled by the "marginal unit of social justice").

83/Namely, any production frontier in diagram 42. This model, according to Prof. Buchanen, was not considered by Ricardo, but was definitely considered by Marshall.

84/And how about "social justice"?

of labor. Under these assumptions, "marginal labor theory of value" follows. For, apparently, the ratio of marginal physical products of labor, in the production of the two commodities (which are the reciprocals of the marginal labor costs — or labor embodied) necessarily equals the inverse of product price ratio (otherwise laborers will move from one sector to another until this equality is satisfied,) hence "marginal labor theory of value"

Without the other blade of the scissor - the relative strength of demand - the problem can never be solved (as long as there are two or more commodities). And when this problem is solved, there is no place for the trivial "marginal labor theory of value" and still less for the erroneous "embodied labor theory of value". Ricardo definitely has saved himself from the embarrassing situation only by obscurity.

We can now see, incidentally, the difference between the Austrian model and the Ricardian model. While in the Jevonian model, both labor and land embodied in the marginal units of the two products, can be taken as an "indicator" (not determinant) of exchange value, this cannot be done in the Ricardian Model - for in this model land has no alternative use for the viewpoint of the farming sector. This might have caused one to believe that "rent" is a "surplus" (an effect) while wage (or labor cost) is the "cause" — but in fact, marginal labor costs are indicators too. 85/

85/In other words, labor has an opportunity cost from the "sector" viewpoint, while land has no such opportunity cost. But in the static Ricardian model, to say that rent is the "surplus" (effect) in the sense that labor is the cause is a completely unintelligent, mysterious, and erroneous assertion. If, in the short run, both the supplies of labor and land are perfectly inelastic, it is more meaningful to say that, in the Ricardian model (and the Jevonian model) the relative strength of demand is the "cause" of the ups and downs of rent (rate or share).

This whole "bundle" of pain cost theory was inherited by the "exploitation school". Even more mysterious philosophical properties have to be ascribed to the factor "labor". 86/ The quality of the theories

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86/In a sympathetic review of the theories of Marx, Prof. P. M. Sweezy (The Theory of Capitalistic Development, Oxford University Press, N.Y. 1942) emphasized the Marxian distinction of "the qualitative value problem" and the "quantitative value problem" (page 25 "The great originality of Marx's value theory lies in its recognition of these two elements of the problem..."). Prof. Sweezy further pointed out (page 34) "That critics of Marx have concentrated their attention of this prospect of the theory (note: the"Quantative" aspect, rather than the "qalitative aspect"), and at that one-sidedly, is no accident; their attitude towards the value problems has disposed them to a preoccupation with exchange ratios to the neglect of the character of the social relations which lie hidden beneath the surface". The writer frankly admits that he is "one-sided" (in launching the attack in the text) because he is preoccupied with the problems of "exchange ratios".
With respect to the qualitative value problem, Prof. Sweezy wrote: (page 29) "We may sum up the qualitative relation of value to labor with the following statement: On the one hand all labor is, speaking phisiologically, an expenditure of human labor power, and in its character of identical abstract human labor, it creates and forms the values of commodities. On the other hand, all labor is the expenditure human labor power in a special form and with a definite aim, and in this, its character of concrete useful labor, it produces use values". It seems to the writer that the first half of the statement of Prof. Sweezy (i.e. "on the one hand") expresses a philosophical (or religous) belief. (Historically, the role of the prime creator of value had been variously assigned to "land" to "God" etc). The second half of the statement (i.e. "on the other hand"), seems to express the idea that the "concrete useful" labor, in a "special form", is embodied in the "use values" that labor creates — which is, nonetheless, a philosophical belief too.
In view of the fact that Prof. Sweezy had taken the trouble to defend the quantitative value theory of Marx (in later chapter - i.e. Chapter III) in a language understandable to "those brought up in the main tradition of economic thought", we can safely conclude that the philosophical observations on the nature of labor (i.e. those pertaining to the qualitative value problem cited above) are largely irrelevant to the analysis of the "exchange ratio" of a capitalistic system. (As indeed it should be; since the concept of "value", as such, is a philosophical one if it does not refer to exchange value — which is something to be explained by analysis).
The apology of Prof. Sweezy for the labor theory value of Marx clearly indicated that the theory is only an approximation (page 42, "AS a first approximation Marx assumes that there is an exact correspondence between exchange ratios and labor-time ratios") and "exceptions" will

of the "exploitation school" could have been greatly improved without the pain cost theory of value. 87/

3. Dynamic Ricardian Model

We can criticize labor theory of value again, in the background of the historical growth process. Labor theory of value unconditionally holds only in the earlier period when there was a superabundant supply of land. (Referring to diagram 42, this is in region A, or the "stage of growth" number A, e.g. points included in OfC). In this stage of growth, land was like air and water; no matter how strong is the demand for the land intensive commodity (food) rent could not arise. Laborers

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Fn. 86 cont'd.
have to be made if this rigid form of labor theory of value is adhered to (page 44) "It is clearly not difficult to think of cases which violate this assumption...for example, opera singers, star baseball players, mathematicians...But these are exceptional cases). (We see, incidentally, that the category of phenomenon which Prof. Sweezy wanted to rule out as exceptions, is precisely those relating to "scarcity value". In other words, the quantity value theory of Marx, is hardly any more advanced than the Old Classical Theories.) Prof. Sweezy quoted Prof. Schumpeter and cited Prof. Keynes (pages 51-52) to support the thesis that it is legitimate to neglect the relative strength of demand (page 52 "We see that Marx's relative neglect of the problems of consumer's choice finds ample support in recent trends in economic thinking"). This is true precisely because of the fact that the problem of exchange ratio should not be the preoccupation of the theorists concerned. (page 52, "Schumpeter in effect admits that for the problems in which he is interested - business cycles and the developmental tendencies of the capitalist system - the theory of consumers' choice is of little or no relevance). In other words, the labor theory of value <u>is</u> an inferior theory after all - if one <u>is</u> concerned with the problem of exchange ratio - and no mythology can save it.

87/ Mrs. J. Robinson has attempted a reconstruction of the exploitation theories by first pointing out that the exploitation theories can do without this bundle of labor theory of value. See J. Robinson, An Essay on Marxian Economics, Macmillan chapter 3 (on page 27 Mrs. Robinson wrote "I hope that it will become clear, in the following pages that no point of substance in Marx's argument depends upon the labor theory of value.")

were the general bottleneck factors. In this region exchange ratio is completely governed by the quantities of labor embodied in the various commodities.88/

But after the edge of the land-absolutely free region (A) is approached, (e.g. OQ_i intersects the straight line Cc' at point C), land may or may not be superabundant — depending upon the relative strength of demand of the two commodities. Had the relative strength of demand been stronger in favor of food (e.g. the growth path were OQ_h rather than OQ_i), the land would no longer be a free good at the same size of population. In other words, the size of population corresponding to the margin of superabundance of land depends upon the relative strength of demand.

After the land relatively free region (B) is passed (i.e. the growth path passes point E' on fc' and reaches the "third" stage of growth (C)) land will definitely not be a free good any more — and the trivial

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88/This state of affairs was clearly recognized by Ricardo when he wrote (34) "On the first settling of a country in which there is an abundance of rich and fertile land, a very small proportion of which is required to be cultivated for the support of the actual population...there will be no rent, for no one would pay for the use of land when there was an abundant quantity not yet appropriated, and therefore, at the disposal of whosoever might choose to cultivate it." That the ratio of exchange is governed by labor embodied in this stage, he wrote (37) "The most fertile and most favorably situated land will be first cultivated, and the exchangable value of its produce will be adjusted in the same manner as the exchangeable value of all other commodities (note: Ricardo apparently meant "all other commodities - produced in the industrial sector - for the production of which the land has no use) by the total quantity of labor necessary...to produce it and bring it to market." (Notice, in this stage both the "marginal labor theory of value" and the "embodiment labor theory of value" hold. They will fall down together in the next stage of growth.)

marginal labor theory of value which has been criticized above, comes into force (the embodied labor theory of value fails).

Hence, it is clearly seen that the labor theories of value are only special cases, from the analytical viewpoint, of the more general "general equilibrium theory".89/

4. The long-run Marshallian Model

In our long run Marshallian model, we arrived at an interesting conclusion: relative price of commodities are independent of the relative strength of demand.90/ We, irristably, recall the often quoted assertion of Marshall:91/

> "Thus we may conclude that, as a general rule, the shorter the period which we are considering, the greater must be the share of our attention which is given to the influence of demand on value; and the longer the period, the more important will be the influence of cost of production on value."

What is more interesting, we have been able to corroborate this assertion of Marshall, even in a more confirmed tone, (and words for words)

89/Our historical approach of the value problem probably represents the psychology of Ricardo: his famous second chapter on rent was written after the first chapter on value and was supposedly an elaboration on the value theory. (His first chapter corresponds to our "land free region" (stage one and partly two of growth) and his second chapter corresponds to our "non-free-region" (the second and the third stages of growth).

90/See page 214 above.

91/Marshall, Principle page 349. This is the main reason that we choose to call the "long run stationary state of Ricardo" a "Marshallian model". In other words, in this way, we can support the assertion of Marshall from the viewpoint of the general equilibrium theory. (In addition to this reason, the nature of the "long run" equilibrium model, also reminds us of the "stationary state" of Marshall. However, we choose to call our model Marshallian temporarily, only to reject it in the remainder of this section).

as a result of our diagrammatical analysis.[92] Can we claim that we have told in diagrammatic language what Marshall wanted to say?

In order to answer this question we have to answer, first of all, the question: why have we been able to reach such a conclusion - i.e. product price ratio is independant of demand in the Marshallian model?

In the first place, we arrived at this conclusion because we have made the assumption of iron law of wages — so that, in the long run, the size of population is governed by the relative strength of demand for the two commodities. Did Marshall assume iron law of wages? The answer is "yes", according to Professor Harros:[93]

> "We know well how lovingly he (i.e. Marshall) treasured all the bits and pieces of traditional theory...Even the iron law of wages reappears; its guise is softened and rendered kindly, but it is there all the same."

On this account, we probably can claim that our model gain a certain "Marshallian" flavor.

But on a closer examination, we found that we have assumed, not only the iron law of wages (defined in terms of a <u>vague level</u> of minimum wage), but we have actually assumed a <u>rigid composition of wage good</u> for the "minimum wage". This amounts to the assumption that the <u>marginal</u>

92/A comparison of the conclusions which we have arrived at (see above pages 214) of the short run Austrian model and the long run Marshallian model confirms this statement. In the Marshallian model, the relative commodity prices are completely independent of the relative strength of the demand of the two commodities; and in the short run Austrian model, the product price ratio is largely governed by the relative strength of demand.

93/Harrod, <u>op.cit</u>. page 15

<u>physical productivity of labor, in both industries, should remain absolutely the same</u>, in the long run. Given the assumption of production function with constant returns to scale, this means that the "input-ratios" in both industries should remain the same; and that the size of population changes whenever the two <u>fixed input ratios</u> are not maintained. If this is the case, then the commodity prices ratio will inevitably be maintained in the long run.[94/]

So the crucial question becomes: did Marshall postulate a fixed composition of wage good? Moreover, did he postulate the "minimum wage" in terms of a fixed composition of wage good, which will be maintained in the stationary state such that the size of population fluctuates in response to a change of the relative strength of demand for the two commodities? In the "well-rounded" exposition of Marshall, such a proposition is simply too "acute" and "novel" for Marshall. Marshall would be the first one to accuse us if we try to interpret him in this way, for:

94/From diagram39 , (and comparing with diagram 42) these assertions will become immediately obvious. We might point out: the "<u>fixed-input ratios in both industries</u>", "<u>the fixed product-price ratio</u>", and the "<u>fixed marginal physical productivities in both industries</u>", "<u>the fixed composition of wage goods</u>" are uniquely correlated to each other. This is true by the assumption of constant returns to scale.

Further, these assertions will remain to be true even though we will postulate a <u>variable</u> (rather than fixed) <u>supply of land</u> — provided a fixed stock of land, where determined in size, becomes completely inelastic in supply. This can be easily demonstrated by the method of box diagrams. We can then call "land" in our long run Marshallian model the "<u>capital</u>" and give our model an additional dimension (i.e. a variable amount of capital) enabling it to resemble, more, the Marshallian position. (after all, land <u>is</u> capital for the Classical School). We, then, will have more right to call our long run model "Marshallian".

> "In this world (note in the realistic world) every plain and simple doctrine as to the relations between cost of production, demand and value is necessarily false and the greater the appearance of lucidity which is given to it by skillful exposition, the more mischievous it is." (368)

And no one can deny that we have given our long run Marshallian model a "great appearance of lucidity" (although it may not be very skillful), and hence, very mischievous, indeed. But in the imaginary stationary state, for which more skillful exposition is supposedly permitted, and in which, in the words of Marshall:

> "(In a stationary state)...the plain rule would be that cost of production governs value" (367)

we found; that it is a "state":

> "...in which <u>population is stationary</u>" (367)

This spoils all our fun of "hunting". What is represented in our long run "Marshallianmodel" is a "run" much more longer than the stationary state of Marshall — because we postulated an ultimate adjustment of the size of population. And, in his "mild form of iron law of wages" Marshall used such "well rounded" expression:

> "Turning next to the growth of wealth, we observed how every inctease of wealth tends in many ways to make a greater increase more easily than before" 95/ (314)

We could find little support for our assertion of a fixed-composition of wage goods defined as the pivoting minimum wage. We must then conclude: despite all our efforts to support the Marshallian assertion (or the Classical assertion in general), namely, that in the long run cost of production (rather than demand, <u>or</u> the "<u>two</u>" blades of scissors) governs value, our model is <u>not</u> "Marshallian".

95/What Marshall had in mind is what was discussed earlier on page 180-192 in the "Principles" on such topics as "marriage-rate", "birth-rate"...etc.

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