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Motivated by the deterministic wireless information flow model introduced by Avestimehr, Diggavi and Tse [2], we introduce a deterministic flow model based on combining *polylinking systems*, a notion introduced by Schrijver [4] and closely related to polymatroids. Given is a set of vertices that can be partitioned into layers $V_1, \dots, V_r$ and flow is sent across consecutive layers. There is no notion of edges, and how flow can be sent from one layer to the next is described by polylinking systems. Thanks to the abstract nature of polylinking systems, this new model is very general and gives large freedom in specifying how flow can be sent through the network. In particular, the new model includes the classical flow model of Ford and Fulkerson restricted to acyclic graphs as well as a wireless information flow model introduced by Avestimehr, Diggavi and Tse [2] to approximate the Shannon capacity of Gaussian relay networks. In the framework of this ADT model, we can aggregate all vertices within a relay, and obtain a more compact and natural formulation of this model based on our polylinking flow model. This has also the advantage of allowing for non-integral capacities and flows.

We define a natural notion of source-destination cut which can be seen as a generalization of the notions used in the flow models mentioned above. Using results from polylinking systems we show that the value of a maximum flow is equal to the value of a minimum source-destination cut. Despite the generality of the new flow model, using submodular flow algorithms, one can find a maximum flow, a minimum source-destination cut as well as a minimum cost flow in polynomial time if an oracle for evaluating a function associated with the polylinking systems used in the description of the network is available. In the case of the aggregated ADT model, evaluating this function can be done very efficiently by using a Fast Fourier Transform.

We give a description of the flow polytope which is the set of all feasible flows. Unlike the classical Ford and Fulkerson flow model, the flow polytope has in general an exponential number of facets. However, using the reduction to the submodular flow polyhedron, we show that the flow polytope is integral if the underlying polylinking systems are integral. This generalizes the integrality property of classical flows.

In the unit-capacity case the notion of polylinking systems reduces to the notion of linking systems and our algorithmic approach reduces to matroid partition. For this special case, our flow model corresponds to a *cascading system* introduced by Schrijver [4] and discussed in [3]. The matroid partition approach provides an efficient algorithm for finding a maximum flow and a minimum cut in the ADT flow model, see [3].

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References

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