Structural Behaviour of Cable-stayed Bridges

by

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Abstract

Cable-stayed bridges have emerged as the dominant structural system for long span bridge crossings during the past thirty years. That success is due to a combination of technical advancements and pleasing aesthetics attributes. The interaction of the various structural components results in an efficient structure which is continuously evolving and providing new methods to increase span lengths. The objective of this thesis is to describe in detail the basic structural behaviour of each of the components of cable-stayed bridges, and to present the analysis of a specific cable-stayed bridge which was proposed for the Charles River Crossing.

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Chapter 1

Introduction

Cable stayed bridges date back many centuries; the system was used by Egyptians for their sailing ships. Early Chinese people used the cable-stayed system to construct suspension bridges out of hemp rope and iron chains [6]. There are many other examples of ancient cable-stayed bridge systems found around the world.

The first modern bridge structures were a combination of a suspension and stayed system. They were constructed at the end of the eighteen-century in the United States and in England [5].

F. Dischinger identified the need to increase the stress in the cables so as to reduce the sag effect in the stiffness. This advancement gave the impulse to modern cable based structures. In 1955 he built the Stromsund bridge, located in Sweden, which is considered the first modern cable stayed bridge [5][9]. Another important factor in the evolution of cable-stayed bridges was the employment of superstructure sections that act as a continuous girder along the longitudinal axis. With these improvements, modern cable-stayed bridges became very popular in the last thirty years. Significant illustrated milestones are the Theodor Heuss bridge in 1958, the Schiller-Steg footbridge which was constructed in Germany in 1961, the Maracabio Bridge constructed in Venezuela in 1962 [5]. The major cable-stayed bridges of the world are listed in table 1.1[1].

[1] From: Karoumi Raid, 'Dynamic Response of Cable-Stayed Bridges Subjected to Moving Vehicles', Licentiate Thesis, Dept. of Structures Eng., Royal Institute of Technology.

Bridge name	Country	Center span (m)	Year of completion	Girder material
Tatara	Japan	890	1999	Steel
Pont de Normandie	France	856	1994	Steel
Yangpu	China (Shanghai)	602	1993	Composite
Xupu	China (Shanghai)	590	1996	Composite
Meiko Chuo	Japan	590	1997	Steel
Skarnsund	Norway	530	1991	Concrete
Tsurumi Tsubasa	Japan	510	1995	Steel
Ikuchi	Japan	490	1991	Steel
Higashi Kobe	Japan	485	1994	Steel
Ting Kau	Hong Kong	475	1997	Steel
Annacis Island	Canada (Vanco.)	465	1986	Composite
Yokohama Bay	Japan	460	1989	Steel
Second Hooghly	India (Calcutta)	457	1992	Composite
Second Severn	England	456	1996	Composite
Dartford	England	450	1991	Composite
Rama IX	Thailand (Bangk.)	450	1987	Steel
Chang Jiang Second	China (Sichuan)	444	1995	Concrete
Barrios de Luna	Spain	440	1983	Concrete
Tonglin Cangjiang	China (Anhui)	432	1995	Concrete
Kap Shui Mun	Hong Kong	430	1997	Composite
Helgeland	Norway	425	1991	Concrete
Nanpu	China (Shanghai)	423	1991	Composite
Hitsushijima	Japan	420	1988	Steel
Iwagurujima	Japan	420	1988	Steel
Yuanyang Han Jiang	China (Hubei)	414	1993	Concrete
Meiko-Nishi Ohashi	Japan	405	1986	Steel
S:t Nazarine	France	404	1975	Steel
Elorn	France	400	1994	Concrete
Vigo-Rande	Spain	400	1978	Steel
Dame Point	USA (Florida)	396	1989	Concrete
Baytown	USA (Texas)	381	1995	Composite
Luling, Mississippi	USA	372	1982	Steel
Flehe, Duesseldorf	Germany	368	1979	Steel
Tjörn (new)	Sweden	366	1981	Steel
Sunshine Skyway	USA (Florida)	366	1987	Concrete
Yamatogawa	Japan	355	1982	Steel

Table 1.1: Major cable-stayed bridges in the world.

Chapter 2

Brief Description of Cable Stayed Bridges

Cable stayed bridges are indeterminate structures. The superstructure behaves as a continuous beam elastically supported by the cables, which are connected to one or two towers. The structural system consists of three main structural sub-systems: Stiffening girder, tower, and inclined cables. The interrelation of these components makes the structural behaviour of cable-stayed bridges efficient for long-span structures, in addition to providing an aesthetic pleasant solution.

The cable-stayed system has become a very effective and economical system during the last century. It is mainly used to cover large spans. The development of this structural system is due to advances in materials, engineering analysis and design, and construction methodology.

The structural components of a cable-stayed system behave in the following manner: The stiffening girder transmits the load to the tower through the cables, which are always in tension. The stiffening girder is subjected to bending and axial loading. The tower transmits the load to the foundation under mainly axial action. The design of cable-stayed bridges, in comparison with the normal bridges, is controlled by the construction sequence, and the construction loads tend to be the dominant design loading.

2.1 Cable types

Different types of cables are used in cable-stayed bridges; their form and configuration depend on the way individual wires are assembled. The steel used for the cables is stronger than ordinary steel. A strand is generally composed of seven wires, helically formed around a center wire; the wire diameter is between 3 and 7 mm. The strands are closely packed together and typically bounded with an helical strand.

Cables are the most important elements in cable-stayed bridges; they carry the load from the superstructure to the tower and to the backstay cable anchorages. In addition to high tensile strength, they must also have high fatigue resistance and corrosion protection.

Cables are classified according to the following descriptions [2]:

1. Helically-wound galvanized strands.

   Ultimate Tensile Stress $\sigma_u = 670$ MPa

   Young Modulus E = 165 000 MPa

2. Parallel wire strands.

   Ultimate Tensile Stress $\sigma_u = 1860$ MPa

   Young Modulus E = 190 000 MPa

3. Strands of parallel wire cables.

   Ultimate Tensile Stress $\sigma_u = 1600$ Mpa

   Young Modulus E = 200 000 MPa

4. Locked coil strands.

   Ultimate Tensile Stress $\sigma_u = 1500$ MPa

   Young Modulus E = 170 000 MPa

The Allowable Stress under dead load effect for the cables under dead load is :

$$\sigma_d = 0.40 * \sigma_u \qquad (2\text{-}1)$$

Each cable type has advantages and disadvantages. For example, locked coil strands have variable stress-strain behaviour and low fatigue strength at the sockets. Therefore, they are less frequently used. It is better to choose a type of cable where the modulus of elasticity is high and constant. The parallel wire strand is the most commonly cable type [4].

2.1.1 Cable arrangements

Cable-stayed systems are classified according to the different longitudinal and transverse cable arrangements. Cable layout is fundamental issue that concerns cables stayed bridges. It not only affects the structural performance of the bridge, but also the method of erection and the economics [5].

Longitudinal Arrangement

The arrangement of the cables involves a number of considerations. It depends on the bridge requirements, site conditions and aesthetics appearance. The longitudinal arrangements are classified as follows [4]:

Harp or parallel system:

The cables are parallel to each other and are connected to the tower at different heights. The aesthetics of this kind of configuration are very pleasant. However, the compression in the girder is higher than the others patterns, and the tower is subjected to bending moments (Figure 2.1) [6].

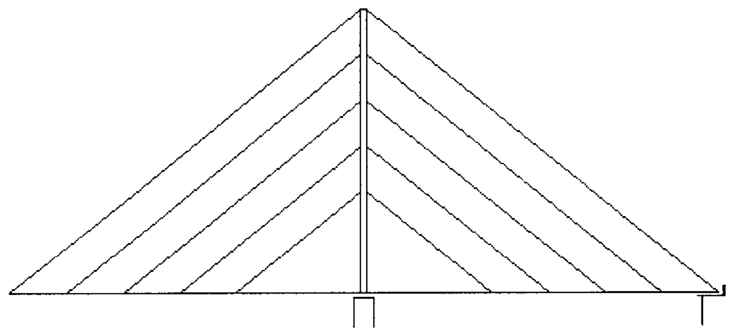

Figure 2.1: Harp System

Fan System:

This is a modification of the harp system; the cables are connected at the same distance from the top of the tower. The fan system is attractive for a bridge where the longitudinal layout is a single-plane, because the cable slope is steeper, it needs and consequently the axial force in the girder is smaller [7] (figure 2.2).

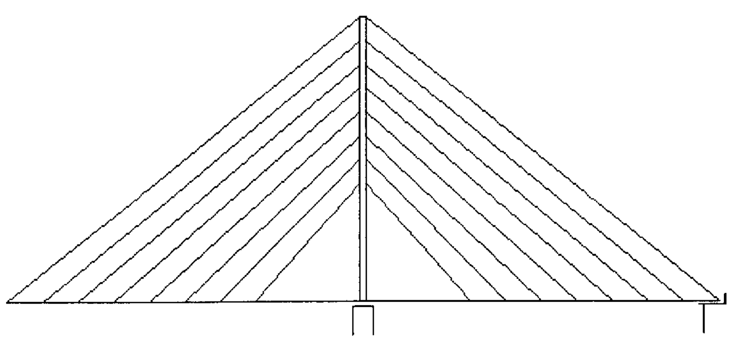

Figure 2.2: Fan System

Radial System:

With the radial configuration, all the cables connect to the top of the tower. This is a convenient cable configuration because all the cables have their maximum inclination; therefore the amount of material required in the girder is reduced [6]. However, this configuration may cause congestion problems and the detailing may be complex (figure 2.3).

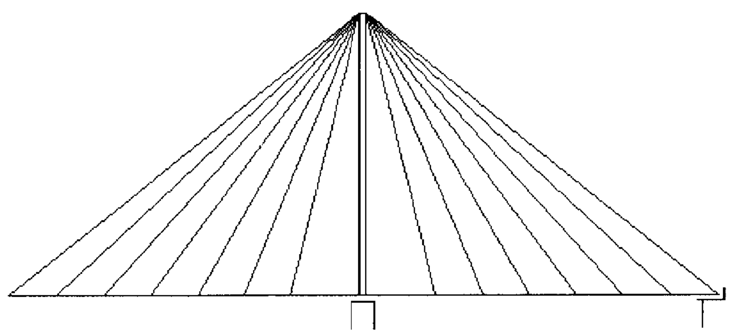

Figure 2.3: Radial System

Transverse Arrangement

For the transverse arrangement the classification is made according to the positioning of the cables in different planes. Two basic classifications follow [4]:

Single-plane system:

This system is composed of a single cable layout along the longitudinal axis of the superstructure. This kind of layout is governed by torsional behaviour. The forces are created by unsymmetrical loading on the deck. The main girder must have adequate torsional stiffness to resist the torsion force.

Two-plane system:

If the tower is of the shape of an H-Tower, the layout is a two-plane vertical system. If only one tower is provided in the middle of the superstructure, then the layout is a two-plane, inclined system.

The transverse layout has two options for the anchorage. The anchorage is located either outside of the deck structure or inside the main girder.

The spacing of the cables varies according the chosen layout and the aesthetics requirements. The current trend is to employ many cables [5]. Increasing the number of cables reduces the required stiffness of the girders, and results in more slender superstructure sections. Consequently, the load in each cable decreases, and the construction process is simplified.

2.2 Deck Types

The most common deck type for these bridges is the orthotropic deck, which consists of longitudinal ribs resting on cross-girders. Orhtotropic decks are a very light, efficient, superstructure solution for long span bridges [1][7]. Concrete deck systems, steel deck systems and composite deck systems are also widely used in cable-stayed systems. Steel decks are about twenty percent lighter than concrete decks. Concrete decks are more common in multiple stay bridges. The choice of the material is in function of the required stiffness, the method of erection, and the economics [5].

Deck systems are chosen according to the cable layout, the span dimensions, the material utilized, and the special requirements of the bridge. The most common types of deck are shown in figure 2.4 [4]. The qualities required for the deck also depend on the nature of the structure and its service requirements (road or rail bridge) [5].

Of the deck types shown in figure 2.4, the most frequently used deck system is the box section deck because it provides convenient anchorages, and has significant torsional properties. It is common to utilize diagonal bracing and frame-type diaphragms to increase the rigidity of the box section [1]. When selecting a deck, it is also important to consider maintenance and deflections limits [5].

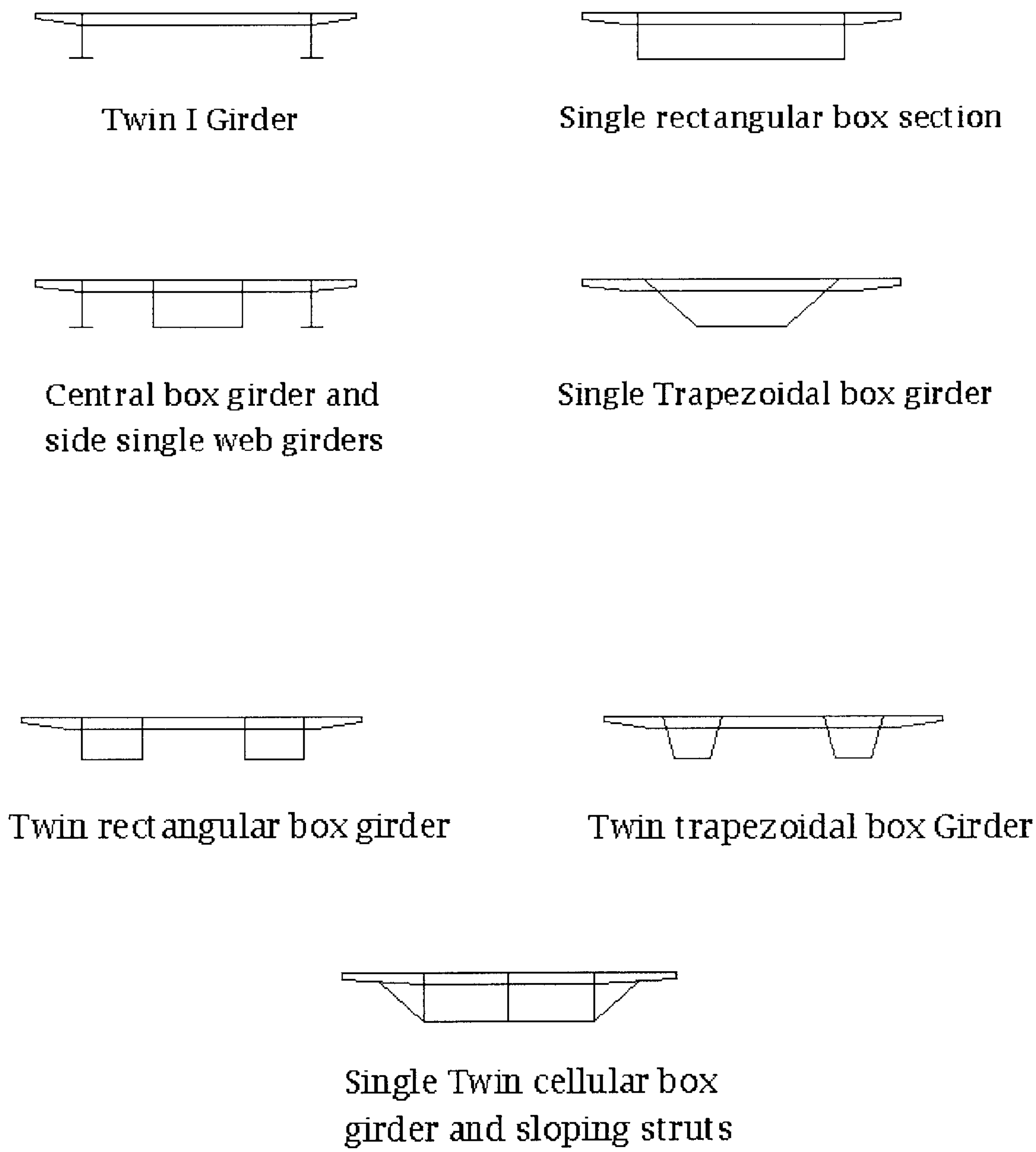

Figure 2.4: Deck Types

2.3 Tower types

The tower shape is mainly selected for aesthetic reasons, and is refined based on proportions, materials, and restrictions associated with the tower design. A considerable variety of tower shapes exist. In general, the shape of the tower is governed by the required height and the environmental loading conditions, such as seismic zones and wind criteria. The towers are classified according to the basic forms shown in figure 2.5 [4].

The towers are subjected to axial forces. Thus they must provide resistance to buckling. A more sophisticated analysis of the tower includes the non-linear P-Delta effect [1]. In addition, the tower strength also has to be checked under lateral loads and second-order effects (non-linearity) produced by tension in the cables.

Box-sections are most frequently used for the towers. They can be fabricated out of steel or reinforced or prestressed concrete. Concrete towers are more common than steel towers because they allow more freedom of shaping, and are more economical [5].

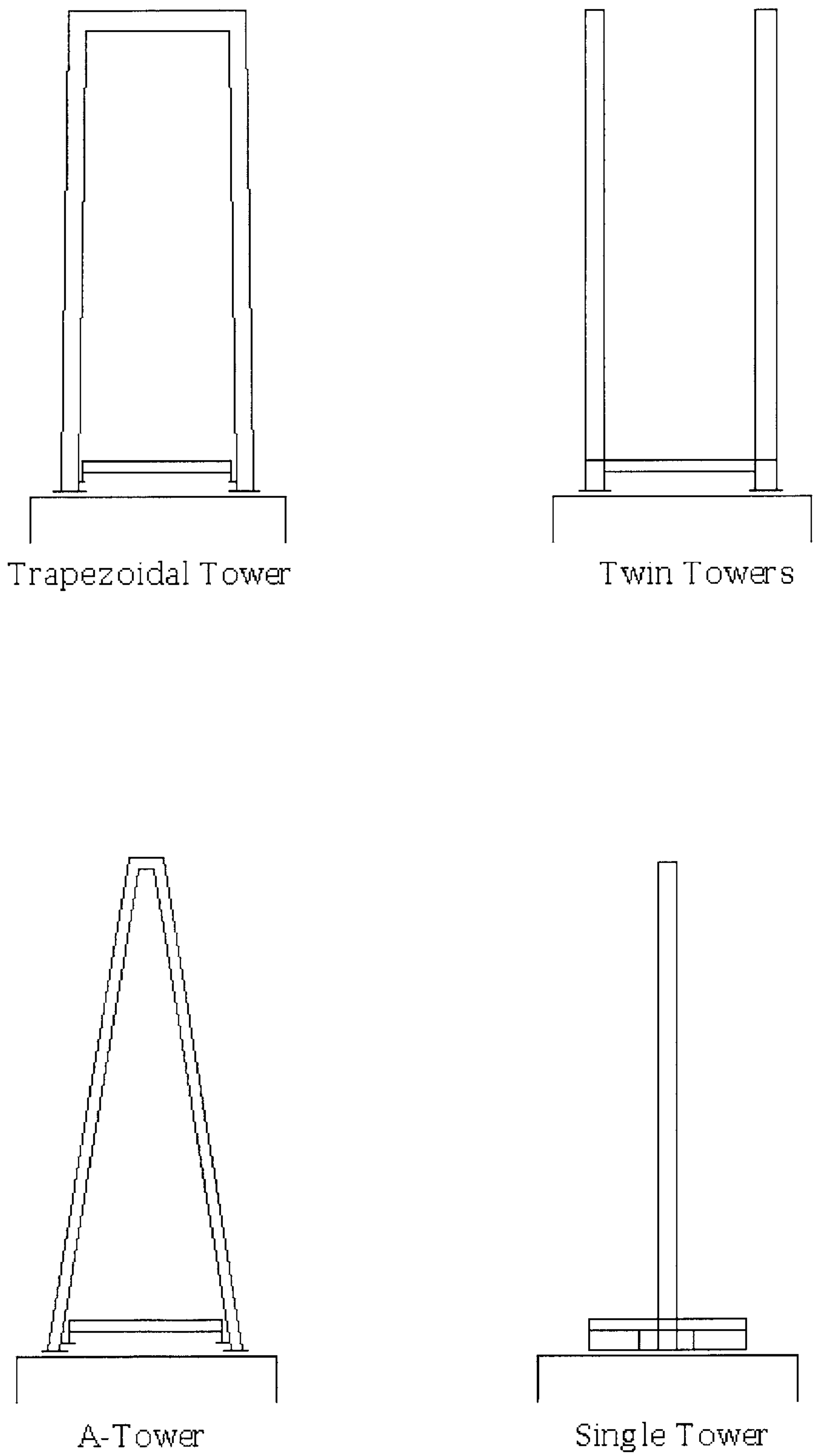

Figure 2.4: Tower Types

Figure 3.1: Charles River Crossing

Chapter 3

Geometric description of the Charles River Crossing

The Charles River Crossing bridge is a single tower cable stayed bridge with the tower placed on one side (figure 3.1). It has a main span of 230 meters and a back span of 115 meters. The tower rises 115 meters above the ground surface (figure 3.2). The total width of the bridge is 24.6 meters, and consists of one 3 meter shoulder, one 1.2 meter shoulder, four 3.5 meter traffic lanes, and two 3 meter pedestrian paths (figure 3.3).

The structural system selected has a single tower with a fan longitudinal cable stayed system, and a two plane inclined transversal system. Backstay cables anchored to back span piers provide support for the tower.

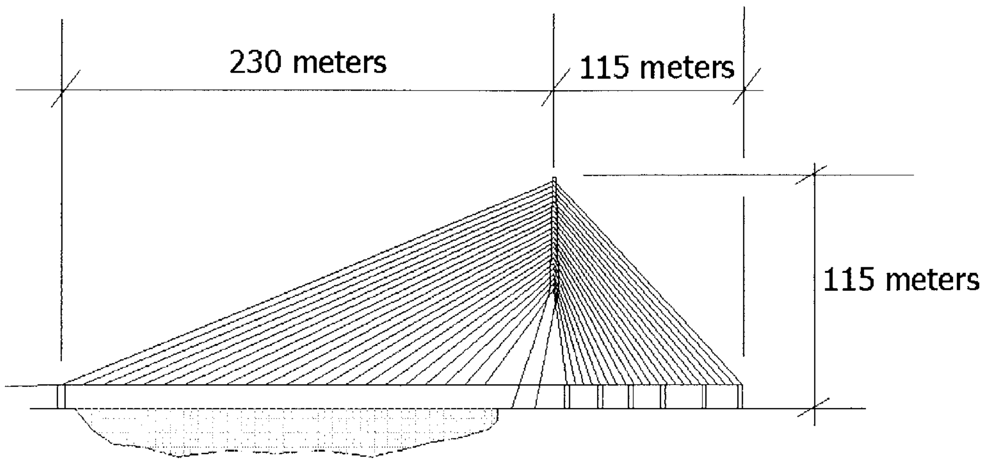

Figure 3.2 : Bridge dimensions – Longitudinal

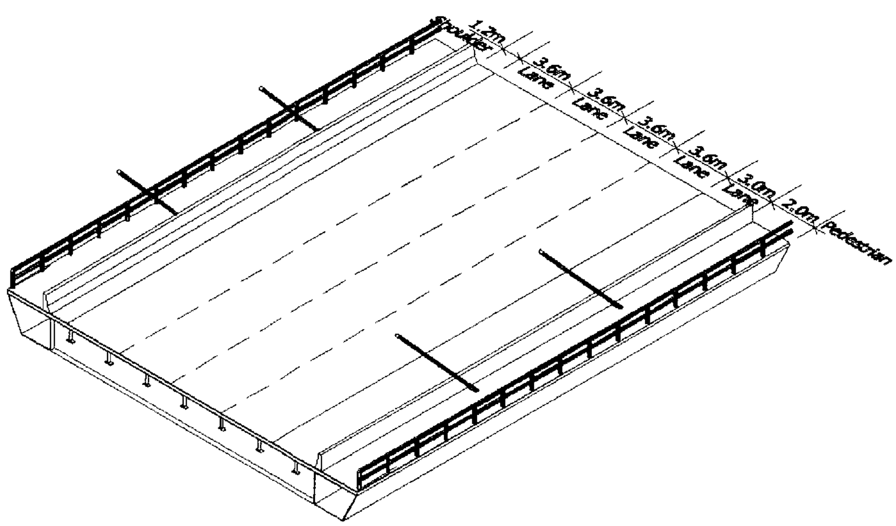

Figure 3.3 : Bridge dimensions - Transverse

3.1 Superstructure Description

The superstructure is configured to provide an effective and simple load path. The deck system is a 20 cm thick plate made of composite material. The deck is supported by longitudinal steel rolled I-beams which span 10 meters between built up steel floor beams. The steel floor beams are in turn supported by closed steel box girders which act as the main longitudinal members for the bridge (figure 3.4).

Thus, the superstructure is composed of the following:

-Overlay

-Composite Deck

-Longitudinal Beams

-Transverse Floor Beams

-Longitudinal Box Girders

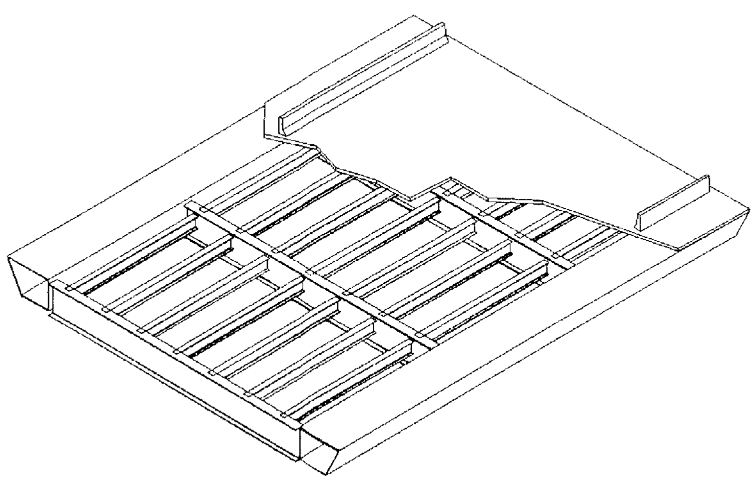

Figure 3.4 : Superstructure

The composite deck material is non-corrosive fiber reinforced polymer (FRP) composite deck. The deck section is composed of hexagon and double trapezoid elements that are bonded together with a high-strength adhesive under controlled conditions in the manufacturing plant. This assemblage is installed transverse to the direction of traffic. It is connected to the longitudinal beam by a high performance adhesive (figure 3.5)(table 3.1).

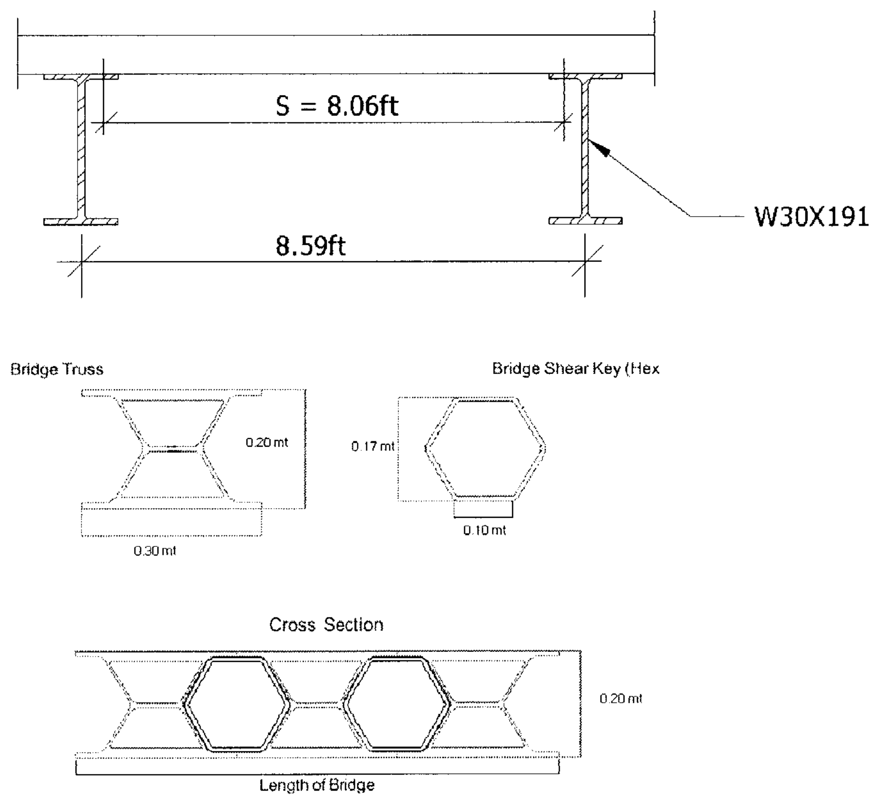

Figure 3.5 : Geometry of the composite deck.

Cross Section Properties of the Bridge Truss		
Area	1.29E-02	mt2
Ix	9.09E-05	m4
Iy	7.32E-05	m4
Sx	8.95E-04	m3
Sy	4.83E-04	m3
Rx	8.37E-02	mt
Ry	7.52E-02	mt

Cross Section Properties or the Bridge Shear Key		
Area	4.64E-03	mt2
Ix	1.86E-05	m4
Iy	1.86E-05	m4
Sx	2.09E-04	m3
Sy	1.84E-04	m3
Rx	6.32E-02	mt
Ry	6.32E-02	Mt

Table 3.1: Composite deck properties.

Longitudinal beams are W30X191 rolled steel sections spanning 10 meters between transverse floor beams. They are placed parallel to the direction of traffic at a spacing of 2.65 meters and directly support the composite deck directly.

The transverse beams are built up sections with top and bottom flanges each measuring 0.5 meters by 0.05 meters. The web measures 1.9 meters by 0.05 meters. The transverse floor beams span 18.3 meters between longitudinal box girders and support the longitudinal beams. Transverse floor beams are spaced every 10 meters.

The longitudinal box girders are built up sections. The dimensions of the girder are shown in figure 3.6. The composite deck is used in place of a concrete deck. The box sections are supported by cables every 10 meters. The box sections provide all the stiffness required by the deck. This dimensions take into account of the situation where a cable may need to be replaced (table 3.2).

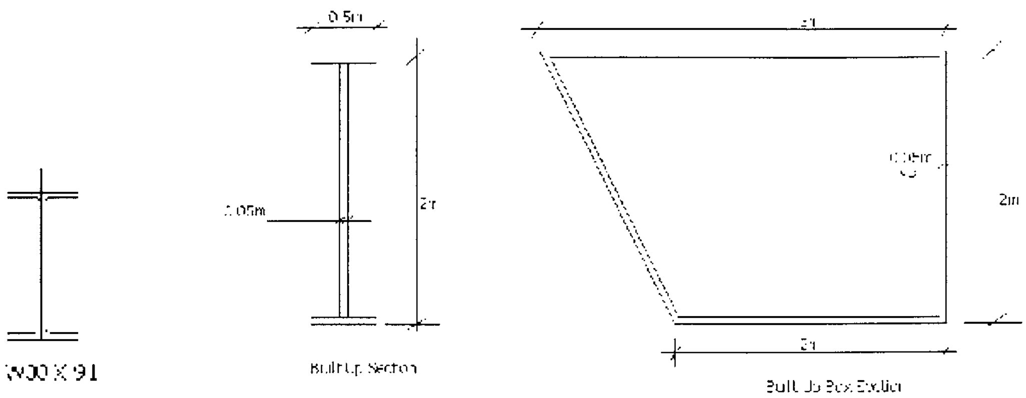

Figure 3.6 : Longitudinal beams and box girder.

The back span has the same structural system as the main span (figure 3.7). However, since the back span assists the tower in resisting the forces from the main span and anchors the backstay cables, it needs to be heavier than the main span. To increase the weight of the back span, the steel box sections are filled with concrete. To provide a better connection between the steel and concrete, shear studs are placed on the bottom flange, and on the web.

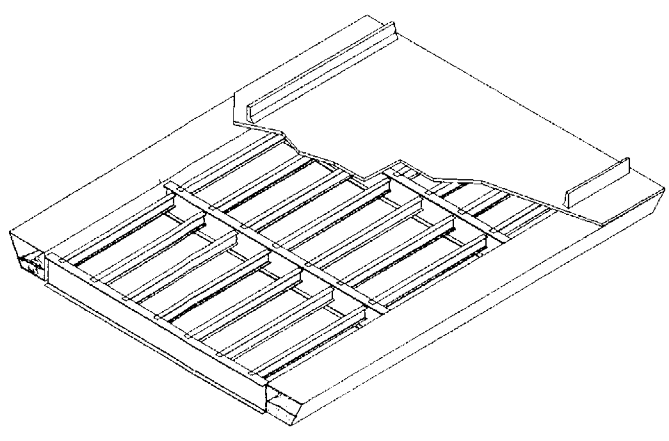

Figure 3.7 : Back Span.

3.2 Cables

The cables are made of bundled seven wire high tensile strength stands having a diameter of 15 mm. Several of these seven wire strands are assembled together in a hexagonal format. They are hot dipped galvanized to protect them against corrosion. The monostrands are sheathed with a tight high-density polyethylene coating to prevent it from corrosion (figure 3.7-A) . The bundled monostrands are supplemented with an outer sheathing to reduce the wind and rain effects on the cable anchorage connections (figure 3.8-B). The cables have passive connections to the tower and an active connection to the longitudinal box girder [13]

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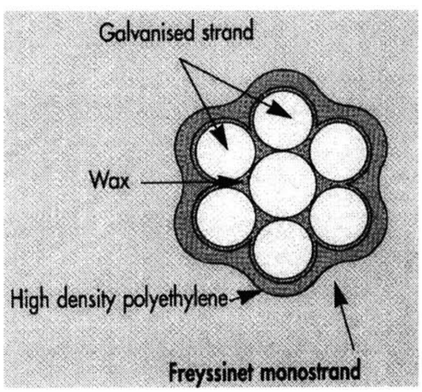
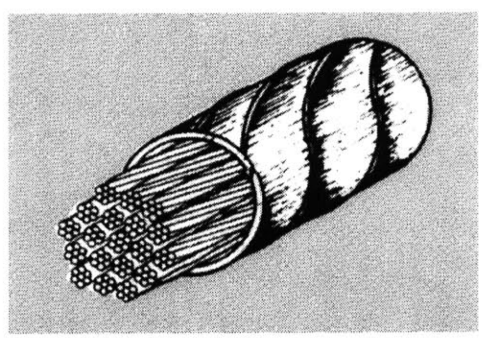
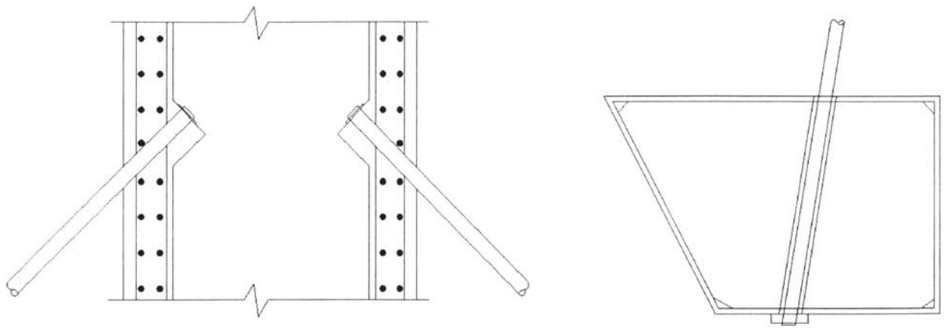


A



B

Figure 3.8 : Cables.



A B

Figure 3.9 : Cable Anchorages.

3.3 Sub Structure Description

The substructure for the bridge includes the following:

-Tower

-Tower Foundation

-Back Span Piers

-Back Span Pier Foundations

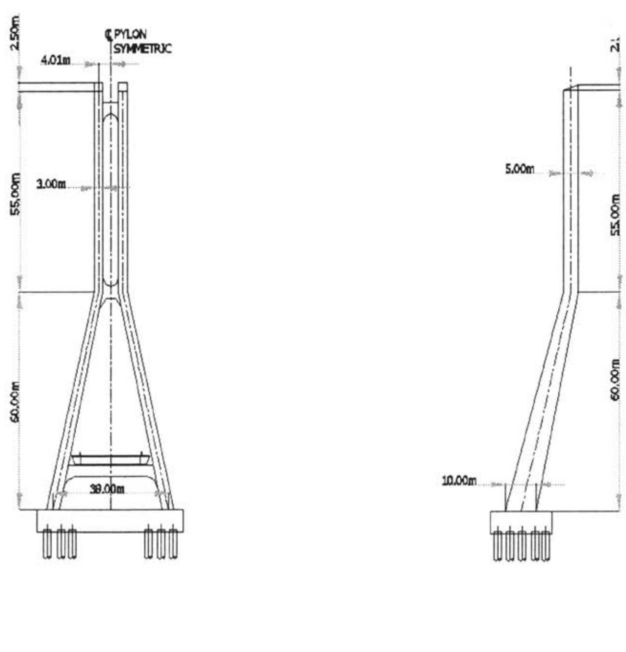


Figure 3.10 : Tower Configuration.

When viewed from either the north or the south direction, the tower has an inverted Y configuration. When viewed from east or west direction, it has a bent configuration for the first 65 meters and a vertical configuration above that distance. The upper portion of the tower, where the cables are anchored, is reinforced with a steel plate.

The tower has a box cross-section. At the base, the cross-section is 10 meters by 4 meters by 0.5 meters thick and tapers to 5 meters by 3 meters by 0.5 meters thick section where the bent section meets the straight portion. The straight portion also has a box cross-section measuring 5 meters by 3 meters by 0.5 meters (figure 3.10).

Horizontal struts are placed in three locations along the height of the tower. The first strut is used to support the superstructure. The support sits on two bearing located underneath each box girder. The second strut is placed where the bent portion of the tower meets the straight portion. Putting a strut at that location makes all the members work together to resist the loads. The third strut is located towards the top of the tower. It has both structural and architectural values. Structurally it ties together the two slender legs of the tower. Architecturally it provides a graceful finish to the vertical elevation of the towers.

The tower is made of reinforced concrete. Vertical post-tensioning is used on the tension side of the bent portion and on both sides of the straight portion. The straight portion of the tower, where the cables are attached, is post-tensioned horizontally. The steel plate reinforces the concrete wall against bursting forces. The post-tensioning closes or reduces any tension cracks that may potentially create corrosion problems.

The type of concrete used is high performance concrete with a 28-day compressive strength of 10,462 tons per sq meter. It also has features which qualifies it as "smart" concrete. The concrete is smart because it utilizes state of the art technology and innovation in concrete mix design that allows the monitoring of its internal condition through its own characteristics.

The foundation of the bridge consists of a pile cap resting on drilled shafts. The plan dimensions of the pile cap are established so as to avoid physical conflicts around the site. The depth and the reinforcement of the pile cap were designed to resist moment and shear forces applied by the tower. Five piers are provided at the back span. The foundation layout consists of a pile cap sitting on drilled shafts that go down to bedrock.

3.4 Summary of geometric properties

	Girder	Top of the tower	Bottom of the tower
Cross-Section Area (m4)	0.44	7.00	28.00
Moment of Inertia (m4)	0.42	8.58	57.33
Section Modulus (m3)	0.33	5.72	28.66

		Concrete	Steel
Modulus of elasticity	(ton/m2)	2.403	7.833
Weight per unit volume	(ton/m3)	2.53E+06	2E+07
Poisson's ratio		0.2	0.3

		Cables
Modulus of Elasticity	(ton/m2)	2E+07
Ultimate Stress	(ton/m2)	186000
Allowable Stress	(ton/m2)	74400

Table 3.2 Sections and materials properties

3.5 Loads

For preliminary modeling stage only dead and live load were considered. For final design, a site specific response spectra analysis was done using SAP2000.

3.5.1 Dead Loads

-Composite Deck	1.20 ton/m^3
-Concrete Overlay	2.25 ton/m^3
-Steel	0.55 ton/m^3
-Barrier	1.80 ton/m^3
-Normal Weight Concrete	2.25 ton/m^3
-Connection	7% of Steel Load

The dead loads obtained for this bridge are then:

Overlay	3.139Ton/mt
Concrete Barriers	1.800Ton/mt
Composite wearing surface	6.432Ton/mt
Steel Girders	1.100Ton/mt
Floor Beam	1.148Ton/mt
Steel Box Girder	9.087Ton/mt

Total	***22.705Ton/mt***

3.5.2 Live Loads

AASHTO HS25-44 Truck and Lane loading was used (figure 3.11 and 3.12) [12].

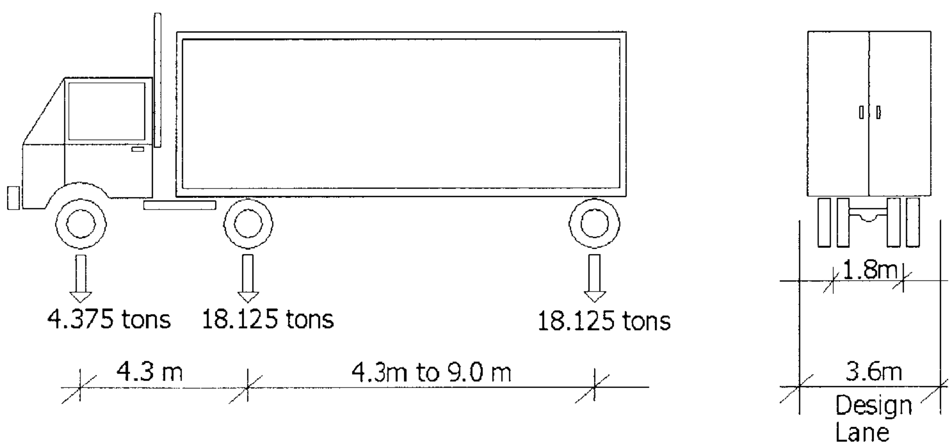


Figure 3.11 : AASHTO Truck Loading.

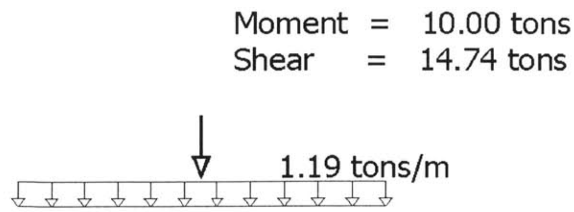


Figure 3.12 : AASHTO Lane Loading.

The beams need to satisfy the L/800 deflection requirement specified by AASHTO [12]. The live load amounted about 25% of the dead load.

3.5.3 Seismic Loads

The response spectra specified for the Boston area is defined in figure 3.13.

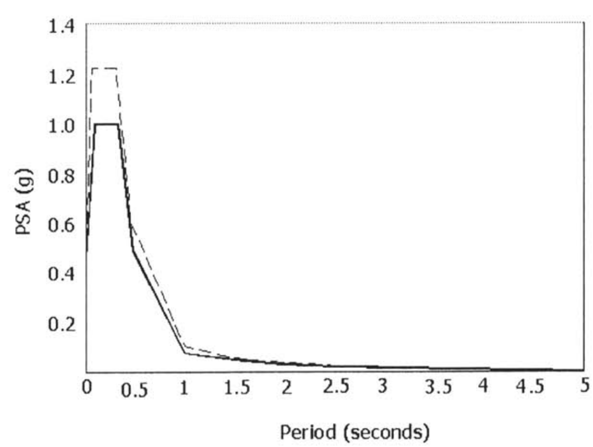


Figure 3.13 : Site Specific Response Spectra.

Chapter 4

Structural modeling of the cable-stayed bridge

Certain structures require an in depth, three-dimensional frame model due to their complexity and high degree of indeterminacy; such is the case of a cable-stayed bridge. The structural development of a cable-stayed bridge is described in the following flow chart (figure 4.1). As in any structure, the analysis and design is based in an iteration process until the desired stresses and deformations are achieved. In this chapter each structural component - cables, girder and tower- is studied individually and their relation is analyzed in the next chapter.

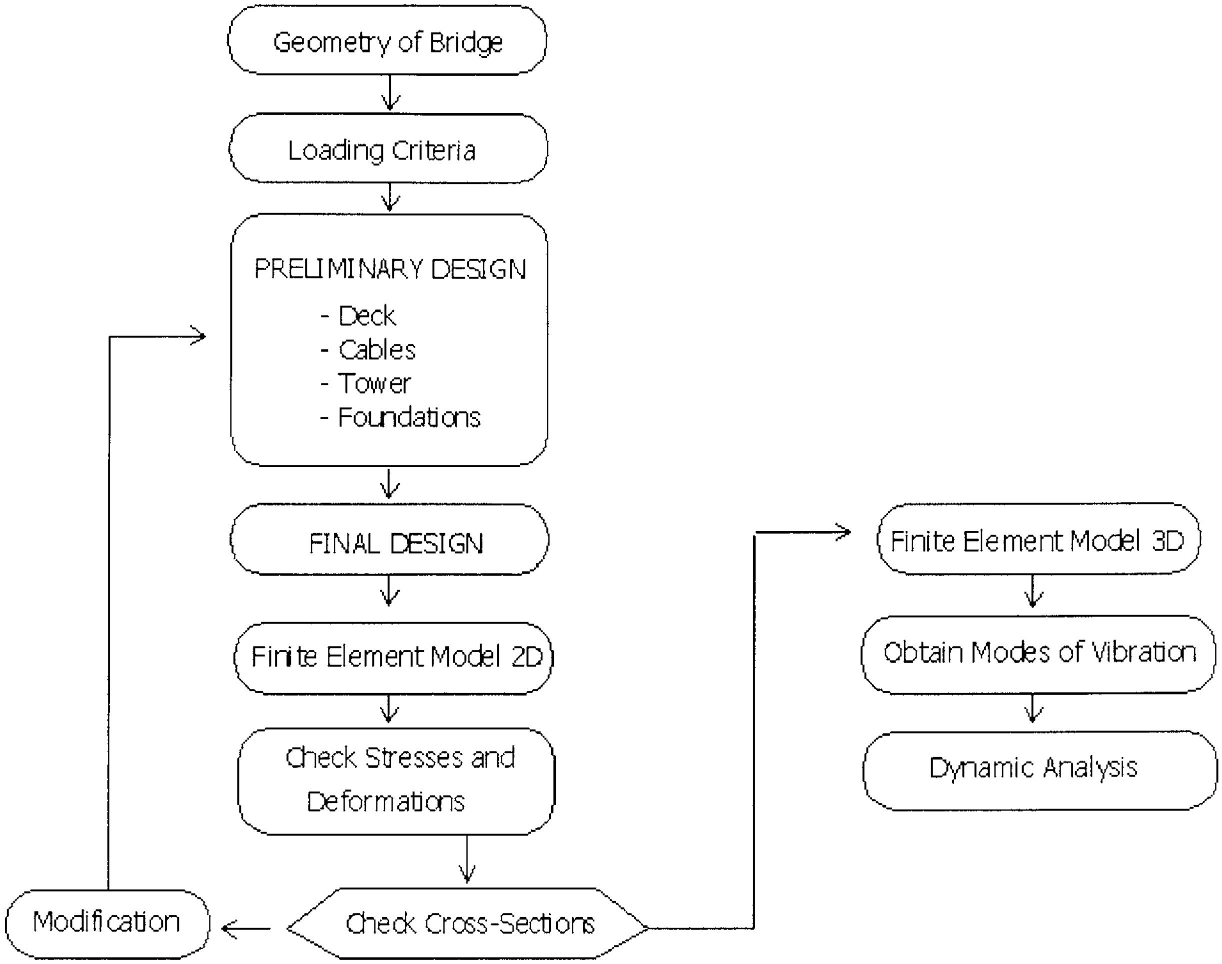


Figure 4.1: Structural development of a cable stayed bridge.

The preliminary modeling, which is a very important stage, can be summarized by the following three steps:

1. Establish preliminary sections for each structural system (girder-deck-tower).
2. Analysis with static methods, and comparison of maximum stresses and deformation requirements.
3. New set of properties to satisfy (2).

The preliminary modeling assumes linear behaviour, thus deflections are obtained by the classical theory of structures and linear superposition can be utilized. Non-linear behaviour is an important characteristic of this type of structures. Geometrical non-linearity is defined by the sag effect in the cables and by the effect of the axial deformation of the tower. Material non-linearity is in function of the type of material utilized, steel, concrete or composite material. For this case, only vertical loads are considered. The behaviour of each structural part will be discussed in this chapter, and only hand calculations will be performed. The preliminary design, in general gives the first approximation for the sections required in the plane and space models as the input data.

4.1 Structural behaviour of cables

The cables are tension members that are modified by different factors. Their performance is classified as non-linear behaviour. These factors are the change in the axial tension and reduction of the stiffness due to the sag effect. By switching the original modulus of elasticity to an effective modulus of elasticity called Ernst's equivalent modulus of elasticity [4], the non-linear behaviour of the cables is considered. There are two approaches to determine the cable properties in the preliminary modeling stage; the strength based approach and the stiffness based approach, the first one takes into account the cable resistance as the first concern, while the second one considers the displacement limitations.

The cables have to provide an initial tension obtained from the dead load of the superstructure to maintain static equilibrium requirements. In addition, due to cables low bending stiffness, its own weight is only balanced by taking a catenary form [5] (figure 4.3).

The general behaviour of a single cable is such that when a tension is applied to it, it elongates a certain amount u_1; the cable self-weight causes a negative movement u_2. Then, the final deformation of the cable is equal to: $u_B = u_1 - u_2$ (Figure 4.2).

This concept is described by the Ernst's modulus of elasticity, which relates the effect of the tensile stress of the cable to the stiffness of the cable. The derivation of the Ernst modulus of elasticity follows:

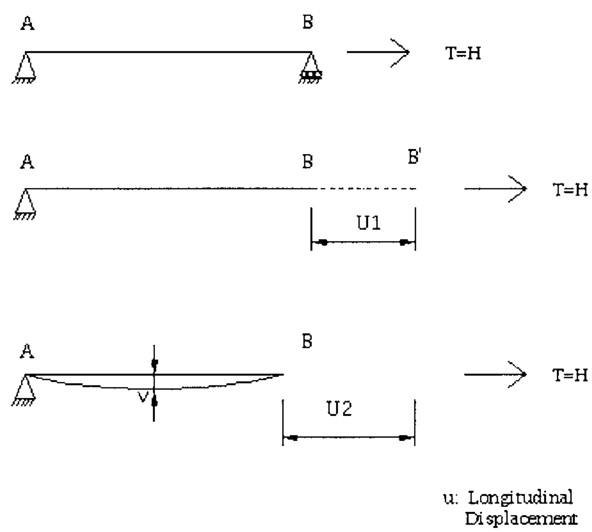


Figure 4.2: Incremental formulation of cables

$$u_1 = \frac{TL_o}{AE}$$

L_o	: Initial length of the cable.	(4-1)
T	: Initial tension of the cable	
A	: Area of the cable	
E	: Original modulus of elasticity	

$$u_2 = \frac{1}{2}\int_0^{Lo} v_{,x}^2 dx \qquad \text{Where } v_{,x} = \frac{W}{T}x \tag{4-2}$$

W : Weight of the cable

Thus,

$$u_2 = \left(\frac{W}{T}\right)^2 \frac{L_o{}^3}{24} \tag{4-3}$$

Hence, the corresponding change is:

$$u_B = L_o\left[\frac{T}{AE} - \frac{1}{24}\left(\frac{WL}{T}\right)^2\right] \tag{4-4}$$

T is incremented by an amount of ΔT . By assuming ΔT to be small with respect to T and differentiate with respect to T:

$$du_B = L_o\left[\frac{1}{AE} + \frac{1}{12}\frac{(WT)^2}{T^3}\right]dT \tag{4-5}$$

The increment in the tension force is then determined by:

$$dT = K_B du_{\mathbf{B}} \tag{4-6}$$

Where K_B is defined as the tangent stiffness

$$K_B = \frac{\dfrac{AE}{L_o}}{1 + \dfrac{1}{12\left(\dfrac{AE}{T}\right)\left(\dfrac{WL_o}{T}\right)^2}} \tag{4-7}$$

$$K_B = \frac{AE^{eff}}{L_o}$$ E^{eff} : Effective modulus of elasticity (4-8)

$$E^{eff} = \frac{E_o}{1 + \dfrac{1}{12}\dfrac{AE}{T}\left(\dfrac{W \cdot L_o}{T}\right)^2} \tag{4-9}$$

Equation 4-9 corresponds to the Ernst's modulus of elasticity in which the tension force and the area required by each cable is a function of the angle θ (See figure 4.3)

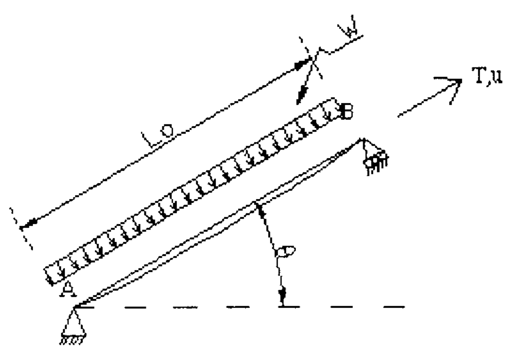


Figure 4.3: Inclined cable

If the loading increases, the sag effect decreases, consequently the stiffness of the cable increases. Equation 4-9 shows that by increasing the tension, the Ernst modulus approaches the original modulus of elasticity. In addition, if the length of the cable increases the sag effect increases (figure 4.4).

In conclusion, by taking the effective modulus of elasticity, it is possible to model the cables as straight members between the anchorage points.

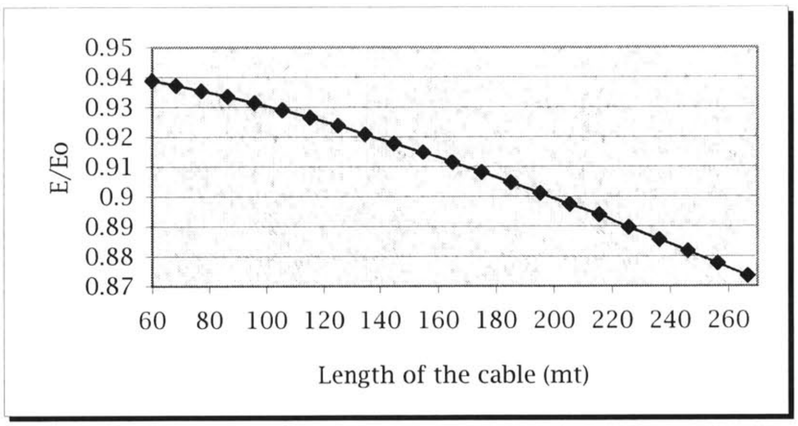


Figure 4.4: Effective modulus of elasticity vs. original modulus

4.1.1 Strength based approach

During the first modeling stage of the bridge, it is possible to visualize the cables as rigid supports to the girder. The backstays balance the horizontal component of the corresponding cable due to the load. The area of each cable is then determined by defining the allowable stress of the cable (Equation 2-1).

The original tension in the cable is determined based on the inclination of the cable, the weight of the girder and the separation of the cables (figure 4.5):

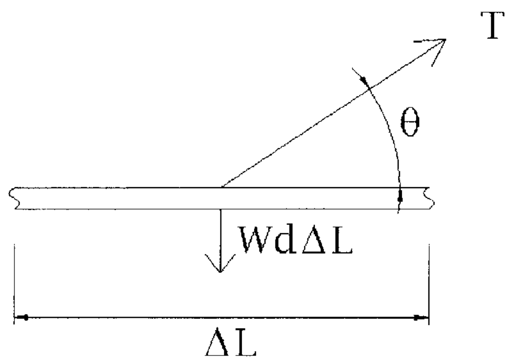


Figure 4.5: Strength based approach

$$T = \frac{w_d \cdot \Delta l}{\sin\phi} \tag{4-10}$$

W_d: Dead Load of the Deck

Δl: Separation of cables

ϕ : Inclination of cables

And the cable area corresponding to this particular segment is:

$$A = \frac{1}{\sigma_d} \cdot T \tag{4-11}$$

The equivalent vertical spring is defined as follows (figure 4-6):

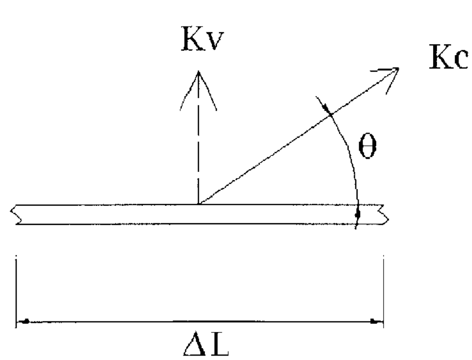


Figure 4.6: Equivalent vertical stiffness

Vertical Stiffness:

$$Kv = Kc \cdot \sin^2 \cdot \theta \tag{4-12}$$

$$Kv = \frac{A \cdot E^{eff}}{L} \cdot \sin^2 \theta \tag{4-13}$$

Distributed Stiffness:

$$kv = \frac{Kv}{\Delta L} = \frac{E^{eff} \cdot W_d}{\sigma_d \cdot L} \cdot \sin \theta \tag{4-14}$$

4.1.2 Stiffness based approach

The stiffness based approach is related to the desired displacement (v*), leading to a desired vertical constant stiffness:

$$v^* = \frac{W_d}{k^*} \tag{4-15}$$

$$Kv = \Delta L \cdot k^* \tag{4-16}$$

$$Kv = \frac{A \cdot E^{eff}}{L} \cdot \sin^2 \theta \tag{4-17}$$

Thus, the area corresponding to this stiffness is:

$$Ac = \frac{k \cdot \Delta L}{E^{eff}} \cdot H \cdot \left[1 + \left(\frac{x}{H}\right)^2\right]^{3/2} \qquad (4\text{-}18)$$

For this case, the limitation for the displacement under dead load is 1/400 of the main span.

Once the corresponding area for the stiffness approach is obtained, it is necessary to check with the strength approach to verify the dominating cable area. Live load needs to be introduced into this revision (figure 4.7).

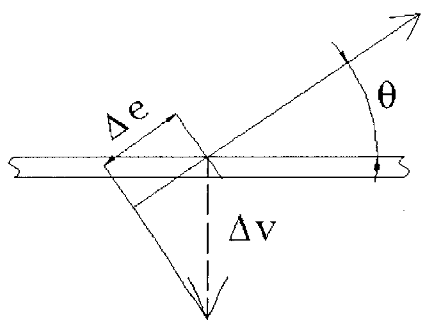


Figure 4.7: Increment due to live load

$$\Delta e_c = \Delta v \cdot \sin\theta \qquad \Delta v\text{: Deformation due to live load} \qquad (4\text{-}19)$$

The increment in force due to live load is then:

$$\Delta T_{ll} = Kc \cdot \Delta e \qquad (4\text{-}20)$$

$$\Delta T_{ll} = \left(\frac{AE^{eff}}{L}\sin\theta\right)\Delta v \qquad (4\text{-}21)$$

$$\Delta T_{ll} = \left(\frac{K_v^* \cdot \Delta L}{\sin\theta} \right) \Delta v \tag{4-22}$$

The total force of the cable is the sum of the tension due to dead load and the increment due to live load:

$$T_{total} = T_{dl} + \Delta T_{ll} \tag{4-23}$$

$$T_{total} = \frac{\Delta L}{\sin\theta} \left(\frac{W_d}{\sigma_d} + K_v^* \Delta v \right) \tag{4-24}$$

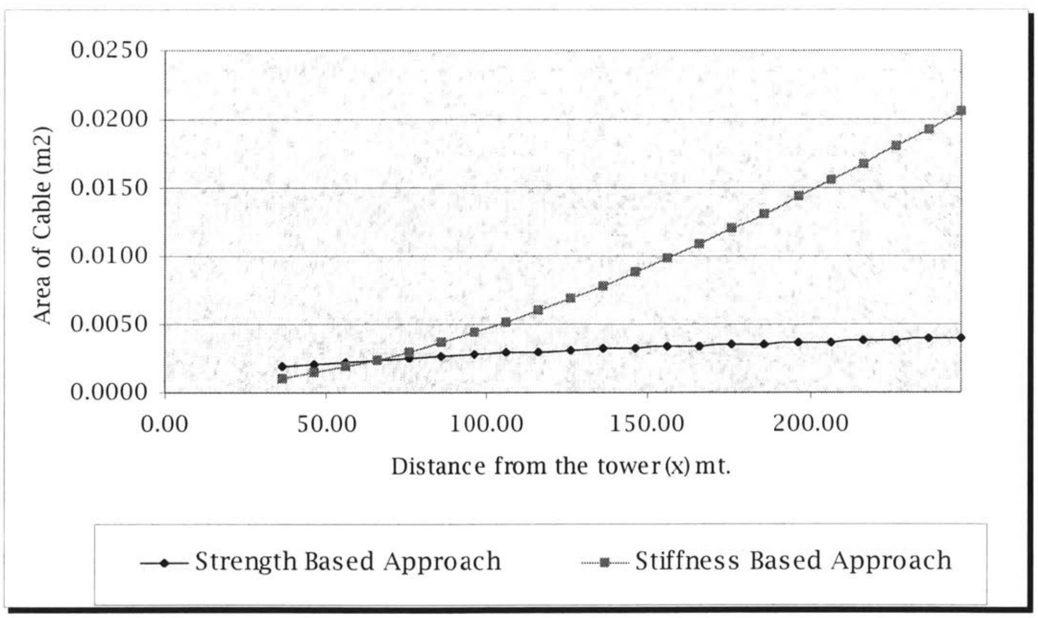


Figure 4.8: Required area per cable.

Cable No	θ (degrees)	Sine(θ)	Length	x	H	Tension (tons)	Area Of Cable (m2)	
							Strength Approach	Stiffness Approach
1	22.72	0.3862	266.69	246.00	103.00	293.88	0.0040	0.0206
2	23.04	0.3914	256.46	236.00	100.38	289.98	0.0039	0.0193
3	23.39	0.3970	246.24	226.00	97.75	285.90	0.0038	0.0180
4	23.77	0.4031	236.25	216.21	95.22	281.59	0.0038	0.0168
5	24.19	0.4097	225.82	206.00	92.52	277.02	0.0037	0.0155
6	24.61	0.4164	215.64	196.05	89.80	272.55	0.0037	0.0143
7	25.14	0.4248	205.46	186.00	87.29	267.16	0.0036	0.0131
8	25.69	0.4335	195.31	176.00	84.67	261.82	0.0035	0.0120
9	26.30	0.4431	185.17	166.00	82.04	256.17	0.0034	0.0109
10	26.98	0.4537	175.06	156.01	79.42	250.18	0.0034	0.0098
11	27.75	0.4656	164.97	146.00	76.81	243.76	0.0033	0.0088
12	28.61	0.4788	154.92	136.00	74.18	237.03	0.0032	0.0078
13	29.60	0.4939	144.91	126.00	71.58	229.78	0.0031	0.0069
14	30.73	0.5110	134.94	115.99	68.95	222.12	0.0030	0.0060
15	32.04	0.5305	125.04	105.99	66.34	213.94	0.0029	0.0051
16	33.57	0.5530	115.22	96.00	63.71	205.26	0.0028	0.0043
17	35.39	0.5791	105.49	86.00	61.09	195.98	0.0026	0.0036
18	37.58	0.6099	95.89	75.99	58.48	186.11	0.0025	0.0030
19	40.24	0.6460	86.46	66.00	55.85	175.70	0.0024	0.0024
20	43.55	0.6890	77.27	56.00	53.24	164.73	0.0022	0.0019
21	47.74	0.7401	68.40	46.00	50.62	153.36	0.0021	0.0014
22	53.13	0.8000	60.00	36.00	48.00	141.88	0.0019	0.0011

Table 4.1: Stiffness based approach vs. Strength based approach.

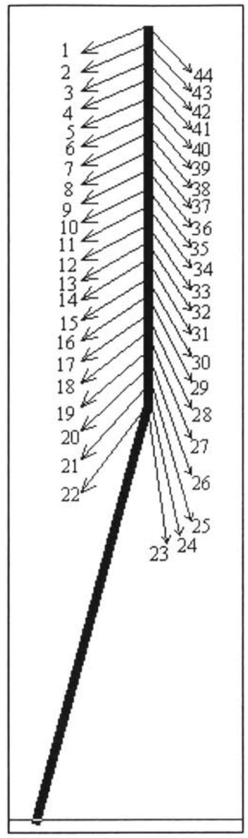


Figure 4.9: Cable layout

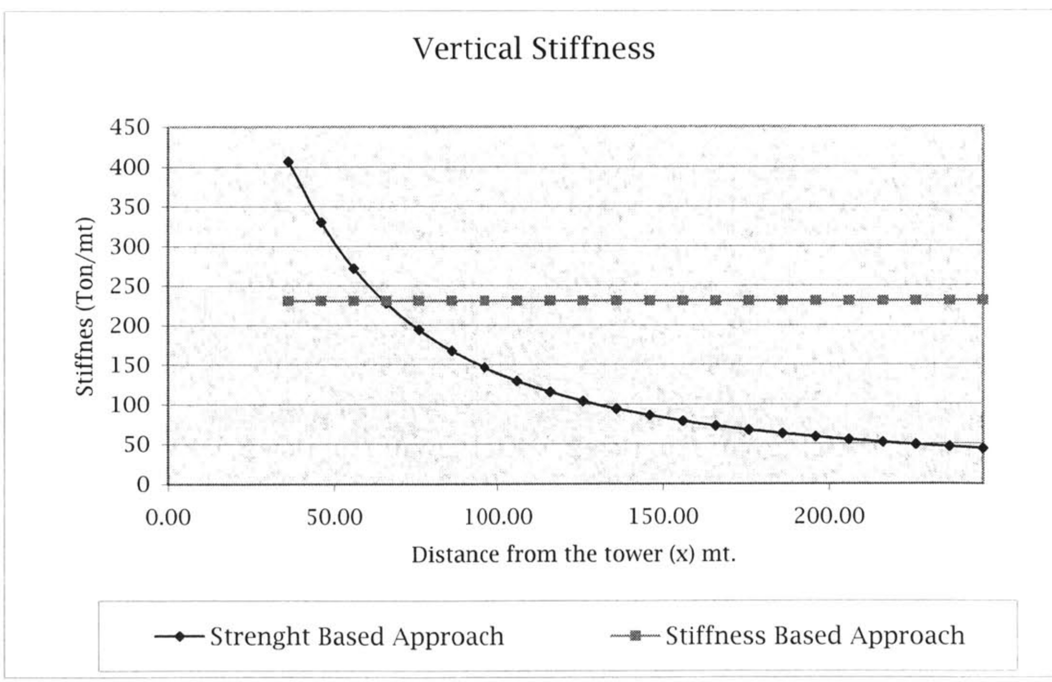


Figure 4.10: Equivalent vertical stiffness.

It is observed in table 4-1 that the strength approach governs for a distance of 66 meters from the tower, beyond that distance the cables are governed by the stiffness based approach (figure 4.8). Therefore, it is necessary to take into account both approaches. The vertical stiffness found from the strength approach ranges from 50 Ton/mt to 450 Ton/mt, while the constant value from the stiffness based approach is on the order of 230 Ton/mt (figure 4.10).

With respect to the effective modulus of elasticity, the stiffness of the cable decreases as the sag increases. This is also related to the inclination of the cables and consequently with the height of the tower.

The stiffness based approach is reliable for the conceptual design, because it allows one to control the displacements under vertical loads [2].

4.2 Structural behaviour of the girder

The behaviour of the girder is defined by the cables arrangement and inclination, which provide elastic supports to the girder. The stiffening girder is generally subjected to two kinds of stresses; bending moments from vertical loads and normal forces from the horizontal components of the loads in the cables.

The structural behaviour of the girder is also a function of the supported conditions in the tower connection, which can be either fix or simple-supported. The girder can be subjected to tension, compression or both [4]. In general, its behaviour is described by three properties: axial stiffness, bending stiffness, and in some cases torsional stiffness.

4.2.1 Girder properties

Axial Stiffness

In the case where the cables have a significant inclination, a shallow cable, the axial force in the girder will be higher compared with the tension force produced in the cable.

For this case, the girder is a continuous girder with movables bearings at one end and at the tower connection. The back span girder rests on tension piers. The girder is subjected to the tower to this compressive force F (figure 4.11), and the system is classified as a self-anchored system [4].

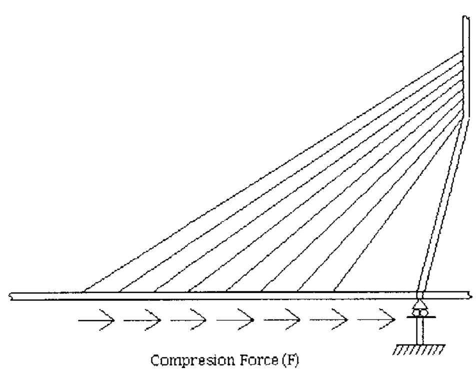


Figure 4.11: Compressive force acting in the girder.

Flexural Stiffness

The flexural stiffness in the vertical direction transfers the vertical loads. The vertical flexural stiffness of the girder follows the load pattern. Therefore, it carries the load locally when floor beams or steel trusses are provided, assists the cable to carry the load globally, and distributes the concentrate loads between the number of cables [4] (figure 4.12). For the bridge the total flexural stiffness is I=0.44 m4 (table 3.2).

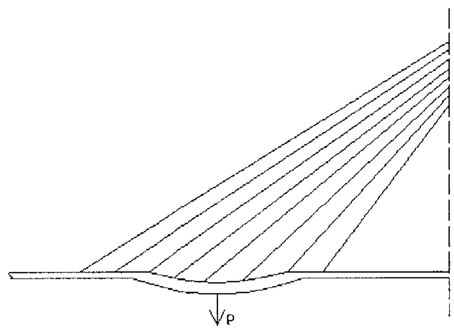


Figure 4.12: Vertical load acting in the girder.

The flexural stiffness in the transverse direction takes the effect of wind and earthquake loads. For the torsional stiffness, due to the fact that the bridge has two cable planes, the torsion is not the governing behaviour. However, revisions for additional torsion due to live load have to be analyzed.

Two cases have to be considered for the design of the girder; the girder as a beam on rigid supports, and the girder as a beam on elastic supports.

4.2.2 Beam or rigid support

The beam on rigid supports covers the primary stage of the bridge in which the erection and prestressing of the cables under dead load take place

(Table 4.2). There are no deformations in the girder under this load condition [5].

The dead load will give the first approximation of the force T in the cables. However, an additional tension in the cables needs to be adjusted during the erection process as each segment is installed (figure 4.13). Redundancy considerations need to be taken into account in order to prevent the case when a cable needs to be replaced.

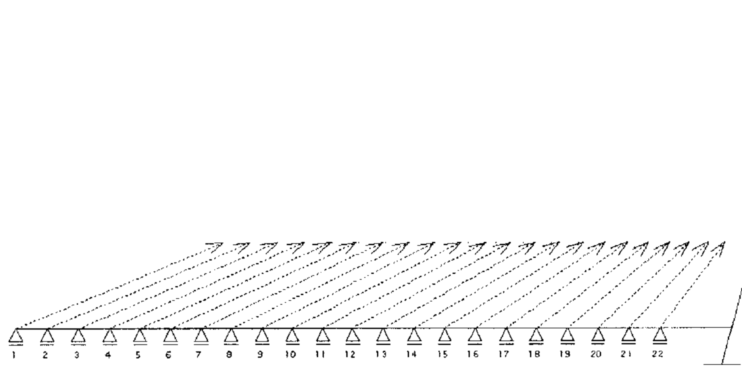


Figure 4.13: Beam on rigid supports

Distributed Dead Load

Wd = 11.35Ton/mt

Segment Length

ΔL = 10.00Mt

Rigid Support	θ (degrees)	Tension (tons)
1	22.42	297.61
2	23.04	289.98
3	23.39	285.90
4	23.77	281.59
5	24.19	277.02
6	24.61	272.55
7	25.14	267.16
8	25.69	261.82
9	26.30	256.17
10	26.98	250.18
11	27.75	243.76
12	28.61	237.03
13	29.60	229.78
14	30.73	222.12
15	32.04	213.94
16	33.57	205.26
17	35.39	195.98
18	37.58	186.11
19	40.24	175.70
20	43.55	164.73
21	47.74	153.36
22	53.13	141.88

Table 4.2: Original tension from the beam rigid supports

4.2.3 Beam on elastic supports

The beam on elastic supports considers the behaviour of the bridge under dead and live loads. This theory takes into consideration the elastic deflection of the cables, and the displacements of the tower. The cables provide a uniform vertical stiffness. When many cables are provided the area and the inertia modulus is reduced, thus the construction is simplified. However, congestion on the tower may appear and the connections may become a problem.

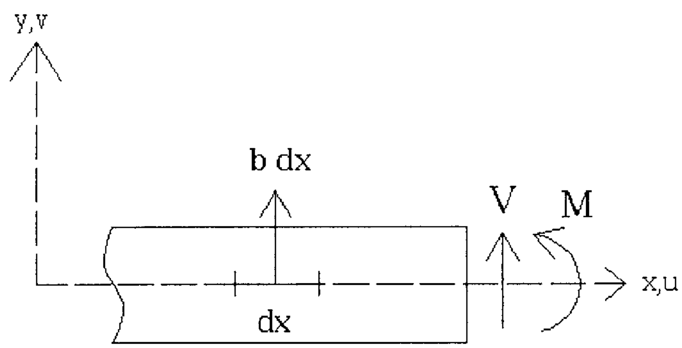


Figure 4.14: General beam

Equilibrium equation:

$$\frac{dV}{dx} + b = 0 \qquad (4\text{-}25)$$

$$\frac{dM}{dx} + V = 0 \qquad (4\text{-}26)$$

Deformation relation:

$$\gamma = v_{,x} - \beta = \frac{V}{D_T}$$ D_T : Shear rigidity (4-27)

$$\beta_{,x} = \frac{M}{D_B} \qquad D_B \quad \text{: Bending rigidity} \tag{4-28}$$

Where γ is the shear transverse deformation, which is neglected, and β the bending deformation. Then, the following equations result:

$$V = -\frac{d}{dx}\left(D_B v_{,xx}\right) \tag{4-29}$$

$$-\frac{d}{dx^2}\left(D_B v_{,xx}\right) + b = 0 \tag{4-30}$$

For the analytical procedure, the stiffness of the cables is assumed to be constant. The elastic formulation corresponds to the "Winkler Formulation Model" (figure 4.15). The formulation assumes that the restraining force at point x, corresponds to displacement $v(x)$.

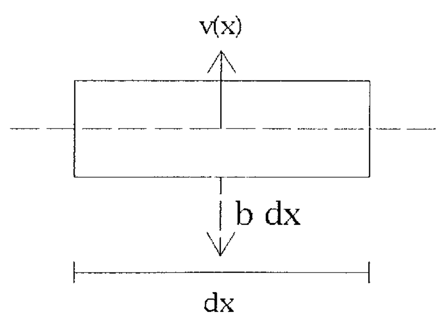


Figure 4.15: Winkler formulation model.

$$b = -k_s \cdot v \tag{4-31}$$

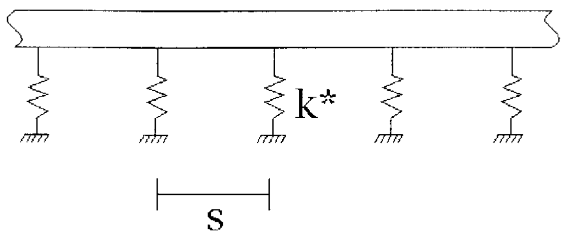


Figure 4.16: Beam on elastic supports

$k_s = \frac{k^*}{s}$ $\qquad$ k_s : Constant stiffness equally spaced. (4-32)

Equilibrium equation:

$b = -k_s \cdot v + \bar{b}$ $\qquad$ $\bar{b}$: Prescribed loading. (4-33)

Thus the governing equation is:

$$\bar{b} = \frac{d^2}{dx^2} \cdot \left(D_B \cdot v_{,xx} \right) + k_s \cdot v \tag{4-34}$$

By assuming the beam rigidity D_B, the foundation stiffness k_s, and the prescribed load $\bar{b}$ to be constants, the solution is:

$$v = e^{-\lambda x}\left(C1 \cdot \sin \lambda x + C2 \cdot \cos \lambda x \right) + e^{\lambda x}\left(C3 \cdot \sin \lambda x + C4 \cdot \cos \lambda x \right) + v_{particular} \tag{4-35}$$

C1,C2,C3,C4: Integration constants based in the boundary conditions.

$$v_p = \frac{\bar{b}}{k_s} \tag{4-36}$$

$$4\lambda^4 = \frac{k_s}{D_B} \qquad \text{Thus:} \qquad \lambda = \sqrt[4]{\frac{k_s}{4 \cdot E^{eff} \cdot I}} \tag{4-37}$$

Where λ is known as the degree of flexibility [6]. Analyzing the general solution, $e^{-\lambda x}$ decays when x increases, and $e^{\lambda x}$ increases when x increases. So, it is convenient to establish a characteristic length L_b in which the solution differs from the particular solution.

If $\lambda x \rangle 3$, then $e^{-\lambda x} \approx 0$

$$\therefore L_b = \frac{3}{\lambda} = 3 \cdot \left[\frac{4 \cdot D_B}{k_s}\right]^{1/4} \tag{4-38}$$

Hence, for $x \rangle L_b$, the $e^{-\lambda x}$ term can be ignored.

In summary the solution can be approximated by:

$$0 \langle x \langle L_b \qquad v = e^{-\lambda x} \cdot (C1 \cdot \sin \lambda x + C2 \cdot \cos \lambda x) + v_{particular} \tag{4-39}$$

$$L_b \langle x \leq L - L_b \qquad v = v_{particular} \tag{4-40}$$

$$L - L_b \langle x \leq L \qquad v = e^{\lambda x} \cdot (C3 \cdot \sin \lambda x + C4 \cdot \cos \lambda x) + v_{particular} \tag{4-41}$$

The solution is then a two-end zone condition and an internal zone condition when the member length is greater than $2 \cdot L_b$ (Figure 4.17).

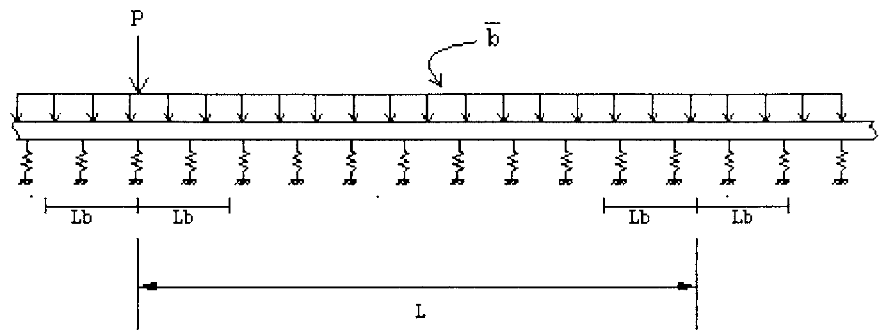


Figure 4.17: Characteristic length

Thus, influence lines, bending stresses and deformations are determined under these basis just presented.

4.3 Structural behaviour of the tower

The tower behaviour is governed by the axial force coming from the vertical reaction of the cables, and from the weight of the tower. The main and back span cables horizontal reactions are in equilibrium with each other. However, the tower is subjected to deformations due to live load and construction accuracy. Thus, it is necessary to analyze second order effects in the tower, P-Delta effects assuming a possible range of eccentricities [4].

As for the stiffening girder the support conditions were governing factors for its behaviour. The tower behaviour is also in function of the support conditions. Because this is a self-anchored system, and the tower is fixed to the pier, the effective column height that must be considered for the design is 0.70 of the height of the tower [4].

Although the governing condition for the tower is the vertical load, it may be loaded under other conditions such as longitudinal loads (i.e. wind load, saddle eccentricity), transverse loads (i.e. wind load, saddle eccentricity, second stresses due to geometrical non-linearity, temperature load) and seismic loads [1].

4.3.1 Longitudinal Stiffness

The main constraint on the tower is that under dead load, the displacements in the tower are controlled, and the bending in the tower is negligible. On the other hand, when the tower is subjected to live load its longitudinal stiffness is governed by the longitudinal properties of bridge (the superstructure weight, and the cable layout), the support conditions, and the symmetry or anti-symmetry of the bridge [4].

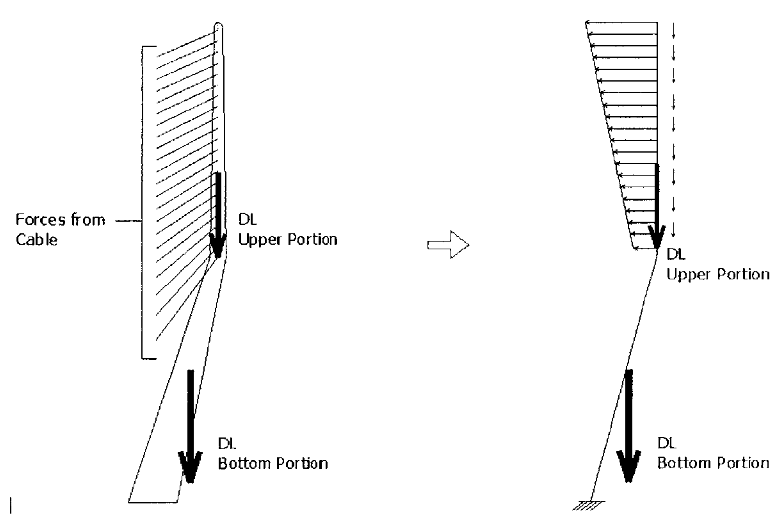


Figure 4.18: Dead load acting in the tower.

The general analysis for the tower must has the following two considerations:

- The maximum vertical load and the corresponding displacement of the tower under dead load and live load.
- The maximum displacement of the tower and the corresponding vertical load due to live load applied in the mid span. This is the critical case for the design of the tower [1].

The tower behaves like an elastic column; when the vertical load (V) of the tower is less compared with the critical load the column can resist, the tower tends to return to its vertical position, the cable system transmits a horizontal force, and the tower remains in a deflected shape. However, when the vertical load (V) reaches the critical load, the tower does not have any more resistance to maintain the displacement at the top, so this system can be considered like a tower with a longitudinally movable bearing at the top (figure 4.19) [1].

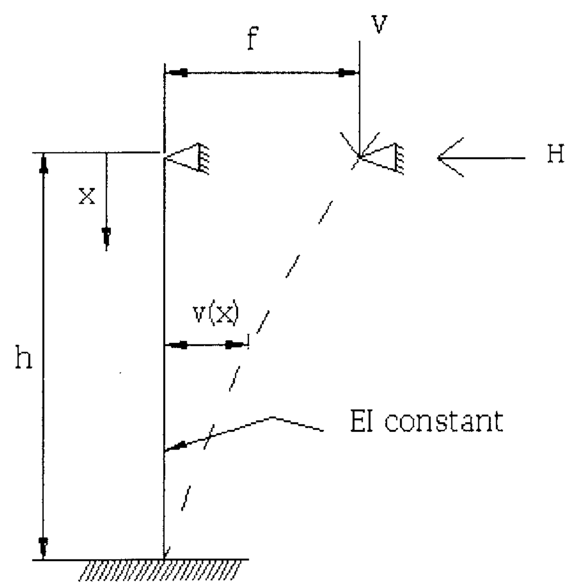


Figure 4.19: Tower deformation

Moment at section x:

$$M(x) = Hx - V[f - v(x)] \tag{4-42}$$

Equilibrium equation:

$$EI\frac{d^2v(x)}{dx^2} + M(x) = 0 \tag{4-43}$$

Substituting:

$$EI\frac{d^2v(x)}{dx^2} + Vv(x) = Vf - Hx \tag{4-44}$$

Define,

$$\frac{V}{EI} = \alpha^2 \tag{4-45}$$

$$\frac{d^2v(x)}{dx^2} + \alpha^2 v(x) = \alpha^2 f - \frac{H}{EI}x \tag{4-46}$$

The general solution for this equation is then:

$$v(x) = C1\cos\alpha x + C2\sin\alpha x + f - \frac{Hx}{V} \tag{4-47}$$

Where C1 and C2 are determined by the boundary conditions:

At x=0 $\quad v(x) = f \quad$ (4-48)

At x=h $\quad v(x) = 0 \quad \frac{dv}{dx} = 0$ (4-49)

Hence,

$$C1 = 0 \qquad C2 = \frac{1}{\sin\alpha h}\left(\frac{Hh}{V} - f\right) \tag{4-50}$$

$$H = \frac{V\alpha f\cos\alpha h}{\alpha h\cos\alpha h - \sin\alpha h} \tag{4-51}$$

And substituting the values, the equations becomes:

The displacement at x:

$$v(x) = f\,\frac{\sin\alpha h - \sin\alpha x - (h-x)\alpha\cos\alpha h}{\sin\alpha h - \alpha h\cos\alpha h} \tag{4-52}$$

The moment at x:

$$M(x) = -fV\,\frac{\sin\alpha x}{\sin\alpha x - \alpha x\cos\alpha x} \tag{4-53}$$

The moment at the fixed based:

$$M(h) = -fV\,\frac{\sin\alpha h}{\sin\alpha h - \alpha h\cos\alpha h} \tag{4-54}$$

It is necessary to first determine the value of αh and find the corresponding moment. In conclusion, important characteristics that define the behaviour of the tower are [5]:

- The value of the ultimate load is highly affected by the geometrical and material non-linearity.
- The cables apply a horizontal restraining force in the deformed state.
- The vertical load is introduced progressively along the axis (figure 4.15)
- The tower is subjected to bending stresses under live load.
- The tower is a generally hyperstatic system.

Chapter 5

The interrelation of the structural components.

The interrelation between the main structural components of a cable-stayed bridge, the cables, girder, and tower, is important because it gives the overall behaviour of the system. The total stiffness of the cable-stayed bridge is obtained by the interaction of the individual stiffness.

Three limit states can be considered to generalize this type of bridges [5]:

- Very stiff girder: Reduces the number of cables, reduces the cross-section of the tower but the construction cost is extremely high.

- Very stiff tower: The tower takes all the longitudinal moments, thus, the cross-section of the girder decreases. This is a very convenient solution for multi-span bridges.

- Inclination and separation of the cables. The cables stabilize the system itself. The use of backstays, counterweight or tension piers is essential.

5.1 Final Modeling

As mentioned in the previous chapter, a cable-stayed bridge has a high degree of indeterminacy; therefore it is necessary to analyze it with the help of two-dimensional and three-dimensional models. In general, computer models are necessary to obtain the final stresses in the bridge as a unique assembled structure. Two-dimensional models are helpful for the basic design stage, they provide a simplified model with a high degree of accuracy, and then a spatial model can carry out a more detail analysis and consider dynamic effects.

5.1.1 2D model SAP2000

For the 2-D model, it is assumed that the bridge is supported by one of the main box girders, and by symmetry this constitutes one half of the cable-stay system and does not collaborate with the other one.

For the analysis of the model, the cables are modeled as rigid bars, with certain amount of tension, greater than the possible compression forces originated by dead load condition. The model for this bridge is showed in figure 5.1, it includes elements for the steel box girder, the cables and the tower. It consists of 68 nodes, 44 cable elements, 23 tower elements and 44 girder elements. It is convenient to modify the modulus of elasticity of the cable to simulate the non-linear behaviour of the cables due to the sag effect (chapter 4).

The cable-stayed system has to be in equilibrium under its own weight. The interrelation between the cable and the girder are such that the cable has to provide elastic foundation for the girder, thus the superstructure needs to be a light superstructure. When applying the preliminary sections for the cables, they interact with the tower and the girder deformations, which affect the preliminary defined tension of the cables (chapter 4). Therefore, the procedure to adjust the tension in the cables is an iterative process based on the following [5]:

1. The tower is fixed in the horizontal direction. The tension of the cables must present no deformation in the girder.
2. The restrains of the tower are released, and the backstays take the reactions.

To consider the live load, influence lines were first obtained, and then by applying the load specified in chapter 3, the envelopes were calculated.

As a self-anchored system, the maximum compression force is found in the tower. Considering the live load envelopes increases around 20 % of the dead load stresses (figure 5.2). The moment diagram is shown in figure 5.3, where the highest moments are found near the first support of the bridge. The maximum deflection under dead load and live load is of the order of 0.30 meters (figure 5.4).

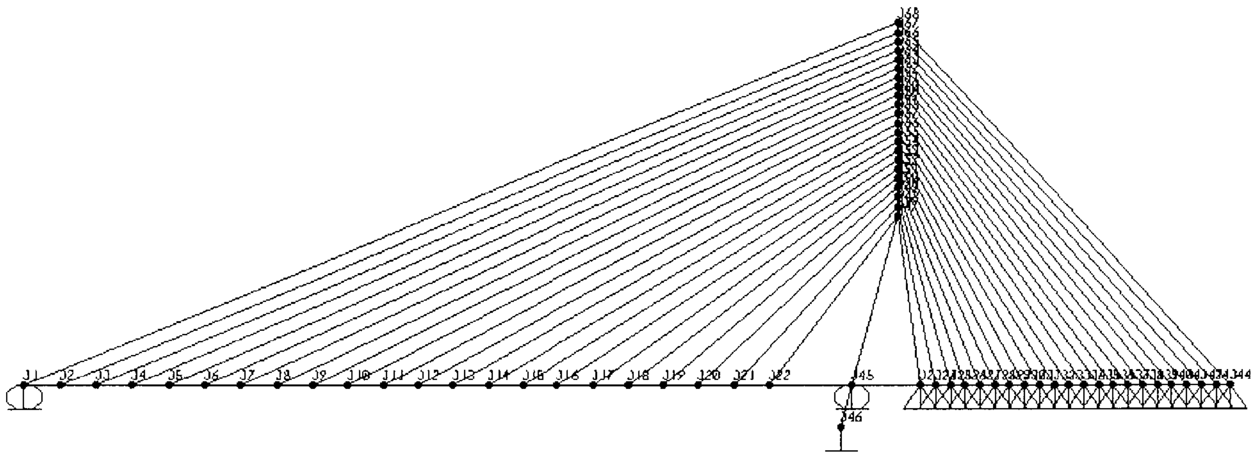


Figure 5.1-A: 2D- Model -Joints

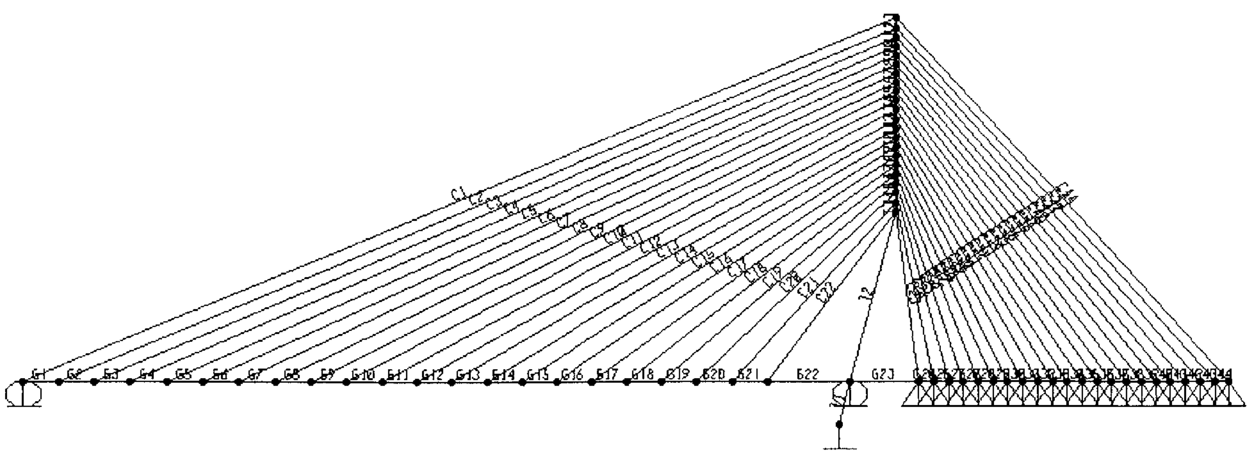


Figure 5.1-B: 2D- Model -Elements

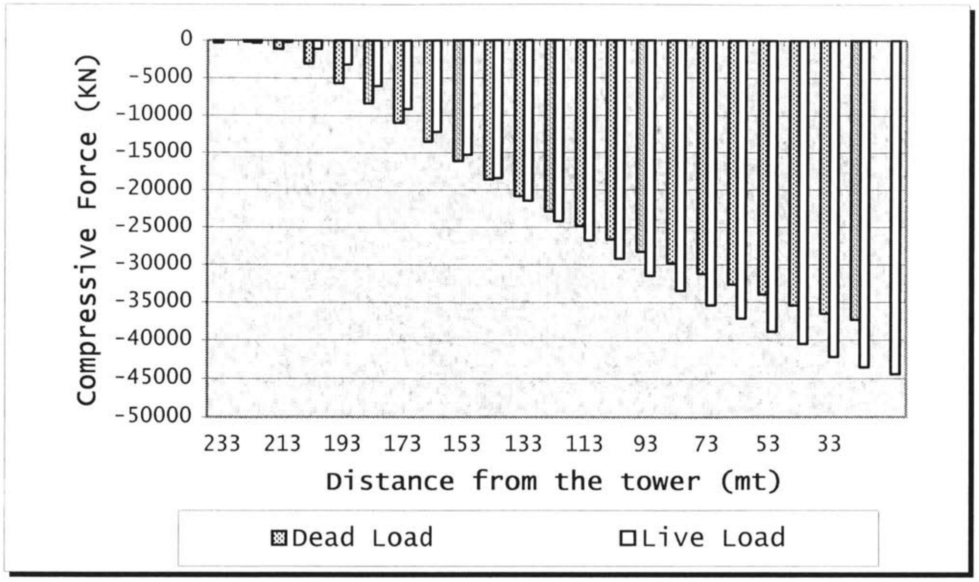


Figure 5.2: Compressive forces in the main span

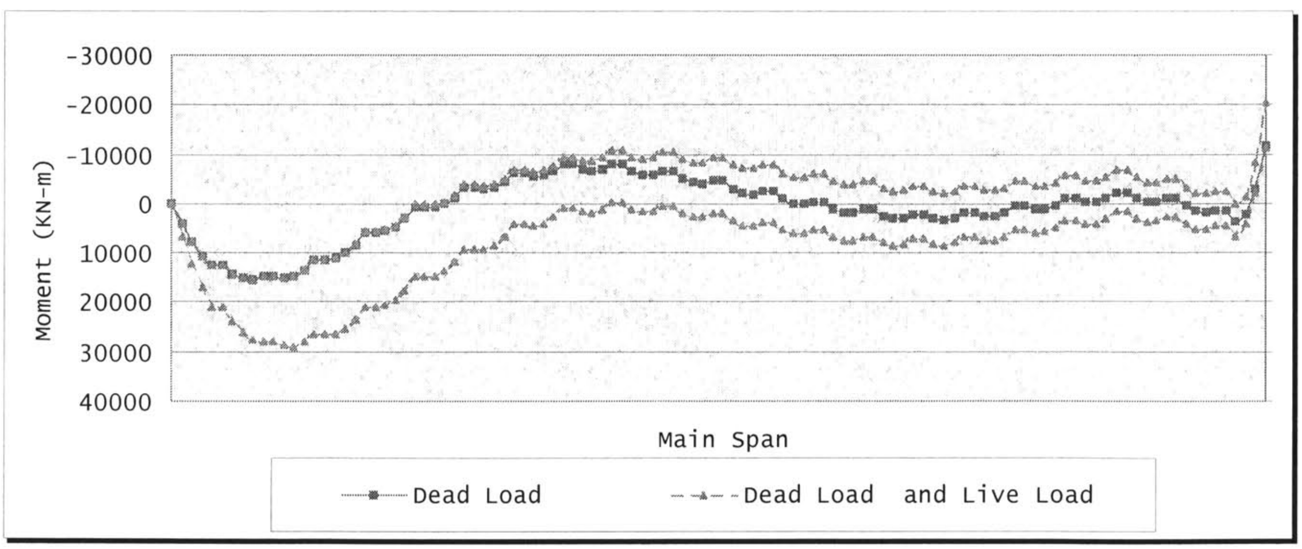


Figure 5.3: Moments in the main span

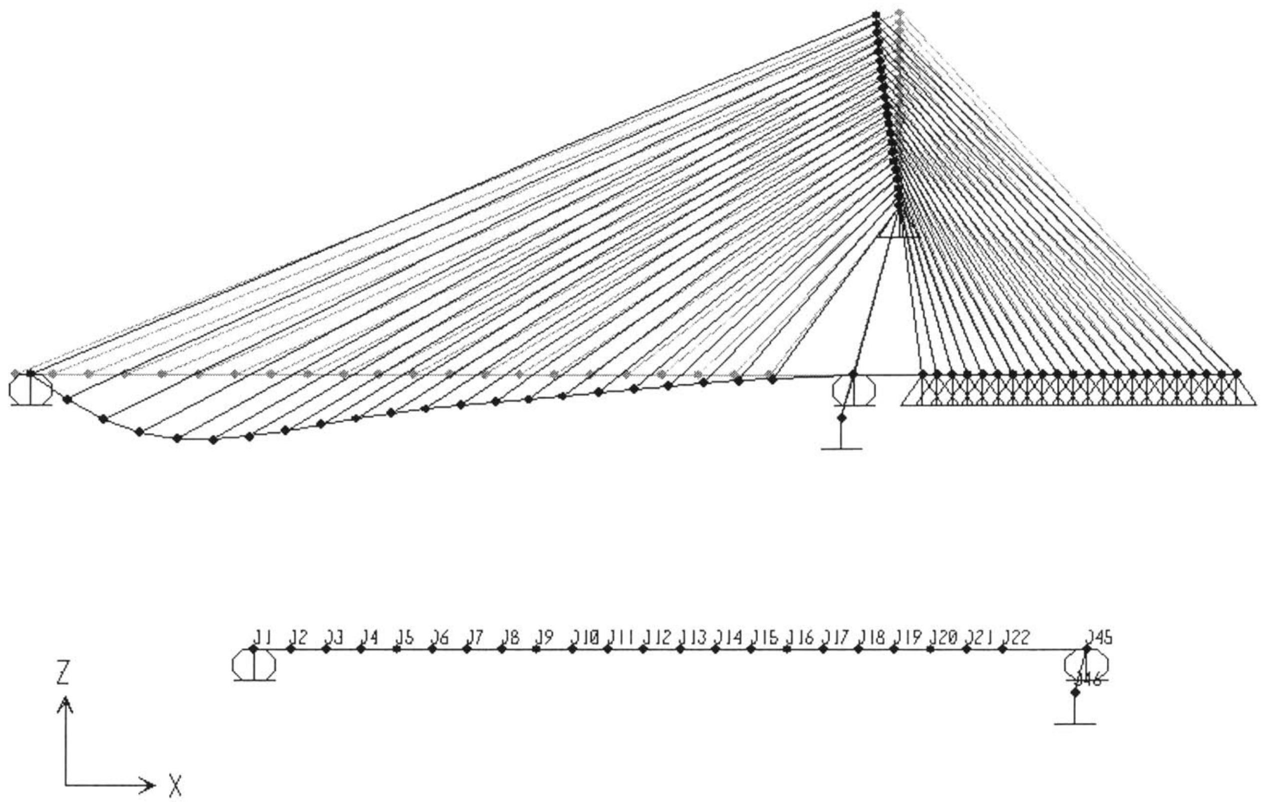


Figure 5.4-A: Deformed shape due to live load.

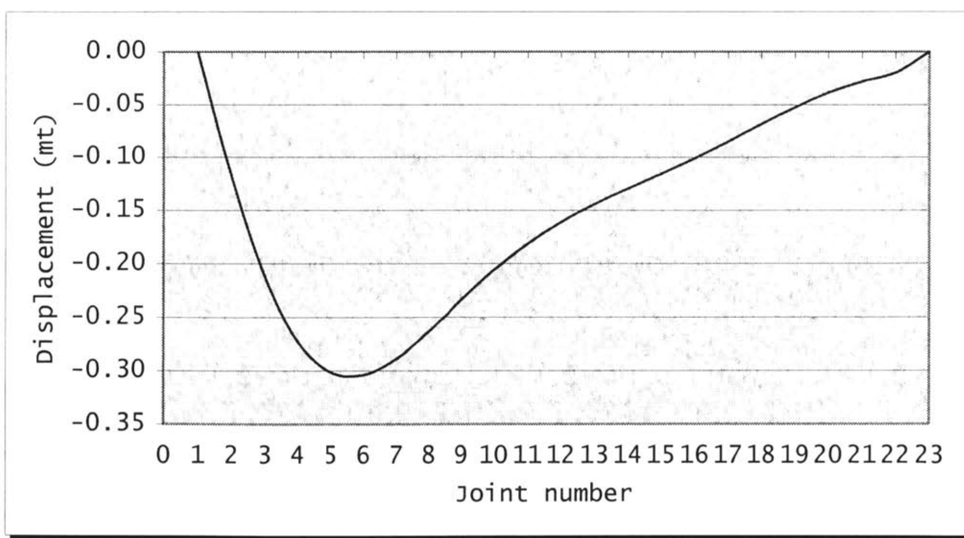


Figure 5.4-B: Displacements due to dead load and live load in the mid span.

5.1.2 3-D model SAP2000

The spatial modeling of the bridge helps to understand the behaviour under lateral loads, such as wind and earthquake loads. The natural modes of vibration are shown (figure 5.5). The study of lateral stiffness of the bridge is beyond the scope of this study. A site-specific response spectra is applied to this model (Chapter 3), and the mode shapes are the followings:

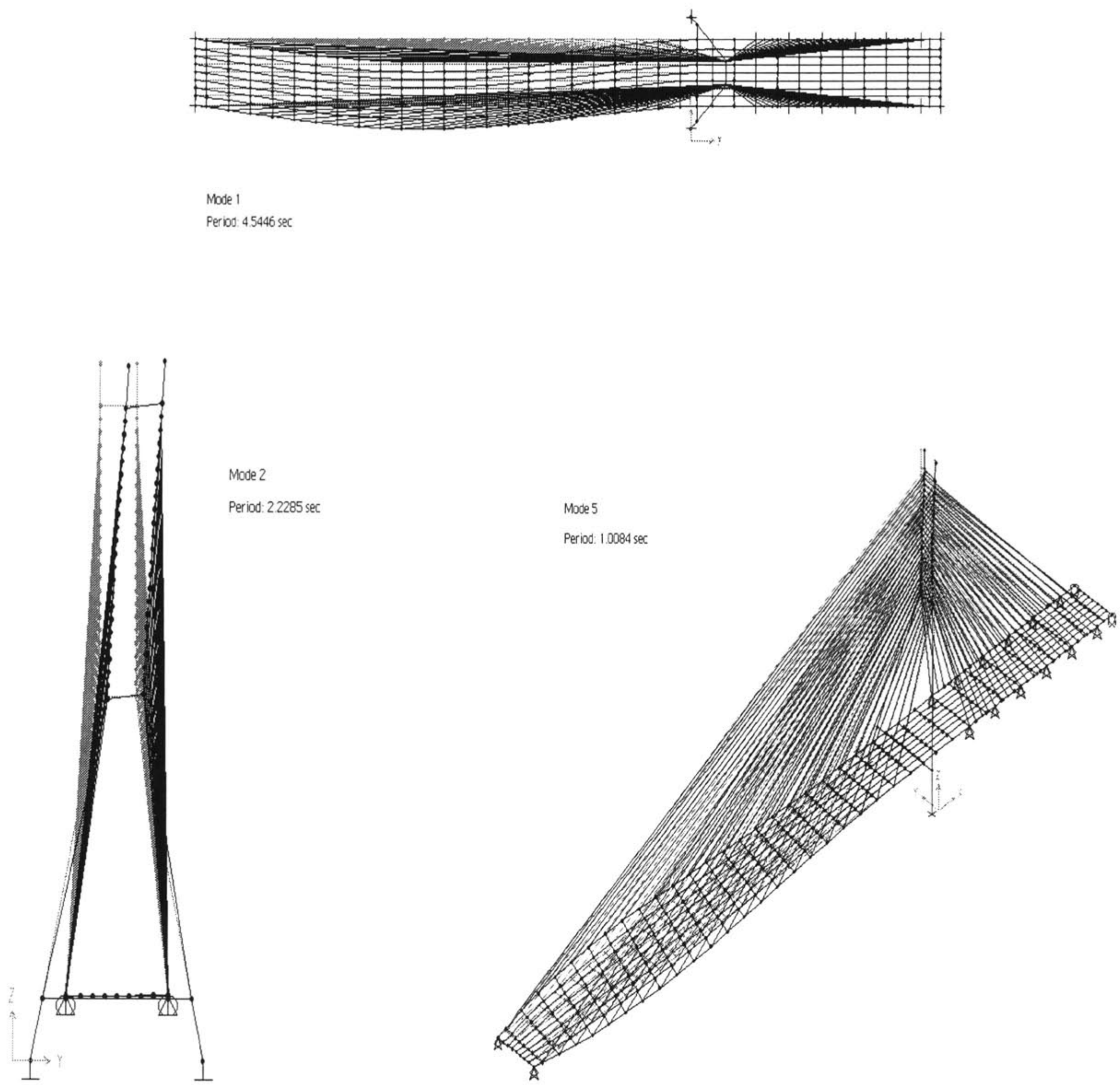


Figure 5.5: Principal mode shapes of the cable-stayed bridge

5.2 Conclusions

From the previous chapters, it is concluded that the main parameters that governs the behaviour of a cable-stayed bridge are:

- The number and configuration of the cables.
- Geometric proportions.
- Support conditions
- Stiffness of the main structural components.

The height of the tower constitutes an important issue because of its relation with the cables and the girder. The height of the tower determines the inclination of the cables, and the normal force acting in the girder. Thus, the height of the tower affects directly the stiffness of the bridge system. From parametric studies, the optimal behaviour of the cable-stayed bridges is found when the ratio between the tower height and the dimension of the central span lies between 0.20 and 0.25 [2].

With respect to the cables, if the cable tends to be more inclined, the stresses in the cable decrease, and the tower requires a smaller cross-section. However, if the length of the cables increases, the deformation increases, and the cables require more material. Considering these two factors, the recommended inclination for the cables ranges between 25 degrees and 65 degrees. 45 degrees is the optimal inclination [1]. The number of cables is also an important factor to consider the larger the number of cables the better the behaviour. If only few cables are provided, the forces in the cables are bigger, the anchorages more difficult, and the cross-section required for the girder is increased. By providing a large number of cables, the overall behaviour tends to approximate a continuous elastic support, the anchorage is simplified, and the cross-section required in the girder decreases.

For the support conditions, the best performance is achieved when the girder is considered as a continuous girder with a movable bearing in the tower connection instead of a rigid connection, which increases the moment at this joint [2].

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