Inventory Redistribution Optimization in the Fast Fashion Industry

by

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Submitted to the MIT Sloan School of Management and the Engineering Systems Division in Partial Fulfillment of the Requirements for the Degrees of

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Abstract

Zara is the world's leader in the fast-fashion industry and introduces over 10,000 unique designs to their stores each year. Zara's parent company, Inditex, reported profits of $2.2 billion in 2012, an increase of 27% for the year. They opened 360 new stores in over 50 markets across all of their brands last year. Zara contributes approximately 60% of these profits and has around 1,720 stores in over 80 countries worldwide.

Zara is committed to meeting the needs of their customer through continuous improvement of their processes and systems. Though they continually improve their already advanced forecasting and distribution methods, there is significant variability in demand that remains challenging to predict. Due to this uncertainty in demand and the short life cycle of trendy clothing articles, it is imperative that Zara is able to quickly respond to changing demand patterns. After initial distribution, inventory can be redistributed among stores in order to satisfy their customers' demand and maximize sales. This critical step in the distribution process is known as inventory transfers.

The purpose of this project was to develop a demand forecast model, optimization model and operational process to optimize and standardize these inventory transfers among the complex network of thousands of Zara stores. The key performance indicator was an increase in profit of at least three percent. The research process was first to identify the key decision-making criteria and variables affecting transfer decisions; second to use that criteria to build an optimization model to propose optimal redistribution of articles among stores; and third to prepare the roll-out and integration of the new approach in the existing operational process and IT system. This project required integration with Zara stakeholders across many functions including product management, buying, distribution and information technology. Crucial to the success of the project was remaining focused on these stakeholder needs to ensure the model would be easily adopted and fully implemented while also considering demand, costs, logistics, feasibility and many other factors. The new model provides a profit increase of 21 percent for those articles transferred and is the first model of this kind applied in retail supply chain management.

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Table of Contents

List of Figures

List of Tables

List of Equations

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Chapter 1 Introduction

Each year, Zara introduces over 10,000 new articles to their approximately 1,720 stores in 80 countries. This complex network of stores and inventory creates a challenging environment for managing inventory redistribution. This section provides details on this challenge and motivation for the thesis along with an overview of the proposed solution and key objectives.

1.1 Problem Statement and Motivation: Difficult inventory transfers

Zara is capable of responding quickly to changing fashion trends through efficient supply-chain operations and advanced forecasting and distribution methods. However, the demand for these trendy clothing articles is unpredictable with short life cycles, often less than four weeks. Additionally, as part of their commercial strategy, Zara keeps only minimal amounts of inventory on-hand in central distribution centers. In this world of short, uncertain, highly varying customer demand with minimal excess stock, it is essential to quickly and efficiently respond to demand fluctuations by optimally redistributing clothing between stores. This aspect of the supply-chain is referred to as inventory transfers and involves reallocating units directly from one store to another in lieu of or in combination with shipping units to stores from the central distribution centers. As a simplified example, there are ten Zara stores in the New York City area. Once those stores have received a new article and displayed that article for a week, the demand is significently better understood based on the actual realized sales. Stores with low sales of that article can transfer units to stores with high sales. This can occur with articles that are still being replenished from central distribution or when articles are stocked out at central warehouse. Further detail regarding inventory transfers is provided in Chapter 2.

Currently Zara manages inventory transfers in a highly manual, non-standard and labor-intensive process. Product Managers must analyze large amounts of data to make inventory transfer decisions, which consumes significant amounts of time. They must assess inventory levels, key performance metrics, transportation logistics and store performance. There is no analytical model focused on optimizing these transfer decisions to support the Product Managers in these decisions.

1.2 Project Description, Context and Proposed Solution

The purpose of this project is to optimize and standardize inventory transfers among Zara's complex network of stores to ensure the right articles of clothing are in the right stores at the right time to satisfy quickly changing customer demand and increase overall profit. We propose a solution model illustrated in Figure 1, consisting of an updated demand forecast based on realized sales and a mixed-integer, multi-period optimization model. The optimization model, forecast model and inputs are discussed in detail in Chapter 5 and Chapter 8. The output of this model is quantities to be transferred along routes (Store A to Store B). The output is summarized for the end-user in a tool, which supports the standard process and allows end-user flexibility. The model implementation, process and tool are discussed in Chapter 7.

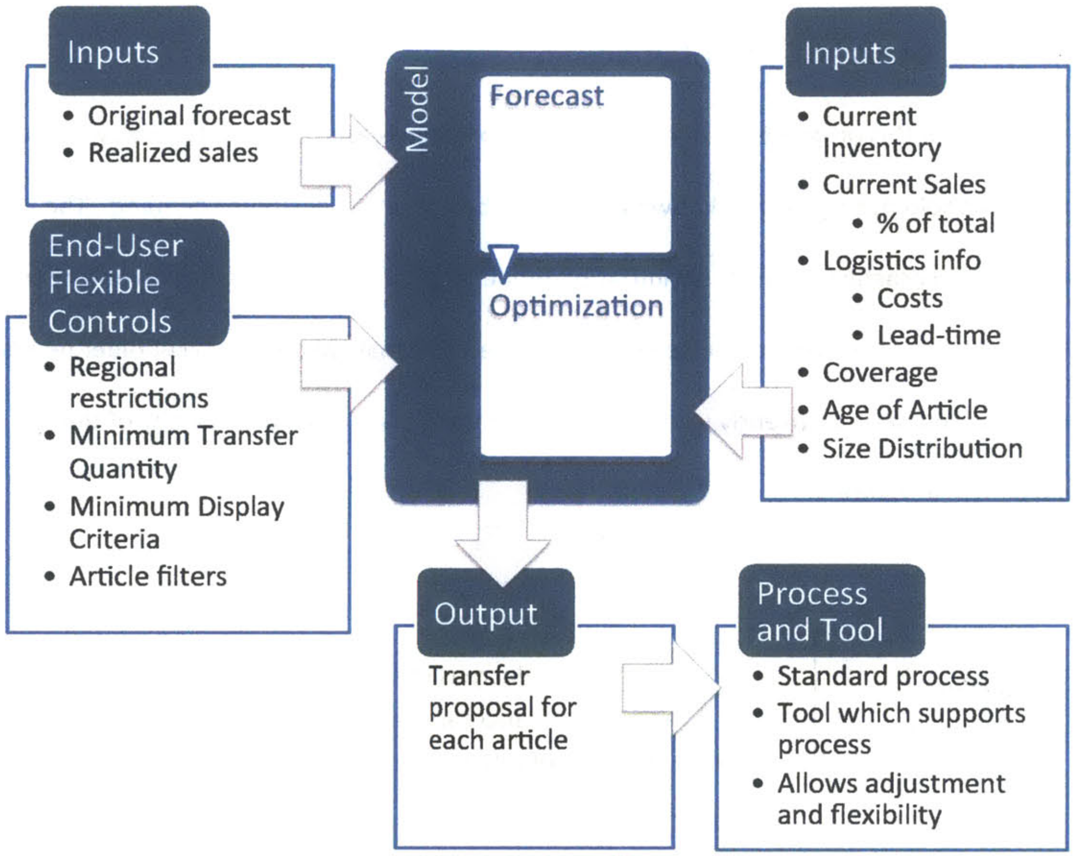

Figure 1. Proposed Solution

1.3 Objectives, Goals and Contributions

The main objective of this project is to increase profit through increased sales and thereby reduce the number of articles sold during the markdown period and better satisfy customer demand. The key deliverables are a forecasting method, optimization model and standard process for inventory transfers among all Zara stores. To achieve these goals, it was imperative that we define stakeholder needs and requirements and define the currently used heuristics for transfers. A visual hierarchy of these objectives is shown in Figure 2.

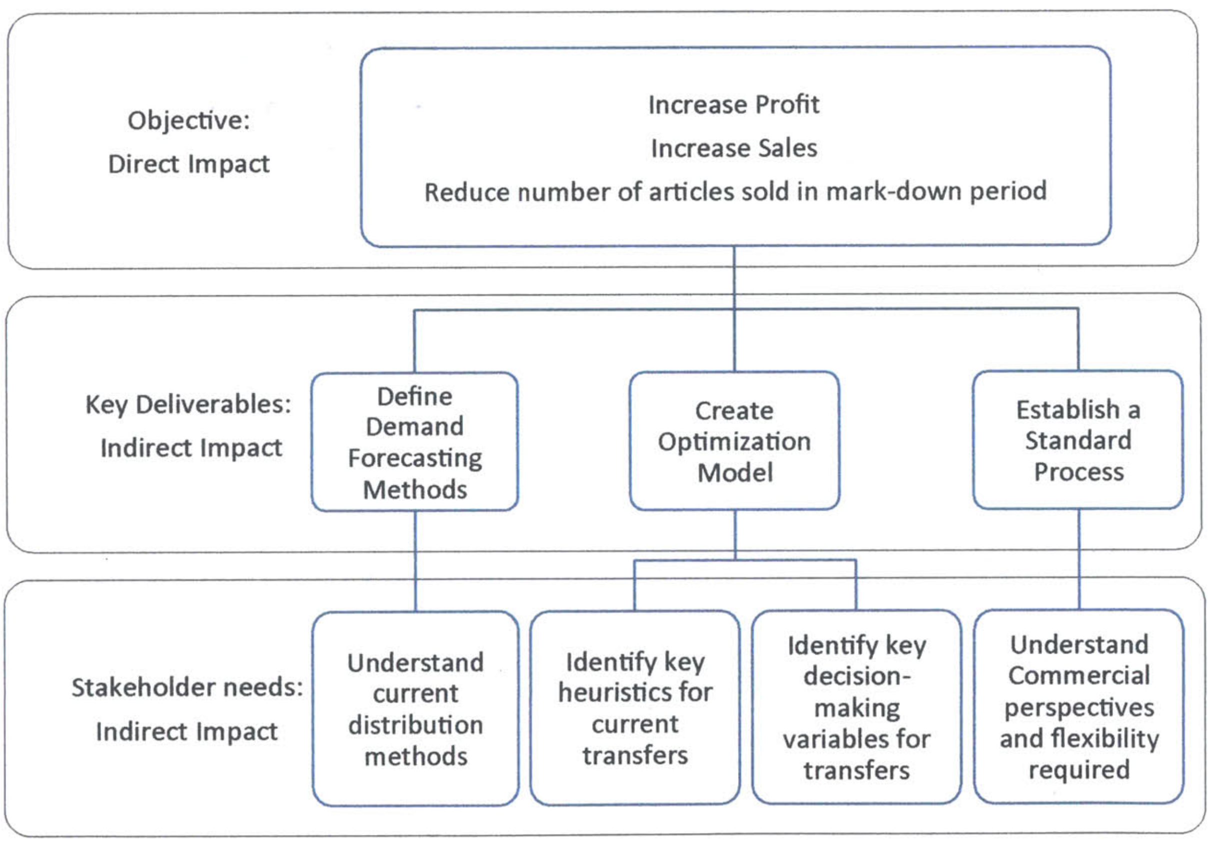

Figure 2. Hierarchy of Objectives

The specific contributions to the challenge of transfer at Zara are a model and process for proposing transfers, which increases profit on articles transferred by 21 percent. This represents greater than 18 million dollars in additional profits. To our knowledge, this is the first model of this kind applied in retail supply chain. We present two multi-period optimization models, which propose transfers: (1) a deterministic mixed-integer optimization model, which optimizes the mean expected profit and (2) a robust, stochastic optimization model, which minimizes the total cost given the uncertainty of the mean expected profit. The second formulation is an advancement of the first formulation to consider the uncertainty of the demand forecast. The demand forecast is a key input to the model and is based on the currently utilized forecasting method at Zara, updated to incorporate the most recent real-sales data.

The process uses the optimization model and inputs from Product Managers to propose transfers, while also providing a reasonable level of flexibility to incorporate the expertise of Product Managers. The process of proposing transfers based on a complex optimization model provides a meaningful proposal while also alleviating the work required by Product Managers. The goal was to propose a process that is no longer manual and labor-intensive. The model accounts for demand, demand uncertainty, logistics, business rules and key metrics.

1.4 Phased Research Approach

We approached the challenge of optimal inventory transfers in five phases: (1) Understanding the current state, (2) Clarifying the scope and objectives, (3) Iterative modeling, (4) Pilot testing and (5) Implementation planning, as outlined in Figure 3. Phases (1) and (2) were focused on understanding stakeholder needs, mapping the current process and documenting requirements and scope. During these phases, we worked closely with stakeholders and end users to define and document the current process and heuristics and the requirements for transfers. During phase (3) we used the requirements and understanding of stakeholder needs to create, test and validate a model to address the challenge of inventory transfers. The modeling process was iterative, using testing and validation to improve and refine the model. In this thesis, we present two models, as mentioned in the previous section: a stochastic optimization model and a robust formulation, which is an iterative advancement of the first.

Upon completion of testing and validation, the project transitioned to phase (4) to use real data to test and review outputs with the end user to refine the model. Phase (5) was focused on defining user-interface and tool requirements along with implementation planning. Figure 3 summarizes this phased approach.

Chapter 4	Phase 1: Understanding the Current State	Define the current transfer process and key stakeholders
		Define types of transfers
		Document key heuristics for transfers
Chapter 4	Phase 2: Clarify Scope and Objectives	Document scope, objectives and requirements
		Review with key stakeholders
Chapters 5, 8	Phase 3: Solution and Modeling	Define key variables for optimization model
		Define forecasting methods
		Design optimization model and iterate
Chapter 6	Phase 4: Testing and validation	Test and validate model with data
		Simulation Testing
		Review results with stakeholders
Chapter 7	Phase 5: Implementation planning	Define flexibility and tool requirements
		Develop implementation plan

Figure 3. Research Approach

1.5 Thesis overview

This thesis is organized into nine chapters as outlined below.

Chapter 1 describes the problem statement and motivation within the context of Zara along with the objective, goals and contributions of this thesis.

Chapter 2 provides an overview of Zara in the context of fast fashion, culture, distribution operations and transfers specifically.

Chapter 3 offers a review of relevant literature relating to inventory redistribution and management and robust optimization.

Chapter 4 outlines the inventory redistribution process within Zara including requirements, stakeholders, current process and heuristics.

Chapter 5 presents the business rules, demand forecast and optimization model including the detailed formulation, which comprise the main workings of the proposed solution to the inventory transfers challenge.

Chapter 6 provides a description of results and detailed model outputs along with testing and performance analysis.

Chapter 7 gives an overview of the model implementation challenges and the process and tool design.

Chapter 8 presents additional features and improvements to the model including a discussion of a multi-article model and a stochastic optimization model considering demand uncertainty.

Chapter 9 concludes this thesis with a summary of the contributions of this thesis and their applicability beyond inventory transfers along with concluding remarks.

Chapter 2 Background

This section provides a brief overview of the fast fashion industry, and Zara's place in that industry along with a summary of Zara's culture and operational methods.

2.1 Fast Fashion Industry and Zara

The fast fashion industry has applied the fast response manufacturing and supply chain models to the fashion industry to radically improve the speed at which new trends appear in stores. The industry is comprised of well-known brands such as H&M, Forever 21, Topshop and Zara, among others. These companies compete to rapidly introduce new articles representing the latest fashion trends at affordable prices. The fast fashion industry is pushing the limits of the apparel industry, and creating a world of fashion with many more collections than the traditional two seasons.

Zara has been a pioneer in the industry and can design, manufacture and display in stores ready-to-wear versions of the latest trends in as little as four weeks after a designer fashion first appears. Zara fully owns and manages the majority of their stores to maintain a high level of oversight and control. Additionally they design all of their own software and logistics and distribution models, allowing them to retain tight control over their supply chain [10].

2.2 Zara Company Culture

Zara emphasizes the importance of analytical models and data to drive their distribution and operations. Seven theses have been written by MIT in partnership with Zara, as well as multiple technical papers using complex analytical models to address various operational challenges within Zara. Several of these technical works are references in the Literature Review in Chapter 3.

Zara has created a culture of continuous improvement, where change is common. This enables them to implement these models and improve their operations to remain the leader in the fast-fashion industry.

Employees are encouraged to consider alternate and novel ways of performing their tasks and have open channels to share and communicate those ideas. Even the open floor plan encourages collaboration with almost no distinction given to hierarchy or authority level [10].

Additionally, Zara understands well the balance between model development and the art of retail management. Employees are given flexibility to experiment outside of the normal process. Their culture is a combination of appreciating analytical methods and using practical and human intuition centered on continuous improvement.

2.3 Zara Operations: Transfers as Part of Distribution

Zara's distribution can be summarized in three phases: (1) suppliers to distribution centers, (2) distribution centers to stores, (3) Transfers between stores, as outlined in Figure 4. Suppliers send bulk orders to the central distribution centers in Spain. The Buyers, who are responsible for determining and placing orders, drive this phase. The amount ordered can vary in quantity from a few hundred to a few thousand articles. The distribution centers are then responsible for sending initial shipments of new articles and replenishments of existing articles to all stores worldwide. Most stores receive shipments from the distribution centers twice weekly. The Distribution Group is responsible for determining the quantities of each article for each store based on advanced forecasting and optimization methods along with input from Product and Stores Managers. This second phase in distribution is a clearly defined process and decisions are made according to the same process and models for all stores.

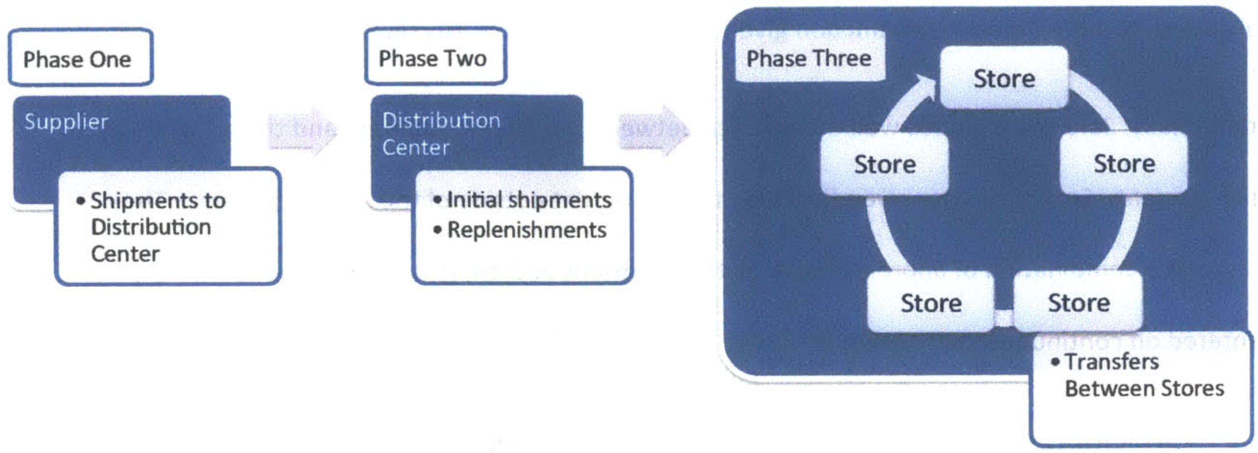

Figure 4. Summary of Distribution Phases

Inventory Transfers is the third phase in the distribution process and consists of transferring inventory directly from and to stores. Transfers have a less rigid process and vary widely in scheduling: day of week and times per month, quantities, types and methodology. They are managed by Product Managers, who are responsible for central management of groups of stores, in a highly manual and labor-intensive process, which is described in more detail in Chapter 4.

2.4 Two Types of Transfers

Transfers between stores can occur in two main settings: (1) in parallel with replenishments from central distribution and (2) in isolation from central distribution. In setting one, stores transfer inventory directly among one another while also possibly receiving inventory replenishments from central distribution. In setting two, stores transfer inventory directly among one another while no longer receiving replenishments most commonly due to a stock out at the central distribution center. We refer to setting one as "Redistribution" and setting two as "Consolidation". Figure 5 provides a visual explanation of these two types of transfers, which comprise the third phase of distribution.

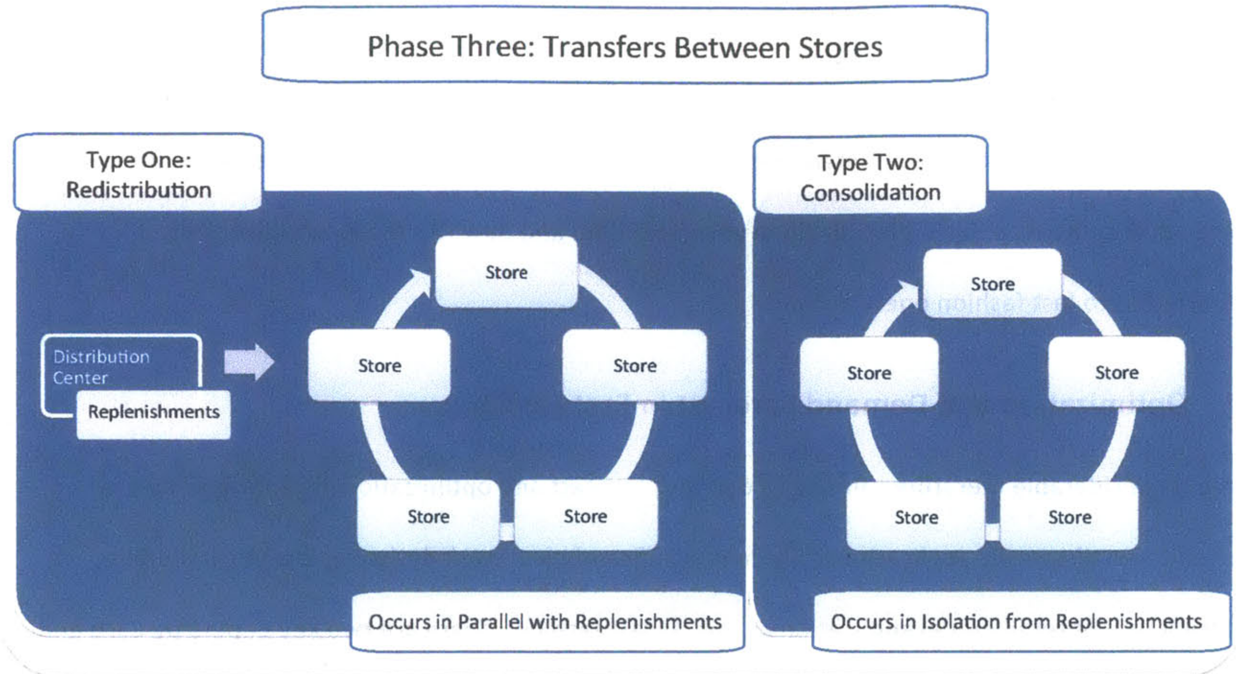

Figure 5. Distribution Phase Three: Two Types of Transfers

The main goal of Redistribution is to rebalance inventory levels among stores according to evolving demand. Low-selling stores can transfer to high-selling stores. The stores receiving articles via transfer then require fewer articles from central distribution replenishments. This allows more articles to sell while also preserving inventory in the distribution centers. Redistribution requires a model and process, which considers the central replenishment system as they occur in parallel.

The main goal of Consolidation is to rebalance and concentrate to the correct stores the remaining inventory already distributed to stores. When articles are not being replenished, they often have broken or skewed size runs and a few units are spread across many stores. These articles then need to be consolidated to a few stores with the best chance to sell the remaining units. Consolidation requires a model that considers size distributions and quantities.

Chapter 3 Literature Review

This chapter provides a review of published literature relevant to the work in this thesis. First we discuss deterministic optimization and demand forecasts in the industry of fast fashion. Then we outline some of the advancements in optimization given uncertain data and how this thesis applies robust optimization to fast fashion operations.

3.1 Optimization and Demand Forecast in Fast Fashion

There is considerable literature relating to demand forecast and optimization in general or even as applied to retail-specific applications. There is significantly less work designing analytical models explicitly for the retail model application known as fast fashion. There are two key papers by Caro and Gallien, which address the allocation of inventory according to demand by implementing a dynamic assortment model [6], [8]. Additionally there are seven theses, which address operational challenges specific to fast fashion such as new product demand forecasting by Andres Garro [11] and clearance pricing by Rodolfo Carboni Borrase [5].

The work by Juan Correa in his thesis sets forth a by article demand forecast for Zara's global network of stores [9]. Andres Garro built on this work to apply a demand forecast to new product distribution [11]. Caro and Gallien discuss the importance of size distributions in the demand forecast in their paper on dynamic of assortment [8]. Tokatli summarizes the affect of shorter lead-time on increased forecast accuracy in his work on global sourcing within Zara [16].

This thesis builds on this prior work to create a forecast and optimization model to address the operational challenge of inventory redistribution within the fast fashion industry.

3.2 Robust Optimization

Linear optimization models, which provide an optimal solution around the mean data, may not provide the optimal solution when the input data is uncertain. In real-world applications with significant data uncertainty, it is definite that in some cases a deterministic optimization model will not provide the best solution, as described by Ben-Tal and Nemirovski in their paper addressing real world application challenges of linear programs [1]. Significant work has been performed to address this challenge of incorporating demand uncertainty. The first model, designed in early the 1970's by Soyester, still uses a linear optimization model but with a solution feasible for all data in a convex set [12].

These solutions, however, are overly conservative, and significant strides have been made to advance the field of robust optimization. Bertsimas and Sim set forth a model in their paper titled, "The Price of Robustness." This paper provides a solution considering demand uncertainty with optimality, while allowing full control over the degree of conservatism [4]. Adida and Perakis present a robust optimization model for inventory management in their paper tited, "A Robust Optimization Approach to Dynamic Pricing and Inventory Controls with no Backorders." They demonstrate that their robust formulation is no more complex than the nominal problem, and provide a reasonable means of adapting a robust formulation considering demand uncertainty from a deterministic model [1].

Previous work in the fast fashion arena has generally used dynamic programming to account for uncertainty of demand, see the Carro and Gallien papers [7][8]. This thesis adapts the new approach set forth by Bertsimas and Thiele in their paper titled, "A Robust Optimization Approach to Inventory" [3]. Their methodology addresses the challenge of optimal solutions given stochastic demand. The model optimizes the solution while guaranteeing optimality in the worst-case scenario [3]. This thesis applies their approach to the application of inventory redistribution within Zara as described in Section 8.2.

Chapter 4 Inventory Transfers Within Zara

This section provides an overview of transfers within the Zara organization. First, the key requirements for addressing the challenge of transfers are defined. Then the current state of transfers is explored by defining the stakeholders, current process and tool, regional considerations, transfer heuristics and types of transfers.

4.1 Requirements for Transfers

To understand how to address the challenge of transfers, we must define the requirements for transfers in the organization. Figure 6 defines these requirements and illustrates how each of them relates to components of the proposed solution model described in this thesis. The current state modeling is defined in this section. The demand forecast and optimization models and article filters are explained in Chapter 5. The process and tool are explained in Chapter 7. The multi-article considerations and robust model are described in Chapter 8.

Aspect of Transfers Model Addressing Requirements	Current state modeling	Demand Forecast	Optimization Model						Article Filters	Process and Tool				Mutli-article Model	Robust Model
			Objective	Transportation Cost	Opportunity Cost	Constraints				Flexible Parameters	Flexible output	Global View	Prioritization Metric		
						Coverage Constraints	Size and color level minimums	Size preferences							
Requirements															
Goal															
Increase profit		X	X												
Transfers															
Understand current process and types	X														
Stakeholder needs	X														
Triggers for when to consolidate or redistribute						X	X								X
Logistics															
Transportation				X											
Lead time of transfer				X											
Regional Considerations				X						X					
Demand															
Demand Time horizon		X													X
Demand Uncertainty		X													X
Business rules															
Visual Merchandising							X	X							
Disruption to store					X							X			
Priority of articles													X		
Size granularity															
Minimum time in store									X						
Article interdependencies														X	
End User Needs															
End-user flexibiltiy	X									X	X				
Ease woarkload	X								X			X	X		

Figure 6. Matrix of Requirements

4.2 Stakeholders

There are four primary groups of stakeholders associated with inventory transfers: Product Managers, Buyers, Distribution and Store and District Management, which are summarized in Figure 7. Product Managers, Buyers and Distribution are all centrally located at Zara headquarters. Store Management and District Management are located in the area or region of their stores.

Figure 7. Summary of Stakeholders

Product Managers are organized by groups of stores, which are usually geographically organized groups. Each Product Manager is responsible for the performance of his or her respective group of stores and acts as a liaison between Store and District Management and Distribution. They currently make most transfer decisions for their stores and sometimes for a transfer, which spans across groups, requiring coordination between Managers.

Buyers are responsible for making buying decisions for all Zara products including type, quantity and sizes and are accountable for the sales performance of those particular articles. They are organized by product collections: Woman, Basic, Weekend Wear, Knitwear, etc. Buyers sometimes make transfer recommendations, which are very collection-specific, usually involving large, cross-regional transfers.

Store Managers oversee a single store and District Managers oversee a region of stores; both are responsible for the operations, success and management of their stores. Store Managers receive transfer orders from Product Managers and are responsible for ensuring the tasks involved in transferring are performed: (compiling inventory, boxing, shipping, etc.). Store Managers and District Managers sometimes propose very store-specific transfers in coordination with Product Managers.

Understanding stakeholder responsibilities, challenges and needs is critical to the project. It was imperative to design a model and tool to fit within the organization while also optimizing transfer decisions. To ensure successful design and implementation we remained focused on Zara stakeholders throughout the project.

4.3 Current Process and tool

The current process is highly varied across Product Managers and regions, which is in part due to varying operational requirements across regions described in further detail in Section 4.4. However, some of this variance is due to the lack of a model that proposes reasonable transfers to support the Product Managers' transfer decisions. The operating rhythm for ordering transfers can vary from once a week to every few months. Some Product Managers have a specific day each week when transfers are ordered; others order transfers less regularly or only when a specific need arises.

There is a tool currently used which requires significant manual adjustment and oversight. The Product Managers must manually choose which articles and which stores to consider based on data and their best understanding of which articles should be transferred. The tool then proposes transfers based on coverage and success ratio, defined in Equations (1) and (2), and will propose the destination store that has the highest value of accumulated sales times success.

$$Success = \frac{Accumulated\ Sales}{Total\ Sent} \quad (1)$$

$$Coverage = \frac{Store\ Stock}{Weekly\ Sales} \quad (2)$$

While this approach provides some support to the Product Managers to manage their decisions, it does not consider sizes, forecasted demand or other key metrics and does not provide an optimal proposal. The tool only proposes transfers of the entire quantities of an article, which is not always a reasonable transfer. Product Managers spend large amounts of time manually verifying and adjusting each proposal. Due to this labor-intensive, highly complex process, the Product Managers have different operating rhythms and strategies for performing transfers.

4.4 Regional considerations

For initial shipments and replenishments all shipping routes originate from the central distribution center, creating a finite set of routes. For transfers, the network could be more complex: considering all 1,720 stores worldwide creates 2,956,680 possible routes between all stores. Because of the complexity of managing such a large network, it is essential for managers to organize those stores into smaller regions. These regions are organized based on existing transportation infrastructure, transportation costs, geographic proximity and organizational responsibility of Product Managers. An example of a country with established regions for Transfers is shown in Figure 8.

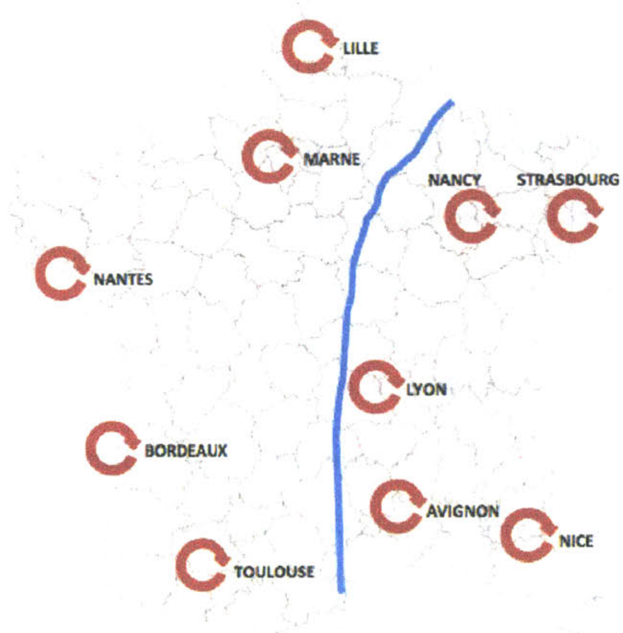

Figure 8. Example of Regional Transfers in France

For some countries, these regions are firmly set due to long lead times or infeasible shipping. For example, it can be challenging to ship from one city to another in Russia, especially in winter when road conditions are poor. However, transfers between stores within the same city in Russia are common. In other countries, these regions are more fluid and transfers are frequently made across regions. For instance, it is common to transfer both within and between cities in the United States. Due to the long lead-time, costs associated with customs agencies, it is rare to transfer between countries. Transfers such as these are performed outside of the normal process and usually return to central distribution prior to redistribution, unlike other transfers.

4.5 Transfer Heuristics

Product Managers currently use a variety of heuristics in a highly manual process to manage inventory transfers. These heuristics can be categorized in the following groups, which are described below: article-specific performance metrics, store information, logistics, inventory levels, visual presentation considerations and seasonality.

The key performance metrics considered when deciding which articles to transfer and where are: success, coverage and sales metrics. Success is defined as the ratio of articles sold to articles sent. It is meant to be a measure of how successful an article has been in a store. Coverage is the amount of days of inventory on hand, according to the expected demand. Both are defined in Equations (1) and (2). Sales metrics are analyzed in a variety of ways including total sales, recent days of sales and sales as a percentage of collection or store. Additionally, articles, sub-families or entire collections are prioritized according to their sales importance in a store or region. Product Managers ask themselves, "Has this article performed well in some stores? Is this an important article or collection for a specific store? Are there stores that have low coverage in an article which has performed well?" The quality of this information is also considered based on how long an article has been in a store and whether it has had significant days with stock outs.

Stores themselves are analyzed using heuristics such as the category of the store, the overall sales performance and performance by collection. Product Managers consider the location and type of customer for their stores along with capacity of both the selling floor and stock rooms.

Logistics are an important aspect for all transfer decisions. Product Managers ask, "Are there existing transportation modes? What are the shipping times? Are the shipping costs low or high?" Additionally, they consider logistics within the stores themselves. Employees in the sending store must obtain and package articles to send. Employees in the receiving store must receive, unpack and display articles outside of the normal replenishment process. Additionally, there are paperwork and electronic tracking tasks at both the sending and receiving end of the transfer. All of these tasks create some disruption in normal store operations and can decrease time employees are on the sales floor.

Inventory levels are a key aspect of transfer heuristics. Product Managers consider total article quantities and size distributions along with inventory levels in central distribution and whether an article will be replenished or not. They analyze stock-outs and how frequently they have occurred. They consider future articles and whether they are similar enough to cannibalize sales.

Zara follows unique visual presentation heuristics, which are guidelines such as display quantities and sizes, collection pairings and specific images of stores. As a strategy, Zara maintains focus on visual presentation of their stores. They do not want stores to appear over-crowded with large amounts of inventory on the floor. While this creates more work for employees to continually restock the sales floors, it also provides a better customer shopping experience and creates a sense of urgency to buy. Additionally, Zara does not want to display articles that do not have a minimum quantity to display or do not have a full range of sizes so that articles do not appear to be haphazardly displayed or disorganized. These visual presentation guidelines are focused on enhancing the shopper experience.

Product managers consider these guidelines in the context of transfers and ask, "Are the minimum display quantities and main sizes available? Is the item part of an important pairing or collection that should be displayed and transferred with another article? Is this store an important or flagship store which needs to display the latest trends?"

Product Managers also consider seasonal-related topics such as weather, holidays or regional events. They ask, "Is the winter season ending in one region but not another? Is there are holiday or large event coming which could affect the demand?"

Chapter 5 Optimization Model

This chapter first provides a detailed explanation of the business rules that shape aspects of the model design. Then it provides a detailed explanation of the demand forecast, which is a key input for the optimization model. Next we describe the methodology for proposing transfers using this updated demand forecast and mixed integer optimization model. Then we provide an overview of the key differences in the two transfer scenarios of consolidation and redistribution in the context of the formulation. Lastly, the section closes with a detailed description of the deterministic optimization model formulation.

5.1 Business Rules

5.1.1 Merchandising and Size Considerations

As described in Section 4.5, Zara has a unique strategy for visual merchandising and display considerations. These business rules have been incorporated in the model in three ways: (1) imposing minimum display quantities, (2) associating possible sales realized with quantity of important sizes and (3) including preferences for balanced size distributions. The first two methods are imposed only in the consolidation model, since articles are not being replenished. During redistribution, articles that do not have enough display quantity, will be replenished according to demand and do not need to be consolidated. Essentially these two criteria are triggers for the model to know when to consolidate based on the color level inventory quantity and proportion of important sizes. The color level inventory is the total quantity of an article including all sizes. The third method is to ensure that transfers proposed balance the size distributions in both consolidation and redistribution

For the first method of imposing display quantity threshold, the model considers a color level minimum quantity to determine if sales can be realized. If the minimum quantity is not met, the model will consolidate to meet these minimums according to the demand situation. An example of these parameters is illustrated in Table 1. The values are determined using commercial perspectives and merchandising display guidelines, which are based on providing a positive customer experience. For example, a product that is available in only one or two sizes can be displayed with a lower minimum quantity than an article that is available in five or six sizes. Additionally to maintain the quality of the visual display, hanging articles displayed on racks have different minimum quantities than folded articles displayed on tables.

The threshold values can vary depending on the quantity of sizes and whether an item is displayed on hanging racks or folded on tables. The minimum values can be calibrated to suit the region and specific Product Manager needs. A higher minimum makes the model more aggressively consolidate.

Table 1. Minimum Display Quantity Parameters

# Sizes	Color level Min Hanging	Color level Min Folded
1	4	6
2	4	6
3	4	8
4	5	8
5	6	8
6	7	8

For the second method of associating possible realized sales with quantity of important sizes, we must first define important sizes. Important sizes are those sizes, which comprise the top 50 to 60 percent of the size weights. Size weights are measured for each subfamily and store and are defined as the proportion of the total of the sales of one size for each article, a, in the subfamily of store, i, to the total sales, S of all sizes, s, shown in Equation (3). A subfamily is a specific category of article, such as folded pants or dresses. An illustration of determination of important sizes is shown in Table 2.

$$Size\,Weight,\,W_{i,s} = \frac{\sum_{a=1}^{T} S_{a,i,s}}{\sum_{a=1}^{T}\sum_{s=1}^{N} S_{a,i,s}} \quad (3)$$

Where a is the article in the set T of articles, i is the store, s is the Size in the set N of sizes. $S_{a,i,s}$ is the sales of article, a, in store, i, of size s.

Table 2. Example of Important Sizes Determination

Size	Size Weight	Important Size
34	0.127871	No
36	**0.239178**	**Yes**
38	**0.30212**	**Yes**
40	0.176678	No
42	0.091431	No
44	0.0627	No

The model then considers the percentage of the inventory of a specific article, which is comprised of these important sizes. If less than half the stock is in the important sizes, then the store can only realize sales for a percentage of the total inventory. An example of this calculation for a specific level of inventory and size composition is shown in Table 3. The exact formulation is explained in Section 5.4, Equation (30). This calculation encourages the model to propose transfers that create sets containing the important sizes, when possible.

Table 3. Illustration of Sales Markdown According to Important Sizes

Size	Important Size	Inventory	Sales Possible
34	No	10	Function of total inventory and inventory in important sizes
36	**Yes**	**2**	
38	**Yes**	**2**	
40	No	5	
42	No	5	
44	No	5	
Total		**40**	**24**

The third method of incorporating merchandising into the model is to incorporate two costs, which measure the imbalance of the size distributions. The first of these, Imbalance Cost One, measures the imbalance in each store according to the stock at the size level compared to the demand at the size level for each size. This encourages the model to transfer in such a way that the size distribution in all stores is as ideal as possible. The second cost, Imbalance Cost Two, measures only the imbalance in the store of origin. This second cost only applies in redistribution when partial transfers are common and ensures that any remaining quantity left in the origin represents a reasonable size distribution.

5.1.2 Disruption to store

Because there is a level of disruption to a store when performing a transfer, there are necessary business rules defining reasonable transfers. The first of these is minimum transfer quantities. It is not reasonable for a store to perform the work of gathering and boxing only a few articles. At a per-article level, this has been incorporated in the model by imposing transfer quantity limitations as described further in Section 5.5. The second aspect of disruption is the number of destinations for one origin. Each unique destination for a store represents another box and administrative paperwork increasing the workload to the store. This per-article aspect has been included in the model by imposing a maximum number of destinations per origin store based on the quantity being transferred, described in detail in Section 5.5. Both of these criteria are only fully meaningful when assessing the total proposal for all articles. Since the model is a single-article model, these aspects of disruption to the store are incorporated through the process and total output methodology, described in Chapter 7.

Additionally, there is an opportunity cost to the store per transfer, which is part of the objective function defined in Section 5.5 and is approximated at a per article level. This opportunity cost is charged per transfer and is approximated based on labor hours, time employees are not on sales floor and general disruption to sales tasks.

5.1.3 Transportation Cost and Lead-Time Approximation

Transportations costs are included in the profit calculation for the objective function of the model. These costs are approximated at a per article level for each possible route based on existing transportation methods.

As described previously, transportation times vary significantly across different routes. This variability is captured in the model by including a cost of lead-time for each route. This cost of lead-time is considered a function of both the opportunity cost of lost sales and changing demand in the destination during the shipping period.

The cost of lead-time is incorporated in the model in the cost of transportation along each route by increasing the cost of transportation as a function of lead time so that the model prefers routes with both lower lead times and lower shipping costs as illustrated in Table 4. The exact multipliers are calibrated based on sensitivity analysis and validation of outputs.

Table 4. Example of Transportation Cost as a Function of Lead-time

Trans Cost (per article)	Lead- Time	Multiplier	Total Cost per Route (per article)
1.25	0-3 days	1	1.25
1.25	3-7 days	2	2.5
1.25	7-14 days	3	3.75
1.25	14-21 days	4	5
1.25	21-28 days	5	6.25

5.1.4 Importance of Articles in Stores

In some stores that are small, have narrowly focused clientele or low turnover of stock, there are a few articles, which comprise a large percentage of total sales for those stores. When an article is in the top 10% of total sales for these stores, the model will prevent removal of these articles from these stores by filtering the routes leaving that store for that article, shown in Table 5.

Table 5. Example of Route Filter Based on Sales

Article	% Sales	Sum % Sales	
Article 1	5%	5%	Top 10% of sales of store: model prevents from removing these articles from this store
Article 2	2%	7%	
Article 3	2%	9%	
Article 4	1%	10%	
Article 5	0.90%	10.90%	Other articles, no restriction
Article 6	0.80%	11.70%	
Article 7	0.75%	12.45%	
Article 8	0.70%	13.15%	

5.1.5 Re-balance Limitations

The main focus of redistribution is to react to demand fluctuations by redistributing inventory according to an updated demand forecast. However, because these articles are still being replenished, we do not want to simultaneously redistribute in every case of minimal imbalance. Instead, we ensure that the model focuses on high imbalances by imposing two criteria for coverage: (1) coverage minimum threshold in origin store and (2) coverage minimum threshold variation between origin and destination store. For the first criterion, stores with coverage of an article below a minimum number of days of coverage are prevented from transferring out. When demand is zero or very small, the coverage is considered to be some large number, so the coverage criteria will never limit the model from proposing a transfer in this case. The second criterion prevents transfers along a route where a minimum value for the difference in coverage between an origin and destination is not met.

5.1.6 Disruption to Central Distribution

In the case of redistribution, the transfers process is performed in parallel with replenishments from central distribution. When transfers orders are placed, the inventory reported in the system will update accordingly and future replenishments will account for these fluctuations in inventory due to transfers. However, to ensure that there are not multiple orders replenishing the same store through two separate processes, we will prevent transfers from being ordered on the same day as replenishment orders.

Additionally, it is assumed that large fluctuations in inventory received from central distribution are due to business decisions made by distribution such as holiday or seasonal issue. The transfer process will prevent the system from removing articles in stores, which have received greater than half of their inventory in the prior week so as not to affect these business decisions.

5.1.7 Minimum Time in Store

There can be some lag between when a store receives an item and when it is displayed. Additionally, visual merchandisers may try different locations in the store to display new items. To ensure that the sales data is accurate and articles have had a reasonable amount of time on the sales floor to accumulate this data, the model imposes a minimum age of an article. Articles that have not been in a store more than a specified period are not considered for transfer out of that store.

5.2 Demand Forecast formulation

The demand forecast is a key input to the optimization model described in the following subsection. The forecast utilized is an application of previous demand forecast modeling projects at Zara, which are summarized in Section 3.1. The forecasting methodology uses linear regression of historical sales data to understand trends and assign parameters dependent on the category of store and type of article. Certain stores are grouped together dependent on behavior and velocity of sales. The number of sizes in which an article is available is considered. The forecast considers and average of prior weeks sales. The formulation is shown in Equation (4), where D is the average of prior weeks sales information. The w value is parameter which weights the specific store within a large grouping of stores. The beta parameters are the weights placed on the average performance of all stores in the group as compared to the individual store. The FC factor is a factor dependent on the velocity of sales and category of store.

$$\ln\left(D_{ij}^{1}\right) = \beta_1 \ln\left(W_j \frac{\sum_{j\in K} \acute{C}_{ij}^{1}}{\sum_{j\in \widetilde{K}} W_j}\right) + \beta_2\left(\sum_{j\,\in \widetilde{K}} W_j\right) + \ln\left(\mathrm{FC_T}\right) \tag{4}$$

Where j is a store in the set K group of stores. C_{ij} is is the average sales of article i, in store j. W_j is the weight of the article in store j in the set of K.

This methodology is adapted directly from Garro's work in 2011 on demand forecasting and distribution for Zara, which is used by Zara for initial shipments and replenishments from central distribution [11]. The application to the challenge of inventory transfers benefits from more recent realized sales information, thereby providing a more accurate mean forecast.

5.3 Optimization Model Description: Multi-period, Mixed-integer Model

The optimization model maximizes average expected profit by transferring inventory among stores considering variables such as demand, costs, lead-time, inventory levels and display criteria. The output is a proposal of transfer quantities along routes (Store A to Store B) for a single article. The model is applied independently for each article and the total output is discussed in Chapter 8. The model is multi-period as illustrated in Figure 9, to allow the model propose to transfer now or wait until the next period. While a multi-period model increases complexity and number of variables, it also gives the model the ability to consider lost sales in period one. This allows the model to make a more optimal proposal to either transfer now or wait until next period. The intent of this design is to allow the model to see demand over all periods and optimize whether it is better to transfer now or wait until a later period. The proposals for period one will be the actual transfer orders made at the time the model is applied.

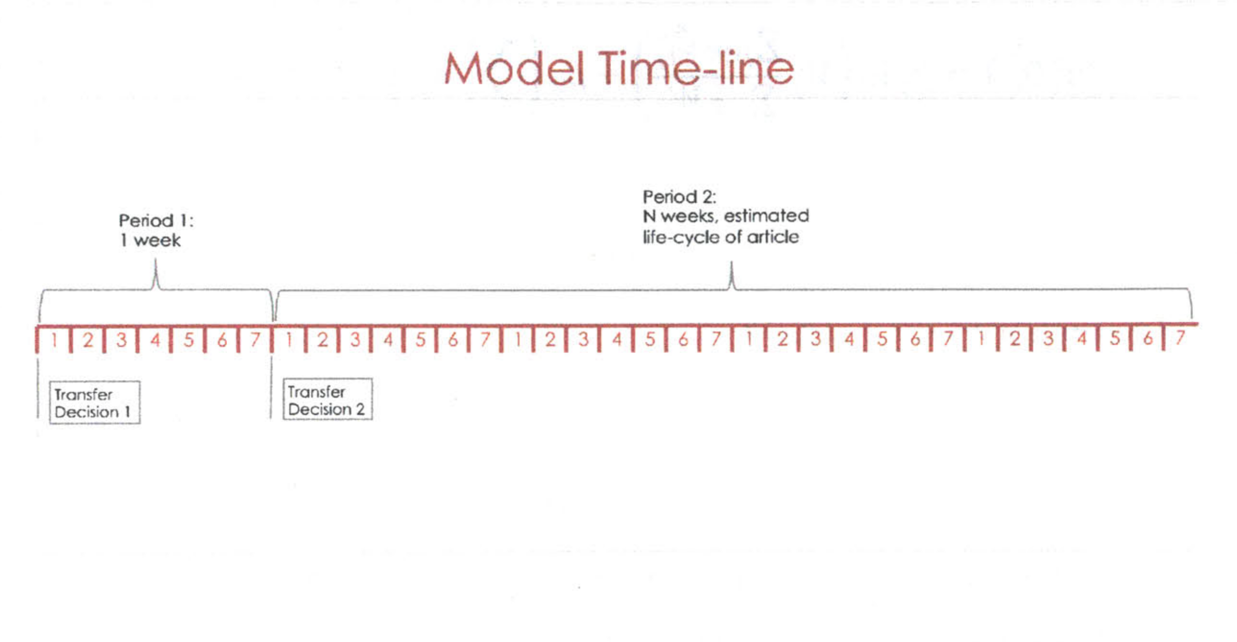

Figure 9. Multi-period Model

There are four key costs included in the objective: transportation cost, opportunity cost, holding cost and size imbalance costs. The transportation cost is an approximation across each route at a per article level and is a function of the shipping cost, lead-time and number of units transferred. The opportunity cost is the cost to the store per transfer and is an approximation of the opportunity cost for an employee to perform the transfer tasks. Holding cost is the cost for holding excess inventory without realizing sales. The size imbalance costs add a preference for the model to propose transfers, which create meaningful size distributions.

5.4 Overview of Consolidation vs. Redistribution in Context of Formulation

There are two distinct formulations to maximize the expected profit to address the scenarios for consolidation (when articles are not being replenished) and redistribution (when articles are being replenished). The decision variables and objective function are similar in the two formulations for each type of transfer, with some unique constraints to address the disparities in the two scenarios. The formulations are described in detail for both scenarios in Section 5.5. Table 6 describes a summary of the redistribution and consolidation models in the context of the formulation.

Table 6. Overview of Consolidation and Redistribution in Context of Formulation

	Consolidation	Redistribution
Scenario	No replenishments	Replenishment
Objective Function	Optimize Profit	
Subject To Constraints:		
Transfer Quantity	Minimum transfer quantity, θ1; 1/2 Inventory or 0.	
Number of Destinations	Stock from one origin can't be sent to multiple destinations unless initial stock is greater than θ3	
Inventory Quantity Post Transfer	Inventory post transfer must be zero or greater than θ2	
No Circular Transfers	Same store can't be destination and origin of same article	
Size Imbalance Overall	Preference for balanced size distributions	
Size Imbalance Origin	---------	Preference for balanced size distributions in origin store for partial transfers
Coverage Threshold	---------	Coverage in Origin>θ4
Coverage Comparison	---------	Coverage in Origin minus Coverage in Destination > θ5
Minimum Display Quantity	Expected salesare correlated to a minimum display at color level	---------
Important Sizes	Expected sales are correllated to proportion of important sizes to total inventory	---------
Subject to Filters:		
% Sales in Origin	Prevent removals from a store of top-selling articles	---------
Age of article	Article must have been in store at least 2 weeks to consider a transfer from that store	
Recent Movements	If a store received greater than half of their inventory in the prior week, do not transfer	
Synch with Replenishments	---------	Do not order transfers simultaneously with replenishment orders

5.5 Formulation: Parameters, Variables, Objective and Constraints

The following is a detailed definition of the deterministic, mixed integer optimization formulation as applied to Zara's transfer challenge. First we define the input data, decision variables and constraint variables. Then the objective function and all constraints are presented.

First we list the input data, which consists of parameters such as inventory levels and threshold values such as minimum transfer quantity.

Input Data

Parameters

- t: Time period in total selling period, *T*.
- Stores $i, j \in N$.
- $I_{i,t=0}$: Initial inventory in store *i* in period zero.
- $I_{i,s,t=0}$: Initial inventory in store *i* in size s in period zero.
- $I'_{i,s,t=0}$: Initial Inventory in important sizes, s', in period zero.
- $D_{i,t}$: Forecasted demand in store *i* for period t.
- $p_{i,t}$: Revenue per unit in store *i* for period *t*.
- *h:* Holding cost for total unsold inventory.
- $C_{1,ij,t}$: Transportation cost per unit from store *i to j* in period *t*.
- $C_{2,ijt}$: Opportunity cost per unit in store *i* in period t.
- $W_{i,s}$: Weight of the demand in store *i* for size s.
- M: Arbitrarily large number.

Threshold Values

- θ_1: Minimum transfer quantity.
- θ_2: Minimum display quantity post transfer.
- θ_3: Minimum inventory level required to send units to multiple destinations.
- θ_4: Coverage threshold (redistribution only).
- θ_5: Coverage comparison threshold (redistribution only).
- θ_6: Minimum display quantity to realize sales (consolidation only).

Next we define the decision variables.

Decision Variables:

- $Y_{ij,t}$: Binary decision variable, $Y_{ij,t} \in \{0,1\}$.
  - $Y_{ij,t} = 1$: Can transfer from store *i* to store *j* in period t.
  - $Y_{ij,t} = 1$: Cannot transfer from store *i* to store *j* in period t.
- $X_{ij,t}$: Number of units transferred from store *i* to store *j* in period t, $X_{ij,t} \geq 0$.
- $S_{i,t}$: Expected sales in store *i* in period t, $S_{i,t} \geq 0$.
- $I_{i,t}$: Inventory in store *i* in period t, prior to transfer decision.
- $I'_{i,t}$: Inventory in store *i* in period t after transfer decision.
- $I'_{i,s'}$: Inventory in important sizes, s', in store *i* in period 1, after transfer decision.
- $E_{1,}$: Size imbalance cost one.
- $E_{2,}$: Size imbalance cost two (redistribution only).

Next we define the dummy variables. These variables are used in the formulation of certain constraints as explained in the following formulation.

Constraint-Specific Dummy Variables

- $a_{1,ij,t}$, $a_{2,ij,t}$: Binary decision variable, ∈ {0, 1} for minimum transfer quantity constraint.
- $b_{1,i}$, $b_{2,i}$: Decision variables used to limit number of destinations.
- $f_{1,i}$, $f_{2,i}$: Decision variables, ∈ {0, 1}, used to limit number of destinations.
- g_i: Counts number of times an origin store receives units.
- $j_{ij,t}$: Counts number of times an origin store receives units.
- $k_{1,i}$, $k_{2,i}$: Decision variables, ∈ {0, 1}, used to set minimum display quantity.

Lastly, we present the optimization model formulation consisting of the objective function and constraints.

Objective Function:

$$Max\ Total\ Profit\ = Revenue - C_{transportation,leadtime} - C_{opportunity} - C_{holding} - C_{size\ imbalances}$$

$$Max \sum_{t=1}^{T}\sum_{i=1}^{N} p_{i,t}s_{i,t} - \sum_{t=1}^{T}\sum_{i\neq j}^{N}\sum_{j=1}^{N} c_{1,ij,t}X_{ij,t} - \sum_{t=1}^{T}\sum_{i\neq j}^{N}\sum_{j=1}^{N} c_{2,i,t}Y_{ij,t} - h\sum_{i=1}^{N} I_i - \sum_{i=1}^{N} E_{1,i} - \sum_{i=1}^{N} E_{2,i}$$

(5)

Subject To:

Inventory Update:

$$I_{i,t+1} = I_{i,t} + \sum_{i \neq j}^{N} X_{ji,t} - \sum_{j \neq i}^{N} X_{ij,t} - S_{i,t} \tag{6}$$

$$I'_{i,t} = I_{t-1} + \sum_{i \neq j} X_{ji,t-1} - \sum_{j \neq i} X_{ij,t-1} \tag{7}$$

$$\forall\, i, t$$

Sales Definition: Expected sales are a function of inventory and demand.

$$S_{i,t} \leq I'_{i,t} \tag{8}$$

$$S_{i,t} \leq D_{i,t} \tag{9}$$

$$\forall\, i, t$$

Coverage Definition: Days of inventory in store.

$$R_{i,t} = \frac{I_{i,t}}{D_{i,t}} \tag{10}$$

$$\forall\, i, t$$

Transfer to Demand at Destination: Transfer quantity must be less than or equal to the overall demand in the destination store.

$$X_{ij,t} \leq \sum_{t=1}^{T} D_{j,t} * Y_{ij,t} \tag{11}$$

$$\forall\, j \neq i, t$$

Transfer Quantity: Number of units transferred must be zero, greater than half of the initial inventory or must be greater than Θ_1.

$$X_{ij,t} \geq \theta_1 * a_{1,ij,t} \tag{12}$$

$$X_{ij,t} \geq \frac{1}{2} I'_{i,t} a_{2,ij,t} \tag{13}$$

$$X_{ij,t} \leq M * Y_{ij,t} \tag{14}$$

$$a_{1,ij,t} + a_{2,ij,t} = Y_{ij,t} \tag{15}$$

$$a_{1,ij,t}, a_{2,ij,t} \{0, 1\}$$

$$\forall j \neq i, t$$

Number of Destinations: For one origin to send to multiple destinations, initial inventory must be greater than Θ_3.

$$b_i = \sum_{i=1}^{N} \sum_{t=1}^{T} Y_{ij,t} \tag{16}$$

$$b_i = \leq \frac{I_i}{\theta_3} + 1 \tag{17}$$

$$\forall i$$

Inventory Quantity Post Transfer: Inventory in an origin store after a transfer is performed must be 0, greater than Θ_2, a minimum display quantity.

$$I_i^{'} \leq f_{1,i} * M \quad (18)$$

$$\theta_2, a * \sum_{i \neq j}^{N} \sum_{t=1}^{T} Y_{ij,t} - I_{i,1}^{'} \leq f_{2,i} * M \quad (19)$$

$$f_{1,i} + f_{2,i} = 1 \quad (20)$$

$$f_{1,i}, f_{2,i} \{0, 1\}$$

$$\forall i$$

No Circular Transfers: One store cannot be both an origin and destination.

$$g_i \geq \frac{\sum_{t=1}^{T} \sum_{j=1}^{N} Y_{ij,t}}{M} \quad (21)$$

$$j_i \geq \frac{\sum_{t=1}^{T} \sum_{j=1}^{N} Y_{ji,t}}{M} \quad (22)$$

$$g_i + j_i \leq 1 \quad (23)$$

$$\forall i$$

Size Imbalance Overall: For each store, E1 is the maximum imbalance of sizes measured as the maximum difference between the size with the most excess stock and the size with the greatest under stock.

$$E_{1,i} \geq [I'_{i,s1} * (1 - W_{i,s1}) - I'_{i,s2} * (1 - W_{i,s2})] - [D_{i,s1} * (1 - W_{i,s1}) - D_{i,s2} * (1 - W_{i,s2})] \quad (24)$$

$$\forall\, i, s1 \neq s2$$

Size Imbalance Origin: When a transfer is performed, E2 measures the maximum

(Redistribution Only) imbalance of a size in the origin store.

$$E_{2,i} \geq -M * (1 - g_i) + \sum \sum_{s=1}^{N} |I'_{i,s} - D_{i,s}|^{1} \quad (25)$$

$$\forall\, i$$

Coverage Threshold: In an origin store, if the current coverage is less than Θ_4, then no

(Redistribution Only) transfer can be made from origin store i.

$$Y_{ij,t} \leq \frac{R_{i,t}}{\theta_4} \quad (26)$$

$$\forall\, j \neq i, t$$

Coverage Comparison: If the difference in coverage of an article between two stores is less

(Redistribution Only) than Θ_5, then no transfer can be made from store i to store j.

$$Y_{ij,t} \leq \frac{max(0, R_{i,t} - R_{j,t})}{\theta_5}^{2} \quad (27)$$

$$\forall\, j \neq i, t$$

[1] The absolute value is non-linear but is used to simplify the notation and representation of the actual constraint, which uses multiple dummy variables to linearize this same function

[2] The 'max' function is non-linear but is used to simplify the notation and representation of the full constraint, which uses multiple dummy variables to linearize this same function.

Minimum Display Quantity: (Consolidation Only)

For sales to be realized, a store must have a minimum display quantity of Θ_6.

$$\sum \sum_{t=1}^{T} S_{i,t} \leqslant k_{1,i} * M \qquad (28)$$

$$\theta_6 - \sum_{t}^{T} \leq k_{2,i} * M \qquad (29)$$

$$k_{1,i} + k_{2,i} = 1 \qquad (30)$$

$$k_{1,i}, k_{2,i} \{0, 1\}$$

$$\forall\, i$$

Important sizes: (Consolidation Only)

If the inventory in the 'important sizes' is less than half the total inventory, then the sales realized must be less than the total inventory.

$$\sum_{t=1}^{N} S_{i,t} \leq \sum_{s'}^{S'} I'_{i,s,1} + \frac{1}{2} I'_{i,1} \qquad (31)$$

$$\forall\, i, t$$

Non-negativity Constraints:

$$I_{i,t} \geq 0 \qquad (32)$$

$$I'_{i,t} \geq 0 \qquad (33)$$

$$S_{i,t} \geq 0 \qquad (34)$$

$$X_{ij,t} \geq 0 \qquad (35)$$

Next, we discuss some of the key aspects of the model formulation and how they influence the transfer proposal of the model.

The Transfer to Demand at Destination constraint, in Equation (11), allows the model to transfer to demand and also relates the decision variable Y to the transfer quantity, X. This way, if another constraint forces the decision variable to be 0, the transfer quantity will also be zero.

The Transfer Quantity restrictions shown in Equations (12) through (15) require the model to propose transfers of quantities, which are of reasonable quantity to transfer. The formulation allows three quantity options: (1) zero, (2) more than half the initial inventory, or (3) more than a minimum threshold. To require solely a minimum transfer quantity would prevent transfers out of stores containing levels of inventory below that minimum transfer quantity. When necessary, the model must transfer these small quantities, so the constraint formulation includes the option to transfer greater than half of the initial inventory. For example, in the case of a store with only two units of an article remaining, the store would have to transfer both units or none.

The number of destinations per origin increases with addition of the merchandising constraints, which encourage the model to optimally redistribute the remaining inventory. To balance this with a reasonable workload for one origin in packing and shipping, we introduce a minimum Number of Destinations constraint, as shown in Equations (16) and (17). Since it is reasonable to split the inventory among multiple destinations when there are large amounts of excess inventory, the constraint is formulated as a restriction based on the number of units of initial inventory.

The Inventory Quantity Post Transfer restriction, shown in Equations (18) through (20) ensures the inventory remaining in an origin store after a transfer is of a minimum quantity to display. Since the model is deterministic it can leave one or two units in a store to transfer exactly to demand. This constraint requires the model to either transfer the entire inventory or leave at least a minimum quantity to display.

The No Circular Transfers constraint in Equations (21) and (22) prevents the model from transferring back and forth between stores. When shipping costs are low, the model has a tendency to transfer to and from the same store in different periods to more optimally distribute sizes. For example, store A could send to Store B size Medium, and store B could send size Small to store A. Since this creates an unreasonable number of transfers and routes used for the total output, we prevent this situation with the restriction as formulated.

The Size Imbalance Overall is a cost, shown in Equation (24), calculated as the maximum imbalance of sizes for an article in each store. The sum over all stores is then subtracted as a cost from the revenue in the objective function, shown in Equation (5). This encourages the model to reallocate inventory with the best possible size distributions in all stores. Since transfers must be made with the inventory available, not an ideal full order of inventory, this formulation is not a hard restriction or requirement but rather a cost, which encourages the best possible situation with the inventory available.

The Size Imbalance in Origin cost, shown in Equation (25) is a similar constraint as the overall size balance but is specific to the redistribution formulation and is only counted for origin stores that send inventory. In the redistribution situation, it is common to make partial transfers. This formulation ensures that the model proposes a transfer, which leaves a reasonable size distribution in any remaining inventory in the origin.

The Coverage Threshold constraint, presented in Equation (26), is the threshold at which a store can send inventory. If a store has a large amount of coverage (meaning excess inventory), that store can be an origin. When a store has a demand of zero, the coverage is calculated to be some large number so that in the case of zero demand, a store can always be an origin.

The Coverage Comparison constraint, shown in Equation (27), considers the difference in the coverage for an origin and destination. This prevents the redistribution formulation from proposing small rebalances, to instead focus on the large disparities. Both of the coverage constraints apply only in redistribution when articles are being replenished. For consolidation, we allow small rebalances of inventory, as the inventory will not be replenished.

The Minimum Display Quantity constraint, shown in Equations (28) through (30), requires the model to obtain a minimum quantity to realize sales. This is based on the visual merchandising guidelines: when an article is below a minimum level it is most likely removed from the sales floor making the possibility of sales zero. This constraint only applies to consolidation, since in redistribution articles below the minimum which are selling well, will be replenished from the central distribution center.

The Important Sizes constraint, shown in Equation (31), relates the expected sales to the ratio of inventory comprised of the important, or most highly demanded sizes. This constraint is not a binding restriction requiring a minimum quantity of main sizes, but instead allows the model to partially relate sales to the important sizes. As mentioned previously, in the consolidation situation, it is not always possible to have the most ideal composition of important sizes to total inventory, so a hard constraint would be overly restrictive.

Chapter 6 Results and Testing

This section contains a discussion of results highlighting various input scenarios and constraints and how they affect the output of the model described in Chapter 5. Additionally, there is a description of sensitivity analysis and testing including simulation, pilot and stakeholder reviews.

6.1 Results

These results provide an understanding of the various features of the model and how the proposals vary given different input scenarios.

6.1.1 Two Period Transfer Experiment

As described in Section 5.3, the optimization model is a multi-period model, consisting of two periods: (period 1) now and (period 2) all later periods. Table 7 shows a result summarized both by transfer routes and by store, where transfers are proposed in two periods. The stores that transfer out in period one (Stores: A, D, F, G, J, K, L, S) either have surplus inventory greater than the total forecast demand or they have no demand. The stores that transfer out in period two (Stores: B and C) have substantial demand in period one and two and little or no surplus. The model recognizes that by waiting to transfer, these stores can realize sales in period one. Stores P and Q receive shipments in both periods. They need the inventory they receive in period one to meet their period one demand. Stores P and Q have higher demand than stores B and C, but Stores P and Q receive enough inventory to meet the demand for period one. Therefore, B and C can wait to send until there is additional need in period two and sales have been realized for period one. This allows the model to account for the possibility of lost sales in period one. Additionally, when the model is run again for period two, another week of actual sales will be realized, so that the new demand forecast will be further improved.

Table 7. Example of Multi-Period Transfer Decisions

Origin	Destination	Transer Quantity	
		Period 1	Period 2
A	O	6	0
B	P	0	26
C	Q	0	14
D	Q	9	0
F	P	2	0
G	P	1	0
H	R	9	0
I	Q	0	6
J	O	1	0
K	P	3	0
L	O	5	0
S	P	2	0

Store	Initial Inventory	Forecast Demand		Transfer Quantity	
		Period 1	Period 2	Period 1	Period 2
A	6	0	0	-6	0
B	29	3	4	0	-26
C	15	1	1	0	-14
D	9	0	0	-9	0
F	10	3	7	-2	0
G	1	0	0	-1	0
J	1	0	0	-1	0
K	3	0	0	-3	0
L	5	0	0	-5	0
P	3	7	28	8	26
S	2	0	0	-2	0
Q	1	10	30	9	20
I	7	1	1	0	-6
O	10	12	8	12	0

6.1.2 Explanation of Shipping Costs

The optimization model includes transportation and lead-time costs along each route. These costs encourage the model to minimize both the number of articles transferred and the number of routes used. Additionally, when applying the model to an entire country, the model will prefer transferring within smaller regions where lead times and costs are lower. Table 8 shows two model results for the same scenario given two different estimations of by article transfer costs. In scenario 1, the costs are very low and the model ships small quantities across the country even when it could rebalance within smaller regions. Region 1 sends to Region 3. In scenario 2, the costs have been calibrated so that the model prefers to rebalance within a region when possible. Region 1 and Region 2 both rebalance the inventory without proposing a cross-regional transfer. During the testing of the model, the model was applied to many country-wide scenarios to calibrate the estimation of per article transfer costs as a function of actual route costs and lead times.

Table 8. Calibration of Shipping Costs

Scenario 1						
Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior	Inventory Post
Region 1						
A	-9	0	4500.0	0.0	9	0
Region 2						
C	6	3	0.0	2.0	0	6
Region 3						
D	-6	0	3000.0	0.0	6	0
E	-2	0	1000.0	0.0	2	0
F	-6	1	15.0	9.0	15	9
G	-9	0	4500.0	0.0	9	0
H	26	5	1.2	6.4	6	32
I	-2	0	1000.0	0.0	2	0
J	2	4	0.3	0.8	1	3

Scenario 2						
Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior	Inventory Post
Region 1						
B	9	3	3.3	6.3	10	19
A	-9	0	4500.0	0.0	9	0
Region 2						
D	-6	0	3000.0	0.0	6	0
E	-2	0	1000.0	0.0	2	0
F	-9	1	15.0	6.0	15	6
G	-9	0	4500.0	0.0	9	0
H	13	5	1.2	3.8	6	19
I	-2	0	1000.0	0.0	2	0
J	9	4	0.3	2.5	1	10
K	6	2	5.0	8.0	10	16

6.1.3 Regional Transfers Results

When the model is applied at the country level, it will propose transfers across regions when there is no option to redistribute within a region. Table 9 shows a result where two regions transfer cross-regionally. Regions 2 and 3 both have zero demand in any of their stores and can only send to another region. Regions 1 and 4 both have little inventory and high demand and cannot rebalance within the region. It is important to note here that Regions 1 and 4 would redistribute within the region if possible and Regions 2 and 4 would then keep their inventory, even though they have zero demand. Additionally, the model proposes these cross-regional transfers when the benefit of gained sales outweighs the cost. If these regions were in a country with inflated shipping costs and/or lead times the threshold for the required additional sales is higher, and the model will, therefore, only propose cross-regional transfers for large quantities or high priced articles.

Table 9. Cross-Regional Transfers

Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior	Inventory Post
Region 1						
A	11	4	4.0	6.8	16	27
Region 2						
B	-12	0	6000.0	0.0	12	0
C	-5	0	2500.0	0.0	5	0
Region 3						
D	-11	0	5500.0	0.0	11	0
Region 4						
E	5	8	3.0	3.6	24	29
F	12	9	4.8	6.1	43	55

6.1.4 Typical Redistribution Results

Table 10 displays a typical model result in the case of redistribution, when an article is still being replenished from central distribution. In Store D, the article has not performed well and is sent to Store A with higher demand. Store C has demand for the article but significant excess inventory and coverage of 22 weeks. The model rebalances this inventory and sends to a store with only two weeks of coverage.

Table 10. Typical Redistribution Results

Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior						Inventory Post					
					Total	S 1	S 2	S 3	S 4	S5	Total	S 1	S 2	S 3	S 4	S5
B	12	26	2	5	8	0	2	3	2	1	20	0	2	11	6	1
A	6	22	5	7	15	3	2	5	4	1	21	3	3	8	6	1
C	-12	10	22	10	22	1	3	11	6	1	10	1	3	3	2	1
D	-6	0	3000	0	6	0	1	3	2	0	0	0	0	0	0	0

6.1.5 Coverage Thresholds Experiment

The coverage thresholds prevent the redistribution model from performing small rebalances. Table 11 shows a result scenario without rotation thresholds. The model sends from store G, which only has six weeks of rotation. Because articles are still being replenished in redistribution and the demand is highly uncertain, the rotation thresholds prevent small rebalances and the model would not propose the transfer shown below.

Table 11. Rotation Thresholds in Redistribution

Store	Transfer Quantity	Inventory Prior	Inventory Post	Forecast Demand	Prior Coverage	Post Coverage
A	0	42	42	86	2.47	2.47
B	0	37	37	80	2.31	2.31
C	15	12	27	71	0.86	1.93
D	0	42	42	56	3.82	3.82
E	0	16	16	49	1.60	1.60
F	0	31	31	37	4.43	4.43
G	-15	70	55	55	6.36	5.00
H	0	30	30	87	1.76	1.76
I	0	31	31	39	3.88	3.88

6.1.6 Size Imbalance Cost Experiment

In the redistribution situation, partial transfers are commonly proposed. The size imbalance cost in the origin is applied to ensure the remaining inventory in a store that performs a transfer is comprised of a good size distribution according to the size level demand. Table 12 shows the model results with and without a size imbalance cost in origin (Imbalance Cost Two). In scenario 2 with the cost, the size distribution of the remaining inventory in Store B is significantly enhanced when compared to scenario 1.

Table 12. Size Imbalances in Origin Store

Scenario 1: No Size Imbalance Cost In Origin																			
Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior							Inventory Post							
					Total	S 1	S 2	S 3	S 4	S 5	S 6	Total	S 1	S 2	S 3	S 4	S 5	S 6
A	49	163	2.7	4.8	62	21	3	25	8	5	0	111	24	27	43	10	7	1
B	-49	49	14.0	7.0	98	13	27	53	2	2	1	49	10	3	35	0	0	0

Scenario 2: Size Imbalance Cost in Origin																			
Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior							Inventory Post							
					Total	S 1	S 2	S 3	S 4	S 5	S 6	Total	S 1	S 2	S 3	S 4	S 5	S 6
A	49	163	2.7	4.8	62	21	3	25	8	5	0	111	27	17	54	8	5	0
B	-49	49	14.0	7.0	98	13	27	53	2	2	1	49	7	13	24	2	2	1

6.1.7 Consolidation Vs. Redistribution Results

Table 13 illustrates the difference between the results for consolidation and redistribution given the same set of stores for the same article in a given week. The consolidation model transfers because merchandising rules are not met and the stores will not be replenished. Note here that the consolidation scenario proposes a transfer even out of a store that does not have excess inventory so that merchandising rules can be met and the article can be sold in more stores. The redistribution result does not transfer because the imbalances are not high enough and the low quantities will be replenished.

Table 13. Consolidation and Redistribution Comparison of Results

Scenario 1: Consolidation															
Store	Transfer Quantity	Forecast Demand	Inventory Prior						Inventory Post						
			Total	S 1	S 2	S 3	S 4	S 5	Total	S 1	S 2	S 3	S 4	S 5
A	11	74	7	1	0	5	1	0	18	1	4	12	1	0
B	0	71	17	1	1	6	6	3	17	1	1	6	6	3
C	10	54	5	0	0	3	2	0	15	3	0	3	9	0
D	-11	40	34	3	13	14	4	0	23	3	9	7	4	0
E	-10	22	10	3	0	0	7	0	0	0	0	0	0	0

Scenario 2: Redistribution															
Store	Transfer Quantity	Forecast Demand	Inventory Prior						Inventory Post						
			Total	S 1	S 2	S 3	S 4	S 5	Total	S 1	S 2	S 3	S 4	S 5
A	0	74	7	1	0	5	1	0	7	1	0	5	1	0
B	0	71	17	1	1	6	6	3	17	1	1	6	6	3
C	0	54	5	0	0	3	2	0	5	0	0	3	2	0
D	0	40	34	3	13	14	4	0	34	3	13	14	4	0
E	0	22	10	3	0	0	7	0	10	3	0	0	7	0

6.1.8 Typical Consolidation Results

In the consolidation situation, the model removes from low demand, consolidates small quantities to meet minimums and improve size distributions to the extent possible with the existing inventory. Table 14 displays a typical result for consolidation. Stores Q, M, P and T all send their inventory because there is no longer demand. Store C sends its inventory because it does not meet minimum display quantities with only one remaining unit. Store B sends a partial transfer of excess inventory to stores that need inventory to meet demand and quantity minimums.

Table 14. Typical Consolidation Results

Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior							Inventory Post					
					Total	S 1	S 2	S 3	S 4	S 5	S 6	Total	S 1	S 2	S 3	S 4	S 5
J	0	51	0.7	0.7	6	0	0	6	0	0	0	6	0	0	6	0	0
K	0	45	1.9	1.9	13	2	1	7	3	0	0	13	2	1	7	3	0
D	25	41	0.1	3.7	1	0	0	1	0	0	0	26	3	8	10	5	1
L	0	35	1.3	1.3	8	2	4	0	0	2	0	8	2	4	0	0	2
I	4	35	0.5	1.2	3	0	1	2	0	0	0	7	0	1	5	1	0
H	0	30	1.8	1.8	9	0	2	6	1	0	0	9	0	2	6	1	0
B	-35	29	13.8	6.8	69	13	22	25	7	2	0	34	9	11	12	2	0
G	0	19	3.3	3.3	10	1	0	8	1	0	0	10	1	0	8	1	0
N	12	16	0.3	4.3	1	0	0	1	0	0	0	13	2	3	5	2	1
A	1	16	1.7	2.0	5	0	0	5	0	0	0	6	0	0	6	0	0
O	12	15	0.5	6.5	1	0	0	1	0	0	0	13	3	2	7	1	0
S	1	11	2.5	3.0	5	0	0	5	0	0	0	6	1	0	5	0	0
C	-1	10	0.5	0.0	1	0	0	1	0	0	0	0	0	0	0	0	0
Q	-2	7	1000.0	0.0	2	0	0	0	2	0	0	0	0	0	0	0	0
M	-12	6	6000.0	0.0	12	3	2	6	1	0	0	0	0	0	0	0	0
P	-1	1	500.0	0.0	1	1	0	0	0	0	0	0	0	0	0	0	0
T	-4	0	2000.0	0.0	4	0	0	3	1	0	0	0	0	0	0	0	0

6.1.9 Size Distributions Experiment

A critical feature of the model is the consideration of overall size distributions across all stores, especially for consolidation when few units could be spread across many stores and size runs are broken. In Table 15 we see the model results for the same article and set of stores both with and without an overall cost of size imbalance (Imbalance Cost One). Note that in scenario 2, with an overall cost of imbalance included in the model, the inventory by size after the transfer demonstrates a clear improvement in the size distributions as compared to scenario 1.

Table 15. Size Imbalance Results

Scenario 1: No Size Imbalance Cost Overall																			
Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior							Inventory Post							
					Total	S 1	S 2	S 3	S 4	S 5	S 6	Total	S 1	S 2	S 3	S 4	S 5	S 6
A	36	67	3.8	7.6	37	6	6	17	7	0	1	73	15	7	27	21	2	1
B	32	59	3.2	6.9	27	4	5	10	4	3	1	59	16	7	28	4	3	1
C	23	52	3.8	6.9	28	8	4	12	0	2	2	51	8	4	35	0	2	2
D	-40	49	12.6	7.0	89	14	27	22	24	0	2	49	7	12	14	14	0	2
E	20	45	1.9	4.9	12	3	0	3	3	3	0	32	3	7	9	10	3	0
F	20	32	1.8	6.1	8	0	2	2	2	2	0	28	7	10	3	5	2	0
G	-55	31	19.6	7.2	87	16	10	54	5	2	0	32	4	8	13	5	2	0
H	-36	21	19.4	7.1	57	13	7	16	17	4	0	21	4	6	6	3	2	0

Scenario 2: Size Imbalance Cost Overall																			
Store	Transfer Quantity	Forecast Demand	Prior Coverage	Post Coverage	Inventory Prior							Inventory Post							
					Total	S 1	S 2	S 3	S 4	S 5	S 6	Total	S 1	S 2	S 3	S 4	S 5	S 6
A	31	67	3.8	7.1	37	6	6	17	7	0	1	68	19	10	29	7	1	1
B	32	59	3.2	6.9	27	4	5	10	4	3	1	59	11	12	17	15	3	1
C	23	52	3.8	6.9	28	8	4	12	0	2	2	51	11	8	23	5	3	2
D	-40	49	12.6	7.0	89	14	27	22	24	0	2	49	14	15	8	12	0	0
I	20	41	2.1	5.5	12	4	0	6	2	0	0	32	4	8	10	9	0	1
J	20	41	1.4	4.8	8	2	3	0	0	3	0	28	2	8	10	5	3	1
G	-54	31	19.6	7.4	87	16	10	54	5	2	0	33	0	2	31	0	0	0
H	-32	21	19.4	8.5	57	13	7	16	17	4	0	25	6	0	9	6	4	0

6.1.10 Important Sizes Experiment

For consolidation, the model not only considers the overall size distributions but also the important sizes (or sizes which comprise the majority of the demand for an article in each store). Table 16 illustrates the behavior of the model results according to the important sizes, which are highlighted for each store. Stores A and B both need additional inventory to satisfy demand. Given that there is only enough inventory to send to one of these two stores, Store A receives inventory because it only has two units in its important sizes, and Store B has 6 units. Store I has enough demand but transfers its inventory to a better store because it contains no inventory in the main size.

Table 16. Important Size Results

Store	Transfer Quantity	Forecast Demand	Inventory Prior						Inventory Post					
			Total	S 1	S 2	S 3	S 4	S 5	Total	S 1	S 2	S 3	S 4	S 5
A	10	51	5	0	0	3	***2***	***0***	15	3	0	3	***9***	***0***
B	0	48	6	0	0	***4***	***2***	0	6	0	0	***4***	***2***	0
C	11	47	17	1	***1***	***6***	6	3	28	1	***5***	***13***	6	3
D	0	47	9	0	0	***5***	***3***	0	9	0	0	***5***	***3***	0
E	0	45	12	0	2	***8***	***2***	0	12	0	2	***8***	***2***	0
F	0	28	12	2	0	***5***	***3***	2	12	2	0	***5***	***3***	2
G	0	23	12	0	0	***11***	1	0	12	0	0	***11***	1	0
H	-11	23	34	3	***13***	***14***	4	0	23	3	***9***	***7***	4	0
I	-10	14	10	3	0	***0***	7	0	0	0	0	***0***	0	0

6.1.11 Number of Destinations Limitation Experiment

Optimal redistribution according to quantities and size distributions increases the number of routes used by the model. The model contains a limitation on the number of destinations so that the model will optimize size distributions within a reasonable number of transfers. Table 17 illustrates three model result scenarios. In Scenario A, the model has multiple destinations for one origin for small amounts of inventory. In Scenario B, the number of destinations limitation has been imposed and the result is only one destination for each origin store. Scenario C is a new article, where inventory levels are higher. For store B, the minimum criteria is met and the store is allowed to split its inventory between two destination stores.

Table 17. Number of Destinations Limitation Results

Scenario A: No constraint							
Origin	Destination	Transer Quantity					
		Total	S 1	S 2	S 3	S 4	S 5
A	B	8	1	0	4	3	0
C	D	1	1	0	0	0	0
E	G	5	0	2	0	3	0
F	D	4	1	1	1	1	0
H	I	1	0	1	0	0	0
K	D	5	1	3	0	1	0
L	B	1	0	0	1	0	0
M	D	3	1	2	0	0	0
	N	4	0	1	3	0	0
O	I	1	0	1	0	0	0
	G	4	0	4	0	0	0
P	J	18	3	8	7	0	0
Q	D	3	1	0	0	2	0

Scenario B: Imposing Number of Destinations Limitation							
Origin	Destination	Transer Quantity					
		Total	S 1	S 2	S 3	S 4	S 5
A	B	8	1	0	4	3	0
C	D	2	2	0	0	0	0
E	G	4	0	2	0	2	0
F	J	4	1	1	1	1	0
H	J	1	0	1	0	0	0
K	J	5	1	3	0	1	0
L	B	1	0	0	1	0	0
M	I	7	1	3	3	0	0
O	G	5	0	1	4	0	0
P	D	18	3	8	7	0	0
Q	J	3	1	0	0	2	0

Scenario C: Number of Destinations Limitation Satisfied							
Origin	Destination	Transer Quantity					
		Total	S 1	S 2	S 3	S 4	S 5
Q	N	2	0	0	0	2	0
B	N	10	2	3	4	0	1
	D	25	3	8	9	5	1
T	I	4	0	0	3	1	0
C	A	1	0	0	1	0	0
M	O	12	3	2	6	1	0
P	S	1	1	0	0	0	0

6.1.12 Key Insights

Below we provide a summary of the key insights gained through the experiments described in the previous sections.

- The model includes two selling periods: next week and all future periods, which allow the model to account for the possibility of lost sales by choosing to transfer now or wait.
- Shipping costs are a per-article estimation of actual shipping costs. Due to these shipping costs, the model prefers to transfer within smaller regions, where shipping costs are lower and lead times are shorter.
- The model transfers across regions when there is no option to reallocate within a smaller region or when the sales gained are significant compared to the shipping cost.

- The redistribution formulation rebalances inventory between stores with high disparities in coverage.
- An important aspect of the redistribution formulation is the size imbalance cost in the origin store, which ensures that the inventory remaining in the origin store is comprised of a good size distribution.
- The consolidation formulation proposes transfers in certain cases when the redistribution formulation proposes zero transfers. Since the consolidation formulation assumes no replenishments, the model transfers to meet merchandising minimums.
- The consolidation model removes from low demand stores and consolidates small quantities to satisfy merchandising criteria.
- An important aspect of the consolidation formulation is the balance of size distributions across all stores due to the overall cost of size imbalance, which encourages the model to consolidate articles in such a way that all stores with inventory have a good size distribution.
- Consolidation considers expected sales as a function of the ratio of important sizes to total inventory.
- Due to the behavior of the model according to the size and display criteria, the model contains a limitation on the number of destinations per origin for a given article.

6.2 Sensitivity Analysis and Calibration

To properly calibrate the model, we must understand how the model reacts as input data and thresholds are varied. This subsection provides an overview of sensitivity analysis and calibration to provide a comprehensive understanding of the model.

6.2.1 Opportunity Cost

In this single article model, the opportunity cost per store is highly sensitive around the price of the article:

- Opportunity Cost > Price: limits transfers of small quantities.
- Opportunity Cost < Price: does not limit.

The cost is therefore calibrated as a function of the price of the article, so that all articles behave the same way in the model. This calibration is performed by running multiple experiments for both types of transfers similar to the experiment previously described in Section 6.1.2. Since redistribution and consolidation are unique scenarios, the function is slightly different for each, where redistribution limits the transfer of small quantities and consolidation does not.

- Redistribution Opportunity Cost = β_1*Price, where $\beta_1 >$ Price , which limits transfers of small quantities.
- Consolidation Opportunity Cost = β_2*Price, where $\beta_2 <$ Price, which does not limit transfers of small quantities.

Another approach for calibrating the opportunity cost can be to fix the cost at a set value, C, for all articles, understanding that the behavior of the model is sensitive around this cost. All articles with a price less than cost, C will behave similarly with respect to the opportunity cost.

6.2.2 Thresholds

In the redistribution formulation, two key thresholds are the coverage threshold and the coverage comparison (Θ_4 and Θ_5), as shown in the experiment in Section 6.1.5. The model output is highly sensitive around these thresholds as they are the triggers for when stores can redistribute inventory. These thresholds require calibration based on business rules for acceptable coverage levels and by examining multiple outputs.

In consolidation, the key threshold is the minimum display quantity to realize sales, Θ_6. This threshold is based partially on the visual merchandising business rules and partially on calibration of outputs. Since this threshold acts as a trigger for the model to know when it is time to consolidate an article, it is a critical threshold.

6.2.3 Demand Sensitivity

As the demand is varied around the mean forecast, the profit and transfer cost also vary. The profit reaches a maximum point when demand is greater than inventory and no additional sales can be made. The transfer cost increases as demand decreases as more expensive routes can be required to sell articles. The opportunity cost decreases as demand increases because the number of transfers required decreases.

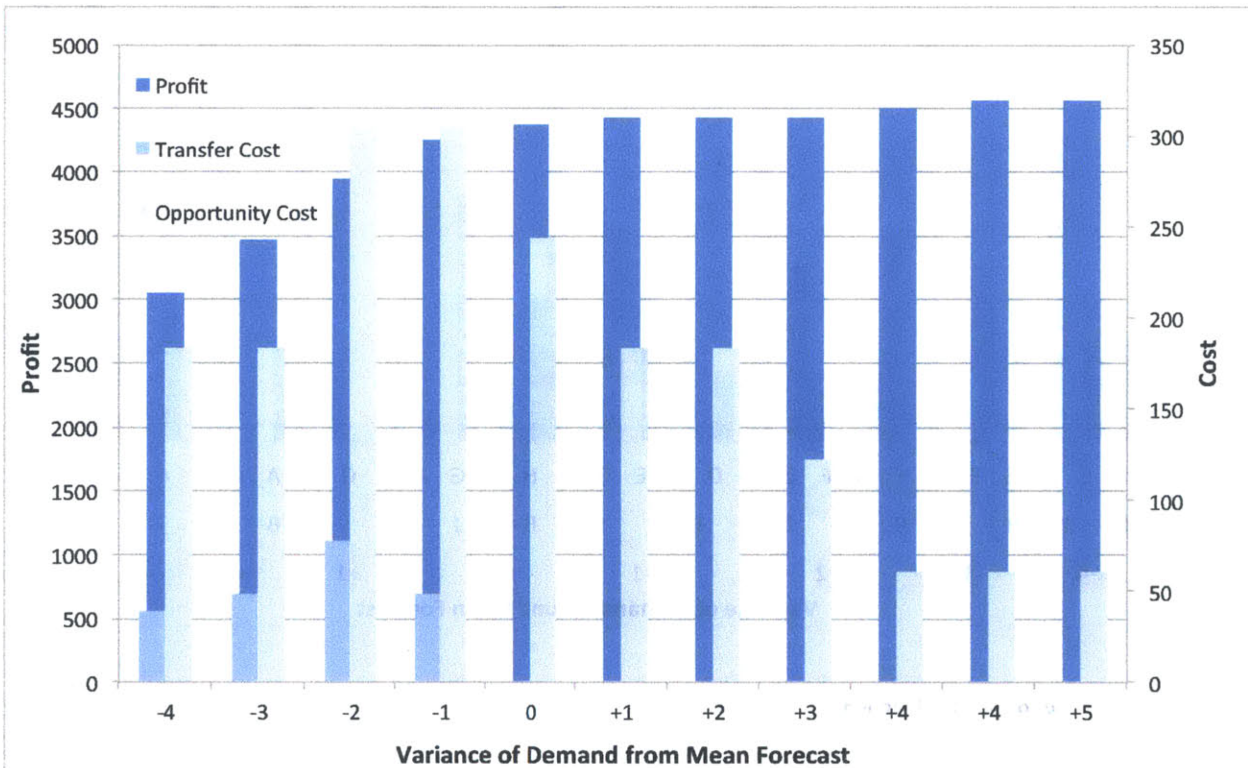

Figure 10. Demand Sensitivity

Additionally, the transfer proposal both in quantity and origin to destination is highly sensitive around the demand forecast. Figure 11 illustrates the change in the proposal of transfer quantities along routes as the forecast is varied around the mean. The origin and destination stores are designated as letters below the x-axis. The x-axis is the variation of the actual demand around the mean forecast, and the y-axis is the transfer quantity. This sensitivity analysis gives an understanding of the importance of an accurate demand forecast updated based on realized sales. Additionally, the analysis provides the basis for further modeling described in Section 8.2, to include demand uncertainty in the optimization model.

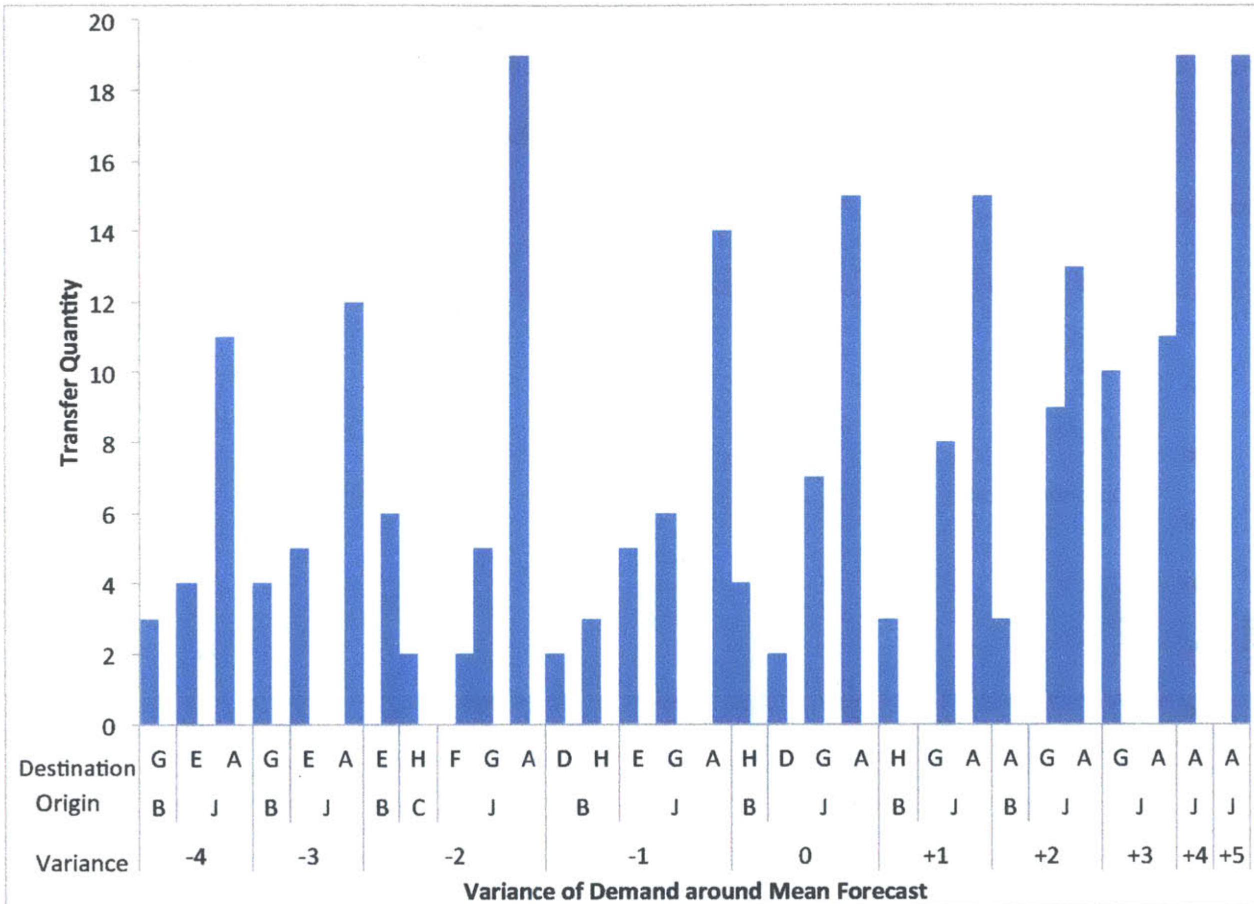

Figure 11. Transfer Proposals as a Function of Varied Demand

Given the high level of sensitivity of the model output relative to demand variance, it is imperative that we further the study of transfer optimization according to demand uncertainty. Section 8.2 proposes a robust formulation of the model presented in Chapter 5 which incorporates uncertainty of the demand forecast.

6.3 Testing and Performance Analysis

6.3.1 Simulation Analysis

Since demand is uncertain, it is important to analyze the results of given the possible distribution of actual demand around the forecasted mean. For this analysis, all seven Zara stores in the New York region are considered. A sample of ten articles is taken among various types of articles (pants, shirts, skirts, etc. with varying amounts of trendiness) to reflect the overall distribution of articles in a given store.

Using the inventory, demand, costs and available shipping mechanisms, the optimization model is applied to each article to propose transfers across all stores. Then the profit is calculated for two scenarios: (1) without transfers (2) with transfers

The profit is calculated for the first case assuming no transfers were performed (inventory is in its existing situation). A demand forecast updated with the prior week's realized sales information for each articles is used as the assumed demand for the next week. The baseline profit is calculated as the minimum between demand or inventory in each store, multiplied by the price for each article in the sample plus a marked down price times any excess inventory. So if the demand is greater than or equal to the existing inventory in a store, all articles are assumed to sell at full price. If the demand is less than the inventory, all articles up to the demand are sold at full price and 50% of the remaining inventory is assumed to sell at a 50% discount. This is an approximation of the estimated average outcome of a complex markdown period.

The profit is calculated for the second case given that the proposed transfers were performed using the same markdown sales assumption as the case without transfers and subtracting a cost per article transferred and per transfer.

The results below reflect the two scenarios using Monte-Carlo Simulation with 1,000 simulations of possible actual demand distributed around the forecasted mean demand. Table 18 shows the metrics for the performance of the model. Performing transfers according to the model increases profit by an estimated 21% when compared to the case without transfers. This reflects a profit increase of 7,600 dollars for the ten articles considered. If we consider the articles transferred as a reasonable subset of all articles in Zara stores, this profit increase represents greater than 18 million dollars of additional Zara profits.

Table 18. Simulation Testing Results

Mean Profit Increase	$ 7,600.00
% Profit Increase	21%
Benefit-Cost Ratio	5.25
Maximum Cost to any store	$ 424.00
Total Cost incurred on system	$ 1,448.00

Figure 12 reflects the cumulative distribution function of the simulation results for each of the two scenarios. Figure 13 displays histograms of the simulation results for the two scenarios.

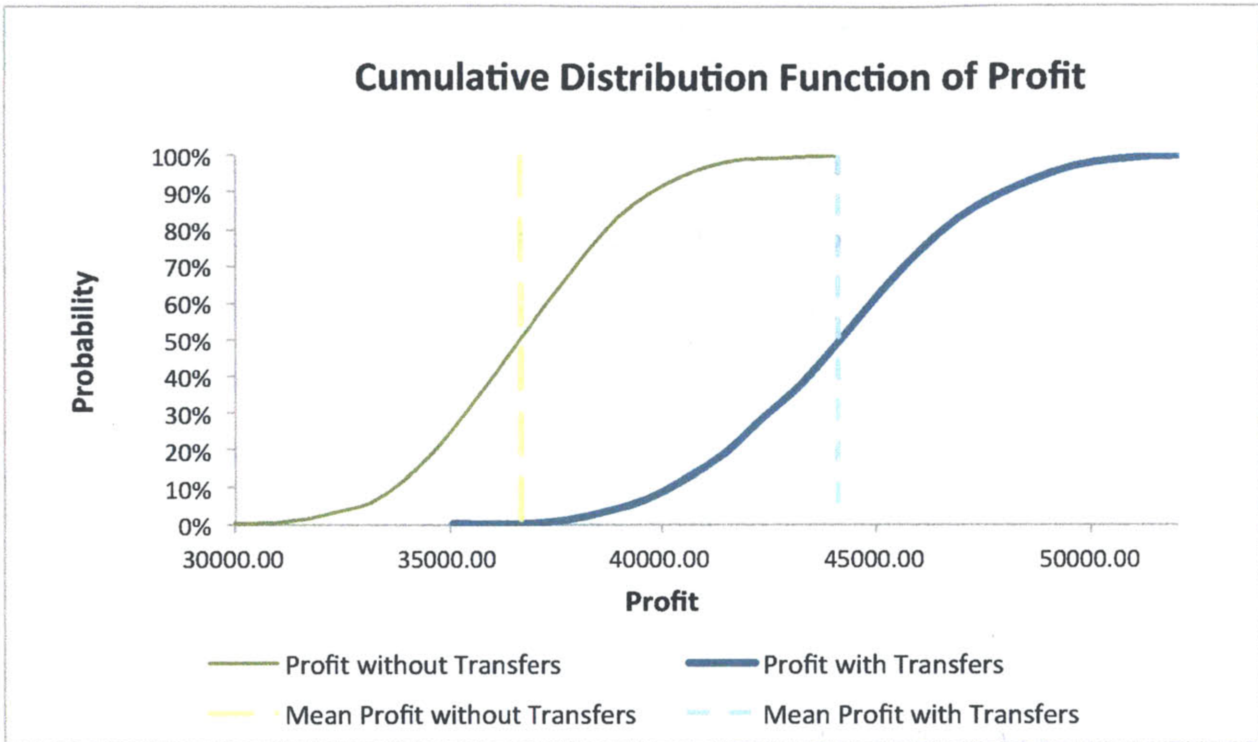

Figure 12. Simulation Cumulative Distribution Function

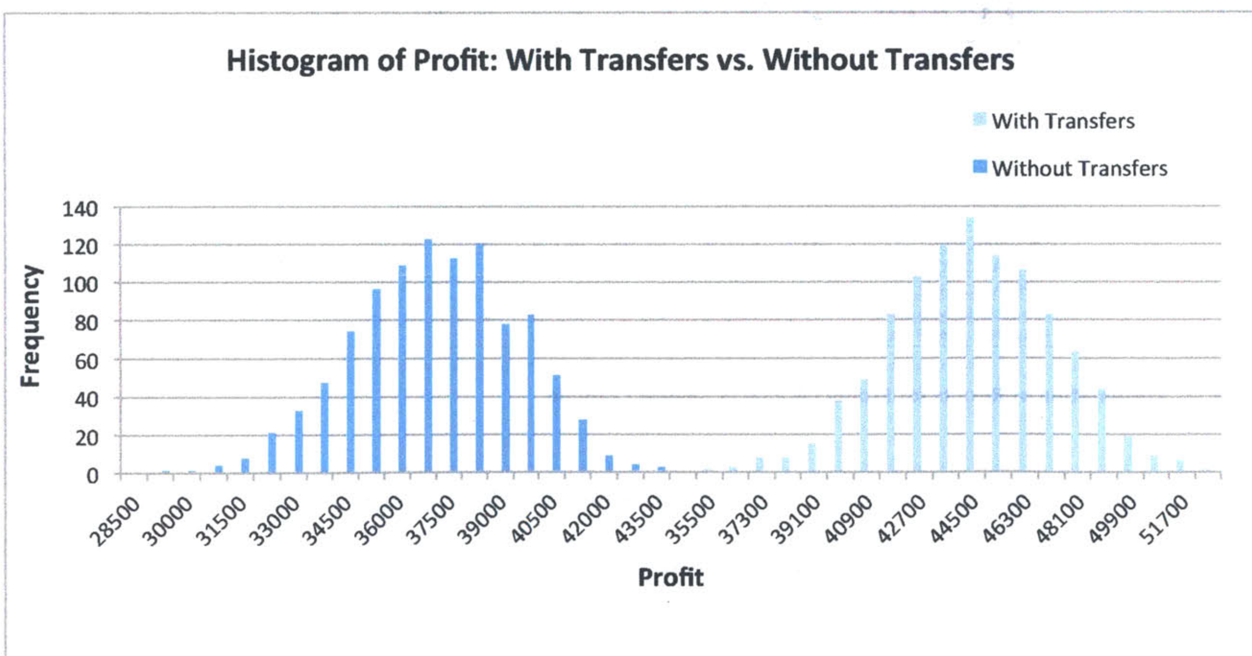

Figure 13. Histogram of Simulation Results

6.3.2 Product Manager Reviews and Pilot Testing

Product Managers are the key stakeholders for transfer proposals and have knowledge experience and expertise in making quality transfer decisions. It is imperative that we review and validate the model results with them. The models for both consolidation and redistribution were applied to multiple articles in a variety of regions and countries including Portugal, France and USA. The Product Managers responsible for the respective regions then reviewed the results and provided feedback that helped to shape the formulation and calibration of thresholds. Figure 14 is an example of results formatted for review with Product Managers containing key metrics and a view of the article proposed.

Tiendas Ajustadas	Demanda	Stock Tienda (uds)							Mov.	Rotation	
		Total	34	36	38	40	42	44		Prior	Post
326PAR-PASSY PL	**68**	**73**	**15**	**7**	**27**	**21**	**2**	**1**	**36**	**4**	**8**
326PAR-PASSY PL	54	37	6	6	17	7	0	1	-------	-------	-------
355PAR-ELYSEES	21	36	9	1	10	14	2	0	-36	19	7
3239PAR-HAUSSMANN	**59**	**59**	**16**	**7**	**28**	**4**	**3**	**1**	**32**	**3**	**7**
3239PAR-HAUSSMANN	59	27	4	5	10	4	3	1	-------	-------	-------
3414PAR-H.DE VILLE	31	32	12	2	18	0	0	0	-32	20	7
3022PAR-CC FORUM	**52**	**51**	**8**	**4**	**35**	**0**	**2**	**2**	**23**	**4**	**7**
3022PAR-CC FORUM	52	28	8	4	12	0	2	2	-------	-------	-------
3414PAR-H.DE VILLE	31	23	0	0	23	0	0	0	-23	20	7
3064PAR-DESGOSSF	**45**	**17**	**1**	**3**	**7**	**6**	**0**	**0**	**20**	**2**	**5**
3064PAR-DESGOSSF	45	32	3	7	9	10	3	0	-------	-------	-------
3128PAR-LAFAYETTE	49	20	0	7	6	7	0	0	-20	13	7
315PAR-CC ARCADES	**32**	**28**	**7**	**10**	**3**	**5**	**2**	**0**	**20**	**2**	**6**
315PAR-CC ARCADES	32	8	0	2	2	2	2	0	-------	-------	-------
3128PAR-LAFAYETTE	49	20	7	8	1	3	0	0	-20	13	7

Tiendas Sin Ajustar y Parcials	Demanda	Stock Tienda (uds)							Mov.
		Total	34	36	38	40	42	44	
3128PAR-LAFAYETTE	49	49	7	12	14	14	0	2	-40
3414PAR-H.DE VILLE	31	32	4	8	13	5	2	0	-55
355PAR-ELYSEES	21	21	4	6	6	3	2	0	-36
3006PAR-DEFENSE	46	26	8	7	4	3	1	3	0
304PAR-OPERA	45	42	6	7	15	10	4	0	0
3003PAR-CCROSNY-DOS	44	25	3	8	4	4	3	3	0
328PAR-CC BELLE EP	41	8	2	3	0	0	3	0	0
9232AUB-LE MILLENAI	41	12	4	0	6	2	0	0	0
3242ORLE-PLACE DARC	40	12	2	6	0	4	0	0	0
....	....	....	......	......	.....	.....	.....	.....	......

Figure 14. Presentation of Results for Product Manager Review Example

In addition to validating with real data, we reviewed results based on generated scenarios to understand specific heuristics and behaviors given different situations.

Chapter 7 Model Implementation

Solving the challenge of inventory transfers requires both an analytical approach as described in Chapter 5 and also a focus on end user needs to ensure smooth implementation and adoption into the organization. This Chapter describes aspects of the implementation planning including complexity simplifications, the process and tool and the current phase of the project.

7.1 Complexity, Runtime and Simplifications

For initial shipments and replenishments from the central distribution center, all shipping routes originate at the central distribution center, creating a finite set of routes. For transfers, the network could be more complex: considering all 1720 stores worldwide creates 2956680 possible routes between all stores. In the optimization model, each route has multiple variables both linear and integer, which creates a large and complex model. For forty stores, the runtime with the full model can be upwards of 2 minutes per article. This sub section focuses on simplifications to improve the run time and decrease complexity.

7.1.1 Linear relaxation

In this mixed integer optimization model formulation, certain decision variables are defined as integers such as $X_{ij,t}$ (transfer quantity), $Y_{ij,t}$ (transfer decision) and $S_{i,t}$ (expected sales). These integer variables are the single largest contributor to run time in the model. For example, in a case of forty stores, the model can take up to ten minutes to propose a solution with 99.5 percent of an optimal solution. To decrease run-time, all or some integer variables can be defined as real numbers. This is a linear relaxation of the model. This relaxation does affect the output of the model, for example if the transfer quantity, X, is taken to be linear, the transfer quantities could be decimal values and proposals would then need to be rounded to be meaningful, as only whole units can be transferred. In the redistribution formulation, for this transfer quantity variable, X, the relaxation output with rounding is within .5 percent of profit realized compared the integer result with significantly reduced runtimes of approximately ninety percent. For example, in the same case of forty stores, the runtime with relaxed variables can be as little as one minute, compared to ten minutes using integer variables.

However for consolidation, the quality of output is somewhat reduced due to the behavior of the model according to the size constraints. In most cases, it is more ideal for the model to propose transfers with partial units to multiple stores to satisfy size minimums and display criteria in multiple locations and increase locations to sell and ultimately profit. When rounding these proposals, the actual integer solution is not ideal. In this case, only a subset of the variables can be relaxed while still maintaining the quality of the output.

7.1.2 Reduction of Routes: Filter and Global Simplification

Decreasing the number of routes can also significantly decrease the run-time. We have investigated two methods to reduce the number of routes. The first is to define a metric to filter certain routes from the model. For example, in redistribution any routes from origin to destination could be filtered from the model when coverage in the destination is larger than the origin. Remember that coverage is a measure of how many weeks of inventory are on hand according to expected sales, as defined in Equation (2). So, in redistribution we never send inventory to a destination store with higher coverage than the origin store. Additionally, in consolidation, routes could be filtered across all stores where size minimums are met and coverage is below a threshold.

The second method to reduce the number of routes in the model is to use a two-step route simplification based on regional routes, as illustrated in Figure 15. First step: run one model for a country using only one route between each region, where the regions are viewed as the sum of all the stores in that region. Second Step: run another model for all stores, where any routes not utilized in step one are filtered from the model. For example, if in step one, region 2 and 3 and region 1 and 3 saw no cross-regional transfers; in step two, all routes between stores in regions 2 and 3 would be filtered.

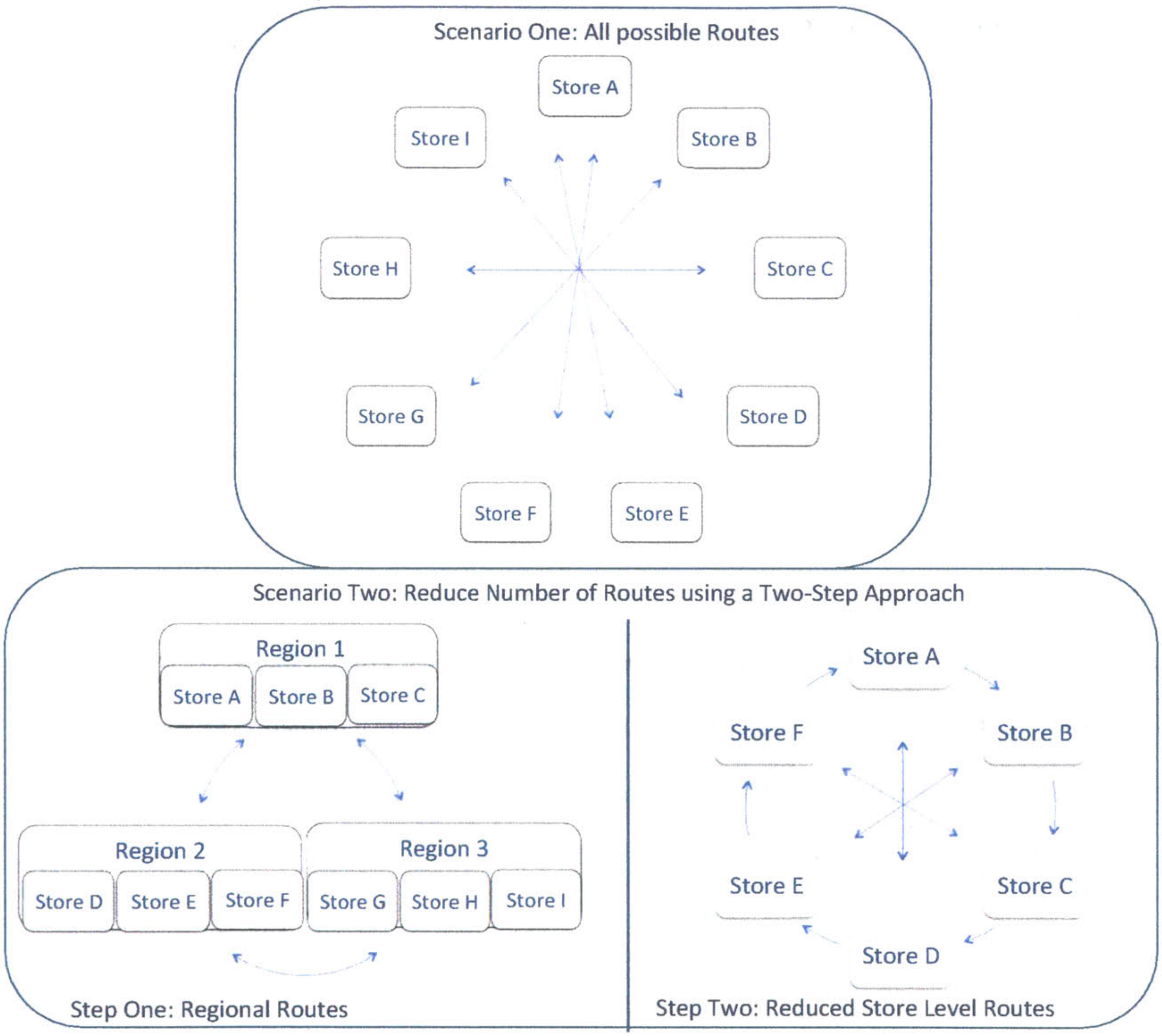

Figure 15. Route Simplification using Regional Routes

7.1.3 Article Filter

To decrease the total runtime for all articles, we can decrease the number of articles to be optimized by prioritizing the articles. A metric can be used to define the most important articles, based on the total imbalance, demand in the region, and number of stores with 'broken' articles. These articles will be run in order of priority based on this metric.

7.2 Process and Tool

The process for utilizing and applying the optimization model for transfers is outlined in Figure 16. The Product Managers will first define the general configuration—what are the possible routes within the region, costs and flexible parameters such as the aggressiveness. Second the Product Managers will choose a region or country. Additionally they can choose a specific collection of articles to analyze. Next the articles will be prioritized according to the metric. The tool will then run the articles in order of priority where routes already used will be preferred by future articles. Product Managers will then be able to view the global results summary for all articles as well as detailed information for each article. In the detail view they can make adjustments and approve the proposals, which will then be sent to the stores to perform the transfer tasks.

Figure 16. Transfers Process

7.2.1 Global View

In a single article model, the optimization does not have a comprehensive view of the total output for all articles. The model makes transfer decisions based on each article independently and does not consider transportation costs in total along one route or total quantities along a route or leaving/entering one store. To address the comprehensive output for all articles we have designed the tool and process of running the single article model such that the global output is optimized.

For each store, each distinct destination store creates additional paperwork and packaging tasks. It is, therefore, important for the model to minimize the total number of destinations for one origin. We have designed the process of running the model so that routes already used are preferred in subsequent single article runs. This preference is added by increasing route costs along unused routes.

Once the model is applied to the articles, the tool provides a global view of the results. Certain scenarios signal a warning to the user for small transfer quantities along one route or large quantities leaving or entering one store. Additionally, certain articles, which are part of a collection will be displayed together so that Product Managers can decide if those articles should be transferred collectively. An example of this global view is illustrated in Figure 17, which is a screenshot of the pilot tool. This view shows a matrix summary of the total units proposed to be transferred across all routes. The left column shows the origin stores. The top row is the destination stores. The numbers are the total units and dollar amount to be transferred.

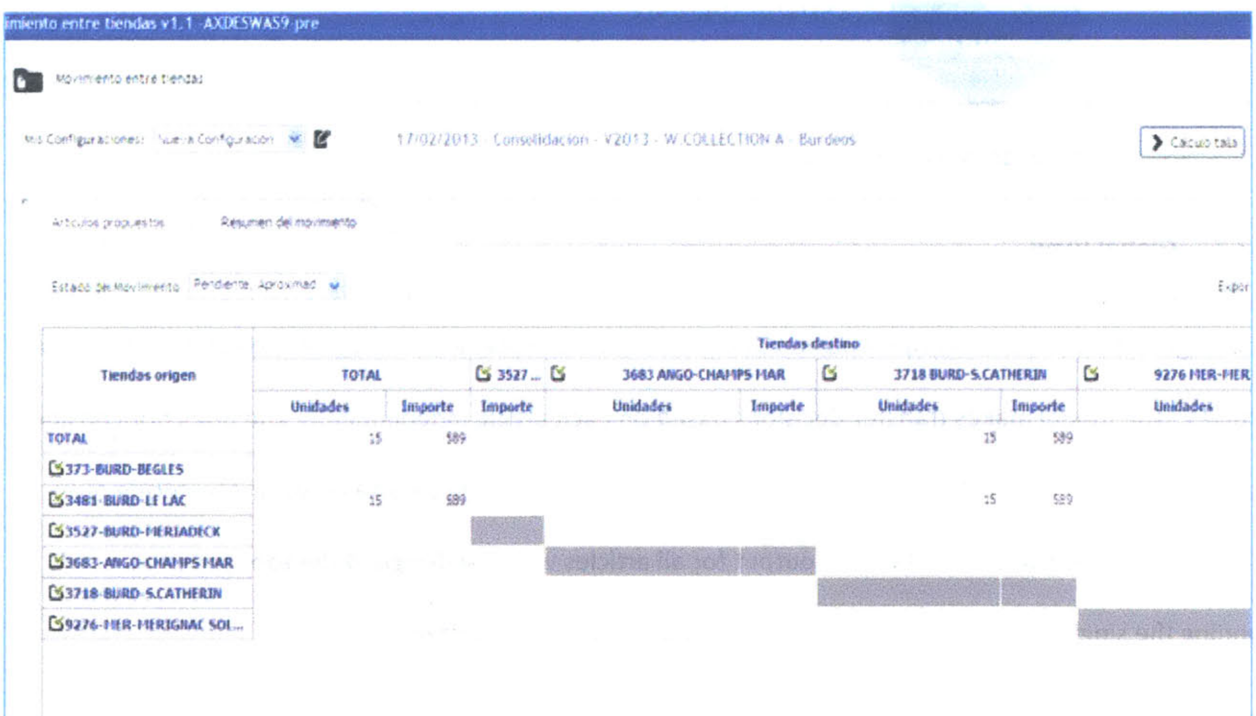

Figure 17. Global View

7.2.2 User Flexibility

As described in Section 2.2, Zara understands the importance of implementing analytical models and utilizing expertise in balance to drive operation excellence. The Product Managers have knowledge and experience that cannot be fully captured in an analytical model. It is, therefore, imperative to retain a reasonable level of flexibility in the process and tool to capture this expertise and experience of the Product Managers. The tool is designed so that certain parameters can be adjusted as inputs to the optimization model. Users can choose the region of stores to optimize as logistics change or when there is a need to transfer between specific stores, such as a store remodel or holiday season. Product Managers can also filter certain routes, which are not reasonable to consider during such situations as a postal strike in an area. Additionally, as the markdown period approaches, Product Managers can choose to increase the aggressiveness of the model by increasing display minimums to consolidate faster and prepare for reduced pricing. An example of the screen to adjust the configuration of the model is shown in Figure 18. This view shows the user a matrix of all the route options. The user can block certain routes as not possible due to logistical constraints. The red checks represent blocked routes and the green checks are possible routes. Additionally, the user can adjust the transfer price across those routes if the cost of shipping changes. This view can be saved and only updated and adjusted when there are changes in the logistics of transfers.

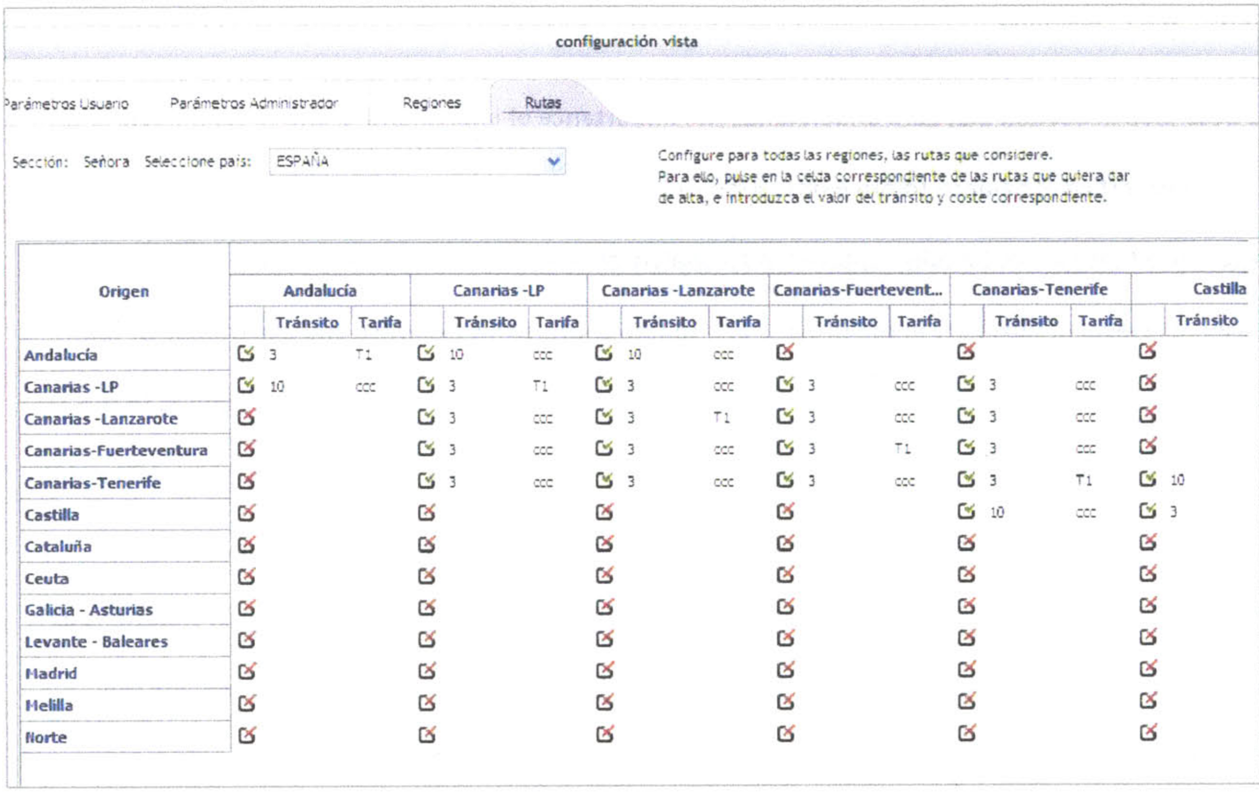

Figure 18. Configuration of Model

Product Managers have the final input and control over transfer orders and can review and adjust proposals as needed. An example of the detail view for review and adjustment is shown in Figure 19. The specific article being transferred is shown in the upper left corner. To the right of that is a summary of the article: how many stores it was sent to, how much stock is available in central distribution, etc. Below that, in the center of the screen is a summary of the transfer proposal, with the origin store in the top row and all destinations below. At the bottom of the screen, any stores in the region with no proposed transfers are listed.

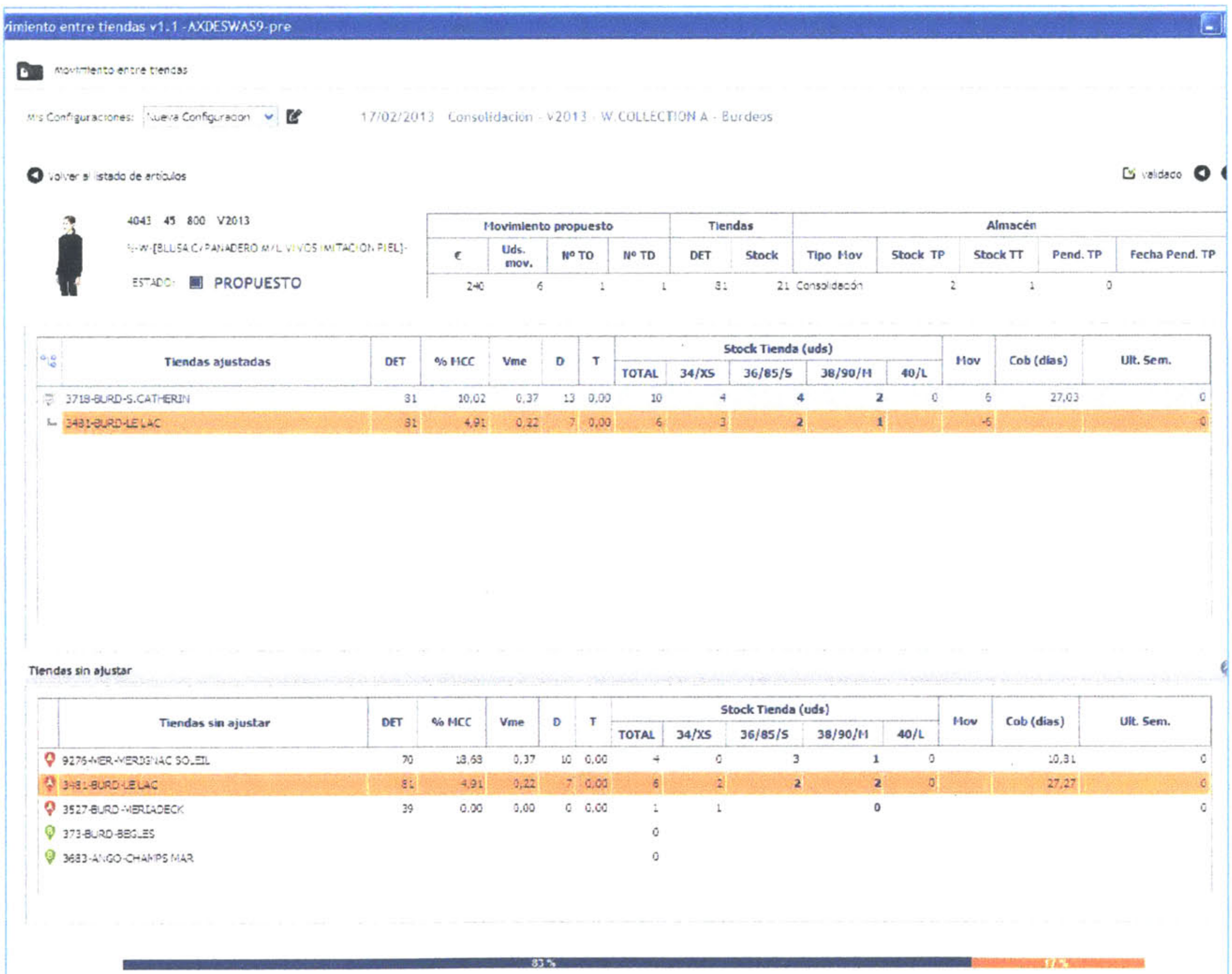

Figure 19. Article Detail View

7.3 Current Phase of Project

The inventory transfer model, process and tool described in this thesis are planned for full implementation during the third quarter of 2013. The project is in the fifth phase of the project plan as described in Section 1.4: implementation planning. Currently, Zara is making final adjustments to the tool design and calibrating parameters. A pilot project is currently being implemented for a subset of regions and articles to test both the output of the model and the usability for the Product Managers.

Chapter 8 Additional Features

This chapter presents two additional features to further address the challenge of inventory transfers and to enhance the model described in Chapter 5. First we discuss a multi-article model and then we propose and investigate a robust optimization formulation, which incorporates uncertainty of the demand data.

8.1 Multi-article model

Given the complexity of the Zara network with thousands of stores and articles around the world, it is not feasible to implement a model to run all articles together in a reasonable amount of time. However, it is sensible to consider the priority of articles as described in the prior section and run only a limited number of articles in a specific region in a multi-article model. The formulation outline in Section 5.5, can be applied to multiple articles by adding an article dimension to the variables. The benefit of this methodology is that the model would optimize true profit given the cost of transferring all of the articles without a need to approximate at a per article level. Currently the process is designed to account for the global view using a metric and subsequent cost in a single-article model, which is described in Section 7.2.1. Additionally, the model could then capture certain interdependencies between articles such as with similar articles or articles which comprise a specific visual collection.

8.2 Demand uncertainty

The model described in Chapter 5 is deterministic and proposes transfers based on the optimal mean expected profit and demand forecast. Nevertheless, demand can be highly varying around the forecasted mean, and as the demand varies so does the output of the optimization model, which is demonstrated in the sensitivity analysis in Section 6.2.3. Therefore, it is important to further study optimal transfers by incorporating demand uncertainty into the optimization model. This section describes a robust optimization formulation, which incorporates demand uncertainty in a stochastic optimization. The methodology is adapted from prior work creating a stochastic optimization model for inventory management by Bertsimas and Thiele as described in Section 3.2 [3].

8.2.1 Robust Formulation

The following is a detailed definition and explanation of the stochastic optimization formulation as evolved from the deterministic model set forth in Chapter 5. First we define the input data and decision variables. Next we provide an explanation and proof of the formulation and its evolution using duality. Lastly we present the final formulation consisting of the objective function and constraints.

Here we define the input data and decision variables.

Input Data

- t: Time period in total selling period, *T*.
- $i, j \in N$: Stores in the total set N.
- $I^{pre}_{i,t=0}$: Initial inventory in store *i* in period zero.
- $I^{post}_{i,t=0}$: Initial inventory in store *i* in size s in period zero.
- $D_{i,t}$: forecasted demand in store *i* for period t.
- $p_{i,t}$: Revenue per unit in store *i* for period *t*.
- $h_{i,t}$: Holding cost for in store i in period t.
- $C_{ij,t}$: Transfer cost per unit from store *i to j* in period *t*.
- $\widetilde{d_{it}}$: Exogenous demand.
- $\bar{d}_{ik}$: Mean demand.

- $\hat{d}_{ik}$: Demand noise.
- $\Gamma_{i,t}$: Maximum noise in store i in period t.

Decision Variables:

- $Y_{ij,t}$: Mismatch cost (overstock and understock).
- $X_{ij,t}$: Number of units transferred from store *i* to store *j* in period t, $X_{ij,t} \geq 0$.
- $I^{pre}_{i,t}$: Inventory in store *i* in period t, prior to transfer decision.
- $I^{post}_{i,t}$: Inventory in store *i* in period t after transfer decision.
- Z_i: Noise in store i in period t.
- $u_{i,t}$: Dual decision variable one, $u_{i,t} \geq 0$.
- $v_{i,t}$: Dual decision variable two, $v_{i,t} \geq 0$.

Next, we discuss the methodology behind the final formulation using duality to reformulate the problem.

The forecasted demand term can be expanded to include mean demand and the possible variation of the mean demand. This exogenous demand, $\widetilde{d_{it}}$, is defined in Equation (36) for store i, in period t. $\overline{d}_{it}$ is the mean demand term and $\widehat{d}_{it} z_{it}$ is the demand noise term, where z is the maximum noise and is less than or equal to one.

$$\widetilde{d_{it}} = \overline{d}_{it} + \widehat{d}_{it} z_{it} \qquad |z_{it}| \leq 1 \tag{36}$$

Considering these two demand terms, the objective to maximize profit can be demonstrated in terms of revenue based on expected demand, transfer cost and overstock and understock costs, as illustrated in Figure 20.

Maximize expected profit = Price x E[demand] – TF cost – Overstock cost – Understock Cost = price x E[$\tilde{d}$] – TF cost – Overstock cost – Understock Cost = price x $\bar{d}$ – TF cost – Overstock cost – Understock Cost

Figure 20. Robust Objective

Since price and mean demand are constant, the objective function can be written to minimize the total cost, shown in Equation (37), subject to constraints shown in Equations (38) -(43), where y is a decision variable representing the mismatch cost over overstock versus understock.

$$Minimize\ Total\ Cost\ = C_{transfer} + C_{overstock} + C_{understock}$$

$$\text{Minimize} \quad \sum_{i}^{n}\sum_{t}^{T} y_{it} + \sum_{i,j\neq i}^{n}\sum_{t}^{T} c_{ijt}x_{ijt} \tag{37}$$

$$C_{overstock} = max\left\{I'_{i,t} - \tilde{d}_{i,t}, 0\right\} \tag{38}$$

$$C_{understock} = max\left\{\tilde{d}_{i,t} - I'_{i,t}, 0\right\} \tag{39}$$

$$y_{it} \geq h_{it}(I_{it}^{post} - \tilde{d}_{it}) \qquad \forall i, \forall t = 1, \ldots, T \tag{40}$$

$$y_{it} \geq -p_{it}(I_{it}^{post} - \tilde{d}_{it}) \qquad \forall i, \forall t = 1, \ldots, T \tag{41}$$

$$I_{it}^{post} = I_{it}^{pre} + \sum_{j\neq i} x_{jit} - \sum_{j\neq i} x_{ijt} \quad \forall i, \forall t = 1, \ldots, T \tag{42}$$

$$I_{it+1}^{pre} = I_{it}^{post} - \tilde{d}_{it} \qquad \forall i, \forall t = 1, \ldots, T-1 \tag{43}$$

Recall that $\widetilde{d_{it}}$, defined in Equation (36) represents a two-term demand function, so that Equations (40) and (41) can be expressed in terms of mean demand and uncertainty shown in Equations (44) through (46). Z is a decision variable representing the noise and gamma is the maximum noise. We want these constraints to hold true for all values of Z, and therefore hold true for the optimal maximum value.

$$y_{it} \geq h_{it}(I_{i1}^{pre} + \sum_{k=1}^{t}\sum_{j\neq i}(x_{jik} - x_{ijk}) - (\sum_{k=1}^{t}\bar{d}_{ik} + \sum_{k=1}^{t}\hat{d}_{ik}z_{ik})) \tag{44}$$

$$\sum_{k=1}^{t}|z_{ik}| \leq \Gamma_{it} \qquad \forall i, \forall t = 1,\ldots,T \tag{45}$$

$$|z_{it}| \leq 1 \qquad \forall i, \forall t = 1,\ldots,T \tag{46}$$

The worst case of demand uncertainty can be formulated from dual constraints, where Equation (47) is the cost of overstock and Equation (48) is the cost of understock.

$$y_{it} \geq h_{it}(I_{i1}^{pre} + \sum_{k=1}^{t}\sum_{j\neq i}(x_{jik} - x_{ijk}) - (\sum_{k=1}^{t}\bar{d}_{ik} + \sum_{k=1}^{t}\hat{d}_{ik}z_{ik})) \tag{47}$$

$$y_{it} \geq -p_{it}(I_{i1}^{pre} + \sum_{k=1}^{t}\sum_{j\neq i}(x_{jik} - x_{ijk}) - (\sum_{k=1}^{t}\bar{d}_{ik} + \sum_{k=1}^{t}\hat{d}_{ik}z_{ik})) \tag{48}$$

From these worst cases of demand uncertainty, we use duality to reformulate the problem, using new decision variables to represent the constraints. Figure 21 expresses this translation.

Primal Form

Maximize $\sum_{k=1}^{t} \hat{d}_{ik} z_{ik}$

s.t. $|\sum_{k=1}^{t} z_{ik}| \leq \Gamma_{it}$

$|z_{it}| \leq 1 \quad \forall k \leq t$

$\Rightarrow$

$\sum_{k=1}^{t} z_{ik} \leq \Gamma_{it} \quad u_{it}$

$0 \leq z_{ik} \leq 1 \quad v_{itk}$

Dual Form

Minimize $\Gamma_{it} u_{it} + \sum_{k=1}^{t} v_{itk}$

s.t. $u_{it} + v_{itk} \geq \hat{d}_{ik}$

$u_{it} \geq 0$

$v_{itk} \geq 0 \quad \forall k \leq t$

Figure 21. Demonstration of Duality Translation for Robust Optimization

This duality reformulation is the basis for the stochastic robust formulation as applied to inventory transfers. Lastly, we present a summary of the final stochastic robust formulation as applied to inventory transfers including the objective function and constraints shown in Equations (37) and (49) – (51).

Objective Function:

$$Minimize\ Total\ Cost\ = C_{transfer} + C_{overstock} + C_{understock}$$

$$\text{Minimize} \qquad \sum_{i}^{n}\sum_{t}^{T} y_{it} + \sum_{i,j\neq i}^{n}\sum_{t}^{T} c_{ijt}x_{ijt} \qquad (37)$$

Subject To:

Cost of Overstock: The decision variable, y, must be greater than or equal to the cost of overstock (the holding cost of excess unsold inventory).

$$y_{it} \geq h_{it}(I_{i1}^{pre} + \sum_{k=1}^{t}\sum_{j\neq i}(x_{jik} - x_{ijk}) - \sum_{k=1}^{t}\bar{d}_{ik} + \Gamma_{it}u_{it} + \sum_{k=1}^{t}v_{itk}) \qquad (49)$$

$\forall\, i, t$

Cost of Understock: The decision variable, y, must be greater than or equal to the cost of understock (the cost of missed sales due to stock-out).

$$y_{it} \geq -p_{it}(I_{i1}^{pre} + \sum_{k=1}^{t}\sum_{j\neq i}(x_{jik} - x_{ijk}) - \sum_{k=1}^{t}\bar{d}_{ik} - \Gamma_{it}u_{it} - \sum_{k=1}^{t}v_{itk}) \qquad (50)$$

$\forall\, i, t$

Demand Worst Case: Consideration of uncertainty of demand using duality to reformulate.

$$u_{it} + v_{itk} \geq \hat{d}_{ik} \qquad (51)$$

$\forall\, i, t \qquad \forall k \leq t$

8.2.2 Robust Model Results

This section provides a discussion of the results of the robust model, which captures the uncertainty of demand. The results are discussed in comparison to the deterministic model results.

8.2.2.1 Robust Experiment: Removing More than Surplus

In Table 19, the robust formulation proposes a transfer out of Store A that is greater than the inventory surplus, which leaves Store A with less inventory than demand. This result is due to the large difference between demand and inventory and noise in destination Store C. The deterministic model does not remove more than the surplus of inventory in store A.

Table 19. Robust Results: Removing More than Surplus

Store	Forecast Demand		Initial Inventory	Robust				Deterministic	
				Demand Noise		Transfer Quantity		Transfer Quantity	
	Period 1	Period 2		Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
A	50	10	80	10	10	-22	0	-20	0
B	100	10	50	20	20	0	0	0	0
C	100	10	20	10	10	22	0	20	0

8.2.2.2 Robust Experiment: Allocating Inventory According to Demand Noise

When proposing transfer quantities, the deterministic model divides inventory according to mean demand. The robust formulation divides both to mean demand and the uncertainty. Table 20 demonstrates the two results. The robust result sends more inventory to the store with higher demand uncertainty.

Table 20. Robust Results: Divides Inventory According to Noise

Store	Forecast Demand		Initial Inventory	Robust				Deterministic	
				Demand Noise		Transfer Quantity		Transfer Quantity	
	Period 1	Period 2		Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
A	50	10	80	10	10	-22	0	0	-20
B	50	10	50	20	20	13	0	0	10
C	50	10	50	10	10	8	0	0	10

8.2.2.3 Robust Experiment: High Uncertainty in Origin

Table 21 illustrates model results when there is high demand uncertainty in the store, which has surplus demand. The robust formulation retains the excess inventory in Store A to account for the uncertainty. The deterministic model transfers the exact amount of surplus.

Table 21. Robust Results: High Noise Prevents Transfer

Store	Forecast Demand		Initial Inventory	Robust				Deterministic	
				Demand Noise		Transfer Quantity		Transfer Quantity	
	Period 1	Period 2		Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
A	50	20	80	50	50	0	0	0	-10
B	50	10	50	20	20	0	0	0	0
C	50	10	50	20	20	0	0	0	10

8.2.2.4 Robust Experiment: High Uncertainty in All Stores

In Table 22, all stores have high demand uncertainty. The robust formulation transfers some, but not all, of the excess inventory from Store A. The Deterministic model transfers all of the excess inventory and splits between stores B and C.

Table 22. Robust Results: High Uncertainty in All Stores

Store	Forecast Demand		Initial Inventory	Robust				Deterministic	
				Demand Noise		Transfer Quantity		Transfer Quantity	
	Period 1	Period 2		Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
A	10	20	90	50	50	-38	0	-60	0
B	50	10	10	50	50	0	0	20	0
C	50	10	10	50	50	38	0	40	0

8.2.2.5 Robust Experiment: Low Uncertainty in Origin Store

When the store with excess inventory has low uncertainty compared to the other stores, the robust formulation will transfer more than the surplus inventory, as shown in Table 23. The deterministic model transfers exactly according to demand same as the previous scenario.

Table 23. Robust Results: Low Noise in Origin Store

Store	Forecast Demand		Initial Inventory	Robust				Deterministic	
				Demand Noise		Transfer Quantity		Transfer Quantity	
	Period 1	Period 2		Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
A	10	20	90	5	5	-76	0	-60	0
B	50	10	10	50	50	0	0	20	0
C	50	10	10	50	50	76	0	40	0

8.2.2.6 Robust Experiment: Varied Uncertainty in Destination Stores

The scenario shown in Table 24 is similar to the previous, but the destination stores now have different levels of demand uncertainty. In this case, the robust formulation splits the inventory between the two destination stores B and C.

Table 24. Robust Results: Destination Stores with Varied Uncertainty

Store	Forecast Demand		Initial Inventory	Robust				Deterministic	
				Demand Noise		Transfer Quantity		Transfer Quantity	
	Period 1	Period 2		Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
A	10	20	90	5	5	-76	0	-60	0
B	50	10	10	60	60	27	0	20	0
C	50	10	10	10	10	48	0	40	0

8.2.2.7 Robust Experiment: Transfer Greater than Surplus

Table 25 illustrates the results when all stores have at least enough inventory to satisfy the mean demand forecast. The robust formulation still transfers to the store with the highest uncertainty to retain surplus where there is a high level of uncertainty. The deterministic model performs no transfers.

Table 25. Robust Results: Transfer Quantity Greater than Surplus

Store	Forecast Demand		Initial Inventory	Robust				Deterministic	
				Demand Noise		Transfer Quantity		Transfer Quantity	
	Period 1	Period 2		Period 1	Period 2	Period 1	Period 2	Period 1	Period 2
A	50	20	100	5	5	-39	0	0	0
B	30	10	40	60	60	41	0	0	0
C	30	10	40	10	10	-2	0	0	0

8.2.2.8 Key Insights of Robust Formulation

In what follows, we provide a summary of the key insights gained through the experiments and results of the stochastic formulation described in the previous sections.

- The robust formulation prefers to have more inventory than demand in stores with high demand and high uncertainty of demand. This means transferring more than the surplus out of a store, and/or transferring into a store more units than needed if considering solely the demand forecast. The deterministic model only transfers very small amounts greater than the demand due to size and merchandising constraints.
- When presented with multiple options for destination stores, the robust formulation will allocate the inventory according to the demand forecast and the uncertainty of the forecast. In low uncertainty stores, the model prefers to allocate up to the forecast. In high uncertainty stores, the model prefers to allocate more than the forecast. The deterministic model will allocate the inventory with equal preferences to destination stores, according to demand.
- In an origin store with surplus inventory and high uncertainty of demand, the robust formulation will seek to keep all or some of the excess inventory due to the possibility of higher sales. The deterministic model will transfer out of the origin exactly the amount of surplus if there is a need for inventory in another store.

Chapter 9 Conclusions

This chapter concludes the thesis by summarizing the contributions. Additionally, it provides a discussion of the applicability of this thesis to situations beyond inventory transfers in fast fashion along with concluding remarks.

9.1 Contributions

Zara has a large network of 1,720 stores in over 80 countries worldwide. This creates a complex problem for optimization modeling. A portion of this research has been focused on simplifying the model to decrease runtimes while maintaining the integrity of the transfer proposal. Additionally, in an environment where thousands of new articles are introduced each year, it is imperative to focus the inventory transfers on the most important articles by prioritizing articles using a metric based on coverage imbalances.

Zara emphasizes the importance of analytical models and data to drive operations, which must be delicately balanced with the art of retail management through capturing expertise and human intuition. During this project, we maintained a focus on stakeholder needs to ensure their expertise was captured in the model through flexible controls and adjustable proposals.

Zara is currently implementing the solution model set forth in this thesis to address the problem of inventory transfers. We expect full-scale implementation in 2013. The key contributions to Zara's business are highlighted in the following list.

- Increase profit by 21 percent of articles transferred, which represents greater than 18 million dollars of additional profit per year.
- Increased ability to respond to demand fluctuations through rapid response inventory transfer decisions.

- Documentation of heuristics, business rules and key decision-making variables for inventory transfers.
- Determination of forecasting methodology for transfers based on previous MIT research and current distribution forecast.
- Deterministic, mixed-integer, multi-period optimization model.
- Robust, stochastic optimization model, advancing the deterministic approach by accounting for the uncertainty of the demand forecast.
- Process for transfers which is based on the optimization model proposal and is no longer labor intensive and manual, alleviates Product Managers to focus on other key tasks.
- Implementation planning: considerations of end-user flexibility need, tool requirements and incorporation of commercial perspectives.

9.2 Applicability to other Situations

The challenge of inventory transfers at Zara is common across many fast-fashion applications as well as in the more traditional apparel industry. Most fashion retailers currently use simple metrics to manually manage transfer decisions, oftentimes only once or less per season. The methodology set forth in this thesis could increase revenue and improve the ability to respond to changing demand.

This challenge of responding to uncertain and evolving demand is not unique solely to the fashion industry. Many retailers such as specialty stores, department stores or warehouse-based online retailers could benefit from the methodology proposed in this thesis. For example, supermarkets could reduce the inventory loss due to expirations by implementing an analytical approach to inventory transfers.

Additionally, many LGO partner companies could apply this methodology for inventory transfers. National Grid has high inventory costs and must ensure the right equipment is in the right locations to respond quickly and efficiently to maintenance and outage issues. Boeing maintains high levels of standard parts across many fabrication, assembly and supplier locations and could benefit from an analytical approach to supply chain and inventory management of standard parts transfers.

9.3 Conclusions

In summary, this thesis proposes a solution model focused on stakeholder and business needs to address the challenge of inventory transfers in the fast-fashion context at Zara. To our knowledge, it is the first of its kind applied in the fast-fashion industry. We present a demand forecast methodology and multi-period, mixed-integer deterministic optimization model along with a robust, stochastic model advancement incorporating the uncertainty of the demand forecast. We expect the model to increase profit of articles transferred by an average of 21 percent, representing greater than 18 million dollars of additional profit. Additionally we provide a process and model, which allows flexibility to capture Product Manager expertise and human intuition. This thesis presents not only a model successfully applied in this specific fast-fashion context but also a methodology, which can be applied across companies and industries.

Glossary of Terms

Buyers: Personnel responsible for buying decisions, located in central headquarters

Distribution Department: Personnel responsible for all central distribution operations and shipment decisions to stores, located in central headquarters

Consolidation: A type of inventory transfers occurring in isolation from replenishments from central distribution

Coverage: Amount of inventory on hand, calculated as inventory divided by weekly demand forecast

Initial shipment: First shipment of a new article to stores from central distribution centers

Inventory Transfers: Transfers of inventory directly between stores, not involving central distribution centers

Product Collections: Groups of articles such as Woman, Basic, Weekend-wear and Knitwear

Product Managers: Personnel responsible for groups of Zara stores; act as a liaison between store management and central management, located in central headquarters

Redistribution: A type of inventory transfers between stores occurring in parallel with replenishments from central distribution

Replenishment: Repeated shipments of an existing article to stores from central distribution centers

Sub-families: Categories of types of articles such as folded pants or dresses

Success: Number of articles sold divided by number of articles sent

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