Network Protection with Service Guarantees

by

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Abstract

With the increasing importance of communication networks comes an increasing need to protect against network failures. Traditional network protection has been an "all-or-nothing" approach: after any failure, all network traffic is restored. Due to the cost of providing this full protection, many network operators opt to not provide protection whatsoever. This is especially true in wireless networks, where reserving scarce resources for protection is often too costly. Furthermore, network protection often does not come with guarantees on recovery time, which becomes increasingly important with the widespread use of real-time applications that cannot tolerate long disruptions. This thesis investigates providing protection for mesh networks under a variety of service guarantees, offering significant resource savings over traditional protection schemes.

First, we develop a network protection scheme that guarantees a quantifiable minimum grade of service upon a failure within the network. Our scheme guarantees that a fraction q of each demand remains after any single-link failure, at a fraction of the resources required for full protection. We develop both a linear program and algorithms to find the minimum-cost capacity allocation to meet both demand and protection requirements.

Subsequently, we develop a novel network protection scheme that provides guarantees on both the fraction of time a flow has full connectivity, as well as a quantifiable minimum grade of service during downtimes. In particular, a flow can be below the full demand for at most a maximum fraction of time; then, it must still support at least a fraction q of the full demand. This is in contrast to current protection schemes that offer either availability-guarantees with no bandwidth guarantees during the downtime, or full protection schemes that offer 100% availability after a single link failure. We show that the multiple availability guaranteed problem is NP-Hard, and develop solutions using both a mixed integer linear program and heuristic algorithms.

Next, we consider the problem of providing resource-efficient network protection that guarantees the maximum amount of time that flow can be interrupted after a failure. This is in contrast to schemes that offer no recovery time guarantees, such as IP rerouting, or the prevalent local recovery scheme of Fast ReRoute, which often over-

provisions resources to meet recovery time constraints. To meet these recovery time guarantees, we provide a novel and flexible solution by partitioning the network into failure-independent "recovery domains", where within each domain, the maximum amount of time to recover from a failure is guaranteed.

Finally, we study the problem of providing protection against failures in wireless networks subject to interference constraints. Typically, protection in wired networks is provided through the provisioning of backup paths. This approach has not been previously considered in the wireless setting due to the prohibitive cost of backup capacity. However, we show that in the presence of interference, protection can often be provided with no loss in throughput. This is due to the fact that after a failure, links that previously interfered with the failed link can be activated, thus leading to a "recapturing" of some of the lost capacity. We provide both an ILP formulation for the optimal solution, as well as algorithms that perform close to optimal.

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Contents

List of Figures

List of Tables

Chapter 1

Introduction

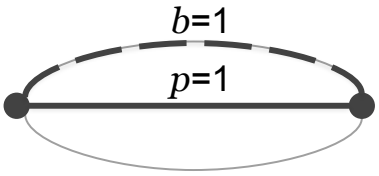
Communications across data networks has become vital in global operations. As data rates continue to rise, the failure of a network line element or worse, a fiber cut, can result in severe service disruptions and large data loss, potentially causing millions of dollars in lost revenue [2]. With this increased strain on network resources, there comes an increased need to provide cost and resource efficient protection [3], which will include a variety of service guarantees that satisfy the multiple needs and demands for protection that various network services may require. In this thesis, we investigate providing network protection with various service guarantees, with a focus on resource efficiency.

Traditional protection schemes focus on recovering all lost traffic after any network failure [3, 4]. This is typically accomplished by providing a primary route for data traffic before a failure, and then providing a protection route that is *failure disjoint*[1] from the primary route [5]. Due to the cost of providing full protection, many service providers offer no protection whatsoever. This is especially true in wireless networks, where the scarcity of shared frequency space often makes the cost of traditional protection schemes prohibitive. Furthermore, full recovery schemes often do not consider the amount of time needed to recover from a network failure. With the proliferation of real-time services such as video and voice [6], as well as the migration

[1]No links and/or nodes of the primary and backup routes overlap, such that after a failure in the primary route, the backup routes would still be active.

towards services being located in the "cloud" [7], time-sensitive restoration becomes paramount.

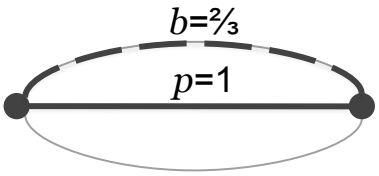
Consider the following motivating example to demonstrate how simply applying traditional full protection schemes does not necessarily optimally utilize resources. In Figure 1-1, a single unit of traffic is routed from the source to the destination, and a disjoint backup path is routed to protect against a failure in the primary path.



Primary	Backup	Total
1	1	2

Figure 1-1: Full protection

Suppose that a network service does not need full protection during a failure; only a fraction, say $\frac{2}{3}$, of the service must be maintained. This is not unreasonable considering that network failures are relatively uncommon, and are on average repaired quickly [4, 8, 9]. Since full protection restores all traffic during a failure, it is not a resource efficient method to protect against a failure when only $\frac{2}{3}$ of the traffic must be maintained during an outage. A simple modification to the full protection scheme is shown in Figure 1-2, where the backup path now has $\frac{2}{3}$ capacity allocated to it.

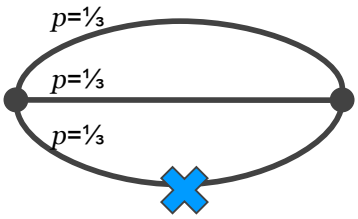


Primary	Backup	Total
1	⅔	1⅔

Figure 1-2: Modified version of full protection to support $\frac{2}{3}$ flow after a failure

While modifying the backup path does reduce the total amount of resources utilized from 2 units of allocated capacity in the full protection scheme to $1\frac{2}{3}$ in the modified version, it does not capture the redundancy and inherent self-protection that the network structure allows. By spreading resources across multiple paths, risk is distributed, and the amount of traffic that is disrupted after a failure is reduced. Figure 1-3 shows such a routing. By allocating $\frac{1}{3}$ units of capacity to each link, no additional backup capacity is needed; after any failure, a flow of $\frac{2}{3}$ is maintained.

Using this routing to meet the protection requirements, which takes advantage of risk distribution over the network, the total resource utilization is only 1 unit of capacity, which is no greater than what was needed to meet the demand without protection.



	Primary	Backup	Total
Before	1	0	1
After	⅔	0	⅔

Figure 1-3: Risk distribution that further reduces resources needed to maintain a flow of 1 before a failure, and $\frac{2}{3}$ after a failure

We note that the previous example is just one of many that further motivate our work in both the wired and wireless setting, and shows the need to examine the various service guarantees that a network may wish to utilize. The service guaranteed protection schemes we investigate in this thesis are as follows.

1. **Guaranteed Partial Protection**: As opposed to full restoration, we develop a mesh network protection scheme that guarantees a quantifiable minimum grade of service upon a failure within the network (as demonstrated by the example above).

2. **Protection with Multiple Availability Guarantees**: We develop a novel network protection scheme that provides guarantees on both the fraction of time a flow has full connectivity, as well as a quantifiable minimum grade of service during downtimes.

3. **Protection with Guaranteed Recovery Times**: We consider the problem of providing network protection that guarantees the maximum amount of time that flow can be interrupted after a failure.

4. **Protection in Multi-Hop Wireless Networks**: We develop a novel formulation to the problem of providing resource-efficient protection against failures in wireless networks subject to the constraints of limited shared resources (i.e. interference constraints).

1.1 Background

Traditional network protection consists of two main approaches: restoration and protection [5]. Restoration and protection differ by when they allocate resources for failure recovery. Restoration seeks to find unused resources in the network *after* a network failure occurred in order to reroute the failed connections. Protection, on the other hand, allocates resources for backup *prior* to a link failure. A notable example of a restoration scheme is IP rerouting, where after a link failure occurs, the network is updated with the new set of shortest paths between node pairs, and then a new path is selected [10]. This is both slow (sometimes on the order of minutes) [11], and does not necessarily guarantee that bandwidth will be available for the new path [12, 13]. Restoration is not limited to the IP layer, and has been utilized in other settings as well [14, 15]. Protection on the other hand allocates resources for recovery prior to any link failure; this guarantees that backup resources are available upon a failure. Additionally, since backup resources are already allocated for network protection, no time is needed to "discover" unused capacity for recovery, which significantly reduces the time to recover after a failure. Pre-allocating resources comes at the expense of additional complexity and resource utilization, but offers guarantees that restoration cannot provide. In this thesis, we focus on network protection with service guarantees, as opposed to network restoration, which cannot offer any such guarantees.

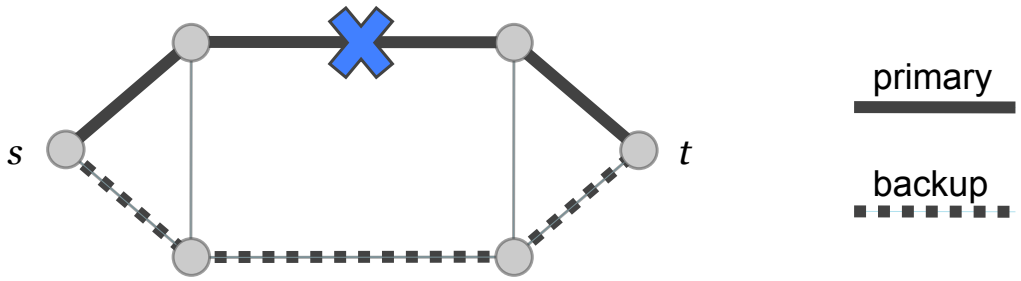


Figure 1-4: Example of $1 + 1$ protection

Guaranteed network protection has been studied extensively [5, 16–26]. The most common in backbone networks is guaranteed path protection [3], which provides an

edge-disjoint backup path for each primary path, resulting in 100% service recovery after any link failure. This scheme is typically referred to as $1+1$ protection [27], and has one primary path to route traffic before a failure, and one backup path for traffic after a failure. An example of $1+1$ protection is shown in Figure 1-4.

Optimization theory is a tool that we extensively use to investigate network protection with service guarantees. Network related optimization problems are oftentimes formulated as linear programs [28, 29]. A classic network flow problem is to find the shortest path between a source node s, and destination node d. This basic formulation to find the shortest path between s and d for some given graph G, with a set of edges E and a set of vertices V, is shown below.

$$\text{Objective: } \min \sum_{\{i,j\} \in E} x_{ij} \tag{1.1}$$

$$\text{Subject to: } \sum_{\{i,j\} \in E} x_{ij} - \sum_{\{j,i\} \in E} x_{ji} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases}, \quad \forall i \in V \tag{1.2}$$

$$x_{ij} = \{0, 1\}, \quad \forall \{i,j\} \in E \tag{1.3}$$

The variables x_{ij} indicate whether or not an edge $\{i,j\}$ in the network is used (Constraint 1.3). The objective is to minimize the amount of flow across all of the edges in order to route a unit of flow from s to d (Equation 1.1). The network flow constraints are given by Constraint 1.2, which indicate that at any given node, the flow into that node must be equal to flow out, except for the source and destination node that will have 1 unit of flow in, and 1 unit of flow out, respectively. We note that in this particular case, the linear program is in fact an *integer* linear program, since the values of x_{ij} can only be 0 or 1.

Numerous shortest path algorithms exist that do not use a linear programming formulation, such as Dijkstra or Bellman-Ford [29, 30], While these algorithms are efficient, they typically do not allow any additional parameters or constraints to the problem. Consider a modification to the shortest path problem that adds a service

guarantee: instead of simply finding the shortest path, we wish to find a shortest path such that the total traversal time across all of the edges in that path do not exceed some parameter T. It is not entirely clear how this can be done using one of the aforementioned shortest path algorithms since they do not take into account any such additional parameters. If each edge $\{i, j\}$ has a traversal time of t_{ij}, then we can simply modify the integer linear program above by adding the additional constraint.

$$\sum_{\{i,j\} \in E} t_{ij} x_{ij} \leq T \tag{1.4}$$

Constraint 1.4 ensures that the sum of all of the edges used in a network will not exceed the maximum path traversal time T. While integer linear programs are typically inefficient to solve directly [31], such formulations allow us to develop optimal solutions which include additional service guarantees, and allow us to begin analyzing the problem and develop efficient algorithms for its solution. An algorithm for the time-guaranteed shortest path (more commonly known as the constrained shortest path problem) was developed using this exact approach in [32].

To further emphasize the utility of linear programming approaches to formulating and solving a problem, we consider another important example: finding the shortest-pair of disjoint paths, which as discussed above, is one of the primary schemes used to protect networks. A naive approach would be to find a shortest path using one of the many available algorithms, remove those edges, and then find another shortest path. If a solution is returned, it will indeed be a pair of disjoint paths; unfortunately, this approach may yield a non-optimal solution, or in some cases, no solution whatsoever when one in fact does exist. Consider the network below in Figure 1-5.

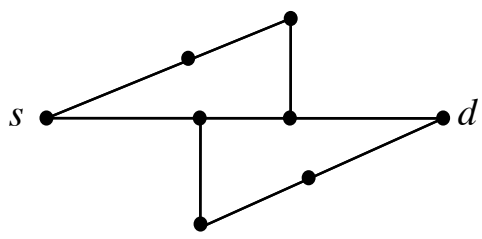
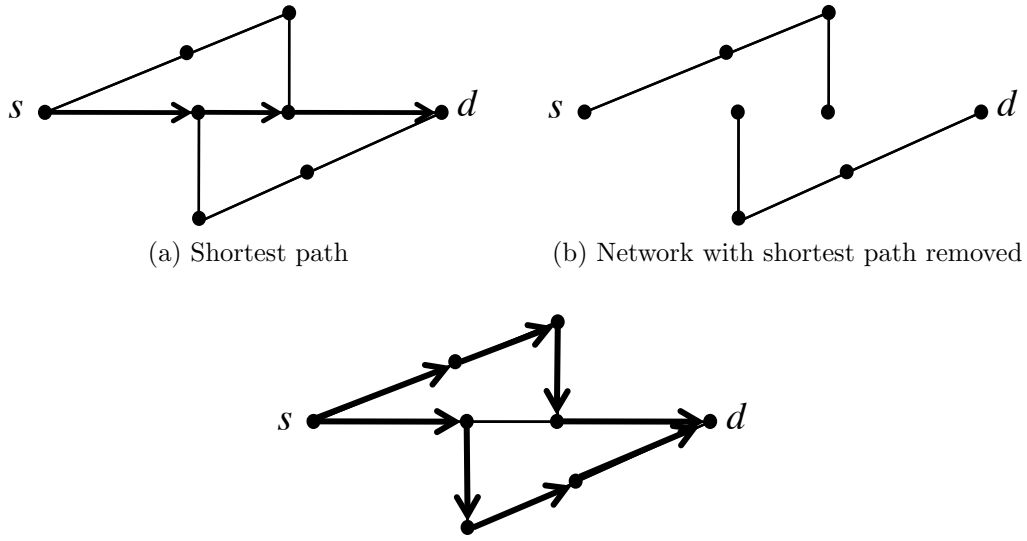


Figure 1-5: Example network for disjoint paths

If we used the approach described above (find the shortest path, remove those edges, find the next shortest path), we see in Figures 1-6a and 1-6b that no second path exists. But it is clear that two disjoint paths *do* exist, as seen in Figure 1-6c. This example is a well known network commonly referred to as the "trap" topology [33].



(a) Shortest path

(b) Network with shortest path removed

(c) Network with two disjoint paths

Figure 1-6: Finding disjoint paths in the trap topolgoy

If in the above linear program, Constraint 1.2 is modified to Constraint 1.5 (seen below), then 2 units of flow must traverse from the source to destination. Since any edge can only have a flow of 0 or 1 (Constraint 1.3), no two paths can use the same edge, and two disjoint paths are guaranteed when solving the integer linear program.

$$\sum_{\{i,j\}\in E} x_{ij} \; - \sum_{\{j,i\}\in E} x_{ji} = \begin{cases} 2 & \text{if } i = s \\ -2 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases} \quad , \quad \forall i \in V \tag{1.5}$$

Examining the structure of the above linear program allows us to further understand the problem, which then allows for efficient algorithmic solutions, as was done

for the shortest pair of disjoint paths problem in [21, 34]. Linear programs, and in particular integer linear programs, are often inefficient to solve. Powerful tools do exist for solving linear and integer linear program [35], though their running times are not guaranteed. But formulating our problems in such a fashion allows us to consider additional service guarantees, and then analyze the effects that these additional guarantees/constraints have on our problem. This approach often times allows us to find efficient algorithmic solutions to the problem that otherwise may have seemed intractable.

We next consider wireless networks and the additional challenges they impose. As opposed to wired networks, two nodes in a wireless network that are within close proximity of one another cannot transmit simultaneously, or else those transmissions will interfere. So, in addition to finding a route, a schedule of link transmissions needs to be specified. Consider the example network shown in Figure 1-7, where each node has a transmission radius of r.

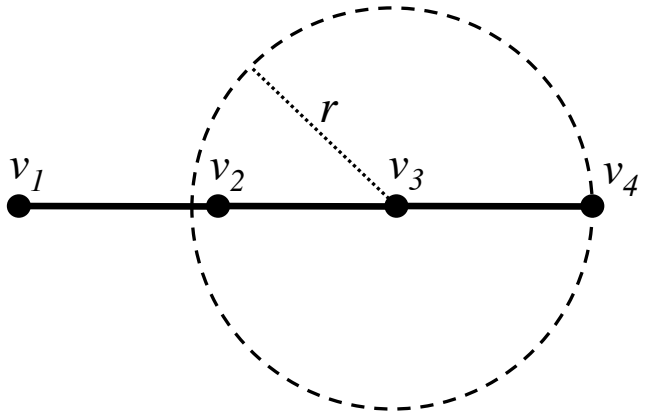


Figure 1-7: Example wireless network

We wish to route a packet from node v_1 to v_4, and in this example, the route the packet will take will be v_1, v_2, v_3, and then v_4. In addition to finding a path for the packet to take, we need to consider sharing the resources of the common transmission medium. As seen in Figure 1-7, when v_3 is transmitting to v_4, that transmission is also heard at v_2. Hence, if v_1 was trying to communicate to v_2 at the same time that v_3 was transmitting to v_4, the messages would interfere at v_2, and communication could

not occur. Similarly, it can be seen that v_2 cannot be transmitting to v_3 while v_3 is transmitting to v_4. In order to have successful communication without interference, links need to be scheduled to transmit during non-overlapping time slots, such that no two links within transmission range will communicate simultaneously. For our example, if we divide time into three time slots, link $\{v_1, v_2\}$ will be active during the first time slot, $\{v_2, v_3\}$ will be active during the second time slot, and $\{v_3, v_4\}$ will be active during the third time slot. With such a transmission schedule, interference-free communication is possible.

The key hurdle to finding an interference-free schedule is that the complexity that is added is substantial. Not only do we wish to find a route and a schedule, but we typically want to find a minimum-length schedule. The smaller fraction of time that a link can transmit, the lower its overall throughput will be. Hence, finding a minimum-length schedule is akin to finding a schedule that maximizes throughput. Because of these considerations, wireless routing and scheduling belongs to the class of non-polynomial time solvable problems [36] known as NP-hard [37]. It is within these additional set of interference constraints that we try to find resource-efficient protection for wireless networks.

1.2 Contributions

We now give a greater overview of the problems considered and the contributions of the thesis.

1.2.1 Guaranteed Partial Protection

In Chapter 2, we develop a novel mesh network protection scheme that guarantees a quantifiable minimum grade of service upon a failure within the network. Typically, networks fully guarantee service after a single-link failure, which is often an over-provisioning of resources to maintain essential traffic for the infrequent event of a failure. Our scheme guarantees that a fraction q of each demand is maintained after any single-link failure, at a small fraction of the cost of full protection.

An example of the partial protection service guarantee was presented earlier in Figures 1-1, 1-2, and 1-3, which demonstrates the significant savings that can be achieved by taking advantage of the redundancy and self-protection that is inherently available in mesh networks.

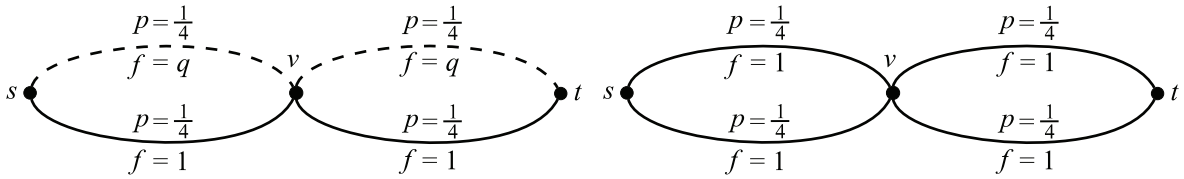
A linear program is developed to find the minimum-cost capacity allocation to meet both the demand and protection requirements. For a partial protection requirement of $q \leq \frac{1}{2}$, an exact algorithmic solution for the minimum-cost routing and capacity allocation is developed using multiple shortest paths. For $q > \frac{1}{2}$, an algorithm is developed based on disjoint path routing that performs, on average, within 1.4% of optimal, and runs four orders of magnitude faster than the minimum-cost solution achieved via the linear program. Furthermore, we demonstrate that our algorithm is guaranteed to give a solution whose cost is at most twice that of the optimal solution. The partial protection strategies developed in this chapter achieve reductions of up to 83% in spare capacity as compared to traditional full protection schemes.

The contribution we make in this chapter is developing a "theory" for partial protection that includes optimal algorithms for capacity allocation, as well as explicit expressions for the amount of required additional backup capacity. In Section 2.2, the partial protection model is described. In Section 2.3, the partial protection problem is formulated as a linear program with the objective of finding the minimum-cost allocation of primary and backup capacity. In Section 2.4, solutions for partial protection without the use of backup capacity sharing are developed, including a simple path based routing for an optimal solution when $q \leq \frac{1}{2}$, and when $q > \frac{1}{2}$, properties of an optimal solution for a network of disjoint paths are determined and used to develop a time-efficient algorithm. In Section 2.5, backup capacity sharing is considered, and an algorithm is developed for the case of dynamic (one-at-a-time) arrivals.

1.2.2 Protection with Multiple Availability Guarantees

In Chapter 3, we develop a novel network protection scheme that provides guarantees on both the fraction of time a flow has full connectivity, as well as a quantifiable

minimum grade of service during downtimes. In particular, a flow can be below the full demand for at most a maximum fraction of time; then, it must still support at least a fraction q of the full demand. This is in contrast to current protection schemes that offer either availability-guarantees with no bandwidth guarantees during the downtime, or full protection schemes that offer 100% availability after a single link failure.



(a) $1 + q$ partial protection (b) $1 + 1$ full protection

Figure 1-8: Comparison of Multiple Availability Guaranteed Protection (MAGP) vs. traditional protection schemes

To further motivate the problem, consider the example in Figure 1-8, with link failure probabilities and flow allocations as labeled (p and f respctively). A unit demand needs to be routed from s to t, and this connnection can drop to its partial protection requirement q after a failure with at most a probability of $\frac{1}{4}$. In Chapter 2, we introduced a simple partial protection scheme called $1 + q$ protection that routes the primary demand on one path and the partial protection requirement onto another edge-disjoint path. After any failure along the primary path, the partial protection requirement is met. This routing is shown in Figure 1-8a, with the solid line carrying the primary flow of 1 and the dotted line carrying the protection flow of q. However, in this routing, the maximum failure probability is exceeded: after a failure, the flow drops below the unit demand between s and t with a probability of $\frac{1}{2}$ (because the failure of either of the primary links would drop the demand below its full capacity). A naive alternative would be to simply allocate another path for protection, which would be identical to the $1 + 1$ full protection scheme (shown in Figure 1-8b), and utilize a total of 4 units of capacity. After any failure, the full flow of 1 unit is maintained; thus, the user will face no downtime, which meets and exceeds the maximum probability of failure requirement of $\frac{1}{4}$.

If we allow different levels of protection on different segments of the primary path, then a more resource efficient allocation is possible. Consider keeping the primary flow on the same bottom two edges as before, but instead of allocating an end-to-end backup path along the top two edges, 1 unit of flow is allocated to protect against the failure of $\{s, v\}$ and q units of flow to protect against the failure of $\{v, t\}$ (shown in Figure 1-9). If after some disruption either of the $\{s, v\}$ edges fail, 1 unit of flow will still remain from s to t. By fully protecting the primary $\{s, v\}$ edge, there is zero probability that its failure will cause the flow to drop below the full demand. The probability that the flow will drop below 1 after some failure is $\frac{1}{4}$, which meets the requirement that flow can drop to q with at most a probability of $\frac{1}{4}$ after a failure. This routing only needs $3 + q$ units of capacity, as opposed to the 4 units of capacity that full protection requires.

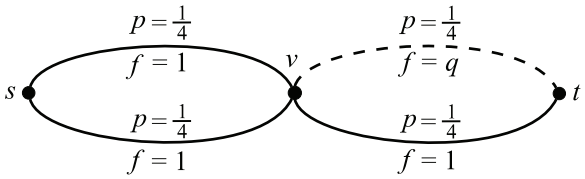


Figure 1-9: Routing with a probability of $\frac{1}{4}$ for the flow to drop to q after a failure

The novel contributions of Chapter 3 include the development of a framework for Multiple Availability Guaranteed Protection (MAGP), and the development of associated algorithms for both the cases when protection resources can and cannot be shared. We show that the multiple availability guaranteed problem is NP-Hard, and develop an optimal solution in the form of an MILP. If a connection is allowed to drop to 50% of its bandwidth for just 1 out of every 20 failures, then a 24% reduction in spare capacity can be achieved over traditional full protection schemes. Allowing for more frequent drops to partial flow, additional savings can be achieved. Algorithms are developed that provide multiple availability guarantees for both the sharing and non-sharing case. For the case of $q = 0$, corresponding to the standard availability constraint, an optimal pseudo-polynomial time algorithm is presented.

Chapter 3 is organized as follows. In Section 3.2, the model for Multiple Availability Guaranteed Protection (MAGP) is described. In Section 3.3, MAGP is shown

to be NP-Hard, and the minimum-cost solution to MAGP is formulated as an MILP. In Section 3.4, optimal solutions and algorithms for MAGP are developed when protection resources cannot be shared, and in Section 3.5, an algorithm is developed for when protection resources can be shared.

1.2.3 Protection with Guaranteed Recovery Times using Recovery Domains

In Chapter 4, we consider the problem of providing resource-efficient network protection that guarantees the maximum amount of time that flow can be interrupted after a failure. This is in contrast to schemes that offer no recovery time guarantees, such as IP rerouting, or the prevalent local recovery scheme of Fast ReRoute (FRR), which often over-provisions resources to meet recovery time constraints. To meet these recovery time guarantees, we provide a novel and flexible solution by partitioning the network into failure-independent "recovery domains", where within each domain, the maximum amount of time to recover from a failure is guaranteed.

The most common protection scheme that tries to ensure fast recovery times is a local recovery scheme known as Fast ReRoute (FRR) [38]. In FRR, after a failure, traffic is routed away from the node directly upstream from a fault, and reconnects the rerouted traffic with the original path at some downstream node. An example is shown in Figure 1-10.

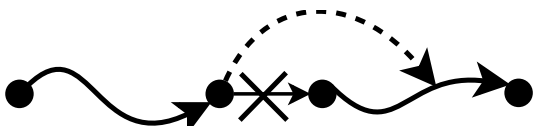


Figure 1-10: Fast ReRoute (FRR)

In Fast ReRoute, since each possible failure has its own dedicated protection path, resources are often over-provisioned beyond what is needed to meet recovery time guarantees. Consider the network shown Fig. 1-11, where the propagation delay for each link is 10 ms, and switching delays are assumed to be negligible. A flow

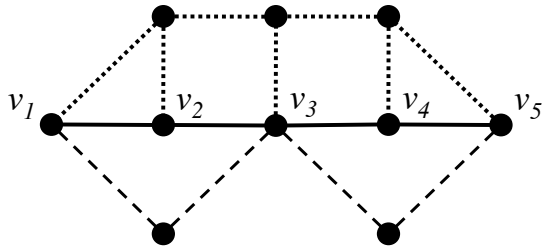


Figure 1-11: Time guaranteed recovery examples

needs to be routed from v_1 to v_5 such that the maximum time that the flow can be disrupted after a failure is 50 ms, which is the typical recovery time for backbone networks [11]. A primary path is already allocated on the solid lines from v_1 to v_5. A solution to FRR local recovery is to use all of the links above the primary path: after a link failure in the primary path, a fault notification is sent to the immediate upstream node of that failed link, and the flow is then switched to an alternate path from that node back towards the destination. This protection scheme requires 7 edges to be used for backup.

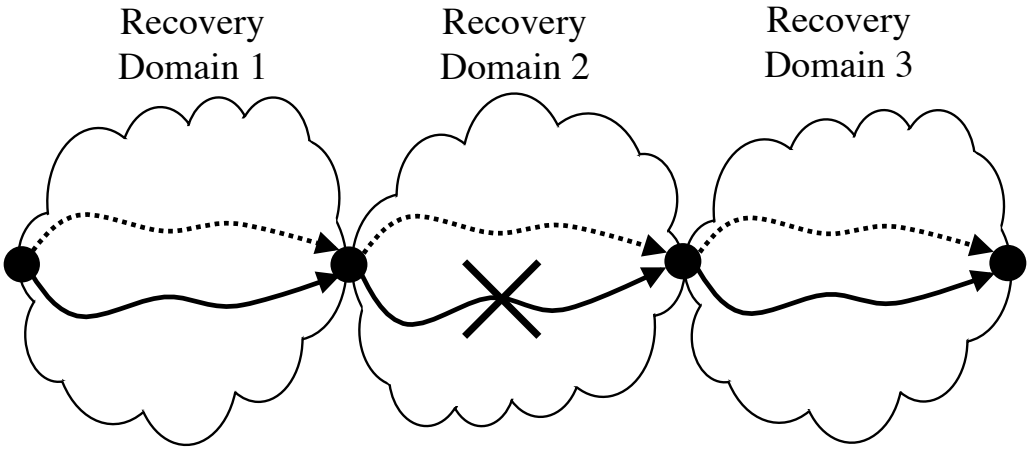


Figure 1-12: End-to-end routing using recovery domains

More recently, the new IETF standard for a backbone network protection framework calls for the creation of "recovery domains" [39]. Recovery domains are defined to be non-overlapping path segments, such that after a failure within a segment, flow is restored using a back-up path between the end-points of that segment. Moreover, recovery domains connect to one another via their respective "reference" end-points,

forming an end-to-end protected flow. An example is shown in Fig. 1-12: after the failure of an edge in the primary path located within Recovery Domain 2, the recovery domain's upstream end-point redirects flow onto the backup path, which then reconnects at that recovery domain's downstream end-point, bypassing the failure.

Now consider an alternative protection routing for the network in Figure 1-11 using the recovery domain model. Two recovery domains are created by using the links below the primary path as the backup paths: one recovery domain between nodes v_1 and v_3, and one between v_3 and v_5. If link $\{v_2, v_3\}$ fails, it would take up to 20 ms for the fault notification to propagate to v_1, and then 20 ms for the data that was switched to the protection route to reconnect with the primary path at node v_3. The other recovery domain will have a similar recovery time after a failure. In this example, only 4 additional links are needed to meet protection guarantees when using recovery domains, as opposed to the 7 needed for FRR.

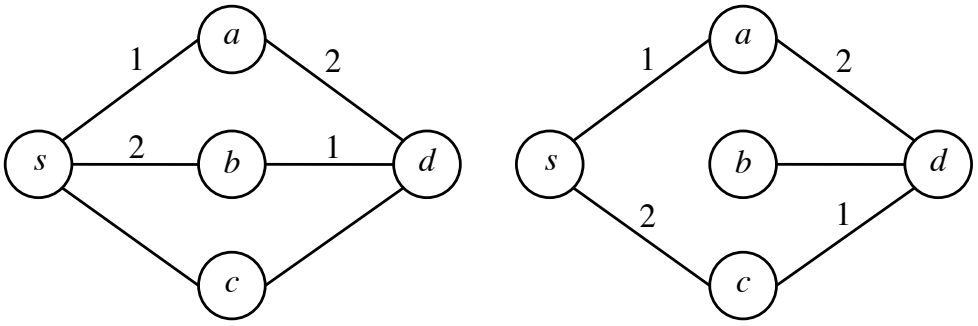
To the best of our knowledge, this work is the first to investigate Guaranteed Recovery Times using Recovery Domains (GRT-RD). The outline of Chapter 4 is as follows. We first present a model of the problem in Section 4.2. We then show in Section 4.3 that the recovery domain problem is NP-Hard, and formulate the optimal solution using an MILP. This provides protection with guaranteed recovery times using up to 45% less protection resources than local recovery. In Section 4.4, we decompose the end-to-end recovery domain problem into more tractable subproblems, which allows us to more easily construct a solution for the end-to-end problem. This allows for the development of flexible and efficient solutions, including an optimal algorithm using Lagrangian relaxation, which simulations show to converge rapidly to an optimal solution. In Section 4.5, an algorithm is developed for the case when backup sharing is allowed. For dynamic arrivals, this algorithm performs better than the solution that tries to greedily optimize for each incoming demand.

1.2.4 Providing Protection in Multi-Hop Wireless Networks

In Chapter 5, we consider the problem of providing protection against failures in wireless networks subject to interference constraints. Typically, protection in wired

networks is provided through the provisioning of backup paths. This approach has not been previously considered in the wireless setting due to the prohibitive cost of reserving limited resources for backup capacity. However, we show that in the presence of transmission interference, protection can often be provided with no loss in throughput. This is due to the fact that after a failure, links that previously interfered with the failed link can be activated, thus leading to a "recapturing" of some of the lost capacity.

The addition of interference constraints makes the protection problem in a wireless setting fundamentally different from the ones found in a wired context. After a failure in a wireless network, links that could not have been used due to interference with the failed link become available, and can be used to recover from the failure.



(a) Before a failure (b) After $\{s, b\}$ fails

Figure 1-13: Time slot assignment for protection in a wireless network

Consider allocating a protected flow between nodes s and d for the network shown in Figure 1-13. We assume an interference model where any two links that have a node in common cannot be active at the same time (often referred to as the 1-hop interference model [40]). Additionally, we assume unit capacity links. Before any failure, the maximum flow from s to d is 1, which can be achieved by scheduling the network into two distinct time slots such that transmissions can occur without interfering with one another, with the time slot assignment shown in Figure 1-13a. At any given point in time, only one outgoing link from s can be active, and similarly, only one incoming link to d can be active. Wireless links $\{s, c\}$, and $\{c, d\}$ cannot be used prior to the failure of $\{s, b\}$, but become available after $\{s, b\}$ fails. After the

failure of $\{s, b\}$, flow can be routed from s to c during time slot 2, and from c to d during slot 1, as shown in Figure 1-13b. Similar schedules can be found for failures of the other links. The maximum flow from s to d is 1 for both before *and* after a failure; i.e., there is no reduction in maximum throughput when allocating resources for a protection route on $\{s, c\}$ and $\{c, d\}$: protection can be assigned for "free". This is in contrast to a wired network where the maximum throughput without protection from s to d is 3, and the maximum throughput when assigning a protection route on $\{s, c\}$ and $\{c, d\}$ is 2, which amounts to a $\frac{1}{3}$ loss in throughput due to protection.

The novel contribution of Chapter 5 is in introducing the Wireless Guaranteed Protection (WGP) problem in multi-hop networks with interference constraints. We show that the general problem of optimal routing and scheduling with protection is NP-hard, and provide both an ILP formulation for the optimal solution, as well as algorithms that perform close to optimal. More importantly, we show that providing protection in a wireless network uses as much as 72% fewer protection resources as compared to similar protection schemes designed for wired networks, and that in many cases, no additional resources for protection are needed.

Chapter 5 is outlined as follows. In Section 5.2, the model for WGP is presented. In Section 5.3, properties of an optimal solution are examined for a single demand with 1-hop interference constraints, which are then used to motivate the development of a time efficient algorithm that guarantees a solution with 1.5 of optimal. In Section 5.4, an optimal solution is developed via a mixed integer linear program for general interference constraints. In Section 5.5, time-efficient algorithms are developed that perform within 4.5% of the optimal solution.

Chapter 2

Guaranteed Partial Protection

2.1 Introduction

Mesh networks with ever-increasing data rates are being deployed to meet the increasing demands of the telecommunication industry. As data rates continue to rise, the failure of a network line element or worse, a fiber cut, can result in severe service disruptions and large data loss, potentially causing millions of dollars in lost revenue [2]. Currently, there exist few options for protection that offer less than complete restoration after a failure. Due to the cost of providing full protection, many service providers offer no protection whatsoever. Additionally, since fiber cuts are relatively uncommon and are on average repaired quickly [4, 9], service providers may wish to only support essential traffic after a network failure. By defining varying and quantifiable grades of protection, service providers can protect vital services without incurring the cost of providing full protection, making protection more affordable and better suited to user/application requirements. The protection scheme developed in this chapter provides "partial protection" guarantees, at a fraction of the cost of full protection, with each session having its own differentiated protection guarantee.

Guaranteed network protection has been studied extensively [5, 16–20]. The most common in backbone networks today is guaranteed path protection [3], which provides an edge-disjoint backup path for each primary path, resulting in 100% service restoration after any link failure. Best effort protection is still loosely defined, but

generally offers no *guarantees* on the amount of protection provided. In best effort protection, a service will be protected, if possible, with any unused capacity after fully protecting all guaranteed services [4, 41]. Best effort protection can also be referred to as partial capacity restoration, since a service will be restored within existing unused capacity, typically resulting in less than 100% restoration.

Many users may be willing to tolerate short periods of reduced capacity to protect only essential services if data rate guarantees can be made at a reduced cost. In this chapter, we consider an alternate form of guaranteed protection, where a fraction of a demand is guaranteed in the event of a link failure. If provided at a reduced cost, many users may opt for partial protection guarantees during network outages.

A quantitative framework for deterministic partial protection in optical networks was first developed in [42]. In this work, a minimum fraction q of the demand is guaranteed to remain available between the source and destination after any single link failure, where q is between 0 and 1. When q is equal to 1, the service is fully protected, and when q is 0, the service is unprotected. More recently, [43] examines the savings that can be achieved by guaranteeing part of the demand in the event of a link failure, as opposed to full protection. It shows that the amount of protection that can be guaranteed depends on the topology of the network. In [44], the partial protection problem on groomed optical WDM networks is studied, under the assumption that flows must traverse a single path.

In this chapter, we further expand upon the framework developed in [42]. We develop a "theory" for partial protection that includes optimal algorithms for capacity allocation, and explicit expressions for the amount of required additional backup capacity. Routing strategies that allocate working and backup capacity to meet partial protection requirements are derived. Similar to [43], flow bifurcation over multiple paths is allowed. Bifurcation reduces the amount of additional backup capacity needed to support the protection requirements. In fact, we show that depending on the value of q, it may be possible to provide protection without any additional backup capacity at all.

A linear program is developed to find the optimal minimum-cost capacity alloca-

tion needed to guarantee partial protection in the event of a link failure. Without backup capacity sharing, a routing and capacity assignment strategy based on shortest paths is shown to be optimal for $q \leq \frac{1}{2}$. For $q > \frac{1}{2}$, an efficient algorithm based on disjoint path routing is shown to have a cost that is at most twice the optimal minimum-cost solution, and in practice only slightly above optimal. For the case with backup capacity sharing, we show that depending on the value of q, it may be possible to provide protection at minimal allocation cost, i.e. the shortest path routing. We consider two cases: preemptive and non-preemptive partial protection. For the preemptive case, primary resources available prior to a link failure may be preempted to provide backup for other demands, as long as all protection requirements are met after the failure. For the non-preemptive case, only demands that are directly affected by the link failure drop to the rates guaranteed under partial protection.

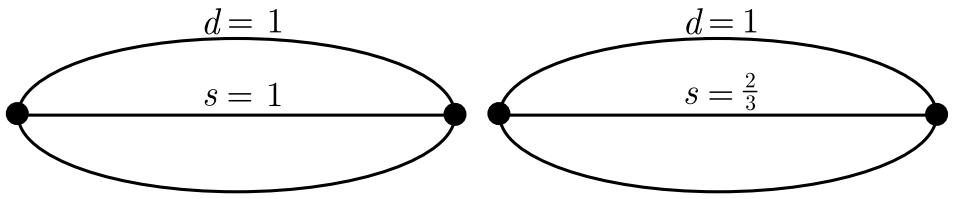
In Section 2.2, the partial protection model is described. In Section 2.3, the partial protection problem is formulated as a linear program with the objective of finding the minimum-cost allocation of primary and backup capacity. In Section 2.5, solutions for partial protection without the use of backup capacity sharing are developed, including a simple path based routing for an optimal solution when $q \leq \frac{1}{2}$, and when $q > \frac{1}{2}$, properties of an optimal solution for a network of disjoint paths are determined and used to develop a time-efficient algorithm. In Section 2.5, backup capacity sharing is considered, and an algorithm is developed for the case of dynamic (one-at-a-time) arrivals.

2.2 Partial Protection Model

The objective of partial protection is to find an allocation that ensures that enough capacity exists to support the full demand before a link failure and a fraction q of that demand afterward. We assume that the graph G, with a set of vertices V and edges E, is at least two-connected. Each link has a fixed cost of use: c_{ij} for each edge $\{i, j\} \in E$. We consider only single link failures. Both primary traffic and protection flows (defined as the flow after a failure) can be bifurcated to traverse

multiple paths between the source and destination. Without loss of generality, we assume unit demands, unless noted otherwise.

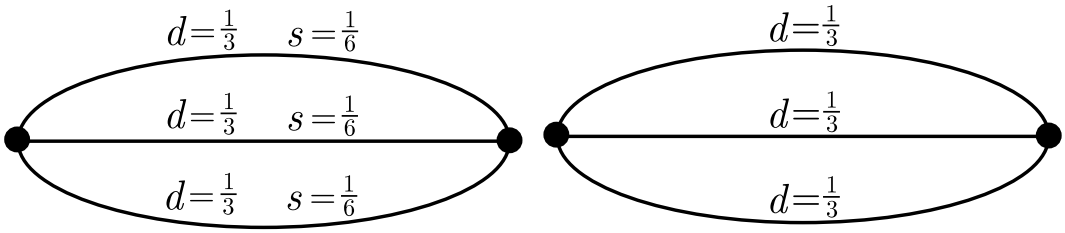
To begin with, assume that link costs are all 1; in the next section we will consider non-uniform link costs. With uniform link costs, the objective is to minimize the total capacity needed to support the flow and the partial protection requirements.



(a) $1 + 1$ protection (b) $1 + q$ protection, $q = \frac{2}{3}$

Figure 2-1: Standard protection schemes

One routing strategy for providing backup capacity is to use a single primary path and a single backup path similar to the $1+1$ guaranteed path protection scheme. Consider the network shown in Figure 2-1. With $1+1$ protection, one unit of capacity is routed on a primary path and one unit of capacity on a backup (Figure 2-1a). Upon a link failure, 100% of the service can be restored via the backup path. Now, consider a partial protection requirement to provide a fraction $q = \frac{2}{3}$ of backup capacity in the event of a link failure. A simple protection scheme similar to $1 + 1$ protection would be to route one unit along the primary path and $\frac{2}{3}$ along a disjoint protection path, as shown in Figure 2-1b. We will refer to this protection scheme as $1 + q$ protection. If the primary path fails, sufficient backup capacity remains to provide service for $\frac{2}{3}$ of the demand.



(a) $q = 1$ (b) $q = \frac{2}{3}$

Figure 2-2: Protection using risk distribution

For both partial and full protection requirements, in many cases capacity savings can be achieved if the risk is distributed by spreading the primary allocation across multiple paths. For example, by spreading the primary allocation across the three available paths, as shown in Figure 2-2a, any single link failure results in a loss of at most $\frac{1}{3}$ of the demand. To fully protect this demand against any single link failure (i.e. $q = 1$), additional spare capacity allocation[1] of $s = \frac{1}{6}$ needs to be added to each link. With this strategy, a total of 1.5 units of capacity are required, as opposed to the total of 2 units needed by $1 + 1$ protection. If instead the protection requirement was $q = \frac{2}{3}$, no spare allocation is needed since after any failure $\frac{2}{3}$ units are guaranteed to remain. By spreading the primary and backup allocation across the multiple paths between the source and destination, the risk is effectively distributed and the fraction of primary allocation lost by a link failure is reduced.

2.3 Minimum-Cost Partial Protection

In this section, a linear program is developed to achieve an optimal minimum-cost solution to the partial protection problem. The objective of the linear program is to find a minimum-cost routing strategy to meet demand d and partial protection requirement q for a set of demands. In particular, a demand's full flow requirement must be routed before any failure, and in the event of any link failure, a fraction q of that flow must remain. Backup capacity sharing is utilized to further reduce the capacity allocation (and cost) needed to meet demand and protection requirements. If two demands' primary paths are edge disjoint, then under a single link failure model, only one demand can fail at a time. Hence, backup capacity can be shared between the two since at most one demand will need to use it at any given point in time. The linear program to solve for the optimal routing strategy, denoted LP_{PP}, is defined below. We start by considering the case where only primary demands that are directly affected by a failure are switched to their respective protection flows (no

[1] We define spare capacity allocation to be the capacity that must be allocated in addition to the necessary capacity used to support the primary demand before a link failure.

preemption). Afterwards, the linear program is modified to allow for primary capacity to be preempted after a failure to route protection flows, so long as all demands have their protection requirements met.

2.3.1 Linear Program to Meet Partial Protection: LP_{PP}

The following values are given:

- $G = (V, E, C)$ is the graph with its set of vertices, edges, and costs
- d^{st} is the total demand between nodes s and t
- q^{st} is the fraction of the demand between s and t that must be supported on the event of a link failure
- c_{ij} is the cost of link $\{i, j\}$

The LP solves for the following variables:

- x_{ij}^{st} is primary flow on link $\{i, j\}$ for demand (s, t), $x_{ij}^{st} \geq 0$
- $f_{ij,kl}^{st}$ is the protection flow on link $\{i, j\}$ after the failure of link $\{k, l\}$ for demand (s, t), $f_{ij,kl}^{st} \geq 0$
- $y_{ij,kl}^{st}$ is the spare capacity for demand (s, t) on link $\{i, j\}$ for failure of link $\{k, l\}$, $y_{ij,kl}^{st} \geq 0$
- w_{ij} is total primary flow on link $\{i, j\}$, $w_{ij} \geq 0$
- s_{ij} is total spare allocation on link $\{i, j\}$, $s_{ij} \geq 0$

The objective of LP_{PP} is to minimize the cost of allocation over all links:

$$\min \sum_{\{i,j\} \in E} c_{ij}(w_{ij} + s_{ij}) \tag{2.1}$$

Subject to the following constraints:

- Flow conservation constraints for primary flow: route primary traffic to meet the set of demands.

$$\sum_{\{i,j\}\in E} x_{ij}^{st} - \sum_{\{j,i\}\in E} x_{ji}^{st} = \begin{cases} d^{st} & \text{if } i = s \\ -d^{st} & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i \in V,\ \forall (s,t) \in (V,V) \quad (2.2)$$

- Partial protection constraint: route flow to meet partial protection requirement q^{st} after failure of link $\{k,l\}$:

$$\sum_{\substack{\{i,j\}\in E \\ \{i,j\}\neq\{k,l\}}} f_{ij,kl}^{st} - \sum_{\substack{\{j,i\}\in E \\ \{j,i\}\neq\{k,l\}}} f_{ji,kl}^{st} = \begin{cases} d^{st}q^{st} & \text{if } i = s \\ -d^{st}q^{st} & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i \in V,\ \forall \{k,l\} \in E,\ \forall (s,t) \in (V,V) \quad (2.3)$$

- Primary capacity on link $\{i,j\}$ must meet all primary flows before a link failure

$$\sum_{(s,t)\in(V,V)} x_{ij}^{st} = w_{ij},\ \forall \{i,j\} \in E \quad (2.4)$$

- Primary and spare capacity on link $\{i,j\}$ for each demand meets partial protection requirements after failure of link $\{k,l\}$:

$$f_{ij,kl}^{st} \leq x_{ij}^{st} + y_{ij,kl}^{st}, \quad \begin{matrix} \forall \{i,j\}\in E,\ \forall \{k,l\}\in E \\ \forall (s,t)\in(V,V) \end{matrix} \quad (2.5)$$

- Spare capacity on link $\{i,j\}$ satisfies all protection flows after failure of link

$\{k, l\}$:

$$\sum_{(s,t)\in(V,V)} y_{ij,kl}^{st} \leq s_{ij}, \quad \begin{matrix}\forall\{i,j\}\in E \\ \forall\{k,l\}\in E\end{matrix} \tag{2.6}$$

A minimum-cost solution will provide flows to meet all primary demands before a link failure and flows to meet their respective partial protection requirements after any single-link failure. Protection capacity sharing is captured in constraint (2.6): for all demands that use link $\{i, j\}$ for protection after the failure of link $\{k, l\}$, enough spare capacity is allocated in addition to those demands' primary capacity to meet protection flow requirements. The spare capacity allocated to link $\{i, j\}$ will be the maximum needed for all possible link failures and will be shared amongst all the demands. To allow for preemption, constraints (2.5) and (2.6) can be replaced by constraint (2.7).

- After failure of link $\{k, l\}$, all protection flows that use link $\{i, j\}$ can use any available primary and spare allocation:

$$\sum_{(s,t)\in(V,V)} f_{ij,kl}^{st} \leq w_{ij} + s_{ij}, \quad \begin{matrix}\forall\{i,j\}\in E \\ \forall\{k,l\}\in E\end{matrix} \tag{2.7}$$

With bifurcation, each of the flows may be routed over multiple paths. An interesting characteristic of the optimal solution given by the linear program is that, at each node, flow conservation for the primary flow is maintained, but the total allocation for primary plus spare capacity, given by $(w_{ij} + s_{ij})$ for edge $\{i, j\}$, does not necessarily maintain flow conservation. Consider the example demonstrated in Figure 2-3. For $q = 1$ between s and t, each of the the two links between nodes s and v will need 1 unit of allocation, and each of the links between nodes v and t will need $\frac{1}{2}$ unit of allocation. It is easily verified that after any link-failure, 1 unit of flow will always remain between s and t. However at node v, there is a total of 2 units of flow going in and 1.5 units going out. Prior to a link failure, the primary path between s and t will use one edge between s and v, and between v and t, two links will be used,

each with a capacity allocation of $\frac{1}{2}$. After a link failure, similar allocations will be used to maintain full flow. Hence the total flow to support the demand before and after the link failure is conserved, however the capacity used to achieve this flow is not conserved at v.

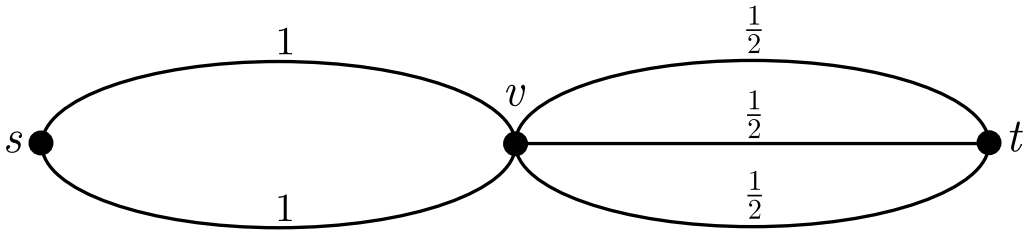


Figure 2-3: Example of flow not being conserved at node v

2.3.2 Comparison to Standard Protection Schemes

To compare the optimal solution to alternative protection schemes, two simulations are run: one where backup capacity sharing is not allowed, and one where it is. For the case without backup capacity sharing, 1000 random graph topologies are generated, each containing 50 nodes with an average node degree of 3.1, and having random link costs. Two nodes are randomly chosen from each graph to be the source and destination. The minimum-cost partial protection routing, as found by LP_{PP}, is compared to the standard scheme of $1+1$ protection, as well as $1+q$ protection. By not allowing flow to bifurcate, i.e. $x_{ij}^{st} \in \{0,1\},\ \forall\{i,j\} \in E$, the resulting scheme would be $1+q$ protection (and hence is now a mixed integer linear program). The linear programs are solved by using the CPLEX solver. Suurballe's algorithm [34] for the shortest pair of disjoint paths were used to solve for $1+1$ protection.

The average cost to route the demand and protection capacity using the different routing strategies are plotted in Figure 2-4 as a function of q. The top line, showing capacity requirements under $1+1$ protection, remains constant for all values of q. The next two lines from the top are $1+q$ and LP_{PP}, respectively. As expected, both meet demand and protection requirements using fewer resources than $1+1$, however, the minimum-cost solution produced by the partial protection linear program that

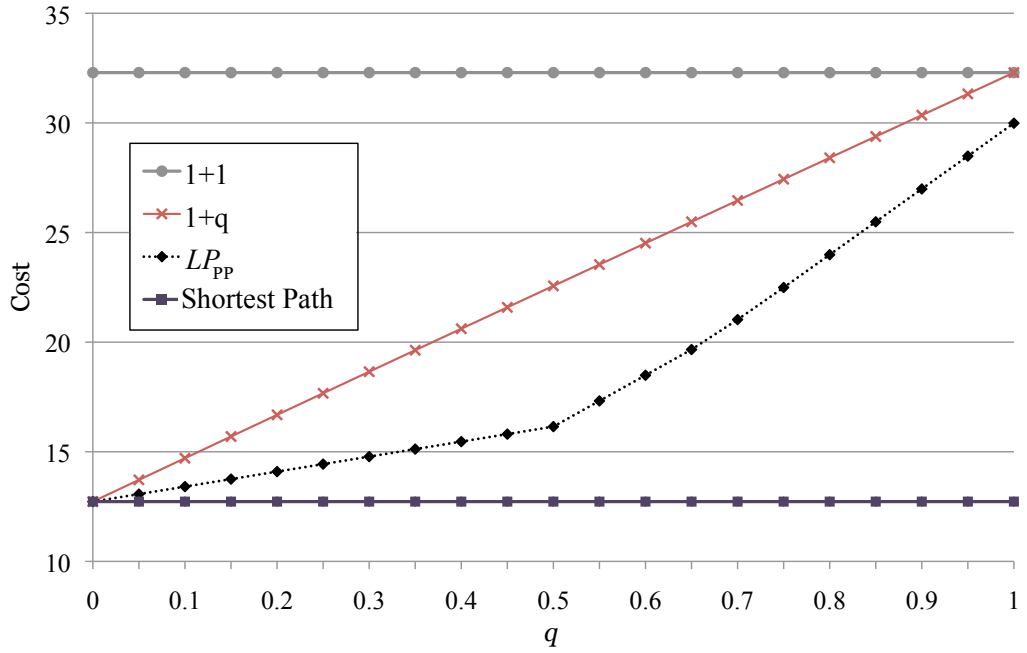


Figure 2-4: Without protection sharing: capacity cost vs. q

allows flow to bifurcate uses significantly less capacity. A lower bound on the capacity requirement is the shortest path routing, which provides no protection (shown in the bottom line of the figure). The cost of providing partial protection q is the difference between the cost of the respective protection strategies and the shortest path routing. Our partial protection scheme achieves reductions in excess resources of 82% at $q = \frac{1}{2}$ to 12% at $q = 1$ over $1 + 1$ protection, and 65% at $q = \frac{1}{2}$ to 12% at $q = 1$ over $1 + q$ protection.

For the case when backup capacity sharing is allowed, we compare both preemptive and non-preemptive partial protection with the $1 + 1$ and $1 + q$ protection schemes, which now allow for backup capacity sharing. The various partial protection schemes are compared via simulation using the NSFNET topology (Fig. 2-6) with 100 random unit demands. The protection requirement, q, for each demand has a truncated normal distribution with standard deviation $\sigma = \frac{1}{2}$. The mean of q is varied between 0 and 1 for each iteration.

The average costs to route the demand and protection capacity using the different routing strategies are plotted in Fig. 2-5 as a function of the expected value of q. Once again, the shortest path routing without protection considerations is used as a

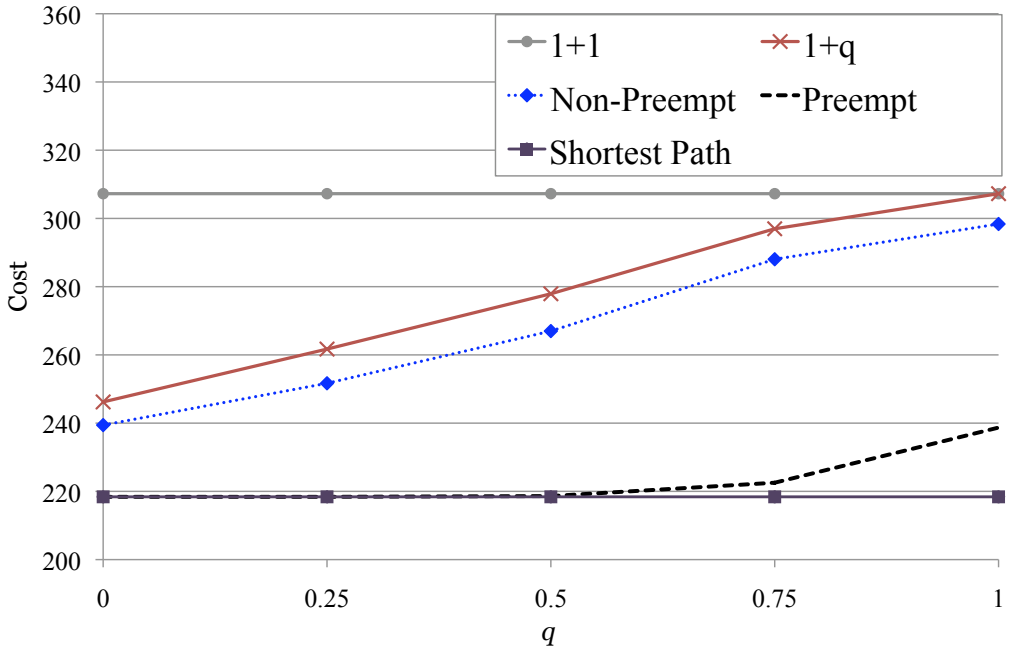


Figure 2-5: With protection sharing: capacity cost vs. q

lower bound for the allocation cost. In this simulation, preemptive partial protection is able to meet requirements using only the capacity needed for the shortest path routing for $q \leq \frac{1}{2}$, and only an additional increase in total capacity of 2% for $q \leq \frac{3}{4}$. When considering savings in excess resources, preemptive partial protection achieves reductions of 83% at $q = 1$ over both $1 + 1$ protection and $1 + q$ protection. Non-preemptive shared partial protection, at $q = \frac{1}{2}$, achieves reductions in excess resources of 59% over $1 + 1$ shared protection and 19% over $1 + q$ shared protection.

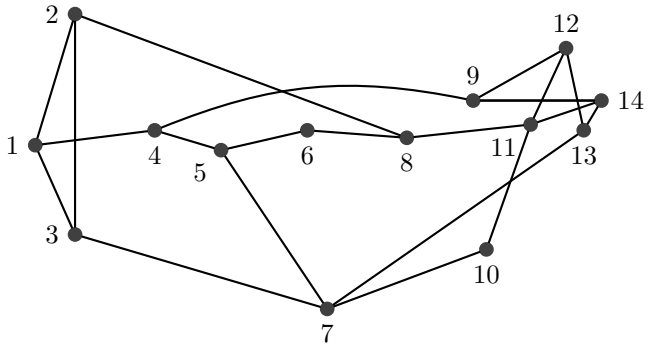


Figure 2-6: 14 Node NSFNET backbone network

2.4 Solutions without Backup Capacity Sharing

In this section, we provide insights on the structure of the solution to the minimum-cost partial protection problem when backup capacity sharing cannot be utilized. In Section 2.4.1, we are able to derive an exact algorithmic solution to the partial protection problem for $q \leq \frac{1}{2}$, which runs in polynomial time using a simple series of shortest paths. When $q > \frac{1}{2}$, we analyze solutions for a simpler two-node networks in Section 2.4.2. Using these insights for a two-node network for $q > \frac{1}{2}$, combined with the exact solution for $q \leq \frac{1}{2}$, a time-efficient algorithm is developed in Section 2.4.3 for general mesh networks. In Section 2.5, the case when backup capacity sharing is allowed is considered.

2.4.1 Solution for $q \leq \frac{1}{2}$

As mentioned in Section 2.2, the total primary and spare allocation coming in and out of any given node for an optimal solution does not necessarily maintain flow conservation. Without this property, most network flow algorithms do not apply [29] and analysis of the linear program becomes difficult. We show that all minimum-cost solutions for $q \leq \frac{1}{2}$ will never need spare allocation, hence allowing us to formulate the partial protection problem using standard network flow conservation constraints. This then allows us to derive a simple path-based algorithmic solution. All proofs for this section are provided in Chapter Appendix Section 2.7.1.

We begin by demonstrating that spare capacity is never needed for an optimal solution if the primary capacity on an edge is less than or equal to $(1 - q)$. Hence, any time a link fails, at least q remains in the network.

Lemma 2.1. *No spare capacity is needed to satisfy the flow and protection requirements if and only if the primary capacity on each link is less than or equal to* $(1-q)$.

In Section 2.4.2, we show routings with zero spare allocation are not necessarily lowest cost for all values of q. However, Lemma 2.2 shows that when $q \leq \frac{1}{2}$, the minimum-cost solution will never use spare allocation.

Lemma 2.2. *Given a demand between nodes s and t with a protection requirement of $q \leq \frac{1}{2}$, all minimum-cost solutions have no spare capacity on any edge: $s_{ij} = 0,\ \forall\{i,j\} \in E$.*

Combining Lemmas 2.1 and 2.2, it can be seen that a minimum-cost solution exists that does not use any spare allocation for $q \leq \frac{1}{2}$, and that $x_{ij} \leq (1-q),\ \forall\{i,j\} \in E$. Since the problem can now be formulated for $q \leq \frac{1}{2}$ using no spare allocation, flow conservation at each node is preserved. The linear program can now be written using a standard flow formulation without the use of spare allocation. The modified linear program, referred to as $LP_{q\leq.5}$, routes the flows on the paths in a manner that minimizes total cost and ensures that no edge carries more than $(1-q)$ of flow.

$$LP_{q\leq.5} : \min \sum_{\{i,j\}\in E} c_{ij}x_{ij} \tag{2.8}$$

$$\sum_{\{i,j\}\in E} x_{ij} \ - \sum_{\{j,i\}\in E} x_{ji} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}, \ \forall i \in V \tag{2.9}$$

$$x_{ij} \leq (1-q), \quad \forall\{i,j\} \in E \tag{2.10}$$

The above linear program achieves a minimum-cost routing in a network by using only primary allocation to meet the demand. $LP_{q\leq.5}$ is a network flow problem with directed and capacitated edges, which is recognized as a minimum-cost flow problem [29], for which algorithmic methods exist for finding an optimal solution. In Theorem 2.1, we show that an optimal solution for $q \leq \frac{1}{2}$ uses at most three paths with allocation q on each of the shortest pair of disjoint paths and allocation $(1-2q)$ on the shortest path.

Consider a directed graph $G = (V, E)$ with a source s and destination t. Let p_0

be the cost of the shortest path, p_1 and p_2 be the costs of the two shortest pair of disjoint paths, f_0 be the flow on the shortest path, f_1 and f_2 be the flows on each of the two shortest pair of disjoint paths, respectively, and $\mathcal{T}_{st}(q)$ be the cost of the allocation needed to meet demand and protection requirements between s and t for a value of q.[2]

Theorem 2.1. *Given a source s and destination t in a two-connected directed network $G = (V, E)$ with $q \leq \frac{1}{2}$, there exists a minimum-cost solution meeting primary and partial protection requirements with $f_0 = (1 - 2q)$ and $f_1 = f_2 = q$, giving a total cost $\mathcal{T}_{st}(q) = (1 - 2q)p_0 + q(p_1 + p_2)$, where path 0 is the shortest path and paths 1 and 2 are the shortest pair of disjoint paths.*

2.4.2 Solutions for $q > \frac{1}{2}$

When $q \leq \frac{1}{2}$, no spare allocation is needed when spare capacity cannot be shared, and the minimum-cost routing to meet the demand and protection requirements can be found using a series of shortest paths. When $q > \frac{1}{2}$, it may be necessary to use spare allocation to meet all requirements. Since the overall allocation of primary plus spare capacity does not necessarily meet flow conservation at any particular node, it may not be possible to provide a simple flow-based description of the optimal solution, as was done when $q \leq \frac{1}{2}$.

If we consider N disjoint paths between the source and destination, with the i^{th} path having cost p_i, we see that this is equivalent to a two-node network with N links where the i^{th} link has cost p_i. Hence, we investigate the properties of minimum-cost solutions for two-node networks in order to gain insight on solutions for general networks. These insights are then extended to develop a time-efficient algorithm for general mesh networks in Section 2.4.3.

A two-node network is defined as having a source and destination node with N links between them. Each link has a fixed cost of use, c_i. We first note that a solution that uses no spare allocation is not necessarily a minimum-cost allocation

[2]It is possible that the shortest path is also one of the shortest pair of disjoint paths.

when unequal link costs are considered. Consider the example in Figure 2-7 and let $q = \frac{2}{3}$. Allocating a capacity of $\frac{1}{3}$ onto each link does not use any spare capacity and has total cost of $\frac{1}{3}(1+2+6) = 3$. In contrast, consider using the two lowest cost links with the addition of spare capacity, with each link having an allocation of $\frac{2}{3}$. The protection requirement is met, and the total cost is reduced to $\frac{2}{3}(1+2) = 2$, which is less than the cost of the allocation that uses zero spare capacity.

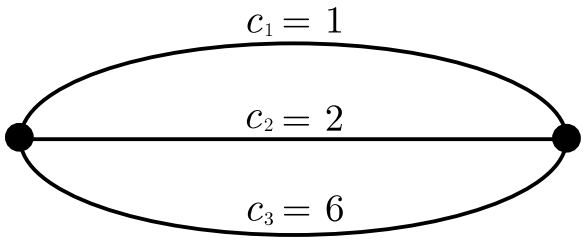


Figure 2-7: Two-node network with link costs

For two-node networks, we order the edges such that $c_1 \leq c_2 \leq ... \leq c_N$. Define x_i as the allocation on the i^{th} edge. We note that if $M \leq N$ edges are used for a minimum-cost allocation satisfying requirements in a two-node network, then the M lowest cost edges are the ones that are used (otherwise, the edge allocations could always be rearranged to produce a lower-cost solution). From our analysis, we are able to define a value K, which will be important for evaluating two-node networks: $K = argmax_{K=2..N}(c_K \leq \frac{1}{K-1}\sum_{i=1}^{K} c_i)$ (demonstrated in the proof for Lemma 2.4). K is the maximum number of links such that the incremental cost of using an additional link would not improve the solution. We now present the minimum-cost capacity allocation for a two-node network. All proofs for this section are provided in the Chapter Appendix Section 2.7.2.

We develop results on the condition when spare capacity is needed, which edges are active (i.e. have non-zero allocation) in the minimum-cost solution, and what the capacity allocation is across that set of active edges. Recall that spare capacity is the capacity that is allocated in addition to the capacity needed to route the primary demand.

Lemma 2.3. *A minimum-cost allocation for a two-node network uses spare capacity*

allocation if and only if $q > \frac{K-1}{K}$.

When spare capacity is needed for a minimum-cost solution (i.e. $q > \frac{K-1}{K}$), *exactly* the K lowest cost edges will be active, which is demonstrated in Lemma 2.4.

Lemma 2.4. *When spare allocation is needed, the minimum-cost solution for a two-node network uses exactly the K lowest cost edges, where $K = argmax_{K=2..N}(c_K \leq \frac{1}{K-1}\sum_{i=1}^{K} c_i)$.*

An interesting result that can be seen from Lemma 2.4 is that if $q > \frac{K-1}{K}$, then the number of edges used in an optimal solution when spare allocation is needed is no longer dependent on the partial protection requirement q.

Next, we demonstrate in Lemma 2.5 that when spare capacity is needed, an even allocation across the K lowest-cost edges is optimal.

Lemma 2.5. *A minimum-cost allocation when spare capacity is needed will be an even allocation of $q\frac{1}{K-1}$ on the K lowest cost edges, and no allocation on the remaining edges.*

Lemmas 2.4 and 2.5 both assume that spare allocation is needed for a minimum-cost solution to meet demand and protection requirements. We now show that when a solution does not use spare allocation, at most the K lowest cost edges will be used, where the K edges are those used in a solution for when spare allocation is required.

Lemma 2.6. *When spare allocation is not needed, a minimum-cost solution will use at most the K lowest cost edges, where K is the number of edges used when spare allocation is needed.*

When $q > \frac{K-1}{K}$, spare capacity is needed, and an even distribution across K edges meeting the above conditions is the optimal minimum-cost solution. When spare allocation is not needed, an even capacity allocation across the edges is no longer necessarily the optimal solution. When $q \leq \frac{1}{2}$, the optimal solution is given by Theorem 2.1. We now present the optimal solution for when $\frac{1}{2} < q < \frac{K-1}{K}$, which is the case when no spare capacity is needed.

Theorem 2.2. *The minimum-cost allocation when* $\frac{1}{2} < q \leq \frac{K-1}{K}$ *will be non-zero allocation on edges* 1 *to* J*, where* J *is the integer satisfying* $\frac{J-2}{J-1} < q \leq \frac{J-1}{J}$*. Moreover, the minimum-cost allocation when* $q \leq \frac{K-1}{K}$ *is:* $x_i = (1-q),\ \forall i = 1..(J-1)$*;* $x_J = (J-1)q - (J-2)$*;* $x_i = 0,\ \forall i = (J+1)..N$*.*

2.4.3 Time-Efficient Heuristic Algorithm

Consider a mesh network with N disjoint paths between the source and destination, and let p_i be the cost of the i^{th} path. By treating these N disjoint paths as a two-node network with N links, the results from Section 2.4.2 can be applied to develop a time-efficient algorithm for general mesh networks for the case of $q > \frac{1}{2}$. Recall that for $q \leq \frac{1}{2}$, the optimal minimum-cost solution for general mesh networks was derived in Section 2.4.1.

The algorithm is based on finding the k-shortest edge-disjoint paths for $k = 2$ to $k = N$, where N is the maximum number of edge-disjoint paths and the length of each path is its cost. The set of shortest disjoint paths can be found using Suurballe's algorithm [34]. For each set of k disjoint paths, we look to see if spare allocation is needed, i.e. $q > \frac{k-1}{k}$, and use the minimum-cost allocation given by Lemma 2.5 and Theorem 2.2. From the different possible disjoint path routings (from $k = 2$ to $k = N$ disjoint paths, where N is the maximum number of disjoint paths available), the allocation of minimum-cost is chosen. We call this algorithm the Partial Protection Disjoint Path Routing Algorithm (PP-DPRA), which is a combination of the optimal algorithm for when $q \leq \frac{1}{2}$, and the optimal solution across disjoint paths, as described in Section 2.4.2, for when $q > \frac{1}{2}$. Theorem 2.3 gives a bound on PP-DPRA's performance.

Theorem 2.3. *PP-DPRA produces a routing meeting demand and protection requirements with a cost that is at most twice the optimal minimum-cost.*

Proof. The cost to allocate capacity for $q = \frac{1}{2}$ is given by Theorem 2.1 as $\frac{1}{2}(p_1 + p_2)$, where p_1 and p_2 are the cost of each of the shortest pair of disjoint paths. Doubling the allocation on each of the shortest pair of disjoint paths will strictly double the

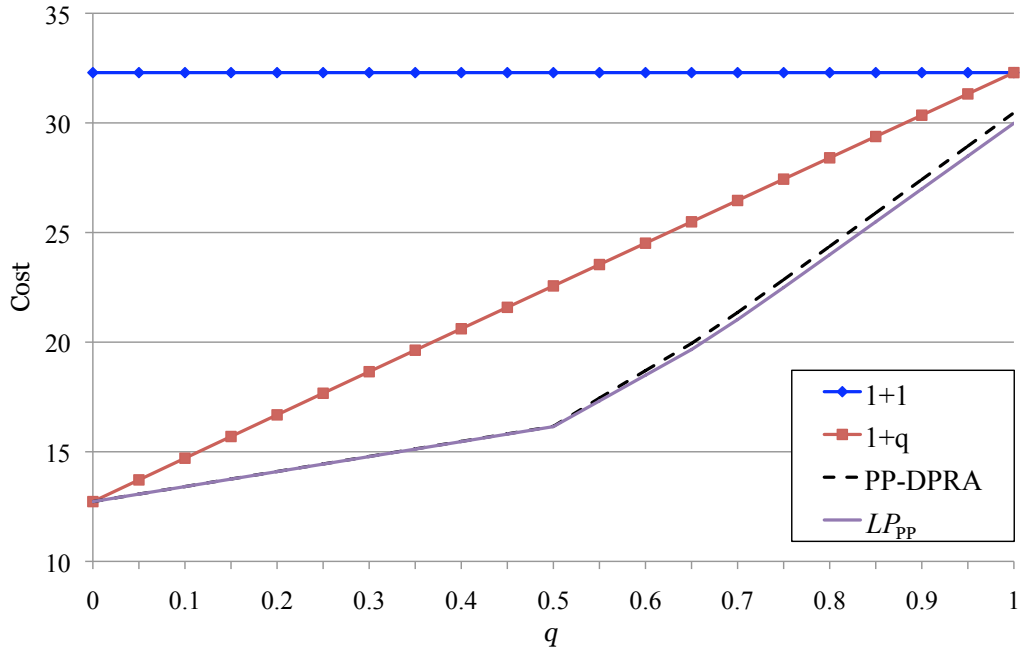


Figure 2-8: Algorithm comparison: cost vs. q

total cost. We note that this allocation is sufficient to provide protection for all $q \leq 1$; so the cost for protecting all $q \leq 1$ using a pair of disjoint paths is at most double that of $q = \frac{1}{2}$. The minimum-cost to provide protection is monotonically non-decreasing with respect to the value of the partial protection requirement q; this can be clearly seen because if there existed a solution for a demand for some q_2 that has a lower cost than that of some q_1, where $q_2 > q_1$, then the solution for q_2 would be used to protect for q_1 as well. Since the optimal solution is monotonically non-decreasing with respect to q, we know that routing $\frac{1}{2}$ unit of flow onto each of the shortest pair of disjoint paths is a lower bound, and routing 1 unit of flow onto each of the disjoint paths will be an upper bound. Hence, routing onto the shortest pair of disjoint paths is at most twice the cost of the optimal solution for any $q > \frac{1}{2}$. Using more disjoint paths, if possible, can only lower the total cost needed to meet demand and protection requirements. □

To assess PP-DPRA's performance, PP-DPRA is compared to $1 + 1$, $1 + q$, and LP_{PP}. The simulation is similar to the one run in Section 2.3.2 for the case when backup capacity sharing is not possible. PP-DPRA is implemented in C. The average

costs to meet demand and protection requirements over all random graphs are plotted in Figure 2-8. Simulation results show that for $q \leq \frac{1}{2}$, as anticipated, the routing given by Theorem 2.1 matches the optimal routing produced by LP_{PP}. For $q > \frac{1}{2}$, the average cost is greater than the minimum-cost solution by 1.4% on average. Additionally, on average, the running time for routing a demand with PP-DPRA was 10^{-3} seconds, while with the linear program LP_{PP} it was 22 seconds. This reduction in running time of four orders of magnitude makes the algorithm suitable for networks that require rapid setup times for incoming demands.

2.5 Solutions with Backup Capacity Sharing

In Section 2.3, a linear program that finds the minimum-cost solution for the partial protection problem utilizing backup capacity sharing was presented. A linear program is often not an efficient method of finding a solution, and in Section 2.4, an efficient algorithm was presented for the case without backup capacity sharing. These results offer a fundamental understanding of the partial protection problem, and are useful for networks that do not allow protection sharing. But often, networks do utilize backup sharing, and significant savings can often be achieved. In this section, a time-efficient algorithm for partial protection in general mesh networks using backup capacity sharing is presented.

If two primary flows for two different demands are edge-disjoint from one another, then under a single-link failure model, at most one can be disrupted at any given point in time. Since at most one demand will need to be restored after a failure, two failure-disjoint flows can share backup capacity.

Determining how much backup capacity can be shared for 1+1 guaranteed protection was investigated in [18, 19]. They consider the case of dynamic (one-at-a-time) arrivals, and we use a similar model for the development of our algorithm. In those papers, conflict sets were used to determine potential backup sharing on an edge by examining of how much backup capacity was allocated on one edge to protect against the failure of another. If more backup capacity is already allocated than is needed on

some edge to protect for the failure of another edge, then that edge's backup capacity can be shared. This model can be extended to partial protection by guaranteeing that any particular demand has its partial flow requirement met after a failure.

For the case of one-at-a-time routing, previous works offer heuristics to jointly optimize the primary and backup path for each incoming demand, as was done in [18, 19]. We instead choose a simple strategy of using the shortest path for the primary route. Our simulations show that using the shortest path for the primary route in fact performs better than jointly optimizing the primary and backup paths for each incoming demand. We call our algorithm Dynamic Shared Partial Protection (DSPP).

We compare, via simulation, DSPP to $1+1$, $1+q$ and the non-preemptive LP (LP_{PP}), each of which jointly optimizes the primary and backup paths for each incoming demand (a "greedy optimal" approach). Demands arrive dynamically and are served one-at-a-time in the order of their arrival. Performance of the strategies is compared using the NSFNET topology (Fig. 2-6) with 100 random unit demands. The protection requirement, q, for each demand has a truncated normal distribution with a standard deviation $\sigma = \frac{1}{2}$. The mean of q is varied between 0 and 1 for each iteration.

The costs to route the demand and protection capacity are plotted in Fig. 2-9 as a function of the expected value of q. It is seen that when demands are routed one at a time with a greedy optimal approach, the partial protection scheme offers significant savings over $1+1$ routing over a wide range of q, with LP_{PP} achieving even greater gains than $1+q$ protection because of its use of flow bifurcation. With dynamic arrivals, we find that DSPP performs *better* than the greedy schemes that jointly optimize the primary and backup paths. The greedy optimal approach of jointly optimizing the primary and backup routes will often take a longer primary path for an incoming demand in order to take advantage of backup sharing. The longer primary path makes it more difficult for future demands to find disjoint primary routes, thus lowering their ability to share protection resources. While other works have focused on finding heuristics to jointly optimize the primary and backup paths

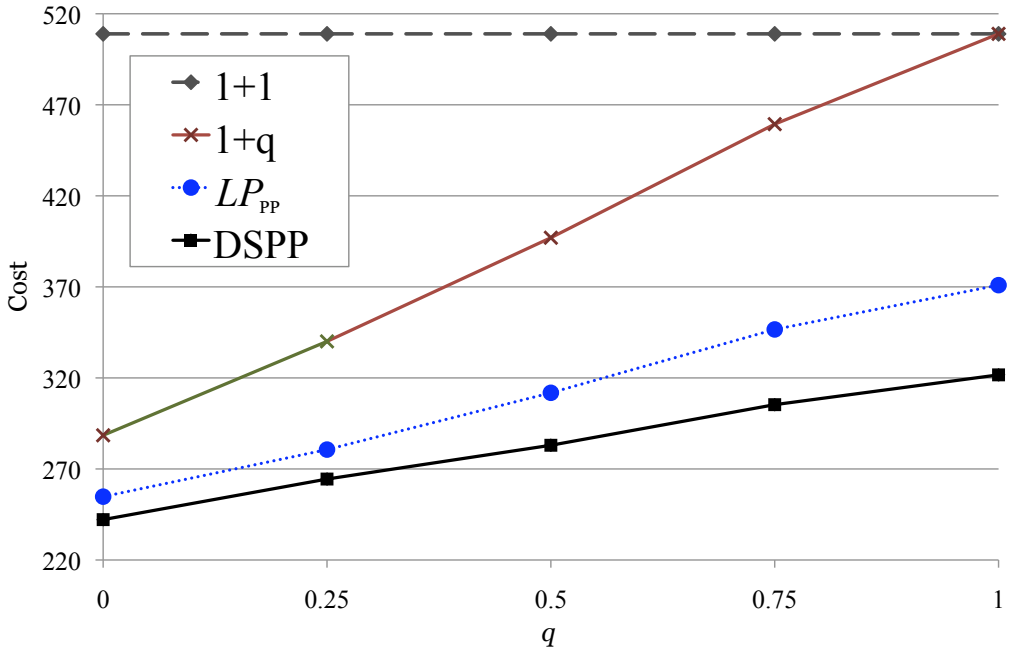


Figure 2-9: Sharing algorithm comparison: cost vs. q

for each incoming demand, it appears to be a better approach to simply take the shortest path for the primary route.

2.6 Conclusion

In this chapter we developed a mathematical model to provide an alternative form of guaranteed protection in networks: partial protection, which guarantees that a fraction q of a demand remains after a network failure. A linear program was formulated to find a minimum-cost solution for both the case with and without backup capacity sharing. Simulations show that this LP offers significant savings over the most common protection schemes used today. For the case with backup sharing, both a preemptive and non-preemptive scheme are developed, with the preemptive scheme able to offer protection for a wide range of partial protection requirements without the use of any additional resources beyond all the demands' shortest path routings. Algorithms for both the cases with and without protection sharing were developed. Without protection sharing, simulation results show that the algorithm comes within 1.4% of optimal on average and runs four orders of magnitude faster than the linear

program. For the case with protection sharing, the algorithm developed actually performs better than jointly optimizing the primary and backup paths for each incoming demand.

2.7 Chapter Appendix

2.7.1 Proofs for Section 2.4.1

Proof of Lemma 2.1. First we show that if there is no spare capacity on any link, i.e. $s_{ij} = 0,\ \forall\{i,j\} \in E$, and all flow and protection requirements are satisfied, then it must be the case that $x_{ij} \leq (1-q),\ \forall\{i,j\} \in E$. Assume all requirements are satisfied, and that there exists an edge $\{k,l\}$ such that $x_{kl} > (1-q)$ with $s_{ij} = 0,\ \forall\{i,j\} \in E$. After the failure of $\{k,l\}$, less than q of the flow will remain between the source and destination, which is below the partial protection requirement of q. This implies spare allocation on some edge will be needed to meet all requirements, which contradicts our original assumption.

We now consider the other direction: if $x_{ij} \leq (1-q),\ \forall\{i,j\} \in E$, then the required spare capacity is zero on all edges: $s_{ij} = 0,\ \forall\{i,j\} \in E$. This is straightforward to see since after the failure of any edge, at most $(1-q)$ of flow between the source and destination is disrupted, leaving at least q, which meets parital protection requirements. □

Proof of Lemma 2.2. For some $q \leq \frac{1}{2}$, assume there exists a minimum-cost solution with an edge $\{i,j\}$ that has $s_{ij} > 0$. Since a minimum-cost solution has an edge with spare capacity allocated to it, according to Lemma 2.1, there must exist some edge $\{k,l\}$ with primary capacity allocation greater than $(1-q)$, i.e. $x_{kl} = 1-q+\epsilon,\ \epsilon > 0$.

To meet protection requirements, after the failure of edge $\{k,l\}$, the remaining flows between s and t must have a total capacity of q. The amount of primary flow remaining after the failure of the edge carrying $(1-q+\epsilon)$ is $(q-\epsilon)$, which means that at least ϵ flow of spare allocation will be necessary along some of the protection paths. If this spare allocation was instead used as primary traffic, the primary flow

on $\{k, l\}$ would decrease from $1 - q + \epsilon$ to $1 - q$, which by Lemma 2.1 implies that no spare allocation is necessary. This maintains the total flow from s to t at 1; hence, the primary demand and protection requirements are met. Clearly, the total cost of the allocation without spare capacity is less than the cost with spare since the primary capacity on $\{k, l\}$ is reduced and the spare allocation on edge $\{i, j\}$ can be removed. □

Proof of Theorem 2.1. The modified linear program $LP_{q \leq .5}$ seeks to find a minimum-cost routing with capacitated edges. This is recognized to be a *minimum-cost flow* formulation, which is defined as finding a flow of lowest cost between a source and destination in a network that has both edge costs and edge capacities [29]. Algorithms exist for finding optimal solutions to minimum-cost flow problems. One such algorithm is the successive shortest paths (SSP), which successively finds the shortest path and routes the maximum flow possible on that path [29]. This is repeatedly done until the desired flow between the source and destination is routed. After SSP terminates, a set of edge allocations representing the minimum-cost flow will be returned. The paths that these edges represent, and those paths' respective flows, can be found by using the path decomposition algorithm [29].

Before we specify the details of SSP, we first define a residual graph, which is commonly used in maximum flow algorithms [29]. If edge $\{i, j\}$ has a capacity and cost of (u_{ij}, c_{ij}) in a graph G with a flow of $x_{ij} \leq u_{ij}$ on it, then the residual graph G^r will have two edges $\{i, j\}^r$ and $\{j, i\}^r$ with respective costs and capacities $(u_{ij} - x_{ij}, c_{ij})$, and $(x_{ij}, -c_{ij})$. Any flow in a residual graph preserves all node conservation constraints in the original graph [29].

The successive shortest paths algorithm is as follows: for a total demand of d needing to be routed from node s to t, first find the shortest path and route flow equal to the lowest capacity edge in that path. If the limiting edge has capacity u^*, then $d - u^*$ remains to be routed (assuming that $d > u^*$). Next, a residual graph is created from the allocation routed on the shortest path. To route the remaining $d - u^*$ of flow, the shortest path in the newly formed residual graph is found. If $d - u^*$ is less than the lowest capacity edge in that path, the algorithm is completed

by routing the remaining $d - u^*$ flow from s to t. Otherwise, flow equal to the lowest capacity edge in shortest path from s to t is routed, the residual graph is updated, and the process is repeated until all of the required d flow has been routed.

In our original network G, every edge has capacity $(1 - q)$; hence, the lowest capacity edge in any path is $(1 - q)$. We find the shortest path in G, with a set of edges E_0 and having a total cost p_0; $(1 - q)$ of flow is routed on the set of edges E_0. In the residual graph, each edge $\{i, j\} \in E_0$ no longer has capacity and is removed, and a new set of edges $E_0^r = \{\{j, i\}^r : \{i, j\} \in E_0\}$ is added, with each edge having capacity $(1 - q)$. All other edges that were not part of the shortest path remain in the residual graph with a capacity of $(1-q)$. Since $(1-q)$ was routed on the shortest path, a flow of q remains to be routed from s to t in the residual graph. All of the edges in the residual graph have a capacity of $(1 - q)$, and since we assume $q \leq \frac{1}{2}$, we know that $q \leq (1 - q)$. Hence, the next shortest path from s to t in the residual graph, with the set of edges E_1 and having a total cost of p_1, has sufficient capacity to satisfy the q amount of remaining flow that needs to be routed.

Two cases are possible: (1) if the second path does not use edges created in the residual graph by the initial shortest path, i.e. $E_1 \cap E_0^r = \emptyset$, and (2) if the second path does use those edges, i.e. $E_1 \cap E_0^r \neq \emptyset$.

For case 1, the first and second path do not overlap, and hence happen to be the shortest pair of disjoint paths in the network between s and t. Since $(1 - q)$ was routed onto the first shortest path, and q routed onto the next shortest disjoint path, we have the same flow as if routing $(1-2q)$ onto the shortest path, and q onto each of the shortest pair of disjoint paths. This yields a total allocation cost of $p_0(1-q)+p_1q$.

For case 2, the set of edges $\{j, i\}^r \in E_1 \cap E_0^r$ those edges where the second shortest path in the residual graph overlaps with the initial shortest path that was found in the original graph. Any allocation on $\{j, i\}^r \in E_1 \cap E_0^r$ "cancels" original flow allocated on edge $\{i, j\}$ from the first shortest path found, i.e., if 1 unit of flow is routed on edge $\{i, j\}$ in the original graph, and $\frac{1}{2}$ unit of flow is allocated on $\{j, i\}^r$ in the residual graph, then the flow in the original graph on edge $\{i, j\}$ is $\frac{1}{2}$. The flow on the edges where the two paths overlapped is $x_{ij} = (1-q)-q = (1-2q)$, $\forall\{j, i\} \in E_1 \cap E_0^r$, which

is non-negative since $q \leq (1-q)$. The remaining edges that did not overlap maintain their original flow values: if $\{i, j\}$ belonged to E_0, then it's value is $x_{ij} = (1-q)$, and if $\{i, j\}$ belonged to E_1, then it's value is $x_{ij} = q$.

To recover the paths, we use flow decomposition [29], which is to repeatedly find a path from source to destination, and subtract the flow equivalent to the minimum edge capacity until all flow from the network has been assigned to some path (almost a successive shortest paths in reverse). We first find the edges of the shortest path, which now has a maximum flow of $(1-2q)$. After removing this flow from the network, we are left with two disjoint paths of q flow each, and costs of p_1 and p_2, respectively. These disjoint paths are by definition the minimum-cost pair of disjoint paths: if there existed a lower cost pair of disjoint paths, then we can produce a lower cost flow by routing $(1-2q)$ onto the shortest path and q onto each of the lower cost pair of disjoint paths, which is a feasible flow and would give a lower cost routing, which is not possible since the successive shortest paths algorithm found the minimum-cost solution. Hence, we are left with a minimum-cost solution having a cost of $(1-2q)p_0 + q(p_1 + p_2)$. □

2.7.2 Proofs of Section 2.4.2

The following is used throughout all proofs in this section: $\mathbf{A_N}$ is the N x N matrix of 1's with the identity matrix subtracted from it (an all 1's matrix with a diagonal of zeros), $\mathbf{c_N}$ and $\mathbf{x_N}$ are the cost and edge allocation row vectors for N edges, respectively, and $\mathbf{e_N}$ is a column vector of N 1's. The expression $C = [\mathbf{A}\ \mathbf{B}]$ denotes a concatenation of matrices $\mathbf{A}$ and $\mathbf{B}$. Throughout these proofs, results from optimization theory are used, with pertinent details being found in [28].

We note that first the proofs for Lemmas 2.4, 2.5, and 2.6 are given before the proof for Lemma 2.3.

Proof of Lemma 2.4. For a given set of edges E with the i^{th} edge having a cost of c_i, the linear program for a two-node network needing to route a unit-demand with a

partial protection requirement of q can be written as follows:

$$LP_2 : \min \mathbf{x'_N c_N} \tag{2.11}$$

$$\text{s.t. } \mathbf{A_N x_N} \geq q\mathbf{e_N} \tag{2.12}$$

$$\sum_{i \in E} x_i \geq 1 \tag{2.13}$$

$$x_i \geq 0, \ \forall i \in E \tag{2.14}$$

Constraint 2.13 specifies that at least one unit of flow must be routed across the set of links, and Constraint 2.12 indicates that after any particular link fails, at least q flow must remain across the remaining set of links.

Since we assume that spare allocation is needed, the total allocation across the set of edges is strictly greater than 1. Hence, Constraint 2.13, which indicates that flow must be at least 1, is not tight, and can be disregarded.

When solving the primal, LP_2, we know generally that some $K \leq N$ edges are active, and $N - K$ edges are not, i.e. if edge i is active, then $x_i > 0$, and vis versa. We note again that the solution will clearly use the K lowest cost edges, otherwise capacity can be shifted from higher to lower cost edges, yielding a lower cost solution. With the K lowest cost variables being active, and the $N - K$ highest cost variables being zero, constraints $K + 1$ through N all have the form $\sum_{i=1}^{K} x_i \geq q$. Summing the first K constraints, we find $\sum_{i=1}^{K} x_i = q\frac{K}{K-1}$; constraints $K+1$ through N are all clearly no longer active (and they also no longer linearly independent), and they can be disregarded. By removing the constraints from LP_2 that are not tight, we are left with K variables and K constraints. To solve for K variables, all K constraints must be used, and can be set to equality. The primal is rewritten as follows:

$$LP_{2K} : \min \mathbf{x'_K c_K} \tag{2.15}$$

$$\text{s.t. } \mathbf{A_K x_K} = q\mathbf{e_K} \tag{2.16}$$

$$x_i \geq 0, \quad \forall i = 1..K \tag{2.17}$$

This solution is straightforward to find, and is an even distribution of $x_i = q\frac{1}{K-1},\ \forall i = 1..K$.

We now use an inductive approach to show that all edges that satisfy the requirement $c_j < \frac{1}{K-1}\sum_{i=1}^{K} c_i$ are in fact part of the minimum-cost solution, where $K = argmax_{K=2..N}(c_K \leq \frac{1}{K-1}\sum_{i=1}^{K} c_i)$. Assume that a minimum-cost solution uses the $J-1$ lowest cost edges, and that the J^{th} edge has cost $c_J < \frac{1}{J-1}\sum_{i=1}^{J} c_i$. If $J-1$ edges are used, then as shown above, the solution will be an even distribution across those $J-1$ edges of $x_i = q\frac{1}{J-2},\ \forall i = 1..(J-1)$. The total cost of the assumed optimal solution is $q\frac{1}{J-2}\sum_{i=1}^{J-1} c_i$. We now consider the solution that uses edge J, which was previously excluded. With J edges being used, each edge will have an even allocation of $x_i = q\frac{1}{J-1},\ \forall i = 1..J$, and the total cost across the J edges will be $q\frac{1}{J-1}\sum_{i=1}^{J} c_i$. Using some algebraic manipulation, we see that the solution using J edges will have higher cost only if $c_J > \frac{1}{J-1}\sum_{i=1}^{J} c_i$, but we assumed otherwise. Hence, a lower-cost solution can be obtained if all J edges are used. Inductively, we see that this approach can be continued until we find the K lowest-cost edges such that $K = argmax_{K=2..N}(c_K \leq \frac{1}{K-1}\sum_{i=1}^{K} c_i)$. □

Proof of Corollary **??**. The set of K edges that are active in a minimum cost solution when spare capacity is needed is given by the result in Lemma 2.4: $K = argmax_{K=2..N}(c_K \leq \frac{1}{K-1}\sum_{i=1}^{K} c_i)$, which does not depend on q. □

Proof of Lemma 2.5. In the proof for Lemma 2.4, a solution that optimally solved the two-node network was found for the primal formulation LP_{2K}, which uses K edges, by setting the the constraints to equality. This solution was an even distribution of $x_i = q\frac{1}{K-1},\ \forall i = 1..K$. We can perform an additional check that our solution is optimal by verifying complementary slackness conditions (not done here for brevity). □

Proof of Lemma 2.6. When spare allocation is not needed, the total allocation across

all of the edges is 1.

$$LP_2 : \min \mathbf{c'_N x_N} \tag{2.18}$$

$$\text{s.t. } \mathbf{A_N x_N} \geq q\mathbf{e'_N} \tag{2.19}$$

$$\sum_{i=1}^{N} x_i = 1 \tag{2.20}$$

$$x_i \geq 0, \ \forall i \in E \tag{2.21}$$

The corresponding dual is as follows:

$$LP_{2d} : \max \sum_{i=1..N} qp_i + p_{N+1} \tag{2.22}$$

$$\text{s.t. } \mathbf{p'_{N+1}}[\mathbf{A_N \ e_N}] \leq \mathbf{c'_N} \tag{2.23}$$

$$p_i \geq 0, \quad \forall i = 1..(N+1) \tag{2.24}$$

We note that the dual variable, p_{N+1}, corresponding to the primal constraint $\sum_{i=1..N} x_i = 1$, is no longer necessarily zero, as it would be if $\sum_{i=1..N} x_i > 1$.

We initially find an optimal solution to the problem as if it requires spare capacity, which assumes Constraint 2.20 is not active. When Constraint 2.20 is set to equality, then the current solution is still dual feasible if p_{N+1} is set to zero. The solution with spare capacity (given in Lemma 2.5) uses an even distribution of $q\frac{1}{K-1}$ on each of the K lowest cost edges, where $K = argmax_{K=2..N}(c_K \leq \frac{1}{K-1}\sum_{i=1}^{K} c_i)$. Since the K lowest cost edges are used, then the first K constraints will be active in the dual. Additionally, since the first K constraints are tight in LP_2, the first K dual variables will be used to solve for an optimal dual solution. Hence, the solution to the dual will have $p_i > 0, \ \forall i = 1..K$, and $p_i = 0, \ \forall i = (K+1)..N$.

We now consider the case when spare capacity is not used. The solution for when spare capacity *is* used remains dual feasible when the spare allocation is *not* used if p_{N+1} is initially set to 0. This solution, while being feasible, is not necessarily optimal. We will use this initial dual feasible solution as our starting point. The initial dual feasible solution has the first K constraints tight (at equality); each of

these constraints contain the variables p_{K+1} to p_{N+1}, which are all initially equal to zero.

We wish to find a lower cost solution to the primal, which means finding a higher value for the objective of the dual. Due to the structure of the problem's linear program and its subsequent dual, if any dual variable p_i, $i = 1..N$, has value greater than zero, then the corresponding edge i (primal variable x_i) will be non-zero. We wish to show that when a solution does not use spare capacty, that at most the K lowest cost edges will be used, where the K edges are those used for the minimum-cost allocation when spare allocation is needed. Hence, we want to show that raising the value of any dual variable p_{K+1} through p_N will not raise the objective function. In the current dual feasible solution, where p_{N+1} is set to zero, all constraints 1 through K are tight. To increase the value of any dual variable that is currently at zero (p_{K+1} through p_{N+1}), the sum of the dual variables that currently have value must be decreased.

Assume we wish to raise p_{K+1} through p_{N+1} by some amount δ, i.e. $\sum_{i=K+1}^{N+1} p_i = \delta$. Since the first K constraints are tight, we must decrease p_1 to p_K by at least δ. Consider the j^{th} tight constraint for some $j \leq K$. The j^{th} constraint has the following form: $p_1 + p_2 + ... + 0p_j + ... + p_K + \sum_{i=K+1}^{N+1} p_i = c_j$, where p_j is multiplied by zero to show that it does not appear in the j^{th} constraint. To raise $\sum_{i=K+1}^{N+1} p_i$ by δ, the $K-1$ dual variables that are greater than zero in the j^{th} constraint must each be reduced by $\frac{1}{K-1}\delta$. When we consider all K tight constraints, it can be easily shown that the only feasible solution to raise $\sum_{i=K+1}^{N+1} p_i$ by δ is to lower each dual variable p_1 through p_K by $\frac{1}{K-1}\delta$. Hence, to achieve the increase of δ across the variables p_{K+1} through p_{N+1}, the total decrease across the first K dual variables is $\frac{K}{K-1}\delta$, which we note is greater than δ.

Looking at the objective function (Equation 2.22), it can be seen that dual variables p_1 through p_N all have the same cost of q. Any increase in the dual variables p_{K+1} through p_N will result in a larger decrease of the dual variables p_1 through p_K, which will bring down the total value of the objective. Hence, raising the value of p_{K+1} through p_N will not find a new maximum for the objective. The cost of the dual

variable p_{N+1} in the objective is 1, and $q \leq 1$. So, it may be possible to raise p_{N+1} while decreasing p_1 throught p_K, and also increase the objective function. The dual variable p_{N+1} appears in each of the first K tight constraints. Again, consider the j^{th} tight constraint for some $j \leq K$, and exclude the dual variables p_{K+1} through p_N. The j^{th} constraint has the following form: $p_1 + p_2 + ... + 0p_j + ... + p_K + p_{N+1} = c_j$. Raising p_{N+1} will simply result in a strict decrease across the first K dual variables. Furthermore, since p_{K+1} through p_N also appear in each of the first K tight constraints, there is never a reason to raise some p_i, $(K+1) \leq i \leq N$, since a larger increase in the objective can be achieved by shifting any allocation that would go to p_i to p_{N+1} instead. Hence, if p_{N+1} were raised, at most the original dual variables p_1 through p_K will be non-zero, which will yield a solution using at most the K lowest cost edges. □

Proof of Lemma 2.3. First, assume there exists a minimum-cost solution that uses spare capacity when $q \leq \frac{K-1}{K}$. Since we assume that spare capacity is used for a minimum-cost solution, we know that the results from Lemmas 2.4 and 2.5 hold. The total capacity allocated for a minimum-cost solution across the K links is $q\frac{K}{K-1}$. Since we assumed that $q \leq \frac{K-1}{K}$, the total allocation across all of the links is $q\frac{K}{K-1} \leq \frac{K-1}{K}\frac{K}{K-1} = 1$, which means that the total allocation either uses no spare allocation or is insufficient to meet the primary demand. Hence, we have a contradiction.

Next, assume there exists a minimum-cost solution using no spare capacity if $q > \frac{K-1}{K}$. Continuing the proof from Lemma 2.5, we consider the objective function of the dual, max $\sum_{i=1..N} qp_i + p_{N+1}$. If the dual variable p_{N+1} is greater than zero, then its corresponding primal constraint (Constraint 2.20) must be tight (by way of complementary slackness). If Constraint 2.20 is tight, then no spare allocation is used. In the proof for Lemma 2.5, a dual feasible solution was considered, and the conditions for increasing the objective were found. To increase the dual variable p_{N+1} by δ, the first K dual variables must be decreased by a total of $\delta\frac{K}{K-1}$. The cost of each of the first K dual variables is q, while the cost of p_{N+1} is 1. The dual objective can only be raised if the decrease in cost from lowering the first K dual variables is offset by an even larger increase in the objective by raising p_{N+1}. A δ increase in p_{N+1} increases

the objective by δ, but decreases the first K dual variables by $\delta\frac{K}{K-1}$, which lowers the objective by $q\delta\frac{K}{K-1}$. We assume that $q > \frac{K-1}{K}$. The decrease in the objective from the first K dual variables is $q\delta\frac{K}{K-1} > \delta\frac{K-1}{K}\frac{K}{K-1} > \delta$. Hence, when $q > \frac{K-1}{K}$, a better solution can be found by having p_{N+1} be greater than zero, which means that no spare allocation is used, thus contradicting our original assumption. □

Proof of Theorem 2.2. We continue the proof from Lemma 2.3. As was shown, when $q \leq \frac{K-1}{K}$, the objective of the dual function can be raised by increasing p_{N+1}. The initial dual feasible solution when $q < \frac{K-1}{K}$ has $p - N + 1$ set to zero. Solving for the dual variables in the set of linear equations of our initial solution (while p_{N+1} is at 0), we get $p_j = \sum_{i=1}^{K} c_i - \frac{2K-3}{K-1}c_j$. By definition, $c_1 \leq c_2 \leq ... \leq c_N$, so the solution to the dual variables are $p_K \leq p_{K-1} \leq ... \leq p_1$. To increase p_{N+1} by some value δ, p_1 to p_K each decrease by $\delta\frac{1}{K-1}$, for a total decrease of $\delta\frac{K}{K-1}$.

We now increase δ by increments, until one of the dual variables goes to zero. Since each dual variable decreases by the same amount $\delta\frac{1}{K-1}$, the dual variable with the lowest value will go to zero first. The dual variable p_K will be the first to go to zero, and the corresponding K^{th} constraint is no longer active, giving a solution where the K^{th} edge has zero allocation; this was the most expensive edge that was in use, so it matches intuition that it would be the first to go to zero. Currently, p_1 to p_{K-1} are greater than zero, and p_{N+1} is also greater than zero. We wish to see if we can further increase p_{N+1} and continue raising the objective value. Without p_K, which is now zero, we have the following objective function: max $\sum_{i=1}^{K-1} qp_i + p_{N+1}$. Using a similar process as above, we get the condition that we will increase δ (which is p_{N+1}) only if $q < \frac{K-2}{K-1}$. This process can be repeated until raising the value of p_{N+1} further does not increase the objective. Inductively, we stop increasing p_{N+1} when we have J active dual variables such that $\frac{J-1}{J-2} < q \leq \frac{J}{J-1}$, where J is an integer. By complementary slackness, there are J active constraints in the primal, which yields a solution using J variables, which will be the J lowest cost edges. We get the following set of J independent equations: $\mathbf{x'_J}[\mathbf{A_{J-1}}\ \mathbf{e_J}] = [q\mathbf{e_J}\ 1]'$. We can solve this set of linear equations, and obtain the results in Theorem 2.2. □

Chapter 3

Protection with Multiple Availability Guarantees

3.1 Introduction

In Chapter 2, we considered providing protection that guarantees a minimum-grade of service after a failure, which we termed "partial protection". This partial protection scheme offered significant savings over the full protection scheme, but it allowed flow to drop to its lower grade of service after any link failure that disrupted service.

An alternative approach is to provide a guarantee on the maximum time a connection can be disrupted. This is known as an "availability guarantee", and it is a bound on the fraction of time or probability that a connection can be disrupted. However, these disruptions (downtimes) may be unacceptably long; thus, many service providers opt for the more resource intensive full protection. In this chapter, we propose a novel protection scheme with multiple availability guarantees. In addition to the traditional availability guaranteed protection, which maintains the full demand for at least a guaranteed fraction of time, we guarantee partial connectivity at all times (similar to the partial protection scheme of Chapter 2). Thus, our approach is a hybrid between the traditional availability guarantees and full protection schemes.

Full protection schemes have been studied extensively [2, 4, 5, 17–19, 46]. The most common scheme in backbone networks today is $1+1$ guaranteed path protection

[3], which provides an edge-disjoint backup path for each working path, and guarantees the full demand to be available at all times after any single link failure. There has also been a growing body of literature for backup provisioning to meet availability guarantees [47–53]. In all of these, primary and backup flows are allocated such that the connection is disrupted for at most a specified fraction of time or probability. During these down-states, the service is completely disrupted. A version of availability guarantees is considered in [54], where an end-to-end flow having a certain expected capacity, based on link availabilities, is found; no guarantees on flow are provided. In this chapter, a flow is guaranteed to be at least a fraction q of the full demand at all times, which is known as "partial protection". Our novel approach is the first to combine the traditional availability guarantee and partial protection guarantee to allow the user to specify flows with different availability guarantees. Moreover, it is particularly applicable to IP-over-WDM networks where MPLS tunnels are used to provision resources.

The partial protection framework was first introduced in [42]. In Chapter 2, we developed a "theory" of partial protection such that after *any* single link failure, the flow can drop to the partial protection requirement. In Chapter 2, a fraction q of the demand is guaranteed to remain available between the source and destination after any single link failure, where q is between 0 and 1. When q is equal to 1, the service will have no disruptions after any single failure, and when q is 0, there will be no flow between the two nodes during the down state. In our work, flows can drop below the full demand for at most a specified fraction of time, and maintain at least q of that demand at all times. Similar to [49–53], the probability of simultaneous failures is assumed to be negligible and we only consider single-link failures.

The novel contributions of this chapter include a framework for Multiple Availability Guaranteed Protection (MAGP) and providing associated algorithms for both the cases when protection resources can and cannot be shared. Moreover, in the $q = 0$ case, corresponding to the previously studied scenario where full availability is guaranteed for a fraction of time, we develop an optimal pseudo-polynomial algorithm.

This chapter is outlined as follows. In Section 3.2, the model for MAGP is de-

scribed. In Section 3.3, MAGP is shown to be NP-Hard, and the minimum-cost solution to MAGP is formulated as an MILP. In Section 3.4, optimal solutions and algorithms for MAGP are developed when protection resources cannot be shared, and in Section 3.5, an algorithm is developed for when protection resources can be shared.

3.2 Multiple Availability Guaranteed Protection

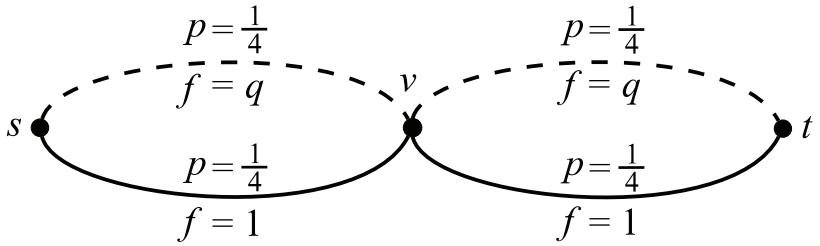
In this chapter, routing strategies are developed and analyzed to minimize the total cost and capacity allocation required to satisfy each demand's protection and availability requirements. A demand needs to be routed from its source s to destination t such that the flow can drop below the full demand for at most some specified downtime, and maintain at least a fraction q of that full demand at all times. To simplify the analysis, a "snapshot" model is used: the network state is considered after a failure has occurred. Let p_{ij} be the conditional probability that edge $\{i, j\}$ failed given that a network failure has occurred. For ease of exposition, instead of a maximum downtime, the Maximum Failure Probability (MFP) is considered, and its value is denoted by P. After some network failure, the flow can be below the full demand, but at least a fraction q of the demand, with at most probability P. The maximum failure probability is the conditional probability that the network is in a downstate given some link disruption. This value can be easily related to the maximum downtime by accounting for expected time between failures and mean time to repair (e.g., see [54]).

We assume that the graph G, with a set of vertices V, edges E, and edge failure probabilities $\mathcal{P}$, is at least two-connected. Since only single-link failures are considered, edge failures are disjoint events, which gives $\sum_{\{i,j\}\in E} p_{ij} = 1$. Similar to previous works, the primary flow is restricted to a single path. After the failure of a link, a network management algorithm reroutes the traffic along the allocated protection paths.

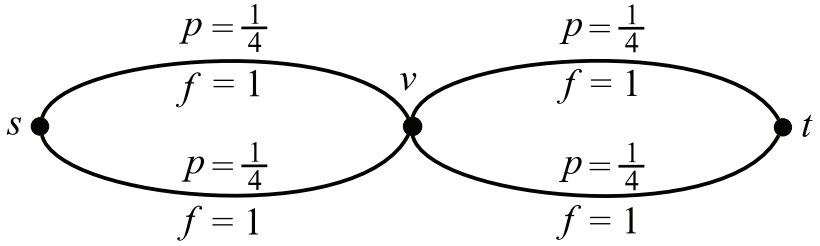
Consider the example in Fig. 3-1, with link failure probabilities and flow allocations as labeled (p and f respctively). A unit demand needs to be routed from s to

t with $P = \frac{1}{4}$ and partial protection requirement q. In Chapter 2, a simple partial protection scheme called $1+q$ protection was developed, which routes the primary demand on one path and the partial protection requirement onto another edge-disjoint path. After any failure along the primary path, the partial protection requirement is met. This is shown in Fig. 3-1a with the solid line carrying the primary flow of 1 and the dotted line carrying the protection flow of q. However, in this example the maximum failure probability is exceeded for the $1+q$ routing: after a failure, the flow drops below the unit demand between s and t with a probability of $\frac{1}{2}$ (because the failure of either of the primary links would drop the demand below its full capacity). A naive alternative would be to simply allocate another path for protection, which would be identical to the $1+1$ full protection scheme (shown in Fig. 3-1b), and utilize 4 units of capacity. After any failure, the full demand of 1 is maintained; thus, the user will face no downtime, which meets and exceeds the maximum probability of failure requirement of $\frac{1}{4}$.

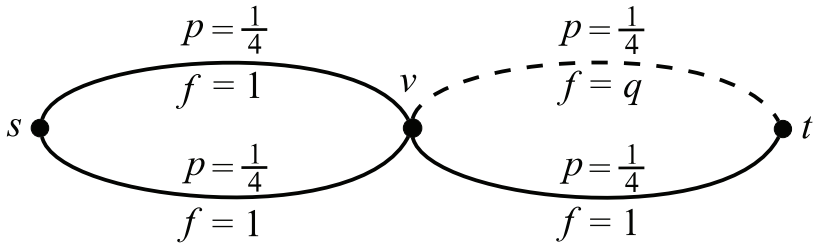
If we allow different levels of protection on different segments of the primary path, then a more resource efficient allocation is possible. Consider keeping the primary flow on the same bottom two edges as before, but instead of allocating an end-to-end backup path along the top two edges, 1 unit of flow is allocated to protect against the failure of $\{s, v\}$ and q units of flow to protect against the failure of $\{v, t\}$ (shown in Fig. 3-1c). If after some disruption either of the $\{s, v\}$ edges fail, 1 unit of flow will still remain from s to t. By fully protecting the primary $\{s, v\}$ edge, there is zero probability that its failure will cause the flow to drop below the full demand. The probability that the flow will drop below 1 after some failure is $\frac{1}{4}$, which meets the MFP requirement. This routing only needs $3+q$ units of capacity, as opposed to the 4 units that full protection requires.



(a) $1 + q$ partial protection



(b) $1 + 1$ full protection



(c) MAGP with $P = \frac{1}{4}$

Figure 3-1: Comparison of MAGP and traditional protection schemes

3.3 Minimum-Cost Multiple Availability Guaranteed Protection

This section investigates minimum-cost allocations for Multiple Availability Guaranteed Protection (MAGP). We first define the MAGP problem. Then, the MAGP problem is shown to be NP-Hard. Subsequently, in Section 3.3.1 an MILP is formulated to find a minimum-cost routing that meets a demand's partial protection and availability requirements. In Section 3.3.2, MAGP is compared to the $1 + 1$ full protection scheme.

We assume a graph G, with a set of vertices V, edges E, and edge failure probabilities $\mathcal{P}$. Each edge $\{i, j\}$ has an associated cost c_{ij}. After a single-link failure, the network can enter a downstate (a flow that is below the full demand) with at

most a Maximum Failure Probability (MFP) of P, where the MFP is the conditional probability that the network is in a downstate given some link disruption. During a downstate, the flow between a source and destination may fall below the full demand, but must always remain at a minimum guaranteed fraction q of the demand. We now show that finding the minimum-cost solution to MAGP is NP-Hard.

Theorem 3.1. *The Minimum-cost Multiple Availability Guaranteed Protection problem is NP-Hard.*

Proof. To demonstrate NP-Hardness of MAGP, a reduction from the 1-0 knapsack problem [37] is performed. See Chapter Appendix Section 3.7.1 for the complete proof. □

3.3.1 Mixed Integer Linear Program to Meet Multiple Availability Guaranteed Protection

Since finding a minimum-cost solution for MAGP is NP-Hard, in this section a mixed integer linear program (MILP) is developed to solve for the minimum-cost solution. For a connection request between two nodes s and t, the flow can drop to a fraction q of the demand with at most probability P. Again, the snapshot model is used, and the set of link failure probabilities $\mathcal{P}$ are conditional given a network failure has occurred.

For the MILP, the following values are given:

- $G = (V, E, C, \mathcal{P})$ is the graph with its set of vertices, edges, costs, and edge failure probabilities, respectively
- d^{st} is the required demand between nodes s and t
- q^{st} is the fraction of the demand between s and t that must be supported in the event of a link failure
- c_{ij} is the cost of link $\{i, j\}$

- p_{ij} is the probability that link $\{i, j\}$ has failed given a network failure has occurred

- P^{st} is the maximum allowable probability that the service between s and t falls below its full demand after some network failure

The following variables will be solved:

- x_{ij}^{st} is the primary flow for demand (s, t) on link $\{i, j\}$, $x_{ij}^{st} \geq 0$

- w_{ij} is the total primary flow assigned on link $\{i, j\}$, $w_{ij} \geq 0$

- s_{ij} is the spare capacity assigned on link $\{i, j\}$, $s_{ij} \geq 0$

- z_{kl}^{st} is 1 if the failure of link $\{k, l\}$ causes the flow between s and t to drop below the primary demand of 1; 0 otherwise

- $f_{ij,kl}^{st}$ is the flow on link $\{i, j\}$ after the failure of link $\{k, l\}$ for demand (s, t), $f_{kl}^{ij} \geq 0$

- $y_{ij,kl}^{st}$ is the spare capacity on link $\{i, j\}$ for failure of link $\{k, l\}$ for demand (s, t), $y_{kl}^{ij} \geq 0$

Objective:

- Minimize the cost of allocation over all links.

$$\text{minimize} \sum_{\{i,j\} \in E} c_{ij}(w_{ij} + s_{ij}) \tag{3.1}$$

Subject to:

- Flow conservation constraints for primary flow: route primary traffic to meet

the set of demands.

$$\sum_{\{i,j\}\in E} x_{ij}^{st} - \sum_{\{j,i\}\in E} x_{ji}^{st} = \begin{cases} d^{st} & \text{if } i = s \\ -d^{st} & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i \in V,\ \forall (s,t) \in (V,V) \quad (3.2)$$

- Full demand availability constraint: The probability that the flow between s and t drops below 1 after a failure is simply the sum of the failure probabilities of the individual edges causing the flow to drop below 1. The sum of these failure probabilities cannot exceed P.

$$\sum_{\{k,l\}\in E} p_{kl} z_{kl}^{st} \leq P^{st}, \quad \forall (s,t) \in (V,V) \quad (3.3)$$

- Flow conservation constraints for partial service: if the failure of link $\{k,l\}$ causes the flow to drop below d^{st}, route q^{st} from s to t; otherwise, maintain the full flow of d^{st}. Let $\mathcal{F}_{kl}^{st}$ be the expression $(1 - z_{kl}^{st}) + q^{st} z_{kl}^{st}$.

$$\sum_{\substack{\{i,j\}\in E \\ \{i,j\}\neq\{k,l\}}} f_{ij,kl}^{st} - \sum_{\substack{\{j,i\}\in E \\ \{j,i\}\neq\{k,l\}}} f_{ji,kl}^{st} = \begin{cases} \mathcal{F}_{kl}^{st} d^{st} & \text{if } i = s \\ -\mathcal{F}_{kl}^{st} d^{st} & \text{if } i = t \\ 0 & \text{otherwise} \end{cases}, \quad \forall i \in V,\ \forall (s,t) \in (V,V),\ \forall \{k,l\} \in E \quad (3.4)$$

- Working allocation is enough on link $\{i,j\}$ for all demands.

$$\sum_{(s,t)\in(V,V)} x_{ij}^{st} = w_{ij}, \quad \forall \{i,j\} \in E \quad (3.5)$$

- Capacity allocation: primary and spare capacity assigned on link $\{i,j\}$ meets

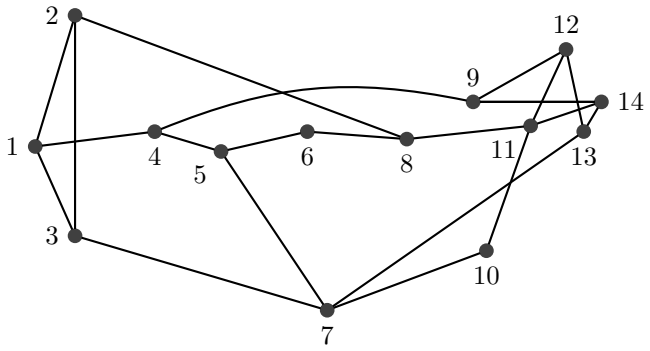


Figure 3-2: 14 Node NSFNET backbone network

protection requirements after the failure of link $\{k, l\}$.

$$f_{ij,kl}^{st} \leq x_{ij}^{st} + y_{ij,kl}^{st}, \quad \begin{matrix} \forall\{i,j\}\in E,\ \forall\{k,l\}\in E \\ \forall(s,t)\in(V,V) \end{matrix} \tag{3.6}$$

- Spare allocation is enough on link $\{i, j\}$ for all demands after failure of edge $\{k, l\}$

$$\sum_{(s,t)\in(V,V)} y_{ij,kl}^{st} \leq s_{ij}, \quad \begin{matrix} \forall\{i,j\}\in E \\ \forall\{k,l\}\in E \end{matrix} \tag{3.7}$$

For some demand between nodes s and t, a minimum-cost solution will provide an edge capacity allocation such that the flow drops to a fraction q^{st} of that demand with at most probability P^{st}.

3.3.2 Comparison to Full Protection

Multiple availability guaranteed protection is compared to the $1 + 1$ full protection scheme via simulation. The performance of the strategies is compared using the NSFNET topology (Fig. 3-2) with 100 random unit demands. The protection requirement q is set to $\frac{1}{2}$ for all demands. All link costs are set to 1, and the probability of failure for any link is proportional to its length, which is reasonable since a longer fiber will have a higher likelihood of being accidently cut. The maximum failure probability P is varied from 0 to .3 by .05 increments. While the main focus of this

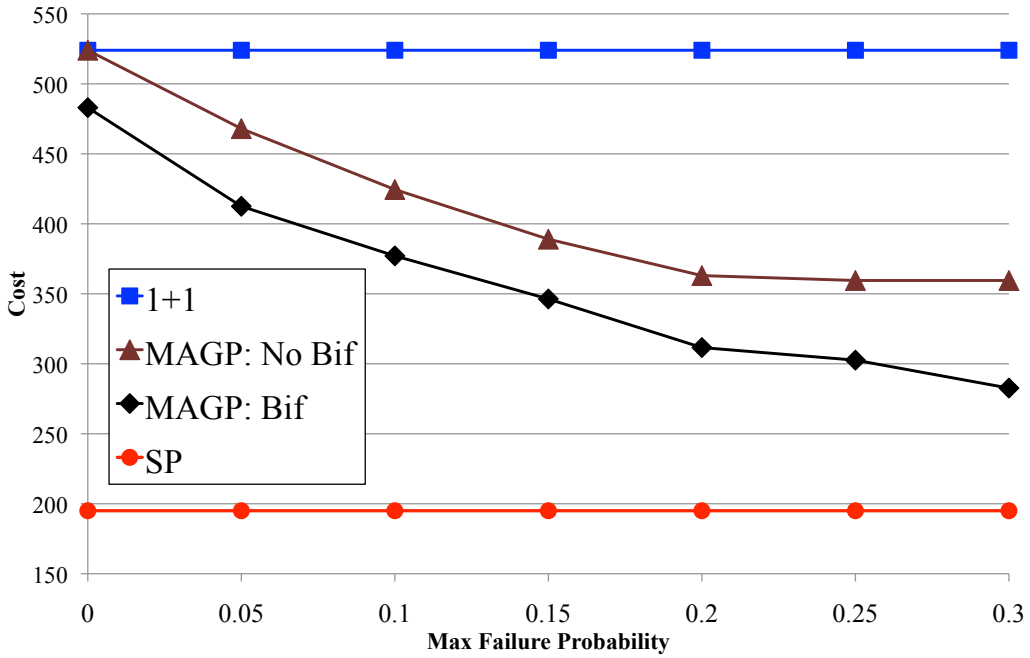


Figure 3-3: Capacity cost vs. MFP with $q = \frac{1}{2}$

chapter is the case where the primary flow is restricted to a single path, this simulation also considers allowing the primary flow to be bifurcated. Bifurcation reduces the loss of flow after any edge failure, thereby reducing the total allocation needed to meet requirements. Relaxing the integer constraint on the primary flow variables in the MILP corresponds to enabling bifurcation of the primary flow. Routing solutions for MAGP were determined using CPLEX to solve the MILP. Due to the length of running time of the MILP, each demand is run individually, which is the case without protection resource sharing. Protection resource sharing is considered in Section 3.5, where a one-at-a-time routing scheme is utilized. The shortest pair of disjoint paths were used for $1 + 1$ protection [34].

The average cost to route the demand and protection capacity using the different routing strategies is plotted in Fig. 3-3 as a function of the maximum failure probability P. The shortest path routing without protection considerations is used as a lower bound. The cost of providing incremental protection with parameters q and P is the difference between the cost of the respective protection strategies and shortest path routing.

Note that allowing the primary flow to bifurcate allows requirements to be met using a lower cost allocation. This is because splitting the primary flow distributes the risk so that upon an edge failure, less primary flow is disrupted, which then requires less protection resources. If the flow is allowed to drop to $\frac{1}{2}$ for 1 out of 20 failures (5% of the time), then a savings of 24% in protection capacity is realized for the case with bifurcation, and 17% without bifurcation as compared to $1 + 1$ protection. As the flow is allowed to drop more often to its partial protection requirement after a failure, savings increase. For $P = 0.1$, a savings of 45% and 30% is seen for MAGP with and without bifurcation, respectively. For $P = 0.2$, the savings are 65% and 49%. Further increases in P result in only small additional savings; hence, the simulations were stopped at $P = 0.3$.

3.4 Optimal Solution and Algorithms without Backup Capacity Sharing

While the MILP presented in the previous section finds an optimal solution to the multiple availability guaranteed protection problem, it is not a computationally efficient method of finding a solution, nor does it provide insight into why a solution is optimal. In this section, we analyze the MAGP problem to help develop efficient algorithms and heuristics for finding a minimum-cost routing when backup capacity sharing in the network is not allowed. The MAGP problem requires identifying a primary path such that segments of it are protected in a way that after a link failure, the flow drops to q with probability of at most P.

The case of $q = 0$ is explored in Section 3.4.1. When $q = 0$, there is no partial protection requirement, so there is only a single availability guarantee. This is the traditional availability guarantee, which has been examined in previous works. An optimal pseudo-polynomial algorithm is presented to solve MAGP with $q = 0$, which to the best of our knowledge is the first such algorithm. In Section 3.4.2, the case of $q > 0$ is examined. We show that finding a feasible solution to the closely related

problem of Singly Constrained Shortest Pair of Disjoint Paths is strongly NP-Hard (there exists no pseudo-polynomial or ϵ-approximation algorithm), and conjecture that the MAGP problem with $q > 0$ is also strongly NP-Hard. Hence, a heuristic for solving MAGP with $q > 0$ is developed. Multiple Availability Guaranteed Protection with the use of backup capacity sharing is examined in Section 3.5.

3.4.1 Availability Guarantees with $q = 0$

When $q = 0$, the partial protection requirement is removed and no flow is needed during the downtime. To solve this problem, a primary path needs to be found such that segments of it are protected, and after a failure, the flow can drop to 0 with probability of at most P. First, a restricted version of the problem is considered where we try to meet availability requirements without the use of spare allocation. It can be shown that the solution to the restricted problem is the constrained shortest path (CSP) problem [29]. Next, the problem without restrictions on spare allocation is studied. We transform this unrestricted problem to an instance of the restricted one, and use CSP to find an optimal pseudo-polynomial algorithm for MAGP when $q = 0$.

Availability Guarantees Without Spare Allocation

First, we consider finding the lowest-cost path between s and t that meets the availability guarantee without the use of spare allocation. In other words, we want to find the lowest-cost path such that the sum of all the failure probabilities in that path are less than P. This problem is recognized to be the constrained shortest path problem (CSP) [29], which is NP-Hard. A dynamic program exists that finds the minimum-cost solution to CSP in pseudo-polynomial time [56], with a running time of $O(n^2P)$, where n is the number of nodes in the network; the P factor is what makes this running time pseudo-polynomial. CSP assumes all inputs are integer, so instead of the failure probabilities being between 0 and 1, we multiply P and all p_{ij} values, $\forall\{i, j\} \in E$, by the smallest factor that makes all the values integer (all inputs

are assumed to be rational). Thus, for the remainder of this section, P and p_{ij} are assumed to be integer.

In general, a path may not exist from the source to the destination that can meet the availability requirement. Furthermore, if one exists, it is not necessarily of lowest cost. We next examine augmenting the flow with spare allocation to find a minimum-cost solution that meets requirements.

Availability Guarantees With Spare Allocation

We now examine allowing the use of spare allocation to protect segments of the primary path in a manner that ensures the entire end-to-end path meets availability guarantees. If a failure of a segment in the primary path does not cause a disruption in the end-to-end flow, then that segment is considered protected. A routing that meets guarantees will be a concatenation of protected and unprotected segments. Fig. 3-4 shows a sample solution for a unit demand between v_1 and v_6 with $P = 0.2$, which illustrates how the use of spare allocation enables meeting availability guarantees. The probabilities of link failure are as labeled, and all lines represent a unit flow.

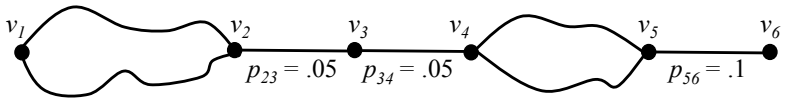


Figure 3-4: Routing to meet $P = 0.2$ with $q = 0$ from v_1 to v_6

The primary segments between node pairs (v_2, v_4) and (v_5, v_6) are unprotected, and their total probability of failure must be at most the maximum failure probability of $P = 0.2$. The primary segments between node pairs (v_1, v_2) and (v_4, v_5) are completely protected with the primary path segment being protected by a disjoint backup path. After a failure of either of these protected primary segments, one unit of flow still remains; they contribute a total failure probability of zero to the routing. In this example, disjoint paths were used for the protected segments between node pairs (v_1, v_2) and (v_4, v_5). In fact, the lowest cost allocation to form a protected segment between any two nodes i and j is the minimum-cost pair of disjoint paths between

the two, as demonstrated in Lemma 3.1.

Lemma 3.1. *The minimum-cost protected segment between nodes i and j is the minimum-cost pair of disjoint paths.*

Proof. For a segment between i and j to be protected, 1 unit of flow must remain between the source and destination after any single edge failure in the primary path. No backup edge will have an allocation greater than 1 because the primary flow will have 1 unit, and exactly 1 unit will need to be restored after any primary failure. An equivalent problem is to find the lowest-cost routing for 2 units of flow between i and j in a network where every edge has a maximum capacity of 1. After any single edge failure, at least 1 unit of flow will remain. This is a minimum-cost flow problem [29], whose solution has integer flows when given integer inputs. Since every edge has a capacity of 1, there will be two distinct edge-disjoint flows of 1 unit each. Clearly, the lowest-cost solution has these flows routed on the minimum-cost pair of disjoint paths. □

Using Lemma 3.1, every possible protected segment between any two nodes can be transformed to a single edge with a failure probability of 0 and a cost equivalent to the minimum-cost pair of disjoint paths between the nodes. We denote the cost and probability of the minimum-cost pair of disjoint paths between nodes i and j as $\hat{c}_{ij}$ and $\hat{p}_{ij} = 0$, respectively. Now, any protected segment between some node pair (i, j) can be represented as a single edge between i and j in the network. This edge contains the primary and spare allocation that would be used if a protected segment between i and j was needed. Adding an edge for every possible protected segment transforms the problem back to the restricted version where no spare allocation was allowed. This problem can now be solved using the constrained shortest path algorithm.

Our proposed algorithm is as follows. We take the graph G, where every edge has a cost and a failure probability associated with it, and augment the graph with an edge between every pair of nodes (i, j) such that the cost of that edge is the minimum-cost pair of disjoint paths between i and j, and the probability of failure for that edge is zero. Thus the new augmented graph has two kinds of edges between

nodes i and j: unprotected edges corresponding to the original edges in graph G (if such an edge existed) with a cost c_{ij} and failure probability p_{ij}, and protected edges with a cost $\hat{c}_{ij}$ and failure probability $\hat{p}_{ij} = 0$, where $\hat{c}_{ij}$ is the cost of the shortest pair of disjoint paths between i and j. We next run the constrained shortest path algorithm on the augmented graph to find the minimum-cost solution. We call this algorithm the Segment Protected Availability Guaranteed Algorithm (SPAG).

Theorem 3.2. *SPAG will return a minimum-cost routing, if one exists, and has a running time of* $O(n^4 log(n) + n^2 P)$.

Proof. To meet availability requirements, a solution will have a primary path that consists of a combination of protected and unprotected segments. As shown in Lemma 3.1, a protected segment between any two nodes is the shortest pair of disjoint paths between those nodes. Using the above graph augmentation, an edge is added for every feasible protected segment. The constrained shortest path algorithm then evaluates every possible combination of protected and unprotected segments to find the lowest cost solution between the source and destination.

For the running time, the $O(n^4 log(n))$ component comes from $O(n^2)$ iterations of the shortest pair of disjoint paths algorithm (there are $O(n^2)$ node pairs), which takes $O(n^2 log(n))$ time per iteration [34]. The recursion for the constrained shortest path problem runs in $O(n^2 P)$ time. □

A simulation similar to that of Section 3.3.2 was used to compare SPAG to the optimal solution without bifurcation for $q = 0$. Simulation results show that SPAG is in fact optimal for all tested demands.

3.4.2 Meeting Availability Requirements with $q > 0$

Next, we examine the case of $q > 0$. The problem now has multiple availability guarantees: after an edge failure in the primary path, the flow either remains at 1 or, with at most a probability of P, drops to q. Consider a sample solution shown for a unit demand between nodes v_1 and v_6 with a maximum failure probability of 0.2 in

Fig. 3-5, which consists of alternating fully-protected and q-protected segments (the dotted line being the q flow). Between node pairs (v_1, v_2) and (v_4, v_5), the primary segments are fully protected, and a failure in those primary segments will not cause a drop in end-to-end flow. Between node pairs (v_2, v_4) and (v_5, v_6), the primary segments have q flow routed on a segment that is edge-disjoint; after a failure in the primary segment, flow will drop to q with a probability of at most 0.2. We conjecture that the multiple availability guaranteed protection problem with $q > 0$ is *strongly* NP-Hard[1] by demonstrating the complexity of a related disjoint paths problem, previously unexplored in the literature, to be strongly NP-Hard. Using this result, we present an efficient heuristic for solving the $q > 0$ case.

A sample solution is shown in Fig. 3-5, which consists of alternating fully-protected and q-protected segments (the dotted line being the q flow).

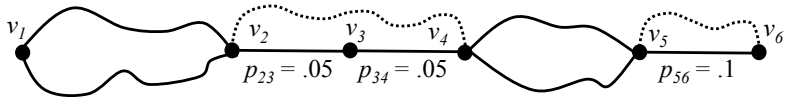


Figure 3-5: Routing to meet $P = 0.2$ and $q > 0$ from v_1 to v_6

We consider protecting a q-protected segment by finding a pair of disjoint paths between i and j such that one of them is constrained: the primary segment is constrained to have a probability of failure of at most P. We call this problem the Singly Constrained Shortest Pair of Disjoint Paths (SCSPD). There has been work trying to find the shortest pair of disjoint paths such that each path is constrained by the same parameter [57]. The authors of [57] found that this doubly constrained problem, while NP-Hard, has an ϵ-approximation algorithm. Their problem is distinct from ours in that SCSPD only constrains one of the two paths. Clearly, a solution to the doubly constrained problem is a feasible solution to the singly constrained one, but it is not necessarily optimal, and a lack of a solution to the former does not imply the non-existence of a solution to the latter. In fact, we show that when the constraint is relaxed for one of the paths, SCSPD becomes *strongly* NP-Hard, which means

[1]In addition to being NP-Hard, a problem that is Strongly NP-Hard indicates that there exists no pseudo-polynomial or ϵ-approximation algorithm for finding a solution [37].

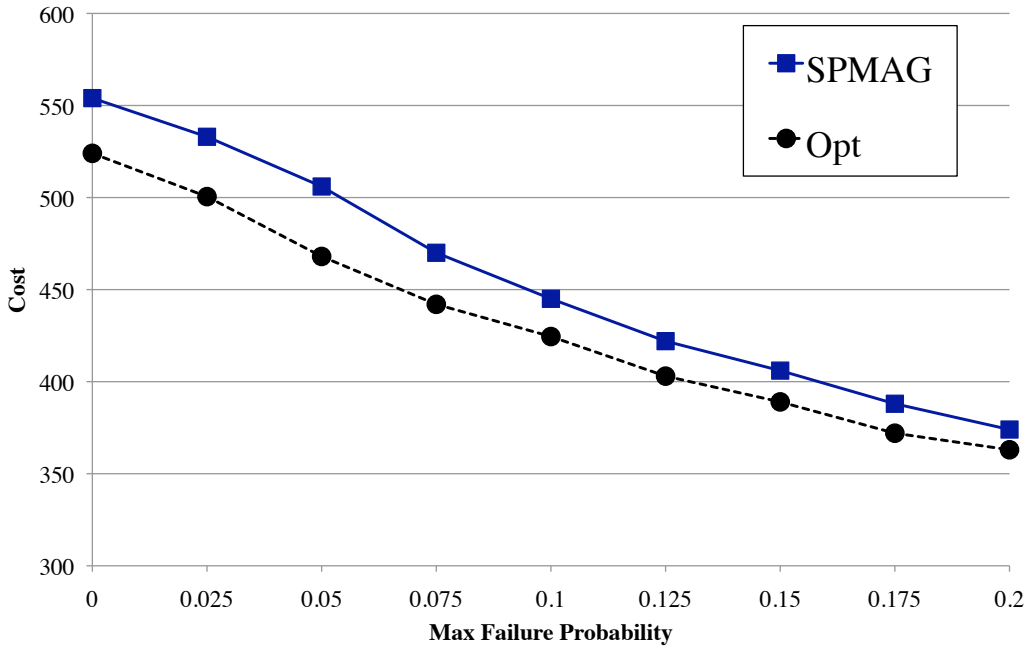


Figure 3-6: SPMAG capacity cost vs. MFP with $q = \frac{1}{2}$

that a solution cannot be ϵ-approximated, nor can there be any pseudo-polynomial algorithm for optimality [37].

Theorem 3.3. *The Singly Constrained Shortest Pair of Disjoint Paths problem is strongly NP-Hard.*

Proof. To demonstrate strong NP-Hardness of SCSPD, a reduction from the 3SAT problem [1] is performed. See Chapter Appendix Section 3.7.2 for the complete proof. □

Since SCSPD is strongly NP-Hard, the dynamic programming approach used to solve for $q = 0$ does not work when $q > 0$. We conjecture that the multiple availability guaranteed protection when $q > 0$ is in fact also strongly NP-Hard, thereby necessitating a heuristic approach to solve the problem. Our proposed heuristic augments the $q = 0$ algorithm: after an optimal solution for $q = 0$ is found, find the shortest disjoint path for the unprotected segments and allocate a flow of q to them. We call this algorithm the Segment Protected Multiple Availability Guaranteed Algorithm (SPMAG).

A simulation similar to that of Section 3.3.2 was run comparing SPMAG and the optimal solution to MAGP without flow bifurcation; the results are plotted in Figure 3-6. On average, SPMAG performs within 6% of the optimal solution to the multiple availability guaranteed protection problem.

3.5 Algorithm with Backup Capacity Sharing

In the previous section, efficient algorithms were presented without the use of backup capacity sharing, including an optimal algorithm for the case of $q = 0$, which corresponds to the standard availability constraint. These results are useful for a basic understanding of the Multiple Availability Guaranteed Protection problem (MAGP), and for networks that do not allow protection sharing. But many times, networks do utilize backup sharing, and significant savings can often be achieved. In this section, a time-efficient algorithm for MAGP using backup capacity sharing is presented.

If two primary flows for two different demands are edge-disjoint from one another, then under a single-link failure model, at most one can be disrupted at any given point in time. Since at most one demand will need to be restored after a failure, two failure-disjoint flows can share backup capacity.

Determining how much backup capacity can be shared for 1+1 guaranteed protection was examined in [18] and [19]. These papers used conflict sets to determine potential backup sharing on an edge by keeping track of how much backup capacity was allocated on one edge to protect against the failure of another. If more backup capacity is already allocated on some edge than is needed to protect for the failure of another edge, then that edge's backup capacity can be shared. This model can be extended to the partial protection framework by guaranteeing that any particular demand has its partial flow requirement met after a failure. An example without probabilistic availability guarantees is given in Fig. 3-7. Define the variable h_{ij}^{kl} to be the number of units of capacity used on edge $\{i, j\}$ to protect against the failure of edge $\{k, l\}$. The maximum number of units allocated on edge $\{i, j\}$ to protect against any edge failure is the total spare allocation needed on $\{i, j\}$. In Fig. 3-7,

two demands with $q^1 = 1$ and $q^2 = \frac{1}{2}$ are routed. Both demands use edge $\{i, j\}$ for protection with 1 unit being needed after the failure of $\{k, l\}$ and $\frac{1}{2}$ unit being needed after the failure of $\{m, n\}$. In this example we have $h_{ij}^{kl} = 1$ and $h_{ij}^{mn} = \frac{1}{2}$.

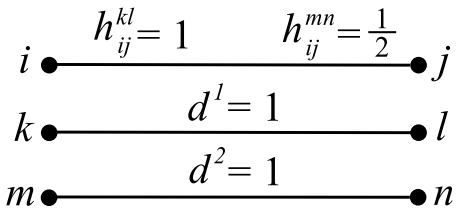


Figure 3-7: Example of a conflict set with partial protection

Now, consider a new demand with $q^3 = \frac{1}{2}$. If this demand were to have its primary flow routed on edge $\{k, l\}$ and use $\{i, j\}$ for protection, h_{ij}^{kl} will increase by $\frac{1}{2}$ unit. Since the amount of spare allocation on an edge is the maximum capacity needed to protect against any edge failure, the total allocation will increase by $\frac{1}{2}$. Alternatively, if the demand were to use $\{m, n\}$ instead of $\{k, l\}$, h_{ij}^{mn} will increase by $\frac{1}{2}$, and the maximum number of units needed to protect against any edge failure will still only be 1. No additional resources are required for protection on $\{i, j\}$ under this routing scenario. For the exact implementation of conflict sets for protection resource sharing, see [18, 19].

We now consider meeting probabilistic availability guarantees. Given some primary path between s and t, certain segments will be fully-protected, and others will be partially protected with a flow of q. For each edge in the primary path, the cost of using 1 or q units on edge $\{i, j\}$ for backup is calculated using conflict sets. For a primary path with a set of edges S, let $B(S, 1)$ be the cost of backup edges for fully protecting any edge, and $B(S, q)$ be the cost of backup edges that partially protect an edge with a flow of q.

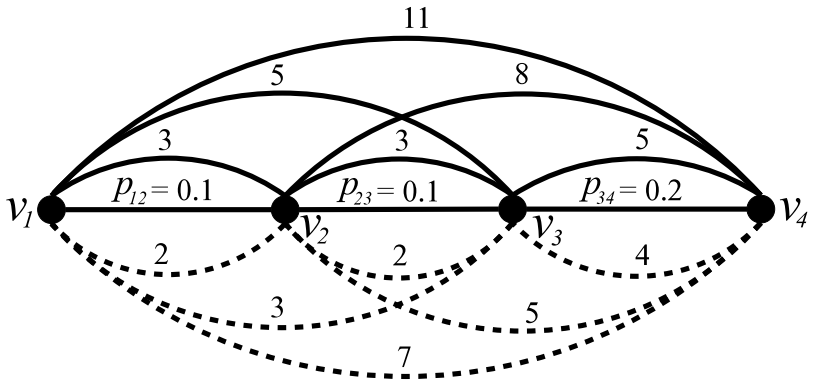
Next, we calculate the cost of protecting each possible segment of a given primary path with either full or partial protection; if there are v nodes in the primary path, then there are $\frac{v(v-1)}{2}$ segments contained within that path. We construct a new graph G_S^{st} with two edges between every pair of nodes in the primary path; these two edges correspond to fully or partially protecting the primary segment between nodes

i and j. For every primary segment in the primary path, we find two paths that are disjoint to that segment: one that fully protects that segment, and one that partially protects it. For full protection, the edge between nodes i and j in G_S^{st} is the shortest disjoint path to primary segment (i, j) using the set of edge costs $B(S, 1)$; for partial protection, the edge between i and j in G_S^{st} is the shortest disjoint path to primary segment (i, j) using $B(S, q)$. The cost of the edge in G_S^{st} to fully protect primary segment (i, j) is c_{ij}^1, and has failure probability $p_{ij}^1 = 0$. The cost of the edge in G_S^{st} to partially protect primary segment (i, j) is c_{ij}^q, and has failure probability p_{ij}^q equal to the failure probability of the primary path segment between nodes i and j. Once G_S^{st} is fully constructed, we find the constrained shortest path in G_S^{st} from s to t with a maximum failure probability of P^{st}. This path will be the backup, which meets all partial protection and availability requirements when combined with the initial primary path.

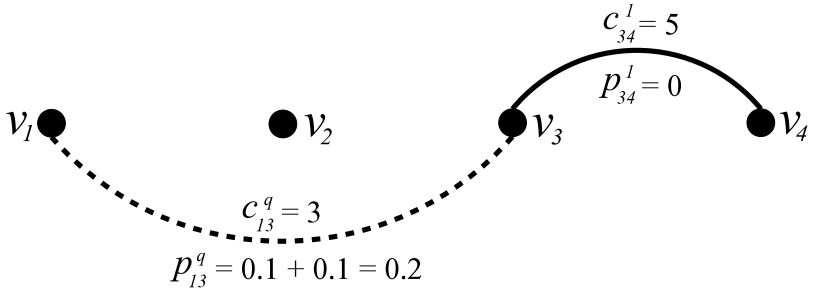
Since jointly optimizing the primary and protection path with backup sharing is NP-Hard [18], we choose a simple strategy of using the shortest path for the primary. This is in contrast to [18, 19] that offer a heuristic approach to jointly optimize the primary and protection path for each incoming demand. Our simulations (presented at the end of this section) show that using the shortest path for the primary route actually performs better than jointly optimizing the primary and backup paths for each incoming demand.

The algorithm for MAGP to route a demand using backup capacity sharing is as follows. For an arriving demand between s and t, find the shortest path between those two nodes; S^{st} will be the set of edges in that shortest path. We then construct the graph G_S^{st} using the backup capacity sharing procedure discussed above, and then we find the lowest-cost shared backup to meet protection and availability requirements. This algorithm is called Dynamic Multiple Availability Guaranteed Segment Protection (DMAGSP).

An example is shown in Fig. 3-8. A unit demand needs to be routed between v_1 and v_4, with a maximum failure probability of $P = 0.2$. For this example, the shortest path between v_1 and v_4 has already been found, $v_1 - v_2 - v_3 - v_4$. This shortest path



(a) All possible protection paths



(b) Final path chosen for backup

Figure 3-8: Example of algorithm with $P = 0.2$

is chosen to be the primary route; failure probabilities for the edges of that primary path are as labeled in the figure.

For demonstration purposes, the protection paths protecting each possible segment of the primary path are already computed, and are shown in Fig. 3-8a. For each segment of the primary path, two arcs are constructed between that segment's two end nodes: one that fully protects against a failure in that segment, having probability of failure $p_{ij}^1 = 0$, and one that q-protects that segment, with probability of failure p_{ij}^q equal to the sum of the edges' failure probabilities in that primary segment. The arcs above the primary path are the lowest-cost full protection paths for each segment of the primary, and the arcs below the path are the lowest-cost partial protection paths. Costs of protecting the primary path segments with 1 or q units of flow are labeled on the arcs.

The protection paths for each of the primary path segments (the arcs above and below the primary path), form the new graph G_S^{st}. The constrained shortest path

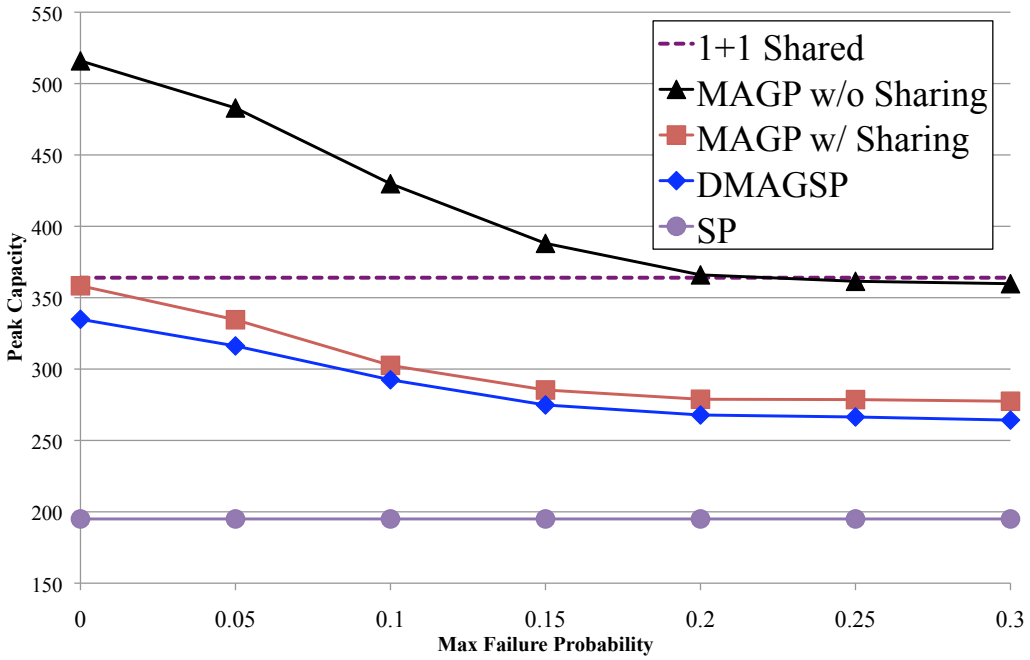


Figure 3-9: Peak capacity cost vs. MFP with $q = \frac{1}{2}$

algorithm is run on G_S^{st} between v_1 and v_4 with a maximum failure probability of P, which returns the final backup path. The backup path for this example with $P = 0.2$, as shown in Fig. 3-8b, meets all protection and availability requirements when combined with the primary path found previously. In this example, the segment between v_1 and v_3 is q protected, whereas the segment between v_3 and v_4 is fully protected.

A simulation similar to that of Section 3.3.2 was run, this time with demands arriving dynamically at random and serviced one-at-a-time in the order of their arrival. The protection requirement q for each demand is drawn from a truncated normal distribution with mean of $q = \frac{1}{2}$ and standard deviation $\sigma = \frac{1}{2}$. The maximum failure probability P has a truncated normal distribution with a standard deviation $\sigma = .025$; the mean of P is varied between 0 and 0.3. We compare multiple availability guaranteed protection with and without sharing (which jointly optimizes the primary and backup path for each incoming demand), DMAGSP, and 1+1 protection with sharing.

The capacity needed to route the demand and protection flows are plotted in Fig.

3-9 as a function of the expected value of P. Again, the shortest path routing without protection considerations is used as a lower bound for the capacity allocation. MAGP with backup sharing, which jointly optimizes the primary and backup paths for each incoming demand, achieves an average reduction in excess resources of 42% over $1+1$ protection with sharing, and a reduction of 51% over MAGP without sharing.

A notable result is that Dynamic Multiple Availability Guaranteed Segment Protection (DMAGSP) in fact performs *better* than the greedy optimal solution with dynamic arrivals. This can be explained by observing that the algorithm takes the simple strategy of the shortest path as the primary for each connection, as opposed to jointly optimizing the primary and backup routes, which may take a longer primary path to take advantage of backup sharing. This longer path makes it potentially more difficult for future demands to find disjoint primary routes, lowering their ability to share protection resources. While other works have focused on finding heuristics to jointly optimize the primary and backup paths for each incoming demand, it appears a better approach is to simply take the shortest path for the primary route.

3.6 Conclusion

In this chapter, a novel network protection scheme with multiple availability guarantees was introduced. In particular, the multiple availability guarantees will maintain the full demand for at least a guaranteed fraction of time and guarantee a partial flow during the downtime. If the demand is allowed to drop to 50% of its flow for only 1 out of every 20 failures, a 24% reduction in excess resources can be realized over the traditional $1+1$ full protection scheme. For the $q = 0$, which corresponds to the previously studied scenario where full availability is guaranteed for a fraction of time, we developed an optimal pseudo-polynomial algorithm. For the case of $q > 0$, we developed a time-efficient heuristic (Segment Protected Multiple Availability Guaranteed Protection) that performs within 6% of the optimal solution to the multiple availability guaranteed protection problem. We then extended the Multiple Availability Guaranteed Problem (MAGP) to the case where backup capacity sharing is utilized

to lower the total amount of resources needed to meet protection and availability requirements. An algorithm for MAGP with protection sharing was developed for dynamic arrivals, which in fact performs better than jointly optimizing the primary and backup paths for each incoming demand.

3.7 Chapter Appendix

3.7.1 Proof of NP-Hardness for Multiple Availability Guaranteed Protection

To demonstrate NP-Hardness of MAGP, the 1-0 knapsack problem [37] will be reduced to MAGP. The knapsack problem finds the maximum value subset of k items, with the i^{th} item having cost c_i and weight p_i, such that the maximum weight P of the knapsack is not exceeded.

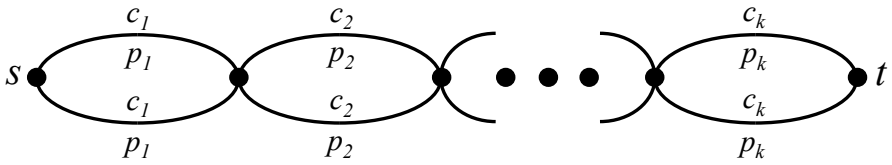


Figure 3-10: Sample network for MAGP NP-Hardness proof

Consider the network shown in Fig. 3-10 with link costs and probabilities denoted by c_i and p_i, respectively. We wish to find a minimum-cost routing for a unit demand from s to t with a maximum failure probability P and partial protection requirement $q = 0$. After any link failure, the network will either maintain its full flow of 1 unit, or have no flow with a probability of at most P. There are k distinct link groups, where each of the two links in any group have the same probability of failure and cost. Primary flow has to be allocated onto at least one of these links, otherwise the primary demand cannot be met. If the network maintains full connectivity after a primary failure in the k^{th} link group, then each link in that group will have an allocation of 1 unit. If there is no flow after a link failure, then only one link has an allocation of 1, and the other 0. So, every link group has at least one link with a flow

of 1, which is a fixed cost regardless of protection allocation.

To find the lowest cost protection allocation to meet availability guarantees, we need to find the lowest cost combination of the remaining links after the primary flow is allocated such that the sum of the failure probabilities for the links that have no allocation are less than P. Our objective is min $\sum_{i=1}^{k} c_i(1 - z_i)$, subject to the constraint of $\sum_{i=1}^{k} p_i z_i \leq P$, with $z_i \in \{0, 1\}\ \forall i \in 1, ..., k$. The objective can be rewritten to maximize the cost of the links that do not have allocation: min $\sum_{i=1}^{k} c_i(1 - z_i) = \max \sum_{i=1}^{k} c_i z_i$. We now recognize this to be the NP-Hard 1-0 knapsack problem with a maximum weight of P, and cost and weight of the i^{th} item being the cost and probability, respectively, of each pair of links in the i^{th} link group. If there existed a polynomial time solution to MAGP, then there would exist one for the 1-0 knapsack problem. Therefore, MAGP is at least as hard as the 1-0 knapsack problem. In addition, we note that our problem is clearly in NP.

3.7.2 Proof of Strong NP-Hardness for Singly Constrained Shortest Pair of Disjoint Paths

To prove that SCSPDP is strongly NP-Hard, we borrow a reduction that demonstrates the NP-Hardness of a different, but similar, problem [1] and adapt it to the SCSPD problem. The authors of [1] attempt to find the "min-min" disjoint pair of paths, which is defined as the minimum-cost pair of disjoint paths that contains, over all sets of possible disjoint paths, the minimum-cost shorter path.

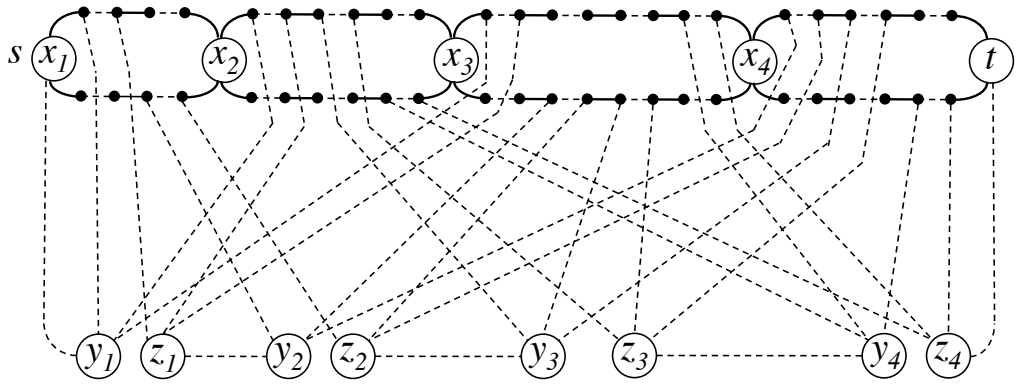


Figure 3-11: Sample network to solve an instance of $3SAT$ from [1]

To demonstrate this problem is NP-Hard, they construct a mapping of the $3SAT$ problem to a graph where a solution to their problem will simultaneously solve the $3SAT$ problem. A solution to $3SAT$ determines if there exists a 1/0 assignment to the variables that will make a specific boolean expression true [37]. The graph in Fig. 3-11 is a sample network corresponding to the instance of the $3SAT$ problem of $(x_1 \vee x_2 \vee x_3) \wedge (\hat{x}_1 \vee \hat{x}_3 \vee x_4) \wedge (x_2 \vee \hat{x}_3 \vee x_4) \wedge (\hat{x}_2 \vee x_3 \vee \hat{x}_4)$ [1]. Without going into the specifics of the reduction (see [1] for details), a generalized version of their result is: if two disjoint paths can be found between s and t such that one of them uses only the dotted lines, then that solution is also a solution to the $3SAT$ problem (see [1] for the proof). To demonstrate strong NP-Hardness, one needs to show the problem remains NP-Hard after the value of all inputs to the system have been bounded by some polynomial [37].

We will first show the problem to be NP-Hard by assigining costs and probabilities to the edges of the above network such that solving SCSPD will also solve the $3SAT$ problem. Then, we will demonstrate that SCSPD is in fact strongly NP-Hard. Assume there exists D dotted edges and L solid edges in the $3SAT$ reduced graph. Since we can assign parameters of our choosing to the edges, assign a cost of 0 for the dotted edges and a cost of 1 to the solid edges. We choose the failure probability of each dotted edge to be $\frac{\alpha}{D}$ and the probability of each solid edge to be $\frac{1-\alpha}{L}$, such that $\alpha \leq \frac{1-\alpha}{L} \rightarrow \alpha \leq \frac{1}{1+L}$. Additionally, choose a maximum probability of failure P such that $\alpha \leq P < \frac{1-\alpha}{L}$. Since using any solid edge will make that path violate the maximum failure probability P, the only feasible solution to SCSPD on this network is for the constrained path to use only dotted edges. But if such a solution could be found, it would solve the $3SAT$ problem, which is NP-Hard. The problem in fact remains NP-hard when all input values are polynomial bounded: L and D are polynomial bounded by the number of inputs from the $3SAT$ problem, and α can be chosen to be polynomial bounded. If all input parameters to a problem are bounded by some polynomial, and the problem remains NP-Hard, then the problem is *strongly* NP-Hard [37]. Finding any feasible solution to SCSPD on this bounded graph will still solve the 3SAT problem. Therefore, SCSPD is strongly NP-Hard.

Chapter 4

Protection with Guaranteed Recovery Times using Recovery Domains

In the previous Chapters 2 and 3 of this thesis, we considered offering alternatives to guaranteed full protection that allowed for a demand to be at a guaranteed level of partial service either after any service disruption, or only with a certain probability after a service disruption. Instead of the service guarantee of partial service, in this chapter, we consider the service guarantee of maximum recovery time after a failure.

4.1 Introduction

As the importance of time-sensitive internet traffic continues to rise, there is an increasing need to offer network protection that provides guarantees on the amount of time flow can be disrupted after a failure. Examples of time-sensitive traffic include voice-over-IP and video streaming, which would be rendered unusable with high latency delays. Many networks employ protection techniques that offer no recovery time guarantees whatsoever. Alternatively, networks typically provide local recovery schemes, which reroute a connection at the point of failure; such schemes typically over-provision resources to meet time recovery constraints. In this chapter, we a

present novel solution that provides guarantees on the maximum amount of time that flow can be disrupted after a failure, which is both flexible and efficient. We refer to these time guarantees as the recovery time of the network.

Protection in the internet has traditionally been accomplished using a real-time rerouting approach: after a link failure occurs, the network is updated with the new set of shortest paths between node pairs, and then a new path is selected. This is both slow (sometimes on the order of minutes) [11], and does not necessarily guarantee that bandwidth will be available for the new path [12, 13]. Despite its slow recovery time, this approach is still commonly used in backbone networks [8]. In the past decade, Multi-Protocol Label Switching (MPLS) has been developed to support constraint based routing, which allows connections to be made with guarantees on parameters such as bandwidth, latency, and recovery time [59]. Because of its flexibility and traffic engineering capabilities, MPLS has become the leading packet transport network technology in backbone networks [60]. To handle these fast recovery times, the Fast ReRoute (FRR) framework was developed to be used within MPLS [38]. FRR is a local recovery scheme where traffic is routed away from the node directly preceding a fault, which is known as the point of local repair (PLR), and reconnects with the original path at the merge point (MP). Various implementations of local recovery have been previously examined [61–65].

More recently, the new IETF standard for the MPLS Transport Profile (MPLS-TP) Protection Framework calls for the creation of "recovery domains" [39]. Recovery domains are defined to be non-overlapping path segments, such that after a failure within a segment, flow is restored using a back-up path between the end-points of that segment. Moreover, recovery domains connect to one another via their respective "reference" end-points, forming an end-to-end protected flow. An example is shown in Fig. 4-1: after the failure of an edge in the primary path located within Recovery Domain 2, the recovery domain's upstream end-point redirects flow onto the backup path, which then reconnects at that recovery domain's downstream end-point, by-passing the failure. The recovery domain model can be used to provide recovery time guarantees.

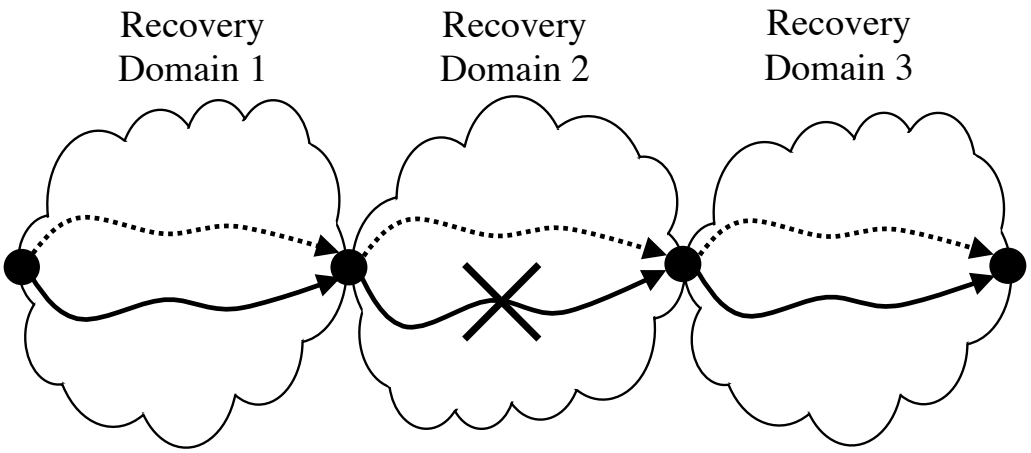


Figure 4-1: End-to-end routing using recovery domains

In local recovery, since each possible failure has its own dedicated protection path, resources are often over-provisioned beyond what is needed to meet recovery time guarantees. Consider the example shown Fig. 4-2; the propagation delay for each link is 10 ms, and switching delays are assumed to be negligible. A flow needs to be found from v_1 to v_5 such that the maximum time that the flow can be disripted after a failure is 50 ms, which is the typical recovery time for MPLS networks [11]. A primary path is already allocated on the solid lines from v_1 to v_5. A solution to FRR local recovery is to use all of the links above the primary path: after a link failure in the primary path, a fault notification is sent to the immediate upstream node of that failed link, and the flow is then switched to an alternate path from that node back towards the destination. This protection scheme requires 7 edges to be used for backup.

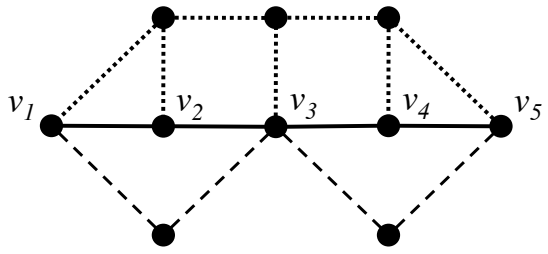


Figure 4-2: Time guaranteed recovery examples

Now consider an alternative protection routing using the recovery domain model.

Two recovery domains are created by using the links below the primary path as the backup paths: one recovery domain between nodes v_1 and v_3, and one between v_3 and v_5. If link $\{v_2, v_3\}$ fails, it would take up to 20 ms for the fault notification to propagate to v_1, and then 20 ms for the data that was switched to the protection route to reconnect with the primary path at node v_3. The other recovery domain will have a similar recovery time after a failure. In this example, only 4 additional links are needed to meet protection guarantees when using recovery domains, as opposed to the 7 needed for FRR.

Previous work has examined providing differentiated reliability for connections, including maximum recovery times. These papers build off of the segment protection framework [66], where segments of a primary path are individually protected, but can overlap by any number of edges. Various heuristics have been considered for segment protection with recovery time considerations [67–69], and an integer linear program (ILP) was presented in [70]. In contrast, recovery domains simplify recovery by separating a flow into disjoint protection regions, where each region guarantees protection for the flow between its end-points. This flow partitioning approach allows the network to be decomposed into a set of individual recovery domains, simplifying capacity allocation and flow control. To the best of our knowledge, the guaranteed recovery time problem using recovery domains has not yet been examined.

Our novel formulation to provide Guaranteed Recovery Times using Recovery Domains (GRT-RD) allows for a general and efficient set of solutions and algorithms. We first present a model of the problem in Section 4.2. We then show in Section 4.3 that the recovery domain problem is NP-Hard, and formulate the optimal solution using an MILP. In Section 4.4, we decompose the end-to-end recovery domain problem into more tractable subproblems, which allows us to more easily construct a solution for the end-to-end problem. This allows for the development of flexible and efficient solutions, including an optimal algorithm using Lagrangian relaxation, which simulations show to converge rapidly to an optimal solution. In Section 4.5, an algorithm is developed for the case when backup sharing is allowed.

4.2 Model and Problem Description

In this chapter, solutions to the problem of Guaranteed Recovery Time using Recovery Domains (GRT-RD) are developed and analyzed. The objective is to provide a primary and protection path for each demand, such that the amount of time flow is disrupted after a failure is guaranteed to be no greater than some maximum value. In order to meet these recovery time guarantees, we separate an end-to-end flow into "recovery domains", where within each recovery domain, the maximum time to recover from a failure cannot exceed a given value. We borrow the definition of recovery domains from [39]: "A recovery domain is defined between two recovery reference end-points which are located at the edges of the recovery domain... To guarantee protection in all situations, a dedicated recovery entity should be pre-provisioned using disjoint resources in the recovery domain, in order to protect against a failure of a working entity."

Adding the constraint for time guaranteed recovery, we implement GRT-RD as follows: an end-to-end recovery domain routing is a set of recovery domains connecting at their respective end-points such that they form a path from the source to the destination, and that the maximum amount of time a flow can be disrupted after any single-link failure is guaranteed. An example was shown in Fig. 4-1. We implement guaranteed protection within a recovery domain using the 1+1 protection scheme, which provides an edge-disjoint backup path for each primary path, and guarantees the full flow to be available at all times after any single-link failure [5, 18].

The following network model is used for the remainder of the chapter. A graph G has a set of vertices V and edges E. We assume a single-link failure model. An end-to-end recovery domain routing will comprise of a set of recovery domains, connected via their reference end-nodes, which form an end-to-end flow from a source to its destination. The maximum recovery time T will be the maximum time data flow can be interrupted: after a link failure of the primary path in some recovery domain, the length of time for the primary flow to be rerouted from the upstream reference end-node to the downstream reference end-node cannot exceed the maximum recovery

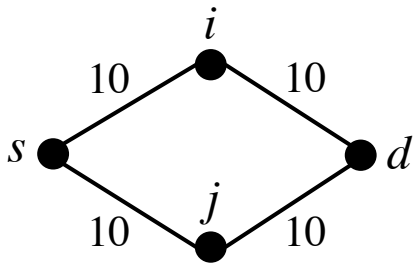


Figure 4-3: Recovery domain example

time of T. This includes the time for fault detection, switching delays, and the time necessary for the flow to reach the end-point of the recovery domain. For ease of exposition, we assume that the link traversal times include all the various switching and detection delays.

Consider the recovery domain in Fig. 4-3, with link traversal times (in ms) as labeled. After a failure occurs on link $\{j, d\}$, the failure will be seen at j in at most 10 ms, and then it will take another 10 ms for that information to reach s. The primary flow will take an additional 20 ms to reach d through i, which gives a total recovery time of 40 ms. Hence, for a recovery domain, the maximum recovery time will be the sum of all the link traversal times within that domain (which includes switching and detection delays).

4.3 A Minimum-Cost Formulation

This section investigates minimum-cost allocations to find a routing that offers protection after a failure with a guaranteed recovery time by using recovery domains (GRT-RD). Each edge $\{i, j\}$ will have an associated cost c_{ij} and link traversal time t_{ij}. The set of link costs and traversal times will be labeled C and $\mathcal{T}$, respectively. We begin by demonstrating that finding a minimum-cost end-to-end routing using recovery domains with recovery time guarantees is NP-Hard. Subsequently, in Section 4.3.1 a mixed integer linear program (MILP) is formulated to find a minimum-cost solution to the end-to-end recovery domain problem with recovery time guarantees. In Section 4.3.2, GRT-RD is compared to the common MPLS local recovery scheme

of Fast ReRoute (FRR).

We first show that solving for an individual recovery domain with a guaranteed recovery time is NP-hard. Afterwards, the end-to-end problem is shown to be NP-hard as well.

Theorem 4.1. *Finding the minimum-cost allocation for a pair of disjoint paths between nodes k and l such that the sum of the link traversals across the two paths does not exceed some maximum time T is NP-hard.*

Proof. We reduce the NP-hard Constrained Shortest Path (CSP) problem [29] to ours. In CSP, each edge $\{i, j\}$ has two associated values: c_{ij} and t_{ij}. In our case c_{ij} is the cost of the edge, and t_{ij} is the link traversal time. If x_{ij} is a binary flow variable for edge $\{i, j\}$, then the objective is to find a path from k to l that minimizes $\sum_{\{i,j\}\in E} c_{ij}x_{ij}$, such that $\sum_{\{i,j\}\in E} t_{ij}x_{ij} \leq T$. To find a minimum-cost solution to CSP by solving GRT-RD with a maximum recovery time of T, add a path between k and l with a total traversal time of zero and a cost of $-(\sum_{\{i,j\}\in E} |c_{ij}| + 1)$. A minimum-cost solution to GRT-RD, if it exists, will always use this path as one of the two disjoint paths, and the other path will be the solution for CSP from k to l with a maximum traversal time of T. In addition, we note that our problem is clearly in NP. □

Next, we show that finding an end-to-end routing using recovery domains that guarantees the maximum length of flow interruption is NP-hard by using a similar proof to Theorem 4.1. An end-to-end recovery domain routing can have a series of recovery domains in sequence, each guaranteeing recovery time for its respective flow.

Theorem 4.2. *Finding the minimum-cost allocation for a protection routing between nodes s and d that guarantees a maximum recovery time of T by using end-to-end recovery domains is NP-hard.*

Proof. We add a path between s and d with a total traversal time of zero and a cost of $-(\sum_{\{i,j\}\in E} |c_{ij}| + 1)$. Any minimum-cost solution to the end-to-end GRT-RD will use this negative cost path if a constrained shortest path (CSP) from s to d exists with

maximum traversal time of T. Hence, if the minimum-cost solution to the end-to-end problem uses this negative cost path, then the CSP problem has also been solved. If the negative cost path is not used, there exists no solution to CSP on the given network. In addition, we note our problem is clearly in NP. □

4.3.1 MILP to find a Minimum-Cost Solution

Since finding a minimum-cost solution for the guaranteed recovery time problem using recovery domains is NP-hard, in this section a mixed integer linear program (MILP) is developed to solve for the minimum-cost solution. Two versions are developed: one for when backup capacity sharing is not allowed, and one when it is. Backup capacity can be shared between two demands if their primary path edges are disjoint under a single-link failure model. If a failure occurs, and the two primary paths are disjoint, then at most one demand can fail. Hence, at most one demand will need backup protection at a time, and the two demands can share backup protection resources. Due to space constraints, only the MILP for the non-sharing case is presented here. The formulation for when protection sharing is allowed can be found in the Chapter Appendix Section 4.7.1.

Solving for an end-to-end protection routing between nodes s and d using recovery domains relies on the following observations: each pair of nodes in the network is a potential recovery domain, and an end-to-end recovery domain routing will be some subset of these recovery domains. Additionally, in an end-to-end recovery domain routing, recovery domains connect only via their reference end-points, and each recovery domain consists of a pair of edge-disjoint paths. Hence, an end-to-end recovery domain routing is, in fact, a pair of edge-disjoint paths between s and d (potentially connected at certain nodes). Using these observations, the MILP is structured as such: a pair of disjoint paths is found from s to d, where each edge from that pair of disjoint paths is associated with exactly one recovery domain. Additionally, any active recovery domain (i.e., part of the solution) must itself consist of a pair of disjoint paths between the end-nodes of that recovery domain, and the sum of the link traversal times of that recovery domain may not exceed the maximum recovery time.

Without loss of generality, a unit demand between s and d is assumed.

The following values are given:

- $G = (V, E)$ is the graph with a set of vertices and edges
- s is the source and d is the destination
- c_{ij} is the cost of link $\{i, j\}$
- t_{ij} is the traversal time for link $\{i, j\}$
- T is the maximum recovery time

The following variables will be solved for:

- x_{ij} is 1 if flow is assigned on link $\{i, j\}$, and 0 otherwise
- R^{kl} is 1 if the recovery domain with end nodes k and l is active (part of the solution), 0 otherwise
- r_{ij}^{kl} is 1 if link $\{i, j\}$ is in recovery domain (k, l), 0 otherwise

The objective is to minimize the total cost of allocating capacity for an end-to-end routing between s and d using recovery domains such that the maximum recovery time of T is not exceeded.

$$\text{minimize} \sum_{\{i,j\} \in E} c_{ij} x_{ij} \tag{4.1}$$

Subject to the following constraints:

- Find two edge-disjoint paths between the source and destination. Since x_{ij} is strictly 0 or 1, routing two units between s and d will result in two edge-disjoint paths.

$$\sum_{\{i,j\} \in E} x_{ij} - \sum_{\{j,i\} \in E} x_{ji} = \begin{cases} 2 & \text{if } i = s \\ -2 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases}, \ \forall i \in V \tag{4.2}$$

- If an edge has allocation, it belongs to exactly one recovery domain.

$$\sum_{(k,l)\in(V,V)} r_{ij}^{kl} = x_{ij}, \quad \forall\{i,j\} \in E \tag{4.3}$$

- Mark active recovery domains

  - If any edge in a recovery domain is active, then that recovery domain is marked as active.

$$\sum_{\{i,j\}\in E} r_{ij}^{kl} \leq |E| \cdot R^{kl}, \quad \forall(k,l) \in (V,V) \tag{4.4}$$

  - If no edge in a recovery domain is active, then that recovery domain is marked as not active.

$$\sum_{\{i,j\}\in E} r_{ij}^{kl} \geq R^{kl}, \quad \forall(k,l) \in (V,V) \tag{4.5}$$

- For each active recovery domain, find two edge-disjoint paths between its respective end-nodes k and l.

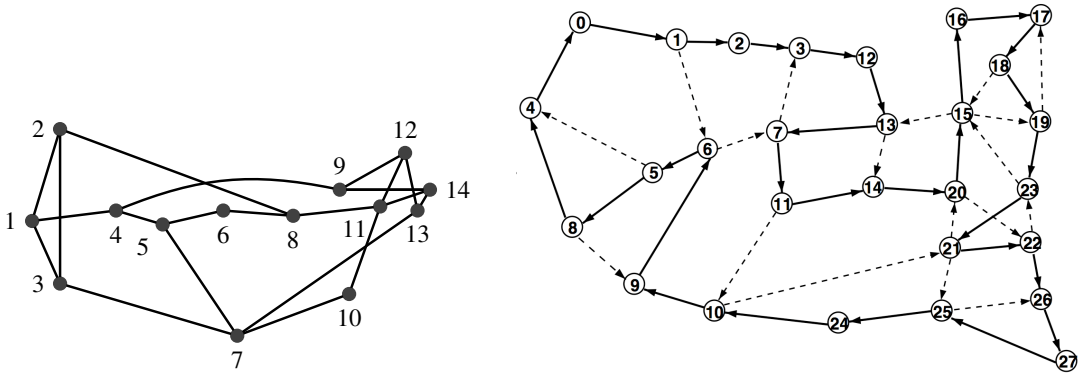
$$\sum_{\{i,j\}\in E} r_{ij}^{kl} - \sum_{\{j,i\}\in E} r_{ji}^{kl} = \begin{cases} 2R^{kl} & \text{if } i = k \\ -2R^{kl} & \text{if } i = l \\ 0 & \text{otherwise} \end{cases}, \quad \forall i \in V,\ (k,l) \in (V,V) \tag{4.6}$$

- The sum of the traversed time delays of the edges in a given recovery domain cannot exceed the maximum recovery time.

$$\sum_{\{i,j\}\in E} r_{ij}^{kl} t_{ij} \leq T, \quad (k,l) \in (V,V) \tag{4.7}$$

4.3.2 Simulation Results for GRT-RD

The minimum-cost solution found by the MILP for the guaranteed recovery time problem using recovery domains is compared to the common MPLS local recovery scheme of Fast ReRoute (FRR). The simulations were run using both the NSFNET and Lata 'X' topologies (Fig. 4-4) with 100 random unit demands. Each link's traversal time was set to be the propagation delay of that link, plus a 3 ms switching delay. Two versions of the network were tested: one with all unit costs, and one with random integer link costs of uniform distribution between 1 and 5. Both the non-sharing and sharing cases were tested. A dynamic model for routing demands was used: connections are serviced in the order of their arrival (in this case, the 100 demands were randomly ordered), and once a connection is routed, it can no longer be changed. The maximum recovery time was set to the MPLS standard of 50 ms [11]. Fast ReRoute (FRR) is implemented using an MILP, which can be found in Chapter Appendix Section 4.7.2.



(a) NSFNET (b) Lata 'X'

Figure 4-4: Network topologies used for simulations

Guaranteed recovery time using recovery domains (GRT-RD) is compared to Fast ReRoute (FRR) for the cases when protection resources can and cannot be shared. The additional cost of spare resources[1] needed to meet protection constraints for the two schemes are compared, with the percent savings of GRT-RD over FRR being

[1]Spare resources is the capacity needed beyond that of the shortest path routing to meet protection guarantees.

shown in Table 4.1.

	No Sharing		Sharing	
Edge Costs	NSFNET	Lata 'X'	NSFNET	Lata 'X'
Random	30%	27%	38%	45%
Deterministic	32%	35%	40%	36%

Table 4.1: Percent savings of spare resources of GRT-RD over FRR

As can be seen from Table 4.1, significant savings in the cost of protection resources over the MPLS local recovery scheme of Fast ReRoute are achieved when using GRT-RD. The savings for the case without backup sharing with random edge costs was 30% for NSFNET and 27% for Lata 'X'; with unit edge costs, the savings were 32% and 35% for NSFNET and Lata 'X', respectively. When backup sharing is allowed, the savings were larger: with random edge costs, the savings were 38% for NSFNET and 45% for Lata 'X', and with unit edge cost, 40% and 36%. Further discussion of why backup sharing is more flexible for recovery domain routing (and hence, larger potential savings over other protection schemes) can be found in in Section 4.5. Overall, GRT-RD offers significant savings in cost over FRR, while providing the same level of resiliency.

4.4 Efficient Algorithms for Guaranteed Recovery Times using Recovery Domains

In the previous section, an MILP was presented to find a minimum-cost solution for GRT-RD, which is not generally computationally efficient. In this section, efficient and flexible algorithms are presented to solve GRT-RD. A gradient algorithm is developed that converges rapidly to an optimal solution, and polynomial time heuristics are developed that offer bounds on their solution with respect to the optimal time and cost. In Section 4.4.1, we demonstrate how finding the optimal solution to the end-to-end recovery domain problem can be solved by decomposing the problem into a set of more tractable and easier to solve individual recovery domain problems. In

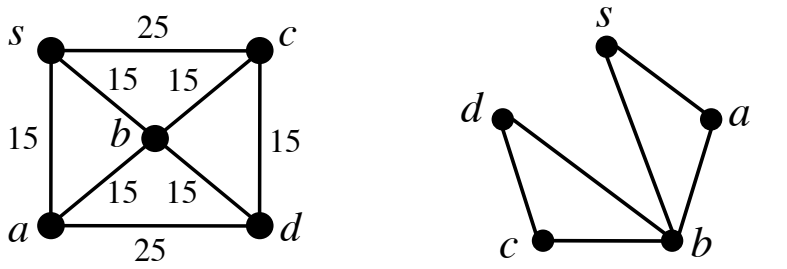
Section 4.4.2, a gradient algorithm using Lagrangian relaxation is developed, which simulations show converges rapidly to an optimal solution. In Section 4.4.3, polynomial timed heuristics that offer bounds with respect to the optimal solution are presented. The different algorithms developed are compared to the optimal solution found by the MILP in Section 4.4.4. Since it is NP-hard to simply determine if a feasible solution exists for the guaranteed protection problem when backup sharing is used [18], we first consider the case without backup sharing. Recovery domain routing that guarantees recovery times with the use of backup capacity sharing is examined in Section 4.5.

4.4.1 Decomposing the End-to-End Recovery Domain Problem

An optimal end-to-end recovery domain routing requires optimizing across the set of possible recovery domains such that a flow from the source to destination is found where the maximum amount of time the flow can be interrupted after a failure is guaranteed. Each individual recovery domain is wholly responsible for protecting against a failure between its respective end-points, and for ensuring that recovery time guarantees are met. This requirement not only allows for easier rerouting after a failure, but, as we demonstrate, allows the end-to-end problem to be simplified by considering only the more tractable individual recovery domains.

The key observation that allows flexibility for finding a solution is that each pair of nodes in the network marks the end-points of a potential recovery domain. Since an end-to-end recovery domain routing will be a series of recovery domains connected via their end-points, the optimal solution will be the lowest cost subset of individual recovery domains that form a flow from the source to the destination. We note that there are $O(|V|^2)$ potential recovery domains in a network.

Consider the network G in Fig. 4-5a, with link traversal times as labeled (in ms), and unit cost links. We wish to find a minimum-cost recovery domain routing from node s to d, with a maximum recovery time of 50 ms. For each pair of nodes in the network, a minimum-cost recovery domain is found. A new network G^R is constructed



(a) Original network G (b) Transformed network G^R

Figure 4-5: Decomposing G into its individual recovery domains

with the same set of nodes as the original network G, and with an edge being placed between any two nodes i and j if there exists a recovery domain between those nodes that meets recovery time guarantees. The cost of all the edges that comprise the recovery domain between nodes i and j is the cost of edge $\{i, j\}$ in G^R.

The graph transformation G^R is shown in Fig. 4-5b. An edge between a pair of nodes represents a recovery domain between those same two nodes in G that meets recovery time guarantees. For example, edge $\{s, a\}$ in G^R has a cost of 3, and is a recovery domain consisting of the following edge in G: $\{s, a\}$, $\{s, b\}$, and $\{b, a\}$. It is easy to verify that for this example, each link in G^R has a cost of 3. A shortest path is found in G^R from the source s to the destination d, where each edge of that path represents a recovery domain that is used in the optimal end-to-end recovery domain solution. The shortest path from s to d in G^R is $\{s, b\}$ and $\{b, d\}$, which represents the recovery domains in G between s and b (consisting of the edges $\{s, a\}$, $\{a, b\}$, and $\{s, b\}$ in G), and b and d (consisting of the edges $\{b, c\}$, $\{c, d\}$, and $\{b, d\}$ in G).

This algorithm is labeled `end_to_end_RD`. We note that this approach does not preclude two recovery domains from sharing edges between them, but the essence of recovery domain routing is preserved: the end-nodes of a recovery domain maintain full responsibility for protecting against a failure for the primary flow contained within it, and for guaranteeing that the amount of time that flow can be interrupted does not exceed the maximum allowed. This allows us to now focus on solving the individual recovery domain problem, i.e., for a given a pair of nodes, finding the shortest-pair of disjoint paths between those nodes that meet time constraints. With such an

algorithm at hand, one can use the above approach to solve for the end-to-end recovery domain routing with guaranteed recovery times.

4.4.2 Optimal Algorithm

In this section, an optimal algorithm using a Lagrangian relaxation is presented. The individual recovery domain problem is first formulated as an MILP, where we wish to find a minimum-cost pair of disjoint paths between a source s and destination d that have some maximum total link traversal time. The MILP is formulated as such: the objective is to find a routing of minimum cost, such that two units of flow are routed between the end-nodes s and d, with flow variables x_{ij} being binary, and the total link traversal time not exceeding the maximum recovery time of T. Since an edge will have strictly a flow of 0 or 1, two paths between s and d cannot overlap; hence, a minimum-cost pair of disjoint paths will be found subject to the time constraints. We refer to this as the constrained shortest-pair of disjoint paths problem.

$$\text{minimize} \sum_{\{i,j\}\in E} c_{ij}x_{ij} \tag{4.8}$$

$$\sum_{\{i,j\}\in E} x_{ij} - \sum_{\{j,i\}\in E} x_{ji} = \begin{cases} 2 & \text{if } i = s \\ -2 & \text{if } i = d\ ,\ \forall i \in V \\ 0 & \text{o.w.} \end{cases} \tag{4.9}$$

$$\sum_{\{i,j\}\in E} t_{ij}x_{ij} \leq T \tag{4.10}$$

$$x_{ij} \in \{0,1\}, \quad \forall\{i,j\} \in E \tag{4.11}$$

The Lagrangian dual is obtained by relaxing the maximum recovery time con-

straint (Constraint 4.10), and placing it into the objective:

$$\begin{aligned} L(\mu) &= \min \sum_{\{i,j\}\in E} c_{ij}x_{ij} + \mu\Bigg(\sum_{\{i,,j\}\in E} t_{ij}x_{ij} - T \Bigg) \\ &= \min \sum_{\{i,j\}\in E} x_{ij}(c_{ij} + \mu t_{ij}) - \mu T \end{aligned} \tag{4.12}$$

For a fixed value of μ, the cost of edge $\{i, j\}$ becomes $c_{ij} + \mu t_{ij}$. The recovery domain problem now becomes a *minimum-cost flow* problem, which is defined as finding a flow of lowest cost between a source and destination in a network that has both edge costs and edge capacities [29]. An important characteristic of minimum-cost flows is when given strictly integer inputs (edge costs and capacities), then the solution will always be a set of integer flows [29]; the flow variables an be relaxed of their integer constraints and still guarantee a solution with strictly integer flows (integer constraints are not necessary for integer flows). Hence, by relaxing the binary flow variables x_{ij}, we can write the Lagrangian dual as a linear program, which becomes polynomial time solvable [28].

$$\sum_{\{i,j\}\in E} x_{ij} \; - \sum_{\{j,i\}\in E} x_{ji} = \begin{cases} 2 & \text{if } s = i \\ -2 & \text{if } t = i \\ 0 & \text{o.w.} \end{cases}, \quad \forall i \in V$$

$$0 \leq x_{ij} \leq 1, \quad \forall \{i, j\} \in E$$

For a fixed value of μ, the Lagrangian relaxation simply becomes finding the shortest-pair of disjoint paths with respect to the edge costs $c_{ij} + \mu t_{ij}$, $\forall \{i, j\} \in E$. The shortest-pair of disjoint paths can be found in polynomial time using Suurballe's algorithm [34].

To solve the Langrangian relaxation, we wish to find $L^* = L(\mu^*) = \max_{\mu\geq 0} L(\mu)$. The constrained shortest-pair of disjoint paths has similarities to the constrained shortest path (CSP) problem, for which Lagrangian relaxation techniques have been considered [32, 72–74]. In [32], a geometric approach is proposed for solving the

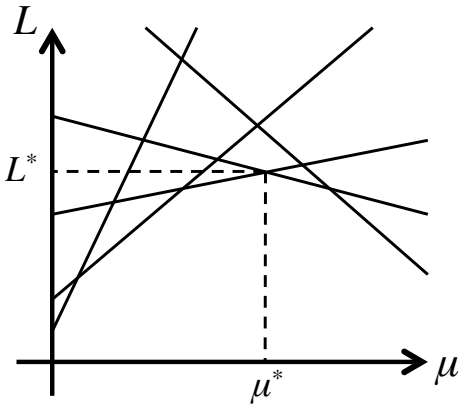


Figure 4-6: Pair of disjoint paths mapped to lines in $L - \mu$ space

Lagrangian relaxation for CSP. While not considered beyond a single constrained path, we demonstrate that the geometric approach can be extended to the constrained shortest-pair of disjoint paths problem. $L(\mu)$ can be rewritten as $L(\mu, \mathbf{x}) = f(\mathbf{x}) + \mu g(\mathbf{x})$, where $f(\mathbf{x}) = \sum_{\{i,j\} \in E} c_{ij} x_{ij}$ and $g(\mathbf{x}) = \sum_{\{i,j\} \in E} t_{ij} x_{ij} - T$. Each path becomes a line in $L - \mu$ geometric space, where $g(\mathbf{x})$ is the slope of the line, and $f(\mathbf{x})$ is the L axis crossing point. Furthermore, since $g(\mathbf{x}) = \sum_{\{i,j\} \in E} t_{ij} x_{ij} - T$, the paths associated with $g(\mathbf{x}) \leq 0$ meets time requirements, where $f(\mathbf{x})$ is the cost of those disjoint paths.

We see that the mapping to a geometric space is in fact not specific to CSP; any discrete optimization problem that takes the form $L(\mu, \mathbf{x}) = f(\mathbf{x}) + \mu g(\mathbf{x})$ can be represented as a line in $L - \mu$ space. In our case, each pair of disjoint paths becomes a line in $L - \mu$ space, and the lower envelope of lines gives the optimal value for μ^*. A visualization is shown in Fig. 4-6 [74].

Since any discrete optimization problem taking the form of $L(\mu, \mathbf{x}) = f(\mathbf{x}) + \mu g(\mathbf{x})$ can be represented in geometric space, including the constrained shortest-pair of disjoint paths, a similar method for finding the optimal μ^* as in [32] can be applied. The key is being able to solve for the optimal discrete variable assignments $\mathbf{x}$ for a fixed μ in polynomial time. In the case of constrained shortest-pair of disjoint paths, Suurballe's algorithm [34] can be used, as previously discussed. The algorithm to find the dual optimal solution is labeled `rd_dual_opt`, and is presented in Algorithm 1.

Algorithm 1 $\mu^* = \texttt{rd\_dual\_opt}(s, d, T, V, E, C, \mathcal{T})$

1: Begin with two pairs of disjoint paths from s to d: $L(0)$ is the shortest pair without consideration to time, and $L(\infty)$ is the shortest time pair, without consideration to cost.
2: The intersection point is the first guess for μ (call this μ'). $L(\mu')$ is the new upper bound on the dual-optimal solution. Any intersection point will be defined by two lines: one with $g(\mathbf{x}_1) \leq 0$ and the other $g(\mathbf{x}_2) > 0$. Since $g(\mathbf{x}) = \sum_{\{i,j\} \in E} t_{ij} x_{ij} - T$, the disjoint paths associated with $g(\mathbf{x}_1) \leq 0$ meets the time requirements.
3: Solve for the shortest-pair of disjoints paths with edge costs $c_{ij} + \mu' t_{ij}, \forall \{i, j\} \in E$. This will give a new pair of disjoint paths $\mathbf{x}$", with some value for $f(\mathbf{x}'')$ and $g(\mathbf{x}'')$.
4: If $f(\mathbf{x}'') + \mu' g(\mathbf{x}'') = L(\mu')$, then the optimal value for μ has been found.
5: **while** $f(\mathbf{x}'') + \mu' g(\mathbf{x}'') \neq L(\mu')$ **do**
6: Add a new line $f(\mathbf{x}'') + \mu' g(\mathbf{x}'')$ in $L - \mu$ space.
7: The next estimate for μ' is the new intersection point on the lower envelope of the lines that gives the max $L(\mu')$.
8: If $f(\mathbf{x}'') + \mu' g(\mathbf{x}'') = L(\mu')$, then the optimal value for μ has been found.
9: **end while**
10: Return $\mu^* = \mu'$. The pair of dual optimal paths $\mathbf{x}^*$ at μ^* that meet time constraints are those associated with $g(\mathbf{x}^*) \leq 0$.
The algorithm concludes with an upper and lower bound on solution. Since $f(\mathbf{x}^*)$ represents the cost for feasible paths in the network that meet time constraints, it is an upper bound on the optimal solution. The lower bound is the dual-optimal value $L(\mu^*) = f(\mathbf{x}^*) + \mu^* g(\mathbf{x}^*)$.

The runtime for `rd_dual_opt` is not specified in [32], but a binary search approach for solving the same problem was presented in [74] with a polynomial runtime.

Since the original problem we are trying to solve has integer constraints (the flow variables), the dual optimal solution may have a gap in cost between itself and the actual optimal solution, which is known as a duality gap [28]. To close this gap, we iterate through the k-shortest pair of disjoint paths with respect to the dual-optimal edge costs $c_{ij} + \mu^* t_{ij}$, $\forall \{i, j\} \in E$, closing the gap with each iteration until the optimal solution is found. The authors of [32] step through the k-shortest paths, using an algorithm [75] that cannot be extended to the k-shortest pair of disjoint paths. Instead, we use an alternate technique proposed in [76], which finds the k best solutions for a discrete optimization problem. The run time to find the k^{th} best discrete optimization solution is polynomial with respect to the time to solve the discrete optimization problem, which for the case of shortest-pair of disjoint paths

is also polynomial [34]. Details of the algorithm are omitted for brevity, and can be found in [76]. We label the final algorithm, which closes the duality gap, `rd_opt`; it is presented in Algorithm 2.

Algorithm 2 $(P_1, P_2) = \texttt{rd\_opt}(s, d, T, V, E, C, \mathcal{T})$

1: Find the initial dual optimal solution:
$\mu^* = \texttt{rd\_dual\_opt}(s, d, T, V, E, C, \mathcal{T})$

2: Find the k^{th} shortest pair of disjoint paths $\mathbf{x}^k$ for each $k = 1, 2, ...$ with respect to the dual-optimal edge costs $c_{ij} + \mu^* t_{ij}$.

- For each k, $L(\mu^*, \mathbf{x}^k)$ is the new lower bound on the optimal feasible solution, and is a non-decreasing function with respect to k.
- An upper bound U^k is maintained: $U^k = \min_{1..k} f(\mathbf{x}^k)$ for all $\mathbf{x}^k$ such that $g(\mathbf{x}^k) \leq 0$ (which means that $\mathbf{x}^k$ is a solution that meets time constraints). U^k is a non-increasing upper bound.

3: Continue until $U^k \leq L(\mu^*, \mathbf{x}^k)$, at which point the optimal solution has been found.

- Since we individually step through the shortest-pair of dual-optimal disjoint paths, which are a lower bound on the solution, until their cost is greater than the upper bound, which is a feasible solution, the solution must be optimal. Proof is shown in [32].

4: Return the pair of disjoint paths P_1 and P_2 associated with the upper bound U^k, which meets time constraints.

Since the number of possible disjoint paths can be potentially exponential with respect to the number of edges, the number of iterations to close the duality gap is not necessarily polynomial bounded. But, our simulations indicate that the number of iterations to close the duality gap is in fact minimal: on average, only 1.46 iterations are needed.

4.4.3 Polynomial Timed Heuristics

In this section, two algorithms are presented that run in polynomial time to solve the guaranteed recovery time problem using recovery domains. The first is a "fastest paths" algorithm, where link costs are ignored, and paths are found with respect to time only. This algorithm is the simplest to implement, and is most useful when link

costs are either not considered, or are proportional to the link traversal times. The second is a fully polynomial time approximation scheme (FPTAS) that guarantees a solution to be within a factor of 1.5 of the optimal cost and $1.5(1+\epsilon)$ of the maximum recovery time.

The fastest paths algorithm is straightforward to implement using the shortest-pair of disjoint paths algorithm [34]. Instead of finding a pair of disjoint paths of minimum-cost, a pair of disjoint paths of minimum time are found. This algorithm is called `rd_fastest`, and has the same complexity as finding the shortest-pair of disjoint paths.

For the approximation scheme, we use an algorithm presented in [57]. In that work, the authors try to solve for disjoint QoS (quality-of-service) paths, such that each path is bounded by some time requirement D. They are not looking at recovery times or recovery domains, but are instead interested in ensuring that the end-to-end primary and backup paths do not exceed some QoS specification. To solve their problem, they relax each path's individual time constraint and try to find a pair of disjoint paths such that the sum of the link traversal times for both paths does not exceed $2D$. By replacing the time requirement $2D$ with the maximum recovery time requirement T, we can solve for a recovery domain with a bound on the optimal cost and time. We label this approximation algorithm `rd_approx`; details of the algorithm are omitted for brevity, and can be found in [57].

4.4.4 Algorithm Simulations

In this section, the algorithms developed in the previous sections for guaranteed recovery times using recovery domains are compared to the end-to-end optimal solution found by the MILP from Section 4.3. A similar simulation to Section 4.3.2 was run, using the Lata 'X' topology (Fig. 4-4b). Table 4.2 shows the percent that each of the algorithms differed in total cost of resources used over the minimum-cost solution found by the MILP.

As expected, the optimal algorithm `rd_opt` did not differ from the optimal solution found by the MILP. Additionally, the average number of iterations needed to

Protection Scheme	Unit Edge Costs	Random Edge Costs
rd_opt	0	0
rd_fastest	1%	6%
rd_approx	5%	2%

Table 4.2: Difference for the algorithms from optimal

close the duality gap was 1.46 over all simulated demands. Both the fastest paths and approximation algorithm also performed close to optimal. When edge costs were uniform, the fastest paths approach in fact gave slightly better results. But when edge costs were changed to be random, the approximation algorithm performed better since it tries to optimize with respect to cost, and the fastest paths algorithm does not.

4.5 Algorithm with Backup Capacity Sharing

In the previous section, efficient algorithms that offer bounds with respect to the optimal solution were presented without the use of backup capacity sharing. These results are useful for a basic understanding of the guaranteed recovery time problem using recovery domains (GRT-RD), and for networks that do not allow protection sharing. But many times, networks do utilize backup sharing, and significant savings can often be achieved. In this section, a time-efficient algorithm for GRT-RD using backup capacity sharing is presented.

If two primary flows for two different demands are edge-disjoint from one another, then under a single-link failure model, at most one can be disrupted at any given point in time. Since at most one demand will need to be restored after a failure, two failure-disjoint flows can share backup capacity. An interesting feature of recovery domain routing is that two demands can share backup capacity even if their two primary flows are not failure disjoint. Traditionally, in path protection schemes, two paths can only share protection resources if their primary paths are disjoint. However, in the recovery domain setting, sharing can take place between two recovery domains, so long as the primary segments in those recovery domains are disjoint. Thus, end-to-end primary

paths that are not entirely disjoint may still share backup resources.

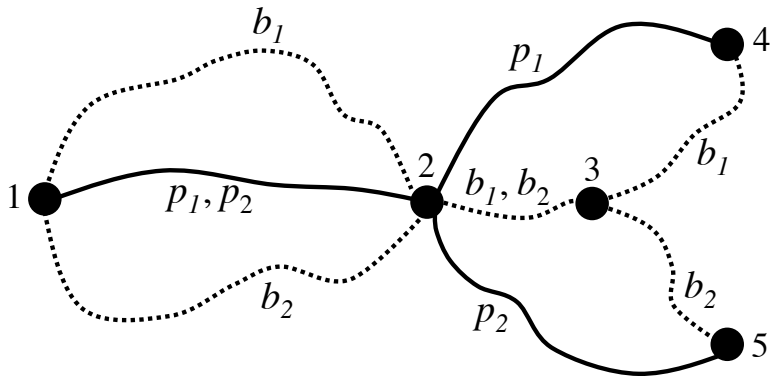


Figure 4-7: Sharing protection resources in a recovery domain

An example of an end-to-end recovery domain routing for two demands is shown in Fig. 4-7. Demand 1 is routed from node 1 to node 4, and demand 2 is routed from node 1 to node 5. The primary paths are labeled p_1 and p_2 for demands 1 and 2, respectively, and backup paths are labeled b_1 and b_2. Two recovery domains are used for each demand: demand 1 uses recovery domains with the end-points of $(1, 2)$, and $(2, 4)$; demand 2 uses recovery domains with the end-points of $(1, 2)$, and $(2, 5)$. The two primary paths overlap in the first recovery domain on the path segment between nodes 1 and 2; hence, they cannot share protection resources within that domain, and each demand has its own dedicated backup path. The primary segments for the two recovery domains starting at node 2 and going to their respective destinations are failure disjoint. Even though the two primary paths overlap between nodes 1 and 2, the two demands can share protection resources for their recovery domains after node 2. On the path segment between nodes 2 and 3, only one unit of backup capacity allocation is needed to protect against a failure for either demand's primary path in recovery domains $(2, 4)$ or $(2, 5)$.

Since a failure is local to a recovery domain, and the primary flow outside of that recovery domain is not affected, a similar approach can be used as was done previously for the algorithms without backup capacity sharing: for every pair of nodes, we find the recovery domain routing that guarantees recovery time and utilizes backup capacity sharing. Then, an end-to-end recovery domain routing is constructed from

a subset of those recovery domains using `end_to_end_RD` (Section 4.4.1).

Conflict sets are used to determine how much backup capacity can be shared by each incoming demand [18]. A conflict set indicates how much backup sharing is possible on an edge by examining how much backup capacity it already has to protect against any particular edge failure. If some edge has more backup capacity already assigned to it than is needed to protect against a particular edge failure, then those resources can be used at no additional cost. For example, let some edge $\{i, j\}$ have one unit of backup capacity allocated to it to protect against the failure of $\{k, l\}$, and with edge $\{i, j\}$ not being scheduled to protect against any other link failures. Now consider some new connection with a primary flow that uses some other edge $\{u, v\}$. Edges $\{k, l\}$ and $\{u, v\}$ can never fail simultaneously under a single-link failure model; thus, the new connection can use the backup capacity allocated to $\{i, j\}$ for protecting against the failure of $\{u, v\}$ without incurring additional cost. Further details of protection routing using conflict sets can be found in [18].

We consider routing demands dynamically (one-at-a-time), where once a connection is routed, it can no longer be changed; this model is similar to those used in path protection schemes [18, 68]. These path protection schemes offer heuristics to jointly optimize the primary and backup path for each incoming demand. We instead choose the simple strategy of using the shortest path for the primary route. After the primary path is found, the cost to use the remaining edges to protect that path can be determined (i.e., find out if edges can utilize protection resource sharing at no additional cost). If the maximum recovery time is T, and the shortest path has traversal time t_s, then a backup path is a constrained shortest path (CSP) that has a traversal time of at most $(T - t_s)$. To solve for the CSP, an optimal solution can be found in pseudo-polynomial time [56]; if T is rational and polynomial bounded with respect to the input parameters, the algorithm becomes polynomial. We label this algorithm `rd_sharing`. Our simulations show that using the shortest path for the primary route in fact performs better than jointly optimizing the primary and backup paths for each incoming demand.

To test the performance of `rd_sharing`, a similar simulation to Section 4.3.2

Protection Scheme	NSFNET	Lata 'X'
Local Recovery with Sharing	583	3459
Greedy Optimal Recovery with Sharing	351	2677
End-to-end `rd_sharing`	349	2605

Table 4.3: Cost of allocation for different protection schemes

was run. The simulations were run using both the NSFNET and Lata 'X' topologies (Fig. 4-4) with 75 random unit demands. Each link's traversal time was set to be the propagation delay of that link, plus a 3 ms switching delay. Edges have random integer cost with uniform distribution between 1 and 5, and the maximum recovery time is set to the MPLS standard of 50 ms [11]. Three different schemes were tested: the optimal recovery domain routing with sharing, which jointly optimizes the primary and backup path for each incoming demand (using the MILP from Chapter Appendix Section 4.7.1), local recovery (FRR) with sharing (found in Chapter Appendix Section 4.7.2), and the end-to-end `rd_sharing` algorithm developed in this section. A dynamic model was used: connections are serviced in the order of their arrival, and once a connection is routed, it can no longer be changed. Table 4.3 shows the total cost of allocation needed to route all 75 demands using the aforementioned protection schemes.

As anticipated, the optimal recovery scheme performs better than the local recovery scheme. Interestingly, the end-to-end `rd_sharing` algorithm performs *better* than the supposed optimal recovery with sharing. This can be explained by observing that the algorithm takes the simple strategy of the shortest path as the primary for each connection. This is as opposed to the optimal recovery scheme that jointly optimizes the primary and backup routes for each incoming demand, which may take a longer primary path to take advantage of backup sharing. By greedily optimizing every incoming demand, the potentially longer primary path makes it more difficult for future demands to find failure disjoint routes, lowering their ability to share protection resources, and thus increasing the overall cost.

4.6 Conclusion

In this chapter, we examined the problem of providing network protection with guaranteed recovery times using recovery domains (GRT-RD). The network is partitioned into failure-independent "recovery domains", where within each domain, the time to recover from a failure is guaranteed. To meet these guarantees, we provide an optimal solution in the form of an MILP. We demonstrate that the network-wide optimal solution can be decomposed into a set of more tractable and easier to solve subproblems. This allows for the development of flexible and efficient solutions, including an optimal algorithm using Lagrangian relaxation, which simulations show to converge rapidly to an optimal solution. Low complexity heuristics are developed for both with and without backup sharing. For dynamic arrivals, the algorithm utilizing backup sharing and using shortest paths performs better than the solution that greedily tries to optimize for each incoming demand.

4.7 Chapter Appendix

4.7.1 Guaranteed Recovery Time using Recovery Domains with Backup Capacity Sharing

The following values are given:

- V is the set of vertices, and E is the set of edges
- f^{sd} is the amount of flow that need to be routed from s to d, assumed to be integer
- c_{ij} is the cost of link $\{i, j\}$
- t_{ij} is the traversal time for link $\{i, j\}$
- T is the maximum recovery time

The following variables will be solved for:

- x_{ij}^{st} is 1 if flow is assigned on link $\{i,j\}$ or demand (s,t), and 0 o.w.
- $p_{ij,kl}^{st}$ is 1 if protection flow is assigned on link $\{i,j\}$ after the failure of link $\{k,l\}$ for demand (s,t), and 0 o.w.
- $y_{ij,kl}^{st}$ is 1 if spare capacity is assigned on on link $\{i,j\}$ for failure of link $\{k,l\}$ for demand (s,t) , and 0 o.w.
- w_{ij} is total primary flow on link $\{i,j\}$, $w_{ij} \geq 0$
- s_{ij} is total spare allocation on link $\{i,j\}$, $s_{ij} \geq 0$
- R_{kl}^{sd} is 1 if the recovery domain with end nodes k and l for demand (s,d) is active (part of the solution), 0 o.w.
- $r_{ij,kl}^{sd}$ is 1 if link $\{i,j\}$ is in recovery domain (k,l) for demand (s,d), 0 o.w.

The objective is to:

- Minimize the cost of allocation over all links:

$$\min \sum_{\{i,j\}\in E} c_{ij}(w_{ij} + s_{ij}) \tag{4.13}$$

Subject to the following constraints:

- Route primary traffic between s and d:

$$\sum_{\{i,j\}\in E} x_{ij}^{sd} - \sum_{\{j,i\}\in E} x_{ji}^{sd} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d\,, \\ 0 & \text{o.w.} \end{cases} \quad \begin{matrix} \forall i \in V \\ \forall (s,d)\in (V,V) \end{matrix} \tag{4.14}$$

- Route flow to protect after the failure of link $\{k,l\}$:

$$\sum_{\substack{\{i,j\}\in E \\ \{i,j\}\neq\{k,l\}}} p_{ij,kl}^{sd} - \sum_{\substack{\{j,i\}\in E \\ \{j,i\}\neq\{k,l\}}} p_{ji,kl}^{sd} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d\,, \\ 0 & \text{o.w.} \end{cases} \quad \begin{matrix} \forall i \in V,\ \forall \{k,l\}\in E \\ \forall (s,d)\in (V,V) \end{matrix} \tag{4.15}$$

- Primary capacity on link $\{i, j\}$ must meet all primary flows before a link failure

$$\sum_{(s,d)\in(V,V)} f^{sd} x_{ij}^{sd} \leq w_{ij}, \ \forall \{i, j\} \in E \tag{4.16}$$

- Primary and spare capacity on link $\{i, j\}$ must be sufficient for all protection flows after failure of link $\{k, l\}$:

$$p_{ij,kl}^{sd} \leq x_{ij}^{sd} + y_{ij,kl}^{sd}, \quad \begin{matrix} \forall \{i,j\}\in E, \ \forall \{k,l\}\in E \\ \forall (s,d)\in(V,V) \end{matrix} \tag{4.17}$$

- Spare capacity on link $\{i, j\}$ satisfies all protection flows after failure of link $\{k, l\}$:

$$\sum_{(s,d)\in(V,V)} f^{sd} y_{ij,kl}^{sd} \leq s_{ij}, \quad \begin{matrix} \forall \{i,j\}\in E \\ \forall \{k,l\}\in E \end{matrix} \tag{4.18}$$

- Mark active recovery domains

  - If any edge in a recovery domain is active, then that recovery domain is active

$$\sum_{\{i,j\}\in E} r_{ij,kl}^{sd} \leq |E| \cdot R_{kl}^{sd}, \quad \begin{matrix} \forall (k,l)\in(V,V) \\ \forall (s,d)\in(V,V) \end{matrix} \tag{4.19}$$

  - If no edge in a recovery domain is active, then that recovery domain is not active

$$\sum_{\{i,j\}\in E} r_{ij,kl}^{sd} \geq R_{kl}^{sd}, \quad \begin{matrix} \forall (k,l)\in(V,V) \\ \forall (s,d)\in(V,V) \end{matrix} \tag{4.20}$$

- For each active recovery domain, find two disjoint paths between its respective

end nodes k and l

$$\sum_{\{i,j\}\in E} r^{sd}_{ij,kl} - \sum_{\{j,i\}\in E} r^{sd}_{ji,kl} = \begin{cases} 2R^{sd}_{kl} & \text{if } i = k \\ -2R^{sd}_{kl} & \text{if } i = l \\ 0 & \text{o.w.} \end{cases}, \quad \begin{matrix} \forall i\in V,\ (k,l)\in(V,V) \\ \forall (s,d)\in(V,V) \end{matrix} \tag{4.21}$$

- The sum of the traversed time delays of the edges in a given recovery domain cannot exceed the maximum recovery time

$$\sum_{\{i,j\}\in E} r^{sd}_{ij,kl} t_{ij} \leq T, \ \begin{matrix} \forall (k,l)\in(V,V) \\ \forall (s,d)\in(V,V) \end{matrix} \tag{4.22}$$

4.7.2 MILP for Optimal Local Recovery (Fast ReRoute)

The following is an MILP for optimal local recovery for a single demand requiring unit flow. Sharing can be accomplished by using similar techniques as the MILP for GRT-RD with sharing, shown in Chapter Appendix Section 4.7.1.
The following values are given:

- V is the set of vertices and E is the set of edges
- c_{ij} is the cost of link $\{i, j\}$
- (s, d) is the source and destination

The following variables will be solved for:

- x_{ij} is 1 if primary flow is assigned to link $\{i, j\}$, and 0 otherwise
- f^{ij}_{kl} is 1 flow is assigned to link $\{i, j\}$ to protect after the failure of link $\{k, l\}$, 0 otherwise
- s_{ij} is 1 if protection flow is assigned to link $\{i, j\}$, and 0 otherwise

The objective is to minimize the total cost of allocation:

$$\text{minimize} \sum_{\{i,j\}\in E} c_{ij}(x_{ij} + s_{ij}) \tag{4.23}$$

Subject to the following constraints:

- Route primary traffic to meet demand before a failure

$$\sum_{\{i,j\}\in E} x_{ij} - \sum_{\{j,i\}\in E} x_{ji} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases}, \ \forall i \in V \tag{4.24}$$

- Route flow from point of local repair k to destination d after the failure of link $\{k,l\}$ if link $\{k,l\}$ carried flow

$$\sum_{\substack{j\in V \\ j\neq l}} f_{kl}^{ij} - \sum_{\substack{j\in V \\ j\neq l}} f_{kl}^{ji} = x_{kl}, \ \forall\{k,l\} \in E \tag{4.25}$$

$$\sum_{\substack{j\in V \\ j\neq k}} f_{kd}^{jd} - \sum_{\substack{j\in V \\ j\neq k}} f_{kd}^{tj} = -x_{kd}, \ \forall\{k,d\} \in E \tag{4.26}$$

$$\sum_{\substack{\{i,j\}\in E \\ \{k,l\}\neq\{i,j\} \\ i\neq k\vee d}} f_{kl}^{ij} - \sum_{\substack{\{j,i\}\in E \\ \{k,l\}\neq\{j,i\} \\ i\neq k\vee d}} f_{kl}^{ji} = 0, \quad \substack{\forall i\in V \\ \forall\{k,l\}\in E} \tag{4.27}$$

- Primary and spare capacity assigned on any link meets flow requirements after the failure of link $\{k,l\}$

$$f_{kl}^{ij} \leq x_{ij} + s_{ij}, \quad \substack{\forall\{i,j\}\in E \\ \forall\{k,l\}\in E} \tag{4.28}$$

Chapter 5

Providing Protection in Multi-Hop Wireless Networks

In the previous chapters of this thesis, we investigated service guarantees for wired networks. We now consider guaranteeing service in wireless networks that takes into account the additional challenges of having interference-free communications across a shared transmission space.

5.1 Introduction

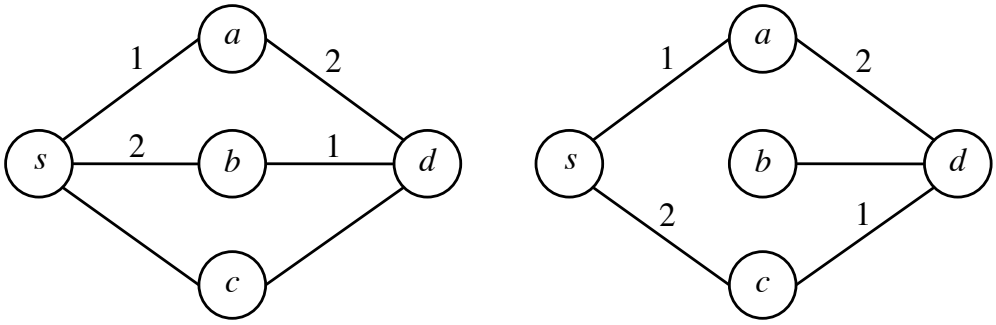
Multi-hop wireless mesh networks have become increasingly ubiquitous, with wide-ranging applications from military to sensor networks. As these networks continue gaining in prominence, there is an increasing need to provide protection against node and link failures. In particular, wireless mesh networks have recently emerged as a promising solution for providing Internet access. Since these networks will be tightly coupled with the wired Internet to provide Internet services to end-users, they must be equally reliable. Wired networks have long provided pre-planned backup paths, which offer rapid and guaranteed recovery from failures. These protection techniques cannot be directly applied to wireless networks due to interference constraints. As opposed to wired networks, two wireless nodes in close proximity will interfere with one another if they transmit simultaneously in the same frequency channel. So, in

addition to finding a backup route, a schedule of link transmissions needs to be specified. In this work, we consider the problem of providing guaranteed protection in wireless networks with interference constraints via pre-planned backup routes, as well as their corresponding link transmission schedules.

Guaranteed protection schemes for wired networks have been studied extensively [2, 3, 5, 17, 18], with the most common scheme being $1+1$ guaranteed path protection [3]. The $1+1$ protection scheme provides an edge-disjoint backup path for each working path, and guarantees the full demand to be available at all times after any single link failure. Protection schemes optimized for wireless networks with interference constraints have not yet been considered. Typically, an approach for resiliency in wireless networks (in particular sensor networks) is to ensure that there exists "coverage" for all nodes given some set of link failures [78, 79]. This approach to resiliency does not consider routing and scheduling with respect to interference constraints, and assumes that there exists some mechanism to find a route and schedule at any given point in time. Furthermore, there is no guarantee that sufficient capacity will be available to protect against a failure. The idea of applying $1 + 1$ protection in wireless networks is briefly mentioned in [80]. However, [80] does not study the specific technical details of such an approach to wireless protection. The goal of this chapter is to study protection mechanisms for wireless networks with a particular focus on the impact of wireless interference and the need for scheduling.

The addition of interference constraints makes the protection problem in a wireless setting fundamentally different from the ones found in a wired context. After a failure in a wireless network, links that could not have been used due to interference with the failed link become available, and can be used to recover from the failure. In fact, it is often possible to add protection in a wireless setting without using any additional resources.

Consider allocating a protection route for the following example, shown in Fig. 5-1. The wireless network operates in a time-slotted fashion, with equal length time slots available for transmission. Any two nodes within transmission range have a link between them, and each link's time slot assignment is shown in the figures. We



(a) Before a failure (b) After $\{s, b\}$ fails

Figure 5-1: Time slot assignment for protection in a wireless network

assume a 1-hop interference model where any two links that have a node in common cannot be active at the same time. Additionally, we assume unit capacity links. Before any failure, the maximum flow from s to d is 1, and can be achieved using a two time slot schedule, as shown in Fig. 5-1a. At any given point in time, only one outgoing link from s can be active, and similarly, only one incoming link to d can be active. Wireless links $\{s, c\}$, and $\{c, d\}$ cannot be used prior to the failure of $\{s, b\}$, but become available after $\{s, b\}$ fails. After the failure of $\{s, b\}$, flow can be routed from s to c during time slot 2, and from c to d during slot 1, as shown in Fig. 5-1b. Similar schedules can be found for failures of the other links. The maximum flow from s to d is 1 for both before *and* after a failure; i.e., there is no reduction in maximum throughput when allocating resources for a protection route on $\{s, c\}$ and $\{c, d\}$: protection can be assigned for "free". This is in contrast to a wired network where the maximum throughput without protection from s to d is 3, and the maximum throughput when assigning a protection route on $\{s, c\}$ and $\{c, d\}$ is 2, which amounts to a $\frac{1}{3}$ loss in throughput due to protection.

The novel contributions of this chapter is introducing the Wireless Guaranteed Protection (WGP) problem in multi-hop networks with interference constraints. In Section 5.2, the model for WGP is presented. In Section 5.3, properties of an optimal solution are examined for a single demand with 1-hop interference constraints, which are then used to motivate the development of a time-efficient algorithm. In Section 5.4, an optimal solution is developed via a mixed integer linear program for general

interference constraints. In Section 5.5, time-efficient algorithms are developed that perform within 4.5% of the optimal solution.

5.2 Model and Problem Description

In this chapter, solutions to the guaranteed protection problem for multi-hop wireless networks subject to interference constraints are developed and analyzed. Our goal is to provide protection in a manner similar to what has been done in the wired setting. Namely, after the failure of some network element, all connections must maintain the same level of flow that they had before the failure. In order to do so, resources are allocated and scheduled in advance on alternate (backup) routes to protect against failures.

In wired networks, two adjacent nodes can transmit simultaneously because they do not interfere with one another; if capacity exists on a set of links, a path can be routed using that capacity. Wireless networks are different; interference constraints must be considered. A set of links in close proximity cannot transmit simultaneously on the same frequency channel; only one link from that set can be active at a time, or else they will interfere with one another. Not only must a path between the source and destination be found with available capacity, but also a schedule of link transmissions needs to be determined. This is known as the routing and scheduling problem [36, 80–87], which is known to be NP-Hard [36].

The addition of interference constraints adds complexity to the traditional wired protection problem, but also presents an opportunity to gain protection from failures with minimal loss of throughput. After a failure in a wireless network, links that could not have been used due to interference with the failed link become available, and can be used to recover from the failure. In fact, it is often possible to add protection in a wireless setting without any loss in throughput.

The following network model is used for the remainder of the chapter. A graph G has a set of vertices V and edges E. An interference matrix $\mathcal{I}$ is given, where $I_{ij}^{kl} \in \mathcal{I}$ is 1 if links $\{i, j\}$ and $\{k, l\}$ can be activated simultaneously (do not interfere with

each other), and 0 otherwise. The interference matrix is agnostic to the interference model used (i.e., it can be used to represent nearly any type of link interferences). For the remainder of this work, we focus on the 1-hop interference model (any two links that share a node cannot be activated simultaneously), but our schemes can be adapted to the K-hop [40] interference model as well. Our goal in this chapter is to develop a framework for routing and scheduling with protection under interference constraints. We assume nodes are fixed, links are bidirectional, and that the network uses a synchronous time slotted system, with equal length time slots; the set of time slots used is $\mathcal{T}$. Only link failures are considered, and a single-link failure model is assumed; it is straightforward to apply the solutions developed in this chapter to node failures as well. For now, we assume centralized control; the algorithms presented can be modified to work in a distributed fashion, as done in [88]. Additionally, we only consider a single frequency channel.

5.3 Efficient Algorithm for a Single Demand

In this section, we aim to achieve insight into providing protection for wireless networks with interference constraints by examining the solution for a single demand under a basic set of network parameters: 1-hop interference constraints and unit capacity links. In Section 5.3.1, properties of an optimal solution are examined for routing and scheduling with and without protection. In Section 5.3.2, a time efficient algorithm is developed that finds a maximum throughput guaranteed to be within 1.5 of the optimal solution. In Chapter Appendix 5.7.2, a polynomial time algorithm is presented that tightens this bound and finds a solution guaranteed to be within $\frac{6}{5}$ of the optimal solution.

5.3.1 Solution Properties

In this section, properties of an optimal solution for WGP for a single demand are examined. First, we look at routing and scheduling without protection, and then those results are extended to the protection setting.

Lemma 5.1. *The maximum flow that can be routed and scheduled between the source s and destination d under 1-hop interference constraints without protection is 1.*

Proof. Under 1-hop interference constraints, only one link exiting the source node can be active at time. Since each link has unit capacity, the maximum flow that can leave the source (or enter the destination) is 1. □

While Lemma 5.1 indicates that a flow of 1 is possible, it does not necessarily mean that a flow of 1 can be achieved. We note that if the source s and destination d are adjacent, then a maximum flow of 1 can always be achieved by using one edge between the two. We assume for the remainder of this section that s and d are at least two hops apart. We now give the properties of maximum flows for a single demand in a unit-capacity wireless network under 1-hop interference constraints.

Lemma 5.2. *To achieve the maximum flow of 1, there must exist at least two node-disjoint paths from s to d.*

Proof. Assume otherwise: there are no node-disjoint paths, and there is a maximum flow of 1 possible from s to d. If there are no node-disjoint paths between s and d, then by Menger's theorem there exists a single node j whose removal will separate s and d [29]; hence, all paths from s to d must pass through j. In order for a flow of 1 to exist between s and d, node j must have a total of 1 unit of flow coming in, and 1 unit of flow going out. This is only possible if node j is both receiving and transmitting the entire time, which is not possible under the 1-hop interference model since node j cannot be both receiving and transmitting simultaneoulsy. □

Corollary 5.1. *If two node-disjoint paths from s to d do not exist, then the maximum flow is $\frac{1}{2}$.*

Proof. In the proof for Lemma 5.2, it was shown that if no node-disjoint paths exist between s and d, all paths must cross some node j. Node j cannot be receiving and transmitting at the same time under the 1-hop interference model. The maximum flow that j can support is to be recieving half of the time, and transmitting the other

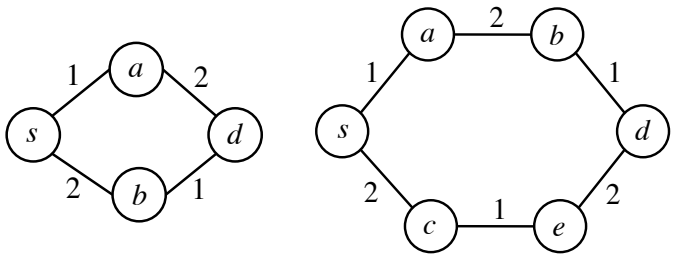
half. Increasing the amount of time j is transmitting will reduce the amount of time it can transmit, which reduces the overall flow. The same is seen if the amount of time j is receiving is increased. Hence, j will have incoming flow half of the time, and outgoing flow for the other half. Since each edge has unit capacity, the maximum flow possible through node j is $\frac{1}{2}$. □

Any loop-free path from s to d (when s and d are greater than one hop apart from one another) can have an interference-free schedule by alternating between time slots 1 and 2 for each edge of the path; hence, any edge of the path will only be active for half of the time, and the path will support a flow of $\frac{1}{2}$. If two or more node-disjoint paths do exist, then a maximum flow is dependent on the total number of edges in the disjoint paths.

Lemma 5.3. *If there exists two node-disjoint paths between s and d with an even total number of edges over both paths, then the maximum flow of 1 is achievable.*

Proof. When both paths have an even total number of edges, an interference-free schedule using two time slots is possible by alternating time slot assignments on each path. If each path has an even number of edges, then path 1 will begin with time slot 1 and end with time slot 2, and path 2 will begin with time slot 2 and end with time slot 1. Each unit-capacity edge will be active for half of the time; hence, each path carries a total of $\frac{1}{2}$ unit of flow, giving the maximum flow of 1 using both paths. A similar schedule can be shown for the case when each path has an odd number of edges. □

To help see Lemma 5.3, two examples are shown in Fig. 5-2, with the time slot assignments for the links labeled in the figures. In Fig. 5-2a and 5-2b, there are two node-disjoint paths from the source s to destination d that have an even total number of edges. In Fig. 5-2a, each path has an even number of edges, and in Fig. 5-2b, each path has an odd number of edges. An interference-free schedule for the two paths can be found using two time slots. Each link is active for $\frac{1}{2}$ of the time; hence, each path can support a flow of $\frac{1}{2}$, giving a total flow of 1 from s to d.



(a) Max flow of 1 (b) Max flow of 1

Figure 5-2: Node-disjoint paths with an even total number of edges

Corollary 5.2. *If there exists more than two node-disjoint paths between s and d, a maximum flow of 1 is always achievable.*

Proof. If there are more than two node-disjoint paths, then there always exists a pair of node-disjoint paths that has an even total number of edges. □

If the total number of edges in the two node-disjoint paths is odd, then the two-time slot schedule used in Fig. 5-2 to achieve the maximum flow of 1 over the two paths is not possible; additional time slots are needed. While a maximum flow of 1 may not be possible, a minimum flow of $\frac{2}{3}$ is always feasible over two node-disjoint paths if a third time slot is used.

Lemma 5.4. *For a pair of node-disjoint paths that have an odd total number of edges, a flow of $\frac{2}{3}$ is always possible between s and d using three time slots.*

Proof. Remove one of the edges from one of the paths; there is now an even number of edges in the two paths. Schedule the two paths using two time slots as if they were a pair of node-disjoint paths with an even number of edges. After this step, all of the scheduled edges can operate without interference. Now reintroduce the removed edge. This reintroduced edge is adjacent to two edges that each have a time slot assignment of 1 and 2, respectively. Clearly, time slot 1 or 2 cannot be assigned to the reintroduced edge, so assign it time slot 3. With three time slots, each link is active for $\frac{1}{3}$ of the time. Since each link has unit capacity, each path carries $\frac{1}{3}$ flow, and the total flow over both paths is $\frac{2}{3}$. □

An example demonstrating Lemma 5.4 is shown in Fig. 5-3. The two node-disjoint paths have an odd total number of edges and it is not possible to schedule the two paths using only two time slots. A third time slot is added, and a feasible schedule is now possible. Using these three time slots, each link is active for $\frac{1}{3}$ of the time, and each path can support a flow of $\frac{1}{3}$, which gives a total flow of $\frac{2}{3}$.

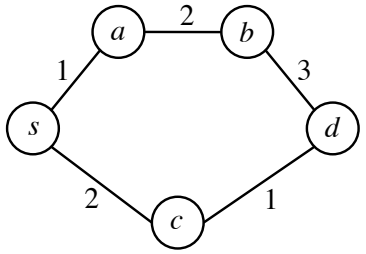


Figure 5-3: Node-disjoint paths with an odd total number of edges supporting a flow of $\frac{2}{3}$

We note that is in fact possible to construct schedules using more than three time slots to achieve higher throughput on node-disjoint paths that have an odd number of edges. These results are given in Chapter Appendix 5.7.2.

These results can be extended to the case where protection is required. For protection against any single link failure in a graph $G = (V, E)$, consider each subgraph after a link failure: $G^e = (V, E \setminus e),\ e \in E$; all the previous results still apply to each of these new subgraphs. To find the maximum possible protected flow, the maximum flow is found after each edge is individually removed (each possible edge failure). The *minimum* of these flows is the maximum protected flow.

5.3.2 Time Efficient Algorithm

Using the different properties of a solution for a single demand under 1-hop interference constraints, we develop an algorithm to solve the problem efficiently. If there exists two node-disjoint paths with an even total number of edges, then the maximum flow is 1 between the source and destination. If there are no node-disjoint paths, then the maximum flow is $\frac{1}{2}$. If there exists only a pair of disjoint paths that has an odd total number of edges, then a flow of $\frac{2}{3}$ can be guaranteed. To find the maximum protected flow between nodes s and d in a graph $G = (V, E)$, the maxi-

mum flow is found for each link failure by using a subgraph with each link e removed: $G^e = (V, E \setminus e),\ e \in E$. The minimum of these maximum flows is the maximum protected flow possible for the demand.

The key to finding the maximum protected flow is to be able to identify node-disjoint paths between s and d with either an even or odd total number of edges. If there are at most two node-disjoint paths, then the maximum flow can only be found if it is possible to find a pair of paths with an even total number of edges. Hence, we focus on trying to find a pair of node-disjoint paths that have an even total number of edges over both of the paths. There has been limited work on trying to identify shortest paths with an even number of edges [89], but no work looking at such an algorithm for disjoint paths.

Development of the optimal algorithm is as follows: we first find the shortest pair of *edge*-disjoint paths with an even number of total edges, and then we extend this algorithm to find the shortest pair of *node*-disjoint paths with an even number of total edges.

Shortest pair of *edge*-disjoint paths with an even number of total edges

To find the shortest pair of edge-disjoint paths with an even number of edges, we begin by considering the more general case without the even-edge restriction (the paths can have any number of edges), which was previously considered in [34]. We use a different formulation for the problem by using *minimum-cost flows*, which are defined as finding a flow of minimum cost between a source and destination in a network that has both edge costs and edge capacities [29]. Minimum-cost flows have the property that when given all integer inputs (for edge costs and capacities), they will have all integer solutions (integer flows). We solve the shortest disjoint pair of paths problem by solving the following optimization problem: find a flow of minimum cost to route two units from s to d in a graph with unit capacity and unit cost edges. This will find the shortest pair of disjoint paths since two units of flow need to be routed, no edge can have more than a single unit of flow, and with integer inputs, the solution will be integer, which will be two edge-disjoint paths of unit flow and

minimum cost.

One algorithm to solve the minimum-cost flow problem is the successive shortest paths (SSP) algorithm [29]. SSP finds the shortest path, and routes the maximum flow possible onto that path. This repeats until the desired flow between the source and destination is routed. SSP runs in polynomial time to solve the minimum-cost flow formulation for the shortest pair of disjoint paths; further details of SSP can be found in [29].

Using SSP to solve for a minimum-cost flow requires the use of some shortest path algorithm. Assume there exists a shortest path algorithm that is capable of finding a path with an even or odd number of edges; label these algorithms Even and Odd Shortest Path (ESP and OSP, respectively). Using SSP to solve for the shortest pair of disjoint paths with either ESP or OSP as the shortest path function will always yield a pair of disjoint paths with an even total number of edges (if they exist). We call this the Even Shortest Pair of Edge-Disjoint Paths algorithm.

Lemma 5.5. *The Even Shortest Pair of Edge-Disjoint Paths algorithm will find, if it exists, the shortest pair of disjoint paths with an even total number of edges.*

Proof. First, we provide more detail for the shortest successive paths (SSP) algorithm. For each iteration of SSP, flow is routed on the residual graph, which allows new flows to cancel existing flows; flows in residual graph are known as "augmenting paths". The residual graph is defined as follows: if edge $\{i, j\}$ has a capacity and cost of (u_{ij}, c_{ij}) with a flow of $f_{ij} \leq u_{ij}$ on it, the residual graph will have two edges $\{i, j\}$ and $\{j, i\}$ with respective costs and capacities ($u_{ij} - f_{ij}$, c_{ij}), and (f_{ij}, $-c_{ij}$) [29]. Finding augmenting paths on the residual graph maintains node conservation constraints; after each iteration of SSP, the residual graph is updated.

To find the shortest pair of disjoint paths, two iterations of SSP on a unit capacity graph are needed. The first pass will find a path with m_1 number of edges, which depending on if ESP or OSP was used, will be even or odd. The second pass, which is done on the residual graph, will find a path with m_2 edges. If the second path uses any residual flow from the first path, its flow will completely cancel the first path's

flow, effectively canceling the usage of the edge in the final set of disjoint paths. Call the number of edges that are cancelled m_x. Since each path used a cancelled edge, when that edge is removed, both paths will no longer traverse the cancelled edge. The total number of edges in the final set of disjoint paths is $m_1 + m_2 - 2m_x$, which is always even. □

In order to use SSP to find the shortest pair of disjoint paths with an even number of edges, a shortest path algorithm is needed that can find a path with an even or odd number of edges. The algorithm in [89] that finds the shortest path with an even number of edges cannot be easily extended to find the shortest pair of disjoint paths with an even number of edges. Hence, we first focus on developing an algorithm to find the Even Shortest Path (ESP), and then extend ESP to find the Odd Shortest Path (OSP).

We modify the standard Bellman-Ford recursion [29] to search for only paths with an even number of edges, which is shown in Equation 5.1. We label $S_z(s,k)$ to be the minimum-cost path from node s to k using at most $2z$ edges. The cost of edge $\{i,j\}$ is c_{ij}; in our case $c_{ij} = 1$, $\forall\{i,j\} \in E$. Instead of checking if a path from s to j plus edge $\{j,k\}$ is of lower cost than the existing path from s to k, we check to see if the path from s to i plus two edges $\{i,j\}$ and $\{j,k\}$ are of lower cost than the existing path from s to k.

$$S_z(s,k) = \min[\min_{\substack{\{i,j\}\in E \\ \{j,k\}\in E \\ \{i,j\}\neq\{j,k\}}} (S_{z-1}(s,j) + c_{ij} + c_{jk}),\ S_{z-1}(s,k)],$$

$$\forall z = 1..|V|,\ \forall k \in V \quad (5.1)$$

To find the shortest path from the source s with an odd number of edges, we run ESP from all neighboring nodes of s (nodes that are one hop from s). The lowest cost path leading back to the source is the solution to OSP.

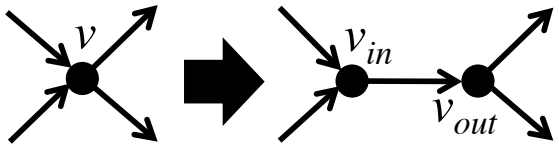


Figure 5-4: Node splitting to find node-disjoint paths

Shortest pair of *node*-disjoint paths with an even number of total edges

In order to optimally solve for routing and scheduling under 1-hop interference constraints, a pair of *node*-disjoint paths with an even number of edges must be found. The Even Shortest Pair of Edge-Disjoint Paths algorithm finds the shortest pair of *edge*-disjoint paths with an even number of edges. To use the *edge*-disjoint algorithm to solve the *node*-disjoint case, each node is transformed into two separate nodes with an edge of zero cost between them: one node has all incoming edges, and the other all outgoing (as shown in Fig. 5-4). If there existed multiple edge-disjoint paths that intersected at node v, they would no longer be able to be edge-disjoint in the transformed network, because then they would all have to share the edge $\{v_{in}, v_{out}\}$.

Running the Even Shortest Pair of Edge-Disjoint Paths algorithm on the transformed network will find node-disjoint paths, but not necessarily achieve the desired result of a pair of disjoint paths with an even number of edges. With the addition of zero-cost edges to the transformed network, finding a pair of disjoint paths with an even number of edges in the transformed network may not yield paths with an even number of edges in the original network. A modification to the algorithm must be made to account for the new edges: in the transformed network, when choosing between an existing path from s to k, or some new path s to i plus a segment i to k, consider only segments that have an even number of "original" edges. This will ensure that a final path in the original network will have an even number of edges. The algorithm now begins to more closely resemble the Floyd-Warshall algorithm [29], which considers joining segments to find a shortest path. This new algorithm is called Even Shortest Pair of Node-Disjoint Paths.

These results can be extended to solve Wireless Guaranteed Protection problem

with a single demand under 1-hop interference constraints. The maximum flow is found after every possible edge failure for each subgraph $G^e = (V, E \setminus e)$, $\forall e \in E$. The minimum of these maximum flows is the maximum protected flow. For each instance, we first see if there exists a pair of node-disjoint paths with an even total number of edges. If this exists, then a maximum flow of 1 is possible. If not, we check to see if there exist node-disjoint paths with an odd total number of edges (by running the standard edge-disjoint path routing algorithm on the transformed graph). If this exists, then a flow of $\frac{2}{3}$ is possible using three time slots. If no node-disjoint paths exist, then find some path from s to d, which can support a flow of $\frac{1}{2}$.

5.4 An Optimal Formulation for Wireless Guaranteed Protection

In the previous section, an optimal solution for routing and scheduling with protection for a single demand was presented. While this provides insight, typical networks will need to simultaneously handle multiple connections. Additionally, many networks have interference constraints other than the 1-hop model. This section provides a mathematical formulation to the optimal solution for the Wireless Guaranteed Protection (WGP) problem with general interference constraints. In particular, for a set of demands, a route and schedule needs to be found such that after any link failure, all end-to-end connections maintain their same level of flow. For general interference constraints, the routing and scheduling problem was demonstrated to be NP-Hard [36]. We conjecture that adding protection constraints preserves NP-hardness; hence, a mixed integer linear program (MILP) is formulated to find an optimal solution to WGP.

In wired networks, a typical objective function for protection is to minimize the total allocated capacity needed to satisfy all demands. A similar objective cannot be clearly defined for wireless networks since the concept of capacity changes in the presence of interference constraints. Consider some active link $\{i, j\}$. An adjacent link $\{j, k\}$ cannot be used simultaneously with $\{i, j\}$ because of interference; hence, simply

adding additional link capacity (in a wired sense) will not allow its use. Another time slot must be allocated to allow a connection to use $\{j, k\}$ such that it does not interfere with $\{i, j\}$. Adding an additional time slot will reduce the time that each individual time slot in the schedule is active, which reduces the overall throughput of the network [36, 80, 84]. For example, consider a network with two time slots and a connection that supports a flow of 1 using these two time slots. If a third time slot is added to the schedule, then the original two time slots are only active for $\frac{2}{3}$ of the total time, and that flow's scheduled throughput is reduced from 1 to $\frac{2}{3}$. Thus, the objective we consider is to use a minimum number of time slots to route and schedule each demand with protection.

Finding a protection route and schedule using the minimum number of time slots allows for a simple comparison to existing wired and wireless protection schemes. The difference between the number of time slots necessary to route and schedule a set of demands before and after adding protection will be considered the reduction of the maximum throughput. To be consistent with the wireless protection scheme mentioned in [80], wireless flows are restricted to single paths (no flow splitting allowed). For ease of exposition, the MILP assigns the same throughput to all demands; see Chapter Appendix Section 5.7.1 for the formulation with different throughput requirements.

For the MILP, the following values are given:

- $G = (V, E)$ is the graph with a set of vertices and edges
- D is the set of flow requirements
- u_{ij} is the capacity of link $\{i, j\}$
- $\mathcal{I}$ is the interference matrix, where $I_{ij}^{kl} \in \mathcal{I}$ is 1 if links $\{i, j\}$ and $\{k, l\}$ can be activated simultaneously, 0 otherwise
- $\mathcal{T}$ is the set of time slots in the system, $\mathcal{T} \subset \mathbb{Z}^+$

The MILP solves for the following variables:

- x_{ij}^{sd} is a routing variable and is 1 if primary flow is assigned for demand (s,d) on link $\{i,j\}$, 0 otherwise
- $y_{ij,kl}^{sd}$ is a routing variable and is 1 if protection flow is assigned on link $\{i,j\}$ for the demand (s,d) after the failure of link $\{k,l\}$, 0 otherwise
- $\lambda_{ij}^{sd,t}$ is a scheduling variable and is 1 if link $\{i,j\}$ can be activated in time slot t for the demand (s,d), 0 otherwise
- $\delta_{ij,kl}^{sd,t}$ is a scheduling variable and is 1 if link $\{i,j\}$ can be activated in time slot t after failure of link $\{k,l\}$ for the demand (s,d), 0 otherwise
- s_t is 1 if time slot t is used by any demand, and 0 otherwise

The objective function is to minimize the number of time slots (the length of the schedule) needed to route all demands with protection:

$$\textbf{Objective:} \quad \min \sum_{t \in \mathcal{T}} s_t \tag{5.2}$$

The following constraints are imposed to find a feasible routing and scheduling with protection.

Before a link failure:

- Flow conservation constraints for the primary flow: route primary traffic before a failure for each demand.

$$\sum_{\{i,j\} \in E} x_{ij}^{sd} - \sum_{\{j,i\} \in E} x_{ji}^{sd} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases}, \quad \substack{\forall i \in V \\ \forall (s,d) \in D} \tag{5.3}$$

- In any given time slot, for a given demand, only links that do not interfere with one another can be activated simultaneously.

$$\sum_{(s,d) \in D} \lambda_{ij}^{sd,t} + \sum_{(s,d) \in D} \lambda_{kl}^{sd,t} \leq 1 + I_{kl}^{ij}, \quad \substack{\forall \{i,j\} \in E,\ \forall \{k,l\} \in E \\ \{i,j\} \neq \{k,l\},\ \forall t \in \mathcal{T}} \tag{5.4}$$

- Only one demand can use a given link at a time.

$$\sum_{(s,d)\in D} \lambda_{ij}^{sd,t} \leq 1, \quad \substack{\forall \{i,j\}\in E \\ \forall t \in \mathcal{T}} \tag{5.5}$$

- Ensure enough capacity exists to support the necessary flow for demand (s,d) on edge $\{i,j\}$ for the length of time that the link is active.

$$x_{ij}^{sd} \leq \sum_{t\in\mathcal{T}} \lambda_{ij}^{sd,t} u_{ij}, \quad \substack{\forall \{i,j\}\in E \\ \forall (s,d)\in D} \tag{5.6}$$

- Mark if slot t is used to schedule a demand before a failure.

$$\lambda_{ij}^{sd,t} \leq s^t, \quad \substack{\forall \{i,j\}\in E \\ \forall t\in\mathcal{T},\ \forall (s,d)\in D}$$

After a link failure:

- Flow conservation constraints for protection flow: route protection traffic after each link failure $\{k,l\} \in E$.

$$\sum_{\substack{\{i,j\}\in E \\ \{k,l\}\neq\{i,j\}}} y_{ij,kl}^{sd} - \sum_{\substack{\{j,i\}\in E \\ \{k,l\}\neq\{j,i\}}} y_{ji,kl}^{sd} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases}, \quad \substack{\forall i\in V,\ \forall \{k,l\}\in E \\ \forall (s,d)\in D} \tag{5.7}$$

- In any given time slot after the failure of link $\{k,l\}$, only links that do not interfere with one another can be activated simultaneously.

$$\sum_{(s,d)\in D} \delta_{ij,kl}^{sd,t} + \sum_{(s,d)\in D} \delta_{uv,kl}^{sd,t} \leq 1 + I_{uv}^{ij}, \quad \substack{\forall \{i,j\}\in E,\ \forall \{k,l\}\in E \\ \forall \{u,v\}\in E,\ \forall t\in\mathcal{T} \\ \{i,j\}\neq\{k,l\}\neq\{u,v\}} \tag{5.8}$$

- Only one demand can use a given link at a time after the failure of link $\{k,l\}$.

$$\sum_{(s,d)\in D} \delta_{ij,kl}^{sd,t} \leq 1, \quad \substack{\forall \{i,j\}\in E,\ \forall \{k,l\}\in E \\ \forall t\in\mathcal{T}} \tag{5.9}$$

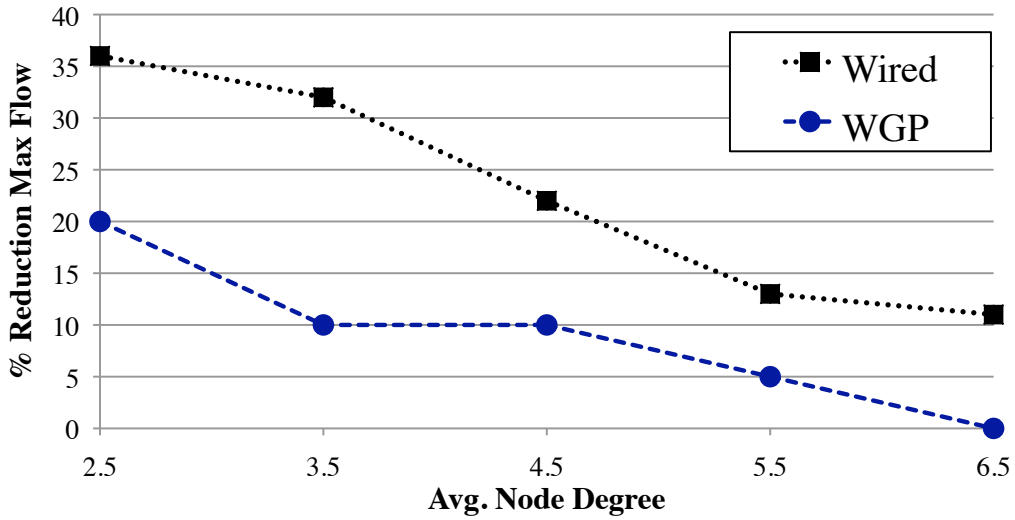


Figure 5-5: Reduction of throughput when adding protection

- Ensure enough capacity exists after the failure of link $\{k,l\}$ to support the necessary flow on edge $\{i,j\}$ for the length of time that the link is active.

$$y_{ij,kl}^{sd} \leq \sum_{t \in \mathcal{T}} \delta_{ij,kl}^{sd,t} u_{ij}, \qquad \begin{matrix} \forall \{i,j\} \in E,\ \forall \{k,l\} \in E \\ \forall (s,d) \in D \end{matrix} \tag{5.10}$$

- Mark if time slot t is used to schedule a demand after the failure of link $\{k,l\}$.

$$\delta_{ij,kl}^{sd,t} \leq s^t, \qquad \begin{matrix} \forall \{i,j\} \in E,\ \forall \{k,l\} \in E \\ \forall t \in \mathcal{T},\ \forall (s,d) \in D \end{matrix}$$

To demonstrate how protection can be added to wireless networks with minimal reduction of throughput, WGP is compared to both the wired (without interference) and wireless protection (with interference) schemes. One hundred random graphs were generated with 25 nodes each. Nodes that are physically within a certain transmission range of one another are considered to have a link, and the transmission range is varied to give different desired average node degrees. The node degree is varied from 2.5 to 6.5, and for each graph, ten source/destination pairs are randomly chosen to be routed concurrently. All links have unit capacity; 1-hop interference constraints were used for the wireless networks. The simulation results are found in Fig. 5-5.

For comparison to wired protection, we use the same network topologies, however,

in the wired case we do not enforce the interference constraints (i.e., all links can be activated simultaneously). For wired protection, we compute the reduction in throughput as the reduction in maximum flow after protection is added. As compared to the wired protection scheme, WGP has a lower reduction in throughput for all node degrees examined. For node degree 2.5, both the WGP and the wired protection schemes have larger reductions in throughput: 20% for WGP and 37% for wired. This is because at lower node degrees, there are fewer available end-to-end paths, and therefore after a failure, there are fewer routing options available. As the node degree increases, and there are more available end-to-end paths, the reduction in throughput decreases when adding protection. In fact, it is often possible for WGP to have no reduction in the throughput between the protected and unprotected setting. For an average node degree of 3.5, WGP only loses about 10% of throughput when adding protection, while the wired scheme loses 32%. For 20% of the simulations at node degree 3.5, there was no loss in throughput for WGP. When the node degree goes to 6.5, WGP no longer has any loss in flow, while the wired setting still has a loss of 11%.

We compare WGP to a wireless 1+1 protection scheme. In particular, wireless 1+1 protection applies the wired 1+1 protection scheme to wireless networks (as mentioned in [80]): i.e., find a schedule for the shortest pair of disjoint paths in the network between the source and destination, with the primary flow before a failure routed onto one path, and the backup flow routed onto the other. To compare WGP to wireless 1+1, the number of time slots needed beyond the non-protection routing are compared; these are the time slots needed to meet the protection requirements. Table 5.1 shows the percent reduction in number of time slots needed to provide protection using WGP over wireless 1+1. When the average node degree is 2.5, WGP has up to a 72% reduction of time slots needed to meet protection requirements. The reason for this is that wireless 1+1 is scheduling two paths for each demand, a primary and a backup, and not trying to recapture any capacity after a failure; this in turn causes a significant increase in interference between connections. As the node degree increases, there is increased path diversity and more opportunities to find interference-

Avg. Node Degree	% Reduction of Protection Time Slots
2.5	72
3.5	63
4.5	60
5.5	52
6.5	46

Table 5.1: WGP vs. Wireless 1+1

free routings; hence, wireless 1+1 has better performance. But at all times, wireless 1+1 needs significantly more time slots to provide protection for all of the demands than WGP does, which is able to recapture capacity after a failure.

5.5 Algorithms for Providing Wireless Protection

In the previous section, an MILP was presented to find an optimal solution to Wireless Guaranteed Protection (WGP), which is not a computationally efficient method of finding a solution. In this section, two time-efficient algorithms are presented to solve the Wireless Guaranteed Protection problem for a set of demands. Similar to the previous section, primary and backup flows are restricted to single paths, and the objective is to minimize the length of the schedule to route all demands with protection. We first show that this problem is NP-Hard under 1-hop interference constraints. Next, algorithms are developed assuming unit demands, unit capacity edges, and a single link failure model; the algorithms can be modified to reflect other values of demand and capacity. The algorithms are developed for dynamic (one-at-a-time) arrivals: an incoming demand needs to be routed and scheduled over an existing set of connections; the existing set cannot have their routings or schedules changed. A 1-hop interference model is used, but the algorithms can be extended to a generic K-hop interference model, with the extensions detailed in the end of Section 5.5.2. We find that when compared to the optimal batch case (all connections are routed and scheduled simultaneously), the dynamic routing performs within a few percentage points of optimal.

First, in Section 5.5.1, we demonstrate WGP to be NP-Hard under 1-hop interference constraints when flows are restricted to a single path. Next, in Section 5.5.2, an algorithm to find a shortest 1-hop interference-free path using a minimal number of time slots is presented. This serves as the building block for the next two algorithms that are developed. In Section 5.5.3, an algorithm for finding a minimal length schedule for WGP is presented, where a backup route and schedule is found for each possible failure. This approach has drawbacks in that after any failure, a new route is found; hence, a route and schedule for each failure event needs to be stored. To overcome this, an algorithm is developed in Section 5.5.4 using disjoint paths such that only two paths are needed: a primary and a backup. In Section 5.5.5, the performance of the two algorithms are compared to the optimal MILP formulation.

5.5.1 Complexity Results under 1-hop Interference Constraints

Without protection, the routing and scheduling problem is NP-Hard under general interference constraints [36]. But if flows for each demand are allowed to be split, a polynomial timed algorithm is possible for 1-hop interference constraints [82]. We demonstrate that when flows cannot be split, the routing and scheduling problem becomes NP-Hard under 1-hop interference constraints.

Theorem 5.1. *Finding the minimum length schedule to route a set of demands under 1-hop interference constraints when flow splitting is not allowed is NP-Hard.*

We first consider the following necessary and sufficient condition for routing a set node-disjoint[1] pairs, (s_1, d_1), ..., (s_N, d_N), in only two time slots without flow splitting.

Lemma 5.6. *Under 1-hop interference constraints, a set of demands that are node-disjoint can be routed and scheduled using two time slots without flow splitting if and only if there exists node-disjoint paths between each of the node pairs.*

Proof. If there exists a node-disjoint path between every node pair in the set of demands, then a schedule using two slots is possible. A proof can be accomplished by

[1] A node is a source or destination for at most one demand.

construction. Any individual path can be scheduled in two time slots by using alternating time slots. If all paths are node-disjoint, then there exists no conflicts between paths under 1-hop interference constraints. Therefore, all paths can be scheduled using the same two-time slots.

For the other direction, assume otherwise: a two slot schedule is possible without there being a node-disjoint path between every pair of nodes in the set of demands. There exists a node v that has $m \geq 2$ paths crossing it. There are m paths coming into v and m paths out that need to be scheduled. Under 1-hop interference constraints, this will require at least $2m$ time slots to produce an interference free schedule. □

Using Lemma 5.6, Theorem 5.1 can be quickly demonstrated.

Proof of Theorem 5.1. We reduce the Disjoint Connecting Paths Problem (DCPP) [37] to ours. DCPP asks the following question: given a graph $G = (V, E)$ and a collection of N node-disjoint pairs (s_1, d_1), ..., (s_N, d_N), does G contain N mutually node-disjoint paths, one connecting s_i and d_i for each i, $1 \leq i \leq N$? We can ask an equivalent question for our routing and scheduling problem: can a set of N node-disjoint pairs be routed and scheduled using the minimal number of time slots (two) under 1-hop interference constraints without flow splitting? If yes, then by Lemma 5.6 that means we have found N mutually node-disjoint paths, one connecting s_i and d_i for each i, $1 \leq i \leq N$, which solves DCPP. An answer of no means a solution to DCPP does not exist. □

Next, we extend this complexity result to the case when protection is required.

Theorem 5.2. *Finding the minimum length schedule to route a set of demands with protection under 1-hop interference constraints without flow splitting is NP-Hard.*

The proof can be found in Chapter Appendix Section 5.7.3.

5.5.2 Minimum Schedule for an Interference Free Path

We begin by developing an algorithm to find a shortest interference-free path using the minimum number of time slots under the 1-hop interference model. This algorithm

will be a building block for the two protection algorithms that will be discussed in the upcoming sections. We consider an incoming demand for a connection between nodes s and d. Connections already exist in the network, with the set of $\mathcal{T}$ time slots already in use. Based on how the current connections are routed and scheduled, a set of edge interferences $\mathbf{I}$ can be constructed, where for every edge $\{i, j\}$, $I_{ij} \in \mathbf{I}$ is the set of time slots that cannot be used on that edge because either that time slot is already used by $\{i, j\}$, or using that time slot on $\{i, j\}$ will interfere with another edge using it at that time. The set of edge interferences $\mathbf{I}$ can be constructed in polynomial time, and will be given as an input to the algorithm.

First, we wish to determine the shortest interference free path without using any additional time slots beyond the set $\mathcal{T}$, and without rescheduling or rerouting existing connections. Each edge $\{i, j\}$ has a set of free time slots during which it can be used: $\tau_{ij} = \mathcal{T} \setminus I_{ij}$. Let P be the set of edges used in a path. If each edge of a loop-free path P has at least two free time slots, then that path can be scheduled without interference using the existing time slot allocation $\mathcal{T}$.

Lemma 5.7. *For 1-hop interference, a loop-free path P can be scheduled without interference if $|\tau_{ij}| \geq 2$, $\forall \{i, j\} \in P$.*

Proof. If $|\tau_{ij}| \geq 2$, $\forall \{i, j\} \in P$, then each edge in P has an available time slot that does not interfere with its adjacent set of edges. Since the path is loop-free, any two edges that use the same time slot will never be less than one hop apart from one another, and therefore never interfere with each other. □

Using the result from Lemma 5.7, the following algorithm is constructed to find a 1-hop interference-free path using only the set of time slots $\mathcal{T}$: remove all edges in G that have $|\tau_{ij}| \leq 1$, find the shortest path P_{sd} between s and d, and assign time slots to the edges in P_{sd} such that it has an interference-free schedule.

An improvement can be made to the algorithm by attempting to maximize the number of free time slots on any edge, so that future connections will be less likely to require additional time slots to find an interference-free path. Currently, edges that have many free time slots are not given any preference. If an edge has only the

minimal number of free time slots, it may be selected for use in a path. This may hurt finding interference-free paths for future connections by limiting the number of available time slots on an edge, thus necessitating new time slots. We assign a cost for each edge to be equal to the number of time slots that that edge interferes with: $c_{ij} = |I_{ij}|$. With respect to these new edge costs, a minimum-cost interference-free path is found. The more time slots an edge is in conflict with, the more expensive that edge will be, and the less likely it will be used in a route. We refer to this algorithm as `int_free_path`, which will return the edges and schedule of a path between s and d.

To find an interference free path that tries to minimize future conflicts, and using minimum additional time slots, we first find a minimum-cost interference free path for the current set of time slots assigned in the network, $\mathcal{T}$. If such a path does not exist, increase the set of available time slots by 1, and repeat. We note that the set of time slots will never increase by more than two since a feasible schedule can be found for any path with two free time slots. We call this algorithm `find_path`.

5.5.3 Minimum Length Schedule for Wireless Protection

In this section, an algorithm is developed that tries to find the minimum length schedule for the Wireless Guaranteed Protection problem, with an approach that is similar to the optimal solution found by the MILP in Section 5.4. The problem is broken up into $|E| + 1$ subproblems. First, the minimum length schedule is found to route the set of demands before a failure. Then, for each possible failure, the minimum length schedule is found to route the set of demands on a failure graph $G^{kl} = (V, E \setminus \{k, l\}), \forall \{k, l\} \in E$ (i.e., the graph that remains after the failure of edge $\{k, l\}$). Each of the solutions to these subproblems represents the route and schedule necessary to meet the protection requirements for the set of demands before and after any link failure.

The *maximum* of any of these minimum length schedules will be the length of the schedule needed to add protection to set of demands in a wireless network. The algorithm is called `minimum_protect`; it will return the set of paths and schedules

for each demand, indicating which path and schedule to use after any link failure.

5.5.4 Disjoint Path Wireless Guaranteed Protection

In Section 5.5.3, an algorithm was described to find the minimum number of time slots to route and schedule a set of demands with protection. After any failure, a new route is found; hence, many possible routing configurations exist, and a route and schedule for each failure event needs to be saved. A more desirable approach may be to limit the number of paths needed to only two: a primary and a backup. Before continuing with the development of the algorithm, a complexity result is presented regarding using disjoint paths to provide protection in a wireless network with 1-hop interference constraints. For a set of time slots $\mathcal{T}$, simply determining if any solution exists to WGP using disjoint paths is NP-Complete.

Theorem 5.3. *For an incoming connection between s and d, using disjoint paths to provide protection in a wireless network with 1-hop interference constraints for the set of time slots $\mathcal{T}$ is NP-Complete.*

A reduction is performed from the Dynamic Shared-Path Protected Lightpath-Provisioning (DSPLP) [18]. The proof can be found in Chapter Appendix Section 5.7.4.

Our approach for developing an algorithm to solve WGP using disjoint paths is similar to the wireless $1 + 1$ protection scheme described earlier; however, we take advantage of the time slot reuse that is possible before and after a failure, as well as the opportunity to share protection resources between failure disjoint demands. If an edge in a primary path P uses time slot t, then for 1-hop interference, all edges adjacent to that edge also cannot use t. After the failure of an edge in the primary path, the time slots used to route that path are no longer needed (since they are not being used). The time slots on the edges of the primary path that did not fail now can be reused for protection; furthermore, the time slots on the edges that interfered with the failed primary path also become free to use for protection.

Protection resource sharing can also allow for time slot reuse. If two primary

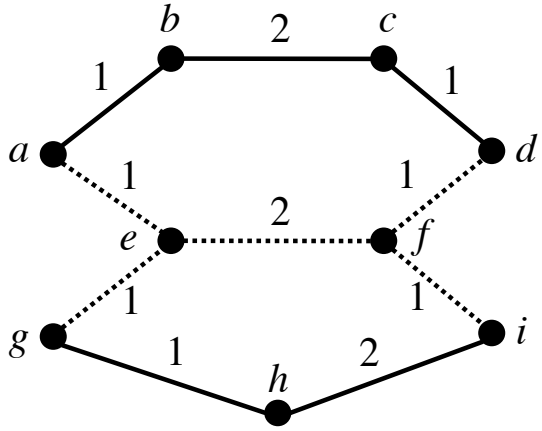


Figure 5-6: Disjoint path routing and scheduling with protection

paths are failure disjoint under a single link failure model, only one will fail at a time. Hence, a time slot t on adjacent edges can be shared for protection between the two failure disjoint connections, since the two adjacent edges will never be activated simultaneously.

An example is shown in Fig. 5-6. Two demands need to be routed under 1-hop interference constraints: one from a to d, and another from g to i. Each edge is assigned a time slot, with the time slot labeling shown in the figure. The edges used for primary flow are indicated by solid lines, and the edges used for protection are dotted lines. After the failure of edge $\{a, b\}$, the entire primary path between a and d is no longer active, and its time slots will no longer be in use; hence, edges $\{a, e\}$ and $\{f, d\}$ can use time slot 1, even though they would have conflicted with $\{a, b\}$ and $\{c, d\}$ before the failure. Similarly, $\{g, e\}$ is assigned time slot 1, even though primary edge $\{g, h\}$ is assigned the same time slot. Since both primary paths are failure disjoint, time slot 2 on $\{e, f\}$ is shared between the two connections for protection. Additionally, because at most one backup path will be used at a time, protection edges $\{g, e\}$ and $\{a, e\}$ can both be assigned time slot 1; they will never interfere with one another. Similarly, $\{f, i\}$ and $\{f, d\}$ can be both assigned time slot 1.

This idea of time slot reuse after a failure forms the basis for the the disjoint path wireless protection algorithm, which we label `disjoint_protect`. We consider an incoming demand requesting a connection between nodes s and d. Connections

already exist in the network, with the set of $\mathcal{T}$ time slots already in use. A interference-free primary path between s and d, P_{sd}, is found using `find_path`. Once a primary path fails, none of the time slots needed for that path, or on the edges that interfered with that path, are needed, and they become available to be used for protection. Next, a backup path B_{sd} is found that is disjoint to P_{sd}, and does not interfere with any of the other connections that did not fail. Additionally, the backup path B_{sd} will not interfere with the protection routings for the different existing demands that would fail if an edge in P_{sd} fails (i.e., B_{sd} will not interfere with the protection paths for demands whose primary paths are not disjoint with P_{sd}). The algorithm is detailed in Algorithm 3.

Algorithm 3
$(\mathbf{P}_{sd}, \boldsymbol{\mathcal{I}}, \mathcal{T}) =$ `disjoint_protect`$(G, \boldsymbol{\mathcal{I}}, \mathcal{T}, s, d)$

Find route and schedule before a failure:
$(P_{sd}, \mathcal{T}, \mathbf{I}') =$ `find_path`$(G, \mathbf{I}, \mathcal{T}, s, d)$
Construct new network without the edges in path P_{sd}:
$G^F = (V, E \setminus P_{sd})$
Once an edge for that demand fails, none of the slots needed to support it are used and become available. Construct a failure interference set using the interference set for the primary routes before that demand was routed, and the failure interference sets for each edge $\{k, l\}$ in P_{sd}
$\mathbf{I}^F = (\cup_{\{k,l\} \in P_{sd}} \mathbf{I}^{kl}) \cup \mathbf{I}$
Find a disjoint path, and schedule it:
$(P_{sd}^F, \mathcal{T}, \mathbf{I}'') =$ `find_path`$(G^F, \mathbf{I}^F, \mathcal{T}, s, d)$
Update interference sets:
Before a failure: $\mathbf{I} = \mathbf{I}'$
After each primary path failure: $\mathbf{I}^{kl} = \mathbf{I}''$, $\forall \{k, l\} \in P_{sd}$
$\boldsymbol{\mathcal{I}}$ = Set of $\mathbf{I}$ and all $\mathbf{I}^{kl}$
$\mathbf{P}_{sd}$ = Set of P_{sd} and all P_{sd}^{kl}
Return $(\mathbf{P}_{sd}, \boldsymbol{\mathcal{I}}, \mathcal{T})$

5.5.5 WGP Algorithm Simulations

The algorithms `minimum_protect` and `disjoint_protect` are compared to the optimal solution found by the MILP in Section 5.4. A similar simulation setup is used as that in Section 5.4. One hundred random graphs were generated with 25 nodes each. The node degree is varied from 2.5 to 6.5, and for each random graph, ten

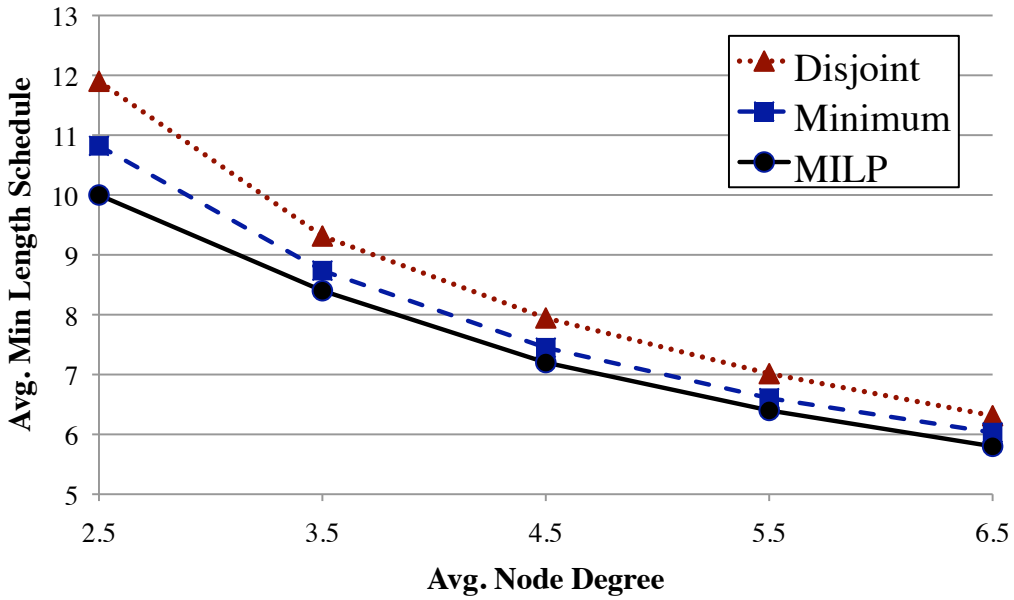


Figure 5-7: Avg. time slots needed for WGP

source/destination pairs are randomly chosen to be routed concurrently, each with a unit demand. All links have unit capacity, and 1-hop interference constraints were used. The algorithms route and schedule demands one-at-a-time, while the MILP optimizes the route and schedule for all demands together (in batch). To compare the two, the algorithms randomly order the set of demands, and then solves for each demand one-at-a-time. The simulation results are found in Fig. 5-7.

Similar to the previous simulation, as node degree increased, the average minimum length schedule decreased. This is because of the increased diversity in possible number of end-to-end path, which leads to a greater opportunity of finding interference free paths. On average, `minimum_protect` needed only 4.5% more time slots to meet all requirements than the optimal MILP needed, and `disjoint_protect` needed 10.1% more time slots than the MILP.

5.6 Conclusion

In this chapter, the problem of guaranteed protection in a multi-hop wireless network is introduced. Because of link interference, resources that were unavailable prior to a failure can be used for protection after the failure. In fact, protection can often

be provided using no additional resources. For the case of a single demand with 1-hop interference constraints, properties of an optimal solution are presented, and a time-efficient algorithm is developed that solves the problem of wireless routing and scheduling with and without protection, guaranteeing a maximum throughput that is within 1.5 of optimal. For general interference constraints and multiple concurrent demands, an optimal solution is developed for the protection problem via a mixed integer linear program. When compared to using traditional wired protection schemes on a wireless network, our Wireless Guaranteed Protection (WGP) scheme uses as much as 72% less protection resources to achieve the same level of resiliency. Two low-complexity algorithms to solve WGP are developed, and on average, these algorithms perform close to the optimal solution. A future direction for our work is to adapt the schemes developed in this chapter to a distributed setting.

5.7 Chapter Appendix

5.7.1 MILP for WGP with Different Throughputs

Some demand between nodes s and d has its own throughput requirement f^{sd}. The objective of the MILP is to minimize the number of times slots needed to schedule every demand. Since each demand has a throughput requirement, finding a minimum length schedule will be with respect to keeping the ratio of the different demands' throughputs constant. We assume f^{sd} is integer $\forall(s,d) \in (V,V)$. If necessary, the demands and link capacities can be scaled by the smallest integer that makes all demand values integer (hence, f^{sd} is assumed to be at the very least rational, $\forall(s,d) \in (V,V)$).

For the MILP, the following values are given:

- $G = (V, E)$ is the graph with a set of vertices and edges
- D is the set of flow requirements
- f^{sd} is the flow required between nodes (s,d); $f^{sd} \in \mathbb{Z}$

- u_{ij} is the capacity of link $\{i, j\}$

- $\mathcal{I}$ is the interference matrix, where $I_{ij}^{kl} \in \mathcal{I}$ is 1 if links $\{i, j\}$ and $\{k, l\}$ can be activated simultaneously, 0 otherwise

- $\mathcal{T}$ is the set of time slots in the system, $\mathcal{T} \subset \mathbb{Z}^+$

The MILP solves for the following variables:

- x_{ij}^{sd} is a routing variable and is 1 if primary flow is assigned for demand (s, d) on link $\{i, j\}$, 0 otherwise

- $y_{ij,kl}^{sd}$ is a routing variable and is 1 if protection flow is assigned on link $\{i, j\}$ for the demand (s, d) after the failure of link $\{k, l\}$, 0 otherwise

- $\lambda_{ij}^{sd,t}$ is a scheduling variable and is 1 if link $\{i, j\}$ can be activated in time slot t for the demand (s, d), 0 otherwise

- $\delta_{ij,kl}^{sd,t}$ is a scheduling variable and is 1 if link $\{i, j\}$ can be activated in time slot t after failure of link $\{k, l\}$ for the demand (s, d), 0 otherwise

- s_t is 1 if time slot t is used by any demand, and 0 otherwise

The objective function is to minimize the number of time slots (the length of the schedule) needed to route all demands with protection:

$$\textbf{Objective:} \quad \min \sum_{t \in \mathcal{T}} s_t \tag{5.11}$$

The following constraints are imposed to find a feasible routing and scheduling with protection.

Before a link failure:

- Flow conservation constraints for the primary flow: route primary traffic before a failure for each demand.

$$\sum_{\{i,j\}\in E} x_{ij}^{sd} - \sum_{\{j,i\}\in E} x_{ji}^{sd} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases}, \quad \substack{\forall i\in V \\ \forall (s,d)\in D} \tag{5.12}$$

- In any given time slot, for a given demand, only links that do not interfere with one another can be activated simultaneously.

$$\sum_{(s,d)\in D} \lambda_{ij}^{sd,t} + \sum_{(s,d)\in D} \lambda_{kl}^{sd,t} \leq 1 + I_{kl}^{ij}, \quad \substack{\forall \{i,j\}\in E,\ \forall \{k,l\}\in E \\ \{i,j\}\neq\{k,l\},\ \forall t\in\mathcal{T}} \tag{5.13}$$

- Only one demand can use a given link at a time.

$$\sum_{(s,d)\in D} \lambda_{ij}^{sd,t} \leq 1, \quad \substack{\forall \{i,j\}\in E \\ \forall t\in\mathcal{T}} \tag{5.14}$$

- Ensure enough capacity exists to support the necessary flow f^{sd} for demand (s,d) on edge $\{i,j\}$ for the length of time that the link is active.

$$f^{sd} x_{ij}^{sd} \leq \sum_{t\in\mathcal{T}} \lambda_{ij}^{sd,t} u_{ij}, \quad \substack{\forall \{i,j\}\in E \\ \forall (s,d)\in D} \tag{5.15}$$

- Mark if slot t is used to schedule a demand before a failure.

$$\lambda_{ij}^{sd,t} \leq s^t, \quad \substack{\forall \{i,j\}\in E \\ \forall t\in\mathcal{T},\ \forall (s,d)\in D}$$

After a link failure:

- Flow conservation constraints for protection flow: route protection traffic after each link failure $\{k,l\} \in E$.

$$\sum_{\substack{\{i,j\}\in E \\ \{k,l\}\neq\{i,j\}}} y_{ij,kl}^{sd} - \sum_{\substack{\{j,i\}\in E \\ \{k,l\}\neq\{j,i\}}} y_{ji,kl}^{sd} = \begin{cases} 1 & \text{if } i = s \\ -1 & \text{if } i = d \\ 0 & \text{otherwise} \end{cases}, \quad \substack{\forall i\in V,\ \forall \{k,l\}\in E \\ \forall (s,d)\in D} \tag{5.16}$$

- In any given time slot after the failure of link $\{k,l\}$, only links that do not interfere with one another can be activated simultaneously.

$$\sum_{(s,d)\in D} \delta_{ij,kl}^{sd,t} + \sum_{(s,d)\in D} \delta_{uv,kl}^{sd,t} \leq 1 + I_{uv}^{ij}, \quad \begin{matrix} \forall\{i,j\}\in E,\ \forall\{k,l\}\in E \\ \forall\{u,v\}\in E,\ \forall t\in\mathcal{T} \\ \{i,j\}\neq\{k,l\}\neq\{u,v\} \end{matrix} \tag{5.17}$$

- Only one demand can use a given link at a time after the failure of link $\{k,l\}$.

$$\sum_{(s,d)\in D} \delta_{ij,kl}^{sd,t} \leq 1, \quad \begin{matrix} \forall\{i,j\}\in E,\ \forall\{k,l\}\in E \\ \forall t\in\mathcal{T} \end{matrix} \tag{5.18}$$

- Ensure enough capacity exists after the failure of link $\{k,l\}$ to support the necessary flow f^{sd} on edge $\{i,j\}$ for the length of time that the link is active.

$$f^{sd} y_{ij,kl}^{sd} \leq \sum_{t\in\mathcal{T}} \delta_{ij,kl}^{sd,t} u_{ij}, \quad \begin{matrix} \forall\{i,j\}\in E,\ \forall\{k,l\}\in E \\ \forall(s,d)\in D \end{matrix} \tag{5.19}$$

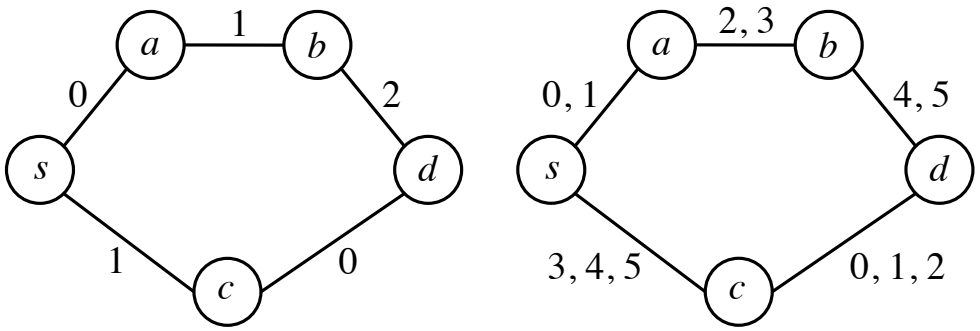
- Mark if time slot t is used to schedule a demand after the failure of link $\{k,l\}$.

$$\delta_{ij,kl}^{sd,t} \leq s^t, \quad \begin{matrix} \forall\{i,j\}\in E,\ \forall\{k,l\}\in E \\ \forall t\in\mathcal{T},\ \forall(s,d)\in D \end{matrix}$$

5.7.2 Schedules for Higher Throughput on Node-Disjoint Paths with an Odd Number of Edges

In Section 5.3.1, for a pair of node-disjoint paths whose total number of edges is odd, a schedule using three time slots was used to achieve a flow of $\frac{2}{3}$ between the source and the destination. Each link is assigned one of three time slots, and since each link is active for only $\frac{1}{3}$ of the time, each path supports a flow of $\frac{1}{3}$, and the total end-to-end flow is $\frac{2}{3}$. An example is shown on the five edge network in Figure 5-8a.

By using additional time slots, it is in fact possible to increase the end-to-end throughput. For the same five edge network, a maximum flow of $\frac{5}{6}$ is possible using six time slots. A schedule that achieves this flow is shown in Figure 5-8b. On the shorter path, three time slots are assigned to each link. Since there are a total of six



(a) Flow of $\frac{2}{3}$ (b) Flow of $\frac{5}{6}$

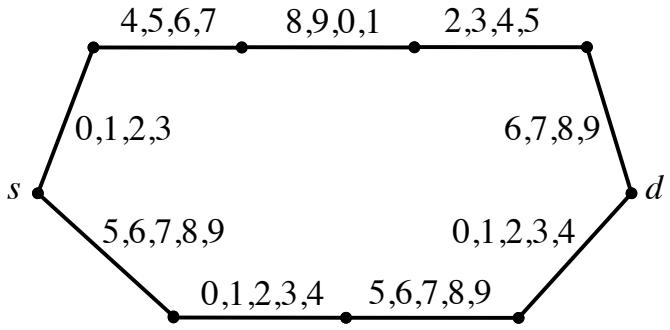
Figure 5-8: Node-disjoint paths with an odd total number of edges supporting a flow of $\frac{2}{3}$ and $\frac{5}{6}$.

time slots in use, each link is active for half of the total time, and is supporting a flow of $\frac{1}{2}$. On the longer path, each link is assigned two time slots. These two time slots represent $\frac{1}{3}$ of the total time; hence the longer path supports a flow of $\frac{1}{3}$. The total flow on both paths is $\frac{1}{2} + \frac{2}{3} = \frac{5}{6}$.

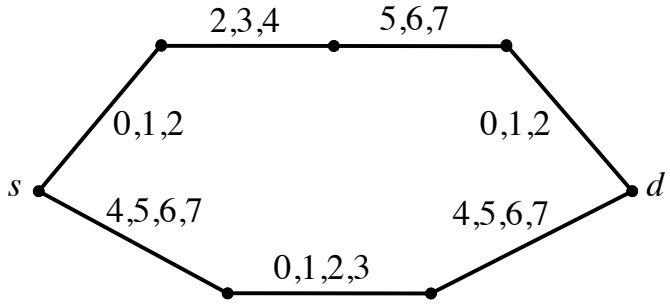
If the longer of the two node-disjoint paths has K edges, then it is in fact possible to always achieve a throughput over the two paths of $\frac{2K-1}{2K}$ by employing the following scheduling scheme that uses $2K$ time slots. On the shorter path, we assign half of the time slots to each edge: K to $(2K-1)$ on the first edge, 0 to $(K-1)$ on the second edge, and alternate between those two assignments for each subsequent edge for the remainder of the path. Since each edge of the shorter path uses half of the time slots, each edge is active $\frac{1}{2}$ of the time, and the shorter path carries a flow of $\frac{1}{2}$.

On the longer path (having K edges), assign $(K-1)$ time slots to each edge in the following fashion: For the j^{th} edge, where edge 0 leaves the source and edge $K-1$ enters the destination, assign time slots $mod[j(K-1), 2K]$ through $mod[(j+1)(K-1)-1, 2K]$. The notation $mod[a, b]$ represents the modulo function whose value is the integer remainder when a is divided by b. Each edge has $(K-1)$ time slots assigned to it and is active for $\frac{K-1}{2K}$ of the time, allowing the longer path to support a flow of $\frac{K-1}{2K}$. The total flow across both paths is $\frac{K}{2K} + \frac{K-1}{2K} = \frac{2K-1}{2K}$. This scheduling scheme can always achieve a throughput of $\frac{2K-1}{2K}$, which is demonstrated in Lemma 5.8.

Two examples are shown in Figure 5-9. In the first network, shown in Figure 5-9a, the longer path has five edges ($K = 5$), and the shorter has four. Ten time slots are used in total, with the shorter path supporting a flow of $\frac{1}{2}$, and the longer path supporting a flow of $\frac{4}{5}$, resulting in an end-to-end flow of $\frac{9}{10}$. It is straightforward to see that if the shorter path had two edges instead of four, the same throughput would have been achievable using the same ten time slots. In the second network, shown in Figure 5-9b, the longer path has four edges ($K = 4$), and the shorter has three; eight time slots are used. The shorter path supports a flow of $\frac{1}{2}$, the longer path supports a flow of $\frac{3}{8}$, and the total end-to-end flow is $\frac{7}{8}$.



(a) $K = 5$, and a flow of $\frac{9}{10}$



(b) $K = 4$, and a flow of $\frac{7}{8}$

Figure 5-9: Node-disjoint paths with an odd number of edges supporting flows of $\frac{2K-1}{2K}$

Lemma 5.8. *For a pair of node-disjoint paths with an odd number of edges, where the longer path has K edges, a schedule exists that achieves a throughput of $\frac{2K-1}{2K}$ over the two paths.*

Proof. We demonstrate the scheduling scheme presented in this section always achieves the desired rate of $\frac{2K-1}{2K}$. We consider two cases: K is odd, and K is even.

We first examine the case where K is odd; an example was shown in Figure 5-9a. Since the longer path has an odd number of edges, the shorter path must have an even number. On the shorter path, half the time slots are assigned to each edge, alternating between time slots K to $2K-1$ on the first edge, time slots 0 to $K-1$ on the second edge, and so forth until the final edge. Since there is an even number of edges, the final edge of the shorter path entering the destination will be assigned time slots 0 to $K-1$. On the longer path, time slots $mod[j(K-1), 2K]$ through $mod[(j+1)(K-1)-1, 2K]$ are assigned to the j^{th} edge, with edge 0 leaving the source and edge $K-1$ entering the destination. This results in $K-1$ time slots assigned to each edge. By construction, the edges leaving the source for each path do not interfere with one another. We need to verify that the final edges entering the destination also do not interfere. The final edge of the path entering the destination is numbered $j = K-1$; the time slot assignment for that edge is $mod[(K-1)(K-1), 2K]$ through $mod[K(K-1)-1, 2K]$. The value of $mod[a, b]$ is equal to $a - b\lfloor \frac{a}{b} \rfloor$ [90], where $\lfloor q \rfloor$ is the integer floor of some value q. The final time slot assigned to edge $K-1$ is:

$$\begin{aligned} mod[K(K-1)-1, 2K] &= K(K-1) - 1 - 2K\left\lfloor \frac{K(K-1)-1}{2K} \right\rfloor \\ &= K(K-1) - 1 - 2K\left\lfloor \frac{K-1}{2} - \frac{1}{2K} \right\rfloor \end{aligned}$$

Since K is odd, $\frac{K-1}{2}$ is integer; hence, we get:

$$\begin{aligned} mod[K(K-1)-1, 2K] &= K(K-1) - 1 - 2K\left(\frac{K-1}{2} + \left\lfloor -\frac{1}{2K} \right\rfloor\right) \\ &= K(K-1) - 1 - 2K\left(\frac{K-1}{2} - 1\right) \\ &= K(K-1) - 1 - K(K-1) + 2K \\ &= 2K - 1 \end{aligned}$$

The final edge ($j = K-1$) of the longer path is assigned time slots: $(K+1)$ through $(2K-1)$. The final edge of the shorter path was assigned time slots 0 to $K-1$. Hence, when K is odd, this scheduling scheme will not cause interference between

adjacent edges, and will achieve an end-to-end flow of $\frac{2K-1}{2K}$.

We next demonstrate a similar result for when K is even; an example network was shown in Figure 5-9b. The longer path has an even number of edges, and the shorter path has an odd number. Again, on the shorter path, half the time slots are assigned to each edge, alternating between time slots K to $2K-1$ on the first edge, time slots 0 to $K-1$ on the second edge, and so forth until the final edge. Since there is an odd number of edges, the final edge of the shorter path entering the destination will be assigned time slots K to $2K-1$. We now consider the final edge entering the destination of the longer path, which will be assigned time slots $mod[(K-1)(K-1), 2K]$ through $mod[K(K-1)-1, 2K]$. The final time slot for this edge will have the value $mod[K(K-1)-1, 2K] = K(K-1)-1-2K\lfloor\frac{K-1}{2} - \frac{1}{2K}\rfloor$. Since K is even, $\frac{K-1}{2}$ is *not* integer, but $\frac{K}{2}$ is; hence, we get:

$$\begin{aligned} mod[K(K-1)-1, 2K] &= K(K-1)-1-2K\left\lfloor \frac{K-1}{2} - \frac{1}{2K}\right\rfloor \\ &= K(K-1)-1-2K\left(\frac{K}{2} + \left\lfloor -\frac{1}{2} - \frac{1}{2K}\right\rfloor\right) \\ &= K(K-1)-1-2K\left(\frac{K}{2} - 1\right) \\ &= K^2 - K - 1 - K^2 + 2K \\ &= K-1 \end{aligned}$$

The final edge of the longer path is assigned time slots 1 through $(K-1)$. The final edge of the shorter path was assigned time slots K to $2K-1$. Therefore, when K is even, this scheduling scheme will not cause interference between adjacent edges at the destination, and will achieve an end-to-end flow of $\frac{2K-1}{2K}$. □

Using the scheme described above, the minimum throughput that can be guaranteed on a pair of node-disjoint paths with an odd number of edges is $\frac{5}{6}$, which is greater than the $\frac{2}{3}$ flow described in Section 5.3.1. The minimum guaranteed throughput of $\frac{5}{6}$ is independent of K.

Lemma 5.9. *For a pair of node-disjoint paths, a schedule can always be found that*

guarantees a flow of at least $\frac{5}{6}$ from the source to the destination.

Proof. We consider only the case when the source and destination are more than one hop apart; otherwise, only one edge needs to be activated between the two nodes, carrying the maximum flow of 1 without the use of node disjoint paths.

When there are an even number of edges over the two node-disjoint paths, then the maximum flow of 1 can be achieved using two time slots, as was shown in Lemma 5.3.

When there are an odd number of edges over the two node-disjoint paths, where the longer path has K edges, then a flow of $\frac{2K-1}{2K}$ can always be achieved, which was demonstrated in Lemma 5.8. We now show that the minimum K is 3, hence the minimum flow is $\frac{5}{6}$. Since the source and destination are more than one hop apart, the minimum number of edges over both paths is 4. With 4 total edges, the two paths have an even number of edges, and a maximum flow of 1 is achievable using two time slots. The next smallest number of edges for both paths is 5. Since the source and destination cannot be one hop apart, this means the longer path has 3 edges, and the shorter has 2. Hence, the smallest value of K possible is 3, which gives an achievable throughput of $\frac{2K-1}{2K} = \frac{5}{6}$. Any value of K that is greater than 3 will result in a higher achievable throughput. □

If only a pair of node-disjoint paths exist with an odd number of edges between the source and destination, a flow of $\frac{2K-1}{2K}$ can found using $2K$ time slots. But this does preclude the possibility of higher feasible throughputs existing that use additional edges. Consider the example in Figure 5-10.

The pair of node-disjoint paths between nodes s and d are shown using the solid edges, and additional edges that connect with one of the node-disjoint paths are shown using the dotted edges. In this network, all possible pairs node-disjoint paths have an odd number edges. The longer path has 4 edges; if we scheduled according the scheme described earlier, a flow of $\frac{7}{8}$ can be achieved between s and d. But by using the dotted edges, in addition to the solid edges, a schedule can be found that achieves the maximum flow of 1, as shown in Figure 5-10. This shows that the flow

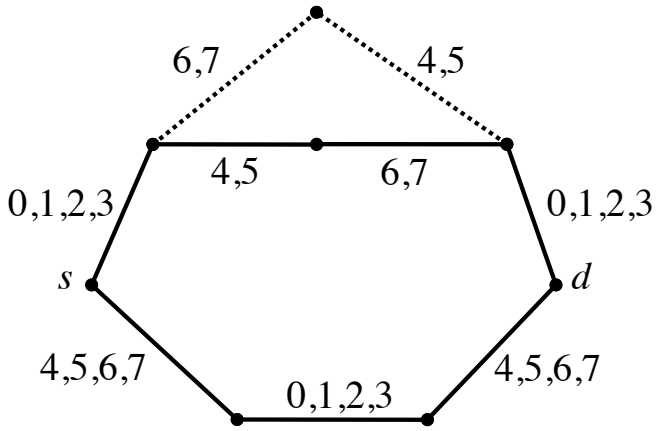


Figure 5-10: Node-disjoint paths with additional edges supporting a flow of 1

$\frac{2K-1}{2K}$ is strictly a lower bound on the maximum flow that can be found when only a pair of node-disjoint paths exist that have an odd number of edges. We do not currently consider the problem of determining whether or not a maximum flow of 1 exists in this case, and we leave it as future work.

5.7.3 Proof for Theorem 5.2

Theorem 5.2: Finding the minimum length schedule to route a set of demands with protection under 1-hop interference constraints without flow splitting is NP-Hard.

Proof. We prove the protection version of the problem to be NP-hard by reducing the non-protection version to it. We begin by taking a graph $G = (V, E)$ and transforming it to some other graph $G' = (V', E')$. Graph G' will be constructed such that any feasible solution in G using two time slots for node-disjoint paths for a set of node-disjoint demands (i.e., a solution for the non-protection problem in G) will have an immediate solution that includes protection using two time slots for the same set of node-disjoint demands.

Consider an edge $\{i, j\}$ of some path between s and d in G, which is node-disjoint from any other path and is scheduled to use time slot 1. Three edges, $\{i, k\}$, $\{k, l\}$, and $\{l, j\}$, can be added to protect $\{i, j\}$ without needing any additional time slots, with time slot assignments as shown in Fig. 5-11. Edge $\{i, k\}$ and $\{l, j\}$ are assigned time slot 1, and since they will only be activated after the failure of edge $\{i, j\}$,

they do not conflict with the time slot assignment for $\{i, j\}$. Furthermore, the edge on the path in G directly preceding and directly following edge $\{i, j\}$ will be time slot 2 (because the schedule only consisted of two time slots); so, after $\{i, j\}$ fails, the protection routing of $\{i, k\}$, $\{k, l\}$, and $\{l, j\}$ will not interfere with the existing scheduled edges of the path. Additionally, since we are currently considering a feasible solution of node-disjoint paths for the set of node-disjoint demands, no node will have more than a single path crossing it, so the protection path of $\{i, k\}$, $\{k, l\}$, and $\{l, j\}$ will not interfere with any other demands.

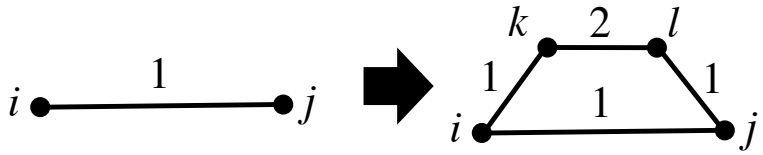


Figure 5-11: Edge transformation for NP-hardness proof

To begin, it is clear that any solution for a non-protection routing in two time slots for the set of node-disjoint demands in G will immediately give a protection routing and schedule using two time slots in G' for the same set of demands. We now consider the other direction: for a set of node-disjoint demands, does a solution that uses two time slots for the protection problem on G' give a solution for the same set of demands using two time slots for the non-protection problem in G? If the protection problem returns a solution for a routing and scheduling using two slots, then that means that before any failure, and after any failure, the set of node-disjoint demands can be routed in two time slots. So, if a routing and schedule *is* found, then we take the route and schedule from before a failure (which is in two time slots), and transfer any flow that may have been routed onto $\{i, k\}$, $\{k, l\}$, and $\{l, j\}$ to edge $\{i, j\}$, with $\{i, j\}$ having the same time slot assignment as $\{i, k\}$. Because of how our transformation was performed, this will always yield a feasible solution for the set of node-disjoint demands in G. In general, it is not necessarily the case that no solution to the protection problem indicates no solution to the non-protection problem. But for our particular graph transformation, this is the case; we know that if a two time slot solution exists in G, then a protection routing must exist that uses two time slots.

Hence, if the minimum schedule to the protection problem for the set of node-disjoint demands in G' uses more than two time slots, then the schedule for the set of demands in G must use more than two time slots.

We complete the proof by noting that our problem is clearly in NP, and that the graph transformation of G to G' can be accomplished in polynomial time. □

5.7.4 Proof for Theorem 5.3

Theorem 5.3: For an incoming connection between s and d, using disjoint paths to provide protection in a wireless network with 1-hop interference constraints for the set of time slots $\mathcal{T}$ is NP-Complete.

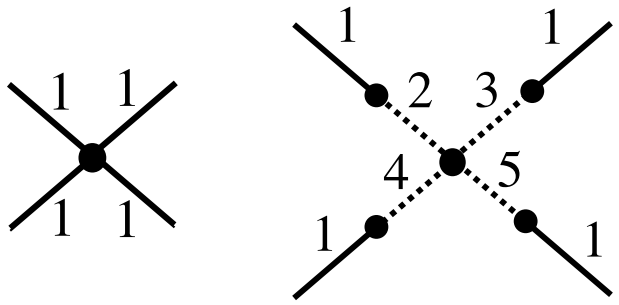
Proof. We reduce the Dynamic Shared-Path Protected Lightpath-Provisioning (DSPLP) [18] to our problem. We first begin by giving details of DSPLP.

DSPLP has the following set of parameters: W is the set of possible wavelengths on any link. $L-1$ paths are routed, where (w_i, b_i) is the i^{th} working and backup path, respectively. The question DSPLP asks is: does there exist a (w_L, b_L) from s to d that satisfies shared path protection constraints? Those constraints being: (1) w_L and b_L are link disjoint; (2) w_L and w_i, $1 \leq i < L$, do not utilize same wavelength on any common link; (3) w_L and b_i, $1 \leq i < L$, do not utilize same wavelength on any common link; (4) b_L and b_i, $1 \leq i < L$, can share a wavelength on a common link if w_L and w_i are link disjoint. The following is provided by DSPLP: graph G with vertices and edges (V, E); W is the set of possible wavelengths on any link. λ_{ij} is the set of wavelengths used on edge $\{i, j\}$ for primary paths; λ_{ij}^{kl} is the set of wavelengths used on edge $\{i, j\}$ to protect against the failure of $\{k, l\}$. $T = |W|$. More simply put, some new incoming demand that needs a disjoint primary and protection path, can share backup resources with some other demand if the two primary paths are failure disjoint.

There is a clear parallel between the wavelength multiplexing scheme that DSPLP is based on and our wireless protection scheme that uses time slots: time slots used for routing/scheduling on a link are similar to wavelengths used on a link for routing

and protection. In [18], NP-Completeness is shown for a network with $T = 1$; hence, it is sufficient for us to demonstrate that if our problem can solve an instance of DSPLP with only one wavelength, our problem is also NP-Complete.

If wavelengths are considered as timeslots, they will interfere with one another. Clearly more than one time slot must exist in order to find a feasible routing and schedule in a wireless network with 1-hop inference constraints. So, we "extend" a "new" link from each node, and we increase T (the number of time slots in the wireless network) such that there exists sufficient time slots to change existing paths that use one wavelength in the original network into interference free schedules using the "new" edges in G^W. It is easy to see that the number of new time slots that need to be added will be the maximum node degree of the network. An example is shown in Fig. 5-12. The node degree is 4, and each edge has a path routed on it using the existing wavelength. We extend "new" edges out of the node, and increase the number of time slots to any value above 5. Now an interference free schedule can be assigned to allow the edges that used wavelength 1 (now time slot 1) to continue routing those paths using time slot 1.



(a) Node in DSPLP network (b) Node with "new" edges

Figure 5-12: Time slot assignment for extended "new" edges

We modify the network G to G^W in the following manner. We "extend" a "new" link from each node, and increase T^W (the number of time slots in the wireless network) by some "large enough" value, such that the "new" edges in G^W will never interfere with one another, or with existing demands using wavelenth/time slot 1. "Old" links in G^W that used wavelength 1 to support a lightpath in G will have no

available time slots in G^W. Since the primary links using wavelength 1 in G will no have no free time slots in G^W, every future incoming demand in G^W will be edge-disjoint from the existing primary demand that use wavelength/time slot 1, and hence can share backup capacity with existing demands. All links in G that use wavelength 1 for protection will have only one free time slot (time slot 1) available for use in G^W, and only as the backup path for a future demand in G^W. All other "old" links in G^W will have only one time slot available, time slot 1, available for use for the primary or protection path.

Consider some new incoming demand in G^W that needs to be routed and scheduled with a disjoint primary and protection path. The way the network G^W was constructed will ensure that if a solution exists for the new demand, then a solution exists for DSPLP in G. The new demand in G^W will only be able to use time slot 1 on the "old" links, which is wavelength 1. Any solution for wireless protection using disjoint paths in G^W can be converted to a solution for DSPLP in G by removing the "new" edges. If no solution exists for wireless protection using disjoint paths in G^W, then it is clear that no solution exists for DSPLP in G. It is also clear that any solution for DSPLP in G will solve wireless protection using disjoint paths in G^W (with the trivial addition of the "new" edges).

To complete the proof, we note that our problem is clearly in NP. □

Chapter 6

Conclusion and Future Directions

In this thesis, we investigated providing network protection with a variety of service guarantees that offer significant resource savings over traditional full protection schemes. Because of the high cost of full protection, many network operators provide no protection whatsoever. In addition, traditional full protection typically offer no guarantees on recovery time, which is becoming increasingly important with the widespread use of real-time applications that cannot tolerate long disruptions. Furthermore, network protection has not been adequately examined in wireless networks because of the scarcity of resources available for backup and the complexity of allocating these resources. By examining various service guarantees, we allow operators to offer resource-efficient protection that fit the particular needs of their network.

In Chapter 2, we developed a network protection scheme called Partial Protection that guarantees a quantifiable minimum grade of service upon a failure within the network. In Chapter 3, we built upon the partial protection scheme developed in Chapter 2 by adding the additional constraint of only allowing flow to drop to its reduced service level for at most a certain probability after a network failure. In Chapter 4, we switch focus from examining guaranteed partial flows after a failure to considering the problem of providing network protection that guarantees the maximum amount of time that flow can be interrupted after a failure. In Chapter 5, we move from the wired setting towards examining protection in wireless networks; in particular, we investigate the problem of providing resource-efficient guaranteed

protection against failures in wireless networks subject to interference constraints.

We believe that the work on protection with various service guarantees is just beginning. As communications continues to evolve, differing types of protection will be necessary to fit the particular needs of future networks and services. One future direction is to continue adapting the models and algorithms developed in this thesis. One example is distributing these algorithms across the network such that there is no need for a central planner. Another example is to study protocol design that will allow the service guaranteed protection frameworks developed within this thesis to be easily adapted into already existing networks. This work considered providing protection for already existing networks. Another approach that can be taken is from the network planning perspective: How can networks be designed such that they are most conducive for protection with respect to the different service guarantees? This question is particularly important for wireless networks that have limited resources available for protection.

As data networks continue to rapidly change, there will never be a lack of future directions for service guaranteed protection. Twenty years ago, the concept of "services" that may have differing needs was not even considered; ten years ago, wireless, cloud, and real-time services were too new to begin thinking of offering customized protection; in ten years from now, new services will exist that we cannot predict today, and they too will need protection to meet their particular needs. This thesis offers a starting point on how to meet the current and future challenges of service guaranteed protection.

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