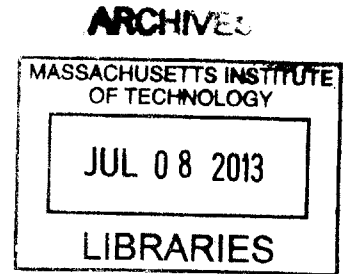# Increasing Supply Chain Robustness through Process Flexibility and Strategic Inventory

by

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Abstract

When a disruption brings down one of company's manufacturing facilities, it can have a ripple effect on the entire supply chain and threaten the company's ability to compete. In this thesis, we develop an effective disruption mitigation strategy by using both process flexibility and strategic inventory. The model is focused on a manufacturer with multiple plants producing multiple products, where strategic inventory can be held for any product. We propose a new metric of supply chain robustness, defined as the maximum time that no customer demand is lost regardless of which plant is disrupted.

Using this metric, we analyze K-chain flexibility designs in which each plant is capable of producing exactly K products. It is demonstrated that a 2-chain design, which is known to be effective for matching supply with demand when there is no disruption, is not robust when there is both disruption and demand uncertainty. However, it is shown that a 3-chain design is significantly more robust and achieves the same robustness as full flexibility under high uncertainty level.

We then extend the model to an assembly system and find that investment in process flexibility designs changes the optimal inventory placements. In particular, when the degree of flexibility is high, more inventory is allocated to standard components, i.e. components used by multiple products, but when the degree of flexibility is low, more inventory is allocated to non-standard components.

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Chapter 1

Introduction

On March 11, 2011 a 9.0-magnitude earthquake, among the five most powerful on record, struck off the coast of Japan. Tsunami waves in excess of 40 meters high traveled up to 10 kilometers inland and as a result, three nuclear reactors at Fukushima Daiichi experienced Level 7 meltdowns. The impact of this combined disaster was devastating, with over 25,000 people dead, missing or injured (Schmidt and Simchi-Levi, 2012). The event was not only a humanitarian disaster, but also an economic crisis for the Japanese industry in general, and the automotive industry in particular. For example, Toyota's production in Japan declined 31.4% in the first six months after the earthquake, as compared with its 2011 forecast. Indeed, "Toyota's consolidated unit sales for the first half of its current fiscal year decreased by 689,000 units to 3,026,000 units, compared with the same period last year, mainly due to the earthquake disrupting the production and supply chain in Japan.[1]"

To safeguard against future disruptions, Toyota is working on changes to its supply chain so they can **recover within two weeks** from any major disruption to one of its facilities. In an interview on March 2, 2012, Toyota's Executive Vice President in charge of purchasing, Shinichi Sasaki, explained that part of their strategy is to make sure that when "one factory is hit, the same part could be manufactured elsewhere...and to ask suppliers further down the chain to hold enough inventory.[2]" This

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[1]Standard & Poor's, December 31, 2011.

[2]Chang-Ran Kim, "Toyota says supply chain will be ready by autumn for next quake", Reuters, March 2, 2012.

suggests that Toyota is focusing on a combination of **process flexibility** and **strategic inventory** as a way to satisfy demand during the two-week recovery period.

Process flexibility has been extensively studied as a strategy for demand uncertainty, but few papers consider it as a tool to mitigate supply uncertainty. The objective of this research is to understand the effectiveness of the hybrid approach of process flexibility and strategic inventory for supply chain disruptions. Specifically, our objective is to understand the impact of process flexibility on the total level of strategic inventory required in the supply chain such that the firm can continue to serve its customers during a period of a plant disruption.

1.1 Strategic Inventory and Process Flexibility

Strategic inventory is the additional inventory that is dedicated to supply chain disruption and hence is independent of lead time, the review policy or the details of the inventory management policy used on a day-to-day basis. Holding strategic inventory beyond cycle stock and safety stock has been identified in a number of papers (see literature review in Section 1.3) as an important tool for dealing with supply chain disruption. Unfortunately, holding a large amount of strategic inventory can be costly or risky and hence negatively effects financial performance.

Process flexibility is defined as the ability to build different types of products in the same manufacturing plant or on the same production line at the same time (Jordan and Graves, 1995), see Figure 1-1. For example, in **full flexibility**, each plant is capable of producing all products while in a **dedicated,** or no-flexibility, strategy, each plant is capable of producing just a single product. With process flexibility the firm is in much better position to match available capacity with variable demand. Unfortunately, implementing full flexibility can be very expensive since each plant needs to be capable of producing all products (Simchi-Levi, 2010), as a result, **partial** flexibility is considered. In such strategy, each plant is capable of producing just a few products. One specific partial flexibility design analyzed extensively in the literature (see Chou et al., 2008) is the **long chain** where each plant produces exactly two

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products and the design connects all plants to all products in a single cycle. While process flexibility can help the firm to safeguard against disruption, it is only viable when the firm has a lot of excess capacity in its plants. However, when plants are highly utilized, process flexibility is no longer an effective strategy to mitigate the impact of disruptions.
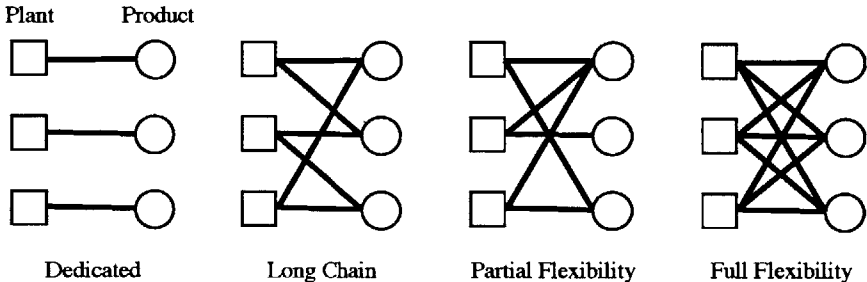
Figure 1-1: Process flexibility designs.

Because both strategic inventory and process flexibility have their limitations as a disruption mitigation strategy, this research is focused on developing a methodology that combines process flexibility and strategic inventory to increase supply chain robustness. We measure supply chain robustness through the concept of **Time-to-Survive** (TTS), the maximum time that customer demand is guaranteed to be satisfied no matter which single plant is disrupted. The longer the TTS, the more robust the supply chain is. For example, in the Toyota new supply chain strategy, if TTS is greater than or equal to two weeks, than the firm will be able to maintain cash flow (sales) even if one of its plants is down.

Observe that our definition of supply chain robustness ignores probability distribution on the likelihood of a major disruption to the supply chain due to the unpredictable nature of such events. As argued in Simchi-Levi (2010), there is little experience to draw on to prepare for natural megadisasters like hurricane Katrina in 2005, the Iceland Volcano eruption in 2010, or the Japanese Tsunami in 2011. Similarly, a viral epidemic like the 2003 SARS can shutdown the flow of products from a plant but is difficult to prepare for because of lack of data. Simchi-Levi (2010) refers to these types of risks as the *Unknown-Unknown*.

1.2 Overview

The Time-to-Survive (TTS) model is formally defined in Chapter 2, under the assumption that customer demand is deterministic. We show that TTS can be solved by a linear program. This result makes a detailed analysis of TTS possible, as we will see in the following chapters. It also shows that TTS can be easily implemented by managers, who can also modify the TTS model to address practical problems in supply chain robustness design.

Chapter 3 investigates the interplay between process flexibility and strategic inventory in supply chain disruption mitigation. We provide two major insights. First, a fully flexible supply chain needs significantly less inventory than a dedicated, or no flexibility, supply chain to achieve the same level of robustness (measurable by TTS). Second, it is possible for partially flexible supply chain designs, such as long chain and 3-flexibility, to have exactly the same robustness as that of full flexibility. Importantly, this implies that for these designs, not only total inventory is the same as that of full flexibility, but also product by product inventory is the same as that of full flexibility. However, the condition for long chain to achieve full flexibility is much more restrictive than that of 3-flexibility.

Chapters 4, 5 and 6 are focused on non-deterministic demand. In Chapter 4, we analyze the case where product demand belongs to an uncertainty set, with the objective of minimizing inventory to achieve a given TTS under the worst-case demand. We show that when demand is highly uncertain, there is a significant gap between the robustness of the long chain and that of full flexibility. However, increasing the degree of flexibility such that each plant produces exactly three products achieve the same robustness as full flexibility under a much larger uncertainty level.

In Chapter 5, the TTS model is further extended to an assembly network. This is motivated by cases where manufacturers use **assemble-to-order** strategy because they cannot afford to hold much inventory of final products, but they can ask suppliers to stock inventory for components. In this setting, we find out that process flexibility and product standardization are substitutes of each other to achieve supply

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chain robustness. (We can view product standardization as product design flexibility because it allow one component to used in many products.) Moreover, depending on whether suppliers have high or low process flexibility, the company should implement completely different inventory decisions. With low level of process flexibility (e.g., dedicated, long chain), more strategic inventory should be allocated to components with high demand volatility. But with high level of process flexibility (e.g., full flexibility), strategic inventory should be stocked for those components with low demand uncertainty.

Chapter 6 studies stochastic demand with known distributions, where we use the expected TTS as a metric for robustness. Numerical tests show that the same insights developed in previous chapters also hold for the stochastic demand case.

1.3   Literature Review

The literature on process flexibility, also referred to as "mix flexibility" or "product flexibility" first began in the 1980s. Earlier research focused on fully flexible systems, see the survey of Sethi and Sethi (1990) for reviews of research circa 1990. The study of of partial flexibility started with the seminal work of Jordan and Graves (1995). In their paper, Jordan and Graves propose the "long chain" structure, and empirically observe that while in the long chain each plant is capable of producing just a few products, this strategy has almost the same expected sales as that of full flexibility. Numerous papers have extended this concept to other settings, such as multistage supply chains, cross-training, queuing networks and call centers (e.g. Graves and Tomlin, 2003; Hopp et al., 2004; Iravani et al., 2005; Wallace and Whitt, 2005). For a more complete review of applications of process flexibility, we refer readers to the survey of Chou et al. (2008). Only recently, however, new theory has been developed to explain the effectiveness of the long chain design when the system size is large (Chou et al., 2010), or for finite size system (Simchi-Levi and Wei, 2012).

In parallel, the academic community has also investigated the (optimal) mix between dedicated and fully flexible resources (e.g., Fine and Freund, 1990; Van Mieghem,

1998; Bish and Wang, 2004; Chod and Rudi, 2005; Bish et al., 2005; Tomlin and Wang, 2005; Goyal and Netessine, 2007, 2011). Other papers have considered characteristics and properties of more general flexible resource structures (e.g., Van Mieghem, 2007; Bassamboo et al., 2010).

Of interest to us is the research that observed flexibility as an effective tool to safeguard against supply disruption. For example, Sodhi and Tang (2012) lists flexible manufacturing processes as one of the eleven robust supply chain strategies. Also, Tang and Tomlin (2008) investigates five types of flexibility strategies and in particular, suggests process flexibility as a useful tool to mitigate supply chain disruptions.

Similarly, there has been extensive research on the use of inventory to mitigate against supply disruptions. Many of these papers assume that once a facility is disrupted all production of that facility stops and it takes a certain amount of time, typically random with known distribution, for the facility to recover. When the facility is down, inventory can be used to satisfy customer demand. We refer readers to Parlar and Berkin (1991), Song and Zipkin (1996), Moinzadeh and Aggarwal (1997), Parlar (1997), Arreola-Risa and DeCroix (1998), Qi et al. (2010) for details. Tomlin and Wang (2011) review the impact of holding extra inventory as one of the strategies to protect against supply disruptions. They suggest that inventory mitigation is easy to implement because it does not involve coordination with suppliers and customers, but for long period disruptions, huge amount of inventory is needed and the cost can be substantial.

Another important supply mitigation strategy considered in the literature, and applied in practice, is the ability to order the same component or product from multiple sources. This implies that when one supplier is down the firm can either switch to another supplier, use inventory or both. By exploiting this hybrid approach, the firm can reduce the amount of inventory used for supply chain disruption. For example, Parlar and Perry (1998) considers ordering a single product from multiple suppliers where each supplier's uptime (i.e., normal operations) and down time (i.e., disruption) forms a continuous-time Markov chain. Gürler and Parlar (1997) assumes a more general distribution of uptime and down time but limits the model to two

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suppliers. Both of these papers assume that suppliers have identical costs and infinite capacity. These assumptions are relaxed in Tomlin (2006), where the focus is on a discrete time model with two suppliers having different ordering costs, capacity constraints and reliability levels. The paper concludes that as the expected length of disruptions increases, sourcing from the more reliable but more expensive supplier is more cost-effective than holding extra inventory.

Our paper is related to the multiple sourcing/inventory mitigation literature in the following sense: A supply chain design where multiple plants are able to produce the same product can be viewed as a multiple sourcing strategy. However, unlike previous literature on the use of inventory for supply disruption, our model involves **multiple products**. Similarly, papers considering process flexibility as a mitigation strategy for supply disruption, do focus on multiple products but do not include the ability to hold strategic inventory.

To the best of our knowledge, our paper is the first to consider process flexibility and strategic inventory as a way to mitigate against supply disruption. Strategic inventory plays an important role in our paper as this inventory is dedicated to mitigate unpredictable disruptions, as opposed to tactical inventory used to balance recurrent supply fluctuations (e.g., random yield, delivery delays) and demand uncertainty. This approach is supported by observations made by Chopra et al. (2007), which shows that the firm should decouple recurrent supply fluctuation and supply disruptions.

Chapter 2

The Time-to-Survive Model

Consider a network consisting of N plants and M products. Plant i, $1 \le i \le N$, has constant capacity (maximum production rate) of c_i, and let plant N be the one with the largest capacity. Assume that the demand for product j, $1 \le j \le M$, is constant at a rate of d_j per unit time. We will relax the assumption of deterministic demand later on in Chapter 4. In our model, a plant may have the ability to produce more than one product, and a **flexibility design** specifies the products that each plant can produce.

We assume that we can express capacities and demands in common units so that for every product, one unit of demand can be satisfied by one unit of capacity. A flexibility design can be presented as a bipartite directed network, where a link (or arc) between plant node i and product node j means that plant i is able to produce product j. We refer to the set of such links $\mathcal{F}$ as a flexibility design. For a given flexibility design $\mathcal{F}$ and a subset of product indices $Y \subseteq \{1, \cdots, M\}$, we define $\delta_{\mathcal{F}}(Y) = \{i : (i,j) \in \mathcal{F}, j \in Y\}$ as the plants that can produce at least one of these products.

Inventory of finished products is stocked up to protect the system from disruptions; we refer to such inventory as **strategic inventory**. Let r_j be the amount of strategic inventory for product j, and assume that the total amount of inventory cannot exceed a given constant R. In what follows, we assume that a disruption would bring at most one of the plants down. This assumption is valid if plants are located at different

geographical regions, and a disruption cannot affect more than one region.[1] After the disruption, the rest of the plants can adjust their productions, but demand continues at the original rate. Of course, demand can be satisfied either from production or inventory.

We define **Time-to-Survive** (TTS) associated with the given flexibility design and allocation of the R units of inventory between the different products as the maximum time that demands are guaranteed to be satisfied, no matter where disruption happens. To be specific, let $t_j^{(n)}$ be the time the system can satisfy demand of product j when plant n is down. Then the TTS is

$$t = \min_n \min_j t_j^{(n)}.$$

By definition, TTS is closely related to the Time-to-Recover (TTR) of the supply chain, which is defined as the time needed for the facility to restore operations after disruption. For example, in the Toyota case, if the supply chain can recover within two weeks after disruptions, and the TTS is longer than two weeks, then all the demand can be met during the recovery period. In general, one can use TTS as a benchmark against TTR to evaluate system robustness.

Clearly, the larger the TTS, the more robust the supply chain is. Given a flexibility design, $\mathcal{F}$, our objective is to allocate R units of inventory to maximize TTS.

$$t^* = \max_{r_j, x_{ij}^{(n)}} \quad t \tag{2.1}$$

$$\text{s.t.} \quad t \le t_j^{(n)} := \frac{r_j}{d_j - \sum\limits_{i:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)}}, \quad \forall 1 \le n \le N, 1 \le j \le M \tag{2.2}$$

$$\sum_{j:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)} \le c_i, \quad \forall 1 \le i, n \le N \tag{2.3}$$

$$\sum_{j:\,(n,j)\in\mathcal{F}} x_{nj}^{(n)} = 0, \quad \forall 1 \le n \le N \tag{2.4}$$

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[1] Indeed, Toyota is "making each region independent in its parts procurement so that a disaster in Japan would not affect production overseas." See Chang-Ran Kim, "Toyota says supply chain will be ready by autumn for next quake". Reuters, March 2, 2012.

$$\sum_{j=1}^{M} r_j \le R, \tag{2.5}$$

$$r_j, x_{ij}^{(n)} \ge 0.$$

In the above formulation, $x_{ij}^{(n)}$ denotes the production of product j by plant i when plant n is down, and constraint (2.2) is the definition of the TTS. Constraint (2.3) ensures production does not exceed plant capacity, constraint (2.4) shows that production at plant n is stopped after disruption, and constraint (2.5) ensures that total strategic inventory does not exceed R.

We exclude the flexibility investment cost and production cost from the model for several reasons. First and foremost, instead of making investment decisions, the intention of the paper is to understand the effectiveness of process flexibility and strategic inventory to safeguard against supply disruptions. To this end, we develop a simple model that captures the basic relationship between process flexibility and strategic inventory, intended to show insights that can affect practical management decisions. Second, because probability distributions of disruption frequency and recovery time are not known, it is impossible to compute expected lost sales caused by disruptions, so it is difficult to compare different flexibility designs in terms of total cost or profit. Finally, in addition to disruption mitigation, flexibility provides other benefits to the supply chain, most importantly, the ability to match available capacity with variable customer demand (Jordan and Graves, 1995) or the ability to manage exchange rate risk (Huchzermeier and Cohen, 1996). Indeed, most companies that implement flexibility strategies, take into account these benefits as well, see Simchi-Levi (2010).

It is easily verified that TTS increases linearly with the total amount of strategic inventory R: If the strategic inventory for each product increases with the same proportion, TTS also increases proportionally. Therefore, instead of fixing R and trying to maximize TTS, we can set a target TTS, e.g. one time unit, and minimize total inventory R. This approach turns out to be convenient because we are essentially dealing with a linear programming model. Let s_j be the inventory allocated to product

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j to achieve one time unit of TTS. The resulting model is the following linear program, which is referred to as the Strategic Inventory Problem (Problem SI) hereafter.

$$\textbf{Problem SI:} \qquad s^* = \min_{s_j} \ \sum_{j=1}^{M} s_j \qquad\qquad\qquad (2.6)$$

$$\text{s.t.} \quad d_j - \sum_{i:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)} \le s_j, \quad \forall 1 \le n \le N, 1 \le j \le M$$

$$(2.7)$$

$$\sum_{j:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)} \le c_i, \quad \forall 1 \le i, n \le N \qquad\qquad (2.8)$$

$$\sum_{j:\,(n,j)\in\mathcal{F}} x_{nj}^{(n)} = 0, \quad \forall 1 \le n \le N \qquad\qquad (2.9)$$

$$s_j, x_{ij}^{(n)} \ge 0.$$

It is readily verified that Problem SI is equivalent to the original TTS model by replacing variables $s_j := r_j/t$. For notational simplicity, we use bold letters to denote a (multi-dimensional) vector in Problem SI. For example, $\mathbf{x} \in \mathbb{R}^{N^2 M}$ denotes the vector with entries $x_{ij}^{(n)}$, $\forall 1 \le i, n \le N$ and $1 \le j \le M$.

Last but not least, we would like to convince our readers that TTS is a versatile tool with many applications. For example, it sometimes not required that customer demand is satisfied by 100% during disruption. If companies decide to satisfy partial demand of product j, they can just replace d_j with the portion that needs to be satisfied in Problem SI. In other cases, companies might want product-specific TTS. Problem SI can be easily modified to suit such requirement. We do not elaborate on product-specific TTS in this thesis; but we include some discussions in Appendix A.

Chapter 3

Analysis of Flexibility Designs and Inventory Levels

3.1 Full Flexibility

In the full flexibility design, every plant is able to produce all products, so we can regard them as one giant plant with capacity $\sum_{i=1}^{N} c_i$. If one of the plants is disrupted, the remaining capacity is $\sum_{i=1}^{N} c_i - \max_{1 \leq k \leq N} c_k = \sum_{i=1}^{N-1} c_i$ in the worst case (recall that we suppose plant N has the largest capacity). Note that if $\sum_{j=1}^{M} d_j \leq \sum_{i=1}^{N-1} c_i$, then strategic inventory is not required ($s_i = 0$ for all $1 \leq i \leq N$). Thus, throughout the section, we will only consider the case where $\sum_{j=1}^{M} d_j \geq \sum_{i=1}^{N-1} c_i$.

Clearly, $\sum_{j=1}^{M} d_j - \sum_{i=1}^{N-1} c_i$ is a lower bound on inventory needed, because it utilizes all plants capacities. Also note that for any $\mathbf{s} = (s_1, \ldots, s_N)$ satisfying

$$0 \leq s_j \leq d_j, \quad j = 1, \ldots, M \tag{3.1}$$

$$\sum_{j=1}^{M} s_j = \sum_{j=1}^{M} d_j - \sum_{i=1}^{N-1} c_i, \tag{3.2}$$

there exists some vector $\mathbf{x}$ feasible to Problem SI (2.6)–(2.9), hence it is also the optimal value. To recapitulate,

Proposition 1. *The inventory needed for full flexibility design equals to* $\sum_{j=1}^{M} d_j -$

$\sum_{i=1}^{N-1} c_i$. *For any* s *that satisfies* (3.1) *and* (3.2), *there exists* x *such that* (x, s) *is an optimal solution to Problem SI. Thus, equations* (3.1) *and* (3.2) *characterize all optimal solutions of strategic inventory for full flexibility.*

Conditions (3.1) and (3.2) state that under full flexibility, the firm has a lot of leeway in allocating inventory to different products. Therefore, to achieve the best TTS, it is not necessary for the firm to stock inventory for all the products, but just a few of them. This result offers further opportunities to reduce inventory cost.

By contrast, the inventory needed for dedicated flexibility design is equal to $\sum_{j=1}^{M} d_j$, which can be much larger than $\sum_{j=1}^{M} d_j - \sum_{i=1}^{N-1} c_i$. For example, if $N = M$, each plant has capacity $c_i = 1$ and each product has demand rate $d_j = 1$, the dedicated network requires N units of inventory while the full flexibility network needs only one unit of inventory. In other words, if the inventory levels are the same for both networks, the full flexibility design has a TTS that is N times longer than that of a dedicated design.

As the number of plants, N, increases, the difference between the robustness (defined by TTS) of the two systems can be substantial. This difference reveals the power of process flexibility to increase supply chain robustness, or equivalently, TTS. Unfortunately, industry is reluctant to implement full flexibility because of the enormous investment required to make sure that every plant is capable of producing all products, see Simchi-Levi (2010). Therefore, in the next subsection we shift our attention to partial flexibility.

3.1.1 A Sufficient Condition for Full Robustness

We say that a flexibility design is **fully robust** if given s that satisfies (3.1) and (3.2), there exists some production vector x feasible to Problem SI. This implies that the flexibility design has exactly the same TTS and the same freedom in allocating inventory to different products as that of full flexibility. Thus, if a flexibility design is fully robust, it must have the same TTS as full flexibility. However, a flexibility design having the same TTS as full flexibility is not necessarily fully robust, because

it may have more constraints on inventory allocation.

Next, we provide a sufficient condition for a flexibility design to be fully robust. If the strategic inventory $\mathbf{s} = (s_1, \ldots, s_M)$ satisfying equations (3.1) and (3.2) is given, then $\mathbf{x}^{(n)} = (x_{ij}^{(n)})_{N \times M}$ is feasible for Problem SI if and only if,

$$d_j - \sum_{i: (i,j) \in \mathcal{F}} x_{ij}^{(n)} \leq s_j, \quad \forall 1 \leq j \leq M \tag{3.3}$$

$$\sum_{j: (i,j) \in \mathcal{F}} x_{ij}^{(n)} \leq c_i, \quad \forall 1 \leq i \leq N \tag{3.4}$$

$$\sum_{j: (n,j) \in \mathcal{F}} x_{nj}^{(n)} = 0, \tag{3.5}$$

$$x_{ij}^{(n)} \geq 0. \tag{3.6}$$

Without loss of generality, assume $d_j \geq s_j$, otherwise, there is no benefit to stock product j higher than its demand. It turns out that finding some $\mathbf{x}^{(n)}$ satisfying (3.3)–(3.6) is equivalent to solving a max-flow problem. This is stated in the next lemma.

Lemma 1. *If plant n is disrupted, there exists some $\mathbf{x}^{(n)}$ satisfying the system of linear inequalities (3.3)–(3.6) if and only if the linear program defined by Equation (3.7) has optimal value $f^* = \sum_{j=1}^{M}(d_j - s_j)$.*

$$f^* = \max_{f_{ij}} \sum_{(i,j) \in \mathcal{F}} f_{ij} \tag{3.7}$$

$$\text{s.t.} \sum_{i: (i,j) \in \mathcal{F}} f_{ij} \leq d_j - s_j, \quad \forall 1 \leq j \leq M \tag{3.8}$$

$$\sum_{j: (i,j) \in \mathcal{F}} f_{ij} \leq c_i, \quad \forall 1 \leq i \leq N,$$

$$\sum_{j: (n,j) \in \mathcal{F}} f_{nj} = 0,$$

$$f_{ij} \geq 0, \quad \forall (i,j) \in \mathcal{F}.$$

Proof. Suppose there exists an $\mathbf{x}^{(n)}$ feasible for (3.3)–(3.6). If all M inequalities in

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(3.3) attain equalities, let $f_{ij} = x_{ij}^{(n)}$, then f_{ij}'s are feasible for (3.7) and $\sum_{(i,j) \in \mathcal{F}} f_{ij} = \sum_{j=1}^{M} (d_j - s_j)$. By constraint (3.8), adding over j, $\sum_{(i,j) \in \mathcal{F}} f_{ij} \leq \sum_{j=1}^{M} (d_j - s_j)$, so $\sum_{j=1}^{M} (d_j - s_j)$ is the optimal value of the objective function. If there exists j such that the sign in (3.3) is strict, we can drive $x_{ij}^{(n)}$ down to attain equality, without violating (3.4)–(3.6), and the conclusion still holds. Conversely, if the optimal value, f^*, satisfies $f^* = \sum_{j=1}^{M} (d_j - s_j)$, then the inequality in (3.8) must hold as equality, so $x_{ij}^{(n)} = f_{ij}$ is feasible for (3.3)–(3.6). $\qquad \square$

To see that problem (3.7) is indeed a max-flow problem, consider a bipartite directed graph G_n with $N - 1$ nodes $A_n = \{a_1, \cdots, a_{n-1}, a_{n+1}, \cdots, a_N\}$ representing the $N - 1$ undisrupted plants, and M nodes $B = \{b_1, \cdots, b_M\}$ representing the products, plus source node s and sink node t. For each $(i, j) \in \mathcal{F}$, $i \neq n$, construct an uncapacitated arc from a_i to b_j in graph G_n. For each $1 \leq i \leq N$, $i \neq n$, construct an arc from s to a_i with capacity c_i, and for $1 \leq j \leq M$, construct an arc from b_j to t with capacity $d_j - s_j$, which can be assumed to be nonnegative because the optimal solution must have $s_j \leq d_j$. It is easy to check that the max-flow problem on G_n is equivalent to (3.7).

Applying the well-known max-flow min-cut theorem, we have that (3.7) has a max-flow of value $\sum_{j=1}^{M} (d_j - s_j)$ if and only if for all $X \subseteq A_n$,

$$\sum_{i:\, a_i \in A_n \backslash X} c_i + \sum_{j:\, b_j \in N_{G_n}(X)} (d_j - s_j) \geq \sum_{j=1}^{M} (d_j - s_j) \tag{3.9}$$

where $N_{G_n}(X)$ denotes the neighbors of X in G_n. Now, we are ready to present a condition for flexibility designs to be fully robust.

Theorem 1. *Suppose that for any $Y \subseteq \{1, \cdots, M\}$ such that $\delta_{\mathcal{F}}(Y) \subsetneq \{1, \cdots, N\}$, it holds that,*

$$\sum_{j \in Y} d_j \leq \sum_{i \in \delta_{\mathcal{F}}(Y)} c_i - \max_{k \in \delta_{\mathcal{F}}(Y)} c_k \tag{3.10}$$

where $\delta_{\mathcal{F}}(Y) = \{i : (i, j) \in \mathcal{F}, j \in Y\}$ is the set of plants that can produce at least one product labeled by Y. Then, $\mathcal{F}$ is fully robust.

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Proof. To show that $\mathcal{F}$ is fully robust, we prove that for any fixed $\mathbf{s}$ that satisfies equation (3.1)–(3.2), and any $1 \leq n \leq N$, there exists some $\mathbf{x}^{(n)}$ that satisfies the system of linear inequalities (3.3)–(3.6).

Fix any $1 \leq n \leq N$, by Lemma 1 and the max-flow min-cut theorem, there exists some $\mathbf{x}^{(n)}$ that satisfies the system of linear inequalities (3.3)–(3.6) if and only if for all $X \subseteq A_n$,

$$\sum_{i:\, a_i \in A_n \setminus X} c_i + \sum_{j:\, b_j \in N_{G_n}(X)} (d_j - s_j) \geq \sum_{j=1}^{M} (d_j - s_j).$$

If $X = \varnothing$, we have

$$\sum_{j=1}^{M} s_j = \sum_{j=1}^{M} d_j - \sum_{i=1}^{N-1} c_i$$

$$\implies \sum_{i=1}^{N-1} c_i = \sum_{j=1}^{M} (d_j - s_j)$$

$$\implies \sum_{1 \leq i \neq n \leq N} c_i \geq \sum_{j=1}^{M} (d_j - s_j)$$

$$\implies \sum_{i:\, a_i \in A_n \setminus \varnothing} c_i + \sum_{j:\, b_j \in N_{G_n}(\varnothing)} (d_j - s_j) \geq \sum_{j=1}^{M} (d_j - s_j).$$

If $X = A_n$, the left hand side and right hand side are equal, so the inequality trivially holds.

Otherwise, suppose $X \neq \varnothing$ and $X \subsetneq A_n$, let $Y = \{j : b_j \notin N_{G_n}(X)\}$. Note that if $i \in \delta_{\mathcal{F}}(Y)$, then $a_i \in A_n \setminus X$ when $i \neq n$. Hence, $\delta_{\mathcal{F}}(Y) \subsetneq \{1, \cdots, N\}$ and

$$\sum_{i:\, a_i \in A_n \setminus X} c_i \geq \sum_{i \in \delta_{\mathcal{F}}(Y) \setminus \{n\}} c_i \geq \sum_{i \in \delta_{\mathcal{F}}(Y)} c_i - \max_{k \in \delta_{\mathcal{F}}(Y)} c_k$$

$$\implies \sum_{i:\, a_i \in A_n \setminus X} c_i + \sum_{j:\, b_j \in N_{G_n}(X)} (d_j - s_j)$$

$$\geq \sum_{i \in \delta_{\mathcal{F}}(Y)} c_i - \max_{k \in N_{G_n}(Y)} c_k + \sum_{j:\, b_j \in N_{G_n}(X)} (d_j - s_j).$$

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By assumption in the statement of Theorem 1,

$$\sum_{i \in \delta_{\mathcal{F}}(Y)} c_i - \max_{k \in \delta_{\mathcal{F}}(Y)} c_k \geq \sum_{j \in Y} d_j,$$

thus

$$\sum_{i:\, a_i \in A_n \setminus X} c_i + \sum_{j:\, b_j \in N_{G_n}(X)} (d_j - s_j) \geq \sum_{j \in Y} d_j + \sum_{j:\, b_j \in N_{G_n}(X)} (d_j - s_j)$$

$$= \sum_{j \in Y} d_j + \sum_{j \notin Y} (d_j - s_j)$$

$$\geq \sum_{j=1}^{M} (d_j - s_j).$$

Thus, for any any fixed s that satisfies equation (3.1)–(3.2), and any fixed $1 \leq n \leq N$, there exists some $\mathbf{x}^{(n)}$ that satisfies the system of linear inequalities (3.3)–(3.6). $\square$

3.2 Long Chain

Sparse flexibility designs are those with significantly fewer flexibility arcs than a fully flexible network. For example, in a 2-flexibility design, each plant can produce only two products. In this section, we pay much attention to a special 2-flexibility design called the **long chain** (see Figure 1-1). As mentioned in the introduction and the literature review, the long chain design is proven to be very effective in matching supply with demand in various applications. This motivates us to examine the robustness of long chain when the supply chain is subject to disruptions.

As is common in the analysis of the long chain, we assume, throughout the section, that the number of plants is equal to the number of products ($M = N$). Plant capacity may very from plant to plant.

Applying Theorem 1, we immediately have the following result for the long chain.

Corollary 1. *If the demand of each product does not exceed the capacity of either of*

its neighboring plant, that is

$$d_i \leq \min\{c_{i-1}, c_i\}, \quad \forall 1 \leq i \leq N \tag{3.11}$$

(by convention $c_0 = c_N$), then the long chain is fully robust. The optimal solution of **s** *is given by (3.1) and (3.2).*

Proof. All we need to prove is that (3.11) infers inequality (3.10). For this purpose, consider Figure 3-1.

For any set $Y \subseteq \{1, \cdots, N\}$, if $\delta_{\mathcal{F}}(Y) \neq \{1, \cdots, N\}$, we must have $Y \subsetneq \{1, \cdots, N\}$. Then Y either has one segment of consecutive nodes or can be partitioned into a few segments, where for each segment i to j, $i - 1 \notin Y$ and $j + 1 \notin Y$ (which could be the same node).
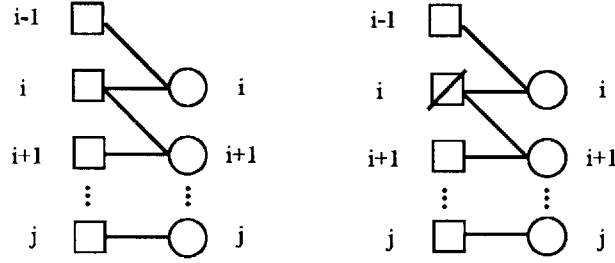

Figure 3-1: Illustration of segments of Y.

In the right hand side of (3.10), the summation should exclude a neighboring plant node with maximum capacity $\max\{c_k : k \in \delta_{\mathcal{F}}(Y)\}$. If the maximum capacity node is not in the segment from $i - 1$ to j, as the left hand side of Figure 1 shows, (3.11) implies $d_i \leq c_i$, so $\sum_{k=i}^{j} d_k \leq \sum_{k=i}^{j} c_k \leq \sum_{k=i-1}^{j} c_k$. If the maximum capacity node is in the segment from $i - 1$ to j, without loss of generality we assume it to be i. Then from condition (3.11), $d_i \leq c_{i-1}$, and $d_k \leq c_k, \forall k = i + 1, \ldots, j$. So $\sum_{k=i}^{j} d_k \leq c_{i-1} + \sum_{k=i+1}^{j} c_k$.

Summing up the inequalities over all segments, we have inequality (3.10). By Theorem 1, therefore, the long chain is fully robust. □

The challenge now is to characterize the performance of the long chain when

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inequality (3.11) does not hold. This is what we show in the next proposition, which is derived by applying Corollary 1.

Proposition 2. *Suppose plant N has the largest capacity. The inventory needed for the long chain is*

$$\max\left\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i, \sum_{i=1}^{N} \Delta d_i\right\}. \tag{3.12}$$

where $\Delta d_i := \max(d_i - c_i, d_i - c_{i-1}, 0)$. In addition, strategic inventory allocation is characterized by

$$\Delta d_j \leq s_j \leq d_j, \quad \forall j = 1, \ldots, N \tag{3.13}$$

$$\sum_{j=1}^{N} s_j = \max\left\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i, \sum_{i=1}^{N} \Delta d_i\right\}. \tag{3.14}$$

Proof. Let the inventory needed for the long chain be s^*. For $j = 1, \ldots, N$, let $\Delta d_j = \max\{d_j - \min\{c_{j-1}, c_j\}, 0\}$, and $d'_j = d_j - \Delta d_j$, with the convention that $c_0 = c_N$.

First, we prove that (3.12) is a lower bound on the inventory level. By Proposition 1, $(\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i)$ is just the inventory level associated with full flexibility, we thus have

$$s^* \geq \sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i.$$

Consider product j. If $d_j \geq \min\{c_{j-1}, c_j\}$, given the possibility that either plant $j-1$ or plant j may fail, one should keep at least $d_j - \min\{c_{j-1}, c_j\} = \Delta d_j$ unit of inventory for product j in order to achieve one time unit of TTS. So the total inventory needed is at least $\sum_{j=1}^{N} \Delta d_j$, which means

$$s^* \geq \sum_{j=1}^{N} \Delta d_j.$$

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Therefore

$$s^* \geq \max\left\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i, \sum_{j=1}^{N} \Delta d_j\right\}.$$

Next we prove that (3.12) is an upper bound on the inventory level. Consider a modified problem with demand $d'_j = d_j - \Delta d_j$ and capacity $\mathbf{c} = [c_1, c_2, ..., c_N]$. If $\sum_{j=1}^{N} d'_j - \sum_{i=1}^{N-1} c_i \geq 0$, then by Corollary 1, $\mathbf{d'}$ and $\mathbf{c}$ satisfy condition (3.11), so there exists a feasible solution $\mathbf{s'}$ such that

$$0 \leq s'_j \leq d'_j, \tag{3.15}$$

$$\sum_{j=1}^{N} s'_j = \sum_{j=1}^{N} d'_j - \sum_{i=1}^{N-1} c_i. \tag{3.16}$$

Otherwise, if $\sum_{j=1}^{N} s'_j = \sum_{j=1}^{N} d'_j - \sum_{i=1}^{N-1} c_i < 0$, the inventory needed for the modified problem is 0. To see this, replace any of the d'_j with some larger $\tilde{d}'_j$ so that (3.16) equals 0. By Corollary 1, this problem has a feasible solution with $\mathbf{x}$ and $\sum_{j=1}^{N} s'_j = 0$.

Let $s_j = s'_j + \Delta d_j$, $\forall j = 1, \ldots, N$, which leads to (3.13) and (3.14). It is easy to verify that $\mathbf{s}$ and $\mathbf{x}$ are feasible for Problem SI with demand $\mathbf{d}$ and capacity $\mathbf{c}$. Also, because

$$\sum_{j=1}^{N} s'_j = \max\{\sum_{j=1}^{N} d'_j - \sum_{i=1}^{N-1} c_i, 0\}, \tag{3.17}$$

we have

$$\sum_{j=1}^{N} s_j = \max\{\sum_{j=1}^{N} d'_j - \sum_{i=1}^{N-1} c_i, 0\} + \sum_{j=1}^{N} \Delta d_j$$

$$= \max\{\sum_{j=1}^{N} d'_j - \sum_{i=1}^{N-1} c_i + \sum_{j=1}^{N} \Delta d_j, \sum_{j=1}^{N} \Delta d_j\}$$

$$= \max\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i \sum_{j=1}^{N} \Delta d_j\}.$$

Therefore, $s^* \leq \max\left\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i, \sum_{j=1}^{N} \Delta d_j\right\}$, which completes the proof. $\square$

Proposition 2 has a number of implications. First, instead of solving Problem SI,

the proposition provides a simple way compute the inventory level of the long chain, as well as the optimal inventory allocation.

Second, it implies that long chain is not always the best design among two flexibility designs. Consider the example shown in Figure 3-2. With the given capacities and demands, there are only two possible long-chain designs; but by Proposition 2, both need more inventory than the short-chain design on the left. The result is contrary to the observations in a lot of literature that long chain is superior to short chains. The intuition here is that plants with very different capacities should not connected (in the example, one plant has a capacity 9 times greater than another). Otherwise, if the large-capacity plant is down, we have limited ability to shift its production to the small-capacity plant, thus reduces the functionality of chaining.
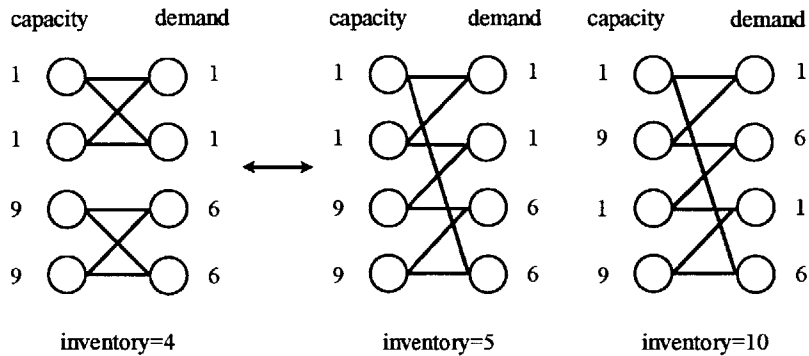

Figure 3-2: An example where short chains is better than long chains.

Third, Proposition 2 suggests that for any long chain to be fully robust, we must have $\Delta d_i = 0$ for all i. This implies that in that case, $0 = \Delta d_i := \max(d_i - c_i, d_i - c_{i-1}, 0)$ $\forall 1 \leq i \leq N$, or $d_i \leq \min\{c_{i-1}, c_i\}$, $\forall 1 \leq i \leq N$ which is exactly the sufficient condition of Corollary 1. Thus, inequality (3.11) is a necessary and sufficient condition for the long chain to be fully robust.

Finally, even if $\sum_{i=1}^{N} \Delta d_i > 0$, it is possible that the long chain may require the same total inventory as that of full flexibility. This is true as long as $\sum_{i=1}^{N} \max(d_i - c_i, d_i - c_{i-1}, 0) \leq \sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i$. However, in that case the long chain design does lose some leeway in the inventory allocation, as (3.13) implies. In other words, extra flexibility, i.e., full flexibility, provides some advantage in the inventory allocation.

3.2.1 Design of the Long Chains

Evidently, as we have seen in the example in Figure 3-2, there are many different ways to form a long chain. Suppose the dedicated network $\mathscr{D} = \{(i,i)|1 \leq i \leq N\}$ is a "base network" for which the firm can add process flexibility to form a long chain. In particular, in the base network, plant i is already matched with product i, and the objective is to find secondary product that each plant will make.

Note that there are $(N-1)!$ different long chains which contains $\mathscr{D}$. Observe that each long chain can be represented by $LC(\sigma) = \mathscr{D} \cup \{(\sigma(i),\sigma(i+1))|1 \leq i \leq N\}$, for some σ such that $\sigma(1),\sigma(2),...,\sigma(N-1)$ is a permutation of numbers from 1 to $N-1$ and $\sigma(N) = N, \sigma(N+1) = \sigma(1)$. The strategic inventory required to achieve a unit of TTS clearly depends on the specific long chain. We are interested in the question of which long chain requires the lowest inventory level.

In particular, we define two special permutations. When $\sigma(i) = i$ for $1 \leq i \leq N-1$, the nodes are connected in increasing order, so we define such permutation as σ^+. Similarly, when $\sigma(i) = N-i$ for $1 \leq i \leq N-1$, the nodes are connected in decreasing order, and we define it as σ^-.

Using the result of Proposition 2, for any fixed σ, the inventory level for $LC(\sigma)$ is

$$\max\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i, \sum_{i=1}^{N} \max\{d_i - c_i, d_{\sigma(i+1)} - c_{\sigma(i)}, 0\}\}.$$

Therefore, finding the long chain with the lowest inventory requirement is equivalent to minimizing $\sum_{i=1}^{N} \max\{d_i - c_i, d_{\sigma(i+1)} - c_{\sigma(i)}, 0\}$.

We start by considering the case where the capacities and demands are matched in $\mathscr{D}$, or $d_i = c_i, \forall i = 1, \ldots, n$. Thus,

$$\max\{d_i - c_i, d_{\sigma(i+1)} - c_{\sigma(i)}, 0\} = \max\{d_{\sigma(i+1)} - c_{\sigma(i)}, 0\}.$$

Without loss of generality, suppose $d_1 \leq d_2 \leq \cdots \leq d_N$. Let t be the index such that

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$\sigma(t) = 1$, then

$$\sum_{i=1}^{N} \max\{d_{\sigma(i+1)} - c_{\sigma(i)}, 0\} \geq \sum_{i=t}^{N-1} \max\{d_{\sigma(i+1)} - c_{\sigma(i)}, 0\}$$

$$\geq \sum_{i=t}^{N-1} (d_{\sigma(i+1)} - d_{\sigma(i)})$$

$$= d_{\sigma(N)} - d_{\sigma(t)}$$

$$= d_N - d_1.$$

It is easy to check that when $\sigma = \sigma^+$ or σ^-, the equality is achieved. While $LC(\sigma^+)$ and $LC(\sigma^-)$ are the long chains with the lowest inventory level, this level does not necessarily equal to the level of full flexibility.

The above result motivates a more general analysis for the case when d_i and c_i are not necessarily equal. Suppose $c_1 \leq c_2 \leq ... \leq c_N$, $d_1 \leq d_2 \leq ... \leq d_N$, and $c_i \geq d_i$ for $1 \leq i \leq N$. The condition essentially states that in the base network $\mathscr{D}$, demand can be satisfied when there is no disruption, and the plants with high capacities produce high volume products. We prove that $LC(\sigma^+) = \mathscr{D} \cup \{(i, i+1) | 1 \leq i \leq N\}$ achieves the minimum inventory among all permutations.

Proposition 3. *In a system with $c_1 \leq c_2 \leq ... \leq c_N$, $d_1 \leq d_2 \leq ... \leq d_N$, and $c_i \geq d_i$ for $1 \leq i \leq N$, the inventory level for $LC(\sigma^+)$, associated with a given TTS, is less than or equal to that of any other long chain design with the same TTS.*

We start the proof with two technical lemmas.

Lemma 2. *For any σ where $\sigma(1), \sigma(2), ..., \sigma(N-1)$ is a permutation of numbers from 1 to $N-1$ and $\sigma(N) = N, \sigma(N+1) = \sigma(1)$,*

$$\sum_{i=1}^{N} \max\{d_{\sigma(i+1)} - c_{\sigma(i)}, 0\} \geq \sum_{t=1}^{T} \max\{d_{y_t} - c_{y_{t-1}}, 0\},$$

where $1 = y_0 < y_1 < y_2 < \cdots < y_T = N$.

Proof. Let i_0 be the integer such that $\sigma(i_0) = 1$. For each $t \geq 1$, if $i_{t-1} < N$, let i_t be the smallest integer satisfying $i_t > i_{t-1}$ and $\sigma(i_t) > \sigma(i_{t-1})$. Such i_t exists

because N is always a candidate. We also have $\sigma(i_t - 1) \leq \sigma(i_{t-1})$. Otherwise, if $\sigma(i_t - 1) > \sigma(i_{t-1})$, index $i_t - 1$ will contradict the fact that i_t is the smallest integer satisfying the condition.

As sequence i_t is increasing with t, it will finally end up with an integer T such that $i_T = N$. Then

$$\sum_{i=1}^{N} \max\{d_{\sigma(i+1)} - c_{\sigma(i)}, 0\} \geq \sum_{t=1}^{T} \max\{d_{\sigma(i_t)} - c_{\sigma(i_t-1)}, 0\}$$

$$\text{(because } \sigma(i_t - 1) \leq \sigma(i_{t-1})) \quad \geq \sum_{t=1}^{T} \max\{d_{\sigma(i_t)} - c_{\sigma(i_{t-1})}, 0\}$$

Let $y_t = \sigma(i_t)$ for $0 \leq t \leq T$, and we are done. $\qquad\square$

Lemma 3. *Given $c_1 \leq c_2 \leq ... \leq c_N$, $d_1 \leq d_2 \leq ... \leq d_N$, and $c_i \geq d_i$ for $1 \leq i \leq N$,*

$$\max\{d_b - c_a, 0\} \geq \sum_{i=a}^{b-1} \max\{d_{i+1} - c_i, 0\}$$

for any positive integers $1 \leq a < b \leq N$.

Proof. Let $a \leq i_1 < i_2 < ... < i_T \leq b - 1$ be all the integers such that $d_{i+1} - c_i > 0$. If $T = 0$, the result trivially holds. If $T = 1$,

$$\sum_{i=a}^{b-1} \max\{d_{i+1} - c_i, 0\} = d_{i_1+1} - c_{i_1} \leq d_b - c_a \leq \max\{d_b - c_a, 0\}$$

and if $T \geq 2$,

$$\sum_{i=a}^{b-1} \max\{d_{i+1} - c_i, 0\}$$

$$= \sum_{t=1}^{T} (d_{i_t+1} - c_{i_t})$$

$$= (d_{i_T+1} - c_{i_T}) + \sum_{t=2}^{T-1} (d_{i_t+1} - c_{i_t}) + (d_{i_1+1} - c_{i_1})$$

$$\text{(because } i_{t-1} + 1 \leq i_t) \quad \leq (d_b - c_{i_{T-1}+1}) + \sum_{t=2}^{T-1} (d_{i_t+1} - c_{i_{t-1}+1}) + (d_{i_1+1} - c_a)$$

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$$\leq (d_b - d_{i_{T-1}+1}) + \sum_{t=2}^{T-1} (d_{i_t+1} - d_{i_{t-1}+1}) + (d_{i_1+1} - c_a)$$

$$= d_b - c_a.$$

This completes the proof. □

Combining Lemma 2 and Lemma 3, we show that $LC(\sigma^+)$ requires the lowest inventory level among all $LC(\sigma)$.

Proof of Proposition 3. Fix some permutation σ that defines a long chain. By Proposition 2, the inventory needed for $LC(\sigma)$ is equal to

$$\max\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i, \sum_{i=1}^{N} \max\{d_{\sigma(i+1)} - c_{\sigma(i)}, 0\}\}.$$

On the other hand, the inventory for $LC(\sigma^+)$ is equal to

$$\max\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i, \sum_{i=1}^{N-1} \max\{d_{i+1} - c_i, 0\}\}.$$

Thus, it is sufficient to prove that

$$\sum_{i=1}^{N} \max\{d_{\sigma(i+1)} - c_{\sigma(i)}, 0\} \geq \sum_{i=1}^{N-1} \max\{d_{i+1} - c_i, 0\}.$$

By Lemma 2, there exists some positive integer T and a sequence $1 = y_0 < y_1 < y_2 < ..., < y_T = N$, such that

$$\sum_{i=1}^{N} \max\{d_{\sigma(i+1)} - c_{\sigma(i)}, 0\} \geq \sum_{t=1}^{T} \max\{d_{y_t} - c_{y_{t-1}}, 0\}$$

$$\text{(by Lemma 3)} \quad \geq \sum_{t=1}^{T} \sum_{i=y_{t-1}}^{y_t-1} \max\{d_{i+1} - c_i, 0\}$$

$$= \sum_{i=1}^{N-1} \max\{d_{i+1} - c_i, 0\}.$$

□

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By the proposition, when $c_1 \leq c_2 \leq ... \leq c_N$, $d_1 \leq d_2 \leq ... \leq d_N$, and $c_i \geq d_i$ for $1 \leq i \leq N$, the best long chain, $LC(\sigma^+)$, requires

$$\max\left\{\sum_{j=1}^{N} d_j - \sum_{i=1}^{N-1} c_i, \sum_{i=1}^{N} \max(d_i - c_{i-1}, 0)\right\}. \tag{3.18}$$

units of inventory to achieve one time unit of TTS.

If the condition of Proposition 3 does not hold, our numerical experiments suggest that a rule of thumb is to add flexibility links where capacity and demand are close to each other. The intuition is that, if the capacity is much larger than the demand, the exceeding capacity is wasted; if the capacity is much lower than product demand, because each product is produced by only two plants, large amount of strategic inventory is required in case the other plant is disrupted.

While (3.18) is equal to the inventory needed for full flexibility when $d_i - c_{i-1}$ is small for $1 \leq i \leq n$, it can be strictly greater than the inventory needed for full flexibility in some instances. However, in the next section, we show that there exists a specific long chain for which adding one degree of flexibility to each of the plant provides a design that is fully robust.

3.3   Other Sparse Flexibility Designs

In this section, we again consider a system with $c_i \geq d_i$ for $1 \leq i \leq N$ and $d_1 \leq d_2 \leq ... \leq d_N$. We show that in this setting, there exists a fully robust sparse design where each plant node is incident to at most 3 arcs.

Proposition 4. *In a system with N plants and N products, suppose $c_i \geq d_i$ for $1 \leq i \leq N$ and $d_1 \leq d_2 \leq ... \leq d_N$. Let $\mathcal{F} = LC(\sigma^-) \cup \{(i, N)|1 \leq i \leq N - 1\}$. Then $\mathcal{F}$ is fully robust.*

Proof. Fix any $Y \subset \{1, 2, ..., N\}$ such that $\delta_{\mathcal{F}}(Y) \subsetneq \{1, 2, ..., N\}$. Because $\delta_{\mathcal{F}}(\{N\}) = \{1, 2, ..., N\}$, we must have $N \notin Y$, or $Y \subseteq \{1, \cdots, N - 1\}$. Notice that for any $i \in \{1, \cdots, N - 1\}$, $c_i \geq d_i$, $c_{i+1} \geq d_i$. Thus, we can apply the same proof as in

Corollary 1 and get

$$\sum_{j \in Y} d_j \leq \sum_{i \in \delta_{\mathcal{F}}(Y)} c_i - \max_{i \in \delta_{\mathcal{F}}(Y)} c_i.$$

Apply Theorem 1, and we have that Proposition 4 holds. $\square$

Proposition 4 further justifies the claim that for any system, when products demands are not volatile, sparse flexibility design is enough for mitigating supply chain risks.

We note that under some circumstances, full robustness can be achieved by adding less flexibility than what is described in Proposition 4. Hence, Proposition 4 is not necessarily the most effective method of adding flexibility to increase supply chain robustness, but rather a justification of the insight that it is possible to achieve full robustness with sparse design.

Chapter 4

Uncertain Demand

In this chapter, we relax the assumption that products demand is deterministic. Instead, we assume all possible demand values belong to an **uncertainty set**, and the exact value is realized after the disruption happens. The firm is able to change the production level after disruption, but inventory decisions are made before demand is realized. The objective is to maximize the TTS in the worst case. We choose a worst-case analysis for demand uncertainty to make it consistent with the definition of TTS, which itself is a worst-case risk measurement.

Our interest in this chapter is to understand the effectiveness of flexibility under demands with different levels of variations. Throughout the chapter, we will consider uncertainty set $\mathcal{U}$, with the form

$$\mathcal{U} = \{(d_1, \cdots, d_n) \mid \sum_{j=1}^{M} d_j = D, l \leq d_j \leq u\}.$$

We also suppose $l = (1 - \alpha)D/M$ and $u = (1 + \alpha)D/M$, where the parameter α indicates the level of uncertainty. We note that while the uncertainty sets under consideration are highly stylized, it contains enough freedom that allows us to study the effectiveness of flexibility with different levels of demands uncertainties. A more general form of uncertainty sets would lead to unnecessarily complex analysis or even intractable mathematical models.

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4.1 Uncertain Demand Time-to-Survive Model

Following the notation in Chapter 2, the problem can be formulated as follows, which is referred to as the Uncertainty Strategic Inventory Problem (Problem U-SI) hereafter.

Problem U-SI:
$$s^* = \min_{s_j} \max_{d_j} \min_{x_{ij}^{(n)}} \sum_{j=1}^{M} s_j \tag{4.1}$$

$$\text{s.t.} \quad d_j - \sum_{i:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)} \le s_j, \quad \forall 1 \le n \le N, 1 \le j \le M$$

$$\sum_{j:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)} \le c_i, \quad \forall 1 \le i, n \le N$$

$$\sum_{j:\,(n,j)\in\mathcal{F}} x_{nj}^{(n)} = 0, \quad \forall 1 \le n \le N$$

$$\sum_{j=1}^{M} d_j = D,$$

$$l \le d_j \le u, \quad s_j, x_{ij}^{(n)} \ge 0.$$

To avoid unnecessary complication, we suppose that the number of products $M = 2m$ is even. The result when $M = 2m + 1$ is odd can be solved similarly, and can be found in Appendix B. As we did in for the deterministic demand, we will compare the TTS of full flexibility design and long chain design in the uncertainty case.

4.2 K-Flexibility Designs

In this section, we consider a **symmetric** K-flexibility design where plant 1 produces product 1 to product K, plant 2 produces product 2 to product $K+1$, and in general plant i produces products $i, i+1, \ldots, i+K-1$. We assume here again that $M = N$, M is an even number, and the capacity of each plant equals to c. Both full flexibility $(K = M)$ and long chain $(K = 2)$ are special cases of this type of design.

To solve Problem U-SI (4.1) for K-flexibility designs, we first prove the following lemma.

Lemma 4. *Suppose for any inventory allocation* $\mathbf{s} = (s_1, s_2, \ldots, s_N)$, *the rearranged allocation* $\sigma(\mathbf{s}) = (s_2, s_3 \ldots, s_N, s_1)$ *(stock s_2 units of product 1, etc.) achieves the same TTS for Problem USI, then the optimal TTS can be achieved by allocating R units of inventory equally among all products.*

Proof. If two inventory allocations $\mathbf{s} = (s_1, s_2, \ldots, s_N)$ and $\mathbf{s}' = (s_1', s_2', \ldots, s_N')$ both have TTS greater than or equal to one time unit with any demand in the uncertainty set $\mathcal{U}$, their convex combination (of $\mathbf{s}$ and $\mathbf{s}'$) also has TTS greater than or equal to one. This is easy to see from the model of Problem USI (4.1): for any demand in $\mathcal{U}$, if the production $\mathbf{x}$ is feasible for $\mathbf{s}$, and $\mathbf{x}'$ is feasible for $\mathbf{s}'$, then $\lambda\mathbf{x} + (1 - \lambda)\mathbf{x}'$ is feasible for inventory allocation $\lambda\mathbf{s} + (1 - \lambda)\mathbf{s}'$. By assumption, if the inventory allocation $\mathbf{s}$ achieves the maximum TTS, so do $\sigma(\mathbf{s}), \sigma^2(\mathbf{s}), \cdots, \sigma^{N-1}(\mathbf{s})$. Therefore, their convex combination $\bar{\mathbf{s}} = (\bar{s}, \bar{s}, \ldots, \bar{s})$, where $\bar{s} = \sum_{i=1}^{N} s_i/N$, achieves a TTS at least as good. Therefore, $\bar{\mathbf{s}}$ also achieves the optimal TTS. □

The symmetry of K-flexibility designs certainly satisfies Lemma 4, so we can assume $s_j = s$ for all product j. We can there characterize the inventory level for K-flexibility designs under uncertain demand.

Proposition 5. *Suppose $K \leq N/2$. An equal inventory allocation $\mathbf{s} = (s, s, \ldots, s)$ in a K-flexibility design will achieve a unit time of TTS for all demands in uncertainty set $\mathcal{U}$ if and only if it does so for such a demand instance: $d_i = l, \forall i = 1, \ldots, N/2$, $d_i = u, \forall i = N/2 + 1, \ldots, N$.*

Proof. Since $\mathcal{U}$ is polyhedral, Problem USI (4.1) suggests that $\bar{\mathbf{s}}$ can achieve TTS of one for all demands in $\mathcal{U}$ if it can do so for all vertices of $\mathcal{U}$. A vertex of $\mathcal{U}$ has the following characterization: Half of the products have upper bound demand u and the rest have lower bound demand l. So we only need to focus on these demand instances.

For a given demand and inventory vectors, inequality (3.9) tells us that the system achieves a unit TTS if and only for any subset of plants X and the products they

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produce $\delta(X)$, it holds

$$\sum_{i \notin X} c_i - \max_{i \notin X} c_i + \sum_{j \in \delta(X)} (d_j - s_j)^+ \geq \sum_{j=1}^{N} (d_j - s_j)^+. \qquad (4.2)$$

In this case, $c_i = c$, $s_j = s$, $d_j = u$ or l.

We start by identifying the range of the uncertainty level, α, where the total inventory needed in K-Flexibility equals to $D - (N - 1)c$, the inventory level for full flexibility in the deterministic case. Because K-flexibility is a superset of the long chain and a subset of full flexibility, $c/D \leq \alpha \leq (N - 1)c/D$. In this range, $u - s \geq c \geq l - s \geq 0$, so the condition (4.2) can be simplified as

$$\sum_{j \in \delta(X)} (d_j - s) \geq \sum_{i \in X} c_i = c|X|.$$

The minimum of $f(X) = \sum_{j \in \delta(X)} (d_j - s) - c|X|$ is reached when $\delta(X)$ only contains products with demand l. Otherwise, if there is a product $j \in \delta(X)$ with $d_j = u$, we can delete the plant in X that produces j to reduce the value $f(X)$. So we can let $f(X) = (l - s)|\delta(X)| - c|X|$.

Observe that by definition of K-Flexibility, $|\delta(X)| \geq |X| + K - 1$, and equality holds when plants in X are clustered, that is plants indices in X are consecutive. Also, because $l - s \leq c$, the minimum of $f(X)$ is reached when $\delta(X)$ contains all products with demand l. Hence, the "worst" demand happens when $N/2$ consecutive products have demand l. The inequality in this case is $\alpha \leq (2K - 3)c/D$.

If $\alpha > (2K - 3)c/D$, let $\Delta\alpha = \alpha - (2K - 3)c/D$. The inventory level $s = D/N - (N-1)c/N + \Delta\alpha D/N = (1+\alpha)D/N - (N+2K-4)c/N$ is enough to achieve a unit TTS, because it offsets any demand amount that exceeds the threshold. Thus, $(1 + \alpha)D/N - (N + 2K - 4)c/N$ is an upper bound on the amount of inventory required for a unit TTS. To see that this is also as lower bound, let $\delta(X)$ be the $N/2$ consecutive products with demand l in condition (4.2). We have

$$s \geq (1 + \alpha)D/N - (N + 2K - 4)c/N.$$

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So the proof is complete. □

4.3 Results and Comparisons

Proposition 5 implies that the strategic inventory for a K-flexibility design ($K \leq N/2$) is

$$\text{Strategic Inventory} = \begin{cases} D - C & \text{if } 0 \leq \alpha \leq (2K - 3)c/D, \\ (1 + \alpha)D - (N + 2K - 4)c & \text{if } (2K - 3)c/D < \alpha \leq 1. \end{cases} \tag{4.3}$$

Recall that $D - C = \sum_{j=1}^{M} d_j - \sum_{i=1}^{N-1} c_i$ is the total inventory required for K-flexibility when demand is deterministic, which is consistent with the solution when $\alpha = 0$. As α increases, the level of demand uncertainty increases. (4.3) suggests that for low degree of uncertainty, i.e., $0 \leq \alpha \leq (2K - 3)c/D$, the inventory level is unchanged. However, as the level of uncertainty increases beyond a certain value, a K-flexible system needs more inventory to achieve a unit of TTS.

Example 1: The Long Chain Design.

By equation (4.3), for long chain designs, the inventory needed is given by

$$\text{SI} = \begin{cases} D - C & \text{if } 0 \leq \alpha \leq c/D, \\ (1 + \alpha)D - Nc & \text{if } c/D < \alpha \leq 1. \end{cases} \tag{4.4}$$

Example 2: The Full Flexibility Design.

When $K \geq N/2 + 1$ (including full flexibility), it is easy to check that equally allocating the total inventory of $(N/2+1)$-flexibility design is feasible for K-flexibility to achieve a unit TTS. Therefore, the total inventory needed for K-flexibility design ($K \geq N/2 + 1$) to achieve a unit of TTS is the same as that of full flexibility (given by (4.5)).

$$\text{SI} = \begin{cases} D - C & \text{if } 0 \leq \alpha \leq C/D, \\ (1 + \alpha)D - 2C & \text{if } C/D < \alpha \leq 1. \end{cases} \tag{4.5}$$

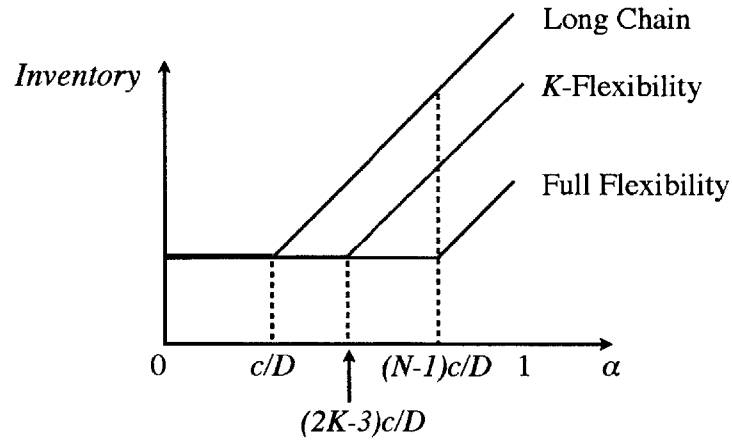To summarize, the result is shown in Figure 4-1.

Figure 4-1: Inventory needed to achieve 1 time unit of TTS with uncertain demand.

We can now compare the strategic inventory needed for full flexibility with that of the long chain, for a system with n plants having identical production capacity c. When the uncertainty level α, is small, i.e., $0 \le \alpha \le c/D$, there is no difference between the two systems, and in fact the inventory level is the same as in the deterministic one. As α increases above c/D, the performance of the long chain decreases and long chain needs more inventory than full flexibility.

Note that c is the capacity of an individual plant and C is the total capacity of all but one plants, so the threshold c/D for long chain is much smaller than $C/D = (n-1)c/D$ of full flexibility, as Figure 4-1 shows. In addition, as the number of products increases, c/D would decrease but C/D would increase, so the difference between long chain and full flexibility becomes even larger.

The result shows that with high uncertainty in demand, the long chain design may not be enough and the firm may need to invest in additional flexibility to keep inventory levels low. For example, when $K = 3$, the threshold for 3-flexibility design is $3c/D$, a substantial improvement over c/D, the threshold of the long chain.

One way to explain this improvement achieved by 3-Flexibility is that the chaining strategy in the long chain design is very effective in satisfying uncertain demand (see Jordan and Graves, 1995). However, a disruption breaks the chain, and hence reduces

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the ability to satisfy uncertain demand. On the other hand, a 3-flexibility design retains the chaining structure even if one plant is down, and hence it has a much better performance than the long chain design.

Chapter 5

Assembly Networks

Motivated by the Toyota case in the introduction part of Chapter 1, we include suppliers into the TTS model. Sometimes the company cannot afford to hold inventory of expensive final products. Therefore, it is worthwhile to apply TTS to an **assemble-to-order** manufacturing system, where suppliers can stock inventory of components (e.g., auto components), but assembly plants cannot stock inventory of final products (e.g., automobiles).

5.1 The Assembly Network Model

To be specific, we consider a two-stage manufacturing network shown in Figure 5-1. There are N suppliers, represented by squares on the left-hand side, producting M components, represented by circles. The components are then made into L final products, represented by triangles, by assembly plants. The manufacturer can stock inventory for components, but cannot do it for final products. Also, suppliers are subject to disruptions, where any of them can be down. Because assembly plants do not hold inventory, we assume they will not be disrupted. (Or, more realistically, we can assume the company has extra capacity for assembly plants, so that even if one of them is down, normal production can resume; there we dismiss the cases where assembly plants are disrupted.) The objective is to allocate inventory of components to maximize the TTS of final products.
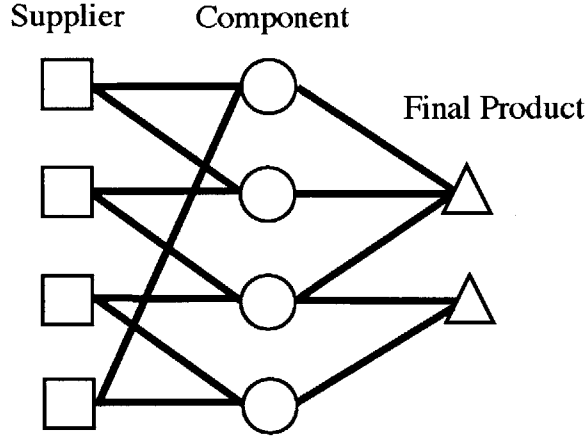
Figure 5-1: An illustration of assembly system.

For convenience, we introduce bill of materials (BOM), which is represented by a matrix A. An element of A, a_{jk}, means that final product k requires a_{jk} units of components j. Suppose that vector d is the demand of final products, and that vector y is the corresponding demand of components, then we immediately know that $y = Ad$.

Suppose $\mathcal{U}$ is the uncertainty set for final product demands. The Assembly-Network Strategic Inventory Problem (Problem A-SI) can be formulated as follows.

Problem A-SI :

$$s^* = \min_{s_j} \quad \sum_{j=1}^{M} s_j \tag{5.1}$$

$$\text{s.t.} \quad a_j^T d(\omega) - \sum_{i:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)}(\omega) \leq s_j, \quad \forall 1 \leq n \leq N, 1 \leq j \leq M, d(\omega) \in \mathcal{U}$$

$$\sum_{j:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)}(\omega) \leq c_i, \quad \forall 1 \leq i, n \leq N, d(\omega) \in \mathcal{U}$$

$$\sum_{j:\,(n,j)\in\mathcal{F}} x_{nj}^{(n)}(\omega) = 0, \quad \forall 1 \leq n \leq N, d(\omega) \in \mathcal{U}$$

$$s_j, x_{ij}^{(n)}(\omega) \geq 0.$$

In the above problem, $d(\omega)$ is an instance in the demand uncertainty set $\mathcal{U}$; $x_{ij}^{(n)}(\omega)$ is corresponding production element, representing the units of component j produced

by supplier i when plant n is disrupted. a_j^T is the jth row of matrix A, representing the units of jth components needed for each final product.

It is immediately clear that when the demand is deterministic, Problem A-SI reduces to Problem SI discussed in Chapter 3: if demand of final products is d, one only need to consider a one-stage network including only suppliers and components, where the demand of components is given by $y = Ad$. Therefore, we shall focus on cases where demand is uncertain.

5.2 Uncertainty Set for Components

In Chapter 4, we assume that $\mathcal{U}$ is a polytope. Since $\mathcal{U}$ is a polytope, Problem U-SI (4.1) suggests that strategic inventory s can achieve TTS of one for all demands in $\mathcal{U}$ if it can do so for all vertices of $\mathcal{U}$. Indeed, given inventory level s, if demand d can be satisfied by feasible production vector x, demand $\tilde{d}$ can be satisfied by $\tilde{x}$, then $\alpha d + (1 - \alpha)\tilde{d}$ can be satisfied by $\alpha x + (1 - \alpha)\tilde{x}$, which is also feasible. Therefore, for Problem U-SI, one only need to focus on the vertices of $\mathcal{U}$.

The result can be extended to Problem A-SI (5.1). Suppose the demand of final products d lies in a bounded polyhedral uncertainty set $\mathcal{U}$. The bill of materials is defined by a matrix A, so that the demand of components is $y = Ad$. All possible demands of components lie in the set

$$\mathcal{Y} = \{y \mid y = Ad, d \in U\},$$

is also a bounded polyhedron (a proof is given by Fourier-Motzkin elimination). By the same argument as before, one only need to focus on the vertices of Y.

Note that if d is a vertex of U, Ad is not necessarily a vertex of $\mathcal{Y}$. However, the following result assures that focusing on the vertices of U is enough.

Proposition 6. *If y is a vertex of $\mathcal{Y}$, there exists a vertex d of $\mathcal{U}$ such that $y = Ad$.*

Proof. Suppose that $\{d(k) \mid k = 1, \ldots, K\}$ is the set of vertices of $\mathcal{U}$. Let $y(k) = Ad(k), \forall k = 1, \ldots, K$. If there exists k such that $y = y(k)$, the proof is complete. If

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not, we can express y as $y = Ad$, and $d = \alpha_1 d(1) + \alpha_2 d(2) + \cdots + \alpha_K d(K)$, where $\sum_{k=1}^{K} \alpha_k = 1$, $0 \le \alpha_k \le 1$, $\forall k$, and at least one $a(k) \in (0,1)$. This implies that $y = A(\alpha_1 d(1) + \alpha_2 d(2) + \cdots + \alpha_K d(K)) = \alpha_1 y(1) + \alpha_2 y(2) + \cdots + \alpha_K y(K)$ is a convex combination of $y(K)$, which contradicts to the fact that y is a vertex of $\mathcal{Y}$. $\square$

By Proposition 6, if all vertices of final product uncertainty set $\mathcal{U}$ are known, the inventory level to achieve a unit of TTS of the assembly network under uncertain demand can be calculated as follows.

$$s^* = \min_{s_j} \sum_{j=1}^{M} s_j \qquad (5.2)$$

$$\text{s.t.} \quad a_j^T d(k) - \sum_{i:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)}(k) \le s_j, \quad \forall 1 \le n \le N, 1 \le j \le M, 1 \le k \le K$$

$$\sum_{j:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)}(k) \le c_i, \quad \forall 1 \le i, n \le N, 1 \le k \le K$$

$$\sum_{j:\,(n,j)\in\mathcal{F}} x_{nj}^{(n)}(k) = 0, \quad \forall 1 \le n \le N, 1 \le k \le K$$

$$s_j, x_{ij}^{(n)}(k) \ge 0.$$

In the above problem, $\{d(k) \mid k = 1, \ldots, K\}$ is the set of vertices of $\mathcal{U}$, and a_j^T is the jth row of matrix A, representing the number of jth components needed for each final product.

5.3   Total Amount of Strategic Inventory

It is evident that in Problem A-SI, allocating strategic inventory equally among all components is often not optimal. Therefore, Problem A-SI is much harder to solve than Problem U-SI discussed in Chapter 4. Despite this difficulty, TTS of assembly network can be characterized analytically when there are only two final products.

Suppose there are two final products with demand that belongs to the uncertainty set

$$\mathcal{U} = \left\{ (d_1, d_2) \mid (1 - \alpha)\bar{d} \le d_i \le (1 + \alpha)\bar{d}, \forall i = 1, 2;\ d_1 + d_2 = 2\bar{d} \right\}.$$

There are only two vertices in $\mathcal{U}$: $\mathbf{d}(1) = ((1-\alpha)\bar{d}, (1+\alpha)\bar{d})$ and $\mathbf{d}(2) = ((1+\alpha)\bar{d}, (1-\alpha)\bar{d})$. It is easy to see that for the $\mathbf{d}(1)$, the associated demand for component i is $y_i = (a_{i1}+a_{i2})\bar{d} - \alpha(a_{i1}-a_{i2})\bar{d}$ and similarly for $\mathbf{d}(2)$, $y_i' = (a_{i1}+a_{i2})\bar{d} + \alpha(a_{i1}-a_{i2})\bar{d}$. We denote by $\bar{y}_i = (a_{i1}+a_{i2})\bar{d}$ the average demand for component i. Let $\Delta y_i = (a_{i1}-a_{i2})\bar{d}$ and hence

$$y_i = \bar{y}_i - \alpha\Delta y_i, \quad y_i' = \bar{y}_i + \alpha\Delta y_i,$$

which implies that Δy_i can be viewed as demand variation.

If component i is a standard component, that is, a component used by multiple products, see for example the third component in Figure 5-1, $|\Delta y_i|$ is likely to be small because of the risk pooling effect. On the other hand, if component i is used by only one product, $|\Delta y_i|$ is likely to be large.

For the following two propositions, we suppose the number of suppliers equals the number of components, i.e., $M = N$. As usual, $C = \sum_{i=1}^{N} c_i - \max_{i=1}^{N} c_i$ is the worst case total suppliers capacity under disruption.

Proposition 7. *Suppose suppliers are fully flexible, i.e., each supplier can produce all the components. Then the strategic inventory required to achieve one time unit of TTS is given by*

$$SI = \begin{cases} \sum_{i=1}^{N} \bar{y}_i + \alpha \left| \sum_{i=1}^{N} \Delta y_i \right| - C, & \text{if } \alpha \leq \alpha_{FF}, \\ \sum_{i=1}^{N} \bar{y}_i + \alpha_{FF} \left| \sum_{i=1}^{N} \Delta y_i \right| - C + (\alpha - \alpha_{FF}) \sum_{i=1}^{N} |\Delta y_i|, & \text{if } \alpha > \alpha_{FF} \end{cases} \quad (5.3)$$

for some constant α_{FF}.

Proof. For a full flexibility network, the maximum total demand is given by $\sum_{i=1}^{N} \bar{y}_i + \alpha \left| \sum_{i=1}^{N} \Delta y_i \right|$. Therefore, the total inventory needed is at least

$$\sum_{i=1}^{N} \bar{y}_i + \alpha \left| \sum_{i=1}^{N} \Delta y_i \right| - C, \quad (5.4)$$

where $C = \sum_{i=1}^{N} c_i - \max_{i=1}^{N} c_i$. With the total inventory given by equation 5.4, the

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minimum total demand is automatically satisfied if

$$\sum_{i=1}^{N} \bar{y}_i - \alpha \sum_{i=1}^{N} |\Delta y_i| \geq \sum_{i=1}^{N} \bar{y}_i - \alpha \left| \sum_{i=1}^{N} \Delta y_i \right| - C,$$

or

$$\alpha \leq C / (\sum_{i=1}^{N} |\Delta y_i| - \left| \sum_{i=1}^{N} \Delta y_i \right|).$$

Beyond this threshold, the inventory needed increases at slope $\sum_{i=1}^{N} |\Delta y_i|$ with α. But this part may not exist, e.g., when $\sum_{i=1}^{N} |\Delta y_i| = \left| \sum_{i=1}^{N} \Delta y_i \right|$. □

Proposition 8. *Suppose each supplier can produce two components, and the process flexibility of suppliers forms a 2-chain design. Then the strategic inventory required to achieve one time unit of TTS for a 2-chain two-product assembly network is given by*

$$SI = \begin{cases} \sum_{i=1}^{N} \bar{y}_i + \alpha \left| \sum_{i=1}^{N} \Delta y_i \right| - C, & \text{if } \alpha \leq \alpha_{LC}, \\ \sum_{i=1}^{N} \max \{ \bar{y}_i + \alpha |\Delta y_i| - c_i, \bar{y}_i + \alpha |\Delta y_i| - c_{i-1}, 0 \}, & \text{if } \alpha > \alpha_{LC} \end{cases} \tag{5.5}$$

for some constant $\alpha_{LC} < \alpha_{FF}$.

Proof. By Proposition 2, the network has one unit of TTS (under uncertainty) if and only if

$$\sum_{i=1}^{N} (y_i - s_i)^+ \leq C, \quad \sum_{i=1}^{N} (y_i' - s_i)^+ \leq C, \tag{5.6}$$

$$(y_i - s_i)^+ \leq c_i, \quad (y_i' - s_i)^+ \leq c_i, \tag{5.7}$$

$$(y_i - s_i)^+ \leq c_{i-1}, \quad (y_i' - s_i)^+ \leq c_{i-1}. \tag{5.8}$$

Equations (5.7) and (5.8) are equivalent to the following condition

$$s_i \geq \max \{ \bar{y}_i + \alpha |\Delta y_i| - c_i, \bar{y}_i + \alpha |\Delta y_i| - c_{i-1}, 0 \}, \forall i.$$

Equation (5.6) implies that inventory needed for the 2-chain is at least that of full

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flexibility, which is given by equation (5.3), i.e.,

$$\sum_{i=1}^{N} s_i \geq \sum_{i=1}^{N} \bar{y}_i + \alpha \left| \sum_{i=1}^{N} \Delta y_i \right| - C,$$

which is an lower bound provided by the inventory of full flexibility design. □

In Figure 5-2 we apply the two propositions to compare strategic inventory levels in full flexibility and 2-chain designs for a two-product assembly system.
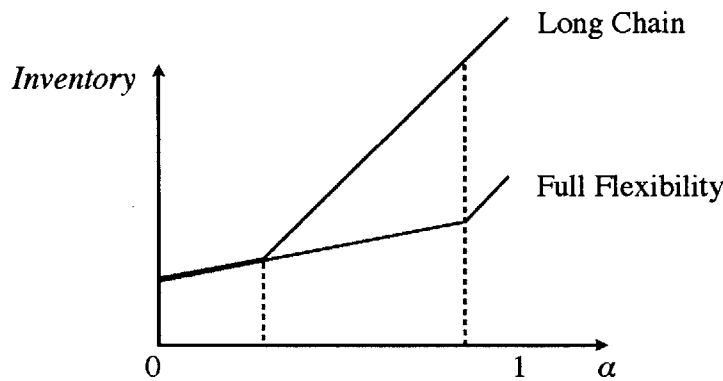

Figure 5-2: Illustration of assembly system

The steeper slope in this Figure equals to $\sum_{i=1}^{N} |\Delta y_i|$ while the flatter slope equals to $\left| \sum_{i=1}^{N} \Delta y_i \right|$. Again, the figure is similar to Figure 4-1, implying that a 2-chain design does not provide enough flexibility when demand has high uncertainty.

According to Figure 5-2, to reduce strategic inventory for a 2-chain assembly network, it is more important to reduce $\sum_{i=1}^{N} |\Delta y_i|$. As observed in the beginning of this subsection, $|\Delta y_i|$ represents risk pooling of component i, implying that if this component is used in multiple products, $|\Delta y_i|$ is likely to be small. Therefore, for a 2-chain design, one way to reduce strategic inventory is to standardize components so that they can be used in many final products.

To reduce strategic inventory for a full flexibility assembly network, it is more important to reduce $\left| \sum_{i=1}^{N} \Delta y_i \right|$ (rather than $\sum_{i=1}^{N} |\Delta y_i|$). In this case, it is less important to standardize components since positive and negative Δy_i's just cancel each other out.

To sum up these observations, **process flexibility** and **product design flexibility** (e.g., the ability for a component to be used in multiple products) are substitutes to each other. A network with low degree of process flexibility (e.g., dedicated or 2-chain designs) needs high level of product design flexibility (e.g., standard components) to reduce strategic inventory. By contrast, a network with high degree of process flexibility may need little strategic inventory even with low level of product design flexibility.

5.4 Allocation of Strategic Inventory

We now consider how to allocate inventory among components in a two (finished) product assembly system. Suppose that for some uncertainty level α, we have $\alpha_{LC} \leq \alpha \leq \alpha_{FF}$. That is, a 2-chain is at the steeper segment in Figure 5-2 while full flexibility is at the flatter segment. We know that this is the case for most α.

In such a case, by equation (5.3), a full flexibility design requires $\sum_{i=1}^{N} \bar{y}_i + \alpha \left| \sum_{i=1}^{N} \Delta y_i \right| - C$ units of inventory. That optimal value can be achieved if

$$s_i \leq \bar{y}_i - \alpha |\Delta y_i|, \tag{5.9}$$

so that no strategic inventory is wasted in either demand instance.

By equation (5.5), for a 2-chain network,

$$s_i = \max \left\{ \bar{y}_i + \alpha |\Delta y_i| - c_i, \bar{y}_i + \alpha |\Delta y_i| - c_{i-1}, 0 \right\}. \tag{5.10}$$

Comparing equations (5.9) and (5.10), we have some surprising results: 2-chain and full flexibility designs often need **completely different** inventory allocation strategies. For a 2-chain design, **more** inventory should be allocated to component i if $|\Delta y_i|$ is large, i.e., if variability faced by this component is high, which is consistent with classical inventory theory. By contrast, in a full flexibility design, **less** inventory should be allocated if $|\Delta y_i|$ is large, which is not what one expects by following classical inventory theory.

The above counter-intuitive observation can be explained in the following way. In a 2-chain design, much like in a dedicated network, there is limited capacity for each component and as a result inventory is used to mitigate against demand variability. Hence more inventory is required for components facing high variability. By contrast, in full (or high degree) flexibility design, the system has enough capacity to hedge against demand variability and hence the concern is to make sure the system does not have too much inventory. Thus, in this case, it is appropriate to stock more strategic inventory for products with stable demand. This observation is confirmed by numerical studies in Chapter 6.

Chapter 6

Stochastic Demand

In many cases, firms cannot predict disruptions on the supply side, but they are able to estimate the distribution of product demand using historical data. Thus, in this section, we consider the TTS model, when products demands are stochastic and their distributions are known. The purpose for considering the stochastic demand setting is twofold: First, in the stochastic model we are able to remove the restrictions in Chapter 4 where capacities of different plants are equal and demands of different products have the same range; Second, we would like to evaluate the TTS of different flexibility structures in the stochastic demand model, and find out whether the results in Chapter 4 and 5 still agree with the results under stochastic demand model.

6.1   Stochastic Demand Model

Suppose demands for different products are independently distributed. Demand of product j is a random variable D_j drawn from a distribution on the interval $[(1 - \alpha)d_j, (1 + \alpha)d_j]$, where d_j is the expectation of D_j, and the distribution is symmetric around d_j. Thus, following the notation in Chapter 4, α is proportional to the coefficient of variation of product demand.

The objective of the firm is to maximize the expected TTS. However, it is possible that for some low demand instances, even if one of the plants fails, the remaining capacity is still greater than demand, so that the TTS becomes infinity. To avoid this

difficulty, we consider the case where the firm sets a **target** TTS. A situation where the actual TTS falls below the target is considered unfavorable. Suppose the actual TTS is t (a random variable since demand is random), and the target being T. We assume that the firm is maximizing $E[t \wedge T]$, where $t \wedge T = \min\{t, T\}$.

The sequence of decisions is similar to the uncertainty model of Chapter 4. The firm first decides the amount of strategic inventory for each product, according to the given demand distribution. Once a disruption happens, realized demands are observed, and the firm decides the production level at each plant.

In sum, the problem can be formulated as a two-stage stochastic program as follows.

$$t^* = \max_{r_j} \quad E[t^\omega] \tag{6.1}$$

$$\text{s.t.} \quad \sum_{j=1}^{M} r_j \leq R,$$

$$r_j \geq 0, \quad \forall j = 1, \dots, M.$$

where the second stage problem is

$$t^\omega = \max \quad t \tag{6.2}$$

$$\text{s.t.} \quad t \leq \frac{r_j}{d_j^\omega - \sum\limits_{i:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)}}, \quad \forall 1 \leq n \leq N, 1 \leq j \leq M$$

$$\sum_{j:\,(i,j)\in\mathcal{F}} x_{ij}^{(n)} \leq c_i, \quad \forall 1 \leq i, n \leq N$$

$$\sum_{j:\,(n,j)\in\mathcal{F}} x_{nj}^{(n)} = 0, \quad \forall 1 \leq n \leq N$$

$$t \leq T,$$

$$x_{ij}^{(n)} \geq 0.$$

Here, d_j^ω is the demand of product j, which is realized at the second stage. We also rewrite the objective function $E[t \wedge T]$ by adding a constraint $t \leq T$ in the second stage problem.

Finally, it is easy to verify that the second stage problem (6.2) can be reformulated as a linear program, by introducing new variables $y_{ij}^{(n)} := t \cdot x_{ij}^{(n)}$. This fact would be helpful in the numerical tests below.

6.2   Numerical Examples for One-Stage Networks

In the following examples, we assume the firm produces five products from five plants. Plants capacities are chosen uniformly in $[0.8, 1.2]$. The expectations of demands are chosen uniformly in $[0.7, 1.1]$. For each Product j, the demand D_j is assumed to have independent discrete uniform distribution between $[(1 - \alpha)d_j, (1 + \alpha)d_j]$, where d_j is the expectation of D_j, and α is a constant between $[0, 1]$, measuring the variation of demands.

In the first example, we compare the expected TTS of four different flexibility designs under different demand variation levels. The four structures are the long chain; the long chain with additional flexibility (defined in Section 3.3), we refer to this design as **long chain additional**; 3-flexibility (defined in Section 4.2), and full flexibility.

To evaluate the different designs, we start with a given level of inventory R in (6.1), and determine the expected TTS of each of the four designs. In our computational tests, we choose R as the total inventory of the long chain required to achieve one unit time of TTS in the determinstic case. In this example, it is also the inventory for full flexibility in the deterministic case, so all structures have a TTS of one when $\alpha = 0$. Finally, we set the target TTS in (6.2) to be 1.

The result is shown in Figure 6-1. As expected, when the variation of demand increases, the performance of long chain drops quickly compared to full flexibility, indicating the need of extra flexibility. It may be surprising, though, that the 3-flexibility structure performs exceptionally well. The reason is that there are only five plants in this case, so 3-flexibility is close to full flexibility, that is, 5-flexibility.

Of course, when the number of plants increases, the gap (in expected TTS) between 3-flexibility and full flexibility is expected to increase. For example, consider

Figure 6-2 where we focus on a model with ten plants and ten products. As demand variation increases, the expected TTS of 3-flexibility drops quickly compared to full flexibility, but it still has significant advantage over the long chain. These examples of the stochastic demand model support our conclusion in Section 4 that extra flexibility is needed to protect the system from disruptions when the level of demand uncertainty is high.
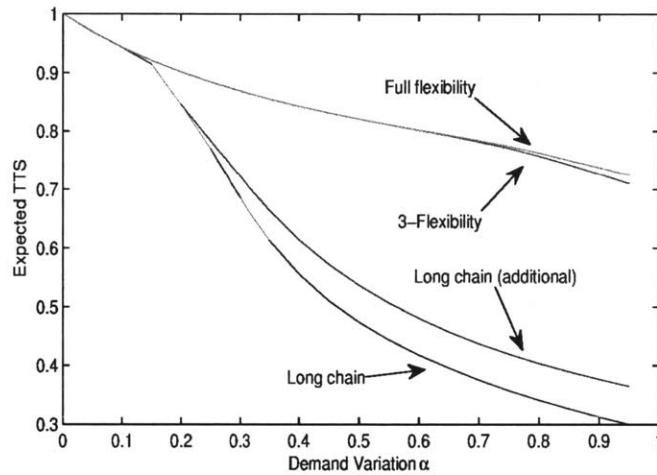

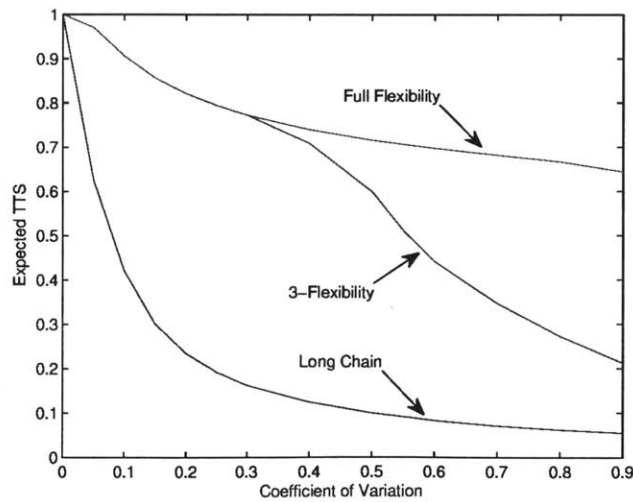Figure 6-1: Expected TTS against demand variations with 5 plants.

Figure 6-2: Expected TTS against demand variations with 10 plants.

Finally, instead of assuming that total strategic inventory is fixed, we can also consider a case where the firm needs to decide the amount of strategic inventory so that the expected TTS reaches a predetermined level. Figure 6-3 shows the results for the five plants, five products model. It shows the amount of inventory needed such that the expected TTS is 95% of the target TTS (of one unit time). It is insightful to compare these results with the results of Figure 4-1 for the uncertain demand model. Again, the figure shows that the inventory needed for the long chain rises more quickly than that of full flexibility as demand variability increases.
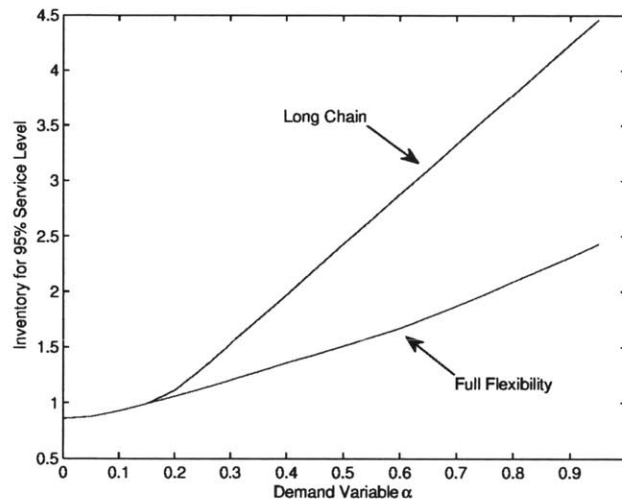

Figure 6-3: Inventory against demand variations.

6.3   Numerical Example for Assembly Networks

We consider an assembly network described in Chapter 5. In the network, there are eight suppliers producing eight different components, which are assembled into three final products. Two of the eight components are standard components, that is, components used in all three final products. Each of the remaining components is used in a single product–these are referred to as non-standard components. Final products expected demand and demand distribution as well as the suppliers production capacities are chosen in the same way as in the previous subsection.

First, we fix the total strategic inventory level, and determine the expected TTS for different flexibility designs, as was done in the previous example. Figure 6-4 depicts TTS as a function of the coefficient of variation of demand, α. Similar to the observations made for a one-stage network, in an assembly network, a 3-chain design is significantly more robust than a 2-chain design when demand variation is high.
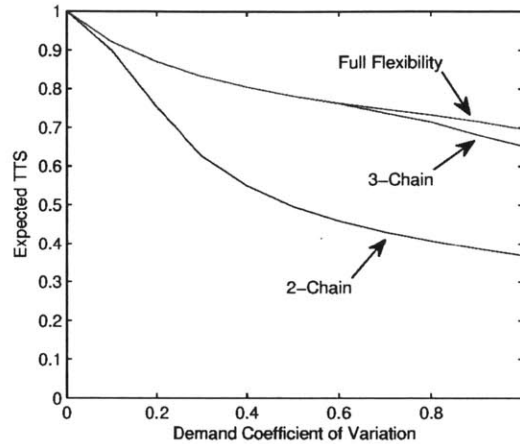

Figure 6-4: Expected TTS in an assembly network.

Next, we compare inventory for standard and non-standard components for different flexibility designs. Following the discussion of standard components in Subsection 5.4, we consider the average inventory level for standard components (equal to the sum of inventory levels for standard components divided by the number of standard components. We do the same for non-standard components. These values of strategic inventory levels (average for standard and non-standard components) are divided by expected TTS so that all cases achieve approximately one time unit of expected TTS. The results are consistent with the conclusions and insights in Subsection 5.4 that were developed analytically for an assembly system with only two final products. These insights suggest that different degrees of flexibility require different inventory allocation strategies.

1. The proportion of total inventory allocated to standard components in full flexibility design is higher than the proportion of inventory allocated to standard component in a 2-chain design. This implies that the proportion of inventory

allocated to non-standard components is higher in a 2-chain design than that of full flexibility.

2. As demand variation increases, more inventory is needed to achieve one time unit of TTS. For a 2-chain design, the increment of inventory is mainly allocated to non-standard components. However, for a full-flexibility design, the increment is allocated to standard components. In this particular example, we even observe a decrease in inventory level for non-standard components when the demand variation increases.

3. The inventory level of a 3-chain design falls between a 2-chain design and a full flexibility design for both standard and non-standard components. In addition, for non-standard components, 3-chain requires almost the same inventory level as full flexibility design, but a 2-chain design requires much more inventory.
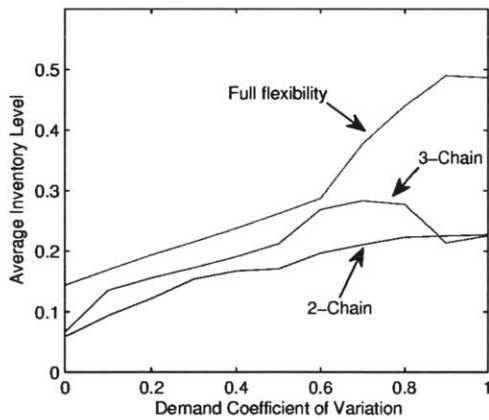

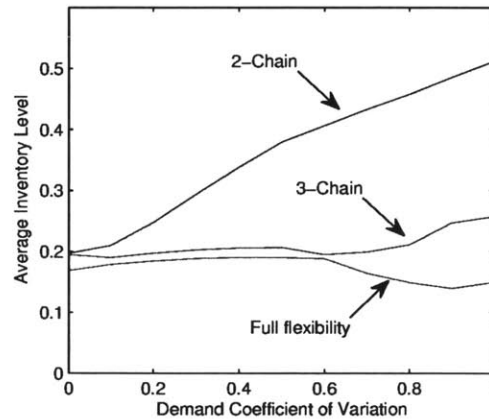Figure 6-5: The average inventory levels for standard components.

Figure 6-6: The average inventory levels for non-standard components.

Chapter 7

Conclusions

In this thesis, we present the concept of Time-to-Survive (TTS), a metric that we apply to measure supply chain robustness. TTS is defined as the longest time that (product or component) demand is guaranteed to be satisfied independent of which plant is disrupted. We consider two well-studied supply chain strategies, process flexibility and strategic inventory, that can be applied to increase TTS. Our goal is to understand the impact of process flexibility and strategic inventory on TTS, including the relationship between the degree of flexibility and the total amount of strategic inventory.

To develop insights on the impact of process flexibility and strategic inventory on TTS, we first consider the case where demand is deterministic. In this case, we show that given a fixed inventory level and a flexibility design, TTS can be calculated by solving a linear program. This implies that TTS increases linearly with total inventory level, but the rate of increase depends on the given flexibility design. We then proceed to study the inventory required to achieve a fixed TTS for different flexibility designs, and provide two insights. First, a full flexibility design needs significantly less strategic inventory than a system with no flexibility, and in addition, it enables a great amount of freedom in inventory placement. Second, when product demands are equal to each other than a 2-chain flexibility design is as effective as full flexibility. On the other hand, when product demands vary significantly, a 2-chain flexibility design may not be an effective one. However, a little more flexibility, where the degree of each plant

note is no more than three, is as effective as full flexibility.

Next, we model uncertainties in product demands using an uncertainty set, and study the level of inventory required to achieve a fixed TTS for all demand instances in the uncertainty set. Interestingly, we find that when there is uncertainties in the demand, there is a big gap between the inventory needed for a 2-chain design and that of full flexibility. However, the amount of strategic inventory decreases when a 2-chain flexibility design is replaced by a 3-chain flexibility design. In particular, a 3-chain design achieves the same robustness as full flexibility under a much larger range of uncertainty level.

We then extend the TTS model to an assembly system. We consider an OEM which applies an assemble-to-order strategy and does not hold inventory for final products, but can ask its suppliers to stock inventory for components. By considering an assembly system with two final products, we make a few surprising observations. First, our result suggests that the degree of process flexibility effects the need for standard components. When the degree of process flexibility is low, as in dedicated or 2-chain designs, it is important to have standard components to reduce strategic inventory. However, standard components are less critical in reducing inventory under high degree of flexibility. Second, our result suggests that having flexibility can greatly affect the allocation of strategic inventory between the different components. In particular, under the dedicated or 2-chain flexibility designs, more inventories should be allocated to components with high demand volatility, i.e., the non-standard components, which is consistent with the classical inventory theory. However, for full flexibility design, more inventory should be allocated to components with low demand volatility, such as standard components. This dichotomy further explains why a 2-chain design may not be a very robust process flexibility strategy, as it displays behaviors similar to a dedicated design than to a full flexibility design.

Finally, to ensure that these findings are not restricted to worst-case demand, we performed a numerical study to compute the expected TTS of flexibility designs when demand is stochastic, and find that the simulation results agree with the theoretical findings.

Appendix A

A More General TTS Framework

Suppose the survival time is divided into K periods. The length of period $1 \le k \le K$ is $t(k)$. No matter which plant is disrupted, during period k, the company wants to satisfy a demand rate of $d_j(k)$ for product j. Suppose $x_{ij}^{(n)}(k)$ is the production rate of product j in plant i during period k when plant n is down. Other assumptions and notations are the same as in Chapter 2. It is easy to see that product-specific TTS is a special case of this problem setting.

With some modification of Problem SI, the problem can be formulated as the following LP.

General TTS:

$$s^* = \min_{s_j} \quad \sum_{j=1}^{M} s_j \tag{A.1}$$

$$\text{s.t.} \quad \sum_{k=1}^{K} t(k)\Big(d_j(k) - \sum_{i:(i,j)\in\mathcal{F}} x_{ij}^{(n)}(k)\Big) \le s_j, \quad \forall 1 \le n \le N, 1 \le j \le M \tag{A.2}$$

$$\sum_{j:(i,j)\in\mathcal{F}} x_{ij}^{(n)}(k) \le c_i, \quad \forall 1 \le i, n \le N, 1 \le k \le K \tag{A.3}$$

$$\sum_{j:(n,j)\in\mathcal{F}} x_{nj}^{(n)}(k) = 0, \quad \forall 1 \le n \le N,, 1 \le k \le K \tag{A.4}$$

$$s_j, x_{ij}^{(n)}(k) \ge 0.$$

Appendix B

Uncertain Demand when Number of Products is Odd

In this appendix, we study the same uncertainty strategic inventory model as Section 4, and present a summary of the total inventory for full flexibility, long chain and K-flexibility when the number of products is odd ($M = 2m + 1$). Like Section 4, we let $\mathcal{U} = \{(d_1, \cdots, d_n)| \sum_{j=1}^{M} d_j = D, l \leq d_j \leq u\}$, where $l = (1 - \alpha)D/M$ and $u = (1 + \alpha)D/M$. Also, we suppose that all plants have the same capacity c and $C = (n - 1)c$.

The total inventory of full flexibility with different α is:

$$
\text{SI} = \begin{cases} D - C & \text{if } 0 \leq \alpha \leq C/D, \\ (1 + \frac{m}{m+1}\alpha)D - \frac{2m+1}{m+1}C & \text{if } C/D < \alpha \leq 1. \end{cases}
\tag{B.1}
$$

The total inventory of the long chain with different α is:

$$
\text{SI} = \begin{cases} D - C & \text{if } 0 \leq \alpha \leq c/D, \\ (1 + \alpha)D - Nc & \text{if } c/D < \alpha \leq 1. \end{cases}
\tag{B.2}
$$

For any integer $1 \leq K \leq m + 1$, the total inventory of K-flexibility with different α

is:

$$\mathrm{SI} = \begin{cases} D - C & \text{if } 0 \leq \alpha \leq (\frac{2m+1}{m}K - \frac{3m+2}{m})c/D, \\ (1+\alpha)D - (\frac{2m+1}{m}K + \frac{2m^2-3m-2}{m})c & \text{if } (\frac{2m+1}{m}K - \frac{3m+2}{m})c/D < \alpha \leq 1. \end{cases}$$

(B.3)

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