

**Essays in the Economics of Retirement Income Security and
Household Decision-Making**

by

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Submitted to the Department of Economics
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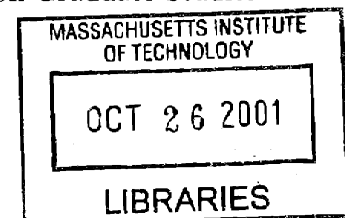
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Abstract

The first essay of this thesis studies within-family decision making regarding investment in income protection for surviving spouses using a simple and tractable Nash-bargaining model. A change in US pension law (the Retirement Equity Act of 1984) is used as an instrument to derive predictions from the bargaining model and to contrast these with the predictions of the classical single-utility-function model of the household. In the empirical part of the essay, the predictions of the classical model are rejected in favor of the predictions of the Nash-bargaining model.

Second essay studies married couple's dynamic investment and consumption choices under the assumption that the couple cannot commit across time to not to renegotiate their decisions. The inefficiencies that can arise are characterized. Efficiency properties of different divorce asset division regimes are examined. The effect of inability to commit across time on the savings level is examined under a tractable special case of the model.

Third essay is coauthored with Professor Peter Diamond and Professor John Geanakoplos. In this essay we extend Arrow's analysis of portfolio choice in a one-period model to savings and portfolio choice in a two-period model.

Thesis Supervisor: Peter A. Diamond
Title: Institute Professor

Thesis Supervisor: Jonathan Gruber
Title: Professor of Economics

Biographical Note

I was born in Espoo, Finland on January 6th 1971. I am a proud product of the excellent Public Schools and Public Universities of Finland: I received High-School Diploma from Tapiola High School in May 1991 and Master's and Licentiate's Degrees in Economics from University of Helsinki in May 1996 and 1997 (respectively). During High School, I spent one year as an exchange student in La Sarre, Québec and between my first and second year of my undergraduate studies I did an internship in Névers, France. In the past I have held various positions including warehouse worker and postal clerk. More recently I was researcher at Government Institute for Economic Research (VATT) in Helsinki and researcher at the Research Unit for Economic Structures and Growth (RUESG) of the University of Helsinki. I am married to Heidi Maria Aura and we have two lovely children: Matias Ensio Aura (born 13th of August 1997) and Saara Emilia Aura (born 12th of December 1999).

Acknowledgments

During the thesis project I have benefitted greatly from the advice of two most remarkable economists: Peter Diamond and Jonathan Gruber. They were both kind enough to be my advisors and I am very grateful for their guidance.

Jon was very influential in making the first chapter of this thesis much better than it would have otherwise been. He taught me on how to give up on ideas that did not work and he saved the chapter by giving new fresh ideas on how to refocus the paper just when all the work seemed to amount to nothing. Without his inspiration and ideas I would probably still be chasing after new datasets trying to answer questions that cannot be answered. He provided realism, insight and experience on the empirical work that I desperately needed to have a paper that was focused and doable.

However, no one else have had similar impact on this thesis as Peter. He was really the faculty member in charge of the whole thesis. With him I decided on the general topic of the thesis (back then vaguely "new household economics and public economics") and with him I ended up coauthoring one of the papers of this thesis (John Geanakoplos from Yale is the third author of that paper and he deserves thanks for his guidance on that paper). Without Peter forcing me work hard and rethink my ideas (especially the theoretical ideas) this thesis would have never become what it is now. I am truly lucky to have had the opportunity to work with both Peter and Jon, this experience has clearly taught me enormously about economics and research in general.

Other faculty members at MIT and elsewhere also deserve to be acknowledged here. James Poterba, the third member of the outstanding Public Economics team at MIT, acted as a third

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I also would like to thank my classmates and other fellow students at MIT. I have found some exceptionally good friends during my years at MIT and I will miss all of them (including all the members of the 1997 entering class of doctoral students). Some of them deserve individual acknowledgments.

Botond Köszegi, who shared an office with me during my third year, was very insightful in the beginning of this project. Like a true scholar, he never accepted half-thought arguments and always challenged them, making my thought-process much sharper in the end. Marko Terviö was always there to discuss anything with me, including my research. Tracy Seslen provided invaluable help with Survey of Income and Program Participation (SIPP) datafiles and it was no fault of hers that the analysis with SIPP never amounted to anything. Emek Basker provided comments, help and encouragement during the whole process, and deserves much credit for this.

Several outstanding Finnish economists also need to be acknowledged here. Without the guidance and encouragement of Seppo Honkapohja, Erkki Koskela, Ilpo Suoniemi, Matti Tuomala and Yrjö Vartia I would have never ended up where I am now. Roope Uusitalo and Tuomas Takalo provided some insightful comments during their visits to MIT.

Naturally, I would also like to thank my family. My parents are to claim much of the credit and the blame on what I have become. My wife, Heidi, to whom I dedicate this thesis, could not have been more understanding during these years at graduate school. Without her support and understanding, this thesis would not have been possible. My son Matias and my daughter Saara have been a source of inspiration and energy during these years.

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All views expressed in this thesis are mine. Naturally I am responsible for all the remaining errors. The following disclaimer also applies: “The research reported herein was supported by the Center for Retirement Research at Boston College pursuant to a grant from the U.S. Social Security Administration funded as part of the Retirement Research Consortium. The opinions and conclusions are solely those of the author and should not be construed as representing the opinions or policy of the Social Security Administration or any agency of the Federal Government, or the Center for Retirement Research at Boston College.”

To Heidi

Chapter 1

Does the Balance of Power Within a Family Matter? The Case of the Retirement Equity Act

1.1 Introduction

Most economic theory assumes that household behavior is determined by a fully rational agent maximizing a single household utility function. While for most purposes this assumption has been proven to be an extremely powerful way of describing actual behavior, in recent decades there have been at least two sets of challenges to this model. The first, behavioral economics, challenges the rationality assumption. The second set of challenges questions the notion that the behavior of multi-person household can be described as decisions made by a (possibly benevolent) dictator within a household. It posits an alternative view: the decisions taken by a household can be characterized by a more complicated process that takes explicitly into account the multi-decision maker structure of the household. Both sets of challenges share the view that there are situations that merit analysis beyond this simple paradigm. This paper considers one such application, in which the single-utility function model is unsatisfactory: the analysis of a government policy intended to redistribute resources within a family.

The specific issue analyzed in this paper is a married couples' choice of the amount of survivor protection to be provided to a surviving spouse after the death of her partner.¹ The potential conflict of interest between spouses rises from the fact that providing protection to a surviving spouse is costly (e.g. life insurance is not free). This means that the more survivor protection is provided, the less resources the household has available in other states of the world. This simple observation, while potentially compatible with the single-utility-function framework, illustrates the potential for conflicting interests between spouses. More generally, many decisions within a household have a potential for conflict between spouses or between other members of the household.²

The application studied in this paper is the spousal signature requirements of the Retirement Equity Act (REA) of 1984. This requirement mandated that a married pension plan participant, when retiring, must choose his pension payment in a form of a joint-and- $\frac{1}{2}$ survivor annuity unless his spouse signs a notarized consent form waiving her right to this survivor protection.³ The mandate affected only pension plan participants who started receiving their pensions after January 1, 1985.

In the theoretical part of the paper, a Nash-bargaining model of family decision making is used to analyze the specific effects of this law change for the selection of survivor annuities, life insurance holdings and savings. The law change is interpreted as having changed spouses' relative outside options. The model predicts that the law change would increase the selection of the survivor annuities and increase life insurance holdings for most households. The effect on the savings behavior is indeterminate. These predictions of the Nash-bargaining model are contrasted with the stark prediction of the classical model that the law would have had no

¹From now on, we will use convention that the husband is the spouse who, having been the primary earner, is more likely to die earlier. While the reverse situation is relevant for some couples, this is still (especially for the cohorts used in the empirical analysis) overwhelmingly more typical. Furthermore, the law change that is studied in this paper, while written in gender-neutral terms, was explicitly targeted to increase the protection of widows after the death of their husbands.

²Some examples studied in the literature are labor supply and labor force participation decisions of spouses, consumption allocations between different goods, health and educational investments, bequests and child labor.

³A joint-and- $\frac{1}{2}$ survivor annuity is an annuity that pays a fixed income stream as long as the primary annuitant (the pension plan participant) is alive and 50% of this stream as a survivor benefit for his spouse after his death as long as she is alive. A typical alternative to the survivor annuity is a single life annuity that pays a higher fixed income stream during the participant's lifetime. The terms "joint annuity" and "survivor annuity" are used interchangeably in this paper.

effect since the household budget set is unchanged.⁴ Thus this exogenous law change provides a well-identified empirical strategy for testing the predictions of the bargaining model against the predictions of the classical model.⁵

In the empirical part of the paper, several cross-section datasets are used to study these predictions. The effect on the survivor annuity selection is studied using the Current Population Survey (CPS) December 1989 Pension Benefit Survey and combination of the Health and Retirement Survey (HRS) and the Assets and Health Dynamics Among the Oldest Old (AHEAD).⁶ These results show that the law change increased the selection of survivor annuities by approximately 7 percentage points (a 10 percent increase). Results from HRS-AHEAD data indicate that the affected households increased their life insurance holding by approximately \$5,000 (this corresponds to approximately 25% of median life insurance holdings of the affected group). These joint-annuitization and life insurance findings support the Nash-bargaining theory over the classical single utility maximization model.

1.2 Survivor protection: legislation and economic evidence

Protection of surviving spouses can be provided by several instruments: privately purchased annuities, survivor annuities from private pensions, public pensions (Social Security), life insurance and savings. It is worth noting that many of these instruments can be used for motives

⁴Since the joint-and- $\frac{1}{2}$ survivor annuity was in the budget set by law (ERISA of 1974).

⁵Most of the existing literature that tries to test between alternative models of household behavior use as their identification sources variables that could easily be interpreted as being endogenous to the decision (like the relative income shares of the husband and wife). Thus, the rejections of the classical model in these papers can be due to this problem of identification strategy. This point is powerfully extended in Duflo (2000). Two studies that use similar natural experiment strategies as this paper are the Duflo paper and Lundberg, Pollak and Wales (1996). In the former the natural experiment was an expansion of pension benefits in South Africa. In the latter the natural experiment was a policy change in the UK, which changed the Child Benefit from tax credits to a direct payment to the mother. Interesting more structural tests of the model are derived in several papers by Chiappori and co-authors (e.g, Chiappori and Browning 1998). All three papers reject the classical single utility function view of the household.

⁶When used together these datasets will be referred as HRS-AHEAD data. Preliminary release data from HRS wave 1998 is used and therefore the following disclaimer applies: "This analysis uses HRS Preliminary Release data. These data have not been cleaned and may contain errors that will be corrected in the final Public Release version of the dataset."

other than survivor protection. Several authors have argued that bequest motives are important explanations for wealth accumulation (savings behavior) and for life insurance holdings (e.g., Bernheim 1991, Brown 1999, Kotlikoff 1998). Most households rely substantially on Social Security, which provides a real joint-annuity for married retirees. The surviving spouse in a typical married couple receives two thirds of the couple's Social Security benefits.⁷

Prior to REA, all private-sector and union pension plans in the United States were affected by the Employment Retirement Income Security Act (ERISA) of 1974.⁸ With respect to survivor annuities, ERISA required that if the pension plan's primary form of pension payout was an annuity, then the default option for married participants must be a joint-and- $\frac{1}{2}$ survivor annuity.⁹ Pension plan participants were free to choose other pay-out options without consulting their spouses. Holden and Nicholson (1998), using New Beneficiary Survey data, show that ERISA increased the selection of survivor annuities by married male pension plan participants from 48.1% to 63.9%. Unfortunately these data cannot be used to disentangle the two effects of ERISA: the mandate that survivor benefits must be an option (increased availability) and the effect of the default choice.¹⁰

The Retirement Equity Act (REA) of 1984 was a major revision of the original ERISA legislation. While it affected vesting requirements, minimum age requirements for pension plan participation, years of service calculations and other more administrative aspects of the covered pension plans, it also included two provisions that were explicitly meant to redistribute resources within a family.¹¹ It mandated the provision of pre-retirement and post-retirement survivor annuities unless the spouse affected signed a consent form in the presence of a notary

⁷This is the case when the Social Security benefits, both before and after the death of primary earner, are based on the earnings record of only one of the spouses. In that case the couple gets 150% of the Primary Insurance Amount (PIA) while the survivor gets 100% of the PIA. The replacement rate is lower for a two-earner couple.

⁸ERISA created standards for several aspects of pension plans including fiduciary duty, vesting requirements and reporting requirements. Compliance with the ERISA regulations is required for a pension plan to enjoy beneficial tax treatment.

⁹Before ERISA pension plans were not required to provide survivor annuities.

¹⁰A recent paper by Madrian and Shea (2000) provides evidence on the effect of the default choice on the investment decision made by 401(k) plan participants. They find that the choice of default option has a significant effect on retirement related investment decisions.

¹¹In the public discussion around that time, the Retirement Equity Act was also dubbed as the "Women's Pension Law".

public or a pension plan administrator.

The pre-retirement survivor annuity provision required that pension plans provide survivor coverage for a spouses if the participant died before his retirement unless the spouse waived this benefit. The decision to decline the pre-retirement annuity could be made at any time after the year of participant's 35th birthday and before his death. While the effects of the mandate to provide pre-retirement annuities is interesting, it is beyond scope of this paper.¹²

This paper will focus on the post-retirement survivor-annuities mandate. This requirement specified that employers must provide married participants with a notice form explaining the choice (typically between a single life annuity and a joint-and-survivor annuity; in some cases lump-sum payment is also offered) and the rights of the parties involved at least 90 days before start of the pension payments. The mandated default form of joint-and-survivor annuity provided 50% of the benefit received when the participant was alive to his spouse after his death. This requirement affected all defined-benefit plans and most defined-contribution plans.¹³ For the empirical part of the paper, it is important to note that state and local government pension plans were not affected by REA. The federal government pension plan had a similar requirement change effective at same time thanks to the Civil Service Retirement Spouse Equity Act of 1984.

For a typical retiring worker, the effect of selecting a joint-and-survivor annuity over a single life annuity is a reduction in pension benefits of approximately 10% (based on TIAA-CREF annuity pricing table, from TIAA-CREF 1996). Pensions where the payments had started before January 1, 1985, were unaffected by these survivor annuity requirements.

¹²One justification for this choice is that the post-retirement annuity selection situation is such that the selection will have an immediate effect on household income, while the decision on the pre-retirement annuity would only affect the household income through change in pension benefits perhaps as late as 30 years from the selection date.

¹³Among defined-contribution plans, all the money-purchase pension plans were affected by the provision. Under certain limited circumstances, profit-sharing and stock-bonus plans were not affected (Schechter 1985). It is also worthwhile to notice that the defined benefit plans that did not provide the option to annuitize the pension wealth were not affected by this requirement.

1.3 Theoretical models of household decision making and the Retirement Equity Act

Three simple models of household decision making are compared in this section with respect to their predictions on the effects of the signature requirement of the Retirement Equity Act. The models are the classical single utility function model, an “almost dictatorial” model and a Nash-bargaining model. Because each of these three models gives different predictions regarding the effects of the signature requirement they can be tested empirically.

1.3.1 The economic environment

For simplicity a two-period structure of the world is assumed. In the first period, both spouses are alive with probability one. In the second period, the husband is alive with probability $(1-p)$ and the wife is alive with probability one. Period 1 in the model presents early retirement and period 2 late retirement. In the first period the household must decide how much of its endowment to consume now, and how much to allocate to different states of the world in period 2. A complete market structure is assumed, so the household can use risk-free bonds and life insurance to transfer resources to period 2 with no short-selling constraints.¹⁴ Also for simplicity, the household decision-making process is assumed to be fully rational and the utility functions of household members (or the household utility function in the case of the classical model) are assumed to be of the time-separable expected utility form. Furthermore, household members are assumed to have no bequest motives.

The household is assumed to have an exogenously given endowment W . The complete and perfect nature of financial markets with no short-selling constraints imply that the source of the endowment is irrelevant to the choice set; only the actuarial present value of all income streams matters. Thus, for example, the fact that Social Security provides survivor annuities

¹⁴In this environment, the same state space is spanned either by life-insurance and risk-free bonds or by single-life and joint-life annuities. In the real world the situation is more complicated, due to the lumpiness of the pension annuity selection, differences in the pricing of annuities and life insurance and rationing/non-linear pricing in the life insurance market.

is irrelevant in this environment, since this can be fully undone by short sales of life insurance on the husband's life.¹⁵

1.3.2 Classical single utility function maximization

In this case, the household maximizes

$$\begin{aligned}
 U(c_1, B, I) &= u(c_1) + (1 - p)v(B) + p\tilde{v}(B + I) \\
 \text{s.t. } W &= c_1 + qI + \frac{1}{R}B,
 \end{aligned}
 \tag{1.1}$$

where c_1 is the current period consumption, B is the amount of the safe bond and I is the amount of life insurance, q is the price of life insurance and R is the real discount rate. Note that the utility function over consumption in period 2 is allowed to be state-dependent.

This model makes a stark prediction with respect to the signature requirement: it should have absolutely no effect on the decisions that the household makes since the budget set remains unaffected.¹⁶

1.3.3 “Almost Dictatorial” model

A slight generalization of the classical model has a (possibly altruistic) husband make all the economic decisions in the family, while the wife has her own utility function (that is irrelevant to the household's decisions). So while she has her own preferences, we observe only the choices made according to husband's preferences. Since the husband has ample tools in this model

¹⁵This implicitly assumes that the same pricing of social security is available in the market.

¹⁶This prediction holds as long as the law change does not change the utility function of the household. An alternative way to specify the decision problem is to parametrize the utility function differently in two states of the world: a separate utility function depending on whether the signature requirement is in effect. In first instance this looks similar to the classical introductory economics example on how preferences over sunlotion and umbrellas might change depending on the weather forecast. However, in this particular application the separately-parametrized utility function would be just an alternative way to parametrize the notion of bargaining power.

to undo any increase in joint annuitization by cancelling life insurance (or selling it short) he may choose not to bother to get his wife's signature to forego the survivor annuity. Instead he can completely offset this increase in survivor protection by cancelling his life insurance (or by short-selling life insurance). This simple model provides justification for the investigation, in the empirical part of the paper, of possible offsetting behavior on the life-insurance margin.

An extension of this model to the case in which the wife has a veto-right after the legislation on the annuity choice (but no say on any other decision of the household) and in which the short-selling constraints could bind, predicts that the legislation might have a real effect on the allocation of households resources, since the husband might not be able to completely undo the effect of increased annuitization. In this case, in the event of accepting the default allocation, life insurance holdings decline either fully offsetting the change or reaching zero. This assumes the same pricing in pension alternatives and in market pricing of life insurance.

1.3.4 Bargaining model

The Nash bargaining models (Manser and Brown 1980, McElroy and Horney 1981) and, more generally, efficient contracting models (Chiappori 1988a) of household behavior have been a topic of research in several areas of economics in past fifteen years. The applications of these models or other more general models of household decision making to retirement-related topics are rare, two notable exceptions, being Browning (2000) on the theory side and Lundberg and Ward-Batts (2000) on the empirical side.¹⁷ The basic tenets of these models are that households have at least two decision makers with separate utility functions, and that the choices households make are Pareto-efficient.

This section presents a Nash-bargaining model, but the model presented can easily be understood to be just a special case of a more general efficient contracting model. All the

¹⁷Browning (2000) models the decisions similar as studied in this paper as a non-cooperative game. Under the assumptions used, he finds that the Nash-equilibrium of the game can be Pareto-efficient. Lundberg and Ward-Batts (2000), on the other hand, find that variables plausibly correlated with the respective bargaining powers of the spouses (such as spouses' respective education levels and age difference between spouses), have an effect on the net worth of households in the first wave of the HRS.

results presented continue to hold in these more general models. In this sense, this paper does not engage in the debate of the relative merits of the Nash-bargaining assumption versus the efficient-contracting approach (Chiappori 1988b, McElroy and Horney 1990, Chiappori 1991). While the formal model is presented and solved in the Appendix, this section provides an informal discussion of its main assumptions and results.

In the Nash-bargaining model, both spouses are assumed to have utility functions over their own consumptions in different states of the world. While altruistic linkages between spouses are assumed away in this analysis, the results presented here apply as long as the altruistic linkages are not too strong.¹⁸ Furthermore, it is assumed that the decision negotiated in period 1 is honored in period 2, so there are no commitment problems across time-periods.

The key determinant of the Nash-bargaining solution is the outside option. This is defined as the utility level that the agent would attain should negotiations break down. In most of the Nash-bargaining literature on family decision making, the outside options are considered to be the spouses' respective utility levels in the case of divorce, given the institutional arrangement on the sharing of household resources, as in the original McElroy and Horney (1981) contribution. Lundberg and Pollak (1993) introduced the notion of a non-cooperative marriage as the outside option. In the context of this application, the non-cooperative marriage is the preferred interpretation. A non-cooperative marriage is interpreted in this context as a situation in which both spouses separately consume the income streams over which they have property rights, and do not optimally divide household chores. Although household chores are not explicitly modelled here, they represent one of the wife's bargaining chips: the threat of not providing household services to the husband is a potential instrument in her bargaining strategy.

In the bargaining context, the signature requirement of the Retirement Equity Act changes the relative outside options of the spouses by redistributing property rights on the income stream provided by the survivor annuity to the wife. Before the requirement, she does not have

¹⁸As long as on the margin both spouses would weakly prefer more of their own consumption in any state of the world over more of their partner's consumption in any state, the analysis goes through, although with more notation. Similarly, household public goods (a limit case of altruism) would only increase notation, but would not change the results presented here.

a claim on that income stream. This is equivalent to a redistribution from the husband's outside option to the wife's outside option.

Proposition 1. *REA increases the utility of wife and decreases the utility of husband.*

Proof: See Appendix.

This result is a direct consequence of the redistribution of outside options by REA. Since her outside option is higher when REA is in effect, her utility in the Nash-bargaining solution will be higher. Moreover, this result holds whether or not the household would have chosen the survivor annuity without REA. This is a general property of the standard Nash-bargaining solution: outside options always matter to the solution.

Proposition 2. *REA increases the amount of money transferred to the survivor state (the sum of survivor annuities and life insurance) and increases the wife's private consumption in periods 1 and 2. The effect on savings is ambiguous.*

Proof: See Appendix.

Proposition 2 basically restates Proposition 1. Since the bargaining power is tilted towards the wife with REA, the family will consume more items that enter positively into her utility function. Since life-insurance holdings and survivor annuities are perfect substitutes in this model, the prediction is only on the sum of these two.

In reality the choice of survivor annuity is a discrete choice the possible choices typically being no survivor benefits in the annuity and some selected levels of survivor annuity (say, 50% or 100% survivor annuity). Consider the following example: the 50% survivor annuity is the only form of survivor annuity available, and households continuously adjust their life-insurance holdings to arrive at the optimum.¹⁹ Households can then be divided into three groups based

¹⁹In reality the pricing of survivor annuities and life insurance differs. Annuities from pension benefits (both single-life and survivor annuities) are calculated using unisex life tables. Life-insurance cost on the other hand is a function of several factors (including health status, gender and access to group life insurance plans). For many households the survivor annuities available through their pension plans are cheaper (especially for couples with no access to group life-insurance markets), so for these households foregoing survivor annuities in favor of holding similar amount of survivor protection through life insurance would not be rational unless there is a strategic reason for this. One strategic reason for holding life insurance instead of survivor benefits might be the fact that the husband could unilaterally cancel his life insurance policies in the future without consulting his wife whereas he would need his wife's consent to cancel the survivor annuity.

on whether they would have chosen survivor benefits without REA and with REA. Table 1 Summarizes REA’s effects on life-insurance holdings.

Table 1. The effect of REA on life-insurance holdings for different types of households

Without REA	With REA	Effect on life insurance
No survivor annuity	No survivor annuity	Positive
No survivor annuity	Survivor annuity	Ambiguous
Survivor Annuity	Survivor annuity	Positive

All these effects are derived from the effect of REA on the amount of resources transferred to the widowhood state (the sum of life insurance holdings and the survivor annuity).²⁰ This unambiguously increases due to REA, since it increases wife’s threat point on the negotiation. The case of the group that would have not chosen a survivor annuity before REA nor after REA is the most intuitive. In this case, a story consistent with the bargaining model is that by promising more life insurance (and also more consumption in both periods) the husband “buys” the consent signature of his wife. In the second group, the effect on REA on the outside options is large enough to change their annuity choice. As shown in Appendix, it is perfectly possible that the redistribution of this outside option leads to a large change in the solution of the bargaining game.²¹ Thus the effect on the life insurance holdings of the second group is ambiguous. For the third group, who would have already chosen survivor annuities without REA, the effect on life insurance comes also from the increased outside option of the wife. Even though she was sufficiently well off in the case of negotiation breakdown even before REA to force the husband to select an survivor annuity, the fact that REA gave her new property rights in the negotiation breakdown state, makes her even better off in the solution. This implies that

²⁰In the empirical part of the paper this narrow notion of wealth available (sum of survivor annuities and life insurance) in the survivor state is also used. In practice, many of the household’s assets (most wealth components) in the typical household would be available to the widow. However, in practice we don’t know what happens to the different components of the households assets after the husband’s death, so using the narrow definition ensures that they are two state contingent assets that deliver income in the widowhood state.

²¹This can be illustrated by the following example: say that the value of the survivor annuity from the pension is \$10. The signature requirement redistributed this amount of outside option wealth from husband to wife. It is perfectly possible that the value of wife’s aggregate consumption across different states in the solution of the bargaining game changes by more than \$10. This can happen when the outside option utilities are very wealth elastic.

the life insurance holdings must go up since her consumption goes up.²²

1.4 Empirical Results

The predictions of the Nash-bargaining model are tested against the predictions of the classical single utility maximization model and the “almost dictatorial” model in this section. Outcomes studied from cross-sections of married couples include survivor annuity choice and life insurance holdings.²³ Identification strategy in these regressions is based on either the husband’s birth year or the start-date of his pension. Where the data permit, households in which husband is not receiving pensions (and will not receive in the future) are used as a control group. This allows us to use both standard first-difference and difference-in-differences empirical strategies.

The data-sets used in this section include Health and Retirement Survey and Assets (HRS), Health Dynamics Among the Oldest Old (AHEAD), Current Population Survey (CPS) December 1989 Pension Benefit Survey and CPS March files from several years. Evidence from published tables on the annuity selections of TIAA-CREF participants are also presented.

1.4.1 Outcomes, population studied and empirical identification

The outcomes studied in this section are:

1. The probability (conditional on the husband having a pension) that the husband’s pension provides survivor benefits;

²²No claim is made that Nash-bargaining or efficient-contracting models are the only sensible models that have these predictions. An alternative, more psychological oriented, model having similar predictions is the following. Suppose that before the REA’s signature-requirement some husbands declined the survivor annuities without really acknowledging the consequences of the choice they made. Assume that the REA’s signature requirement conveyed the information to the pension holder that the choice of survivor annuity was very important and forced him to seriously think about the consequences of his potential death to his wife. Thus instead of an effect through bargaining the REA might have had an effect by making agents to take more efficient action through providing more information.

²³Results on household net worth were also studied. No effects of the legislation were found. This is not surprising, given that none of the models in the previous section make strong predictions about the savings behavior and that net worth is not necessarily very good proxy for savings behavior.

2. The probability that the wife receives life insurance payments should her husband die;
3. The amount of life insurance protection.

The classical single utility function model gives a stark prediction that the law should not have affected any of these outcomes. The Nash-bargaining model predicts an increase in joint annuitization. Furthermore, for most households²⁴ the Nash-bargaining model predicts increase in the life insurance holdings (so also the second and third outcomes should increase due to the legislation).²⁵

The population studied is married couples whose husband was born between 1916-1919 (old group) or between 1924-1931 (young group). The reason for the omission of the 1920-1923 birth cohort is described later in this section. The sample (where the data allow this) was further restricted to married couples who were married when the husband turned 60 to eliminate couples who might have not been married at the time of the annuity selection. Due to relatively low number of couples marrying after age 60, this criterion has very little effect on the sample.

The receipt of pension income by the husband is the key quantity in the analyses.²⁶ When annuity choice is studied, only those who had pension income are included in the sample. When other outcomes are studied, the pension variable is used to divide the data into control and experimental groups. While using non-pension holders as a control group for pension holders is not an ideal control strategy, the inclusion of a rich set of covariates (like career high earnings of the spouses) should mitigate problems related to the differences between these groups.²⁷ Furthermore, the set of non-affected household in the control group includes many participants

²⁴That is, for households that did not change their annuity choice because of the law change. According to the estimates of this section, these households constitute approximately 93% of the households in which the husband has pension income.

²⁵It would also be interesting to study the effects of the legislation on the outcomes of widows using data on transitions to widowhood. This was attempted using the Survey of Income and Program Participation panels from years 1985 to 1996 (10 different panels), but even these large data sets would give only 686 transitions that satisfy all the necessary age restrictions to be informative about the effects of the legislation. Unfortunately, with such small sample size, no statistically significant (nor of any consistent sign for the legislative effect) results were obtained.

²⁶The cohorts studied are sufficiently old in the data (at least 66) to be very likely to have already started their pensions, which is important for our purposes, since it implies that they have already made their annuity selection.

²⁷One would expect that the pension coverage is more likely for those who worked in highly unionized industries or in high paying professions.

of defined contribution plans who chose to have their pension distributed as a lump sum instead of an annuity and who are a priori in many aspects more similar to the experimental group than the households where husband never had any pension coverage.

For all outcomes the following regression is estimated using only households where the husband had pension income:

$$Y = \alpha + \beta * young + \eta * Z + \varepsilon, \quad (1.2)$$

where, Y is the dependent variable (either an indicator for survivor benefits from pension, or an indicator for having life insurance, or the amount of life insurance holdings), $young$ is an indicator for the husband being young enough to be affected by the legislation and Z is the set of covariates.²⁸ The full covariate set in all these regressions includes non-linear controls for age differences between spouses, third-order polynomials in career high earnings of the spouses (including interaction terms), an indicator for wife having no work history, educational and race indicators and an indicator for the wife having a pension (or expecting one) of her own. The regressions are estimated with a varying number of covariates in the model. The model estimated is the standard first difference model (where the difference is with respect to birth cohort of the husband) and where β is the estimated effect of the legislation.

For outcomes other than annuity selection the following model using all the observations are estimated:²⁹

$$Y = \alpha + \beta_0 * young + \beta_1 * pension + \beta_2 * (young * pension) + \eta * Z + \varepsilon, \quad (1.3)$$

where $pension$ is an indicator for husband having a pension and β_2 is the program effect. Here the full covariate set also includes indicators for the husband's birth year (since the identification is

²⁸Even though the dependent variable is in many cases a binary variable, these regressions were estimated using OLS. As a robustness check, they were also estimated as Probits. All the qualitative results of the analysis were unaffected by the choice between Probit and the linear probability model estimated by OLS.

²⁹The control group did not face the annuity selection, so they cannot be used as a control for annuity selection.

no longer based solely on the birth year).³⁰ This estimation strategy can be seen as a standard difference-in-differences strategy, where the first difference is between cohorts and the second is between pension statuses of the husband.

An exception to the empirical strategies described above is the annuity selection equation estimated from the CPS 1989 Pension Benefit Survey.³¹ There an indicator variable for the pension starting after 1985 was used instead of the cohort proxy. This specification allows the use of the husband’s birth year in the covariate set.

As described above, in most regressions the birth year of the husband is used as a proxy for whether the husband’s pension started before January 1985. The empirical justification for this comes from the yearly March CPS files from 1976 to 1998. The probability of a pension income receipt from federal or private pensions is calculated for married males as a function of their age. These results are presented in the Figure 1. From these results we can deduce the information presented in the Table 2.

Table 2. Approximate propability of having a pension income as a function of the husband’s age

Pension	Percent
At age 54	4%
At age 62	15%
At age 66	38%
Ever	40%

The evidence from this relatively-stable relationship across years seem to suggest that most of pension starts happen when husband is between 62 and 65. Based on that information the observations were divided into three categories according to the husband’s age. These categories are presented in Table 3. Only the cohorts listed were used in the analysis.

³⁰When birth year indicators are included in the regression they replace the variable *young* . However, due to the limited sample size, the interaction terms with *pension* -variable are not estimated separately for each birth year even in this case. Instead the interaction *young * pension* is still used.

³¹The described husband’s birth cohort restrictions do not apply to this case either, since we do not have to rely on the birth cohort for identification with this data.

Table 3. Age cohorts.

Husband's Age on January 1, 1985	Status
54-61	"After" group (young)
62-65	Omitted middle group
66-69	"Before" group (old)

1.4.2 Data Sources

The Current Population Survey (CPS) December 1989 Pension Benefit Survey is a special supplement to the CPS collected for the purpose of analyzing the effects of the Retirement Equity Act. It includes detailed pension information (including the start date of benefits) and information on whether each pension provides survivor benefits. Because this survey was collected only four years after the law change it is subject to fewer sample selection problems than other datasets such as HRS-AHEAD.³² Ideally one would like to analyze a question like selection of survivor annuities from flow data (or from a panel that tracks individuals as they make their choices). The downside of CPS 1989 data is that it does not include information on life insurance holdings, but it does include information on the career high earnings, which is a key covariate in the regressions.

The Health and Retirement Survey (HRS) and the Assets and Health Dynamics among Oldest of the Old (AHEAD) panels are the main sources of data in this paper. HRS and AHEAD started as separate panels in 1992 and 1993. AHEAD started as a panel survey of households in which at least one of the members was over 70 years old at the time of the first interview (born in 1923 or earlier). HRS started as a panel of households having at least one member born between 1931-1941. Before 1998 there was one additional AHEAD wave (1995)

³²The mechanism for sample selection is the following: suppose husbands have private information on their life expectancy and this information enters into the decision whether to select survivor annuity (with these more likely to die soon more prone to select survivor annuities). Then any cross-section attempt to estimate the effect of the start year (or birth cohort group) on annuity selection will be biased since the earlier the start year is (or the older the birth cohort is), the higher the proportion of those who chose survivor annuities because of the private negative information on their life expectancy and have died before reaching the data collection. This leads to a bias towards the finding that among later pension starter a higher fraction choose survivor annuities.

and two additional HRS waves (1994 and 1996). In the HRS 1998 these two panels were merged and additional cohorts were included in the panel to have a representative sample of households where at least one member was born before 1947. It is important to note that most individuals who are young enough to be affected by the law change are only part of the HRS 1998 data. The advantage of HRS-AHEAD data is the detailed information on the work histories, pensions and life insurance and assets holdings.

The HRS-AHEAD data were used in two separate ways in the empirical analysis. A cross-sectional estimation of the effect of legislation uses the HRS 1998. However, the mortality bias for the older group could be significant in this approach, since the members of the older group would have to have lived to be 79-82 to be included in the sample. For this reason the effects of the law change are also estimated using data for the older group from the first wave of AHEAD and data from HRS 98 for the younger group. Although not a perfect solution, this approach could reduce the mortality bias substantially. Ideally one would like to compare similarly aged individuals at different times, but this is not possible given the existing datasets. The ages of the older and younger group at the times of different surveys is reported in Table 4.

Table 4. Age of the compared cohorts in the first wave of AHEAD and in the HRS 98. The preferred comparison is between the off-diagonal cells of this table

Data-set	Age of the young	Age of the Old
AHEAD 1	62-69 (not part of the sample)	74-77
HRS 98	67-74	79-82

1.4.3 Results

Survivor Annuity Selection

Evidence on annuity selection are presented from three different data sources. The first data source is a published table of annuity choices by TIAA-CREF participants (TIAA-CREF 1996), where the data is tabulated according to the start year of the pension. The other data-sources are the CPS 1989 December Supplement and the HRS-AHEAD data, described above.

The evidence from TIAA-CREF flow data is presented in Figure 2. Between 1978 and 1994 selection of survivor annuities went from 56.5% to 74%, an increase of 17.5 percentage points. More than half of the total change (9 percentage points) occurred between 1984 and 1986.. This suggest that while there was an pre-existing trend in the data, the legislation had a substantial effect on the selection of survivor annuities. Two caveats are in order here: this data include also non-married participants and the workers from state universities (and certain church-run universities) who were not affected by the legislation.

The CPS 1989 December Supplement was used in a General Accounting Office Report (GAO 1992) studying the effects of REA on the selection of survivor annuities. Based on simple tabulation, the GAO estimated that the selection of survivor annuities increased by 15 percentage points after the legislation. Two caveats on the GAO analysis are in order. First, the GAO included all the observations in the analysis regardless of how long ago the selection was made, which can lead to a mortality bias. This choice would also attribute any effects of possible existing trends in the data to REA, while the exclusive use of more recent years (where data appears relatively stationary except for the effect of REA), should at least mitigate this problem. Secondly, the 15 percentage points increase in survivor annuity selection is based on a comparison of the highest after-legislation fraction of choosing survivor annuities (for years 1988-1989) with the average fraction of all before legislation choices. The use of this latest and highest value is only correct if the legislation took some time to have an effect on the choices. Since the sample sizes are small enough for the data to be fairly noisy, a more conservative estimate would use all years after 1985 to estimate the effect of the legislation and not the arbitrary highest.

Our estimates from CPS 1989 December Supplement are presented in Tables 5 and 6, and in Figure 3. The sample used differs from the GAO's study in that only observations in which the husband started his pension between 1979-1989 are used and federal employees are included in the sample since the law affecting them changed at the same time as REA was enacted (as explained in Section 2).³³ Across different specifications these results suggest that the selection of survivor annuities went up by 7 percentage points after the law change. This result is robust

³³The inclusion of federal employees does not affect the qualitative estimates.

to the choice of covariate set and is statistically significant.³⁴

The results from the HRS-AHEAD data are presented in Table 7. These estimates suggest that signature requirement increased the selection of survivor annuities by 5-10 percentage points. These results range from statistically insignificant to significant at the 1% level depending on the data used and the specification estimated.

The magnitude of estimated effect is not very large. This natural given that already before the legislation a majority of married husband were choosing survivor benefits (approximately 70 percent).³⁵

Among the covariates, it is interesting to note that if the wife has a pension on her own or is expecting a pension, her husband is less likely to provide survivor benefits through his pension. This holds for both datasets and is statistically significant in both datasets.

Having higher education level (both for husband and wife) in general seem to imply that the selection of survivor benefits is more likely. These education effects are not always significant. In the preferred models (regression with all the covariates and in case of HRS-AHEAD comparing between AHEAD Wave 1 and HRS 98), the point estimate for the effect of education is approximately 13 (CPS) or 20 (HRS-AHEAD) percentage points increase in the probability of selecting survivor benefits when the comparison is between a couple where neither finished high school and a couple where both graduated college.

These results on increased joint-annuitization are consistent with the bargaining model and with the “almost dictatorial” model. They constitute a rejection of the standard single-utility function model of the household. However, the magnitude of estimated effect is not very large.

³⁴A difference-in-differences strategy with CPS 1989 data was also tried, where the control group was state and local government workers who were not affected by the law changes. The estimates of the program effect were similar in magnitude as the first difference estimates but not statistically significant. This is probably due to the small sample of state and local workers in the data (328). It is thus not very surprising that the effects are made non-significant by the inclusion of the control group, since in practice the difference-in-differences estimator just subtracts a very noisy measure of change in proportion for state and local workers from the first difference estimate discussed in the preceding paragraph.

³⁵In the HRS-AHEAD data the proportion selecting survivor benefits is on the average approximately 5 percentage points less than in CPS 1989. One possible interpretation for at least some of this difference is that HRS-AHEAD data, having been collected later than CPS 1989, suffers more from the mortality bias.

This natural given that already before the legislation a majority of married husband were choosing survivor benefits (approximately 70 percent).

Probability of having life insurance protection for the wife

From HRS-AHEAD first-difference and difference-in-differences models were estimated for the probability that the husband has life insurance policy in which the wife is listed among the beneficiaries. With the exception of the first-difference estimator across HRS and AHEAD data (in which the old group is from AHEAD Wave 1), no significant effect on this margin is found. The point estimates from difference-in-difference models are slightly negative, but given their standard errors, they are consistent also with large positive effects. The results are presented in Table 8.

It is interesting to note that the probability of having life insurance is significantly higher across all specifications for households in which husband has pension income. This might reflect better access to the group life insurance market through former employers. Other significant result is that husband being Hispanic has a huge negative effect on the probability of having life insurance. This is consistent with the results of Bernheim et al (1999) who find using the first wave of the HRS (1992) that non-white households are more likely to be underinsured.³⁶ However, the effect of husband being African-American on the probability of having life insurance is approximately zero.

The results on education are mostly non-significant, with high school education (either for the wife or the husband) increasing the probability of having life insurance. The effect of having college education is more mixed, but given the precision of the effects-of-education coefficients, these results are only tentative in nature. In the preferred model (regression with all the covariates, comparing between AHEAD Wave 1 and HRS 98), the point estimate for the effect of education is approximately 9 percentage points increase in the probability of life insurance

³⁶Underinsured in their vocabulary means having less life insurance protection than their behavioral model would predict being optimal. Thus the use of the term underinsurance carries a normative judgement.

protection when the comparison is between a couple where neither finished high school and a couple where both graduated college.

The Amount of life insurance Protection

From HRS-AHEAD both first difference and difference in differences models were estimated for the value of life insurance protection of life insurance plans where the wife is listed as a beneficiary (note that zero values were included since that is a valid amount of life insurance protection).³⁷ Three different statistical models were estimated: standard linear regression by OLS, median regression and a version of robust regression that uses biweight-weighting scheme to downweight outliers (Hamilton 1991). The latter two models are estimated to ensure that the results are not driven by small number of outliers. The results are presented in Tables 9 and 10.

The results for life insurance protection seem to imply that the law change increased by approximately \$5,000 (preferred median regression estimate).³⁸ The magnitude and significance of this estimate varies across different models, the OLS estimates being significantly larger but statistically non-significant and the robust regression results being smaller than those of the median regression.³⁹ The results from the median regressions are always statistically different

³⁷Missing values in the face value of life insurance pose an interesting problem in the analysis. For many observations in the HRS-AHEAD data, we know that they have life insurance, but we do not know the face value of the plan. The results reported here don't use these observations. However, similar results were obtained by imputing the median of the positive life insurance values for the missing value. This suggest that the tendency not to report value of life insurance seems to be uncorrelated with the treatment variable *young * pension*.

³⁸This would correspond roughly to 10 times the husband's median monthly pension.

³⁹The fact that we find a positive results on the average life insurance holdings, but no effect on the probability of having life insurance is not consistent with the Nash-bargaining model if we take the point estimates to be the true values. This observation is not very damaging to the empirical validity of the Nash-bargaining model and could be just due to sample variation (remember that the point estimate for the effect on the probability of having life insurance is very imprecise). The Nash-bargaining model predicts that every household (with the potential exception of the approximately 7% of the households who changed their annuity choice) should increase their life insurance holdings. For many households the non-negativity constraint of life insurance holdings is binding in reality (this constraint was not taken into account in the model). It is possible that for many of them the change brought on by REA is small enough not to make them hold positive amounts of life insurance. For the households already holding life insurance, the Nash-model predicts that all of them should increase their life insurance holdings. Thus it should be easier to detect the change in the latter variable. In statistical terms this is similar as saying that converting life insurance holdings into a binary variable uses less efficiently the sample information available on the life insurance holdings, since the conversion discards relevant information.

from zero as are the results from the robust regressions when the model is estimated using both AHEAD Wave 1 and HRS 98. This result of increased life insurance protection is consistent with the Nash-bargaining model of household behavior and inconsistent with both the classical model and the “almost dictatorial model”.⁴⁰

Similar results as in the previous section regarding the covariates hold for these regressions. Husband being Hispanic has a negative effect on the amount life insurance protection (approximately \$7,000 less in the median regression case). This effect is significant across specifications. Now also having African-American husband is weakly related to less insurance coverage. In the preferred formulation, where the AHEAD Wave 1 is used for the older cohort, this effect is not statistically significant. When the pure cross section model from HRS 98 is estimated, the effect is significant. The magnitude of this effect varies across specifications in the median regression case from \$100 less insurance to \$3,500 less insurance coverage. As in the previous section, these results are consistent with the findings of Bernheim et al (1999).

The effect of education of the spouses on the amount of life insurance holdings is generally positive (although not all the coefficients estimated are significantly different from zero). In the preferred model (median regression with all the covariates, comparing between AHEAD Wave 1 and HRS 98), the point estimate for the effect of education is approximately \$16,000 increase in the life insurance holdings when the comparison is between a couple where neither finished high school and a couple where both graduated college.

A general point about all the models estimated (survivor annuity choice, and the probability and the amount of life insurance) is that the covariates have surprisingly little effect on either the magnitude or the significance of the estimated program effect. This is evidence for the

⁴⁰Some authors (e.g. Turner 1988) have argued that the increase in selection of joint annuities around 1985 was not caused by the requirements of Retirement Equity Act, but by the Supreme Courts Decision in 1983 to require the use of unisex life table in the calculation of the annuity payments from employer-provided pensions. For male pension participants this made joint life annuities relatively less expensive as compared with the single life annuity. However, the increase in life-insurance holdings would be hard to justify based on the unisex decision reasoning, since life insurance and survivor annuities are near-perfect substitutes. In a classical model of household behavior, if we assume that life insurance and survivor annuities are Hicksian substitutes, then life insurance holdings should go down when the price of survivor annuities goes down, unless the income effect dominates. Furthermore, the price shock for most pension plans was likely to be small since the unisex decision called for the use of unisex tables within each pension plan, so that the unisex table used could incorporate the gender composition of the risk pool in the calculation.

control strategy used here being successful, at least in the observable-characteristics-space, the differences (or differential trends) between control group and experimental group do not seem to be driving our results.

1.5 Conclusion

This paper presented a tractable Nash-bargaining model for household decision making over survivor protection. The model made two specific predictions regarding the effects of the Retirement Equity Act on the choices that households make: the selection of joint-annuities would increase, and, for most households, life-insurance holdings would increase. These predictions are in stark contrast with the predictions of the classical single-utility-function maximization model. Using several microdata sources it is shown the predictions of the Nash-bargaining model are confirmed. This constitutes a rejection of the single-utility maximization model of household behavior in this decision-making realm.

These results imply that the change in the selection of survivor annuities were not the only effect of the Retirement Equity Act. The increase in life-insurance holdings through the household bargaining mechanism, while increasing income security for widows, was neither foreseen nor intended by the legislation. In this, there is an important lesson for policy making that targets the resource allocation within a family. Because we do not yet fully understand the decision-making dynamics in the family, policies can have unanticipated effects due to the household decision-making process. The model and the empirical results presented here take the literature one step closer to understanding this process and its implications.

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Appendix to Chapter 1: Results relating to the Nash-bargaining model

The bargaining solution maximizes (subject to the household budget constraint):

$$\begin{aligned} & \left(u^f(c_1^f) + (1-p)v^f(c_2^f) + p\bar{v}^f(\hat{c}_2^f) - h^f \right) * \\ & \left(u^m(c_1^m) + (1-p)v^m(c_2^m) - h^m \right), \end{aligned} \tag{1.4}$$

where h^f and h^m are the outside options of the wife and husband and c_1^f, c_2^f and \hat{c}_2^f are respectively, first-period consumption, second-period consumption in the state where husband is alive, and second-period consumption in the wife's widowhood. In all proofs below we assume that utility functions are concave and twice differentiable and that there is some marital surplus to be shared in the optimum (so the outside options do not bind).⁴¹

⁴¹The assumption that the family members' utility functions are of time-separable expected utility form is not crucial to the results. An alternative way to obtain the results of this appendix would be to assume that each partner has a general utility function over their own consumption (with no within family externalities) and to break down the process into two components: wealth sharing between spouses and individual utility maximization for given individual wealth share. The additional assumption needed is that the individual utility functions are such that the utility maximization for a single individual having same preferences generates demand functions that are increasing in lifetime wealth.

Lemma 1 states an obvious feature of the Nash-bargaining solution.

Lemma 1. *Each agent's utility is increasing in his outside option and decreasing in his partner's outside option.*

Proof: Define the utility possibility frontier by $g^f = K(g^m)$, where g^f and g^m are the utility levels of the wife and husband respectively. Now the maximization can be written as

$$\max_{g^m} (K(g^m) - h^f)(g^m - h^m). \quad (1.5)$$

From the first order condition and from the fact that $K' \leq 0$ it follows that g^m is increasing in h^m and decreasing in h^f .

To save on notation we will introduce a notational device that also highlights the link between Nash-bargaining solution and the efficient-contracting solution by writing the problem in the general form (originally from Chiappori 1988a).

Lemma 2. *Any Nash-bargaining solution in which the outside options do not bind is also a solution to maximization of a weighted sum of utilities:*

$$\begin{aligned} & u^f(c_1^f) + (1-p)v^f(c_2^f) + p\tilde{v}^f(\tilde{c}_2^f) + \\ & \lambda (u^m(c_1^m) + (1-p)v^m(c_2^m)), \end{aligned} \quad (1.6)$$

where

$$\lambda = \frac{g^m - h^m}{g^f - h^f} \quad (1.7)$$

and g^m and g^f are now the utility levels attained by the husband and wife in the Nash-bargaining solution.

Proof: The maximization of the Equation 1.6 has same first-order condition as the Nash-bargaining solution.

Lemma 3. *In the solution of maximization of Equation 1.6, the husband's utility is increasing in λ .*

Proof: Similar to the proof of Lemma 1.

Proposition 1. *REA increases the utility of wife and decreases the utility of husband.*

Proof: Since REA increases the wife's outside option and decreases the husband's outside option, the results follows from Lemma 1.

Since from Lemmas 1 and 3 it follows that increasing the wife's (husband's) outside option has qualitatively similar effect as a decrease (increase) of λ Proposition 2 will be proved using the weighted sum of utilities form of the problem.

Proposition 2. *REA increases the amount of money transferred to the survivor state (the sum of survivor annuities and life insurance) and increase the wife's private consumption in periods 1 and 2. The effect on savings is ambiguous.*

Proof: Consider the maximization of Equation 1.6 subject to a budget constraint. The first-order conditions can be written as:

$$\begin{aligned}
 u^{f'} &= \frac{v^{f'}}{p(c_2)} & (1.8) \\
 u^{f'} &= \frac{\hat{v}^{f'}}{p(\hat{c}_2)} \\
 u^{f'} &= \lambda u^{m'} \\
 u^{f'} &= \lambda \frac{v^{m'}}{p(c_2)},
 \end{aligned}$$

where first-period consumption is the numeraire and the p function gives the price of each consumption good in terms of first-period consumption. For concave u and v functions and for an unchanged budget set, an decrease in λ will unambiguously increase c_1^f, c_2^f and \hat{c}_2^f and also decrease c_1^m and c_2^m . The effect on savings (defined as resources not consumed during period 1) depends on the relative magnitudes of the second derivatives of u^f and u^m functions, since

these quantities determine whether the wife's consumption in period 1 will increase by more than the husband's consumption in period 1 will decrease.⁴²

Lemma 4: *A redistribution of wealth between spouses in the state where the negotiation breaks down can lead to changes in the allocation of total consumption that is either smaller or larger than the amount redistributed.*

Proof: As in lemma 1, write the optimization problem as

$$\max_{g^m} (K(g^m) - h^f((1 - \alpha)W))(g^m - h^m(\alpha W)), \quad (1.9)$$

where now the outside options are functions of the wealth-sharing rule of the household should the negotiation break down. Redistributing wealth towards the husband is an increase in α . Now it is clear that the comparative statics of g^m with respect to α depend (for a given utility possibility frontier) on the magnitude of the first derivatives of the h^f and h^m functions. The larger these derivatives are, the more g^m (and the consumption components of the husband) will respond. The response is zero if the derivatives are zero and increases without bound when the derivatives jointly go to infinity, so the magnitude of the effect of redistribution is not restricted by our assumptions.

Lemma 4 provides justification for the result that households which, due to the REA, changed their annuity selection, could also have increased the life-insurance holdings (so that provision of a given amount of survivor protection in the case of negotiation breakdown can lead to a larger increase of survivor protection in the optimum). With our assumptions, the effect of a small redistribution of the of outside options can have any size effect on the solution, from negligible to huge.

⁴²An alternative way to prove the first part of Proposition 2, that also highlights the structure of the model considered here, is the following. Since the utility functions of the spouses are not interdependent and all the solutions considered are Pareto-efficient, we can consider the solution as a two-stage procedure, where in stage 1 the wealth is distributed within the family and in stage 2 both spouses make independently their consumption and investment decisions. Now the effect of the REA can be seen as a transfer of income to the wife. Given the additive separable structure of the individual utility functions, all the components of the individual demands are normal (increasing in wealth). Hence all the husband's demand components decrease (since husband gets less money) and all the wife's demand components increase.

However, there exists a special case where we can derive stronger result than the ambiguity result of Lemma 4. Let g^m , g^f , h^m and h^f still be the utility levels, but now expressed as indirect utilities over wealth that the spouses command in the solution of the bargaining and in the negotiation-breakdown-case. Furthermore assume that $h^m(W^m) = g^m(W^m) - a^m$ and $h^f(W^f) = g^f(W^f) - a^f$, where a^m and a^f are just additive constant (additive utility of being in the cooperative marriage).⁴³ Now it is a relatively easy calculation to show that if $g^m(W^m) = W^m$ and $g^f(W^f) = W^f$, then a redistribution of \$1 in the outside option case leads to a redistribution of \$1 in the solution. Furthermore, since all demand components are normal, it leads to a change in every demand component that is less than \$1. Thus under this special case, the REA should decrease life insurance holdings for the households that change their annuity selection.

⁴³The assumption that indirect utility function could be expressed as simply the wealth level is, as long as we don't consider price changes, only an assumption about the right cardinalization of the preferences for the Nash-bargaining solution and not an assumption about the underlying preference structure over the different consumption goods.

Table 5: Probability that husband's pension provides survivor benefits
CPS 89 December Supplement

Start year of pension	Raw series	Regression adjusted			
79	65%	65%	65%	64%	64%
80	74%	75%	74%	73%	73%
81	67%	67%	66%	66%	66%
82	66%	67%	66%	66%	66%
83	71%	72%	70%	69%	69%
84	71%	71%	71%	71%	71%
85	68%	68%	67%	67%	67%
86	72%	73%	72%	72%	72%
87	76%	77%	75%	74%	74%
88	84%	85%	83%	84%	84%
89	81%	82%	79%	79%	79%
Age dummies	No	Yes	Yes	Yes	Yes
Race	No	No	Yes	Yes	Yes
Education	No	No	Yes	Yes	Yes
Income variables	No	No	No	No	Yes

Table 6: Results on probability that husband's pension provides survivor benefits (CPS 89 December Supplement)

1985 or after	0.069 (0.025)**	0.071 (0.025)**	0.070 (0.025)**	0.063 (0.025)*
Husband black		0.027 (0.060)	0.042 (0.058)	0.046 (0.059)
Husband hispanic		-0.012 (0.081)	0.017 (0.084)	0.014 (0.084)
High-school husband			-0.022 (0.058)	-0.018 (0.057)
High-school wife			0.012 (0.046)	0.023 (0.046)
High-school both			0.036 (0.068)	0.033 (0.068)
College husband			0.118 (0.039)**	0.096 (0.039)*
College wife			0.088 (0.064)	0.106 (0.063)
College both			-0.102 (0.086)	-0.110 (0.085)
Wife has pension				-0.081 (0.038)*
Age controls	No	Yes	Yes	Yes
Income variables	No	No	No	Yes
N	1540	1540	1540	1540
R2	0.0060	0.026	0.0409	0.0581

N.B ** and * indicate significance of coefficient at 5% and 1% confidence levels. Below the coefficient is its estimated standard error of the coefficient.

Table 7: Results on the probability that husband's pension provides survivor benefits
From HRS-AHEAD

	Comparison between			Crossection in 1998		
	Crossection 1993 and 1998			Crossection in 1998		
Husband young	0.048 (0.038)	0.082 (0.043)	0.077 (0.047)	0.047 (0.044)	0.091 (0.048)	0.101 (0.050)*
Husband black		0.081 (0.060)	0.105 (0.062)		0.051 (0.063)	0.073 (0.065)
Husband hispanic		-0.040 (0.097)	-0.046 (0.097)		-0.122 (0.103)	-0.122 (0.103)
High-school husband		0.072 (0.053)	0.076 (0.054)		0.121 (0.053)*	0.120 (0.055)*
High-school wife		0.142 (0.057)*	0.162 (0.056)**		0.097 (0.060)	0.111 (0.060)
High-school both		-0.113 (0.074)	-0.136 (0.074)		-0.076 (0.079)	-0.089 (0.079)
College husband		0.079 (0.052)	0.077 (0.053)		0.014 (0.055)	0.012 (0.056)
College wife		-0.081 (0.075)	-0.077 (0.076)		-0.080 (0.078)	-0.073 (0.081)
College both		0.086 (0.091)	0.094 (0.091)		0.141 (0.094)	0.144 (0.096)
Wife has pension			-0.101 (0.039)*			-0.082 (0.040)*
Age controls	No	Yes	Yes	No	Yes	Yes
Income variables	No	No	Yes	No	No	Yes
N	1001	1001	999	913	913	911
R2	0.0021	0.0403	0.0579	0.0014	0.0541	0.0707

Probability that husband's pension provides survivor benefits			
Age in 1985			
54-61	69.1%		
66-69	64.3%	from 1993 data	
66-69	64.5%	from 1998 data	

Table 8: Results on life insurance holding probability (HRS-AHEAD)

	Across 1993 and 1998 crossections				1998 Cross section comparison			
	OLS D	OLS DD	OLS DD	OLS DD	OLS D	OLS DD	OLS DD	OLS DD
Husband young	0.081 (0.032)*	0.111 (0.041)**	n.a.	n.a.	0.036 (0.035)	0.057 (0.047)	n.a.	n.a.
Husband has pension		0.156 (0.047)**	0.129 (0.046)**	0.126 (0.047)**		0.147 (0.054)**	0.128 (0.054)*	0.116 (0.056)*
Young*Pension		-0.030 (0.052)	-0.018 (0.051)	-0.020 (0.052)		-0.021 (0.059)	-0.014 (0.058)	-0.007 (0.060)
Husband black			-0.005 (0.042)	0.008 (0.042)			-0.025 (0.044)	-0.010 (0.044)
Husband hispanic			-0.398 (0.048)**	-0.391 (0.049)**			-0.359 (0.054)**	-0.343 (0.055)**
High-school husband			0.048 (0.034)	0.042 (0.035)			0.048 (0.035)	0.038 (0.036)
High-school wife			0.044 (0.036)	0.048 (0.036)			0.093 (0.034)**	0.093 (0.034)**
High-school both			-0.020 (0.048)	-0.033 (0.048)			-0.068 (0.047)	-0.077 (0.048)
College husband			0.037 (0.035)	0.034 (0.036)			0.029 (0.036)	0.031 (0.037)
College wife			-0.005 (0.044)	0.003 (0.045)			-0.016 (0.042)	-0.009 (0.044)
College both			-0.028 (0.058)	-0.035 (0.059)			-0.047 (0.058)	-0.057 (0.059)
Wife has pension				-0.018 (0.026)				-0.004 (0.025)
Age indicators	No	No	Yes	Yes	No	No	Yes	Yes
Age difference controls	No	No	Yes	Yes	No	No	Yes	Yes
Income variables	No	No	No	Yes	No	No	No	Yes
N	1039	1952	1951	1937	951	1775	1775	1761
R2	0.0088	0.0347	0.0768	0.0867	0.0014	0.0258	0.0678	0.0759

Across 1993 and 1998 crossections

% Insured:	Young	Old
Pension	84.23%	76.15%
No Pension	71.64%	60.58%

1998 Cross section comparison

% Insured:	Young	Old
Pension	84.23%	80.63%
No Pension	71.64%	65.98%

Table 9: Life insurance amount results using 1993 and 1998 crosssections (HRS-AHEAD)

	OLS		OLS		OLS		Median reg		Median reg		Robust Reg		Robust Reg		Robust Reg	
	D	DD	DD	DD	D	DD	DD	DD	DD	D	DD	DD	DD	DD	DD	DD
Husband young	37,185.462 (5,905.807)**	24,956.330 (9,661.471)**	n.a.	n.a.	15,659.852 (841.741)**	10,773.234 (959.363)**	n.a.	n.a.	12,031.057 (1,581.502)**	3,739.170 (1,148.021)**	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.
Husband has pension			-14,727.409 (8,523.627)	-12,816.467 (8,825.748)		2,000.000 (1,118.612)	-39.188 (2,129.841)	-661.479 (1,107.338)		1,647.565 (1,350.176)	268.599 (1,570.093)	268.599 (1,570.093)	-153.718 (1,637.515)			
Young*Pension			12,229.132 (11,323.758)	14,155.032 (11,130.498)		4,886.617 (1,282.542)**	6,237.144 (2,444.490)*	4,845.562 (1,272.804)**		3,684.131 (1,562.734)*	4,421.906 (1,813.430)*	4,421.906 (1,813.430)*	4,383.408 (1,890.033)*			
Husband black			-9,363.551 (5,012.271)	-4,046.618 (5,147.707)			-2,039.188 (2,067.775)	-178.680 (1,087.196)					-1,975.167 (1,494.719)			
Husband hispanic			-21,884.450 (8,611.852)*	-14,158.959 (9,052.596)			-7,968.092 (2,189.340)**	-7,831.364 (1,182.851)**					-7,037.358 (1,685.539)**			
High-school husband			3,129.930 (5,971.746)	-3,331.329 (6,702.440)			5,928.904 (1,803.552)**	4,161.788 (945.250)**					5,136.367 (1,373.905)**			5,283.787 (1,430.461)**
High-school wife			13,432.459 (10,238.713)	10,139.216 (10,876.410)			2,925.263 (1,856.171)	3,084.158 (961.626)**					2,768.627 (1,374.732)*			3,417.458 (1,442.147)*
High-school both			3,169.180 (11,344.515)	1,980.287 (11,403.460)			-3,814.979 (2,490.935)	-1,959.553 (1,284.063)					-4,760.755 (1,863.863)*			-5,931.171 (1,940.228)**
College husband			28,442.115 (8,692.671)**	25,321.739 (8,985.464)**			11,659.309 (1,902.612)**	9,822.155 (992.255)**					3,977.370 (1,430.883)**			3,733.811 (1,489.322)*
College wife			-838.795 (10,192.748)	3,018.680 (9,649.425)			-1,149.473 (2,330.926)	-1,563.503 (1,217.864)					-1,430.924 (1,723.367)			-1,556.881 (1,818.470)
College both			18,275.738 (19,346.756)	10,059.038 (18,145.503)			4,029.352 (3,164.895)	2,746.726 (1,639.574)					515.159 (2,354.479)			674.868 (2,449.064)
Wife has pension			-12,774.010 (5,716.142)*	0.1348			84.218 (715.171)						-223.803 (1,097.241)			
Age indicators	No	No	Yes	Yes	No	No	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes
Age difference controls	No	No	Yes	Yes	No	No	Yes	Yes	No	No	No	No	Yes	Yes	Yes	Yes
Income variables	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No	No
N	912	1714	1713	1700	912	1714	1713	1700	912	1714	1714	1713	1700	1713	1700	1700
R2	0.0281	0.0215	0.0967	0.1348												

Insurance holdings:		Young	Old
(Average)	Pension	52374.449	15188.989
	No Pension	50041.19	25084.86

Insurance holdings:		Young	Old
(median)	Pension	20659.847	5000
	No Pension	13773.23	3000

Table 10: Life insurance amount results using 1998 crosssection (HRS)

	OLS D	OLS DD	OLS DD	OLS DD	OLS DD	Median reg DD	Median reg DD	Median reg DD	Median reg DD	Robust reg D	Robust reg DD	Robust reg DD	Robust reg DD	Robust reg DD
Husband young	28,202.546 (6,760.103)**	-585.300 (17,465.504)	n.a.	n.a.	12,395.911 (2,954,140)**	6,886.617 (1,416.107)**	n.a.	n.a.	n.a.	6,873.738 (2,345.072)**	2,195.847 (1,822.806)	n.a.	n.a.	n.a.
Husband has pension														
Young*Pension														
Husband black														
Husband hispanic														
High-school husband														
High-school wife														
High-school both														
College husband														
College wife														
College both														
Wife has pension														
Age indicators	No	No	Yes	Yes	No	No	Yes	Yes	Yes	No	No	Yes	Yes	Yes
Age difference controls	No	No	Yes	Yes	No	No	Yes	Yes	Yes	No	No	Yes	Yes	Yes
Income variables	No	No	No	No	No	No	No	No	No	No	No	No	No	Yes
N	814	1545	1545	1545	814	1545	1545	1545	1545	814	1545	1545	1545	1532
R2	0.01	0.005	0.0753	0.1325										

	Young	Old
Insurance holdings: (Average)	52374.4498	24171.9
Pension	50041.1898	50626.49
No Pension		

	Young	Old
Insurance holdings: (median)	20659.851	8263.94
	13773.234	6886.617

Figure 1: Probability of private, union or federal pension receipt for married men as function of age

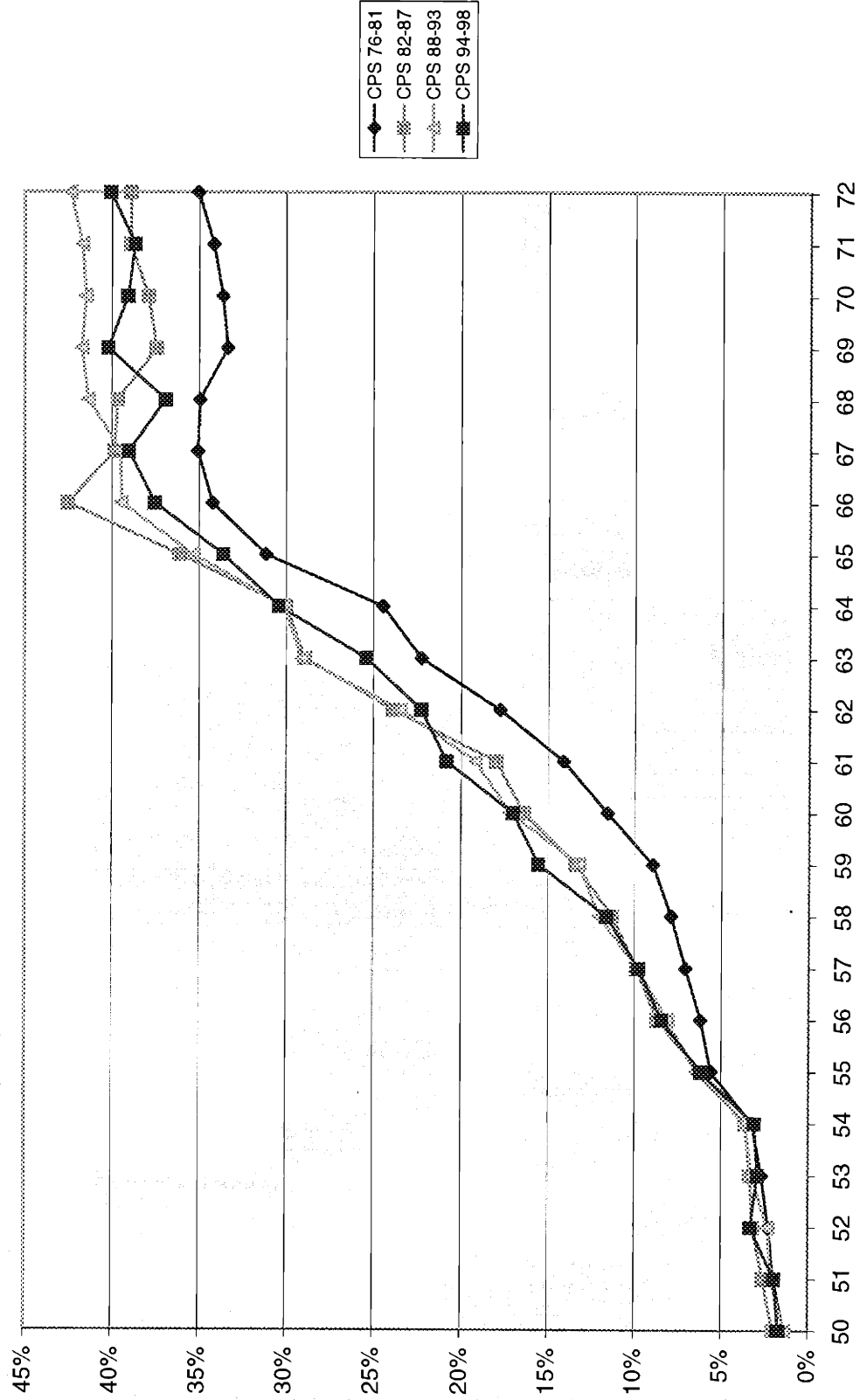


Figure 2: Percentage of male TIAA-CREF Participants choosing survivor annuities as a function of the year when started receiving benefits

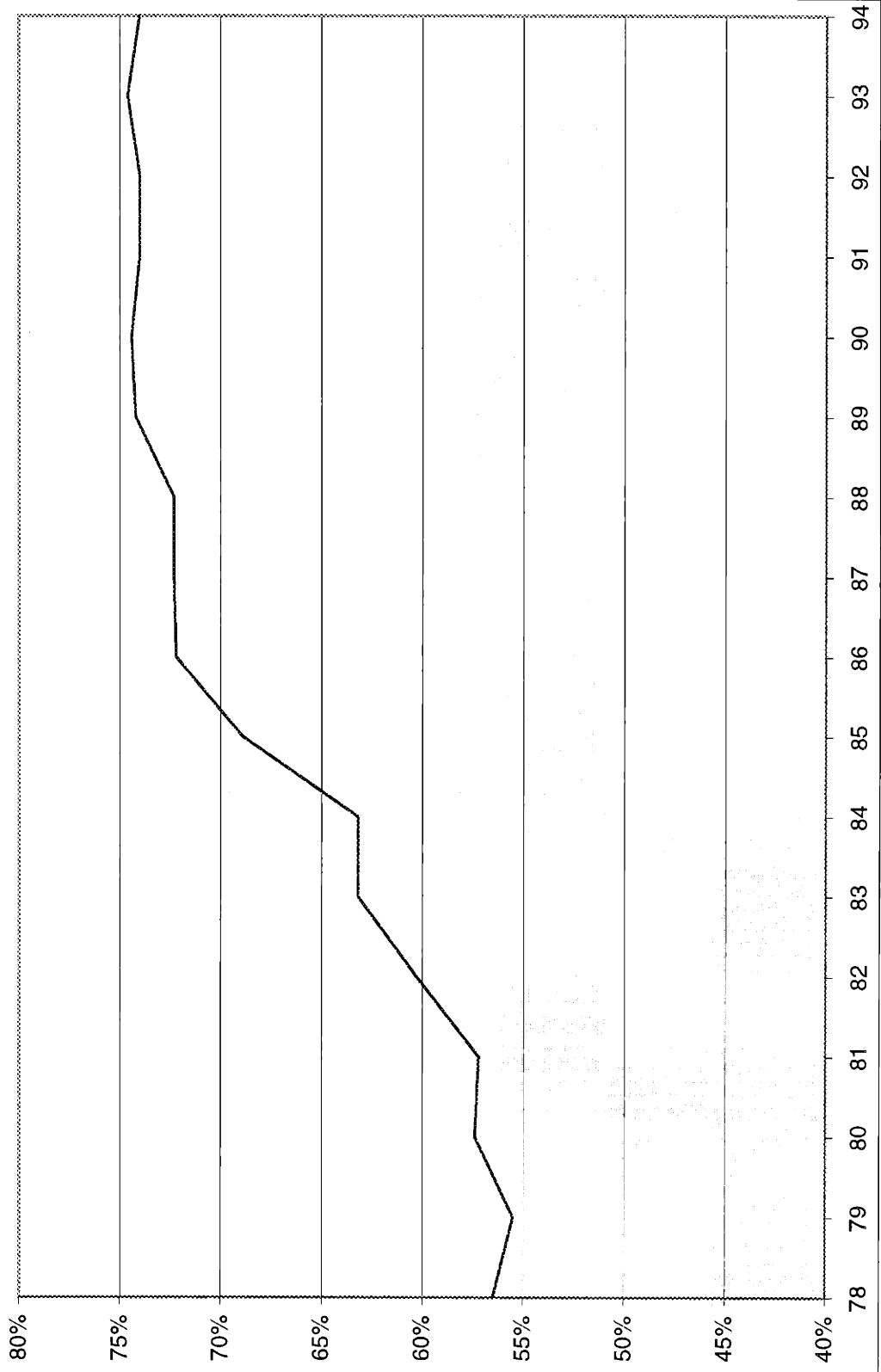
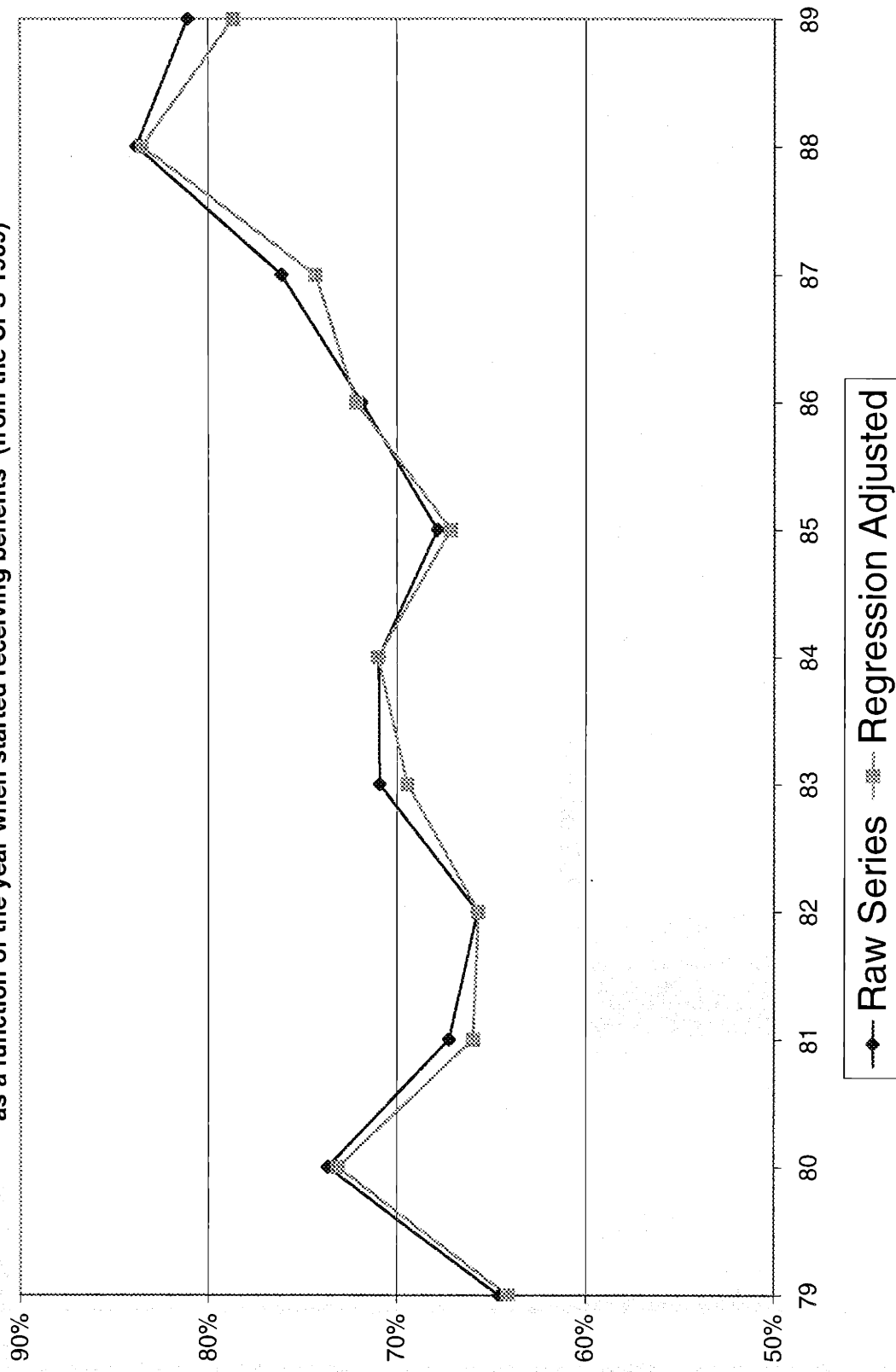


Figure 3: Percentage of married man choosing survivor annuities as a function of the year when started receiving benefits (from the CPS 1989)



Chapter 2

Uncommitted Couples: Some Efficiency and Policy Implications of Marital Bargaining

2.1 Introduction

This paper studies the implications of inability to commit across time for economic efficiency and decision-making. While there exists a vast literature on the effects of the inability to commit to a given solution in many applications (including investment decisions and monetary policy), the implications of this imperfectness have not been studied much in the context of a household. This paper asks how the fact that a (married) couple cannot necessarily credibly commit to a future consumption division affects the decision made today.

The setup of this paper is simple. A couple needs to decide today on how to divide current consumption between spouses and how much to save. They cannot credibly agree on how to divide consumption in the future, since they cannot commit not to renegotiate the current agreement in the future. Furthermore, the "balance of power" in the family might be different in the future, so one spouse might have comparative advantage in the family now that will decay in the future. This potential disparity of "balance of power" is shown to lead to an economic

inefficiency within the household since the spouses cannot complete Pareto-improving trades between themselves across periods due to lack of commitment.

This paper remains mostly agnostic about the sources of the difference of "balance of power" across time periods by not explicitly modelling the phenomenon in most cases. It is only assumed that for some reason, the relative "welfare weight"¹ in the household's objective function varies across time periods. The reason why current objective function is not necessarily aligned with the future objective can be justified e.g. by assuming that the spouses engage in period-by-period bargaining. If, loosely speaking, the relative outside options of the spouses differ from period to period, then welfare weights will differ too. Reasons for nonconstant relative outside options abound: dynamic effects of labor force attachment of spouses (through human capital acquisition), spouse specific educational investments, the details divorce laws, remarriage prospects, health shocks² and time-varying differing relative attachment to the community outside the family.

A major exception to the agnostic attitude toward the "balance of power" phenomenon is taken in the section dealing with divorce. This paper makes two contributions to theory of divorce: First it shows how divorce rules can have an effect on savings for families that stay married through their effect on the marital bargaining. In a tractable repeated Nash-bargaining model (a special case of the general model), this effect can be positive or negative depending on the elasticity of intertemporal substitution of the utility functions. This effect is completely separate from the traditional "insurance against bad outcome" effect of divorce on savings considered in Cubeddu and Ríos-Rull (1997).

The second contribution to the theory of divorce is efficiency comparison between different divorce regimes. It is shown in the section 4 that a stylized common-law property regime attains full efficiency under special conditions while the community property regime is unlikely to lead to full efficiency. Thus the choice between common-law and community property regimes involves potentially an equity-efficiency trade-off.

¹This corresponds to the ratios of marginal utilities of consumption on a given period between spouses in the optimum solution.

²This paper, for most part, does not deal with uncertainty, which is an integral part of health shocks.

This paper is a part of the growing literature of models of family that models the family not as a single aligned entity, that can be modelled as if it were an individual agent, but as group of agents whose preferences are not necessarily completely aligned. While this line of inquiry goes back at least to Becker (1973), the papers that initiated the more recent interest in this research topic are papers on Nash bargaining models of family (Manser and Brown 1980, McElroy and Horney 1981) and papers on the efficient contracting models of family (starting from Chiappori 1988). This paper can be seen as a simple extension of the framework set out in the series of papers by Chiappori and his coauthors into dynamic setting. Mazzocco (2000) extends similar ideas as this paper, although its focus is different. Mazzocco's analysis of risk is more general than in this paper, but he restricts his analysis of investment decisions into cases where one household member holds all the property rights on the assets.

There are two separate branches of literature that are closely related to this paper. First, a new and a small branch is the one which this paper is also belongs to: the literature on savings decisions and family bargaining. Three papers are worth mentioning here: Browning (2000), Lundberg and Ward-Batts (2000), and Lundberg, Startz and Stillman (2001). Browning (2001) presents a two-period game-theoretic model, where he shows that under certain assumption, the non-cooperatively made savings and investment decisions yield full Pareto efficiency. Papers by Lundberg and coauthors on the other hand are mostly empirical papers trying to shed light on retirement related issues (like the drop in consumption at retirement) using household bargaining logic and models relatively similar to, although simpler than, the ones presented in this paper.

The other related literature is the growing literature on informal insurance arrangements. Two examples of this are Attanasio and Ríos-Rull (2000) and Ligon, Thomas and Worrall (2000). This literature emphasizes the same issue as this paper: the inability to commit across time (and states of the world). However, since most of this literature is on agricultural communities in developing countries, they do not see savings as the most interesting aspect (since the crop cannot be stored indefinitely) and emphasize risk-sharing almost solely. Ligon et al. do consider savings, but while coming close to the results of the first part of this paper, they do not emphasize the role of savings in their model.

The paper is organized as follows: Section 2 presents the basic model. Section 3 analyzes the effect of lack of commitment on the households savings level. Section 4 analyzes efficiency properties of different divorce regimes. Section 5 considers the problem of life-insurance protection for surviving spouses. Section 6 concludes.

2.2 The basic model and results

Can a married couple commit to a given consumption path and sharing rule across time? One reason to think that the answer to this question might be negative is that the outside options (outcomes, should they divorce) of the spouses might evolve in time. This might make the agreement based on yesterday's balance of power unsustainable today. If this is the case, then in today's decision making the couple has to take into account the effect of today's choices on future decisions. This dynamic linkage, through the fact that future behavior is constrained by the outcome of future renegotiation process and the fact that today's choices affect this process, is the central attention of the study undertaken in this paper. In order to proceed, this paper makes three further assumptions on the decision making process.

First, the within-period decision-making is assumed to Pareto-efficient with respect to the constraints. This can be seen either as a simple modelling choice or as a statement about possible interactions within a couple. If we assume that no motives of malice guide the decision-making (and even these could be incorporated into the utility functions) it is hard to believe that a partner in a couple would turn down suggested change that would make both partners better off while fully taking into account the constraints of the problem. At least as long as the partners are sufficiently patient and the random divorce risk³ is sufficiently low, it is a justifiable working

³The simplistic view on divorce taken in this paper is the following. Each period two states of the world are possible: under the normal circumstances, the couple continues together if their utility from sticking together under the cooperative regime is higher than in the case of divorce. However, in each period, there is a remote chance of a random shock that irrespective of economic variables makes the couple incompatible with each other (say one partner is caught cheating). The latter (an unmodelled phenomenon) is what is called the risk of divorce in this paper. Also, for most of the paper the break-up of relationship is interchangeable with divorce, since unless different legal environments are explicitly modelled, it does not matter whether the couple is legally married or not for the decision making problem.

assumption⁴ to assume that the couple can overcome prisoner's dilemma type problems that they might face.

The second, related assumption, is a very strong assumption on rationality. In the models that follow, it is assumed that in making decisions about savings and investment (these could broadly be viewed to include decisions about human capital investment) the strategic element of each of these decisions in the future is fully understood and incorporated in the decision making. By the structure of the models that follow (say repeated Nash-bargaining), this means that the couple is able to understand the very complicated dynamic effects of current period decisions.

Third important assumption is that of full information. While many of the results presented here are or could be extended to one type of uncertainty (exogenous risks) the issues relating to asymmetric information (hidden knowledge or hidden actions) within marriage are not considered. This can be viewed as an important caveat in modelling families, since it might be argued that one of the spouses often has informational advantage over families finances (the one who takes care of the day-to-day finances). This possible extension is left to further research.

To illustrate the decision-making problem that the couple⁵ faces, when it cannot commit to a time-consistent solution, consider the following problem. Let the world last for T -periods. Assume that there is no uncertainty and that the only decision the couple faces at period t is how much to save and how to divide the current consumption between spouses. Assume that the life-time utility functions of respective spouses are defined as time-separable utilities over their own consumption only:⁶

⁴Formally justifiable by Folk-Theorem type arguments.

⁵The assumption that there are exactly two members of the family whose utility functions are relevant for the decision making will matter for the results that follow. So this paper's results do not necessarily extend to families with teenage or grown-up children or other family members participating in the decision process. The key is that children can be included through the effect of their consumption through parent's utility functions, but not as someone exerting any power in the decision-making.

⁶So there are no externalities nor altruistic motives. These (completely related) extensions are considered later.

$$U^m = \sum_{t=1}^T u_t^m(c_t^m)$$

and $U^f = \sum_{t=1}^T u_t^f(c_t^f),$

where superscripts m and f refer to husband and wife respectively.

The inability to commit means that future behavior is taken into account as a constraint on the constrained Pareto-efficient decision program at every period. Starting from last period this means that at period T the problem that couple solves is:⁷

$$V_T^m(A_T) = \max_{c_T^m, c_T^f} u_T^m(c_T^m)$$

subject to $A_T = c_T^m + c_T^f$

and $u_T^f(c_T^f) = V_T^f(A_T),$

for some function $V_T^f(A_T)$ that represent wife's period specific utility level at time T in the optimum.⁸ The $V_T^m(A_T)$ and $V_T^f(A_T)$ functions are reduced form representations of the household decision-making process, representing the point in the utility possibility frontier that the household will choose. A leading example of structural form representation that could be characterized by these functions is Nash-bargaining between spouses (being a member of the class of Pareto efficient decision-making processes).

Working backwards, at period $T - 1$ the household now has three choice variables, current consumption of respective spouses and the assets level at period T . The inability to commit

⁷As long as we assume that there is only one private consumption good the period T maximization is trivial.

⁸This is nothing more than the usual characterization of Pareto-efficient choice, except that for generality it is assumed that the wife's utility level is a function of the wealth holdings of the household at period T . The dependence of V_T^f on the wealth holding makes it possible to characterize general efficient forms of household decision-making (like Nash-bargaining or social welfare function maximization) where an increase of wealth available in the period T will in general have effect on the utilities and consumptions of both spouses.

across time is captured by the fact that they cannot contract on the respective consumption levels of spouses at period T . Instead, they have to take the decision process in the period T as a constraint while making decisions at time $T - 1$. One possible way to characterize this constrained decision problem is:

$$\begin{aligned} V_{T-1}^m(A_{T-1}) &= \max_{c_{T-1}^m, c_{T-1}^f, A_T} u_{T-1}^m(c_{T-1}^m) + V_T^m(A_T) \\ \text{subject to } A_{T-1} &= c_{T-1}^m + c_{T-1}^f + \frac{A_T}{(1+r_{T-1})} \\ \text{and } u_{T-1}^f(c_{T-1}^f) &= V_{T-1}^f(A_{T-1}) - V_T^f(A_T), \end{aligned}$$

where, as earlier, $V_{T-1}^f(A_{T-1})$ represents the sum of the wife's period-specific utilities at period $T - 1$ and T in the optimum. The functions $V_{T-1}^m(A_{T-1})$ and $V_{T-1}^f(A_{T-1})$ give us a reduced form representation of the decision making process without commitment.

Using backward induction this leads to following characterization of the problem:

$$\begin{aligned} V_t^m(A_t) &= \max_{c_t^m, c_t^f, A_{t+1}} u_t^m(c_t^m) + V_{t+1}^m(A_{t+1}) \\ \text{subject to } A_t &= c_t^m + c_t^f + \frac{A_{t+1}}{(1+r_t)} \\ \text{and } u_t^f(c_t^f) &= V_t^f(A_t) - V_{t+1}^f(A_{t+1}). \end{aligned}$$

The first constraint is the usual budget constraint, where A_t is the remaining life-time wealth of the couple at period t .⁹ The second constraint is the usual Pareto-efficiency requirement altered to take into account that the couple is constrained by the process characterizing their decision-making in the future and cannot commit to (potentially better) future consumption allocations that are incompatible with that process.¹⁰ Naturally, a completely equivalent

⁹All the analysis in this paper would go through when A_t is construed as the net wealth of the couple at time t , and in each period each partners gets an additional amount (possible negative) of income I_t^m and I_t^f respectively. While completely equivalent characterizations of the dynamic budget constraint in the current environment, in the extensions this allows for the respective I_t -processes to be contingent on the divorce or widowhood states. Thus the results do extend to a more general specification of the household's state-contingent budget constraint.

¹⁰This is a very reduced form characterization of the decision-making process. A more specific structural model

characterization of the process is the maximization of the wife's life-time utility subject to constraint on the husbands utility level. This interchangeability will become useful in providing short proofs for the theorems.

A key assumption in the definition of V_t -functions is that they are functions of the current assets holdings only. Thus, while this will be extended in one direction to handle multiple assets, it precludes complicated dynamic dependencies from past actions. This assumption can be defended on tractability grounds and by its intuitive appeal.¹¹

It is important to understand that the above is meant to be a characterization of the optimum in a same way as Pareto-frontier characterizes possible optima in a usual exchange economy setting. This means that while we can study properties of the optimum using this characterization, we cannot derive comparative statics with respect to changes of the economic parameters without specifying the structural process by which the couple arrives into the solution.¹² Thus, without further specification of the process we can study types of inefficiencies that can arise and if we find policy intervention that can lead to a first-best solution, we can claim that it will make at least one member of the couple better off. However, without getting into the black box of family decision making, the approach followed in this section cannot be used to say anything about within family redistributive effects. This is an important qualification, since some of efficiency-enhancing policy recommendations of this paper might have huge distributive impacts (like the desirability of common-law divorce asset division regime over community property regime purely on efficiency grounds).

To characterize the efficiency properties and to describe the nature of inefficiencies that can arise because of the lack of commitment across time periods, the following assumption will be made.

of repeated Nash-bargaining (a special case of this general model) is presented later.

¹¹However, this excludes the possibility that the assets division in case of divorce might be affected by e.g. a spending spree prior to divorce by one of the spouses.

¹²A reduced-form way of doing this would be to parametrize the V_t^f -function family.

An analogy of this into the exchange economy setting is that while we can study the properties of the Pareto-frontier, in order to say how the equilibrium changes with respect to the outside parameters, we need to make additional assumptions (like price-taking and utility maximization) to have predictions about the effects of parameter changes.

Assumption 1 (More wealth is better for both in every period) $V_t^{m'} \geq 0$ and $V_t^{f'} \geq 0$ for all time periods in the optimum solution.

The justification for assumption 1 is that it bounds the bargaining effect of wealth to not dominate the intuitive effect of more wealth (the expansion of the budget set in the future effect dominates the bargaining effect of more wealth). A problem with assumption 1 is that it is in terms of both $V_t^{m'}$ and $V_t^{f'}$. While intuitively appealing, this is not completely satisfactory, since only one of the V_t -functions should be taken as fundamental of the problem (i.e. reduced form presentation of the bargaining process), the other being a quantity derived from the optimization solution. While the author feels that assumption 1 is a reasonable restriction on the class of admissible models under many circumstances, this might not be satisfactory to all the readers. Therefore it is also necessary to give fundamental sufficient conditions that would yield Assumption 1. A set of alternative conditions guaranteeing that assumption 1 holds is given as lemma 0.

Lemma 0. Any one of the following assumptions is sufficient for Assumption 1 to be satisfied:

- a) Value functions are independent of wealth for one of the spouses: $V_t^{f'} = 0 \forall t, A_t$ or $V_t^m = 0 \forall t, A_t$.
- b) Two period world and wealth good for both in second period: let $T = 2$ and let the second period solution be characterized by a sharing rule for wealth: $c_2^m = \psi_2(A_2)$ and $c_2^f = A_2 - \psi_2(A_2)$. Furthermore let $0 \leq \psi_2 \leq 1$.
- c) T-period world, with last period described as in b) and letting the series of functions $V_t^f(A_t)$ satisfy following conditions: $V_t^{f'} \geq 0, V_t^{f''} \leq 0 \forall t, A_t$ and $(1 + r_t)V_{t+1}^{f'}(A) \geq V_t^{f'}(A) \forall t, A$.
- d) Like c), but applying the restrictions on $V_t^m(A)$ -functions.
- e) CRRA-utilities and outside options in repeated Nash-bargaining: Let Assumptions 2-5 hold of the section 3 hold.

f) $V_t^{m'} \geq 0$ and $V_t^{m'} \leq u_t^{m'} \forall t$ in the solution; or equivalently $V_t^{f'} \geq 0$ and $V_t^{f'} \leq u_t^{f'} \forall t$ in the solution.

Proof: See Appendix.

The point of Lemma 0 is to illustrate that Assumption 1 covers a large class of interesting problems. Some of these alternative conditions need further commenting: first f) is nothing more than restatements of Assumption 1 using envelope theorem to derive more interpretable conditions. It suffers from the same weakness as Assumption 1, it refers to quantities (marginal utilities of consumption) that are defined in the optimum.

Condition a) provides an interesting special case, where one of the spouses, say the husband, has constant outside options. This means, that in the optimum, the wife will attain full efficiency in her consumption plan even though the husband's consumption plan might be distorted.

Condition e) provides an example of a structural model that satisfies the Assumption 1.

Conditions b), c) and d) are the most fundamental. Condition b) uses two-stage budgeting in the last period as the starting point (the sharing rule of assets). This can be viewed as fundamental description of the bargaining and not as an outcome.¹³ Conditions c) and d) extend this with relatively strong assumptions into T-period setting. However, the wide class of problems that will satisfy Assumption 1 will become evident from the proof of condition c). From that proof we can see that condition c) is just a very strong sufficient condition that can be violated while Assumption 1 still holds.¹⁴ Thus Assumption 1 is rather general.

Definition 1. (Undersaving and oversaving) Define the situation where $u_t^{m'} < (1 + r_t)u_{t+1}^{m'}$ and $u_t^{f'} < (1 + r_t)u_{t+1}^{f'}$ as undersaving and situation where both inequalities are reversed as oversaving.

¹³Unfortunately this two-stage budgeting does not extend to any other than last period. In all other periods there are three goods, one of which is a public good (assets in the next period).

¹⁴Condition c) is too strong in two senses. First, it requires that the inequality holds for all values of possible values of A . However, this is only done to avoid a (circular) reference to quantities relating to the optimum. Another sufficient condition is that $(1 + r_t)V_{t+1}^{f'}(A_{t+1}) \geq V_t^{f'}(A_t)$, where A_{t+1} and A_t are the quantities chosen in the optimum (this form is implied by the for all values of A condition, concavity of $V_t^{f'}$ and by the fact that $A_t \geq A_{t+1}$). Even this is not necessary, since this inequality condition is also just a sufficient condition, that may not be satisfied while Assumption 1 still holds.

The justification for the definition of undersaving is that it defines a situation where a both spouses would prefer to transfer current consumption into next period consumption.¹⁵ It is noteworthy, that the definition here applies to consecutive time periods only.

Theorem 1. Let assumption 1 be satisfied. In the optimum there cannot be undersaving nor oversaving.¹⁶

Proof: Let μ_t and λ_t be the Lagrange-multipliers for the budget constraint and the wife's utility level constraint on the period t -suboptimization. The first order conditions for the optimization are:

$$\begin{aligned} u_t^{m'} - \mu_t &= 0 \\ \lambda_t u_t^{f'} - \mu_t &= 0 \\ V_{t+1}^{m'} - \frac{\mu_t}{1+r_t} + \lambda_t V_{t+1}^{f'} &= 0. \end{aligned}$$

Using the envelope theorem and the first order condition with respect husband's consumption the intertemporal first order condition can manipulated as:

¹⁵Note that the inequalities in Definition 1 are strict inequalities, so the requirement is that an increase in savings would be a strict Pareto-improvement. A situation, where one of the inequalities in Definition 1 holds with equality is not defined to be under- or oversavings since no series of intrapersonal intertemporal trades can lead to a strict Pareto-improvement. An interpersonal intertemporal trade would lead to a strict Pareto improvement (by lowering consumption of the spouse whose decision are distorted in the low marginal utility period and increasing the consumption in the high marginal utility period and increasing consumption of the other spouse).

However, the situation, where one spouse satisfies the dynamic efficiency condition on marginal utilities with equality and the other does not can only arise in the optimum if one of the next period value function for one of the spouses is independent of wealth. Outside this limiting case the distinction on whether series of intrapersonal intertemporal trades could lead to a strict or a weak Pareto-improvement is irrelevant.

¹⁶Note that the non-negativity constraints on consumptions are ignored in the specification. While this is mostly to save on notation, it is not completely without loss of generality. An assumption that would guarantee an interior optimum with respect to consumption is that $\lim_{c_t^q \rightarrow 0} u_t^{q'}(c_t^q) = \infty \forall t$ where $q \in \{m, f\}$. Otherwise Theorem 1 might not hold for if the non-negativity constraint on consumption are imposed and at least one of these constraints is binding in the optimum.

$$\begin{aligned}\mu_{t+1} - \lambda_{t+1}V_{t+1}^{f'} - \frac{\mu_t}{1+r_t} + \lambda_t V_{t+1}^{f'} &= 0 \Leftrightarrow \\ u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} &= (\lambda_{t+1} - \lambda_t) V_{t+1}^{f'}.\end{aligned}$$

By considering the same problem from wife's perspective (and using relations between the Lagrange coefficients in the two problems), the intertemporal first order condition for the wife can be written as:

$$u_{t+1}^{f'} - \frac{u_t^{f'}}{1+r_t} = \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) V_{t+1}^{m'}.$$

By applying the envelope theorem, the dynamic first order conditions can be now written as:

$$\begin{aligned}u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} &= (\lambda_{t+1} - \lambda_t) V_{t+1}^{f'} \\ u_{t+1}^{f'} - \frac{u_t^{f'}}{1+r_t} &= u_{t+1}^{m'} \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) \left(1 - \frac{V_{t+1}^{f'}}{u_{t+1}^{f'}} \right),\end{aligned}$$

or equivalently as

$$\begin{aligned}u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} &= u_{t+1}^{f'} (\lambda_{t+1} - \lambda_t) \left(1 - \frac{V_{t+1}^{f'}}{u_{t+1}^{f'}} \right) \\ u_{t+1}^{f'} - \frac{u_t^{f'}}{1+r_t} &= \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) V_{t+1}^{m'}.\end{aligned}$$

The result is immediate from the above. ■

It is well worth noting, that this result does not carry over to non-consecutive time periods: an counterexample for more general result will be given in the next section. The following are constructed examples of undersaving and oversaving, when Assumption 1 is not satisfied:

Example 1 (Undersaving). Consider a two period world with no discounting. Let both

spouses have identical separable log utility functions over first and second period consumption. Let total lifetime wealth of the couple be 4. Let the outcome of the second period negotiation between spouses be characterized¹⁷ by $c_2^f = 1 - 0.1 * A_2$ and let the first period required utility level for wife be zero (achievable e.g. by consuming 1 each period). In the optimum (approximately) $c_1^m = 1.00$, $c_2^m = .96$, $c_2^f = 1.21$ and $c_1^f = .82$.

Example 2 (Oversaving). Like example 1, but let the outcome of the second period negotiation be characterized by $c_2^f = 2 - 0.1 * A_2$. In the optimum (approximately) $c_1^m = .81$, $c_2^m = .86$, $c_2^f = .57$ and $c_1^f = 1.7$.

Corollary 2. (Multiple goods) Let period t consumption vector be divided into three separate components for each period: c_t^m , c_t^f and c_t^p (husband's consumption, wife's consumption and within household public goods consumption respectively). Let prices of goods within-period be normalized to 1, and let the series of rates of interest be r_t (so there are no good specific interest rates). Theorem 1 holds for between any arbitrary pairs of husband's and wife's private consumption.

Proof of Corollary 2. Similar to theorem 1 with additional notation. ■

Unfortunately, the Euler equation for the household public goods does not yield any interesting economic intuition.

Theorem 1 and its extension would provide an interesting starting point for empirical investigation. Since Theorem 1 is very robust to additions to the model, it could be used either in panel or repeated cross-section setting to test the theory of efficient decision-making without commitment. The data requirement for this kind of exercise are not simple. Like most other test of new models of household in economics, it would require data on consumption items that are assignable to spouses (pure private goods in the household, or at least goods that have arguable much stronger effect on one spouse's utility). This empirical application is left for further research.

¹⁷Equivalently this can stated as $V_2^f(A_2) = \log(1 - 0.1 * A_2)$. This means that $V_2^{f'} < 0$.

2.3 Effect of lack of commitment on the level of savings

Beyond the question of efficiency (i.e. would more or less savings be Pareto-improvement), it is interesting to also ask, whether lack of commitment causes the couple to save more or less than in a world where they could commit from day one to a consumption plan. As explained earlier, this is a question that cannot be asked in the most general framework, since this change in environment will almost always include redistribution effects too. Hence, one needs more specific model to answer this question. The model presented in this section will give one answer to this question: in this model, the effect of inability to commit is similar to the effect of as decrease of the assets return. In the end of this section, it is shown how a small change in the assumptions can be used to generate a completely different answer. This counterexample should not be viewed as nothing more than a proof that the result does not admit arbitrary generalizations, since the model presented in this section still has intuitive appeal as a possible characterization of the household's decision-making process.

This section makes also general point about the feedback of divorce rules to savings behavior. In previous literature (like in Cubeddu and Ríos-Rull 1997) the effect of divorce on the savings behavior is an insurance effect: the savings are positively linked to higher probability of divorce because the marginal utilities of consumption are assumed to be higher in divorce state. This effect could be trivially incorporated to the model of this section, but to save on notation it is omitted. The feedback effect of divorce rules on savings that is presented in this section operates through a different channel: the future divorce property division rules affect bargaining power in the future and this can have (positive or negative) effect on savings in a model where decisions are renegotiated each period.

Assumption 2. (CRRA utilities) Let both spouses have life time utility function that can be written in the time separable CRRA-form:

$$\begin{aligned}
U^m &= \frac{1}{1-\theta^m} \sum_{k=1}^T (\beta^m)^{k-1} (c_k^m)^{1-\theta^m} \\
U^f &= \frac{1}{1-\theta^f} \sum_{k=1}^T (\beta^f)^{k-1} (c_k^f)^{1-\theta^f}
\end{aligned}$$

Assumption 3. (Nash-Bargaining with divorce outside options) Let the household decision making process be repeated Nash-bargaining without commitment across time periods, so the period t decision making process can be characterized as:

$$\begin{aligned}
\max_{c_t^m, c_t^f, A_{t+1}} & \left(\left(\frac{1}{(1-\theta^m)} (c_t^m)^{1-\theta^m} + V_{t+1}^m(A_{t+1}) \right) - \tilde{V}_t^m(A_t) \right) \\
& * \left(\frac{1}{(1-\theta^f)} \left((c_t^f)^{1-\theta^f} + V_{t+1}^f(A_{t+1}) \right) - \tilde{V}_t^f(A_t) \right)
\end{aligned}$$

$$\text{subject to } A_t = c_t^m + c_t^f + \frac{A_{t+1}}{(1+r_t)},$$

where $V_{t+1}^m(A_{t+1})$ and $V_{t+1}^f(A_{t+1})$ are the value functions characterizing the utility value of future periods for a given asset level in period $t+1$ and where $\tilde{V}_t^m(A_t)$ and $\tilde{V}_t^f(A_t)$ characterize the outside options of spouses should the negotiation break down and divorce occur. Thus, the outside option functions incorporate all the relevant information on the institutions (e.g. divorce assets division rules) and environment (e.g. remarriage prospects) relevant to spouses should they divorce. Furthermore, let us assume that once divorce happens, it is final.

Assumption 4. Let the outside option functions be of the form:

$$\begin{aligned}
\tilde{V}_t^q(A_t) &= \max_{\{\tilde{c}_i^q\}_{i=t}^T} \frac{1}{1-\theta^q} \sum_{k=t}^T (\tilde{\beta}^q)^{k-t} (\tau_k^q \tilde{c}_k^q)^{1-\theta^q} \\
\text{subject to } \psi_t^q(A_t) &= \sum_{k=t}^T \frac{\tilde{c}_k^q}{\prod_{j=t}^k (1+r_j)^{j-t}},
\end{aligned}$$

where $q \in \{m, f\}$. Parameters τ_k^q (typically < 1) presents how much utility loss is there in each period from being divorced. Furthermore assume linear sharing rule of property in case of divorce so that

$$\psi_t^q(A_t) = \begin{cases} \alpha_t A_t, & \text{if } q = m \\ (1 - \alpha_t) A_t, & \text{if } q = f \end{cases}$$

Assumption 5. (Identical discount factor and CRRA parameter) Let $\theta^m = \theta^f \equiv \theta$ and $\beta^m = \beta^f \equiv \beta$.

Theorem 3 Let assumptions 2-5 hold. The inability to commit across periods implies higher wealth holdings in every period after initial period (more savings) if $\theta > 1$. For $\theta = 1$ (log utility) the level of savings is unaffected by the inability to commit. For $\theta < 1$ the inability to commit decreases savings in every period.

Proof: In the appendix.

Theorem 3 provides a special case where inability to commit across time-periods acts analogously as decrease of the return on the asset. Thus, when elasticity of intertemporal substitution is higher than 1 (i.e. $\theta < 1$) the inability to commit decreases savings. When elasticity of intertemporal substitution is less than 1 (i.e. $\theta > 1$) the inability to commit increases savings.

How good a guide is Theorem 3 for our intuition about the effects of inability to commit to savings level in more general cases? The following is an artificial counterexample to show that changing just one of the assumptions can turn this conclusion around.

Example 3 (Property rights regimes matter for the savings level). Let both spouses have time separable log utilities with no discounting. Let the lifetime wealth of the couple be 4 and let the interest rate be zero. Let the decision be made through repeated Nash-bargaining. Let the first period outside option be characterized like in assumption 3 (with $\tau_1^m = \tau_1^f = \tau_2^m = \tau_2^f = 0.8$). Let the wife have second period outside option equal to zero ($=\log(1)$, meaning that husband has to provide her with $1/0.8 = 1.25$ in the case of divorce in the second period) and hence the husband's second period outside option is to consume the rest of the assets after providing his wife's after divorce. In the solution, $c_1^m = 0.99$, $c_2^m = 0.96$, $c_1^f = 0.83$ and

$c_2^f = 1.22$. Thus the wealth at the start of the second period is 2.18, which is larger than what solution with commitment would be ($= 2$, since the efficient solutions involve both consuming 1 in each time period).

Examples 4 and 5 use the models of this section to show that Theorem 1 holds only for consecutive periods even if Assumption 1 is satisfied.

Example 4. (Undersaving) Consider three times repeated Nash-bargaining problem as described in this section. Let $\theta^m = \theta^f = .5$, $\alpha^1 = .75$, $\alpha^2 = .65$, $\alpha^3 = .95$ and $\tau_1^m = \tau_1^f = \tau_2^m = \tau_2^f = \tau_3^m = \tau_3^f = .8$. Let the couple have life-time wealth of 6. In the solution, $c_1^m = 1.70$, $c_2^m = .90$, $c_3^m = 1.45$, $c_1^f = .43$, $c_2^f = 1.29$ and $c_3^f = 0.22$. Thus, an increase of consumption in the period 3 for both spouses and a decrease in period 1 consumption for both spouses would be Pareto improving.

Example 5. (Oversaving) Let $\theta^m = \theta^f = 2$, $\alpha^1 = .9$, $\alpha^2 = .8$, $\alpha^3 = .95$ and $\tau_1^m = \tau_1^f = \tau_2^m = \tau_2^f = \tau_3^m = \tau_3^f = .8$. Let the couple have life-time wealth of 6. In the solution, $c_1^m = 1.68$, $c_2^m = 1.11$, $c_3^m = 2.11$, $c_1^f = .20$, $c_2^f = .65$ and $c_3^f = 0.24$.

Example 6 below makes a point that is rather general to the class of models considered in this paper: an increase of one partner's (say wife's) outside option in the future can be bad for both partners by making pre-existing dynamic distortions worse. So while in general redistribution towards wife should be good for her this efficiency effect of redistribution effect can dominate the positive future redistribution effect under some circumstances. This observation has an application to changes in pension legislation: say that the rights of non-working spouses (say wives') on their husband's pension are enhanced (like in the Retirement Equity Act of 1984 analyzed in the first chapter of this thesis) and this legislation becomes as an unexpected shock. The effects on the couples who are at the "retirement" period are straightforward: the wives benefits on their husbands expense. However, the effect of such a legislation can be welfare deteriorating on young couples: it is possible that for them, both are worse off because this future redistribution makes the pre-existing distortions worse. The opposite might also hold for young couples, the legislation might decrease the pre-existing distortions and make both spouses better off.

Example 6. (Future redistribution can be welfare decreasing for both spouses) Consider twice repeated Nash-bargaining model as in this section, with no discounting and log-utilities. Let $\tau_1^m = \tau_1^f = \tau_2^m = \tau_2^f = .8$ and let the life-time wealth of the couple be 4. The table below characterizes the life-time utilities attained by prospective spouses under three different divorce property division arrangement:

Table 1. Life-time utilities of spouses under different property regimes.

α_1	α_2	U^m	U^f
0.5	0.5	0	0
0.5	0.4	-0.0694	0.0289
0.5	0.25	-0.2461	-0.0517

2.4 Divorce

This section continues to consider the case where divorce outcomes define the outside options of the spouses. The question asked in this section is whether the family of models considered here can give strong policy recommendations, i.e. whether changes in divorce laws could be used to restore first-best solutions. The answer to this shown to depend crucially on how one views divorce. If divorce is just an off-equilibrium path event, then the first-best solution can be easily restored. However, adding (even small) additional risk of getting into the divorce state will destroy this conclusion. Perhaps surprisingly it is shown that in general even “divorce insurance”-products could not restore efficiency.

It is interesting to compare the results of this section to the real world existing divorce property division regimes. The two-assets model of this section can be viewed as stylized common-law property regime, where both spouses can own assets while married without them becoming jointly owned. A special case of single assets model is a community property regime where the property accumulated during marriage would be split 50-50 in case of divorce. Generally, community property regime is viewed as more progressive and more “pro-women” or “pro-weaker spouse” (Weitzman 1990). Dnes (1999) argues that a community property regime

is likely to be more efficient, since it provides lesser incentives for costly litigation in case divorce than community property and since, at least in England, the discretion that judges have in common-law property regime creates excess uncertainty.¹⁸

The results of this section are not supportive of Dnes' conclusion (although, they do not consider the effects that Dnes emphasizes). Consider community property regime through the following description of marriage. In the beginning of marriage, the spouses both own some assets. These assets can be covered by prenuptial contract to assign permanent property rights on them. The assets accumulated during marriage¹⁹ are divided 50-50 in case of divorce. Now if the assets are the only thing that matter for relative outside options (like in the examples in the previous sections) there is no reason to expect that the community property regime does not yield the first-best solution in the simple model that has no exogenous divorce risk. However, other factors also can affect the outside options and these factors can lead to a non-constant time path of relative outside options. These factors can include remarriage prospects, attachment to community outside the couple and human capital accumulation that are unlikely to be included (at least perfectly) in the community property valuation. Also, if the community property valuation is not perfectly forward-looking and the pre-marital assets are not divided equally in the prenuptial contract, then this potentially leads to a non-constant path of relative outside options even if the assets are sole factor affecting outside options. In contrast, Theorem 4 states that under a stylized common-law setting the couple can take care off these disparities themselves by trading future property rights to current consumption and attain first-best efficiency.

The point of Theorem 4 is not to say that common-law regime is necessarily more desirable than community property regime. Instead Theorem 4 is meant to highlight the possible efficiency-equity trade-off between these two regimes. Under the common-law regime the couple attains a point in the unrestricted life-time utility possibilities frontier. This point does not

¹⁸In the US community property states are: Arizona, California, Idaho, Louisiana, Nevada, New Mexico, Texas, Washington and Wisconsin. In addition, Puerto Rico is a community property jurisdiction. In UK, Scotland has community property regime, while Wales and England have common-law regime (Dnes 1999). Many continental European countries have community property regime (Dnes 1999).

¹⁹In the US, depending on the State, the asset accumulation that is considered community property can include human capital components like acquired degrees (Weitzman 1990).

necessarily Pareto-dominate the solution under community property. Thus, by forcing the community property rules on the couple the government can possibly attain distributional goals, but since this typically involves a departure from the first-best intertemporal and intrapersonal allocation of consumption within the couple, this possible equity gain comes with an efficiency cost. Whether a common-law regime is more desirable than community property regime therefore depends on the magnitudes of these gains and losses and how these gains and losses are weighted in the social objective.

2.4.1 No-equilibrium path divorce

Consider the environment of Theorem 1 with the following modifications:

Assume that there are more than one asset (without loss of generality, we can assume that there are only two assets), with possibly different rates of return.²⁰ Assume that what drives the power in decision-making is the outcome for the spouses should they divorce. An obvious structural model having this implication is repeated Nash-bargaining with divorce as outside option.

The decision problem can now be written as:

$$\begin{aligned}
 V_t^m(A_t^m, A_t^f) &= \max_{c_t^m, c_t^f, A_{t+1}^m, A_{t+1}^f} u_t^m(c_t^m) + V_{t+1}^m(A_{t+1}^m, A_{t+1}^f) \\
 \text{subject to } A_t^m + A_t^f &= c_t^m + c_t^f + \frac{A_{t+1}^m}{(1+r_t^m)} + \frac{A_{t+1}^f}{(1+r_t^f)} \\
 u_t^f(c_t^f) &= V_t^f(A_t^m, A_t^f) - V_{t+1}^f(A_{t+1}^m, A_{t+1}^f),
 \end{aligned}$$

where A_t^m (A_t^f respectively) represents the assets that has stronger effect on the husband's (wife's) utility in the future since he has larger marginal claim on this assets in case of divorce. As always in this paper, assume that there are no liquidity constraints. The first-order conditions characterizing the optimal choice are:

²⁰For simplicity, it is assumed that the rates of return are non-stochastic.

$$\begin{aligned}
u_t^{m'} - \mu_t &= 0 \\
\lambda_t u_t^{f'} - \mu_t &= 0 \\
\frac{\partial V_{t+1}^m}{\partial A_{t+1}^m} - \frac{\mu_t}{1+r_t^1} + \lambda_t \frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} &= 0 \\
\frac{\partial V_{t+1}^m}{\partial A_{t+1}^f} - \frac{\mu_t}{1+r_t^2} + \lambda_t \frac{\partial V_{t+1}^f}{\partial A_{t+1}^f} &= 0.
\end{aligned}$$

Now, make the following two assumptions.

Assumption 6. (Different marginal effects of assets on outcomes) There does not exist a pair of A_t^m, A_t^f for any t such that $\frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} = \frac{\partial V_{t+1}^f}{\partial A_{t+1}^f}$.

Assumption 7. (Equal rates of return) Let $r_t^m = r_t^f \equiv r_t$ for all t .

Assumption 6 can be justified in a case of divorce as outside option Nash-bargaining, where the marginal rights to assets in case of divorce are different, since while increase in each asset has similar effect on the budget set, they will have differing effect on the outside options in the next period.

Theorem 4. Let assumption 6 and 7 hold and let an interior optimum exist. The resulting outcome without commitment is fully efficient.

Proof: The first order conditions can be rearranged to yield:

$$\left(\frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} - \frac{\partial V_{t+1}^f}{\partial A_{t+1}^f} \right) (\lambda_t - \lambda_{t+1}) = 0.$$

Since this is true for all time-periods, this means that $\lambda_t = \lambda_{t'} \quad \forall t, t'$. This combined with the basic dynamic first-order conditions

$$\begin{aligned}
u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} &= (\lambda_{t+1} - \lambda_t) \frac{\partial V_{t+1}^f}{\partial A_{t+1}^m} \\
u_{t+1}^{f'} - \frac{u_t^{f'}}{1+r_t} &= \left(\frac{1}{\lambda_{t+1}} - \frac{1}{\lambda_t} \right) \frac{\partial V_{t+1}^m}{\partial A_{t+1}^m},
\end{aligned}$$

will yield the fully efficient solution:

$$\begin{aligned}u_t^{m'} &= (1 + r_t)u_{t+1}^{m'} \\u_t^{f'} &= (1 + r_t)u_{t+1}^{f'}. \blacksquare\end{aligned}$$

The intuition for theorem 4 comes from incomplete markets analogy. With just one asset, the couple is constrained in the way it can transform current consumption to future consumption. With two assets and assumptions 6 and 7, they have two assets that span the whole space of required transactions.

Three major caveats are in order, before one thinks of Theorem 4 having strong public policy implications. While Theorem 4 seems to be saying that the efficient divorce laws would let the married couple (while still married) decide on who owns what portions of their net worth in case of divorce (by having “his” and “hers” accounts), this does not take into account that moving from a single-asset world (community property) to a stylized common-law regime would imply a redistribution of welfare within family. As is typical, a move from one inefficient regime to a Pareto-efficient regime is not necessarily a Pareto-improvement. Also, consideration of independent divorce risk will alter the conclusion of the Theorem 4.

Third major caveat to Theorem 4 is that unlike many of the theorems in this paper it does not extend straightforwardly to the case where there is any asset return risk or other risk (like health shocks) that affects the V -functions. If this extension is taken into account, then one needs to consider the incomplete risk sharing aspect of lack of intertemporal commitment. This question and its implications on the desirability of common-law versus community property regime is left for further research.

An interesting point to note about this model (now for generality, assume that there can be more than two assets) is that the model is consistent with interior optimum without any constraints in cases where one assets return is dominated by some other assets return. This is obvious from the first-order conditions, if the property rights (i.e. the effects on the next period’s value functions) are different, an interior optimum with positive holdings of dominated

assets is a possibility. If some of the investment possibilities are interpreted to be human capital investments (like in Wells and Maher 1998) this model is compatible with inefficient human capital investment because of the bargaining effects in the future periods.

2.4.2 Independent risk of divorce

The results of theorem 7 changes if one adds little bit of empirical relevance to the model. In real life, divorces do occur. This section takes another simplistic view (ignoring the link of past actions to divorces) by assuming that divorce is a random event that strikes the couple with an exogenous probability. This could be called “suddenly the love died out” view of divorce: in the beginning of each period a random event (no divorce or divorce) is realized. After that the couple renegotiates their allocation (e.g. by Nash-bargaining) still taking into account the divorce outcomes as threat points in their decision making, since they always have the option to divorce if the negotiations breaks down.

Now, for the sake of argument, assume that there are three assets available to the couple: an asset that pays in the case of couple not divorcing and divorce insurance accounts (which can have also negative balances) for both spouses. A divorce insurance is an insurance product, that will deliver income to each spouse in the case of divorce (regardless of whether divorce happens because of the exogenous shock or because of mutual welfare maximization).²¹

With divorce risk the decision problem can be written as:

$$\begin{aligned}
 V_t^m(A_t, I_t^m, I_t^f) &= \max_{\substack{c_t^m, c_t^f, A_{t+1}, \\ I_{t+1}^m, I_{t+1}^f}} u_t^m(c_t^m) + (1-p)V_{t+1}^m(A_{t+1}, I_{t+1}^m, I_{t+1}^f) + p\tilde{V}_{t+1}^m(I_{t+1}^m) \\
 \text{subject to } A_t &= c_t^m + c_t^f + \frac{(1-p)A_{t+1}}{(1+r_t)} + \frac{p(I_{t+1}^m + I_{t+1}^f)}{(1+r_t)} \\
 u_t^f(c_t^f) &= V_t^f(A_t, I_t^m, I_t^f) - (1-p)V_{t+1}^f(A_{t+1}, I_{t+1}^m, I_{t+1}^f) - p\tilde{V}_{t+1}^f(I_{t+1}^f)
 \end{aligned}$$

²¹The huge moral hazard and asymmetric information problems related to divorce insurance accounts are ignored in this section. The reason why divorce insurance is considered in this section is to illustrate even the perfect insurance markets would not restore full efficiency.

where \tilde{V}_{t+1}^m and \tilde{V}_{t+1}^f represent the indirect utilities of spouses in case of divorce and p is the probability of divorce.

Theorem 5. (Inefficiency theorem) With independent divorce risk, generically the decision will not be efficient.

Proof: Full efficiency requires that

$$\begin{aligned} u_t^{m'} &= (1 + r_t)u_{t+1}^{m'} = (1 + r_t)\tilde{V}_{t+1}^{m'} \\ u_t^{f'} &= (1 + r_t)u_{t+1}^{f'} = (1 + r_t)\tilde{V}_{t+1}^{f'}. \end{aligned}$$

This means that full efficiency is a condition on 6 objectives (marginal utilities to be equated). The couple only has five choice variables to achieve this, so generically it cannot do this while satisfying the budget constraint. ■

The sense in which word “generically” is used in Theorem 5 is that starting from a model where the solution of the problem without commitment is fully efficient; by slightly perturbing the problem (e.g. by changing how the divorce threat points affects current bargaining) we will always find solutions that are not Pareto-efficient. Conversely, if we start with a solution that is not fully efficient, a slight perturbation of the problem will not lead to a solution that is fully efficient.

The basic problem in the model with independent divorce risk is that the divorce insurance is used to do two things: to equate individual marginal utilities between current consumption and tomorrow’s divorce state; and to equate the ratios of marginal utilities of spouses between today’s consumption and tomorrow’s consumption.

The point about divorce insurance in this section was not to be a realistic description of reality. Instead, the divorce insurance was considered to illustrate the following point. With divorce insurance, the couple has set of assets that span the future state-space. Since even with the complete spanning they cannot always reach Pareto-efficient allocation, they generally cannot do that with less complete assets selection.

2.5 Death

Besides sharing of the current consumption, one of the potential points of contention couples face is how much protection to provide for each spouse in the case of the death of the other spouse. Since providing survivor protection is costly, spouses (even if one allows for altruistic motives) can have differing views on the optimal level of survivor protection. This section illustrates that under complete markets the survivor protection is similar to private consumption. Thus, under the class of models considered here and with complete and perfect markets, there cannot be independent concern about the lack of survivor protection without concern for the economic circumstances of the individual spouses while both partners are alive.²²

With perfect insurance markets, the decision problem becomes:

$$V_t^m(A_t) = \max_{\substack{c_t^m, c_t^f, A_{t+1}, \\ I_{t+1}^m, I_{t+1}^f}} u_t^m(c_t^m) + (1 - p_{t+1}^m - p_{t+1}^f - p_{t+1}^{mf})V_{t+1}^m(A_{t+1}) + p_{t+1}^m \tilde{V}_{t+1}^m(I_{t+1}^m)$$

subject to

$$A_t = c_t^m + c_t^f + \frac{(1 - p_{t+1}^m - p_{t+1}^f - p_{t+1}^{mf})A_{t+1}}{(1 + r_t)} + \frac{p_{t+1}^m I_{t+1}^m}{(1 + r_t)} + \frac{p_{t+1}^f I_{t+1}^f}{(1 + r_t)}$$

$$u_t^f(c_t^f) = V_t^f(A_t) - (1 - p_{t+1}^m - p_{t+1}^f - p_{t+1}^{mf})V_{t+1}^f(A_{t+1}) - p_{t+1}^f \tilde{V}_{t+1}^f(I_{t+1}^f),$$

where p_{t+1}^f is the probability of the wife becoming a widow (and being alive), p_{t+1}^{mf} is the probability of both spouses dying before next period, I_t^f is life insurance protection protecting the wife and A_t is now an annuity that ceases to pay when one partner dies.²³ Note, that for simplicity, the altruistic motives are assumed away from the specification.²⁴

²²This does not mean that the view that couples seem to choose insufficient amounts of survivor protection is irrational. The results here depend on several assumptions. The key assumption, that the couples have access to actuarially fair insurance, is especially questionable for a large part of the population.

²³The choice of A_t as an annuity product instead of regular savings account is just to save on notation. As always, the choice of which group of assets to characterize a completely spanned state-space allocation is completely irrelevant for substantive purposes.

²⁴If altruistic motives for survivor protection were to be included, then also altruistic motives for current period consumption should be included for avoid biasing the model. With altruistic motives, the interpretation of results would always hinge on whether the altruistic motives between spouses are stronger when spouses are both alive or for survivor protection.

Theorem 6. (Limited efficiency of survivor protection) In the optimum, neither spouse would like to trade his or her current period consumption for more survivor protection.

Proof: Follows trivially from the first order condition for the optimum. ■

The key intuition that drives the difference between mortality and divorce risk is that once spouses make their decisions about their consumption allocations at period $t + 1$ the survivor protection that they had for that period $t + 1$ is just a sunk investment that did not pay off, while the divorce allocation can be seen as affecting the allocation (e.g. through threat points like in Nash-bargaining).

One of the key assumptions of the partial efficiency result is the ability to adjust the level of life-insurance protection and annuity holdings each period with actuarially fair pricing. A key feature in the real world of these life-contingent insurance products is that the required transactions to do this are not necessarily available. Typically an annuity or life-insurance contract is a long-term contract that cannot be undone in the next period without non-trivial financial penalties.²⁵ Unfortunately, the analysis of longer term contracts is very complicated in the current setting because a purchase of long term contract implies non-negativity constraints on the future allocations (since the purchase of more life-insurance protection is a possibility in the future but the reverse transaction might not be available). Whether (a modified version of) Theorem 6 holds with long-term contracts is left as an open research question.

2.6 Conclusion

This paper provides starting-point for further research. The theoretical results identified in this paper could be taken to data to test the model of family decision-making presented here against more restrictive models (single-utility-function view of household) or more general models (non-cooperative models of household). The results could also easily be extended to consider risk more generally than is done in this paper.

²⁵A purchase of annuity is often done in relation of conversion of pension wealth to an annuity stream at retirement. This choice cannot often be undone. Adverse selection concerns are one obvious plausible reason why the perfect access period-to-period markets with actuarially fair pricing are not necessarily very accurate description of the reality for many households.

The main conclusion of this paper is that at least in theoretical model of family decision-making the inability to commit across time matters for economic efficiency. The use of seemingly dominated assets can be explained as an attempt to overcome the problems related to incomplete commitment. Theorem 3 also provides an added justification why divorce as a phenomenon can lead couples to save more than they would otherwise do. This justification has nothing to do with the traditional “saving for the rainy day” argument for increased savings (self-insuring against divorce). Instead, it is shown that the fact that the divorce threat-points affect the balance of power within the family while still married can lead to higher saving through an effect that is analogous to decrease in the return on the assets. In Theorem 4, it is shown that in a simple model a stylized common-law divorce property division regime is likely to lead to an efficient solution. This results can be viewed as an extension of Coase’s Theorem: in absence of transaction cost assigning property rights leads to a Pareto-efficient outcome. However, this results is shown to depend on the assumption of no exogenous divorce risks. Taking into account this caveat and also taking into account the possible differing distributional impact of different divorce regimes means that the superiority of common-law over community property regime in the theoretical model should be taken only as a tentative result. Further research on the optimal divorce property division regimes is clearly needed.

2.7 References

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2.8 Appendix to Chapter 2: Proofs of Selected Propositions

2.8.1 Proof of Lemma 0

Lemma 0. Any one of the following assumptions is sufficient for Assumption 1 to be satisfied:

- a) Value functions are independent of wealth for one the spouses: $V_t^{f'} = 0 \forall t, A_t$ or $V_t^m = 0 \forall t, A_t$.
- b) Two period world and wealth good for both in second period: let $T = 2$ and let the second period solution be characterized by a sharing rule for wealth: $c_2^m = \psi_2(A_2)$ and $c_2^f = A_2 - \psi_2(A_2)$. Furthermore let $0 \leq \psi_2' \leq 1$.
- c) T-period world, with last period described as in b) and letting the series of functions $V_t^f(A_t)$ satisfy following conditions: $V_t^{f'} \geq 0, V_t^{f''} \leq 0 \forall t, A_t$ and $(1 + r_t)V_{t+1}^{f'}(A) \geq V_t^{f'}(A) \forall t, A$.
- d) Like c), but applying the restrictions on $V_t^m(A)$ -functions.
- e) CRRA-utilities and outside options in repeated Nash-bargaining: Let Assumptions 2-5 hold.
- f) $V_t^{m'} \geq 0$ and $V_t^{m'} \leq u_t^{m'} \forall t$ in the solution; or equivalently $V_t^{f'} \geq 0$ and $V_t^{f'} \leq u_t^{f'} \forall t$ in the solution.

Proofs:

- a) Direct consequence of envelope theorem and the first order conditions yielding $V_t^{m'} = u_t^{m'} - \frac{u_{t+1}^{m'}}{u_{t+1}^{f'}} V_t^{f'}$.
- b) Under the assumptions, $V_2^m(A_2) = u_2^m(\psi_2(A_2))$ and $V_2^f(A_2) = u_2^f(A_2 - \psi_2(A_2))$. Therefore $V_2^{m'} \geq 0$ and $V_2^{f'} \geq 0$ iff $0 \leq \psi_2' \leq 1$.
- c) By induction. Since last period is like period 2 in b) the claim holds for last period. Now, the first order condition of the problem yields $u_{t+1}^{m'} - \frac{u_t^{m'}}{1+r_t} = (\lambda_{t+1} - \lambda_t) V_{t+1}^{f'}$. Using the fact

that by induction assumption $V_{t+1}^{f'} \leq u_{t+1}^{m'}$ this can be manipulated to yield $\frac{u_t^{f'}}{1+r_t} \geq V_{t+1}^{f'}$. Now using the assumption on the inequalities that V^f -functions will satisfy, concavity of V^f and the fact $A_{t+1} \leq A_t$ yields $u_t^{f'} \geq (1+r_t)V_{t+1}^{f'}(A_{t+1}) \geq V_t^{f'}(A_{t+1}) \geq V_t^{f'}(A_t)$. By the calculation done in a) this means that claim holds.

- d) Same as c), except with roles of m and f reversed.
- e) Follows from the positivity of γ_j and δ_j constants in Lemma A2.
- f) Follows from the same calculation as a).

2.8.2 Proof of Theorem 3

Theorem 3 is proved in by first stating and proving two lemmas.

Lemma A1. Let assumptions 2-4 be satisfied. The Nash-Bargaining problem in period t can be written as:

$$\max_{c_t^m, c_t^f, A_{t+1}} \left(\left(\frac{1}{(1-\theta^m)} (c_t^m)^{1-\theta^m} + \sum_{k=t+1}^T (\beta^m)^{k-t} (\gamma_k A_{t+1})^{1-\theta^m} \right) - A_t^{1-\theta^m} \tilde{v}_t^m \right) \\ * \left(\frac{1}{(1-\theta^f)} (c_t^f)^{1-\theta^f} + \sum_{k=t+1}^T (\beta^f)^{k-t} (\delta_k A_{t+1})^{1-\theta^f} \right) - A_t^{1-\theta^f} \tilde{v}_t^f$$

$$\text{subject to } A_t = c_t^m + c_t^f + \frac{A_{t+1}}{(1+r_t)},$$

for some constants \tilde{v}_t^m and \tilde{v}_t^f and series of constants γ_k and δ_k .

Proof: Consider first the outside for the husband:

$$\tilde{V}_t^m(A_t) = \max_{\{\tilde{c}_i^m\}_{i=t}^T} \frac{1}{1-\theta^m} \sum_{k=t}^T (\tilde{\beta}^m)^{k-t} (\tau_k^m \tilde{c}_k^m)^{1-\theta^m} \\ \text{subject to } \alpha_t A_t = \sum_{k=t}^T \frac{\tilde{c}_k^m}{\prod_{j=t}^k (1+r_j)^{j-t}}.$$

By rewriting the objective as

$$\tilde{V}_t^m(A_t) = \max_{\{\tilde{c}_i^m\}_{k=t}^T} A_t^{1-\theta^m} \frac{1}{1-\theta^m} \sum_{k=t}^T (\tilde{\beta}^m)^{k-t} \left(\tau_k^m \frac{\tilde{c}_k^m}{A_t} \right)^{1-\theta^m}$$

it is easy to see that $\tilde{V}_t^m(A_t) = A_t^{1-\theta^m} \tilde{V}_t^m(\alpha_t)$. The outside option value function for the wife is handled similarly. The rest of the proof is a simple induction argument. Consider the Nash-bargaining problem in period T :

$$\max_{c_t^m, c_t^f} \left(\left(\frac{c_T^m}{(1-\theta^m)} \right)^{1-\theta^m} - A_T^{1-\theta^m} \tilde{v}_T^m \right) \left(\left(\frac{c_T^f}{(1-\theta^f)} \right)^{1-\theta^f} - A_T^{1-\theta^f} \tilde{v}_T^f \right)$$

$$\text{subject to } A_T = c_T^m + c_T^f.$$

By the similar homogeneity argument as in the case of the outside options, the optimum solution for $\{c_T^m, c_T^f\}$ is just a linear scaling of the optimal solution in the case where $A_T = 1$. The induction step, assuming that the claim holds for period $t+1$, and showing that the claim holds for period t follows using exactly the same homogeneity of the objective function argument as in the period T .

Lemma A2. Let the assumptions 2-4 hold. Consider maximization of the linear combination of life time utilities of spouses (with weight μ_t on wife's utility, this being one characterization of a Pareto-efficient solution subject to constraints):

$$W(A_t, \mu_t) = \max_{A_{t+1}, \{c_i^m, c_i^f\}_{i=t}^T} \frac{1}{1-\theta} \left(\sum_{k=t}^T \beta^{k-t} (c_k^m)^{1-\theta} + \mu_t \sum_{k=t}^T \beta^{k-t} (c_k^f)^{1-\theta} \right)$$

$$\text{subject to } A_t = c_t^m + c_t^f + \frac{1}{1+r_t} A_{t+1}$$

$$c_j^m = \gamma_j A_{t+1}$$

$$c_j^f = \delta_j A_{t+1},$$

where $j = t+1, \dots, T$ and where the series of constants γ, δ satisfy the budget constraint

for future periods $1 = \sum_{k=t+1}^T \left(\frac{\gamma_k + \delta_k}{\prod_{j=t+1}^k (1+r_j)^{j-t-1}} \right)$. For $\theta > 1$ the savings A_{t+1} are higher than in the optimal unconstrained solution. For $\theta = 1$ (log utility) the savings A_{t+1} are always at the first best level. For $\theta < 1$ the savings A_{t+1} are lower than in the optimal unconstrained solution.

Proof: Substitute the constraints for future consumption into the objective and consider the first order condition:

$$\begin{aligned} c_t^m &= \mu_t^{-1/\theta} c_t^f \\ c_t^m &= (1+r_t)^{-1/\theta} A_{t+1} * \omega_t^{-1/\theta} \\ \text{where } \omega_t &= \sum_{k=t+1}^T \beta^{k-t} \gamma_k^{1-\theta} + \mu_t \sum_{k=t}^T \beta^{k-t} \delta_k^{1-\theta} \end{aligned}$$

The second part of the claim is immediate from above: in the case of $\theta = 1$ the choice of the coefficients does not affect the savings level.²⁶ Next consider the case where $\theta > 1$ and consider ω_t as a function of $(\gamma_k, \delta_k)_{k=t+1}^T$. Subject to the constraint $1 = \sum_{k=t+1}^T \left(\frac{\gamma_k + \delta_k}{\prod_{j=t+1}^k (1+r_j)^{j-t-1}} \right)$, the function ω_t has a unique minimum (since it is a convex function) at the choice $(\gamma_k, \delta_k)_{k=t+1}^t$ that correspond to the unconstrained optimal solution of joint utility maximization. This fact, budget constraint and the first order conditions imply that for any feasible choice of $(\gamma_k, \delta_k)_{k=t+1}^t$ the wealth holdings in period $t + 1$ will be higher than in the unconstrained optimum. For $\theta < 1$, similar reasoning will imply that the capital stock in period $t + 1$ will be lower than in the unconstrained optimum (since ω_t now is a concave function).■

Theorem 3 (from the main text): Let assumptions 2-5 hold. The inability to commit across periods implies higher wealth holdings in every period after initial period (more savings) if $\theta > 1$. For $\theta = 1$ (log utility) the level of savings is unaffected by the inability to commit. For $\theta < 1$ the inability to commit decreases savings in every period.

Proof: Application of Lemmas A1 and A2. By Lemma A1 the problem in any period can be written as Nash-Bargaining with future consumption allocation being a linear transformation

²⁶Naturally, a separate treatment of the log-utility would confirm this result.

of tomorrow's wealth. Since any Nash-bargaining solution is Pareto-efficient (with respect to constraints) Lemma A2 applies here for some μ_t . To prove the theorem, it now suffices to notice that under the assumption of equal discount rates and θ -parameters for spouses, any fully Pareto-efficient solution will imply same levels of wealth holdings, so the fact that the fully efficient (with commitment) solution corresponds to a (potentially) different welfare μ_t becomes irrelevant. ■

Chapter 3

Savings and Portfolio Choice in a Two-Period Two-Asset Model (joint work

with Peter Diamond and John Geanakoplos)

Arrow (1971) analyzed portfolio choice in a one-period model with one safe and one risky asset. He showed that with decreasing absolute risk aversion (DARA), the demand for the risky asset is increasing in wealth. He also showed that with increasing relative risk aversion (IRRA), the elasticity of demand for the safe asset with respect to wealth is greater than one. Thus, with both DARA and IRRA, both asset demands are normal. Furthermore, in any two-good model, the goods must be Hicksian substitutes. Following Arrow, Sandmo (1968) analyzed a two-period model with a safe and risky asset, and with intertemporally additive utility for consumption in both periods. He showed that with DARA and IRRA, both asset demands are normal, as is demand for first-period consumption.

This note shows that in Sandmo's two-period economy, DARA and IRRA also guarantee that each of the goods is a Hicksian substitute for each of the others (Proposition 6). Moreover, if the safe interest rate goes up and the price of stocks goes up, expected utility constant, then first-period consumption decreases (savings increase) (Proposition 7).¹ In passing, we also

¹We have used such a model to study savings and portfolio choice in a two-asset economy in the context of

rederive Sandmo's original result on normal demands (Proposition 5).

Similar results are proved for more general preferences of the form $U(x, y, z) = f(x) + g(y, z)$. We proceed by first considering this general case. In Proposition 1 we show that x and y are Hicksian substitutes if and only if y is a normal good. In Proposition 2 we show that y and z are Hicksian substitutes if increasing y decreases the marginal utility of z ($g_{yz} \leq 0$). In Proposition 3 we show that y is a normal good if and only if y is a normal good for the utility function $V(x, y, z) = g(y, z)$. In Proposition 4, we show that an increase in the price of y together with a decrease in the price of z that leaves utility constant increases demand for x if and only if the income elasticity of demand for good y is larger than the income elasticity of demand for good z . After proving these propositions, we apply the results to the savings-and-portfolio-choice problem.

In Proposition 8, and its discussion, we partially extend Propositions 1–3 to multiple consumption goods, multiple assets, and multiple periods.

3.1 Preferences of the Form $U(x, y, z) = f(x) + g(y, z)$

We begin by relating the Hicksian cross-price derivative to the Marshallian income derivative.

Let $U : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a strictly increasing, strictly concave, twice differentiable utility function. Let p_x, p_y, p_z, W all be strictly positive and define

$$V(p_x, p_y, p_z, W) = \max_{x, y, z} U(x, y, z) \text{ subject to } p_x x + p_y y + p_z z \leq W.$$

The solution $x(p_x, p_y, p_z, W) = x(p, W)$, $y(p, W)$, $z(p, W)$ to the utility maximization problem is called the Marshallian demand, and V is called the indirect utility function. When all three of x, y, z are strictly positive, then it is well-known that Marshallian demand is differentiable. Furthermore, Roy's identity holds that $\partial V(p, W) / \partial p_\alpha = -\alpha(p, W) [\partial V(p, W) / \partial W]$ for any good

social security trust fund investment in risky securities (Diamond and Geanakoplos, 1999).

$\alpha \in \{x, y, z\}$. Good α is called normal if $\partial\alpha(p, W)/\partial W > 0$. The income elasticity of good α is defined by $(W/\alpha)(\partial\alpha(p, W)/\partial W)$.

Similarly, define the expenditure function

$$e(p, U) = \min_{x, y, z} p_x x + p_y y + p_z z \text{ subject to } U(x, y, z) \geq U.$$

The solution $x(p, U)$, $y(p, U)$, $z(p, U)$ to the expenditure minimization problem is called the Hicksian or compensated demand. When all three of x , y , z are strictly positive, it is well-known that Hicksian demand is differentiable. Furthermore, $\partial e(p, U)/\partial p_\alpha = \alpha(p, U)$ for any good $\alpha \in \{x, y, z\}$. A pair of goods α and β are called Hicksian substitutes if

$$\partial\alpha(p, U)/\partial p_\beta = \partial^2 e(p, U)/\partial p_\alpha \partial p_\beta = \partial\beta(p, U)/\partial p_\alpha > 0.$$

Proposition 1 *Let the utility function $U : \mathbb{R}^3 \rightarrow \mathbb{R}$ be additively separable into the form $U(x, y, z) = f(x) + g(y, z)$, where both f and g are twice-differentiable, strictly increasing and strictly concave. Then when demands x , y , z are strictly positive,*

$\partial x(p, U)/\partial p_y = -(f'/f'')[\partial y(p, W)/\partial W]$, where the first term is the compensated cross derivative and the last term is the income derivative.

Proof Define the expenditure function as

$$e(p, U) = \min_{x, y, z} p_x x + p_y y + p_z z \text{ subject to } f(x) + g(y, z) \geq U.$$

The first order condition for expenditure minimization and the envelope theorem give:

$$\frac{\partial e(p, U)}{\partial U} = \frac{p_x}{f'(x)}. \quad (1)$$

Differentiation of (1) with respect to p_y yields:

$$\frac{\partial y(p, U)}{\partial U} = \frac{\partial^2 e(p, U)}{\partial U \partial p_y} = -\frac{p_x f''(x)}{(f'(x))^2} \frac{\partial x(p, U)}{\partial p_y}. \quad (2)$$

Differentiating the identity between compensated and ordinary demands

$$y(p, U) = y(p, e(p, U)) \quad (3)$$

with respect to U and using (1) gives

$$\frac{\partial y(p, U)}{\partial U} = \frac{\partial y(p, W)}{\partial W} \frac{\partial e(p, U)}{\partial U} = \frac{\partial y(p, W)}{\partial W} \frac{p_x}{f'(x)}. \quad (4)$$

Substituting (2) into (4) gives the result. ■

Proposition 1 shows that x and y are Hicksian substitutes if and only if y is normal. Note that the proof would hold with z being a vector.

We now consider the Hicksian cross-price derivative between goods y and z .

Proposition 2 *Suppose $U(x, y, z) = f(x) + g(y, z)$, where both f and g are twice-differentiable, strictly increasing and strictly concave. If $\partial^2 g(y, z) / \partial y \partial z \leq 0$, then goods y and z are Hicksian substitutes, whenever x, y, z are positive..*

Proof Consider again the expenditure minimization problem defined in the proof of Proposition 1. Suppose a solution is achieved at the Hicksian or compensated demands $(x, y, z) \gg 0$. Let p_y increase, and suppose, contrary to what we would like to prove, that the compensated demand for z stays the same or declines. Since Hicksian own effects are negative, y must decline. In order to maintain the same utility with y decreasing and z nonincreasing, x must rise, since utility is increasing in each variable. So the marginal utility of x falls (since f is strictly concave), and the marginal utility of z does not fall (since y and z fall or stay the same, and $\partial^2 g / \partial y \partial z \leq 0$). This is a contradiction, since p_x and p_z are the same. ■

Next, we consider the relationship among income derivatives in the full maximization and the submaximization over just y and z . Consider together the utility maximization problem

$$\max_{x, y, z} f(x) + g(y, z) \text{ subject to } p_x x + p_y y + p_z z \leq W \quad (5)$$

and the sub-maximization problem

$$\max_{y,z} g(y, z) \text{ subject to } p_y y + p_z z \leq I. \quad (6)$$

Denote by $v(I)$ the maximum value of (6) for a fixed price vector. Problem (5) can be separated into first maximizing (6) and then choosing I optimally from

$$\max_{x,I} f(x) + v(I) \text{ subject to } p_x x + I \leq W. \quad (7)$$

Proposition 3 *Let $f(x)$ and $g(y, z)$ satisfy the same restrictions as in Proposition 1, twice-differentiable, strictly increasing and strictly concave. Demand for good y (or z) is increasing in wealth in sub-problem (6) if and only if it is increasing in wealth in problem (5). In particular, at least one of y or z must be increasing in wealth in problem (5).*

Proof From the strict concavity of $g(y, z)$ it follows that $v(I)$ is a strictly concave function of I . From (7) it is easy to see that given strict concavity of $f(x)$ and $v(I)$, both x and I are increasing in W . Hence y is increasing in W in problem (5) if and only if it is increasing in I in problem (6). ■

Note that the proof would hold with z being a vector.²

We turn next to simultaneous changes in the prices of y and z that leave utility constant.

Proposition 4 *Let the utility function be $U(x, y, z) = f(x) + g(y, z)$, where both f and g are twice-differentiable, strictly increasing and strictly concave. Let p_x and W be fixed. If p_y increases and p_z decreases so that the indirect utility V is constant, then the demand for good x increases (decreases) if the income elasticity of demand for good y is larger (smaller) than the income elasticity of demand for good z .*

²Given Proposition 3, intuition for the sign structure in Proposition 1 becomes clear. Consider the effect of a compensated price increase of x on the demand for I (the composite expenditure on y and z). Since this is a two-good problem the compensated effect on I must be positive, and the resulting effect on y and z depends on normality in the suboptimization. Given the additive separable structure of the problem this is the only channel for the compensated price change to affect the demand for y and z .

Proof Define $p_z(p_y)$ so that utility is constant as a function of p_y , given p_x and income W . Then, by Roy's identity, we have

$$p'_z(p_y) = -(\partial V/\partial p_y)/(\partial V/\partial p_z) = -y/z.$$

Taking the total derivative of x with respect to p_y , with p_z varying according to $p_z(p_y)$, leaves both income and utility fixed. By Proposition 1, we have

$$\frac{dx}{dp_y} = \frac{\partial x(p, W)}{\partial p_y} - \frac{y}{z} \frac{\partial x(p, W)}{\partial p_z} = \frac{\partial x(p, U)}{\partial p_y} - \frac{y}{z} \frac{\partial x(p, U)}{\partial p_z} = -\frac{f'}{f''} \left(\frac{\partial y(p, W)}{\partial W} - \frac{y}{z} \frac{\partial z(p, W)}{\partial W} \right)$$

or

$$\frac{dx}{dp_y} = -\frac{y f'}{W f''} \left(\frac{W \partial y(p, W)}{y \partial W} - \frac{W \partial z(p, W)}{z \partial W} \right). \quad \blacksquare$$

In Proposition 3 we saw the relationship between income derivatives in the full optimization and the suboptimization, $\partial y(p, W)/\partial W = [\partial y(p, I)/\partial I](\partial I/\partial W)$. From this relationship, we have a corollary using the income elasticities in the suboptimization.

Corollary *If p_y increases and p_z decreases so that expected utility is constant, then the demand for good x increases (decreases) if the income elasticity of demand for good y is larger (smaller) than the income elasticity of demand for good z in the suboptimization over y and z .*

3.2 Savings and Portfolio Choice

Next, consider a special case where $g(y, z) = E\{u(y + zR)\} = \int u(y + zR)dF(R)$, where u is strictly increasing, twice-differentiable and strictly concave. The utility maximization problem, (5), using this subutility function, is the savings-and-portfolio-choice problem described in the introduction. The goods x, y, z are, respectively, first-period consumption, the safe investment good, and the risky investment good. R is the random payoff of the risky investment. (We suppose R only takes on nonnegative values.) We now present the applications of Propositions 1–4 to this problem.

Proposition 5 *In the savings-and-portfolio-choice problem, if the second period utility function, u , is concave, then (i) first-period consumption is a normal good. If u globally satisfies decreasing absolute risk-aversion (DARA), then (ii) risky investment is a normal good. Furthermore, if u satisfies increasing relative risk-aversion (IRRA) then (iii) safe investment is a normal good. In short, if u satisfies both DARA and IRRA, then both investment goods are normal goods.*

Proof The concavity of u implies the concavity $g(y, z)$. This gives (i) and the applicability of Proposition 3. Applying Arrow's original result in the second period sub-maximization and using Proposition 3 yields (ii) and (iii). ■

Proposition 6 *Let the second period utility function, u , globally satisfy decreasing absolute risk-aversion (DARA). Then the compensated demand for first-period consumption is increasing in the price of the risky investment good, as is the compensated demand for the safe investment good. Furthermore, if second period utility satisfies increasing relative risk-aversion (IRRA), then the compensated demand for first-period consumption is increasing in the price of the safe investment good, as is the compensated demand for the risky investment good. In short, if u satisfies both DARA and IRRA, then each good is a Hicksian substitute for each of the others.*

Proof From Propositions 1 and 5, the compensated demand for first period consumption is increasing in the price of the risky good. Since increased holdings of either asset decreases the marginal utility of the other asset, Proposition 2 guarantees that the compensated demand for the safe good is increasing in the price of the risky good. This proves the first half of the proposition. The proof that the compensated demands for both first period consumption and risky investment are increasing in the price of the riskless investment good is handled in exactly the same way. ■

Proposition 7 *Let the second period utility function, u , globally satisfy decreasing absolute risk-aversion (DARA) and increasing relative risk-aversion (IRRA). An increase in the interest rate accompanied by an increase in the price of stocks that just leaves expected utility constant decreases first period consumption.*

Proof From Arrow, we know that the income elasticity of the demand for the safe asset in the portfolio choice problem exceeds one, and so must be larger than the income elasticity of the demand for the risky asset. Together with the Corollary to Proposition 4, this completes the proof. ■

3.3 Two Examples

While Proposition 6 refers to compensated demands, using normality, we can also sign the response of the ordinary demand for first-period consumption (and so savings) to a change in the price of an asset for some settings of initial endowments. Consider a budget constraint in which income is derived from the sale of initial endowments, so that utility maximization is:

$$\max_{x,y,z} f(x) + g(y, z) \text{ subject to } p_x x + p_y y + p_z z = p_x x_0 + p_y y_0 + p_z z_0, \quad (8)$$

where 0-subscripted goods are initial endowments. From the Slutsky equation, we know that if good i is a normal good and the consumer is a net seller of good j , then the Marshallian demand derivative $\partial x_i / \partial p_j$ must be positive if the Hicksian demand derivative $\partial x_i^c / \partial p_j$ is positive. We consider two examples of such a setting.

Consider a consumer whose income is derived entirely from the sale of endowments, as in (8), and whose utility satisfies the assumptions of Proposition 6. We can think of the risky investment good z as a proxy for stocks, and the safe investment good y as a proxy for government bonds. Suppose the FED lowers the interest rate on bonds. Propositions 5 and 6 imply that if the consumer was a borrower to begin with, $p_y y < p_y y_0$, then the consumer will respond to the lower interest rate (stock prices held constant) by borrowing more, increasing both investment in the stock market and first period consumption. His savings (first-period income, x_0 , less first-period consumption, x) must go down.³ Notice that a drop in the interest rate is equivalent to a rise in the price of the safe investment good, and a borrower is made richer by the fall in the interest rate.

³We define savings in the manner corresponding to national income accounting. We do not claim that the total value held of stocks and bonds would go down.

If instead the consumer had been a seller of stocks (say a middle aged investor winding down his accounts as old age approached), and the stock market suddenly ran up in price, with no change in the expected payoffs of stocks or the interest rate, then the consumer would respond by increasing his demand for immediate consumption and his investment in bonds

3.4 Extensions to Multiple Assets and Multiple Periods

Consider now a multiperiod model with uncertainty, where at each date-event the agent consumes and adjusts his portfolio, in an effort to maximize lifetime expected utility. For the moment, we continue to assume that there are two assets available in the first period, but now suppose that in each state of nature in the second period there is an opportunity to consume and a new opportunity to trade. We can write lifetime expected utility as $f(x) + g(y, z)$ where $g(y, z) = E\{u_R(y + zR)\} = \int u_R(y + zR)dF(R)$, where u_R is the Bellman value function for income available starting in period 2 in the state of nature where the return on the risky asset was R . While u_R is strictly increasing, twice-differentiable and strictly concave, there is no guarantee that it satisfies DARA and IRRA even if the utility of consumption does. Thus Propositions 1–4 apply, but not Propositions 5–7.

Consider the situation where an agent has chosen an optimal lifetime plan, and then period 1 prices change, but the spot prices for period 2 and onward stay the same in all states of nature, as do subjective probabilities. Thus, the Bellman value functions u_R do not change. Suppose the agent is a short-term borrower at date 1. Then by Proposition 2, if the FED lowered the interest rate at date 1 (leaving it unchanged at every other date-event), he would invest more in the stock market. We could not be sure if he would consume more in date 1, or borrow more, even if his utility u satisfied DARA and IRRA, because there is no guarantee that the expectation of u_R satisfies DARA and IRRA.

However, we know by Proposition 1 that if the safe asset were a normal good, then the increase in its price (which is equivalent to the drop in the interest rate) would indeed raise

period 1 consumption, and so reduce savings, and increase borrowing. Finally, we know from Proposition 3 that either the safe asset or the risky asset must be normal. Thus we can say that in a multiperiod, two asset economy, a rise in either asset price in period 1 (all else equal) will raise the Hicksian demand for the other asset. Furthermore, if the consumer is a net seller of both assets, then for at least one of the assets, a rise in its price will increase consumption and so reduce savings.

We turn next to settings with explicit attention to more consumption goods and more assets. As noted above, the proofs of Propositions 1 and 3 did not use the property that z was a scalar, and so extend to the case where z is a vector. Proposition 2 also admits extensions. Moreover, a version of the results carries over when x is a vector.

Proposition 8 *Let $U : R^k \times R^\ell \rightarrow R$ be a utility function which is additively separable into the form $U(x, y) = f(x) + g(y)$ where both f and g are twice-differentiable, strictly increasing and strictly concave, and x and y are vectors. If some x_i is a normal good for the utility function $F(x, y) = f(x)$, and some y_j is a normal good for the utility function $G(x, y) = g(y)$, then (i) x_i and y_j are normal goods for the utility function U , and (ii) x_i and y_j are Hicksian substitutes. Moreover, if y_j is a normal good, then (iii) increasing p_{y_j} increases the compensated aggregate expenditure on goods x . Whether or not y_j is normal, if all the mixed partials of g are less than or equal to zero, then (iv) y_j and some y_k are Hicksian substitutes.*

Proof Consider the subproblem:

$$\max_x f(x) \text{ subject to } p_x x \leq I. \quad (9)$$

Denote by $v(I)$ the maximum value of (9) for a fixed price vector. The full maximization can be separated into first maximizing (9) and then choosing I optimally from

$$\max_{y, I} g(y) + v(I) \text{ subject to } p_y y + I \leq W. \quad (10)$$

Since v is strictly concave, the proof of Proposition 3 applies to (10) and to the problem with the roles of x and y reversed. This proves (i).

The proof of Proposition 1 also applies to (10). Hence an increase in the price of any y_j increases the compensated demand for I . This proves (iii).

Moreover, if x_i is normal in (9), this increase in the compensated demand for I increases the compensated demand for x_i in the original problem, proving (ii).

Finally, the proof of Proposition 2 can be extended in a straightforward manner to prove that some y_k must be a Hicksian substitute for y_j in (10), which proves (iv). ■

In one interpretation, we can think of x as the vector of first period consumption goods, and y as a vector of assets in a two-period problem. If g is the expected utility of the aggregate asset payoffs, and if the return from each asset is nonnegative, then Proposition 8 applies. If any asset is a normal good, then it is a Hicksian substitute for first period aggregate consumption and a Hicksian substitute for at least one other asset. This can be extended to Marshallian demands in some settings. If we knew, say, that investments in mortgage derivatives increased with income, then an increase in the price of mortgage derivatives, all else equal, would raise aggregate consumption in the first period and increase investment in at least one other asset, for any investor who maximized expected utility and was a net seller of mortgage derivatives.

In another interpretation, we can think of y as two assets paying off in some states in period 2, x_i as first-period consumption, and the rest of the x variables as assets paying off only in period 2 states disjoint from those in which the y assets pay off, or paying off in later periods (provided intertemporal preferences are additive and there is no additional trading at the time of consumption of the proceeds of assets y). Then again Proposition 8 applies. From separability of f , we know that first period consumption is a normal good. Using the von Neumann expected utility interpretation of g , the two assets y are Hicksian substitutes and, if asset y_j is a normal good, then it is a Hicksian substitute for first-period consumption.

3.5 References

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