224

Economic Development
Italy Project
C/56-19 (Revised C/56-13)

THE NEOCLASSICAL ANALYSIS OF EXTERNAL ECONOMIES AND DISECONOMIES

Louis Lefeber

The author is indebted for constructive suggestions and valuable criticism to Professors R. L. Bishop, R. S. Eckaus, and R. M. Solow. Responsibility for the correctness of the ideas propounded in the paper rests, of course, with him. The paper itself is the first part of the Ph.D. thesis of the author.

Center for International Studies
Massachusetts Institute of Technology
Cambridge, Massachusetts
June 28, 1956

PART I

THE NEOCLASSICAL ANALYSIS OF EXTERNAL ECONOMIES AND DISECONOMIES

Introduction

The concept of external economies is one which frequently arises in connection with the theory of economic development. The reason for the importance attached to it is that the presence of external economies under certain circumstances can drive a wedge between social and private marginal products, resulting in deviations from the social optimum in a competitive system. This, in turn, may call for corrective governmental interference.

The concept of external economies originated with Marshall who used it as an analytical device to explain production at falling unit costs. With the advent of the theory of monopolistic competition, however, a more legitimate analytical tool was created to handle such phenomena, while the concept of external economies took on new importance in arguments concerning optimal allocation in a purely competitive state and in socialist or planned economies. An increasing weight was given to the divergences of social and private marginal cost as an obstructive phenomenon to the decentralized achievement of the ideal, and there was a tendency to refer loosely to divergences of this nature as external economies or diseconomies.

The terms external economies and diseconomies properly include a good number of phenomena which may or may not give rise to the above mentioned divergences. Such are, for instance, situations involving the Marshallian forward falling supply curve, technical interconnections among firms, externalities operating directly on the consumer and even the simple adjustment mechanism of the purely competitive general equilibrium model.

It should be mentioned at the very outset of this paper that the recognition of the existence of certain types of externalities, far from being helpful, has in the past been something of a "red herring" in the analysis of optimal allocation. This is the case of pecuniary external economies and diseconomies.

Pecuniary external effects operating on a firm in a general equilibrium framework are, under all circumstances, the manifestations of the workings of the general equilibrium system itself. In other words, when the conditions underlying the general equilibrium system change, the maximising mechanism grinds out a new solution together with a new set of prices. The change in prices, of course, will have an external effect on the decision-making of the competitive firm.

Pecuniary external economies and diseconomies[1] refer to just one variety of an infinity of possible outcomes of the adjustment mechanism. As such, there is no reason whatsoever to suppose that they ever would lead to misallocation,[2] barring the cases where the adjustment mechanism

---

[1] A more detailed treatment will be given below.

[2] That pecuniary external diseconomies are but transfer costs in a purely competitive system, being instrumental rather than detrimental for the achievement of the optimum, was the outcome of the well-known controversy between A. C. Pigou and A. A. Young. For a detailed discussion of this argument, see H. S. Ellis and W. Fellner: "External Economies and Diseconomies," in the American Economic Review, Vol. XXXIII, 1943, pp. 493-511.

is already crippled by a more basic factor, namely, output restriction. In this latter case the blame should be properly allocated to those elements of the system that are responsible for output restrictions: monopolies and their competitive "pseudo-counterparts," firms operating with technically interconnected production functions.[1]

Those who attach a great significance to the role of external pecuniary economies in a developing economy operate under the implicit assumption that the process of economic development necessarily has to create a significant number of socially uncontrollable monopolies and external technological effects. This, of course, may be a valid assumption, but the proper conclusion is that greater attention should be given to the real cause of the divergences between marginal private and social products such as the different types of output restrictions.

A last word of caution is in order in connection with external pecuniary effects. The discussion of these effects involves the notion of a straightforward functional relationship between the output of the different industries and factor prices. For expository purposes—and with silent theorising—such functional relationships are used in partial equilibrium analysis. They vanish, however, in a general equilibrium setting, where depending on the nature of the shifts involved, "anything can happen."

Since the concept of external economies, used in a vague sense rather than in a rigorously defined form, is gaining importance in the theory of economic growth, a precise analytical formulation of the issue would be necessary to eliminate the present ambiguities.

---

[1]A more detailed treatment follows below.

In the following an attempt will be made to present the concept of external economies as conceived by neoclassical economists. This will be done both in the context of partial equilibrium analysis and welfare theory.[1]

---

[1] The exposition relies on considerable literature which, however, due to limitations imposed by space, cannot be discussed. In addition to those who are noted in the text, the following important contributors should be mentioned: W. J. Baumol, R. L. Bishop, H. S. Ellis, W. Fellner, A. Lerner, P. A. Samuelson, and T. de Scitovsky.

I.1 The Concepts of Internal and External Economies and Diseconomies of Scale

1.1 Internal Economies and Diseconomies

Though the aim of this part is to clarify problems created by external economies and diseconomies, it seems necessary at this point to refer back to the forces underlying the slope of the unit cost curve. This has to be done for two reasons. One is the fact that the presence of external economies in one firm may be the manifestation of internal economies in another firm. Secondly, by integration, horizontal or vertical, economies previously external to one or more firms can be incorporated into the cost function (a phenomenon frequently referred to as internalisation of external economies), and vice versa.

1.2 It is customary to refer to the average cost curve of a firm in its generalized form as a U-shaped one. It is an empirical observation that as a firm gradually increases its output, its average cost decreases due to factors which are commonly referred to as economies of scale. There is, however, a critical output (more realistically a critical range of outputs) where the economies of scale are just balanced by diseconomies which accompany increases in production. From this critical point (or range) average cost increases as production increases, economies of scale being increasingly outweighed by the diseconomies.

1.3 Non-linearities in the cost function can be attributed to the characteristics and properties of inputs and processes. Most of the causes underlying economies and diseconomies of scale can be described in terms of

a. indivisibility (lumpiness) of factors of production

b. interaction of production units (both physical and organisational)

c. purely technological facts.

The above categories are by no means completely independent of each other. Indivisibility, for instance, is clearly a technological fact. Nevertheless, it is useful to differentiate between technological facts of this nature, on the one hand, and those that are concerned with phenomena such as the relation between diameters and volumes or relative heat losses of a furnace.

The group of factors producing economies of scale, classified under "interaction of production units," refers to the indivisibility of certain productive processes, the law of diminishing returns, organizational advantages and disadvantages, etc. To illustrate, let us take a few examples. Economies of scale derived from the division of labor would originate clearly from the causes listed under (a) and (b). Labor is not finely divisible;[1] consequently, a productive operation has to reach a certain size before specialization can begin. Specialisation in itself is not enough, however, to result in economies

---

[1] Labor (as all inputs and outputs) is finely divisible if counted in units per time. This is, however, irrelevant in the context of specialisation where physical divisibility for any given time instance is required.

of scale. It has to be stipulated that the interaction of specialized labor with the other productive factors will result in a more efficient productive process than that of the non-specialized labor.

Diseconomies of scale will always be present (sooner or later) when one or more productive factors are fixed in quantity while others are variable. Also organisational complications will arise as the productive process becomes more complicated (through specialization) and the volume of production increases. Here again a differentiation can be established between purely technical causes and interaction: diseconomies of interaction would manifest themselves in the increasing inefficiency of the bureaucracy, while purely technical factors would act through phenomena such as the quadratic increase of communication lines as communication points increase.

1.4 External Economies and Diseconomies

The definition given by neoclassical economists to external economies and diseconomies refers to the effects of the total output of the industry on the cost curve of the individual firm. More specifically, in the case of external economies the cost curves of individual firms within the industry will shift downward as industry output increases, whereas in the presence of external diseconomies the shift is in the opposite direction.

An objection can be made at this point against the use of the industry concept in this context. On the one hand, the concept of industry suffers from ambiguity; on the other hand, it seems unduly restrictive to limit the analysis of external economies to the reactions of individual cost curves to the output of the industry alone and to exclude firms outside the industry.

1.5 The shift in the cost curve, necessary for external economies or diseconomies, is induced by a change in the data from which the cost curve is derived. Cost curves are based on the current state of technology and the prevailing market prices of the productive factors. A purely competitive firm (acting as such both in the product and factor markets) will not be able to affect the prices of inputs by its individual decisions. As a consequence, its manager accepts the prevailing factor prices as given and bases his maximising decisions on them. If the price of an input changes, the optimal factor combinations will change, and the cost curve shifts upward or downward (usually, however, not in a parallel fashion).

Another alternative is that some existing interrelation between the production functions of two or more firms, operating on the efficiency of the productive inputs, will have an effect on the output of the individual firm. Whenever one of the technically interconnected firms changes its scale of operation (output or factor quantities), the output of the other firms also changes. The result is a shift of the individual cost curves (even if factor prices remain constant).

Based on these alternative ways by which cost curves may shift, Viner[1] classified economies and diseconomies as pecuniary and technological.

---

[1] J. Viner, "Cost Curves and Supply Curves," in A.E.A. Readings in Price Theory (Chicago: Richard D. Irwin, Inc., 1952) reprinted from Zeitschrift fuer Nationaloekonomie, Vol. III (1931).

Accordingly, pecuniary external economies and diseconomies operate through changes in factor prices induced by shifts of the total demand for productive factors in response to a change in the output of the industry (or any other firm in the economy). Technological external economies and diseconomies operate through a change in the efficiency of the productive inputs in response to a change in the total output of the industry (or any other firm), even if the prices of productive factors remain constant.

It might be more appropriate, however, to reformulate the definition of external technological economies and diseconomies in terms of the productive factors rather than the output of the firm whose activities give rise to these effects. In subsequent parts of the study this formulation will be adopted, unless otherwise specified. Such a conception broadens the generality of the notion by including all technological interconnections--those which operate through the output of a firm and also the ones where the effects are the result of a different choice of factor combinations. Accordingly, external technological economies and diseconomies operate through a change in efficiency of the productive inputs of a firm in response to a change in the total quantities of productive inputs employed by the industry (or any other firm), even if the prices of productive factors remain constant.

The difference between pecuniary and technological external effects is readily demonstrated by the use of a mathematical model. Let c_i be the average cost of the i-th firm producing the commodity x_i, V_j^i the j-th factor employed in the production of x_i, P_j the price of the j-th factor and x_k the product of the k-th firm. Then, using vector notation, write

$$c_i = c_i[x_i(V_j^i, V_h^k), P_j(x_k)]$$

$$i = 1, \ldots k, \ldots n \,; \; j = 1, \ldots h, \ldots m \,;$$

$$V_j^i = V_1^i, \ldots \ldots V_j^i, \ldots V_m^i \,;$$

$$P_j = P_1, \ldots \ldots P_j, \ldots P_m \,;$$

$$x_k = x_1, \ldots \ldots x_k, \ldots x_n \,.$$

This relationship states that the average cost of the i-th firm is a function of its output (function of technology) and the prices of the productive inputs. The production function, however, stipulates in addition to productive inputs also the inputs employed in the production of the output of the k-th firm. The rationale is that if V_h^k changes, x_i will also change, either because of the direct influence of V_h^k (or x_k) on the production of x_i or, because a change in V_h^k has an effect on the form of the production function of x_i which, in turn, warrants a reorganisation of the productive factors in order to fulfill the requirements of least cost production.

Prices of productive inputs are shown as functions of the outputs of the other firms in the economy.[1] If the total demand for inputs

---

[1] It is important to realise that the functional relationship $P_j = P_j(x_k)$ is based on a good amount of implicit theorising and serves only expositional purposes. In the general multi-good case we have no knowledge of what the properties of this function would be either in a purely competitive or in a monopolistic situation. The same remarks do not refer to the function describing technological interrelations. Here, at least under ideal conditions, the exact form and properties of the function can be stipulated.

changes as a consequence of the activity of the other firms, the prices of the productive factors used by the i-th firm will change.

The total change in c_i due to external effects (other factors being held constant) is given by

$$dc_i = \frac{\partial c_i}{\partial x_i} \cdot \sum_{k,h} \frac{\partial x_i}{\partial V_h^k} \cdot dV_h^k + \sum_{j,k} \frac{\partial c_i}{\partial P_j} \frac{\partial P_j}{\partial x_k} \cdot dx_k .$$

The first half of the right-hand side sum refers to technological, the second half to pecuniary external effects.

Pecuniary external economies will be present if

$$\frac{\partial c_i}{\partial P_j} \cdot \frac{\partial P_j}{\partial x_k} < 0 \qquad i \neq k$$

and pecuniary external diseconomies will be present if

$$\frac{\partial c_i}{\partial P_j} \cdot \frac{\partial P_j}{\partial x_k} > 0 \qquad i \neq k .$$

These conditions hold since the first member of the product is always positive and the second by assumption is negative for external economies and positive for diseconomies.

The case of external technological effects is somewhat less straightforward. The sign of $\frac{\partial c_i}{\partial x_i}$ will depend on whether production takes place at falling or rising unit cost and is independent of external effects. For this reason we can consider only the change of x_i with respect to V_h^k. Thus technological external economies will operate if

$$\frac{\partial x_i}{\partial v_h^k} > 0 \qquad i \neq k$$

and external technological diseconomies will operate if

$$\frac{\partial x_i}{\partial v_h^k} < 0 \qquad i \neq k .$$

Notice that in the case of external pecuniary diseconomies setting i equal to k, we get monopsony power within the i-th firm.

1.6 Technological external economies and diseconomies are usually considered unimportant and rare in the neoclassical literature. The examples given refer to the overcrowding of an unappropriated natural resource (such as fishing) or excessive use of a public highway. Further examples are given by Meade which are, in Scitovsky's terminology, somewhat bucolic in nature--concerning themselves with apples and bees and forests and grain.

A somewhat different case of external technological economies is the reorganization of the industry when a certain size is reached (such as the creation of a sales agency). In this case the production functions of the individual firms undergo a change. In order to minimize cost for any given production, an appropriate change will be necessary in the selection of the quantities of productive factors.

As opposed to the technological category greater weight was attached to the pecuniary class, particularly to external pecuniary diseconomies. Even in the case of pure competition where individual producers are confronted with horizontal factor supply curves, the industry in its entirety may well be able to affect factor prices causing the cost curves of the individual producer to shift. This is even more so if the factor in question is specific to the industry in the sense that the industry might be employing a substantial part of the total factor supply.

Pecuniary external economies refer to situations where an increased demand for a factor results in a lower factor price. It is usual to think in this context of factors (intermediate goods) which are produced at falling unit cost, implying the presence of monopoly. It is, however, objected--and justly so--that in addition to decreasing cost the shift of the demand curve has to be such as to induce an increase in output and a willingness on the monopolist's part to share the benefits of falling cost with the purchasing firm.[1]

Integration (horizontal or vertical) will transform external economies or diseconomies into internal ones. It is interesting to note, however, that integration may change the character of the economies or diseconomies in question. Thus, a <u>pecuniary</u> external economy can be transformed into a <u>technological</u> internal economy by vertical integration. The opposite is also true. <u>Technological</u> internal economies can be changed into <u>pecuniary</u> external economies by vertical disintegration.

---

[1] It can be easily demonstrated that a rightward shift of the demand curve may have perverse effects on the monopolist's profit maximising decision in the sense that the new output may be smaller rather than bigger relative to the previous quantity produced.

It is worthwhile to mention at this point the possible significance of this change of character for the theory of economic development. The process of development can be thought of--in an abstract and limited way--as a process of continuous externalisation of economies and diseconomies through vertical disintegration. If one single firm is established in an unindustrialised country, all stages of production will have to be realised by this firm. It will have to create its own power, service departments, etc. As more and more firms operate, an increasing number of service and utility industries will take over the production of those operations which previously constituted--to a varying degree--an integral part of the productive activities of each individual firm. The process itself is vertical disintegration. The result is a continuous change in cost structure where internal <u>technological</u> economies and diseconomies are transformed into external <u>pecuniary</u> ones. Notice, however, that such a view, abstract as it is, takes the creation of monopolies for granted.[1]

1.7 External economies or diseconomies shift the cost curves of the individual firms upward or downward depending on their relative strength, and only in the borderline case, when diseconomies exactly offset economies, would the cost curves maintain the same position. The net manifestation of external economies and diseconomies in the industry supply curve is in its downward or upward sloping character.

---

[1] The term monopoly in this context is not necessarily identical to the concept as used in connection with Western capitalism. Monopolies may arise in underdeveloped countries due to the lack of vigorous entrepreneurship rather than the wilful exclusion of competitors from the market.

A horizontal-industry supply curve, however, does not necessarily imply the absence of external economies and diseconomies; neither is an upward sloping industry supply curve a necessary implication of the presence of external diseconomies (cost curves of individual firms may not necessarily be at an equal level).

1.8 Reversible and Irreversible External Economies

The concepts of irreversibilities and reversibilities were conceived by Marshall.[1] His notion was that certain external economies once achieved by an increased industry output would remain effective, even if output contracted again. Clearly such would be the case if more extensive industrial activity resulted in the bettering of the skills of labor or in other technical progress.

External economies manifest themselves in a downward-sloping industry supply curve. In this case an increase in demand decreases the equilibrium market price. Such a representation, however, makes the handling of the irreversibility problem difficult in a static framework. It may be more illustrative, even if not more legitimate, to conceive of a horizontal or positively sloped supply curve shifting in response to a shift in demand in such a way that the new equilibrium intersection is at a lower price than previously. If this operation works in both directions, then the process is reversible (which it

[1] The terminology itself is post-Marshallian. The concepts are scattered widely throughout Marshall's Principles of Economics (8th ed.; London: Macmillan & Co., Ltd., 1920), pp. 266, 271, 284, 317, 615, 808, etc.

really has to be under strict statics). If it works only towards the right, then it is irreversible.

Though Marshall was sufficiently vague about forward-falling supply curves and irreversibilities, it is clear that he worried not only about finding an explanation for production at falling unit cost alone but also for the irreversible gains of urbanization, the coalescence of markets, the clustering of producers around a labor pool, induced versus autonomous technological progress, and other phenomena necessarily dynamic in character. There is little doubt that the importance of external economies lies along precisely these lines and that the bucolic and other examples which fit into the framework provided by the assumptions of the neoclassical analysis understate the significance of external economies.

I.2 Distortions Caused by External Effects in Optimal Allocation

Much of the discussion concerning external economies and diseconomies is centered around the problem of divergences between private and social marginal cost. Under what conditions will the different external effects give rise to these divergences? The answer is provided by investigating each external effect in the context of welfare analysis. For this purpose the assumptions on which the welfare system is constructed should be spelled out carefully. In so doing, much of the misunderstanding frequently encountered in the literature can be clarified.

2.1 The Assumptions of the Neoclassical Welfare Analysis

Let us adopt the usual neoclassical procedure by separating the problem of efficient production allocation from exchange conditions.

The usual neoclassical assumptions for dealing with problems of production are:

1. given, fixed supplies of productive factors, invariant to changes in factor prices which are allocated by firms according to the rules of least cost production, subject to limitations provided by the production functions;
2. Given production functions have a convex surface;

3. Pure competition prevails in all markets;

4. Technological external effects do not exist.[1]

Based upon the given factor supplies and production functions, we construct an "Edgeworth box" where the tangency of the isoquants will provide the locus of efficient production points (generalised contract curve). The meaning of efficiency in this context is that holding all products but one at a constant level, it should not be possible to produce more of that one good by a suitable rearrangement of the productive factors.

Along the contract curve the ratio of the marginal product of a factor in the first line of production to that in a second line of production will be equal to the ratio of the marginal product of any other factor in the first line of production to that in the second line of production. The reciprocal of this ratio is further equal to the ratio of the marginal cost of the first good to that of the second good.

It is a theorem that under the assumptions stated, the conditions of a purely competitive optimum, based on the principle of profit maximization, always coincide with those of efficient production, i.e., the firms will produce at a point on the generalised contract curve, where any increase in the output of one good necessarily results in a decrease in the output of one or more other goods.

---

[1] Condition 4 also excludes the existence of unappropriated factors of production.

A somewhat simpler device than the generalised contract curve is the transformation curve (derived from the generalised contract curve). Production which takes place at any point other than those defined by the transformation curve is inefficient or impossible. Pure competition will always lead to efficient production, i.e., at a point on the transformation function itself.

2.2 Differences between Private and Social Marginal Products

With the assumptions listed above no difference can exist between private and social marginal costs. This follows from the fact that each firm's output depends only on the quantities of its own factors.

Let us now relax some of the assumptions listed under 2.1 and investigate the ways by which discrepancies between the social and private marginal product come into existence. Assume the existence of some technological interaction between two productive processes. The examples given below will show their effects on the maximising decisions of the managers of the processes.

Take first the following case where x_1 and x_2 stand for the two outputs, V_1 and V_2 for two different types of factors employed in the two processes respectively. The factor V_1 is not employed by the second process; it is, however, shown in the production function of x_2 as its presence has a beneficial influence on the production of x_2. The same with V_2 for the production of x_1.

Then the following equations will denote the two processes:

$$x_1 = F_1(V_1; V_2)$$

$$x_2 = F_2(V_2; V_1) .$$

It is immediately obvious that each factor will have a marginal product in both processes, whether hired or not. This very fact is the cause of the divergence between private and social marginal product.

In pure competition factors are so hired that the value of their marginal product should equal their respective price. In our case, each manager hires only one of the factors; the other factor, the presence of which exerts a beneficial influence on the process, is hired by the other firm respectively. In so doing--and in the absence of collusive practices--each manager will calculate the value of the marginal product of his hired factor based on its marginal productivity in the process where it is employed, and he will neglect its marginal productivity in the other process.

If P_x stands for product prices and P_v for factor prices, then the value of the factors' social marginal productivity (VSMP) is defined by

$$VSMP_1 = P_{x_1} \frac{\partial F_1}{\partial V_1} + P_{x_2} \frac{\partial F_2}{\partial V_1}$$

$$VSMP_2 = P_{x_2} \frac{\partial F_2}{\partial V_2} + P_{x_1} \frac{\partial F_1}{\partial V_2}$$

and optimal allocation is achieved when

$$P_{V_1} = VSMP_1$$

$$\text{and } P_{V_2} = VSMP_2 \ .$$

But hiring is done, as stated previously, to fulfill the conditions[1]

$$P_{V_1} = P_{X_1} \frac{\partial F_1}{\partial V_1} < VSMP_1$$

$$P_{V_2} = P_{X_2} \frac{\partial F_2}{\partial V_2} < VSMP_2 \ .$$

It follows that the quantity of factors employed in both processes is less than the socially desirable amount.

A different example, similar to the one provided by Meade is the following case.[2]

---

[1]In the case of external technological diseconomies the inequalities are reversed, and we are confronted with higher employment in both processes than socially desirable.

[2]J. E. Meade, "External Economies and Diseconomies in a Competitive Situation," the Economic Journal, Vol. LXII, March 1952, pp. 54-67. Actually, Meade gives an example for his reciprocal interaction model which fits more the case of the interacting factors described above than his own model. He conceives of an orchard and a bee-keeping firm interacting in such a way that an increase in the output of one affects the output of the other, which, in turn, has its effect on the first, etc., in an infinite converging series. It is, however, the factors of production and not the respective outputs which interact. If the beekeeper increases the number of hives, more apple blossoms will be fertilised, but the resulting increase in the output of apples will not in any way affect the output of honey. The opposite is also true.

(footnote continued on next page)

Two production functions are given and the favorable effect of the output of the other process is taken into account explicitly in each. If x_1 and x_2 stand for the respective outputs and $V_j^{1,2}$ for the factors employed, then

$$x_1 = F_1(V_j^1 \, ; \, x_2)$$

$$x_2 = F_2(V_j^2 \, ; \, x_1) \quad .$$

Holding every factor constant but one in the first industry and permitting the reciprocal technical interaction to operate freely, the marginal physical product of the one variable factor is given by the following expression[1]

$$\frac{dx_1}{dV_j^1} = \frac{\dfrac{\partial F_1}{\partial V_j^1}}{1 - \dfrac{\partial F_1}{\partial x_2} \cdot \dfrac{\partial F_2}{\partial x_1}} \quad .$$

(footnote continued) There is no infinite interaction series, though—as in the case discussed above—each factor, whether employed in bee-keeping or in the orchard, will have a marginal productivity in <u>both</u> processes. Though Meade's "real life example" does not quite fit <u>his</u> model, his analysis of the model is accurate and stimulating. A comment is in order, however, on the second part of his paper which refers to what he calls the "creation of atmosphere." There is no real distinction between his model of creating atmosphere and his more general models: it is nothing but a special case of the latter (having the general form $x = F(L, C)\, A(Z)$, where $A(Z)$ is the "atmosphere factor"). The feature of the model, which lends it special character, is in the form of interaction, resulting in a change in the affected production function such that the marginal rates of substitution between factors remain unchanged in the process. Actually, there is no reason to believe that a "change in atmosphere" would have no effect on the marginal rates of substitution.

[1]The expression is obtained by totally differentiating both production functions. The partials involving the factors held constant are zero. The rest follows easily.

A similar expression can be obtained for the total change in x_2 with respect to the same variable factor employed in the first process:

$$\frac{dx_2}{dV_j^1} = \frac{\frac{\partial F_2}{\partial x_1} \cdot \frac{\partial F_1}{\partial V_j^1}}{1 - \frac{\partial F_2}{\partial x_1} \frac{\partial F_1}{\partial x_2}} \quad .$$

The above expressions are nothing but instantaneous multipliers summing the infinite series of repercussions of reciprocal interaction set into motion by hiring an additional unit of the variable factor. To derive further conclusions, we have to assume now that the system is stable. In other words, the growth of outputs due to interaction must converge to a finite limit.[1]

The social marginal product of the factor V_j^1 will consist of its marginal product, taking repercussions into account, <u>both</u> in the production of x_1 and the production of x_2. The relevant equilibrium condition for a social optimum is then

$$P_{V_j^1} = P_{x_1} \frac{dx_1}{dV_j^1} + P_{x_2} \frac{dx_2}{dV_j^1} = VSMP_{V_j^1} \quad .$$

---

[1] The assumption of convergence and of favorable interaction imposes the following boundary condition:

$$0 < \frac{\partial F_1}{\partial x_2} \cdot \frac{\partial F_2}{\partial x_1} < 1 \quad .$$

But each manager, with no concern for what good the factor will do for the other process equates $P_{V_j^1}$ to the value of the marginal product of the factor based on its marginal productivity in the process where it is hired. Thus

$$P_{V_j^1} = P_{x_1} \frac{dx_1}{dV_j^1} < VSMP_{V_j^1}$$

and the amount hired of the factor will be less than the socially desirable amount for proper allocation.[1]

[1]Notice that if either $\frac{\partial F_1}{\partial x_2}$ or $\frac{\partial F_2}{\partial x_1}$ are zero in the multiplier-marginal product expression, then $\frac{dx_1}{dV_j^1} = \frac{\partial F_1}{\partial V_j^1}$. In this case there is no mutual interaction. However, the fact that one of the partials is not zero indicates that there is a one way interaction. It follows that some factors must have marginal productivities also in the process where they are not employed. The analysis of this case is similar to the one presented in the first example.

It should be mentioned that Svend Laursen in his paper "External Economies in Economic Development" (presented at an Evening Seminar of the Center for International Studies, M.I.T., on December 16, 1954) is in error when he claims that the divergence between the social and private marginal product is found in the fact that $\frac{dx_1}{dV_j^1}$ is greater than $\frac{\partial x_1}{\partial V_j^1}$ (see p. 9). Actually, in the case of one-sided interactions the two are equal; in reciprocal interaction $\frac{\partial x_1}{\partial V_j^1}$ loses its significance and the true private marginal product of the factor is $\frac{dx_1}{dV_j^1}$, reflecting the multiplier relation.

2.3 The example of the technical interaction clarifies the problem in the framework of welfare allocation.

It was stated earlier that the condition of efficient allocation is the equality of the rates of substitution, these being equal to the reciprocal ratio of the corresponding marginal costs. In a competitive equilibrium each firm equates price to marginal cost. It is then tautological that the price ratio of two commodities has to be equal to the ratio of the corresponding marginal costs. The ratio of the marginal costs defines the slope at each point of the transformation curve, and it follows that the price line has to be tangent to the transformation curve under conditions of efficient allocation. This condition is automatically achieved by universally operating pure competition (under the stated assumptions) since the conditions of competitive equilibrium are identical with those defining the equality of rates of substitution for each individual and each factor of production. This implies, in effect, that marginal social costs have to be equal to marginal private costs or, in other words, it is indifferent whether we equate the ratio of marginal social costs or marginal private costs to the respective price ratios, since the optimum will be reached in exactly the same way. The conclusion, as stated earlier, holds <u>only</u> if the proper assumptions are valid.

We have demonstrated the way divergences between marginal private and social products come about when technological interaction exists among two firms. Similar divergences will be created by monopoly and monopsony influences which will be discussed in paragraphs below. The cause of the divergences is then in the system's inability to compensate or penalise those marginal activities of individuals (executed wilfully as in the case of monopoly practices or unwittingly as in the case of external technological economies) which are desirable or undesirable for the social whole.

The divergence and its effect can be represented in the simplified case of a two-dimensional diagram where the price line cuts across the transformation curve rather than being tangent to it.

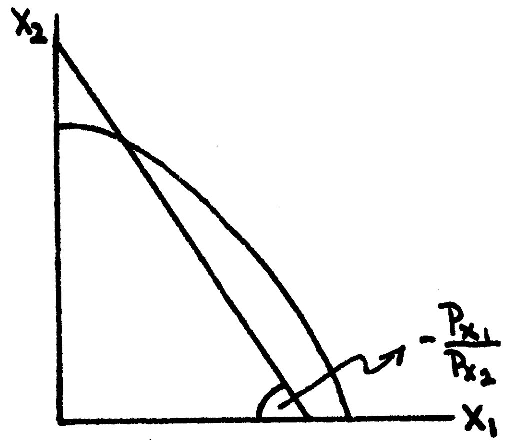
Assume now that production takes place at a point on the transformation schedule[1] and that the good on the horizontal axis is being produced under conditions where its marginal private cost is relatively higher than its social equivalent (as was the case in the example given under 2.2). The price line under these conditions will be steeper than the slope of the transformation curve at the point of production (see Figure 1A). If the marginal social cost, on the other hand, is higher than private marginal cost, the price line is less steep than the slope of the transformation curve at the point of intersection (see Figure 1B).

Though under these assumptions there will be no unemployment and factor allocation is efficient in the purely technical sense (one of the goods being held constant, the other one is maximised), however, the national product is not maximised in terms of value (i.e., the commodity composition of output could be improved from the point

---

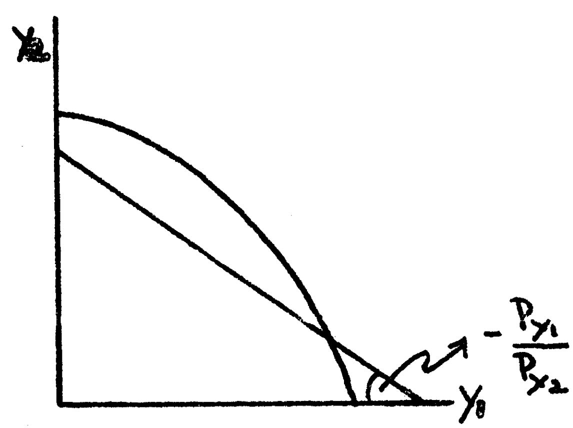
[1]This will be the case if the factor market is competitive.

Figure 1A



External Technological Economies

Figure 1B



External Technological Diseconomies

of view of individual satisfaction).

Mathematically if the transformation curve is defined as

$$T(x_1, x_2) = 0$$

the ratio of the social marginal costs will be

$$\frac{T_1}{T_2} = \frac{\partial x_2}{\partial x_1}$$

where T_1 and T_2 represent the derivatives $\frac{\partial T}{\partial x_1}$ and $\frac{\partial T}{\partial x_2}$.

The optimum output will be achieved where

$$\frac{T_1}{T_2} = \frac{P_1}{P_2} .$$

At this point factors are allocated efficiently and the value of the total product is maximized. Should the private marginal cost of x_1 exceed its social marginal cost, then

$$\frac{T_1}{T_2} < \frac{P_1}{P_2} .$$

If private marginal cost is smaller than social marginal cost, then

$$\frac{T_1}{T_2} > \frac{P_1}{P_2} .$$

Having reached this far in the analysis, we shall now complete the system by including demand conditions.[1] The price ratio $\frac{P_1}{P_2}$, of

---

[1]Assume independent consumer tastes and rationality in maximising satisfaction for each and every individual.

course, is not a given but the outcome of the general equilibrium mechanism. Equilibrium will be reached at a point where the marginal rate of transformation (the slope of the transformation curve) is equal to the marginal rate of substitution (the slope of indifference curves) for each and every individual.[1] The slope of the budget line being tangent to both the transformation curve and the highest attainable indifference curve of each individual defines the price ratio $\frac{P_1}{P_2}$. Under such conditions each person's satisfaction is maximised.

It is again a theorem that universally operating pure competition will lead to a result which satisfies the conditions of double tangency.

It was demonstrated that technological external effects within the system violate the optimal production conditions. It follows that the condition of double tangency cannot hold either with the result that the satisfaction of some individuals could be increased without diminishing the satisfaction of the others. This is so even if the price ratio is tangential to each person's indifference curve at the point where the price line cuts through the transformation function (so that factor allocation is efficient technologically).

2.4 It was mentioned at the very outset of this part that external pecuniary effects by themselves do not give rise to divergences

---

[1]The aggregation of firms implies the assumption of constant returns to scale in production.

between the private and social marginal ratios. Notice also that the absence of external pecuniary effects was not required among the assumptions listed under 2.1; a social optimum could still be reached by competitive means. Certain types of external pecuniary effects were assumed away implicitly: those which are based on the production of an intermediate good at falling unit costs or on monopsony power of individual firms are ruled out by the assumption of universal competition.

Other external pecuniary effects still remain and operate without being detrimental to the competitive achievement of the social optimum. For instance, an increase of output in one of the goods in response to a change in demand will probably result in bidding up the prices of some of the inputs needed for the production of the increased output (especially the price of the factor specific to the production of the good in question). The changes in factor prices are necessary to the transfer of factors from one occupation into the other. Thus pecuniary external diseconomies operating under such assumptions are instrumental rather than detrimental to the achievement of the optimum.[1] (It may be mentioned that should the contracting industries release every factor at the same rate as the expanding one employs them, no change in factor prices needs to take place.)

2.5 If the adjustment of the "real" factors of the economy are hampered by the output distortions caused either by technical

---

[1]See page 3, footnote 2.

interconnections or monopoly practices, the change in the price system (resulting in external pecuniary effects) reflects the fact that the equilibrium, subsequent to a change in basic data, is not optimal. Since external pecuniary effects of the detrimental kind are usually associated with monopolies, let us briefly examine their effect on the equilibrium system.

It is a well-known fact that a monopolist restricts his output. In deciding the proper quantities of factors to be hired, he equates the marginal revenue product (marginal revenue times the marginal product) of the factor to its price. The consequence is lower employment than would be desirable from the social point of view.

The diagrammatic representation of monopoly effects (assume that the good on the horizontal axis is produced under monopoly practices) is identical to that of external technological economies (see Figure 1A above).[1] If production takes place under conditions

---

[1]Since a monopolist can produce in a phase of increasing returns and the presence of external technological effects may imply that purely competitive industries have a forward-falling supply curve, it is relevant and important to investigate the effects of increasing returns on the transformation curve. It is customary to show the presence of increasing returns by the concavity of the transformation curve partially or throughout its whole range. It is also customary to refer to a convex transformation curve as one reflecting increasing costs. The convexity of the transformation curve implies, however, increasing costs in terms of <u>one</u> commodity which has to be given up in increasing quantities when the <u>other</u> commodity's production is increased by equal increments (assuming technically efficient allocation). It reveals, however, only little about the properties of the underlying production functions both of which <u>can</u> be in a phase of increasing returns to scale in spite of the convexity of the transformation function. A simplified proof of this proposition is given in the Appendix of Part I.

(footnote continued on next page)

of efficient technical allocation, the price line will be steeper than the slope of the transformation curve at the point of intersection. The intersection itself will take place in that section of the transformation function which is bounded on one side by the axis of the good produced by the pure competitor and on the other side by the point of optimum output and at that point where the monopolist's profits are maximized.

Notice the striking similarity between wilful monopoly behavior and the unwitting practice of technically interconnected firms. Both employ less of their factors of production than the quantities necessary for optimal allocation, guided by the equation of the price of the factor to marginal magnitudes which are <u>not</u> the socially desirable ones.

(footnote continued) Given a concave transformation curve, pure competition can maintain a stable equilibrium at other than vertex points <u>only</u> if the concavity is the result of technological external interactions. Otherwise, pure competition would always result in rushing to one or the other vertex points. It follows that in the case of technological external economies, the mechanics of pure competition may result in minimization rather than maximization. To avoid the added complications created by concave transformation curves in welfare analysis, all situations discussed in the following pages refer to convex transformation curves (even though the underlying functions may operate in a phase of increasing returns to scale).

I.3 The Functioning of External Effects in a General Equilibrium System

3.1 Technological External Economies and Diseconomies

When increasing costs or constant returns prevail throughout the system, external pecuniary effects caused by increasing returns cannot exist.[1] Technological external economies, however, can well be present. As demonstrated earlier, the quantities produced will be under the socially desirable amounts in the presence of these economies unless communal management is possible.[2] This, in turn, may result in monopolistic practices. The question can be posed whether monopoly activity or competitive production is preferable under these circumstances. In both cases the respective outputs are below the socially desirable amount. Monopoly may be desirable if the monopolist's output is nearer to the optimum than would be the output of the pure competitors. This is the more so since monopoly can be made to "behave" with relatively simple measures

---

[1]This statement does not imply, however, that monopolies now do not have to be excluded to avoid pecuniary externalities. Shifts of demand for the monopolists product can be such that an increase in quantity produced may be sold at a reduced price, even though the monopolist produces at increasing average cost. The statement thus refers to the traditional type of external pecuniary economies which may exist if an intermediate good used by a competitive firm is produced on the falling segment of the average cost curve.

[2]See paragraph 2.2, pp. 20-25.

whenever the intersection of demand and marginal cost is above the unit cost[1] (which is the case now by the exclusion of increasing returns).

An alternative possibility is that one of the outputs is produced under monopoly, and the other under pure competition with the presence of external technological economies. If the assumption is made that the level of total employment is fixed in society, then the output restriction practices of the monopolist may increase the competitive outputs towards the optimum.

There are several combinations possible, even within the framework of a two-output economy, of the interaction of monopoly and external technological economies. While some of them will further increase the deviation from the optimum, others may well act to offset already existing discrepancies between the marginal social and private products.

3.2 External Pecuniary Economies and Diseconomies

If one of the processes works in a phase of increasing returns, we know that the maximising decisions of the firm are of the monopolistic kind. It is usually in this connection that external pecuniary economies are attributed the ability to drive a wedge between social and private marginal ratios. Notice, however, that the wedge is there from the very beginning in the form of monopoly activities. The external pecuniary economies are nothing but the

---

[1]The problem of subsidies does not arise.

manifestations of an already crippled equilibrium mechanism. If the monopolist is made to "behave" (equate price to marginal cost), the external pecuniary effects are also made harmless.

In order to investigate the nature of the case, we write that the cost of i-th firm is a function of its own output and of the price of the intermediate good (P_z) which, in turn, depends on the quantity of z produced.[1] The assumption is made that shifts in demand for z are such which bring about an increase of production and a lowering of prices with the monopolist. Accordingly,

$$c_i = c_i(x_i; P_z)$$

$$\text{with } \frac{\partial c_i}{\partial P_z} > 0 .$$

It should be explained that P_z, even without specifying it as in the equation above, is already implicitly present in the function being one of the parameters. Bringing it into the relationship explicitly, we establish P_z as a regular variable. A change in P_z, of course, will result in a different choice of factor proportions. The final effect is the change in the shape of the cost curve accompanied by a vertical shift.

---

[1] For the economies to operate, two industries have to expand their outputs simultaneously which is incompatible with efficient technological allocation in the two good case. The analysis can be extended to include a third good. Here the expansion of one of the industries accompanied by the industry producing an intermediate good will be executed with a simultaneous decrease in the production of the third good. However, this is a system whose properties are not known to us.

Under competitive long-run equilibrium, each individual firm will produce at the minimum unit cost point. From the relation $C_i = xc_i$ we find[1] that

$$\frac{dC_i}{dx_i} = c_i + x_i\frac{dc_i}{dx_i} .$$

At the minimum unit cost point $\frac{dc_i}{dx_i} = 0$ and marginal cost equals average cost.

With the admission of one of the factor prices, $\underline{P_s}$, as a variable, the minimum unit cost and the position of the marginal cost curve becomes a less straightforward proposition. Thus

$$\frac{dc_i}{dx_i} = \frac{\partial c_i}{\partial x_i} + \frac{\partial c_i}{\partial P_s}\ \frac{dP_s}{dx_i} .$$

Substituting into the marginal cost relationship, we have

$$\frac{dC_i}{dx_i} = c_i + x_i \left[\frac{\partial c_i}{\partial x_i} + \frac{\partial c_i}{\partial P_s}\ \frac{dP_s}{dx_i}\right].$$

We immediately see that at the point where $\frac{\partial c_i}{\partial x_i}$ equals zero, marginal cost will not equal unit cost. We know that $\frac{\partial c_i}{\partial P_s}$ must be positive and by previous assumption $\frac{dP_s}{dx_i}$ has to be negative. The sign

---

[1] C_i represents total cost and c_i average cost.

of the product consequently is negative. It follows that "true" marginal cost must be smaller than unit cost at the point where $\frac{\partial c_i}{\partial x_i}$ equals zero. Also, where marginal cost is smaller than unit cost, the latter must have a negative slope.

The competitive output will not under these circumstances result in an optimal quantity. Each individual firm will equate price to the "partial" marginal cost. But even if every firm in the industry would recognise the existence of external economies, they would not increase their output to the proper quantity unless subsidies were provided to offset the losses.

We have seen in the case of technological external economies that in the absence of direct government interference, monopoly may be preferable to pure competition. This is decidedly not the case with pecuniary external economies. The purely competitive industry restricts its output; nevertheless, industry output will expand until the profit of the individual firms is zero.

Substituting a monopoly, the equilibrium output would necessarily be smaller (assuming profit maximizing behavior). This must be so since the monopolist would equate marginal revenue to marginal cost instead of price to average cost (as done by the purely competitive industry).

Since two industries in the system produce below the socially desirable amount, it follows that, assuming technologically efficient allocation, the third industry will produce more than the optimum.

If this industry is transformed into a monopoly by horizontal integration, the subsequent restriction of output and release of factors may result in the expansion of the other two industries in the direction of the social optimum.

A similar case to that of monopoly can be established for the presence of monopsony. Monopsony in itself will create a discrepancy between the private and social marginal product. Care should be taken, however, to differentiate between monopsony power exercised by a single firm as against a purely competitive industry's ability to affect factor prices. In the case of the latter, no harm is done to the attainment of the social optimum: each firm within the industry equates the value of marginal product of the factor to its price. The monopsonist--one single firm--faced individually with a rising factor supply curve will take advantage of the situation by restricting employment to the intersection of the value of marginal product of the factor to its marginal cost (which is higher than its price). The restriction in employment gives rise to distortions to the social optimum.

Monopsony practices may result in external pecuniary diseconomies to a firm which obtains an intermediate good from the monopsonist. This, however, is by no means certain and it will depend on the elasticities of demand and on the nature of the shift of demand involved.

Monopsony can be and usually is exercised in conjunction with monopoly. The following inequalities given by Joan Robinson[1] sum

[1]The terminology used here is <u>not</u> Mrs. Robinson's.

up the different alternatives:

$$VMP_F \geq MRP_F = MC_F \geq P_F \quad .$$

Where VMP_F stands for the value of the marginal product of a factor, MRP_F for its marginal revenue product (marginal physical product times marginal revenue at the corresponding level of production), MC_F for the marginal cost of the factor and P_F for its market price or wage. The left-hand side refers to the monopolistic exploitation of the market, whereas the right-hand side refers to the monopsonistic one.

Though the different combinations may give rise to several ways by which discrepancies between the social and private marginal product can be created, which, in turn, may have external effects on other firms in the system, the subject will not be further discussed. The analysis involved is similar to that presented for external pecuniary economies created by monopolies, complicated by the assumptions which have to be made about demand elasticities.

I.4 External Effects, Changing Skills, and Changing Production Functions

The previous discussion referred to situations where the total supply of each factor and the state of technology was held invariant. Let us consider cases now where both the total supply of factor and production functions may undergo changes. Under these assumptions, if universally pure competition prevails (and technical interconnections between production functions do not exist), the mechanics of pure competition will lead to the social optimum much the same way as in the previous situation.

4.1 One of the important externalities underlying a forward-falling supply curve may be analysed in this connection. It is a well-known argument (which originated with Marshall) that as output increases in the economy, the skill of the labor force also increases. This is, of course, a dynamic argument, but in the subsequent analysis an attempt will be made to present it in a static framework.

The example of increasing skills is an interesting one. It is well suited to demonstrate the essence of the difference between technological and pecuniary external effects and some fallacies to which careless reasoning may lead.

A change in the skill of labor can be analysed in two ways. One alternative is that we assume, production functions given, a fixed physical supply of labor which we convert into "efficiency units" computed by the use of some index based on marginal productivities. When skills increase, the supply curve of labor in efficiency units shifts to the right.

Such an approach, however, is a misleading one (for reasons which will be discussed in a moment) and the notion of efficiency units is ill-defined. So let us turn to the second alternative.

We shall define the production function in such a way that with each change in the skills of the labor supply a new function becomes relevant, while the total supply of the factor remains invariant. This means that as skills change, the isoquants of the function will shift reflecting the various new combinations of inputs which will lead to the production of different specified levels of outputs. In this process isoquants are so rearranged that the corresponding outputs produced by identical factor combinations will be greater after than those before the change in skill. The final result is something similar to Hicks' labor saving innovation.

Let us investigate now what the external effect consists of and how it comes about. Assume that to each level of total output, there corresponds a certain level of skill. When the economy expands (due, say, to a physical increase in the supply of an input), the level of production increases and with it the level of skills of the labor force. It is clear that the totality of firms in the process of increasing the production of the different marketable outputs also produces a nonmarketable output: skills. The creation of skills, in turn, affects each firm's production function in the above discussed manner.

The productive factors engaged by each firm will have a marginal productivity in terms of the good in the production of which they are employed. They also have a marginal productivity in terms of all other products, the production functions of which are affected through the marginal contributions of these factors to the general level of skills.

Each manager, however, as demonstrated in the discussion of external technological economies, will disregard whatever marginal productivity a factor may have in the production of other goods and compute the value of its marginal product based on its marginal productivity in the process where it is employed. The result is that the distribution of labor among the different firms will not be the same as the socially desirable one.

Labor-saving innovations (such as the shift in production functions in response to a change in skills) will have, of course, pecuniary effects. In a one good, two factor economy, it will decrease the relative or absolute share of labor in the distribution of the total product (unless the other factor is redundant). With the change in relative or absolute shares, factor prices will also change which, in turn, will have its effect on the cost curves. Since the shift of cost curves is due to factors external to the firm, the shift can be attributed to external pecuniary economies or diseconomies. The effect of these external pecuniary economies or diseconomies can be analyzed exactly the same way as we have done previously.

Let us consider now what we have demonstrated above. Due to changing skills the firms experience two types of external effects: one is technological (operating on the production function), the other is pecuniary (operating through changes in factor prices).

It is most important to realize that the distortion in welfare allocation is due solely to the distortions caused by the technological external interaction. Whatever distortions the external pecuniary effects may have, they are reflections of the fact that the allocation of factors among the firms is already distorted due to technological interaction. If the latter is offset by appropriate taxation or subsidies, the pecuniary external effects are nothing but the tools of the general equilibrium mechanism adjusting to the shifting of the production functions.

Notice that technological external effects have taken over the role of monopolies in creating conditions which result in pecuniary external effects of the type associated with welfare distortion.

At the outset of this section it was mentioned that there is an alternative way to analyze the problem of increasing skills, and the notion of a supply curve in efficiency units was established. It was also mentioned that the analysis based on the efficiency supply curve (apart from the fact that this is an ill-defined concept) might be misleading. Here is the reason. As the efficiency supply of labor shifts due to a change in total output in the economy, the marginal product curve of labor (in efficiency units) will shift.

This alters factor prices (relative or absolute) and the firm's cost function undergoes a change due to external pecuniary economies or diseconomies. So far, so good. In this type of analysis, however, no mention is made about the technological interconnections among the firms. The careless conclusion is easily reached here that external pecuniary effects are cause for welfare distortions. The distortions caused by the technological misallocation are camouflaged by the shifting efficiency supply curve.

4.2 An essentially similar treatment can be given to the problem of induced technological change. Here an increased productive activity in an industry or the whole economy will result in an improvement of productive techniques manifested by the shifting of production functions. The analysis of the case is not necessary here since it follows exactly the steps of the above reasoning.

Conclusion

The previous part aimed to demonstrate the workings of the pecuniary and technological external effects within the setting of the neoclassical framework. It was demonstrated that pecuniary external effects are no <u>real</u> cause to welfare misallocation, and inasmuch as they lead to distortions, they do so only because the general equilibrium mechanism is already disturbed by the presence of more basic factors. These are monopolies and technological external interconnections among the production functions of the different firms. In the absence of the latter pecuniary external effects are nothing but the manifestations of the general equilibrium adjustment mechanism.

For this reason it seems to be important to approach the problem from the viewpoint of productivity rather than cost. If the social optimum is not achieved, the investigation of the causes should focus on the practices of the monopolies and on the fact that certain productive factors may have marginal productivities--positive or negative--in other than those processes where they are hired.

Monopolies and purely competitive firms operating under the influence of external technological effects have something in common.

Both produce (employ) other than socially desirable quantities. The monopolist does this by equating the marginal revenue product of the factor, rather than the value of its marginal product, to the price of the factor. The pure competitor, on the other hand, does not take into consideration the marginal product of his factor in other lines of production when computing the value of its marginal product. The result is similar.

The solution is in penalizing or rewarding (whatever the case may call for) those marginal activities which lead to misallocation. Benevolent collusion under the circumstances may be desirable. Just as two monopolists (producing complementary goods) can enter into collusive practices which will increase both the welfare of consumers and their own profits, pure competitors by collusive hiring may eliminate the effects of technological interaction.

Appendix to Part I

The production functions of purely competitive industries--under certain assumptions--are homogeneous of the first order. Technical interaction between firms and industries will change the character of the functions by creating increasing returns to scale.

While purely competitive firms cannot produce on a falling segment of the average cost curve, monopolies can. This, in effect, implies that the monopolist may produce in a phase of increasing returns to scale in the production function.

If increasing returns to scale prevail in both industries of a two good, two factor model, what right do we have to assume a convex transformation function?

In the following pages it will be demonstrated that it is possible to have a convex transformation function, even though both production functions underlying it operate in a phase of increasing returns. Before giving a more detailed demonstration, consider the following nonrigorous heuristic argument.

It is a well-known fact that, given two homogeneous functions of the first order, the resulting transformation function may demonstrate strong convexity. By renumbering the isoquants of the two functions in such a way that the resulting increasing returns to scale are infinitesimal in both, the corresponding transformation function will undergo a change. Based on the notion of continuity, however, we instinctively know that a transformation curve with strong convexity will not suddenly assume a concave shape following this operation.

Let us turn now to a more rigorous demonstration of the same proposition by establishing sufficient conditions.

Assume the existence of two production functions, both of them homogeneous of order greater than one, and two factors of production, the supply of which is given.

$x = F(L_x, C_x)$, with the property $\lambda^n x = F(\lambda L_x, \lambda C_x)$; $n > 1$

$y = G(\bar{L}-L_x, \bar{C}-C_x)$ —"—"— $\lambda^n y = G(\lambda(\bar{L}-L_x), \lambda(\bar{C}-C_x))$; $n > 1$

$\bar{L} = L_x + L_y$

$\bar{C} = C_x + C_y$

Let us form the Edgeworth box, the sides of which conform to $\bar{L}$ and $\bar{C}$. The efficient points of production will be defined by the "efficiency locus," i.e., the locus of the points of tangencies of the respective isoquants.

By the assumption of homogeneity, we know that the efficiency locus will be a monotonically increasing function of L_x and will lie on one or the other side of the diagonal of the box without crossing it. It may have, however, an inflection point which, for the sake of simplicity, we rule out.

If $\underline{x}$ is the good produced by labor-intensive methods, then the generalised contract curve is formalised by

$$C_x = \phi(L_x)$$

and the condition of monotonic increase without inflection is given by

$$\frac{C_x}{L_x} < \frac{d\phi}{dL_x} ; \frac{d^2\phi}{dL_x^2} < 0 \text{ for all } L_x \quad .$$

The geometric representation is found in Figure 2. Along the efficiency locus for each capital-labor ratio (defined by the slope of a straight line drawn from the $\underline{x}$ origin to any point of the contract curve), there exists a unique quantity of $\underline{x}$ produced. This follows from the properties of the $\phi(L_x)$ function.

Thus

$$x = x\left(\frac{C_x}{L_x}\right) = x\left[\frac{\phi(L_x)}{L_x}\right].$$

As $\underline{x}$ is a function of $\frac{C_x}{L_x}$ and $\underline{C_x}$ of $\underline{L_x}$, it follows that $\underline{x}$ is a function of $\underline{L_x}$ alone along the contract curve.

Each $\frac{C_x}{L_x}$ designates a unique quantity of $\underline{C_x}$ and $\underline{L_x}$ along the contract curve. Given fixed factor supplies, the residuals will fix for each $\frac{C_x}{L_x}$ unique quantities of $\underline{L_y}$ and $\underline{C_y}$. In effect, for each $\frac{C_x}{L_x}$ there will be a unique quantity of $\underline{y}$ produced along the contract

Figure 2

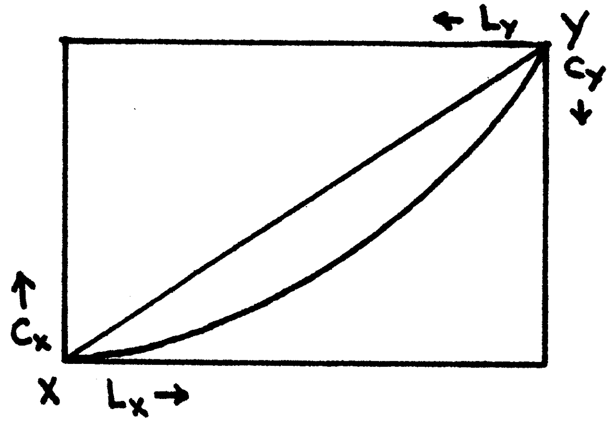


Figure 3A

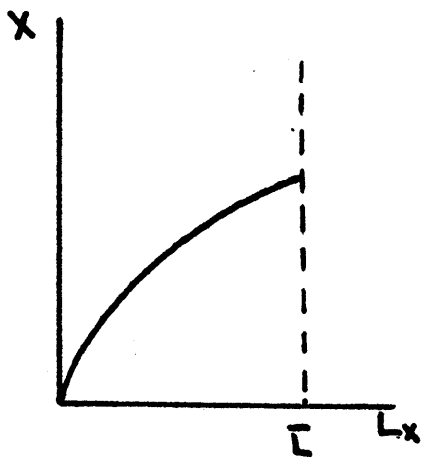
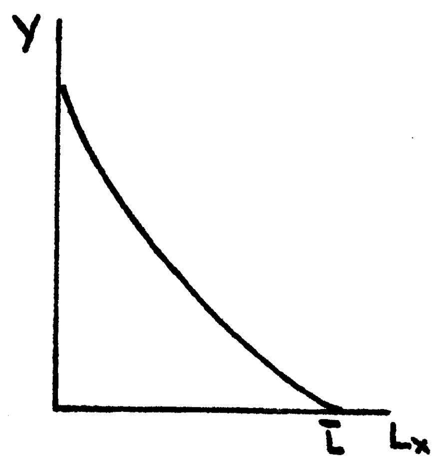


Figure 3B



curve--also a function of L_x. We thus have two parametric equations

$$x = F_1(L_x)$$

$$y = F_2(L_x) \quad .$$

<u>Theorem</u>:

If <u>x</u> is a monotonically increasing convex $(\frac{d^2x}{dL_x^2} < 0)$ function of <u>L_x</u> and <u>y</u> is a monotonically decreasing concave $(\frac{d^2y}{dL_x^2} > 0)$ function of <u>L_x</u>, then the transformation function between <u>x</u> and <u>y</u> has the property that $\frac{d^2y}{dx^2} < 0$. In the limit both <u>x</u> and <u>y</u> may be straight line functions of <u>L_x</u> for the theorem to hold.

The geometric representations are given in Figures 3A and 3B.

To demonstrate the validity of the theorem, two propositions have to be verified.

<u>Lemma I</u>: <u>x</u> and <u>y</u> can be functions of <u>L_x</u> such as described by the theorem, even though the respective production functions are homogeneous of order greater than one.

Lemma II: there exists a relationship between the properties of $F_1(L_x)$, $F_2(L_x)$ and $T(x,y)$ such as to enable us to predict the properties of the latter based on the properties of the former.

To verify Lemma I,[1] let us recall that in our example x is a labor-intensive commodity (there is no loss of generality in this assumption: if x is capital-intensive, an analogous argument can be coined). Let us also remind ourselves that along any ray from the origin x is produced under increasing returns to scale. The contract curve, however, is so defined that for each x there belongs a different $\frac{C_x}{L_x}$ which increases as x increases. In other words, as the production of x increases and factor proportions are so adjusted that the conditions of efficient allocation are observed, the labor intensity of the production is gradually watered down by adopting increasingly capital-intensive methods. As we increase the production of x and with it $\frac{C_x}{L_x}$, the marginal rate of substitution of capital for labor also increases. In effect, with each successive adaptation of a higher $\frac{C_x}{L_x}$, the amount of capital, which one would have to give up in order to stay on the same isoquant if labor were substituted for it in equal increments, increases. If the successive increase in the marginal rate of substitution is significant and the

---

[1] The verification of Lemma I by purely mathematical means is not difficult. Lemma II, however, does not lend itself readily to mathematical treatment. For this reason a verbal proof is retained here for both Lemma I and Lemma II.

order of the homogeneous function is only slightly greater than one, it follows that as L_x increases, x will increase but at a diminishing rate. This proves that part of Lemma I which refers to x as function of L_x.[1]

As the corresponding proof for the statement concerning y is readily found by essentially similar reasoning, the completion of the proof of Lemma I will be omitted.

If we have reached this far, then we are almost home. The proof of Lemma II and with it the proof of the theorem is readily found.

The mark of convexity in the transformation function is that as the production of one of the goods is successively increased by identical quantities, the production of the other good decreases by increasing amounts. (In the continuous case the limit of the increments is taken.)

Does the above property hold in our case? It most certainly does. To demonstrate this the elegant way, assume that x is a straight line function of L_x (the limiting case) and y is a concave ($\frac{d^2y}{dL_x^2} > 0$) function of L_x. The proof of the more general case, in which both functions are nonlinear, readily follows from this special case.

Let us push along the efficiency locus from the origin of x towards the origin of y in successive steps such that the length of each step is determined by an identical increment in L_x.

---

[1]The proof for the limiting case, where x is a straight line function of L_x, follows readily from the above.

In terms of the production of x this will mean identical increments for each successive equal increment in L_x. For the production of y the equal increments in L_x imply successively increasing decrements. This proves the convexity of the transformation function for the special case. What about the general case when x is a convex ($\frac{d^2x}{dL_x^2} < 0$) function of L_x? In this case as L_x is increased by equal increments, the corresponding increments in x will be successively diminishing: a phenomenon which further reinforces the convexity of the transformation function. This completes the proof of the theorem.

Having reached this far, it is worthwhile to point out the following interesting corollary to our theorem.

> Corollary: If the production functions of x and y are homogeneous of order greater than one, and the parametric equations are both straight line functions, then the resulting transformation curve will be also a straight line function. This is so even though the efficiency locus is convex and not a straight line.

The proof of the corollary follows straight from the proof of the theorem and does not need separate treatment.