

A STOCHASTIC REVISED SAMPLED DATA
MODEL FOR EYE TRACKING MOVEMENTS

by

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ABSTRACT

The sampled data model for eye tracking movements of Young is revised to make the pursuit system continuous and proportional to target rate. Target-synchronized and non-synchronized sampler control logic are analyzed. Frequency distributions of latencies to target steps for non-synchronized sampling systems with stochastic intersampling intervals are shown to be strictly non-increasing. Transient responses of the model are shown to agree with classes of observed eye movement including steps, pulses, ramps, and step-ramps. Four experiments are performed in which the occurrence of a target step is synchronized to the occurrence of a saccade. The results show responses corresponding to both synchronized and non-synchronized sampling occurring in the eye movement control system. The sampler control logic in a sampled data model for eye tracking movements, therefore, should have a non-synchronized sampler which can be adapted in phase and frequency depending upon the target movement.

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TABLE OF CONTENTS

<u>Chapter No.</u>		<u>Page No.</u>
CHAPTER 1	INTRODUCTION	1
CHAPTER 2	A STOCHASTIC REVISED SAMPLED DATA MODEL	5
2.1	The Discrete Nature of Saccadic Eye Movement	5
2.2	The Continuous Nature of Smooth Pursuit Eye Movement	7
2.3	A Revised Sampled Data System	9
2.4	The Nature of Sampling	11
2.5	Intersample Distribution for Optimum Unsynchronized Sampled Data Modeling	14
CHAPTER 3	TRANSIENT RESPONSE OF THE REVISED MODEL	26
3.1	Step Response	26
3.2	Pulse Response	28
3.3	Ramp Response	33
3.4	Step-Ramp Response	36
3.5	Ramp-Step Response	39
CHAPTER 4	EXPERIMENTAL PLAN, APPARATUS, AND PROCEDURE	43
4.1	Saccade-Synchronized Data Reduction	43
4.2	The Saccade-Synchronized Inputs	44
4.3	Model Response to Saccade-Synchronized Experiments	47

<u>Chapter No.</u>		<u>Page No.</u>
4.4	Apparatus and Procedures	48
4.5	Target Generation	48
4.6	Eye Position Measurement	50
4.7	Input Generation	51
4.8	Saccade-to-Step Delay Control	53
4.9	Eye Movement Recording	54
4.10	Procedure	55
4.11	Data Reduction	57
CHAPTER 5	RESULTS	58
5.1	Distribution of Initial Saccadic Response	59
5.2	Experiment One - Synchronized Returning Pulse	63
5.3	Experiment Two - Synchronized Stairway Pulse	66
5.4	Experiment Three - Ramp with Synchronized Inward Step	69
5.5	Experiment Four - Ramp with Synchronized Outward Step	72
5.6	Subject Interexperimental Results	72
CHAPTER 6	CONCLUSIONS AND RECOMMENDATIONS	83
6.1	Conclusions	83
6.2	Recommendations for Further Study	85
	REFERENCES	87
APPENDIX A	THE EXPERIMENTAL SYSTEM	91
A.1	The Digital System	92
A.2	The Analog System	97

	<u>Page No.</u>
APPENDIX B EXPERIMENTAL DATA	102
APPENDIX C STOCHASTIC SAMPLED DATA SYSTEM SIMULATION PROGRAMS	127

LIST OF FIGURES

<u>Figure No.</u>		<u>Page No.</u>
2.1	A Revised Sampled Data Model for Eye Tracking Movements	10
2.2	Definition of Symbols Showing Inputs Occurring in Sampling Instants τ_c	16
2.3	Mathematical Descriptions of Latency Histograms	20
2.4	Simulated Latency Histogram for a Constant Sampling Interval System	22
2.5	Simulated Latency Histogram for a System with Sampling Intervals Uniformly Distributed Between 0 and 200 msec.	23
2.6	Simulated Latency Histogram for a System with Sampling Intervals Uniformly Distributed Between 150 and 250 msec.	24
2.7	Simulated Latency Histogram for a System with Sampling Intervals Dual-Uniformly Distributed as Described in Eq. 2.5	25
3.1	Step Input Model Variation as a Function of Synchronization to the Sampling Instants	27
3.2	Beeler's Average Latencies in Response to the First Step of Pulse Inputs	29
3.3	Beeler's Average Latencies in Response to the Second Step of Pulse Inputs	29

<u>Figure No.</u>		<u>Page No.</u>
3.4	Forms of Response Predictable for Pulse Inputs	31
3.5	Percent No Response to Returning Pulse Input	32
3.6	Percent No Response to Stairway Pulse Input	32
3.7	Robinson's 10 deg./sec. ramp response	34
3.8	Step-Ramp Model Response as a Function of Synchronization of Inputs to Sampling Instants	37
3.9	Comparison of Regions of No-Saccade Step-Ramp	40
3.10	Ramp-Step Response	41
4.1	Experimental Inputs	45
4.2	Average Latency Predicted to Saccade-Synchronized Experiments when Presented to Sampled Data Models with different Sampler Control Logic	49
4.3	Typical Eye Monitor Records	52
4.4	Typical Experimental Recording	56
5.1	Saccadic Latency Histograms	60
5.2	Experiment One - All Subjects	64
5.3	Experiment One - Intersubject Means and Model Predictions for Two Sampler Control Logics	65
5.4	Experiment Two - All Subjects	67
5.5	Intersubject Means for Experiment Two and Model Predictions for Two Sampler Control Logics	68
5.6	Experiment Three - All Subjects	70
5.7	Experiment Three - Intersubject Means	71

<u>Figure No.</u>		<u>Page No.</u>
5.8	Experiment Four - All Subjects	73
5.9	Experiment Four - Intersubject Means	74
5.10	Subject CO - All Experiments	76
5.11	Subject NVH - All Experiments	77
5.12	Subject AVH - All Experiments	78
5.13	Subject LN - All Experiments	79
5.14	Subject SF - All Experiments	80
5.15	Subject MK - All Experiments	81
A.1	The Digital System - Program and Input List	93
A.2	Target Movement Circuit	99
A.3	Saccade Detector Circuit	99
A.4	Analog Control Panel Circuits	101
B.1-B.24	Experimental Data for All Subjects to All Experiments	103-126
C.1	Simulation of Constant Sampling Interval System	128
C.2	Simulation of System with Sampling Intervals Uniformly Distributed Between 0 and 200 msec.	129
C.3	Simulation of System with Sampling Intervals Uniformly Distributed Between 150 and 250 msec.	130
C.4	Simulation of System with Sampling Intervals Dual-Uniformly Distributed as Described in Eq. 2.5	131
C.5	Program Notes Applying to all Listings- Line References refer to Figure C.1	132
C.6	Specific Program Notes for each Listing	133

CHAPTER 1

INTRODUCTION

In this thesis, the classes of outputs which arise from varying the timing of an input in relation to the sampling intervals are studied in a refined version of Young's sampled data model for eye tracking movements (23). Target-synchronized and non-synchronized sampler control logic for the model are investigated by probabilistic analysis and simulation of a non-synchronized system, and by four saccade-synchronized eye movement experiments. These investigations reveal that both types of sampler control logic should play a role in a sampled data model for eye tracking movements.

Saccadic eye movements were first distinguished from smooth pursuit movements by Dodge in 1903. A saccadic eye movement or saccade is a burst of muscle force moving the eye rapidly from one angular position in relation to the head, to another. The duration of a saccade is usually less than .1 sec. during which the eye's rotation rate may reach 600 deg./sec. for large saccades (26). The purpose of a saccade in eye tracking movements is to correct position error between the target and the eye.

The discrete nature of the saccadic eye movement control

system has been well established (1, 12, 13, 19, 20, 23). Several discrete systems have been proposed, including pre-programed systems, saturating systems, digitally controlled systems, and sampled data systems. It is questionable whether the minute differences between such systems allow differentiation between them in defining, from present research, the exact nature of the eye movement control system. Any system at all, for example, could be equated to a system pre-programed to give the observed response to each input. Furthermore for inputs requiring only one sample, a target-synchronized sampled data model need only chose stochastic synchronization times to be exactly those of observed latencies minus a constant time delay. This thesis evaluates the assets and limitations of a sampled data model including possible extentions to include in model results, sampler adaptation, and finite sampler width. This modeling does not show that such sampling takes place in the actual saccadic eye movement control system. The model does, however, give a simple analyzable means of predicting saccadic eye movements based on target movement only.

Pursuit eye movement consists of the smooth low velocity eye movements which appear in eye tracking of target movements with low velocity components. Pursuit eye movements usually do not exceed 30 deg./sec. and appear only when the subject is visually tracking a target. The purpose of pursuit movement is to match eye angular velocity to that of the target in order to prevent a build-up of error in eye position between possible saccades.

The experiments of Rashbass (12) and recently of Robinson (14) have lead to a change in the pursuit control system of Young's sampled data model to make it a continuous function of target rate rather than a sampled function of error rate. This revision is discussed in Chapter 2 of this thesis.

If unsynchronized sampling occurs in the eye movement control system, observed latency distributions must be explainable as a function of the stochastic distribution of intersample times. A careful analysis and simulation of the non-synchronized system at the end of Chapter 2 leads to the result that latency distributions must be non-increasing except at the lowest latency where they jump from zero to their maximum value. Actual latency distributions do increase, however, for approximately the first 50 msec. of possible latency. Thus if unsynchronized sampling occurs in the eye movement control system, the eye must be able to shorten the sampling interval if a step input is observed when the eye has not performed a saccade for 0.2 sec.

The transient response of the model is discussed in Chapter 3. In general, the classes of eye movement predicted by varying the timing of input to the first sample agree with observed classes of eye movement.

If an input to the revised sampled data model occurs immediately after a saccade, the sample which caused that saccade and the sample which will cause a response to the input will be separated by only one sampling interval. It follows that the intersampling interval and the sampler control logic would be revealed by the distributions of response form

for inputs occurring at small constant times after a saccade. Four such saccade-synchronized inputs were performed. The results show responses corresponding to both target-synchronized and non-synchronized sampling with the predominant type of control logic changing from experiment to experiment.

The sampler control logic to model the results of these experiments would be a free running non-synchronized sampler which can be influenced in phase and frequency depending upon the difficulty of observed target motion. The trade-off between difficulty and sampler influence, and the complete extension of the model to finite width sampling are topics for further research.

CHAPTER 2

A STOCHASTIC REVISED SAMPLED DATA MODEL

In this chapter, evidence is presented of the discrete nature of saccadic eye tracking movements and the continuous nature of smooth pursuit eye movements. Based on this evidence, a revised sampled data model is presented and its limitations discussed. Target-synchronized and non-synchronized forms of control logic for the sampler are weighed, and the equation for the distribution of non-synchronized intersample times for fit of observed latencies is derived.

2.1 The Discrete Nature of Saccadic Eye Movement

A control system is discrete if it changes its output based on new input information taken in only during very short periods, interspersed by periods of no revision of output no matter how large the error may instantaneously become. The most convincing demonstration of the discrete nature of the saccadic eye movement control system is the response to a target that steps to one side and then returns in 0.2 sec. or less. If the eye responds, and it may not, it responds by making a saccade approximately 0.2 sec. after the first step, despite the fact that this movement is inappropriate since it

takes the eye away from the target. The eye then returns after an additional 0.2 sec. The response predicted by a continuous system with a pure delay would be a pulse of the same duration as that of the target, delayed in time by an amount equal to the reaction time. A continuous system also fails under any circumstance to predict no response. Evidence of these responses have been shown by Westheimer (19) in 1954, Young (23) in 1962, Beeler (1) in 1965, and by the results of this thesis.

Further evidence of the discrete nature of saccadic eye response is found in the results of open-loop and variable feedback experiments. In these experiments, the effective visual feedback is modified by the addition of an external path from measured eye position to target position. In the open-loop experiment, the response to a target step is a staircase of equal amplitude saccades spaced approximately 0.2 sec. apart. The response to variable feedback also shows a refractory time of 0.2 sec. between saccadic responses. These results were found by Young (23) in 1962 and duplicated by Robinson (14) in 1965.

Furthermore, the frequency response of the eye shows an extraneous peak at about 2.5 cps., as would characterize a discrete system with fundamental refractory interval of 0.2 sec. This result was found by Young (23), who recorded subjects tracking a continuous pseudo-random input consisting of a sum of sine waves, and analyzed the response at each of the input frequencies. Young found good agreement with the results of Fender and Nye and others. For an explanation of frequency

response in a discrete system, see Jury (8).

The discrete nature of the saccadic eye movement system also evidences itself in the response to predictive square waves. Young studied subjects tracking a square wave of 0.4 cps., an input known to elicit predictive saccades. He plotted the percent error in the size of the saccadic responses as a function of the time from actual target movement to the saccade. Young found random error even when the target moved before the already commanded saccade by as much as 120 msec. This result was also shown by Horrocks and Stark (6). In this experiment, the eye was unable to modify an already commanded saccade even though new information was available 0.12 sec. before the onset of saccadic movement. The system must be discrete, then, since it cannot take in information over these 0.12 sec. intervals.

One discrete system which exhibits all the properties described above is a sampled data system. Young (23) proposed such a model of the saccadic eye movement control system in which eye position error is sampled and used to command appropriate saccadic eye movements. Young showed that this model agrees well with mean saccadic eye movement responses.

2.2 Continuous Nature of Smooth Pursuit Eye Movement

Rashbass (12) investigated the pursuit eye movement control system by presenting subjects with ramp inputs (constant velocity target movements), and with step-ramp inputs (constant velocity movements preceded by a step movement in the opposite direction). Rashbass found that in response to both ramps

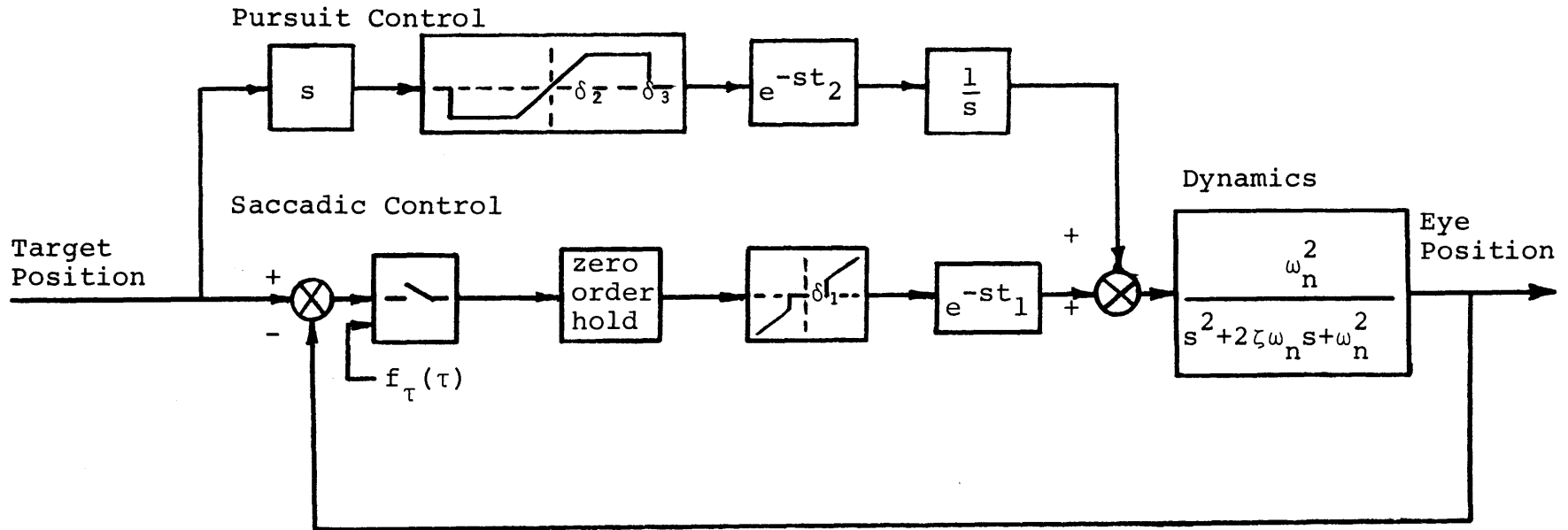
and step-ramps, smooth pursuit eye movement could occur before, during, or after saccadic eye movement. Thus he concluded that the pursuit movement command system was independent of the saccadic movement command system. Since the occurrence of smooth pursuit movement in the direction of the ramp before a saccade in response to the step actually moved the eye away from the target, Rashbass further concluded that the smooth pursuit system was proportional to target rate only. Rashbass also noted that if the step size in degrees is 0.15 to 0.2 times the ramp rate in degrees per second, the eye responded with smooth pursuit motion containing no saccades.

Robinson (14) studied the smooth pursuit system using sequences of the above described step-ramps that elicit no saccades. The responses showed that the pursuit system was capable of making two distinct movements temporally spaced by the same time interval as the input, for spacings as low as 150, 100 and 75 msec. Robinson also studied the pursuit system under conditions of variable feedback by using a differentiator to eliminate saccadic eye movements from the feedback signal. Eye responses to these experiments were always smooth and continuous even in the Tantalus-like chase of positive feedback. Under high (-4 to -8) values of negative feedback, the pursuit system showed sinusoidal oscillations at a frequency of 2.9 - 3.3 cycles/sec. These results are in contrast to variable feedback around the saccadic system which were also shown by Robinson to exhibit discrete responses.

Robinson further showed that discrete changes in pursuit velocity occur at saccadic responses in the same direction. This finding helps to explain why Westheimer (19) and Young (23) considered the possibility of pursuit movements occurring in constant velocity segments between saccades since both their experiments included saccadic responses in the same direction as the pursuit movements. On the strength of these experiments, Robinson concluded that the pursuit eye movement control system was continuous rather than discrete in nature.

2.3 A Revised Sampled Data System

A revised sampled data model with a continuous, target rate sensitive, pursuit loop has been developed by Van Houtte and Forster (4) and is shown in Fig. 2.1. This model retains the sampler of eye position error for the saccadic system and thus preserves the excellent modeling of discrete saccadic response of Young's original model. A model of this type was proposed by Vossius (17) but without any attempt to define parameters. The continuous pursuit loop contains a delay and a differentiator followed by a non-linearity and an integrator. The input to the pursuit system is target position which is differentiated, making the system proportional to target rate only. The pursuit non-linearity exhibits both saturation and a cut-off level. The saturation level of 25-30 deg/sec reflects the actual maximum pursuit velocity of observed eye movement. The cut-off level eliminates pursuit response to small target steps which caused erroneous results in Young's sampled data pursuit system. (see Bleuze (2) and Jury and Pavlidis (9)).



$f_{\tau}(\tau)$ = intersample time distribution

t_1 = saccadic delay (100 msec.)

t_2 = pursuit delay (134 msec.)

δ_1 = foveal dead zone ($.3^{\circ}$)

δ_2 = saturation limit (25 - 30 $^{\circ}$ /sec.)

δ_3 = cutoff limit (100 $^{\circ}$ /sec.)

Dynamics of Westheimer (18)

ω_n = 120 rad./sec.

ζ = .7

Figure 2.1 A REVISED SAMPLED DATA MODEL FOR EYE TRACKING MOVEMENTS

The signal is then delayed before an integrator converts the signal into position commands to the eye dynamics.

This model has the limitation of being a deterministic model of a nondeterministic biological servomechanism. The choice of a deterministic model, however, has the advantage of yielding a system that is simple, analyzable, and easily simulated while retaining all principle characteristics of eye tracking movements. Moreover, by allowing random synchronization of samples, a stochastic distribution of sampling intervals, and a finite pulse width, the variation in model response to the same target motion closely resembles the variation of eye movements actually observed to identical target motions. A study of the nature of the sampler for agreement with observed eye response forms the subject of the following section.

2.4 The Nature of Sampling

In order to retain uniformity of analytic results, Young made the simplifying assumption that, "The synchronization of the modulator must be set to coincide with the beginning of a target motion, if the eye had made no saccadic jumps during the previous 0.2 sec."* Beeler (1) misinterpreted Young to mean the sampler was to remain closed except for the 0.2 sec. immediately following a change in eye position error.

Zuber (28) performed two experiments which suggest that the sampling instants are randomly distributed with respect

* Young (23) page 102

to the input. In the first experiment Zuber presented subjects with ramp-step-ramp inputs in which the magnitude and direction of the ramps were random. He showed that the size of the saccade in response to the step was dependent upon the ramp rate following the step. For ramps moving in the same direction as the step the saccade in response to the step was enlarged, while for ramps in the opposite direction of the step the saccade was smaller than the step. This suggests that some information, available only after the step, was used in the determination of the size of the saccade. This result is consistent with a sample synchronized constant or stochastic time after the step, or a sample unsynchronized to the step, but is not consistent with a sample synchronized instantaneously after the step. This fact is because the movement of the target between the step and the sample would be included in the calculation of the size of the saccade required for all the former systems but not for the latter.

Zuber's second experiment consisted of step inputs followed by periods during which the target was not displayed to the subject. Zuber argued that if the blank target period occurred over a sampling instant, inflated error would result. Zuber allowed a 50 msec. blank period to occur over a range from 10 msec. to 200 msec. after the step, but observed only random oscillations in the error. Thus the sampling must not occur at a constant time after the input but still may be stochastically synchronized or unsynchronized.

Experiments which may indicate a finite pulse width were performed by Wheelless (20) and Beeler (1). Wheelless presented

subjects with pulse-step stimuli consisting of a pulse in one direction with the return step twice the size of the first step, leaving a steady-state error in the opposite direction from the initial pulse. Wheelless noted that in cases in which the eye did not respond to the pulse, the latency in response to the step was inflated by an average of 40 msec. This response indicates that on the average the pulse has not gone completely unnoticed but has delayed the eye in response to the step. For a sampled data system to exhibit this response, the samples would have to be of finite width with some samples occurring during the step from one side to the other, and thus containing both positive and negative error components which cancel each other, resulting in a delay in response until the next sample. When these delays are averaged with all samples which do not cross the positive to negative step, result is an average delay increased by 40 msec. if the pulse width is about 40 msec. Complete interpretation of this experiment must await further study, because, as Wheelless noted, subjects showed inflated latencies to simple step inputs intermixed with the pulse-steps, indicating hesitation in anticipation of a pulse-step pair.

Further evidence of the possibility of a finite sample width comes from Beeler (1). Beeler presented subjects with returning pulses (as described above) and with stairway pulses (second step in the same direction as the first). Beeler observed (see Figs. 3.5 and 3.6) fewer abnormal responses (single response to stairway pulse, no response to returning pulse) in stairway pulses than in returning pulses when the

pulse width was less than 80 msec. This suggests that the eye differentiates in some way between the two and therefore must have sampled over most of the input. The meaning of this experiment is also obscure since the eye may be acting to minimize effort. This criterion would imply the eye is most inclined to not respond, next most inclined to make two smaller saccades, and least inclined to make one large response.

Young noted the possibility of a finite sample width in Young, Stark, and Zuber (26). Further work in this area is necessary to establish the possibility of a finite width sample. Because of the high uncertainty of these results, for the remainder of this thesis models with impulse sampling will not be extended to finite width sampling.

2.5 Intersample Distribution for Optimum Unsynchronized Sampled Data Modeling

If sampling intervals in the eye movement control system are not changed by the occurrence of a target movement, observed latency variations should be explainable on the basis of the distribution of these sampling intervals. This section derives the equation for the required distribution of sampling instants in an unsynchronized sampled data model which would nearly produce observed latency distributions. It is hoped that some link will evidence itself between the model and some control function within the eye movement system.

The following symbols will be used in the derivation:

L = latency predicted by the model

D = delay time from occurrence of the input to the next occurrence of a sampling instant (from this definition, D is strictly non-negative)

t_1 = constant delay time as in Fig. 2.1

τ = time between sampling instants

t = time from last occurrence of sampling instant to input occurrence

$f_x(x)$ = probability density function of the variable x (any of above variables)

Latency predicted by the model is the sum of the time from the input to the next sampling instant plus the constant delay t_1 .

$$L = D + t_1$$

Since D is a random variable, L must also be a random variable and their density functions must be related by:

$$f_L(L) = f_D [D + t_1]$$

The distribution of latencies, therefore, is determined by the distribution of input-to-sample times translated by t_1 along the time axis. This distribution of input-to-sample times ($f_D(D)$) will be computed by assuming the input occurs in an interval of size τ_c , the resulting conditional distribution calculated, and then relieving the condition by integrating the constant τ_c across the entire distribution of intersample times $f_\tau(\tau)$.

If the input occurs within a sampling period of given

width τ_c , D is the difference between the given τ_c and the time t that the input occurs, as seen in Fig. 2.2.

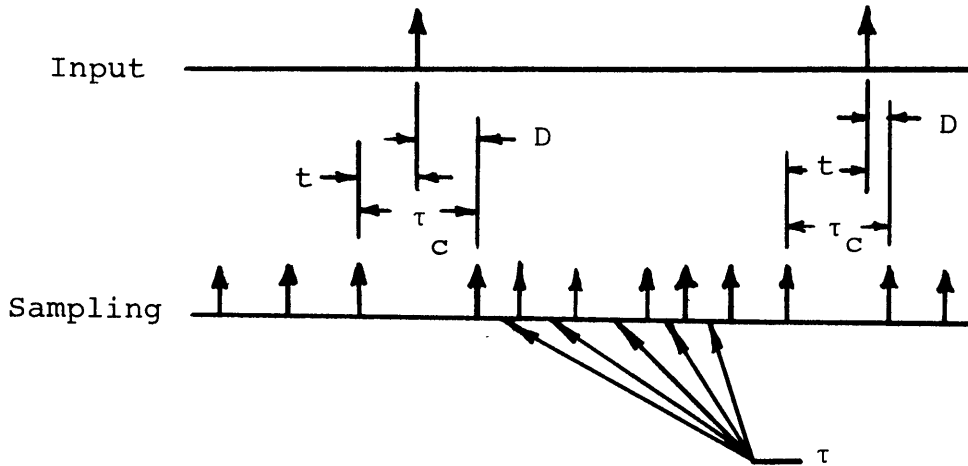


FIG. 2.2: Definition of symbols showing inputs occurring in sampling intervals τ_c .

$$D = \tau_c - t \quad \text{given } \tau_c, \text{ a constant}$$

From the definition of a probability density function:

$$P(a \leq D \leq b/\tau_c) = \int_a^b f_D(D/\tau_c) dD \quad \text{Eq. 2.1}$$

but the left hand member may be written as:

$$\begin{aligned} P(a < D \leq b/\tau_c) &= P(a < \tau_c - t \leq b/\tau_c) \\ &= P(a - \tau_c < -t \leq b - \tau_c/\tau_c) \\ &= P(\tau_c - b \leq t \leq \tau_c - a/\tau_c) \end{aligned}$$

$$= \int_{\tau_c^{-b}}^{\tau_c^{-a}} f_t(t) dt$$

This integral is unconditional because of the assumption that the occurrence of an input does not change the times of occurrence of a sample, and thus t is independent of τ_c . Changing variables to D from the relation $D = \tau_c - t$:

$$\begin{aligned} P(a < D \leq b/\tau_c) &= \int_b^a f_t(\tau_c - D) (-dD) \\ &= \int_a^b f_t(\tau_c - D) dD \end{aligned}$$

by comparison with Eq. 2.1:

$$f_D(D/\tau_c) = f_t(\tau_c - D) \quad \text{Eq. 2.2}$$

This equation states that the probability density function of D , given $\tau = \tau_c$, is dependent only upon the probability density function of t for the difference $\tau_c - D$ which is equal to t .

From the definition of conditional density of a continuous random variable:

$$f_Y(Y) = \int_{-\infty}^{\infty} f_Y(Y/X = X_C) f_X(X) dx \quad (\text{see Papoulis (11)})$$

Thus the conditional nature of Eq. 2.2 may be relieved by integrating across all values of the conditioning variable τ_c , that is:

$$f_D(D) = \int_{-\infty}^{\infty} f_t(\tau - D) f_\tau(\tau) d\tau \quad \text{Eq. 2.3}$$

The function $f_\tau(\tau)$ must be found, then which makes this integral, when translated forward through the time delay t_1 , equal to observed distributions of latency.

If the occurrence of an input does not change the time of a sample, then $f_t(t)$ may be reasonably assumed uniformly distributed over the maximum intersample time τ_{\max} that is:

$$f_t(t) = \begin{cases} \frac{1}{\tau_{\max}} & 0 \leq t \leq \tau_{\max} \\ 0 & \text{otherwise} \end{cases}$$

Then: $f_t(\tau - D) = 0$ for $\tau - D < 0$ i.e. $\tau < D$

And: $f_\tau(\tau) = 0$ for $\tau > \tau_{\max}$

i.e. τ cannot be greater than its maximum so that Eq. 2.3 becomes:

$$f_D(D) = \begin{cases} \frac{1}{\tau_{\max}} \int_D^{\tau_{\max}} f_\tau(\tau) d\tau & \text{for } D \geq 0 \\ 0 & \text{for } D < 0 \end{cases} \quad \text{Eq. 2.4}$$

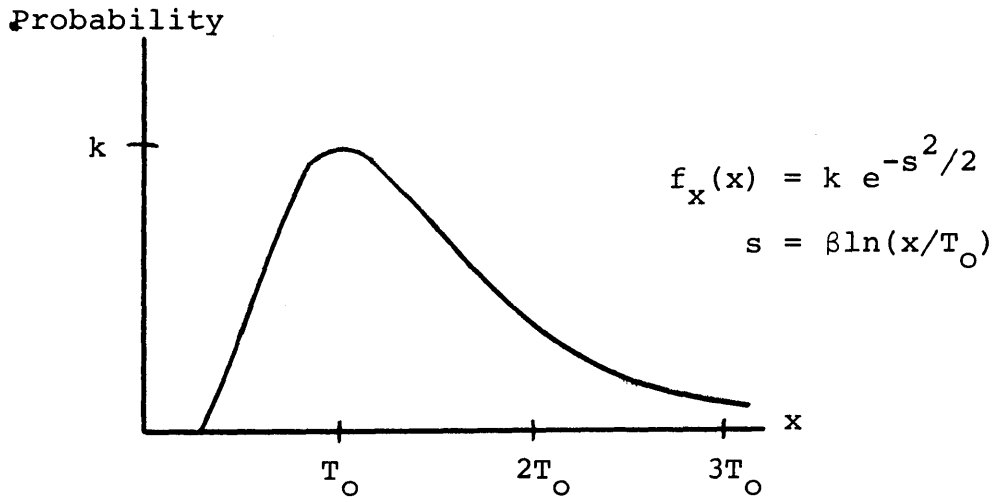
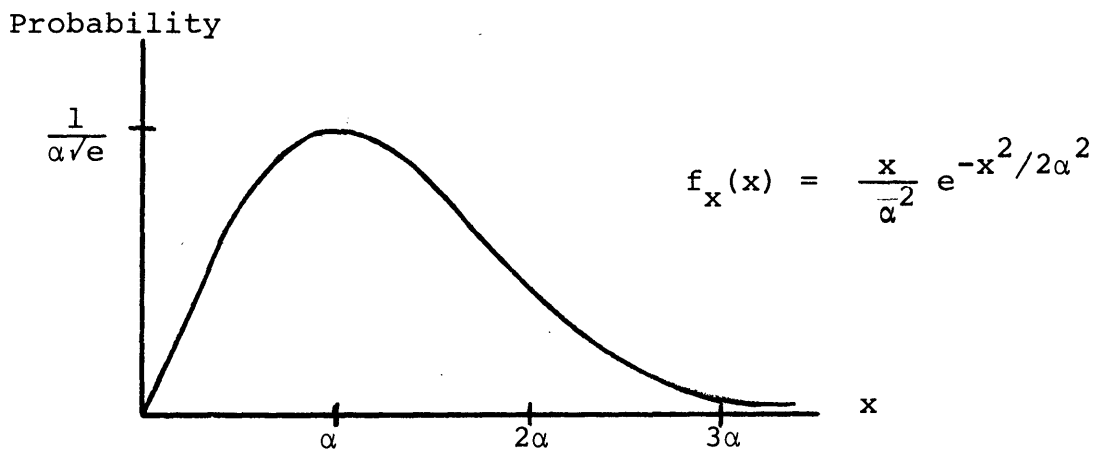
This equation established the dependence upon intersample time of latency distribution for the revised sampled data model as calculated from $f_D(D)$ by translating forward through time delay t_1 , under the assumption that the input is uniformly

distributed over the maximum intersample time. The probability density function $f_{\tau}(\tau)$, however, must be everywhere positive. Thus as D increases, less and less positive area will be included in this integral. Therefore $f_D(D)$ must be strictly non-increasing under the assumption t uniformly distributed over τ_{\max} , except at $D = 0$ where $f_D(D)$ jumps from zero to the total value of the integral:

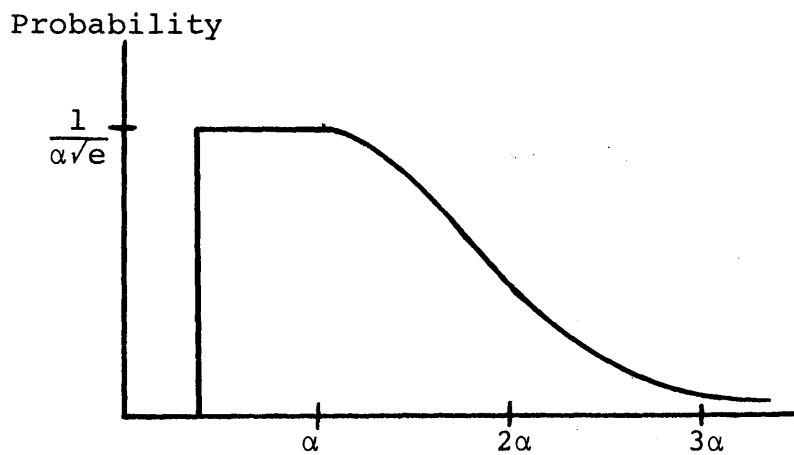
$$f_D(0) = \frac{1}{\tau_{\max}} \int_0^{\tau_{\max}} f_{\tau}(\tau) d\tau$$

Fig. 2.3 shows possible mathematical descriptions of observed latency distributions. Fig. 2.3a shows the log normal distribution used by Beeler (1). Fig. 2.3b shows a Rayleigh distribution. Fig. 2.3c shows the form of a non-increasing $f_L(L)$ which could be matched to the Rayleigh distribution. Actual histograms of step latencies appear in Figs. 5.1a - f.

The result that $f_D(D)$ is strictly non-increasing has been verified by Monte Carlo simulation on the MIT timeshared IBM 7094 computer using standard IBM random number generating subroutines. MAD language listings of the programs used and program notes may be found in Appendix C. The results of this simulation are seen in Figs. 2.4 through 2.7. All these results were obtained under the assumption that the times between inputs were equal to τ_{\max} plus a time uniformly distributed between 0 and τ_{\max} . Each mark represents 5 occurrences of latency in the interval noted in msec. at the left.

a) Log Normal Distribution $\beta = 2$ 

b) Rayleigh Distribution



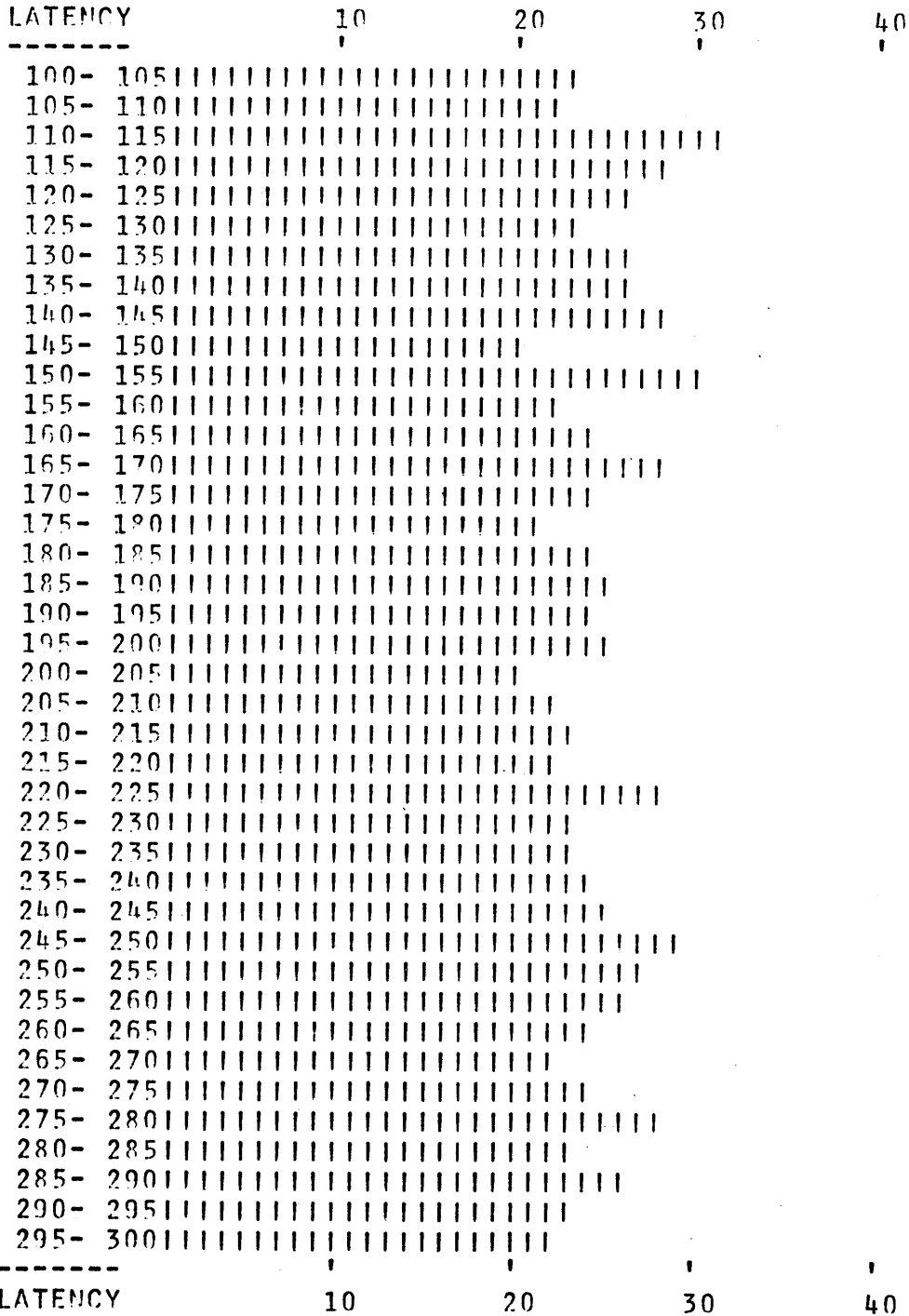
c) Non-increasing fit to Rayleigh Distribution

Figure 2.3 Mathematical Descriptions of Latency Histograms

Fig. 2.4 shows the histogram resulting from taking $f_{\tau}(\tau) = \delta(200 - \tau)$, a constant sampling interval, and a time delay of 100 msec. This histogram should be rectangular but exhibits small variations due to use of a finite number (5000) of computer trials. This effect is seen in all other simulation histograms as well. Fig. 2.5 shows the histogram resulting from taking $f_{\tau}(\tau) = \frac{1}{200}$ for $0 \leq \tau \leq 200$ msec. and a time delay of 100 msec. Fig. 2.6 shows the results of taking $f_{\tau}(\tau) = \frac{1}{100}$ for $150 \leq \tau \leq 250$ and a time delay of 100 msec. Fig. 2.7 shows the result of a dual uniform distribution as described in Eq. 2.5 with a time delay of 140 msec.

$$f_{\tau}(\tau) = \begin{cases} .01333 & 60 \leq \tau \leq 120 \\ .0025 & 120 < \tau \leq 200 \\ 0 & \text{otherwise} \end{cases} \quad \text{Eq. 2.5}$$

All these figures are closely predicted by Eq. 2.4, and show the analytically predicted property of being strictly non-increasing. Similar results were obtained under the assumptions; (1) time between inputs uniformly distributed over a time large with respect to the maximum intersample time τ_{\max} , (2) time between inputs a large constant time plus a time uniformly distributed over the maximum intersample time, and (3) inputs occur at large constant intervals. Actual latency distributions, however, do increase for low latencies. Therefore, if free running sampling occurs in the eye movement control system, the eye must sometimes be able to shorten the sampling intervals when a step is observed. This hypothesis is supported by the short intersample times of the good fit to actual latencies seen in Fig. 2.7.



EXIT CALLED. PM MAY BE TAKEN.
R 8.250+1.700

Figure 2.4 Simulated Latency Histogram for a Constant Sampling Interval System

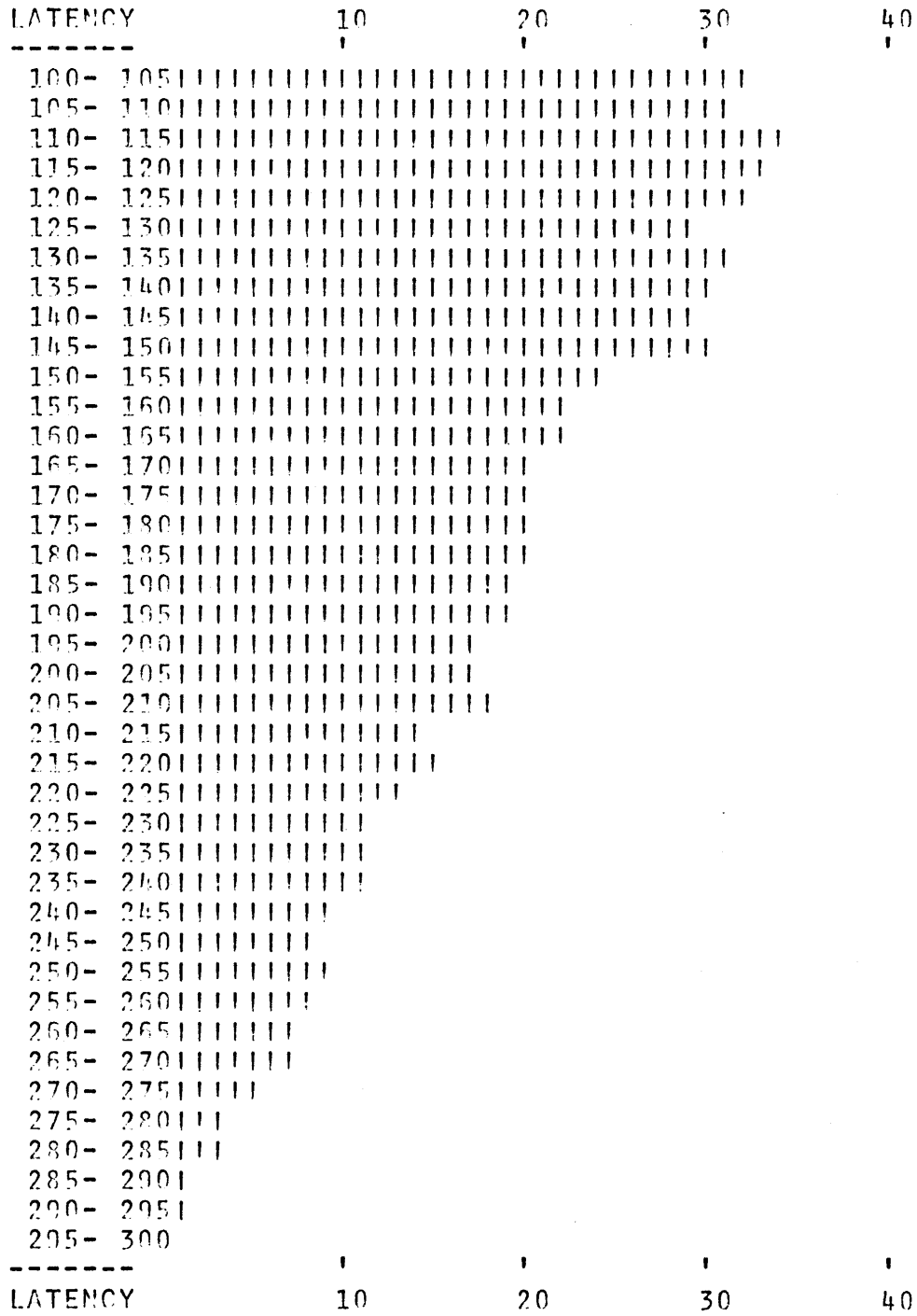


Figure 2.5 Simulated Latency Histogram for a System with Sampling Intervals Uniformly Distributed between 0 and 200 msec.

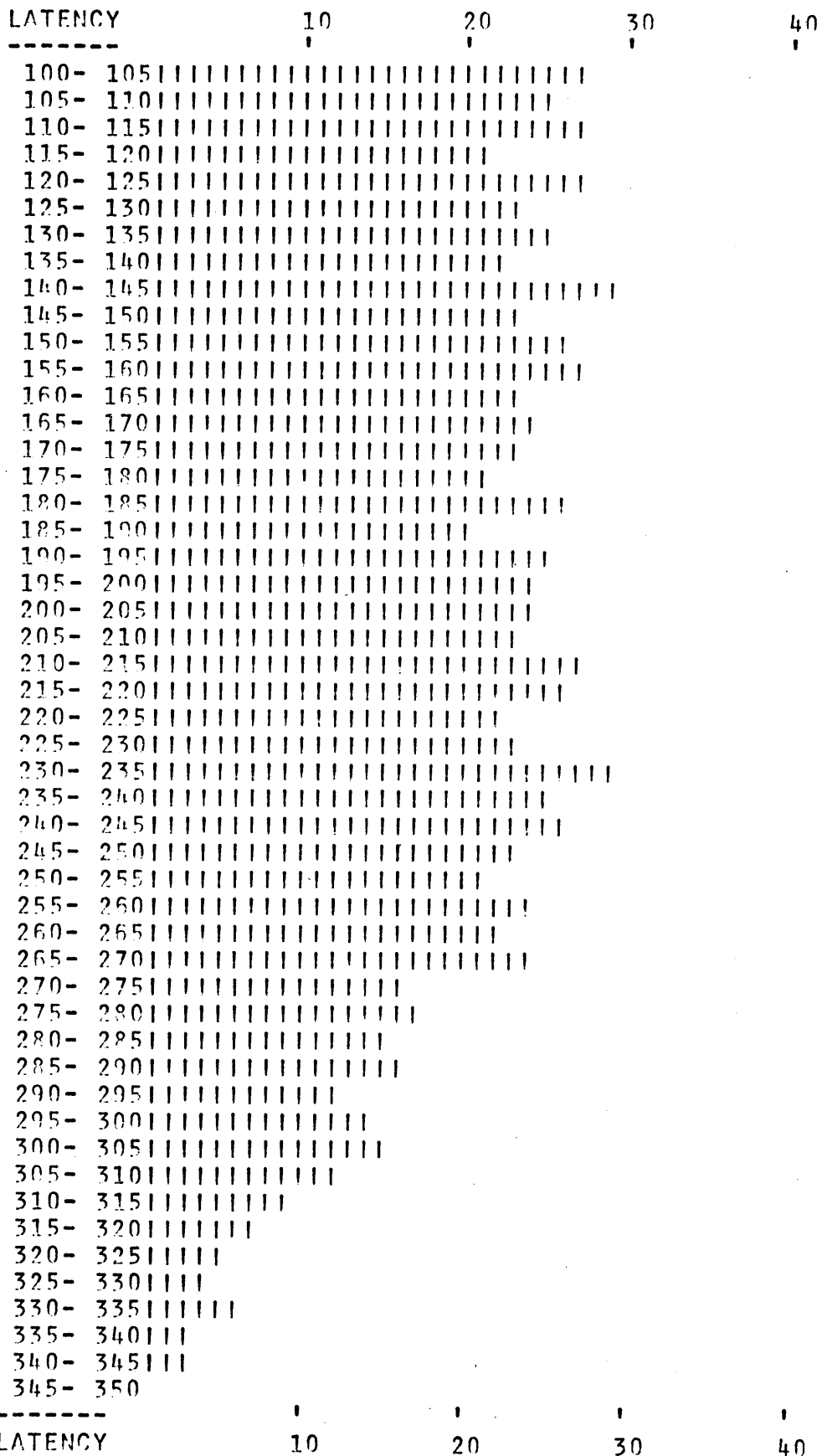


Figure 2.6 Simulated Latency Histogram for a System with Sampling Intervals Uniformly Distributed between 150 and 250 msec.

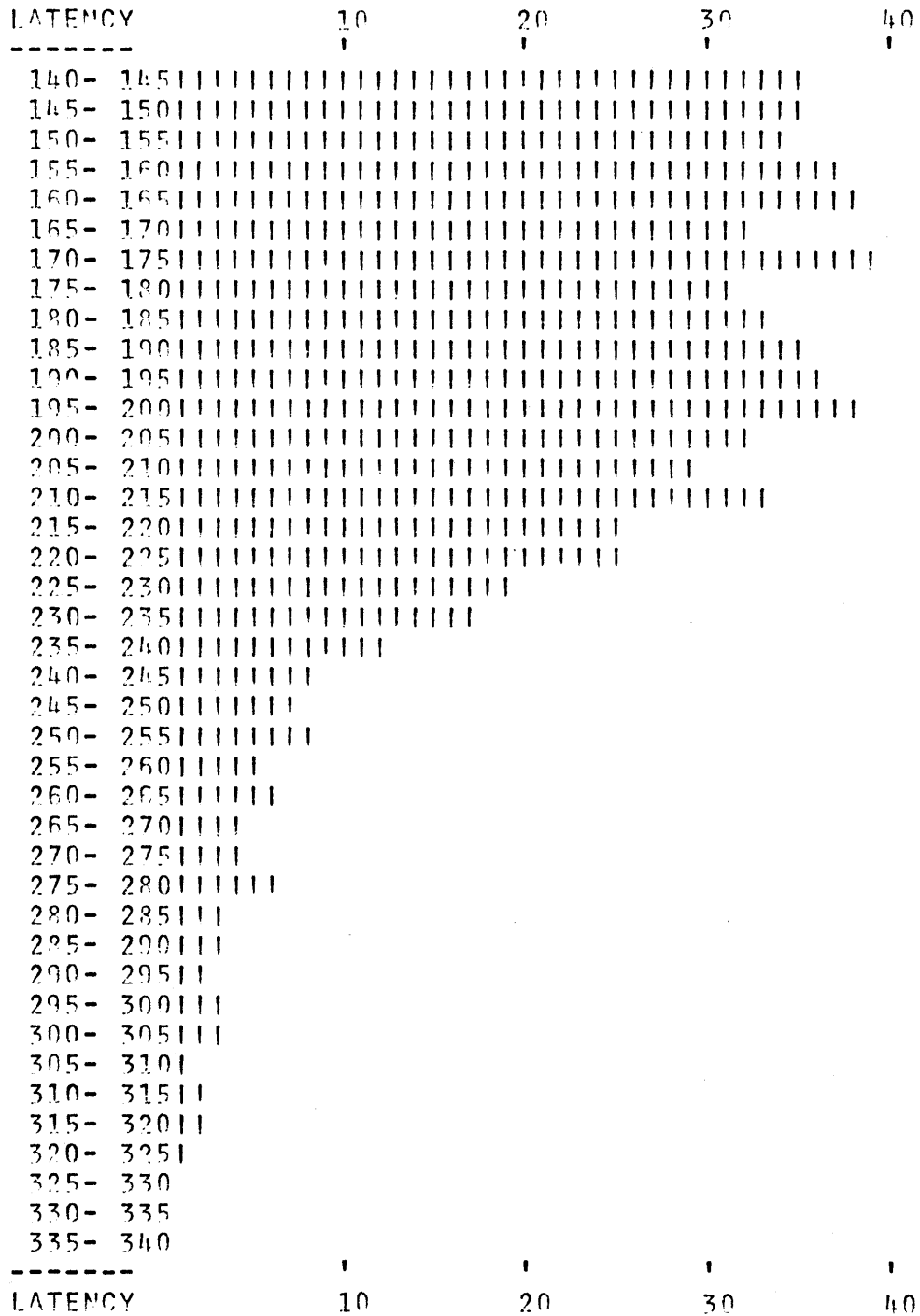


Figure 2.7 Simulated Latency Histogram for a System with Sampling Intervals Dual-Uniformly Distributed as Described in Eq. 2.5

CHAPTER 3

TRANSIENT RESPONSE OF THE REVISED MODEL

In this chapter the response of the revised sampled data model for eye tracking movements presented in the last chapter is analysed for each of six transient inputs. For each input the variation in types of response predicted by the model is examined and the percentage occurrence compared to actual response percentages.

3.1 Step Response

The distribution of latency predicted by the model for a step input has already been discussed at length in the last section of Chapter 2. The synchronization diagrams of Fig. 3.1 show five different latencies obtainable from the revised Young's model by varying the synchronization of input to sampling instants. The minimum latency obtainable is t_1 , here taken to be 0.1 sec. (Fig. 3.1a) when the input occurs immediately before a sampling instant. Fig. 3.1c shows the mean human latency of 0.2 sec. obtained from synchronizing the input 0.1 sec. before a sampling instant, while Fig. 3.1e shows the maximum modelable latency of $\tau_{\max} + t_1$, obtainable by synchronizing the input immediately after a sampling

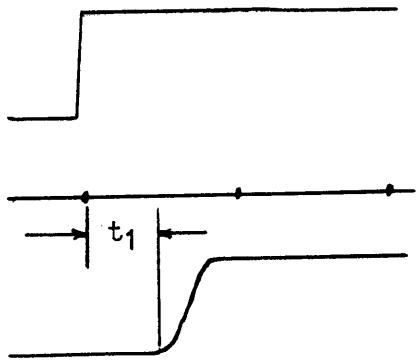


Fig. 3.1a Latency = t_1 (minimum)

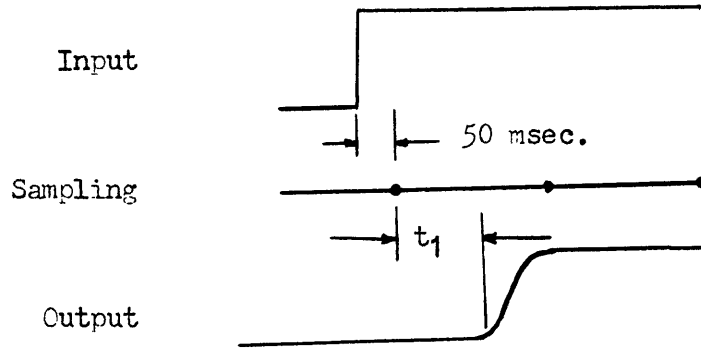


Fig. 3.1b Latency = $50 + t_1$

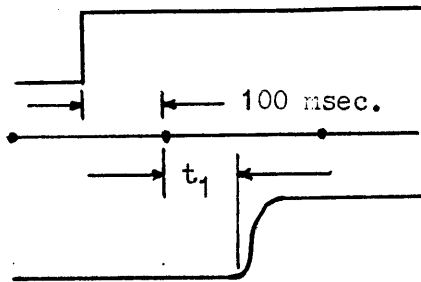


Fig. 3.1c Latency = $100 + t_1$

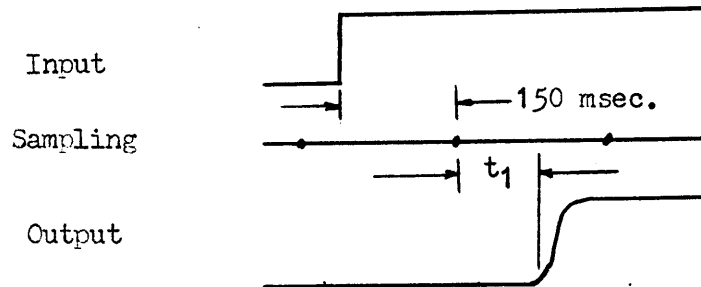


Fig. 3.1d Latency = $150 + t_1$

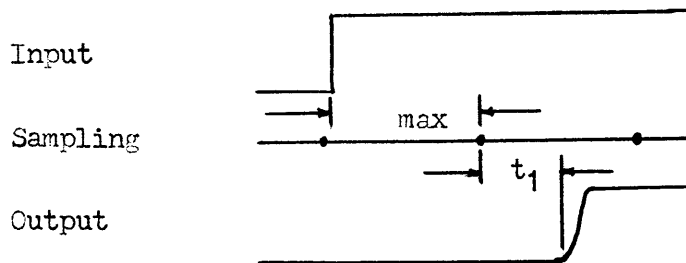


Fig. 3.1e Latency = $\text{max} + t_1$ (maximum)

Figure 3.1 Step Input Model Variation as a Function of Synchronization to the Sampling Instants

instant, and before a maximum length sampling interval. As seen in Chapter 2, the distribution of these latencies is uniquely determined by choice of the distribution of intersample times. For the remainder of the inputs discussed, saccadic responses elicited by sampled error exceeding the dead zone limit will be understood to be distributed in the same manner unless otherwise specified.

Even when tracking a small step the saccadic system often performs inaccurately, and must follow the initial saccade with a second to correct for the inaccuracy. This question of saccadic inaccuracy has not been investigated at length, and therefore, very little can be said with confidence. Both overshoot and undershoot inaccuracies are observed in eye movements. One way of modeling these inaccuracies would be to stochastically vary the shape of the non-linearity in the saccadic system. A line below the shown 45° line would produce undershoot, a line above would produce overshoot. This non-linearity may be a function of latency as well as of error size.

3.2 Pulse Response

Beeler (1) studied eye tracking movements in response to both returning pulses and stairway pulses. His results, as shown in Figs. 3.2 and 3.3, agree with those of Westheimer that, for pulse widths less than 0.2 sec., the eye responds to the two target motions with two saccades, the first averaging 200 msec. from the onset of target motion, the second averaging at least 200 msec. from the first. As seen

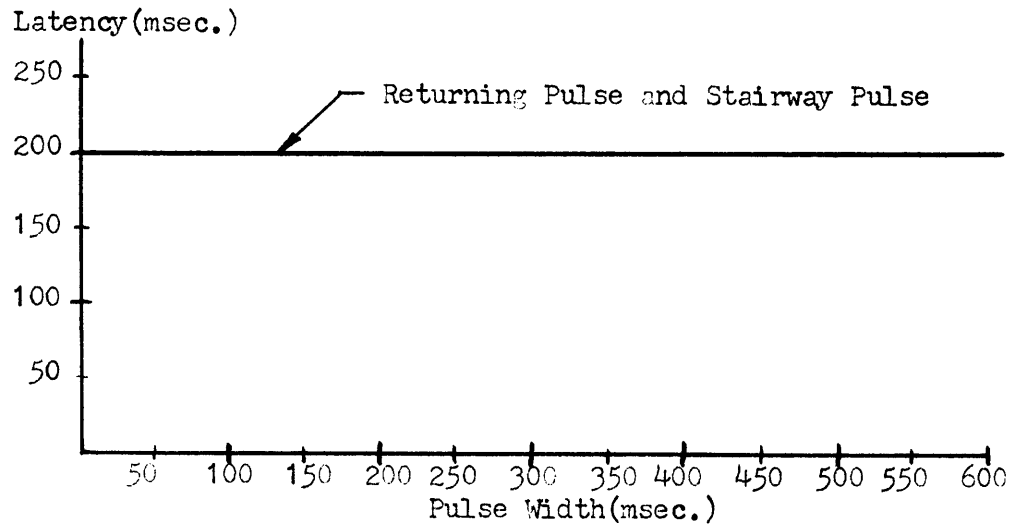


Figure 3.2 Beeler's average latencies in response to the first step of pulse inputs

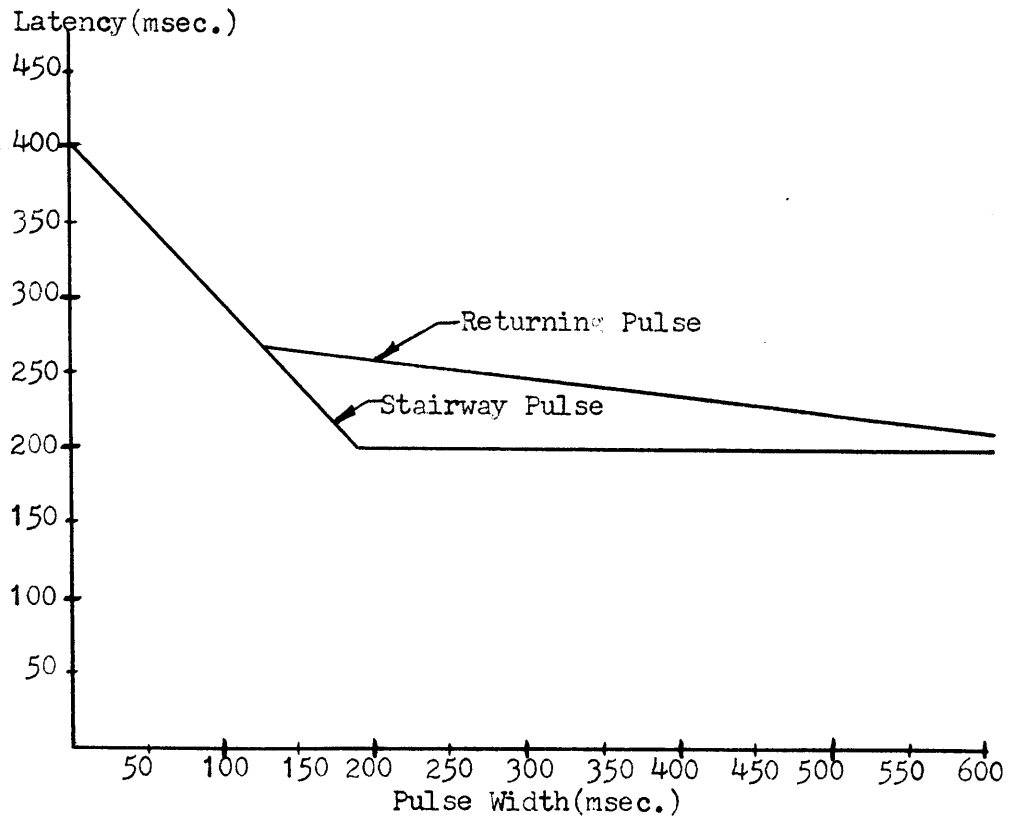


Figure 3.3 Beeler's average latencies in response to the second step of pulse inputs

in Fig. 3.3, Beeler noted inflated average latency to the second step of a returning pulse not evident in stairway pulses, for pulse widths from 125-600 msec. This finding may have been due to the difference in predictability of the two inputs arising from the use of only three target positions, although recent experiments by Saslow (15) suggests this was not the case. Forms of response predicted by the revised sampled data model are shown in Fig. 3.4. For pulse widths (W) less than or equal to 200 msec., the revised sample data model with mean intersample time 200 msec. predicts either abnormal response or two saccadic responses, the first averaging 200 msec., the second averaging 400 msec. For pulse widths greater than 200 msec., it predicts the two saccades at average latencies of 200 msec. and $W + 200$ msec. These predictions are true for both the returning pulse and the stairway pulse since there is no dependence of delay time upon past values of eye position error. Thus the revised sample data model agrees well with Beeler's results for the stairway pulse but predicts slightly too short latencies over the interval $125 < W < 600$ for the returning pulse.

The revised sampled data model also predicts the percent of inputs to which there will be abnormal response as a function of pulse width. This prediction arises because response to the first step is dependent upon the occurrence of a sampling instant within the pulse width. But with a mean intersample time of 200 msec. and the pulse uniformly distributed over this interval, the probability of occurrence

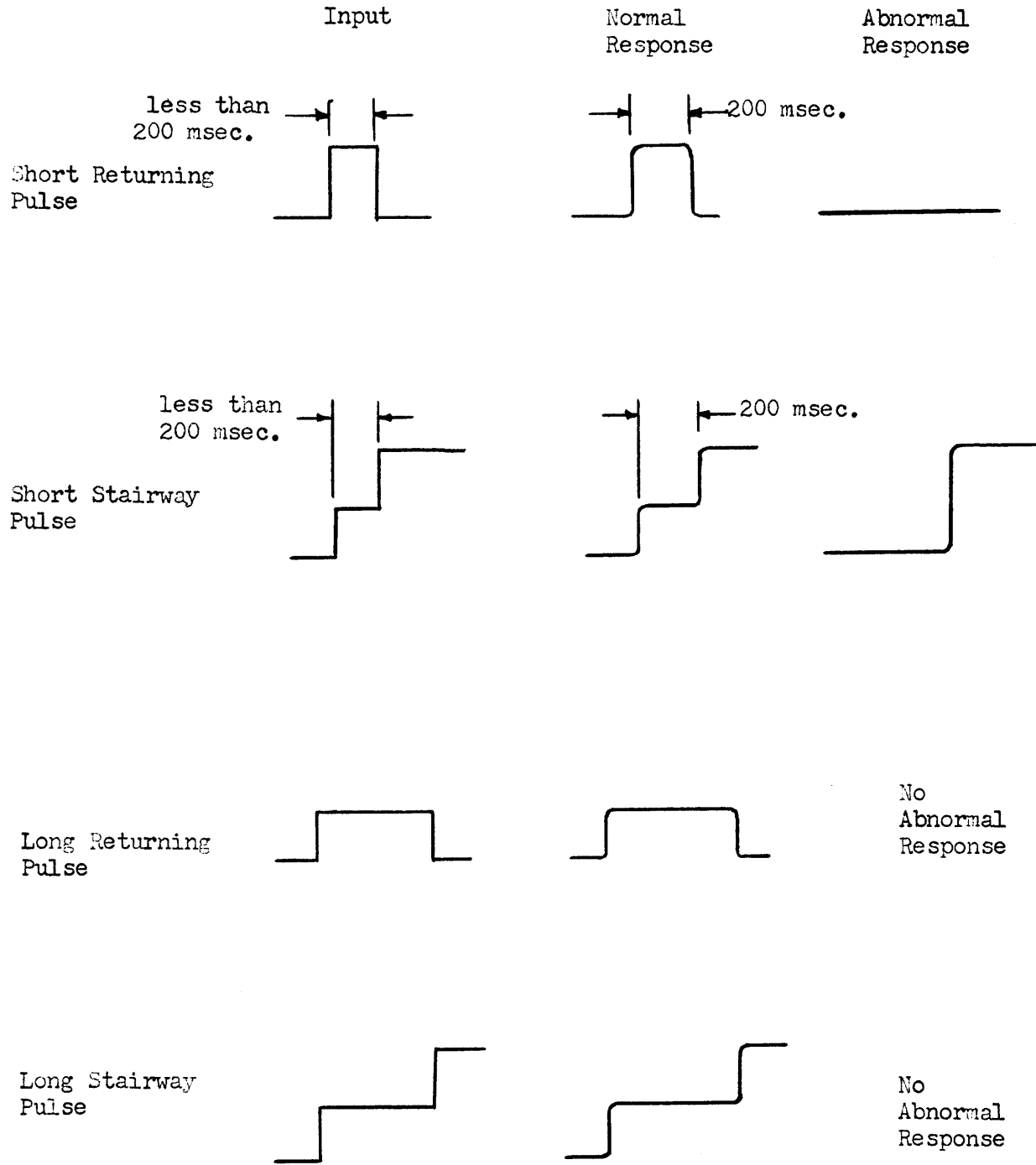


Figure 3.4 Forms of Response Predictable for Pulse Inputs

of a sampling instant in time W is just $W/200$. Thus the probability of no response is $1 - (W/200)$. This linear relationship compares well with the findings of Beeler (1) and Wheelless (20) for returning pulses, as seen in Fig. 3.5.

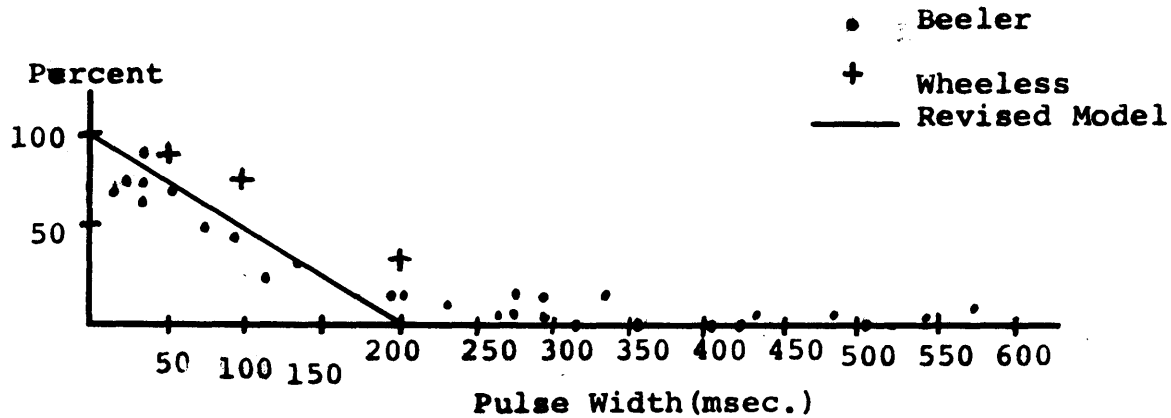


FIG. 3.5: Percent no response to returning pulse input

Beeler's results do not agree as well, however, for the case of the stairway pulse, as seen in Fig. 3.6

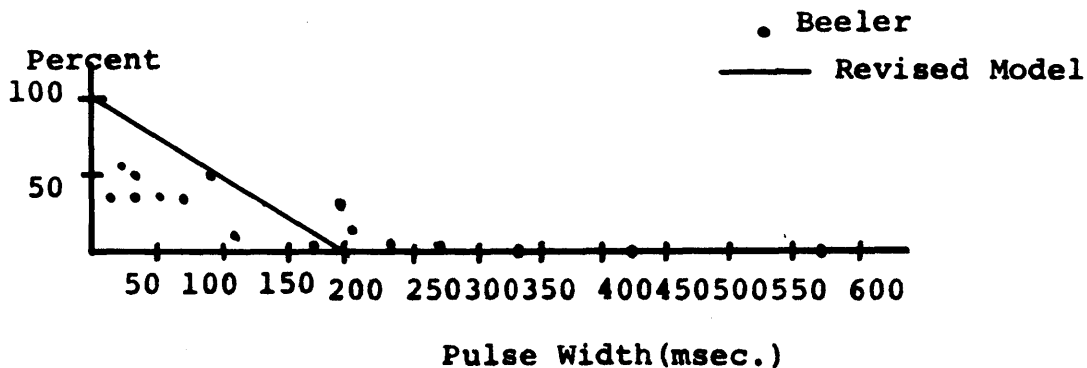


FIG. 3.6: Percent no response to stairway pulse input

This discrepancy has already been mentioned as a possible indication of finite pulse width sampling.

Table 3.1 summarized the findings of this section on the response of the revised sample data model to returning pulse and stairway pulse inputs.

TABLE 3.1

INPUT	LATENCY	PERCENT ABNORMAL RESPONSE
Returning Pulse	Slightly low 125 < W < 600	Excellent
Stairway Pulse	Excellent	Slightly high below 80 msec.

3.3 Ramp Response

Another input which has been studied extensively is the constant velocity or ramp input. Rashbass (12) studied the beginning (0° to 5°) of the ramp response as a function of ramp speed. He reported the entrance of pursuit movement into the response at approximately 150 msec. and a saccadic response at about 200 msec., both measured from the onset of target motion. Rashbass also noted an increase in the size of the saccadic response in direct proportion to the velocity of the input.

Robinson (14) reported three main forms of response to a 10 deg/sec ramp and the percent of observation of each. His results are redrawn in Fig. 3.7. Robinson's most frequent (59% occurrence) response shows smooth pursuit movement beginning at 125 msec. (± 20 S.D.), a saccadic response at 237 msec. (± 30 S.D.), followed by an increase in smooth pursuit velocity to greater than the target velocity in order to reduce the error to zero, after which pursuit velocity

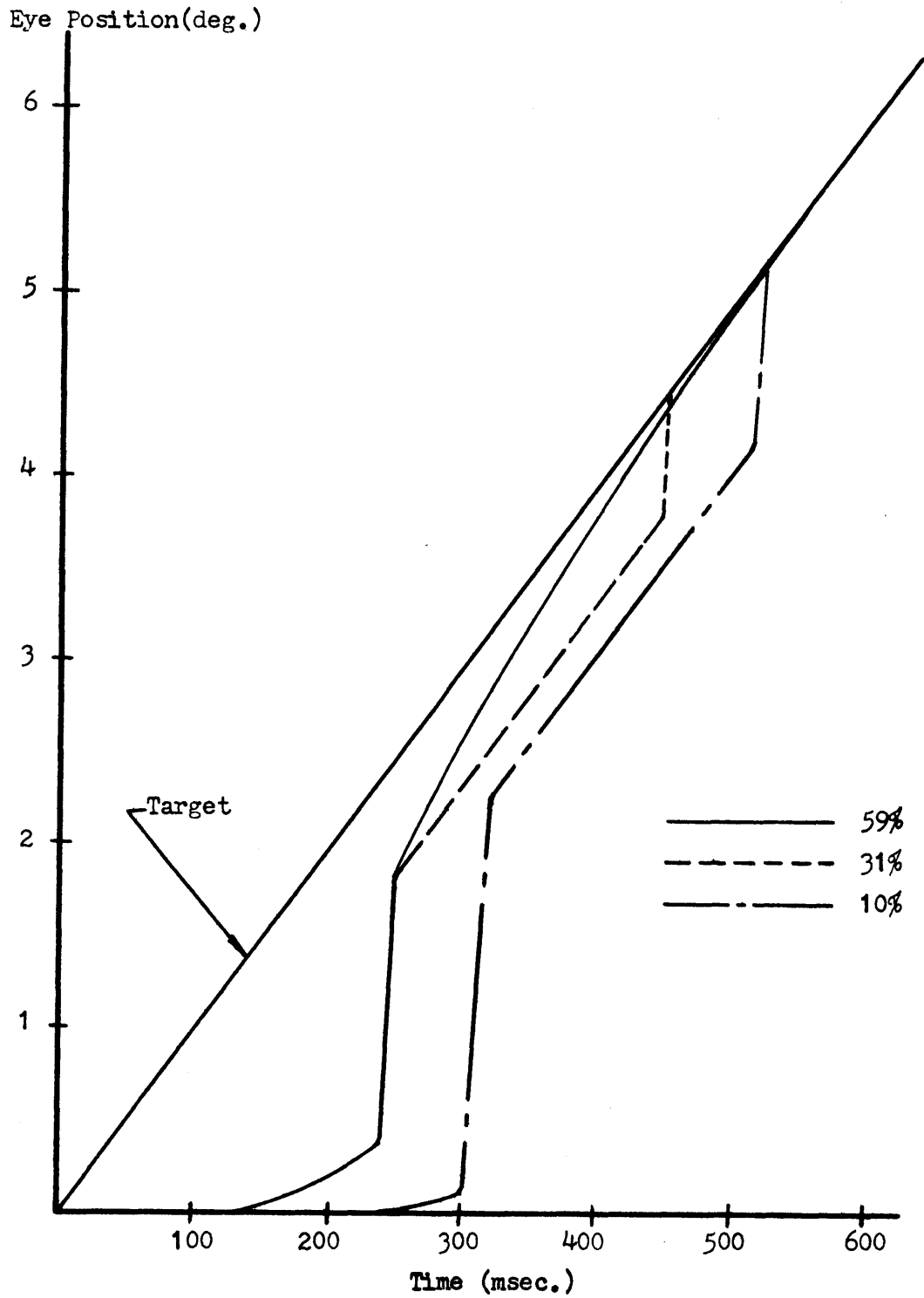


Figure 3.7 Robinson's 10 deg./sec. ramp response

slackens to equal target velocity. The remaining 41% of response contain two saccades to reduce the error to zero. Robinson observed that his data showed a lengthening of saccadic latency over latencies observed in response to step inputs. Weighing Robinson's reported saccadic latencies by his observed percentage of occurrences, the mean latency resulting is 243 msec. This displacement by about 40 msec. of mean saccadic latency in response to a 10 deg/sec ramp input compared with mean latencies to step inputs is substantiated by the experiments of this thesis. (see histograms, Figs. 5.1a-f).

The revised sample data model predicts the entry of pursuit motion invariable at 134 msec. due to the continuous nature of the pursuit system. This latency is the weighted mean of Robinson's data. The model also predicts one saccadic movement, proportional to the speed of the ramp, due to the accumulation of error during the 134 msec. delay in pursuit movement. The magnitude and timing of the saccade are randomly distributed depending on D. Second saccades would be predicted by the saccadic inaccuracy model or an equivalent pursuit inaccuracy model. The saccadic dead zone prevents any sampled error from commanding movement unless it is greater than 0.3° . Thus the distribution of saccadic latencies is displaced by the time required to exceed 0.3° , or 30 msec. for a 10 deg/sec ramp, in good agreement with observed results.

The model does not, however, predict any velocity overshoot in pursuit motion, as observed by Robinson. This suggests

the introduction of slightly underdamped dynamics into the pursuit loop. The other 40% of responses which show no velocity overshoot, however, dictate dependence of these dynamics on the saccadic system. This area of saccadic augmentation of pursuit movement requires further research before transfer functions for it can be assigned. Experiments 3 and 4 of this thesis, for example, illustrate that previously initiated pursuit movement affects saccadic latency.

3.4 Step-Ramp Response

As previously reported, Rashbass (12) showed the independence of the pursuit and saccadic systems by presenting subjects with step-ramp inputs. He observed smooth pursuit movement before, at, and after saccadic movement in the responses. He did not report the percent occurrence of each response. Rashbass also observed that when the step size in degrees was 0.15-0.2 times the ramp rate in degrees per second, most responses contained no saccadic eye movement.

Fig. 3.8 shows three outputs obtainable from the revised model for a 5 degree step - 10 degree/sec ramp combination by assuming different synchronizations of input to sampling instants. Fig. 3.8a shows the model output for an input-to-sample delay of zero. In this example, the pursuit movement enters the response 34 msec. after the saccade. Fig. 3.8b shows the response for an input-to-sample delay of 34 msec. This response represents pursuit movement at a saccade. Figs. 3.8c shows response in which the pursuit movement begins before the saccadic movement.

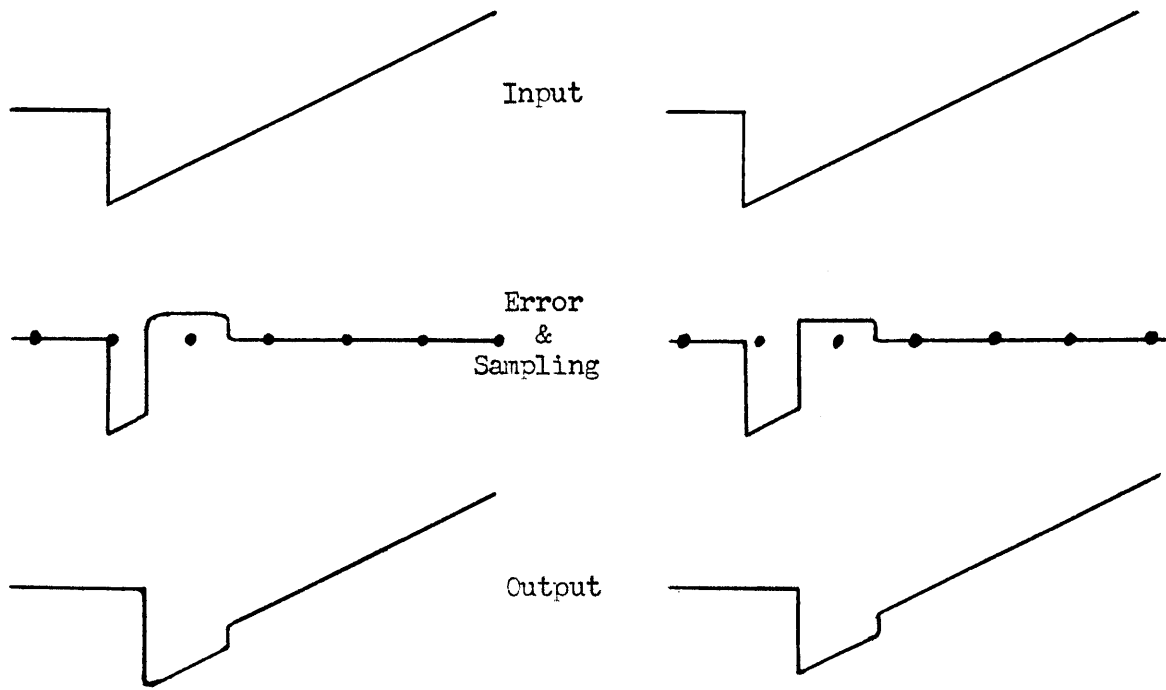
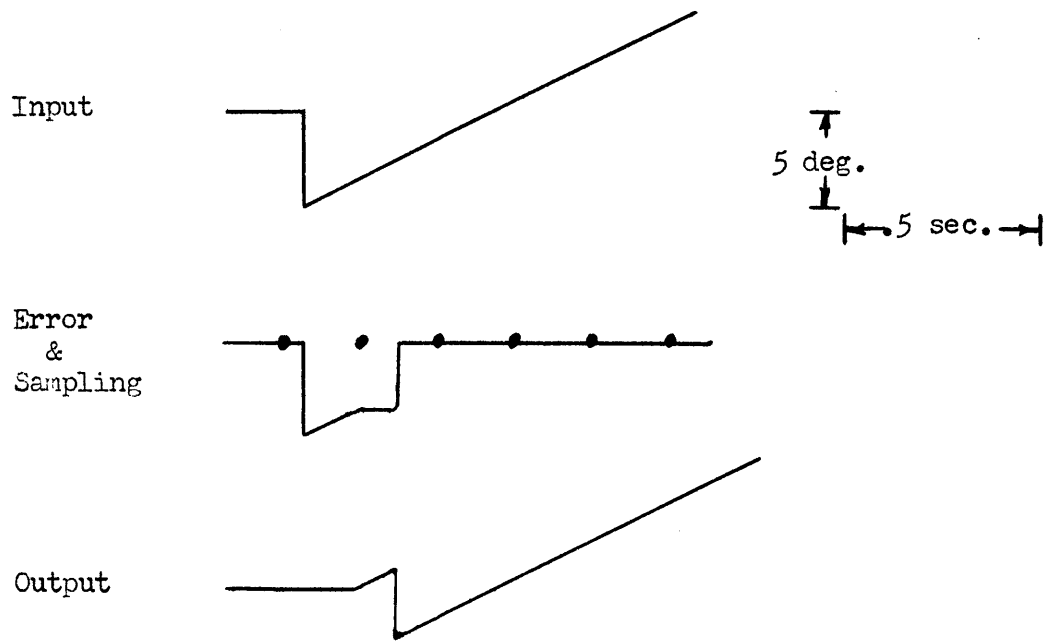
Fig. 3.8a $D = 0$ Fig. 3.8b $D = 34\text{msec.}$ Fig 3.8c $D = 150\text{msec.}$

Figure 3.8 Step-Ramp Model Response as a Function of Synchronization of Input to Sampling Instants

The percent occurrence of these three types of output may be estimated by assuming the distribution of D (input-to-sample delay) derived for the step at the end of Chapter 2, and investigating the delay intervals in which each output is predicted. Taking the parameters of Fig. 2 and assuming pursuit movement unrecognizable until 30 msec. from its start, pursuit motion after the saccade will occur in the interval $0 < D < 34$ msec. or $\int_0^{34} f_D(D) dD$ probability. Similarly, pursuit motion at a saccade is predicted in the interval $34 < D < 64$ msec. with $\int_{34}^{64} f_D(D) dD$ probability, and pursuit motion before a saccade is predicted in the interval $64 < D < 200$ msec. with $\int_{64}^{200} f_D(D) dD$ occurrence. The estimated probability of outputs with smooth pursuit movements at the saccade is probably low due to ignoring the saccadic augmentation of pursuit movement as discussed by Robinson (14). Further investigation is needed in the area of saccadic augmentation of pursuit movement to establish the complete effect of a saccade on smooth pursuit movement.

Young's revised model also predicts a large number of no-saccade responses to step-ramp inputs near the inputs to which Robinson and Rashbass observed no saccadic response experimentally. As seen in Figs. 3.8a-c, once the pursuit system begins following the target, predicted eye position error remains constant until corrected by a saccadic eye movement. If, however, this constant eye position error is within the $\pm 0.3^\circ$, the revised model predicts no saccadic response. In equation form, the relationship necessary between step size and ramp rate in order to elicit pure pursuit response is:

$$\text{Ramp speed} = \frac{\text{Step size} \pm 0.3^\circ}{134 \text{ msec.}}$$

This equation assumes that the foveal error is zero at the initiation of the transient input. The distribution of saccades associated with random uniform distribution of initial image on the fovea is described by Young in Young, Stark, and Zuber (26). Fig. 3.9 shows that this prediction compares favorably in the range below pursuit saturation with observed results of Rashbass and Robinson. Moreover, Wheelless reported this result over wider ranges of step-ramp combinations.

The revised model does not, however, predict the inflated pursuit latency noted by Robinson for this type of input. This indicates the possible presence of prediction even when target motion is unknown and is another area for future research.

3.5 Ramp-Step Response

Experimental observation shows that when the eye returns to the original position after following a ramp, the eye occasionally undershoots by 1-2 degrees, requiring another saccade to bring the eye to the target as shown in Fig. 3.10. The revised model predicts this response, as did Young's original model, when the continuation of the pursuit system moves the eye further than the dead zone from the target, after both the target return and the next sampling instant. This continuation creates an error which must wait until the next sampling instant to be corrected. The model predicts

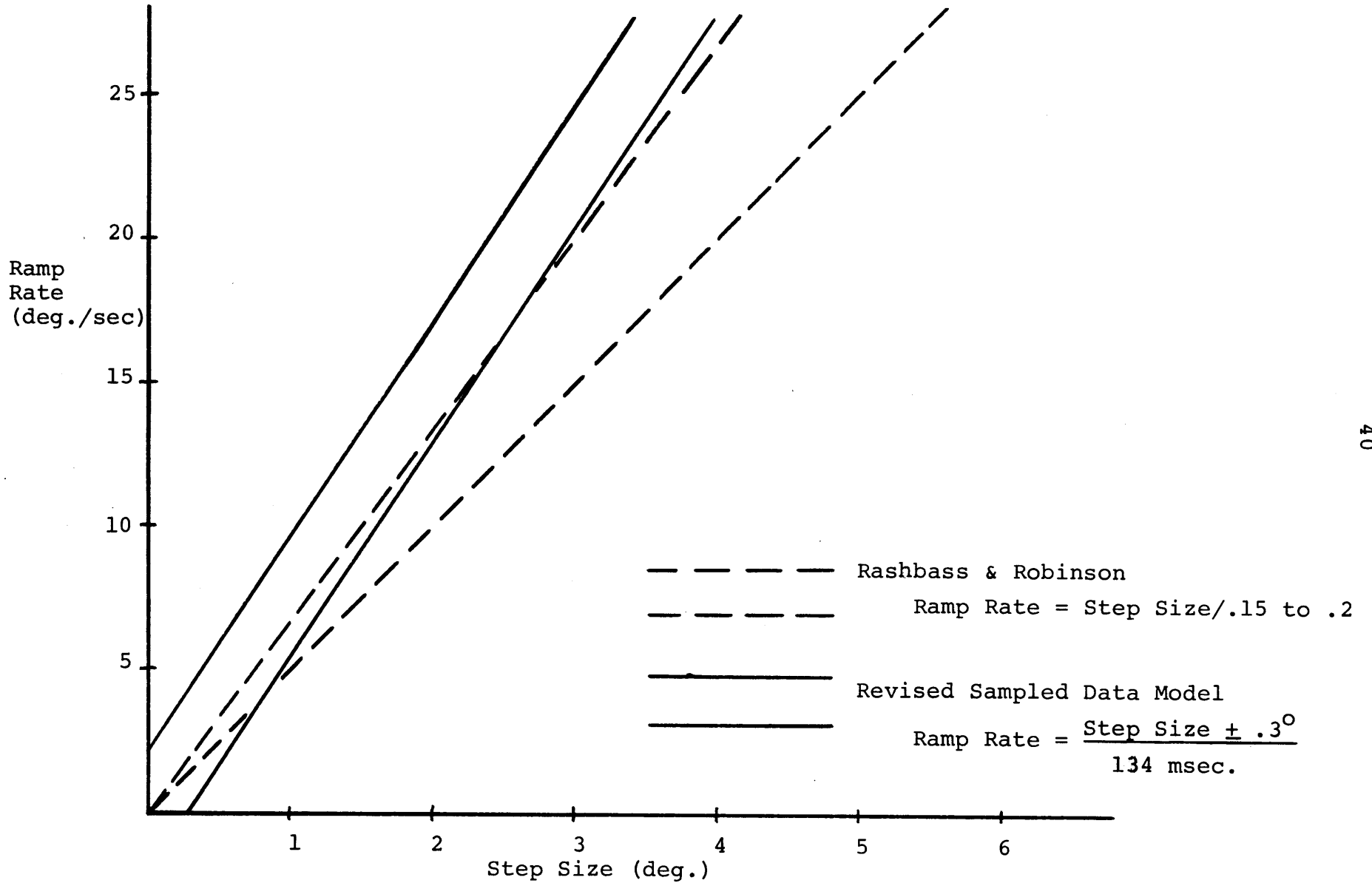


Figure 3.9 Comparison of Regions of No-saccade Step-Ramps

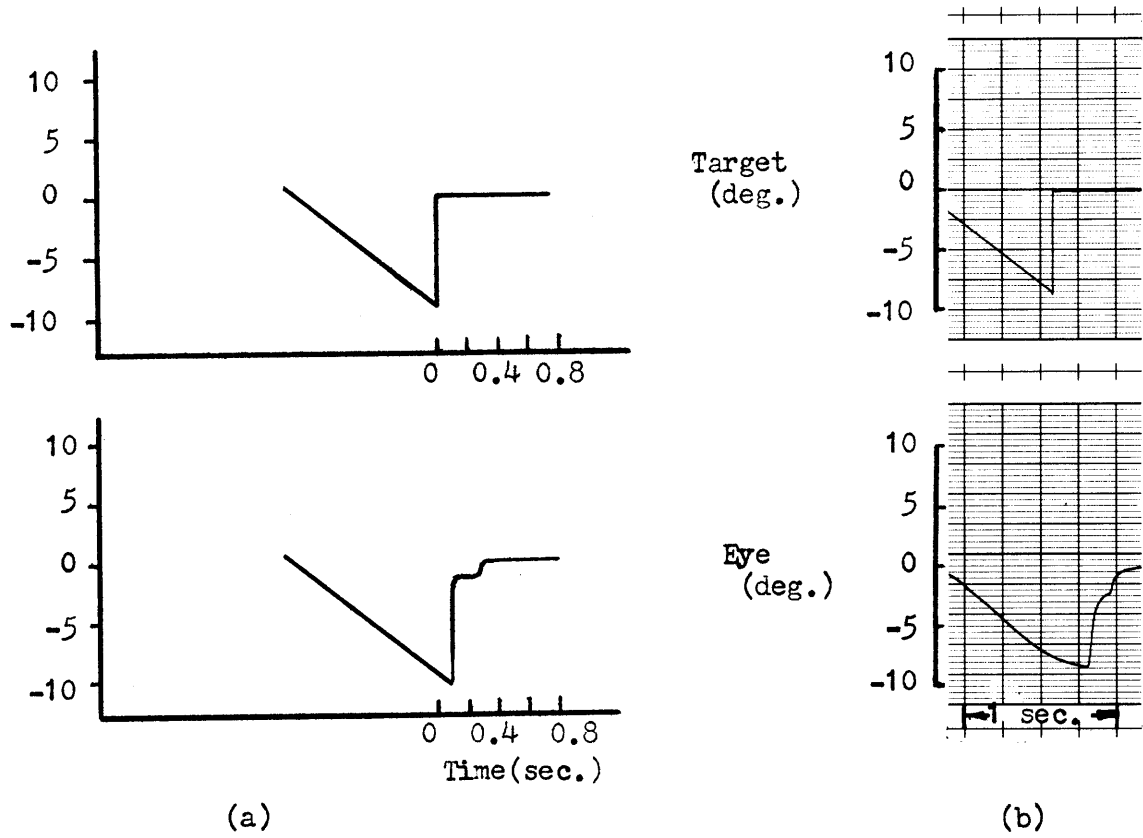


Figure 3.10 Ramp-Step Response: (a) model; (b) experimental

continuation of pursuit tracking for 134 msec. following the ramp termination. For a 10 deg/sec ramp, if the sample occurs in the last 30 of these 134 msec., the error will be less than the dead zone limit. Therefore, the model predicts a second step if the sample occurs in the first $134 - 30 = 104$ msec. or $\int_0^{104} f_D(D) dD$ probability. This type of response would also be predicted by a saccadic inaccuracy model which undershoots the target position.

CHAPTER 4

EXPERIMENTAL PLAN, APPARATUS, AND PROCEDURES

In this chapter, a method of data reduction is proposed which would yield predictable latency tendencies in a sampled data model. Four experiments are proposed to be reduced by this method and the response of synchronized sampling and random sampling sample data models to these four experiments is discussed. The apparatus and procedures used in performing these experiments are then presented. Chapter 5 discusses the observed results of six subjects to these experiments.

4.1 Saccade-Synchronized Data Reduction

From the preceding Chapter's analysis of transient responses, it is evident that two sources of uncertainty contribute to the determination of output in a non-synchronized sampled data model. First, since the onset of target motion is independent of the sampling instants it must be considered uniformly distributed over the maximum intersampling interval. Second, the intersampling intervals are stochastically distributed themselves. Since there is no output at the time of a sampling instant, the only method of estimating the time of occurrence of a sample is from observation of a saccade resulting from the

sample. If a target input occurs immediately after a saccade, the sample which caused that saccade and the sample which will cause a response to the current target motion are separated by only one intersample time. For this type of input only one factor, the intersample times, contributes to the uncertainty of response form. If eye tracking were controlled by a system such as the revised sampled data model, the distribution of intersampling intervals would be revealed by the distribution of forms of response when the input occurs at a constant time, smaller than one intersample time, after a saccade. Experiments to date which have involved inputs near saccades have been reduced only by recording distributions of latency from the beginning of target motion. In order to obtain a measure of synchronization of input to sampling instants, responses must be grouped by constant saccade to input time.

4.2 The Saccade-Synchronized Inputs

The experiments performed were to examine the nature of sampling in the eye movement control system by grouping inputs that occur at identical times after a saccade as described above. In order to facilitate grouping inputs in this way, a saccade detector was used to initiate a delay of controllable length before the introduction of the second part of each target motion.

The four experimental inputs used are illustrated in Fig. 4.1. Experiments 1 and 2 use a step of four degrees to elicit the saccade which initiated the delayed target motion.

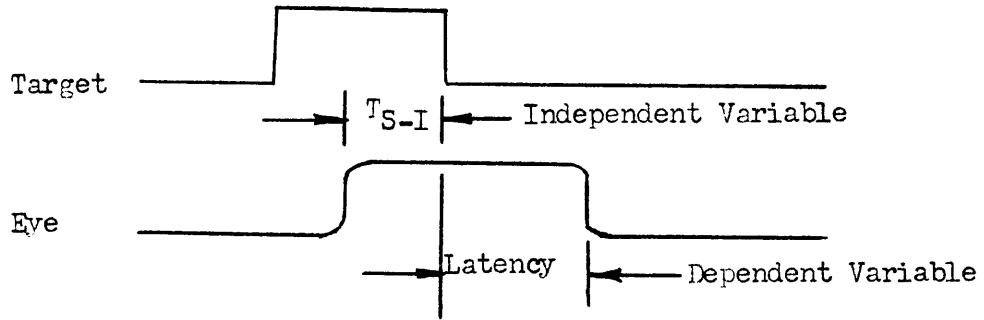


Fig. 4.1a Experiment One

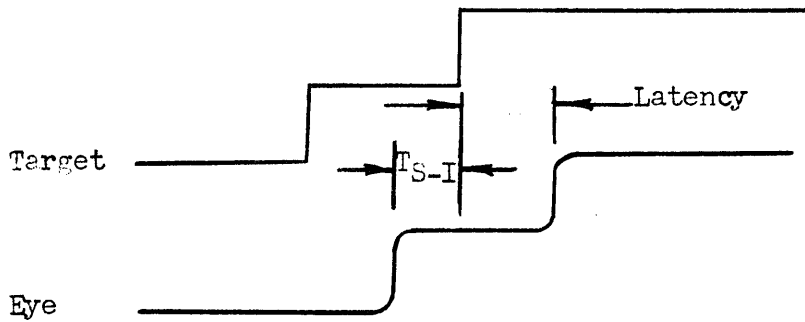


Fig. 4.1b Experiment Two

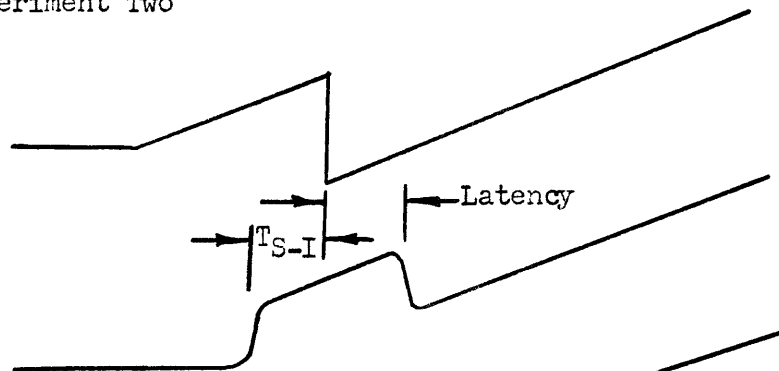


Fig. 4.1c Experiment Three

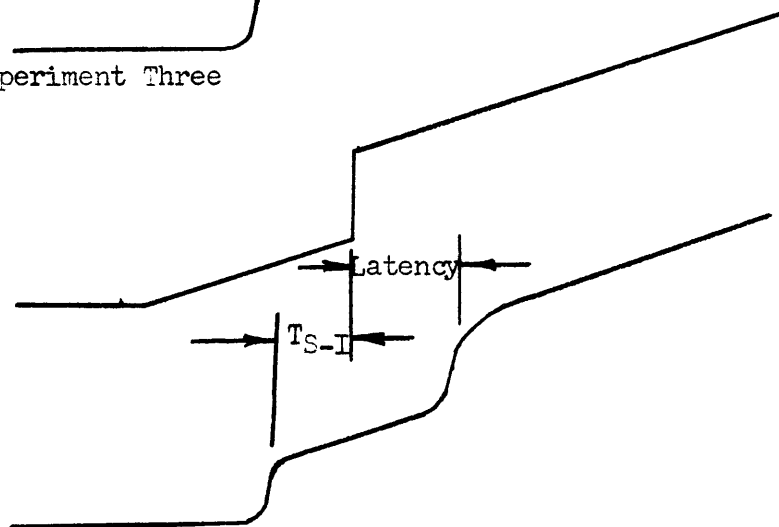


Fig. 4.1d Experiment Four

Figure 4.1 Experimental Inputs

Experiments 3 and 4 used a ramp of 10 deg/sec to elicit the saccade to which target motion is synchronized. The target motion introduced was a step in the opposite direction of the first saccade in Experiments 1 and 3, and a step in the same direction as the first saccade in Experiments 2 and 4. Thus Experiments 1 and 2 consisted of a returning pulse and a stairway pulse respectively, in which the occurrence of the second step was synchronized to the occurrence of a saccade in response to the first step. Experiments 3 and 4 consisted of a ramp into which was introduced, respectively, an inward or outward step, synchronized to the occurrence of a saccade in response to the initial ramp. Due to the dependence of these inputs, on the occurrence of a saccade, the pulse width of any given input was indeterminable before that input was presented.

The only dependent variable in Experiments 1 and 2 was latency of response to the synchronized step. Latencies to the first step were also recorded to assure normal non-predictive tracking. Initially the dependent variable of interest in Experiment 3 was the form of response as seen in Fig. 3.8. Since pursuit movement was previously initiated, however, the effect of pursuit latency was lost and no results of this kind were obtained. Latency of response to the synchronized step was recorded in Experiments 3 and 4 as in Experiments 1 and 2. Thus, the results of these four experiments represent the effect on latency of recently occurred saccades.

4.3 Model Response to Saccade-Synchronized Experiments

The revised sampled data model predicts identical mean latency variation to all four saccade-synchronized experiments. In an unsynchronized sampling revised sampled data model, both saccade-to-input time, T_{S-I} , and distribution of intersample times affect predicted latency. As the saccade-to-input delay increases from zero to the expected time of occurrence of the next sample, average latency decreases due to proportional shortening of time from input to sample (D). As the saccade-to-input delay increases near the mean time of occurrence of a sample, average latency rises since a greater fraction of responses must wait for the second sample after the saccade to cause a response. As input delay increases to near 200 msec., latency again decreases as all outputs must wait for the second sample, and D is decreasing for lengthening delay as for short delays.

If these experiments were presented to a sampled data model with synchronized sampling and time of synchronization stochastically variable, mean latency would not be expected to vary as a function of when the last saccade occurred, since each target step sets off its own independent sample. If this system is to agree with observed pulse response, a further constraint must be placed upon it limiting the minimum intersample time to about 200 msec. With this constraint and a constant delay of 100 msec., this system predicts high mean latency when the input occurs during a saccade, decreasing latency until saccade-to-input delay reaches 100 msec. (for 200 msec. intersample time) and constant normal latency through

larger saccade-to-input delays.

Fig. 4.2 illustrates the variation of average latency predicted by the revised sampled data model presented with these inputs for two different unsynchronized sampling distributions and for synchronized sampling. Fig. 4.2a shows latency for a sampling distribution of a delta function at 200 msec., that is, a constant sampling interval system. This system shows a sharp rise in latency at 100 msec. after the saccade due to the occurrence of the next sample always 100 msec. after the first saccade. Fig. 4.2b shows average latency for a revised sampled data model with sampling interval uniformly distributed between 150 and 250 msec. This system shows a slower rise in average latency over a 100 msec. band of delay times. The important characteristics of these latencies are the rise and fall of latency as T_{S-I} increases. Fig. 4.2c shows latency for synchronized sampling with minimum inter-sample time 200 msec. as discussed above.

4.4 Apparatus and Procedures

The four experiments described were performed at the Man-Vehicle Control Lab of the Massachusetts Institute of Technology on six subjects. The following sections describe the apparatus used and the procedures followed in conducting these experiments.

4.5 Target Generation

Subjects were seated 28.3 inches in front of an Electromec model 565 10" x 12" oscilloscope. A vertical line was generated on the oscilloscope by introducing a 100 kc sawtooth wave

a & b NON-SYNCHRONIZED SAMPLING

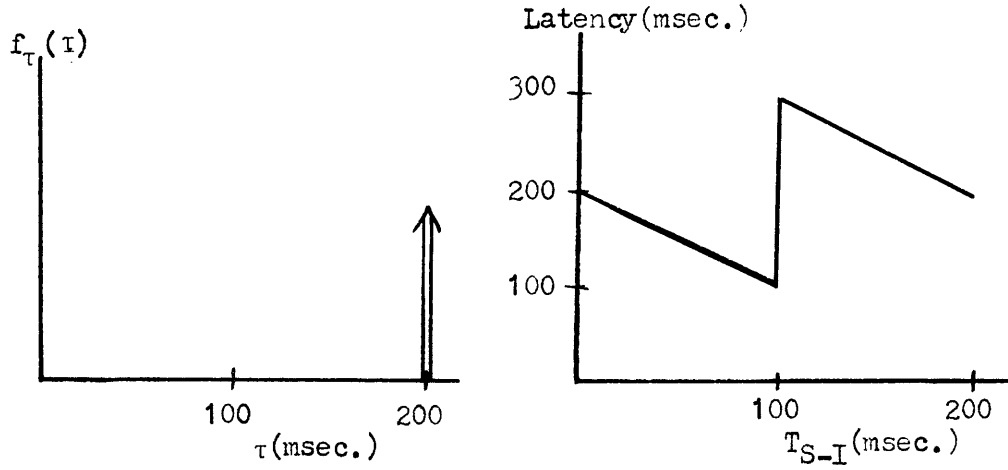


Fig. 4.2a Constant Sampling Interval System

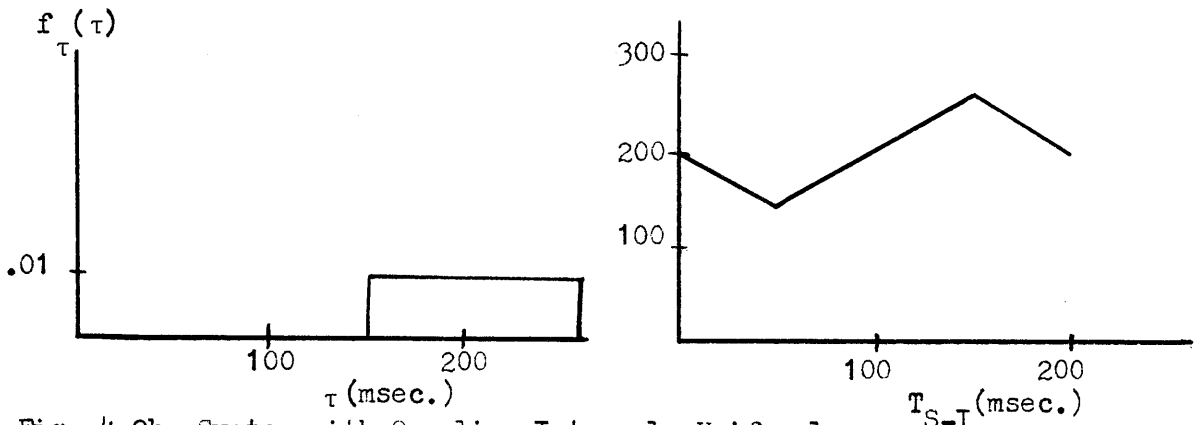


Fig. 4.2b System with Sampling Intervals Uniformly Distributed between 150 and 250 msec.

c SYNCHRONIZED SAMPLING

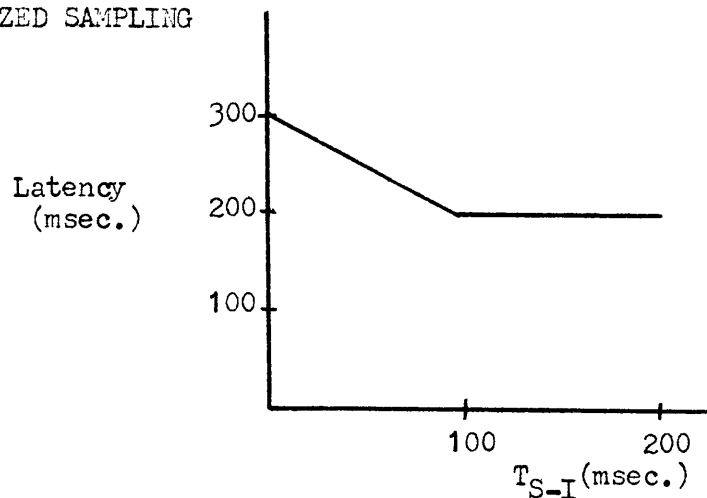


Fig. 4.2c Synchronized Sampling System with minimum intersample time of 200msec.

Figure 4.2 Average Latency Predicted to Saccade-Synchronized Experiments when Presented to Sampled Data Models with different Sampler Control Logic

into the vertical input of the oscilloscope. The line thus displayed was adjusted to 3/4" high by 1/32" wide thus subtending a horizontal visual angle of .0625°. Subjects were allowed to adjust the intensity of the target to a comfortable level which was then maintained throughout the experiments. The target was moved horizontally by introducing voltage signals generated by the lab's hybrid computer into the horizontal input of the oscilloscope. The target moved along the 12" dimension to a maximum of ± 5 inches from center thereby moving through ± 10° of the subjects horizontal visual field. The room was normally lighted and the oscilloscope had fine graticule lines every 1/10" (.2°) and heavier lines every inch (2°). Thus target motion relative to the oscilloscope was easily detected. The P7 phosphor caused a faint after-image but subjects reported no ambiguity in target position at any time.

4.6 Eye Position Measurement

Subject eye position was measured using a Biosystem Inc. GP1 eye movement monitor. This non-contact monitor uses the differential reflection technique to produce a voltage proportional to eye position. The monitor has two photoresistors mounted in a pair of glasses frames and centered in front of the left and right scleral-iris junctions of the left eye. The two photoresistors make up two legs of a wheatstone bridge which may be balanced by adjusting the resistances of the two other legs. The sides of the bridge are introduced into a differential amplifier with filtering of 60 cycle noise.

a & b NON-SYNCHRONIZED SAMPLING

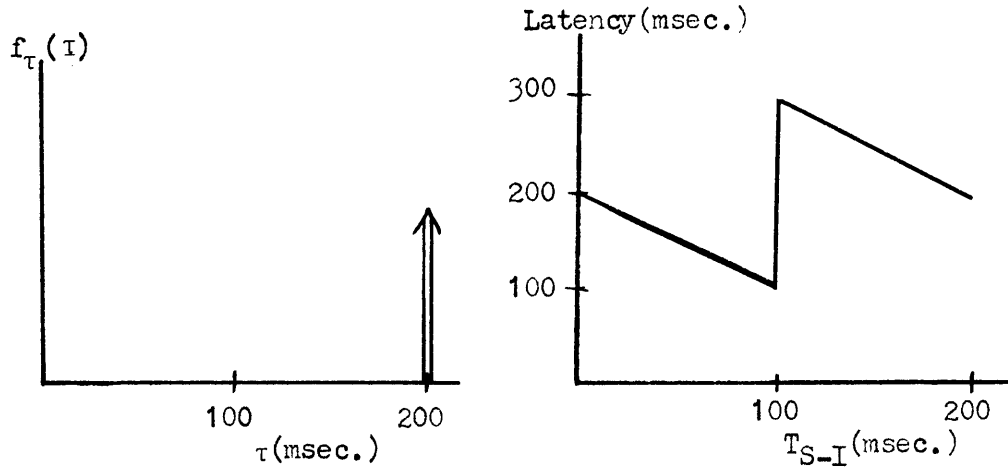


Fig. 4.2a Constant Sampling Interval System

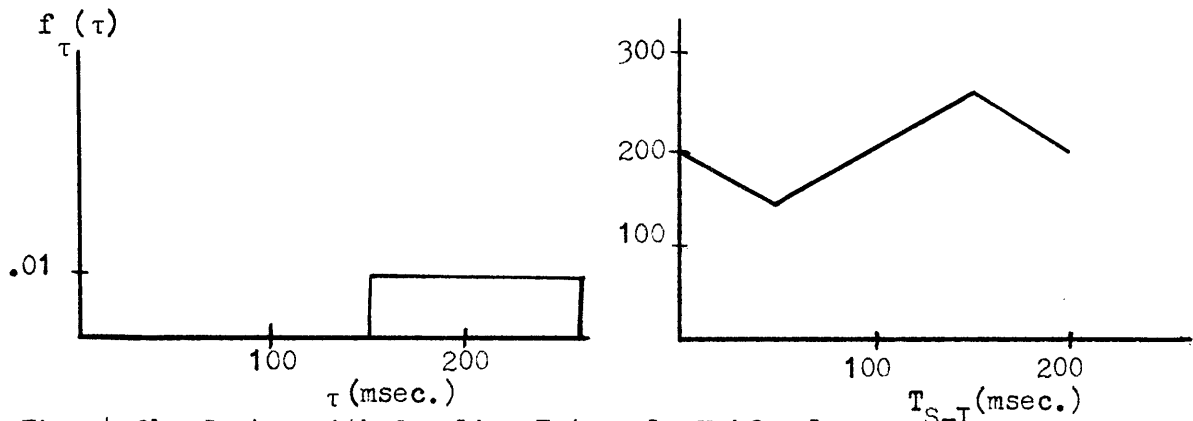


Fig. 4.2b System with Sampling Intervals Uniformly Distributed between 150 and 250 msec.

c SYNCHRONIZED SAMPLING

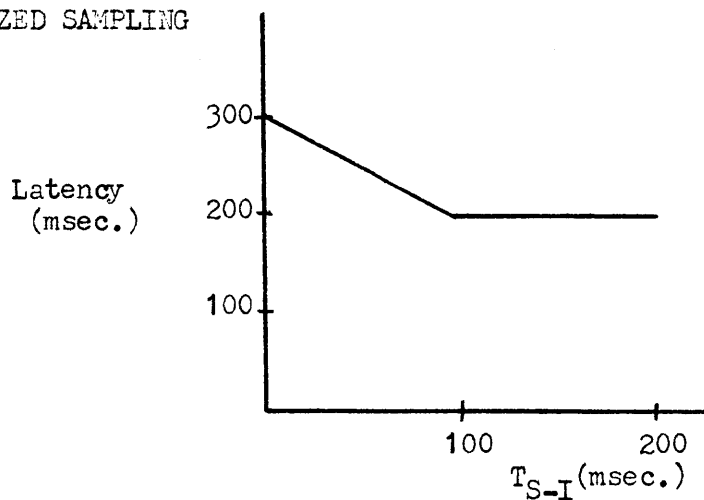


Fig. 4.2c Synchronized Sampling System with minimum intersample time of 200msec.

Figure 4.2 Average Latency Predicted to Saccade-Synchronized Experiments when Presented to Sampled Data Models with different Sampler Control Logic

into the vertical input of the oscilloscope. The line thus displayed was adjusted to 3/4" high by 1/32" wide thus subtending a horizontal visual angle of .0625°. Subjects were allowed to adjust the intensity of the target to a comfortable level which was then maintained throughout the experiments. The target was moved horizontally by introducing voltage signals generated by the lab's hybrid computer into the horizontal input of the oscilloscope. The target moved along the 12" dimension to a maximum of ± 5 inches from center thereby moving through ± 10° of the subjects horizontal visual field. The room was normally lighted and the oscilloscope had fine graticule lines every 1/10" (.2°) and heavier lines every inch (2°). Thus target motion relative to the oscilloscope was easily detected. The P7 phosphor caused a faint after-image but subjects reported no ambiguity in target position at any time.

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Thus as the dark iris moves in front of the photoresistors, the amplifier is biased positively or negatively in proportion to the size of the eye movement. The resulting signal is linear to $\pm 15^\circ$ of horizontal eye motion with a noise level of less than $1/4^\circ$.

The frequency response of the photoresistors was flat to 10 C/S and then falls off at 6 db/octave. Because saccadic responses invariably exceed this low frequency cutoff, records of saccades were characterized by the photoresistors dynamics rather than the dynamics of the subject's eye movements. For this reason the device was only valid for measuring the time of onset of a saccadic movement. The predominance of the same frequencies in every saccade, however, made the task of detection of a saccade easier. Typical records of eye response to the four experiments, recorded using this monitor, are seen in Fig. 4.3.

4.7 Input Generation

The M.I.T. Man-Vehicle Control Laboratory's GPS 290T hybrid computer was used to control the horizontal movement of the target. A complete description of this computer and the programs and circuits used appears in Appendix A. An experimental run consisted of 44 inputs containing 11 each of the four experimental inputs. The saccade-to-step delay was varied from zero to 200 msec. in steps of 20 msec. among the 11 inputs of the same type. The input types and delays were controlled by a 44 entry circular list stored in the digital computer. This list was initially chosen using a

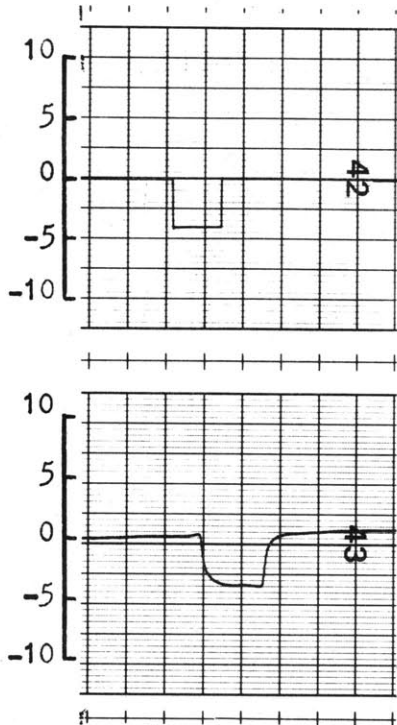


Fig. 4.3a Experiment One

Target
(deg.)

Eye
(deg.)

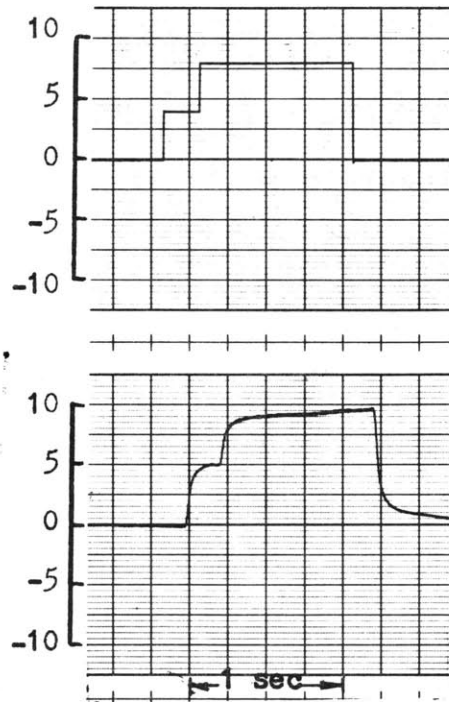


Fig. 4.3b Experiment Two

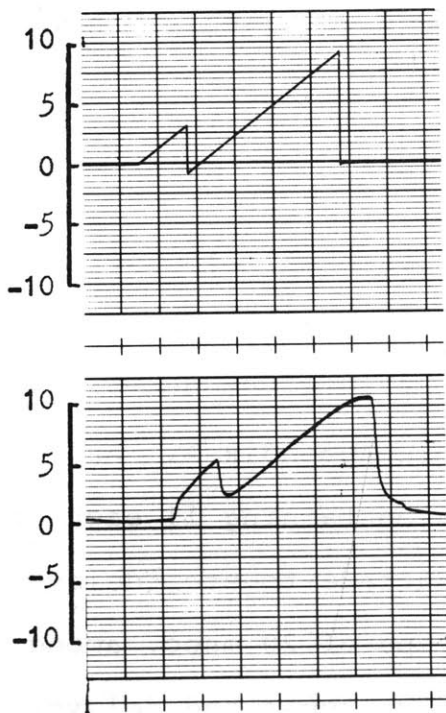


Fig. 4.3c Experiment Three

Target
(deg.)

Eye
(deg.)

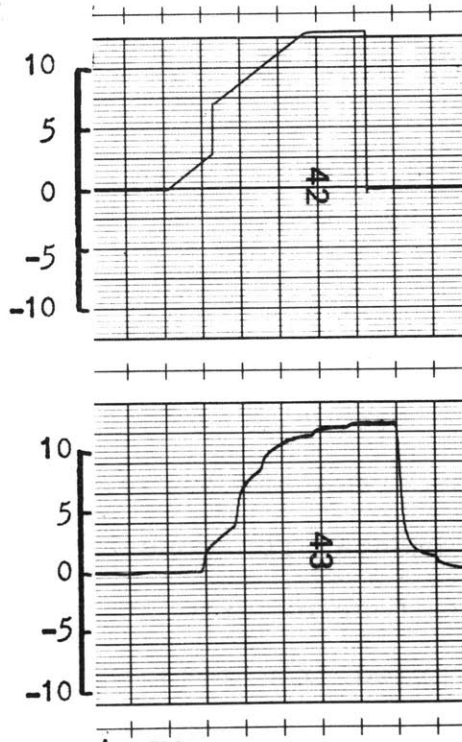


Fig. 4.3d Experiment Four

Figure 4.3 Typical Eye Monitor Records

random number table and remained invariant throughout the experiment. Each list entry was a 12 bit computer word. The digital program tested the three highest order bits to set the input type and direction, then added 7000_8 to the remaining 9 bits and used the result as the initial value of the counter in the digital delay. The starting point in the list was under the control of the experimenter and was varied at his discretion from run to run. When the computer had generated 44 inputs and thus returned to the specified starting point, the run was terminated. The time of initiation of each input was also controlled by the experimenter through a switch on the computer console. Time between inputs was varied from about 2 sec. to about 5 sec. throughout each run. The subjects reported complete inability to predict input beginning, saccade-to-step delay, or input form, throughout the experiment.

4.8 Saccade-to-Step Delay Control

In order to obtain exactly the saccade-to-step delay intervals desired, a saccade detector was used to initiate a digital delay which terminated in the introduction of a 4° step into the target motion. The saccade detector consisted of a pseudo-differentiator to estimate eye rate, which was input to a comparator. The comparator set a digital sense line when the pseudo-differentiator exceeded a preset threshold level just above the level of smooth pursuit movement. Due to the low frequency cut-off of the photoresistors, the best transfer function for the pseudo-differentiator was

found to be $\frac{S}{S+25}$ which was implemented using the circuitry shown in Appendix A. An input sign switching circuit, also shown in Appendix A, was necessitated by the comparator's characteristic of detecting threshold crossings in only one direction, while the saccade to be detected changed direction with the input. The comparator was reset only at the beginning of an input so that just the first saccade was detected.

After initiation of an input by the operator, the digital program repeatedly tested the sense line set by the saccade-detecting comparator. When this sense line was set, the program began incrementing the preset counter once each msec. When the counter reached zero, a control line was set which introduced the 4° step into the input, in the direction preset from the three highest order bits of the current list entry. After introducing the step, the computer waited 1 sec. and then terminated the input by resetting the target position to the center and setting the control lines and counter for the next input from the next list entry.

4.9 Eye Movement Recording

Initially data was recorded and analyzed digitally, however two factors unique to this system made this technique inaccurate. First, variability of time from actual saccade to detection of the saccade ranged from 0 to 10 msec. due to transmission delays, component variability, and variability in eye monitor balance and sensitivity. Second, low-quality control pulses slightly contaminated the high quality analog output signals causing variabilities of ± 10 msec. from commanded

to actual initiation of ramp movements. For these reasons the data reported was taken from the chart recordings made of all responses. The recordings were made at 20 mm/sec and examined closely under a magnifier to an accuracy of ± 5 msec. in all time measurements. A typical 4 channel recording is seen in Fig. 4.4.

4.10 Procedure

Of the six subjects, four were male research assistants in the Man-Vehicle Control Lab and two were female staff members of M.I.T. The subjects were instructed to fixate with both eyes on the target line at all times. All subjects showed typical responses when given experience tracking common transient inputs. In addition to this experience, no experimental data was taken until the subject reported feeling familiar with tracking the actual experimental inputs.

Each subject was presented with at least 10 experimental runs. The runs were presented over the course of an approximately 3 hour experimental session broken by a long rest, which usually included refreshments, halfway through and by shorter, five to ten minute rests between experimental runs.

Each experimental run was preceded and followed by calibration of the eye movement monitor by instructing the subject to look to the right and left edges of the oscilloscope screen which subtended $\pm 12^\circ$ of horizontal visual angle. Subjects were encouraged to terminate a run if their eyes felt fatigued or began to water uncomfortably. Furthermore, eye position was continuously monitored by watching the chart recorder and

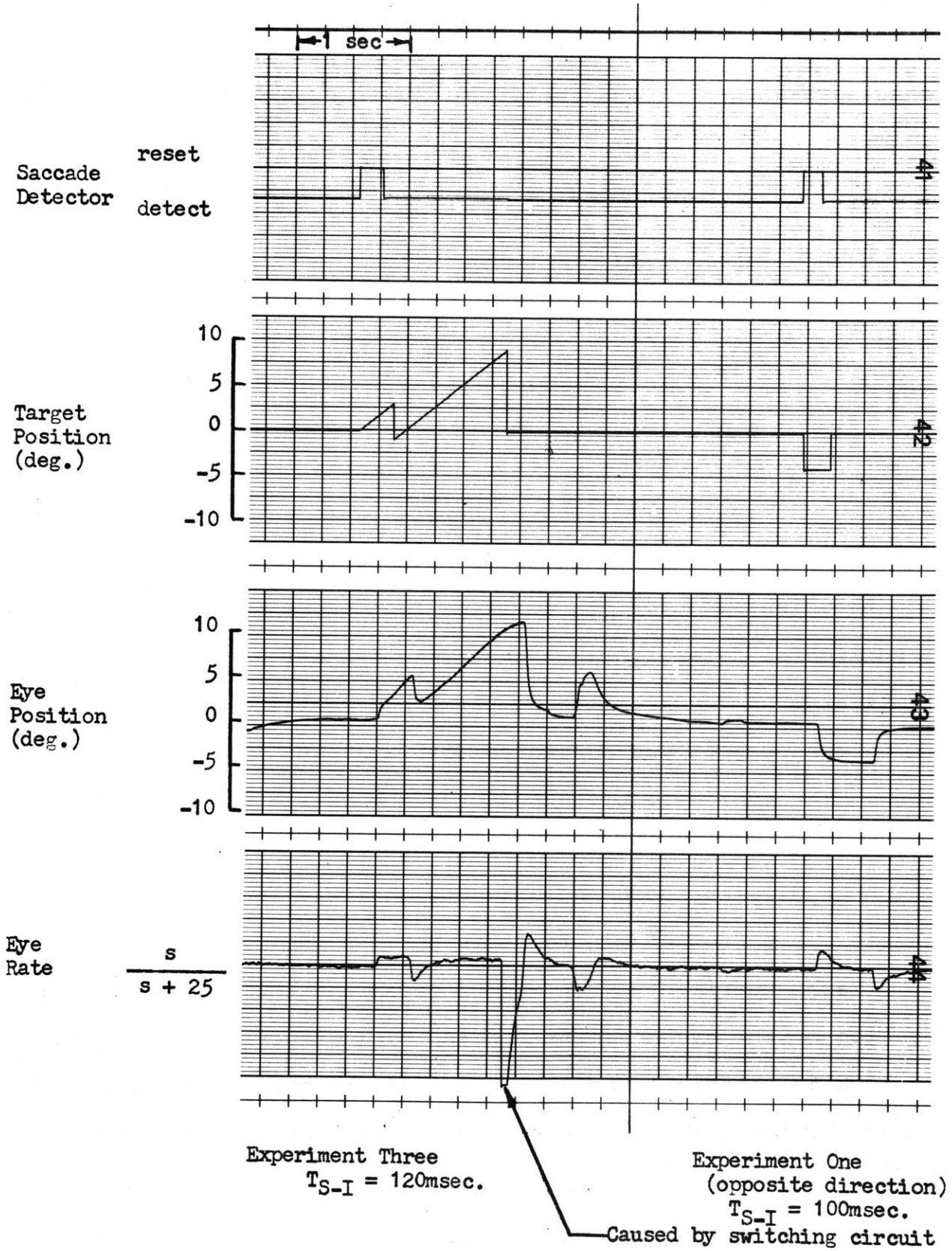


Figure 4.4 Typical Experimental Recording

the run terminated if excessive blinking or drifting was noted. An experimental run of 44 inputs lasted approximately 3 minutes. A total of more than 3000 inputs were reduced.

4.11 Data Reduction

Excepting histograms, which are self explanatory, data was reduced to means and standard deviations of step latencies observed as a function of time after a saccade. Statistical calculations were made from the raw data using a short special purpose computer program written in compiler language for the digital portion of the lab's computer facility. The data from each subject was quantized independently to take into account differences in actual T_{S-I} observed and in the amount of data taken. Intersubject averaging, therefore, occasionally required requantization to insure consistent saccade-to-step delay within each group.

CHAPTER 5

RESULTS

This chapter reports the results of the four experiments discussed in the preceding chapter. Histograms of saccadic latency to the initial step and ramp portions are presented to demonstrate normal non-predictive tracking and to indicate tracking tendencies of each subject. The results are then presented by experiment in the form of curves fit through observed mean latencies. Complete data including mean, standard deviation, and curve fits through means are seen in Appendix B for each subject in response to each experiment. Intersubject differences are then discussed and compared to the response of unsynchronized and synchronized revised sampled data models. At the end of the discussion of each experiment, the mean and standard deviation of intersubject mean latencies are presented and discussed. It should be remembered that this intersubject averaging often covers individual response characteristics and so must not be over-emphasized. The results are then regrouped by subject and the interexperimental response of each subject is discussed.

5.1 Distribution of Initial Saccadic Responses

The first portion of each input was intended to elicit a saccade to which a step would be synchronized. This was done using a 4° step or 10 deg/sec ramp input to either the left or right. The response to these initial inputs should agree with other published results if the subjects are tracking normally in the nonpredictive mode. Histograms of saccadic latency for each of the six subjects in response to the initial step or ramp are seen in Figs. 5.1 a-f. The histograms of step latencies agree very well with those of Beeler (1), Young (23), and others, and thus indicate that all subjects are tracking normally as desired. Most subjects showed a bias in average latency for inputs going in one direction over the other, sometimes by as much as 20 msec. This preference varied from subject to subject and was probably due to inexact center position or undetected glare in the background. This effect accounts for some of the small oscillations in data recorded in Appendix B, due to the predominance of left or right going inputs at some saccade-to-step delay values. Histograms of ramp saccadic latencies agree with Robinson in so far as they show higher means and larger deviations than the step latencies. This effect is modeled by the stochastic sampled data model as previously discussed in Chapter 3. The very low deviation of subject NVH is probably a result of high motivation due to his knowledge of eye movement studies. The secondary distribution of subject AVH was made up almost exclusively of left going

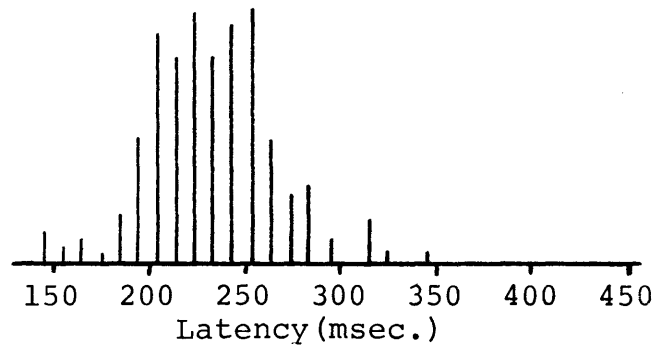
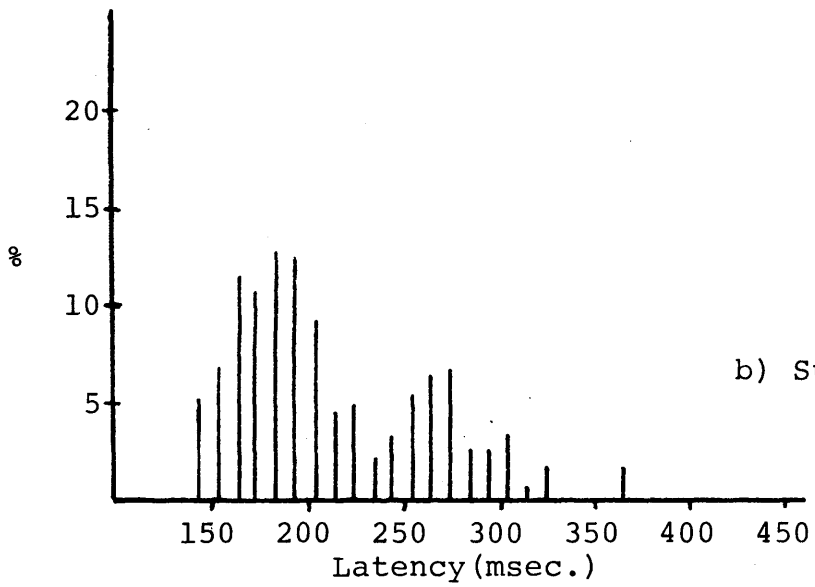
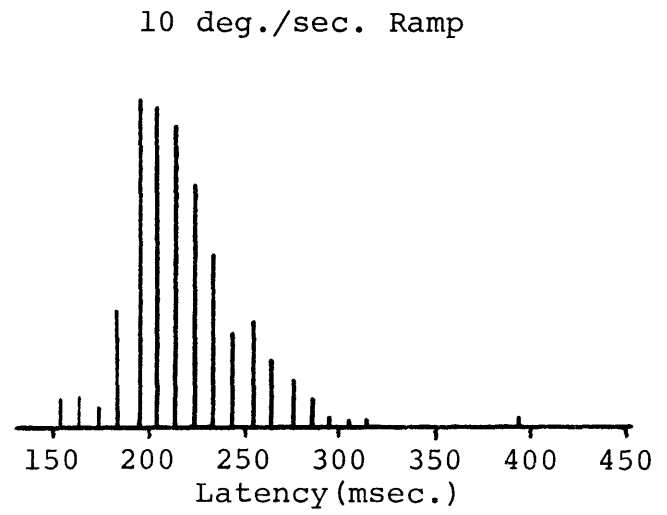
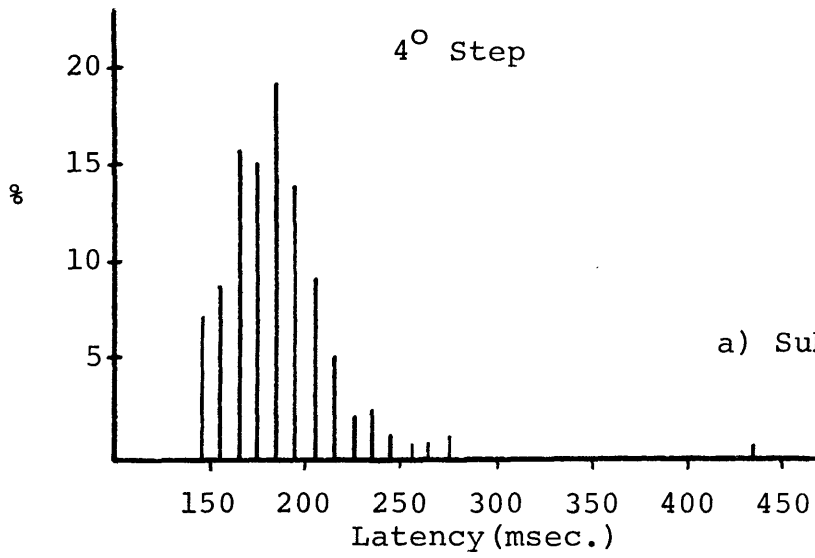


Figure 5.1 Saccadic Latency Histograms

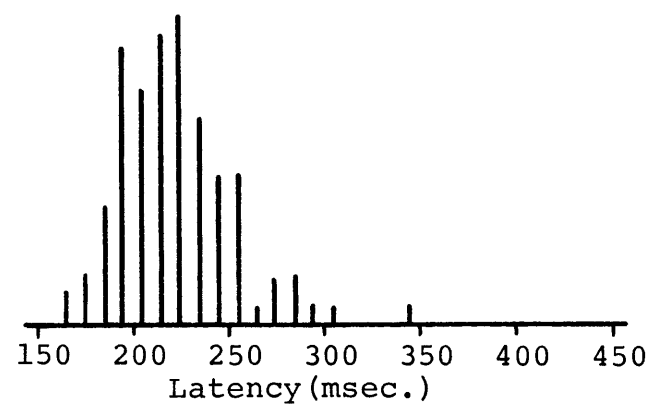
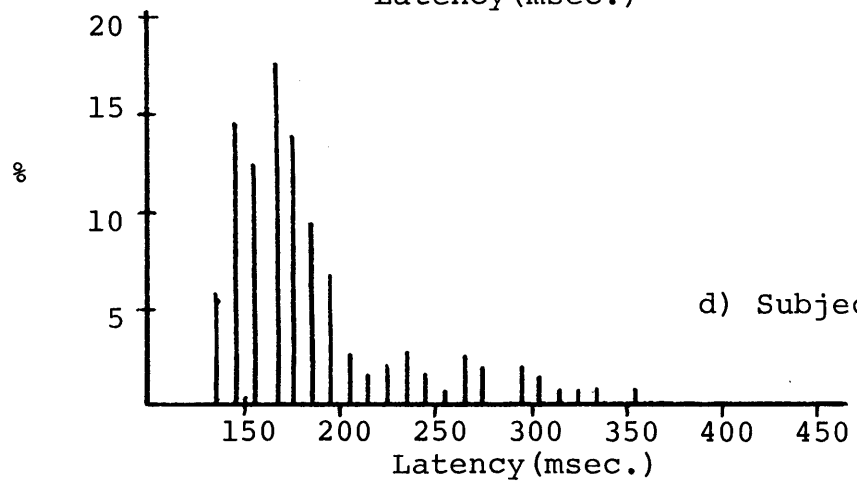
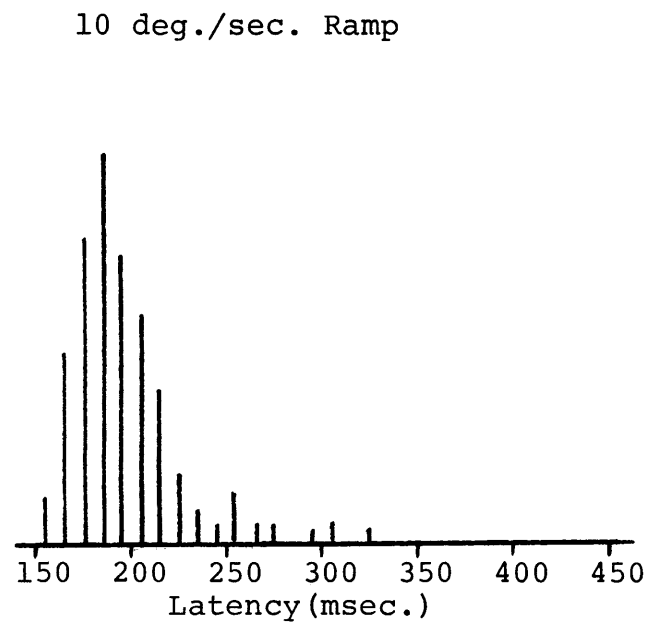
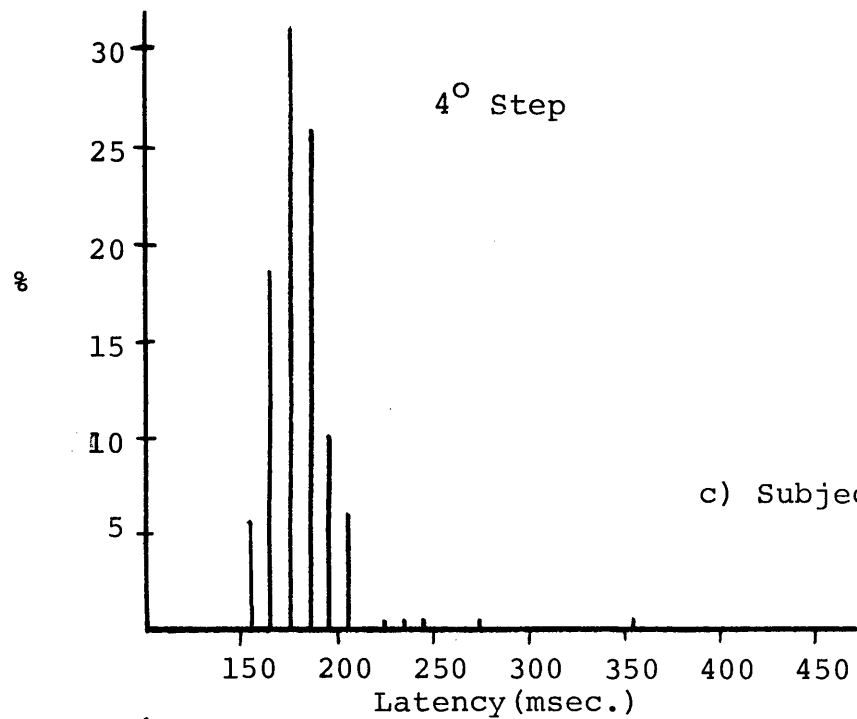


Figure 5.1(Continued) Saccadic Latency Histograms

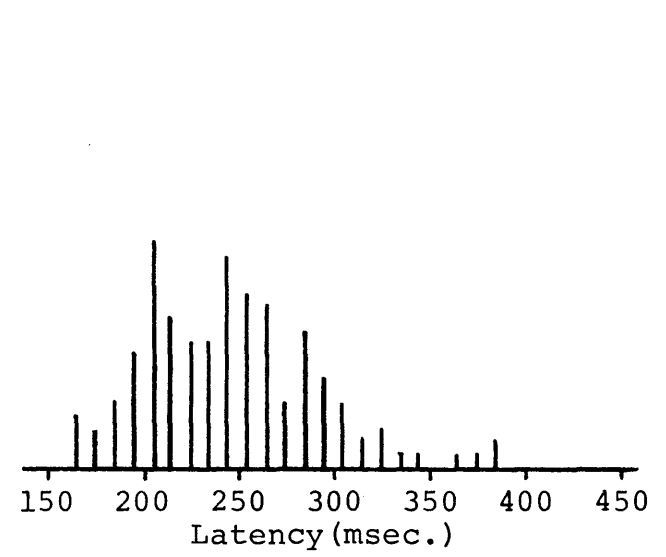
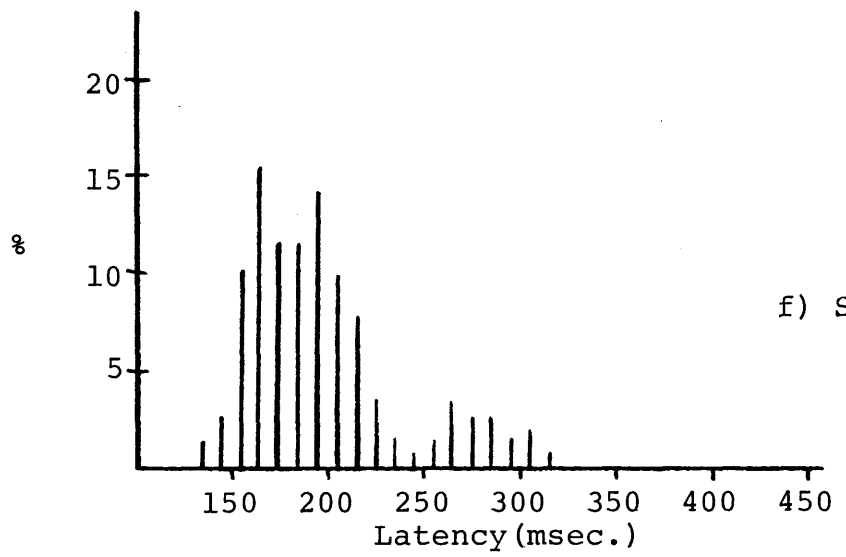
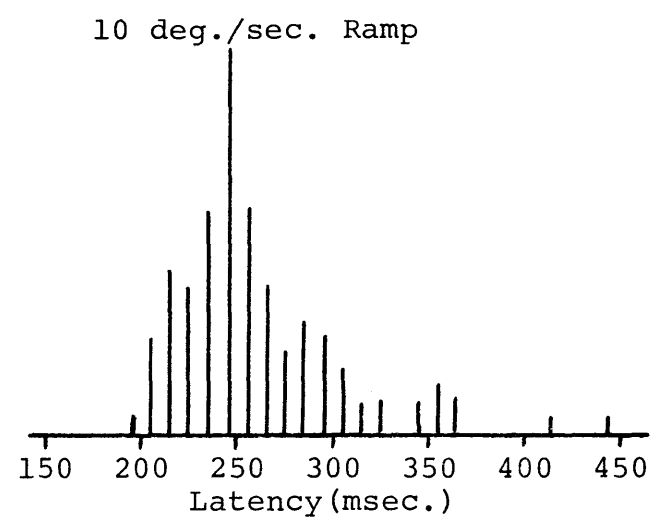
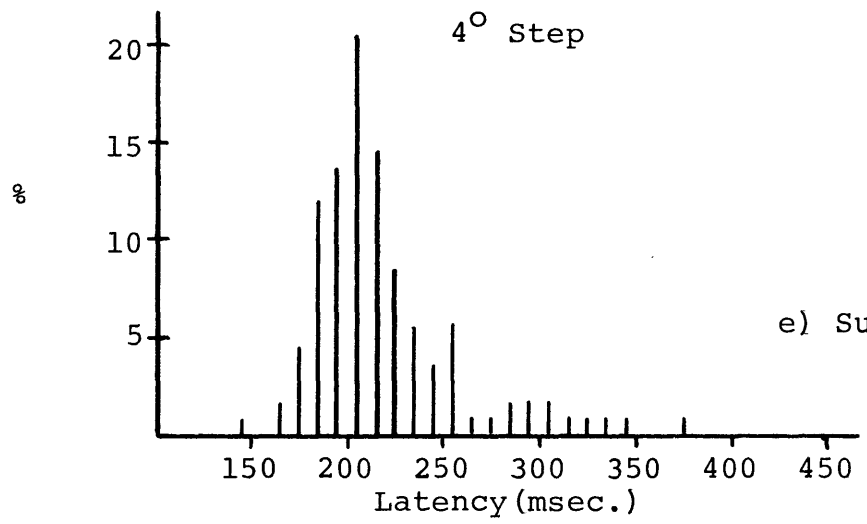


Figure 5.1 (continued) Saccadic Latency Histograms

responses indicating his results may be atypical.

5.2 Experiment 1 --- Synchronized Returning Pulse

Experiment 1 was a returning pulse in which the target return was synchronized to the saccade in response to the first step. Fig. 5.2 shows plots of latency to the second step as a function of T_{S-I} , the time from the saccade in response to the first step, to the occurrence of the second target step. Subjects show variation in latency characterized by two portions, first decreasing latency for shorter T_{S-I} , then little variation in latency. In only two subjects has mean latency decreased to the value of mean latency in response to the initial step. Since latency should decrease to normal as T_{S-I} gets very large, the effect of a saccade on latency must continue even after 200 msec. in many cases. In no subject was there any significant tendency toward increasing latency with increasing T_{S-I} as predicted by the revised sampled data model. This fact suggests synchronization of sampling in response to this input, but with some delay to account for above normal latencies.

Fig. 5.3 shows the average and standard deviation of the intersubject means of Experiment 1 as a function of T_{S-I} , the saccade-to-step delay. The predominant characteristic is that of decreasing latency followed by a leveling off above the value of initial step latency. The two plots on the same graph are the latencies predicted by the revised sampled data model with sampling synchronized to the occurrence of each step and minimum intersample time 200 msec., and the

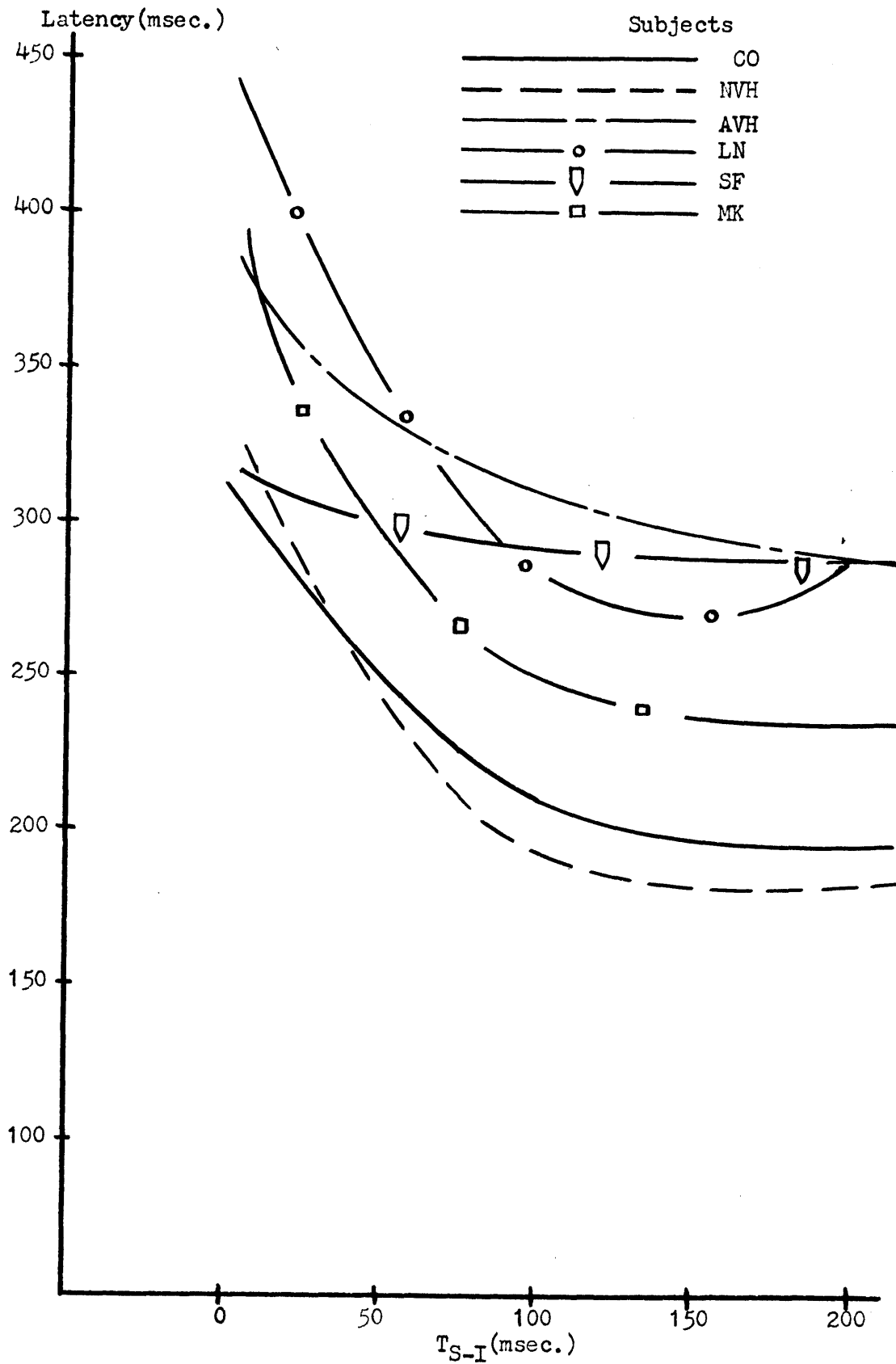


Figure 5.2 Experiment One - All Subjects

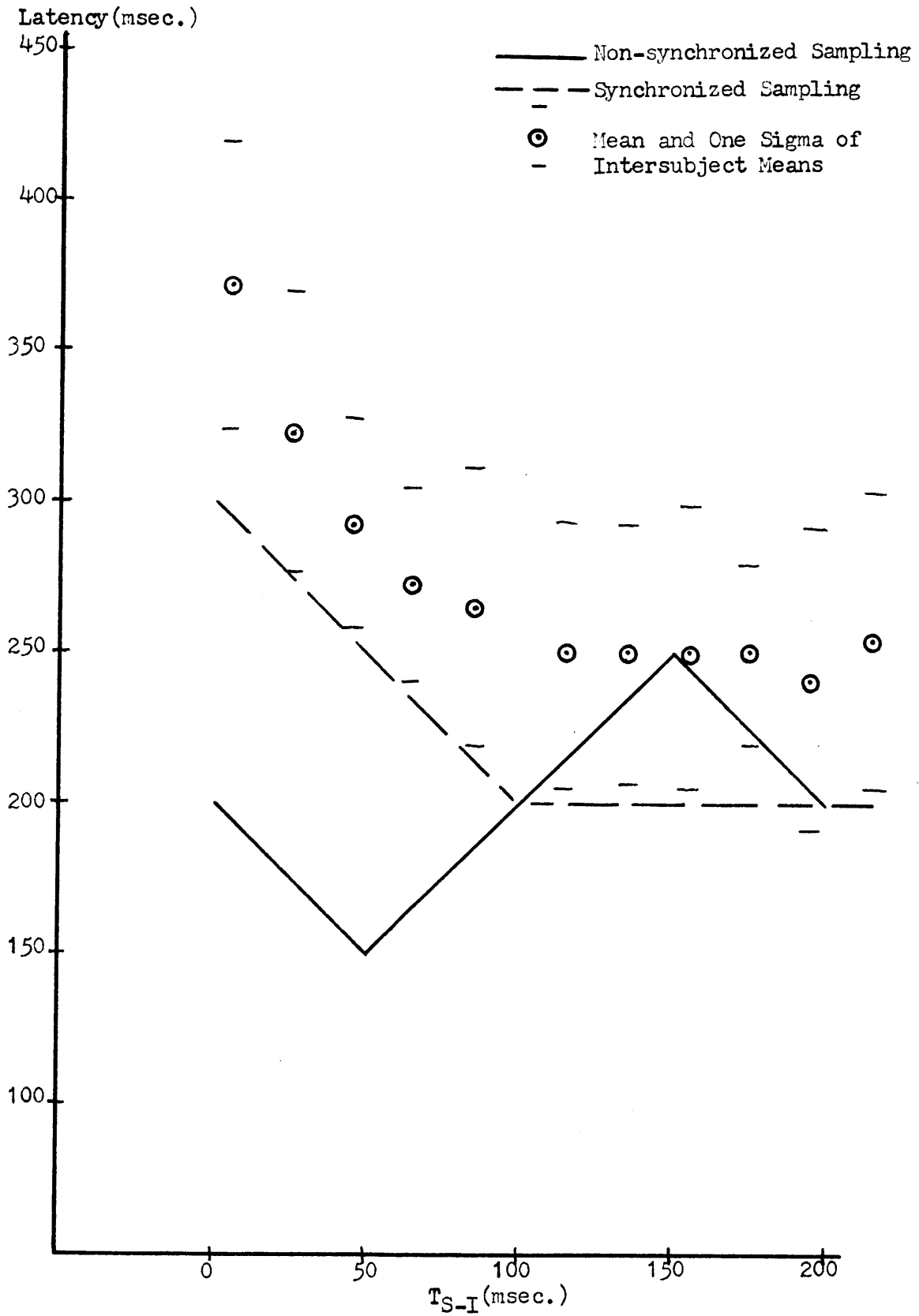


Figure 5.3 Experiment One - Intersubject Means and Model Predictions for two Sampler Control Logics

prediction of the unsynchronized revised sampled data model with sampling intervals uniformly distributed between 150 msec. and 250 msec. The experimental averages follow the synchronized sampling prediction but lie approximately 50 msec. above it throughout. For this experiment, then, the eye delays sampling and synchronizes it to the input.

5.3 Experiment 2 --- Synchronized Stairway Pulse

Experiment 2 consisted of a stairway pulse in which the second step was synchronized to the occurrence of a saccade in response to the first step. Latencies observed as a function of time from the first saccade are seen in Fig. 5.4. Variation of latency in this experiment was less than that of Experiment 1. The decrease in latency for short T_{S-I} delays is observable in the responses of four subjects; however, in Experiment 2 this decrease is from normal latencies to slightly below normal values and occurs over a shorter interval of T_{S-I} delays. Furthermore all subjects exhibit slight rises in latency as predicted by the unsynchronized revised sampled data model, but most follow this rise with non-varying, rather than decreasing latency. In only two subjects does this rise go above normal step latencies. These results, then, show the effects of both synchronized and unsynchronized sampling but with a larger unsynchronized sampling effect than observed in Experiment 1.

Fig. 5.5 shows the mean and standard deviation of inter-subject means observed in this experiment. Plotted on the same graph are the prediction of the revised sampled data

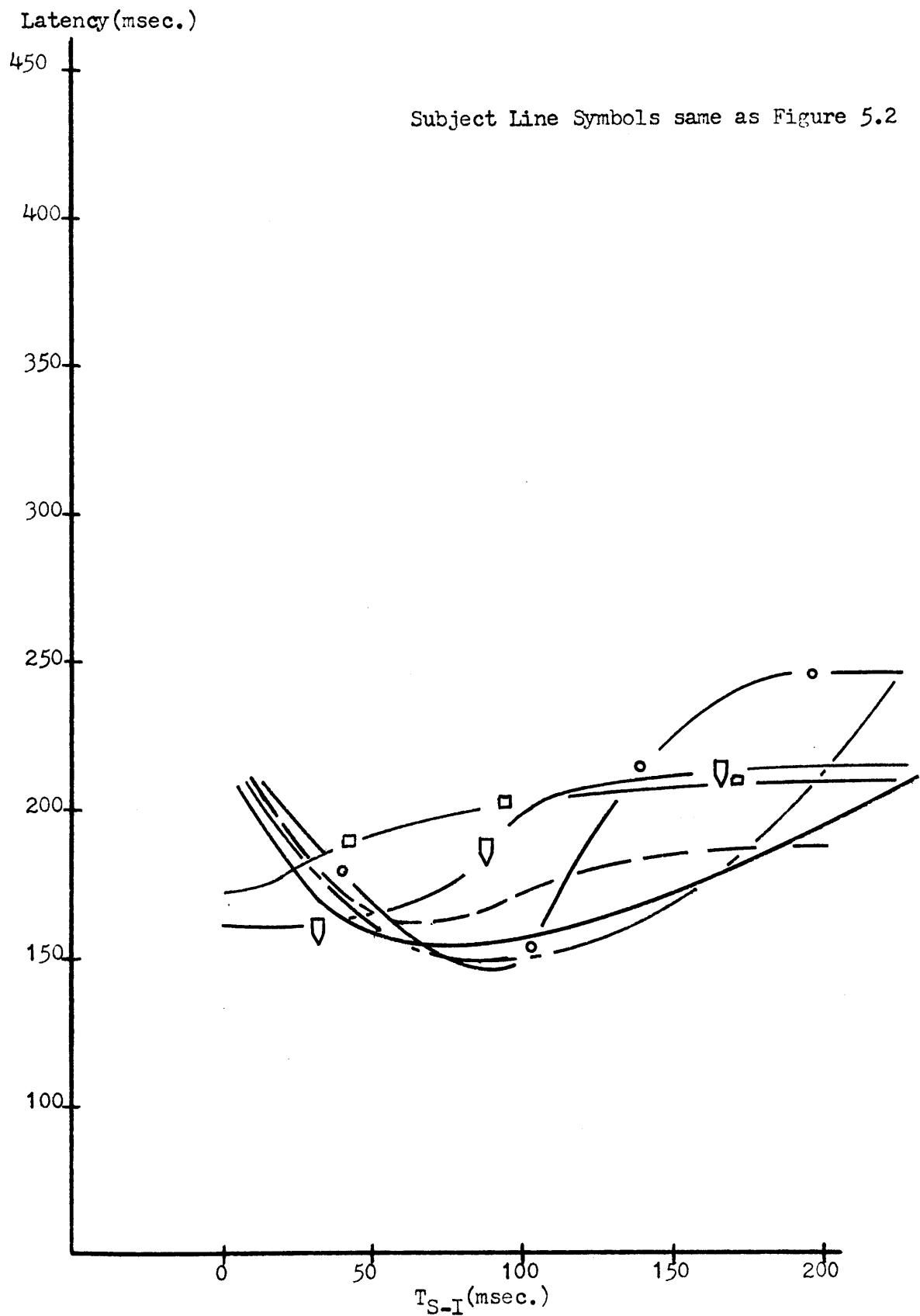


Figure 5.4 Experiment Two - All Subjects

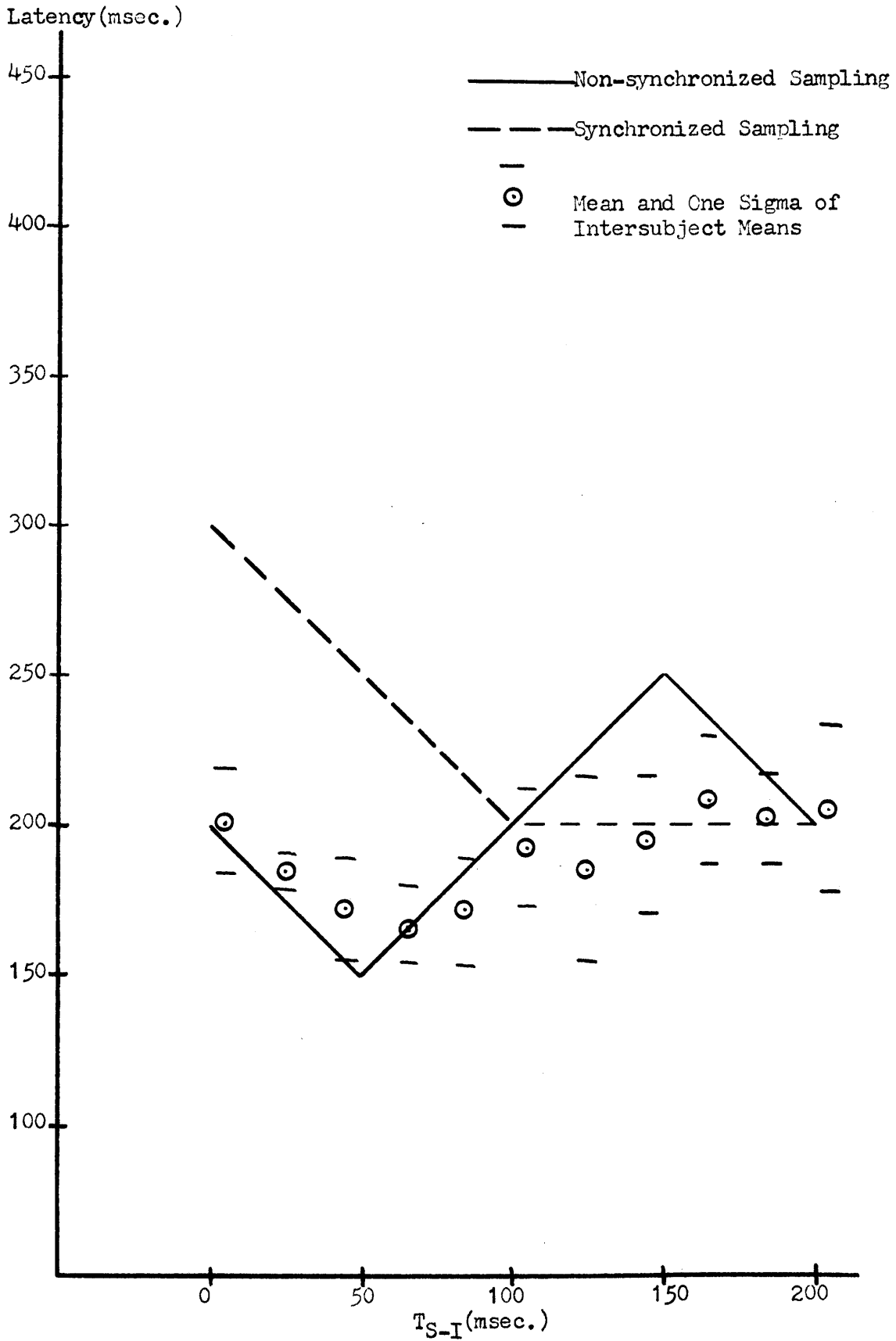


Figure 5.5 Intersubject Means for Experiment Two and Model Predictions for Two Sampler Control Logics

model with synchronized sampling and with unsynchronized sampling interval uniformly distributed between 150 msec. and 250 msec. This data follows the unsynchronized sampling model prediction and thus indicates that in this experiment, the eye exhibits response close to that of an unsynchronized sampler.

5.4 Experiment 3 --- Ramp with Synchronized Inward Step

Experiment 3 consisted of a 10 deg/sec ramp movement into which was introduced a 4° step, synchronized to the occurrence of the first saccade in response to the ramp and in the opposite direction of the ramp movement. Five curves of observed latency in response to the introduced step are seen in Fig. 5.6, as a function of T_{S-I} , the saccade-to-input delay time. The sixth subject showed highly oscillatory responses to which no meaningful fit could be made. As Fig. 5.6 shows, four subjects show slight decreases in latency for short T_{S-I} and little or no variation for longer T_{S-I} . The other subject shows a rise in latency for low T_{S-I} values and decreasing latency for larger values. At large values of T_{S-I} , three subjects exhibit normal step latencies. As in Experiment 1, these results indicate the effects of both synchronized and unsynchronized sampling but with more emphasis on synchronized sampling because of the lack of a rise in mean latency.

Fig. 5.7 shows the intersubject averages for this experiment. Since all variation takes place within one standard deviation, little can be concluded from this result. The

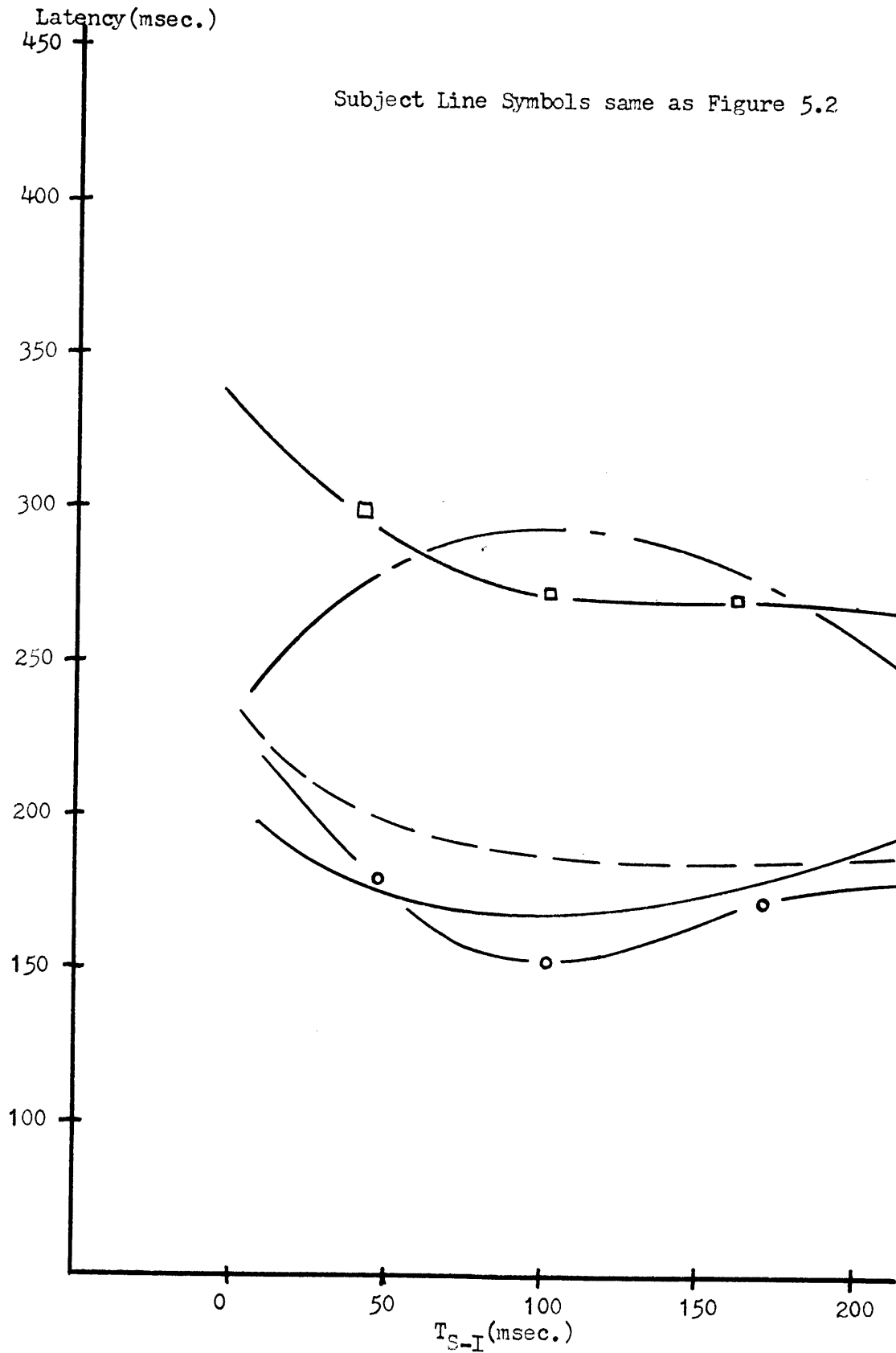


Figure 5.6 Experiment Three - All Subjects

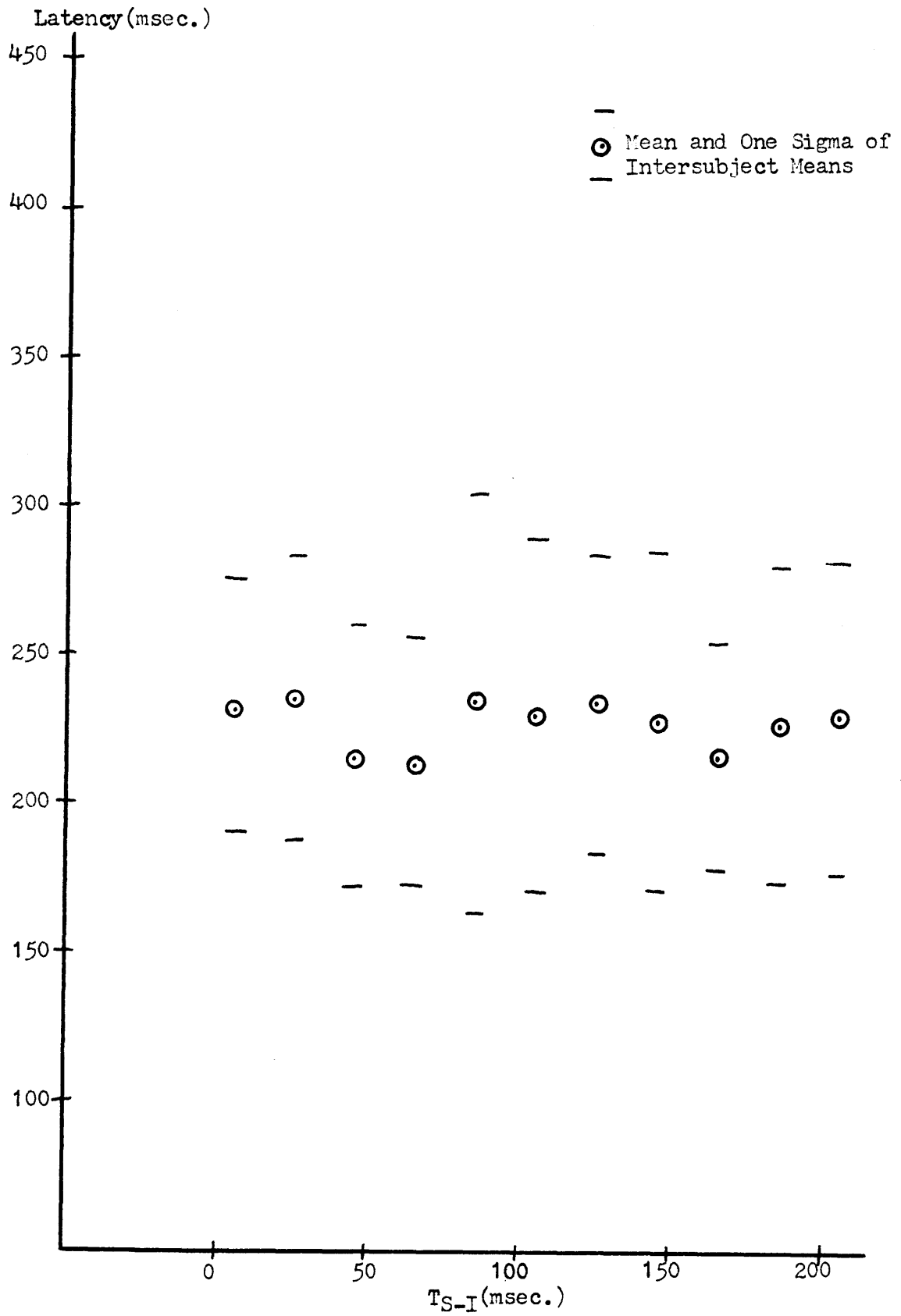


Figure 5.7 Experiment Three - Intersubject Means

fact that all means are above normal tracking latencies indicates that, as might be expected from muscle force considerations, previously initiated pursuit motion in the opposite direction of a saccade has an inflationary effect on latency.

5.5 Experiment 4 --- Ramp with Synchronized Outward Step

Experiment 4 consisted of a 10 deg/sec ramp into which was introduced a 4° step, synchronized to the occurrence of the first saccade in response to the ramp and in the same direction as the ramp. Fig. 5.8 shows the curves of mean latencies observed for this experiment. Four subjects show slight rises of latency while two show a decrease followed by an increase in latency. These six rises indicate predominance of unsynchronized sampling but do not fit the model well. Therefore, little can be concluded about the nature of sampling in this experiment.

Fig. 5.9 presents the intersubject means for this experiment. As in Experiment 3, variations of latency are too small and oscillatory to be meaningful. All latencies, however, are approximately as far below normal step latencies as Experiment 3 was above normal step latencies. This indicates that the effect on step latency of previously initiated pursuit movement in the same direction is to decrease latency about as much as pursuit movement in the opposite direction inflates latency.

5.6 Subject Interexperimental Results

Figs. 5.10, 5.11, 5.12, 5.13, 5.14, and 5.15 each show

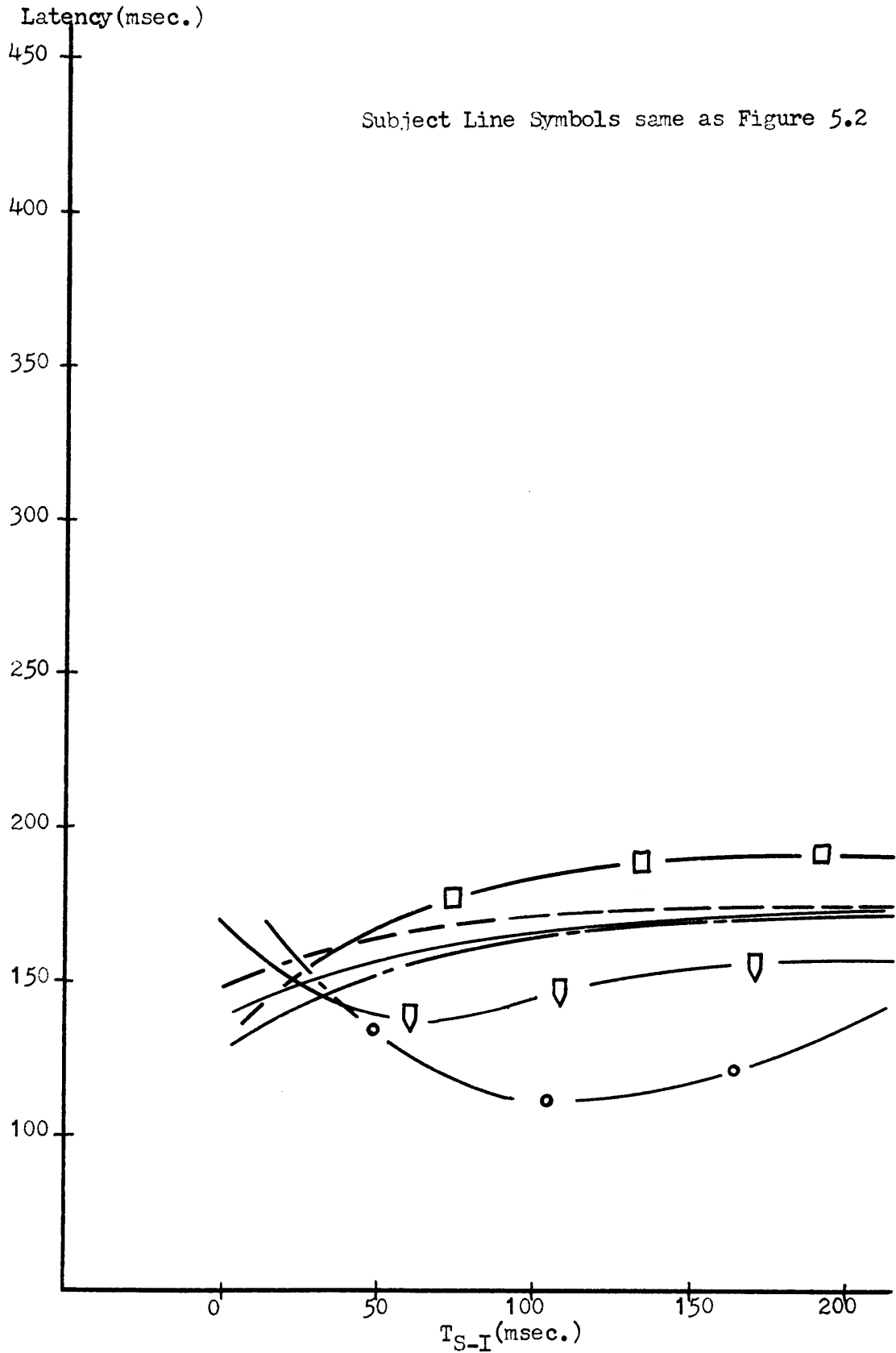


Figure 5.8 Experiment Four - All Subjects

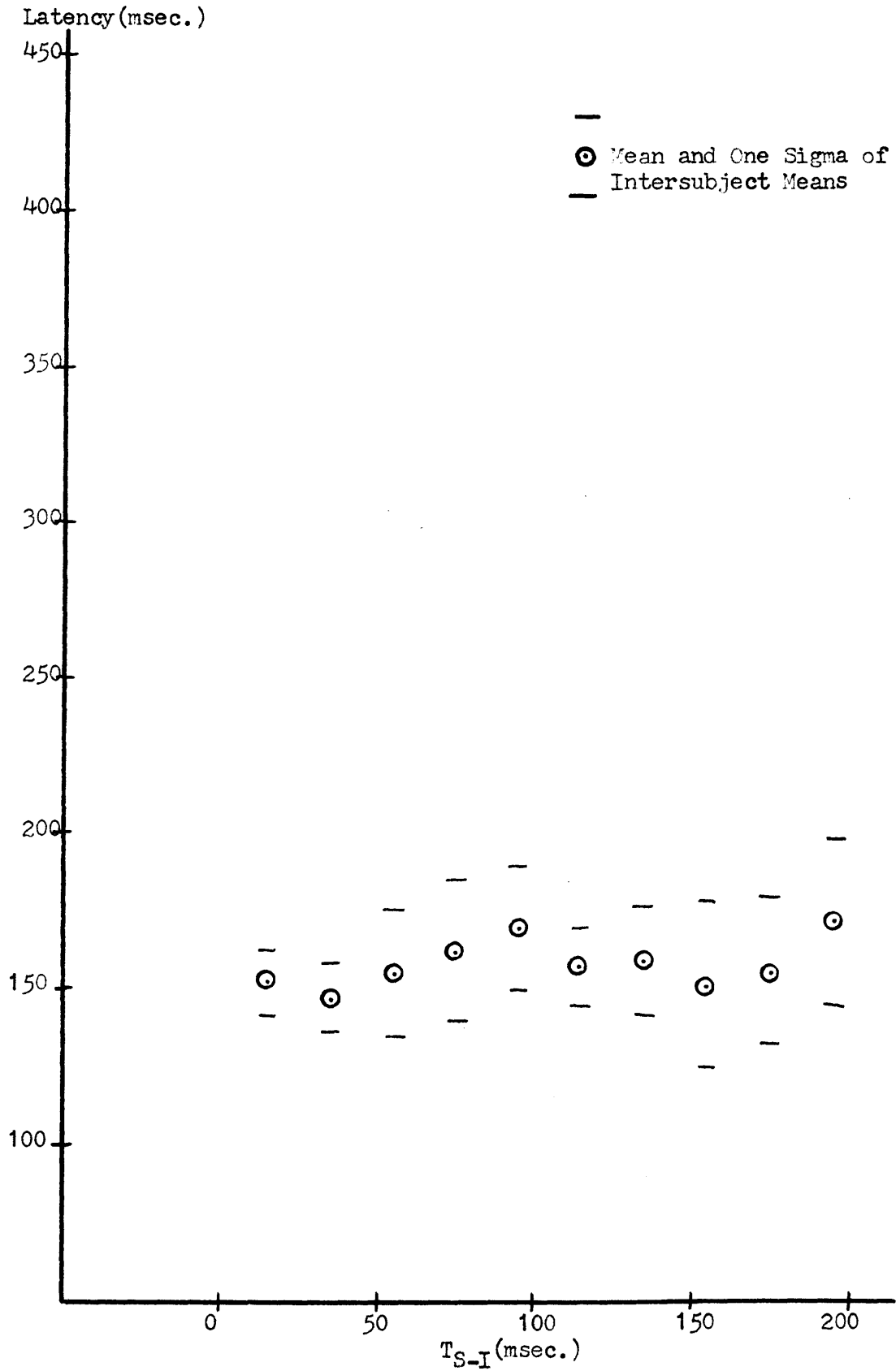


Figure 5.9 Experiment Four - Intersubject Means

the response of a different subject to the four experiments. These figures show that actual mean step latencies in these experiments were not identical as predicted by the revised sampled data model. Experiments 1 and 3 which required reversal of eye movement direction elicited longer latencies throughout than Experiments 2 and 4 which did not. This result agrees with the results of Beeler (1) previously discussed. Experiments 1 and 2 which involved saccadic response only, showed larger variation of mean latency than Experiments 3 and 4 which also involved pursuit movement.

Only one subject (LN) showed a tendency toward increasing latency in all experiments as predicted by the revised sampled data model. However, even these rises end in leveling off of latency instead of again decreasing latency as predicted by the model.

Four subjects show a point of minimum latency in Experiment 2 as do three subjects in Experiment 3 and two subjects in Experiment 4. All these minima occur between 60 and 80 msec. after a saccade and indicate that a sampling instant may have occurred immediately after the target step. With a constant delay of 100 msec., this would indicate a predominance of intersample times slightly higher than 160 to 180 msec., in good agreement with intersample times for realistic model results to other transient inputs as discussed in Chapter 3. Other results which show no minima often show a transition of some sort in the same region of T_{S-I} . These transitions might be attributable to saccadic suppression, but this does not explain the rises of those showing minima.

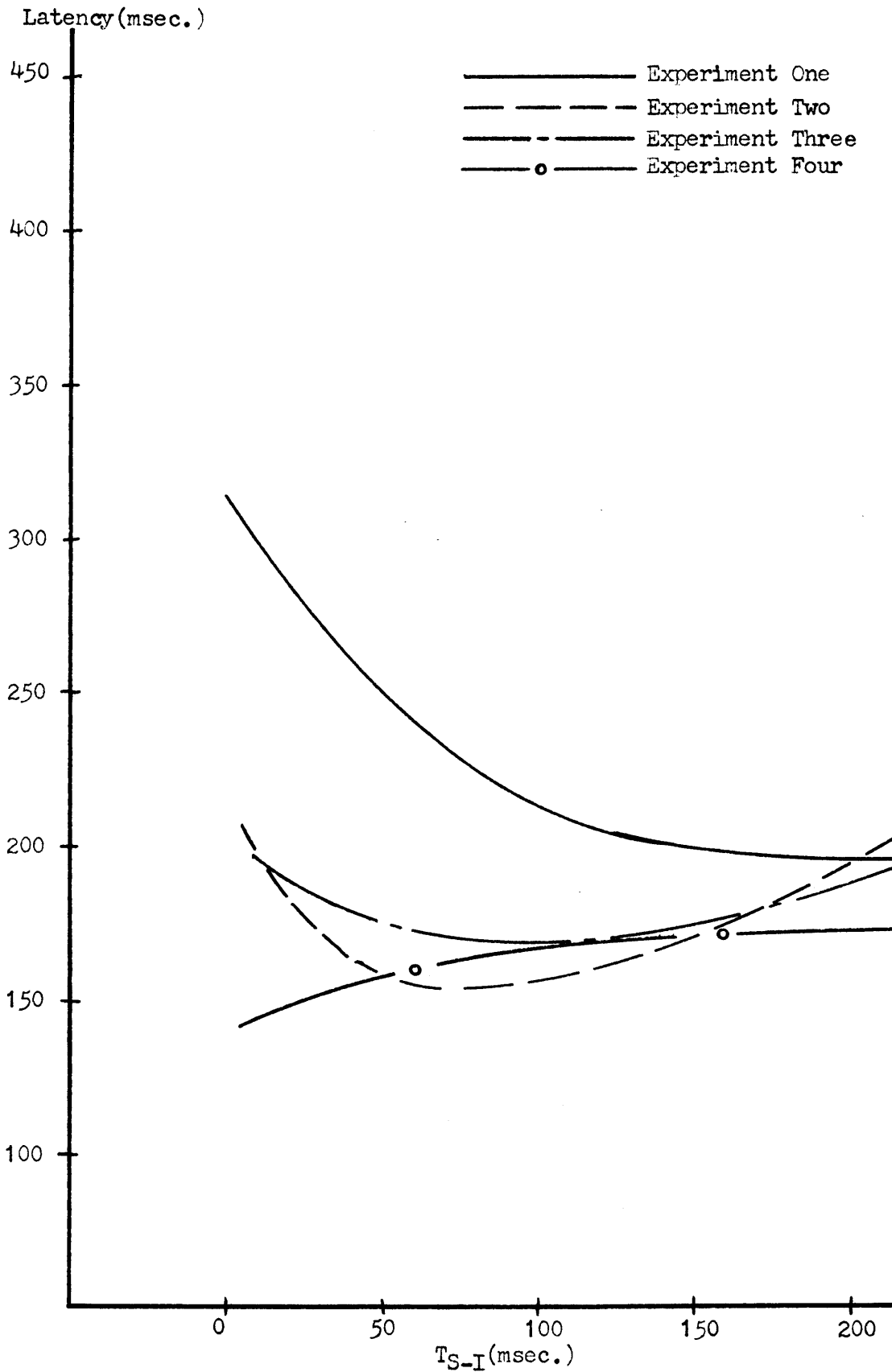


Figure 5.10 Subject CO - All Experiments

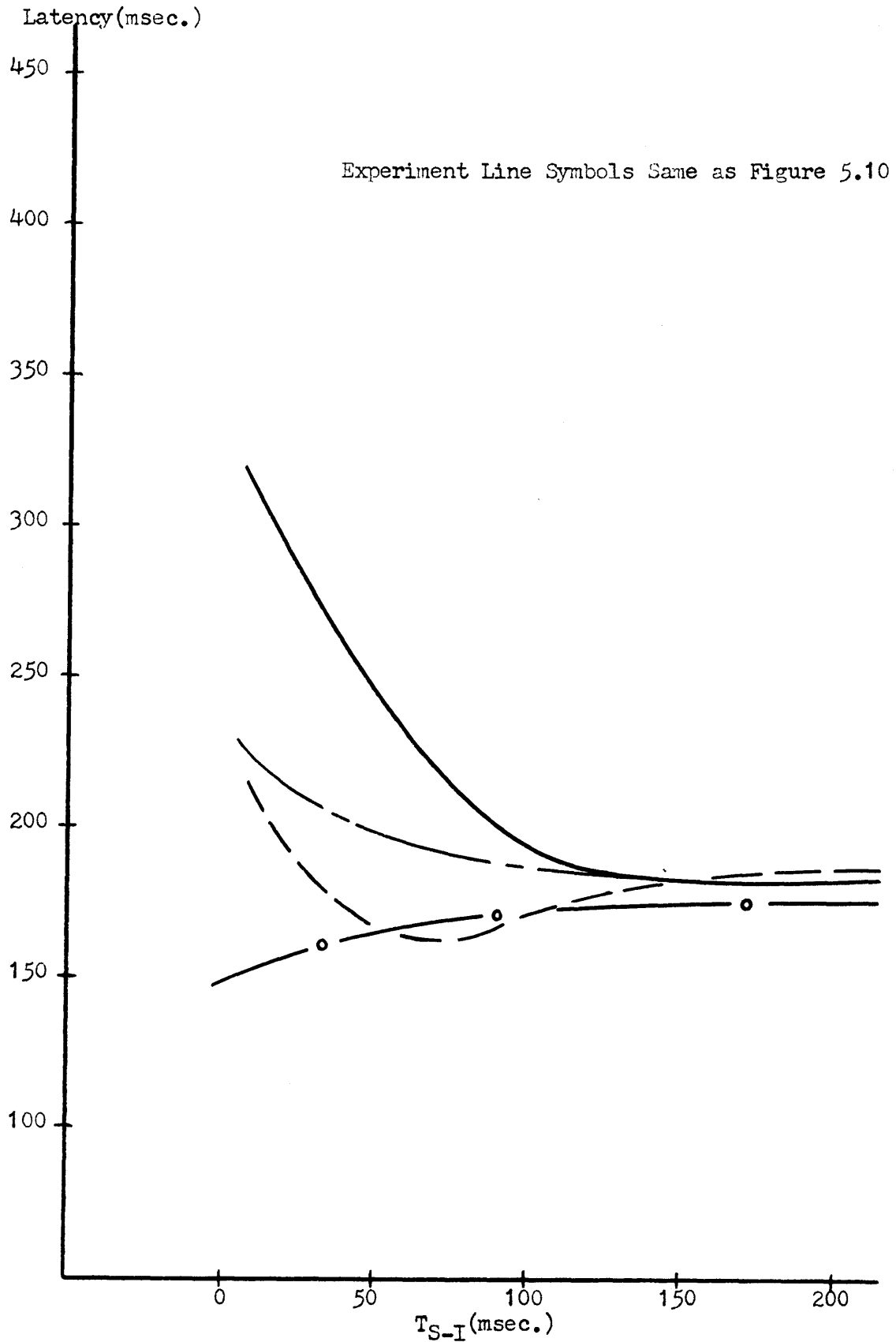
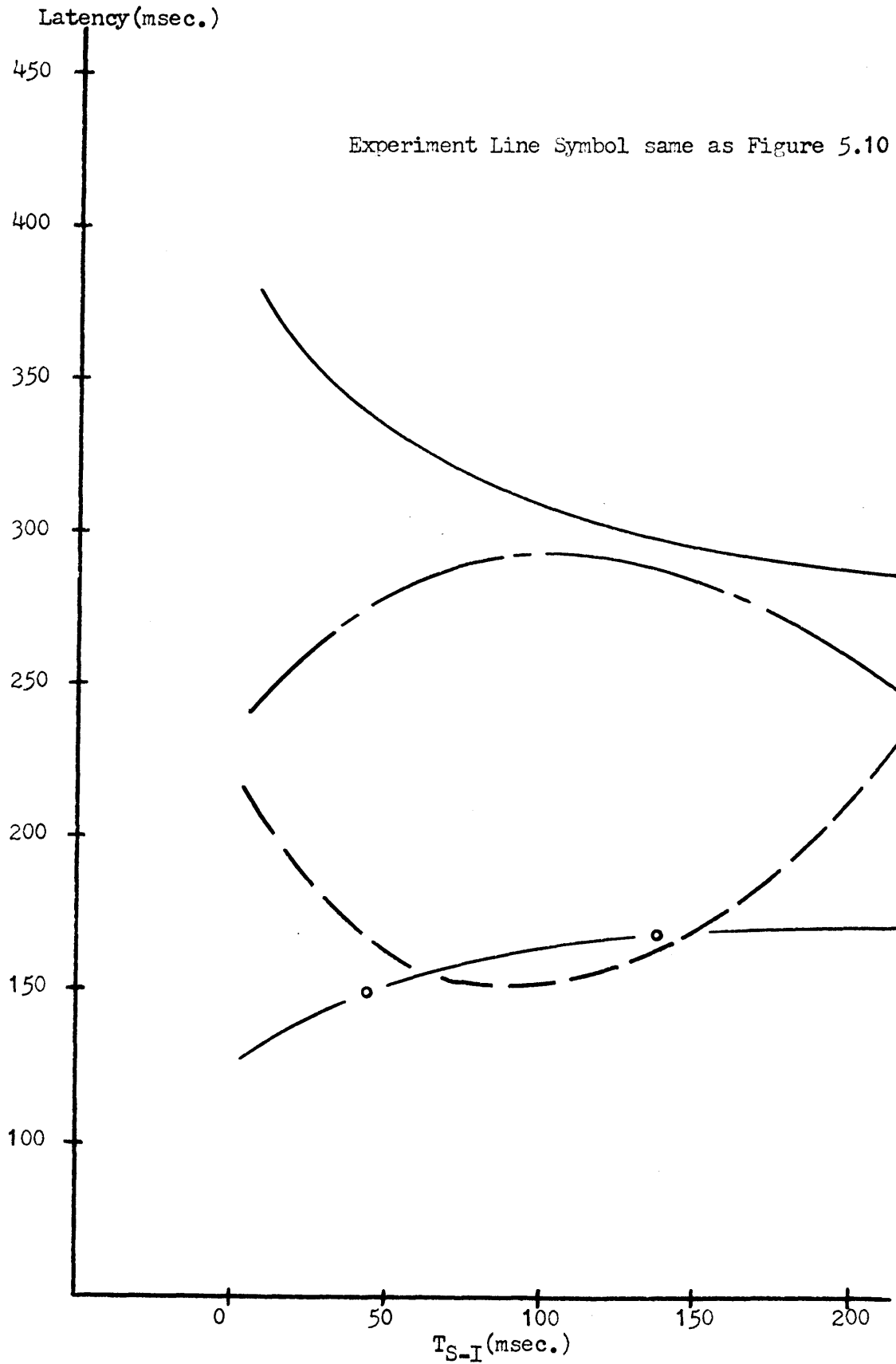


Figure 5.11 Subject NVH - All Experiments



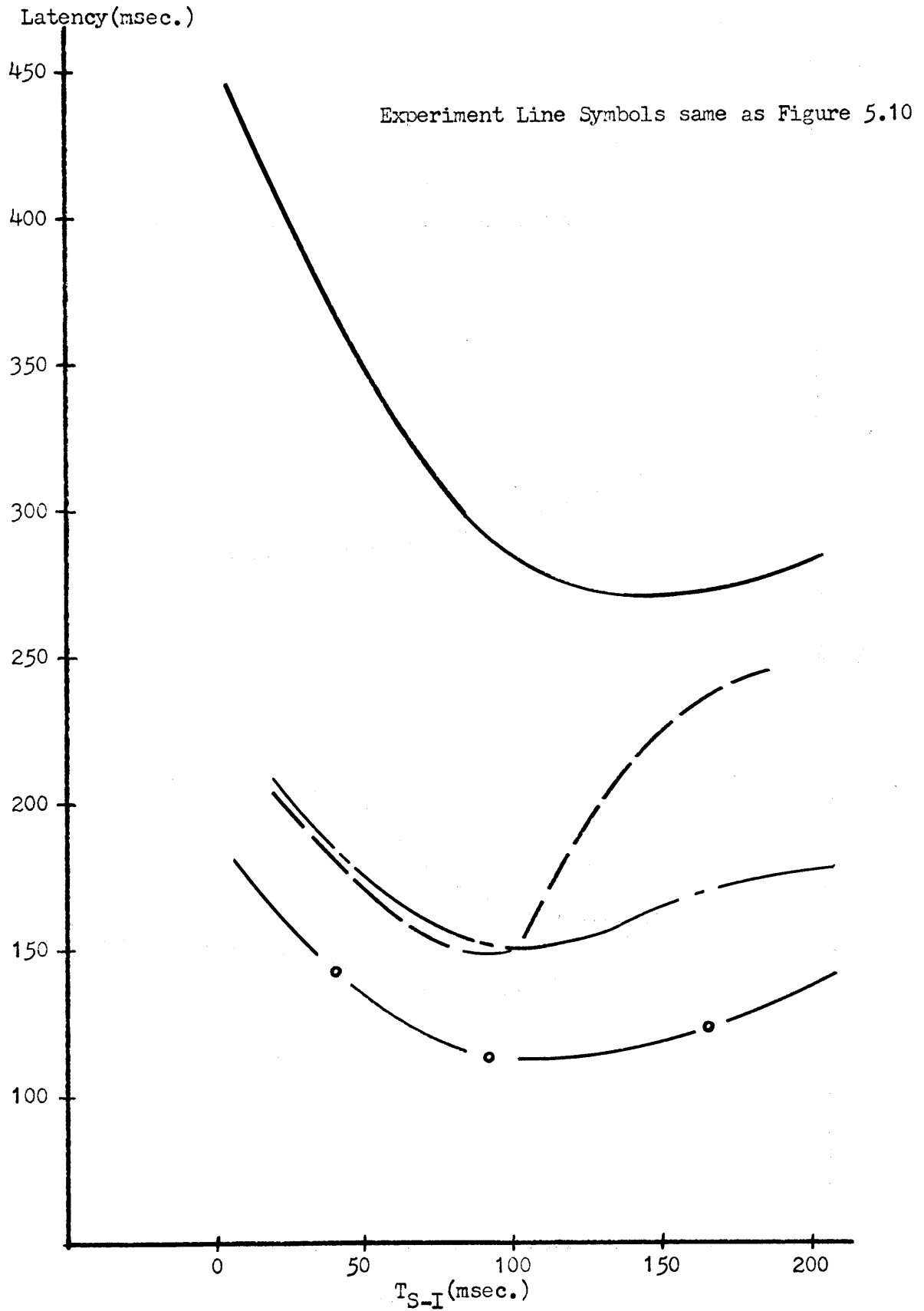


Figure 5.13 Subject LN - All Experiments

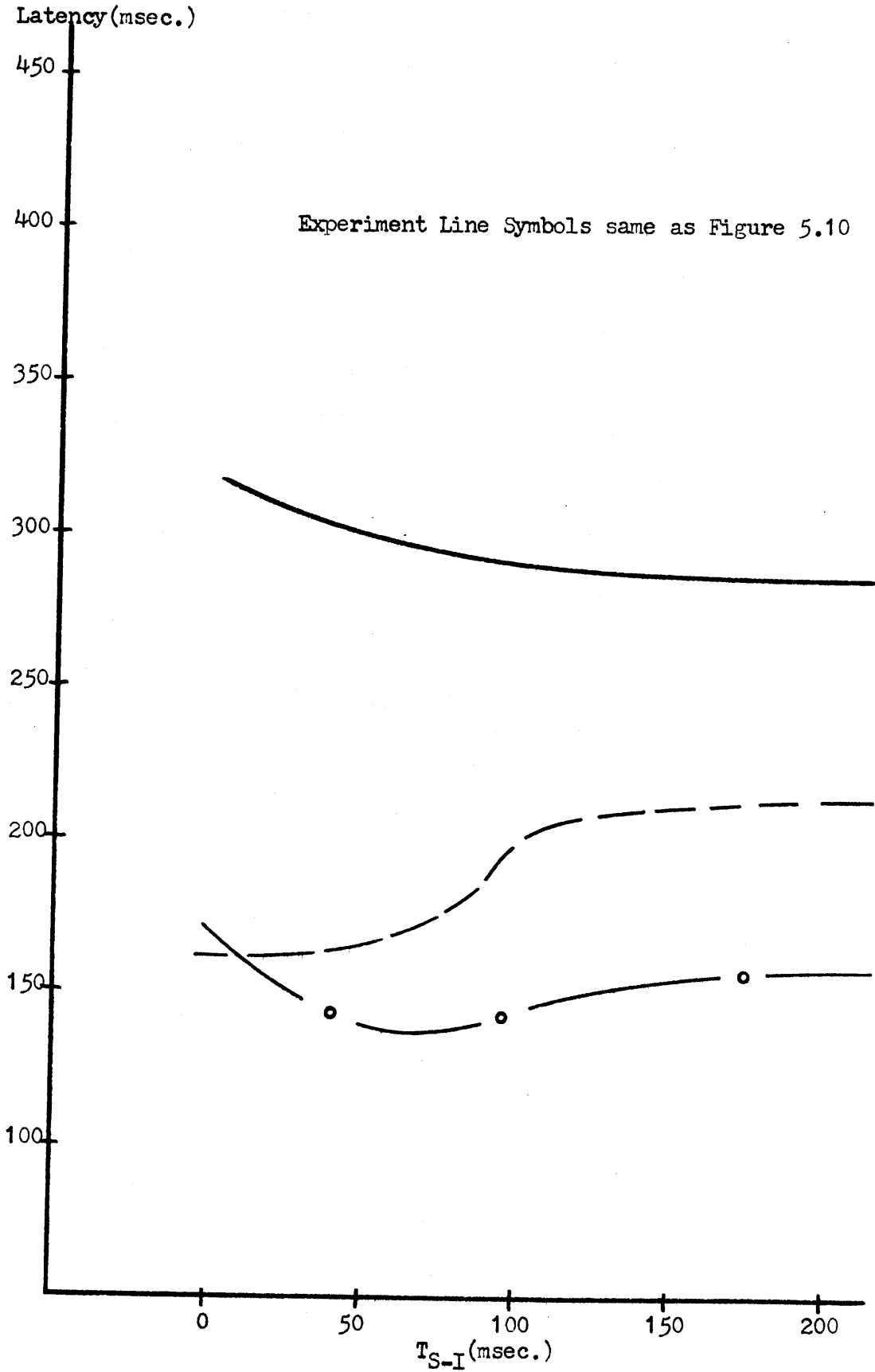


Figure 5.14 Subject SF - All Experiments

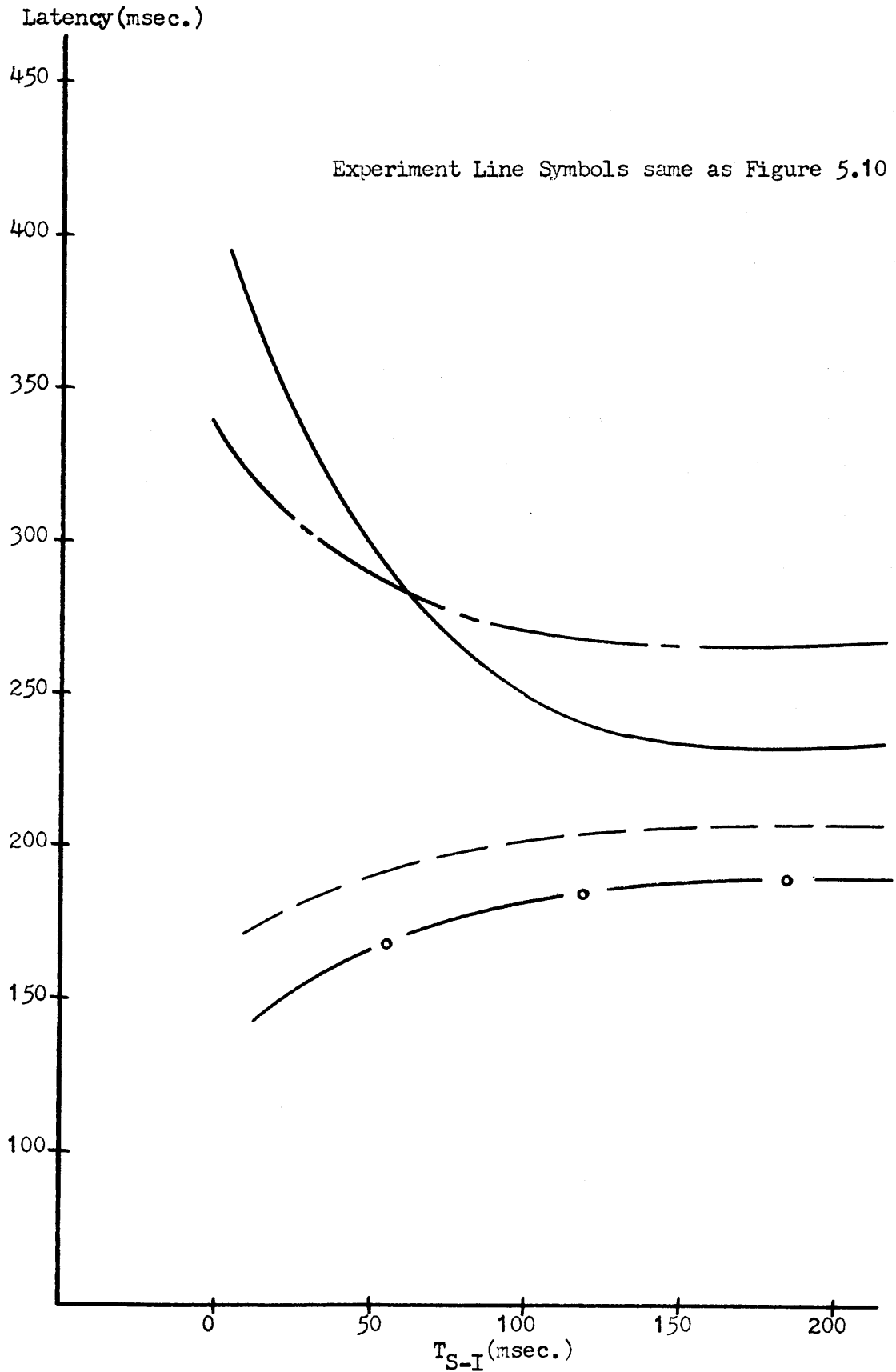


Figure 5.15 Subject MK - All Experiments

The inability of either target-synchronized sampling or unsynchronized sampling to explain the results of these experiments indicates that both types of sampling play a part in eye movement control. One plausible system would be an unsynchronized sampler which may be adapted to sample less frequently when the eye detects difficult movements required to follow the target. Thus reversal of eye movement direction as in Experiments 1 and 3, would show higher latency and more synchronization effects as observed. A system of this sort would also explain the inflated latencies observed by Wheelless (20) to pulse-step combinations.

(see Chapter 2)

CHAPTER 6

CONCLUSIONS AND RECOMMENDATIONS6.1 Conclusions

The revised sampled data model has been shown to exhibit responses which closely resemble eye tracking movements. In particular, classes of responses obtainable from the same input by varying the synchronization of input to sampling intervals resemble actual classes of observed eye movement to identical inputs. The revised sampled data model is, therefore, one simple analyzable deterministic model which may be used to predict classes of human eye tracking movements with remarkable accuracy.

Predicted latency distributions to a single target step movement for an unsynchronized sampled data model are strictly non-increasing as shown at the end of Chapter 2. Actual latency distributions, however, do increase over approximately the first 50 msec. of possible latency. Thus if unsynchronized sampling occurs in the eye movement control system, the eye must be able to adapt at least some of the samples to occur sooner when the target step is observed.

When a target movement occurs immediately after a saccade, the eye movement control system shows tendencies of both

target-synchronized and non-synchronized sampler control logic. The experiments requiring two movements in opposite directions showed the decreasing then constant latencies characteristic of target-synchronized sampling. Most subjects' latencies remained above normal throughout the entire range of saccade-to-step delays used.

The experiments requiring two movements in the same direction showed more of the decreasing, then increasing, then decreasing latency tendencies characteristic of the non-synchronized sampler. Only in the experiment involving just saccadic movement did these tendencies fit the unsynchronized model to any degree. This result indicates that pursuit movement has some effect on sampler control logic.

The experiments involving previously initiated pursuit movement show that saccadic latency to the introduced step is shorter than normal if the step is in the same direction as pursuit movement, and longer than normal if it is in the opposite direction.

The best sampler control logic to model latency tendencies observed in this study is a non-synchronized sampler which could be adapted in phase and frequency depending upon the difficulty of observed target movement. Thus when a simple step movement is observed, the eye could hurry the next sample and show the observed response distributions which are best modeled by short intersample times. In a similar manner when a complex movement is observed, a sample could be delayed and brought into phase with the target movement. This system

would explain observed results to pulse-steps and the wide range of no-saccade step-ramps of Wheelless. This system, however, requires some continuous measure of target movement difficulty and a description of sampling adaptation as a function of this difficulty.

The best non-synchronized sampling system considered in this thesis is the system with intersample time distribution uniformly distributed between 150 and 250 msec. Although this system predicts a trapezoidal step latency distribution it retains the excellent pulse response of a sampled data model with mean intersample time 200 msec. while matching much of the saccade-synchronized experimental results.

6.2 Recommendations for Further Study

1) The saccade-synchronized experiments should be extended to larger values of saccade-to-step time. A step- (step-ramp) experiment should be included in which the step-ramp is saccade-synchronized. In the latter experiment, reconstruction of responses in the manner of Robinson (14) should yield the responses predicted in Figure 3.8 of this thesis.

2) Further study should be made to find combinations of assumptions on $f_D(D)$ and forms of $f_\tau(\tau)$ in the analysis of section 2.5 to exactly model observed latency distributions.

3) Under the assumptions of section 2.5 of this thesis, a polynomial fit to a distribution of intersample times could be constructed to agree with observed step latencies at given

points on the decreasing portion. The jump from zero to the maximum of $f_D(D)$ could then be placed so that the integral over all values of $f_D(D)$ equals unity. This function would be an approximation to the distribution of intersample times to agree with step latencies in a non-synchronized sample data model.

4) The changes in saccades due to pursuit movement and the changes in pursuit movement due to saccades (particularly the latter) need to be further investigated to determine the intersystem control functions necessary in the model.

5) A complete investigation is in order of the phenomenon observed by Beeler and substantiated in this thesis that during a certain short period after an initial saccade the eye can perform a saccade in the same direction as the initial saccade but cannot perform a saccade in the opposite direction. This tendency is probably due to muscle hysteresis nonlinearities and viscoelastic elements in the eye.

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APPENDIX A

THE EXPERIMENTAL SYSTEM

The saccade synchronized experiments of this thesis were performed using the Man-Vehicle Control Laboratory of the Massachusetts Institute of Technology's GPS-290T hybrid computer. The digital portion of this computer is a PDP-8, built by the Digital Equipment Corporation. This computer is a single address, 12 bit machine with 4096 word core memory and a cycle time of 1.5 microseconds. The analog portion consists of a GPS-200T analog computer with amplifiers, integrators, multipliers, comparators, and one-input two-output electronic switches quite compatible with the digital portion since they have a megacycle bandwidth. Two-input one-output switches may be constructed by patching integrators as amplifiers and applying switching signals to the mode control of the integrators. The computer has an analog control panel with individual integrator mode controls, electronic switch controls, two clocks, NAND and NOR gates, flip-flops, digital sense lines and control lines, and program interrupt capability. Only a small portion of these computer capabilities were needed to perform the experiments. This appendix should in no way be considered a complete description of the GPS-290T computer.

The hybrid system developed for the saccade-synchronized experiments of this thesis generates the inputs for the four experiments (Section 4.2 & Figure 4.1) in the order specified by a 44_{10} entry input list. The digital portion sets the switches which control input form, executes the variable time delay, and performs counting functions. The analog portion generates the input voltage signals, detects saccades, and records the experimental variables.

In describing the system developed, a knowledge of basic PDP-8 assembly language and a basic knowledge of analog circuitry are assumed. In the digital portion, variable names and statement labels are capitalized and followed by their octal address in parentheses.

A.1 The Digital System

An assembly language listing of the complete digital portion of the system developed is seen in Figure A.1 together with brief program notes. The starting address of this program is 200. The three instructions preceeding 200 are used to terminate an experimental run with the computer immediately ready to start the next run. The program must be started with the location of entry into the input list set in the switch registers. The program immediately halts with this first input location in the accumulator to allow a check of correct entry. When the CONTINUE key is pressed the computer stores this value in INFROM(307), initializes COUNT(306) which will halt the run after 44_{10} inputs, and enters the main portion at LOOP(205).

```

*175

0175 6452 OUT, CMR
0176 6402 AMC
0177 7402 HLT
0200 7200 CLA / START
0201 7606 LAS HLT
0202 3307 DCA INFROM
0203 1312 TAD CM54
0204 3306 DCA COUNT

0205 6314 LOOP, SCL03 / SELECT INPUT
0206 6322 SCL04
0207 6331 SCL05
0210 1307 TAD INFROM
0211 1314 TAD CM374
0212 7710 SPA CLA
0213 5216 JMP .+3
0214 1315 TAD C320
0215 3307 DCA INFROM
0216 1707 TAD I INFROM / SET SWITCHES
0217 7510 SPA
0220 6312 CCL03
0221 7004 RAL
0222 7510 SPA
0223 6321 CCL04
0224 7004 RAL
0225 7510 SPA
0226 6324 CCL05
0227 7417 LSR /SET DELAY COUNTER
0230 0001 01
0231 0316 AND C777
0232 1317 TAD C7000
0233 3311 DCA DELAY
0234 2307 ISZ INFROM / INCREMENT LIST LOCATION
0235 6311 SCL02
0236 6452 CMR
0237 6401 AMIC
0240 7604 LAS / WAIT FOR SIGNAL
0241 7510 SPA
0242 5240 JMP .-2
0243 4300 JMS ONE
0244 7604 LAS
0245 7700 SMA CLA
0246 5244 JMP .-2
0247 6452 CMR / INITIATE INPUT
0250 6402 AMC
0251 6435 SLR CAC / TEST SACCADE DETECTOR
0252 7700 SMA CLA
0253 5261 JMP PAWS
0254 7604 LAS
0255 7004 RAL
0256 7700 SMA CLA
0257 5251 JMP .-6
0260 5205 JMP LOOP

```

FIGURE A.1 THE DIGITAL SYSTEM - PROGRAM LISTING


```

0261 2311 PAWS,ISZ DELAY / DELAY PRESET TIME
0262 4270 JMS WAIT
0263 6304 CCL02 / INTRODUCE STEP
0264 4300 JMS ONE
0265 2306 ISZ COUNT / FINISHED?
0266 5205 JMP LOOP
0267 5175 JMP OUT

0270 0000 WAIT,0 / WAIT ONE ANALOG FLAG
0271 7344 CLA CLL CMA RAL / RETURN TO INSTRUCTION ABOVE CALL
0272 1270 TAD WAIT
0273 3270 DCA WAIT
0274 6454 CLIF
0275 6461 SKIF
0276 5670 JMP I WAIT
0277 5275 JMP .-2

0300 0000 ONE,0 / DELAY ONE SEC. USING WAIT ROUTINE
0301 1313 TAD CM1750
0302 3310 DCA END
0303 2310 ISZ END
0304 4270 JMS WAIT
0305 5700 JMP I ONE

/ CONSTANTS & SYMBOL TABLE
0306 0000 COUNT,0
0307 0000 INFROM,0
0310 0000 END,0
0311 0000 DELAY,0
0312 7724 CM54,-54
0313 6030 CM1750,-1750
0314 7404 CM374,-374
0315 0320 C320,320
0316 0777 C777,777
0317 7000 C7000,7000
CM1750 0313
CM374 0314
CM54 0312
COUNT 0306
C320 0315
C7000 0317
C777 0316
DELAY 0311
END 0310
INFROM 0307
LOOP 0205
ONE 0300
OUT 0175
PAWS 0261
WAIT 0270

```

FIGURE A.1 (CONTINUED) THE DIGITAL SYSTEM - PROGRAM LISTING

		*320
0320	1633	1633
0321	0777	0777
0322	2537	2537
0323	5753	5753
0324	1467	1467
0325	2727	2727
0326	3563	3563
0327	7703	7703
0330	7513	7513
0331	4657	4657
0332	6607	6607
0333	3467	3467
0334	1537	1537
0335	5607	5607
0336	7753	7753
0337	1727	1727
0340	3633	3633
0341	6657	6657
0342	4513	4513
0343	2777	2777
0344	0563	0563
0345	4703	4703
0346	5657	5657
0347	1777	1777
0350	2563	2563
0351	3727	3727
0352	0467	0467
0353	7607	7607
0354	6703	6703
0355	6513	6513
0356	4753	4753
0357	7537	7537
0360	0633	0633
0361	3777	3777
0362	1563	1563
0363	7657	7657
0364	6753	6753
0365	0537	0537
0366	2633	2633
0367	0727	0727
0370	5513	5513
0371	4607	4607
0372	5703	5703
0373	2467	2467

FIGURE A.1 (CONTINUED) THE DIGITAL SYSTEM - INPUT LIST

Each input generated starts at LOOP(205). The program first resets the control lines to be used to determine the input, and tests INFROM(307) to see if the end of the list has been reached or a value above the list has been accidentally initially specified. In either case, the program resets this pointer to the beginning of the input list by setting INFROM (307) to 320_8 . The program then tests the three highest order bits of the list entry whose address is in INFROM(307), clearing the corresponding control line of any bit that is one. Next 7000_8 is added to the remaining 9 bits and this value stored in DELAY(311) for later use as a counter. Values of the last 9 bits corresponded to counters for delays of 0 to 200 msec. in steps of 20 msec. With the analog flag set to occur at one msec. intervals (as specified in Figure A.4), these counters are constructed by taking the twos complement negative of the number of msec. to be delayed plus one expressed octally. Counter values were placed on the input list at random. The program then puts the analog computer into initial condition mode and prepares control line 2 to introduce the saccade-synchronized step.

Next, the program waits for switch register bit 0 to be zero (down), delays one second, and waits for switch register bit 0 to become one (up), at which time the input is initiated. These instructions (240-246) give the experimenter control over the initiation of an input, with a minimum time between inputs of one second. The instructions could easily be replaced by a constant or random delay.

The program then puts the analog computer into compute mode, thus introducing a step or ramp input to the subject. Next the program continuously tests the sense line which is set by the saccade detector. When a saccade is detected, the sense line is set allowing the program to go to PAWS(261) where it delays the preset time by using the subroutine WAIT(270) and the counter DELAY(311). At the termination of the delay, a step is introduced into the target motion by clearing control line 2. Finally the program delays one second then tests COUNT(306) and goes accordingly to termination at OUT(175) or back for another input to LOOP(205).

Should the saccade detector fail to detect a saccade, the experimenter may return the program to LOOP(205) for the next input by raising switch register bit 1. In this case the input will not be counted in the 44_{10} inputs per run.

This program may be easily altered to produce inputs of known pulse width by changing SLR CAC at location 251 to NOP. If this is done, the pulse width of an input will be exactly the value of the saccade-to step delay of its corresponding list entry.

Future more accurate systems should make possible digital recording and digital data reduction routines to relieve the burden of data reduction by hand. Digital ensemble averaging, however, has been found to conceal typical response characteristics and therefore is not recommended.

A.2 The Analog System

Figure A.2 shows the analog circuit used to generate

target movement in the saccade-synchronized experiments of this thesis. Switches B, C, and D are the switches that control the input form. Table A.1 shows the eight outputs obtainable from the eight different switch settings. Also shown are octal contents of the three highest order bits in the input list to which each input corresponds. Switch A introduces the saccade-synchronized step. Switch E is used to introduce initial steps, and integrator A introduces ramps.

Figure A.3 shows the circuit used for the saccade detector. The circuit for changing the sign of the input was necessary because of the comparator's characteristic of detecting threshold crossings in one direction only, while the saccade to be detected changed direction with the input direction. The $\frac{s}{s + 25}$ transfer function for the pseudo-differentiator was chosen for its good results in conjunction with the low frequency cutoff of the eye monitor used. The comparator threshold level was set just above the level of pursuit eye movement to minimize delay in saccade detection.

Figure A.4 shows the circuits necessary on the analog control panel to interface the analog and digital portions of the experimental system.

All Gains One Unless Otherwise Specified

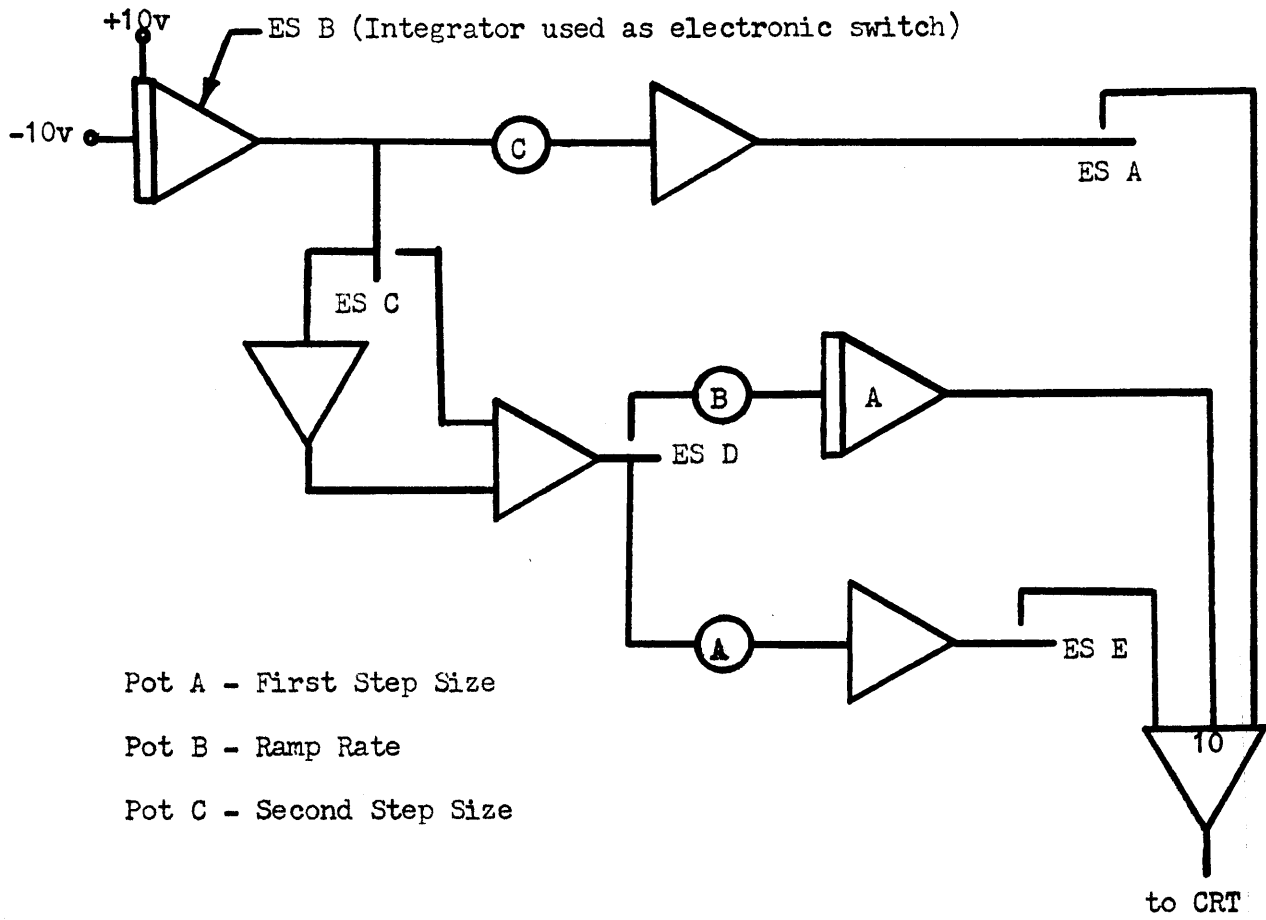


Figure A.2 Target Movement Circuit

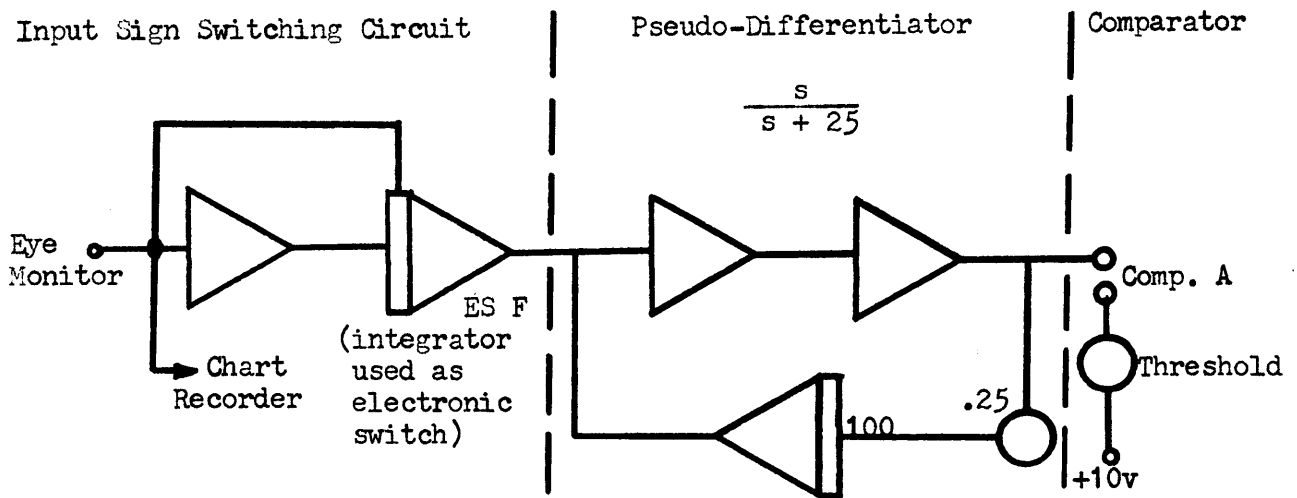
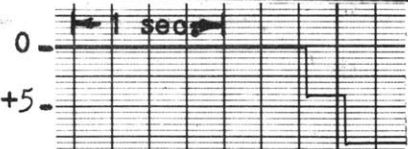


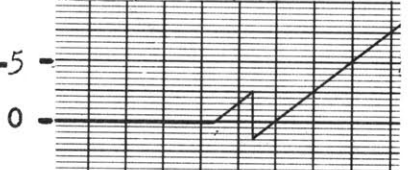
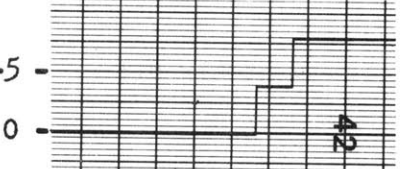
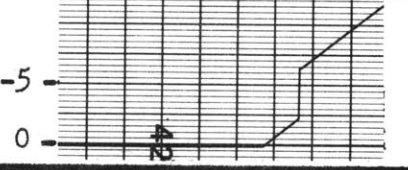
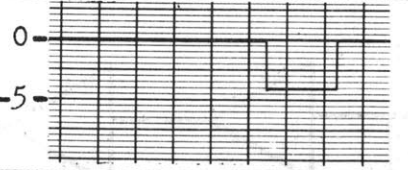
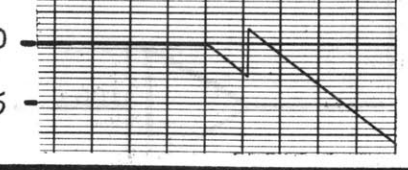


Figure A.3 Saccade Detector Circuit

Table A.1 Inputs Generated by List Entries

Three Highest Order Bits (octal)	Input (deg.)	Description
0		Stairway Pulse to the right
1		Ramp with Outward Step to the right
2		Returning Pulse to the left
3		Ramp with Inward Step to the left
4		Stairway Pulse to the left
5		Ramp with Outward Step to the left
6		Returning Pulse to the right
7		Ramp with Inward Step to the right

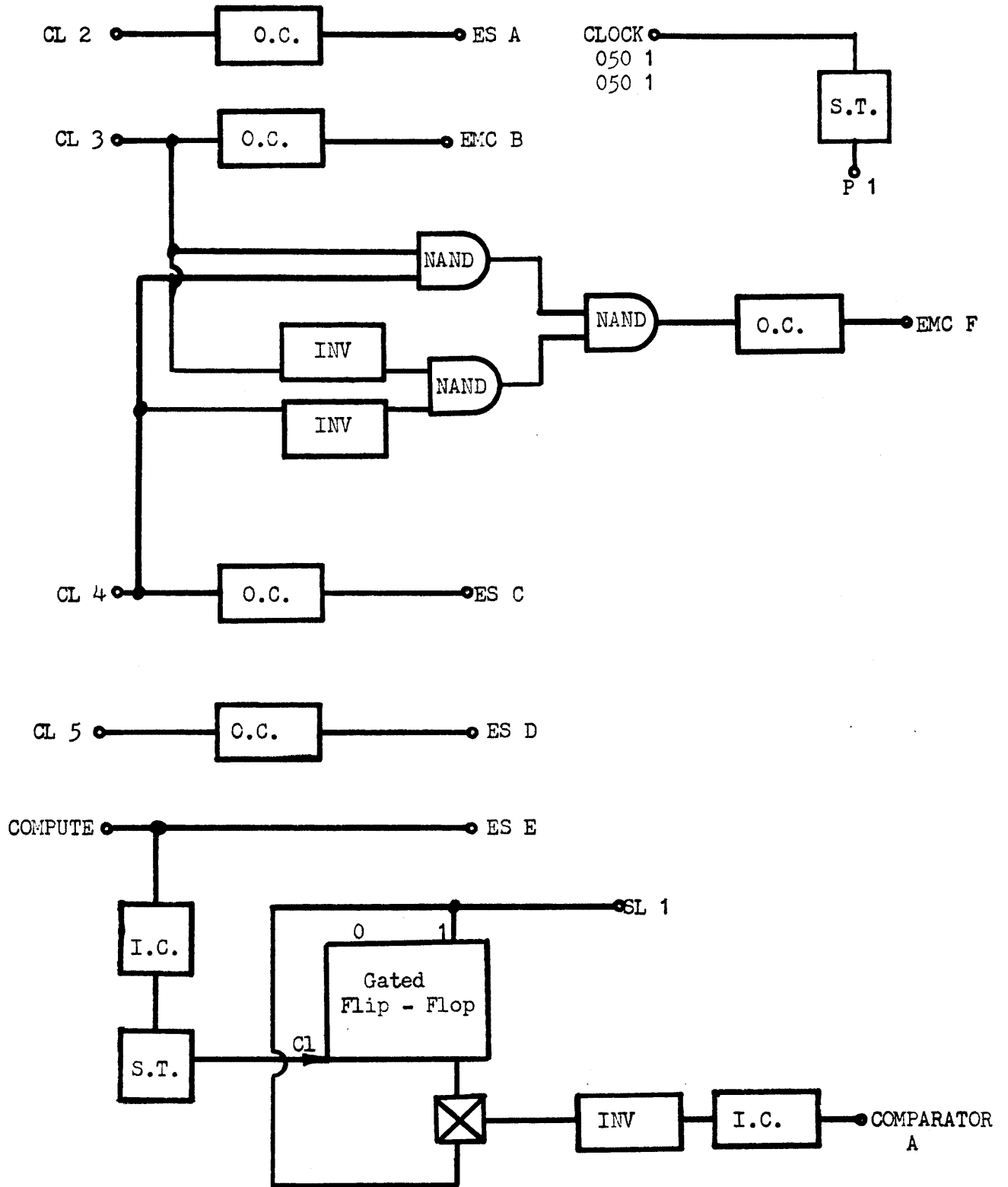


Figure A.4 Analog Control Panel Circuits

APPENDIX B

EXPERIMENTAL DATA

Figures B.1 - B.24 of this appendix present the results of each subject in response to each experiment in the form of means and standard deviations of step latency observed as a function of time from the last saccade to the occurrence of the step (T_{S-I}). The four experimental inputs are described in Section 4.2 of this thesis and are pictured in Figure 4.1. Each graph also shows the curve fit to its means which represents its data in the discussion of results in Chapter 5. The data for each graph were quantized independently to take into account differences in amount and placement of data.

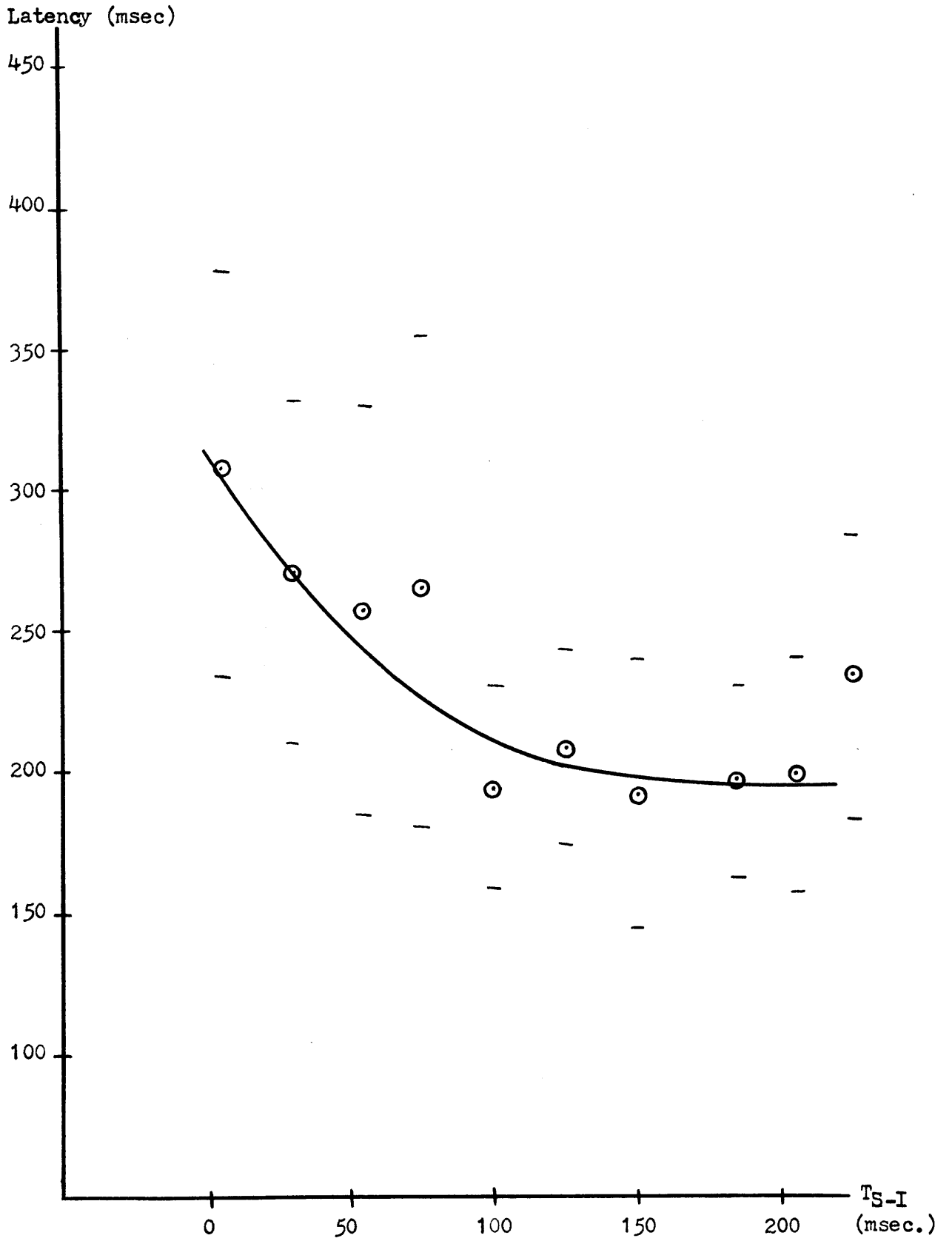


Figure B.1 Exp. One - Subject CO

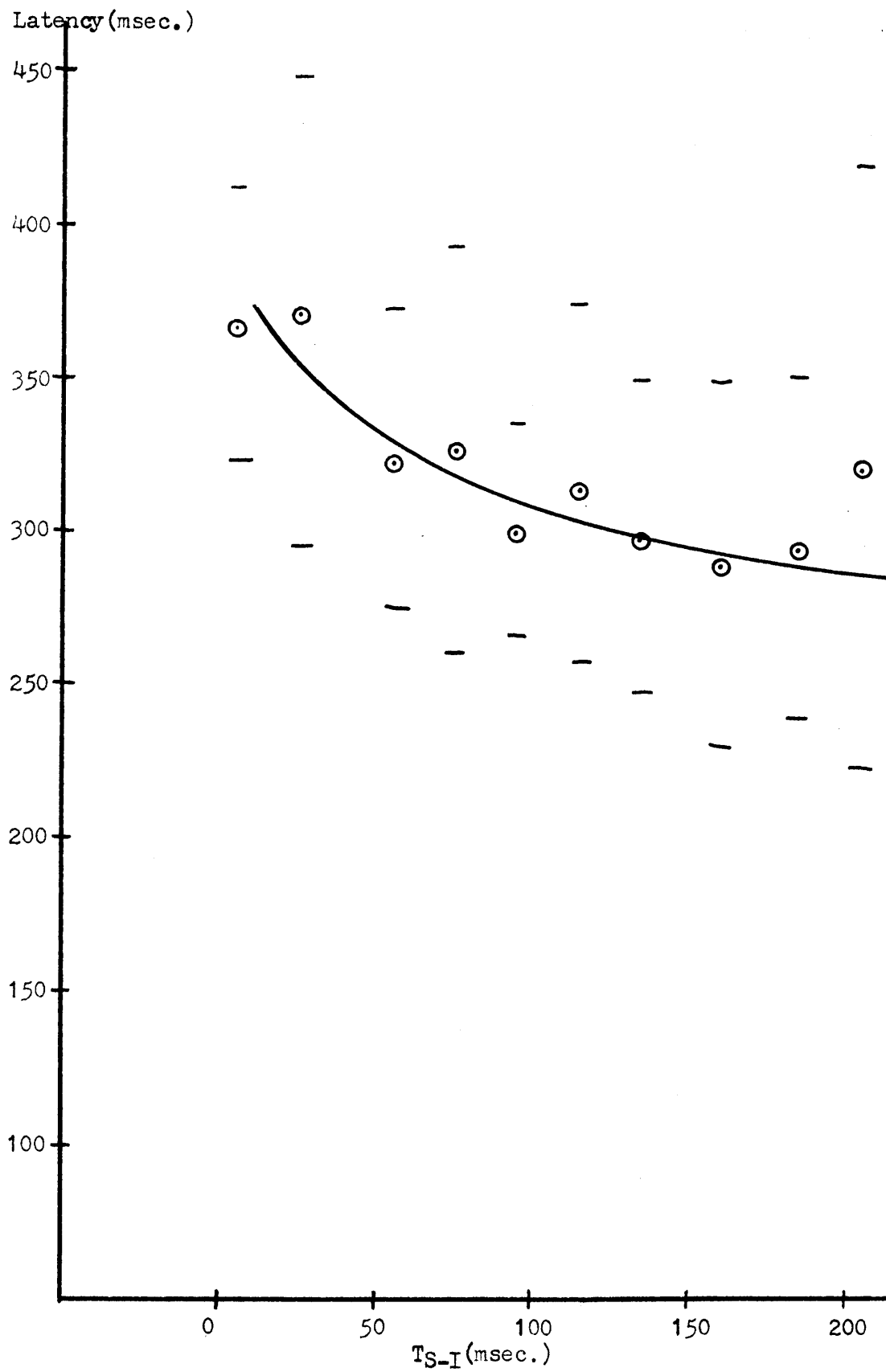


Figure B.2 Subject AVH - Exp. One

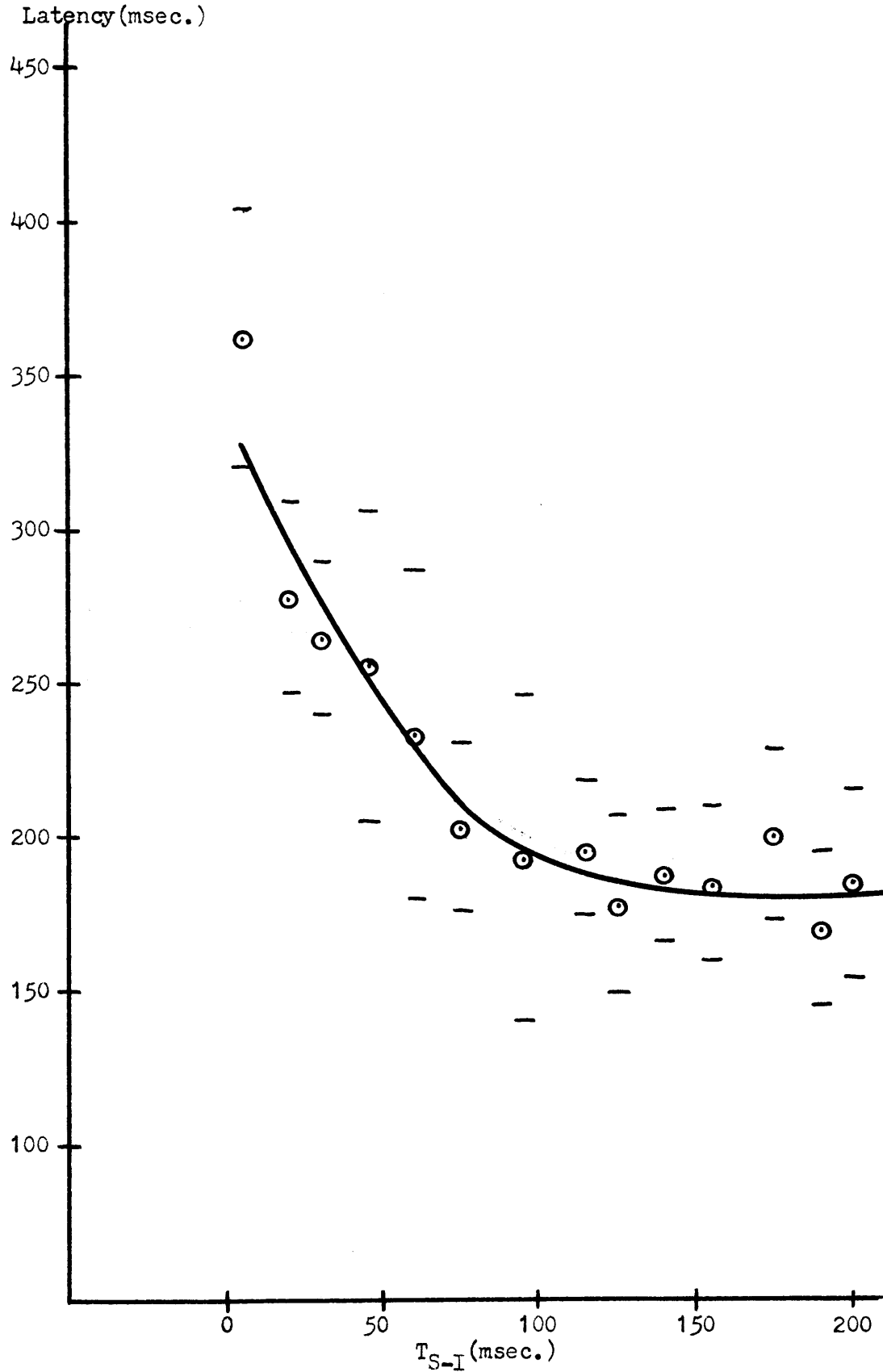


Figure B.3 Exp. One - Subject NVH

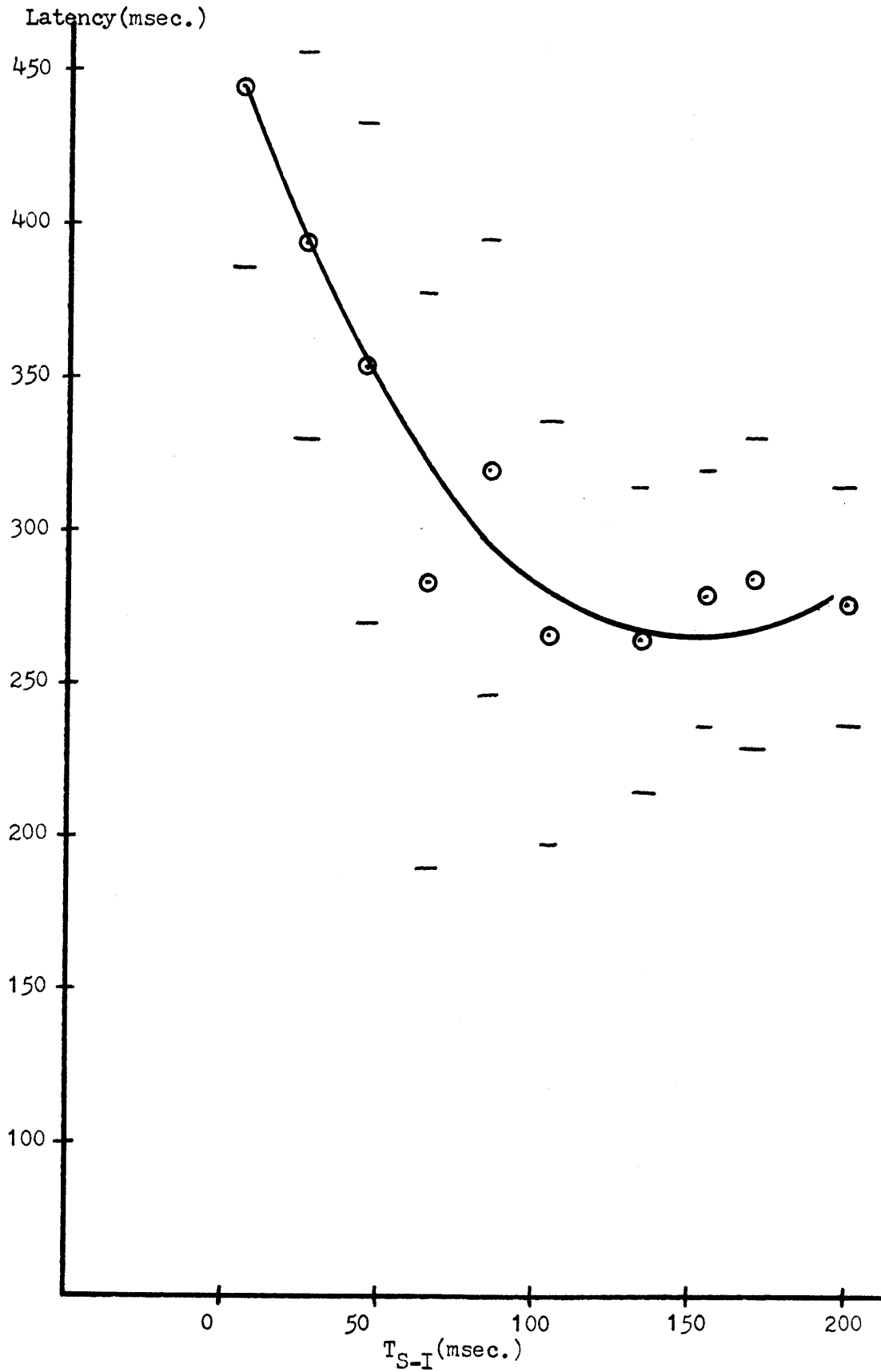


Figure B.4 Exp. One - Subject LN

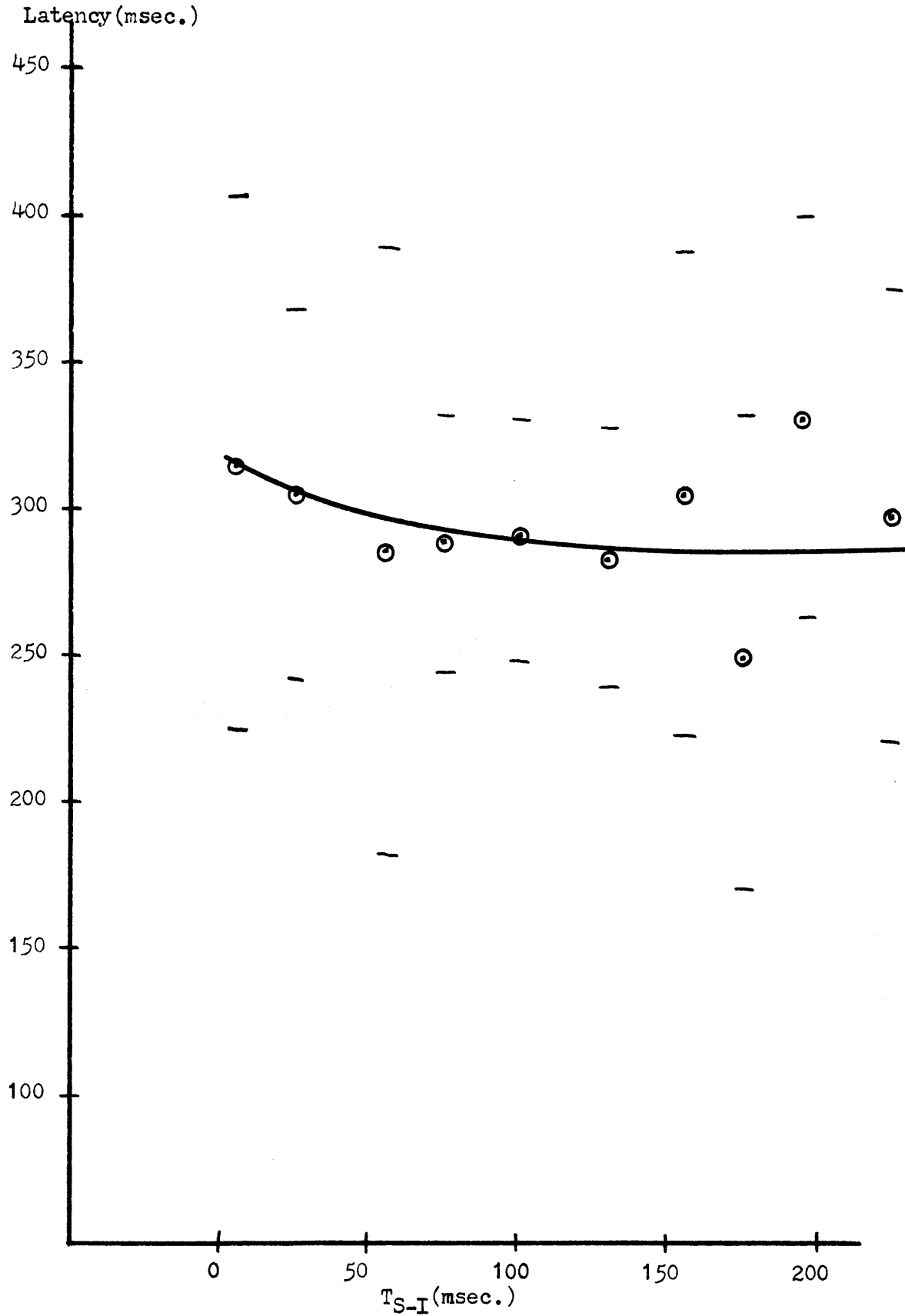


Figure B.5 Exp. One - Subject SF

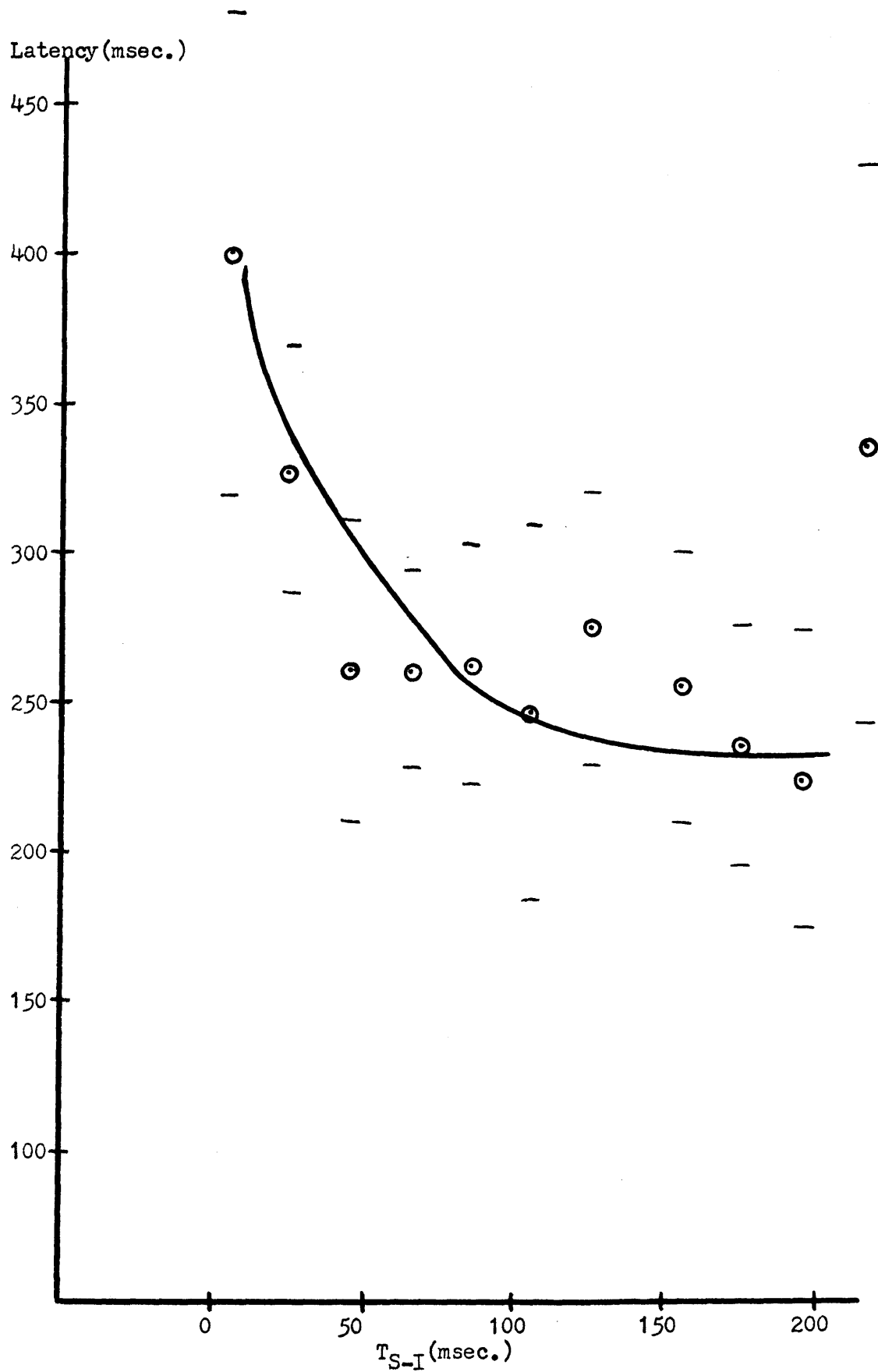


Figure B.6 Exp. One - Subject MK

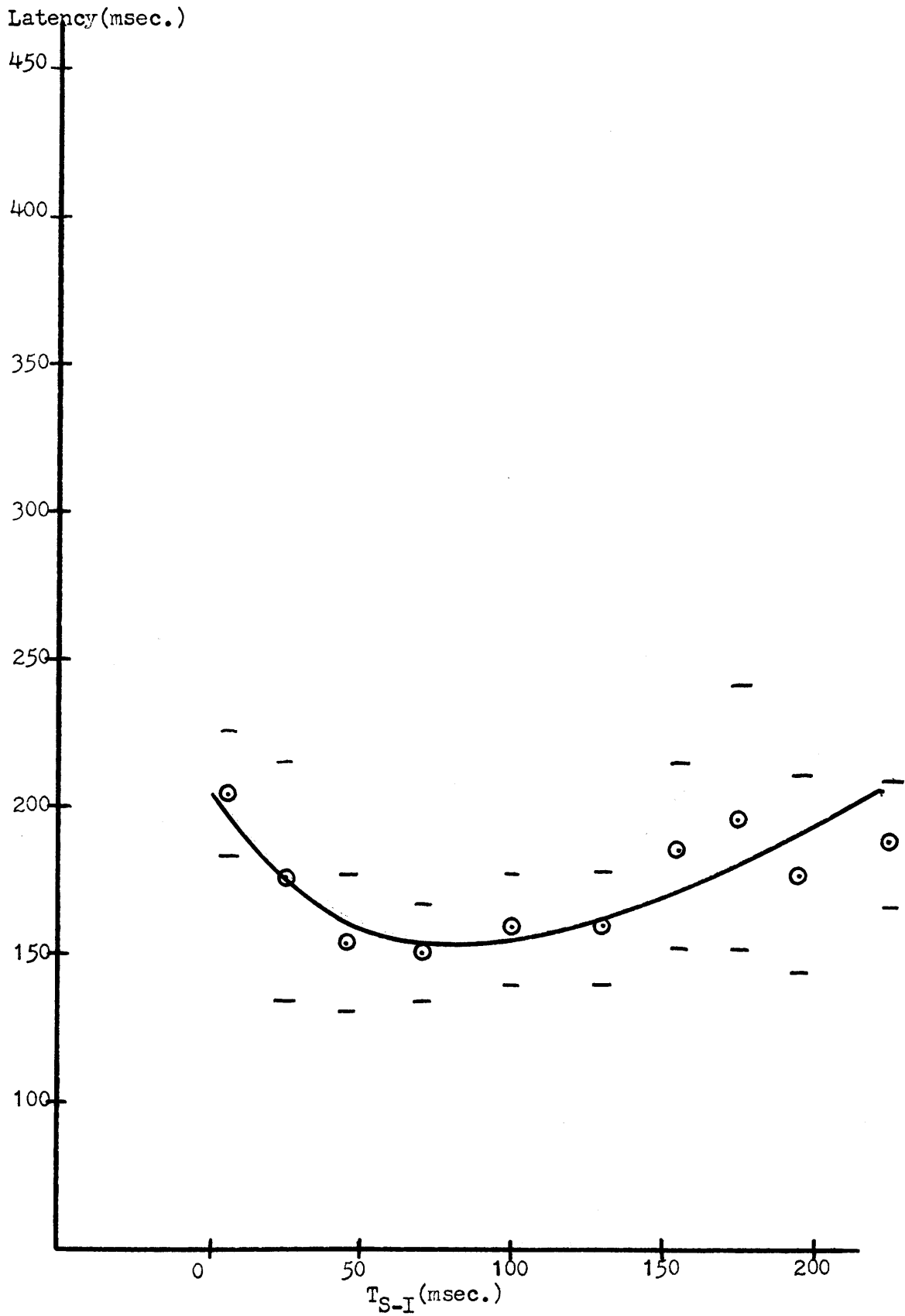


Figure B.7 Exp. Two - Subject CO

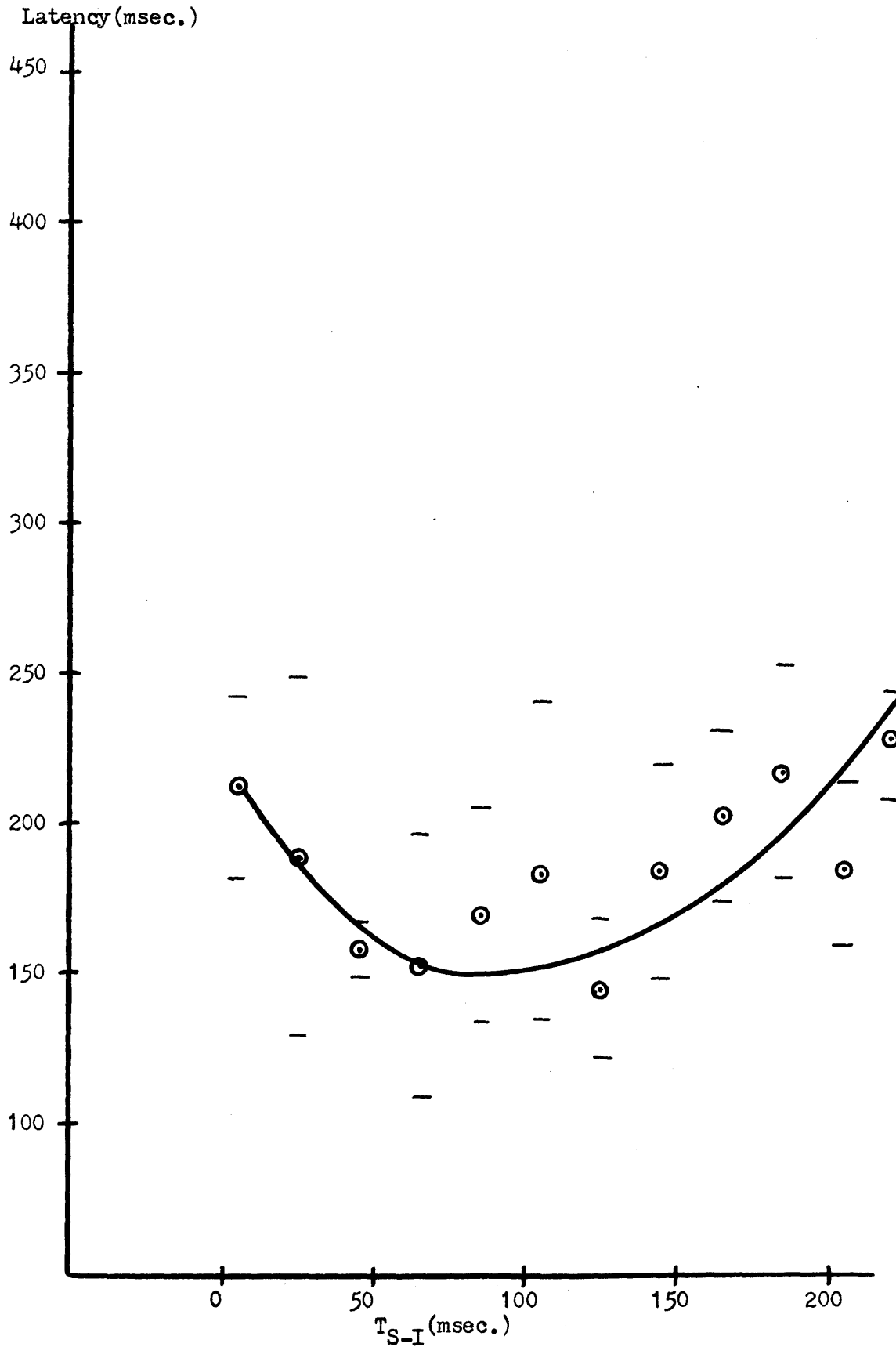


Figure B.8 Exp. Two - Subject AVH

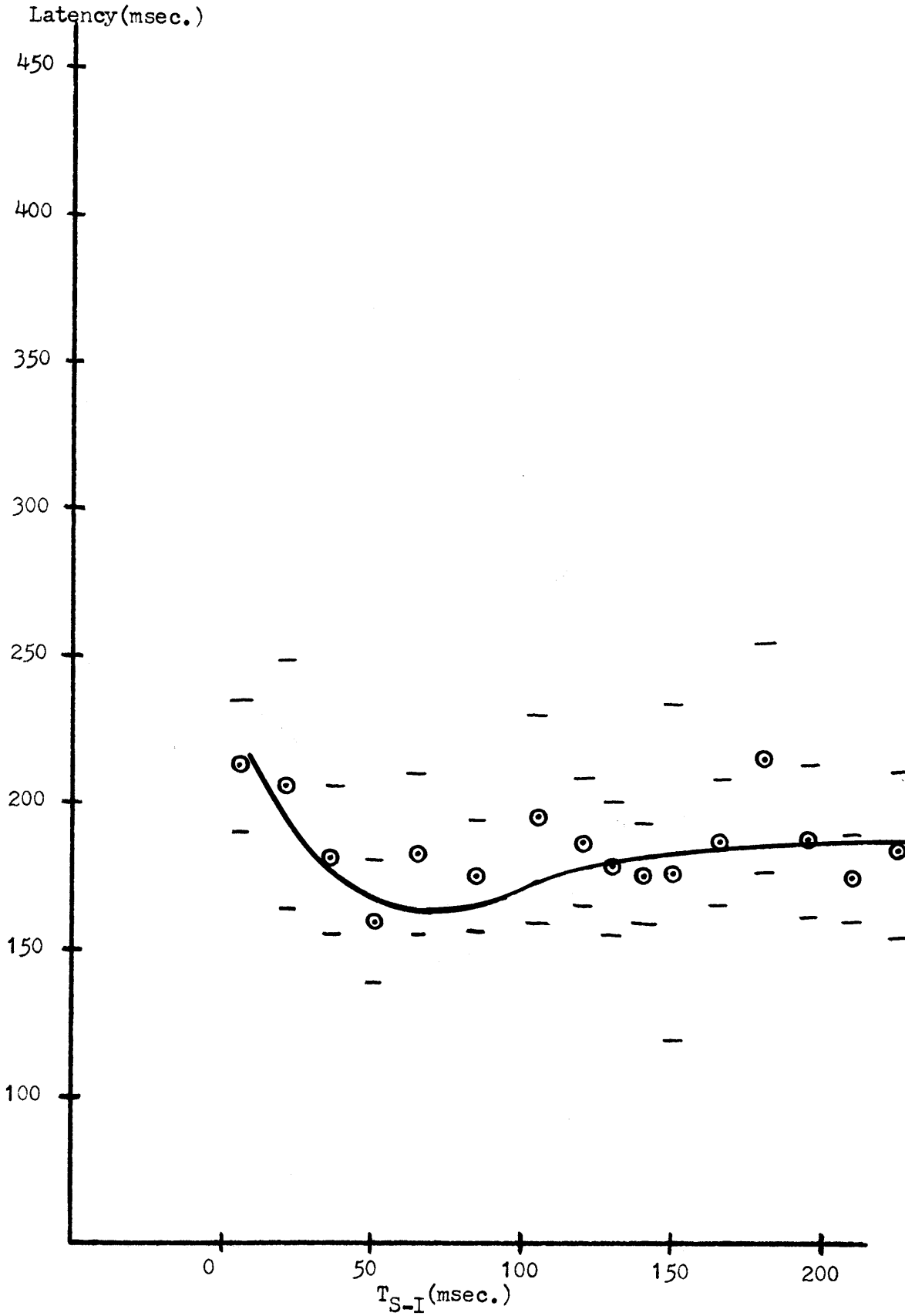


Figure B.9 Exp Two - Subject NVH

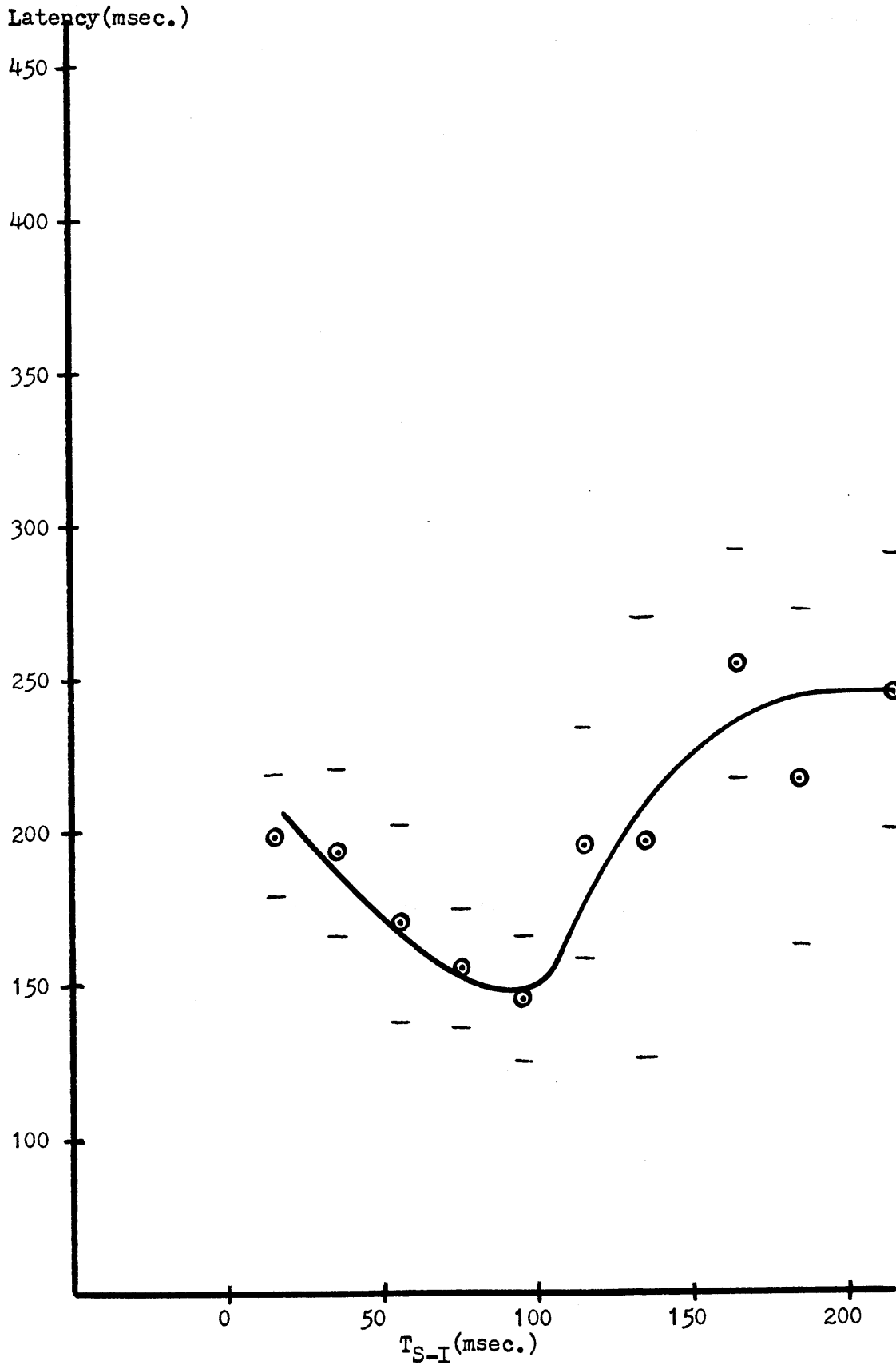


Figure B.10 Exp. Two - Subject LN

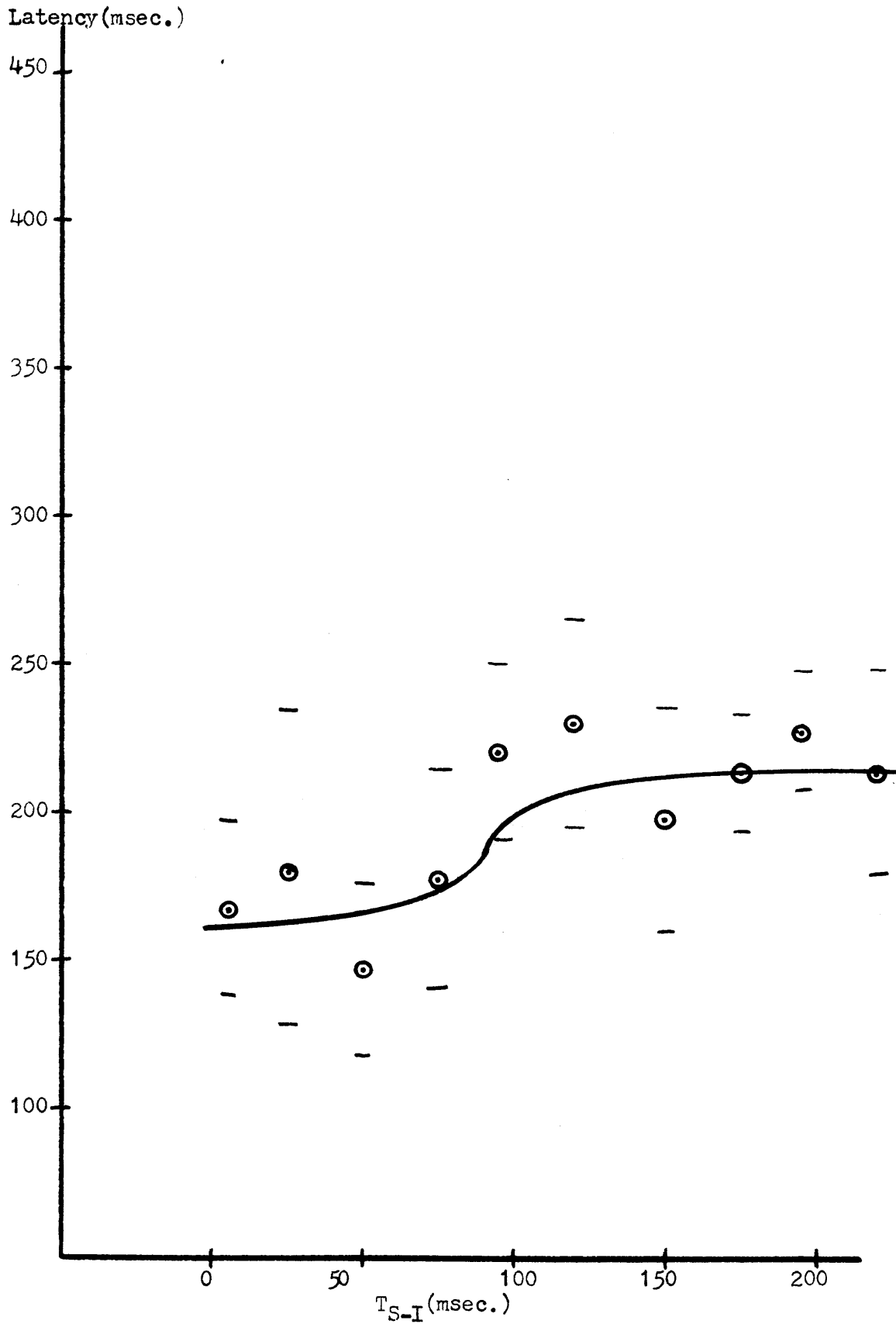


Figure B.11 Exp. Two - Subject SF

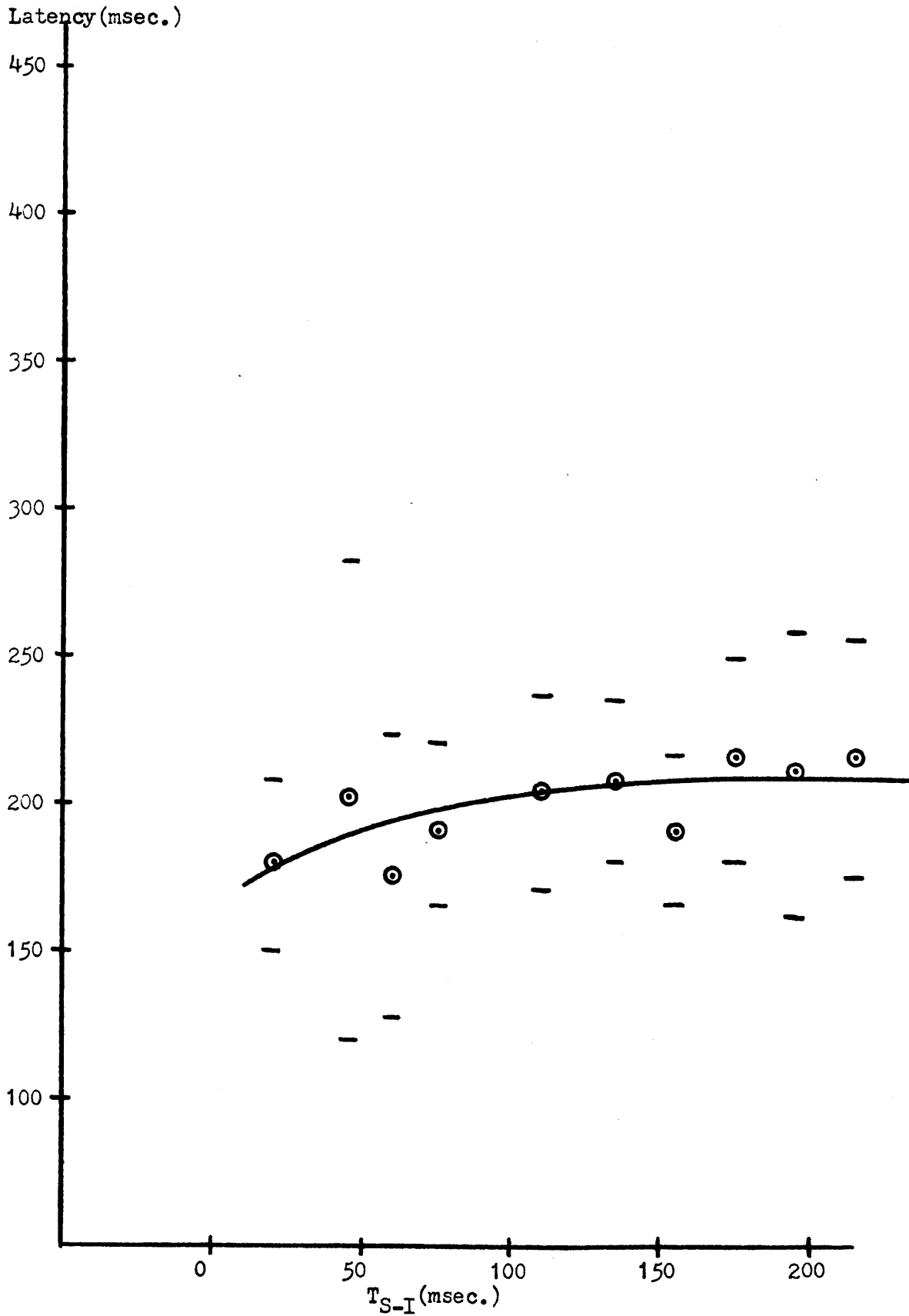


Figure B.12 Exp. Two - Subject MK

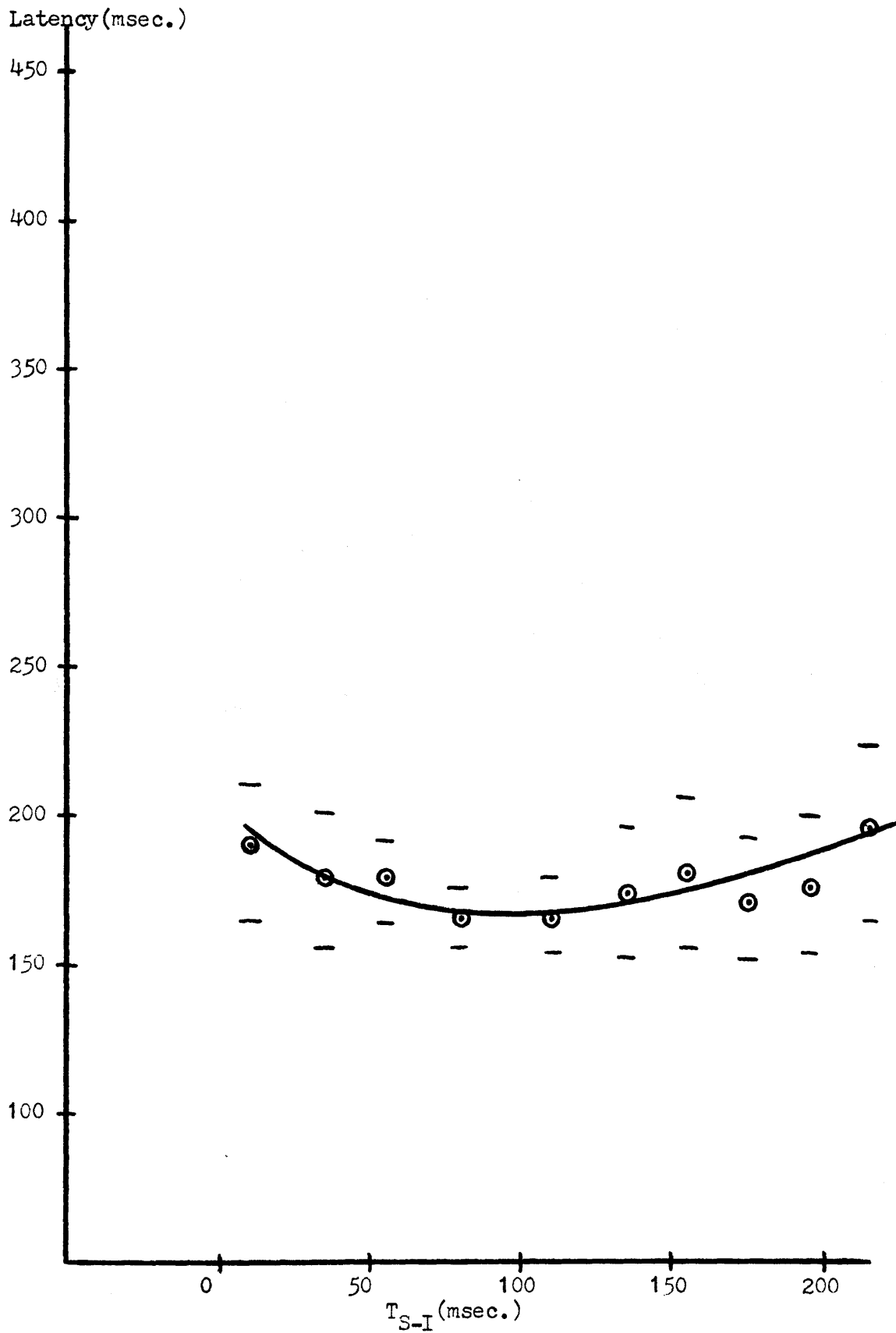


Figure B.13 Exp. Three - Subject CO

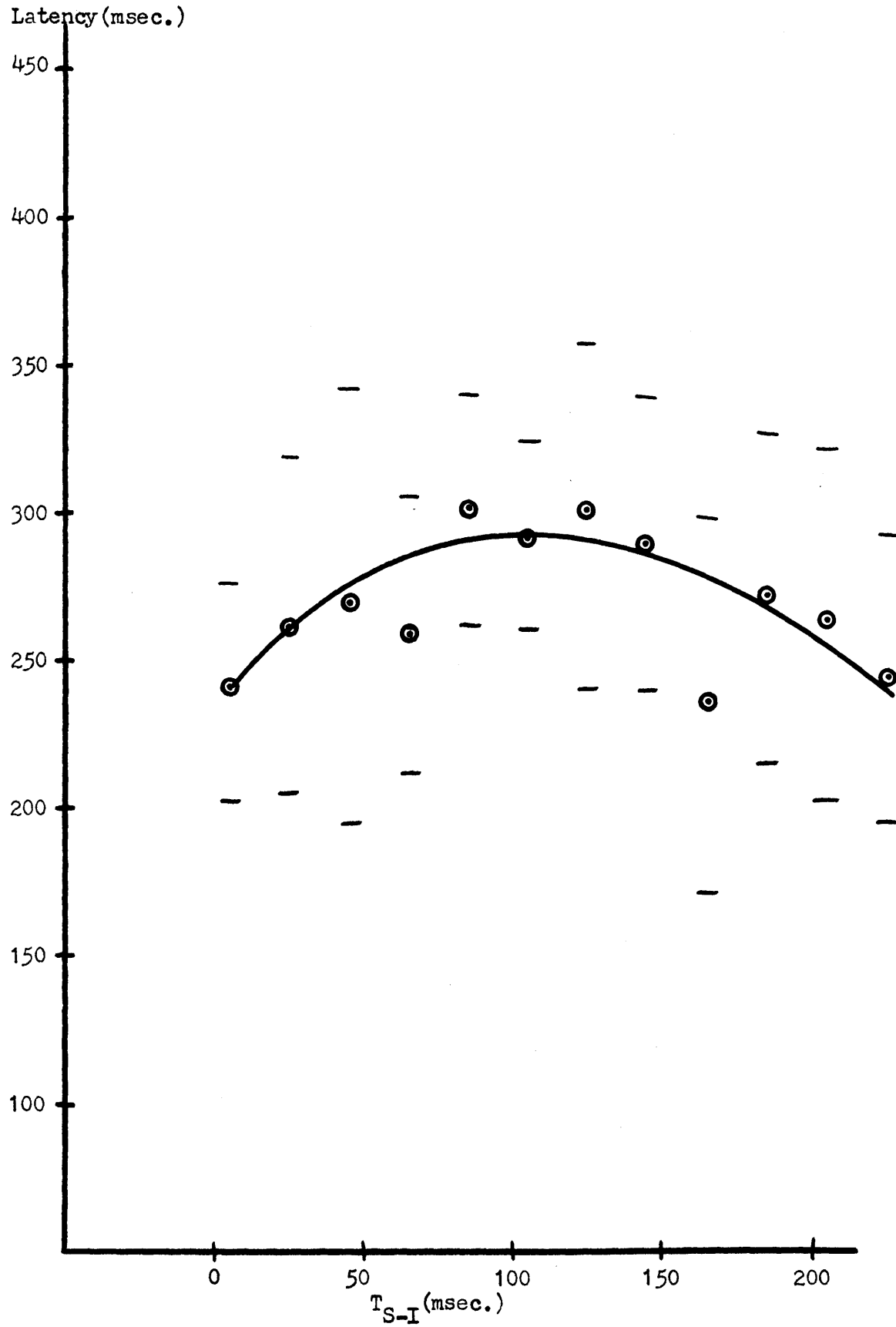


Figure B.14 Exp. Three - Subject AVH

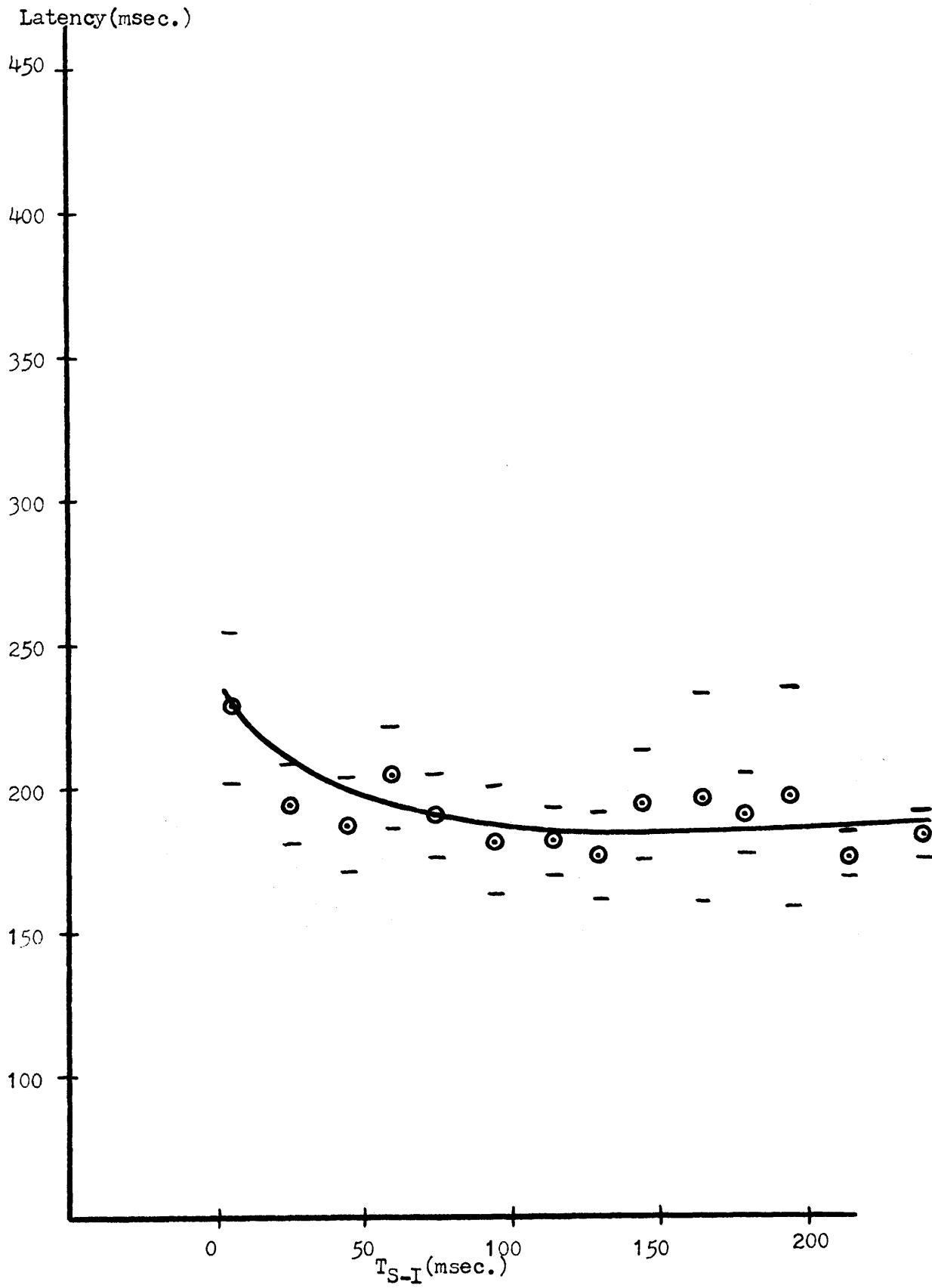


Figure B.15 Exp. Three - SubjectNVH

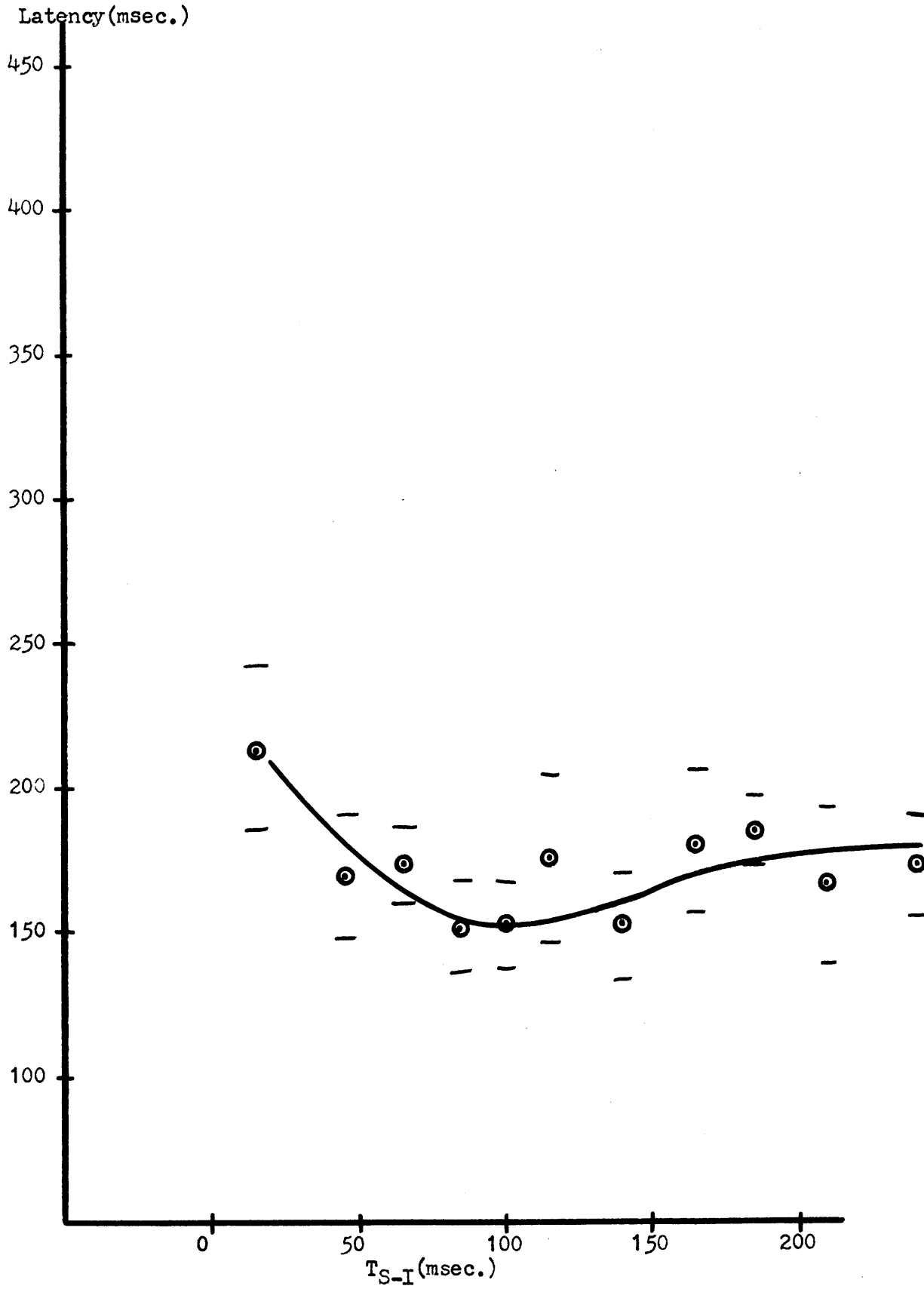


Figure B.16 Exp. Three - Subject LN

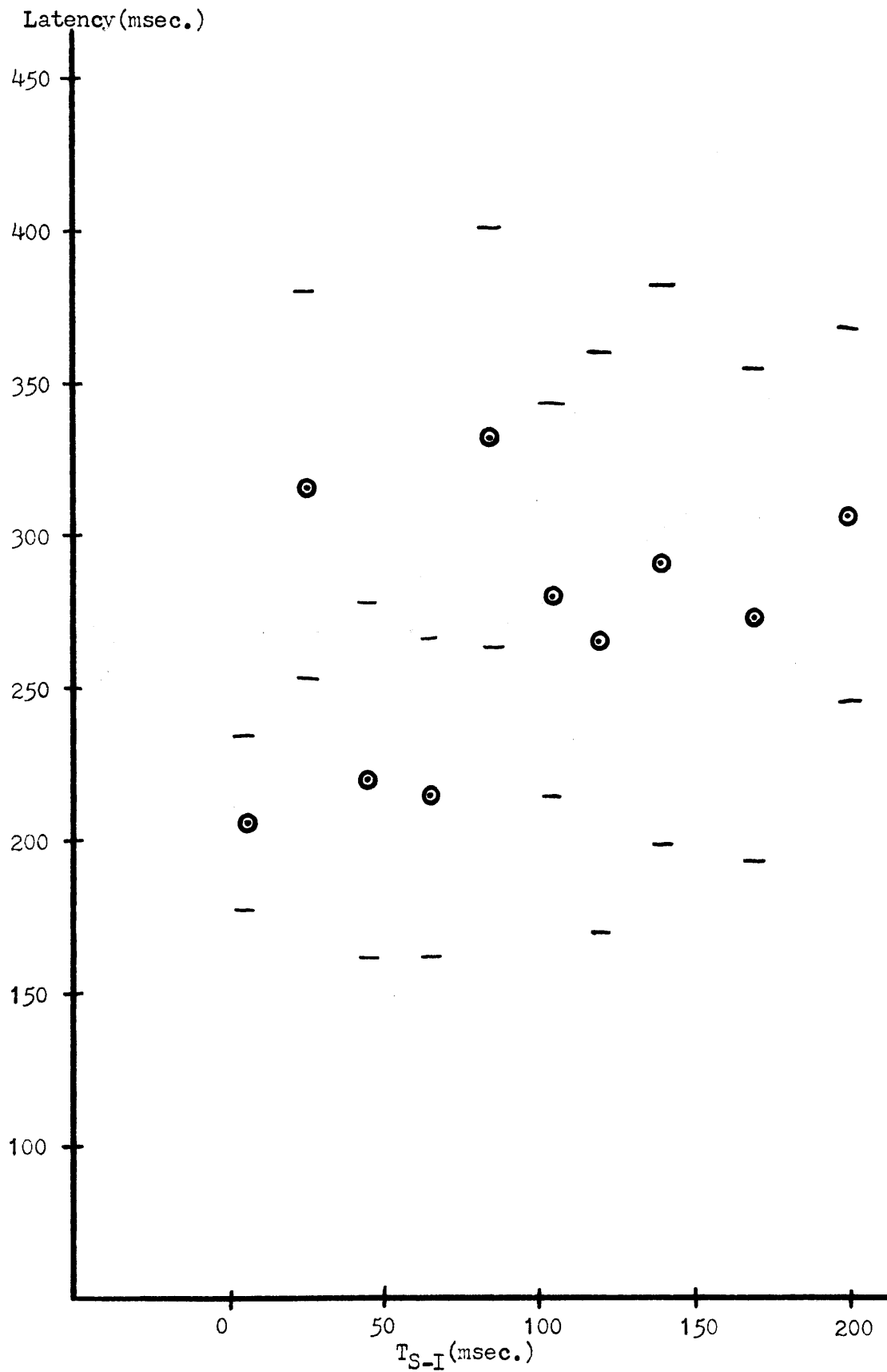


Figure B.17 Exp. Three - Subject SF

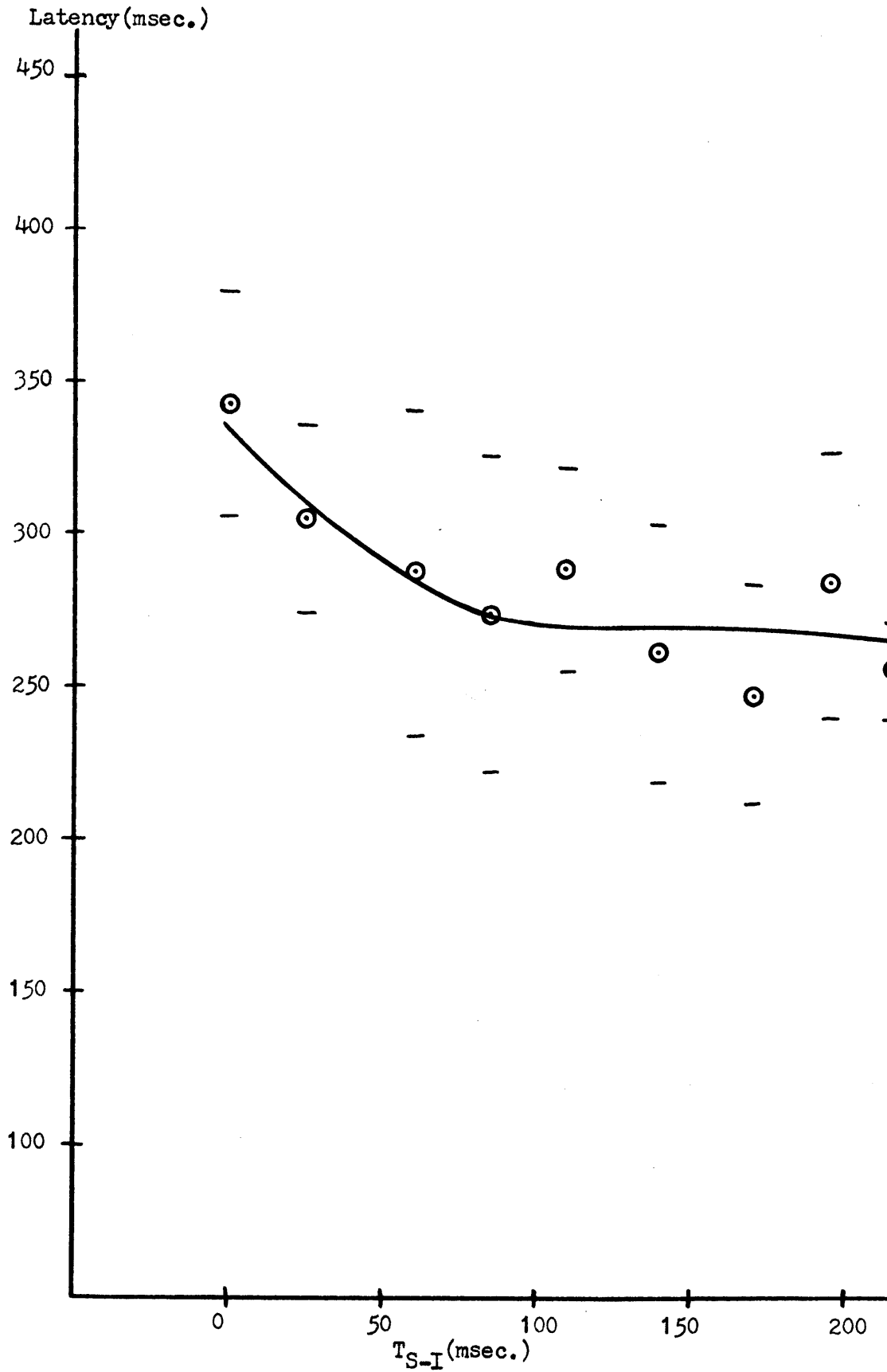


Figure B.18 Exp. Three - Subject MK

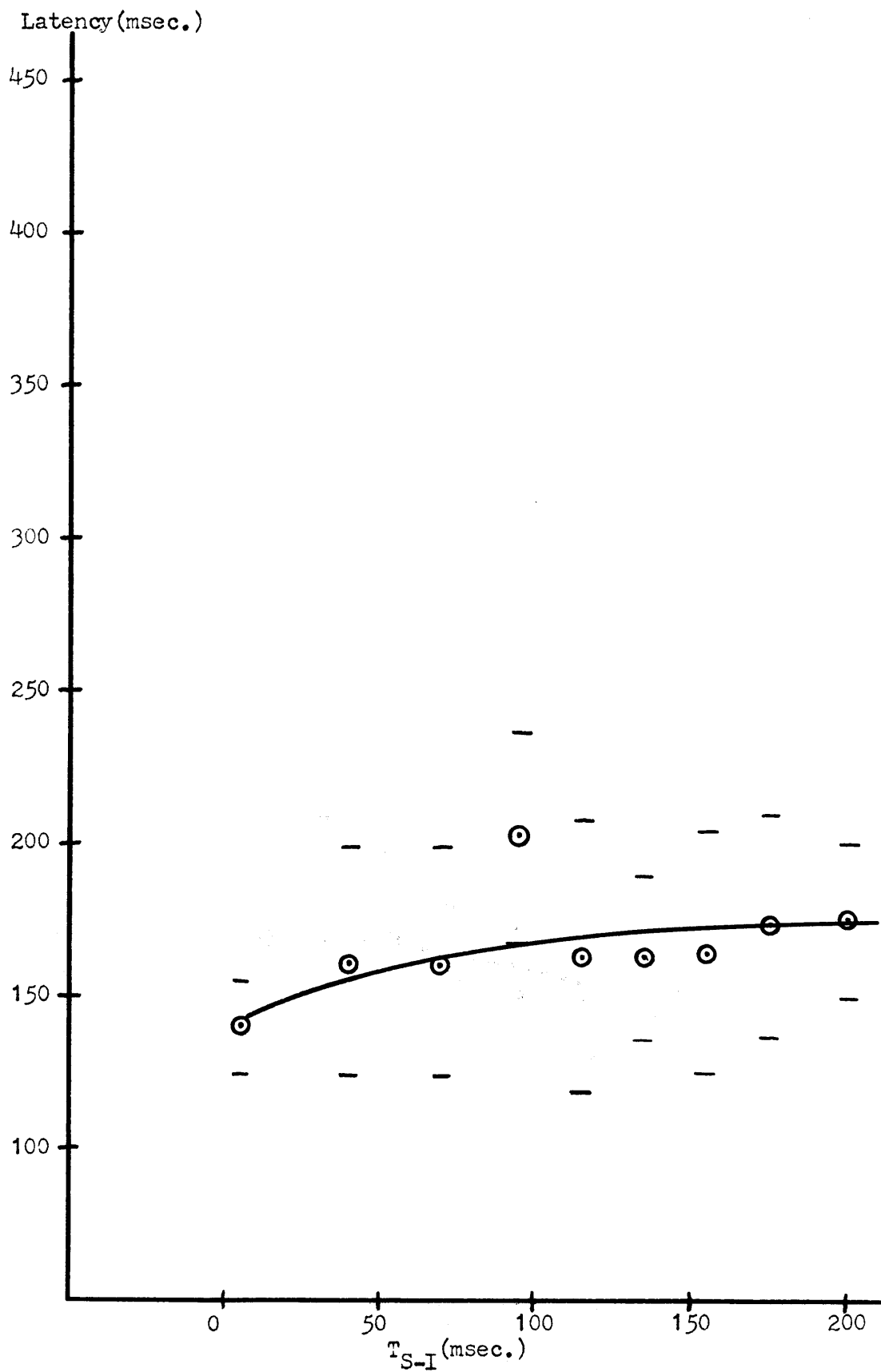


Figure B.19 Exp. Four - Subject CO

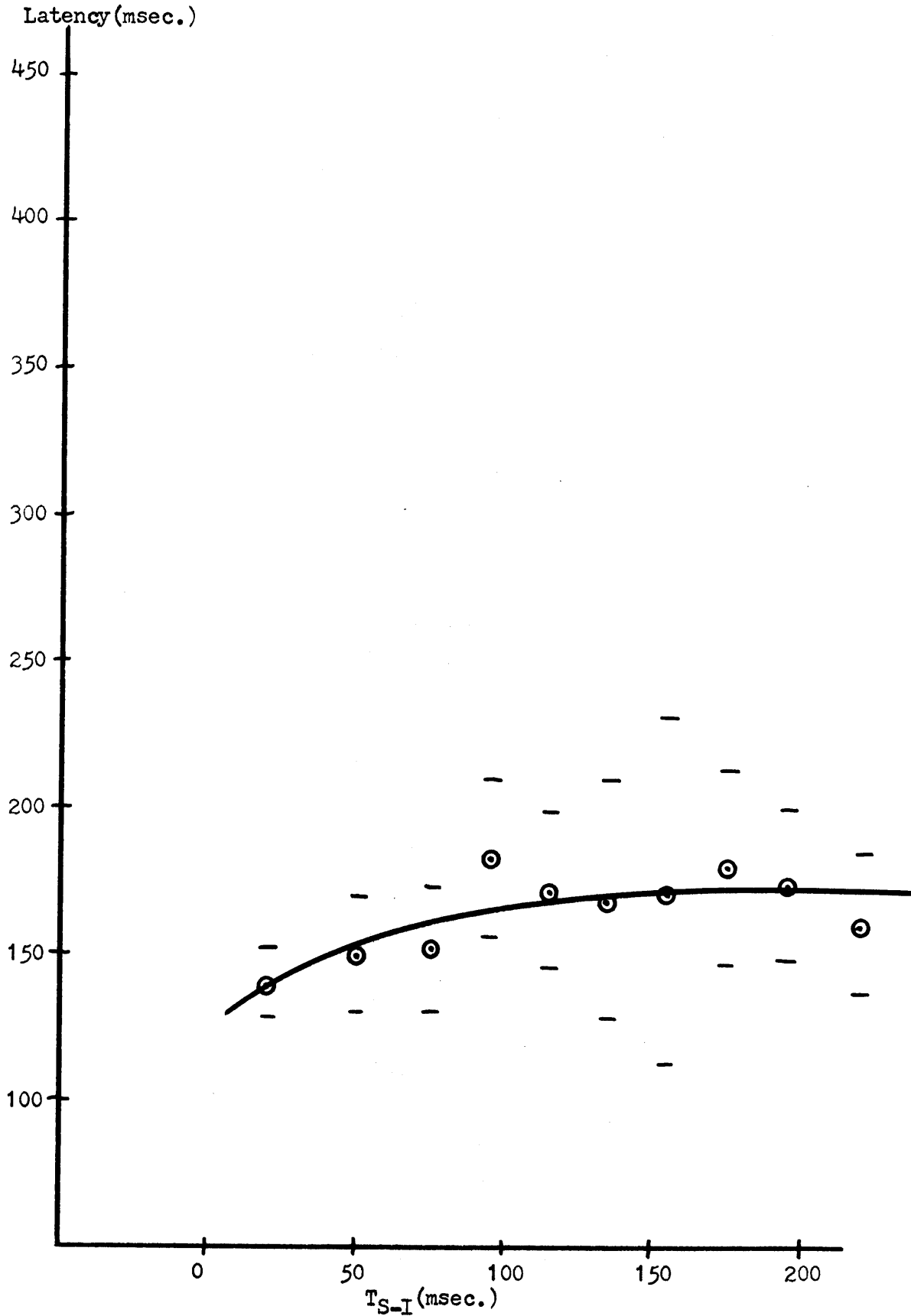


Figure B.20 Exp. Four - Subject AVH

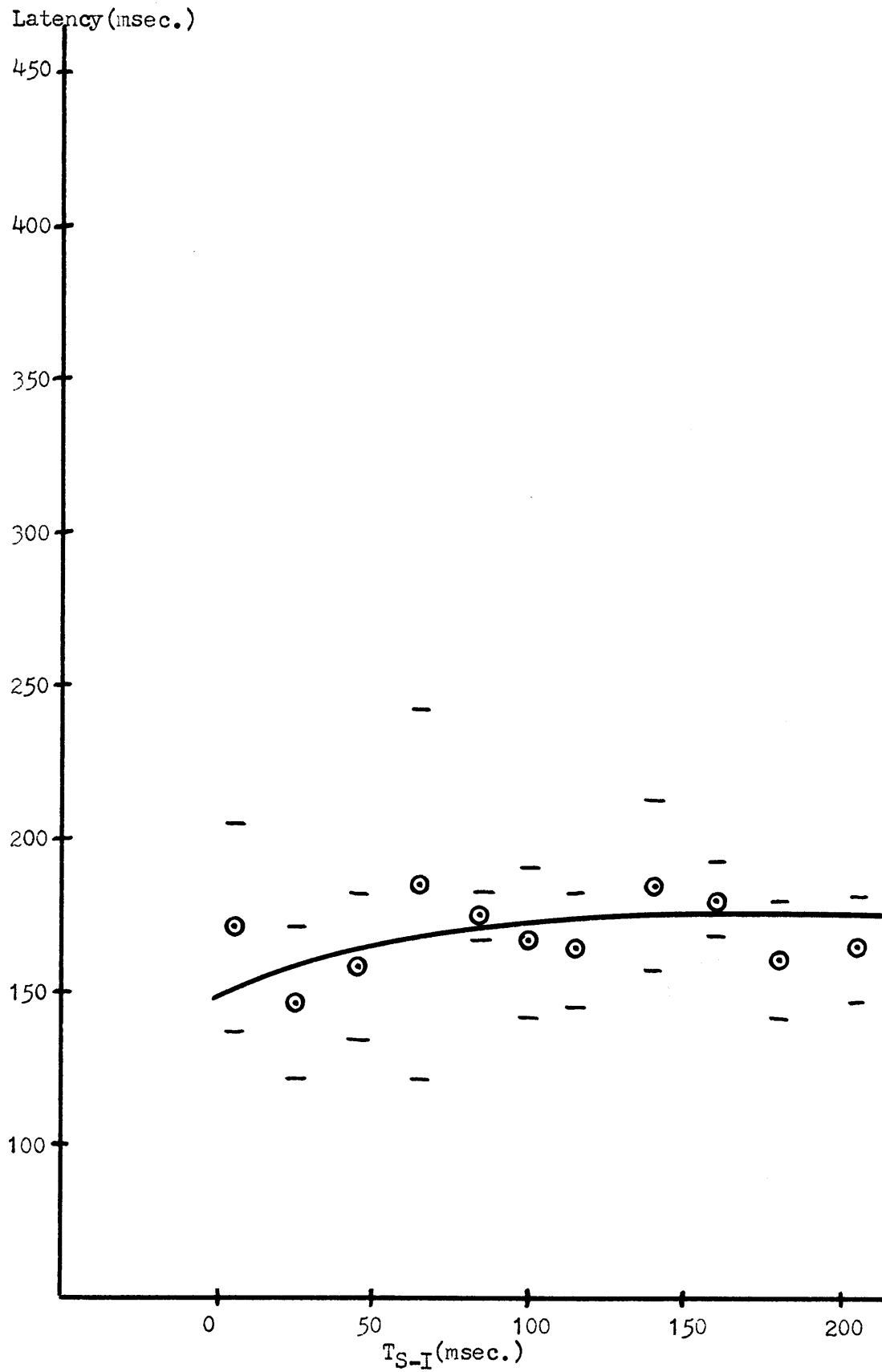


Figure B.21 Exp. Four - Subject NVH

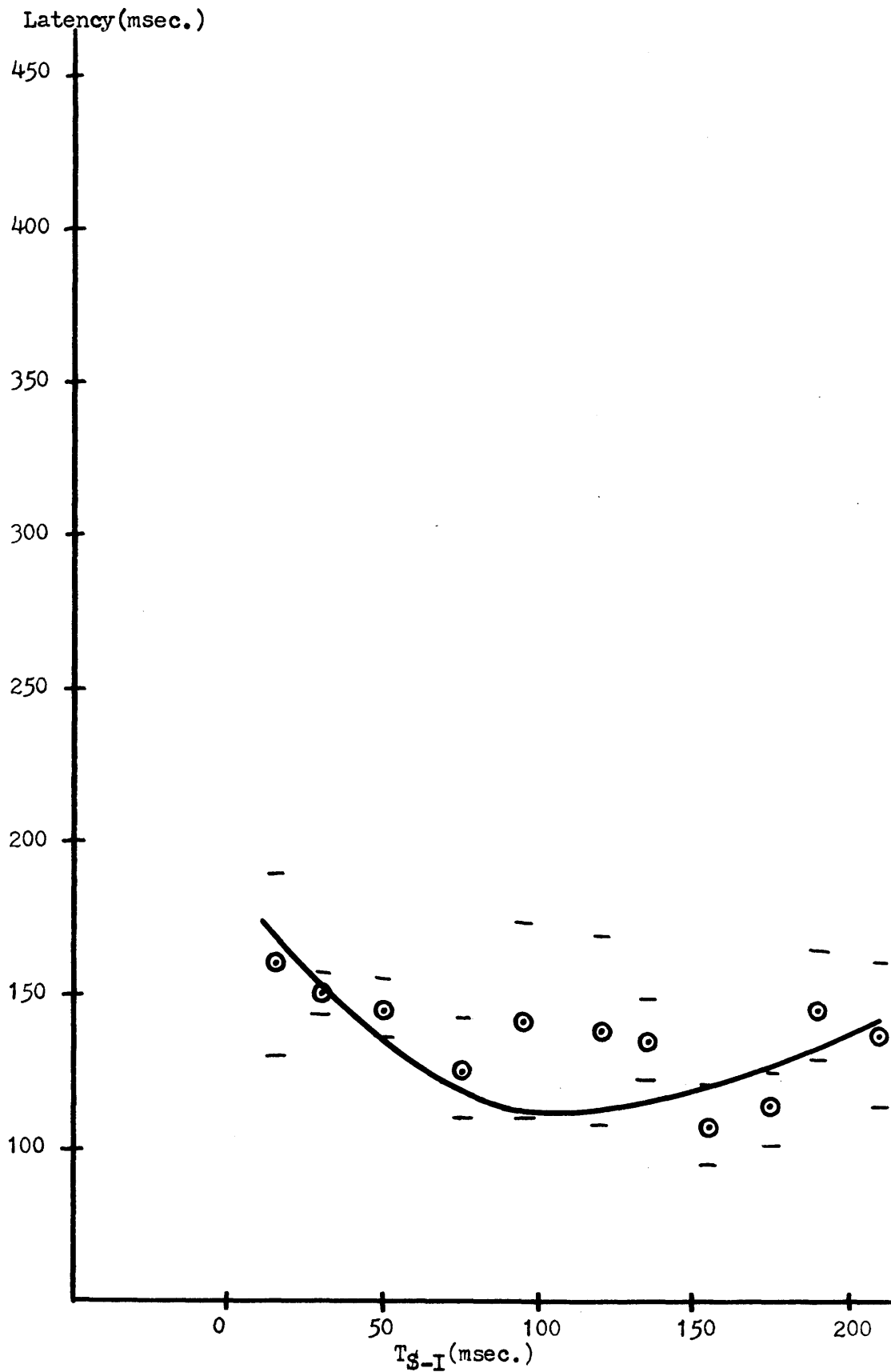


Figure B.22 Exp. Four - Subject LN

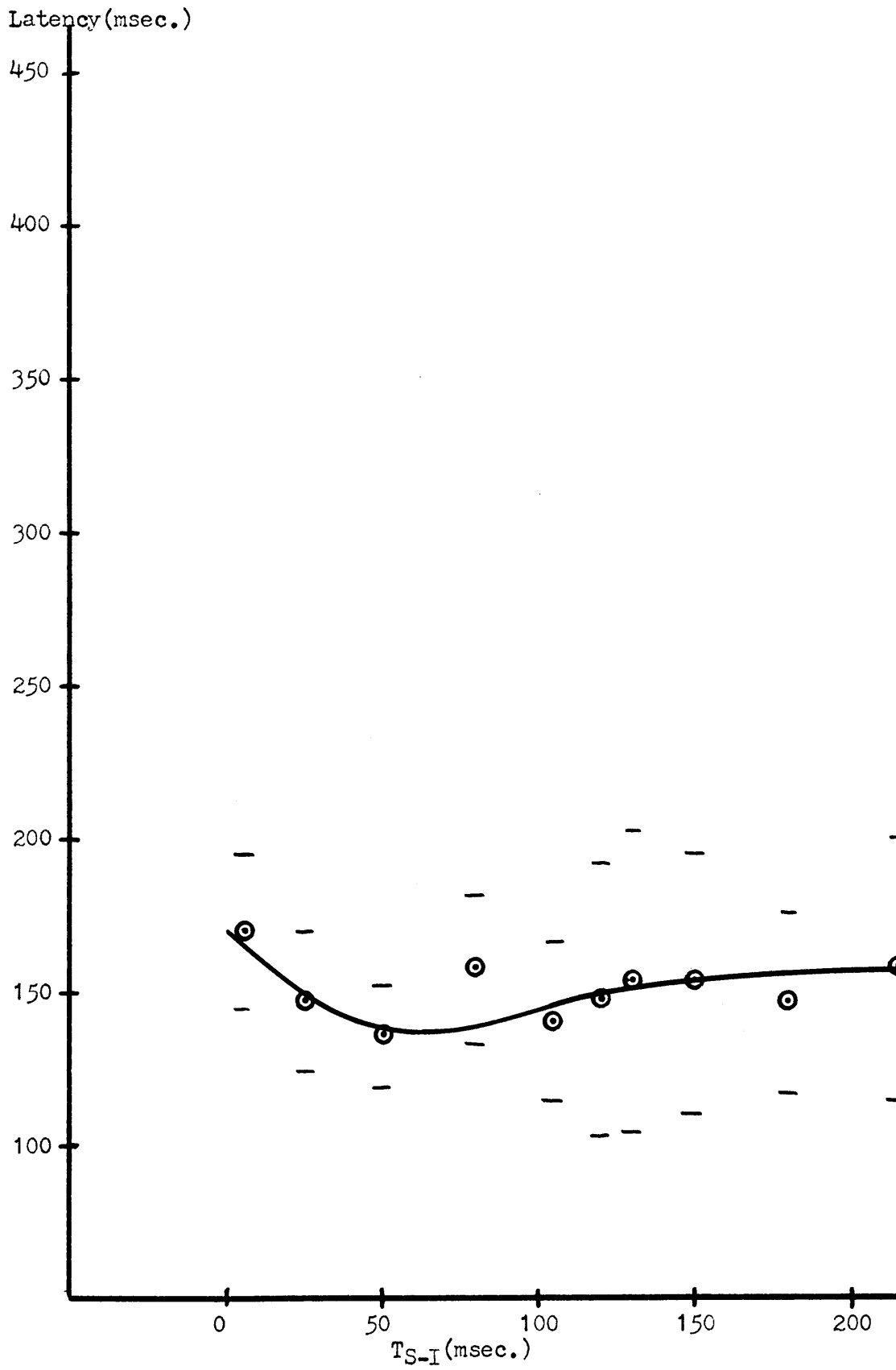


Figure B.23 Exp. Four - Subject SF

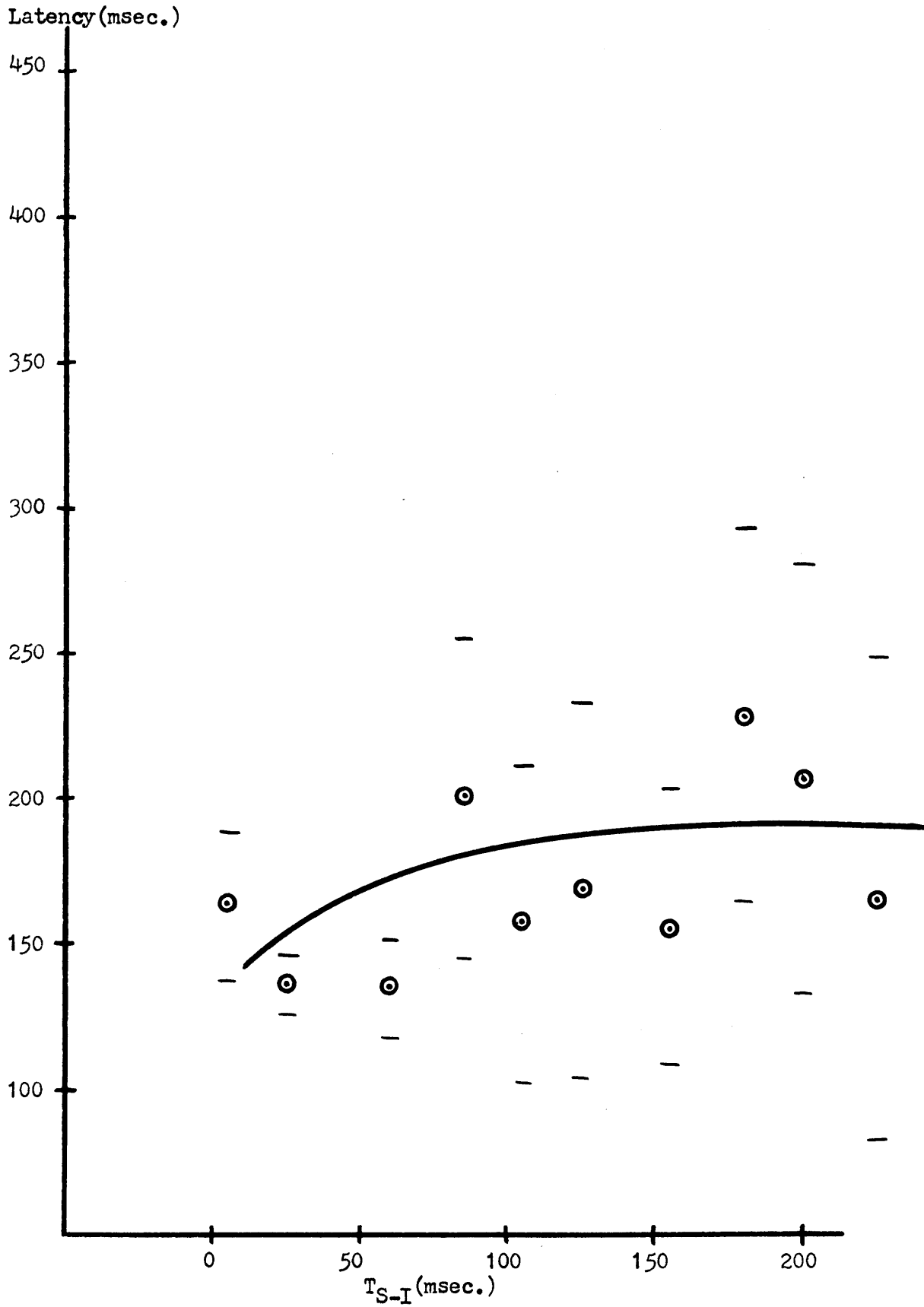


Figure B.24 Exp. Four - Subject MK

APPENDIX C

STOCHASTIC SAMPLED DATA SYSTEM SIMULATION PROGRAMS

Figures C.1 - C.4 of this appendix present MAD (Michigan Algorithm Decoder) listings of the programs used to generate Figures 2.4 - 2.7 in Section 2.5 of this thesis. Figure C.5 presents program notes that apply to all programs but with line references to the listing of Figure C.1. Figure C.6 presents individual program notes which apply to each specific program.

```

printf monty mad
W 1012.6
00010      FORMAT VARIABLE IF
00020      DIMENSION M(250)
00030      PRINT ONLINE FORMAT SIGNAL
00040      VECTOR VALUES SIGNAL=$16HPEADY FOR INPUT *$
00050      READ FORMAT TAKEIN, ID
00060      PRINT ONLINE FOPMAT SIGNAL
00070      READ FORMAT TAKEIN, K
00080      VECTOR VALUES TAKEIN=$18*$
00090      R1=SETU.(ID)
00100      THROUGH LINEA, FOR N=1,1,N.G.10
00110      RAN=RANNO.(Y)
00120  LINEA      CONTINUE
00130      ISAM=0
00140      THROUGH LINER, FOR I=1,1,I.G.250
00160      M(I)=0
00170  LINER      CONTINUE
00180      THROUGH LINED, FOR I=1,1,I.G.K
00190      INPUT=200*RANNO.(Y)+200
00200  LINEC      ISAM=200+ISAM
00210      WHENEVER (ISAM-INPUT).L.0, TRANSFER TO LINEC
00220      M(ISAM-INPUT+1)=M(ISAM-INPUT+1)+1
00230      ISAM=ISAM-INPUT
00240  LINED      CONTINUE
00250      PRINT ONLINE FORMAT A
00260      V'S A=$S10,7HLATENCY,S11,2H10,S8,2H20,S8,2H30,S8,2H40*$
00270      PRINT ONLINE FORMAT B
00280      VECTOR VALUES B=$S10,7H-----,S11,1H',3(S9,1H')*$
00290      THROUGH LINEE, FOR I=1,5,I.G.200
00450      IF=0
00460      THROUGH LINEE, FOR N=1,1,N.G.5
00470      IF=M(I+N-1)+IF
00480  LINEE      CONTINUE
00490      IF=IF/5
00500      K=I+99
00510      J=I+104
00520      PRINT ONLINE FORMAT HISTOC,K,J
00530      VECTOR VALUES HISTOC=$S10,14,1H-,14,'IF'(1H)*$
00540  LINEF      CONTINUE
00550      PRINT ONLINE FOPMAT R
00560      PRINT ONLINE FORMAT A
00570      EXECUTE EXIT.(0)
00580      INTEGER ID,K,N,ISAM,J,I
00590      INTEGER M,INPUT,IF
00600      END OF PROGRAM
P 1.683+1.100

```

Figure C.1 Simulation of Constant Sampling Interval System

```

printf monty mad
M 1058.1
00010      FORMAT VARIABLE IF
00020      DIMENSION M(250)
00030      PRINT ONLINE FORMAT SIGNAL
00040      VECTOR VALUES SIGNAL=$16$READY FOR INPUT *$
00050      READ FORMAT TAKEIN, I0
00060      PRINT ONLINE FORMAT SIGNAL
00070      READ FORMAT TAKEIN, K
00080      VECTOR VALUES TAKEIN=$18*$
00090      R1=SETU.(I0)
00100      THROUGH LINEA, FOR N=1,1,N.G.10
00110      RAN=RANNO.(Y)
00120  LINEA      CONTINUE
00130      ISAM=0
00140      THROUGH LINEB, FOR I=1,1,I.G.250
00160      M(I)=0
00170  LINEB      CONTINUE
00180      THROUGH LINEC, FOR I=1,1,I.G.K
00190      INPUT=200*RANNO.(Y)+200
00200  LINEC      ISAM=200*RANNO.(Y)+ISAM
00210      WHENEVER (ISAM-INPUT).L.0, TRANSFER TO LINEC
00220      M(ISAM-INPUT+1)=M(ISAM-INPUT+1)+1
00230      ISAM=ISAM-INPUT
00240  LINED      CONTINUE
00250      PRINT ONLINE FORMAT A
00260      V'S A=$S10,7H$LATENCY, S11,2H10, S8,2H20, S8,2H30, S8,2H40*$
00270      PRINT ONLINE FORMAT B
00280      VECTOR VALUES B=$S10,7H-----, S11,1H', 3(S0,1H')*$
00290      THROUGH LINEE, FOR I=1,5,I.G.200
00450      IF=0
00460      THROUGH LINEE, FOR N=1,1,N.G.5
00470      IF=M(I+N-1)+IF
00480  LINEE      CONTINUE
00490      IF=IF/5
00500      K=I+99
00510      J=I+104
00520      PRINT ONLINE FORMAT HISTOC, K, J
00530      VECTOR VALUES HISTOC=$S10,14,1H-,14,'IF'(141)*$
00540  LINEF      CONTINUE
00550      PRINT ONLINE FORMAT B
00560      PRINT ONLINE FORMAT A
00570      EXECUTE EXIT.(0)
00580      INTEGER I0, K, N, ISAM, J, I
00590      INTEGER M, INPUT, IF
00600      END OF PROGRAM
R 1.600+.600

```

Figure C.2 Simulation of System with Sampling Intervals Uniformly Distributed Between 0 and 200 msec.

```

printf monty mad
W 1301.5
00010      FORMAT VARIABLE IF
00020      DIMENSION M(250)
00030      PRINT ONLINE FORMAT SIGNAL
00040      VECTOR VALUES SIGNAL=$16HREADY FOR INPUT *$
00050      READ FORMAT TAKEIN, ID
00060      PRINT ONLINE FORMAT SIGNAL
00070      READ FORMAT TAKEIN, K
00080      VECTOR VALUES TAKEIN=$18*$
00090      R1=SETU.(ID)
00100      THROUGH LINEA, FOR N=1,1,N.G.10
00110      RAN=RANNO.(Y)
00120 LINEA  CONTINUE
00130      ISAM=0
00140      THROUGH LINER, FOR I=1,1,I.G.250
00160      M(I)=0
00170 LINER  CONTINUE
00180      THROUGH LINED, FOR I=1,1,I.G.K
00190      INPUT=250*RANNO.(Y)+250
00200 LINEC  ISAM=150+100*RANNO.(Y)+ISAM
00210      WHENEVER (ISAM-INPUT).L.O, TRANSFER TO LINEC
00220      M(ISAM-INPUT+1)=M(ISAM-INPUT+1)+1
00230      ISAM=ISAM-INPUT
00240 LINED  CONTINUE
00250      PRINT ONLINE FORMAT A
00260      V'S A=$S10,7HLATENCY,S11,2H10,S8,2H20,S8,2H30,S8,2H40*$
00270      PRINT ONLINE FORMAT B
00280      VECTOR VALUES B=$S10,7H-----,S11,1H',3(S9,1H')*$
00290      THROUGH LINEF, FOR I=1,5,I.G.250
00450      IF=0
00460      THROUGH LINEE, FOR N=1,1,N.G.5
00470      IF=M(I+N-1)+IF
00480 LINEE  CONTINUE
00490      IF=IF/5
00500      K=I+99
00510      J=I+104
00520      PRINT ONLINE FORMAT HISTOG,K,J
00530      VECTOR VALUES HISTOG=$S10,14,1H-,14,'IF'(1H')*$
00540 LINEF  CONTINUE
00550      PRINT ONLINE FORMAT B
00560      PRINT ONLINE FORMAT A
00570      EXECUTE EXIT.(0)
00580      INTEGER ID,K,N,ISAM,J,I
00590      INTEGER M,INPUT,IF
00600      END OF PROGRAM
R 1.800+.683

```

Figure C.3 Simulation of System with Sampling Intervals Uniformly Distributed Between 150 and 250 msec.

```

printf monty mad
W 1323.4
00010      FORMAT VARIABLE IF
00020      DIMENSION M(250)
00030      PRINT ONLINE FORMAT SIGNAL
00040      VECTOR VALUES SIGNAL=$16HF$ADY FOR INPUT *$
00050      READ FORMAT TAKEIN,I0
00060      PRINT ONLINE FORMAT SIGNAL
00070      READ FORMAT TAKEIN,K
00080      VECTOR VALUES TAKEIN=$18*$
00090      B1=SFTU.(I0)
00100      THROUGH LINEA, FOR N=1,1,N.G.10
00110      RAN=RANNO.(Y)
00120 LINEA  CONTINUE
00130      ISAM=0
00140      THROUGH LINER, FOR I=1,1,I.G.250
00160      M(I)=0
00170 LINER  CONTINUE
00180      THROUGH LIUED, FOR I=1,1,I.G.K
00190      INPUT=200*RANNO.(Y)+200
00200 LINEC  DR=RANNO.(Y)
00210      WHENEVER DR.L.0.8, TRANSFER TO ONE
00220      ISAM=60+140*(3.25*DR-2.25)+ISAM
00230      TRANSFER TO TWO
00240 ONE    ISAM=60+140*3.5*DR/8+ISAM
00250 TWO    WHENEVER (ISAM-INPUT).L.0, TRANSFER TO LINEC
00260      M(ISAM-INPUT+1)=M(ISAM-INPUT+1)+1
00270      ISAM=ISAM-INPUT
00280 LINED  CONTINUE
00290      PRINT ONLINE FORMAT A
00300      V'S A=$S10,7HL$ATENCY,S11,2H10,S8,2H20,S8,2H30,S8,2H40*$
00310      PRINT ONLINE FORMAT B
00320      VECTOR VALUES B=$S10,7H-----,S11,1H',3(S9,1H')*$
00330      THROUGH LINEE, FOR I=1,5,I.G.200
00450      IF=0
00460      THROUGH LINEE, FOR N=1,1,N.G.5
00470      IF=M(I+N-1)+IF
00480 LINEE  CONTINUE
00490      IF=IF/5
00500      K=I+139
00510      J=I+144
00520      PRINT ONLINE FORMAT HISTOG,K,J
00530      VECTOR VALUES HISTOG=$S10,14,1H-,14,'IF'(1H!)*$
00540 LINEF  CONTINUE
00550      PRINT ONLINE FORMAT B
00560      PRINT ONLINE FERNAT A
00570      EXECUTE EXIT.(0)
00580      INTEGER ID,K,N,ISAM,J,I
00590      INTECEP M,INPUT,IF
00600      END OF PROGRAM
P 1.050+1.150

```

Figure C.4 Simulation of System with Sampling Intervals
Dual Uniformly Distributed as Described in Eq. 2.5

Line	Notes
10,20,580,590	Variable mode and format designation
30 - 80	Read in ID (used in initialization of random number generator) and K (number of trials)
90 - 170	Initialize random number generator, ISAM, and M matrix
180 - 240	Main program loop - executes inputs and generates samples until total time between samples exceeds time of input
190	Selects time of occurrence of inputs
210	Tests for total time between samples exceeding input time
220	Increments member of M matrix corresponding to time from input to next sample plus one (one added for easy subscripting in printout)
230	Resets intersample time corresponding to input at zero to avoid accumulation of large numbers where only differences are interesting
250 - 560	Print latency histogram resulting
500 - 510	Add the constant delay to form latency from input to sample time

Figure C.5: Program Notes Applying to All Listings-
Line References refer to Figure C.1

Figure	Line	Notes
C.1	200	Sampling instants occur at 200 msec. intervals
C.2	200	Sampling intervals occur uniformly distributed between 0 and 200 msec.
C.3	200	Sampling intervals occur uniformly distributed between 150 & 250 msec.
C.4	200 - 240	This transformation carries values of DR (uniformly distributed between 0 and 1) that are less than .8 into intersample times uniformly distributed between 60 and 120 msec., and values of DR greater than or equal to .8 into intersample times uniformly distributed between 120 and 200 msec.

Figure C.6: Specific Program Notes for each Listing