Using a Ferro-Fluid Pad to Climb Walls

By
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ABSTRACT

The goal of this thesis is to build a wall climbing system that utilizes the viscosity property of ferrofluids. Ferrofluid viscosity is varies based on the magnetic field applied to it and this property enables ferrofluids to be used as an adhesive. This would allow a human, with a specially designed climbing gripper, to climb up walls by varying the magnetic field on the ferrofluid that sits between the gripping surface and the wall. While this concept sounds feasible, it is completely untested. The goal of this study was to create theoretical models of how a gripper would work, and then build a climbing gripper using the data from the models. We found that it is theoretically possible to build a ferrofluid climbing system that would allow a human to climb a wall. We then used finite element analysis to optimize a permanent magnet array. Finally, we designed, built, and tested a system around our analysis and found that the gripper did not work and the system was unable to carry any load.

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1. Introduction

In this thesis the goal was to design a gripper that would allow humans to climb walls using ferrofluids as an adhesive. A ferrofluid is a special oil that has iron filings mixed into it. A change in viscosity gives the ferrofluid adhesive properties, similar to glue. When there is no magnetic field applied to the ferrofluid, the viscosity of the ferrofluid is low. When a magnetic field is applied, the ferrofluid becomes very viscous giving and the microstructure changes turning it into a Bingham plastic (also known as a yield stress fluid) [1] which can hold shear and static adhesive forces.

A climbing gripper is built using a permanent magnet array that can be actuated so that the user can move it close to and far from the active surface. The gripper has an active surface that is coated with the ferrofluid and is in contact with the wall. When the magnet array is near the active surface, the ferrofluid is at high viscosity and the gripper will adhere to a wall. When the magnet array is moved away from the surface, the ferrofluid transitions to a low viscosity state and the adhesion force between the gripper and the wall disappears. A basic sketch of how the gripper would be configured is shown in figure 1.

![Figure 1: A sketch of the device](image)

We chose to design a gripper that uses a permanent magnet array and the power of a human to actuate the array. The reasoning behind not using an electromagnet is that we wanted to make sure the user would not be endangered by a loss of power. We also chose not to use a powered actuator to move a disk of permanent magnets for similar reasons.

There were three stages in this study. The first was to analyze the feasibility of using ferrofluids as a mechanism to allow humans to climb walls. The second stage was to optimize the permanent magnetic disk array that would be used to actuate the ferrofluids. The third and final stage of my thesis
was to design, build, and test a gripper to confirm its feasibility and to compare its performance to the models created in the other stages.

In the feasibility analysis stage we looked at four different factors that would affect the effectiveness and design of the gripper: the active surface area needed to hold the weight of a climber; the actuation distance required to release the ferrofluid; the force necessary to overcome the magnetic attraction between the magnetic array and the ferrofluid; and the amount of grip force a human needs to apply to actuate the mechanism.

In the magnetic optimization stage we used a combination of four different software packages to model the normal magnetic field applied to the gripping surface by an array of permanent magnets. We used FEMM, a 2D magnetic finite element analysis package, in conjunction with Matlab to test over five hundred different array configurations. We then sorted the results and chose a configuration that best met our criteria. Finally, we designed a disk in Solid Works based on the 2D magnetic finite element analysis. In the final stage we designed, built, and tested a gripper. Using the data we got from the first feasibility analysis and magnetic optimization stages we designed a gripper in Solid Works using parts that could easily be sourced. We then built a gripper and ran tests on it to see if it performed as the models predicted it would.

Before starting this study, we looked at other climbing systems. Previous work was done on the adhesive properties of ferrofluids by a former Masters student at MIT and is compiled in his thesis [1]. It provides a lot of useful data on the specific adhesive properties of ferrofluids.

The other work we examined was the biologically inspired climbing systems, based on geckos, that are being built at Stanford [2]. Their paper had a few key take away points. First, the mass grows as a function of the length scale of the gripping surface cubed while area grows as a function of the length scale squared. This is why it is difficult to scale up climbing systems. Second, there are two main goals in adhesive climbing: engaging every fiber and applying the load evenly. Third, one of the key issues in designing this type of system is peeling as a result of a gripper surface being flexible. The paper gives an equation to calculate the required stiffness of the active surface which can be modeled as a clamped beam with even loading (shown below). Fourth, using a tail or a contact point can help the pressure distribution.

\[ y = \frac{wL^4}{8EI} \quad (1) \]

Another source [3] pointed out that the two main problems in any climbing system are locomotion and adhesion.
There are a few key definitions that this paper refers to that the reader should know. Gripper refers to the whole gripping system shown in figure 1. Gripping surface refers to the surface of the gripper that is in contact with the ferrofluid. The magnet disk refers to the carrier disk that holds the neodymium magnets. The magnetic array refers to the array of neodymium magnets on the magnet disk.

2. Feasibility Analysis

The first step is to determine if using a ferrofluid pad for wall climbing is feasible. The overall design principle is that a disk of magnets, a set number of inches in diameter, will be used to change the viscosity of the ferrofluid. The disk will be moved back and forth utilizing a lever actuated by a user. In the active position the disk has to provide at least 0.15 Tesla of magnetic field onto the ferrofluid, the amount of magnetic field needed to actuate the ferrofluid [1]. In the inactive position the disk has to provide less than 0.04 Tesla of magnetic field onto the ferrofluid.

Before a gripper can be designed, a few feasibility issues and design parameters had to be ascertained. Four key design parameters were considered. First, how much active surface area is needed in order to hold the weight of the climber. Second, how far does the disk need to actuate in order to release the fluid so that the climber can remove the gripper from the wall. Third, how much force does it take to actuate the magnetic array from the active to the inactive position. And fourth, how much force can a human grip provide in order to actuate the device. Once these parameters are determined, we can design a gripper based on them.

Active surface area needed

In order to determine the gripper surface area we made a few assumptions about the conditions under which it would be operated. First, we would like a factor of safety of two. Second, a maximum climber weight is 70 kg. Third, there will be about a 0.15 Tesla B field, the amount of normal magnetic flux needed to sufficiently activate the ferrofluid. This magnetic field, combined with a rough surface finish, will result in a gripping strength of about 20 KPa (this is a conservative number) normal force [1] as shown in fig. 2. Previous research has shown that when a pre-load is applied, the shear force failure point is an order of magnitude larger and thus it can be ignored [4].
Fourth, we can assume that two of the four pads will always be touching the wall. This means that one pad should be able to carry 35 kg with a factor of safety of two, or 70 kg. The fifth assumption is that the normal forces are approximately the same as the load. Fig. 3 and equations two through five show why this is a reasonable approximation when the lever arm is centered and its length is the same as the radius of the disk (note that in this figure the factor of safety is included).
\[
\sum M_z^A = 0 \quad (2)
\]
\[
= F_C \cdot \cos(45) \cdot \sqrt{2}n - F_N \cdot n \quad (3)
\]
\[
= F_C \cdot \frac{\sqrt{2}}{2} \cdot \sqrt{2}n - F_N \cdot n \quad (4)
\]
\[
F_N = F_C \quad (5)
\]

The active surface area is defined as an area where the normal magnetic flux density through the ferrofluid is 0.15 Tesla. We used the above assumptions to calculate the active surface area. This calculation is shown below in equations six through eight.

\[
\text{AreaOf Disk} = \frac{\text{HoldingWeight}}{\text{StrengthofGrip}} \quad (6)
\]
\[
= \frac{80Kg}{20KPa} \quad (7)
\]
\[
= .04m^2 \quad (8)
\]

Based on the above calculation, the magnetic disk's active surface area is four hundred square centimeters. This translates to a disk with a diameter of approximately nine inches.

**Disk Actuation Distance**

In order to design the gripper with the correct amount of travel, we need to find the distance that the disk has to be moved to be put into the fully off position. To fully deactivate the ferrofluid there needs to be a normal magnetic field through the fluid of less than .03 Tesla. We used a series of simulations in FEMM, a 2D magnetic field finite element analysis software, to determine how far the disk should travel to deactivate the magnetic field. WE used the optimal magnetic array design produced by the simulations (see section 4 for the determination of the optimal magnetic array design). From the simulations using the FEMM software WE found that, for the normal magnetic flux in the ferrofluid to be less than .03 Tesla, the magnet should be at least one inch away from the fluid. Conservatively, the magnet array should move one and a half inches away from the active surface to fully deactivate the ferrofluid. The finite element analysis set up for the optimal distance is shown in Fig. 4, the color map of the magnetic flux density is shown in Fig. 5, and the graph of the normal magnetic flux density along the gripping surface is shown in Fig. 6.
An important calculation is the force of the magnetic attraction between the magnetic disk and the ferrofluid on the other side of the active surface. It is needed in order to determine the load that a human operator would be required to overcome to actuate the disk from its off to its on position. This force is difficult to calculate exactly because of the unusual properties of ferrofluids. In our case we
model the ferrofluid as a solid steel plate. This will give us a worst case scenario of the magnetic attraction between the magnetic disk and the ferrofluid. The equations supplied by a magnet vendor [6] that tell us this force are below in equations nine through thirteen.

\[ B_m = \text{magnetic fluxdensity} = 0.15 \]
\[ a_m = \text{Area of the magnet} = 0.04 \]
\[ \text{Force} = \frac{B_m^2 \cdot a_m}{8\pi \times 10^{-7}} \]
\[ = \frac{0.15^2 \times 0.04}{8\pi \times 10^{-7}} \]
\[ = 358N \]

**Force of a human grip**

The last key issue is determining the strength of the human grip, because this will be the actuator. To actuate the disk from the “off” to “on” position, the operator has to overcome the force between the magnetic disk and the ferrofluid. The gripper’s handle should be designed so that the target user can actuate it. To find this number we used human factor data shown in Fig. 7 from SH’s Research [5]. As you can see the ideal number should be around 40kg which would allow fit people to use the gripper pad. We assume that a weaker person would not be using the climbing system.

The user has to input 358N of force and the expected user can put in 390N of force. This means that a lever is not needed. Even though a direct lever will work, more users will be able to use the system with a lever. If a lever is used a 2:1 lever ratio is ideal. The operator will need to provide 3 inches of travel to move the disk this far.

![Figure 7: Grip strength chart [5]](image_url)
3. Magnetic field analysis

A key challenge is determining the magnet configuration. The goal is to obtain a normal magnetic flux of at least 0.15 Tesla on the ferrofluid that is between the gripping surface and the wall with the least amount of neodymium. There are three main optimization parameters. First, we would like to use a minimum amount of neodymium in order to save on weight and cost. Second, we would like to use reasonable sized magnets to compose the array because large neodymium magnets can be dangerous to work with. Third we can only obtain magnets in certain sizes and we do not have the tools to reshape the magnets, as a result we have to use the sizes that are easily obtainable through commercial vendors.

There are three basic magnet configurations. The first option is a single solid magnet, this is not practical due to safety concerns. The second option is a series of spaced magnets with alternating polar directions. The third option is a series of spaced magnets with the same polar direction. Both the second and third options have a significant number of configurations as a result of varying three main parameters: the magnet thickness, the spacing between the magnets, and the dimensions of each individual magnet.

In order to determine the optimum magnetic array configuration, we ran a two stage analysis. In the first stage, we ran a finite element analysis of series of array configurations in two dimensions using Matlab and FEMM. Using an optimization algorithm we chose the best configuration. In the second stage, we designed a disk in Solid Works based on the optimal array configurations that were determined in the first stage and built this disk.

Optimum Array Configuration

There are hundreds of practical permutations of how the magnetic array can be arranged. Before we could build the gripper we had to determine which configuration was optimal. In order to do this we had to run all the practical permutations in a 2D finite element analysis program. We chose to use FEMM to do the analysis because it was compatible with Matlab so we could write code in Matlab that could run all the permutations in FEMM.

The first part was to understand how FEMM works. It is a two dimensional electromagnetic finite element analysis program. Its interface is shown in figures four, five, and six. To use it, we first defined the geometry. Then we defined the material and the polarities of any magnetic materials. Then we ran a mesh and the simulation. After that we examined the normal magnetic flux along the line that represents the gripping surface.
FEMM can be controlled through Matlab. In order to run and sort all the cases we created two functions and a program to run the functions (the code is shown in appendix A). The two functions ran the individual cases; one was for the cases where the magnetic field polarity was alternating, the other was for the cases where all the magnets were polarized in the same direction. They both took the magnet thickness, spacing, and width of the magnets as inputs. They both outputted the percent of the normal magnetic flux on the gripping surface that was over certain thresholds (the thresholds were 0.15 Tesla, 0.175 Tesla, 0.2 Tesla, 0.225 Tesla, 0.25 Tesla, 0.275 Tesla, and 0.375 Tesla).

The two Matlab functions work in the same way. First, they define the geometry in FEMM based on the inputs. They then define the material. After that they run the mesh and the analysis. Then they take a series of points along the gripping surface and look at the normal magnetic flux at each of these points. Finally, they calculate what percentage of these points have a normal magnetic flux above the set thresholds and output these values.

The program takes in a range of values for the magnet thickness, spacing, and width. It does two things. First it runs all the permutations of these parameters in both the functions. After that it sorts the results in order from the highest percent of magnetic flux above the 0.15 Tesla threshold to the lowest.

We looked at magnets with thickness of 0.25, 0.375, and 0.5 inches and width ranging from 0.25 inches to 2 inches in increments of 0.25 inches. We looked at gaps between 1/32 and 0.75 inches in increments of 1/32 inches. This meant we had to test one thousand one hundred fifty two cases.

Finally, using an algorithm shown in equation fourteen we scored each magnet configuration (the results from the Matlab simulation and how they scored are shown in a table in Appendix B). The algorithm is based on data on how the holding strength varies with magnetic field shown in figure 2. We determined that a 0.375 inch thick, 1 inch square magnet spaced 0.0975 inches apart with alternating polarization was ideal. Although some of the half inch configurations scored higher we decided against them because they did not score a lot higher and would add a significant amount of weight. One other 0.375 inch configuration scored slightly higher but it involved 1.25 inch square magnets which are difficult to obtain.
\[ \text{Score} = 15 \times \text{fluxabove}0.15\text{Tesla} \]  
\[ + 5 \times \text{fluxabove}0.175\text{Tesla} \]  
\[ + 2 \times \text{fluxabove}0.2\text{Tesla} \]  
\[ + 2 \times \text{fluxabove}0.225\text{Tesla} \]  
\[ + 2 \times \text{fluxabove}0.25\text{Tesla} \]  
\[ + 2 \times \text{fluxabove}0.275\text{Tesla} \]  
\[ + 2 \times \text{fluxabove}0.3\text{Tesla} \]  

\[ (14) \]

Disk Design and Assembly

In order to design a three dimensional array, we had to take the data from the two dimensional magnetic field analysis that was done in the YZ plane and design a three dimensional array that would fit inside a nine inch diameter disk. To do this we used Solidworks, a three dimensional CAD software. First we sketched a nine inch diameter disk on the XY plane. We then drew a one inch square in the center of the circle. After this the square was patterned in order to create an array of one inch squares spaced three thirty-seconds of an inch apart that were all inside of the array. Then we filled out the areas of the disk which were too small to fit a one inch square but still large with a three quarter inch square. Finally we removed the initial middle square to make room for the attachment point to the actuator that will move the disk inside the gripper.

\[ \text{Figure 8: Solid Model of the magnetic array} \]
The disk itself was designed to be created out of two disks of laser cut acrylic that would be cemented together. Each acrylic disk is the same thickness as the magnets. The top piece would be a solid disk with an attachment point for the actuator in the middle. The bottom piece would be a disk with the array pattern cut out of it; it would also have the same diameter of the top disk. The magnets would fit into the squares cut out of the bottom disk. The final model of the disk is shown below in figure 8.

The next step, after designing the disk, was to build it. We sourced the materials, high grade neodymium magnets, acrylic, and adhesives from online sources. We then laser cut the two acrylic disks and used acrylic cement to bind them together. We then inserted the magnets into the disk and used JB weld adhesive to keep them in place. We checked the polarities of the magnets while inserting them by using a small magnet to make sure they were inserted with alternating polarities. The final disk is shown below in figure 9.

Figure 9: The assembled magnetic array


The final stage of my work involved designing and building a single full scale gripper to test how feasible this system would actually be and how accurate our models were. In order to do this we first designed a gripper in solid works based around the array designed in section three and on what we found in the feasibility analysis. We then built the gripper and tested it on a variety of surfaces for both
holding strength and usability. Finally, using the results of these tests, we created a ninety five percent confidence interval for the mean holding strength on different surfaces.

**Design and Assembly**

The goal of the design was to build a simple gripper around the magnetic disk using mostly sourced parts. This makes the gripper much easier to build. We used solid works to design the gripper. We settled on the design shown in the solid models below as figures 10 & 11. The design concept was to use a piston to activate the array that would otherwise be floating inside a protective housing. The housing consists of a thick acrylic top plate and a thin acrylic bottom plate. The bottom plate also acts as the active surface. The top and bottom plate are connected with a ring of bolts and spacers (the bolts are not shown in the CAD). Around the spacers there is a skirt made out of a flexible, large diameter tube to keep ferrofluids from leaking into the housing. There is a linear bearing in the top plate for the piston to go through. This bearing allows the piston to move smoothly and keeps the array centered in the housing. In order to keep the array in the active position by default we put a spring around the piston (not shown in the CAD) between the array and the upper housing. Attached to the top plate and the piston is a lever that is used to actuate the disk. The handle is also attached to the top plate.

![Figure 10](left): The gripper assembly CAD model in the inactive position (magnetic array is up and the magnetic field on the ferrofluids is low). **Figure 11** (Right): The gripper assembly CAD model in the active position (magnetic array is down and the magnetic field on the ferrofluids is high).
The next step was to build a single gripper to run tests on. We used some sourced and some machined parts. All the acrylic parts were laser cut, the piston and the rounds were turned on the lathe to the correct dimensions, and the brackets were milled. We then assembled the gripper using acrylic cement to attach acrylic parts together, and using bolts to fasten the rest of the assembly together. The assembled gripper is shown below in figure 12.

*Figure 12: The gripper assembled without the skirt (the skirt is the part that keeps ferrofluids from getting inside the housing)*

After assembling the gripper with the lever mechanism we found that it was too unwieldy for a person to use. As a result we decided to abandon the lever for a simpler pull mechanism shown below in figure 13.
Testing

The final part of this project involved testing the gripper for usability, holding strength in shear, and holding strength in tension on multiple surfaces. The usability test consisted of trying to actuate the gripper with one hand. The shear and tension tests both used a spring scale set up to measure the holding force. The configuration for the shear test consists of a spring scale attached to the gripper and pulled by a ratchet strap. This enables the gripper to be actuated slowly so that the force is added gradually. The gripper is tethered to a fixed point so that in case it fails it does not go flying across the room. The shear test configuration is shown below in figure 14.
We planned to test four different surfaces, sandpaper, wood, acrylic, and sanded acrylic. We were also planning on running multiple of each test to set up confidence intervals for mean shear and tensile holding force for each configuration. However, we observed that there was no adhesion between the griper and any of the surfaces. We tried varying the amount of ferrofluid and preload and there was still only negligible adhesion (less than 5kg, the mass of the gripper). While we were unable to get the gripper to grip we were able to make a few interesting observations about the system, and come up with a possible reason why the system did not work.

The first key observation was that the gripper was difficult to impossible to actuate with one hand when a significant amount of ferrofluid was used. This was expected because we switched from the lever actuator to the direct actuator since the lever was too bulky. To address this, future iterations of the gripper should use a lever. However, the design of the lever should be less bulky.

The second key observation is that in the active position the ferrofluid was hard. This means that the viscosity of the fluid did increase when the magnet was in the active position. As a result this system is feasible and the ferrofluid not activating was not the reason for failure.

The final key observation was that in the inactive position the ferrofluid was still more viscous then it was when it was completely away from the magnet. This is probably due to not taking into account the thickness of the spring, resulting in the disk not moving the full inch and a half away from the active surface. We think this is why the gripper did not adhere to any surface. If the fluid was even partially activated before the gripper and the surface came into contact, it would not adhere well.

5. Conclusion and Future Steps

In this project we looked at whether ferrofluids could be used as an adhesive for human wall climbing. In the first stage of this project, we found that it was theoretically feasible. In the second stage, we performed a magnetic field analysis to determine the optimal configuration of a magnetic array. In the final stage, we designed, built and tested a prototype gripper. We found that the gripper did not adhere to any surface. However, it did activate the fluid significantly when it was in the active position. From observation, the most likely reason for failure was that the fluid was not fully deactivated when the gripper was in the inactive configuration. For the next step we would like to take apart and rebuild the gripper so that it fully removes the magnetic field from the ferrofluid. To do this, we will remake all the spacers and the piston to be an inch longer. Then we will reassemble the gripper with the new spacers and piston. Finally, we will test whether the gripper works in this new configuration.
Appendix A

This appendix contains the Matlab code used for the two dimensional finite element analyses. It consists of three main blocks of code, a runner and two functions. The functions run an FEA analysis in FEMM for a specified geometry, they take length, width, and the gap thickness as inputs. The runner is an iterative runner that runs the two functions for specified cases and sorts the results.

Iterative runner

```matlab
% This file contains the Matlab code used for the two dimensional finite
% element analyses. It consists of three main blocks of code, a runner
% and two functions. The functions run an FEA analysis in FEMM for a
% specified geometry, they take length, width, and the gap thickness as
% inputs. The runner is an iterative runner that runs the two functions
% for specified cases and sorts the results.

% Set the initial values
i=8;
j=4;
k=50;
iinitial=i;
jinitial=j;
kinitial=k;
l=[1:i]./4; % range of possible magnet lengths
6lj2i=0.5;
w=[1:k]./32; % range of possible gaps

% Initialize the solutions
solutionsame=zeros(i*j*k,10);

% Run the iterative runner
counter=1;
while (i>0)
    while (j>0)
        while (k>0)
            solutionsame(counter,1)=l(i);
solutionsame(counter,2)=t;
solutionsame(counter,3)=w(k);
[A,B,C,D,E,F,G]=SameSolver(l(i),t,w(k));
solutionsame(counter,4)=A;
solutionsame(counter,5)=B;
solutionsame(counter,6)=C;
solutionsame(counter,7)=D;
solutionsame(counter,8)=E;
solutionsame(counter,9)=F;
solutionsame(counter,10)=G;
counter=counter+1
k=k-1;
end;
% j=j-1;
% k=kinitial;
end
% j=jinitial;
k=kinitial;
i=iinitial;
end
% orders the matrix from highest to lowest
solutionsame=flipdim(sortrows(solutionsame,[4 10]),1);
```

```
j=jinitial;
k=kinitial;
solutionalt=zeros(i*j*k,10);
counter=1;
while(i>0)
  while(j>0)
    while(k>0)
      solutionalt(counter,1)=l(i);
solutionalt(counter,2)=t;
solutionalt(counter,3)=w(k);
      [A,B,C,D,E,F,G]=AltSolver(l(i),t,w(k));
solutionalt(counter,4)=A;
solutionalt(counter,5)=B;
solutionalt(counter,6)=C;
solutionalt(counter,7)=D;
solutionalt(counter,8)=E;
solutionalt(counter,9)=F;
solutionalt(counter,10)=G;
counter=counter+1;
k=k-1;
    end
  j=j-1;
end
k=kinitial;
end
j=jinitial;
i=i-1;

%orders the matrix from highest to lowest
solutionalt=flipdim(sortrows(solutionalt,[4 10]),1);

%row 3 = percent above .15 tessla
%row 4 = percent above .175 tessla
%row 5 = percent above .2 tessla
%row 6 = percent above .225 tessla
%row 7 = percent above .25 tessla
%row 8 = percent above .275 tessla
%row 9 = percent above .3 tessla

Same polarization solver
%uses octave FEMM to set up a 2D magnetic field simulation
%this function sets up a series of magnets with Ferro fluid underneath and
%calculates the normal magnetic field in the ferro fluid
%for more info on FEMM and integrating it with matlab go to
%http://www.femm.info/wiki/OctaveFEMM

function [A, B, C, D, E, F, G] = SameSolver(l, t, w)

% l magnet length
% t magnet thicknesses
% w gap

openfemm; % opens the 2D FEA software (FEMM)
newdocument(0); % sets up a new document
mi_probdef(0, 'inches', 'planar', 1.e-8, 0, 30); % defines the problem
mi_getmaterial('Air') %pulls air's properties from the materials library
mi_getmaterial('NdFeB 52 MGOe') %pulls neodinium's properties from the materials library
mi_getmaterial('LORD MRF 132-DG - mu=6') %pulls the ferro fluid's properties (which was inputed into FEMM from a seperate source)

%%%%%%%%%%%%%%%%%%%%%%%%%%draws the fluid bed
mi_drawrectangle([-5 0; 5 .005]); %defines a rectangle by the cordanets of oposit corners
mi_addblocklabel(0,0.002); %labels the rectangle which it is inside of
mi_selectlabel(0,.002); %selects the closest lable
mi_setblockprop('LORD MRF 132-DG - mu=6', 0, 1, '<None>', 0, 0, 0); %defines the properties of the block which contains the selected lable
mi_clearselected %deselects the block

%%%%%%%%%%%%%%%%%%%%%%%%%%draws the air
mi_drawrectangle([-8 -3; 8 5]);
mi_addblocklabel(-6,2);
mi_selectlabel(-6,2);
mi_setblockprop('Air', 0, 1, '<None>', 0, 0, 0);
mi_clearselected

%%%%%%%%%%%%%%%%%%%%%%%%%%draws neodinium the blocks
counter=1;
y1=.13;
y2=.13+t;
x1=0;
x2=0;
while(x2 < 4.5)
    x1=x2+w;
    x2=x1+1;
    if(x2>4.5)
        x2=4.5;
        if(x2-x1<.25)
            x2=x1+.25;
    end
end
mi_drawrectangle([x1 y1; x2 y2]);
mi_addblocklabel( x1+.01, y1+.01);
mi_selectlabel( x1+.01, y1+.01);
mi_setblockprop('NdFeB 52 MGOe', 0, 1, '<None>', 90, 0, 0);
mi_clearselected

mi_drawrectangle([-x1 y1; -x2 y2]);
mi_addblocklabel( -x1-.01, y1+.01);
mi_selectlabel( -x1-.01, y1+.01);
mi_setblockprop('NdFeB 52 MGOe', 0, 1, '<None>', 90, 0, 0);
mi_clearselected
end

%%%%%%%%%%%%%%%%%%%%%%%%%anilizer
mi_zoomnatural
mi_saveas('SameSolver.fem');
mi_createmesh;
mi_analyze
mi_loadsolution
normal flux density
xe=-4.975:.05:4.975;
ye=zeros(1,length(xe));
bee=mo_getb(xe,ye);
norm=bee(:,2);

A=0;
B=0;
C=0;
D=0;
E=0;
F=0;
G=0;

counter=1;
while(counter<=length(norm))
    val=abs(norm(counter));
    if(val>=0.3)
        A=A+0.5;
        B=B+0.5;
        C=C+0.5;
        D=D+0.5;
        E=E+0.5;
        F=F+0.5;
        G=G+0.5;
    elseif(val>=0.275)
        A=A+0.5;
        B=B+0.5;
        C=C+0.5;
        D=D+0.5;
        E=E+0.5;
        F=F+0.5;
    elseif(val>=0.25)
        A=A+0.5;
        B=B+0.5;
        C=C+0.5;
        D=D+0.5;
        E=E+0.5;
    elseif(val>=0.225)
        A=A+0.5;
        B=B+0.5;
        C=C+0.5;
    elseif(val>=0.2)
        A=A+0.5;
        B=B+0.5;
    elseif(val>=0.175)
        A=A+0.5;
        B=B+0.5;
    elseif(val>=0.15)
        A=A+0.5;
    end
    counter=counter+1;
end
%A = percent above .15 tesla
%B = percent above .175 tesla
%C = percent above .2 tesla
%D = percent above .225 tesla
%E = percent above .25 tesla
%F = percent above .275 tesla
%G = percent above .3 tesla
end

Alternating polarization solver

% uses octave FEMM to set up a 2D magnetic field simulation
% this function sets up a series of magnets with Ferro fluid underneath and
% calculates the normal magnetic field in the ferro fluid
% for more info on FEMM and integrating it with matlab go to
% http://www.femm.info/wiki/OctaveFEMM

function [A, B, C, D, E, F, G] = AltSolver(l, t, w)

% l magnet length
% t magnet thicknesses
% w gap

openfemm; % opens the 2D FEA software (FEMM)
newdocument(0); % sets up a new document
mi_probdef(0, 'inches', 'planar', 1.e-8, 0, 30); % defines the problem

%%%%%%%%%%%%%%%%%%%%%%%%%% gets materials
mi_getmaterial('Air') % pulls air's properties from the materials library
mi_getmaterial('NdFeB 52 MGOe') % pulls neodinium's properties from the
materials library
mi_getmaterial('LORD MRF 132-DG - mu=6') % pulls the ferro fluid's properties
(which was inputed into FEMM from a separate source)

%%%%%%%%%%%%%%%%%%%%%%%%%% draws the fluid bed
mi_drawrectangle([-5 0; 5 .005]); % defines a rectangle by the coordinates of
opposite corners
mi_addblocklabel(0,0.002); % labels the rectangle which it is inside of
mi_selectlabel(0,1); % selects the closest label
mi_setblockprop('LORD MRF 132-DG - mu=6', 0, 1, '<None>', 0, 0, 0); % defines
the properties of the block which contains the selected label
mi_clearselected % deselects the block

%%%%%%%%%%%%%%%%%%%%%%%%%% draws the air
mi_drawrectangle([-8 -3; 8 5]);
mi_addblocklabel(-6,2);
mi_selectlabel(-6,2);
mi_setblockprop('Air', 0, 1, '<None>', 0, 0, 0);
mi_clearselected

%%%%%%%%%%%%%%%%%%%%%%%%%% draws the blocks
counter=1;
y1=.13;
y2=.13+t;
x1=0;
x2=0;

n=1;
while(x2 < 4.5)
x1=x2+w;
x2=x1+1;
if(x2>4.5)
x2=4.5;
if(x2-x1<.25)
x2=x1+.25;
end
end
mi_drawrectangle([x1 y1; x2 y2]);
mi_addblocklabel( x1+.01, y1+.01);
mi_selectlabel( x1+.01, y1+.01);
mi_setblockprop('NdFeB 52 MGOe', 0, 1, '<None>', 90*n, 0, 0);
mi_clearselected

mi_drawrectangle([-x1 y1; -x2 y2]);
mi_addblocklabel( -x1-.01, y1+.01);
mi_selectlabel( -x1-.01, y1+.01);
mi_setblockprop('NdFeB 52 MGOe', 0, 1, '<None>', -90*n, 0, 0);
mi_clearselected

n=n*-1;
end

%正常化
mi_zoomnatural
mi_saveas('SameSolver.fem');
mi_createmesh;
mi_analyze
mi_loadsolution

%正常磁通密度
xe=-4.975:.05:4.975;
ye=zeros(1,length(xe));
bee=mo_getb(xe,ye);
norm=bee(:,2);
A=0;
B=0;
C=0;
D=0;
E=0;
F=0;
G=0;
counter=1;
while(counter<=length(norm))
  val=abs(norm(counter));
  if(val>=0.3)
    A=A+0.5;
    B=B+0.5;
  end
C=C+0.5;
D=D+0.5;
E=E+0.5;
F=F+0.5;
G=G+0.5;
elseif(val>=0.275)
A=A+0.5;
B=B+0.5;
C=C+0.5;
D=D+0.5;
E=E+0.5;
F=F+0.5;
elseif(val>=0.25)
A=A+0.5;
B=B+0.5;
C=C+0.5;
D=D+0.5;
E=E+0.5;
elseif(val>=0.225)
A=A+0.5;
B=B+0.5;
C=C+0.5;
D=D+0.5;
elseif(val>=0.2)
A=A+0.5;
B=B+0.5;
C=C+0.5;
elseif(val>=0.175)
A=A+0.5;
B=B+0.5;
elseif(val>=0.15)
A=A+0.5;
end
end
counter=counter+1;
end
%A = percent above .15 tessla
%B = percent above .175 tessla
%C = percent above .2 tessla
%D = percent above .225 tessla
%E = percent above .25 tessla
%F = percent above .275 tessla
%G = percent above .3 tessla
end
Appendix B

This appendix shows selected results (highest scoring in each thickness and polarization) from the two dimensional finite element analysis. The green highlighted and boldface row is the configuration that we selected. The reason for selecting this configuration over the higher scoring ones is that we wanted to use smaller magnets for safety reasons and this was the smallest configuration that scored above a 20. The first column shows the magnet length (we assume the magnet is square). The second column is the thickness of the selected magnet configuration, the third column in the gap between magnets. The fourth column is the polarization configuration of the magnets. The fifth through eleventh column is the percent of the normal magnetic field on the gripping surface that is above that threshold. The final column is the score using the formula described in equation 14.

\[
1.5gt 0.5kes 0.03125 arnzatin
\]

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<tr>
<th>length</th>
<th>thickness</th>
<th>gap width</th>
<th>polarization</th>
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<th>0.175 Tesla</th>
<th>0.2 Tesla</th>
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