8.044 Statistical Physics I
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044: Statistical Physics I — Spring Term 2008

Problem Set #5

Practice Problem: Exact Differentials (problem not graded, no need to hand in)

Which of the following is an exact differential of a function $S(x, y)$? Find S where possible.

a) $2x(x^3 + y^3)dx + 3y^2(x^2 + y^2)dy$

$$S(x, y) = (2x^5 + 5x^2y^3 + 3y^5)/5 + C$$

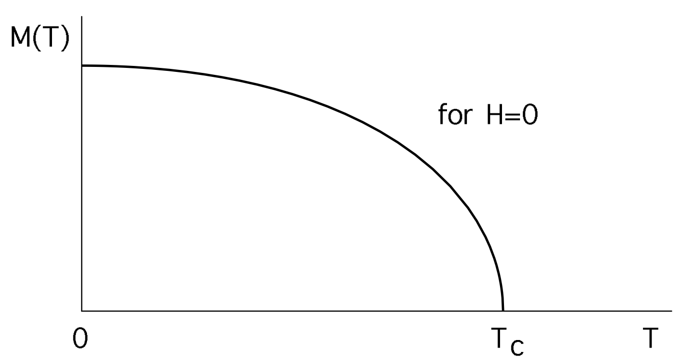
b) $e^y dx + x(e^y + 1)dy$

$S(x, y)$ does not exist.

c) $(y - x)e^x dx + (1 + e^x)dy$

$$S(x, y) = y + (1 + y - x)e^x + C$$

Problem 1: Equation of State for a Ferromagnet



For a ferromagnetic material in the absence of an applied field, $H = 0$, the spontaneous magnetization is a maximum at $T = 0$, decreases to zero at the critical temperature $T = T_c$, and is zero for all $T > T_c$.

For temperatures just below T_c the magnetic susceptibility and the temperature coefficient of M might be modeled by the expressions

$$\chi_T \equiv \left(\frac{\partial M}{\partial H}\right)_T = \frac{a}{(1 - T/T_c)} + 3bH^2$$

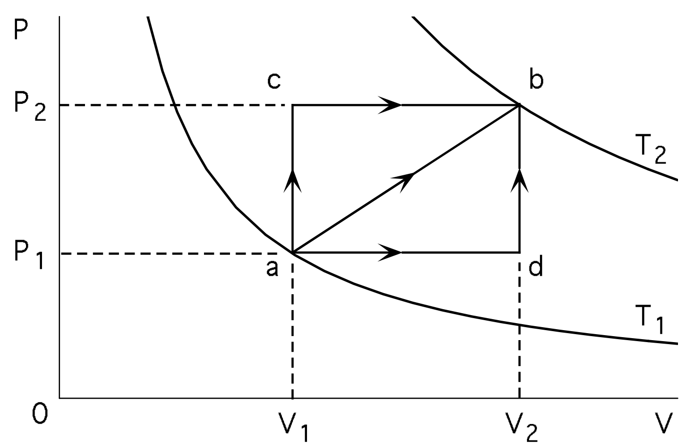
$$\left(\frac{\partial M}{\partial T}\right)_H = \frac{1}{T_c}\frac{f(H)}{(1 - T/T_c)^2} - \frac{1}{2}\frac{M_0}{T_c}\frac{1}{(1 - T/T_c)^{1/2}}$$

where M_0, T_c, a, and b are constants and $f(H)$ is a function of H alone with the property that $f(H = 0) = 0$.

a) Find $f(H)$ by using the fact that M is a state function.

b) Find $M(H, T)$.

Problem 2: Heat Supplied to a Gas



An ideal gas for which $C_V = \frac{5}{2}Nk$ is taken from point a to point b in the figure along three paths: acb, adb, and ab. Here $P_2 = 2P_1$ and $V_2 = 2V_1$. Assume that $(\partial U/\partial V)_T = 0$.

a) Compute the heat supplied to the gas (in terms of N, k, and T_1) in each of the three processes. [Hint: You may wish to find C_P first.]

b) What is the "heat capacity" of the gas for the process ab?

Problem 3: Equation of State and Heat Capacity of a Liquid Surface

The surface tension $\mathcal{S}$ of a liquid is the work required to increase the free surface area of the liquid by one unit of area.

For pure water in contact with air at normal pressure, the surface tension has a constant value $\mathcal{S}_0$ at all temperatures for which the water is a liquid.

Certain surfactant molecules, such as pentadecylic acid, can be added to the water. They remain on the free surface and alter the surface tension. For water of area A containing N molecules, one can measure this effect. Experiments show:

$$\left(\frac{\partial \mathcal{S}}{\partial A}\right)_T = \frac{NkT}{(A-b)^2} - \frac{2a}{A}\left(\frac{N}{A}\right)^2$$

$$\left(\frac{\partial T}{\partial \mathcal{S}}\right)_A = -\frac{A-b}{Nk}$$

where k is Boltzmann's constant and a and b are constants.

a) Find an expression for $\mathcal{S}(A,T)$ that reduces to the result for pure water when $N=0$.

Additional experiments determine the heat capacity at constant area C_A and the change in internal energy with area at constant temperature $(\partial U/\partial A)_T$.

b) Find an expression for the heat capacity at constant surface tension, $C_{\mathcal{S}}$, in terms of $\mathcal{S}$, C_A, $(\partial \mathcal{S}/\partial A)_T$, $(\partial T/\partial \mathcal{S})_A$, and $(\partial U/\partial A)_T$.

Problem 4: Thermodynamics of a Curie Law Paramagnet

Simple magnetic systems can be described by two independent variables. State variables of interest include the magnetic field H, the magnetization M, the temperature T, and the internal energy U. Four quantities that are often measured experimentally are

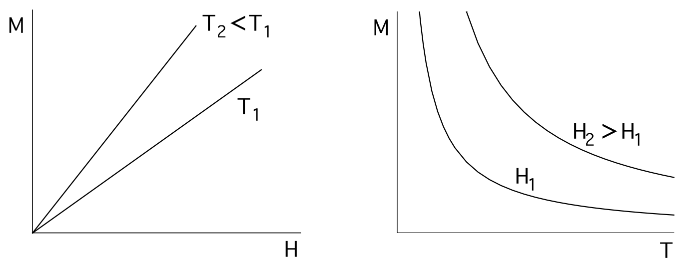
$$\chi_T \equiv \left(\frac{\partial M}{\partial H}\right)_T, \quad \text{the isothermal magnetic susceptibility,}$$

$$\left(\frac{\partial M}{\partial T}\right)_H, \quad \text{the temperature coefficient,}$$

$$C_M \equiv \left(\frac{đQ}{dT}\right)_M, \quad \text{the heat capacity at constant } M\text{, and}$$

$$C_H \equiv \left(\frac{đQ}{dT}\right)_H, \quad \text{the heat capacity at constant } H.$$

A particular example of a simple magnetic system is the Curie law paramagnet defined by an equation of state of the form $M = aH/T$ where a is a constant.



For such a system one can show that $(\partial U/\partial M)_T = 0$ and we shall assume that $C_M = bT$ where b is a constant.

a) Use T and M as independent variables and consider an arbitrary simple magnetic system (that is, not necessarily the Curie law paramagnet). Express C_M as a derivative of the internal energy. Find an expression for $C_H - C_M$ in terms of a derivative of the internal energy, H, and the temperature coefficient. Write an expression for $dU(T, M)$ where the coefficients of the differentials dT and dM are expressed in terms of measured quantities and $H(T, M)$.

b) Find explicit expressions for $C_H(T, M)$ and $U(T, M)$ for the Curie law paramagnet. You may assume that $U(T = 0, M = 0) = 0$.

c) Consider again an arbitrary simple magnetic system, but now use H and M as the independent variables. Write an expression for $dU(H, M)$ where the coefficients of the differentials dH and dM are expressed in terms of the measured quantities and H. Is the coefficient of the dM term the same as in part a)?

d) Find explicit expressons for the coefficients in c) in the case of the Curie law paramagnet. You will need your result from b) for C_H. Convert the coefficients to functions of H and M, that is, eliminate T. Integrate $dU(H, M)$ to find $U(H, M)$. Compare your result with that found in b).

e) Using T and M as the independent variables, find the general constraint on an adiabatic change; that is, find $(\partial T/\partial M)_{\Delta Q=0}$ in terms of a derivative of the internal energy, $H(M,T)$, and $C_M(M,T)$.

f) Evaluate $(\partial T/\partial M)_{\Delta Q=0}$ for the Curie law paramagnet and integrate the result to find the equation of an adiabatic path in the T, M plane through the point T_0, M_0.