8.044 Statistical Physics I
Spring 2008

MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

8.044: Statistical Physics I — Spring Term 2008

Problem Set #9

Problem 1: The Big Bang

Early in the evolution of the universe, when the universe occupied a much smaller volume and was very hot, matter and radiation were in thermal equilibrium. However, when the temperature fell to about 3000K, matter and the cosmic radiation became decoupled. The temperature of the cosmic (black body) radiation has been measured to be 3K now. Assuming adiabatic expansion, by what fraction has the universe increased in volume since the decoupling of cosmic radiation and matter? Hint: Remember the expression we derived for the Helmholtz free energy of thermal radiation,

$$F(T,V) = -\frac{1}{45}\frac{\pi^2}{c^3\hbar^3}(kT)^4V.$$

Note: The cosmic or background radiation is quite different from local radiation fields associated with a star, nebula or planet which certainly can be in equilibrium with the accompanying matter.

Problem 2: Comet Hale-Bopp

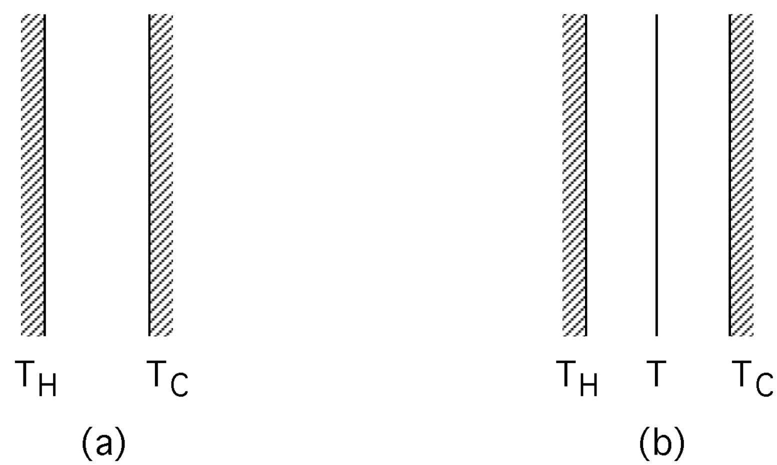
a) Estimate the surface temperature of comet Hale-Bopp when it is at a distance of 200 solar radii from the sun (about the same distance from the sun as the earth). Assume

   1) The surface temperature of the sun is 6000K.
   2) The absorptivity of both the sun's and the comet's surface can be approximated as $\alpha(\nu, T) = 1$.
   3) The comet's temperature changes are only due to its changing distance from the sun.

b) This temperature is a bit high for a comet which is supposed to be a frozen slush of various components, including H_2O, CO_2, and CH_4. What might cause the actual surface temperature to be lower than that calculated in a)?

c) Does the temperature found in a) seem familiar? Do you think this is a coincidence or is it meaningful? How does the temperature found in a) depend on the radius of the object? What simple evidence do we have that the sun contributes to the heating of the earth's surface? What evidence do we have that there is another heating mechanism as well? What is that other mechanism?

Problem 3: Super Insulation



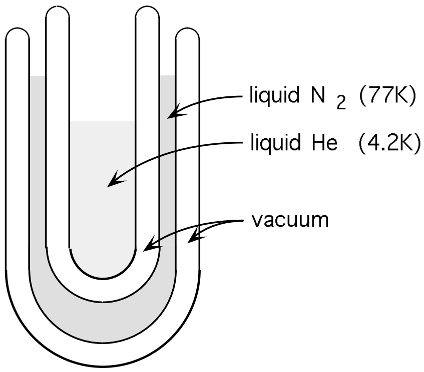
a) Two parallel plates of infinite extent are separated by a vacuum (Fig. a). The temperature of the hotter plate is T_H and that of the colder one is T_C. Find an expression for the heat flux, J, in watts-m^{-2} across the gap. Assume that both surfaces act as black bodies and are maintained at temperatures T_H and T_C.

b) A thin heat conducting sheet (also a black body) is suspended in the vacuum as shown in Fig. b. Heat can be transferred to the sheet only by radiation through the vacuum. Find the steady state temperature of the sheet. Hint: What does the steady state imply about the heat flux from T_H to T and from T to T_C?

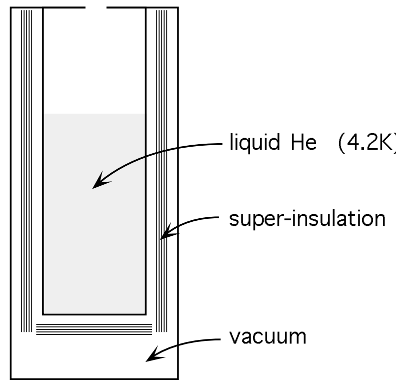
c) The heat flux from T_H to T_C in b) is a fraction $\mathcal{F}$ of that in a). Find $\mathcal{F}$.

d) There is an analytic expression for $\mathcal{F}$ in the case of n non-touching suspended sheets. Find $\mathcal{F}(n)$ by using the fact that the heat flux J between any two adjacent sheets must be the same.

The effect found here, that thermal radiative transfer can be suppressed substantially by interposing free-floating, non-contacting conducting sheets, is used to build liquid-nitrogen-free liquid helium storage containers. The dewar flask filled with liquid nitrogen, commonly surrounding the liquid helium filled dewar flask (Fig. c), is replaced with super-insulation, many layers of aluminum coated mylar film separated by a tenuous mesh of mylar thread (Fig. d).



(c)



(d)

Problem 4: Properties of Blackbody Radiation

For 1 m^3 of blackbody radiation at room temperature (300K) find

a) The heat capacity C_V,

b) The number of atoms of a monatomic gas that would give the same C_V,

c) The rms electric field in volts per centimeter.

Problem 5: Radiation Pressure

The radiation field and a monatomic gas exist in thermal equilibrium in a region of space. If the density of the gas is the STP density of 2.69×10^{19} atoms-cm^{-3} (Loschmidt's number), what must the temperature be so that the gas pressure and the radiation pressure are equal?

In the interior of very high mass stars the radiation pressure dominates the kinetic pressure.