8.044 Statistical Physics I
Spring 2008

8.044 Exam 1

Spring 2008

1. Do all 4 problems.
2. Write your solutions in the exam books.
3. No calculators, books, and notes are permitted.
4. For full credit, show all of your work for each problem.

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Physics Department

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Equation Sheet

Variance of a random variable x:

$$\mathrm{Var}(x) = \langle (x - \langle x \rangle)^2 \rangle = \langle x^2 \rangle - \langle x \rangle^2$$

A properly normalized Gaussian density:

$$p(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{(x - x_0)^2}{2\sigma^2}\right]$$

Work differential for a hydrostatic system:

$$đW = -PdV \quad \text{(work done on system)}$$

First Law of Thermodynamics:

$$dU = đQ + đW$$

Heat capacities:

$$C_V = \left(\frac{đQ}{dT}\right)_V \quad \text{and} \quad C_P = \left(\frac{đQ}{dT}\right)_P$$

For any ideal gas:

$$PV = NkT \quad \text{and} \quad U = U(T)$$

Adiabatic paths for a monatomic ideal gas:

$$\begin{aligned} TV^{\frac{2}{3}} &= \text{constant} \\ PV^{\frac{5}{3}} &= \text{constant} \end{aligned}$$

A possibly useful relation which is valid for any hydrostatic system:

$$\left(\frac{\partial U}{\partial V}\right)_T = T\left(\frac{\partial P}{\partial T}\right)_V - P$$

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Exam #1

Problem 1: (25 points) Particle inside a sphere

Suppose a particle has a uniform probability density for being located anywhere inside a sphere with unit radius. The joint probability density in the position variables x, y, and z is given by:

$$\begin{aligned} p(x, y, z) &= \frac{3}{4\pi} \quad \text{for } x^2 + y^2 + z^2 \leq 1 \\ &= 0 \qquad \text{otherwise} \end{aligned}$$

Calculate $p(x)$, the probability density for the single random variable x. Make a plot of $p(x)$ with carefully labeled axes.

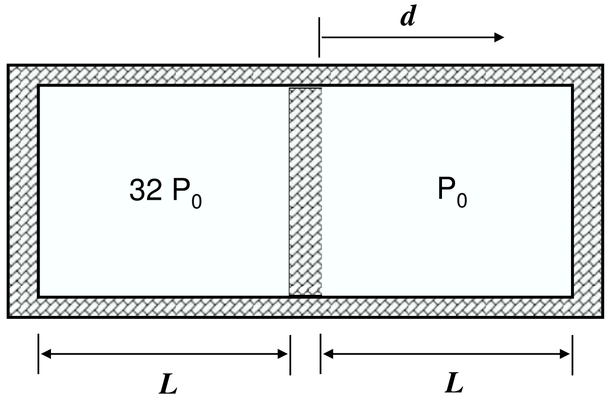
Problem 2: (25 points) Rolling dice

You have a collection of 100 six-sided dice. For a single die, the integers 1 through 6 are all equally likely to result after a roll. Suppose you roll all 100 dice at the same time. What is the approximate probability density for the sum s (the summation of the values on all of the dice)? Even though s is discrete, writing an expression for the continuous envelope function is acceptable.

(Don't worry about reducing messy fractions to simplified form. Just make sure that all of the parameters in your probability density are clearly defined.)

Problem 3: (25 points) Piston in an adiabatic chamber

A cylinder of cross-sectional area A is divided into two chambers by means of a frictionless piston. The walls *and the piston* are made of heat-insulating (adiabatic) material. Initially, each chamber has equal length L, and each is filled with the same number N of monatomic ideal gas particles. However, the initial pressure of the chamber on the left is $32P_0$ whereas the initial pressure of the chamber on the right is P_0 (as shown below). The piston is then released, and the gas in the left chamber pushes the piston to the right in a quasi-static manner (for example, a very heavy piston would move very slowly). The piston then travels a distance d to the point where no net force acts upon it. Find the distance d.



Problem 4: (25 points) Energy of a non-ideal gas

Experiments on a non-ideal gas led to the following observations:

1) $(P + \frac{aN^2}{V^2})(V - Nb) = NkT$

2) $C_V = \frac{3}{2}Nk$

Here, the number of particles N is fixed, and a, b, and k are constants. Derive an expression for the internal energy $U(T, V)$ of the gas.