8.044 Statistical Physics I
Spring 2008

8.044 Exam 2

Spring 2008

1. Do all 3 problems.
2. Write your solutions in the white exam books.
3. No calculators, books, or notes are permitted.
4. For full credit, show all of your work for each problem.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

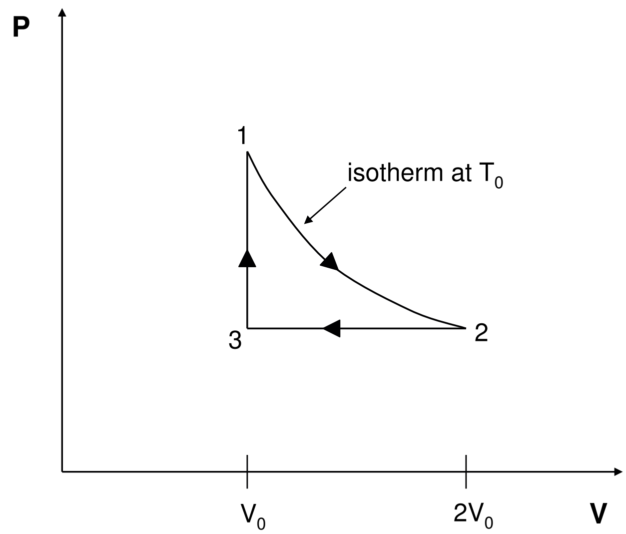
Physics Department

8.044: Statistical Physics I — Spring Term 2008

Exam #2

Problem 1: (30 points) Heat engine

Consider taking a monatomic ideal gas around the closed cycle depicted below. It consists of one isotherm at temperature T_0, one change at constant pressure (from $2V_0$ to V_0), and one change at constant volume. The heat capacity of the gas is $C_V = \frac{3}{2}Nk$.



a) Between which pair(s) of points is heat added to the system? (We'll denote the heat added as Q_H.) Between which pair(s) of points is heat removed from the system? (We'll denote the heat removed as Q_C.)

b) Calculate W_{out}, the work done *by* the gas after one cycle. Your answer should be expressed in terms of the given quantities.

c) Calculate Q_H and Q_C in terms of the given quantities.

d) Calculate the efficiency η of this engine. Compare this to the efficiency of a Carnot engine operating between the highest and lowest temperatures of the cycle. (I realize that most of you are not human calculators, so at least write down a plausible inequality for this comparison involving simplified arithmetic expressions.)

Problem 2: (35 points) Non-ideal gas

The equation of state for a certain *non-ideal* gas is given by:

$$P = \frac{NkT}{V - Nb} - \frac{aN^2}{V^2}.$$

Here, the number of particles N is fixed, and a, b, and k are constants.

a) Using the first and second laws, find the differential of the internal energy dE in terms of dT and dV. Here, we use the notation $E = U$ as the internal energy. You may express your answer in terms of the given quantities, plus V, T, and the heat capacity C_V.

b) Show that C_V is independent of the volume V, that is $\left(\frac{\partial C_V}{\partial V}\right)_T = 0$.

c) Assuming C_V is independent of temperature, find an expression for the entropy $S(T, V)$ of this gas.

Problem 3: (35 points) Nuclei in a solid

A certain solid contains N identical nuclei. Each nucleus can be in any one of three spin states labeled by the quantum number m, where $m = 1, 0, \text{or -1}$. Due to coupling with the electric field environment, the energy of a nucleus depends on its spin orientation in the following way: the nucleus has energy 0 in the $m = 0$ state, and the nucleus has energy ϵ in the $m = 1$ state and the $m = -1$ state.

a) Find an expression, as a function of temperature T, for the nuclear contribution to the average energy $\langle E \rangle$ of the solid. Assume that the N identical nuclei are located in distinct positions in the solid where the fields are identical.

b) Find an expression, as a function of T, of the contribution of these N nuclei to the entropy S of the solid.

c) By considering the total number of accessible states, calculate the nuclear contribution to the entropy of the solid in the $T \to 0$ limit. Compare this with your answer in part (b).

d) Calculate the nuclear contribution to the heat capacity C_V of the solid.

e) Make a plot of the heat capacity C_V as a function of the temperature T. Label the axes with approximate scales. Explain why the low and high temperature limits behave the way they do.

   Note: You can receive full credit for a correct answer here, independent of the correctness of your answer in part d).

PARTIAL DERIVATIVE RELATIONSHIPS

Let x, y, z be quantities satisfying a functional relation $f(x, y, z) = 0$. Let w be a function of any two of x, y, z. Then

$$\left(\frac{\partial x}{\partial y}\right)_w \left(\frac{\partial y}{\partial z}\right)_w = \left(\frac{\partial x}{\partial z}\right)_w$$

$$\left(\frac{\partial x}{\partial y}\right)_z = \frac{1}{\left(\frac{\partial y}{\partial x}\right)_z}$$

$$\left(\frac{\partial x}{\partial y}\right)_z \left(\frac{\partial y}{\partial z}\right)_x \left(\frac{\partial z}{\partial x}\right)_y = -1$$

INTEGRALS

$$\int_0^\infty \frac{x^n}{a}\, e^{-x/a}\, dx = n!\, a^n$$

$$\int_{-\infty}^{\infty} \frac{x^{2n}}{\sqrt{2\pi\sigma^2}}\, e^{-x^2/2\sigma^2} = 1 \cdot 3 \cdot 5 \cdots (2n-1)\sigma^{2n}$$

$$\int e^{ax}\, dx = \frac{1}{a}\, e^{ax}$$

STERLING'S APPROXIMATION

$$\ln K! \approx K \ln K - K \quad \text{when} \quad K >> 1$$

SOLID ANGLE INCREMENT

$$d\Omega = \sin\theta\, d\theta\, d\phi$$

WORK IN SIMPLE SYSTEMS

System	Intensive quantity	Extensive quantity	Work
Hydrostatic system	P	V	$-PdV$
Wire	$\mathcal{F}$	L	$\mathcal{F}dL$
Surface	$\mathcal{S}$	A	$\mathcal{S}dA$
Reversible cell	E	Z	EdZ
Dielectric material	$\mathcal{E}$	$\mathcal{P}$	$\mathcal{E}d\mathcal{P}$
Magnetic material	H	M	HdM

THERMODYNAMIC POTENTIALS

For a system in which the increment of work done on the system is dW = Xdx

Energy	E	dE=TdS+Xdx
Helmholtz free energy	F=E-TS	dF=-SdT+Xdx
Gibbs free energy	G=E-TS-Xx	dG = -SdT-xdX
Enthalpy	H=E-Xx	dH=TdS-xdX