8.044 Statistical Physics I
Spring 2008

8.044 Exam 1

Spring 2007

1. Do all 3 problems.
2. Write your solutions in the exam books.
3. No calculators, books, and notes are permitted.
4. For full credit, show all of your work for each problem.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

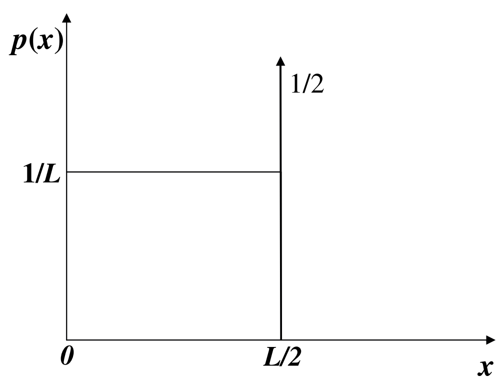
Physics Department

8.044: Statistical Physics I — Spring Term 2007

Exam #1

Problem 1: (30 points) Breaking chalk

Sticks of chalk come in a standard length L. A teacher wants to use a small piece of length $L/2$. He marks a standard stick at that length and breaks it into two pieces. The chalk breaks at the mark with probability 1/2, and breaks at a random location with probability 1/2. The probability density for the length x of the smaller piece is shown below.



An expression for the probability density for the length of the smaller piece may be written as follows:

$$
\begin{aligned}
p(x) &= 1/L + \frac{1}{2}\delta(x - L/2) \qquad \text{for } 0 < x \leq L/2 \\
&= 0 \qquad\qquad \text{otherwise}
\end{aligned}
$$

a) What is the probability that x will be within $\pm L/10$ of the desired length?

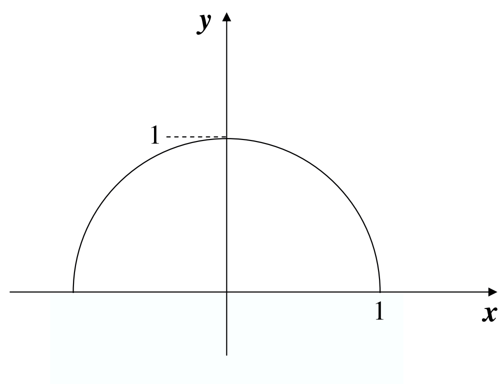
b) Calculate $< x >$, the mean of x.

c) Calculate Var(x)=$< (x - < x >)^2 >$, the variance of x.

d) The teacher breaks 30 sticks of chalk and lays the smallest piece from each endeavor end to end. What is the approximate probability density for the total length D of the line? Indicate the mean and variance for this probability density.

Problem 2: (35 points) Uniform half-circular disk

A particle resides on the surface of a half-circular disk of unit radius as shown below.



The particle has a uniform probability density for being located at any position on the surface. The joint probability density in the variables x and y is given by:

$$\begin{aligned} p(x,y) &= 2/\pi \quad \text{for } x^2+y^2 \leq 1 \text{ and } y > 0 \\ &= 0 \quad \text{otherwise} \end{aligned}$$

a) Calculate $p(x)$, the probability density of the single random variable x. Make a plot of $p(x)$ with carefully labeled axes.

b) Calculate $p(y)$ and make a plot of $p(y)$ with carefully labeled axes.

c) Calculate $p(x|y)$, the conditional probability density of the random variable x for a given value of y. Make a plot of $p(x|y)$ with carefully labeled axes.

d) Calculate $p(y|x)$, the conditional probability density of the random variable y for a given value of x. Make a plot of $p(y|x)$ with carefully labeled axes.

e) Let $r = \sqrt{x^2+y^2}$. Calculate $p(r)$ and make a plot of $p(r)$ with carefully labeled axes.

Problem 3: (35 points) Thermodynamics of a rod

Experiments on an elastic rod are used to determine the following functional forms for derivatives.

$$\left(\frac{\partial L}{\partial f}\right)_T = \frac{1}{3a(L - L_0)^2\, T^2}$$

$$\left(\frac{\partial f}{\partial T}\right)_L = 2a(L - L_0)^3\, T$$

Here, f is the tension of the rod, L is the length of the rod, and a, b, and L_0 are constants.

a) Derive a general equation of state relating f, L, and T. Next, suppose further observations indicate that the tension goes to zero as $L \to L_0$. What is the equation of state in this case?

b) The internal energy of the rod is a known function $U(L, T)$. Write an expression for the heat capacity C_L for the rod *held at constant length* in terms of derivatives of U and other known quantities.

c) Determine an expression for the heat capacity C_f for the rod *held under constant tension* in terms of the derivatives of U and other known quantities.