8.044 Statistical Physics I
Spring 2008

8.044 Exam 2

Spring 2007

1. Do all 3 problems.
2. Write your solutions in the white exam books.
3. No calculators, books, or notes are permitted.
4. For full credit, show all of your work for each problem.

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

Physics Department

8.044: Statistical Physics I Spring Term 2007

Exam #2

Problem 1: (25 points) Carnot heat engine

A reversible (Carnot) heat engine operates between two reservoirs with initial temperatures T_1 and T_2 (where $T_2 > T_1$). The colder reservoir at T_1 can be considered to have infinite mass (that is, T_1 remains constant). However, the hotter reservoir at T_2 consists of a finite amount of ideal gas at constant volume (for which the heat capacity C_V is a given constant).

After the heat engine is operated for some period of time, the temperature of the hotter reservoir is lowered from T_2 to T_1.

a) What is the change in the entropy ΔS of the hotter reservoir during this period?

b) How much work did the engine do during this period?

Problem 2: (40 points) Thermodynamics of a system

In equilibrium, a macroscopic system is found to have the following equation of state related the entropy S, internal energy E, volume V, and number of particles N:

$$S = a[NVE]^{1/3}.$$

Here, N and a may be treated as given constants.

a) Find an expression for $E(S, V)$.

b) Find an expression for $E(T, V)$.

c) Find an expression for the heat capacity $C_V(T)$.

d) Suppose two bodies are composed of material obeying the above equation of state. N and V are the same for both, however one body is initially at temperature T_1 and the other is initially at temperature T_2. They are brought in contact with each other, such that they can exchange energy but perform no work (the volumes remain constant). The combined system is isolated from the rest of the universe. What is the final temperature T_f of the combined system when equilibrium is re-established?

e) For the combined system, is the final internal energy E_f^{total} greater than, less than, or equal to the initial internal energy E_i^{total}? Justify your conclusion in one or two lines. (You need *not* calculate the result.)

f) For the combined system, is the final entropy S_f^{total} greater than, less than, or equal to the initial entropy S_i^{total}? Justify your conclusion in one or two lines. (You need *not* calculate the result.)

Problem 3: (35 points) Properties of a gas

Consider a gas composed of N non-interacting, identical molecules contained within a volume V at temperature T. Suppose each molecule has 4 possible internal states: one ground state and 3 excited states which have the same energy (that is, the excited energy level is 3-fold degenerate). The molecule's internal energy is $\epsilon_{int} = 0$ if the molecule is in the ground state or $\epsilon_{int} = \Delta$ if the molecule is in one of the 3 excited states. Thus, the energy of a single molecule may be written as:

$$\epsilon = \frac{|\vec{p}|^2}{2m} + \epsilon_{int}$$

a) Find the partition function Z_1 for a single gas molecule. In your expression, evaluate any integrals or summations that appear.

b) Find the average energy of a single gas molecule $< \epsilon >$. Note, this may be calculated using Z_1 or using other arguments. If you use other arguments, be sure to explain your steps.

c) Now consider the collection of N molecules. Find an expression for the heat capacity C_V of the gas.

d) Make a plot of the heat capacity C_V as a function of the temperature T. Label the axes with approximate scales. Here, you may assume that V is large enough so that the molecules always move as a free particles (that is, the effects of "particle in a box" quantization can be neglected).

   Note: Think carefully about the limiting behaviors that make sense, and what happens in between. You can receive full credit for a *correct* plot here, independent of the correctness of your answer in part c).