TRANSFORMATION OF WAVE CHARACTERISTICS IN THE REFLECTION-TRANSMISSION PROCESS THROUGH OPEN CHANNELS OF VARYING GEOMETRY

by

Efstathios Lampros Bourodimos

Civil Engineering Degree
The Athens National Technical University of Greece
(1953)

Science Master Degree
Massachusetts Institute of Technology
(1963)

Submitted in Partial Fulfillment of the
Requirements of the Degree of
Doctor of Philosophy
at the
Massachusetts Institute of Technology
(1966)

Signature of the Author____________________________
Department of Civil Engineering
May 12, 1966

Certified by____________________________
Thesis Supervisor

Approved by____________________________
Chairman, Departmental Committee on Graduate Students

ACKNOWLEDGEMENT

The investigation reported was carried out as a research project in the Hydrodynamics Laboratory of the Department of the Civil Engineering at the Massachusetts Institute of Technology.

The research study was sponsored by the Office of Naval Research of the Department of the Navy, United States Government, under Contract Nonr-1841(59) and administered under Project No. DSR 8228 by the Division of Sponsored Research of the Massachusetts Institute of Technology.

The author is grateful to many persons who have in one way or another offered help in the course of this investigation.

Of these the following deserve special mention. Dr. Arthur T. Ippen, Professor of Civil Engineering and Director of the Hydrodynamics Laboratory, who initiated the project and acted as supervisor, for his aid, encouragement, fruitful ideas and time spent for discussion and criticism of all parts of the thesis. Dr. D. R. F. Harleman for his supervision in this study for one year during which Dr. Ippen was absent on sabbatical leave.

Dr. Louis Howard, Professor of the M.I.T. Department of Mathematics with whom the author had many inspiring discussions concerning the theoretical phase of this investigation.

Mr. Joel Brainard who joined the project for four months as research assistant and provided valuable assistance.

The author is deeply grateful for the moral support given by his wife and children during the past three years of this investigation.

ABSTRACT

TRANSFORMATION OF WAVE CHARACTERISTICS IN THE REFLECTION-TRANSMISSION PROCESS THROUGH OPEN CHANNELS OF VARYING GEOMETRY

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The topics of this thesis are a theoretical development and an experimental investigation of the transformation of water-wave characteristics in the reflection and transmission processes through channel transitions of varying geometry, connecting two prismatic channels of constant cross section.

The theoretical developments are based on small amplitude linearized wave theory in an inviscid, homogeneous and incompressible fluid. Two theoretical aspects have been treated:

1. The wave amplitude variation in a channel of constant width for a bottom of arbitrary configuration was obtained for the various characteristics of the oncoming waves. The basis of this development is the energy transmission undiminished by reflection or friction. The general expression of the integral type was solved for two limiting cases: for shallow water waves resulting in Green's law and for the range from deep water to intermediate depth water waves resulting in an exponential formula.

2. Reflection and transmission coefficients were derived for shallow water waves for gradual channel transitions, specifically for four cases:

A - for linearly varying depth and constant width
B - for linearly varying depth and width
C - for linearly varying width and constant epth
D - for parabolic variation of depth and constant width

The experimental part of the thesis is concerned with the determinations of reflection and transmission coefficients and of the energy relations including dissipation for the above cases A, B and C. The experimental range of wave conditions extended from deep water to shallow water waves. The results are compared to previous investigations and to the conventional classical theories. Relations were also found with regard to wave steepness, a factor which cannot be theoretically dealt with so far in channel transitions.

Reflection and transmission coefficients show considerable dependence on wave steepness, the decrease being most pronounced for the former. Reflection coefficients are generally higher than those predicted by Lamb's theory for abrupt transitions, the incrtease being higher for decreasing bottom slopes as verified also by comparison to previous investigations. Transmission coefficients therefore are exhibiting the opposite trend.

Thesis Supervisor: Arthur T. Ippen
Title: Professor of Civil Engineering

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LIST OF NOTATIONS - LOWER CASE

a = amplitude of wave measured from mean surface elevation, ft or cm.

b = amplitude of wave measured from mean surface elevation, ft or cm.

c_I, c_I^*... functions defined at section 3.1.

g = gravitational acceleration = 32.2 ft/sec.

h = total undisturbed water depth, ft.

k = wave number = $2\pi/L$, ft^{-1}.

ℓ = length of the transition, ft.

ℓ_1 = length as defined in Chapter III, ft.

n = dimensionless parameter for wave group velocity

p = pressure intensity, lb/ft^2.

q = rate of flow or flux per unit width, ft^2/sec.

t = time, sec.

u = velocity in x-direction (varying with time) ft/sec.

v = velocity in y-direction (varying with time) ft/sec.

w = velocity in z-direction (varying with time) ft/sec.

x = horizontal direction, in wave propagation, ft.

y = horizontal direction, perpendicular to x-lateral direction, ft.

z = vertical direction, with origin in surface, ft.

LIST OF NOTATIONS - UPPER CASE

A = total cross-sectional area, ft^2.

B = surface width of the channel, ft.

$C_1,...C_6$ = complex constants in the solution of differential equations as defined.

$C = L/T$ = velocity of wave propagation (phase velocity) ft/sec.

$D_1,...D_4$ = terms involving Bessel functions as defined in section 3.3.

C_G = wave group velocity, ft/sec.

E = energy ft. lbs. per square ft.

$F(a,\beta,\gamma,x)$ = hypergeometric functions as solution of Legendre equation.

$F_1, F_2, F_1^*...$ = hypergeometric functions as defined in section 3.5.

J_o, J_1 = Bessel functions of first kind of zero and first order.

$H = 2a$ = wave height - distance from crest to trough, ft.

K_r = reflection coefficient - dimensionless.

K_t = transmission coefficient - dimensionless.

$K_b = L^2a/h^3$ = breaking parameter - dimensionless.

H/L = wave steepness - dimensionless.

S = slope of channel bottom - dimensionless.

$S_p = \sigma^2\ell/g$ = shoaling parameter - dimensionless.

T = wave period, sec.

Y_o, Y_1 = Bessel functions of second kind of zero and first order.

NOTATIONS - SUBSCRIPTS - SUPERSCRIPTS

$-_i$ = incoming wave or energy.

$-_{I,II,III}$ = indicating wave amplitudes in regions I, II, III, as defined in theoretical study (Chapter III).

$-^*$ = indicating dimensionless wave amplitude.

$-_o$ = indicating deep water conditions.

$-_r$ = reflected wave.

$-_t$ = transmitted wave.

$-'$ = indicating derivative and also correction of amplitude for zero end-channel reflection.

$-_{rB}$ = wave energy reflected from the end of the channel.

$-_{rT}$ = wave energy reflected upstream.

$-_T$ = wave energy transmitted downstream.

$-_{1,3}$ = indicating wave conditions in the upstream or downstream region of the channel.

NOTATIONS - GREEK LETTERS - LOWER CASE

$\alpha_1 = \ell \left(\frac{h_1}{h_1 - h_3}\right)^{1/2}$ = dimensional parameter, ft. or cm.

β = wave phase angle, radians.

δ = wave phase angle, radians.

γ = specific weight of water, 62.4 lb/ft^3.

$\varepsilon = \left(1 - \frac{\ell}{\ell_1}\right)^{1/2}$ = dimensionless quantity.

ξ = displacement from mean position in x-direction, ft.

η = vertical displacement of water surface from mean surface elevation, ft.

π = 3.1416.

ρ = density - mass per unit volume = γ/g, slugs/ft^3.

$\sigma = 2\pi/T$ = wave angular frequency, sec^{-1}.

μ = scale factor in strained coordinate, $X = \mu x$.

$\lambda = k_1 \ell_1$ = dimensionless quantity as defined.

NOTATIONS - GREEK LETTERS - UPPER CASE

$\left.\begin{matrix} A_1, A_2, A_3, A_4 \\ B_1, B_2, B_3, B_4 \end{matrix}\right\}$ = terms involving Bessel functions as defined in section 3.2.

$\Gamma_1, \Gamma_2, \Gamma_3, \Gamma_4, \Gamma_5, \Gamma_6$ = terms involving Bessel functions as defined in section 3.3.

$\Delta_1, \Delta_2, \Delta_3, \Delta_4$ = terms involving hypergeometric functions as defined in section 3.5.

$\left.\begin{matrix} \Lambda_1, \Lambda_2, \Lambda_3, \Lambda_4, \Lambda_5 \\ M_1, M_2, M_3, M_4 \end{matrix}\right\}$ = terms involving Bessel functions as defined in section 3.4.

$\theta(X) = \theta(\mu x)$ = function representing influence of bottom change.

θ_X = derivative of $\theta(X)$ ~ wave number, $k = 2\pi/L$.

$\phi, \hat{\phi}, \phi^{(1)} \ldots$ = velocity potential, ft^2/sec.

$S_P = \sigma^2 \ell/g$ = shoaling parameter, dimensionless.

$X = \mu x$ = strained coordinate in x-direction, ft.

Dedicated

to the memory of my father

LAMPROS DEMETRIOS BOURODIMOS
(1885-1956)

A teacher in the quest of truth and
kindness, a guide to integrity under whose
gentle tutelage and love
I first knew freedom.

INSTEAD OF A PREFACE

"Reason, holding in one hand its principles according to which alone concordant appearances can be admitted as equivalent to laws and in the other hand the experiment which it has derived in conformity with these principles must approach nature in order to be taught by it. It must not, however, do so in the character of a pupil who listens to everything that the teacher chooses to say, but of an appointed judge who compels the witnesses to answer questions which he has himself formulated.

Even physics therefore, owes the beneficient revolution in its points in view entirely to the happy thought, that while reason must seek in nature, not fictionly ascribe to it, whatever as not being knowable through reasons's own resources has to be learned, if learned at all, only from nature, it must adopt as its guide, in no seeking, that which it has itself put into nature. It is thus that the study of nature has entered on the secure path of a science after having for so many centuries been nothing but a process of merely random groping"

Immanual Kant, "Critique of Pure Reason"

I. INTRODUCTION

1.1 The Significance of the Problem

The problems of the transformation of wave characteristics by channels of varying geometry are of great practical significance in engineering applications.

Waves encounter rapidly or slowly varying depth during the shoaling process on beaches, in entrances to tidal embayments, in estuaries. In addition to depth changes variations occur in the width of channels with expensions and contractions. In all cases engineers like to obtain information on the wave reflection and transmission processes and the propagation of wave energy for effective planning. Theoretical methods for prediction of the changing wave characteristics in transitions have remained inadequate in spite of this engineering interest. Some experimental evidence exists to suggest that the classical solution for abrupt transitions is not sufficient for the description of the transformation and the partial reflection phenomenon. This report will present an analytical solution extended beyond presently available theory and extensive experimental data on reflection and transmission coefficients for various transitions of linearly varying channel sections.

1.2 The Purpose of the Present Theoretical and Experimental Study

More specifically stated, the theoretical and experimental study reported in the following is concerned with:

1. The analytical wave amplitude variation over a bottom of arbitrary geometry in a channel of constant width.

2. The analytical wave amplitude variation due to reflection and transmission for various cases of channels of linearly varying depth and width. The solutions are restricted to shallow water waves.

3. The experimental amplitude variations for waves of the entire spectrum from deep water to shallow water in channels of linearly varying depth and width.

The purpose of the first phase of the theoretical approach was to find a general expression for the amplitude variation as a function of arbitrary changes in the bottom geometry on the basis of constant energy transport. No reflection is introduced. The amplitude change between two stations of different depth must, of course, result in the same value as obtained from the usual procedure involving constant energy transmission. However, the approach presented results in a general integral expression which may be solved for explicit functions describing the variation of the bottom in the direction of wave transmission. In the limiting case of shallow water waves, the expression reduces to the well-known Green's theorem. At the other extreme, for the transition from deep water to intermediate depths, the amplitude increases exponentially.

The second phase of the theoretical developments is the major one and gives specific solutions for the amplitude changes of shallow water waves over transitions of various geometries with full consideration of reflection from the transition. Again, energy dissipation is neglected. The following cases have been solved analytically determining the amplitudes and phase angles not only upstream and downstream, but also over the extent of the transition itself:

A. The case of a transition of linearly varying depth of arbitrary slope of constant width

B. The case of linearly varying depth and width of arbitrary slopes.

C. The case of constant depth with linearly varying width.

D. The case of constant width with depth varying parabolically.

All solutions are derived on the basis of linearized, small amplitude wave theory applied to shallow water waves. The third phase covers a very extensive program of experimental determinations of reflection and transmission coefficients for a wide range of wave conditions and several cases of linearly varying depth and/or width (see cases A,B,C above). The experimental range of waves was not confined to shallow water waves alone but was broadened to include initial deep water conditions, with predominant emphasis given to intermediate waves between deep and shallow water characteristics. Of necessity the experimental results cannot be compared therefore to the theoretical findings of the second phase. However, the latter results provide convenient limits for comparison as shallow water conditions are approached, while the limits of the other extreme - i.e. deep water conditions - are obviously trivial. For practical applications it was desirable that the wave range covered by experiments be expanded beyond the possibilities of theoretical analysis, which is not susceptible to approaches for intermediate waves.

II. THE STATE OF KNOWLEDGE - WAVE REFLECTION AND TRANSMISSION

The following discussion is concerned with a review of the basic theoretical results obtained up to the present on wave reflection and transmission. It will become apparent that all attempts in this very difficult problem had to be confined by necessity to very limited phases of the problem circumscribed by small-amplitude wave theory. For the purpose of this review therefore the essential features of this theory may be stated here again.

2.1 Summary of Linear, Small Amplitude Theory

The basis of wave theory as derived essentially by Aivy (1) and Stokes (2) is given by the continuity equation and the dynamic equation for the motion of a non-viscous, incompressible, homogeneous fluid. The condition of irrotationality permits the introduction of the velocity potential ϕ. Thus for the two-dimensional problem:

$$u_x + w_z = \phi_{xx} + \phi_{zz} = \nabla^2\phi = 0 \quad \text{from continuity} \tag{2.1}$$

$$-\phi_t + \frac{1}{2}(u^2+w^2) + \frac{p}{\rho} + gz = 0 \quad \text{from dynamic conditions} \tag{2.2}$$

The restriction to small amplitude variations permits the reduction of the dynamic equation to

$$-\phi_t + \frac{p}{\rho} + gz = 0 \tag{2.3}$$

The solution is accomplished by satisfying the essential boundary conditions (figure 1).

$$w = -\phi_z = 0 \quad \text{for } z = -b \tag{2.4}$$

and

$$\eta = \frac{1}{g}(\phi_t)_{z=\eta} \quad \text{for the surface} \tag{2.5}$$

Assuming further in line with the small amplitude condition that (2.5) is approximately satisfied by

$$\eta = \frac{1}{g}(\phi_t)_{z=0} \tag{2.6}$$

Further, small amplitude variations, permit the introduction of the kinematic condition

$$\phi_z \simeq \eta_t \quad \text{for } z=0 \tag{2.7}$$

The solution of equations (2.1) and (2.3) for these constraints results in the well-known harmonic description of surface variations η as a function of space and time

$$\eta = a \sin(kx-\sigma t) \tag{2.8}$$

representing a progressive wave travelling in the positive x direction. Velocity and pressure variations throughout the depth may also be established from the solution for the velocity potential for this case.

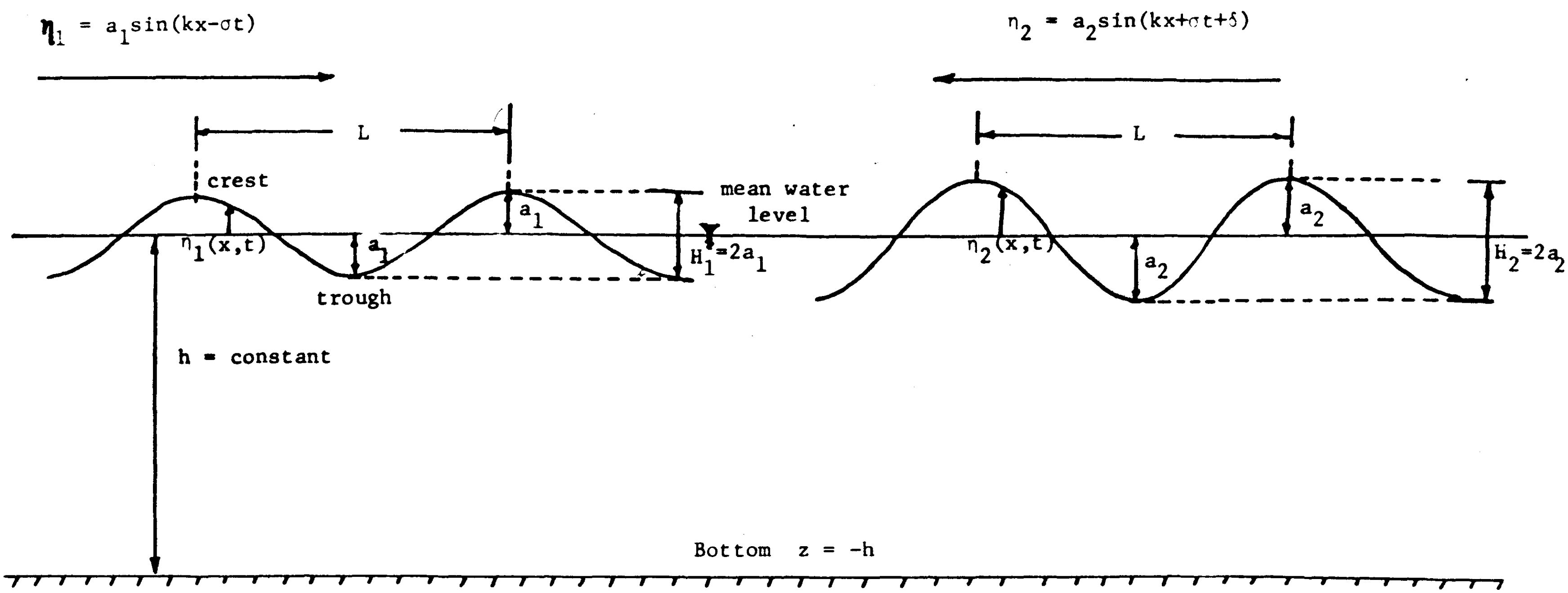


Fig. 1 Small Amplitude Wave System of Two Waves
Travelling in Opposite Directions
Definition Sketch.

$$\phi = \frac{ag}{\sigma} \frac{\cosh k(h+z)}{\cosh kh} \cos(kx-\sigma t) \tag{2.9}$$

For the linear problem dealt with here superposition of such waves is permissible; hence for the problem of reflection, waves travelling in the opposite direction may be superimposed, considering however appropriate phase shifts. Thus for partial reflection the amplitude variation may be given by (figure 1)

$$\eta = a_1 \sin(kx-\sigma t) + a_2 \sin(kx+\sigma t+\delta) \tag{2.10}$$

The phenomenon under consideration in this report is specifically addressed to the complex problem of solving theoretically and experimentally for the characteristics of the reflected wave in relation to certain geometries of the channel transition. Basically this requires the prediction of the reflected wave amplitude a_2 and of the phase shift δ with respect to the incoming wave a_1. The amplitude and phase angle of the portion of the wave continuing in the same direction, the transmitted wave, must also be determined. For these purposes use is made of the conservation of wave energy. The energies of the wave components involved in the process are given by

$$E = \frac{\gamma a^2}{2} \quad \text{average energy per unit of surface area} \tag{2.11}$$

This wave energy is transmitted with the group velocity

$$C_G = C\,\frac{1}{2}[1 + \frac{2\,kh}{\sinh\,2kh}] \tag{2.12}$$

wherein

$$C = \left(\frac{g}{k} \tanh kh\right)^{1/2} \qquad (2.13)$$

Equations (2.11) to (2.13) have also been obtained from the small amplitude solutions of the basic equations cited above, and for the case of constant depth in the field of wave motion.

2.2 The Problem of Channel Transitions and Wave Reflection

(i) Gradual Transitions

A progressive gravity wave entering a region of gradually varying geometry suffers important changes in its basic characteristics, the amplitude and phase angle, depending on the shape of the transition. As a result of the change of the channel geometry there is a partial reflection and transmission of the wave. Both, the transmitted and the reflected wave, have different amplitudes and phase angles with respect to the incoming wave (figure 2).

For very gradual transitions the reflection is very weak and the entire energy is approximately transmitted assuming no loss by bottom friction. This case is represented by Green's Law for long (shallow) waves in very gradual transitions under the assumption of zero reflection and loss. The incoming energy is equal to the transmitted energy and from the balance of the energy flux we obtain;

$$(EBC_G)_{x_1} = (EBC_G)_{x_3} \tag{2.14}$$

and since

$$C = C_G = \sqrt{gh} \quad \text{and} \quad E = \gamma \frac{a^2}{2}$$

the amplitude variation is given by;

$$\left(\frac{a_1}{a_3}\right) = \left(\frac{B_3}{B_1}\right)^{1/2} \left(\frac{h_3}{h_1}\right)^{1/4} \tag{2.15}$$

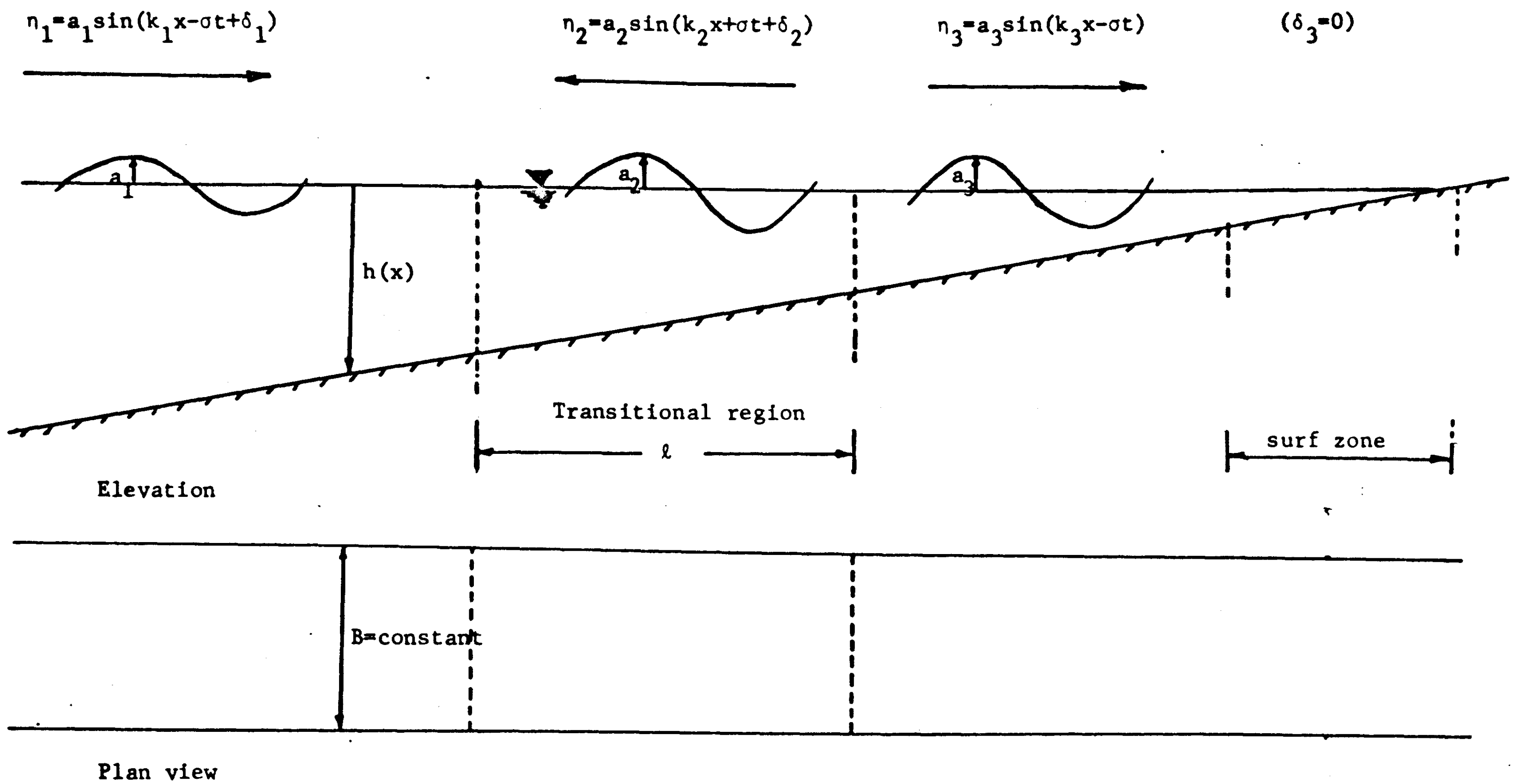


Fig. 2. Wave Partial Reflection and Transmission Process in a Gradual Transition (Gradually Varying Depth - Constant Width).

With steeper bottom slopes reflection must be considered and Green's Law is no longer applicable. The energy transport relation does not furnish any information on phase angles. Also frictional effects may become significant for transitions of considerable length (3). Hence the problem of wave transformation in such transitions becomes quite complex.

(ii) Abrupt Transitions

At the opposite end of the spectrum of transitions which can be approximated by Green's Law are the cases of abrupt transitions. Here reflections must be evaluated. The velocity potential ϕ should be defined subject to the appropriate boundary conditions over the abrupt transition. Such a general potential has not been determined as yet.

However a procedure has been adopted assuming the existence of the following wave system:

$$\text{incoming wave:} \quad \eta_1 = a_1 \sin(k_1 x - \sigma t + \delta_1) \tag{2.16}$$

$$\text{reflected wave:} \quad \eta_2 = a_2 \sin(k_1 x + \sigma t + \delta_2) \tag{2.17}$$

$$\text{transmitted wave:} \quad \eta_3 = a_3 \sin(k_2 x - \sigma t + \delta_3) \tag{2.18}$$

Phase angle δ_3 can be taken as reference angle equal to zero. The water flux at the abrupt transition is continuous at any moment and under the further assumption of continuity and uniformity of the free water surface (4) in the y-direction we obtain the following relations from continuity and energy concepts:

$$\text{From continuity:} \quad \eta_1 + \eta_2 = \eta_3 \quad \text{at the abrupt transition for all time} \tag{2.19}$$

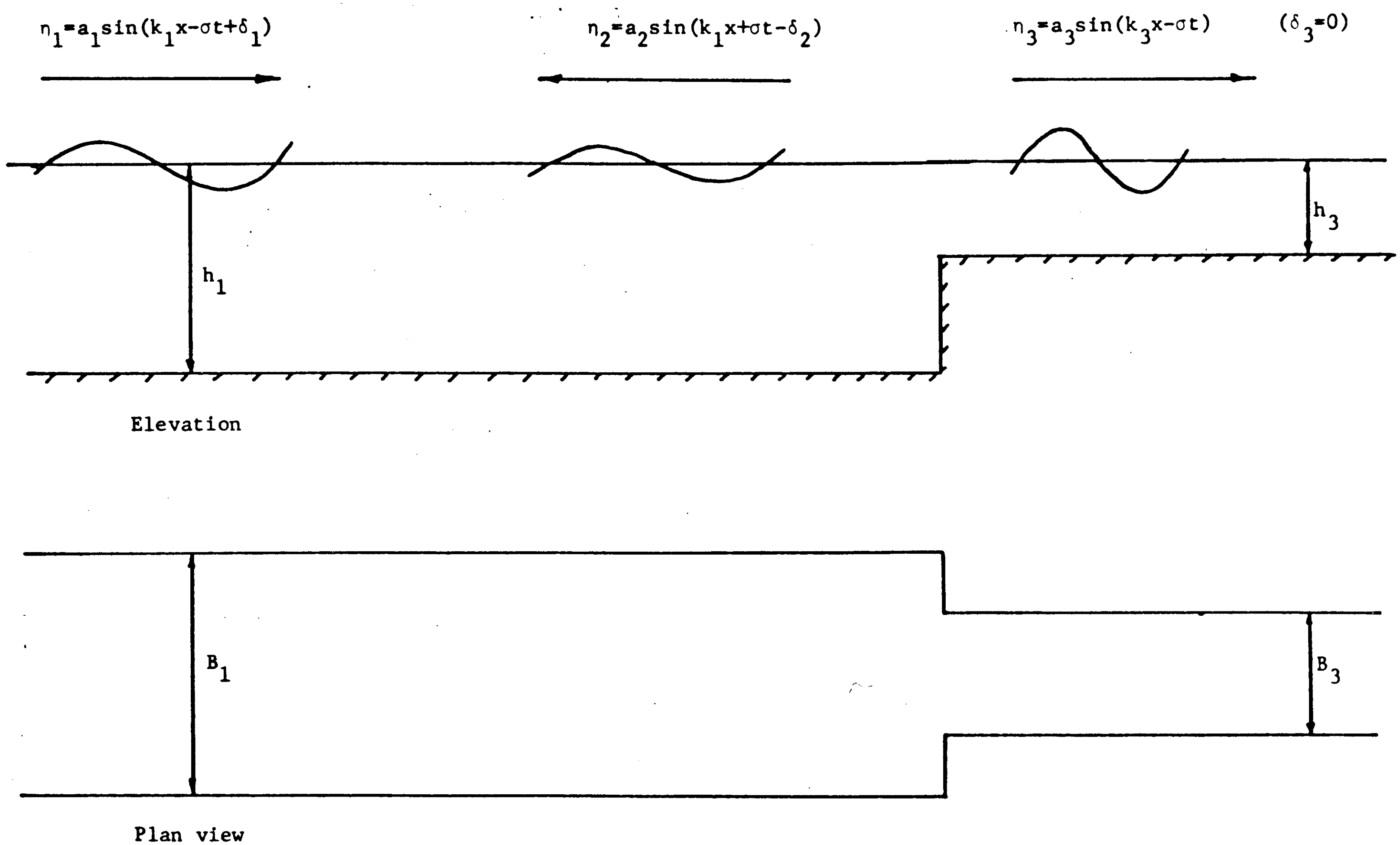


Fig. 3. Wave Partial Reflection and Transmission Process at Channel Discontinuity (Abrupt Transition)

From energy balance: $n_1 E_1 C_1 B_1 = n_2 E_2 C_2 B_1 + n_3 E_3 C_3 B_3$ (2.20)

where C_1, C_2, C_3 are wave celerities and E_1, E_2, E_3 refer to incoming, reflected and transmitted wave energies. Also,

$$n_1 = n_2 = \frac{1}{2}[1 + \frac{2k_1 h_1}{\sinh 2k_1 h_1}]$$

and

$$n_3 = \frac{1}{2}[1 + \frac{2k_3 h_3}{\sinh 2k_3 h_3}]$$

If we substitute equations (2.16) up to (2.18) into (2.19) we receive the relation (4):

$$a_2 \sin\delta_2 = a_3 \sin\delta_3 \qquad (2.21)$$

The phase angles δ_2 and δ_3 are not known.

A solution can be obtained under the assumption that $\delta_2 = \pi$, as in the case of complete reflection from a vertical wall, that implies $\delta_3 = 0$ i.e. the transmitted wave has the same phase angle as the incoming wave.(4)

Under this assumption ($\delta_2 = \pi$):

$$a_1 + a_2 = a_3$$

and the transmission and reflection coefficients can then be defined for deep water and intermediate depth waves as follows:

$$K_r = \frac{a_2}{a_1} = \frac{n_1 L_1 B_1 - n_3 L_3 B_3}{n_1 L_1 B_1 + n_3 L_3 B_3} \tag{2.22}$$

$$K_t = \frac{a_3}{a_1} = 1 + K_r = \frac{2 n_1 L_1 B_1}{n_1 L_1 B_1 + n_3 L_3 B_3} \tag{2.23}$$

In the case of waves which are deep in both channel sections $n_1 = n_2 = n_3 = \frac{1}{2}$ and $L_1 = L_3$ the previous coefficients become:

$$K_r = \frac{B_1 - B_3}{B_1 + B_3} \quad , \quad K_t = \frac{2B_1}{B_1 + B_3}$$

For shallow water waves on the other hand, $n_1 = n_2 = n_3 = 1$, and hence:(5)

$$K_r = \frac{a_2}{a_1} = \frac{C_1 B_1 - C_3 B_3}{C_1 B_1 + C_3 B_3}$$

$$K_t = \frac{a_3}{a_1} = 1 + K_r = \frac{2 C_1 B_1}{C_1 B_1 + C_3 B_3}$$

Experimental tests have been conducted at the M.I.T. Hydrodynamics Laboratory (6, 7) with deep water and intermediate depth waves over abrupt and gradual transitions, and the results have been compared with the various transmission and reflection coefficients defined above. The experimental evidence is in fair agreement with these theoretical definitions.

2.3 Theoretical Solutions for Linear Shallow Wave Theory for Gradual Transitions

The theoretical difficulties encountered in abrupt transitions for the determination of amplitude and phase angles are augmented in the case of gradual transition, when we want to consider reflection, for the following reasons:

(i) The mass flux varies continuously with position and time over the length of transition and only over a full wave cycle is the net storage equal to zero.

(ii) The characteristics (amplitude, wave length, phase angle) of both transmitted and reflected waves vary continuously over the transition with both position and time.

On the basis of small amplitude linear wave theory Takano(8) has solved the general case of transitions with abrupt ends submerged in a uniform rectangular channel. He gives the theoretical transmission and reflection coefficients.

P. Jolas (9a,b) following Takano's theoretical investigation has determined these coefficients experimentally. However in this experimental work no corrections were introduced for reflections from the end of the channel.

Dean and Ursell (10) solved the problem of wave reflection and transmission coefficients and of the force components on a semi-immersed circular cylinder the axis of which is perpendicular to the direction of propagation of the waves.

As in all other wave tank experiments their measured data were influenced by reflection of the transmitted wave from the end of the channel. However they established a mathematically rigorous method by which these data could be modified to the idealized case of an endless channel, in which the transmitted wave suffers no reflection. Their modified experimental data agreed fairly well with the theoretical predictions. They established also the important result that usually the channel-end reflections are not negligible. This was confirmed by the present experimental investigation.

Ursell, Dean and Yu (11) studied also the reflection phenomena on a smooth beach and compared their results to the findings of Miche (12) considering deep-water wave steepness.

Bocco and Gagnon (6) performed experiments for intermediate depth and deep water waves with transitions, i.e. sills with front slopes 1:0.58 (α=60^{o}) and 1:2.75 (α=20^{o}) a horizontal section of finite length and abrupt downstream ends. They analyzed the experimental data according to the method proposed by Dean-Ursell (10) and compared the reflection and transmission coefficients with Lamb's theory for abrupt transitions.

Ippen, Alam, Bourodimos (7) extended the experimental investigation of Bocco and Gagnon for the entire spectrum of wave conditions from deep to shallow depth waves with a transition of slope 1:16 (α=3.57^{o}) between two uniform rectangular regions upstream and downstream with emphasis on the effect of wave steepness on reflection and transmission.

Using linearized small amplitude shallow wave theory, Perroud (13) studied the problem of wave motion in a channel of linear or exponentially varying cross section. However, he simply introduced the usual assumption of a linearized resistance, constant throughout the transition length, and neglected reflection of any type along the transition. Therefore the amplitude of the progressive wave decreases exponentially as in the case of a channel of uniform section.

Kajiura (14) investigated on a rigorous mathematical formulation the same problem for a transition of non-linear variable depth for long waves of small amplitude. The approach is that of a boundary value problem after a linearization of boundary conditions. He found that the transmission and reflection coefficients can be predicted by the theoretical coefficients of Lamb (5) for an abrupt transition for small values of the ratio of the length of transition, ℓ, to the incident wave length, L_1. He confirmed again that Green's Law is valid for weak reflections.

Evangelisti (15) on the basis of small amplitude linear shallow wave theory defined the wave modification in a channel of monotonic variation of width and breadth. He gave a solution in terms of Hankel functions.

The most important contribution in this field in recent years is due to Dean (16). He determined theoretically the wave reflection and transmission coefficients on the basis of linearized shallow wave theory for three linear transitions of rectangular section, each of which is joined to uniform channel segments upstream and downstream. The three cases are: (a) linearly varying depth - constant width, (b) gradually varying depth and width, and (c) gradually varying width - constant depth. His solution is restricted to the reflection and transmission coefficients without consideration of the amplitude variation over the transition and the phase angles in the three regions.

Stoker (17) presents a mathematically rigorous treatment of different cases of wave motion over sloping beaches. In Appendix A a general review of his pertinent contribution is given. In addition other studies are reviewed there, which have less direct connection with the present investigation. This includes a study by Beitinjani and Brater (18) who investigated the refraction of waves in a trapezoidal channel on the basis of Stokian wave theory.

III. THE GENERAL PROBLEM OF WAVE MOTION THROUGH TRANSITIONS OF VARYING GEOMETRY

3.1 THE CASE OF WAVE MOTION OVER A BOTTOM OF CHANGING GEOMETRY - A DEVELOPMENT TO THE FIRST ORDER OF APPROXIMATION

We assume

$$\nabla^2\phi = 0 \tag{3.1}$$

and

$$\phi \sim \overset{*}{\phi} e^{-i\sigma t} \tag{3.2}$$

for irrotational motion of a homogeneous incompressible, non-viscous fluid. The boundary value problem, after linearization of the non-linear B.C. for the linearized wave motion, is:

$$g\eta + \phi_t = 0 \tag{3.3a}$$

$$\eta_t - \phi_z = 0 \tag{3.3b}$$

for $z = 0$ (instead of $z = \eta$).

From the geometry (fig. 4) we have:

$$\frac{dh}{dx} = + \frac{w}{u} = + \frac{\phi_z}{\phi_x} \tag{3.4}$$

for $z = h(x)$.

From (3.3a, b) B.C. after elimination of η_t, we have:

$$\phi_{tt} + g\,\phi_z = 0 \tag{3.5}$$

on $z = 0$.

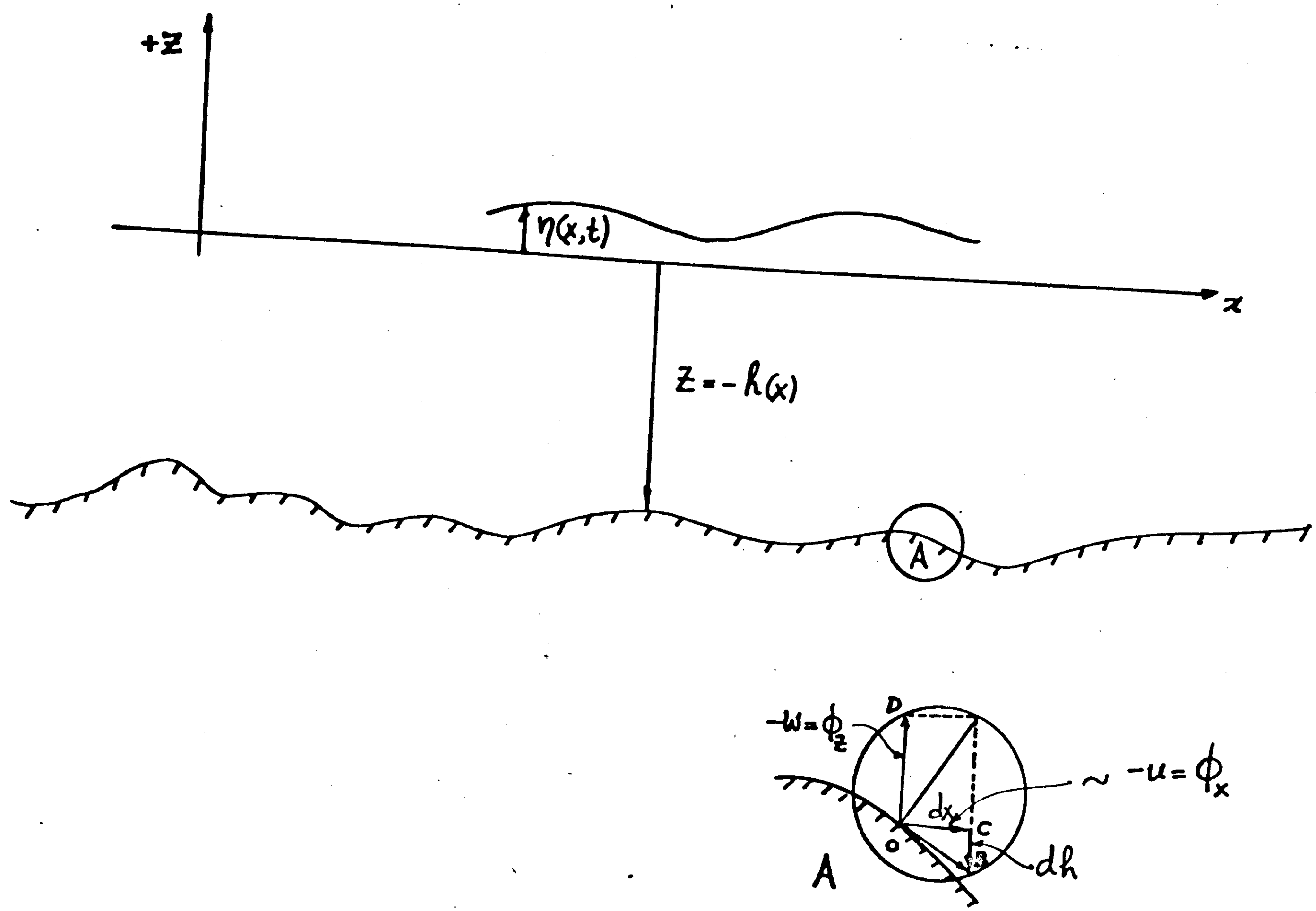


Fig. 4. Wave Motion Over an Uneven Bottom
Definition Sketch

Since $\phi \sim e^{-i\sigma t}$, the (3.5) relation above becomes:

$$\frac{\sigma^2}{g}\phi = \phi_z \qquad (3.6)$$

on z = 0.

The wave problem is now the following:

$$\nabla^2\phi = 0 \qquad (3.1)$$

$$\phi_z + \phi_x \cdot h'_x = 0 \quad (\text{on } z = -h) \qquad (3.4)$$

$$\frac{\sigma^2}{g}\phi - \phi_z = 0 \quad (\text{on } z = 0) \qquad (3.6)$$

A change in the horizontal scale is next introduced, using the method of "strained coordinates" (M. Van Dyke: Perturbation Methods in Fluid Dynamics (25)): $x \rightarrow X$ $\quad x = 1/\mu X$ $\quad \mu \ll 1$

The above equations, (3.1), (3.4), and (3.6), thus assume the form:

$$\phi_{zz} + \mu^2\phi_{XX} = 0 \qquad (3.7)$$

$$\frac{\sigma^2}{g}\phi - \phi_z = 0 \quad (\text{on } z = 0) \qquad (3.8)$$

$$\phi_z + \mu^2\phi_X h'_X(X) = 0 \quad (\text{on } z = -h(X)) \qquad (3.9)$$

since

$$\phi_x = \phi_X X_x = \mu \phi_X$$

$$h'_x(X) = + h_X X_x = \mu h'_X$$

With $\phi = \hat{\phi} e^{\frac{i}{\mu}\theta(X)}$, where $\hat{\phi}(X,z)$, we get:

$$\phi_{zz} = \hat{\phi}_{zz} e^{\frac{i}{\mu}\theta(X)} \tag{3.10}$$

$$\phi_X = (\hat{\phi}_X + \frac{i}{\mu}\theta_X(X)\hat{\phi}) e^{\frac{i}{\mu}\theta(X)} \tag{3.11}$$

$$\phi_{XX} = (\hat{\phi}_{XX} + \frac{i}{\mu}(\theta_X\hat{\phi})_X + \frac{i}{\mu}\theta_X\hat{\phi}_X - \frac{\theta_X^2}{\mu}\hat{\phi}) e^{\frac{i}{\mu}\theta(X)} \tag{3.12}$$

Substituting (3.10), (3.11), (3.12) into (3.7), (3.8), (3.9), we obtain:

$$\hat{\phi}_{zz} - \theta_X^2\hat{\phi} + i\mu[2\theta_X\hat{\phi}_X + \theta_{XX}\hat{\phi}] + \mu^2\hat{\phi}_{XX} = 0 \tag{3.13}$$

$$\frac{\sigma^2}{g}\hat{\phi} = \hat{\phi}_z \quad \text{(on } z = 0) \tag{3.14}$$

$$\hat{\phi}_z + i\mu\theta_X\hat{\phi}h'_X(X) + \mu^2\hat{\phi}_X h'_X(X) \quad \text{(on } z = -h) \tag{3.15}$$

Assuming that ϕ has a development in power series in a perturbation scheme of the form:

$$\hat{\phi} = \hat{\phi}^{(0)} + \mu\hat{\phi}^{(1)} + \mu^2\hat{\phi}^{(2)} + \mu^3\hat{\phi}^{(3)} + \ldots \tag{3.16}$$

we take the terms of zeroth order after the insertion of (3.16) into (3.13), (3.14), (3.15). The terms of zeroth order yield the equations:

$$\hat{\phi}^{(0)}_{zz} - \hat{\phi}^{(0)}\theta_X^2 = 0 \tag{3.17}$$

$$\frac{\sigma^2}{g}\hat{\phi}^{(0)} - \hat{\phi}^{(0)}_z = 0 \quad (\text{on } z = 0) \tag{3.18}$$

$$\hat{\phi}^{(0)}_z = 0 \quad (\text{on } z = -h) \tag{3.19}$$

The terms of first order yield the equations:

$$\hat{\phi}^{(1)}_{zz} - \theta_X^2\hat{\phi}^{(1)} = -\, i[2\theta_X\hat{\phi}^{(0)}_X + \theta_{XX}\hat{\phi}^{(0)}] \tag{3.20}$$

$$\frac{\sigma^2}{g}\hat{\phi}^{(1)} - \hat{\phi}^{(1)}_z = 0 \quad (\text{on } z = 0) \tag{3.21}$$

$$\hat{\phi}^{(1)} + i\theta_X h'_X(X)\hat{\phi}^{(0)} = 0 \quad (\text{on } z = -h) \tag{3.22}$$

Dropping "hats" we have the solution of zeroth order problem (3.17) with B.C. (3.18), (3.19).

$$\phi^{(0)} = a^{(0)}(X)\ \cosh\ \theta_X(z+h) \tag{3.23}$$

Note that θ_X^2 is considered as an "eigen value" since equation $\phi_{zz} - \theta_X^2\phi=0$ contains functions of independent variable z only.

Applying the boundary condition $\phi_z^{(0)} = 0$ on $z = -h$,

$$\phi_z^{(0)} = a^{(0)}(X)\ \sinh \theta_X(-h+z) = 0 \tag{3.24}$$

and for B.C. (3.18)

$$\frac{\sigma^2}{g}\ a^{(0)}(X)\ \cosh \theta_X(h+0) = a^{(0)}(X)\ \sinh \theta_X(h+0)$$

or

$$\frac{\sigma^2}{g}\ a^{(0)}(X)\ \cosh \theta_X h = \theta_X\ a^{(0)}(X)\ \sinh \theta_X h$$

and finally

$$\frac{\sigma^2}{g} = \theta_X\ \tanh\ (\theta_X h) \tag{3.25}$$

Substituting the solution (3.23) into the first order problem we get, for $\phi^{(1)}$ from (3.20):

$$\phi_{zz}^{(1)} - \theta_X^2\phi^{(1)} = -i\left[2\theta_X\left[a_X^{(0)}\ \cosh \theta_X(z+h) + a^{(0)}[(\theta_{XX}z + (\theta_X h)_X\ \sinh \theta_X(z+h)]\right] + \theta_{XX}\ a^{(0)}\ \cosh \theta_X(z+h)\right]$$

$$\phi_{zz}^{(1)} - \theta_X^2\phi^{(1)} = -i[2\theta_X a_X^{(0)} + \theta_{XX}a^{(0)}]\cosh \theta_X(z+h)$$

$$- 2ia^{(0)}\ \theta_X[\theta_{XX}(z+h) + (\theta_X h_X)]\sinh \theta_X(z+h) \tag{3.26}$$

Now the nonhomogeneous second part is mainly a function of z and the whole equation for $\phi^{(1)}$ can be represented as follows:

$$\phi'' - \theta_X^2\phi = c_I \cosh \theta_X X_I + (c_{II} X_I + c_{III}) \sinh \theta_X X_I \quad (3.27)$$

Integrating we get: (Particular Solution)

$$\phi = c_I^* X_I \sinh \theta_X X_I + c_{II}^* X_I^2 \cosh \theta_X X_I + c_{III}^* X_I \cosh \theta_X X_I \quad (3.28)$$

Thus, $(X_I = z+h)$

$$\phi'' - \theta_X^2\phi = 2\theta_X c_I^* \cosh \theta_X X_I + 2c_{II}^* \cosh \theta_X X_I$$

$$+ 4c_{II}^* X_I \theta_X \sinh \theta_X X_I + 2c_{III}^* \theta_X \sinh \theta_X X_I \quad (3.29)$$

$$\phi'' - \theta_X^2\phi = (2\theta_X c_I^* + 2c_{II}^*) \cosh \theta_X X_I$$

$$+ (2c_{III}^* \theta_X + 4c_{II}^* \theta_X X_I) \sinh \theta_X X_I \quad (3.30)$$

with

$$2\theta_X c_I^* + 2c_{II}^* = c_I$$

$$4\theta_X c_{II}^* = c_{II}$$

$$2\theta_X c_{III}^* = c_{III}$$

thus,

$$c^*_{III} = \frac{c_{III}}{2\theta_X} \tag{3.31}$$

$$c^*_{II} = \frac{c_{II}}{4\theta_X} \tag{3.32}$$

$$c^*_{I} = \frac{c_{I}}{2\theta_X} - \frac{c_{II}}{4\theta_X} \tag{3.33}$$

In our case,

$$c_I = -i[2\theta_X a_X^{(0)} + \theta_{XX} a^{(0)}] \tag{3.34}$$

$$c_{II} = -i2a^{(0)}\theta_{XX}\theta_X \tag{3.35}$$

$$c_{III} = -ia^{(0)}[(\theta_X h_X]\theta_X \tag{3.36}$$

Hence

$$c^*_{III} = -\frac{i2a^{(0)}[(\theta_X h_X]\cdot\theta_X}{2\theta_X} = -ia^{(0)}h_X\theta_X$$

$$c^*_{II} = -\frac{2ia^{(0)}\theta_{XX}\theta_X}{4\theta_X} = -\frac{ia^{(0)}}{2}\theta_{XX}$$

$$c^*_I = -i\frac{[2\theta_X a_X^{(0)} + \theta_{XX}a(0)]}{2\theta_X} + \frac{2ia^{(0)}\theta_{XX}\theta_X}{4\theta_X} = -i\frac{2\theta_X a_X^{(0)}}{2\theta_X} = -ia_X^{(0)}$$

So

$$c_I = -ia_X^{(0)} \tag{3.37}$$

$$c_{II} = -\frac{i}{2} a^{(0)} \theta_{XX} \tag{3.38}$$

$$c_{III} = -ia^{(0)} h_X \theta_X \tag{3.39}$$

Thus $\phi^{(1)}$, the first order solution, becomes:

$$\phi^{(1)} = a^{(1)} \cosh \theta_X(z+h) + b^{(1)} \sinh \theta_X(z+h) - ia_X^{(0)}(z+h)\sinh \theta_X(z+h)$$

$$-i \frac{a^{(0)}}{2} \theta_{XX}(z+h)^2 \cosh \theta_X(z+h)$$

$$-ia^{(0)} h_X \theta_X (z+h)\cosh \theta_X(z+h) \tag{3.40}$$

The B.C. on $\phi^{(1)}$ are:

$$\frac{\sigma^2}{g} \phi^{(1)} - \phi_z^{(1)} = 0 \quad (\text{on } z = 0)$$

$$\phi_z^{(1)} + i\theta_X h_X \cdot \phi^{(0)} = 0 \quad (\text{on } z = -h)$$

$$b^{(1)} \theta_X - ia^{(0)} h_X \theta_X + i\theta_X h_X a^{(0)} = 0$$

So

$$b^{(1)} = 0 \quad (\text{since } \theta_X \neq 0)$$

Thus:

$$\phi_z^{(1)}\Big|_{z=0} = a^{(1)}\theta_X \sinh\theta_X h + b^{(1)}\theta_X \cosh\theta_X h - ia_X^{(0)}(\sinh\theta_X h + \theta_X h \cosh\theta_X h)$$

$$-i\,\frac{a^{(0)}}{2}\,\theta_{XX}(2h \cosh\theta_X h + h^2\theta_X \sinh\theta_X h) - ia^{(0)}h_X\theta_X(\cosh\theta_X h + \theta_X h \sinh\theta_X h)$$

or

$$\phi^{(1)}\Big|_{z=0} = [a^{(1)}\cosh\theta_X h - ia_X^{(0)}h \sinh\theta_X h - i\,\frac{a^{(0)}}{2}\,\theta_{XX}h^2 \cosh\theta_X h$$

$$- ia^{(0)}h_X\theta_X h \cosh\theta_X h] \qquad (3.41)$$

Multiplying (3.41) by σ^2/g, we get:

$$\frac{\sigma^2}{g}\,\phi^{(1)}\Big|_{z=0} = \frac{\sigma^2}{g}\,[a^{(1)}\cosh\theta_X h - ia_X^{(0)}h \sinh\theta_X h - \frac{i}{2}\,a^{(0)}\theta_{XX}h^2\cosh\theta_X h$$

$$- ia^{(0)}h_X\theta_X h \cosh\theta_X h] \qquad (3.42)$$

and using the B.C., $\frac{\sigma^2}{g}\,\phi^{(1)} = \phi_z^{(1)}$ at z=0 for the above $\phi^{(1)}$, we have:

$$a_X^{(0)}[-\, i(\sinh\theta_X h + \theta_X h \cosh\theta_X h) + i\sigma^2\,\frac{h}{g}\,\sinh\theta_X h]$$

$$= a^{(0)}[i\,\frac{\theta_{XX}}{2}(2h \cosh\theta_X h + h^2\theta_X \sinh\theta_X h) + ih_X\theta_X(\cosh\theta_X h$$

$$+ \theta_X h \sinh\theta_X h) - i\,\frac{\sigma^2}{2g}\,\theta_{XX}h^2\cosh\theta_X h - \frac{i\sigma^2}{g}\,h\, h_X\theta_X \cosh\theta_X h] \quad (3.43)$$

From (3.25) we have:

$$\coth\theta_X h = \frac{g\theta_X}{\sigma^2}$$

Dividing (3.42) by $\sinh\theta_X h$, multiplying by i and using (3.25), we get:

$$a_X^{(0)}\left[\frac{\sigma^2 h}{g} - 1 - \frac{gh\theta_X^2}{\sigma^2}\right] = a^{(0)}\left[\frac{gh}{\sigma^2}\,\theta_X\theta_{XX} + g\,\frac{h_X\theta_X^2}{\sigma^2}\right] \qquad (3.44)$$

Differentiating (3.25) with respect to X, we get:

$$\left(\frac{\sigma^2}{g} - g\,\frac{\theta_X^2}{\sigma^2}\right)(\theta_X h)_X = \theta_{XX} \qquad (3.45)$$

Substituting this in (3.44), we get:

$$a_X^{(0)}(h_X\theta_X) = -\,a^{(0)} g\,\frac{(h\theta_X)_X}{\sigma^2}\left[h\theta_X\theta_{XX} + h_X\theta_X^2\right]$$

or

$$a_X^{(0)} h_X\theta_X = -\,\frac{a^{(0)}}{\sigma^2}\,g\theta_X\left[(h\theta_X)_X\right]^2$$

or

$$\frac{a_X^{(0)}}{a^{(0)}} = -\,\frac{g}{\sigma^2 h_X}\left[(h\theta_X)_X\right]^2 \qquad (3.46)$$

After integration of (3.46), we get:

$$\ln\frac{a^{(0)}(X)}{a^{(0)}(X_o)} = -\int \frac{g}{\sigma^2 h_X}\left[(h\theta_X)_X\right]^2 dx$$

$$a^{(0)}(X) = a^{(0)}(X_o) \exp\left[-\int_{X_o}^{X} \frac{g}{\sigma^2 h_X} [(h\theta_X)_X]^2 dx\right] \qquad (3.47)$$

The integral clearly indicates that an increment of amplitude, $a^{(0)}$, is obtained since for shallower water $h_X < 0$ and the integral remains positive. For $h_X > 0$ (in the direction towards deeper water) the integral becomes negative and the amplitude decreases.

Special applications of (3.47) are:

a. Limiting case of shallow water waves

b. Deeper part of intermediate wave region

a. Limiting case of shallow water waves

Assuming $\theta_X h$ is small, since $\theta_X \sim \frac{1}{L} \sim k(x)$ is small and when the water is shallow and depth, h, is small, the quantity $\theta_X h = \frac{h}{L}$ becomes smaller and then (3.25) becomes:

$$\frac{1}{\theta_X h} = \frac{gh\theta_X}{\sigma^2 h} \quad \text{or} \quad h\theta_X = \sigma\left(\frac{h}{g}\right)^{1/2}$$

Differentiating with respect to X,

$$(h\theta_X)_X = \frac{\sigma h_X}{2(gh)^{1/2}}$$

Substituting the last two equations into (3.47) and integrating, we get:

$$a^{(0)}_{(X)} = a^{(0)}_{(X_o)} \exp\left[-\int_{X_o}^{X} \frac{g}{\sigma^2 h_X} \frac{\sigma^2 h_X^2}{4gh}\, dx\right]$$

$$a^{(0)}_{(X)} = a^{(0)}_{(X_o)} \exp\left[-\int_{X_o}^{X} \frac{h_X}{4h}\, dx\right] = a^{(0)}_{(X_o)} \exp\left[-\frac{1}{4}\, \ell n\, h\right]_{X_o}^{X}$$

$$a^{(0)}_{(X)} = a^{(0)}_{(X_o)} \exp\left[-\frac{1}{4}\, \ell n \frac{h(X)}{h(X_o)}\right]$$

or

$$\frac{a^{(0)}_{(X)}}{a^{(0)}_{(X_o)}} = e^{-\frac{1}{4}\, \ell n \frac{h(X)}{h(X_o)}} = \exp\left[-\frac{1}{4}\, \ell n \frac{h(X)}{h(X_o)}\right]$$

$$\ell n\left[\frac{a^{(0)}_{(X)}}{a^{(0)}_{(X_o)}}\right] = -\frac{1}{4}\, \ell n \frac{h(X)}{h(X_o)}$$

or

$$\frac{a(X)}{a(X_o)} = \left[\frac{h(X)}{h(X_o)}\right]^{-1/4} \quad \text{If} \quad \begin{array}{l} a^{(0)}_{(X)} \equiv a_3 \text{ and } h(X_o) \equiv h_1 \\ a^{(0)}_{(X_o)} \equiv a_1 \text{ and } h(X) \equiv h_3 \end{array}$$

$$\frac{H_1}{H_3} = \frac{a_1}{a_3} = \left(\frac{h_3}{h_1}\right)^{1/4} \qquad (3.48)$$

This is the well known Green's Law for shallow water waves for the case of pure transmission without reflection or dissipation of energy.

b. Deeper part of intermediate wave region

For the deeper part of the intermediate region when $\tanh \theta_X h \to 1$, (3.25), gives $\theta_X \sim \frac{\sigma^2}{g}$. Then the general formula, (3.47), for the amplitude relation becomes:

$$a(X) = a^{(0)} \exp\left[- \int_{X_o}^{X} \frac{g}{\sigma^2 h_X} \left[\left(\frac{h\sigma^2}{g}\right)_X\right]^2 dx\right]$$

$$a^{(0)}_{(X)} = a^{(0)} \exp[- \int_{X_o}^{X} \frac{g h_X \sigma^4}{\sigma^2 g^2} dx]$$

and, after integration,

$$a(X) = a^{(0)}_{(X_o)} \exp\left[- \frac{\sigma^2}{g} [h(X) - h(X_o)]\right] \tag{3.49}$$

Introducing the transition length between the location $h(X)$ and $h(X_o)$ where $a^{(0)}_{(X)}$ and $a^{(0)}_{(X_o)}$ respectively, we get the variation of amplitudes to the zeroth approximation.

$$\frac{a^{(0)}_{(X)}}{a^{(0)}_{(X_o)}} = e^{-\frac{\sigma^2 \ell}{g}\left[\frac{h(X) - h(X_o)}{\ell}\right]} \tag{3.50a}$$

In the usual notation of upstream and downstream amplitudes $a^{(0)}_{(X)} \equiv a_3$ and $a^{(0)}_{(X_o)} \equiv a_1$ with $h(X) \equiv h_3$ and $h(X_o) \equiv h_1$, we get:

$$\frac{a_3}{a_1} = \exp\left[-\frac{\sigma^2 \ell}{g} \left(\frac{h_3 - h_1}{\ell}\right)\right] \tag{3.50b}$$

Thus the amplitude ratio $\frac{a_3}{a_1}$ is governed by the parameters:

$$\frac{\sigma^2 \ell}{g} = Sp = \text{shoaling parameter}$$

$$\frac{h_3 - h_1}{\ell} = \text{slope of bottom}$$

The amplitude variation was computed as a function of shoaling parameter Sp for different slopes S. These computations are given in graphical form in Fig. 5, 6, and 7.

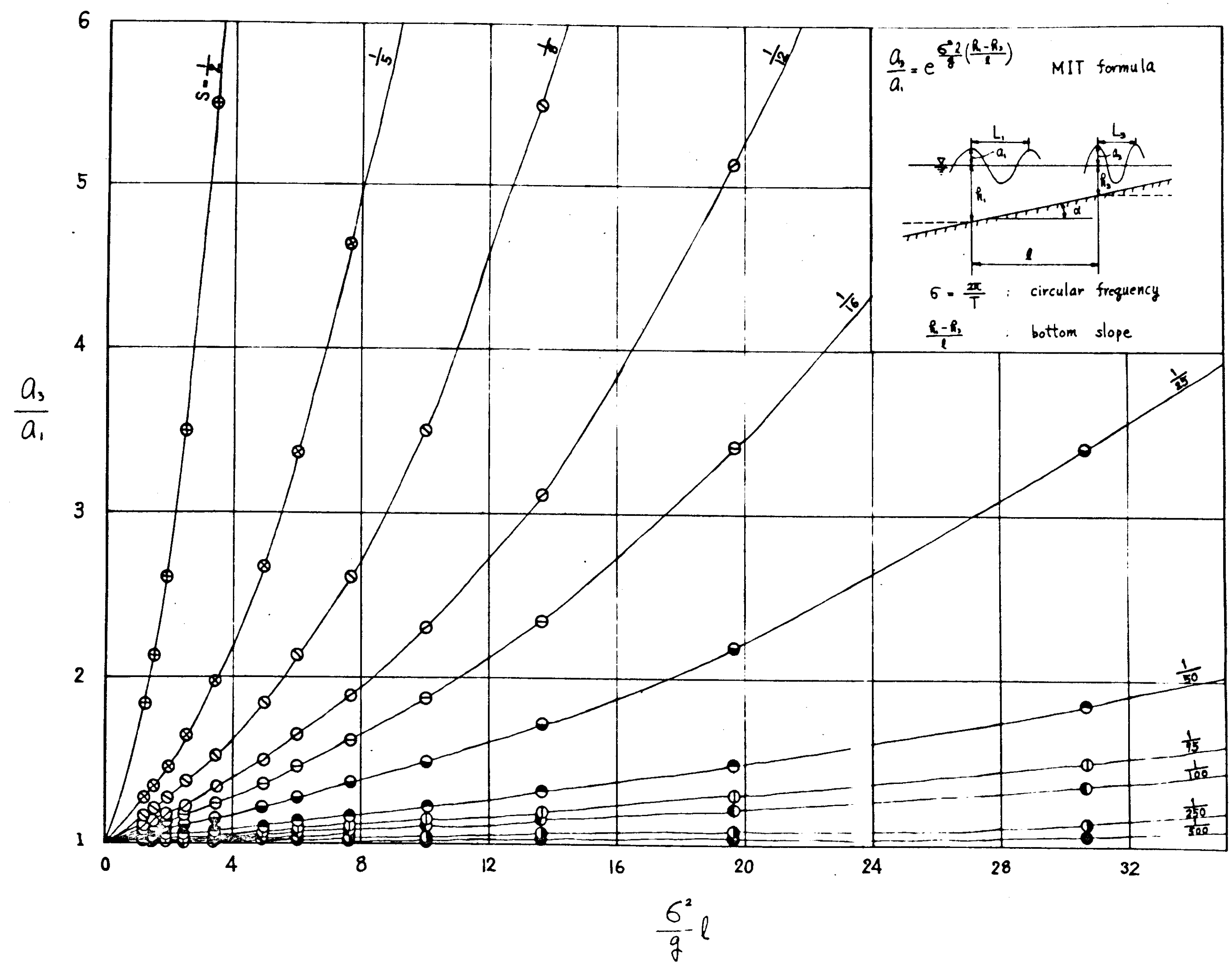


Fig. 5 Wave Amplitude Variation with Shoaling Parameter $\frac{\sigma^2 \ell}{g}$ for Different Slopes

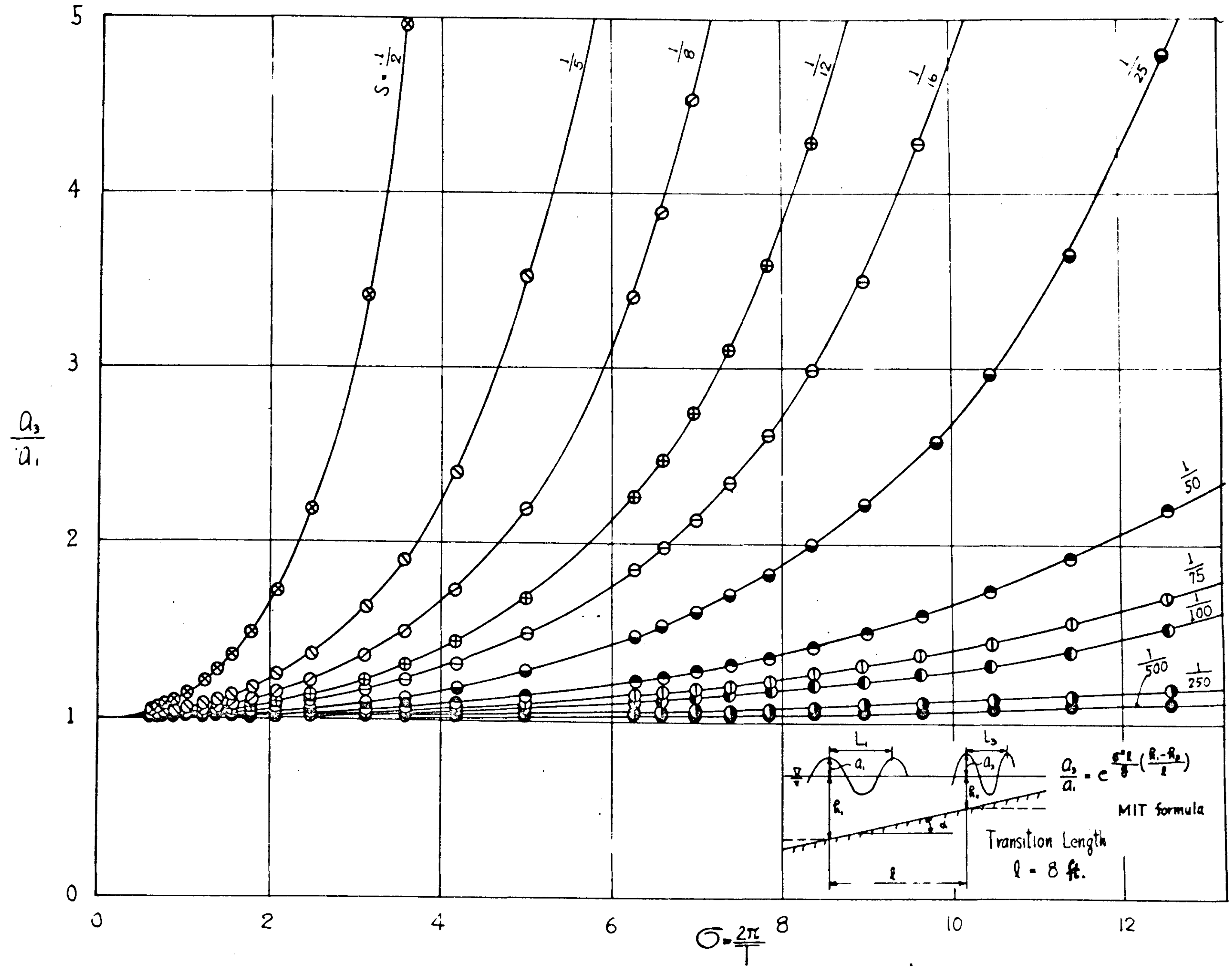


Fig. 6 Wave Amplitude Variation with Circular Frequency for Different Slopes (ℓ = 8 ft.)

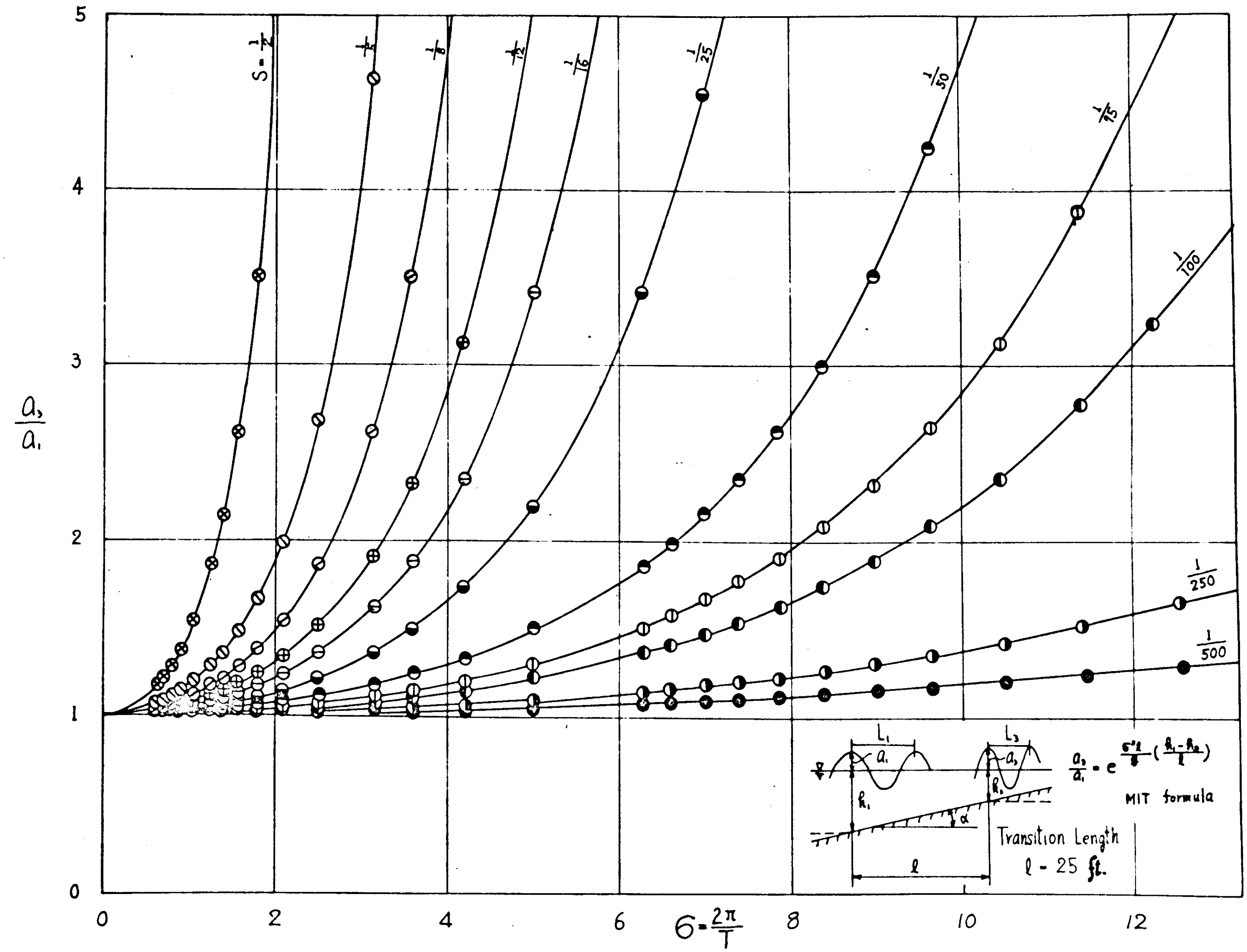


Fig. 7 Wave Amplitude Variation with Circular Frequency for Different Slopes (ℓ = 25 ft.)

3.2 CASE A OF TRANSITION: GRADUALLY VARYING DEPTH - CONSTANT WIDTH

From the geometry of the transition we have:

(i) Region I (Upstream)

$$B = B_1 = B_3 = \text{constant} \qquad +\infty \geqslant x \geqslant +\ell_1$$

$$h = h_1 = \text{constant} \qquad +\infty \geqslant x \geqslant +\ell_1$$

(ii) Region II (Transition)

$$\frac{h(x)}{h_1} = \frac{x}{\ell_1} \quad \text{or} \quad h(x) = \frac{h_1}{\ell_1} x \quad \text{in} \qquad +\ell_1 \geqslant x \geqslant +(\ell_1 - \ell)$$

$$B = B_1 = B_3 = \text{constant} \qquad +\ell_1 \geqslant x \geqslant +(\ell_1 - \ell)$$

(iii) Region III (Downstream)

$$B = B_1 = B_3 = \text{constant} \qquad +(\ell_1 - \ell) \geqslant x \geqslant -\infty$$

$$h = h_3 = \text{constant} \qquad +(\ell_1 - \ell) \geqslant x \geqslant -\infty$$

From these geometrical considerations the area of cross section at distance x over the transition is

$$A(x) = Bh(x) = B\left(\frac{h_1}{\ell_1}\right)x$$

The equations of wave motion of small amplitude, linearized for shallow water, are deduced from the non-linear shallow wave theory in two dimensions as follows:

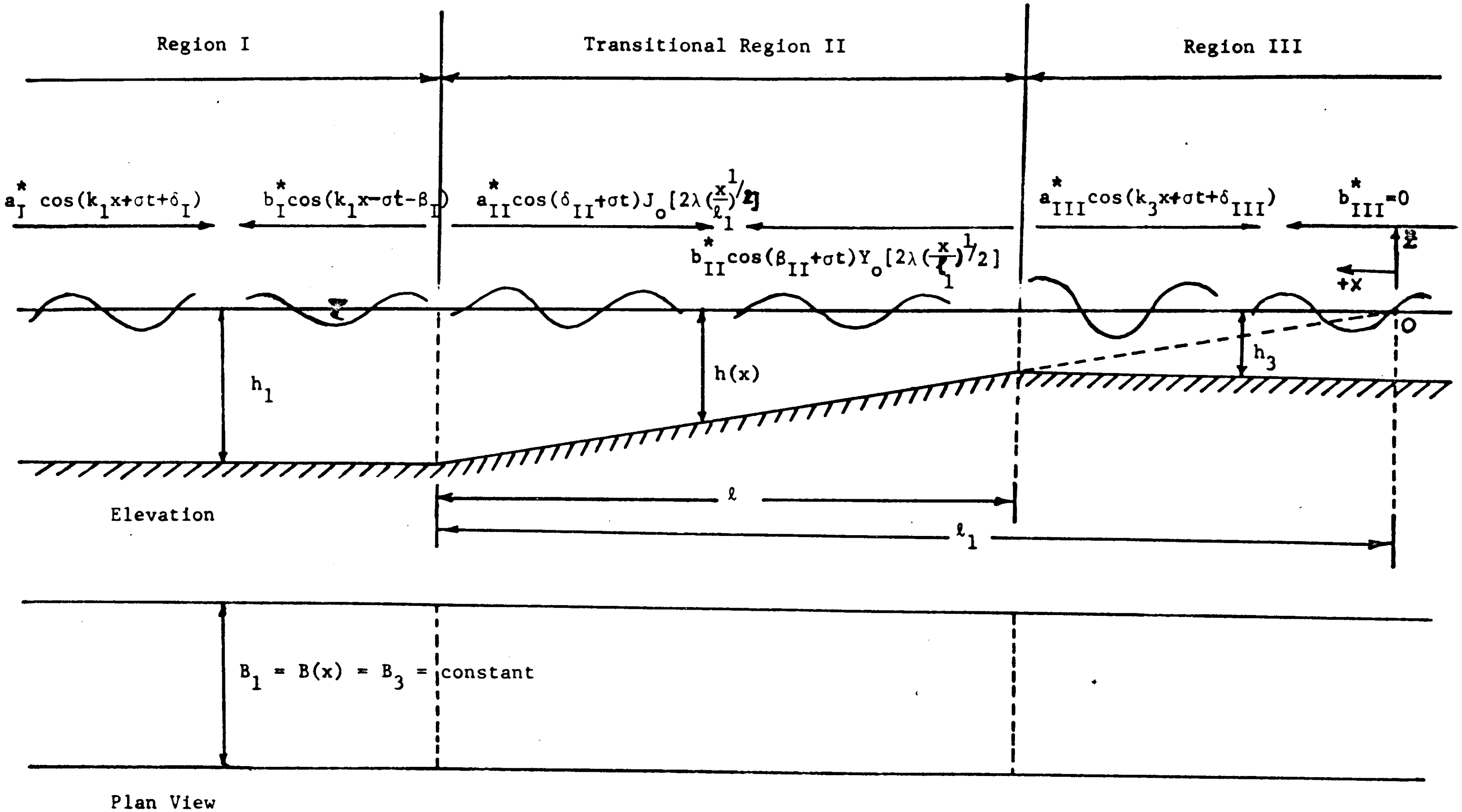


Fig. 8. Schematic Diagram of Case A of Transition Gradually Varying Depth - Constant Width.

$$u_t + g\eta_x = 0 \tag{3.51}$$

$$(uh)_x + \eta_t = 0 \tag{3.52}$$

Equation (3.52) is the continuity equation. Writing this more generally (4) we get:

$$(uA)_x + B\eta_t = 0 \tag{3.53}$$

Denoting by ξ the horizontal displacement $u = \xi_t$, equations (3.51) and (3.52) can be written:

$$\xi_{tt} + g\eta_x = 0 \tag{3.54}$$

$$(A\xi_t)_x + B\eta_t = 0 \tag{3.55}$$

From (3.54) we get, after differentiation:

$$(A\xi_{tt})_x = (-gA\eta_x)_x$$

or

$$(A\xi)_{ttx} + (gA\eta_x)_x = 0 \tag{3.56}$$

and from (3.55)

$$(A\xi)_{ttx} + (B\eta)_{tt} = 0 \tag{3.57}$$

Combining (3.56) and (3.57), we get:

$$(gA\eta_x)_x = (B\eta)_{tt} \tag{3.58}$$

for B = constant

$$B\eta_{tt} - (gh_1 \frac{B}{\ell_1} x\eta_x)_x = 0$$

$$B\eta_{tt} - \frac{h_1}{\ell_1} Bg\eta_x - g\frac{h_1}{\ell_1} Bx\eta_{xx} = 0 \tag{3.59}$$

Under the further assumption of simple harmonic wave motion of the type: $\eta(x,t) = \bar{\eta}(x)e^{+i\sigma t}$, we get from (3.59) the following:

$$\eta_{tt} = (+i)^2\sigma^2 e^{+i\sigma t}\cdot\bar{\eta}(x) \quad , \quad \eta_x = \bar{\eta}_x e^{+i\sigma t} \quad , \quad \eta_{xx} = \bar{\eta}_{xx} e^{+i\sigma t}$$

Substituting these quantities into (3.59) we get:

$$\bar{\eta}_{xx} \frac{gh_1}{\ell_1} x + \frac{gh_1}{\ell_1} \bar{\eta}_x + \sigma^2\bar{\eta} = 0$$

$$x\bar{\eta}_{xx} + \bar{\eta}_x + \frac{\sigma^2\ell_1}{gh_1}\bar{\eta} = 0 \tag{3.60}$$

and taking

$$\frac{\sigma^2\ell_1}{gh_1} = \frac{k_1^2\ell_1^2}{\ell_1} = \frac{\lambda^2}{\ell_1} \quad \text{with} \quad k_1 = \frac{2\pi}{L_1} = \frac{\sigma}{C_1}$$

we get:

$$x\bar{\eta}_{xx} + \eta_x + \frac{\lambda^2}{\ell_1}\bar{\eta} = 0 \qquad (3.61)$$

This equation can be reduced to a Bessel differential equation of zero order under the transformation

$$x = \frac{\omega^2 \ell_1}{4\lambda^2} \qquad (3.62)$$

Using this transformation the equation (3.61) becomes:

$$\frac{d^2\bar{\eta}}{d\omega^2} + \frac{1}{\omega}\frac{d\bar{\eta}}{d\omega} + \bar{\eta} = 0 \qquad (3.63)$$

which belongs to Bessel differential equation of zero order (since p=0) of the type:

$$\omega^2 \frac{d^2\bar{\eta}}{d\omega^2} + \omega\frac{d\bar{\eta}}{d\omega} + (\omega^2 - 0)\bar{\eta} = 0$$

References 19 up to 39 were used generally for the theoretical development of the present and next cases of transitions. The solution of equation (3.63) given by Bessel functions of zero order of first and second kind:

$$\bar{\eta} = C_1 J_o(\omega) + C_2 Y_o(\omega) = C_1 J_o[2\lambda(\frac{x}{\ell_1})^{1/2}] + C_2 Y_o[2\lambda(\frac{x}{\ell_1})^{1/2}] \qquad (3.64)$$

Hence:

$$\eta(x,t) = \left\{ C_1 J_o[2\lambda(\frac{x}{\ell_1})^{1/2}] + C_2 Y_o[2\lambda(\frac{x}{\ell_1})^{1/2}] \right\} e^{+i\sigma t} \tag{3.65}$$

Taking the arbitrary constants of integration in the form: $C_1 = a^*_{II} e^{\delta_{II} i}$ and $C_2 = b^*_{II} e^{\beta_{II} i}$, we obtain:

$$\eta(x,t) = a^*_{II} e^{i(\delta_{II}+\sigma t)} J_o[2\lambda(\frac{x}{\ell_1})^{1/2}] + b^*_{II} e^{i(\beta_{II}+\sigma t)} Y_o[2\lambda(\frac{x}{\ell_1})^{1/2}] \tag{3.66}$$

Considering only the real part of this solution we get as a solution for the Region II:

$$\eta_{II}(x,t) = a^*_{II} \cos(\delta_{II}+\sigma t) J_o[2\lambda(\frac{x}{\ell_1})^{1/2}] + b^*_{II} \cos(\beta_{II}+\sigma t) Y_o[2\lambda(\frac{x}{\ell_1})^{1/2}] \tag{3.67}$$

For the rest of the channel with constant cross section area A=Bh in Regions I and III upstream and downstream from the transition the differential equation becomes:

$$B\eta_{tt} - gA\eta_{xx} = 0$$

$$\eta_{tt} - gh\eta_{xx} = 0 \tag{3.68}$$

which is the well known linear wave equation $\eta_{xx} = \frac{1}{C}\eta_{tt}$ with $C = \sqrt{gh}$. We assume the same simple harmonic motion for Regions I and III of the type $\eta(x,t) = \bar{\eta}(x)e^{+i\sigma t}$. We take the second derivatives with respect to t and x,

$$\eta_{xx} = \bar{\eta}_{xx} e^{+i\sigma t} \quad \text{and} \quad \eta_{tt} = (+i)^2 \sigma^2 e^{+i\sigma t}$$

and after substituting into (3.68) we get for the upstream Region I:

$$\bar{\eta}_{xx} + \frac{\sigma^2}{C_1^{\;2}} \bar{\eta} = 0 \tag{3.69}$$

This homogeneous linear differential equation with constant coefficients (the linear oscillator equation) has as a characteristic equation: $r^2 + \frac{\sigma^2}{C_1^{\;2}} = 0$ with roots: $r_{1,2} = \pm i \frac{\sigma}{C_1} = \pm i\, k_1$ (where $k_1 = \frac{2\pi}{L_1}$) and the general solution is given by:

$$\bar{\eta}(x) = C_1 e^{ik_1} + C_2 e^{-ik_1} \tag{3.70}$$

where C_1 and C_2 are arbitrary constants.

Assuming these constants have a complex form similar to the previous one, we get:

$$\bar{\eta}(x) = a_I^* e^{i\delta_I} e^{ik_1 x} + b_I^* e^{i\beta_I} e^{-ik_1 x} = a_I^* e^{i(k_1 x + \delta_I)} + b_I^* e^{-i(k_1 x - \beta_I)} \tag{3.71}$$

Hence, since $\eta_I(x,t) = \bar{\eta}(x) e^{+i\sigma t}$,

$$\eta_I(x,t) = a_I^* e^{i(k_1 x + \delta_I + \sigma t)} + b_I^* e^{-i(k_1 x - \beta_I - \sigma t)} \tag{3.72}$$

Taking again only the real part of the above expression we get for Region I:

$$\eta_I(x,t) = a_I^* \cos(k_1 x + \sigma t + \delta_I) + b_I^* \cos(k_1 x - \sigma t - \beta_I) \qquad (3.73)$$

With a similar procedure we get for the Region III downstream from the transition:

$$\eta_{III}(x,t) = a_{III}^* \cos(k_3 x + \sigma t + \delta_{III}) + b_{III}^* \cos(k_3 x - \sigma t - \beta_{III}) \quad (3.74)$$

In all three cases the two parts with a_I^*, b_I^*, a_{II}^*, b_{II}^*, a_{III}^*, b_{III}^* as amplitudes and δ_I, β_I, δ_{II}, β_{II}, δ_{III}, β_{III} as phase angles represent two waves, one incoming and one reflecting (partially reflecting) in Regions I, II and III.

Now the boundary conditions are applied in order to determine the twelve arbitrary constants: a_I^*, b_I^*, a_{II}^*, b_{II}^*, a_{III}^*, b_{III}^*, δ_I, β_I, δ_{II}, β_{II}, δ_{III}, β_{III}.

Without loss of generality we can assume 1) that the reflection of the outgoing wave in Region III is zero, $b_{III}^*=0$, (no reflection from the beach which actually is eliminated either with Ursell's method or in reality by a strong absorber) used in the analysis of experimental results and 2) the amplitude of the transmitted wave into Region III, $a_{III}^*=1$, and the phase angle, $\delta_{III}=0$, can be taken as zero. Thus the remaining (8) unknowns can be computed by the matching conditions:

(i) the surface perturbation is continuous

(ii) the flux of water is continuous

This gives:

$$\eta_I|_{x=\ell_1} = \eta_{II}|_{x=\ell_1} \tag{3.75}$$

$$(\eta_I)_{x|x=\ell_1} = (\eta_{II})_{x|x=\ell_1} \tag{3.76}$$

$$\eta_{II}|_{x=\ell_1-\ell} = \eta_{III}|_{x=\ell_1-\ell} \tag{3.77}$$

$$(\eta_{II})_{x|x=\ell_1-\ell} = (\eta_{III})_{x|x=\ell_1-\ell} \tag{3.78}$$

Since the above mentioned boundary conditions give relations valid for all t, we evaluate these for $\sigma t=0$ and $\sigma t=-\frac{\pi}{2}$ and thus we get a system of eight unknowns with eight equations, computing in this way the constants, the amplitudes a_I, b_I, a_{II}, b_{II} and the phase angles δ_I, β_I, δ_{II}, β_{II}. Defining

$$\frac{\ell_1-\ell}{\ell_1} = \frac{h_3}{h_1} = \varepsilon^2$$

and, hence

$$k_3(\ell_1-\ell) = k_3\ell_1(1-\frac{\ell}{\ell_1}) = k_3\ell_1\varepsilon^2$$

we get the following relations:

$$a_I^*\cos(k_1\ell_1+\sigma t+\delta_I) + b_I^*\cos(k_1\ell_1-\sigma t-\beta_I) = a_{II}^*\cos(\delta_{II}+\sigma t)J_o(2\lambda) + b_{II}^*\cos(\beta_{II}+\sigma t)Y_o(2\lambda) \tag{3.79}$$

$$-a_I^* \sin(k_1\ell_1+\sigma t+\delta_I) - b_I^* \sin(k_1\ell_1-\sigma t-\beta_I)$$

$$= a_{II}^* \cos(\delta_{II}+\sigma t)J_o'(2\lambda) + b_{II}^* \cos(\beta_{II}+\sigma t)Y_o'(2\lambda) \qquad (3.80)$$

$$a_{II}^* \cos(\delta_{II}+\sigma t)J_o(2\lambda\varepsilon) + b_{II}^* \cos(\beta_{II}+\sigma t)Y_o(2\lambda\varepsilon)$$

$$a_{III}^* \cos(k_3\ell_1\varepsilon^2+\sigma t) \quad (\text{since } a_{III}^*=1,\ b_{III}^*=0) \qquad (3.81)$$

$$a_{II}^* \frac{\lambda}{\ell_1\varepsilon} \cos(\delta_{II}+\sigma t)J_o'(2\lambda\varepsilon) + b_{II}^* \cos(\beta_{II}+\sigma t)Y_o'(2\lambda\varepsilon)$$

$$= - a_{III}^* \left(\frac{k_3\ell_1\varepsilon}{\lambda}\right) \sin[k_3(\ell_1-\ell)+\sigma t] \qquad (3.82)$$

Dividing in equations (3.79) up to (3.82) through the downstream amplitude $a_{III}^*=1$ and taking the dimensionless amplitudes as follows:

$$a_I = \frac{a_I^*}{a_{III}^*}, \quad b_I = \frac{b_I^*}{a_{III}^*}, \quad a_{II} = \frac{a_{II}^*}{a_{III}^*}, \quad b_{II} = \frac{b_{II}^*}{a_{III}^*}, \quad \frac{a_{III}^*}{a_{III}^*} = 1$$

Now for $\sigma t=0$ and $\sigma t=\frac{-\pi}{2}$, we get (8) relations from the above equations (3.79) to (3.82):

for $\sigma t=0$: $a_I \cos(k_1\ell_1+\delta_I)+b_I \cos(k_1\ell_1-\beta_I) = a_{II}\cos\delta_{II} J_o(2\lambda)+b_{II}\cos\beta_{II} Y_o(2\lambda)$ (3.79a) for $\sigma t = \frac{\pi}{2}$: $a_I \sin(k_1 \ell_1 + \delta_I) - b_I \sin(k_1 \ell_1 - \beta_I)$

$$= a_{II} \sin\delta_{II} J_o(2\lambda) + b_{II} \sin\beta_{II} Y_o(2\lambda) \qquad (3.79b)$$

for $\sigma t = 0$: $-a_I \sin(k_1 \ell_1 + \delta_I) - b_I \sin(k_1 \ell_1 - \beta_I)$

$$= a_{II} \cos\delta_{II} J_o'(2\lambda) + b_{II} \cos\beta_{II} Y_o'(2\lambda) \qquad (3.80a)$$

for $\sigma t = \frac{\pi}{2}$: $a_I \cos(k_1 \ell_1 + \delta_I) - b_I \cos(k_1 \ell_1 - \beta_I)$

$$= a_{II} \sin\delta_{II} J_o'(2\lambda) + b_{II} \sin\beta_{II} Y_o'(2\lambda) \qquad (3.80b)$$

for $\sigma t = 0$: $a_{II} \cos\delta_{II} J_o(2\lambda\varepsilon) + b_{II} \cos\beta_{II} Y_o(2\lambda\varepsilon)$

$$= \cos(k_3 \ell_1 \varepsilon^2) \qquad (3.81a)$$

for $\sigma t = \frac{\pi}{2}$: $a_{II} \sin\delta_{II} J_o(2\lambda\varepsilon) + b_{II} \sin\beta_{II} Y_o(2\lambda\varepsilon)$

$$= \sin(k_3 \ell_1 \varepsilon^2) \qquad (3.81b)$$

for $\sigma t = 0$: $a_{II} \cos\delta_{II} J_o'(2\lambda\varepsilon) + b_{II} \cos\beta_{II} Y_o'(2\lambda\varepsilon)$

$$= -\left(\frac{k_3 \ell_1 \varepsilon}{\lambda}\right) \sin(k_3 \ell_1 \varepsilon^2) \qquad (3.82a)$$

$$\text{for } \sigma t=\frac{-\pi}{2}\text{: } a_{II}\sin\delta_{II}J_o'(2\lambda\varepsilon) + b_{II}\sin\beta_{II}Y_o'(2\lambda\varepsilon)$$

$$= \left(\frac{k_3\ell_1\varepsilon}{\lambda}\right)\cos(k_3\ell_1\varepsilon^2) \tag{3.82b}$$

From (3.81a) and (3.82a) we get for a_{II}:

$$a_{II} = \frac{\cos(k_3\ell_1\varepsilon^2)Y_o'(2\lambda\varepsilon) + \left(\frac{k_3\ell_1\varepsilon}{\lambda}\right)\sin(k_3\ell_1\varepsilon^2)Y_o(2\lambda\varepsilon)}{\cos\delta_{II}[J_o(2\lambda\varepsilon)Y_o'(2\lambda\varepsilon) - J_o'(2\lambda\varepsilon)Y_o(2\lambda\varepsilon)]} \tag{3.83}$$

Defining:

$$A_1 = \cos(k_3\ell_1\varepsilon^2)Y_o'(2\lambda\varepsilon) + \left(\frac{k_3\ell_1\varepsilon}{\lambda}\right)\sin(k_3\ell_1\varepsilon^2)Y_o(2\lambda\varepsilon) \tag{3.84}$$

$$A_2 = [J_o(2\lambda\varepsilon)Y_o'(2\lambda\varepsilon) - J_o'(2\lambda\varepsilon)Y_o(2\lambda\varepsilon)] \tag{3.85}$$

we get, for a_{II}:

$$a_{II} = \frac{A_1}{A_2\cos\delta_{II}} \tag{3.86}$$

In the same way we get, for b_{II}:

$$b_{II} = \frac{-\left(\frac{k_3\ell_1\varepsilon}{\lambda}\right)\sin(k_3\ell_1\varepsilon^2)J_o(2\lambda\varepsilon) - J_o'(2\lambda\varepsilon)\cos(k_3\ell_1\varepsilon^2)}{\cos\beta_{II}[J_o(2\lambda\varepsilon)Y_o'(2\lambda\varepsilon) - J_o'(2\lambda\varepsilon)Y_o(2\lambda\varepsilon)]} \tag{3.87}$$

Defining:

$$A_3 = -[(\frac{k_3\ell_1\varepsilon}{\lambda})\sin(k_3\ell_1\varepsilon^2)J_o(2\lambda\varepsilon) + J_o'(2\lambda\varepsilon)\cos(k_3\ell_1\varepsilon^2)] \quad (3.88)$$

we get:

$$b_{II} = \frac{A_3}{A_2\cos\beta_{II}} \quad (3.89)$$

From (3.81b) and (3.82b) we get the values of a_{II} and b_{II} in a similar procedure:

$$a_{II} = \frac{\sin(k_3\ell_1\varepsilon^2)Y_o'(2\lambda\varepsilon) - (\frac{k_3\ell_1\varepsilon}{\lambda})\cos(k_3\ell_1\varepsilon^2)Y_o(2\lambda\varepsilon)}{\sin\delta_{II}[J_o(2\lambda\varepsilon)Y_o'(2\lambda\varepsilon) - J_o'(2\lambda\varepsilon)Y_o(2\lambda\varepsilon)} \quad (3.90)$$

Defining:

$$A_4 = \sin(k_3\ell_1\varepsilon^2)Y_o'(2\lambda\varepsilon) - (\frac{k_3\ell_1\varepsilon}{\lambda})\cos(k_3\ell_1\varepsilon^2)Y_o(2\lambda\varepsilon) \quad (3.91)$$

we get:

$$a_{II} = \frac{A_4}{A_2\sin\delta_{II}} \quad (3.92)$$

and

$$b_{II} = \frac{(\frac{k_3\ell_1\varepsilon}{\lambda})\cos(k_3\ell_1\varepsilon^2)J_o(2\lambda\varepsilon) - \sin(k_3\ell_1\varepsilon^2)J_o'(2\lambda\varepsilon)}{\sin\beta_{II}[J_o(2\lambda\varepsilon)Y_o'(2\lambda\varepsilon) - J_o'(2\lambda\varepsilon)Y_o(2\lambda\varepsilon)} \quad (3.93)$$

Defining:

$$A_5 = (\frac{k_3\ell_1\varepsilon}{\lambda})\cos(k_3\ell_1\varepsilon^2)J_o(2\lambda\varepsilon) - \sin(k_3\ell_1\varepsilon^2)J_o'(2\lambda\varepsilon) \quad (3.94)$$

we get:

$$b_{II} = \frac{A_5}{A_2 \sin\beta_{II}} \tag{3.95}$$

From (3.86) and (3.92) we get:

$$\frac{A_1}{A_2 \cos\delta_{II}} = \frac{A_4}{A_2 \sin\delta_{II}}$$

or

$$\tan\delta_{II} = \frac{A_4}{A_1} \tag{3.96}$$

and

$$\delta_{II} = \tan^{-1} \frac{A_4}{A_1} \tag{3.97}$$

From (3.89) and (3.95) we get:

$$\frac{A_3}{A_2 \cos\beta_{II}} = \frac{A_5}{A_2 \sin\beta_{II}}$$

or

$$\tan\beta_{II} = \frac{A_5}{A_3} \tag{3.98}$$

and

$$\beta_{II} = \tan^{-1} \frac{A_5}{A_3} \tag{3.99}$$

The A_1, A_2, A_3, A_4, A_5 are all known quantities. From the phase angles δ_{II} and β_{II} the values of amplitudes a_{II} and b_{II} can be computed from (3.86) and (3.89):

$$a_{II} = \frac{A_1}{A_2 \cos\delta_{II}} \quad \text{and} \quad b_{II} = \frac{A_3}{A_2 \cos\beta_{II}}$$

Knowing now the values of a_{II}, b_{II}, δ_{II} and β_{II}, equations (3.79a), (3.79b), (3.80a) and (3.80b) give the values of four unknowns a_I, b_I, δ_I, β_I.

$$a_I \cos(k_1\ell_1+\delta_I) + b_I \cos(k_1\ell_1-\beta_I) = [\frac{A_1}{A_2} J_o(2\lambda) + \frac{A_3}{A_2} Y_o(2\lambda)] = B_1 \quad (3.100)$$

$$a_I \sin(k_1\ell_1+\delta_I) - b_I \sin(k_1\ell_1-\beta_I) = [\frac{A_1}{A_2} \tan\delta_{II} J_o(2\lambda) + \frac{A_3}{A_2} \tan\beta_{II} Y_o(2\lambda)] = B_2 \quad (3.101)$$

$$-a_I \sin(k_1\ell_1+\delta_I) - b_I \sin(k_1\ell_1-\beta_I) = (\frac{A_1}{A_2}) J_o'(2\lambda) + (\frac{A_3}{A_2}) Y_o'(2\lambda) = B_3 \quad (3.102)$$

$$a_I \cos(k_1\ell_1+\delta_I) - b_I \cos(k_1\ell_1-\beta_I) = (\frac{A_1}{A_2}) \tan\delta_{II} J_o'(2\lambda) + (\frac{A_3}{A_2}) \tan\beta_{II} Y_o'(2\lambda) = B_4 \quad (3.103)$$

From (3.100) and (3.101) we get for a_I:

$$a_I = \frac{-[B_1 \sin(k_1\ell_1-\beta_I) + B_2 \cos(k_1\ell_1-\beta_I)]}{[\cos(k_1\ell_1+\delta_I)\sin(k_1\ell_1-\beta_I) + \cos(k_1\ell_1-\beta_I)\sin(k_1\ell_1+\delta_I)]} \quad (3.104)$$

From (3.102) and (3.103) we get for a_I:

$$a_I = \frac{-[B_3 \cos(k_1\ell_1-\beta_I) - B_4 \sin(k_1\ell_1-\beta_I)}{[\sin(k_1\ell_1+\delta_I)\cos(k_1\ell_1-\beta_I) + \sin(k_1\ell_1-\beta_I)\cos(k_1\ell_1+\delta_I)]} \quad (3.105)$$

Setting equations (3.104) and (3.105) equal we get after considerable algebraic reduction as a general solution for phase angle β_I:

$$\tan(k_1\ell_1-\beta_I) = \frac{[B_2+B_3]}{[B_4-B_1]} \tag{3.106}$$

$$\beta_I = k_1\ell_1-\tan^{-1}\left(\frac{B_2+B_3}{B_4-B_1}\right) \tag{3.107}$$

With the same approach we compute the phase angle δ_I. From (3.100) and (3.101) we have for amplitude b_I:

$$b_I = \frac{B_2\cos(k_1\ell_1+\delta_I) - B_1\sin(k_1\ell_1+\delta_I)}{[\cos(k_1\ell_1+\delta_I)\sin(k_1\ell_1-\beta_I) + \cos(k_1\ell_1-\beta_I)\sin(k_1\ell_1+\delta_I)]} \tag{3.108}$$

From (3.102) and (3.103) we get also for b_I:

$$b_I = \frac{-[B_4\sin(k_1\ell_1+\delta_I) + B_3\cos(k_1\ell_1+\delta_I)]}{[\sin(k_1\ell_1+\delta_I)\cos(k_1\ell_1-\beta_I) + \sin(k_1\ell_1-\beta_I)\cos(k_1\ell_1+\delta_I)]} \tag{3.109}$$

Setting equations (3.108) and (3.109) equal we obtain after considerable reduction:

$$\tan(k_1\ell_1+\delta_I) = \frac{B_2-B_3}{B_4+B_1} \tag{3.110}$$

and

$$\delta_I = \tan^{-1}\left(\frac{B_2-B_3}{B_4+B_1}\right)-k_1\ell_1 \tag{3.111}$$

Substituting the values of β_I and δ_I again into (3.104) and (3.108) we get the values for the amplitudes a_I and b_I, hence the reflection coefficient K_r in the upstream region I and the transmission coefficient K_t in the downstream region III can be obtained.

In summary for the transition A of gradually varying depth the values of the amplitudes, phase angles, reflection and transmission coefficients are given explicitly as follows:

(i) amplitudes: reflection and transmission coefficients:

$$a_I = \frac{B_1 \sin(k_1 \ell_1 - \beta_I) + B_2 \cos(k_1 \ell_1 - \beta_I)}{\sin(\delta_i - \beta_I)}$$

$$b_I = \frac{B_2 \cos(k_1 \ell_1 - \delta_I) - B_1 \sin(k_1 \ell_1 + \delta_I)}{\sin(\delta_I - \beta_I)}$$

$$K_r = \frac{b_I}{a_I} = \frac{\cos(\delta_I + k_1 \ell_1) - \frac{B_1}{B_2} \sin(\delta_I + k_1 \ell_1)}{\cos(\beta_I - k_1 \ell_1) - \frac{B_1}{B_2} \sin(\beta_I - k_1 \ell_1)}$$

$$K_t = \frac{1}{a_I} = \frac{\sin(\delta_I - \beta_I)}{B_1 \sin(k_1 \ell_1 - \beta_I) + B_2 \cos(k_1 \ell_1 - \beta_I)}$$

$$a_{II} = \frac{A_1}{A_2 \cos \delta_{II}}$$

$$b_{II} = \frac{A_3}{A_2 \cos \delta_{II}}$$

$$a_{III} = 1 \qquad b_{III} = 0$$

At this point it must be remembered that all amplitudes are stated as ratios with respect to the downstream amplitude a_{III} assumed as unity. Hence, actual amplitudes a_I^*, b_I^*, a_{II}^*, b_{II}^* must be computed by multiplying with a_{III}^*.

(ii) <u>phase angles</u>

$$\delta_I = \tan^{-1}\left(\frac{B_2-B_3}{B_1-B_4}\right)-k_1\ell_1$$

$$\beta_I = k_1\ell_1 - \tan^{-1}\left(\frac{B_2+B_3}{B_4-B_1}\right)$$

$$\delta_{II} = \tan^{-1}\frac{A_4}{A_1}$$

$$\beta_{II} = \tan^{-1}\frac{A_5}{A_3}$$

$$\delta_{III} = \beta_{III} = 0$$

3.3 CASE B OF TRANSITION: LINEARLY VARYING DEPTH AND WIDTH

From the geometry of the transition in case of simultaneous linear change in depth and width we have:

(i) Region I (Upstream)

$$B = B_1 = \text{constant} \qquad +\infty > x \geq +\ell_1$$

$$h = h_1 = \text{constant} \qquad +\infty \geq x \geq +\ell_1$$

(ii) Region II (Transition)

$$\frac{B(x)}{B_1} = \frac{x}{\ell_1} \quad \text{or } B(x) = \frac{B_1}{\ell_1} x \qquad +\ell_1 \geq x \geq +(\ell_1-\ell)$$

$$\frac{h(x)}{h_1} = \frac{x}{\ell_1} \quad \text{or } h(x) = \frac{h_1}{\ell_1} x \qquad +\ell_1 \geq x \geq +(\ell_1-\ell)$$

(iii) Region III (Downstream)

$$B = B_3 = \text{constant} \qquad +(\ell_1-\ell) \geq x > -\infty$$

$$h = h_3 = \text{constant} \qquad +(\ell_1-\ell) \geq x > -\infty$$

Referring to equations (3.66), (3.57) and (3.58) we have as in the case of linearly varying depth:

$$[B(x)\eta]_{tt} = [A(x)g\eta_x]_x$$

and since variation of B(x) is independent of time:

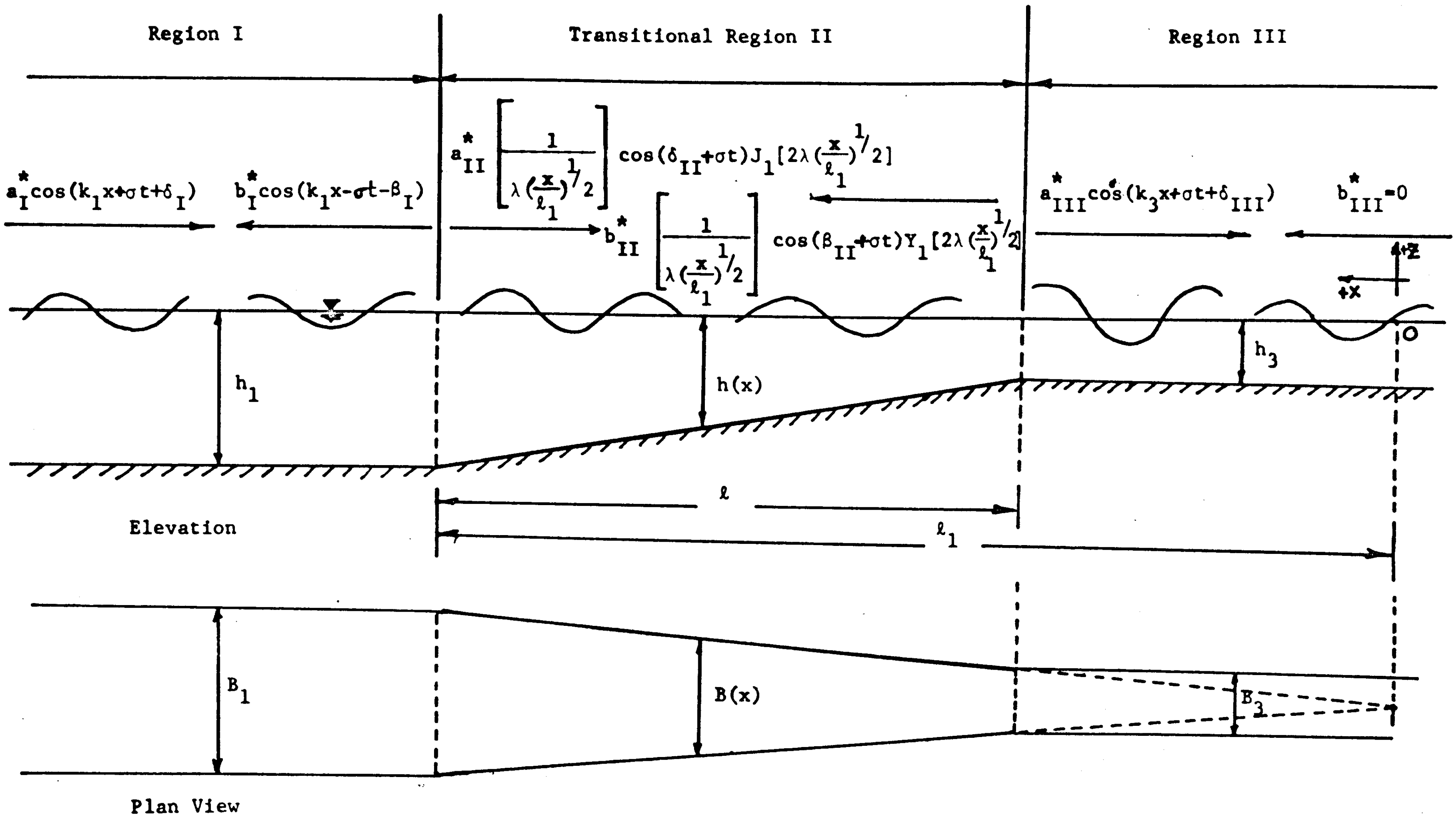


Fig. 9. Schematic Diagram of Case B of Transition
Gradually Varying Depth and Width.

$$\eta_{tt} = \frac{g}{B(x)}\,[A(x)\eta_x]_x \tag{3.112}$$

Assuming again a solution of simple harmonic motion in the form $\eta(x,t) = \bar{\eta}(x)e^{+i\sigma t}$ we get:

$$\eta_{tt} = \bar{\eta}(x)(+i)^2\sigma^2 e^{+i\sigma t} \quad \text{and} \quad \eta_x = \bar{\eta}_x e^{+i\sigma t}$$

Substituting into equation (3.112) we get:

$$\frac{g\ell_1}{B_1 x}\;\frac{B_1 h_1}{\ell_1}\;x^2\bar{\eta}_x{}_{\;x} + \sigma^2\bar{\eta} = 0$$

or

$$x\bar{\eta}_{xx} + 2\bar{\eta}_x + \frac{\sigma^2\ell_1}{h_1 g}\,\bar{\eta} = 0 \tag{3.113}$$

Introducing for

$$\frac{\sigma^2\ell_1}{h_1 g} = \frac{\sigma^2\ell_1^2}{C_1^2}\,\frac{1}{\ell_1} = \frac{\lambda^2}{\ell_1}$$

for the eigen values and substituting according to the transformation $x = \frac{\phi^2\ell_1}{4\lambda^2}$, we get equation (3.113) transformed into: (3.114)

$$\bar{\eta}_{\phi\phi} + \frac{3}{\phi}\,\bar{\eta}_\phi + \bar{\eta} = 0 \tag{3.115}$$

Using the additional transformation $\bar{\eta} = \frac{\omega}{\phi}$ equation (3.115) is transformed into a Bessel differential equation of the first order:

$$\phi^2 \omega_{\phi\phi} + \phi \omega_{\phi} + (\phi^2-1)\omega = 0 \quad (\phi \equiv P=1) \tag{3.116}$$

The solution is given by first and second Bessel functions of first order $\omega = J_1(\phi)$ and $\omega = Y_1(\phi)$. Since $\bar{\eta} = \frac{\omega}{\phi} = \frac{\omega}{2\lambda}\sqrt{\frac{\ell_1}{x}}$ the general solution is:

$$\bar{\eta} = C_I \frac{J_1(2\lambda\sqrt{\frac{x}{\ell_1}})}{(2\lambda\sqrt{\frac{x}{\ell_1}})} + \frac{C_{II} Y_1(2\lambda\sqrt{\frac{x}{\ell_1}})}{(2\lambda\sqrt{\frac{x}{\ell_1}})}$$

or

$$\bar{\eta} = C_1 \frac{J_1(2\lambda\sqrt{\frac{x}{\ell_1}})}{\lambda\sqrt{\frac{x}{\ell_1}}} + C_2 \frac{Y_1(2\lambda\sqrt{\frac{x}{\ell_1}})}{\lambda\sqrt{\frac{x}{\ell_1}}} \tag{3.117}$$

where C_1 and C_2 are arbitrary constants. Hence

$$\eta_{II}(x,t) = \left[\frac{C_1}{\lambda\sqrt{\frac{x}{\ell_1}}} J_1(2\lambda\sqrt{\frac{x}{\ell_1}}) + \frac{C_2}{\lambda\sqrt{\frac{x}{\ell_1}}} Y_1(2\lambda\sqrt{\frac{x}{\ell_1}}) \right] e^{+i\sigma t} \tag{3.118}$$

The solution for the regions I and III upstream and downstream from the transitions where depth and width are constant (h_1, B_1 and h_3, B_3) are given as in previous cases of linearly varying depth from the solution of the differential equation;

$$\bar{\eta}_{xx} + k_1^2\bar{\eta} = 0 \quad \text{for upstream Region I} \tag{3.119a}$$

$$\bar{\eta} + k_2^2\bar{\eta} = 0 \quad \text{for downstream Region III} \tag{3.119b}$$

Since

$$k_1^2 = \frac{\sigma^2}{gh_1} = \frac{\sigma^2}{c_1^2} \quad \text{and} \quad k_3^2 = \frac{\sigma^2}{gh_3} = \frac{\sigma^2}{c_3^2}$$

Hence the solutions are

$$\eta_I(x,t) = [\, C_3 e^{ik_1 x} + C_4 e^{-ik_1 x}]e^{+i\sigma t} \tag{3.120}$$

$$\eta_{III}(x,t) = [C_5 e^{ik_3 x} + C_6 e^{-ik_3 x}]e^{+i\sigma t} \tag{3.121}$$

Assuming that the arbitrary constants of integration have the form

$$C_5 = a^*_{III} e^{i\delta_{III}} \;, \quad C_6 = b^*_{III} e^{i\beta_{III}} \;, \quad \text{etc.,}$$

as in the previous case, and taking only the real part of the exponential expressions, we get for the three regions:

$$\eta_I(x,t) = a_I^* \cos(k_1x+\sigma t+\delta_I) + b_I^* \cos(k_1x-\sigma t-\beta_I) \tag{3.122}$$

$$\eta_{II}(x,t) = a_{II}^* \left(\frac{1}{\lambda\sqrt{\frac{x}{\ell_1}}}\right)\cos(\delta_{II}+\sigma t)J_1\left(2\lambda\sqrt{\frac{x}{\ell_1}}\right)$$

$$+ b_{II}^* \left(\frac{1}{\lambda\sqrt{\frac{x}{\ell_1}}}\right)\cos(\beta_{II}+\sigma t)Y_1\left(2\lambda\sqrt{\frac{x}{\ell_1}}\right) \tag{3.123}$$

$$\eta_{III}(x,t) = a_{III}^* \cos(k_3x+\sigma t+\delta_{III}) + b_{III}^* \cos(k_3x-\sigma t-\beta_{III}) \tag{3.124}$$

Using as in the previous case the boundary conditions of continuity of surface perturbation and water flux at $x=\ell_1$ and $x=\ell_1-\ell$ we get the following system of eight equations and eight unknowns for $\sigma t=0$ and $\sigma t=\frac{-\pi}{2}$ under the assumption that the reflection from the end is zero.

$$a_I^* \cos(k_1\ell_1+\sigma t+\delta_I) + b_I^* \cos(k_1\ell_1-\sigma t-\beta_I) =$$

$$= a_{II}^* \frac{1}{\lambda} \cos(\delta_{II}+\sigma t)J_1(2\lambda) + b_{II}^* \frac{1}{\lambda} \cos(\beta_{II}+\sigma t)Y_1(2\lambda) \tag{3.125}$$

$$a_I^* \sin(k_1\ell_1+\sigma t+\delta_I) + b_I^* \sin(k_1\ell_1-\sigma t-\beta_I) =$$

$$= a_{II}^* \left[\frac{1}{2\lambda k_1\ell_1} J_1(2\lambda) - \frac{1}{k_1\ell_1} J_1'(2\lambda)\right] \cos(\delta_{II}+\sigma t) +$$

$$+ b_{II}^* \left[\frac{1}{2\lambda k_1\ell_1} Y_1(2\lambda) - \frac{1}{k_1\ell_1} Y_1'(2\lambda)\right] \cos(\beta_{II}+\sigma t) \tag{3.126}$$

$$a_{II}^{*} \frac{1}{\lambda\varepsilon} \cos(\delta_{II}+\sigma t) J_1(2\lambda) + b_{II}^{*} \frac{1}{\lambda\varepsilon} \cos(\beta_{II}+\sigma t) Y_1(2\lambda)$$

$$= a_{III}^{*} \cos(k_3 \ell_1 \varepsilon^2 + \sigma t) \quad (3.127)$$

$$a_{II}^{*} \left[\frac{1}{2\lambda\varepsilon k_3(\ell_1-\ell)} J_1(2\lambda\varepsilon) - \frac{1}{k_3(\ell_1-\ell)} J_1'(2\lambda\varepsilon) \right] \cos(\delta_{II}+\sigma t) +$$

$$+ b_{II}^{*} \left[\frac{1}{2\lambda\varepsilon k_3(\ell_1-\ell)} Y_1(2\lambda\varepsilon) - \frac{1}{k_3(\ell_1-\ell)} Y_1'(2\lambda\varepsilon) \right] \cos(\beta_{II}+\sigma t)$$

$$= a_{III}^{*} \sin(k_3 \ell_1 \varepsilon^2 + \sigma t) \quad (3.128)$$

Defining:

$$\frac{1}{\lambda}[J_1(2\lambda)] = \Gamma_1 \quad (3.129a)$$

$$\frac{1}{\lambda}[Y_1(2\lambda)] = \Gamma_2 \quad (3.129b)$$

$$\frac{1}{k_1\ell_1}\left[\frac{1}{2\lambda} J_1(2\lambda) - J_1'(2\lambda)\right] = \Gamma_3 \quad (3.130)$$

$$\frac{1}{k_1\ell_1}\left[\frac{1}{2\lambda} Y_1(2\lambda) - Y_1'(2\lambda)\right] = \Gamma_4 \quad (3.131)$$

$$\frac{1}{\lambda\varepsilon}[J_1(2\lambda)] = \frac{\Gamma_1}{\varepsilon} \quad (3.132a)$$

$$\frac{1}{\lambda\varepsilon}[Y_1(2\lambda)] = \frac{\Gamma_2}{\varepsilon} \quad (3.132b)$$

$$\frac{1}{k_3(\ell_1-\ell)}\left[\frac{1}{2\lambda\varepsilon} J_1(2\lambda\varepsilon) - J_1'(2\lambda\varepsilon)\right] = \Gamma_5 \quad (3.133)$$

$$\frac{1}{k_3(\ell_1-\ell)}[\frac{1}{2\lambda\varepsilon} Y_1(2\lambda\varepsilon) - Y_1(2\lambda\varepsilon)] = \Gamma_6 \qquad (3.134)$$

Dividing by a^*_{III} all terms to become dimensionless then we get;

$$a_I = \frac{a^*_I}{a^*_{III}} \ , \quad b_I = \frac{b^*_I}{a^*_{III}} \ , \quad a_{II} = \frac{a^*_{II}}{a^*_{III}} \ , \quad b^*_{II} = \frac{b^*_{II}}{a^*_{IIII}} \ , \quad \frac{a^*_{III}}{a^*_{III}} = 1$$

the system of equations (3.125) up to (3.128) becomes:

$$a_I\cos(k_1\ell_1+\sigma t+\delta_I)+b_I\cos(k_1\ell_1-\sigma t-\beta_I) = a_{II}\Gamma_1\cos(\delta_{II}+\sigma t)+b_{II}\Gamma_2\cos(\beta_{II}+\sigma t) \qquad (3.135)$$

$$a_I\sin(k_1\ell_1+\sigma t+\delta_I)+b_I\sin(k_1\ell_I-\sigma t-\beta_I) = a_{II}\Gamma_3\cos(\delta_{II}+\sigma t)+b_{II}\Gamma_4\cos(\beta_{II}+\sigma t) \qquad (3.136)$$

$$a_{II}\Gamma_1\cos(\delta_{II}+\sigma t)+b_{II}\Gamma_2\cos(\beta_{II}+\sigma t) = \varepsilon \cos(k_3\ell_1\varepsilon^2+\sigma t) \qquad (3.137)$$

$$a_{II}\Gamma_5\cos(\delta_{II}+\sigma t)+b_{II}\Gamma_6\cos(\beta_{II}+\sigma t) = \varepsilon \cos(k_3\ell_1\varepsilon^2+\sigma t) \qquad (3.138)$$

The above system for equations with eight unknowns is valid for all times and gives for $\sigma t=0$ and $\sigma t=\frac{\pi}{2}$ the eight equations following:

$$a_I\cos(k_1\ell_1+\delta_I)+b_I\cos(k_1\ell_1-\beta_I) = a_{II}\Gamma_1\cos\delta_{II}+b_{II}\Gamma_2\cos\beta_{II} \qquad (3.139)$$

$$a_I\sin(k_1\ell_1+\delta_I)-b_I\sin(k_1\ell_1-\beta_I) = a_{II}\Gamma_1\sin\delta_{II}+b_{II}\Gamma_2\sin\beta_{II} \qquad (3.140)$$

$$a_I \sin(k_1\ell_1+\delta_I)+b_I \sin(k_1\ell_1-\beta_I) = a_{II}\Gamma_3 \cos\delta_{II}+b_{II}\Gamma_2 \cos\beta_{II} \quad (3.141)$$

$$-a_I \cos(k_1\ell_1+\delta_I)+b_I \cos(k_1\ell_1-\beta_I) = a_{II}\Gamma_3 \sin\delta_{II}+b_{II}\Gamma_4 \sin\beta_{II} \quad (3.142)$$

$$a_{II}\Gamma_1 \cos\delta_{II}+b_{II}\Gamma_2 \cos\beta_{II} = \varepsilon \cos(k_3\ell_1\varepsilon^2) \quad (3.143)$$

$$a_{II}\Gamma_1 \sin\delta_{II}+b_{II}\Gamma_2 \sin\beta_{II} = \varepsilon \sin(k_3\ell_1\varepsilon^2) \quad (3.144)$$

$$a_{II}\Gamma_5 \cos\delta_{II}+b_{II}\Gamma_6 \cos\beta_{II} = \sin(k_3\ell_1\varepsilon^2) \quad (3.145)$$

$$a_{II}\Gamma_5 \sin\delta_{II}+b_{II}\Gamma_6 \sin\beta_{II} = \cos(k_3\ell_1\varepsilon^2) \quad (3.146)$$

From (3.143) and (3.144) we obtain:

$$a_{II} = \frac{\varepsilon \sin(\beta_{II}-k_3\ell_1\varepsilon^2)}{\Gamma_1 \sin(\beta_{II}-\delta_{II})} \quad (3.147)$$

From (3.145) and (3.146) we obtain;

$$a_{II} = \frac{-\cos(\beta_{II}-k_3\ell_1\varepsilon^2)}{\Gamma_5 \sin(\beta_{II}-\delta_{II})} \quad (3.148)$$

Setting equations (3.147) and (3.148) equal we have:

$$\frac{\varepsilon \sin(\beta_{II}-k_3\ell_1\varepsilon^2)}{\Gamma_1 \sin(\beta_{II}-\delta_{II})} = -\frac{\cos(\beta_{II}-k_3\ell_1\varepsilon^2)}{\Gamma_5 \sin(\beta_{II}-\delta_{II})}$$

$$\tan(\beta_{II}-k_3\ell_1\varepsilon^2) = -\frac{\Gamma_1}{\varepsilon\Gamma_5}$$

$$\beta_{II} = k_3\ell_1\varepsilon^2 - \tan^{-1}(-\frac{\Gamma_1}{\varepsilon\Gamma_5}) \tag{3.149}$$

Using the same approach for b_{II} from (3.143) and (3.144) we obtain:

$$b_{II} = \frac{\varepsilon \sin(k_3\ell_1\varepsilon^2-\delta_{II})}{\Gamma_2 \sin(\beta_{II}-\delta_{II})} \tag{3.150}$$

and from (3.145) and (3.146);

$$b_{II} = \frac{\cos(k_3\ell_1\varepsilon^2 - \delta_{II})}{\Gamma_6 \sin(\beta_{II}-\delta_{II})} \tag{3.151}$$

Setting equations (3.150) and (3.151) equal we obtain:

$$\tan(k_3\ell_1\varepsilon^2-\delta_{II}) = \frac{\Gamma_2}{\varepsilon\Gamma_6}$$

$$\delta_{II} = k_3\ell_1\varepsilon^2-\tan^{-1}(\frac{\Gamma_2}{\varepsilon\Gamma_6}) \tag{3.152}$$

Thus the values of a_{II}, b_{II} are now known since the values of β_{II} and δ_{II} are given explicitly by (3.149) and (3.152).

With the quantities a_{II}, b_{II}, β_{II} and δ_{II} known we determine the right side part of equations (3.139) up to (3.142). Defining then:

$$a_{II}\Gamma_1\cos\delta_{II}+b_{II}\Gamma_2\cos\beta_{II} = D_1 \tag{3.153}$$

$$a_{II}\Gamma_1\sin\delta_{II}+b_{II}\Gamma_2\sin\beta_{II} = D_2 \quad (3.154)$$

$$a_{II}\Gamma_3\cos\delta_{II}+b_{II}\Gamma_2\cos\beta_{II} = D_3 \quad (3.155)$$

$$a_{II}\Gamma_3\sin\delta_{II}+b_{II}\Gamma_4\sin\beta_{II} = D_4 \quad (3.156)$$

The system of equations (3.139) to (3.142) becomes:

$$a_I\cos(k_1\ell_1+\delta_I) + b_I\cos(k_1\ell_1-\beta_I) = D_1 \quad (3.157)$$

$$a_I\sin(k_1\ell_1+\delta_I) - b_I\sin(k_1\ell_1-\beta_I) = D_2 \quad (3.158)$$

$$a_I\sin(k_1\ell_1+\delta_I) + b_I\sin(k_1\ell_1-\beta_I) = D_3 \quad (3.159)$$

$$-a_I\cos(k_1\ell_1+\delta_I) + b_I\cos(k_1\ell_1-\beta_I) = D_4 \quad (3.160)$$

From (3.157) and (3.158) we get for a_I:

$$a_I = \frac{D_1\sin(k_1\ell_1-\beta_I) + D_2\cos(k_1\ell_1-\beta_I)}{\cos(k_1\ell_1+\delta_I)\sin(k_1\ell_1-\beta_I) + \cos(k_1\ell_1-\beta_I)\sin(k_1\ell_1+\delta_I)} \quad (3.161)$$

From (3.159) and (3.160) we obtain:

$$a_I = \frac{D_3\cos(k_1\ell_1-\beta_I) - D_4\sin(k_1\ell_1-\beta_I)}{\cos(k_1\ell_1-\beta_I)\sin(k_1\ell_1+\delta_I) + \sin(k_1\ell_1-\beta_I)\cos(k_1\ell_1+\delta_I)} \quad (3.162)$$

Setting equations (3.161) and (3.162) equal after significant algebraic reduction and simplification we obtain:

$$\tan(k_1\ell_1-\beta_I) = \frac{D_3-D_2}{D_1+D_4}$$

$$\beta_I = k_1\ell_1-\tan^{-1}\left(\frac{D_3-D_2}{D_1+D_4}\right) \qquad (3.163)$$

Similarly for b_I, from (3.157) and (3.158) we get:

$$b_I = \frac{D_2\cos(k_1\ell_1+\delta_I) - D_1\sin(k_1\ell_1+\delta_I)}{-[\cos(k_1\ell_1+\delta_I)\sin(k_1\ell_1-\beta_I) + \cos(k_1\ell_1-\beta_I)\sin(k_1\ell_1+\delta_I)]} \qquad (3.164)$$

and from (3.159) and (3.160) we obtain:

$$b_I = \frac{D_4\sin(k_1\ell_1+\delta_I) + D_3\cos(k_1\ell_1+\delta_I)}{\sin(k_1\ell_1+\delta_I)\cos(k_1\ell_1-\beta_I) + \sin(k_1\ell_1-\beta_I)\cos(k_1\ell_1+\delta_I)} \qquad (3.165)$$

Setting equations (3.164) and (3.165) equal, after considerable reduction, we obtain:

$$\tan(k_1\ell_1+\delta_I) = \frac{D_3+D_2}{D_1-D_4}$$

$$\delta_I = \tan^{-1}\left(\frac{D_3+D_2}{D_1-D_4}\right)-k_1\ell_1 \qquad (3.166)$$

In summary for the transition B of gradually varying depth and width the values of the amplitudes, phase angles, reflection and transmission coefficients are given explicitly as follows:

(i) Amplitudes, reflection and transmission coefficients:

$$a_I = \frac{D_1\sin(k_1\ell_1-\beta_I) + D_2\cos(k_1\ell_1-\beta_I)}{\sin(2k_1\ell_1+\delta_I-\beta_I)}$$

$$b_I = \frac{D_1\sin(k_1\ell_1+\delta_I) - D_2\cos(k_1\ell_1+\delta_I)}{\sin(2k_1\ell_1+\delta_I-\beta_I)}$$

$$K_r = \frac{b_I}{a_I} = \frac{\sin(k_1\ell_1+\delta_I) - \frac{D_2}{D_1}\cos(k_1\ell_1+\delta_I)}{\sin(k_1\ell_1-\beta_I) + \frac{D_2}{D_1}\cos(k_1\ell_1-\beta_I)}$$

$$K_t = \frac{1}{a_I} = \frac{\sin(2k_1\ell_1+\delta_I-\beta_I)}{D_1\sin(k_1\ell_1-\beta_I) + D_2\cos(k_1\ell_1-\beta_I)}$$

$$a_{II} = \frac{\varepsilon\ \sin(\beta_{II}-k_3\ell_1\varepsilon^2)}{\Gamma_1\sin(\beta_{II}-\delta_{II})}$$

$$b_{II} = \frac{\varepsilon\ \sin(k_3\ell_1\varepsilon^2-\delta_{II})}{\Gamma_2\sin(\beta_{II}-\delta_{II})}$$

$$a_{III} = 1 \quad , \quad b_{III} = 0$$

(ii) Phase angles

$$\delta_I = \tan^{-1}\left(\frac{D_3+D_2}{D_1-D_4}\right)-k_1\ell_1$$

$$\beta_I = k_1\ell_1-\tan^{-1}\left(\frac{D_3-D_2}{D_1+D_4}\right)$$

$$\delta_{II} = k_3\ell_1\varepsilon^2-\tan^{-1}\left(\frac{\Gamma_2}{\varepsilon\ \Gamma_6}\right)$$

$$\beta_{II} = k_3\ell_1\varepsilon^2-\tan^{-1}\left(-\frac{\Gamma_1}{\varepsilon\ \Gamma_5}\right)$$

$$\delta_{III} = \beta_{III} = 0$$

3.4 CASE C OF TRANSITION: LINEARLY VARYING WIDTH - CONSTANT DEPTH

From the geometry of the transition in case of linearly varying width and constant upstream and downstream depth we have:

(i) Region I (Upstream)

$B = B_1 = \text{constant} \qquad + \infty > x \geq + \ell_1$

$h = h_1 = \text{constant} \qquad + \infty > x \geq + \ell_1$

(ii) Region II (Transition)

$\frac{B(x)}{B_1} = \frac{x}{\ell_1} \quad \text{or} \quad B(x) = \frac{B_1}{\ell_1}x \qquad + \ell_1 \geq x \geq + (\ell_1 - \ell)$

$h(x) = h = \text{constant} \qquad + \ell_1 \geq x \geq + (\ell_1 - \ell)$

(iii) Region III (Downstream)

$B = B_3 = \text{constant} \qquad + (\ell_1 - \ell) \geq x > - \infty$

$h = h_3 = \text{constant} \qquad + (\ell_1 - \ell) \geq x > - \infty$

Hence $A(x) = B(x)h_1 = \frac{B_1}{1} h_1 x$

Referring to equations (3.56), (3.57) and (3.58) we start with the basic equation for the wave motion over the transition (3.112):

$$\eta_{tt} = \frac{g}{B(x)} [A(x)\eta_x]_x$$

Assuming again a solution of simple harmonic motion in the form

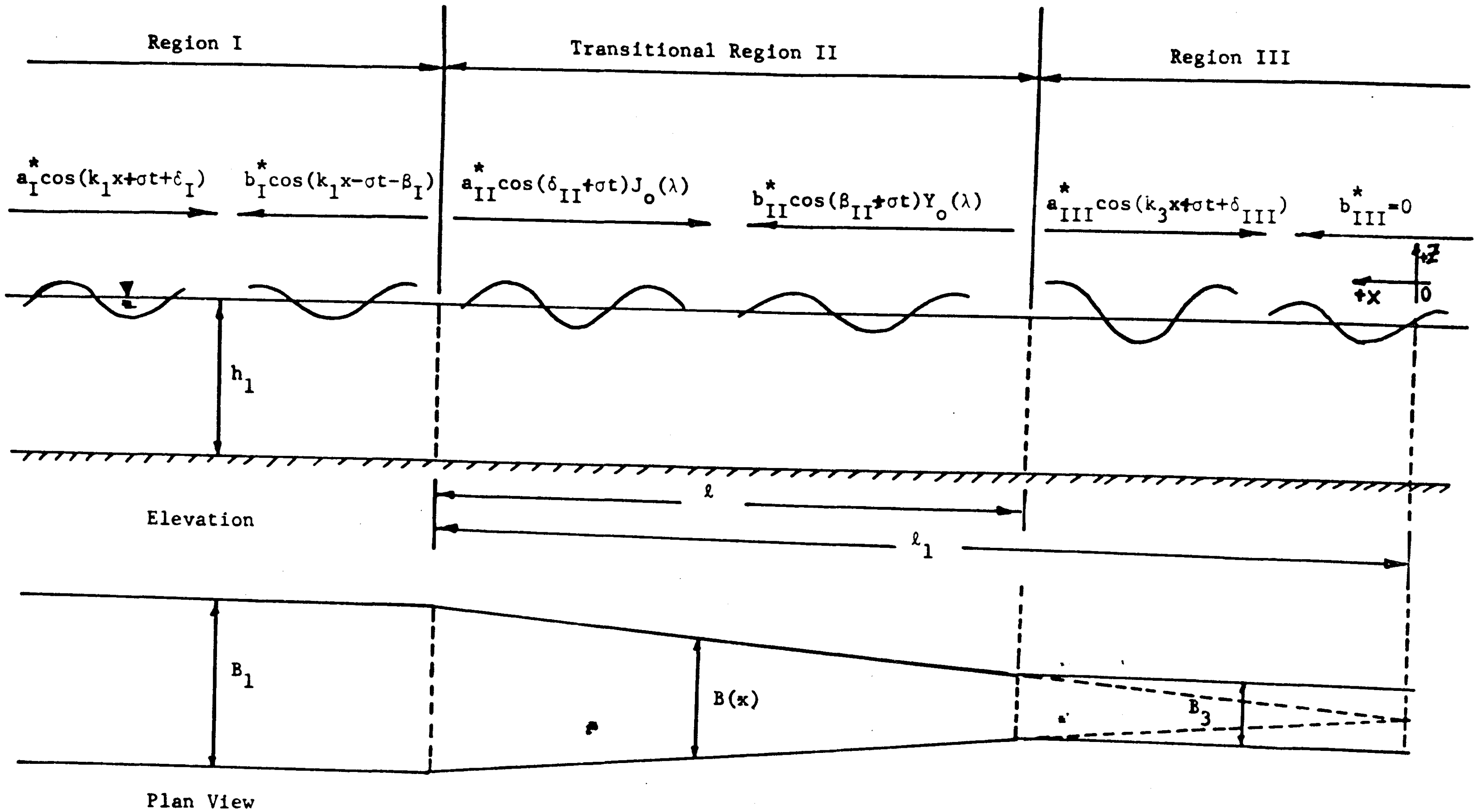


Fig. 10. Schematic Diagram of Case C of Transition Gradually Varying Width - Constant Depth.

$\eta(x,t) = \overline{\eta}(x)e^{+i\sigma t}$, we get:

$$\overline{\eta}_{xx} \frac{B_1 x h_1 g}{\ell_1} + \overline{\eta}_x \frac{B_1 h_1 g}{\ell_1} + B(x)\sigma^2 \overline{\eta} = 0 \qquad (3.167)$$

and since $B(x) = \frac{B_1}{\ell_1} x$, we get finally:

$$x\overline{\eta}_{xx} + \overline{\eta}_x + \frac{\sigma^2}{gh} \overline{x\eta} = 0 \qquad (3.168)$$

where $h = h_1 = h_3$ = constant.

Setting eigen values

$$\frac{\sigma^2}{gh} = \frac{\sigma^2 \ell_1^2}{gh \ell_1^2} = \left(\frac{\sigma^2 \ell_1^2}{C^2}\right) \frac{1}{\ell_1^2} = \frac{\lambda^2}{\ell_1^2} \qquad (3.169)$$

where λ = dimensionless quantity and, taking the transformation $u = \frac{\lambda}{\ell_1} x$, the equation (3.168) becomes:

$$u\overline{\eta}_{uu} + \overline{\eta}_u + u\overline{\eta} = 0 \qquad (3.170)$$

which is a Bessel differential equation of zero order P=0.

The general solution is given by:

$$\overline{\eta}(x) = C_1 J_o(u) + C_2 Y_o(u) = C_1 J_o\left(\frac{\lambda}{\ell_1} x\right) + C_2 Y_o\left(\frac{\lambda}{\ell_1} x\right) \qquad (3.171)$$

and combining the time component $e^{+i\sigma t}$:

$$\eta_{II}(x,t) = [C_1 J_o(\frac{\lambda}{\ell_1} x) + C_2 Y_o(\frac{\lambda}{\ell_1} x)]e^{+i\sigma t}$$

$$\eta_{II}(x,t) = [a^*_{II} e^{\delta_{II} i} J_o(\frac{\lambda}{\ell_1} x) + b^*_{II} e^{\beta_{II} i} Y_o(\frac{\lambda}{\ell_1} x)]e^{+i\sigma t}$$

$$\eta_{II}(x,t) = a^*_{II} e^{i(\delta_{II}+\sigma t)} J_o(\frac{\lambda}{\ell_1} x) + b^*_{II} e^{i(\beta_{II}+\sigma t)} Y_o(\frac{\lambda}{\ell_1} x) \quad (3.172)$$

Taking only the real part of the equation (3.172) we get:

$$\eta_{II}(x,t) = a^*_{II}\cos(\delta_{II}+\sigma t) J_o(\frac{\lambda}{\ell_1} x) + b^*_{II}\cos(\beta_{II}+\sigma t) Y_o(\frac{\lambda}{\ell_1} x) \quad (3.173)$$

For the region I and III upstream and downstream from the transition under the assumption of simple harmonic motion the differential equation expressing the motion is the linear wave equation:

$$\eta_{xx} = \frac{1}{C_2} \eta_{tt} \qquad \text{with } C = \sqrt{gh}$$

This type of equation as in previous cases gives as solutions for:

Region I:

$$\eta_I(x,t) = a^*_{II}\cos(kx+\sigma t+\delta_I) + b^*_{II}\cos(kx-\sigma t-\beta_I) \quad (3.174)$$

Region III:

$$\eta_{III}(x,t) = a^*_{III}\cos(kx+\sigma t+\delta_{III}) + b^*_{III}\cos(kx-\sigma t-\beta_{III}) \quad (3.175)$$

Setting the boundary conditions of continuity of surface perturbation and water flux at $x=\ell_1$ and $x=\ell_1-\ell$ we get the following system of eight equations and eight unknowns for $\sigma t=0$ and $\sigma t=\frac{-\pi}{2}$ and under the assumption that the reflection from the beach-end is zero and that the amplitude of the transmitted downstream wave $a^*_{III} = 1$, hence; for

$$\frac{\ell_1-\ell}{c_1} = (1 - \frac{\ell}{\ell_1}) = \varepsilon^2 \quad \text{and} \quad \lambda = \frac{\sigma\ell_1}{c} = k\ell_1:$$

$$a^*_I\cos(k\ell_1+\sigma t+\delta_I) + b^*_I\cos(k\ell_1-\sigma t-\beta_I)$$

$$= a^*_{II}\cos(\delta_{II}+\sigma t)J_o(\tfrac{\lambda}{\ell}) + b^*_{II}\cos(\beta_{II}+\sigma t)Y_o(\tfrac{\lambda}{\ell}) \tag{3.176}$$

$$-a^*_I\sin(k\ell_1+\sigma t+\delta_I) - b^*_I\sin(k\ell_1-\sigma t-\beta_I)$$

$$= a^*_{II}\cos(\delta_{II}+\sigma t)J'_o(\lambda) + b^*_{II}\cos(\beta_{II}+\sigma t)Y'_o(\lambda) \tag{3.177}$$

$$a^*_{II}\cos(\delta_{II}+\sigma t)J_o(\lambda\varepsilon^2) + b^*_{II}\cos(\beta_{II}+\sigma t)Y_o(\lambda\varepsilon^2)$$

$$= a^*_{III}\cos(k\ell_1\varepsilon^2+\sigma t) \tag{3.178}$$

$$a^*_{II}\cos(\delta_{II}+\sigma t)J'_o(\lambda\varepsilon^2) + b^*_{II}\cos(\beta_{II}+\sigma t)Y'_o(\lambda\varepsilon^2)$$

$$= a^*_{III}\sin(k\ell_1\varepsilon^2+\sigma t) \tag{3.179}$$

Evaluating for $\sigma t=0$ and $\sigma t=\frac{-\pi}{2}$ we get after dividing all terms by a^*_{III} and taking equations in dimensionless form $a_I = \frac{a^*_I}{a^*_{III}}$ etc.:

$$a_I\cos(k\ell_1+\delta_I)+b_I\cos(k\ell_1-\beta_I) = a_{II}\cos(\delta_{II})\,J_o(\lambda)+b_{II}\cos(\beta_{II})\,Y_o(\lambda) \tag{3.180}$$

$$a_I\sin(k\ell_1+\delta_I)-b_I\sin(k\ell_1-\beta_I) = a_{II}\sin\delta_{II}J_o(\lambda)+b_{II}\sin\beta_{II}Y_o(\lambda) \tag{3.181}$$

$$-a_I\sin(k\ell_1+\delta_I)-b_I\sin(k\ell_1-\beta_I) = a_{II}\cos\delta_{II}J'_o(\lambda)+b_{II}\cos\beta_{II}Y'_o(\lambda) \tag{3.182}$$

$$a_I\cos(k\ell_1+\delta_I)-b_I\cos(k\ell_1-\beta_I) = a_{II}\sin\delta_{II}J'_o(\lambda)+b_{II}\sin\beta_{II}Y'_o(\lambda) \tag{3.183}$$

$$a_{II}\cos\delta_{II}J_o(\lambda\varepsilon^2)+b_{II}\cos\beta_{II}Y_o(\lambda\varepsilon^2) = \cos(k\ell_1\varepsilon^2) \tag{3.184}$$

$$a_{II}\sin\delta_{II}J_o(\lambda\varepsilon^2)+b_{II}\sin\beta_{II}Y_o(\lambda\varepsilon^2) = \sin(k\ell_1\varepsilon^2) \tag{3.185}$$

$$a_{II}\cos\delta_{II}J'_o(\lambda\varepsilon^2)+b_{II}\cdot\cos\beta_{II}Y'_o(\lambda\varepsilon^2) = \sin(k\ell_1\varepsilon^2) \tag{3.186}$$

$$a_{II}\sin\delta_{II}J'_o(\lambda\varepsilon^2)+b_{II}\cdot\sin\beta_{II}Y'_o(\lambda\varepsilon^2) = \cos(k\ell_1\varepsilon^2) \tag{3.187}$$

From equations (3.184) and (3.186) we get for a_{II} and b_{II}:

$$a_{II} = \frac{\cos(k\ell_1\varepsilon^2)Y'(\lambda\varepsilon^2) - \sin(k\ell_1\varepsilon^2)Y_o(\lambda\varepsilon^2)}{\cos\delta_{II}[J_o(\lambda\varepsilon^2)Y'_o(\lambda\varepsilon^2)-Y_o(\lambda\varepsilon^2)J'_o(\lambda\varepsilon)]} \tag{3.188}$$

Defining:

$$\Lambda_1 = \cos(k\ell_1\varepsilon^2)Y'(\lambda\varepsilon^2) - \sin(k\ell_1\varepsilon^2)Y_o(\lambda\varepsilon^2) \tag{3.189}$$

$$\Lambda_2 = J_o(\lambda\varepsilon^2)Y_o'(\lambda\varepsilon^2) - Y_o(\lambda\varepsilon^2)J_o'(\lambda\varepsilon^2) \tag{3.190}$$

then

$$a_{II} = \frac{\Lambda_1}{\Lambda_2\cos\delta_{II}} \tag{3.191}$$

and for b_{II}:

$$b_{II} = \frac{\sin(k\ell_1\varepsilon^2)J_o(\lambda\varepsilon^2) - \cos(k\ell_1\varepsilon^2)J_o'(\lambda\varepsilon^2)}{\cos\beta_{II}[J_o(\lambda\varepsilon^2)Y_o'(\lambda\varepsilon^2) - J_o'(\lambda\varepsilon^2)Y_o(\lambda\varepsilon^2)]} \tag{3.192}$$

Defining:

$$\Lambda_3 = \sin(k\ell_1\varepsilon^2)J_o(\lambda\varepsilon^2) - \cos(k\ell_1\varepsilon^2)J_o'(\lambda\varepsilon^2) \tag{3.193}$$

then

$$b_{II} = \frac{\Lambda_3}{\Lambda_2\cos\beta_{II}} \tag{3.194}$$

From (3.185) and (3.187) we obtain for a_{II} and b_{II}:

$$a_{II} = \frac{\sin(k\ell_1\varepsilon^2)Y_o'(\lambda\varepsilon^2) - \cos(k\ell_1\varepsilon^2)Y_o(\lambda\varepsilon^2)}{\sin\delta_{II}[J_o(\lambda\varepsilon^2)Y_o'(\lambda\varepsilon^2)-Y_o(\lambda\varepsilon^2)J_o'(\lambda\varepsilon^2)]} \tag{3.195}$$

Defining:

$$\sin(k\ell_1\varepsilon^2)Y_o'(\lambda\varepsilon^2) - \cos(k\ell_1\varepsilon^2)Y_o(\lambda\varepsilon^2) = \Lambda_4 \tag{3.196}$$

the above equation becomes:

$$a_{II} = \frac{\Lambda_4}{\Lambda_2\sin\delta_{II}} \tag{3.197}$$

Similarly for b_{II}:

$$b_{II} = \frac{\cos(k\ell_1\varepsilon^2)J_o(\lambda\varepsilon^2) - \sin(k\ell_1\varepsilon^2)J_o'(\lambda\varepsilon^2)}{\sin\beta_{II}[J_o(\lambda\varepsilon^2)Y_o'(\lambda\varepsilon^2)-J_o'(\lambda\varepsilon^2)Y_o(\lambda\varepsilon^2)]} \tag{3.198}$$

Defining:

$$\Lambda_5 = \cos(k\ell_1\varepsilon^2)J_o(\lambda\varepsilon^2) - \sin(k\ell_1\varepsilon^2)J_o'(\lambda\varepsilon^2) \tag{3.199}$$

equation (3.198) becomes:

$$b_{II} = \frac{\Lambda_5}{\Lambda_2\sin\beta_{II}} \tag{3.200}$$

Setting equations (3.191) and (3.197) equal we obtain:

$$\tan\delta_{II} = \frac{\Lambda_4}{\Lambda_1} \quad \text{or} \quad \delta_{II} = \tan^{-1}(\frac{\Lambda_4}{\Lambda_1}) \tag{3.201}$$

Similarly from (3.194) and (3.200) we get:

$$\tan\beta_{II} = \frac{\Lambda_5}{\Lambda_3} \quad \text{or} \quad \beta_{II} = \tan^{-1}(\frac{\Lambda_5}{\Lambda_3}) \tag{3.202}$$

Knowing now the values of δ_{II}, β_{II}, a_{II} and b_{II} we substitute into equations (3.180), (3.181), (3.182) and (3.183) and we have the following system:

$$a_I\cos(k\ell_1+\delta_I)+b_I\cos(k\ell_1-\beta_I) = [a_{II}\cos(\delta_{II}-\sigma t)J_o(\lambda)+b_{II}\cos(\beta_{II}-\sigma t)Y_o(\lambda)]=M_1 \tag{3.203}$$

$$a_I\sin(k\ell_1+\delta_I)-b_I\sin(k\ell_1-\beta_I) = [a_{II}\sin\delta_{II}J_o(\lambda)+b_{II}\sin\beta_{II}Y_o(\lambda)]=M_2 \tag{3.204}$$

$$a_I\sin(k\ell_1+\delta_I)+b_I\sin(k\ell_1-\beta_I) = -[a_{II}\cos\delta_{II}J_o'(\lambda)+b_{II}\cos\beta_{II}Y_o'(\lambda)]=M_3 \tag{3.205}$$

$$a_I\cos(k\ell_1+\delta_I)-b_I\cos(k\ell_1-\beta_I) = [a_{II}\sin\delta_{II}J_o'(\lambda)+b_{II}\sin\beta_{II}Y_o'(\lambda)]=M_4 \tag{3.206}$$

defining M_1, M_2, M_3, M_4 as indicated above.

From (3.203) and (3.204) we obtain for a_I and b_I:

$$a_I = \frac{M_1\sin(k\ell_1-\beta_I) + M_2\cos(k\ell_1-\beta_I)}{\cos(k\ell_1+\delta_I)\sin(k\ell_1-\beta_I)+\cos(k\ell_1-\beta_I)\sin(k\ell_1+\delta_I)} \tag{3.207}$$

Similarly for b_I:

$$b_I = \frac{M_1\sin(k\ell_1+\delta_I) - M_2\cos(k\ell_1+\delta_I)}{\sin(k\ell_1-\beta_I)\cos(k\ell_1+\delta_I)+\cos(k\ell_1-\beta_I)\sin(k\ell_1+\delta_I)} \tag{3.208}$$

With a similar procedure we get from (3.205) and (3.206) a_I and b_I:

$$a_I = \frac{M_3\cos(k\ell_1-\beta_I) + M_4\sin(k\ell_1-\beta_I)}{\sin(k\ell_1+\delta_I)\cos(k\ell_1-\beta_I)+\sin(k\ell_1-\beta_I)\cos(k\ell_1+\delta_I)} \qquad (3.209)$$

and for b_I:

$$b_I = \frac{M_3\cos(k\ell_1+\delta_I) - M_4\sin(k\ell_1+\delta_I)}{\sin(k\ell_1+\delta_I)\cos(k\ell_1-\beta_I)+\sin(k\ell_1-\beta_I)\cos(k\ell_1+\delta_I)} \qquad (3.210)$$

Setting equations (3.207) and (3.210) equal after considerable reduction we obtain:

$$\tan(k\ell_1-\beta_I) = \frac{M_2-M_3}{M_1+M_4}$$

or

$$\beta_I = k\ell_1-\tan^{-1}\left(\frac{M_2-M_3}{M_1+M_4}\right) \qquad (3.211)$$

Similarly setting equations (3.209) and (3.211) equal following the same procedure after considerable reduction we obtain:

$$\tan(k\ell_1+\delta_I) = \frac{M_2+M_3}{M_1+M_4}$$

and

$$\delta_I = \tan^{-1}\left(\frac{M_2+M_3}{M_1+M_4}\right)-k\ell_1 \qquad (3.212)$$

In summary for the C transition of gradually varying width the values of amplitudes, phase angles, reflection and transmission coefficients are given explicitly as follows:

(i) Amplitudes, reflection and transmission coefficients:

$$a_I = \frac{M_1 \sin(k\ell_1-\beta_I) + M_2 \cos(k\ell_1-\beta_I)}{\sin(2k\ell_1+\delta_I-\beta_I)}$$

$$b_I = \frac{M_1 \sin(k\ell_1+\delta_I) - M_2 \cos(k\ell_1+\delta_I)}{\sin(2k\ell_1+\delta_I-\beta_I)}$$

$$K_r = \frac{b_I}{a_I} = \frac{\sin(k\ell_1-\beta_I) - \frac{M_2}{M_1}\cos(k\ell_1-\beta_I)}{\sin(k\ell_1+\delta_I) + \frac{M_2}{M_1}\cos(k\ell_1-\beta_I)}$$

$$a_{II} = \frac{\Lambda_1}{\Lambda_2 \cos\delta_{II}}$$

$$b_{II} = \frac{\Lambda_3}{\Lambda_2 \cos\beta_{II}}$$

$$K_t = \frac{1}{a_I} = \frac{\sin(2k\ell_1+\delta_I-\beta_I)}{M_1 \sin(k\ell_1-\beta_I) + M_2 \cos(k\ell_1-\beta_I)}$$

$$a_{III} = 1 \quad , \quad b_{III} = 0$$

(ii) phase angles:

$$\delta_I = \tan^{-1}\left(\frac{M_2+M_3}{M_1+M_4}\right) - k\ell_1$$

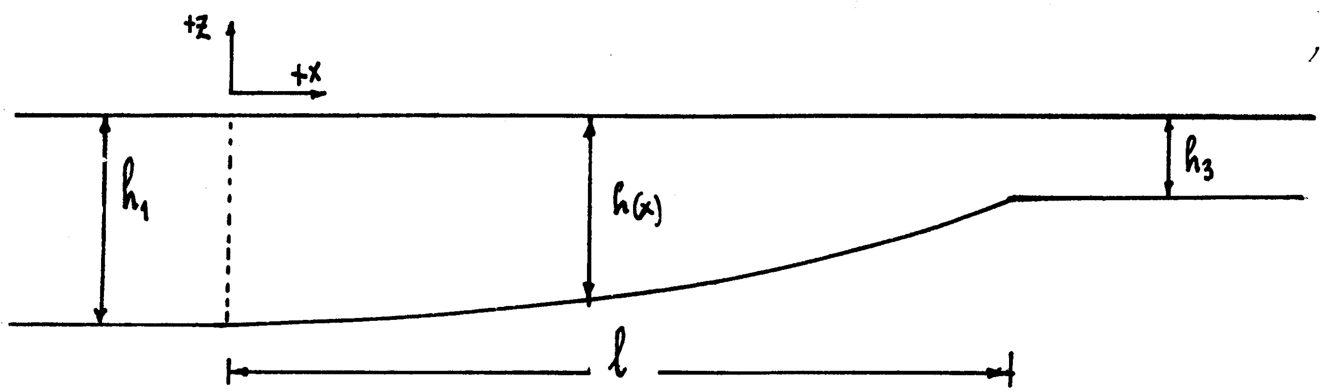
$$\beta_I = k\ell_1 - \tan^{-1}\left(\frac{M_2-M_3}{M_1+M_4}\right)$$

$$\delta_{II} = \tan^{-1}\left(\frac{\Lambda_4}{\Lambda_1}\right)$$

$$\beta_{II} = \tan^{-1}\left(\frac{\Lambda_5}{\Lambda_3}\right)$$

$$\delta_{III} = \beta_{III} = 0$$

3.5 CASE D OF TRANSITION: PARABOLIC VARIATION OF DEPTH - CONSTANT WIDTH



The geometry of the parabolic transition of depth for a channel of constant width may be assumed by:

$$z = C_1 x^2 + C_2 \qquad (3.213)$$

For the determination of C_1 and C_2 we have:

at $x = 0$ $\qquad z = -h_1$

$x = \ell$ $\qquad z = -h_2$

Thus from equation (3.213) we get:

$$C_2 = -h_1$$

$$C_1 \ell^2 + C_2 = -h_2$$

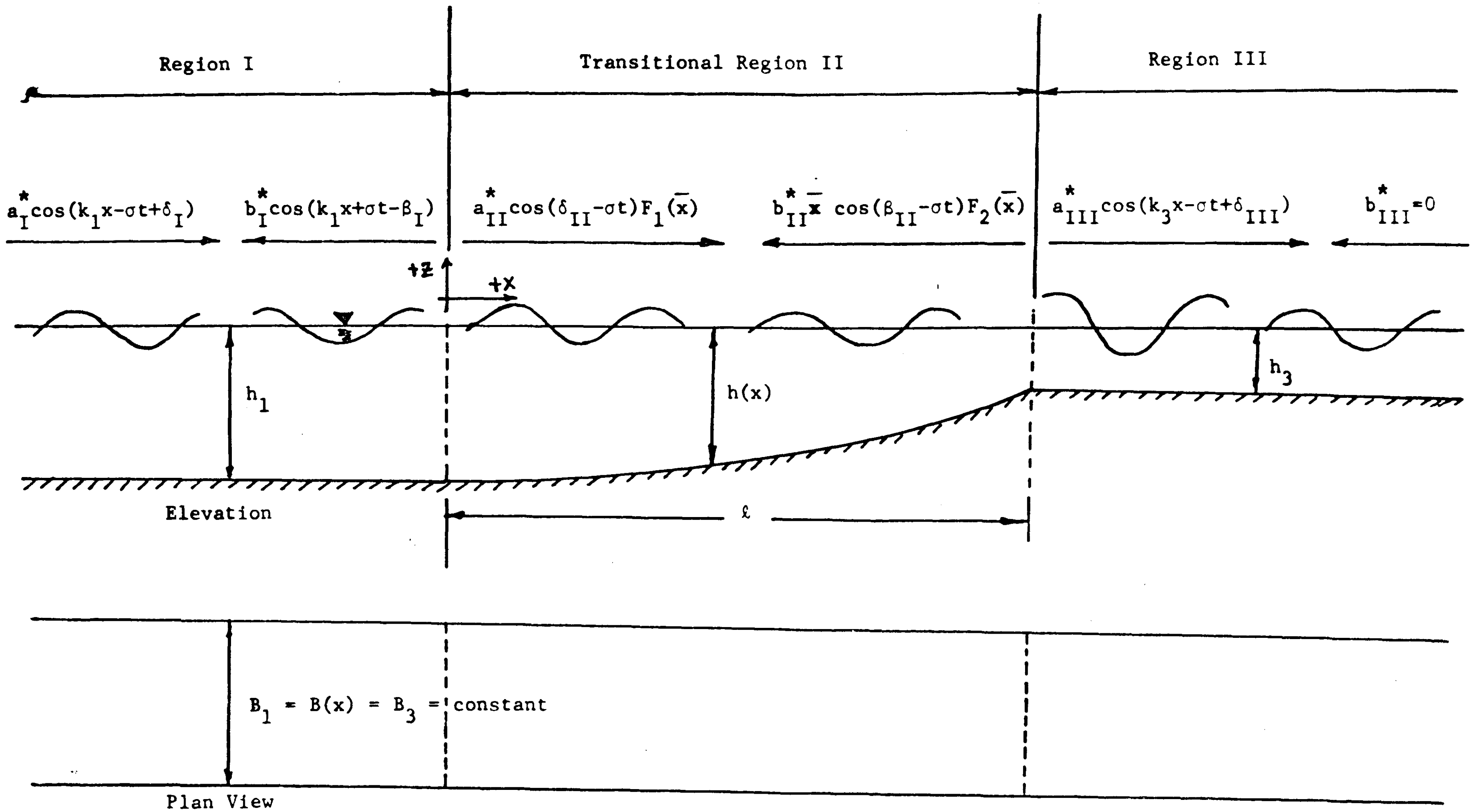


Fig. 11. Schematic Diagram of Case D of Transition Parabolic Variation of Depth - Constant Width.

Thus

$$C_1 = \frac{h_1-h_2}{\ell^2}$$

and since

$$z(x) = -h(x)$$

we get:

$$h(x) = -[\frac{h_1-h_2}{\ell^2} x^2 - h_1] = h_1 - \frac{h_1-h_2}{\ell^2} x^2$$

or

$$h(x) = h_1(1 - \frac{h_1-h_3}{\ell^2 h_1} x^2) \qquad (3.214)$$

or

$$h(x) = h_1(1 - \frac{x^2}{\alpha_1^2}) \qquad (3.215)$$

where $\alpha_1 = \ell \sqrt{\frac{h_1}{h_1-h_3}}$ (3.216)

From the geometry of the transition we have:

(i) <u>Region I (Upstream)</u>

$B = B_1 = B_3 = \text{constant}$ $\quad - \infty < x \leqslant 0$

$h = h_1 = \text{constant}$ $\quad - \infty < x \leqslant 0$

(ii) Region II (Transition)

$$B = B_1 = B_3 = \text{constant} \qquad 0 \leq x \leq \ell \text{ or } 0 \leq \frac{x^2}{\alpha_1^2} \leq \frac{\ell^2}{\alpha_1^2}$$

$$h(x) = h_1(1 - \frac{x^2}{\alpha_1^2}) \qquad \text{or} \qquad 0 \leq \frac{x^2}{\alpha_1^2} \leq (\frac{h_1-h_3}{h_1}) < 1$$

The fact that $\frac{x^2}{\alpha_1^2} < 1$ is of significance in the following development since the convergence of the hypergeometric series of the problem depends on it.

(iii) Region III (Downstream)

$$B = B_1 = B_3 = \text{constant}$$

$$h = h_3 = \text{constant}$$

From the previous developments for the other cases the combined equation of motion and continuity (3.112) is:

$$\eta_{tt} = \frac{g}{B(x)} [A(x)\eta_x]_x$$

Since B is constant throughout:

$$A(x) = Bh_1(1 - \frac{x^2}{\alpha_1^2}) \qquad (3.217)$$

Substituting the expression for A(x) into (3.112) we get:

$$\eta_{tt} = gh_1[(1 - \frac{x^2}{\alpha_1^2})\eta_x]_x \qquad (3.218)$$

Assuming a solution of simple harmonic motion of the type $\eta(x,t) = \bar{\eta}(x)e^{-i\sigma t}$ and substituting η_{xx}, η_x and η_{tt}:

$$\frac{d}{dx}\left[\left(1 - \frac{x^2}{\alpha_1^2}\right)\frac{d\bar{\eta}}{dx}\right] + \frac{\sigma^2}{gh_1}\bar{\eta} = 0 \qquad (3.219a)$$

Putting $\frac{x}{\alpha_1} = \bar{x}$ and $\frac{dx}{d\bar{x}} = \alpha_1$:

$$\frac{d}{d\bar{x}}\left[(1-\bar{x}^2)\frac{d\bar{\eta}}{d\bar{x}}\right] + \frac{\sigma^2\alpha_1^2}{gh_1}\bar{\eta} = 0 \qquad (3.219b)$$

This is a special case of the Sturm-Liouville equation. The solutions when satisfying given boundary conditions are known as eigen functions. In order to solve this equation we restrict the expression of the eigen values $\frac{\sigma^2\alpha_1^2}{gh_1}$ to the values $n(n+1)$ wherein $n = 1,2,3....$ Equation (3.219b) has the following form:

$$(\bar{x}^2-1)\frac{d\bar{\eta}^2}{d\bar{x}^2} + 2\bar{x}\frac{d\bar{\eta}}{d\bar{x}} - n(n+1)\bar{\eta} = 0 \qquad (3.220)$$

This is the Gauss-Legendre differential equation which has solutions in convergent hypergeometric series since $|\bar{x}| = |\frac{x}{\alpha_1}| < 1$.

It should be noted that the restriction imposed on the expression of eigen values:

$$\frac{\sigma^2\alpha_1^2}{gh_1} = n(n+1) \quad \text{or} \quad \sigma = \frac{1}{\alpha_1}\left[n(n+1)gh_1\right]^{1/2} \qquad (3.221)$$

gives the admissible values of frequencies σ for $n = 1,2,3...$, i.e. the different modes of the boundary value problem and therefore the different wave lengths for each channel depth. The solution of equation (3.220) is given by:

$$\bar{\eta}(\bar{x}) = C_1 F_1\left(-\frac{n}{2}, \frac{1+n}{2}, \frac{1}{2}, \bar{x}\right) + C_2 \bar{x} F_2\left(\frac{1-n}{2}, \frac{2+n}{2}, \frac{3}{2}, \bar{x}\right) \quad (3.222)$$

wherein $F_1(\bar{x})$, $F_2(\bar{x})$ are convergent hypergeometric functions defined as follows:

$$F_{1,2}(\alpha, \beta, \gamma, \bar{x}) = 1 + \sum_{\nu=1}^{\infty} \frac{\alpha(\alpha+1)\ldots(\alpha+\nu-1)\ldots\beta(\beta+1)\ldots(\beta+\nu-1)}{!\ \gamma(\gamma+1)\ldots(\gamma+\nu-1)} \bar{x}^{\nu}$$

$$= 1 + \sum_{\nu=1}^{\infty} \frac{\binom{\alpha+\nu-1}{\nu}\binom{\beta+\nu-1}{\nu}}{\binom{\gamma+\nu-1}{\nu}} \bar{x}^{\nu} \quad (3.223)$$

The above series converges for $|\bar{x}| < 1$, since $\bar{x} = \frac{x}{\alpha_1} = \left(\frac{h_1-h_3}{h_1}\right)^{1/2} \cdot \frac{x}{\ell}$ is always smaller than unity. The differentiation of the hypergeometric function (needed in using the B.C.) is permissible.

In the above series we have for F_1 associated with an arbitrary constant of integration C_1:

$$\alpha = -\frac{n}{2}, \quad \beta = \frac{1+n}{2}, \quad \gamma = \frac{1}{2}$$

and for F_2 associated with C_2

$$\alpha = \frac{1-n}{2}, \quad \beta = \frac{2+n}{2}, \quad \gamma = \frac{3}{2}$$

Taking into consideration the time component equation (3.222) is:

$$\eta(\bar{x},t) = \bar{\eta}(\bar{x})e^{-i\sigma t} = [C_1F_1(\bar{x}) + \bar{x}C_2F_2(\bar{x})]e^{-i\sigma t} \qquad (3.224)$$

Considering C_1 and C_2 as complex constants of the type $C_I = a^*_{II}e^{i\delta_{II}}$ the wave surface elevation $\eta(x,t)$ over the transitional region II can be written as:

$$\eta(\bar{x},t) = a^*_{II}\cos(\delta_{II}-\sigma t)F_1(\bar{x}) + b^*_{II}\bar{x}\cos(\beta_{II}-\sigma t)F_2(\bar{x}) \qquad (3.225)$$

For the upstream and downstream regions I and III the solutions are as in the former cases A, B and C:

$$\eta_I(\bar{x},t) = a^*_I\cos(k_I\alpha_1\bar{x}-\sigma t+\delta_I) + b^*_I\cos(k_I\alpha_1\bar{x}+\sigma t-\beta_I) \qquad (3.226)$$

$$\eta_{III}(\bar{x},t) = a^*_{III}\cos(k_3\alpha_1\bar{x}-\sigma t) \qquad (3.227)$$

assuming that there is no reflected wave in the downstream region, i.e. $b^*_{III}=0$. Considering the amplitudes in dimensionless form $a_I = a^*_I/a^*_{III}$, $b_I = b^*_I/b^*_{III}$, etc., and inserting the following B.C.

$$\eta_I(x,t)\Big|_{x=0} = \eta_{II}(x,t)\Big|_{x=0} \qquad \eta_{II}(x,t)\Big|_{x=\ell} = \eta_{III}(x,t)\Big|_{x=\ell}$$

$$[\eta_I(x,t)]_x\Big|_{x=0} = [\eta_{II}(x,t)]_x\Big|_{x=0} \qquad [\eta_{II}(x,t)]_x\Big|_{x=\ell} = [\eta_{III}(x,t)]_x\Big|_{x=\ell}$$

we get after evaluation of these relations for $\sigma t=0$ and $\sigma t=\frac{\pi}{2}$:

$$a_I \cos\delta_I + b_I \cos\beta_I = a_{II} \cos\delta_{II} F_1(0) \tag{3.228}$$

$$a_I \sin\delta_I + b_I \sin\beta_I = a_{II} \sin\delta_{II} F_1(0) \tag{3.229}$$

$$-a_I k_I \alpha_1 \sin\delta_I + b_I k_I \alpha_1 \sin\beta_I = a_{II} \cos\delta_{II} F_1'(0) + b_{II} \cos\beta_{II} F_2(0) \tag{3.230}$$

$$a_I k_I \alpha_1 \cos\delta_I - b_I k_I \alpha_1 \cos\beta_I = a_{II} \sin\delta_{II} F_1'(0) + b_{II} F_2(0) \sin\beta_{II} \tag{3.231}$$

$$a_{II} \cos\delta_{II} F_1(\ell) + b_{II} \cos\beta_{II} F_2(\ell) \frac{\ell}{\alpha_1} = \cos(k_3 \ell) \tag{3.232}$$

$$a_{II} \sin\delta_{II} F_1(\ell) + b_{II} \sin\beta_{II} F_2(\ell) \frac{\ell}{\alpha_1} = \sin(k_3 \ell) \tag{3.233}$$

$$a_{II} \cos\delta_{II} F_1'(\ell) + \left[\frac{\ell}{\alpha_1} F_2'(\ell) + F_2(\ell)\right] b_{II} \cos\beta_{II} = -k_3 \alpha_1 \sin(k_3 \ell) \tag{3.234}$$

$$a_{II} \sin\delta_{II} F_1'(\ell) + \left[\frac{\ell}{\alpha_1} F_2'(\ell) + F_2(\ell)\right] b_{II} \sin\beta_{II} = k_3 \alpha_1 \sin(k_3 \ell) \tag{3.235}$$

Defining

$$\frac{\ell}{\alpha_1} F_2'(\ell) + F_2(\ell) = F_2^*(\ell) \tag{3.236}$$

the system of equations (3.233) up to (3.236) becomes:

$$a_{II} \cos\delta_{II} F_1(\ell) + b_{II} \cos\beta_{II} F_2(\ell) \frac{\ell}{\alpha_1} = \cos(k_3 \ell) \tag{3.237}$$

$$a_{II}\sin\delta_{II}F_1(\ell)+b_{II}\sin\beta_{II}F_2(\ell)\,\frac{\ell}{\alpha_1} = \sin(k_3\ell) \tag{3.238}$$

$$a_{II}\cos\delta_{II}F_1'(\ell)+b_{II}F_2^*(\ell)\cos\beta_{II} = -k_3\alpha_1\sin(k_3\ell) \tag{3.239}$$

$$a_{II}\sin\delta_{II}F_1'(\ell)+b_{II}F_2^*(\ell)\sin\beta_{II} = k_3\alpha_1\cos(k_3\ell) \tag{3.240}$$

Function $F_1(\bar{x})$ evaluated for x=0 is:

$$F_1(0) \equiv F_1(a,\beta,\gamma,\frac{x}{\alpha_1})\Big|_{x=0} = 1 \tag{3.241}$$

and

$$F_1'(0) \equiv F_1'(a,\beta,\gamma,\frac{x}{\alpha_1})\Big|_{x=0} = \frac{a\beta}{\gamma} = -\frac{\frac{n}{2}(\frac{1+n}{2})}{\frac{1}{2}} = -\frac{n}{2}(1+n) \tag{3.242}$$

The same function for $x=\ell$ results in:

$$F_1(\ell) \equiv F_1(a,\beta,\gamma,\bar{x})_{\bar{x}=\frac{\ell}{\alpha_1}} = 1 + \frac{a\beta}{\gamma}(\frac{\ell}{\alpha_1})$$

$$+ \frac{a(a+1)\beta(\beta+1)}{1\cdot2\cdot\gamma(\gamma+1)}(\frac{\ell}{\alpha_1})^2 + \frac{a(a+1)(a+2)\beta(\beta+1)(\beta+2)}{1\cdot2\cdot3\cdot\gamma(\gamma+1)(\gamma+2)}(\frac{\ell}{\alpha_1})^3+\ldots$$

$$\ldots-\frac{a(a+1)(a+2)\ldots(a+\nu-1)\beta(\beta+1)(\beta+2)\ldots(\beta+\nu-1)}{1.2.3\ldots\nu.\gamma(\gamma+1)(\gamma+2)\ldots(\gamma+\nu-1)}(\frac{\ell}{\alpha_1})^{\nu}+\ldots \tag{3.243}$$

and

$$F_1'(\ell) \equiv F_1'(a,\beta,\gamma,\bar{x})\Big|_{\bar{x}=\frac{\ell}{\alpha_1}} = \frac{a\beta}{\gamma} + \frac{a(a+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}.2(\frac{\ell}{\alpha_1})+\ldots$$

$$+\ldots\frac{a(a+1)\ldots(a+\nu-1)\beta(\beta+1)\ldots(\beta+\nu-1)}{1.2.\ldots\nu.\gamma(\gamma+1)\ldots(\gamma+\nu-1)}\,\nu(\frac{\ell}{\alpha_1})^{\nu-1}+\ldots \qquad (3.244)$$

In all above expressions for $F_1(0)$, $F_1'(0)$, $F_1(\ell)$, $F_1'(\ell)$ the coefficients are: $a = -\frac{n}{2}$, $\beta = \frac{1+n}{2}$, $\gamma = \frac{1}{2}$.

For the hypergeometric series the values of coefficients are $a = \frac{1-n}{2}$, $\beta = \frac{2+n}{2}$, $\gamma = \frac{3}{2}$ and can be evaluated as follows:

$$F_2(0) \equiv F_2(a,\beta,\gamma,\frac{x}{\alpha_1})\Big|_{x=0} = 1 \qquad (3.245)$$

$$F_2'(0) \equiv F_2'(a,\beta,\gamma,\frac{x}{\alpha_1})\Big|_{x=0} = \frac{(1-n)(2+n)}{6} \qquad (3.246)$$

$$F_2(\ell) \equiv F_2(a,\beta,\gamma,\bar{x})\Big|_{\bar{x}=\frac{\ell}{\alpha_1}} = 1 + \frac{a\beta}{\gamma}(\frac{\ell}{\alpha_1}) + \frac{a(a+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}(\frac{\ell}{\alpha_1})^2$$

$$+\ldots\frac{a(a+1)(a+2)\ldots(a+\nu-1)\beta(\beta+1)(\beta+2)\ldots(\beta+\nu-1)}{1.2.3.\ldots\nu.\gamma(\gamma+1)(\gamma+2)\ldots(\gamma+\nu-1)}(\frac{\ell}{\alpha_1})^{\nu}+\ldots \qquad (3.247)$$

$$F_2'(\ell) \equiv F_2'(a,\beta,\gamma,\bar{x})\Big|_{\bar{x}=\frac{\ell}{\alpha_1}} = \frac{a\beta}{\gamma} + \frac{a(a+1)\beta(\beta+1)}{1.2.\gamma(\gamma+1)}\,2(\frac{\ell}{\alpha_1})+\ldots$$

$$+\ldots\frac{a(a+1)\ldots(a+\nu-1)\beta(\beta+1)\ldots(\beta+\nu-1)}{1.2.\ldots\nu.\gamma(\gamma+1)\ldots(\gamma+\nu-1)}(\frac{\ell}{\alpha_1})^{\nu-1} \qquad (3.248)$$

Under these determinations for F_1 and F_2 we can solve the system of equations

(3.237) up to (3.240). From (3.237) and (3.238) we get for the amplitude a_{II}:

$$a_{II} = \frac{\frac{\ell}{\alpha_1}[\sin\beta_{II}\cos(k_3\ell)-\sin(k_3\ell)\cos\beta_{II}]}{F_1(\ell)[\cos\delta_{II}\sin\beta_{II}-\sin\delta_{II}\cos\beta_{II}]}$$

$$a_{II} = \frac{\frac{\ell}{\alpha_1}\sin(\beta_{II}-k_3\ell)}{F_1(\ell)\sin(\beta_{II}-\delta_{II})} \qquad (3.249)$$

From equations (3.239) and (3.240) we get for the amplitude a_{II}:

$$a_{II} = \frac{-k_3\alpha_1[\sin(k_3\ell)\sin\beta_{II}+\cos(k_3\ell)\cos\delta_{II}]}{F_1'(\ell)[\cos\delta_{II}\sin\beta_{II}-\sin\delta_{II}\cos\beta_{II}]} = \frac{-k_3\alpha_1\cos(\beta_{II}-k_3\ell)}{F_1'(\ell)\sin(\beta_{II}-\delta_{II})} \qquad (3.250)$$

Setting equations (3.249) and (3.250) equal we get:

$$\tan(\beta_{II}-k_3\ell) = -\frac{k_3\alpha_1^2F_1(\ell)}{\ell F_1'(\ell)}$$

and

$$b_{II} = k_3\ell + \tan^{-1}[-\frac{k_3\alpha_1F_1(\ell)}{\ell F_1'(\ell)}] \qquad (3.251)$$

Similarly for the amplitude b_{II} from (3.237) and (3.238) we get:

$$b_{II} = \frac{\sin(k_3\ell-\delta_{II})}{F_2(\ell)\sin(\beta_{II}-\delta_{II})} \qquad (3.252)$$

From equations (3.239) and (3.240) we get:

$$b_{II} = \frac{k_3\alpha_1\cos(k_3\ell-\delta_{II})}{F_2^*(\ell)\sin(\beta_{II}-\delta_{II})} \tag{3.253}$$

Setting equations (3.252) and (3.253) equal we get:

$$\tan(k_3\ell-\delta_{II}) = \frac{k_3\alpha_1F_2(\ell)}{F_2^*(\ell)}$$

or

$$\delta_{II} = k_3\ell-\tan^{-1}[\frac{k_3\alpha_1F_2(\ell)}{F_2^*(\ell)}] \tag{3.254}$$

With the values of amplitudes a_{II}, b_{II}, and phase angles β_{II} and δ_{II}, known the system of equations (3.228) up to (3.231) can be solved. Defining;

$$a_{II}\cos\delta_{II}F_1(0) = a_{II}\cos\delta_{II} = \Delta_1 \tag{3.255}$$

$$a_{II}\sin\delta_{II}F_1(0) = \Delta_2 \tag{3.256}$$

$$\frac{1}{k_1\alpha_1}[a_{II}\cos\delta_{II}F_1'(0)+b_{II}\cos\beta_{II}F_2(0)] = \Delta_3 \tag{3.257}$$

$$\frac{1}{k_1\alpha_1}[a_{II}\sin\delta_{II}F_1'(0)+b_{II}F_2(0)\sin\beta_{II}] = \Delta_4 \tag{3.258}$$

Hence the system of equations (3.228) up to (3.231) becomes:

$$a_I\cos\delta_I+b_I\cos\beta_I = \Delta_1 \tag{3.259}$$

$$a_I \sin\delta_I + b_I \sin\beta_I = \Delta_2 \tag{3.260}$$

$$-a_I \sin\delta_I + b_I \sin\beta_I = \Delta_3 \tag{3.261}$$

$$a_I \cos\delta_I - b_I \cos\beta_I = \Delta_4 \tag{3.262}$$

From (3.259) and (3.260) we get:

$$a_I = \frac{(\Delta_1 \sin\beta_I - \Delta_2 \cos\beta_I)}{\cos\delta_I \sin\beta_I - \sin\delta_I \cos\beta_I} = \frac{(\Delta_1 \sin\beta_I - \Delta_2 \cos\beta_I)}{\sin(\beta_I - \delta_I)} \tag{3.263}$$

and from (3.261) and (3.262) we get:

$$a_I = \frac{(\Delta_3 \cos\beta_I + \Delta_4 \sin\beta_I)}{-\sin\delta_I \cos\beta_I + \sin\delta_I \cos\beta_I} = \frac{(\Delta_3 \cos\beta_I + \Delta_4 \sin\beta_I)}{\sin(\beta_I - \delta_I)} \tag{3.264}$$

Setting equations (3.263) and (3.264) equal we get:

$$\frac{\Delta_1 \sin\beta_I - \Delta_2 \cos\beta_I}{\Delta_3 \cos\beta_I + \Delta_4 \sin\beta_I} = 1$$

or

$$\frac{\Delta_1 \tan\beta_I - \Delta_2}{\Delta_3 + \Delta_4 \tan\beta_I} = 1$$

or

$$\tan\beta_I = \frac{\Delta_2 + \Delta_3}{\Delta_1 - \Delta_4}$$

$$\beta_I = \tan^{-1}\left(\frac{\Delta_2 + \Delta_3}{\Delta_1 - \Delta_4}\right) \tag{3.265}$$

In a similar procedure we get for b_{II} from (3.259) and (3.260):

$$b_I = \frac{\Delta_2\cos\delta_I - \Delta_1\sin\delta_I}{\sin(\beta_I - \delta_I)} \tag{3.266}$$

and from (3.261) and (3.262):

$$b_I = \frac{(\Delta_4\sin\delta_I + \Delta_3\cos\delta_I)}{\sin(\beta_I - \delta_I)} \tag{3.267}$$

Setting equations (3.266) and (3.267) equal we obtain:

$$\Delta_2\cos\delta_I - \Delta_1\sin\delta_I = \Delta_3\cos\delta_I + \Delta_4\sin\delta_I$$

$$\Delta_2 - \Delta_1\tan\delta_I = \Delta_3 + \Delta_4\tan\delta_I$$

$$(\Delta_4 + \Delta_1)\tan\delta_I = \Delta_2 - \Delta_3$$

$$\tan\delta_I = \frac{\Delta_2 - \Delta_3}{\Delta_1 + \Delta_4}$$

$$\delta_I = \tan^{-1}\left(\frac{\Delta_2 - \Delta_3}{\Delta_1 + \Delta_4}\right) \tag{3.268}$$

In summary for the transition D of parabolic variation of depth the values of the amplitudes and phase angles, reflection and transmission coefficients are given explicitly as follows:

(i) Amplitudes, reflection and transmission coefficients:

$$a_I = \frac{\Delta_3\cos\beta_I + \Delta_4\sin\beta_I}{\sin(\beta_I - \delta_I)}$$

$$b_I = \frac{\Delta_2 \cos\beta_I - \Delta_1 \sin\beta_I}{\sin(\beta_I - \delta_I)}$$

$$K_r = \frac{b_I}{a_I} = \frac{\Delta_2 \cos\delta_I - \Delta_1 \sin\delta_I}{\Delta_3 \cos\beta_I + \Delta_4 \sin\beta_I}$$

$$K_t = \frac{1}{a_I} = \frac{\sin(\beta_I - \delta_I)}{\Delta_3 \cos\beta_I + \Delta_4 \sin\beta_I}$$

$$a_{II} = \frac{\ell}{\alpha_1 F_1(\ell)} \frac{\sin(\beta_{II} - k_3 \ell)}{\sin(\beta_{II} - \delta_{II})}$$

$$b_{II} = \frac{1}{F_2(\ell)} \frac{\sin(k_3 \ell - \delta_{II})}{\sin(\beta_{II} - \delta_{II})}$$

$$a_{III} = 1 \quad , \quad b_{III} = 0$$

(ii) Phase angles:

$$\delta_I = \tan^{-1}\left(\frac{\Delta_2 - \Delta_3}{\Delta_1 + \Delta_4}\right)$$

$$\beta_I = \tan^{-1}\left(\frac{\Delta_2 + \Delta_3}{\Delta_1 - \Delta_4}\right)$$

$$\delta_{II} = k_3 \ell - \tan^{-1}\left(\frac{k_3 \alpha_1 F_2(\ell)}{F_2^*(\ell)}\right)$$

$$\beta_{II} = k_3 \ell + \tan^{-1}\left(\frac{-k_3 \alpha_1^2 F_1(\ell)}{F_1'(\ell)}\right)$$

$$\delta_{III} = \beta_{III} = 0$$

IV. EXPERIMENTAL EQUIPMENT FOR THE TEST PROGRAM

4.1 General Description of the Wave Tank and the Transitions A, B, C

Figure 12 gives a schematic representation of the equipment and the experimental tank used for the present investigation.

The detailed description of the set-up is given in the previous Technical Report No. 72 of the Hydrodynamics Laboratory. Briefly reviewing the essentials, the wave tank is of rectangular cross section with a length of 100 ft., a width of 2.5 ft. and a depth of 3 ft. The wall over the entire length and 40 ft. of the bottom in the upstream section near the wave maker region of the channel are of plate glass. The remaining 60 ft. of the channel bottom consists of steel plates. Two wave makers, a piston type wave maker or a flap type wave maker are available at one end. Energy absorbers were placed normally at the other end of the channel, different in shape and arrangement according to the type of transition A, B or C. Near the wave maker at a distance about 4 ft. an expanded aluminum filter is located to smooth out secondary wave disturbances.

The test program was conducted with three linearly sloping transitions:

(i) Transition A (figure 13) with linearly varying depth with a slope of 1:8 ($\alpha=7.16^{\circ}$) and constant width $B_1=B_3$ upstream and downstream. The depth reduction over the length of the transition is one foot.

(ii) Transition B (figure 14) with linearly varying depth with a slope 1:8 ($\alpha=7.16^{\circ}$) and a symmetrical side wall contraction of 1:12.80 ($\alpha=4.46^{\circ}$)

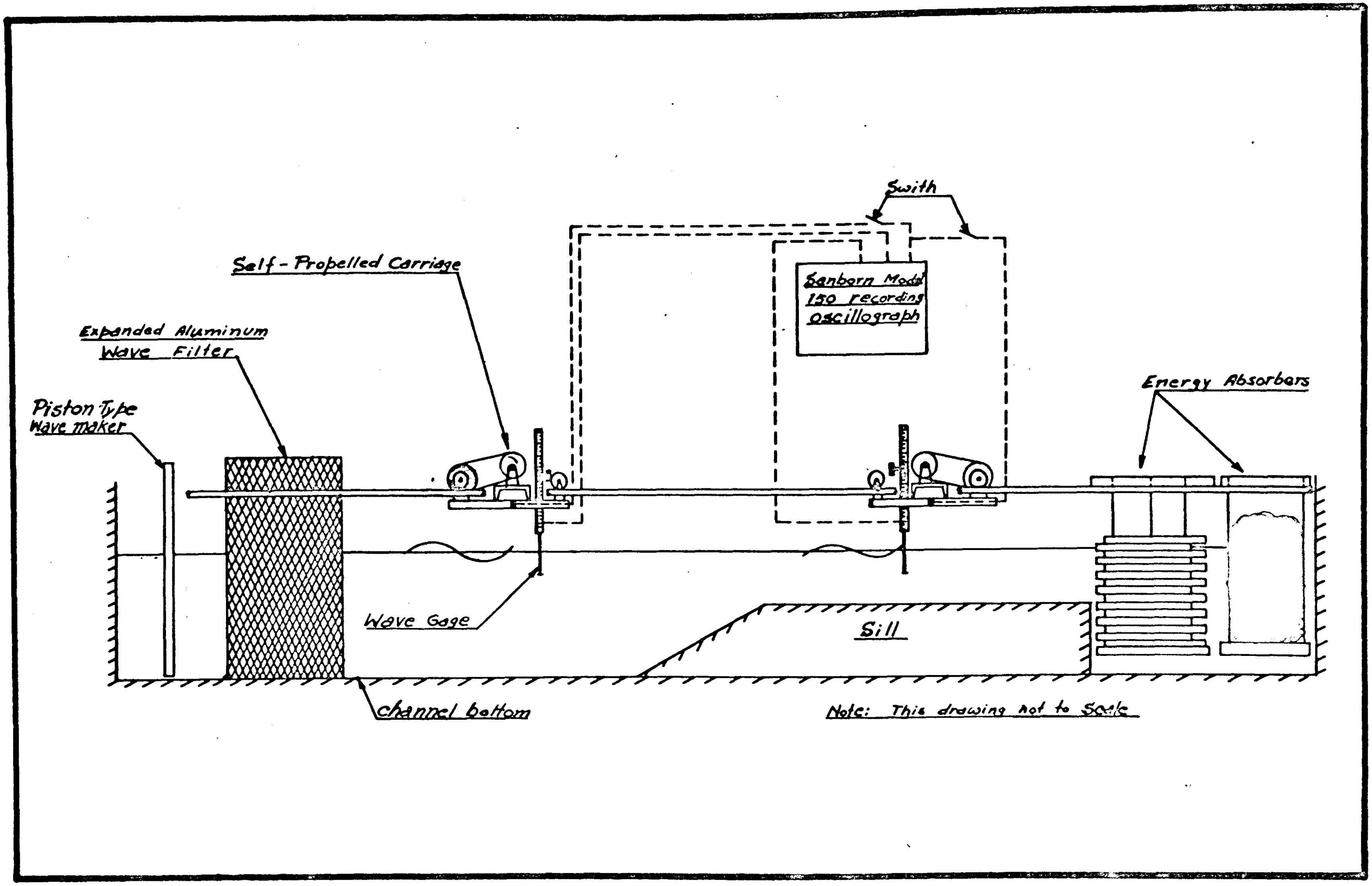


Fig 12 Schematic Diagram of Experimental Equipment

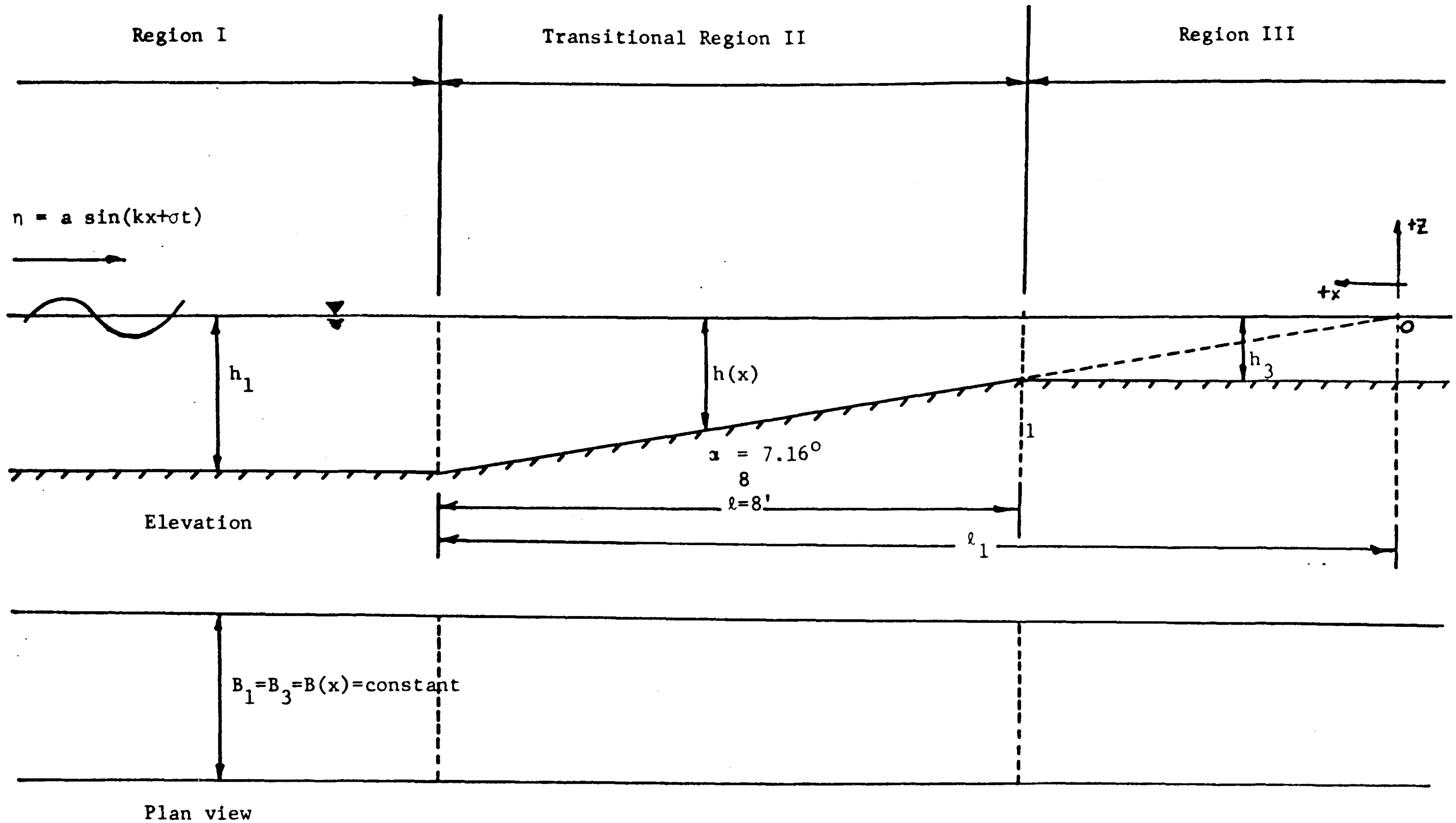


Fig. 13. Schematic Diagram of Case A of Transition Used for the Experimental Study

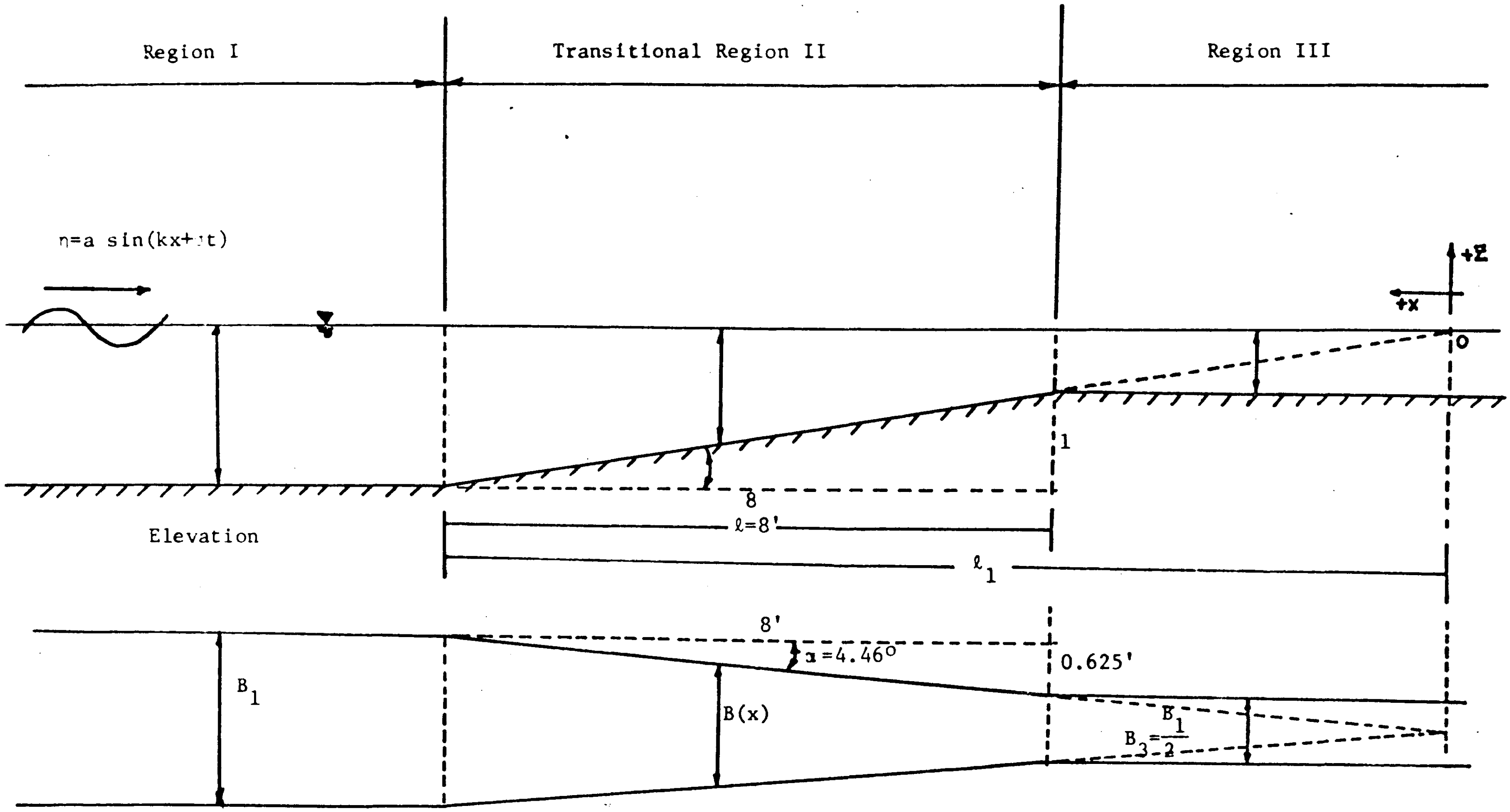


Fig. 14. Schematic Diagram of Case B of Transition Used for the Experimental Study.

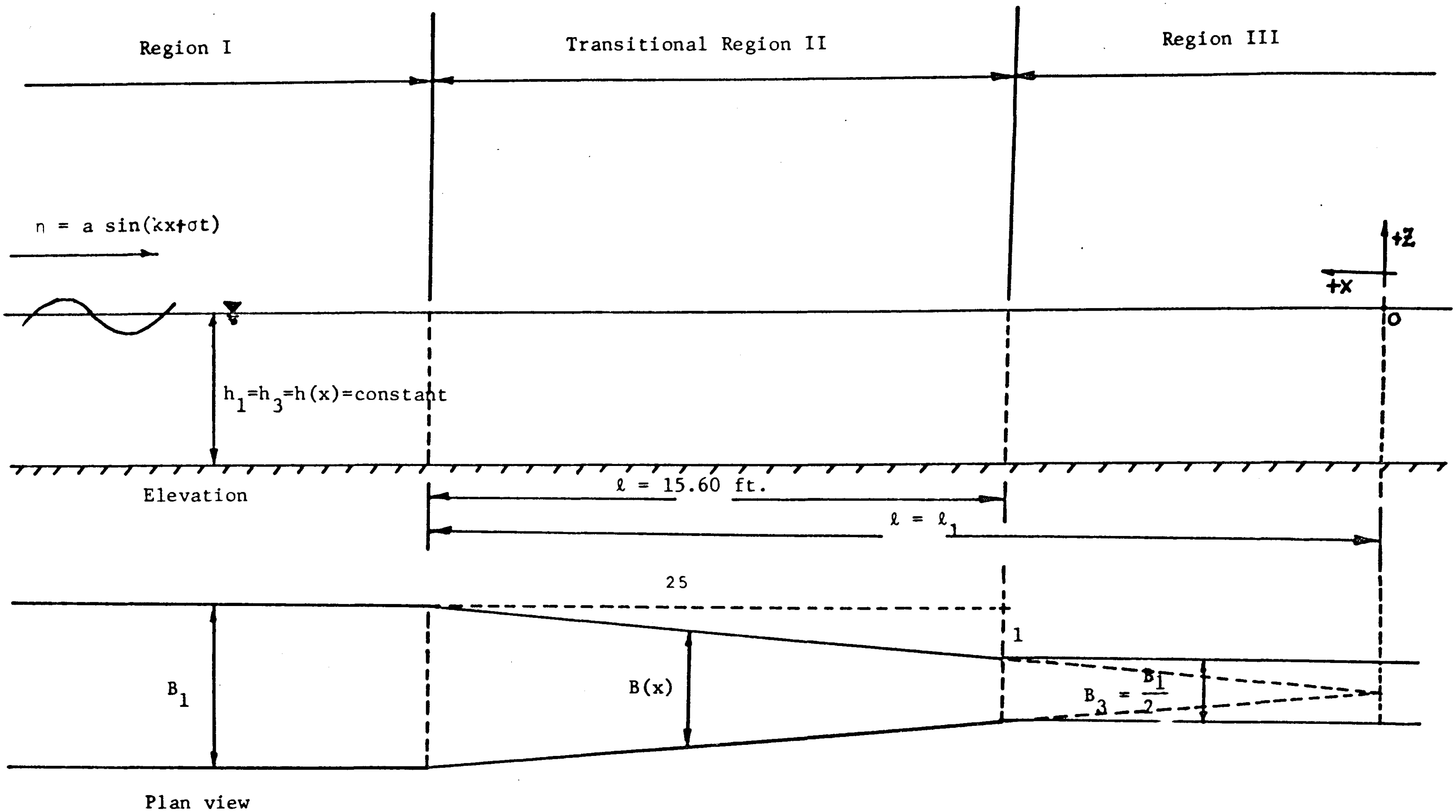


Fig. 15. Schematic Diagram of Case C of Transition
Used for the Experimental Study.

over the distance of 8 ft. from the beginning of the transition.*

(iii) Transition C (figure 16) with linearly contracting walls a rate of 1:25 (α =2.29°). This transition had a length of ℓ=15.60 ft. leaving a downstream width B_3=B1/2. The depth remained $h_3=h_1$=constant.
The toe of each of these three transitions above was 40 ft. from the flap-type wave maker.

The low depth region h_3=constant for the A and B transitions extended to the end of the channel, about 37.55 ft. downstream from the end of the transition.

In both cases the beach at the end was eliminated. In case A the beach used previously as an absorber was replaced by a tubular wave energy dissipator followed by an aluminum wool wave absorber placed in the short end section of larger depth.

In transition B for which the width was one half of that of the upstream region only an aluminum wool wave absorber was placed at the end.

In transition C where also B_3=B1/2 again only an aluminum wool wave absorber was placed at the end.

In transitions A and B both piston and flap type wave makers were used and waves of short (deep water), intermediate and long (shallow water) type were generated.

In transition C only the flap type wave maker was used and short (deep water) and predominantly intermediate waves were generated. The length of this transition ℓ=15.60 ft. reduced significantly the length of the downstream region.

---

*Thus the width of the channel downstream is B_3=B1/2.

The measuring devices, wave gages and carriages, energy dissipators and filters, Sanborn recorder and counters employed for the experiments were the same as in previous investigations (7). Also, the method of the analysis of the experimental results is similar to that used in previous investigations (7). For the C transition experiments were carried out with the newer model of the Sanborn recorder which had better stability and response than the Sanborn used initially in the testing program.

In the case of the A transition, as in previous investigations (7), a tubular breakwater was put ahead of the aluminum wool energy absorber and employed as an additional dissipator (figure 12).

V. PRESENTATION AND DISCUSSION OF RESULTS

5.1 GENERAL SYSTEM OF PRESENTATION

Experimental results are presented in tabular form in Appendix C and in graphical form in this section. The results are generally expressed in terms of the pertinent wave parameters: reflection and transmission coefficients as defined in Section II.

wave steepness H_1/L_1

channel depth ratio h_3/h_1

group velocity ratio C_{G_3}/C_{G_1}

reflected and transmitted wave energies in terms of incoming wave energies

dissipated wave energies in terms of incoming wave energies.

The experimental results are given in two forms;

1. the measured quantities were used to compute the results without correction for reflection from the end of the channel (see Tables I, II and III),
2. the measured quantities were converted to results that may be expected in an endless channel on the basis of the corrections for zero end reflection as developed by Ursell (10) (See Tables IV, V and VI).

For ready reference the tables and graphs are identified with respect to the type of transition, i.e. A, B and C (see Section IV), and with regard to the type of the incoming wave, a) for deep water and intermediate depth waves and b) for shallow water or long waves. Hence, for transition A the tabular presentation is given in Tables Ia, IVa and Ib, IVb; for transition B in Tables IIa, Va and IIb, Vb, etc.

5.2 RANGE OF EXPERIMENTAL CONDITIONS

For Transition A the tests covered Runs A-1 to A-94 for deep water and predominantly intermediate depth incoming waves as listed in Tables Ia and IVa. The frequencies T for these waves range from .766 seconds to 3.96 seconds with wave lengths L_1 = 3.00 ft. and L_1 = 30.92 ft. respectively. For this group of experiments the depths in the approach channel were changed from 27" to 15" in five steps; since the transition height is constant at one foot, the corresponding depth ratios h_1/h_3 varied from 1.80 up to 5.00.

Runs A-95 through A-161 fall into the range of shallow water waves with regard to the incoming wave. Their frequencies as given in Tables Ib and IVb varied from 4.52 seconds up to 12.2 seconds, with corresponding wave lengths from 35.5 ft. to 77.0 ft. for essentially the same range of depth ratios as before.

For Transition B the experiments extended from Runs B-1 to B-82 for deep and intermediate depth incoming waves as listed in Tables IIa and Va. The frequencies T for these waves varied from .903 to 3.44 seconds with wave lengths L_1 = 4.15 ft. to L_1 = 28.10 ft. For this group of tests the depths in the upstream region of the channel were changed from 27" to 18" in four steps with corresponding depth ratios h_1/h_3 from 1.80 up to 3.00.

Runs B-83 through B-121 fall into the range of shallow water waves with regard to the incoming wave. Their frequencies as given in Tables IIb and Vb varied from 4.20 seconds up to 8.62 seconds with corresponding wave lengths from 34.70 ft. up to 59.60 ft. for the same range of depth ratios as in previous cases of the deep and intermediate depth waves.

For <u>Transition C</u> for constant depth the tests covered Runs C-1 to C-52 for deep and intermediate depth incoming waves as listed in Tables III and VI. The frequencies T for these waves range from 1.05 seconds to 3.09 seconds with corresponding wave length $L_1 = L_3 = 5.60$ and $L_1 = L_3 = 25.00$ ft. The depths for this group of tests was varied from 27" to 17" in three steps.

The results for transitions A, B and C are presented also on the basis of corrections made for channel end reflections by means of the Ursell method. These converted values are listed in Tables IV, V and VI for the transitions A, B and C respectively. These computations were carried out by computer programs P_I and P_{II} in Fortran language as given in Appendix D. The P_I program is based on the measurement of the upstream wave length L_1 for the range of deep-water and intermediate depth waves. For shallow water waves the P_{II} program was used on the basis of a measured downstream wave length L_3. The P_I program was verified by analyzing Run A-2 by desk calculation. This confirmation is presented in Appendix B.

5.3 EXPERIMENTAL RESULTS FOR DEEP-WATER AND INTERMEDIATE DEPTH WAVES

a. Reflection and Transmission Coefficients as a Function of Wave Steepness

The figures 16, 17 and 18 represent a summary of the reflection and transmission coefficients as affected by wave steepness of the incoming wave H_1/L_1. All values have been corrected for end reflection and are listed in Tables IV, V and VI. The large scatter common to all such experiments had to be represented by an average line. Nevertheless, a decided decrease is notable, as found previously (7) in the reflection coefficients with increasing wave steepness, which varied from .002 to .06. K_r decreases in this range from .375 to .20 for transition A as shown in figure 16. For transition B this trend is more strongly present in figure 17 with maximum values of K_r decreasing from .60 for corresponding wave steepnesses. This is expected as the transition is one containing decrease in depth as well in width. Transition C shows considerably larger experimental scatter for the K_r values as given in figure 18. However, the trend for the decreasing width of the channel of constant depth is still downward from approximately K_r = .40 within the range of wave steepness tested.

It is to be noted here that wave steepness is not a parameter appearing in the theoretical analysis, hence the reason for the variation of the K_r values, now well confirmed, is not readily apparent. It is possible that some of this effect can be accounted for by the variation of the energy dissipation, which was not considered in the analysis of the experimental K_r values. This concept is followed up in the subsequent presentation of the energy balance in section c.

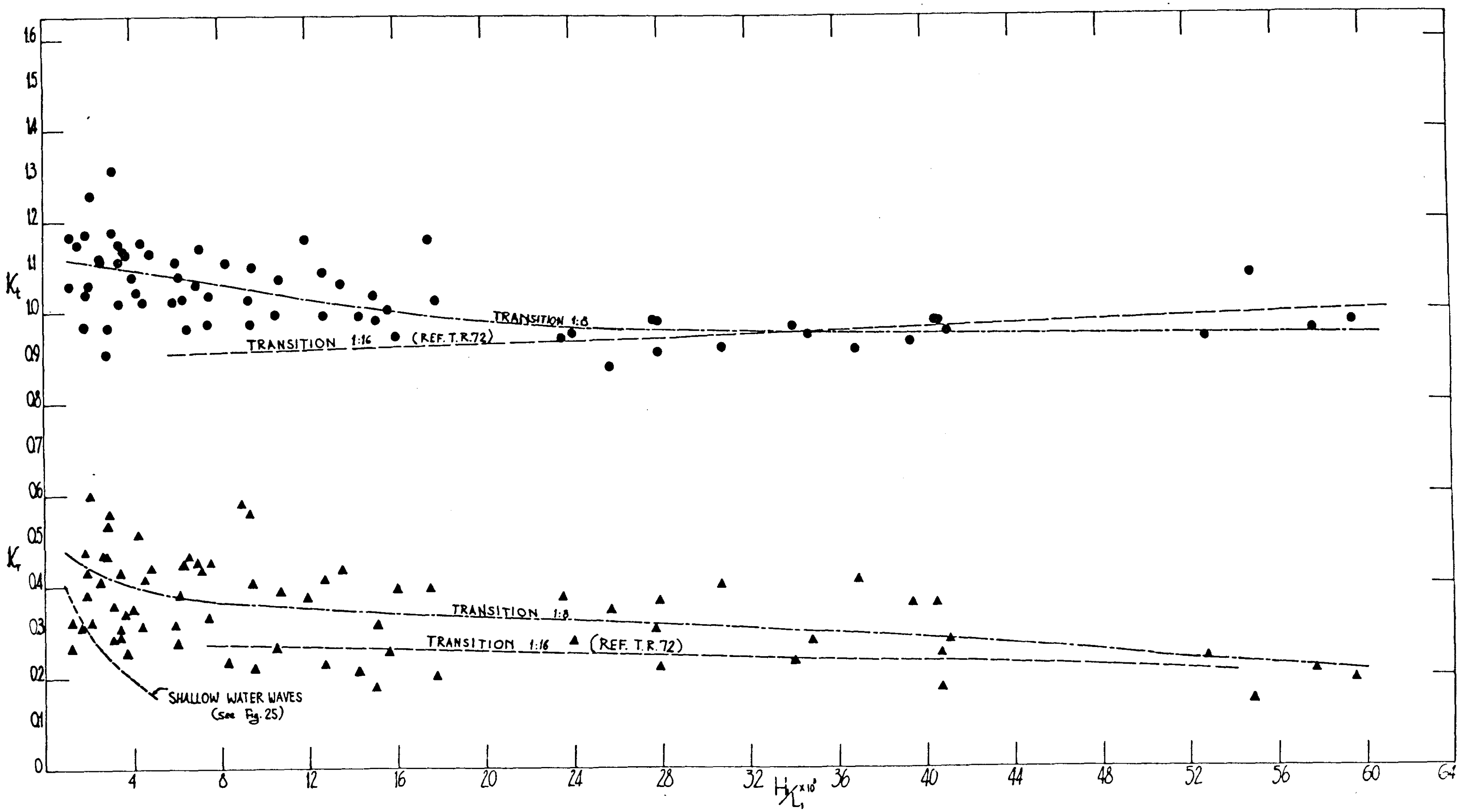


Fig. 16 Reflection and Transmission Coefficients vs. Wave Steepness - Short and Intermediate Waves - Transition A

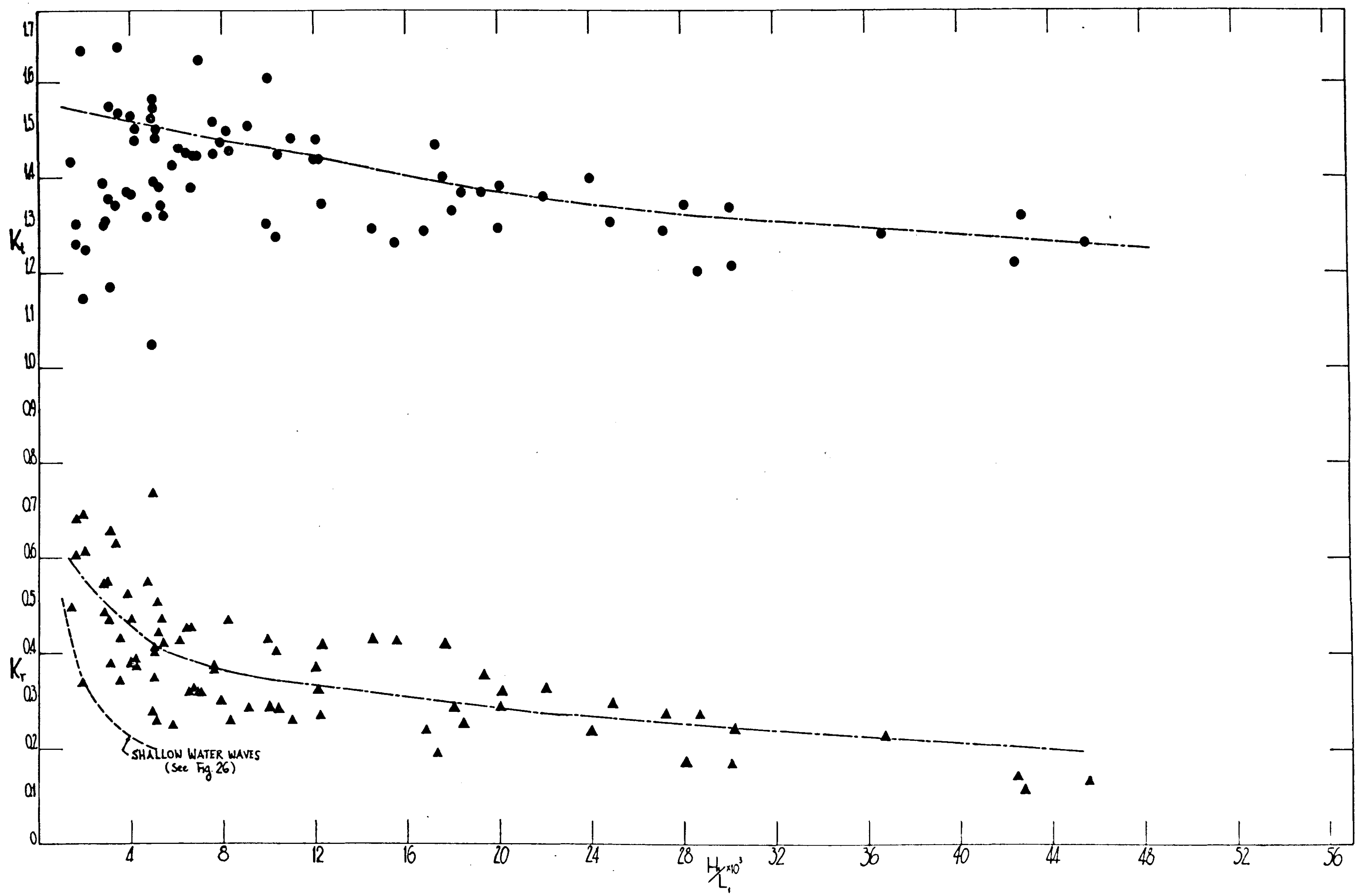


Fig. 17 Reflection and Transmission Coefficients vs. Wave Steepness - Short and Intermediate Waves - Transition B

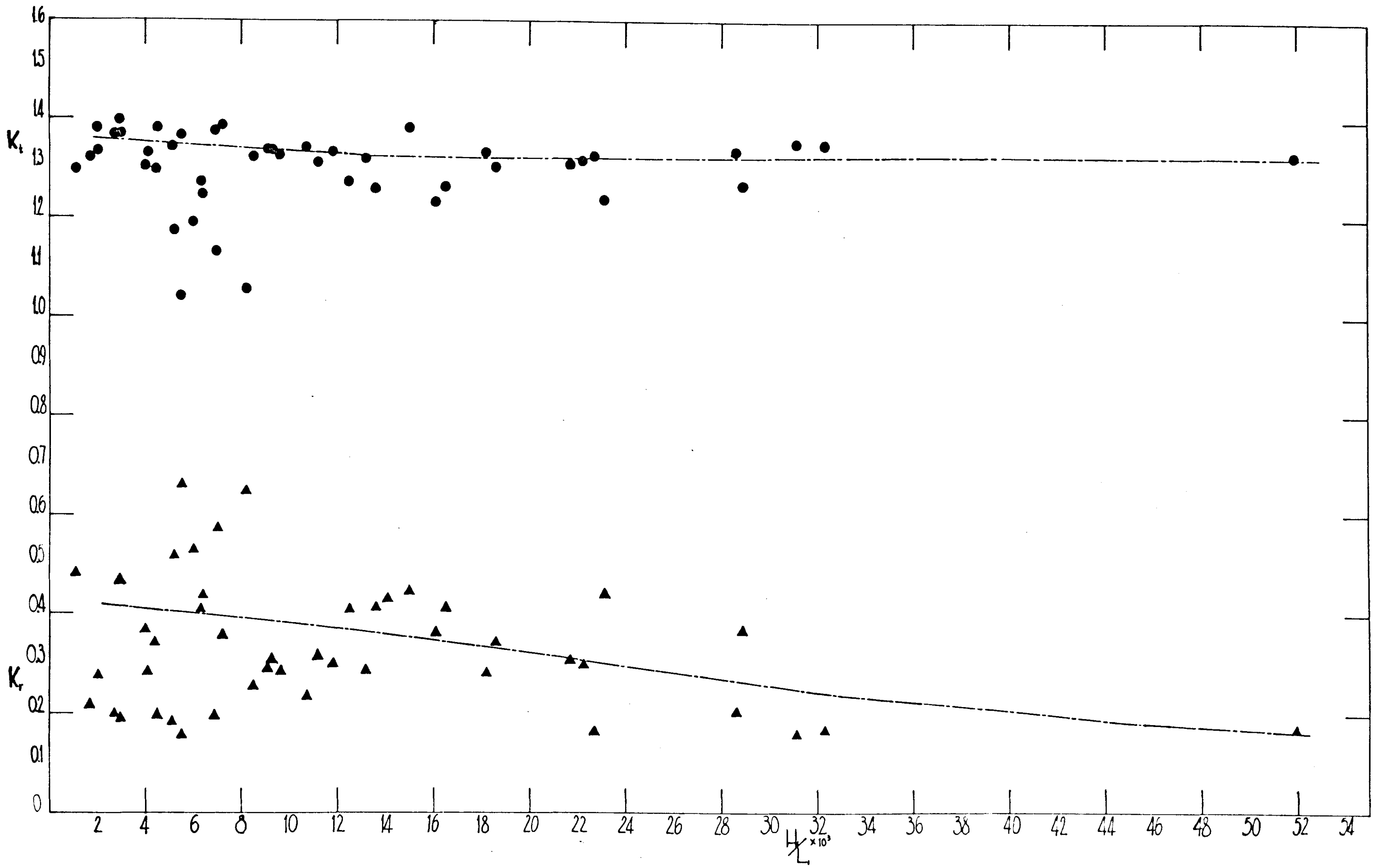


Fig. 18 Reflection and Transmission Coefficients vs. Wave Steepness - Short and Intermediate Waves - Transition C

It should be noted also that the scatter is not related to variations in the depth ratio h_1/h_3 as is evident from the plots presented in section d.

Transmission coefficients presented also in figures 16 to 18 again exhibit a more moderate decrease with increasing wave steepness from values of K_t = 1.10 to .95 for transition A; K_t = 1.50 to 1.25 for transition B; and K_t = 1.35 to 1.30 for transition C. It is clear that the higher values of transitions C and B are primarily due to the channel side contractions B_1/B_3 = 2.

b. Reflection and Transmission Coefficients as a Function of Group-Velocity Ratio.

Figures 19 and 20 present the results for the reflection and transmission coefficients in relation to the wave group velocities in the channel downstream and upstream of the respective transitions A and B. Again the evaluation is hindered by considerable scatter; however, the trend of the values is decidedly downward with increase of the group velocity ratio. It is also confirmed that for more gradual transitions the values lie generally above those given by Lamb's theory for abrupt transitions. It must always be recalled that this theory is not applicable here as it was derived for shallow water waves only, but is is used for reference. It is noted from the numbers at the experimental points that waves of higher steepness are associated generally with higher values of the group velocity ratio. This is due, however, to the limitations imposed by the available wave maker characteristics and is not inherent in the physical process. The higher group velocity ratios are generally associated with the shorter waves in this range of intermediate waves, which were

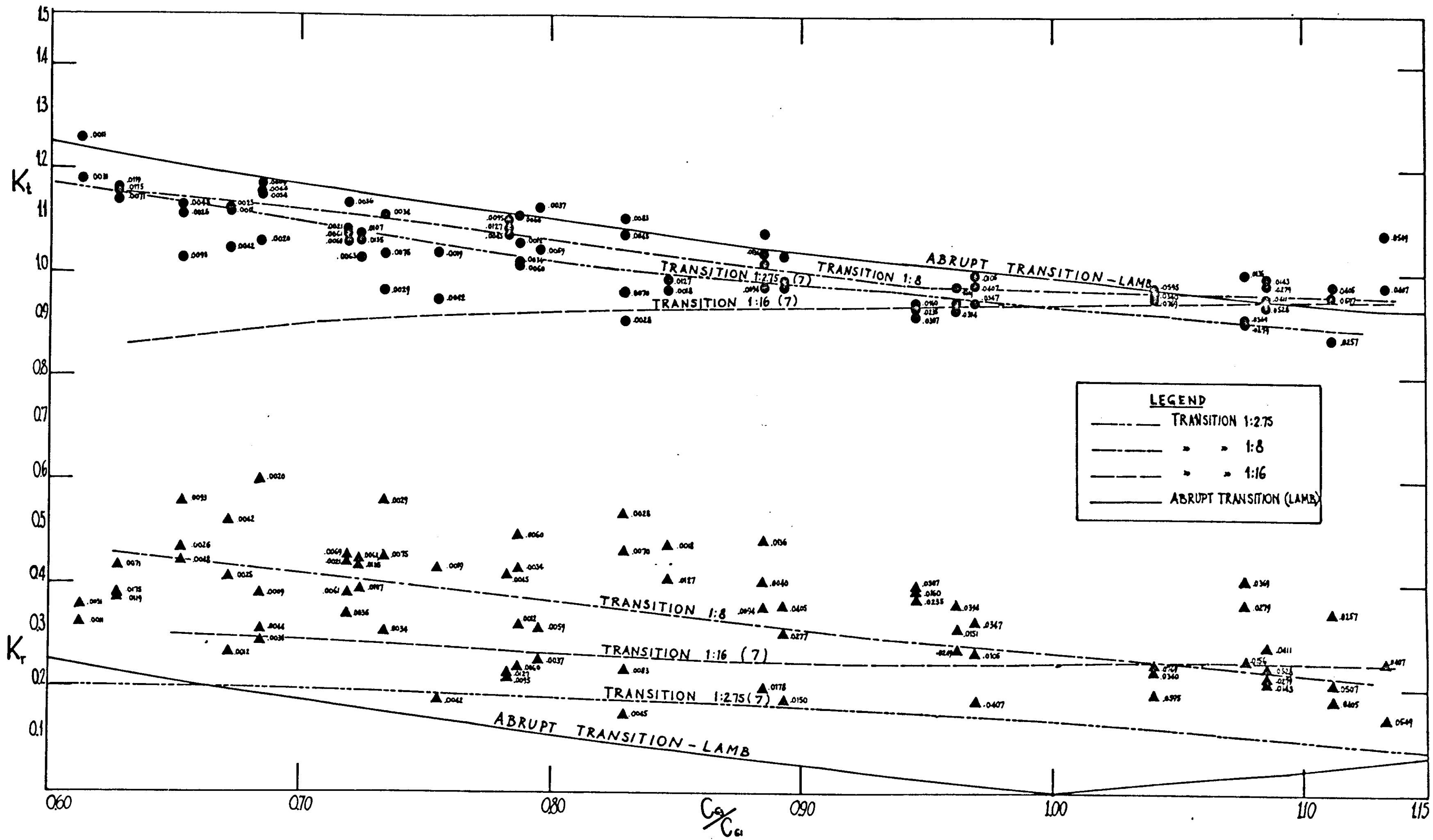


Fig. 19 Reflection and Transmission Coefficients vs. Group Velocity Ratio - Short and Intermediate Waves - Transition A

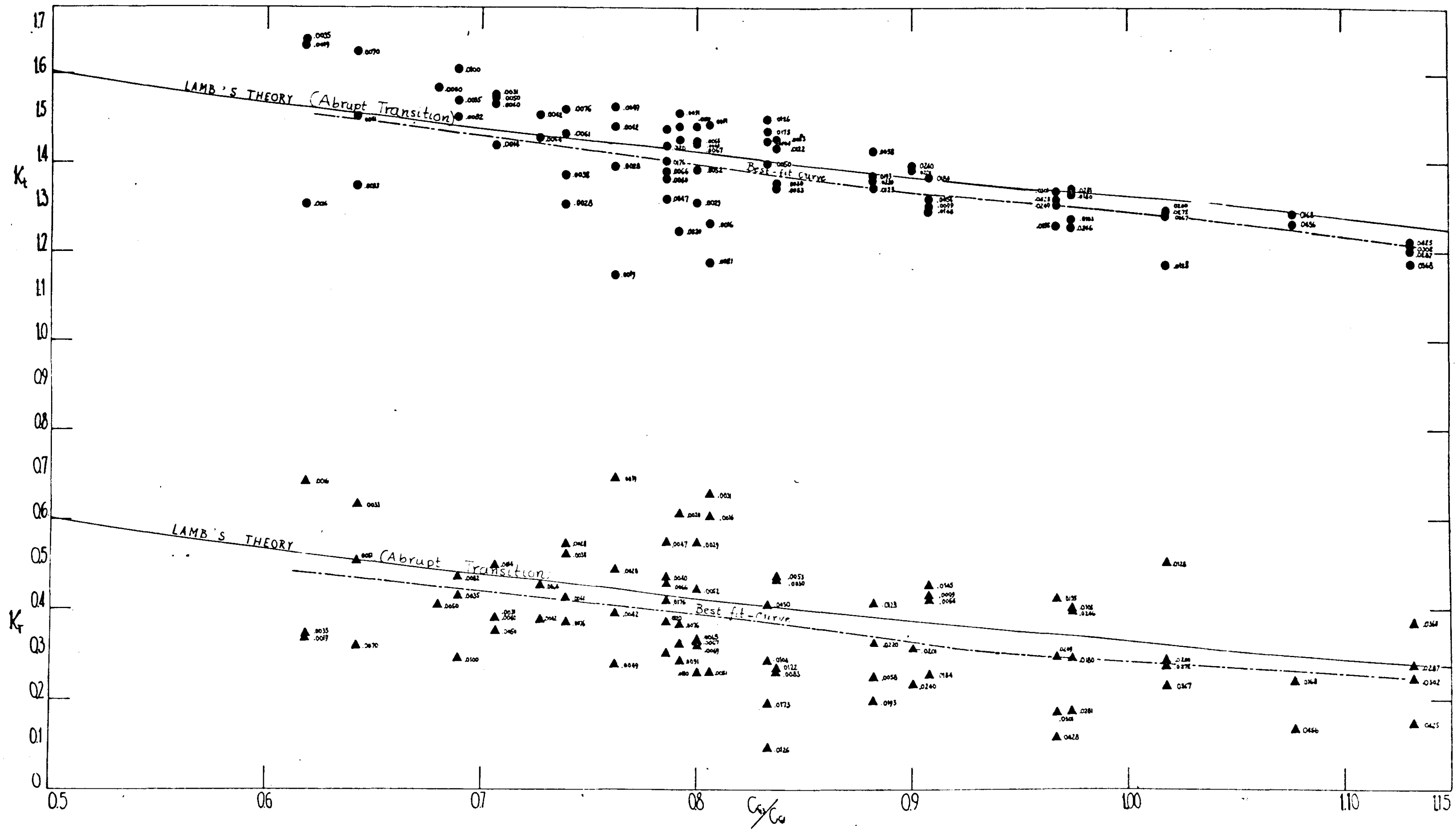


Fig. 20 Reflection and Transmission Coefficients vs. Group Velocity Ratio - Short and Intermediate Waves - Transition B

produced with relatively larger amplitudes. This points to the possibility that the trend for the reflection and transmission coefficients in figures 19 and 20 is influenced to some degree by energy dissipation as already noted under a.

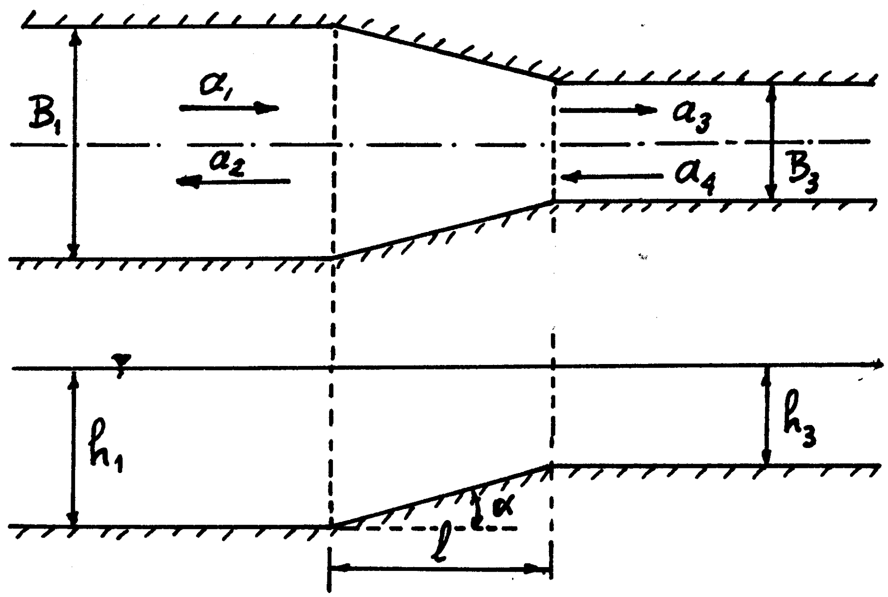
For comparison the previous results by Bocco-Gagnon (6) and Alam (7) for more abrupt and a more gradual transition respectively are shown by their average lines in figure 19. The only general conclusion possible at this time is that gradual transitions result in higher reflection coefficients and lower transmission coefficients as the transition slope decreases. This trend is confined primarily to the reflection process, although here the results for the 1:16 slope lie somewhat below the new values for the 1:8 transition.

In general it may be noted that for the range of intermediate waves considered here the transmission coefficients are only approximately 5% below those that may be computed on the basis of undiminished transmission of the wave energy. This assumption of constant energy transmission would result for the case of transition A in $a_3/a_1 = (C_{G1}/C_{G3})^{1/2}$.

c. Wave Energy Dissipation, Transmission and Reflection as a Function of Wave Steepness.

For the evaluation of the experimental results an attempt was made to analyze the wave energy consitions upstream and downstream of the various transitions termed A, B and C. The following scheme on the energy flux by wave action was adhered throughout employing the usual assumptions of small amplitude, linearized wave theory. These relations hold for deep water and intermediate depth conditions as well as for the shallow water waves considered in Section 5.4.

In accordance with the notations of the sketch



the balance for the energy flux of the wave system can be stated as follows:

$$\overbrace{\gamma \frac{a_1^2}{2} B_1 \, CG_1 + \gamma \frac{a_4^2}{2} B_3 \, CG_3}^{\text{energy inward}} - \overbrace{\gamma \frac{a_2^2}{2} B_1 \, CG_1 - \gamma \frac{a_3^2}{2} B_3 \, CG_3}^{\text{energy outward}} = \gamma \frac{a_\ell^2}{2} B_1 \, CG_1$$

incoming wave energy	reflected from end of channel downstream	reflected upstream	transmitted downstream	energy dissipated in transition

Dividing through by the incoming wave energy flux (first term) the equation is:

$$1 + \left(\frac{a_4}{a_1}\right)^2 \frac{B_3}{B_1} \frac{CG_3}{CG_1} - \left(\frac{a_2}{a_1}\right)^2 - \left(\frac{a_3}{a_1}\right)^2 \frac{B_3}{B_1} \frac{CG_3}{CG_1} = \left(\frac{a_\ell}{a_1}\right)^2$$

[Percent Energy Loss = $\Delta E(\%) = \dfrac{E_{in} - E_{out}}{E_{in}} \times 100\%$]

For convenience this equation is rewritten with alternate notations for the same sequence of terms:

$$1 + \frac{E_{rB}}{E_i} - \frac{E_{rT}}{E_i} - \frac{E_T}{E_i} = \frac{E_{loss}}{E_i}$$

For transition A the variation in channel section is due only to change in depth, hence $B_3/B_1 = 1$. The above individual ratios were evaluated for all the runs from the amplitudes determined from the measured wave envelopes upstream and downstream of the transition. These energy characteristics are plotted as a function of wave steepness in figure 21. Extreme scatter is observed for the lowest values of the wave steepness, which must be attributed to the limits of accurate experimental measurements. In general the variations in energy flux were relatively small; reflected wave energy being only of the order of 2 to 10% of the incoming wave energy. Dissipation as expected is increasing with wave steepness and varies from 1 to 6% over the range covered experimentally. An additional difficulty was encountered through the reflection from the downstream end of the channel. Despite attempts to minimize this reflection by various wave absorbers, the reflected energy from this source exceeded 7% of the incoming wave energy for higher values of wave steepness. In summary, the evidence presented in figure 21 must be viewed primarily as of statistical significance, but it nevertheless indicates correctly the essential trends. This holds also for the following presentations for the other transitions analyzed in the same manner.

In transition B both the channel depth and the width decrease and therefore reflection and transmission phenomena are amplified to some

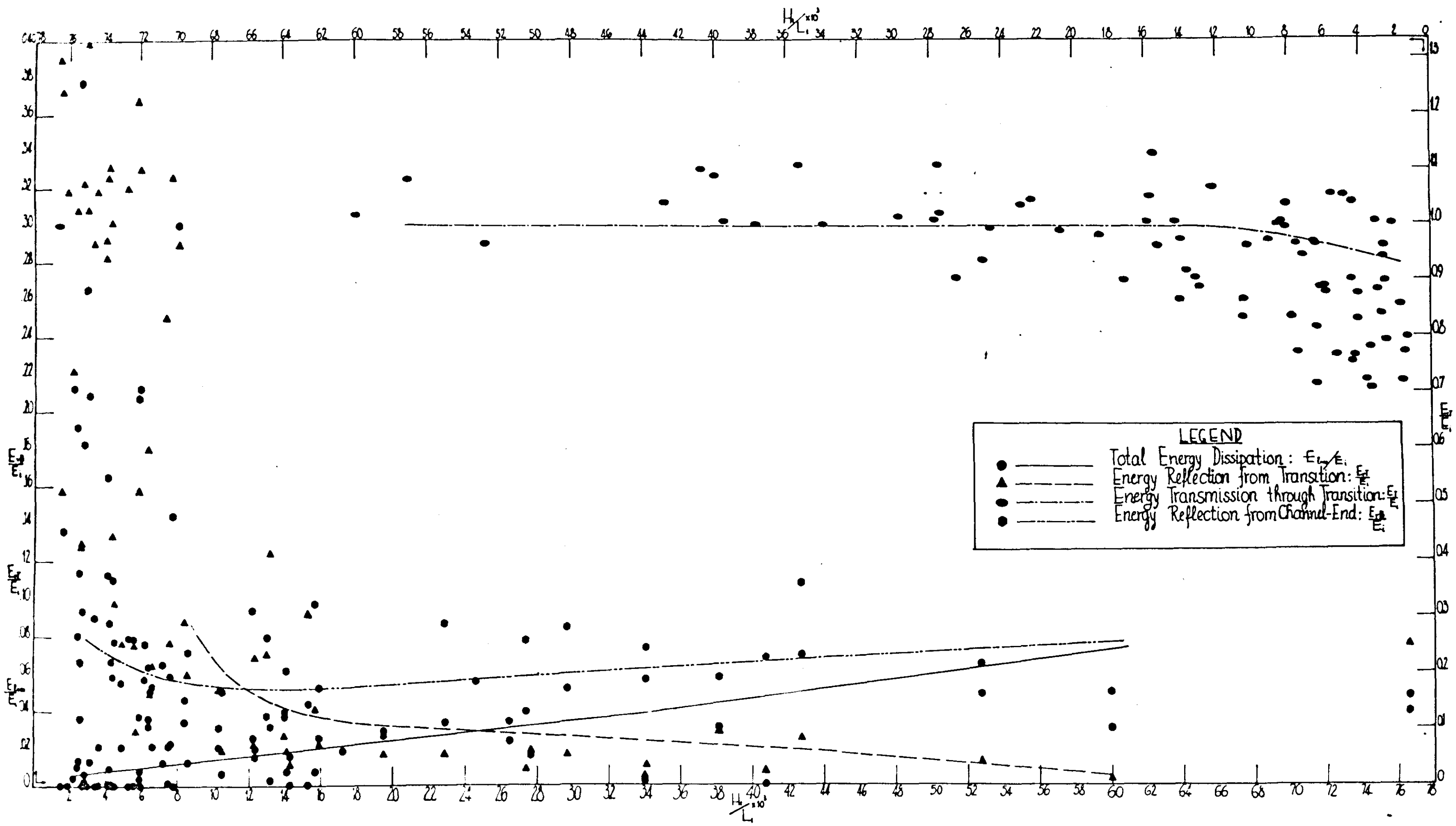


Fig. 21 Wave Energy Dissipation, Transmission and Reflection vs. Wave Steepness - Short and Intermediate Waves - Transition A

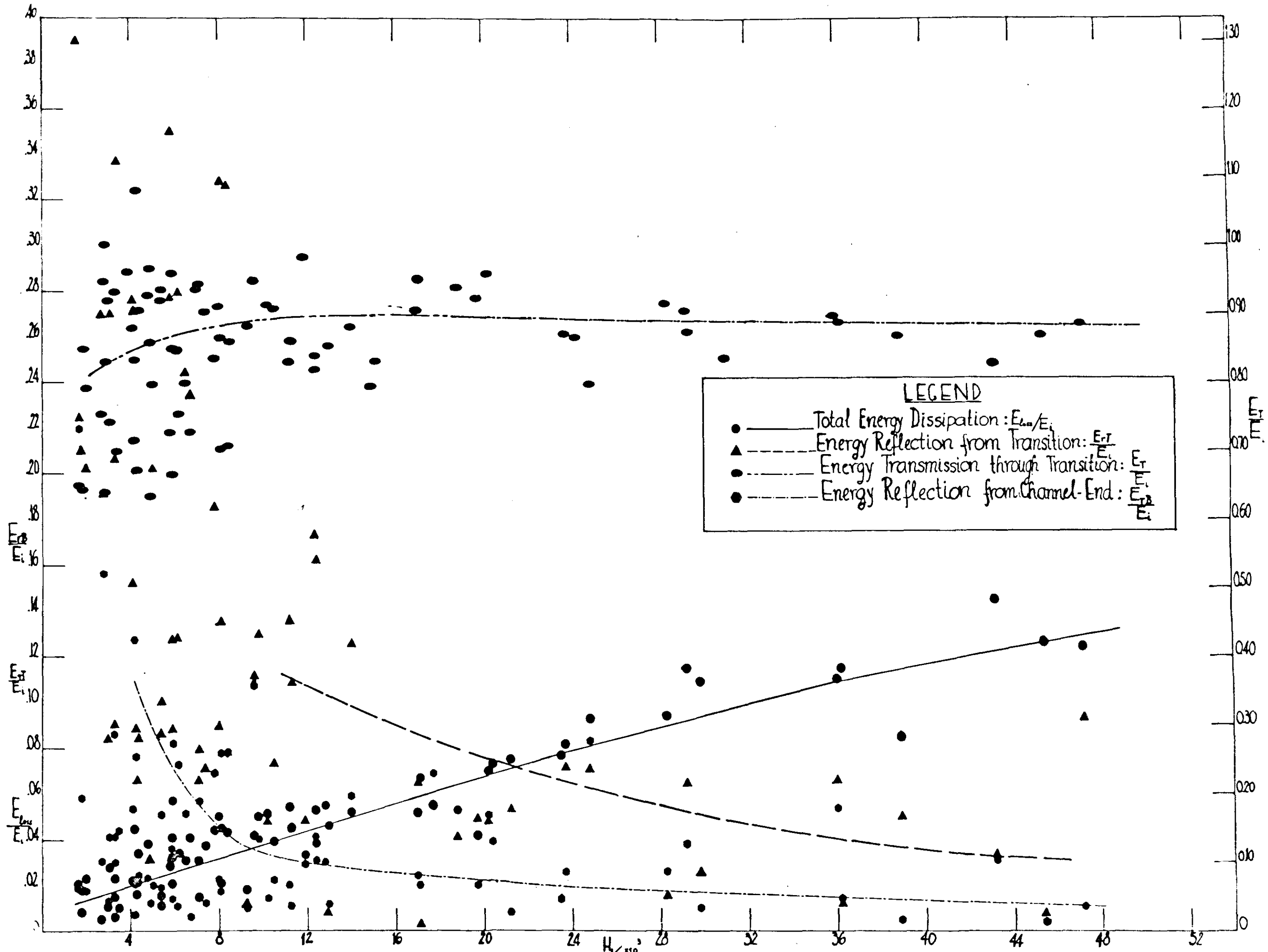


Fig. 22 Wave Energy Dissipation, Transmission and Reflection vs. Wave Steepness - Short and Intermediate Waves - Transition B

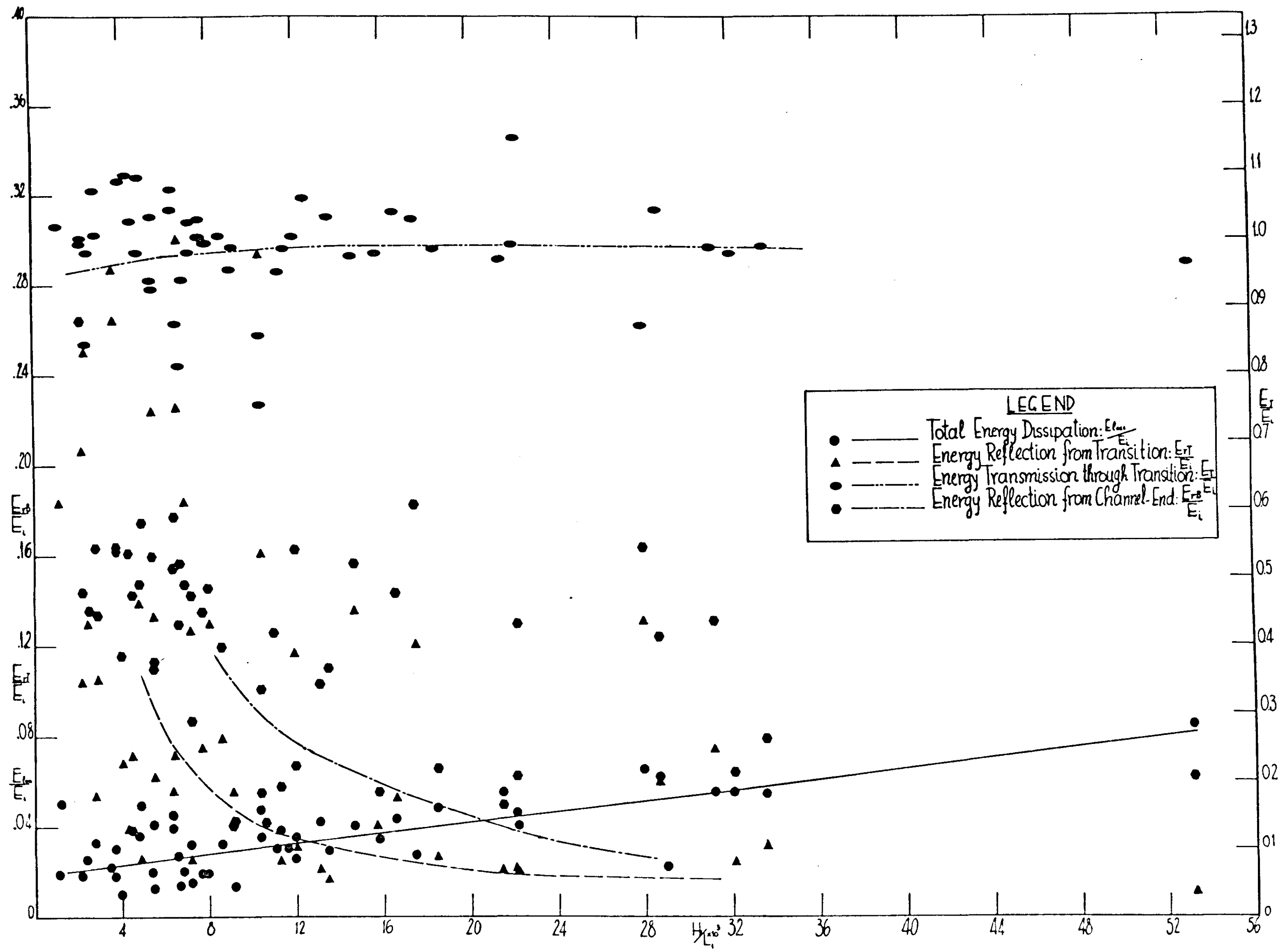


Fig. 23 Wave Energy Dissipation, Transmission and Reflection vs. Wave Steepness - Short and Intermediate Waves - Transition C

moderate extent as seen in figure 22. It must again be kept in mind that the difference in incoming and transmitted wave energy is only of the order of 10% of the incoming energy, i.e. reflection and dissipation are affected heavily by any inaccuracies in amplitude measurements. This difficulty inherent in the experimental results accounts for the large scatter of points which is obviously greatest for the lowest values of wave steepness. Considering this, it is nevertheless seen that again the reflected wave energy is decreasing with increasing steepness, while the dissipation exhibits the reversed trend. It is difficult to judge therefore, in view of the essentially constant values of the transmitted energy flux, whether wave steepness within the range covered, has indeed any marked effect on the transmitted energy flux.

Transition C, involving a contraction of width at constant depth, shows again the same variation in the energy components as transition A. All comments pertaining to the experimental points made for A and B apply also here.

As a general conclusion it can be stated only that for the three geometries of transitions studied the wave reflection and transmission process depends relatively little on the wave steepness and, also, that energy dissipation is markedly increasing with wave steepness relative to the incoming wave energy expressing a resistance coefficient increasing with wave amplitude.

d. Reflection Coefficients for Transition A Compared to Reflection from Beaches.

Miche (12) has given a theory for wave reflection from smooth plane beaches in terms of a critical deep water wave steepness, which is a

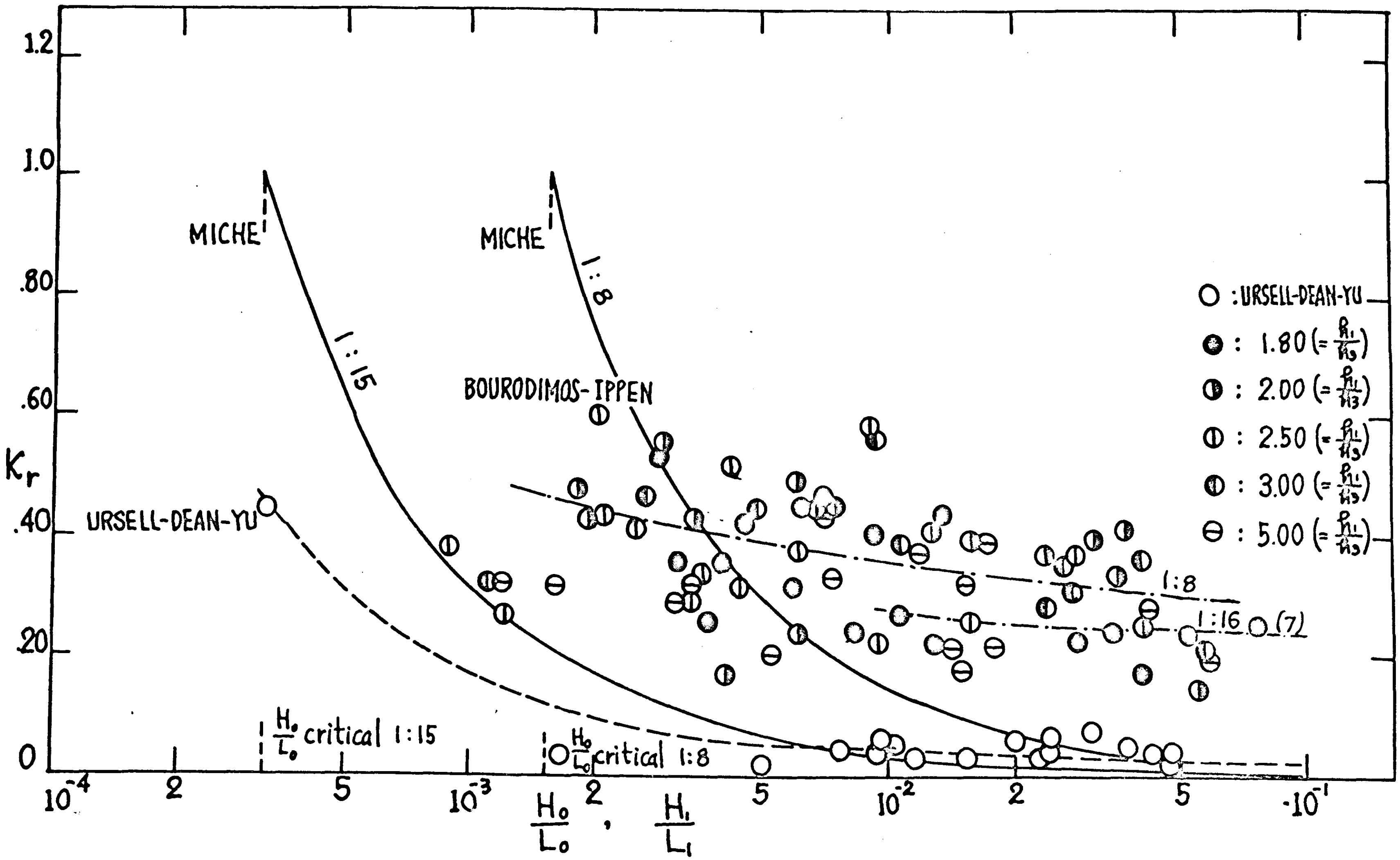


Fig. 24 Reflection Coefficients for Transition - A, Compared to Reflection from Beaches

function of the beach slope β.

$$\left(\frac{H_o}{L_o}\right)_{crit.} = \left(\frac{2\beta}{\pi}\right)^{1/2} \cdot \frac{\sin^2\beta}{\pi}$$

The reflection coefficient K_r is given by him as:

$$K_r = \left(\frac{H_o}{L_o}\right)_{crit.} \cdot \left(\frac{L_o}{H_o}\right)$$

At the critical condition K_r is obviously unity.

It was thought to be of interest to compare the results for the reflection coefficients of transition A with this theory, for which Ursell, Dean and Yu (11) had previously provided some experimental values. This may be justified on the basis that in the limit for $h_3=0$ the two reflection processes become identical. Figure 24 gives the results as previously stated in figure 16, i.e. the reflection coefficients K_r as a function wave steepness H_1/L_1. The curves according to the Miche theory are given for beach slopes of 1:15 and 1:8 for comparison with the experimental results of Ursell, Dean and Yu for a beach of 1:15 slope and of the transition studies 1:16 and 1:8. The results for the latter studies were not reduced to deep water wave steepnesses (H_o/L_o) since the shifting of the points did not seem too important in this context. It is seen that the present studies result in considerably higher reflection coefficients for the lower wave steepness range. The variation in values is not as marked as predicted by Miche. The results for the milder slope of 1:16 are lower in the average than those for the steeper slope.

5.4 EXPERIMENTAL RESULTS FOR SHALLOW WATER WAVES

In line with the usual definition for shallow water waves the experimental results included in this section are those for h_1/L_1 ratios around the values of 1/18 to 1/38. The relatively narrow range of h_1/L_1 is governed by the physical dimensions of the wave tank.

a. Reflection and Transmission Coefficients as A Function of Wave Steepness.

Figure 25 for transition A gives the experimental results for the shallow water waves exhibiting a decrease of the reflection coefficients from .45 to .16 with steepness increasing from 2.10^{-4} to 50.10^{-4}. The wave steepness is smaller in the average than for the intermediate depth waves, for which the comparable results are given in figure 16. The range in magnitude of K_r is not very different from that in figure 16. Generally the scatter is within the extreme values of K_r. However, separating the data essentially according to ranges of H_1/L_1 ratios was held to be meaningful with respect to the effect of this parameter. Hence the scale of steepness in figure 25 was expanded tenfold over the scale of the ordinate in figure 16.

The transmission coefficients are seen to match up very well from figure 25 to figure 16. If the results were combined into the same plot continuously decreasing trend would become more obvious. For waves of lowest steepness in the shallow water range the transmission coefficient K_t has a value of 1.15 in figure 25 decreasing only to 1.07 at the highest steepness for this range. In figure 16 this last value coincides with the value for the lowest steepness on this plot, decreasing further to K_t = 0.95 for the highest steepness reached in the experiments of

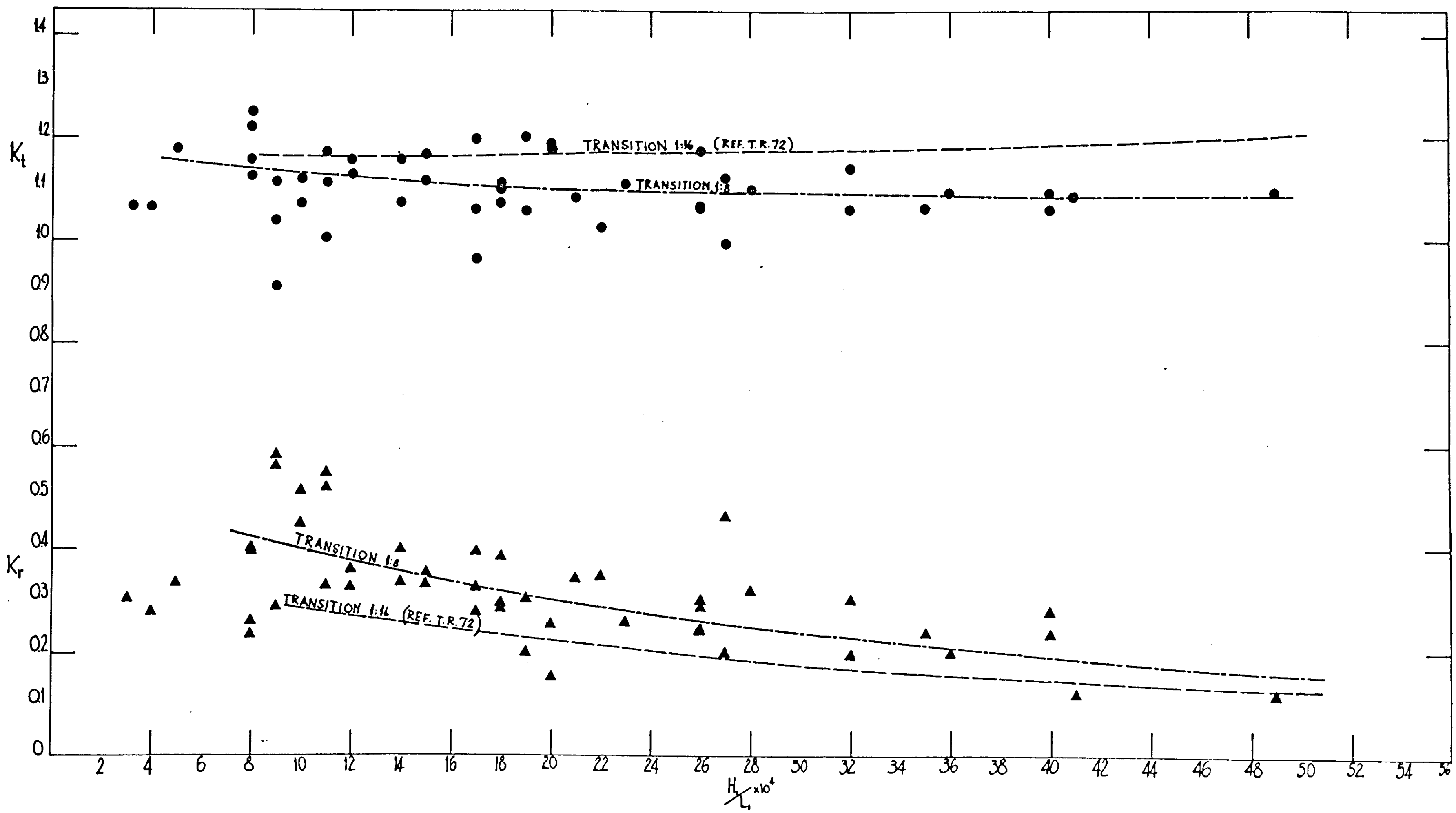


Fig. 25. Reflection and Transmission Coefficients vs. Wave Steepness - Shallow Waves - Transition A.

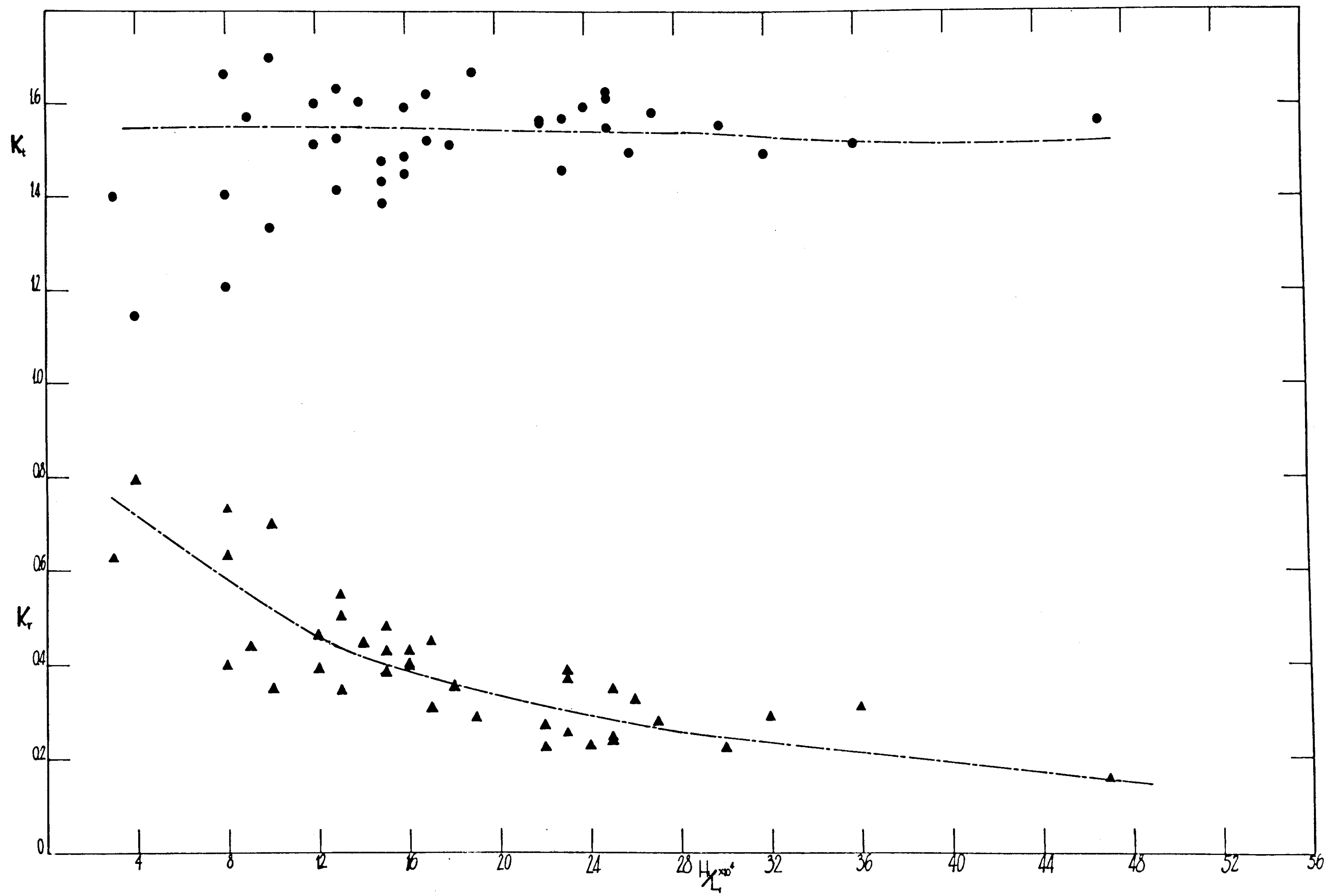


Fig. 26. Reflection and Transmission Coefficients vs. Wave Steepness - Shallow Waves - Transition B.

$H_1/L_1 = 6.10^{-2}$. The total range of the wave steepness is seen to extend from 10^{-4} to 6.10^{-2}.

As is expected the reflection and transmission coefficients for transition B are generally higher than for transition A also in the range of the shallow water waves. This effect is due to the combination of reduction of depth and width. Again the scale for wave steepness has been expanded by a factor of 10 in figure 26 as compared to the ordinate of figure 17. The values of the transmission coefficient again indicate a decrease over the entire range of wave steepness covered by figures 17 and 26 although the absolute change is very much less than for transition A. The reflection coefficients for transition B for shallow water waves are generally lower than for the intermediate depth wave conditions, indicating a dependence not only on wave steepness but also on the relative depth ratio h_1/L_1. The effect of depth is most pronounced for shallow water waves as demonstrated in the correlation of the reflection and transmission processes with the wave velocities in the upstream and downstream sections of the transitions.

b. Reflection and Transmission Coefficients as a Function of Group Velocity Ratio.

The reflection and transmission coefficients in figures 27 and 28 representing results for transitions A and B exhibit the same trends as in figures 19 and 20. Generally the coefficients decrease with increasing values of the group velocity ratio. For the shallow water waves group velocity ratios obviously depend only on the depth ratio and therefore remain below unity. Reflection coefficients for transitions A and B are somewhat lower than for the intermediate depth conditions. Transmission

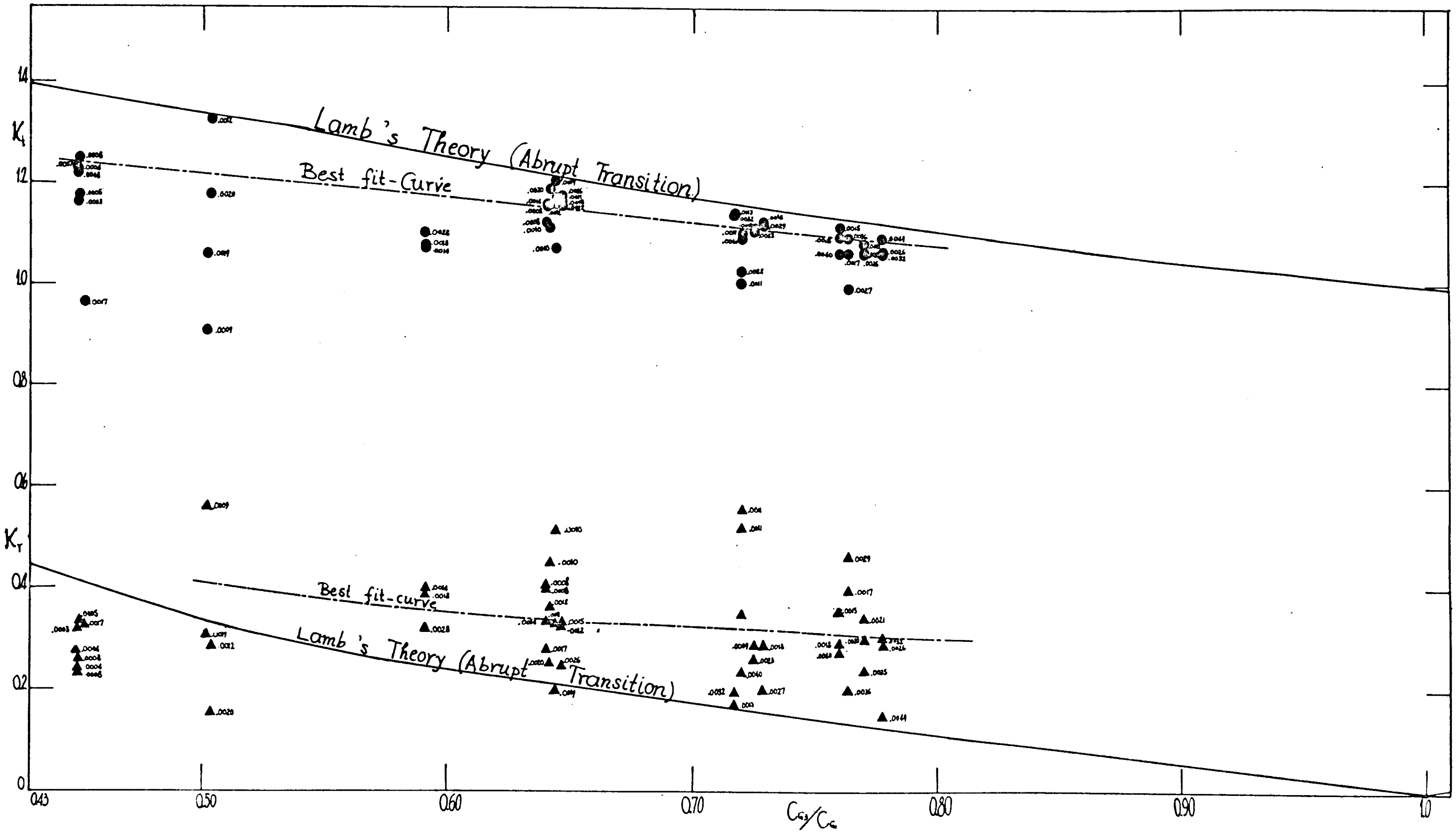


Fig. 27. Reflection and Transmission Coefficients vs. Group Velocity Ratio - Shallow Waves - Transition A.

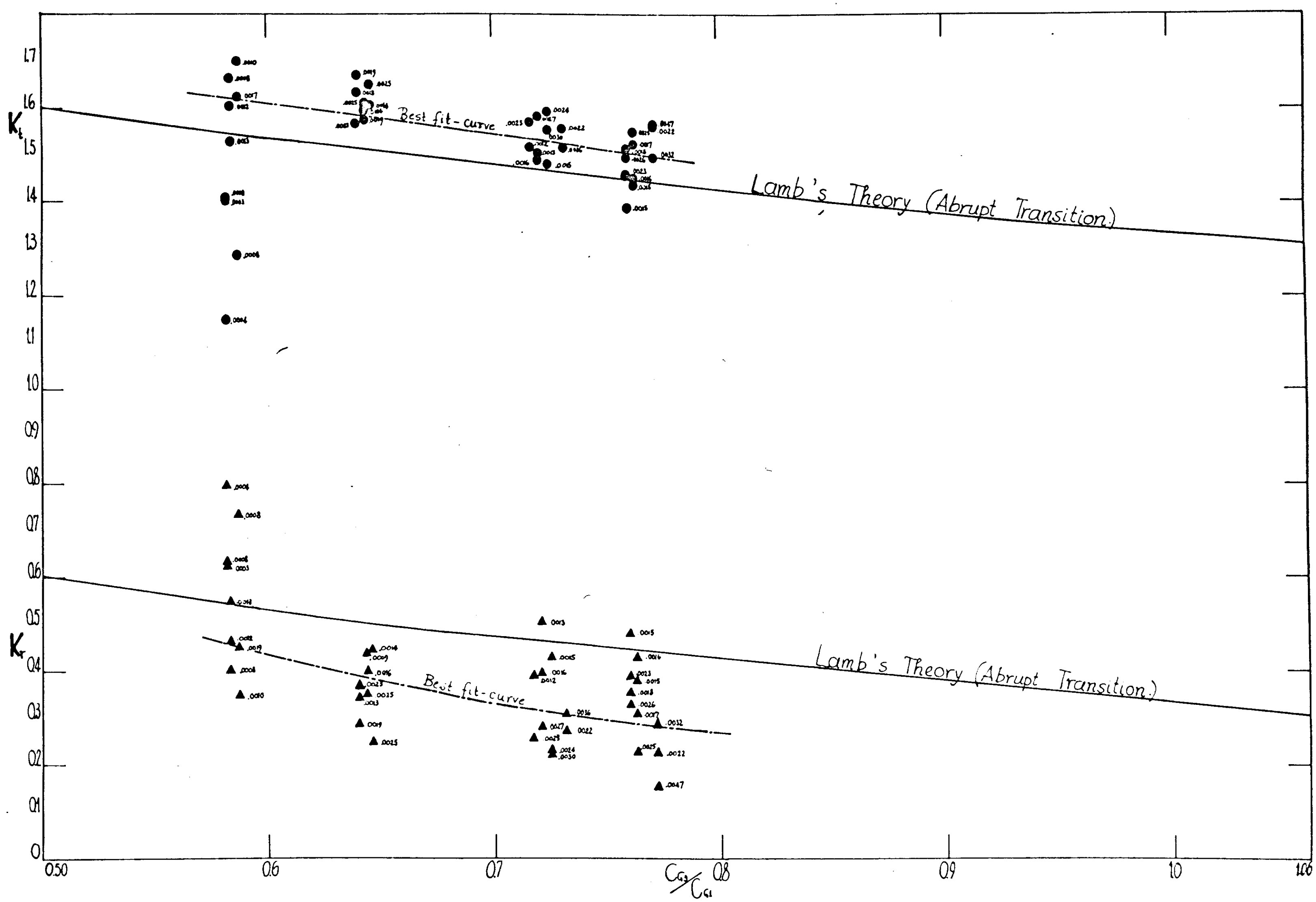


Fig. 28. Reflection and Tranmission Coefficients vs. Group Velocity Ratio - Shallow Waves - Transition B.

coefficients, however, are essentially the same for transition A for both wave ranges. For transition B the transmission coefficients in figure 28 rise above Lamb's solution for abrupt transitions, while figure 20 for intermediate depth waves shows values very close to those for abrupt transitions. For all results again the interpretation of the data is made somewhat difficult in view of the scatter.

c. Wave Energy Dissipation, Transmission and Reflection as a Function of Wave Steepness.

Figures 29 and 30 represent a correlation of the wave energy dissipation, transmission and reflection as affected by the wave steepness of the incoming wave H_1/L_1. The energy flux evaluation was done on the same basis of energy flux analysis as in case (c) of section 5.3. Extreme scatter is observed especially for the lowest values of the wave steepness due mainly to inaccuracies in amplitude measurements. The scale of plotting is obviously too large, but was chosen to be consistent with the earlier plots. For transition A the energy flux transmitted is of the order of 0.850 up to 0.985 of the incoming energy, while the variations in reflection is of the order of 2 to 12% of the incoming wave energy. Dissipation is increasing with wave steepness and varies from 2 to 5% of the incoming energy, as was observed before for intermediate depth waves.

To be noted here is the fact that for many runs of shallow depth range (A-101 to 161) the transmitted waves were in the breaking range. These points were therefore not included in figure 29 in view of the much higher values for the dissipation. These results are plotted separately in figure 31 presenting the percentage of energy dissipated against a breaking parameter defined by Longuet-Higgins (99). This breaking

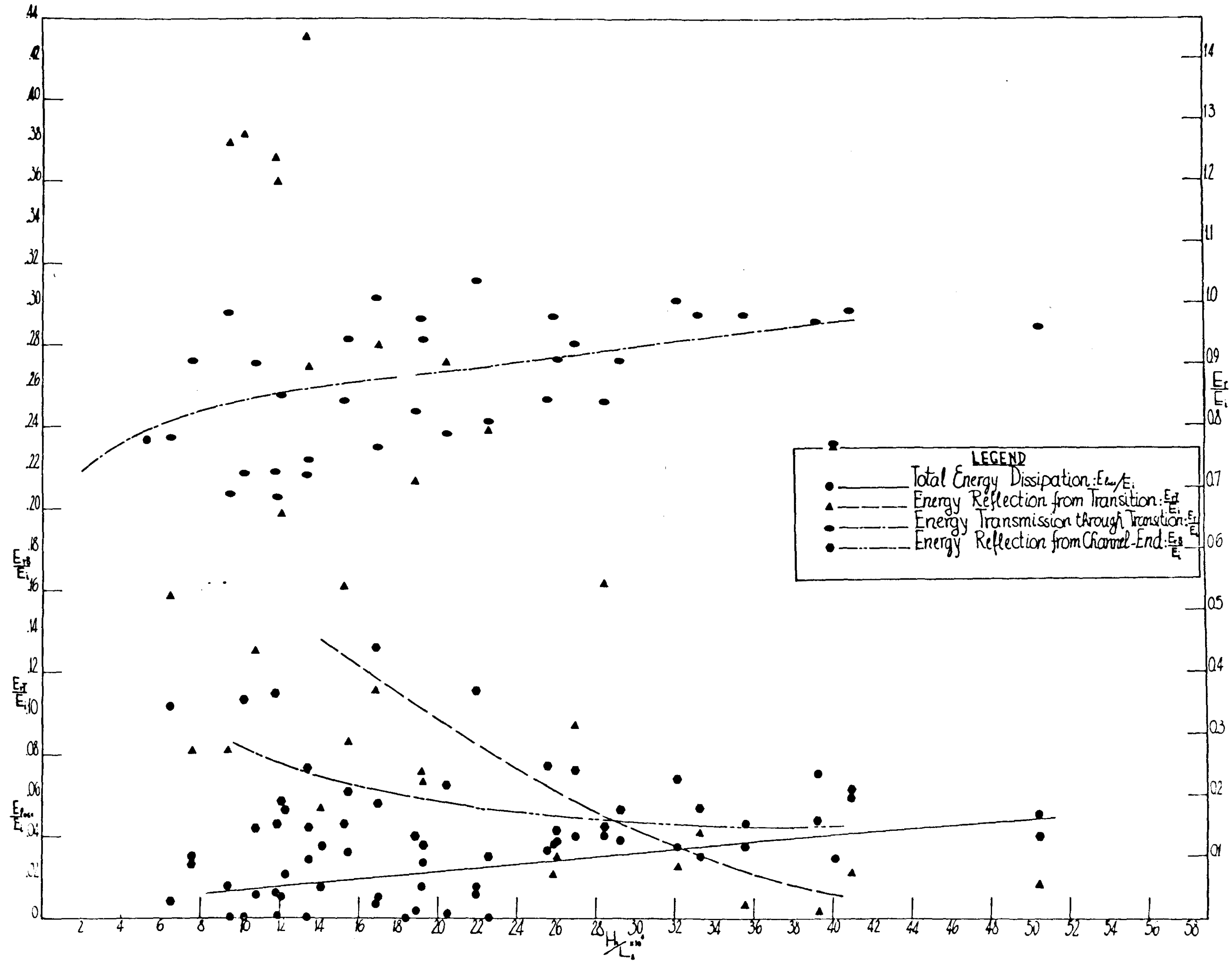


Fig. 29. Wave Energy Dissipation, Transmission and Reflection vs. Wave Steepness - Shallow Waves - Transition A.

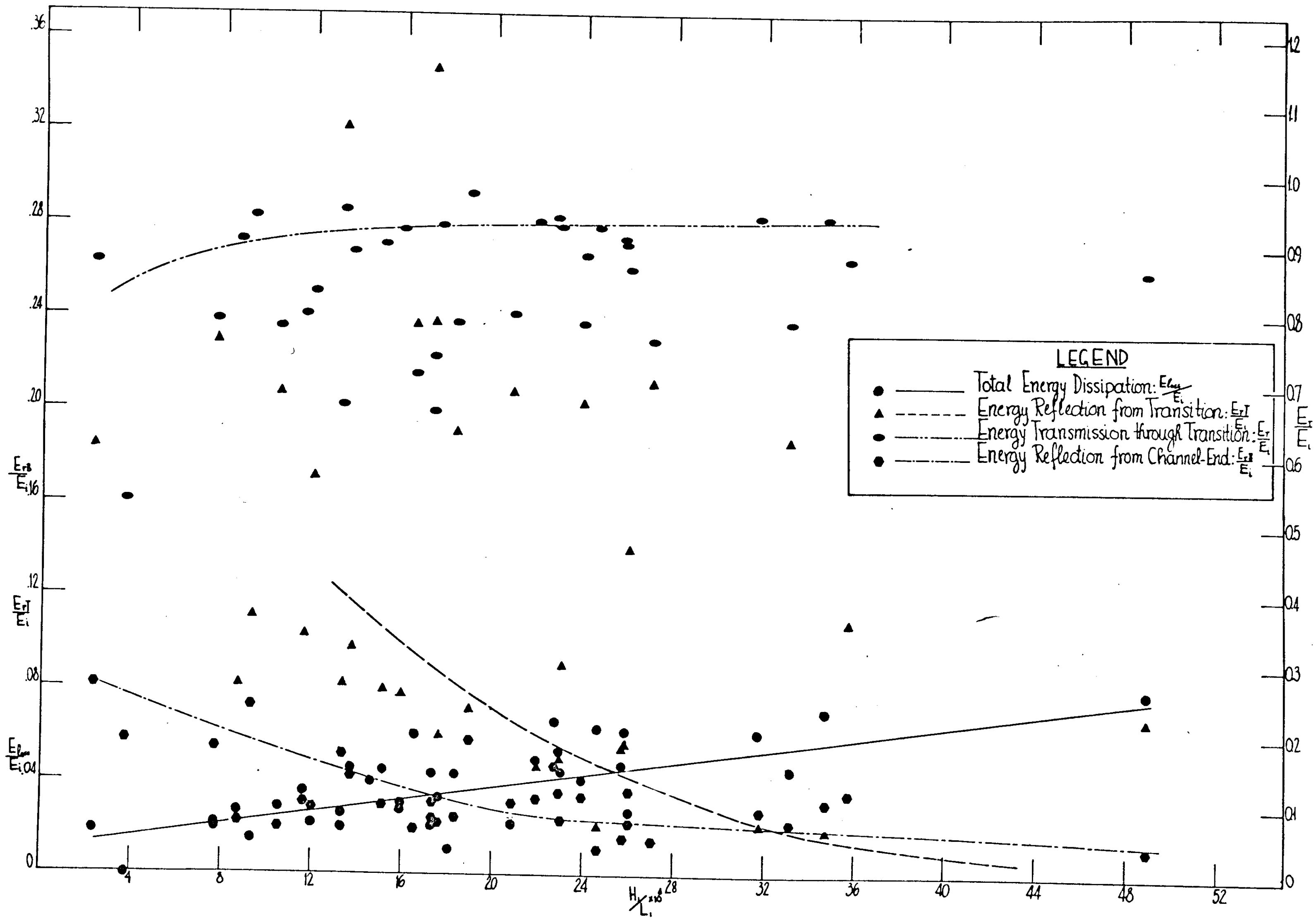


Fig. 30. Wave Energy Dissipation, Transmission and Reflection vs. Wave Steepness - Shallow Waves - Transition B.

parameter is defined as $K_b = L_1^2\ a_1 / h_1^3$, which is equivalent to $248(\frac{H_1}{L_1})(\frac{1}{k_1 h_1})^3$. Longuet-Higgins specifies that this parameter should have values very small as compared to $\frac{16\pi^2}{3} = 52.53$ in order to consider the waves still within the range of the linearized small amplitude wave theory employed in the present analysis. In figure 31 it is seen that breaking waves were observed first for K_b values in upstream region I of 10 as high as 78. Between these limits the energy dissipation evaluated by linearized theory is rising from approximately 15% to 50%. It is appreciated that even for lower values of the K_b parameter the waves can

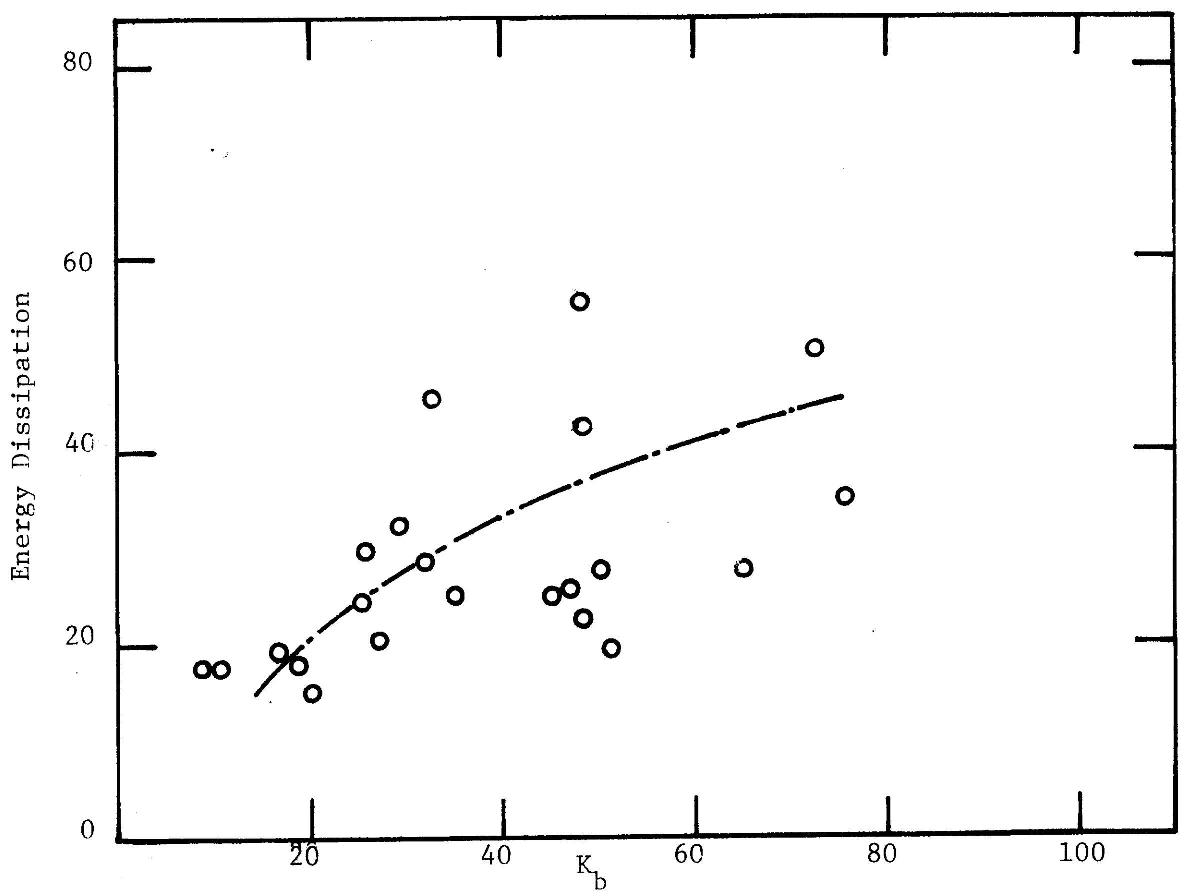


Fig. 31. Energy Dissipation vs. Breaking Parameter K_b

hardly be expected to conform to the wave form assumed in the analysis for the downstream channel, since they generally fall into the range of non-linear, finite amplitude waves. This fact is probably an important additional reason for the observed scatter in figures 29 and 30.

For transition B the flux of reflected and transmitted wave energies are modified as seen in figure 30 over the corresponding values given for transition A. As expected, the reflected energy is somewhat higher and the transmitted energy lower in view of the more rapidly converging section. These effects, however, are not too pronounced. The energy dissipation also is higher, varying from 2 to 8% of the incoming energy.

VI. SUMMARY AND CONCLUSIONS

6.1 Review of Theoretical Development

The theoretical approach is restricted by the difficulty that general treatment on the basis of a velocity potential with appropriate boundary conditions is presently not possible for the problem of the general wave transformation in channel transitions.

However, during the course of the study two limited theoretical treatments pertaining to some restricted phases of the problem became apparent.

1. Making no restrictions with respect to the type of incoming wave a general expression of the integral type was developed on the basis of undiminished transmission of the wave energy. No reflection and dissipation is considered. This expression was solved on the one hand for shallow water waves, the result confirming Green's theory; the other solution was derived on the basis of restriction to intermediate depth waves over the entire transition for which the hyperbolic tangent $\tan\theta_X h$ can still be assumed close to unity. This approximation resulted in an exponential expression for the amplitude of the transmitted wave relative to that of the incoming wave.

2. Restricting the treatment to shallow water waves specific reflection and transmission coefficients were derived using small amplitude linearized theory. Assuming harmonic components of wave motion throughout the transition, solutions were obtained for the following four cases:

A - for linearly varying depth and constant width

B - for linearly varying depth and width

C - for linearly varying width and constant depth

D - for parabolic variation of depth and constant width

The solution resulted in each case in specific expressions for the reflection and transmission coefficients in terms of the parameters of the incoming wave, the geometry of the particular transition and trigonometric functions involving the phase angles of the various wave components.

The expressions involve Bessel functions of zero order for transition A and C, of first order for transition B and hypergeometric (Legendre) functions for transition D, still remain to be evaluated numerically.

Since the experimental investigation in the range of shallow water waves has determined all the pertinent quantities including phase angles a detail comparison of theoretical and experimental coefficients should form an important additional phase of investigation.

6.2 Review of Experimental Results

The experimental part of the program has been conducted to determine the wave reflection and transmission phenomena through channel transitions of varying geometry connecting two prismatic channels of constant cross section. The analysis of the experimental results was conducted within the framework of linearized small amplitude wave theory and the essential experimental results were reduced to correspond to those comparable to an infinite channel of transmission.

The following are some general conclusions that may be drawn from this phase of the investigation.

1. The reflection coefficients decrease considerably with increasing wave steepness for the entire spectrum of wave conditions from deep and intermediate depth to shallow depth water waves for all transitions of linearly varying depth and width, A, B, C.

2. The transmission coefficients as a function of wave steepness exhibit a more moderately decreasing trend with increasing steepness for the entire spectrum of waves. This trend is not always clear due to the presence of considerable scatter, which is normally associated with such tests.

3. The reflection coefficients when correlated with the group velocity ratio for short and intermediate waves have higher values than those given by Lamb for abrupt transitions. The trends established in previous investigations (6,7) were confirmed by this study.

4. The transmission coefficients as a function of group velocity ratio for short and intermediate waves have lower values than those given by Lamb's theory for abrupt transitions. Again a confirmation is given

by this study of the trends explored in previous investigations.

5. The reflection and transmission coefficients for shallow waves follow generally the trend, in relation to the wave velocity ratio, given by Lamb's theory for abrupt transitions. The reflection coefficients are somewhat lower than those for the intermediate depth range, while transmission coefficients are slightly higher.

6. The wave energy dissipation for the entire spectrum of waves from deep to shallow water exhibits an increasing trend with increasing steepness while the reflection from the end of the channel and the transition is very small in the region of larger values of steepness. The transmission rate for wave energy is not materially affected by wave steepness and remains approximately constant around the values of .98-.99 for the higher steepness range.

7. A comparison of reflection coefficients with the Miche theory for transition A indicates a considerably smaller rate of decrease with increasing wave steepness. While this theory is only applicable for beaches of smaller slopes, the comparison was made for reference purposes with regard to results obtained for milder transition slopes than that of transition A.

A few general comments are in order here for the proper assessment of the experimental results.

Since the theory does not take into account energy dissipation, some discrepancies must be expected on that account. This dissipation must be expected of considerable influence not only for shallow water waves but also for the deep and intermediate depth waves in view of the boundary layers on the side-walls. This explains in part the material increase in energy dissipation with wave steepness observed for the entire spectrum of waves tested. An additional source of energy dissipation may be associated with the relative sharp breaks in the bottom slope due to separation particularly at the downstream end of the transition.

Also, the theory considers only small amplitude linearized harmonic waves while in the experiments non-linear waves were often present, especially downstream of the transition. Hence non-linear interactions should influence the results in certain ranges of the wave characteristics. The experimental set-up was to some extent inadequate in the range of long waves when the wave length often was of the order of the distance between wave maker and transition.

The method of correction for zero reflection from the end of the channel was that developed by Ursell in a former study (10). This method does not consider energy dissipation. Since the amplitude of the wave reflected from the end of the channel was often of the order of 15 to 30% of the transmitted wave amplitude this correction has a sizeable effect on the reduced values of the amplitudes upstream and downstream. Dissipation therefore would modify also these corrections.

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APPENDIX A

REVIEW OF WAVE THEORY LITERATURE

1. Exact Non-Linear Wave Theory of Finite Amplitude

The existence of two-dimensional permanent, periodic, progressive, oscillatory, gravity waves of finite amplitude moving unchanged in form in water of infinite depth was mathematically proved by Levi-Civita (40). [A problem formulated by Stokes (2) but never completely solved until 1924.] The Levi-Civita approach was based on the assumption of the irrotationality of an inviscid, incompressible fluid, with the exact non-linear boundary conditions on the free surface of the wave motion and with open particle paths (mass transport predicted). Struik (41) expanded the same problem of two-dimensional progressive wave existence in water of finite uniform depth. The next step was the formulation of the problem for any boundary condition.

This case is extremely difficult to handle mathematically. Stoker (17) gives a rigorous mathematical presentation on this subject. It should be noted here that the assumption of irrotationality of wave motion of an incompressible inviscid fluid was kept in this approach. That implies that if the motion is irrotational at the beginning (impulsive motion) it remains irrotational afterwards. But the difficulty arises in the dynamical and kinematical boundary conditions of the free surface which is moving and gives the non-linearity to the problem. Thus even with the linear Laplace equation expressing the irrotationality of the motion and the incompressibility of the fluid, the problem becomes non-linear, due to the non-linear boundary conditions.

The other crucial points besides the non-linearities of the B.C. are: (i) The free surface of wave motion $z = \eta(x,y,t)$ or $\zeta = z-\eta(x,y,t) = 0$ is not described and known a priori (it has to be determined as an integral part of the solution); (ii) This undetermined and unknown free surface as a boundary, varies with time and thus the region of the velocity potential is not known a priori; (iii) Connected with these difficulties of the problem is the existence of the solution which might be valid for all $t > 0$ of might be valid only for limited time. In the latter case (which is closer to the physical situation in which ocean waves going to the shore steepen in the front and break) consideration must be made of time and space singularities not known a priori (17).

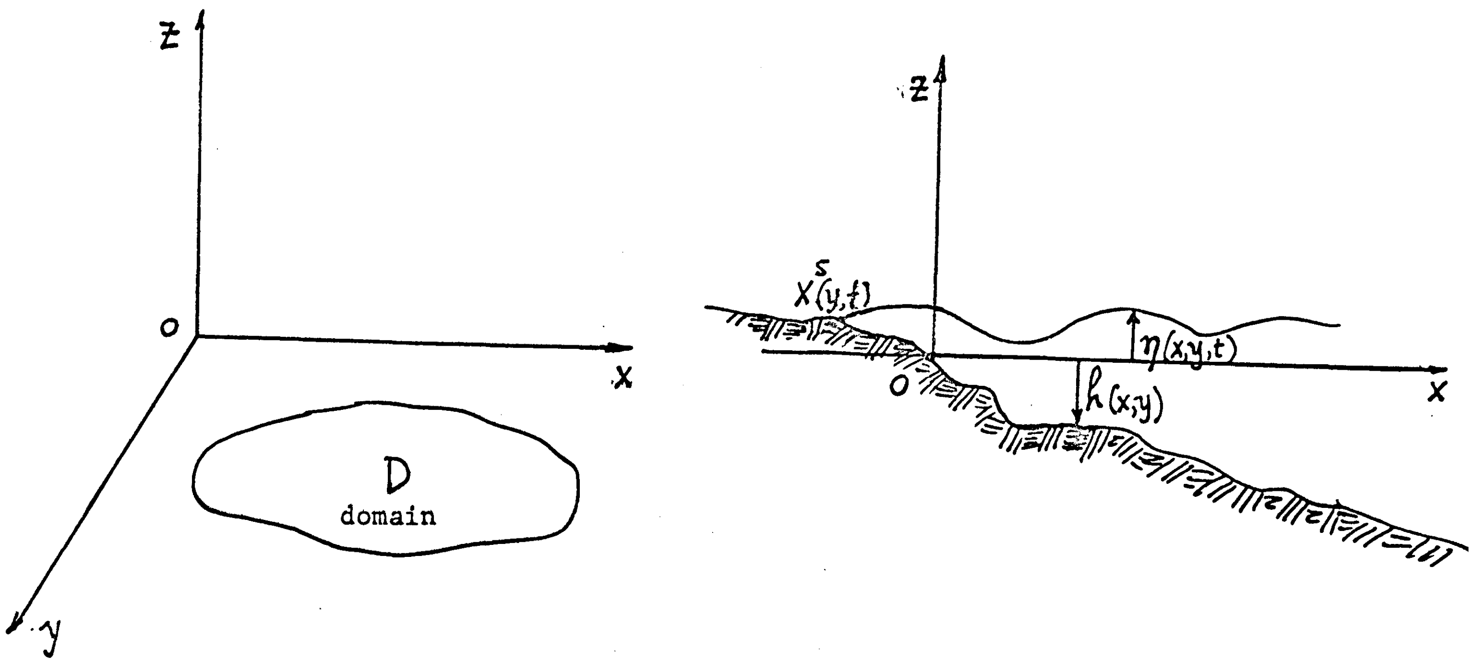
In view of the above the general mathematical formulation of the problem is:

(i) The Laplace equation $\nabla^2\phi = 0$

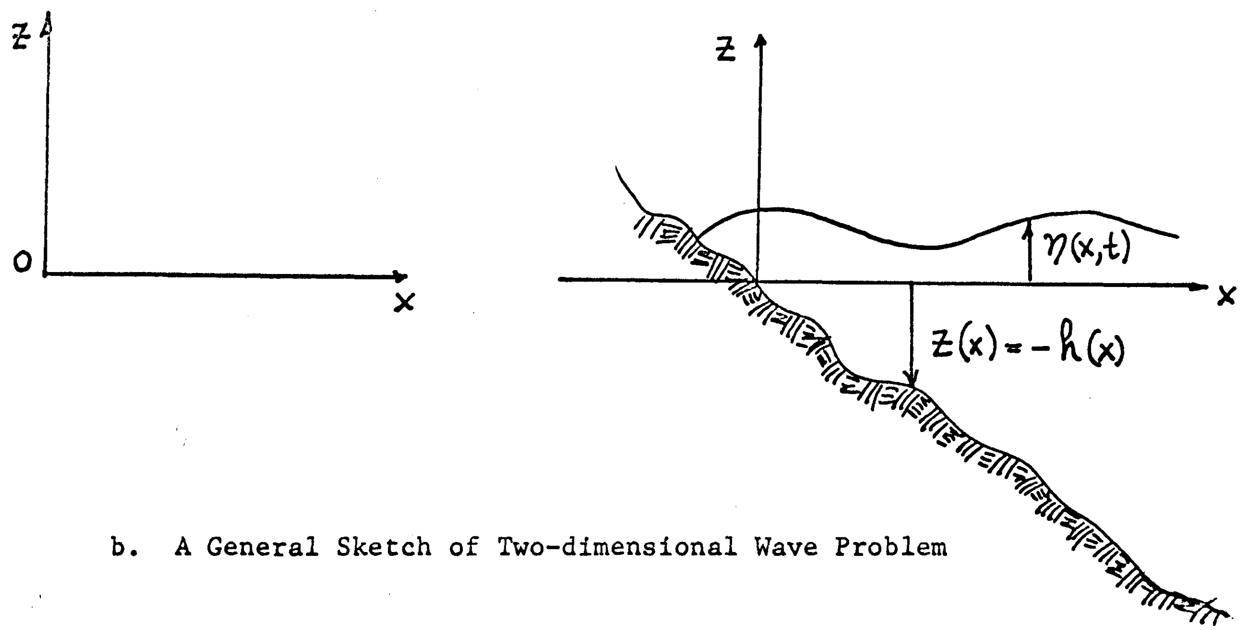
$$\text{for} \quad \begin{cases} x^s(y,t) < x < \infty \\ -h(x,y) < z < \eta(x,y,t) \\ -\infty < y < \infty \end{cases}$$

It has to be assumed here that ϕ is uniformly bounded and its derivatives are regular throughout the above region.

(ii) The boundary condition to be satisfied at the bottom (which is a homogeneous linear B.C.) or at the walls of a container is $\frac{\partial\phi}{\partial\eta} = 0$ for $z = -h(x,y)$. In this case the important thing is that we know in advance



a. A General Sketch of Three-dimensional Wave Problem



b. A General Sketch of Two-dimensional Wave Problem

Fig. 32 (a,b)

the exact boundary, which remains constant with time. This is not the case for the time dependent free surface $\zeta = z-\eta(x,y,t) = 0$. (2)

(iii) The non-linear kinematical condition $\frac{dz}{dt} = 0$ where $\zeta = z-\eta(x,y,t) = 0$ (a consequence of continuum mechanics) says that the "surface ζ" has to be conserved, i.e. a water particle on the free surface remains on it and cannot escape. Thus we have:

$$\phi_x\eta_x-\phi_z\eta_y-\phi_z+\eta_t = 0 \quad \text{for } \zeta=0 \qquad (3)$$

(iv) The non-linear dynamical condition from Bernoulli's law for unsteady non-uniform motion is:

$$g\eta+\phi_t+\frac{1}{2}(\phi_x^2+\phi_y^2+\phi_z^2) = F(x,y,t) \quad \text{on } \zeta=0 \qquad (4)$$

with $F(x,y,t) \equiv 0$ everywhere except over the region where the disturbance is created. For $x \to \infty$ we might prescribe that ϕ and η are finite. If we prescribe the conditions fixing the disturbance, for example, by assuming the pressure P as atmospheric over the disturbed region and thus prescribing the function $F(x,y,t)$, we obtain the dynamical B.C.:

$$g\eta+\phi_t+\frac{1}{2}(\phi_x^2+\phi_y^2+\phi_z^2)+\frac{P}{\rho} = C(t) \quad \text{on } \zeta=0 \qquad (5)$$

(v) Initial Condition. It is assumed that the water is initially at rest and fills the region $-h(x,y) < z < 0$, $-\infty < y < \infty$, $x^s(y,t) < x < \infty$. Initial conditions are then $\eta(x,y,t) = 0$ for $t=0$, $\phi_x=\phi_y=\phi_z = 0$ for $t=0$.

At time $t=0$ a given disturbance is created on the surface of the water (wind, etc.). With all the above assumptions and restrictions we wish to describe mathematically the subsequent physical motion of any finite amplitude.

2. Exact Linear Potential Theory of Small Amplitude

Two different types of approximations to the exact non-linear wave theory can be distinguished: (1) the linear exact theory of small amplitude waves [approximation of Airy (11) and Stokes (2)] and (2) the shallow non-linear water wave theory of finite amplitude based on the assumption of hydrostatic pressure distribution [approximation of Rayleigh (42) and Boussinesg (43)]. The latter contains the linearized shallow water theory of small amplitude waves as a part.

In the case of exact linear theory of small amplitude waves we have: (1) the linearization of the two non-linear B.C., namely the dynamical and kinematical conditions and (2) a wave with a small amplitude. Assuming that $\eta, \phi_x, \phi_y, \phi_z$ are so small that all non-linear terms in the two B.C. can be neglected and that this type of approximation happens to take place not on $z = \eta(x,y,t)$ which is unknown a priori but on z=0, the undisturbed water surface. Then we have: (1) the domain of the solution fixed a priori $\eta = \frac{1}{g}\phi_t$ and (2) the problem is now the linear classical boundary value problem of potential theory.

Assuming that the atmospheric pressure P=0, the two-dimensional problem is now:

$$\nabla^2\phi = 0 \qquad \text{for} \quad x^s(t) < x < \infty \quad -h(x) < z < \eta(x,t) \qquad (6)$$

with (1) B.C.

$$\frac{\partial\phi}{\partial z} = 0 \qquad \text{for} \quad z = -h(x) \qquad (7)$$

$$\eta_t - \phi_z = 0 \qquad \text{for} \quad z=0 \quad \text{(kinematical condition)} \qquad (8)$$

$$g\eta + \phi_t = 0 \qquad \text{for} \quad z=0 \quad \text{(dynamical condition)} \qquad (9)$$

and (ii) initial conditions

$$\eta(x,t) = 0 \qquad \text{for} \quad t=0 \tag{10}$$

$$\phi_x = \phi_z = 0 \qquad \text{for} \quad t=0 \tag{11}$$

From conditions (8) and (9) by elimination of η we get the expression:

$$\phi_{tt} + g\phi_z = 0 \qquad \text{for} \quad z=0 \tag{12}$$

The above now is a dynamical problem of small amplitude oscillation. Assuming a solution of the form $\phi(x,y,t) = \varphi(x,y)e^{i\sigma t}$ with φ a real function we obtain the homogeneous boundary value problems:

$$\nabla^2\varphi = 0 \qquad \begin{array}{l} -h(x) < z < 0 \\ x^s(t) < x < \infty \end{array} \tag{13}$$

$$\varphi_z = 0 \qquad \text{for} \quad z = -h(x) \tag{14}$$

$$\varphi_z - \frac{\sigma^2}{g} = 0 \qquad \text{for } z=0 \tag{15}$$

It is also assumed that φ and φ_z are finite at infinity which implies that the vertical components of the displacements and velocity are bounded at infinity. Arbitrary initial conditions cannot be described since it is assumed that the oscillation is small and of the harmonic type $e^{i\sigma t}\varphi(x,z)$. Thus

$$g\eta+\phi_t = 0 \qquad\qquad \text{for } z=0$$

and gives

$$\eta = \frac{1}{g}\phi_t = -\frac{i\sigma}{g} e^{i\sigma t} \; (x,0)$$

For water of infinite depth the functions satisfying the above are

$$\phi_1 = e^{(kz+i\sigma t)} \cos kx$$

and

$$\phi_2 = e^{(kz+i\sigma t)} \sin kx$$

where

$$k = \frac{2\pi}{L} \quad \left(k \sim \frac{\sigma^2}{g}\right)$$

The general solution, since the problem is linear and homogeneous, takes the form

$$\phi = \phi_1+\phi_2 = e^{(kz+i\sigma t)}(\cos kx + i \sin kx)$$

which is a standing wave solution.

In case of uniform finite depth, h, the above homogeneous boundary value problem gives a solution in the form

$$\phi_1 = e^{i\sigma t}\cosh k(z+h)\cos kx$$

$$\phi_2 = e^{i\sigma t}\cosh k(z+h)\sin kx$$

or

$$\phi = \phi_1 + \phi_2 = \frac{ag}{\sigma} \frac{\cosh k(h+z)}{\cosh kh} \cos(kx \pm \sigma t) \tag{16}$$

where a represents the amplitude of progressive eaves in both directions and k satisfies the dispersion relation $\sigma^2 = gk \tanh kh$. ($\sigma = Ck$) or

$$C = \left(\frac{gL}{2\pi} \tanh \frac{2\pi h}{L}\right)^{1/2}$$

Expanding $\sigma^2 = gk \tanh kh$ in power series of $\tanh \frac{2\pi h}{L}$:

$$\sigma^2 = \frac{2\pi g}{L} \left[\frac{2\pi h}{L} - \frac{1}{3}\left(\frac{2\pi h}{L}\right)^3 + \frac{2}{15}\left(\frac{2\pi h}{L}\right)^5 - \frac{17}{315}\left(\frac{2\pi h}{L}\right)^7 + \ldots\right]$$

from which we can see that $\sigma^2 = \left(\frac{2\pi}{L}\right)^2 gh = k^2 gh$ as $\frac{h}{L} \to 0$ and hence $C \simeq \sqrt{gh}$, if $\frac{h}{L}$ is very small. This is the case of linear shallow wave theory. In case of $\frac{h}{L}$ very big, ($h \to$ infinite), $\tanh \frac{2\pi h}{L} \to 1$ and thus $C = \left(\frac{gL}{2\pi}\right)^{1/2}$. This is the case of deep water waves (4).

This development proves the accuracy of linear shallow water theory. It can be calculated (44) that $C = \sqrt{gh}$ is in error by 5-6% to the exact value if the wave length is ten times the depth and by less than 2% if the wave length is about twenty times the depth. The conclusion is that the variation of the depth has small influence on a progressive wave of small amplitude provided that the depth is more than half of the wave length.

3. Non-Linear Shallow Wave Theory (Non-potential theory)

The other type of approximation to the exact wave theory is the non-linear shallow wave theory based on the fundamental assumption that the water depth is small in comparison with the radius of curvature of water surface, for example, and the pressure is given by hydrostatic law $P = g\rho(\eta - z)$ (17). Thus the horizontal velocity component u is independent of the depth coordinate z for all time t if it was independent at the initial time t=0. No assumption is made in this theory concerning the wave amplitude or the B.C. which are non-linear and give the non-linear form of the wave problem (45, 46, 47).

For the formulation of the non-linear shallow water theory we use the continuity equation (which does not imply irrotationality as in the case of the Laplace equation) and the exact kinematical and dynamical conditions.

Starting from continuity $u_x + w_z = 0$ in two-dimensional (18) form, the free surface kinematical condition for

$$0 = z - \eta(x,t) = \zeta$$

is

$$\frac{dz}{dt} = -\eta_t + u(\eta_x) + w = 0 \quad \text{on } z = \eta$$

or

$$\eta_t + u\eta_x - w \Big|_{z=\eta} = 0 \qquad (19)$$

The free surface dynamical condition is

$$P\Big|_{z=\eta} = 0 \tag{20}$$

The bottom kinematical condition is

$$\frac{dh(x)}{dt} = 0$$

or

$$h_t + uh_x + wh_z = 0$$

or

$$-uh_x - w\Big|_{z=-h} = 0$$

or

$$uh_x + w\Big|_{z=-h} = 0 \tag{21}$$

Integration of (18) with respect to z yields:

$$\int_{-h}^{\eta} (u_x)\,dy + w\Big|_{-h}^{\eta} = 0 \tag{22}$$

Using the relations (19) at z=η and (21) at z=-h gives:

$$\int_{-h}^{\eta} u_x dz + \eta_t + u\Big|_{\eta} \cdot \eta_x + u\Big|_{-h} h_x = 0 \tag{23}$$

Introducing the relation of integration according to the Leibniz rule:

$$\frac{\partial}{\partial x}\int_{-h(x)}^{\eta(x)} u dy = u\Big|_{z=\eta}\eta_x + u\Big|_{z=-h}h_x + \int_{-h}^{\eta} u_x dy \tag{24}$$

Combining with (23) we obtain:

$$\frac{\partial}{\partial x}\int_{-h}^{\eta} u dy = -\eta_t \tag{25}$$

No approximation up to this point has been made. With the assumption of hydrostatic pressure distribution we have:

$$P = \rho g(\eta - z)$$

and differentiating

$$P_x = \rho g \eta_x \tag{26}$$

Thus P_x is independent of z. It follows that the x-component of the acceleration of the water particles is also independent of z for all time, t, if it is at the beginning.

The equation of motion in the x-direction is:

$$u_t + uu_x + uu_z + \frac{1}{\rho} P_x = 0 \tag{27}$$

Since u_z=0 and $P_x = -\rho g \eta_x$, we have:

$$u_t = uu_x = g\eta_x \tag{28}$$

and using (25) we obtain:

$$\frac{\partial}{\partial x}\int_{-h}^{\eta} u\ dz = -\eta_t$$

and considering

$$\int_{-h}^{h} u\ dy = u\int_{-h}^{\eta} dz$$

then

$$\int_{-h}^{\eta} u\ dz = u[z]\Big|_{-h}^{\eta} = u[\eta+h]$$

and

$$\frac{\partial}{\partial x}\int_{-h}^{\eta} u\ dz = -\eta_t = \frac{\partial}{\partial x}(u[\eta+h])$$

or

$$[u(\eta+h)]_x = -\eta_t \tag{29}$$

Thus we have the two first-order differential equations of the non-linear shallow water wave theory:

$$u_t + uu_x + g\eta_x = 0 \qquad (30)$$

$$[u(\eta+h)]_x + \eta_t = 0 \qquad (31)$$

This is the development of shallow non-linear wave theory based on continuity and momentum equations and exact non-linear B.C. without consideration of irrotationality of an incompressible, inviscid fluid (17).

Another formulation of the non-linear shallow water theory based on irrotationality of an inviscid incompressible fluid was given by Friedrichs (48) as the lowest order approximation of a perturbation scheme for the surface wave elevation η, pressure and the velocity potential ϕ. Perturbing all quantities in powers of the ratio $(\frac{h}{L})^2$, Friedrichs derived the non-linear wave theory to any order of approximation (i.e. non-linear shallow, cnoidal, etc.) the equations of non-linear shallow wave theory in three dimensions and in a dimensionalized form are:

$$u_t^{(o)} + u^{(o)}u_x^{(o)} + w^{(o)}u_z^{(o)} + \eta_x^{(o)} = 0 \qquad (32)$$

$$w_t^{(o)} + u^{(o)}w_x^{(o)} + w^{(o)}w_z^{(o)} + \eta_z^{(o)} = 0 \qquad (33)$$

$$\eta_t^{(o)} + [u^{(o)}(\eta^{(o)} + h)]_x + [w^{(o)}(\eta^{(o)} + h)]_z = 0 \qquad (34)$$

It should be noticed here that with the above perturbation approach the non-linear shallow water theory presents complete evidence of what parameter determines the accuracy of the approximation process. This was not so evident in the previous method of hydrostatic law assumption. Another interesting fact pointed out by Stoker (17) is that the above approximation of lowest order to the solution of a problem in potential theory is sought in the form of a solution of a non-linear wave equation. This implies that the solution of a problem of elliptic type is approximated (at least in the lowest order) by the solution of a problem of hyperbolic type.

It should also be noticed that the shallow non-linear wave theory, to the lowest approximation - and only for this approximation - for a constant depth, flat horizontal bottom is similar to gas dynamics analogy (two-dimensional isentropic flow) as pointed out by Riabouchinsky (49, 50) with a polytropic index equal to 2 (specific heat ratio $\gamma=2$).

4. Shallow Linear Water Wave Theory of Small Amplitude

If in addition to the above basic assumptions of hydrostatic pressure distribution of non-linear shallow wave theory we add the further assumptions:

(i) that the particle velocity, u, and the surface elevation, η, are small quantities, and

(ii) that the derivatives of these two quantities, u_x and η_x, are small and thus their squares and products, the non-linear terms, are of negligible importance in comparison to the linear terms of the equations, it follows at once that the above (30) and (31) equations become:

$$u_t + g\eta_x = 0 \qquad (35)$$

$$(uh)_x + \eta_t = 0 \qquad (36)$$

Eliminating η from these two equations we obtain:

$$u_{tt} + g\,\eta_{tx} = 0$$

and

$$(uh)_{xx} + \eta_{tx} = 0$$

$$(uh)_{xx} - \frac{1}{g}\,u_{tt} = 0$$

$$(u_{xx}h) - \frac{1}{g}\,u_{tt} = 0$$

$$u_{xx} = \frac{1}{gh} u_{tt}$$

or

$$u_{xx} = \frac{1}{C^2} u_{tt} \tag{37}$$

This is the linearized version of the non-linear shallow wave theory. The tidal wave phenomena predominantly can be attacked on the basis of linear shallow wave theory. The above equation can be formulated in terms of the wave surface elevation, η.

Thus from (36) we obtain:

$$\eta_{tt} + [(uh)_x]_t = [u_t h + uh_t]_x$$

and using (35)

$$\eta_{tt} + [(uh)_x]_t = [-g\eta_x h + uh_t]_x$$

since $h_t = 0$ ($h=h[x]$), we obtain finally:

$$\eta_{tt} = g(\eta_x h)_x \tag{38}$$

5. Transformation of Gravity Waves over Sloping Beaches on the Basis of Small Amplitude Potential Theory

The problem closely associated with the transformation of waves through a channel of linearly or non-linearly varying geometry is the problem of wave motion over a sloping beach.

This significant problem was studied extensively and solved completely on a theoretical basis. An accurate way of measuring experimentally the amplitude and phase angle of the reflected wave from the beach is still missing. According to the exact approach the partial reflection from the beach was computed and explicit mathematical formulae were found.

This approach is significant as it considers reflection of the waves from the beach, while Green's theory did not. The assumption of no energy dissipation remains.

The first formulation and attack to the problem was done by Miche (12) in France (1944) and its progressive wave solution over sloping beach was found. Following the above attempt a very basic and mathematically elegant approach to the problem was undertaken by the New York University, Courant Institute of Mathematical Sciences. A very sketchy description of these important theoretical studies will be given in the following paragraphs.

The basic assumption by all the investigators of NYU is the linearized small amplitude potential wave motion of an inviscid, incompressible fluid, without consideration of energy dissipation or geostrophic effects.

According to this theory the problem of reflection from a sloping beach for all wave lengths was solved. The shortcoming of the theory is the lack of an explanation of the phenomenon of a probable breaking of the wave on the shore proper to the physical reality. The non-linear phenomenon of wave breaking cannot be handled on the basis of the linear theory with the assumption of a logarithmic singularity of the velocity potential. Wave energy in breaking is transformed through turbulence into heat energy and undertow.

It is true that Miche's (12) approach didn't reach the details and the conclusions of the New York University studies especially the asymptotic solution (48, 51) for the case of beaches of small slope. Miche's considered without any mathematical proof that beaches with a slope smaller than 20^{o} may have little influence on the wave motion, and for practical purposes such an influence can be neglected and the wave speed can be considered as independent of the beach slope.

A rigorous mathematical formulation and proof was given by K. O. Friedrichs (48, 51) to Miche's conclusion, for beaches with very small slope where the determination of the wave motion becomes very complicated for mathematical treatment.

Through an asymptotic approximation and using linearized theory, Friedrichs showed that for sloping beaches of less than 20^{o} the bottom slope did not effect seriously the wave motion. Thus the thesis of small negligible influence of such beaches on the wave motion was very well established.

J. J. Stoker (17) gives a description of the methods followed by different investigators in the attack of the problem of finding standing wave solutions over sloping beaches.

Hanson (52) presented (1926) a type of standing wave linear solution of the problem.

Bondi (53) also found progressive wave solutions and Lewy (54) and Stoker (44) found solutions using the method of linear operators. With this method a complex function F(z), whose real part vanishes at bottom and surface boundary, can be determined uinquely be successive reflections in the entire complex plane. The basic idea is that, once the wave frequency has been established, two different types of standing waves can be obtained: one with infinite amplitude at the shore line - which takes care of the amplitude amplification and breaking - the other of finite amplitude. Thus the two different types of standing waves behave at infinity like the simple standing wave solutions for water of infinite depth. The combination of the standing wave with appropriate factors yield arbitrary simple harmonic progressive wave solutions at infinity.

On a similar line F. John (55), S. C. Lowell (56), G. Kreisel (57) and E. Isaacson (58) for different specific geometries and wave conditions over sloping beaches solved the same problem on the basis of linearized small amplitude wave theory. E. Isaacson (58) obtained the two-dimensional wave solution in an integral form and for all slope angles. A. Heins (59) gave also a solution of a three-dimensional wave in water of uniform finite depth.

M. Roseau (60) used the method proposed by Stoker in solving in a very rigorous mathematical treatment the three-dimensional wave problem over a sloping beach of slope $\alpha = \frac{\pi}{4}$.

A. Peters (61) has completely solved the problem of three-dimensional wave over a sloping beach for any angle of slope. J. Tlapa (62) tried to formulate the problem of two-dimensional wave motion over a sloping beach following a region of horizontal uniform depth on the basis of a potential theory for finite amplitude waves.

Tlapa didn't solve this very difficult problem. The value of his approach lies in the idea of a double scheme perturbation technique - used recently in many other fields of applied mathematics - with the wave amplitude and bottom slope as perturbed parameters. The determination of a potential function by proceeding in the manner of Stokes for the above case is a very difficult subject and has not yet been solved.

K. K. Wong, A. T. Ippen, D. R. F. Harleman (63) presented theoretical solutions of the problem of progressive waves over an idealized continental shelf (gradually varying depth) or a submerged shelf of different geometries under the assumption of linearized small amplitude wave theory for very long waves - the so-called tsunamis waves.

M. C. Shen (64) theoretically investigated the two-dimensional waves generated by a surface pressure disturbance over a sloping beach on the basis of linearized theory. He considered the forced and free waves (steady state and transient case) and reduced using Laplace transform technique to an inhomogeneous differential equation. A theoretical example of an oscillatory point pressure acting on an equilibrium surface of a beach of 45° slope has been solved in detail.

For the sake of completeness of the review and without analysis of the details, the non-linear approach of finite amplitude waves over sloping beaches follows:

The non-linear wave approach of finite amplitude over constant uniform depth was established by Stokes (2), Levi-Civita (40), Rayliegh (42), Struik (41), Nekrassov (65), Michell (66), McCowan (67) and Wilton (68).

The more extensive mathematical treatment recently is that of Penney and Price (69), Taylor (70), Ursell (71), T. Kishi (72), Yamada (73), C. C. Lin and A. Clark (74), Chappelear (75), Borgman and Chappelear (76), Skjelbreia (77), Skjelbreia and Hendrickson (78), Bretschneider (79), Wehausen and Laitone (80), Long (81), J. Luke (82) and S. C. De (83).

In the non-linear approach of finite amplitude waves over sloping beaches investigations were made by Carrier and Greenspan (84), Greenspan (85) and Carrier (86).

Carrier and Greenspan solved in part the difficulty and theoretical inconsistence arising at the shore line development of a progressive wave where according to the linear theory and also according to the shallow non-linear theory of gas dynamics analogy, the wave reaching a critical value of amplitude amplification breaks. According to some aspects of the non-linear approach (84) the wave can climb - under special conditions (wave shape, particle velocity distribution, etc.) - without breaking over a linearly sloping beach in a continuous interchange of potential energy to kinetic energy and vice versa. Thus the reflection coefficient is equal to one as in the case of full reflection into a vertical wall without breaking. However general criterion for the occurrence of breaking of a wave was not established. Greenspan (85) in another study found a very limited mathematical criterion for the wave breaking on the basis of a non-linear wave propagation shoreward into a quiescent water. He proved that a wave with a non-zero slope at the wave front does not climb in the coastline but breaks before reaching it. Experimental justification of the above theories on a thorough basis does not exist.

Carrier (86) tried in a very general mathematical theory to include the influence of bottom topography varying in both horizontal coordinates (one and two-dimensional bottom topograp y) especially in the shallower part of the shelf where the non-linear contributions to the wave dynamics become ery important.

However the problem of reflection of incoming waves is not included in such a theory - and also in the previous ones (84) and (85) - as a basic requirement. Experimental investigation of the above theories on a rigorous basis does not exist.

Kishi (87) has presented the first experimental results on wave run-up of a long wave for 1/30 slope on the basis of non-linear theory. A correlation of the relative run-up height (ratio of the height of the wave over the height at the foot of the transition) with wave steepness agrees fairly well with Kaplan's (88) curve for the solitary wave under the assumption that the solitary wave (Kaplan's) is replaced by the half wave height of the long wave (Kishi). Kishi also supported the idea that his results agree fairly well with Greenspan's non-linear wave solution and also stated that the transformation of above in shoaling water (sloping beach 1:30) in his experiments agree with the theoretical results presented by Keller, Levine, and Whitlau (89). Even the effects of bottom friction is briefly discussed in this experimental study and the problem of wave reflection from a sloping beach is ignored in all such cases. Since reflection from the beach is a major item and till now there is no complete mathematical and experimental treatment of the subject in non-linear wave theory, the results and conclusions are open to such a consideration.

Le Méhauté and C. C. Mei (90) studied theoretically the non-linear shallow wave over an uneven bottom under the assumption of small depth in comparison with wave length (a characteristic horizontal length). They transformed the wave equations to a set of first-order quasi-linear equations with the characteristic curves in x-t plane which have direct relation with the bottom topography and shape.

Amein (91) more concretely studied the propagation of long period waves (shallow waves) by the first-order linear approximation (using Friedrich's (21) asymptotic representation appropriate for small beach slope) for regions far from the shore and he also studied the propagation of shallow waves on the basis of non-linear shallow wave theory for regions near the shore. A rigorous computational program and numerical calculation were made on a digital computer by a finite difference scheme based on the method of characteristics in the usual approach of solution of the first-order hyperbolic differential equation of the non-linear shallow water wave theory.

It could be mentioned here the extensive study of Japanese school (92) on tsunamis run-up along beaches and shores. The papers of Horiwaka, Iida, Iwagaki, Kato, Murota, Takahashi are characteristic of the traditional Japanese contribution to this subject.

Finally extensive theoretical work has been conducted in the investigation of solitary waves modification over sloping beach or in channel with arbitrary cross section. The papers of Keller (93) and Peters (94) are among the most extensive and mathematically rigorous on this subject.

Beitinjani and Brater (18) investigated the refraction of the waves on a trapezoidal channel under the assumptions (i) of pure energy transmission and (ii) of negligible refraction from the side of the trapezoid and reflection from the beach. They presented an expression of wave energy transmission in the form of an integral equation and evaluated it through a computer program.

Phillips (95), M. S. Longuet-Higgins (96) and Benney (97) treated mathematically the dynamics of non-linear interactions and possible resonances associated with these interactions of finite amplitude gravity waves as a mechanism for the transfer of wave energy.

G. A. Tlapa, C. C. Mei, P. S. Eagleson finally (98) treated theoretically an asymptotic expansion to formulate a potential theory for the propagation of waves on beaches of small slope. The form of this asymptotic expansion has the advantage of developing solutions to higher approximations and over the entire range from deep water to breaking.

M. S. Longuet-Higgins (99) established also theoretically the parameter $K_b = \frac{L^2 H}{2h^3}$ and its relation to the breaking of the wave.

APPENDIX B

Verification of the Computer Program for the Reduction of Experimental Data by Elimination of Beach-end Reflection According to Ursell-Dean's Method

We obtain the experimental run A-2 and we analyze it by desk calculations for the elimination of beach-end reflection as follows:

Experimental Run A-2 - Measured Data:

L_1 = 19.00 upstream wave length

h_1 = 2.25 upstream undisturbed water depth

h_2 = 1.25 downstream undisturbed water depth

Upstream amplitudes:

a_1 = 0.047 x_{max} = 33.00 location of maximum value of combined wave amplitude upstream $|\eta_1+\eta_2|$

a_2 = 0.013 x_a = 31.60 position of gauge upstream for simultaneous upstream and downstream maxima.

Downstream amplitudes:

a_3 = 0.053 x_{max} = -7.25 location of maximum value of combined wave amplitude downstream $|\eta_3+\eta_4|$

a_4 = 0.020 x_b = -1.70 position of gauge downstream for simultaneous upstream and downstream maxima.

For h_1/L_1 = 0.1184 we obtain from Wiegel's Gravity Waves - Tables of functions:

for h_1/L_1 = 0.118 we obtain $\frac{h_1}{L_o} = 0.075 \qquad L_o = \frac{2.25}{0.075} = 30.00$ (1)

$$\frac{h_3}{L_o} = \frac{h_3}{h_1} \cdot \frac{h_1}{L_o} = \frac{1.25}{2.25} \cdot \frac{2.25}{30.00} = 0.04166$$

for $h_3/L_o = 0.04166$ we obtain $\frac{h_3}{L_3} = 0.0850$, $L_3 = \frac{1.25}{0.0850} = 14.70$ (2)

We have also:

$$K_1 = \frac{2\pi}{L_1} = \frac{6.28}{19.00} = 0.330$$

$$K_3 = \frac{2\pi}{L_3} = \frac{6.28}{14.70} = 0.427$$

Thus;

$$\delta_1+\delta_2 = (2n+1)\pi - 2K_1\, x_{max} = \pi - 2(0.330)(33.00)$$

$$= 3.14 - 21.78 = -18.64 \qquad (3)$$

The amplitude ratios are:

$$\frac{a_2}{a_1} = \frac{0.013}{0.047} = 0.276$$

$$\frac{a_1}{a_2} = \frac{0.047}{0.013} = 3.615$$

$$\frac{a_4}{a_3} = \frac{0.020}{0.053} = 0.377$$

We compute the phase angle δ_4:

$$\delta_4 = (2n+1) \quad - 2K_3\, x_{max} = 3.14 - 2(0.427)(-7.25) = 3.14+6.19 = 9.33 \text{ rad} \qquad (4)$$

We compute the following quantities for determination of phase angle δ_2:

$$K_1x_a = (0.330)(31.60) = 10.43 \text{ rad} = 4.15 \text{ rad}$$

$$K_3x_b = (0.427)(-1.70) = -0.726 = -0.726 \text{ rad}$$

$$\cos(K_1x_a) = \cos(4.15) = -0.532 \quad \sin(K_1x_a) = \sin(4.15) = -0.845$$

$$\cos(K_3x_b) = \cos\ (-0.726) = 0.754 \quad \sin(K_3x_b) = \sin(-0.726) = -0.663$$

$$\cos(K_1x_a+\delta_1+\delta_2) = \cos(10.43-18.64) = \cos(-1.93) = \cos(4.35) = -0.354$$

$$\sin(K_1x_a+\delta_1+\delta_2) = \sin(-1.93) = \sin(4.35) = -0.898$$

$$\cos(K_3x_b+\delta_4) = \cos(-0.726+9.33) = \cos(8.6) = \cos(2.32) = -0.681$$

$$\sin(K_3x_b+\delta_4) = \sin(2.32) = 0.731$$

Thus the value of the parameter R is:

$$R = \frac{-\cos K_3x_b + \frac{a_4}{a_3}\cos(K_3x_b + {}_4)}{\sin K_3x_b + \frac{a_4}{a_3}\sin(K_3x_b + {}_4)} = \frac{-0.754+0.377(-0.681)}{-0.663+0.377(0.731)}$$

$$= \frac{-0.754-0.256}{-0.663+0.275} = \frac{-1.010}{-0.380} = 2.66$$

Now δ_2 can be computed as follows:

$$\tan \delta_2 = \frac{\cos(K_1x_a+\delta_1+\delta_2) + R\sin(K_1x_a+\delta_1+\delta_2) - \frac{a_2}{a_1}[\cos K_1x_a + R\sin K_1x_a]}{-\sin(K_1x_a+\delta_1+\delta_2) + R\cos(K_1x_a+\delta_1+\delta_2) - \frac{a_2}{a_1}[\sin K_1x_a + R\cos K_1x_a]}$$

$$= \frac{-0.354+2.66(-0.898) - 0.276[-0.532-2.66(-0.845)]}{-(-0.898)+2.66(-0.354)-0.276[-0.845+2.66(-0.532)]}$$

$$= \frac{-0.355-2.39-0.276(-0.532+2.24)}{0.898-0.948-0.276(-0.845-1.415)} = \frac{-2.742-0.473}{-0.050+0.623} = \frac{-3.22}{0.57} = -5.65$$

$$-2.260$$

Thus $\delta_2 = 4.88$ and $\delta_2 = -1.40$ (5)

since

4.88 rad equivalent to -1.40.

Now we obtain:

from $\delta_1+\delta_2 = -18.7$ (6)

then we have $\delta_1 = -17.3$ (7)

Now $\cos(\delta_1-\delta_2+\delta_4) = \cos(-17.3+1.4+9.33) = \cos(-6.57) = \cos(-0.29)$

$$= 0.954$$

The reduced values of wave amplitudes are now obtained as follows:

(i) $$a_1' = a_1[1+(\frac{a_2}{a_1})^2(\frac{a_4}{a_3})^2 - 2(\frac{a_2}{a_1})(\frac{a_4}{a_3})\cos(\delta_1-\delta_2+\delta_4)]^{1/2}$$

$$= 0.047[1+(0.276)^2(0.377)^2 - 2(0.276)(0.377)(0.954)]^{1/2}$$

$$= 0.047[1+(0.0761)(0.142) - 0.198]^{1/2}$$

$$= 0.047[1+ 0.108 - 0.198]^{1/2}$$

$$= 0.047(0.90)^{1/2} \; 0.047(0.94) = 0.044 \qquad (8)$$

(ii) $$a_2' = 0.013[1 + (\frac{a_1}{a_2})^2 (\frac{a_4}{a_3})^2 - 2(\frac{a_1}{a_2})(\frac{a_4}{a_3})\cos(\delta_1 - \delta_2 + \delta_4)^{1/2}$$

$$= 0.013[1+(3.615)^2(0.377)^2-2(3.615)(0.377)(0.954)]^{1/2}$$

$$= 0.013[1+(13.06)(0.142)-2(3.615)(0.359)]^{1/2}$$

$$= 0.013[1+ 1.854 - 2.62]^{1/2} = 0.013(0.234)^{1/2}$$

$$= 0.013(0.484) = 0.00629 \qquad (9)$$

(iii) $$a_3' = a_3[1-(\frac{a_4}{a_3})^2] = 0.053[1-(0.377)^2] = 0.053[1-0.142] = 0.0455 \qquad (10)$$

The reduced values of reflection and transmission coefficients are now obtained as follows:

$$\frac{H'}{L_1} = \frac{2a_1'}{L_1} = \frac{2(0.044)}{19.00} = \frac{0.088}{19} = 0.0046 \qquad (11)$$

$$\frac{H_3}{L_3} = \frac{2a_3'}{L_3} = \frac{2(0.0455)}{14.7} = \frac{0.091}{14.7} = 0.00619 \qquad (12)$$

$$K_t' = \frac{a_r'}{a_1'} = \frac{0.0460}{0.0440} = 1.04 \qquad (13)$$

$$K_r' = \frac{0.0063}{0.0440} = 0.140 \qquad (14)$$

From Wiegel's Table:

$$\text{for } \frac{h_1}{L_1} = 0.118 \qquad n_1 = 0.853$$

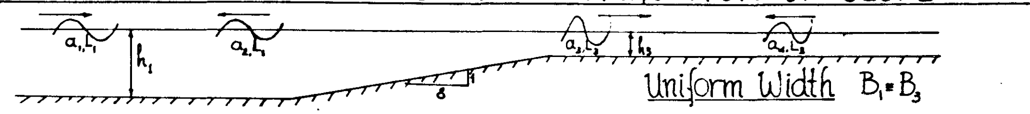
$$\text{for } \frac{h_3}{L_3} = 0.0850 \qquad n_2 = 0.916$$

$$\frac{C_{G3}}{C_{G1}} - \frac{L_3 n_3}{L_1 n_1} = \frac{(14.7)(0.916)}{(19)(0.853)} + \frac{13.46}{16.20} = 0.830 \qquad (15)$$

The values of quantities (1) up to (15) computed by desk calculations are the same as those given by the computer program used in the reduction of experimental data.

APPENDIX C

TABLE Ia TEST SERIES WITH TRANSITION OF SLOPE 1:8



Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	C_1 ft/sec	C_3 ft/sec	C_{g3}/C_{g1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^6$ ft²	$E_r=a_2^2\times10^6$ ft²	$E_t=a_3^2\frac{C_{g3}}{C_{g1}}\times10^6$ ft²	$E_{tr}=a_4^2\frac{C_{g3}}{C_{g1}}\times10^6$ ft²	$(a_2^2+a_3^2\frac{C_{g3}}{C_{g1}})\times10^6$ ft²	$(a_1^2+a_4^2\frac{C_{g3}}{C_{g1}})\times10^6$ ft²	$1-\frac{a_2^2+a_3^2\frac{C_{g3}}{C_{g1}}}{a_1^2+a_4^2\frac{C_{g3}}{C_{g1}}}$ % Loss	E_T/E_i	E_{rT}/E_i	E_{tr}/E_i	$L_1^2a_1/h_1^3$	$L_3^2a_3/h_3^3$	C_1T/a_1	$\frac{\delta^2}{g}(h_1-h_3)$
A-1	2.424	2.25	1.25	30.08	19.00	14.70	0.0026	7.842	6.067	0.8299	0.025	0.009	0.026	0.007	625	81	561	41	642	666	3.6	0.897	0.129	0.066	0.804	2.88	160.36	0.208
2	"	"	"	"	"	"	0.0049	"	"	"	0.047	0.013	0.053	0.020	2209	169	2331	332	2490	2541	2.0	1.055	0.076	0.150	1.520	5.85	404.44	"
3	"	"	"	"	"	"	0.0066	"	"	"	0.063	0.016	0.068	0.016	3967	256	3837	212	4093	4179	2.1	0.967	0.064	0.053	2.030	7.52	301.93	"
4	"	"	"	"	"	"	0.0086	"	"	"	0.082	0.020	0.090	0.024	6724	400	6722	478	7122	7202	1.2	0.999	0.059	0.071	2.640	9.95	231.82	"
5	1.916	"	"	18.79	14.25	11.30	0.0185	7.441	5.900	0.8857	0.132	0.040	0.133	0.023	17424	1600	15667	468	17267	17892	3.5	0.899	0.092	0.027	2.400	9.08	108.00	0.334
6	"	"	"	"	"	"	0.0140	"	"	"	0.100	0.016	0.104	0.021	10000	256	9764	390	10020	10390	3.6	0.976	0.026	0.039	2.900	6.79	142.56	"
7	"	"	"	"	"	"	0.0091	"	"	"	0.065	0.017	0.068	0.014	4225	289	4095	174	4384	4399	0.4	0.969	0.068	0.041	1.890	4.44	219.32	"
8	"	"	"	"	"	"	0.0045	"	"	"	0.031	0.015	0.032	0.007	961	225	886	43	1111	1004	—	0.922	0.234	0.045	0.730	2.22	451.87	"
9	1.533	"	"	12.02	10.50	8.60	0.0130	6.855	5.651	0.9698	0.061	0.024	0.061	0.020	3721	576	3611	387	4187	4108	—	0.970	0.154	0.104	2.080	2.57	172.26	0.521
10	"	"	"	"	"	"	0.0131	"	"	"	0.069	0.059	0.160	0.018	4761	3481	24827	314	28208	4761	—	—	0.731	0.066	2.110	6.05	152.29	"
11	"	"	"	"	"	"	0.0341	"	"	"	0.179	0.015	0.185	0.049	32041	225	33191	2328	33416	35369	5.6	1.035	0.010	0.073	5.450	7.00	58.70	"
12	"	"	"	"	"	"	0.0428	"	"	"	0.225	0.036	0.233	0.075	50625	1296	52649	5455	53945	56080	3.9	1.039	0.025	0.108	6.875	8.82	44.10	"
13	1.190	"	"	7.25	7.00	6.19	0.0143	5.855	5.203	1.0864	0.050	0.009	0.059	0.005	2500	81	2500	25	2581	2525	—	1.000	0.032	0.010	1.264	1.00	139.34	0.865
14	"	"	"	"	"	"	0.0277	"	"	"	0.097	0.013	0.097	0.012	9409	169	10218	156	10387	9565	—	1.085	0.018	0.016	2.810	1.90	71.82	"
15	"	"	"	"	"	"	0.0408	"	"	"	0.143	0.010	0.146	0.036	20449	100	21316	1407	21416	21856	—	1.101	0.007	0.068	4.153	2.82	48.81	"
16	"	"	"	"	"	"	0.0528	"	"	"	0.185	0.020	0.182	0.040	34225	400	35986	1600	36386	37825	—	1.051	0.011	0.047	5.350	3.57	37.66	"
17	0.766	"	"	3.00	3.00	2.97	0.0340	3.921	3.882	1.0414	0.051	0.003	0.052	0.012	2601	9	2704	144	2713	2745	0.2	1.039	0.004	0.055	1.031	0.23	58.88	2.087
18	"	"	"	"	"	"	0.0600	"	"	"	0.090	0.005	0.091	0.020	8100	25	8281	400	8306	8500	2.8	1.020	0.003	0.049	1.821	0.41	33.36	"
19	"	"	"	"	"	"	0.0766	"	"	"	0.115	0.010	0.115	0.025	13225	100	13225	625	13325	13850	3.8	1.000	0.075	0.047	2.300	0.52	26.11	"
20	3.089	"	"	48.82	25.00	19.06	0.0017	8.100	6.174	0.7959	0.021	0.016	0.022	0.016	441	196	385	203	581	644	9.8	0.873	0.444	0.460	0.453	4.09	1191.48	0.128
21	"	"	"	"	"	"	0.0030	"	"	"	0.038	0.024	0.040	0.022	1444	576	1273	385	1849	1829	—	0.881	0.398	0.266	0.824	7.43	658.45	"
22	"	"	"	"	"	"	0.0050	"	"	"	0.063	0.039	0.064	0.028	3969	1521	3260	624	4781	4593	12.0	0.900	0.291	0.353	1.360	12.45	397.16	"
23	"	"	"	"	"	"	0.0065	"	"	"	0.081	0.036	0.083	0.023	6561	1296	5482	421	6778	6982	5.0	0.961	0.049	0.064	1.750	16.53	308.90	"
25	3.064	2.00	1.00	48.06	23.50	16.99	0.0022	7.674	5.551	0.7555	0.026	0.012	0.030	0.014	676	144	679	150	823	826	0.4	1.004	0.213	0.222	0.500	4.42	904.35	0.130
26	"	"	"	"	"	"	0.0031	"	"	"	0.036	0.020	0.039	0.026	1296	400	1148	272	1549	1568	1.3	0.886	0.308	0.209	2.500	11.26	653.14	"
27	"	"	"	"	"	"	0.0044	"	"	"	0.052	0.028	0.054	0.020	2704	784	2203	300	2987	3004	—	1.039	0.133	0.110	3.600	17.46	452.72	"
28	2.415	"	"	29.84	18.00	13.21	0.0019	7.459	5.475	0.7878	0.017	0.009	0.022	0.015	289	81	381	177	462	466	0.9	1.318	0.280	0.612	0.690	3.84	1057.59	0.210
29	"	"	"	"	"	"	0.0041	"	"	"	0.037	0.020	0.039	0.017	1369	400	1198	227	1598	2598	—	0.875	0.292	0.165	1.500	6.81	486.84	"
30	"	"	"	"	"	"	0.0056	"	"	"	0.051	0.014	0.059	0.016	2601	196	2742	201	2938	2801	—	1.054	0.075	0.077	2.060	10.30	353.20	"
31	"	"	"	"	"	"	0.0078	"	"	"	0.070	0.040	0.072	0.030	4900	1600	4083	709	5683	5609	—	0.833	0.326	0.144	2.840	12.56	257.33	"
32	1.872	"	"	17.93	13.25	9.99	0.0027	7.084	5.342	0.8473	0.018	0.012	0.019	0.012	324	144	305	122	449	446	—	0.941	0.444	0.377	0.396	1.89	736.72	0.350
33	"	"	"	"	"	"	0.0060	"	"	"	0.040	0.023	0.041	0.020	1600	529	1424	339	1953	1939	—	0.890	0.330	0.212	0.880	4.07	331.52	"

TABLE Ia CONTINUED

Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	C_1 ft/sec	C_3 ft/sec	C_{G3}/C_{G1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_I=a_1^2\times10^6$ ft²	$E_{rr}=a_2^2\times10^6$ ft²	$E_T=a_3^2\frac{C_{G3}}{C_{G1}}\times10^6$ ft²	$E_{rB}=a_4^2\frac{C_{G3}}{C_{G1}}\times10^6$ ft²	$(a_2^2+a_3^2\frac{C_{G3}}{C_{G1}})\times10^6$ ft²	$(a_1^2+a_4^2\frac{C_{G3}}{C_{G1}})\times10^6$ ft²	$1-\frac{a_2^2+a_3^2\frac{C_{G3}}{C_{G1}}}{a_1^2+a_4^2\frac{C_{G3}}{C_{G1}}}$ % Loss	$\frac{E_T}{E_I}$	$\frac{E_{rr}}{E_I}$	$\frac{E_{rB}}{E_I}$	$\frac{L_1^2 a_1}{h_1^3}$	$\frac{L_3^2 a_3}{h_3^3}$	$\frac{C_1 T}{a_1}$	$\frac{\sigma^2}{g}(h_1-h_3)$
A-34	1.872	2.00	1.00	17.93	13.25	9.99	0.0084	7.084	5.342	0.8473	0.056	0.012	0.061	0.018	3136	144	3152	274	3296	3410	3.4	1.005	0.046	0.087	1.230	6.06	236.80	0.350
35	"	"	"	"	"	"	0.0122	"	"	"	0.081	0.012	0.091	0.027	6561	144	7016	617	7160	7178	2.5	1.069	0.022	0.094	1.78	9.04	163.71	"
36	1.463	"	"	10.95	9.50	7.50	0.0082	6.499	5.130	0.9456	0.039	0.021	0.040	0.022	1521	441	1512	457	1953	1978	1.3	0.994	0.289	0.300	0.44	2.25	243.80	0.572
37	"	"	"	"	"	"	0.0157	"	"	"	0.075	0.015	0.079	0.024	5625	225	5901	544	6126	6169	0.7	1.049	0.040	0.097	0.85	4.44	126.78	"
38	"	"	"	"	"	"	0.0229	"	"	"	0.109	0.014	0.114	0.033	11881	196	12289	1029	12485	12910	3.3	1.034	0.016	0.086	1.23	6.43	87.23	"
39	"	"	"	"	"	"	0.0297	"	"	"	0.141	0.018	0.146	0.042	19881	324	20156	1668	20470	21549	5.1	1.014	0.016	0.084	1.59	8.21	67.43	"
40	0.957	"	"	4.69	4.65	4.23	0.0223	4.860	4.424	1.1338	0.052	0.020	0.050	0.021	2704	400	2825	498	3225	3202	—	1.045	0.148	0.184	0.14	0.90	89.44	1.337
41	0.957	"	"	4.69	"	"	0.0417	"	"	"	0.097	0.012	0.102	0.031	9407	144	11752	1287	11896	10694	—	1.249	0.153	0.136	0.26	1.82	47.95	"
42	"	"	"	"	"	"	0.0610	"	"	"	0.142	0.050	0.152	0.047	20164	2500	26292	2513	28792	22677	—	1.303	0.124	0.125	0.38	2.72	32.75	"
43	1.020	"	"	5.33	5.15	4.11	0.0353	5.051	4.032	1.0777	0.091	0.017	0.092	0.022	8241	289	9121	522	9410	8763	—	1.106	0.035	0.063	0.30	1.61	56.61	1.178
44	"	"	"	"	"	"	0.0275	"	"	"	0.071	0.009	0.072	0.022	5041	81	5586	521	5667	5562	—	1.108	0.016	0.103	0.25	1.22	72.56	"
45	"	"	"	"	"	"	0.0155	"	"	"	0.040	0.008	0.041	0.005	1600	64	1811	27	1875	1627	—	1.131	0.015	0.017	0.13	0.69	128.80	"
46	1.298	"	"	8.63	7.60	5.53	0.0081	5.858	4.265	0.9154	0.031	0.009	0.033	0.012	961	81	997	132	1078	1093	1.4	1.037	0.084	0.137	0.22	1.01	245.26	0.727
47	"	"	"	"	"	"	0.0137	"	"	"	0.052	0.021	0.052	0.015	2704	441	2475	234	2916	2938	0.8	0.915	0.016	0.086	0.37	1.59	146.21	"
48	"	"	"	"	"	"	0.0207	"	"	"	0.079	0.017	0.082	0.016	6241	289	6155	234	6444	6475	0.5	0.986	0.046	0.374	0.57	2.51	96.24	"
49	"	"	"	"	"	"	0.0250	"	"	"	0.095	0.035	0.096	0.027	9025	1225	8436	667	9671	9692	0.3	0.935	0.013	0.074	0.68	2.94	80.03	"
50	1.727	"	"	15.26	11.20	7.65	0.0059	6.490	4.431	0.7832	0.033	0.020	0.034	0.017	1089	400	905	226	1305	1315	0.8	0.831	0.367	0.207	0.52	1.99	339.64	0.411
51	"	"	"	"	"	"	0.0105	"	"	"	0.059	0.025	0.062	0.015	3481	625	3010	176	3635	3656	0.6	0.864	0.018	0.050	0.92	3.63	189.97	"
52	"	"	"	"	"	"	0.0141	"	"	"	0.079	0.033	0.083	0.022	6241	1089	5395	379	6484	6520	0.6	0.864	0.017	0.061	1.23	4.86	141.87	"
53	2.222	"	"	25.26	15.15	10.03	0.0024	6.823	4.515	0.7195	0.018	0.010	0.019	0.006	324	100	259	26	359	350	0.1	0.799	0.308	0.080	0.53	1.91	842.27	0.248
54	"	"	"	"	"	"	0.0043	"	"	"	0.033	0.019	0.034	0.010	1089	361	832	72	1193	1161	—	0.764	0.331	0.066	0.97	3.42	459.42	"
55	"	"	"	"	"	"	0.0062	"	"	"	0.047	0.016	0.053	0.013	2209	256	2020	169	2277	2378	5.7	0.985	0.029	0.076	1.37	5.54	322.57	"
56	"	"	"	"	"	"	0.0076	"	"	"	0.058	0.032	0.059	0.014	3364	1024	2504	196	3528	3560	2.2	0.959	0.076	0.058	1.70	6.74	261.40	"
57	2.823	"	"	40.80	19.80	12.88	0.0017	7.018	4.565	0.6850	0.017	0.013	0.019	0.013	289	169	247	115	416	404	—	0.854	0.584	0.398	0.83	3.15	1165.41	0.154
58	"	"	"	"	"	"	0.0025	"	"	"	0.025	0.018	0.025	0.010	729	225	576	83	801	811	1.3	0.790	0.308	0.114	1.22	4.15	792.48	"
59	"	"	"	"	"	"	0.0042	"	"	"	0.042	0.024	0.044	0.015	1764	576	1326	154	1902	1918	0.9	0.752	0.326	0.087	2.06	7.30	471.71	"
60	"	"	"	"	"	"	0.0053	"	"	"	0.053	0.030	0.056	0.018	2809	900	2148	222	3048	3031	—	0.765	0.320	0.079	2.59	9.29	373.81	"
61	3.224	"	"	53.21	22.85	14.76	0.0016	7.091	4.583	0.6723	0.018	0.011	0.019	0.008	324	121	247	44	368	368	—	0.762	0.373	0.136	1.17	4.16	1270.06	"
62	"	"	"	"	"	"	0.0024	"	"	"	0.028	0.019	0.029	0.015	784	361	565	151	926	935	1.0	0.720	0.460	0.192	1.82	6.35	816.46	0.118
63	"	"	"	"	"	"	0.0033	"	"	"	0.038	0.025	0.039	0.017	1444	625	1022	194	1647	1638	—	0.707	0.433	0.134	2.46	8.51	601.61	"
64	"	"	"	"	"	"	0.0044	"	"	"	0.051	0.028	0.054	0.015	2601	784	1960	151	2744	2752	0.3	0.753	0.301	0.058	3.31	11.82	448.25	"
65	3.192	"	"	52.15	21.50	12.66	0.0034	6.740	3.972	0.6133	0.039	0.021	0.044	0.015	1521	441	1186	137	1627	1658	1.9	0.780	0.290	0.090	2.53	7.05	551.64	0.120
67	3.192	"	"	52.15	21.50	12.66	0.0015	"	"	0.6133	0.016	0.010	0.018	0.007	256	100	198	31	298	287	—	0.773	0.390	0.012	0.93	4.88	1344.62	"
68	2.247	"	"	25.83	14.65	8.82	0.0059	6.325	3.931	0.6534	0.063	0.025	0.073	0.015	3969	625	3482	147	4107	4116	0.3	0.877	0.157	0.037	3.64	5.68	232.73	0.243
69	"	"	"	"	"	"	0.0036	"	"	"	0.039	0.022	0.041	0.007	1521	484	1098	32	1582	1553	—	0.722	0.318	0.021	2.24	3.19	375.95	"
70	"	"	"	"	"	"	0.0019	"	"	"	0.021	0.012	0.022	0.004	441	144	314	10	460	451	—	0.716	0.326	0.022	1.21	1.71	698.19	"

TABLE I_a CONTINUED

Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	C_1 ft/sec	C_3 ft/sec	C_{G3}/C_{G1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^8$ ft²	$E_{rf}=a_2^2\times10^8$ ft²	$E_T=a_3^2\frac{C_{G3}}{C_{G1}}\times10^8$ ft²	$E_{rb}=a_4^2\frac{C_{G4}}{C_{G1}}\times10^8$ ft²	$(a_2^2+a_3^2\frac{C_{G3}}{C_{G1}})\times10^8$ ft²	$(a_1^2+a_4^2\frac{C_{G4}}{C_{G1}})\times10^8$ ft²	$1-\frac{a_2^2+a_3^2\frac{C_{G3}}{C_{G1}}}{a_1^2+a_4^2\frac{C_{G4}}{C_{G1}}}$ % Loss	E_T/E_i	E_{rf}/E_i	E_{rb}/E_i	$L_1^2a_1/h_1^3$	$L_0^2a_0/h_0^3$	C_1T/a_1	$\frac{\sigma^2}{g}(h_1-h_0)$
A-71	1.668	2.00	1.00	14.24	10.30	6.44	0.0064	6.180	3.864	0.7247	0.033	0.014	0.035	0.007	1089	196	888	35	1084	1124	3.6	0.815	0.180	0.032	0.437	1.45	312.36	0.440
72	"	"	"	"	"	"	0.0103	"	"	"	0.053	0.012	0.061	0.011	2809	144	2696	87	2840	2896	1.9	0.959	0.051	0.031	0.705	2.51	194.49	"
73	"	"	"	"	"	"	0.0132	"	"	"	0.068	0.024	0.076	0.014	4624	576	4185	142	4761	4766	0.2	0.905	0.124	0.031	0.900	3.15	151.59	"
74	1.083	"	"	6.00	5.60	3.96	0.0152	5.175	3.661	0.9628	0.043	0.013	0.043	0.008	1849	169	1780	80	1949	1929	—	0.962	0.091	0.043	0.169	0.68	130.33	1.044
75	"	"	"	"	"	"	0.0246	"	"	"	0.068	0.010	0.069	0.019	4624	100	4584	333	4684	4957	5.5	0.991	0.021	0.072	0.268	1.08	82.41	"
76	"	"	"	"	"	"	0.0382	"	"	"	0.107	0.014	0.109	0.026	11449	196	11439	650	11635	12099	2.8	0.999	0.028	0.057	0.421	1.71	52.37	"
77	0.874	1.50	0.50	3.91	3.85	3.03	0.0265	4.409	3.474	1.1236	0.051	0.019	0.045	0.009	2601	361	2268	90	2629	2691	2.3	0.872	0.138	0.034	0.224	3.30	75.55	1.606
78	"	"	"	"	"	"	0.0400	"	"	"	0.071	0.008	0.077	0.008	5929	64	1640	72	6704	6001	—	1.119	0.011	0.012	0.338	5.64	54.27	"
79	"	"	"	"	"	"	0.0571	"	"	"	0.110	0.009	0.108	0.014	12100	81	13105	220	13186	12320	—	1.083	0.067	0.018	0.484	7.92	35.03	"
80	0.921	1.25	0.25	4.34	4.15	2.45	0.0262	4.508	2.666	0.8936	0.057	0.005	0.061	0.016	3249	25	3325	229	3350	3478	3.7	1.023	0.077	0.070	0.503	23.42	72.83	1.445
81	"	"	"	"	"	"	0.0152	"	"	"	0.033	0.010	0.034	0.008	1089	100	1033	57	1133	1146	2.4	1.005	0.023	0.052	0.292	13.44	125.79	"
82	"	"	"	"	"	"	0.0378	"	"	"	0.082	0.022	0.083	0.009	6724	484	6156	73	6640	6797	2.3	0.915	0.072	0.011	0.723	31.87	50.62	"
83	1.363	"	"	9.51	7.45	3.76	0.0134	5.469	2.759	0.6282	0.064	0.019	0.078	0.014	4096	361	3821	161	4182	4261	1.7	0.933	0.088	0.039	1.83	70.59	116.47	0.660
84	"	"	"	"	"	"	0.0097	"	"	"	0.046	0.012	0.060	0.023	2116	144	2261	332	2405	2448	1.8	1.068	0.068	0.015	1.31	54.32	162.04	"
85	"	"	"	"	"	"	0.0059	"	"	"	0.028	0.014	0.031	0.005	784	196	604	17	800	801	0.1	0.770	0.250	0.021	0.80	28.03	266.21	"
92	3.959	2.00	1.00	80.22	30.92	22.13	0.0011	7.815	5.600	0.7347	0.044	0.026	0.047	0.022	1936	625	1623	355	2248	2260	0.6	0.838	0.323	0.183	5.25	23.02	703.16	0.078
93	"	"	"	"	"	"	0.0016	"	"	"	0.064	0.034	0.068	0.025	4096	1156	3398	460	4554	4556	0.1	0.830	0.282	0.112	7.64	33.30	483.42	"
94	"	"	"	"	"	"	0.0027	"	"	"	0.111	0.027	0.129	0.033	12321	729	12231	800	12960	13121	1.2	0.943	0.059	0.065	13.25	63.18	278.93	"
95	"	"	"	"	"	"	0.0010	"	"	"	0.042	0.015	0.048	0.015	1764	225	1693	165	1917	1929	0.2	0.959	0.127	0.098	5.02	23.51	736.64	"

TABLE Ib CONTINUED

Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	C_1 ft/sec	C_3 ft/sec	C_{g3}/C_{g1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^8$ ft²	$E_{rT}=a_2^2\times10^8$ ft²	$E_T=a_3^2\frac{C_{g3}}{C_{g1}}\times10^8$ ft²	$E_{rB}=a_4^2\frac{C_{g3}}{C_{g1}}\times10^8$ ft²	$(a_2^2+a_3^2\frac{C_{g3}}{C_{g1}})\times10^8$ ft²	$(a_2^2+a_3^2\frac{C_{g3}}{C_{g1}})\times10^8$ ft²	$1-\frac{a_2^2+a_3^2\frac{C_{g3}}{C_{g1}}}{a_1^2+a_4^2\frac{C_{g3}}{C_{g1}}}$ % Loss	E_T/E_i	E_{rT}/E_i	E_{rB}/E_i	$L_1^2a_1/h_1^3$	$L_3^2a_3/h_3^3$	C_1T/a_1
A-95	4.516	2.00	1.00	104.36	35.49	25.35	0.0022	7.864	5.618	0.7289	0.039	0.021	0.040	0.012	1521	441	1167	105	1608	1626	1.2	0.767	0.290	0.069	6.14	25.70	910.59
96	″	″	″	″	″	″	0.00293	″	″	″	0.052	0.017	0.058	0.014	2704	289	2452	143	2741	2847	3.8	0.907	0.107	0.053	8.19	37.27	682.94
99	4.953	″	″	125.53	39.05	27.85	0.00169	7.891	5.627	0.7252	0.033	0.011	0.039	0.014	1089	121	1103	143	1224	1232	0.7	1.012	0.111	0.131	6.29	30.25	1184.34
101	″	″	″	″	″	″	0.00256	″	″	″	0.050	0.022	0.054	0.016	2500	484	2114	185	2598	2685	3.3	0.845	0.194	0.074	9.53	41.88	781.68
103	″	″	″	″	″	″	0.00118	″	″	″	0.023	0.014	0.023	0.009	529	196	384	58	580	587	1.2	0.726	0.371	0.110	4.38	17.84	1699.31
104	″	″	″	″	″	″	0.00102	″	″	″	0.020	0.013	0.021	0.008	400	169	319	47	488	488	—	0.723	0.383	0.107	3.81	16.29	1954.20
105	6.644	″	″	225.90	52.79	37.50	0.00155	7.950	5.648	0.7171	0.041	0.012	0.047	0.012	1681	144	1584	103	1728	1784	3.2	0.942	0.086	0.061	14.28	66.09	1288.27
106	″	″	″	″	″	″	0.00333	″	″	″	0.088	0.018	0.103	0.024	7744	324	7604	413	7932	8157	3.0	0.982	0.042	0.053	30.65	144.84	600.22
107	5.854	″	″	175.41	46.39	33.00	0.00134	7.929	5.641	0.7200	0.031	0.021	0.032	0.010	961	441	737	72	478	1096	—	0.720	0.431	0.070	8.34	34.85	1495.77
108	″	″	″	″	″	″	0.00108	″	″	″	0.025	0.009	0.028	0.006	625	81	564	27	645	652	0.1	0.903	0.130	0.043	6.72	30.49	1854.76
109	″	″	″	″	″	″	0.00220	″	″	″	0.051	0.013	0.061	0.020	2601	169	2679	288	2848	2889	1.5	1.030	0.065	0.111	13.72	66.43	907.19
110	″	″	″	″	″	″	0.00410	″	″	″	0.095	0.014	0.109	0.028	9025	196	8854	564	9050	9589	5.8	0.981	0.022	0.063	25.56	118.70	488.10
111	5.582	2.25	1.25	159.47	46.78	35.10	0.00192	8.386	6.292	0.7603	0.045	0.012	0.051	0.013	2025	144	1977	128	2121	2153	1.5	0.976	0.071	0.063	8.63	62.83	1040.24
112	″	″	″	″	″	″	0.00153	″	″	″	0.036	0.015	0.040	0.009	1369	225	1152	62	1377	1431	3.8	0.841	0.161	0.045	6.92	49.28	1300.31
113	″	″	″	″	″	″	0.00205	″	″	″	0.048	0.025	0.049	0.014	2304	625	1825	149	2450	2453	0.2	0.792	0.271	0.065	9.22	60.37	975.23
114	″	″	″	″	″	″	0.00393	″	″	″	0.092	0.005	0.104	0.023	8464	25	8220	402	8245	8866	7.0	0.971	0.003	0.048	17.68	128.13	508.81
115	5.051	″	″	130.55	42.18	31.70	0.00261	8.358	6.281	0.7637	0.055	0.017	0.060	0.013	3025	289	2749	129	3038	3154	3.7	0.909	0.096	0.043	8.59	60.29	769.56
116	″	″	″	″	″	″	0.00184	″	″	″	0.039	0.020	0.040	0.010	1521	400	1222	76	1622	1597	—	0.803	0.263	0.050	6.09	40.40	1082.46
117	″	″	″	″	″	″	0.00356	″	″	″	0.075	0.006	0.085	0.016	5625	36	5517	196	5553	5821	4.6	0.981	0.006	0.035	11.72	85.42	562.80
118	4.372	″	″	97.84	36.29	27.35	0.00270	8.306	6.259	0.7700	0.049	0.015	0.054	0.015	2401	225	2245	173	2470	2574	4.0	0.935	0.094	0.072	5.67	20.68	741.10
119	″	″	″	″	″	″	0.00226	″	″	″	0.041	0.020	0.042	0.008	1681	400	1358	50	1758	1731	—	0.808	0.238	0.030	4.74	16.08	885.71
120	″	″	″	″	″	″	0.00402	″	″	″	0.073	0.035	0.073	0.020	5329	1225	4103	154	5328	5483	2.9	0.770	0.230	0.029	8.44	27.96	497.52
121	3.844	″	″	75.63	31.68	23.95	0.00322	8.246	6.234	0.7774	0.051	0.008	0.058	0.015	2601	64	2615	175	2679	2776	3.5	1.005	0.025	0.067	4.49	17.03	621.52
122	″	″	″	″	″	″	0.00259	″	″	″	0.041	0.006	0.046	0.009	1681	36	1645	63	1681	1744	3.6	0.978	0.021	0.038	3.61	13.57	773.12
123	″	″	″	″	″	″	0.00505	″	″	″	0.080	0.010	0.089	0.018	6400	100	6157	252	6257	6652	6.0	0.962	0.016	0.039	7.05	26.13	396.22
124	5.519	1.67	0.67	155.86	39.99	25.50	0.00135	7.251	4.624	0.6464	0.027	0.014	0.029	0.007	729	196	543	32	739	761	2.8	0.745	0.269	0.044	9.27	62.74	1482.15
125	″	″	″	″	″	″	0.00170	″	″	″	0.034	0.018	0.037	0.010	1156	324	885	64	1209	1220	1.0	0.765	0.280	0.055	11.67	80.05	1177.00
126	″	″	″	″	″	″	0.00285	″	″	″	0.057	0.023	0.065	0.015	3249	529	2731	145	3260	3394	4.0	0.841	0.163	0.045	19.57	140.86	702.09
127	6.130	″	″	192.32	44.15	28.35	0.00094	7.266	4.628	0.6439	0.021	0.006	0.026	0.007	441	36	435	37	471	478	1.5	0.986	0.082	0.084	8.93	69.53	2121.00
128	″	″	″	″	″	″	0.00121	″	″	″	0.027	0.012	0.031	0.008	729	144	619	41	763	770	1.0	0.850	0.197	0.056	11.48	82.91	1649.67
129	″	″	″	″	″	″	0.00193	″	″	″	0.043	0.011	0.052	0.010	1849	121	1741	64	1862	1913	2.7	0.942	0.065	0.035	18.29	139.08	1035.84
130	6.946	″	″	246.93	50.54	32.15	0.00200	7.281	4.632	0.6415	0.050	0.010	0.061	0.010	2500	100	2387	64	2487	2564	3.0	0.955	0.040	0.026	21.27	163.14	1011.48

TABLE Ib CONTINUED

Test No	T sec	h_1 ft	h_2 ft	L_0 ft	L_1 ft	L_2 ft	H_1/L_1	C_1 ft/sec	C_2 ft/sec	C_{g2}/C_{g1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^8$ ft²	$E_{rT}=a_2^2\times10^8$ ft²	$E_T=a_3^2\frac{C_{g2}}{C_{g1}}\times10^8$ ft²	$E_{rB}=a_4^2\frac{C_{g2}}{C_{g1}}\times10^8$ ft²	$(a_2^2+a_3^2\frac{C_{g2}}{C_{g1}})\times10^8$ ft²	$(a_1^2+a_4^2\frac{C_{g2}}{C_{g1}})\times10^8$ ft²	$1-\frac{a_2^2+a_3^2\frac{C_{g2}}{C_{g1}}}{a_1^2+a_4^2\frac{C_{g2}}{C_{g1}}}$ % Loss	$\frac{E_T}{E_i}$	$\frac{E_{rT}}{E_i}$	$\frac{E_{rB}}{E_i}$	$\frac{L_1^2a_1}{h_1^3}$	$\frac{L_2^2a_3}{h_2^3}$	$\frac{C_1T}{a_1}$
A-131	6.946	1.67	0.67	246.93	50.54	32.15	0.00119	7.281	4.632	0.6415	0.030	0.018	0.031	0.008	900	324	616	41	940	941	0.1	0.684	0.360	0.046	16.45	106.62	1685.80
132	"	"	"	"	"	"	0.00123	"	"	"	0.031	0.008	0.038	0.008	961	64	926	50	990	1011	2.1	0.964	0.067	0.052	17.00	130.70	1631.42
133	7.548	"	"	291.56	54.98	34.95	0.00076	7.289	4.634	0.6403	0.021	0.006	0.025	0.004	441	36	400	10	436	451	3.0	0.907	0.082	0.023	13.63	101.63	2619.86
134	"	"	"	"	"	"	0.00095	"	"	"	0.026	0.016	0.027	0.008	676	256	466	40	922	716	—	0.689	0.379	0.059	16.98	109.75	2116.04
135	"	"	"	"	"	"	0.00142	"	"	"	0.039	0.012	0.047	0.012	1521	144	1414	92	1558	1613	3.5	0.930	0.095	0.061	25.31	191.05	1410.69
136	"	"	"	"	"	"	0.00189	"	"	"	0.052	0.024	0.059	0.013	2704	576	2229	108	2805	2812	0.3	0.824	0.213	0.0400	33.75	239.83	1058.02
137	11.226	1.33	0.33	644.98	73.26	36.55	0.00030	6.530	3.258	0.5006	0.011	0.003	0.013	0.002	121	9	84	2	93	123	24.4	0.694	0.074	0.017	25.09	57.78	6664.09
138	"	"	"	"	"	"	0.00038	"	"	"	0.014	0.004	0.017	0.003	196	16	144	5	160	201	20.4	0.735	0.082	0.026	31.94	73.57	5236.07
139	"	"	"	"	"	"	0.00063	"	"	"	0.023	0.007	0.026	0.005	529	49	338	13	387	542	28.6	0.639	0.093	0.025	52.47	115.57	3189.17
140	"	"	"	"	"	"	0.00141	"	"	"	0.052	0.012	0.062	0.009	2704	144	1924	14	2068	2744	24.7	0.711	0.053	0.015	118.63	235.62	1409.71
141	8.726	"	"	389.64	56.86	28.40	0.00084	6.521	3.257	0.5022	0.024	0.007	0.027	0.009	576	49	288	40	337	616	45.3	0.500	0.085	0.069	32.67	1213.2	2370.92
142	"	"	"	"	"	"	0.00123	"	"	"	0.035	0.009	0.036	0.009	1225	81	648	40	729	1265	42.4	0.529	0.066	0.033	48.11	1615.7	1625.79
143	"	"	"	"	"	"	0.00193	"	"	"	0.055	0.013	0.061	0.014	3025	169	1860	98	2029	3123	35.0	0.614	0.064	0.032	75.60	2740.9	1034.58
144	7.300	"	"	292.73	47.50	23.75	0.00084	6.511	3.256	0.5039	0.020	0.006	0.025	0.007	400	36	313	25	349	425	18.0	0.782	0.090	0.063	18.94	785.6	2376.50
145	"	"	"	"	"	"	0.00131	"	"	"	0.031	0.013	0.039	0.008	961	169	504	33	673	994	32.3	0.524	0.175	0.034	29.43	1225.5	1533.23
146	"	"	"	"	"	"	0.00210	"	"	"	0.050	0.014	0.058	0.009	2500	196	1695	41	1891	2541	25.6	0.678	0.078	0.016	47.25	1822.5	950.60
147	5.033	1.50	0.50	129.65	34.53	20.10	0.00144	6.866	3.996	0.5916	0.025	0.009	0.028	0.008	625	81	464	38	545	663	17.8	0.742	0.130	0.061	8.83	90.49	1382.24
148	"	"	"	"	"	"	0.00180	"	"	"	0.031	0.010	0.035	0.008	961	100	724	38	824	999	17.6	0.753	0.106	0.040	10.95	113.12	1114.71
149	"	"	"	"	"	"	0.00278	"	"	"	0.048	0.014	0.054	0.009	2304	196	1725	48	1921	2352	18.4	0.749	0.085	0.021	16.96	174.53	719.92
150	10.674	1.25	0.25	583.07	67.52	30.25	0.00033	6.330	2.836	0.4496	0.011	0.003	0.013	0.001	121	9	76	0.5	85	121	29.8	0.628	0.074	0.004	25.68	76253	6141.45
151	"	"	"	"	"	"	0.00044	"	"	"	0.015	0.005	0.018	0.002	225	25	145	2	170	227	25.2	0.644	0.111	0.009	35.02	1055.83	4503.73
152	"	"	"	"	"	"	0.00065	"	"	"	0.022	0.004	0.029	0.003	484	16	378	4	394	488	19.3	0.781	0.157	0.008	51.35	1701.07	3010.73
153	"	"	"	"	"	"	0.00083	"	"	"	0.028	0.007	0.034	0.002	784	49	519	2	568	786	27.8	0.662	0.063	0.003	65.36	1994.35	2412.71
154	9.546	"	"	466.38	60.35	27.05	0.00040	6.326	2.836	0.4502	0.012	0.003	0.016	0.002	144	9	115	2	124	146	15.1	0.799	0.063	0.014	20.02	735.28	5032.33
155	"	"	"	"	"	"	0.00053	"	"	"	0.016	0.006	0.019	0.002	256	36	162	2	198	258	23.3	0.633	0.141	0.008	29.84	896.90	3794.25
156	"	"	"	"	"	"	0.00086	"	"	"	0.026	0.008	0.032	0.002	676	64	460	2	524	678	32.7	0.680	0.095	0.003	48.49	151056	2322.62
157	7.360	"	"	279.28	46.45	20.85	0.00112	6.314	2.835	0.4523	0.026	0.008	0.023	0.005	676	64	240	9	304	683	55.7	0.355	0.095	0.013	48.49	64093	1787.35
158	"	"	"	"	"	"	0.00116	"	"	"	0.027	0.006	0.033	0.002	729	36	496	2	528	731	27.8	0.674	0.049	0.003	50.35	919.60	1721.15
159	"	"	"	"	"	"	0.00168	"	"	"	0.039	0.010	0.038	0.004	1521	100	653	6	753	1527	50.7	0.429	0.066	0.004	72.73	1058.93	1191.56
160	12.172	"	"	758.26	77.04	34.50	0.00023	6.333	2.836	0.4491	0.009	0.001	0.012	0.002	81	1	65	2	66	83	20.5	0.802	0.012	0.025	27.35	915.58	8555.00
161	"	"	"	"	"	"	0.00038	"	"	"	0.015	0.003	0.019	0.002	225	9	162	2	171	227	25.0	0.720	0.040	0.009	45.58	1449.66	5139.00

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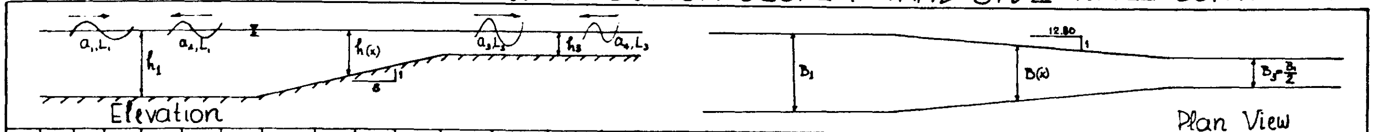
TABLE IIa TEST SERIES WITH BOTTOM SLOPE (1:8) AND SIDE WALL CONTRACTION (1:1280)



Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	C_1 ft/sec	C_3 ft/sec	C_{G3}/C_{G1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i = a_1^2 \times 10^8$ ft²	$E_{rr} = a_2^2 \times 10^8$ ft²	$E_T = a_3^2 \frac{C_{G3}}{C_{G1}} \times 10^8$ ft²	$E_{rB} = a_4^2 \frac{C_{G3}}{C_{G1}} \times 10^8$ ft²	$(a_2^2 + a_3^2 \frac{C_{G3}}{C_{G1}}) \times 10^8$ ft²	$(a_2^2 + a_3^2 \frac{C_{G3}}{C_{G1}}) \times 10^8$ ft²	$1 - \frac{a_2^2 + a_3^2 \frac{C_{G3}}{C_{G1}}}{a_1^2 + a_4^2 \frac{C_{G3}}{C_{G1}}}$ % Loss	E_T/E_i	E_{rr}/E_i	E_{rB}/E_i	$L_1^2 a_1 / h_1^3$	$L_3^2 a_3 / h_3^3$	$C_1 T / a_1$	$\frac{\sigma^2}{g}(h_1 - h_3)$
B-1	2.446	1.75	0.75	30.63	17.25	11.71	0.0044	7.056	4.788	0.3635	0.038	0.017	0.056	0.010	1444	289	1140	36	1421	1480	3.4	0.789	0.200	0.0249	2.11	18.20	454.18	0.205
2	"	"	"	"	"	"	0.0061	"	"	"	0.053	0.019	0.081	0.009	2809	361	2384	30	2745	2839	3.4	0.848	0.128	0.0106	2.94	26.32	325.64	-
3	1.401	"	"	10.05	8.60	6.34	0.0197	6.143	4.520	0.4502	0.085	0.019	0.122	0.018	7225	361	6700	146	7061	7371	4.2	0.927	0.049	0.020	1.17	11.62	101.25	0.624
4	"	"	"	"	"	"	0.0242	"	"	"	0.104	0.025	0.145	0.014	10904	576	9465	88	10090	10904	7.5	0.868	0.053	0.008	1.44	13.81	82.75	-
5	1.056	"	"	5.71	5.50	4.47	0.0170	5.213	4.236	0.5385	0.047	0.012	0.061	0.010	2209	144	2003	54	2147	2263	5.2	0.907	0.065	0.024	0.27	2.89	117.13	1.099
7	"	"	"	"	"	"	0.0389	"	"	"	0.107	0.024	0.136	0.010	11449	576	9960	54	10536	11503	8.5	0.870	0.050	0.005	0.60	6.45	51.45	-
8	"	"	"	"	"	"	0.0454	"	"	"	0.125	0.010	0.159	0.011	15625	100	13614	65	13714	15690	12.6	0.871	0.006	0.004	0.71	7.52	44.04	-
9	1.621	"	"	13.45	10.50	7.49	0.0102	6.482	4.626	0.4164	0.054	0.012	0.080	0.010	2916	144	2662	42	2806	2956	5.1	0.913	0.049	0.014	1.11	10.64	194.57	0.466
10	"	"	"	"	"	"	0.0049	"	"	"	0.026	0.009	0.038	0.007	676	81	601	21	650	697	2.2	0.889	0.012	0.031	0.54	5.04	404.11	-
11	"	"	"	"	"	"	0.0128	"	"	"	0.067	0.006	0.102	0.018	4489	36	4332	135	4368	4624	5.5	0.965	0.008	0.030	1.38	13.55	156.82	-
12	"	"	"	"	"	"	0.0171	"	"	"	0.090	0.004	0.136	0.020	8100	16	7701	166	7717	8266	6.7	0.951	0.002	0.020	1.85	18.08	116.74	-
13	1.823	"	"	17.02	12.20	8.54	0.0181	6.695	4.687	0.3960	0.011	0.005	0.016	0.004	121	25	120	7	127	128	0.8	0.843	0.210	0.058	0.31	2.76	1109.54	0.369
14	"	"	"	"	"	"	0.0074	"	"	"	0.045	0.012	0.068	0.008	2025	144	1831	25	1975	2050	3.7	0.904	0.071	0.012	1.25	11.74	271.22	-
15	"	"	"	"	"	"	0.0093	"	"	"	0.057	0.019	0.085	0.009	3249	361	2861	32	3222	3281	1.8	0.881	0.011	0.010	1.58	14.69	214.12	-
16	"	"	"	"	"	"	0.0119	"	"	"	0.073	0.016	0.113	0.020	5329	256	5056	159	5312	5488	3.3	0.948	0.048	0.029	2.03	19.53	167.19	-
17	2.296	"	"	26.98	16.05	10.95	0.0071	6.995	4.771	0.3688	0.059	0.015	0.094	0.012	3481	225	3258	53	3483	3534	1.5	0.936	0.066	0.015	2.84	26.42	272.22	0.232
18	"	"	"	"	"	"	0.0059	"	"	"	0.048	0.017	0.073	0.008	2304	289	1965	23	2254	2327	3.2	0.852	0.012	0.010	2.31	20.94	334.60	-
19	"	"	"	"	"	"	0.0043	"	"	"	0.035	0.019	0.053	0.024	1225	361	1036	212	1397	1437	3.8	0.845	0.294	0.017	1.68	15.06	458.89	-
20	"	"	"	"	"	"	0.0027	"	"	"	0.022	0.011	0.030	0.005	441	121	331	13	452	454	0.5	0.751	0.270	0.030	1.06	8.52	730.04	-
21	2.858	"	"	41.82	20.50	13.77	0.0016	7.177	4.822	0.3533	0.016	0.010	0.021	0.004	256	100	156	5	256	261	2.0	0.610	0.390	0.019	1.26	9.43	1281.94	0.150
22	"	"	"	"	"	"	0.0030	"	"	"	0.031	0.009	0.050	0.006	961	81	883	13	964	974	1.1	0.918	0.084	0.013	2.43	22.46	661.64	-
23	"	"	"	"	"	"	0.0039	"	"	"	0.040	0.011	0.065	0.014	1600	121	1538	70	1659	1670	0.7	0.961	0.075	0.043	3.14	29.21	512.78	-
24	"	"	"	"	"	"	0.0048	"	"	"	0.050	0.012	0.081	0.013	2500	144	2318	59	2462	2559	3.8	0.927	0.057	0.023	3.92	36.40	410.22	-
25	2.886	2.00	1.00	42.62	22.00	15.96	0.0038	7.629	5.535	0.3811	0.036	0.022	0.048	0.016	1296	484	878	97	1362	1393	2.3	0.677	0.037	0.075	2.18	12.23	611.58	0.147
26	"	"	"	"	"	"	0.0019	"	"	"	0.021	0.013	0.028	0.010	441	169	298	38	467	479	2.6	0.675	0.380	0.086	1.27	7.13	1048.42	-
27	"	"	"	"	"	"	0.0043	"	"	"	0.047	0.014	0.075	0.021	2209	196	2143	168	2339	2377	1.6	0.970	0.088	0.076	2.84	19.11	468.45	-
28	"	"	"	"	"	"	0.0054	"	"	"	0.060	0.028	0.087	0.022	3600	784	2884	184	3668	3784	3.1	0.801	0.217	0.051	3.63	22.16	366.95	-
29	2.252	"	"	25.97	16.60	12.26	0.0059	7.375	5.445	0.4004	0.049	0.027	0.067	0.023	2401	729	1797	211	2526	2612	3.3	0.748	0.303	0.088	1.68	10.07	338.95	0.242
30	"	"	"	"	"	"	0.0078	"	"	"	0.065	0.036	0.087	0.027	4225	1296	3030	291	4326	4516	4.2	0.717	0.185	0.069	2.24	13.07	255.52	-
30	"	"	"	"	"	"	0.0081	"	"	"	0.068	0.039	0.090	0.030	4624	1521	3243	360	4764	4984	4.4	0.701	0.306	0.098	2.34	13.53	244.25	-
30	"	"	"	"	"	"	0.0084	"	"	"	0.070	0.040	0.093	0.031	4900	1600	3463	385	5063	5285	4.3	0.707	0.326	0.098	2.41	13.98	237.27	-
31	"	"	"	"	"	"	0.0029	"	"	"	0.024	0.012	0.034	0.010	576	144	465	40	607	616	1.5	0.803	0.250	0.069	0.83	5.71	612.04	-

TABLE I_a CONTINUED

Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	C_1 ft/sec	C_3 ft/sec	C_{g3}/C_{g1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^8$ ft²	$E_{rT}=a_2^2\times10^8$ ft²	$E_T=a_3^2\frac{C_{g3}}{C_{g1}}\times10^8$ ft²	$E_{rB}=a_4^2\frac{C_{g3}}{C_{g1}}\times10^8$ ft²	$(a_2^2+a_3^2\frac{C_{g3}}{C_{g1}})\times10^8$ ft²	$(a_2^2+a_4^2\frac{C_{g3}}{C_{g1}})\times10^8$ ft²	$1-\frac{a_2^2+a_3^2\frac{C_{g3}}{C_{g1}}}{a_1^2+a_4^2\frac{C_{g3}}{C_{g1}}}$ % Loss	E_T/E_i	E_{rT}/E_i	E_{rB}/E_i	$L_1^2a_1/h_1^3$	$L_3^2a_3/h_3^3$	C_1T/a_1	$\frac{\sigma^2}{g}(h_1-h_3)$
B-32	2.252	2.00	1.00	25.97	16.60	12.26	0.0113	7.375	5.445	0.4004	0.094	0.031	0.137	0.016	8836	961	7582	102	8543	8938	4.5	0.858	0.108	0.011	3.30	20.59	176.69	0.242
33	1.689	-	-	14.60	11.60	8.89	0.0065	6.872	5.266	0.4412	0.038	0.018	0.051	0.013	1444	324	1147	74	1471	1518	3.1	0.794	0.224	0.051	0.64	4.03	305.45	0.429
34	-	-	-	-	-	-	0.0124	-	-	-	0.072	0.029	0.099	0.019	5184	841	4324	159	5165	5343	3.4	0.834	0.162	0.031	1.21	7.83	161.21	-
35	-	-	-	-	-	-	0.0188	-	-	-	0.109	0.022	0.159	0.030	11881	484	11153	397	11637	12278	5.3	0.938	0.041	0.033	1.83	12.57	106.49	-
36	-	-	-	-	-	-	0.0235	-	-	-	0.136	0.060	0.177	0.024	18496	3481	13823	254	17303	18750	7.7	0.747	0.188	0.014	2.29	13.99	85.35	-
37	1.348	-	-	9.80	8.95	7.00	0.0112	6.328	5.066	0.4871	0.049	0.026	0.059	0.010	2401	676	1695	49	2371	2450	3.3	0.706	0.281	0.020	0.47	3.13	174.08	0.674
38	-	-	-	-	-	-	0.0185	-	-	-	0.081	0.025	0.109	0.022	6561	626	5785	235	6412	6796	4.7	0.882	0.095	0.036	0.78	4.85	105.31	-
39	-	-	-	-	-	-	0.0237	-	-	-	0.104	0.028	0.139	0.024	10816	784	9411	280	10195	11096	8.2	0.870	0.072	0.026	0.99	6.81	82.02	-
40	-	-	-	-	-	-	0.0283	-	-	-	0.124	0.015	0.170	0.029	15376	225	14077	409	14302	15785	9.4	0.915	0.015	0.026	1.19	8.33	68.99	-
41	0.903	-	-	4.17	4.15	3.86	0.0298	4.601	4.279	0.5659	0.062	0.010	0.077	0.008	3844	100	3355	36	3455	3880	10.9	0.873	0.026	0.010	0.13	1.15	67.00	1.503
43	0.992	2.25	1.25	5.04	5.00	4.69	0.0292	5.044	4.734	0.5579	0.075	0.014	0.093	0.025	5329	196	4825	348	5021	5677	11.5	0.905	0.037	0.065	0.16	1.05	68.55	1.245
44	-	-	-	-	-	-	0.0360	-	-	-	0.090	0.008	0.116	0.031	8100	64	7507	536	7571	8636	11.0	0.895	0.054	0.066	0.20	1.28	55.60	-
45	-	-	-	-	-	-	0.0472	-	-	-	0.118	0.036	0.148	0.052	13924	1296	12222	1504	13518	15432	12.4	0.877	0.093	0.011	0.26	1.66	42.41	-
46	1.385	-	-	9.81	9.00	7.61	0.0202	6.504	5.496	0.5089	0.091	0.020	0.123	0.029	8281	400	7699	428	8099	8709	7.0	0.929	0.048	0.051	0.65	3.64	98.99	0.639
47	-	-	-	-	-	-	0.0124	-	-	-	0.056	0.023	0.071	0.016	3136	529	2565	130	3094	3266	5.3	0.818	0.168	0.041	0.40	2.10	160.86	-
48	-	-	-	-	-	-	0.0275	-	-	-	0.124	0.032	0.163	0.030	15376	1024	13520	458	14544	15834	7.6	0.795	0.156	0.029	0.88	4.56	72.65	-
49	-	-	-	-	-	-	0.0362	-	-	-	0.163	0.018	0.215	0.027	26569	324	23524	371	23848	26940	11.5	0.885	0.012	0.014	1.16	6.37	55.26	-
50	1.786	-	-	16.33	13.00	10.41	0.058	7.282	5.834	0.4542	0.038	0.020	0.048	0.010	1444	400	1044	45	1490	1489	2.8	0.725	0.277	0.031	0.56	2.66	342.23	0.384
51	-	-	-	-	-	-	0.0098	-	-	-	0.064	0.023	0.088	0.019	4096	529	3517	164	4046	4260	5.0	0.858	0.129	0.040	0.95	4.88	203.22	-
52	-	-	-	-	-	-	0.0149	-	-	-	0.097	0.044	0.128	0.028	9409	1936	7438	356	9374	9765	4.1	0.791	0.205	0.037	1.44	6.97	134.08	-
53	-	-	-	-	-	-	0.0204	-	-	-	0.133	0.060	0.172	0.039	17689	3600	13437	693	17037	18382	7.3	0.759	0.203	0.039	1.97	9.54	99.79	-
54	2.332	-	-	27.83	18.15	14.09	0.0059	7.788	6.045	0.4186	0.054	0.032	0.068	0.016	2916	1024	1935	107	2959	3023	2.1	0.663	0.350	0.036	1.56	6.91	336.33	0.225
55	-	-	-	-	-	-	0.0034	-	-	-	0.031	0.018	0.040	0.010	961	324	669	42	993	1003	1.0	0.696	0.337	0.044	0.90	4.06	585.89	-
56	-	-	-	-	-	-	0.0096	-	-	-	0.087	0.039	0.125	0.044	7569	1521	6540	810	8061	8379	4.2	0.949	0.111	0.107	2.66	13.31	208.96	-
57	-	-	-	-	-	-	0.0140	-	-	-	0.127	0.061	0.173	0.048	16129	3721	12528	965	16249	17094	5.2	0.879	0.125	0.059	3.67	18.68	143.01	-
58	2.832	-	-	41.04	22.70	17.38	0.0062	8.021	6.141	0.4031	0.070	0.037	0.096	0.030	4900	1369	3714	360	5083	5260	3.4	0.758	0.279	0.073	3.17	14.83	324.50	0.153
59	-	-	-	-	-	-	0.0042	-	-	-	0.048	0.025	0.069	0.027	2304	625	1919	294	2544	2598	2.1	0.833	0.271	0.127	2.17	10.66	473.23	-
60	-	-	-	-	-	-	0.0028	-	-	-	0.032	0.014	0.049	0.020	1024	196	967	160	1163	1184	1.8	0.944	0.191	0.156	1.45	7.57	709.54	-
61	-	-	-	-	-	-	0.0017	-	-	-	0.019	0.009	0.030	0.014	361	81	360	79	421	440	0.1	0.999	0.224	0.219	0.86	4.63	1185.53	-
62	2.997	1.50	0.50	45.96	20.10	11.88	0.0020	6.712	3.967	0.3097	0.020	0.009	0.032	0.005	400	81	317	7	398	407	2.3	0.792	0.202	0.017	2.39	36.12	1005.85	0.136
63	-	-	-	-	-	-	0.0033	-	-	-	0.033	0.015	0.055	0.012	1089	225	903	45	1128	1134	0.6	0.829	0.206	0.041	3.95	62.08	609.55	-
64	-	-	-	-	-	-	0.0041	-	-	-	0.041	0.022	0.064	0.017	1681	484	1267	90	1751	1771	2.2	0.877	0.152	0.053	4.91	77.08	490.61	-
65	-	-	-	-	-	-	0.0061	-	-	-	0.016	0.010	0.023	0.006	256	100	164	11	264	267	1.2	0.640	0.390	0.043	1.92	25.16	1257.19	-
66	1.957	-	-	19.61	12.50	7.64	0.0054	6.391	3.905	0.3403	0.034	0.018	0.050	0.008	1156	324	851	22	1175	1178	1.1	0.922	0.086	0.019	1.58	26.12	367.83	0.320
67	1.181	-	-	18.30	12.00	7.36	0.0033	6.351	3.897	0.3444	0.020	0.006	0.033	0.006	400	36	370	12	406	412	1.5	0.925	0.090	0.030	0.85	14.32	629.05	0.312
68	1.957	-	-	19.61	12.50	7.64	0.0081	6.391	3.905	0.3403	0.049	0.018	0.078	0.011	2401	324	2068	41	2392	2442	2.1	0.861	0.135	0.017	2.26	36.44	255.25	0.320

TABLE Ia CONTINUED

Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	C_1 ft/sec	C_3 ft/sec	C_{G3}/C_{G1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^6$ ft²	$E_{rT}=a_2^2\times10^6$ ft²	$E_T=a_3^2\frac{C_{G3}}{C_{G1}}\times10^6$ ft²	$E_{rB}=a_4^2\frac{C_{G3}}{C_{G1}}\times10^6$ ft²	$(a_2^2+a_3^2\frac{C_{G3}}{C_{G1}})\times10^6$ ft²	$(a_1^2+a_4^2\frac{C_{G3}}{C_{G1}})\times10^6$ ft²	$1-\frac{a_2^2+a_3^2\frac{C_{G3}}{C_{G1}}}{a_1^2+a_4^2\frac{C_{G3}}{C_{G1}}}$ % Loss	$\frac{E_T}{E_i}$	$\frac{E_{rT}}{E_i}$	$\frac{E_{rB}}{E_i}$	$\frac{L_1^2 a_1}{h_1^3}$	$\frac{L_3^2 a_3}{h_3^3}$	$\frac{C_1 T}{a_1}$	$\frac{\sigma^2}{g}(h_1-h_3)$
B-69	1.981	1.50	0.50	18.30	12.50	7.64	0.0105	6.391	3.905	0.3403	0.063	0.017	0.103	0.016	3969	289	3610	87	4056	4056	3.9	0.909	0.073	0.022	2.92	48.08	198.52	0.320
70	2.430	"	"	30.23	16.00	9.58	0.0031	6.587	3.943	0.3209	0.025	0.013	0.038	0.009	625	169	464	26	633	651	2.8	0.742	0.270	0.041	1.90	27.92	640.24	0.207
71	"	"	"	"	"	"	0.0050	"	"	"	0.040	0.018	0.063	0.010	1600	324	1274	32	1598	1632	2.1	0.796	0.202	0.020	3.03	46.24	400.15	"
72	"	"	"	"	"	"	0.0071	"	"	"	0.057	0.016	0.098	0.024	3249	256	3073	185	3329	3434	3.1	0.946	0.078	0.057	4.32	71.96	280.81	"
73	1.431	"	"	10.48	8.45	5.45	0.0080	5.908	3.811	0.3929	0.034	0.010	0.051	0.009	1156	100	1022	26	1122	1182	5.0	0.911	0.089	0.022	0.72	12.08	248.65	0.598
74	"	"	"	"	"	"	0.0130	"	"	"	0.055	0.027	0.075	0.012	3025	729	2210	56	2939	3081	4.6	0.852	0.012	0.018	1.16	19.24	153.72	"
75	"	"	"	"	"	"	0.0177	"	"	"	0.075	0.032	0.105	0.010	5625	1024	4332	39	5356	5664	5.5	0.770	0.191	0.069	1.59	24.96	112.72	"
76	1.077	"	"	5.93	5.55	3.94	0.0432	5.157	3.657	0.4835	0.120	0.021	0.157	0.010	14400	441	11918	48	12359	14448	14.5	0.827	0.031	0.033	1.09	15.48	46.28	1.056
77	"	"	"	"	"	"	0.0310	"	"	"	0.086	0.023	0.113	0.012	7396	529	6174	69	6703	7468	10.3	0.834	0.071	0.093	0.79	14.04	64.58	"
78	"	"	"	"	"	"	0.0248	"	"	"	0.069	0.019	0.091	0.009	4761	361	4003	39	4364	4806	9.0	0.786	0.131	0.082	0.63	10.92	80.49	"
79	"	"	"	"	"	"	0.0151	"	"	"	0.042	0.015	0.055	0.007	1764	225	1462	24	1687	1788	5.7	0.828	0.127	0.014	0.38	6.84	132.24	"
80	3.438	2.25	1.25	60.49	28.10	21.32	0.0049	8.179	6.207	0.3929	0.070	0.042	0.089	0.012	4900	1764	3112	56	4876	4956	1.7	0.635	0.360	0.011	4.85	20.69	401.70	0.104
81	"	"	"	"	"	"	0.0042	"	"	"	0.059	0.031	0.078	0.008	3481	961	2390	25	3351	3506	4.5	0.713	0.276	0.007	4.09	18.14	476.59	"
82	"	"	"	"	"	"	0.0067	"	"	"	0.095	0.046	0.129	0.012	9025	2116	6538	56	8654	9025	4.1	0.724	0.234	0.006	6.59	30.00	295.99	"

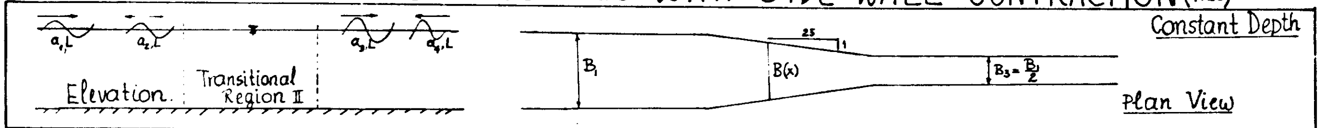
TABLE Ib CONTINUED

Test No	T sec	h_1 ft	h_2 ft	L_0 ft	L_1 ft	L_2 ft	H_1/L_1	C_1 ft/sec	C_2 ft/sec	C_{g2}/C_{g1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^8$ ft²	$E_{rT}=a_2^2\times10^8$ ft²	$E_T=a_3^2\frac{C_{g3}}{C_{g1}}\times10^8$ ft²	$E_{rB}=a_4^2\frac{C_{g4}}{C_{g1}}\times10^8$ ft²	$(a_2^2+a_3^2\frac{C_{g3}}{C_{g1}})\times10^8$ ft²	$(a_1^2+a_4^2\frac{C_{g4}}{C_{g1}})\times10^8$ ft²	$1-\frac{a_2^2+a_3^2\frac{C_{g3}}{C_{g1}}}{a_1^2+a_4^2\frac{C_{g4}}{C_{g1}}}$ % Loss	E_T/E_i	E_{rT}/E_i	E_{rB}/E_i	$L_1^2a_1/h_1^3$	$L_2^2a_3/h_2^3$	C_1T/a_1
B-83	4.194	2.25	1.25	90.00	34.93	26.20	0.00489	8.288	6.252	0.3861	0.085	0.022	0.132	0.023	7225	484	6253	85	6737	7310	7.9	0.866	0.067	0.012	9.00	46.79	408.94
84	"	"	"	"	"	"	0.00317	"	"	"	0.055	0.008	0.086	0.015	3025	64	2855	83	2919	3018	6.1	0.944	0.021	0.027	5.82	30.23	632.00
85	"	"	"	"	"	"	0.00230	"	"	"	0.040	0.012	0.062	0.010	1600	144	1422	37	1566	1637	4.4	0.889	0.090	0.023	4.24	21.79	869.00
86	5.090	"	"	132.58	42.52	31.95	0.00183	8.360	6.282	0.3817	0.039	0.017	0.057	0.010	1521	289	1202	37	1491	1558	4.3	0.790	0.190	0.024	6.19	29.79	1091.08
87	"	"	"	"	"	"	0.00165	"	"	"	0.035	0.017	0.048	0.008	1225	289	879	24	1168	1249	6.0	0.717	0.236	0.020	5.56	28.22	1215.77
88	"	"	"	"	"	"	0.00151	"	"	"	0.032	0.009	0.050	0.009	1024	81	925	30	1006	1054	4.5	0.903	0.079	0.029	5.08	26.13	1329.75
89	"	"	"	"	"	"	0.00258	"	"	"	0.055	0.013	0.086	0.014	3025	169	2736	73	2905	2989	6.2	0.904	0.056	0.024	8.73	44.95	773.67
90	5.566	"	"	158.58	44.64	35.00	0.00176	8.385	6.292	0.3802	0.041	0.010	0.065	0.010	1681	100	1563	37	1663	1718	3.3	0.930	0.060	0.022	7.83	40.77	1138.31
91	"	"	"	"	"	"	0.00219	"	"	"	0.051	0.011	0.080	0.015	2601	121	2433	85	2554	2686	4.9	0.935	0.047	0.033	9.74	50.17	915.12
92	"	"	"	"	"	"	0.00137	"	"	"	0.032	0.010	0.049	0.010	1024	100	913	38	1013	1062	4.6	0.892	0.098	0.037	6.11	30.73	1458.49
93	"	"	"	"	"	"	0.00257	"	"	"	0.060	0.014	0.093	0.012	3600	196	3288	55	3484	3655	4.7	0.913	0.054	0.015	11.45	58.33	777.85
94	5.679	2.00	1.00	165.06	44.97	32.00	0.00173	7.923	5.638	0.3604	0.039	0.019	0.056	0.010	1521	361	1129	36	1490	1557	4.3	0.742	0.237	0.024	9.86	64.47	1153.72
95	"	"	"	"	"	"	0.00133	"	"	"	0.030	0.017	0.041	0.007	900	289	605	18	894	918	2.6	0.672	0.321	0.020	7.58	41.99	1499.03
96	"	"	"	"	"	"	0.00267	"	"	"	0.060	0.012	0.099	0.021	3600	144	3532	159	3676	3759	2.2	0.981	0.040	0.044	15.17	101.38	749.92
97	4.979	"	"	126.86	39.27	28.00	0.00229	7.892	5.628	0.3625	0.045	0.010	0.072	0.014	2025	100	1880	71	1980	2096	5.3	0.928	0.049	0.035	8.67	56.45	893.20
98	"	"	"	"	"	"	0.00173	"	"	"	0.034	0.020	0.046	0.010	1156	400	767	36	1167	1192	2.1	0.663	0.346	0.031	6.55	39.99	1155.71
99	"	"	"	"	"	"	0.00260	"	"	"	0.051	0.019	0.079	0.016	2601	361	2262	93	2623	2694	2.7	0.870	0.139	0.036	9.83	61.94	770.49
100	"	"	"	"	"	"	0.00331	"	"	"	0.065	0.028	0.096	0.022	4225	784	3340	93	4124	4318	4.5	0.791	0.186	0.022	12.53	75.27	604.52
101	4.263	"	"	93.00	33.41	23.90	0.00347	7.844	5.611	0.3658	0.058	0.008	0.093	0.017	3364	64	3162	105	3228	3469	7.0	0.941	0.019	0.031	8.09	53.12	576.53
102	"	"	"	"	"	"	0.00239	"	"	"	0.040	0.018	0.059	0.012	1600	324	1261	53	1585	1653	4.1	0.788	0.203	0.033	5.58	35.42	835.98
103	"	"	"	"	"	"	0.00189	"	"	"	0.030	0.008	0.049	0.012	900	64	878	52	942	952	1.1	0.976	0.071	0.058	4.88	27.99	1114.63
106	6.573	"	"	221.15	52.22	37.10	0.00257	7.949	5.648	0.3586	0.067	0.029	0.100	0.020	4489	841	3586	157	4427	4646	4.8	0.881	0.108	0.035	22.84	144.53	779.84
107	6.216	1.67	0.67	197.75	45.15	28.95	0.00159	7.268	4.628	0.3218	0.036	0.010	0.061	0.011	1296	100	1197	39	1297	1335	2.8	0.924	0.077	0.030	15.76	167.80	1254.94
108	"	"	"	"	"	"	0.00093	"	"	"	0.021	0.007	0.036	0.010	441	49	417	32	466	473	1.5	0.946	0.111	0.073	9.19	97.02	2151.33
109	"	"	"	"	"	"	0.00270	"	"	"	0.061	0.028	0.094	0.013	3721	784	2843	54	3627	3775	4.0	0.764	0.211	0.015	26.90	250.57	740.62
110	5.497	"	"	154.65	39.83	25.40	0.00146	7.250	4.624	0.3232	0.029	0.015	0.045	0.007	900	225	654	16	879	916	4.0	0.726	0.071	0.018	9.88	107.35	1374.24
112	"	"	"	"	"	"	0.00246	"	"	"	0.049	0.007	0.083	0.009	2401	49	2226	26	2275	2427	6.3	0.927	0.020	0.011	16.69	178.20	813.33
113	7.936	"	"	267.99	52.68	33.56	0.00133	7.285	4.633	0.3204	0.035	0.010	0.060	0.014	1225	100	1163	63	1263	1288	2.0	0.949	0.082	0.051	20.86	224.88	1506.11
114	"	"	"	"	"	"	0.00227	"	"	"	0.060	0.011	0.102	0.023	3604	121	3399	169	3520	3769	6.6	0.943	0.034	0.047	35.75	382.30	898.57
115	"	"	"	"	"	"	0.00208	"	"	"	0.055	0.025	0.087	0.017	3025	625	2425	93	3050	3118	2.2	0.802	0.207	0.031	32.77	326.09	958.44
116	6.997	1.50	0.50	250.60	48.29	28.00	0.00116	6.906	4.004	0.2923	0.028	0.009	0.049	0.010	784	81	702	29	783	813	3.6	0.895	0.103	0.037	19.35	307.32	1925.75
117	"	"	"	"	"	"	0.00087	"	"	"	0.021	0.011	0.033	0.006	441	121	318	10	439	451	2.7	0.907	0.816	0.023	14.51	232.08	2301.00

TABLE IIb CONTINUED

Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	C_1 ft/sec	C_3 ft/sec	C_{G3}/C_{G1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i = a_1^2 \times 10^8$ ft²	$E_{rT} = a_2^2 \times 10^8$ ft²	$E_T = a_3^2 \frac{C_{G3}}{C_{G1}} \times 10^8$ ft²	$E_{rB} = a_4^2 \frac{C_{G3}}{C_{G1}} \times 10^8$ ft²	$(a_2^2 + a_3^2 \frac{C_{G3}}{C_{G1}}) \times 10^8$ ft²	$(a_2^2 + a_3^2 \frac{C_{G3}}{C_{G0}}) \times 10^8$ ft²	$1 - \frac{a_2^2 + a_3^2 \frac{C_{G3}}{C_{G1}}}{a_1^2 + a_4^2 \frac{C_{G3}}{C_{G1}}}$ % Loss	E_T/E_i	E_{rT}/E_i	E_{rB}/E_i	$L_1^2 a_1 / h_1^3$	$L_0^2 a_3 / h_3^3$	$C_1 T / a_1$
B-118	6.997	1.50	0.50	250.60	48.29	28.00	0.0012	6.906	4.004	0.2923	0.029	0.012	0.049	0.009	841	144	702	24	846	865	2.2	0.835	0.171	0.029	20.04	307.32	1666.24
119	8.616	"	"	379.91	59.59	34.50	0.00023	6.921	4.007	0.2911	0.007	0.003	0.012	0.004	49	9	43	4	52	53	1.9	0.878	0.184	0.082	7.36	114.28	9887.14
120	"	"	"	"	"	"	0.00037	"	"	"	0.011	0.008	0.015	0.005	121	64	65	7	129	128	—	0.537	0.529	0.058	11.57	142.80	6291.82
121	"	"	"	"	"	"	0.00077	"	"	"	0.023	0.011	0.038	0.010	529	121	420	29	541	558	2.2	0.794	0.229	0.055	24.19	361.84	3009.13
122	6.065	"	"	188.23	41.77	24.25	0.00077	6.892	4.001	0.2936	0.016	0.011	0.021	0.004	256	121	130	4	251	256	2.0	0.508	0.473	0.016	8.27	106.80	2612.50
123	"	"	"	"	"	"	0.00105	"	"	"	0.022	0.010	0.036	0.006	484	100	380	10	480	494	2.9	0.785	0.207	0.021	11.37	169.36	1900.00
124	"	"	"	"	"	"	0.00105	"	"	"	0.034	0.011	0.059	0.009	1156	121	1022	24	1143	1180	3.2	0.884	0.105	0.021	17.58	297.56	1229.41

TABLE III TEST SERIES WITH SIDE WALL CONTRACTION (1:25)



Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	H_3/L_3	$C_1=C_3$ ft/sec	C_{g3}/C_{g1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^8$ ft²	$E_r=a_2^2\times10^8$ ft²	$E_T=a_3^2\times10^8$ ft²	$E_{tr}=a_4^2\times10^8$ ft²	$(2a_2^2+a_3^2)\times10^8$ ft²	$(2a_2^2+a_4^2)\times10^8$ ft²	$1-\frac{2a_2^2+a_3^2}{2a_1^2+a_4^2}$ % Loss	$\frac{E_T}{E_i}=\frac{a_3^2}{2a_1^2}$	$\frac{E_r}{E_i}=\frac{a_2^2}{a_1^2}$	$\frac{E_{tr}}{E_i}=\frac{a_4^2}{2a_1^2}$	$\frac{L_1^2a_1}{h_1^3}$	$\frac{L_3^2a_3}{h_3^3}$	$\frac{C_1T}{a_1}$
C-1	3.089	2.25	2.25	48.82	25.00	25.00	0.0067	0.0086	8.099	0.500	0.084	0.046	0.107	0.047	7056	2116	11449	2209	16001	16321	1.4	0.811	0.300	0.156	4.61	5.87	299.85
2	-	-	-	-	-	-	0.0022	0.0192	-	-	0.028	0.009	0.240	0.015	784	81	1600	225	1762	1793	1.8	1.02	0.103	0.143	1.54	13.18	893.57
3	-	-	-	-	-	-	0.0080	0.0113	-	-	0.100	0.036	0.141	0.054	10000	1296	19881	2916	22473	22916	1.9	0.994	0.129	0.145	5.49	7.74	250.20
4	2.534	-	-	32.86	20.00	20.00	0.0077	0.0111	7.899	-	0.077	0.028	0.111	0.040	5929	441	12321	1600	13203	13458	1.9	1.03	0.074	0.134	2.70	3.90	260.00
5	-	-	-	-	-	-	0.0055	0.0075	-	-	0.055	0.020	0.075	0.026	3025	400	5625	676	6425	6726	4.1	0.927	0.132	0.112	1.93	2.63	364.00
6	-	-	-	-	-	-	0.0025	0.0035	-	-	0.025	0.009	0.035	0.013	625	81	1225	169	1387	1419	2.2	0.980	0.129	0.135	0.88	1.22	800.80
7	1.880	-	-	18.08	13.90	13.90	0.0064	0.0094	7.400	-	0.045	0.012	0.065	0.025	2025	144	4225	625	4513	4675	4.5	1.043	0.071	0.154	0.76	1.10	417.78
8	-	-	-	-	-	-	0.0135	0.0197	-	-	0.094	0.012	0.137	0.044	8836	144	18769	1936	19057	19608	2.9	1.062	0.016	0.109	1.59	2.32	200.00
9	-	-	-	-	-	-	0.0187	0.0272	-	-	0.130	0.018	0.189	0.062	16900	324	35721	3844	36369	37644	3.4	1.057	0.019	0.227	2.20	3.21	144.62
10	1.478	-	-	11.18	9.95	9.95	0.0287	0.0416	6.936	-	0.143	0.011	0.207	0.071	20449	121	42849	5041	43091	45939	6.2	1.048	0.006	0.123	1.24	1.80	69.65
11	-	-	-	-	-	-	0.0221	0.0312	-	-	0.110	0.016	0.155	0.039	12100	256	24025	1521	24537	25721	4.6	0.993	0.021	0.062	0.96	1.35	90.55
12	-	-	-	-	-	-	0.0086	0.0123	-	-	0.043	0.012	0.061	0.021	1849	144	3721	441	4009	4139	3.2	1.006	0.078	0.119	0.37	0.53	231.63
13	1.053	-	-	5.67	5.60	5.60	0.0321	0.0450	5.323	-	0.090	0.014	0.126	0.032	8100	196	15876	1024	16268	17224	5.5	0.980	0.024	0.063	0.25	0.35	62.33
14	-	-	-	-	-	-	0.0532	0.0739	-	-	0.149	0.015	0.207	0.054	22201	225	42849	2916	43299	47318	8.5	0.965	0.010	0.062	0.41	0.57	37.65
15	3.044	-	-	49.42	24.60	24.60	0.0011	0.0016	8.087	-	0.014	0.006	0.020	0.007	196	36	400	49	472	441	1.9	1.020	0.183	0.125	0.74	1.06	1757.14
16	-	-	-	-	-	-	0.0024	0.0032	-	-	0.030	0.015	0.039	0.015	900	225	1521	225	1971	2025	2.6	0.845	0.250	0.125	1.59	2.07	820.00
17	-	-	-	-	-	-	0.0040	0.0059	-	-	0.050	0.013	0.072	0.024	2500	169	5184	576	5522	5576	1.0	1.086	0.067	0.115	2.66	3.83	492.00
18	2.550	-	-	33.28	20.15	20.15	0.0069	0.0095	7.907	-	0.070	0.030	0.096	0.038	4900	900	9216	1444	11061	11244	2.0	0.940	0.183	0.147	2.49	3.42	288.00
19	-	-	-	-	-	-	0.0054	0.0074	-	-	0.055	0.026	0.075	0.031	3025	676	5625	961	6877	7011	2.0	0.930	0.223	0.159	1.96	2.67	366.55
20	-	-	-	-	-	-	0.0022	0.0031	-	-	0.022	0.010	0.031	0.016	484	100	961	256	1161	1224	5.0	0.993	0.206	0.264	0.78	1.10	916.36
21	1.984	-	-	20.16	14.90	14.90	0.0037	0.0050	7.513	-	0.028	0.015	0.037	0.016	784	225	1369	256	1819	1824	0.3	0.873	0.287	0.163	0.55	0.72	590.36
22	-	-	-	-	-	-	0.0092	0.0130	-	-	0.069	0.014	0.097	0.020	4761	196	9409	400	9801	9922	1.3	0.988	0.041	0.042	1.34	1.89	239.57
23	-	-	-	-	-	-	0.0131	0.0189	-	-	0.098	0.014	0.141	0.044	9604	196	19881	1936	20273	21144	4.2	1.035	0.020	0.102	1.91	2.75	168.67
24	1.633	-	-	13.65	11.50	11.50	0.0175	0.0252	7.046	-	0.101	0.035	0.145	0.061	10201	1225	21025	3721	23475	23651	2.7	1.031	0.120	0.182	1.17	1.68	113.96
25	-	-	-	-	-	-	0.0120	0.0170	-	-	0.069	0.012	0.098	0.025	4761	144	9604	625	9892	10147	2.5	1.009	0.030	0.066	0.80	1.14	166.81
26	-	-	-	-	-	-	0.0055	0.0080	-	-	0.032	0.008	0.046	0.015	1024	64	2116	225	2244	2273	1.3	1.033	0.062	0.109	0.37	0.53	357.68
27	1.375	-	-	9.67	8.90	8.90	0.0116	0.0164	6.477	-	0.052	0.009	0.073	0.015	2704	81	5329	225	5491	5663	3.0	0.986	0.030	0.041	0.36	0.51	191.35
28	-	-	-	-	-	-	0.0222	0.0324	-	-	0.099	0.014	0.144	0.049	9801	196	20739	2401	21131	22003	4.0	1.115	0.020	0.129	0.69	1.00	90.00
29	-	-	-	-	-	-	0.0312	0.0440	-	-	0.139	0.038	0.196	0.071	19321	1444	38416	5041	41304	43683	5.5	0.987	0.074	0.130	0.97	1.36	64.10
30	1.964	1.67	1.67	19.74	13.10	13.10	0.0120	0.0171	6.676	-	0.079	0.027	0.112	0.045	6241	729	12544	2025	14002	14507	3.5	1.005	0.116	0.162	2.91	4.13	165.94
31	-	-	-	-	-	-	0.0091	0.0127	-	-	0.060	0.014	0.083	0.017	3600	196	6889	289	7281	7489	3.0	0.956	0.054	0.040	2.21	3.06	218.50
32	-	-	-	-	-	-	0.0045	0.0066	-	-	0.030	0.008	0.043	0.016	900	64	1849	256	1977	2056	3.8	1.027	0.071	0.142	1.10	1.58	437.00

TABLE III CONTINUED

Test No	T sec	h_1 ft	h_3 ft	L_0 ft	L_1 ft	L_3 ft	H_1/L_1	H_3/L_3	$C_1=C_3$ ft/sec	C_{G0}/C_{G1}	a_1 ft	a_2 ft	a_3 ft	a_4 ft	$E_i=a_1^2\times10^6$ ft²	$E_{rT}=a_2^2\times10^6$ ft²	$E_T=a_3^2\times10^6$ ft²	$E_{10}=a_4^2\times10^6$ ft²	$(2a_3^2+a_2^2)\times10^6$ ft²	$(2a_1^2+a_4^2)\times10^6$ ft²	$1-\frac{2a_3^2+a_2^2}{2a_1^2+a_4^2}$ % Loss	$\frac{E_T}{E_i}=\frac{a_3^2}{2a_1^2}$	$\frac{E_{rT}}{E_i}=\frac{a_2^2}{a_1^2}$	$\frac{E_{10}}{E_i}=\frac{a_4^2}{2a_1^2}$	$\frac{L_1^2a_1}{h_1^3}$	$\frac{L_3^2a_3}{h_3^3}$	$\frac{C\cdot T}{a_1}$
C-33	1.486	1.67	1.67	11.30	9.20	9.20	0.0104	0.0128	6.195	0.500	0.048	0.026	0.059	0.021	2304	676	3481	462	4833	5070	4.7	0.755	0.293	0.100	0.87	1.07	191.88
34	″	″	″	″	″	″	0.0147	0.0207	″	″	0.068	0.025	0.095	0.038	4624	625	9025	1444	10275	10692	4.0	0.976	0.135	0.156	1.24	1.73	135.54
35	″	″	″	″	″	″	0.0215	0.0300	″	″	0.099	0.016	0.138	0.031	9810	196	19044	961	19436	20563	5.5	0.992	0.020	0.049	1.80	2.51	93.03
36	1.082	″	″	6.00	5.70	5.70	0.0158	0.0221	5.269	″	0.045	0.009	0.063	0.051	2025	81	3969	225	4131	4275	3.4	0.980	0.040	0.055	0.31	0.44	126.67
37	″	″	″	″	″	″	0.0336	0.0474	″	″	0.096	0.017	0.135	0.038	9216	289	18225	1444	18803	19876	5.4	0.989	0.031	0.078	0.67	0.94	59.37
38	2.989	″	″	45.74	21.05	21.05	0.0043	0.0065	7.046	″	0.046	0.009	0.068	0.026	2116	81	4624	676	4786	4908	2.5	1.097	0.038	0.160	4.38	6.47	457.83
39	″	″	″	″	″	″	0.0029	0.0042	″	″	0.031	0.010	0.044	0.016	961	100	1936	256	2131	2178	2.2	1.007	0.104	0.133	2.95	4.19	679.35
40	2.558	″	″	33.49	17.75	17.75	0.0049	0.0073	6.943	″	0.044	0.007	0.065	0.026	1936	49	4225	676	4323	4548	4.9	1.091	0.025	0.174	2.98	4.40	403.63
41	″	″	″	″	″	″	0.0066	0.0088	″	″	0.059	0.028	0.078	0.030	3481	784	6084	900	7652	7862	2.7	0.874	0.225	0.129	3.99	5.28	301.02
42	3.009	1.42	1.42	46.33	19.65	19.65	0.0037	0.0050	6.536	″	0.037	0.019	0.049	0.021	1369	361	2401	441	3123	3179	1.8	0.877	0.264	0.161	4.99	6.61	531.62
43	″	″	″	″	″	″	0.0028	0.0042	″	″	0.028	0.0065	0.041	0.016	784	42	1681	256	1765	1824	3.3	1.072	0.053	0.163	3.78	5.53	702.50
44	2.277	″	″	26.53	14.50	14.50	0.0048	0.0068	6.373	″	0.035	0.013	0.049	0.019	1225	165	2401	361	2739	2811	3.6	0.980	0.138	0.147	2.57	3.60	414.59
45	″	″	″	″	″	″	0.0064	0.0095	″	″	0.047	0.011	0.069	0.028	2209	121	4761	784	5003	5203	3.9	1.078	0.055	0.177	3.45	5.07	308.92
46	1.938	″	″	15.46	10.60	10.60	0.0166	0.0240	6.102	″	0.088	0.020	0.127	0.047	7144	400	16129	2209	16929	17697	4.3	1.041	0.052	0.143	3.45	4.98	120.59
47	″	″	″	″	″	″	0.0104	0.0136	″	″	0.055	0.022	0.072	0.018	3025	484	5184	324	6152	6374	3.5	0.857	0.160	0.054	2.16	2.83	192.90
48	″	″	″	″	″	″	0.0072	0.0104	″	″	0.038	0.006	0.055	0.016	1444	36	3025	256	3097	3144	1.5	1.024	0.025	0.086	1.49	2.16	279.21
49	1.14	″	″	6.65	6.00	6.00	0.0185	0.0260	5.266	″	0.055	0.009	0.078	0.020	3080	81	6084	400	6246	6560	4.8	0.987	0.026	0.065	0.69	0.98	109.09
50	″	″	″	″	″	″	0.0280	0.0370	″	″	0.084	0.039	0.111	0.048	7056	1521	12321	2304	15363	16416	6.5	0.873	0.130	0.163	1.06	1.40	71.43
51	1.998	″	″	20.43	12.50	12.50	0.0113	0.0157	6.260	″	0.071	0.018	0.098	0.024	5041	324	9604	576	10252	10658	3.8	0.953	0.024	0.057	3.88	5.35	176.20
52	″	″	″	″	″	″	0.0072	0.0101	″	″	0.045	0.016	0.063	0.024	2025	256	3969	576	4481	4625	3.2	0.981	0.126	0.142	2.46	3.44	278.00

APPENDIX C

•M4013-3529,FMS,RESULT,1,5,500,5C0,   BCUROCIMOS

•   XEC   .04

JCB TIME =   .06 MIN.

LIBRARY ENTRY PCINTS,

.SETUF   (CSHM)   (RTN)   (SPHM)   (FIL)   SORT   COS   SIN   ATAN   EXP(2

NAME	CRIGIN	ENTRY	NAME	ORIGIN	ENTRY	NAME	ORIGIN	ENTRY	NAME	ORIGIN	ENTRY	NAME	ORIGIN	ENTRY
MAIN	C0144	C0156	AKEFQ	01616	01623	.SETUP	02075	C2102	(RCPM)	02113	03412	FINBP	02113	02140
(F2EF)	02113	02303	FTNPM	02113	02141	(F2PM)	02113	C2130	(FPT)	07605	07614	(FRM7)	10103	10410
RSTRTN	1C103	10374	TIMLFT	10103	10160	KILLTR	10103	10342	STOPCL	1C103	10202	RSCLCK	10103	10175
JOBTM	1C103	10142	TIMER	10103	10220	(TIME)	10103	1C106	ENDJOB	10477	10563	CLKOUT	10477	10531
EXITM	10477	10505	EXIT	10477	10531	.LOOK	10616	11031	.SCRDS	10616	11033	.READL	10616	10671
.READ	10616	10671	.TAPRD	1C616	10666	(TSHM)	10616	10635	(CSHM)	10616	10634	(TSH)	10616	10645
(CSH)	1C616	10660	IOHSIZ	11341	14703	(RTN)	11341	14553	(FIL)	11341	14536	STQUO	11341	11601
(IOH)	11341	11604	.03311	16214	16216	.C3310	16214	16216	SFDP	16233	16316	DFDP	16233	16322
CCEXIT	16233	16376	DFMP	16233	16270	DFSB	16233	16253	DFAD	16233	16236	(TCO)	16406	16512
(TEF)	16406	16511	(RCH)	16406	16510	(ETT)	16406	16507	(REW)	16406	16506	(WEF)	16406	16505
(BSR)	16406	16504	(WRS)	16406	16503	(RDS)	16406	16502	(IOS)	16406	16413	(TRC)	16406	16513
(EXE)	16551	16560	(IOU)	17411	17416	(TES)	17433	17435	RECDUP	17436	17441	.SPRNT	17444	17670
.PRINT	17444	17550	.TAPWR	17444	17543	.PUNCH	17444	17524	(SCH)	17444	17471	(STHD)	17444	1750[illegible]
(STHM)	17444	17457	(STH)	17444	17460	(SPHM)	17444	17456	(SPH)	17444	17515	(PRNT)	17444	20067
(SCHM)	17444	17466	.FOUT	17444	20404	.CLOUT	17444	20401	.COMNT	17444	17550	.PNCHL	17444	17524
FRRCR	20644	2C650	(WTC)	21C40	21127	(WER)	21040	21054	(BST)	21164	21175	(RDPM)	21224	21320
(RDC)	21224	21303	(RER)	21224	21237	SQR	21327	21333	SQRT	21327	21333	EXP(2	21437	21443
LDUMP	21573	21576	ATN	21602	21604	ATAN	21602	21604	SIN	21731	21744	COS	21731	21733
MOVIE)	22123	22123												

PROGRAM LENGTH = 22500. LOWEST COMMCN = 77461

.22 MINUTES ELAPSED SINCE START OF JOB

EXECUTICN

THE FCLLCWING RUNS HAVE BED SLCPE = .125C00 AND SIDEWALL SLOPE = 1.CCCC00

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P DELT1	A3P DELT2	STEPU DELT4	STEPD HL11	Z1 HL33	XKR	XKT A1	CELAU A2	CELAD A3	CGRAT A4
1	2.424	30.08	19.00	2.25	1.25	1.80	14.70	.0265	.0141	.0241	.0028	.0033	11.91	.5322	.9089	7.842	6.067	.8299
									-19.4	.3	9.3	.12	.09		.025	.009	.026	.007
2	2.424	30.08	19.00	2.25	1.25	1.80	14.70	.0423	.0063	.C455	.0045	.0062	11.91	.1488	1.0746	7.842	6.067	.8299
									-17.3	-1.4	9.3	.12	.09		.047	.013	.053	.020
3	2.424	3C.08	19.00	2.25	1.25	1.80	14.7C	.0668	.0308	.0642	.0070	.0087	11.91	.4616	.9621	7.842	6.067	.8299
									-19.0	-.5	9.1	.12	.09		.063	.016	.068	.016
40	2.424	30.08	19.00	2.25	1.25	1.80	14.70	.0787	.0181	.C871	.0083	.0118	11.91	.2301	1.1066	7.842	6.067	.8299
									-19.6	.7	14.9	.12	.09		.082	.019	.094	.026
5	1.916	18.79	14.25	2.25	1.25	1.80	11.30	.1265	.0254	.1290	.0178	.0228	15.87	.2010	1.0201	7.441	5.900	.8857
									-28.0	.5	3.9	.16	.11		.132	.040	.133	.023

IV$_a$-2

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
6	1.916	18.79	14.25	2.25	1.25	1.80	11.30	.0968	.0047	.0998	.0136	.0177	15.87	.0484	1.0307	7.441	5.900	.8857
									-27.8	.1	9.1	.16	.11		.100	.016	.104	.021
7	1.916	18.79	14.25	2.25	1.25	1.80	11.30	.0671	.0270	.0651	.0094	.0115	15.87	.4030	.9707	7.441	5.900	.8857
									-19.1	-1.5	2.9	.16	.11		.065	.017	.068	.014
8	1.916	18.79	14.25	2.25	1.25	1.80	11.30	.0283	.0099	.0305	.0040	.0054	15.87	.3503	1.0780	7.441	5.900	.8857
									-20.6	1.0	-4.1	.16	.11		.031	.015	.032	.007
9	1.533	12.02	10.50	2.25	1.25	1.80	8.66	.0549	.0144	.0544	.0105	.0126	21.54	.2617	.9918	6.855	5.651	.9698
									-27.4	-.6	-5.2	.21	.14		.061	.024	.061	.020
10	1.533	12.02	10.50	2.25	1.25	1.80	8.66	.0758	.0662	.1580	.0144	.0365	21.54	.8732	2.0843	6.855	5.651	.9698
									-28.5	.8	1.3	.21	.14		.069	.059	.160	.018
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
1	1.533	12.02	10.50	2.25	1.25	1.80	8.66	.1820	.0595	.1720	.0347	.0397	21.54	.3269	.9452	6.855	5.651	.9698
									-28.1	.1	-5.6	.21	.14		.179	.015	.185	.049
12	1.533	12.02	10.50	2.25	1.25	1.80	8.66	.2135	.0371	.2089	.0407	.0483	21.54	.1740	.9781	6.855	5.651	.9698
									-28.1	-.4	2.4	.21	.14		.225	.036	.233	.075
13	1.190	7.25	7.00	2.25	1.25	1.80	6.19	.0500	.0105	.0495	.0143	.0160	32.31	.2095	.9891	5.885	5.203	1.0864
									-37.5	-.4	-8.5	.32	.20		.050	.009	.050	.005
1	1.190	7.25	7.00	2.25	1.25	1.80	6.19	.0978	.0215	.0955	.0279	.0309	32.31	.2196	.9769	5.885	5.203	1.0864
									-37.9	-.7	3.6	.32	.20		.097	.013	.097	.012
15	1.190	7.25	7.00	2.25	1.25	1.80	6.19	.1439	.0400	.1371	.0411	.0443	32.31	.2782	.9527	5.885	5.203	1.0864
									-44.8	-1.0	-4.5	.32	.20		.143	.010	.146	.036
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
16	1.190	7.25	7.00	2.25	1.25	1.80	6.19	.1848	.0443	.1732	.0528	.0560	32.31	.2395	.9373	5.885	5.203	1.0864
									-33.0	-.6	-10.1	.32	.20		.185	.020	.182	.040

17	.766	3.00	3.00	2.25	1.25	1.80	2.97	.0509	.0119	.0492	.0340	.0331	75.40	.2337	.9663	3.921	3.882	1.0414
								-110.2	1.3	-.0	.75	.42			.051	.003	.052	.012
18	.766	3.00	3.00	2.25	1.25	1.80	2.97	.0893	.0169	.0866	.0595	.0583	75.40	.1890	.9701	3.921	3.882	1.0414
								-104.7	1.6	-31.1	.75	.42			.090	.005	.091	.020
19	.766	3.00	3.00	2.25	1.25	1.80	2.97	.1153	.0281	.1096	.0769	.0738	75.40	.2438	.9502	3.921	3.882	1.0414
								-80.1	-.1	-31.3	.75	.42			.115	.010	.115	.025
20	3.089	48.82	25.00	2.25	1.25	1.80	19.06	.0095	.0016	.0104	.0008	.0011	9.05	.1653	1.0949	8.100	6.174	.7959
								-14.6	1.1	9.3	.09	.07			.021	.016	.022	.016
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
21	3.089	48.82	25.00	2.25	1.25	1.80	19.06	.0248	.0031	.0279	.0020	.0029	9.05	.1254	1.1249	8.100	6.174	.7959
								-14.8	1.1	9.6	.09	.07			.038	.024	.040	.022
22	3.089	48.82	25.00	2.25	1.25	1.80	19.06	.0460	.0116	.0517	.0037	.0054	9.05	.2524	1.1255	8.100	6.174	.7959
								-14.6	.7	9.1	.09	.07			.063	.039	.064	.028
23	3.089	48.82	25.00	2.25	1.25	1.80	19.06	.0734	.0230	.0766	.0059	.0080	9.05	.3137	1.0436	8.100	6.174	.7959
								-15.1	.1	9.6	.09	.07			.081	.036	.083	.023
24	3.064	48.06	23.50	2.00	1.00	2.00	16.99	.0468	-.0266	.0139	.0040	.0016	8.56	-.5675	.2978	7.674	5.551	.7555
								-17.3	-.3	13.0	.09	.06			.054	-.034	.016	.006
25	3.064	48.06	23.50	2.00	1.00	2.00	16.99	.0226	.0097	.0235	.0019	.0028	8.56	.4287	1.0393	7.674	5.551	.7555
								-15.3	-1.2	19.6	.09	.06			.026	.012	.030	.014
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
26	3.064	48.06	23.50	2.00	1.00	2.00	16.99	.0263	.0029	.0297	.0022	.0035	8.56	.1120	1.1307	7.674	5.551	.7555
								-12.0	.5	6.4	.09	.06			.036	.020	.039	.019
27	3.064	48.06	23.50	2.00	1.00	2.00	16.99	.0492	.0085	.0466	.0042	.0055	8.56	.1728	.9473	7.674	5.551	.7555
								-16.0	.5	6.4	.09	.06			.057	.023	.054	.020
28	2.415	29.84	18.00	2.00	1.00	2.00	13.21	.0111	.0036	.0118	.0012	.0018	11.17	.3200	1.0572	7.459	5.475	.7878

								-14.0	-.5	7.4	.11	.08			.017	.009	.022	.015
29	2.415	29.84	18.00	2.00	1.00	2.00	13.21	.0310	.0132	.0316	.0034	.0048	11.17	.4275	1.0194	7.459	5.475	.7878
								-13.8	-.7	7.5	.11	.08			.037	.020	.039	.017
30	2.415	29.84	18.00	2.00	1.00	2.00	13.21	.0544	.0267	.0552	.0060	.0084	11.17	.4905	1.0145	7.459	5.475	.7878
								-13.2	-1.5	7.9	.11	.08			.051	.014	.059	.016

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
31	2.415	29.84	18.00	2.00	1.00	2.00	13.21	.0538	.0128	.0595	.0060	.0090	11.17	.2376	1.1068	7.459	5.475	.7878
								-14.1	-1.2	6.8	.11	.08			.070	.040	.072	.030
32	1.872	17.93	13.25	2.00	1.00	2.00	9.99	.0118	.0056	.0114	.0018	.0023	15.17	.4740	.9668	7.084	5.342	.8473
								-20.4	.5	8.8	.15	.10			.018	.012	.019	.012
33	1.872	17.93	13.25	2.00	1.00	2.00	9.99	.0288	.0036	.0312	.0043	.0063	15.17	.1263	1.0849	7.084	5.342	.8473
								-21.1	.4	8.9	.15	.10			.040	.023	.041	.020
34	1.872	17.93	13.25	2.00	1.00	2.00	9.99	.0525	.0045	.0557	.0079	.0111	15.17	.0863	1.0616	7.084	5.342	.8473
								-20.5	.4	8.3	.15	.10			.056	.012	.061	.018
35	1.872	17.93	13.25	2.00	1.00	2.00	9.99	.0839	.0344	.0830	.0127	.0166	15.17	.4098	.9896	7.084	5.342	.8473
								-28.2	1.0	7.9	.15	.10			.081	.012	.091	.027

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
36	1.463	10.95	9.50	2.00	1.00	2.00	7.50	.0376	.0257	.0279	.0079	.0074	21.16	.6833	.7422	6.499	5.130	.9456
								-37.8	-.4	7.2	.21	.13			.039	.021	.040	.022
37	1.463	10.95	9.50	2.00	1.00	2.00	7.50	.0760	.0295	.0717	.0160	.0191	21.16	.3882	.9439	6.499	5.130	.9456
								-22.1	.4	14.5	.21	.13			.075	.015	.079	.024
38	1.463	10.95	9.50	2.00	1.00	2.00	7.50	.1114	.0413	.1044	.0235	.0279	21.16	.3707	.9375	6.499	5.130	.9456
								-22.2	-.5	6.9	.21	.13			.109	.014	.114	.033
39	1.463	10.95	9.50	2.00	1.00	2.00	7.50	.1458	.0576	.1339	.0307	.0357	21.16	.3952	.9185	6.499	5.130	.9456
								-22.2	-.4	6.5	.21	.13			.141	.018	.146	.042

RLN	T	XLO	XI 1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
40	.957	4.69	4.65	2.00	1.00	2.00	4.23	.0436	.0020	.C412	.0188	.0195	43.24	.0451	.9444	4.860	4.424	1.1338
								-50.2	.7	19.5	.43	.24			.052	.020	.050	.021
41	.957	4.69	4.65	2.00	1.00	2.00	4.23	.0947	.0236	.0926	.0407	.0437	43.24	.2494	.9777	4.860	4.424	1.1338
								-51.5	.6	13.5	.43	.24			.097	.012	.102	.031
42	.957	4.69	4.65	2.00	1.00	2.00	4.23	.1277	.0181	.1375	.0549	.0649	43.24	.1418	1.0766	4.860	4.424	1.1338
								-57.5	-1.1	-.6	.43	.24			.142	.050	.152	.047
43	1.020	5.33	5.15	1.67	.67	2.49	4.11	.C950	.0386	.0867	.0369	.0422	32.60	.4067	.9129	5.051	4.032	1.0777
								-32.3	.8	-1.7	.32	.16			.091	.017	.092	.022
44	1.020	5.33	5.15	1.67	.67	2.49	4.11	.0719	.0260	.0653	.0279	.0318	32.60	.3610	.9078	5.051	4.032	1.0777
								-48.5	-.0	.1	.32	.16			.071	.009	.072	.022
45	1.02C	5.33	5.15	1.67	.67	2.49	4.11	.0402	.0102	.0404	.0156	.0197	32.60	.2532	1.0045	5.051	4.032	1.0777
								-55.2	.2	.6	.32	.16			.040	.008	.041	.005
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
46	1.298	8.63	7.60	1.67	.67	2.49	5.53	.0339	.0196	.0286	.0089	.0104	22.09	.5776	.8459	5.858	4.265	.9154
								-25.2	-.3	2.4	.22	.12			.031	.009	.033	.012
47	1.298	8.63	7.60	1.67	.67	2.49	5.53	.0460	.0062	.0477	.0121	.0172	22.09	.1346	1.0371	5.858	4.265	.9154
								-35.0	-.9	2.8	.22	.12			.052	.021	.052	.015
48	1.298	8.63	7.60	1.67	.67	2.49	5.53	.0759	.0061	.0789	.C200	.0285	22.09	.0806	1.0391	5.858	4.265	.9154
								-37.5	1.3	14.0	.22	.12			.079	.017	.082	.016
49	1.298	8.63	7.60	1.67	.67	2.49	5.53	.0859	.0137	.0884	.0226	.0320	22.09	.1599	1.0297	5.858	4.265	.9154
								-25.3	1.4	14.5	.22	.12			.095	.035	.096	.027
50	1.727	15.26	11.20	1.67	.67	2.49	7.65	.0250	.0104	.0255	.0045	.0067	14.99	.4148	1.0207	6.490	4.431	.7832
								-13.7	-.3	12.9	.15	.09			.033	.020	.034	.017

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
51	1.727	15.26	11.20	1.67	.67	2.49	7.65	.0531	.0116	.0584	.0095	.0153	14.99	.2187	1.0984	6.490	4.431	.7832
									-19.7	-.4	13.3	.15	.09		.059	.025	.062	.015
52	1.727	15.26	11.20	1.67	.67	2.49	7.65	.0711	.0161	.0772	.0127	.0202	14.99	.2268	1.0859	6.490	4.431	.7832
									-19.7	-1.1	.1	.15	.09		.079	.033	.083	.022
53	2.222	25.26	15.15	1.67	.67	2.49	10.03	.0158	.0069	.0171	.0021	.0034	11.08	.4362	1.0839	6.823	4.515	.7195
									-7.8	-1.5	5.5	.11	.07		.018	.010	.019	.006
54	2.222	25.26	15.15	1.67	.67	2.49	10.03	.0274	.0093	.0311	.0036	.0062	11.08	.3391	1.1330	6.823	4.515	.7195
									-10.5	1.1	-1.0	.11	.07		.033	.019	.034	.010
55	2.222	25.26	15.15	1.67	.67	2.49	10.03	.0462	.0174	.0498	.0061	.0099	11.08	.3765	1.0771	6.823	4.515	.7195
									-17.5	1.1	4.7	.11	.07		.047	.016	.053	.013
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
56	2.222	25.26	15.15	1.67	.67	2.49	10.03	.0526	.0235	.0557	.0069	.0111	11.08	.4479	1.0594	6.823	4.515	.7195
									-18.0	.5	5.2	.11	.07		.058	.032	.059	.014
57	2.823	40.80	19.80	1.67	.67	2.49	12.88	.0086	.0033	.0101	.0009	.0016	8.48	.3789	1.1706	7.013	4.565	.6850
									-14.6	1.3	9.9	.08	.05		.017	.013	.019	.013
58	2.823	40.80	19.80	1.67	.67	2.49	12.88	.0198	.0119	.0210	.0020	.0033	8.48	.5978	1.0587	7.018	4.565	.6850
									-14.6	1.3	4.0	.08	.05		.025	.018	.025	.010
59	2.823	40.80	19.80	1.67	.67	2.49	12.88	.0338	.0097	.0389	.0034	.0060	8.48	.2864	1.1498	7.018	4.565	.6850
									-15.1	1.6	10.4	.08	.05		.042	.024	.044	.015
60	2.823	40.80	19.80	1.67	.67	2.49	12.88	.0435	.0135	.0502	.0044	.0078	8.48	.3109	1.1535	7.018	4.565	.6850
									-15.1	1.2	3.6	.08	.05		.053	.030	.056	.018
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
61	3.224	53.21	22.85	1.67	.67	2.49	14.76	.0134	.0035	.0156	.0012	.0021	7.35	.2645	1.1665	7.091	4.583	.6723
									-13.9	-.8	6.9	.07	.05		.018	.011	.019	.008

62	3.224	53.21	22.85	1.67	.67	2.49	14.76	.0305	.0250	.0212	.0027	.0029	7.35	.8171	.6957	7.091	4.583	.6723
								-14.7	.4	7.1	.07	.05			.028	.019	.029	.015
63	3.224	53.21	22.85	1.67	.67	2.49	14.76	.0282	.0116	.0316	.0025	.0043	7.35	.4093	1.1190	7.091	4.583	.6723
								-11.1	1.4	12.9	.07	.05			.038	.025	.039	.017
64	3.224	53.21	22.85	1.67	.67	2.49	14.76	.0477	.0245	.0498	.0042	.0068	7.35	.5129	1.0447	7.091	4.583	.6723
								-10.8	.9	6.5	.07	.05			.051	.028	.054	.015
65	3.192	52.15	21.50	1.50	.50	3.00	12.66	.0331	.0118	.0389	.0031	.0061	7.01	.3559	1.1763	6.740	3.972	.6133
								-11.3	.7	6.3	.07	.04			.039	.021	.044	.015

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
66	3.192	52.15	21.50	1.50	.50	3.00	12.66	.1152	-.0776	.0362	.0107	.0057	7.01	-.6739	.3148	6.740	3.972	.6133
								-16.2	.4	19.1	.07	.04			.134	-.101	.038	.009
67	3.192	52.15	21.50	1.50	.50	3.00	12.66	.0121	.0039	.0153	.0011	.0024	7.01	.3198	1.2580	6.740	3.972	.6133
								-13.8	-1.2	12.7	.07	.04			.016	.010	.018	.007
68	2.247	25.83	14.65	1.50	.50	3.00	8.82	.0681	.0379	.0699	.0093	.0159	10.29	.5564	1.0265	6.525	3.931	.6534
								-17.8	1.4	9.9	.10	.06			.063	.025	.073	.015
69	2.247	25.83	14.65	1.50	.50	3.00	8.82	.0353	.0155	.0398	.0048	.0090	10.29	.4387	1.1273	6.525	3.931	.6534
								-16.2	.2	10.3	.10	.06			.039	.022	.041	.007
70	2.247	25.83	14.65	1.50	.50	3.00	8.82	.0192	.0089	.0213	.0026	.0048	10.29	.4664	1.1104	6.525	3.931	.6534
								-16.0	-.0	10.3	.10	.06			.021	.012	.022	.004

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
71	1.668	14.24	10.30	1.50	.50	3.00	6.44	.0327	.0145	.0336	.0063	.0104	14.64	.4429	1.0289	6.180	3.864	.7247
								-20.2	-.1	12.4	.15	.08			.033	.014	.035	.007
72	1.668	14.24	10.30	1.50	.50	3.00	6.44	.0550	.0211	.0590	.0107	.0183	14.64	.3843	1.0730	6.180	3.864	.7247
								-21.6	-.3	11.4	.15	.08			.053	.012	.061	.011
73	1.668	14.24	10.30	1.50	.50	3.00	6.44	.0693	.0299	.0734	.0135	.0228	14.64	.4309	1.0595	6.180	3.864	.7247

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RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P (DELT1)	A2P (DELT2)	A3P (DELT4)	STEPU (HL11)	STEPD (HL33)	Z1	XKR	XKT (A1)	CELAU (A2)	CELAD (A3)	CGRAT (A4)
								-23.1	1.3	10.0	.15	.08			.068	.024	.076	.014
74	1.083	6.00	5.60	1.50	.50	3.00	3.96	.0424	.0133	.C415	.0151	.0210	26.93	.3139	.9787	5.175	3.661	.9628
								-31.7	-.5	4.7	.27	.13			.043	.013	.043	.008
75	1.083	6.00	5.60	1.50	.50	3.00	3.96	.0672	.0185	.0638	.0240	.0322	26.93	.2747	.9483	5.175	3.661	.9628
								-40.7	-.4	-2.4	.27	.13			.068	.010	.069	.019
76	1.083	6.C0	5.60	1.50	.50	3.00	3.96	.11C3	.0395	.1028	.0394	.0519	26.93	.3582	.9317	5.175	3.661	.9628
								-44.9	-1.0	-3.2	.27	.13			.107	.014	.109	.026
77	.874	3.91	3.85	1.50	.50	3.00	3.03	.C494	.0170	.0432	.0257	.0285	39.17	.3436	.8749	4.409	3.474	1.1236
								-42.7	.1	22.8	.39	.16			.051	.019	.045	.009
78	.874	3.91	3.85	1.50	.50	3.00	3.03	.0780	.0138	.0762	.0405	.0502	39.17	.1773	.9771	4.409	3.474	1.1236
								-52.1	1.4	18.1	.39	.16			.077	.008	.077	.008
79	.874	3.91	3.85	1.50	.50	3.00	3.03	.1111	.0231	.1062	.0577	.0700	39.17	.2082	.9554	4.409	3.474	1.1236
								-43.3	-.4	-4.1	.39	.16			.110	.009	.108	.014
8C	.921	4.34	4.15	1.25	.25	5.00	2.45	.0575	.0174	.0568	.0277	.0463	30.28	.3033	.9879	4.508	2.666	.8936
								-35.5	1.4	20.0	.30	.10			.057	.005	.061	.016
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P (DELT1)	A2P (DELT2)	A3P (DELT4)	STEPU (HL11)	STEPD (HL33)	Z1	XKR	XKT (A1)	CELAU (A2)	CELAD (A3)	CGRAT (A4)
81	.921	4.34	4.15	1.25	.25	5.00	2.45	.0310	.0055	.0321	.0150	.0262	30.28	.1759	1.0344	4.508	2.666	.8936
								-35.6	-1.4	21.1	.30	.10			.033	.010	.034	.008
82	.921	4.34	4.15	1.25	.25	5.00	2.45	.0840	.0300	.C820	.0405	.0669	30.28	.3564	.9759	4.508	2.666	.8936
								-35.0	.2	20.0	.30	.10			.082	.022	.083	.009
83	1.363	9.51	7.45	1.25	.25	5.00	3.76	.0652	.0253	.0755	.C175	.0402	16.87	.3879	1.1572	5.469	2.759	.6282
								-17.9	.C	19.9	.17	.07			.064	.019	.078	.014
84	1.363	9.51	7.45	1.25	.25	5.00	3.76	.0442	.0164	.C512	.0119	.0272	16.87	.3707	1.1590	5.469	2.759	.6282
								-17.4	-.3	5.6	.17	.07			.046	.012	.060	.023

RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P DELT1	A2P DELT2	A3P DELT4	STEPU HL11	STEPD HL33	Z1	XKR	XKT A1	CELAU A2	CELAD A3	CGRAT A4
85	1.363	9.51	7.45	1.25	.25	5.00	3.76	.0265	.0114	.0302	.0071	.0161	16.87	.4308	1.1386	5.469	2.759	.6282
								-17.2	1.4	.6	.17	.07			.028	.014	.031	.005
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P DELT1	A2P DELT2	A3P DELT4	STEPU HL11	STEPD HL33	Z1	XKR	XKT A1	CELAU A2	CELAD A3	CGRAT A4
86	2.109	22.77	12.60	1.25	.25	5.00	-5.90	.0158	.0057	.0260	.0031	-.0088	9.97	.2849	1.3125	5.978	2.804	-.5136
								-7.4	-1.2	2.3	.10	-.04			.019	.003	.026	.003
87	2.109	22.77	12.60	1.25	.25	5.00	-5.90	.0327	.0063	.0543	.0052	-.0184	9.97	.1912	1.6602	5.978	2.804	-.5136
								-10.4	1.1	-1.9	.10	-.04			.033	.005	.057	.013
88	2.109	22.77	12.60	1.25	.25	5.00	-5.90	.0469	.0154	.0455	.0074	-.0154	9.97	.3295	.9703	5.978	2.804	-.5136
								-13.0	-1.2	-9.1	.10	-.04			.046	.007	.048	.011
89	2.988	45.69	18.40	1.25	.25	5.00	-8.42	.0317	.0013	.0608	.0034	-.0144	6.83	.0400	1.9180	6.162	2.821	-.4793
								-10.4	.6	-1.9	.07	-.03			.032	.003	.061	.007
90	2.988	45.69	18.40	1.25	.25	5.00	-8.42	.0237	.0043	.0370	.0026	-.0088	6.83	.1824	1.5608	6.162	2.821	-.4793
								-7.7	-1.5	-9.5	.07	-.03			.023	.002	.037	.005
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P DELT1	A2P DELT2	A3P DELT4	STEPU HL11	STEPD HL33	Z1	XKR	XKT A1	CELAU A2	CELAD A3	CGRAT A4
91	2.988	45.69	18.40	1.25	.25	5.00	-8.42	.0143	.0044	.0165	.0016	-.0039	6.83	.3099	1.1482	6.162	2.821	-.4793
								-12.0	.6	-2.8	.07	-.03			.014	.002	.017	.003
92	3.959	80.22	30.92	2.00	1.00	2.00	22.13	.0554	.0457	.0367	.0036	.0033	6.50	.8244	.6622	7.815	5.600	.7347
								-10.2	1.5	1.9	.06	.05			.044	.026	.047	.022
93	3.959	80.22	30.92	2.00	1.00	2.00	22.13	.0530	.0162	.0588	.0034	.0053	6.50	.3067	1.1101	7.815	5.600	.7347
								-8.7	-.5	1.4	.06	.05			.064	.034	.068	.025
94	3.959	80.22	30.92	2.00	1.00	2.00	22.13	.1165	.0523	.1206	.0075	.0109	6.50	4490	1.0350	7.815	5.600	.7347
								-8.5	-.2	12.1	.06	.05			.111	.027	.129	.033
95	3.959	80.22	30.92	2.00	1.00	2.00	22.13	.0449	.0250	.0433	.0029	.0039	6.50	.5575	.9649	7.815	5.600	.7347
								-9.4	1.1	6.4	.06	.05			.042	.015	.048	.015

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•     XEC                  *M4013-3529,FMS,RESULT,1,5,500,500,    BOURODIMOS                .04

JCB TIME =    .06 MIN.

LIBRARY ENTRY PCINTS,
    .SETUP     (CSHM)     (RTN)     (SPHM)     (FIL)     SQRT     COS     SIN     ATAN     EXP(2
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NAME	ORIGIN	ENTRY	NAME	ORIGIN	ENTRY	NAME	ORIGIN	ENTRY	NAME	ORIGIN	ENTRY	NAME	ORIGIN	ENTRY
MAIN	C0144	00156	AKEFQ	01622	01627	.SETUP	02101	C2106	(RCPM)	02117	03416	FTNBP	02117	02144
(F2EF)	02117	02307	FTNPM	02117	02145	(F2PM)	02117	02134	(FPT)	07611	07620	(FRM7)	10107	10414
RSTRTN	1C107	1C400	TIMLFT	10107	10164	KILLTR	10107	10346	STOPCL	10107	10206	RSCLCK	10107	10201
JOBTM	1C107	10146	TIMER	10107	10224	(TIME)	10107	10112	ENDJOB	10503	10567	CLKOUT	10503	10535
EXITM	1C503	10511	EXIT	10503	10535	.LOOK	10622	11035	.SCRDS	10622	11037	.READL	10622	10675
.READ	10622	10675	.TAPRC	10622	10672	(TSHM)	10622	10641	(CSHM)	10622	10640	(TSH)	10622	10651
(CSH)	1C622	10664	IOHSIZ	11345	14707	(RTN)	11345	14557	(FIL)	11345	14542	STQUU	11345	11605
(IOH)	11345	11610	.03311	16220	16222	.03310	1622C	16222	SFDP	16237	16322	DFDP	16237	16326
DCEXIT	16237	16402	DFMP	16237	16274	DFSB	16237	16257	DFAD	16237	16242	(TCO)	16412	16516
(TEF)	16412	16515	(RCH)	16412	16514	(ETT)	16412	16513	(REW)	16412	16512	(WEF)	16412	16511
(BSR)	16412	16510	(WRS)	16412	16507	(RDS)	16412	16506	(IOS)	16412	16417	(TRC)	16412	16517
(EXE)	16555	16564	(IOU)	17415	17422	(TES)	17437	17441	RECOUP	17442	17445	.SPRNT	17450	17674
.PRINT	17450	17554	.TAPWR	17450	17547	.PUNCH	17450	17530	(SCH)	17450	17475	(STHD)	17450	17513
(STHM)	17450	17463	(STH)	17450	17464	(SPHM)	1745C	17462	(SPH)	17450	17521	(PRNT)	17450	20073
(SCHM)	17450	17472	.FOUT	17450	2041C	.CLOUT	17450	20405	.COMNT	17450	17554	.PNCHL	17450	17530
ERRCR	20650	20654	(WTC)	21044	21133	(WER)	21044	21060	(BST)	21170	21201	(RDPM)	21230	21324
(RDC)	21230	21307	(RER)	21230	21243	SQR	21333	21337	SQRT	21333	21337	EXP(2	21443	21447
LDUMP	21577	21602	ATN	21606	2161C	ATAN	21606	21610	SIN	21735	21750	COS	21735	21737
MOVIE)	22127	22127												

PROGRAM LENGTH = 22504. LOWEST COMMCN = 77461

.22 MINUTES ELAPSED SINCE START OF JOB

EXECUTICN

THE FOLLCWING RUNS HAVE BED SLCPE = .125000 AND SIDEWALL SLOPE = 1.CCCC00

RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAC	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
95	4.516	104.36	35.49	2.00	1.00	2.00	25.35	.0327	.0093	.0364	.0018	.0029	5.67	.2844	1.1131	7.864	5.618	.7289
									-6.0	.0	6.0	.06	.04		.039	.021	.040	.012
96	4.516	104.36	35.49	2.00	1.00	2.00	25.35	.0486	.0096	.0546	.0027	.0043	5.67	.1970	1.1229	7.864	5.616	.7289
									-5.4	-.7	4.1	.06	.04		.052	.017	.058	.014
99	4.953	125.53	39.05	2.00	1.00	2.00	27.85	.0292	.0029	.0340	.0015	.0024	5.15	.0990	1.1642	7.891	5.627	.7252
									-5.6	-.8	4.6	.05	.04		.033	.011	.039	.014
1C1	4.953	125.53	39.05	2.00	1.00	2.00	27.85	.0444	.0114	.C493	.C023	.0035	5.15	.2575	1.1099	7.891	5.627	.7252
									-5.5	-.7	4.4	.05	.04		.050	.022	.054	.016
1C3	4.953	125.53	39.05	2.00	1.00	2.00	27.85	.0175	.0050	.C195	.CC09	.0014	5.15	.2862	1.1114	7.891	5.627	.7252
									-8.3	-.8	7.4	.05	.04		.023	.014	.023	.009

RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
1C4	4.953	125.53	39.05	2.00	1.C0	2.C0	27.85	.0173	.0101	.C180	.CCC9	.0013	5.15	.5838	1.0372	7.891	5.627	.7252
								-8.4	-.7	6.8	.05	.04			.020	.013	.021	.008
1C5	6.644	225.90	52.79	2.00	1.C0	2.00	37.5C	.0384	.0064	.C439	.C015	.0023	3.81	.1672	1.1428	7.950	5.648	.7171
								-5.8	.4	6.8	.04	.03			.041	.012	.047	.012
1C6	6.644	225.90	52.79	2.00	1.00	2.00	37.50	.0854	.0166	.C974	.0032	.0052	3.81	.1940	1.1407	7.950	5.548	.7171
								-7.1	1.0	7.2	.04	.03			.088	.018	.103	.024
1C7	5.854	175.41	46.39	2.00	1.00	2.00	33.00	.C260	.0144	.0289	.0011	.0017	4.33	.5545	1.1093	7.929	5.641	.7200
								-2.2	-1.0	1.8	.04	.03			.031	.021	.032	.010
1C8	5.854	175.41	46.39	2.C0	1.00	2.C0	33.0C	.0266	.0138	.C267	.C011	.0016	4.33	.5177	1.0033	7.929	5.641	.7200
								-4.3	.8	1.4	.04	.03			.025	.009	.028	.006
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
1C9	5.854	175.41	46.39	2.00	1.C0	2.C0	33.00	.C506	.0175	.C520	.C022	.0032	4.33	.3452	1.0268	7.929	5.641	.7200
								-2.8	.2	1.5	.04	.03			.051	.013	.056	.015
110	5.854	175.41	46.39	2.00	1.00	2.00	33.00	.C934	.0218	.1018	.0040	.0062	4.33	.2333	1.0902	7.929	5.641	.7200
								-2.8	.4	2.2	.04	.03			.095	.014	.109	.028
111	5.582	159.47	46.78	2.25	1.25	1.80	35.10	.C420	.0016	.0477	.0018	.0027	4.84	.0372	1.1363	8.386	6.292	.7603
								-6.0	1.5	1.1	.05	.04			.045	.012	.051	.013
112	5.582	159.47	46.78	2.25	1.25	1.80	35.1C	.0341	.0120	.0380	.C015	.0022	4.84	.3511	1.1151	8.386	6.292	.7603
								-5.7	.4	.7	.05	.04			.036	.015	.040	.009
113	5.582	159.47	46.78	2.25	1.25	1.80	35.10	.0410	.0119	.C450	.0018	.0026	4.84	.2905	1.0966	8.386	6.292	.7603
								-6.1	1.0	.6	.05	.04			.048	.025	.049	.014
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
114	5.582	159.47	46.78	2.25	1.25	1.80	35.10	.0931	.0253	.C989	.C040	.C056	4.84	.2713	1.0627	8.386	6.292	.7603

									-4.5	-.4	.7	.05	.04		.092	.005	.104	.023
115	5.051	130.55	42.18	2.25	1.25	1.80	31.70	.0575	.0265	.0572	.0027	.0036	5.36	.4600	.9943	8.353	6.281	.7637
									-9.4	1.4	2.2	.05	.04		.055	.017	.060	.013
116	5.051	130.55	42.18	2.25	1.25	1.80	31.70	.0353	.0139	.0375	.0017	.0024	5.36	.3937	1.0633	8.358	6.281	.7637
									-8.9	.4	2.3	.05	.04		.039	.020	.040	.010
117	5.051	130.55	42.18	2.25	1.25	1.80	31.70	.0749	.0149	.0820	.0036	.0052	5.36	.1993	1.0943	8.358	6.281	.7637
									-8.9	.7	1.9	.05	.04		.075	.006	.085	.016
118	4.372	97.84	36.29	2.25	1.25	1.80	27.35	.0470	.0141	.0498	.0026	.0036	6.23	.2996	1.0610	8.306	6.259	.7700
									-5.7	-.5	4.2	.06	.05		.049	.015	.054	.015
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
119	4.372	97.84	36.29	2.25	1.25	1.80	27.35	.0374	.0127	.0405	.0021	.0030	6.23	.3399	1.0834	8.306	6.259	.7700
									-5.5	-.8	4.4	.06	.05		.041	.020	.042	.008
120	4.372	97.84	36.29	2.25	1.25	1.80	27.35	.0634	.0150	.0675	.0035	.0049	6.23	.2368	1.0648	8.306	6.259	.7700
									-5.4	-.9	4.5	.06	.05		.073	.035	.073	.020
121	3.844	75.63	31.68	2.25	1.25	1.80	23.95	.0510	.0154	.0541	.0032	.0045	7.14	.3009	1.0608	8.246	6.234	.7774
									-10.8	.5	.3	.07	.05		.051	.008	.058	.015
122	3.844	75.63	31.68	2.25	1.25	1.80	23.95	.0415	.0119	.0442	.0026	.0037	7.14	.2874	1.0653	8.246	6.234	.7774
									-4.9	-.5	.1	.07	.05		.041	.006	.046	.009
123	3.844	75.63	31.68	2.25	1.25	1.80	23.95	.0782	.0090	.0854	.0049	.0071	7.14	.1148	1.0909	8.246	6.234	.7774
									-1.1	-1.4	.3	.07	.05		.080	.010	.089	.018
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
124	5.519	155.86	39.99	1.67	.67	2.49	25.50	.0237	.0077	.0273	.0012	.0021	4.20	.3247	1.1530	7.251	4.624	.6464
									-2.3	-.4	1.6	.04	.03		.027	.014	.029	.007
125	5.519	155.86	39.99	1.67	.67	2.49	25.50	.0294	.0097	.0343	.0015	.0027	4.20	.3295	1.1661	7.251	4.624	.6464
									-2.4	-.4	1.7	.04	.03		.034	.018	.037	.010

126	5.519	155.86	39.99	1.67	.67	2.49	25.50	.0523	.0127	.0615	.0026	.0048	4.20	.2429	1.1764	7.251	4.624	.6464
								-2.7	-.4	8.0	.04	.03			.057	.023	.065	.015
127	6.130	192.32	44.51	1.67	.67	2.49	28.35	.0226	.0115	.0241	.0010	.0017	3.77	.5113	1.0693	7.266	4.628	.6439
								-4.3	-1.3	5.9	.04	.02			.021	.006	.026	.007
128	6.130	192.32	44.51	1.67	.67	2.49	28.35	.0247	.0080	.0289	.0011	.0020	3.77	.3253	1.1709	7.266	4.628	.6439
								-2.3	-1.5	6.3	.04	.02			.027	.012	.031	.008
RUN	T	XLO	XL1	H1	H3	FH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
129	6.130	192.32	44.51	1.67	.67	2.49	28.35	.0425	.0084	.0511	.0019	.0036	3.77	.1966	1.2029	7.266	4.628	.6439
								-2.4	-1.2	6.3	.04	.02			.043	.007	.053	.010
130	6.946	246.93	50.54	1.67	.67	2.49	32.15	.0499	.0125	.0594	.0020	.0037	3.32	.2503	1.1892	7.281	4.632	.6415
								-5.7	.1	4.3	.03	.02			.050	.010	.061	.010
131	6.946	246.93	50.54	1.67	.67	2.49	32.15	.0259	.0116	.0289	.0010	.0018	3.32	.4467	1.1165	7.281	4.632	.6415
								-5.1	-.5	4.2	.03	.02			.030	.018	.031	.008
132	6.946	246.93	50.54	1.67	.67	2.49	32.15	.0314	.0113	.0363	.0012	.0023	3.32	.3599	1.1572	7.281	4.632	.6415
								-4.7	-.1	2.8	.03	.02			.031	.008	.038	.008
133	7.548	291.56	54.98	1.67	.67	2.49	34.95	.0217	.0087	.0244	.0008	.0014	3.05	.4002	1.1238	7.289	4.634	.6403
								-1.1	-.4	3.0	.03	.02			.021	.006	.025	.004
RUN	T	XLO	XL1	H1	H3	FH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
134	7.548	291.56	54.98	1.67	.67	2.49	34.95	.0213	.0085	.0246	.0008	.0014	3.05	.3972	1.1548	7.289	4.634	.6403
								-2.3	1.0	3.4	.03	.02			.026	.016	.027	.008
135	7.548	291.56	54.98	1.67	.67	2.49	34.95	.0380	.0126	.0439	.0014	.0025	3.05	.3315	1.1554	7.289	4.634	.6403
								.5	-1.3	3.2	.03	.02			.039	.012	.047	.012
136	7.548	291.56	54.98	1.67	.67	2.49	34.95	.0468	.0129	.0561	.0017	.0032	3.05	.2765	1.1989	7.289	4.634	.6403
								-2.5	1.1	3.4	.03	.02			.052	.024	.059	.013

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
137	11.226	644.98	73.26	1.33	.33	4.03	36.55	.0105	.0013	.0130	.0003	.0007	1.83	.1271	1.2434	6.530	3.258	.5006
								-2.4	-1.3	7.8	.02	.01			.011	.003	.013	.002
138	11.226	644.98	73.26	1.33	.33	4.03	36.55	.0133	.0015	.0165	.0004	.0009	1.83	.1151	1.2389	6.530	3.258	.5006
								-1.9	-.5	7.6	.02	.01			.014	.004	.017	.003

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
139	11.226	644.98	73.26	1.33	.33	4.03	36.55	.0217	.0031	.0250	.0006	.0014	1.83	.1417	1.1528	6.530	3.258	.5006
								-2.2	-.6	8.2	.02	.01			.023	.007	.026	.005
140	11.226	644.98	73.26	1.33	.33	4.03	36.55	.0507	.0086	.0611	.0014	.0033	1.83	.1693	1.2033	6.530	3.258	.5006
								-2.1	.1	9.2	.02	.01			.052	.012	.062	.009
141	8.726	389.68	56.86	1.33	.33	4.03	28.40	.0265	.0148	.0240	.0009	.0017	2.35	.5587	.9052	6.521	3.257	.5022
								-.0	-1.6	7.2	.02	.01			.024	.007	.027	.009
142	8.726	389.68	56.86	1.33	.33	4.03	28.40	.0327	.0016	.0340	.0011	.0024	2.35	.0475	1.0408	6.521	3.257	.5022
								-1.3	-.4	7.3	.02	.01			.035	.009	.036	.009
143	8.726	389.68	56.86	1.33	.33	4.03	28.40	.0546	.0165	.0578	.0019	.0041	2.35	.3030	1.0587	6.521	3.257	.5022
								-2.5	.4	7.8	.02	.01			.055	.013	.061	.014

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
144	7.300	272.73	47.50	1.33	.33	4.03	23.75	.0187	.0009	.0230	.0008	.0019	2.82	.0502	1.2329	6.511	3.256	.5039
								-4.0	.4	4.5	.03	.01			.020	.006	.025	.007
145	7.300	272.73	47.50	1.33	.33	4.03	23.75	.0288	.0082	.0381	.0012	.0032	2.82	.2839	1.3213	6.511	3.256	.5039
								-4.4	.7	4.6	.03	.01			.031	.013	.039	.008
146	7.300	272.73	47.50	1.33	.33	4.03	23.75	.0480	.0074	.0566	.0020	.0048	2.82	.1536	1.1795	6.511	3.256	.5039
								-3.6	.6	4.6	.03	.01			.050	.014	.058	.009
147	5.033	129.65	34.53	1.50	.50	3.00	20.10	.0243	.0096	.0260	.0014	.0026	4.37	.3972	1.0704	6.866	3.996	.5916
								-5.9	-.0	4.7	.04	.02			.025	.009	.028	.008
148	5.033	129.65	34.53	1.50	.50	3.00	20.10	.0309	.0119	.0332	.0018	.0033	4.37	.3838	1.0723	6.866	3.996	.5916

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RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									-6.4	.4	5.3	.04	.02		.031	.010	.035	.008
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
149	5.033	129.65	34.53	1.50	.50	3.00	20.10	.0477	.0151	.0525	.0028	.0052	4.37	.3161	1.1002	6.806	3.996	.5916
									-6.6	.6	5.8	.04	.02		.048	.014	.054	.009
150	10.674	583.07	67.52	1.25	.25	5.00	30.25	.0111	.0035	.0129	.0003	.0009	1.86	.3165	1.1618	6.330	2.836	.4496
									-2.5	-.7	5.9	.02	.01		.011	.003	.013	.001
151	10.674	583.07	67.52	1.25	.25	5.00	30.25	.0145	.0035	.0178	.0004	.0012	1.86	.2395	1.2281	6.330	2.836	.4496
									-1.9	-1.6	6.3	.02	.01		.015	.005	.018	.002
152	10.674	583.07	67.52	1.25	.25	5.00	30.25	.0216	.0018	.0287	.0006	.0019	1.86	.0844	1.3236	6.330	2.836	.4496
									-1.8	-1.5	6.5	.02	.01		.022	.004	.029	.003
153	10.674	583.07	67.52	1.25	.25	5.00	30.25	.0278	.0065	.0339	.0008	.0022	1.86	.2320	1.2178	6.330	2.836	.4496
									-2.3	-1.0	6.5	.02	.01		.028	.007	.034	.002
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
154	9.546	466.34	60.35	1.25	.25	5.00	27.05	.0123	.0019	.0157	.0004	.0012	2.08	.1533	1.2808	6.326	2.836	.4502
									-3.4	-1.0	7.9	.02	.01		.012	.003	.016	.002
155	9.546	466.34	60.35	1.25	.25	5.00	27.05	.0160	.0053	.0188	.0005	.0014	2.08	.3305	1.1759	6.326	2.836	.4502
									-2.7	-.9	7.3	.02	.01		.016	.006	.019	.002
156	9.546	466.34	60.35	1.25	.25	5.00	27.05	.0256	.0066	.0319	.0008	.0024	2.08	.2573	1.2475	6.326	2.836	.4502
									-2.5	-1.2	8.0	.02	.01		.026	.008	.032	.002
157	7.360	277.28	46.45	1.25	.25	5.00	20.85	.0242	.0024	.0222	.0010	.0021	2.71	.1008	.9188	6.314	2.835	.4523
									-5.5	1.2	6.5	.03	.01		.026	.008	.023	.005
158	7.360	277.28	46.45	1.25	.25	5.00	20.85	.0266	.0044	.0329	.0011	.0032	2.71	.1654	1.2340	6.314	2.835	.4523
									-5.6	.9	6.3	.03	.01		.027	.006	.033	.002
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4

159	7.360	277.28	46.45	1.25	.25	5.00	20.85	.0357	.0128	.0382	.0017	.0037	2.71	.3227	.9627	6.314	2.835	.4523
									-5.2	.9	3.7	.03	.01		.039	.010	.038	.004
160	12.172	758.26	77.04	1.25	.25	5.00	34.50	.0089	.0002	.0123	.0002	.0007	1.63	.0229	1.3871	6.333	2.836	.4491
									1.4	-.4	4.7	.02	.01		.009	.001	.012	.002
161	12.172	758.26	77.04	1.25	.25	5.00	34.50	.0152	.0041	.0188	.0004	.0011	1.63	.2729	1.2368	6.333	2.836	.4491
									-2.6	-.2	4.6	.02	.01		.015	.003	.019	.002

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*M4013-3529,FMS,RESULT,1,5,500,500,    MCURODIMOS

LIBRARY ENTRY POINTS,
     .SETUP      (CSHM)      (RTN)       (SPHM)      (FIL)       SQRT        COS         SIN         ATAN        EXP(2

      NAME ORIGIN ENTRY         NAME ORIGIN ENTRY         NAME ORIGIN ENTRY         NAME ORIGIN ENTRY         NAME ORIGIN ENTRY
      MAIN   00144 00156        AKEFQ  01616 01623        .SETUP 02075 02102        (RCPM) 02113 03412        FTNBP  02113 02140
     (F2EF)  02113 02303        FTNPM  02113 02141        (F2PM) 02113 02130        (FPT)  07605 07614        (FRM7) 10103 10410
     RSTRTN  10103 10374        TIMLFT 10103 10160        KILLTR 10103 10342        STOPCL 10103 10202        RSCLCK 10103 10175
     JOBTM   10103 10142        TIMER  10103 10220        (TIME) 10103 10106        ENDJOB 10477 10563        CLKOUT 10477 10531
     EXITM   10477 10505        EXIT   10477 10531        .LOOK  10616 11031        .SCRDS 10616 11033        .READL 10616 10671
     .READ   10616 10671        .TAPRD 10616 10666        (TSHM) 10616 10635        (CSHM) 10616 10634        (TSH)  10616 10645
     (CSH)   10616 10660        IOHSIZ 11341 14703        (RTN)  11341 14553        (FIL)  11341 14536        STQUO  11341 11601
     (IOH)   11341 11604        .03311 16214 16216        .03310 16214 16216        SFDP   16233 16316        OFDP   16233 16322
     DCEXIT  16233 16376        DFMP   16233 16270        DFSB   16233 16253        DFAD   16233 16236        (TCO)  16406 16512
     (TEF)   16406 16511        (RCH)  16406 16510        (ETT)  16406 16507        (REW)  16406 16506        (WEF)  16406 16505
     (BSR)   16406 16504        (WRS)  16406 16503        (PDS)  16406 16502        (IOS)  16406 16413        (TRC)  16406 16513
     (EXE)   16551 16560        (IOU)  17411 17416        (TES)  17433 17435        RECOUP 17436 17441        .SPRNT 17444 17670
     .PRINT  17444 17550        .TAPWR 17444 17543        .PUNCH 17444 17524        (SCH)  17444 17471        (STHM) 17444 17507
     (STHM)  17444 17457        (STH)  17444 17460        (SPHM) 17444 17456        (SPH)  17444 17515        (PRNT) 17444 20067
     (SCHM)  17444 17466        .FOUT  17444 20404        .CLOUT 17444 20401        .COMNT 17444 17550        .PNCHL 17444 17524
     ERRCR   20644 20650        (WTC)  21040 21127        (WER)  21040 21054        (BST)  21164 21175        (RDPM) 21224 21320
     (RDC)   21224 21303        (RER)  21224 21237        SQR    21327 21333        SQRT   21327 21333        EXP(2  21437 21443
     LDUMP   21573 21576        ATN    21602 21604        ATAN   21602 21604        SIN    21731 21744        COS    21731 21733
     MOVIE)  22123 22123
PROGRAM LENGTH =  22500. LOWEST COMMON =  77461

       .87 MINUTES ELAPSED SINCE START OF JOB

      EXECUTION
THE FOLLOWING RUNS HAVE BED SLOPE =  .125000 AND SIDEWALL SLOPE =  .052000
RUN    T     XLO    XL1     H1     H3   HH13   XL3    A1P    A2P    A3P   STEPU  STEPD    Z1      XKR     XKT    CELAU  CELAD  CGRAT
                                                    DELT1  DELT2  DELT4   HL11   HL33                     A1      A2     A3     A4
  1  2.446  30.63  17.25  1.75   .75  2.33  11.71  .0360  .0134  .0542  .0042  .0093  10.20   .3711  1.5055  7.056  4.788  .3635

                                                    -9.0    -.7    2.8    .10    .06                      .038   .017   .056   .010

 20  2.446  30.63  17.25  1.75   .75  2.33  11.71  .0551  .0249  .0800  .0064  .0137  10.20   .4511  1.4521  7.056  4.788  .3635

                                                    -8.3   -1.2    4.1    .10    .06                      .053   .019   .081   .009

 30  1.401  10.05   8.60  1.75   .75  2.33   6.34  .0863  .0270  .1193  .0201  .0376  20.46   .3127  1.3834  6.143  4.528  .4502

                                                   -23.0     .4    2.5    .20    .12                      .085   .019   .122   .018

 40  1.401  10.05   8.60  1.75   .75  2.33   6.34  .1033  .0239  .1436  .0240  .0453  20.46   .2316  1.3908  6.143  4.528  .4502

                                                   -23.9    1.0    1.0    .20    .12                      .104   .025   .145   .014

 50  1.056   5.71   5.50  1.75   .75  2.33   4.47  .0462  .0110  .0594  .0168  .0266  31.99   .2386  1.2359  5.213  4.236  .5385

                                                   -32.1   -1.5   10.6    .32    .17                      .047   .012   .061   .010

RUN    T     XLO    XL1     H1     H3   HH13   XL3    A1P    A2P    A3P   STEPU  STEPD    Z1      XKR     XKT    CELAU  CELAD  CGRAT
                                                    DELT1  DELT2  DELT4   HL11   HL33                     A1      A2     A3     A4
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8C	1.056	5.71	5.50	1.75	.75	2.33	4.47	.1254	.0164	.1582	.0456	.0708	31.99	.1308	1.2621	5.213	4.236	.5385
								-33.8	.8	32.4	.32	.17			.125	.010	.159	.011
9C	1.621	13.45	10.50	1.75	.75	2.33	7.49	.0545	.0154	.0787	.0104	.0210	16.76	.2826	1.4461	6.482	4.626	.4164
								-20.1	1.6	.9	.17	.10			.054	.012	.080	.010
1C	1.621	13.45	10.50	1.75	.75	2.33	7.49	.0263	.0108	.0367	.0050	.0098	16.76	.4102	1.3964	6.482	4.626	.4164
								-19.8	-.5	8.5	.17	.10			.026	.009	.038	.007
11	1.621	13.45	10.50	1.75	.75	2.33	7.49	.0659	.0059	.0988	.0126	.0264	16.76	.0889	1.4986	6.482	4.626	.4164
								-20.5	.6	8.5	.17	.10			.067	.006	.102	.018
12	1.621	13.45	10.50	1.75	.75	2.33	7.49	.0906	.0172	.1331	.0173	.0355	16.76	.1897	1.4690	6.482	4.626	.4164
								-20.7	.7	-.3	.17	.10			.090	.004	.136	.020
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P DELT1	A2P DELT2	A3P DELT4	STEPU HL11	STEPD HL33	Z1	XKR	XKT A1	CELAU A2	CELAD A3	CGRAT A4
13	1.823	17.02	12.20	1.75	.75	2.33	8.54	.0120	.0074	.0150	.0020	.0035	14.42	.6138	1.2483	6.695	4.687	.3960
								-15.6	-.8	-1.6	.14	.09			.011	.005	.016	.004
14	1.823	17.02	12.20	1.75	.75	2.33	8.54	.0462	.0168	.0671	.0076	.0157	14.42	.3642	1.4500	6.695	4.687	.3960
								-17.7	1.2	2.7	.14	.09			.045	.012	.068	.008
15	1.823	17.02	12.20	1.75	.75	2.33	8.54	.0557	.0158	.0840	.0091	.0197	14.42	2840	1.5080	6.695	4.687	.3960
								-14.7	-.3	2.7	.14	.09			.057	.019	.085	.009
16	1.823	17.02	12.20	1.75	.75	2.33	8.54	.0740	.0237	.1095	.0121	.0256	14.42	.3207	1.4790	6.695	4.687	.3960
								-14.6	-.9	3.0	.14	.09			.073	.016	.113	.020
17	2.296	26.98	16.05	1.75	.75	2.33	10.95	.0609	.0225	.0925	.0076	.0169	10.96	.3699	1.5180	6.995	4.771	.3688
								-15.2	1.2	.7	.11	.07			.059	.015	.094	.012
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P DELT1	A2P DELT2	A3P DELT4	STEPU HL11	STEPD HL33	Z1	XKR	XKT A1	CELAU A2	CELAD A3	CGRAT A4
18	2.296	26.98	16.05	1.75	.75	2.33	10.95	.0493	.0210	.0721	.0061	.0132	10.96	.4255	1.4629	6.995	4.771	.3688
								-14.9	.8	7.1	.11	.07			.048	.017	.073	.008
19	2.296	26.98	16.05	1.75	.75	2.33	10.95	.0308	.0161	.0421	.0038	.0077	10.96	.5237	1.3695	6.995	4.771	.3688

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RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
								-15.0	.1	7.9	.11	.07			.035	.019	.053	.024
20	2.296	26.98	16.05	1.75	.75	2.33	10.95	.0224	.0121	.0292	.0028	.0053	10.96	.5425	1.3042	6.995	4.771	.3688
								-15.1	1.1	1.9	.11	.07			.022	.011	.030	.005
21	2.858	41.82	20.50	1.75	.75	2.33	13.77	.0141	.0070	.0202	.0014	.0029	8.58	.4955	1.4337	7.177	4.822	.3533
								-6.3	-1.0	5.1	.09	.05			.016	.010	.021	.004
22	2.858	41.82	20.50	1.75	.75	2.33	13.77	.0318	.0120	.0493	.0031	.0072	8.58	.3778	1.5493	7.177	4.822	.3533
								-9.3	.0	.7	.09	.05			.031	.009	.050	.006
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
23	2.858	41.82	20.50	1.75	.75	2.33	13.77	.0405	.0152	.0620	.0040	.0090	8.58	.3753	1.5299	7.177	4.822	.3533
								-9.3	-.5	7.0	.09	.05			.040	.011	.065	.014
24	2.858	41.82	20.50	1.75	.75	2.33	13.77	.0511	.0178	.0789	.0050	.0115	8.58	.3476	1.5444	7.177	4.822	.3533
								-9.2	.0	7.0	.09	.05			.050	.012	.081	.013
25	2.886	42.62	22.00	2.00	1.00	2.00	15.96	.0307	.0149	.0427	.0028	.0053	9.14	.4857	1.3883	7.629	5.535	.3811
								-9.1	-1.3	.8	.09	.06			.036	.022	.048	.016
26	2.886	42.62	22.00	2.00	1.00	2.00	15.96	.0213	.0147	.0244	.0019	.0031	9.14	.6912	1.1459	7.629	5.535	.3811
								-9.3	-.9	.6	.09	.06			.021	.013	.028	.010
27	2.886	42.62	22.00	2.00	1.00	2.00	15.96	.0467	.0181	.0691	.0042	.0087	9.14	.3876	1.4793	7.629	5.535	.3811
								-9.1	-1.3	-.0	.09	.06			.047	.014	.075	.021
RLN	T	XLO	XL1	H1	H3	HH13	XL3	A1P / DELT1	A2P / DELT2	A3P / DELT4	STEPU / HL11	STEPD / HL33	Z1	XKR	XKT / A1	CELAU / A2	CELAD / A3	CGRAT / A4
28	2.886	42.62	22.00	2.00	1.00	2.00	15.96	.0534	.0148	.0814	.0049	.0102	9.14	.2767	1.5243	7.629	5.535	.3811
								-11.7	.8	-.4	.09	.06			.060	.028	.087	.022
29	2.252	25.97	16.60	2.00	1.00	2.00	12.26	.0428	.0189	.0591	.0052	.0096	12.11	.4410	1.3813	7.375	5.445	.4004
								-19.2	1.5	2.6	.12	.08			.049	.027	.067	.023
30	2.252	25.97	16.60	2.00	1.00	2.00	12.26	.0543	.0172	.0786	.0065	.0128	12.11	.3174	1.4491	7.375	5.445	.4004
								-13.8	1.2	2.6	.12	.08			.065	.036	.087	.027

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RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT	DELT1	DELT2	DELT4	HL11	HL33	A1	A2	A3	A4
30	2.252	25.97	16.60	2.00	1.00	2.00	12.26	.0555	.0178	.0300	.0067	.0131	12.11	.3213	1.4425	7.375	5.445	.4004	-13.8	1.2	2.6	.12	.08	.068	.039	.090	.030
30	2.252	25.97	16.60	2.00	1.00	2.00	12.26	.0571	.0182	.0827	.0069	.0135	12.11	.3185	1.4468	7.375	5.445	.4004	-13.8	1.2	2.6	.12	.08	.070	.040	.093	.031
31	2.252	25.97	16.60	2.00	1.00	2.00	12.26	.0237	.0130	.0311	.0029	.0051	12.11	.5478	1.3081	7.375	5.445	.4004	-9.8	-1.3	-2.7	.12	.08	.024	.012	.034	.010
32	2.252	25.97	16.60	2.00	1.00	2.00	12.26	.0912	.0236	.1351	.0110	.0221	12.11	.2584	1.4812	7.375	5.445	.4004	-14.1	.4	2.6	.12	.08	.094	.031	.137	.016
33	1.689	14.60	11.60	2.00	1.00	2.00	8.89	.0334	.0083	.0477	.0058	.0107	17.33	.2493	1.4270	6.872	5.266	.4412	-23.7	.1	4.9	.17	.11	.038	.019	.051	.013
34	1.689	14.60	11.60	2.00	1.00	2.00	8.89	.0711	.0294	.0954	.0123	.0215	17.33	.4142	1.3419	6.872	5.266	.4412	-17.7	1.1	4.9	.17	.11	.072	.029	.099	.019
35	1.689	14.60	11.60	2.00	1.00	2.00	8.89	.1120	.0394	.1533	.0193	.0345	17.33	.3521	1.3690	6.872	5.266	.4412	-17.9	-.2	-3.6	.17	.11	.109	.022	.159	.030
36	1.689	14.60	11.60	2.00	1.00	2.00	8.89	.1279	.0416	.1737	.0220	.0391	17.33	.3255	1.3586	6.872	5.266	.4412	-17.5	-.8	4.2	.17	.11	.136	.060	.177	.024
37	1.384	9.80	8.75	2.00	1.00	2.00	7.00	.0450	.0181	.0573	.0103	.0164	22.98	.4026	1.2731	6.328	5.066	.4871	-26.2	.2	1.3	.23	.14	.049	.026	.059	.010
38	1.384	9.80	8.75	2.00	1.00	2.00	7.00	.0787	.0225	.1046	.0180	.0299	22.98	.2854	1.3280	6.328	5.066	.4871	-33.5	.4	-4.9	.23	.14	.081	.025	.109	.022
39	1.384	9.80	8.75	2.00	1.00	2.00	7.00	.1076	.0430	.1349	.0246	.0385	22.98	.3996	1.2531	6.328	5.066	.4871	-33.9	.4	-5.8	.23	.14	.104	.028	.139	.024

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RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
40	1.384	9.80	8.75	2.00	1.00	2.00	7.00	.1231	.0212	.1651	.0281	.0471	22.98	.1725	1.3405	5.328	5.066	.4971
									-33.6	-1.0	6.3	.23	.14		.124	.015	.170	.029
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
41	.903	4.17	4.15	2.00	1.00	2.00	3.86	.0627	.0152	.0762	.0302	.0395	48.45	.2418	1.2145	4.601	4.279	.5659
									-117.6	1.2	9.7	.48	.26		.062	.010	.077	.008
43	.992	5.04	5.00	2.25	1.25	1.80	4.69	.0717	.0195	.0863	.0287	.0368	45.24	.2713	1.2034	5.044	4.734	.5579
									-48.9	-1.0	5.1	.45	.27		.073	.014	.093	.025
44	.992	5.04	5.00	2.25	1.25	1.80	4.69	.0919	.0313	.1077	.0368	.0459	45.24	.3409	1.1722	5.044	4.734	.5579
									-54.7	-1.2	25.8	.45	.27		.090	.008	.116	.031
45	.992	5.04	5.00	2.25	1.25	1.80	4.69	.1063	.0152	.1297	.0425	.0553	45.24	.1430	1.2204	5.044	4.734	.5579
									-51.6	1.4	2.3	.45	.27		.118	.036	.148	.052
46	1.385	9.81	9.00	2.25	1.25	1.80	7.61	.0900	.0258	.1162	.0200	.0305	25.13	.2863	1.2901	6.504	5.496	.5089
									-30.7	1.2	-7.1	.25	.16		.091	.020	.123	.029
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
47	1.385	9.81	9.00	2.25	1.25	1.80	7.61	.0576	.0290	.0674	.0128	.0177	25.13	.5031	1.1708	6.504	5.496	.5089
									-23.7	-1.2	-7.1	.25	.16		.056	.023	.071	.016
48	1.385	9.81	9.00	2.25	1.25	1.80	7.61	.1224	.0334	.1575	.0272	.0414	25.13	.2729	1.2866	6.504	5.496	.5089
									-24.5	-.6	-6.3	.25	.16		.124	.032	.163	.030
49	1.385	9.81	9.00	2.25	1.25	1.80	7.61	.1651	.0378	.2116	.0367	.0556	25.13	.2291	1.2816	6.504	5.496	.5089
									-28.3	1.1	.7	.25	.16		.163	.018	.215	.027
50	1.786	16.33	13.00	2.25	1.25	1.80	10.41	.0348	.0146	.0459	.0054	.0089	17.40	.4204	1.3183	7.282	5.834	.4542
									-17.9	-.4	-2.0	.17	.12		.038	.020	.048	.010
51	1.786	16.33	13.00	2.25	1.25	1.80	10.41	.0645	.0275	.0839	.0099	.0161	17.40	.4260	1.3016	7.282	5.334	.4542
									-18.1	-.3	-2.7	.17	.12		.064	.023	.088	.019
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
52	1.786	16.33	13.00	2.25	1.25	1.80	10.41	.0945	.0425	.1219	.0145	.0234	17.40	.4504	1.2900	7.282	5.834	.4542
									-20.1	.9	-2.9	.17	.12		.097	.044	.128	.028
53	1.786	16.33	13.00	2.25	1.25	1.80	10.41	.1194	.0299	.1632	.0184	.0313	17.40	.2507	1.3663	7.282	5.834	.4542
									-17.3	-1.3	-2.9	.17	.12		.133	.060	.172	.039
54	2.332	27.83	18.15	2.25	1.25	1.80	14.09	.0479	.0225	.0642	.0053	.0091	12.46	.4705	1.3407	7.788	6.045	.4186
									-11.4	1.1	-.6	.12	.09		.054	.032	.068	.016
55	2.332	27.83	18.15	2.25	1.25	1.80	14.09	.0277	.0130	.0375	.0030	.0053	12.46	.4685	1.3557	7.788	6.045	.4186
									-17.4	1.0	-1.1	.12	.09		.031	.018	.040	.010
56	2.332	27.83	18.15	2.25	1.25	1.80	14.09	.0754	.0195	.1095	.0083	.0155	12.46	.2586	1.4533	7.788	6.045	.4186
									-13.9	-1.5	-.6	.12	.09		.087	.039	.125	.044
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
57	2.332	27.83	18.15	2.25	1.25	1.80	14.09	.1111	.0298	.1597	.0122	.0227	12.46	.2679	1.4375	7.788	6.045	.4186
									-11.3	.5	-1.1	.12	.09		.127	.061	.173	.048
58	2.832	41.04	22.70	2.25	1.25	1.80	17.38	.0584	.0151	.0866	.0051	.0100	9.96	.2589	1.4823	8.021	6.141	.4031
									-12.0	.9	.2	.10	.07		.070	.037	.096	.030
59	2.832	41.04	22.70	2.25	1.25	1.80	17.38	.0557	.0410	.0584	.0049	.0067	9.96	.7358	1.0494	8.021	6.141	.4031
									-10.1	-1.0	.4	.10	.07		.048	.025	.069	.027
60	2.832	41.04	22.70	2.25	1.25	1.80	17.38	.0349	.0230	.0408	.0031	.0047	9.96	.6581	1.1705	8.021	6.141	.4031
									-10.1	-.8	1.0	.10	.07		.032	.014	.049	.020
61	2.832	41.04	22.70	2.25	1.25	1.80	17.38	.0186	.0113	.0235	.0016	.0027	9.96	.6065	1.2607	8.021	6.141	.4031
									-9.4	-1.0	.8	.10	.07		.019	.009	.030	.014
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
62	2.997	45.96	20.10	1.50	.50	3.00	11.88	.0187	.0063	.0312	.0019	.0053	7.50	.3371	1.6661	6.712	3.967	.3093
									-6.1	-.2	6.3	.07	.04		.020	.009	.032	.005

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RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
63	2.997	45.96	20.10	1.50	.50	3.00	11.88	.0305	.0105	.0524	.0030	.0088	7.50	.3431	1.7152	6.712	3.967	.3093
								-6.1	-.1	19.3	.07	.04			.033	.015	.055	.012
64	2.997	45.96	20.10	1.50	.50	3.00	11.88	.0355	.0122	.0595	.0035	.0100	7.50	.3432	1.6750	6.712	3.967	.3093
								-5.9	-.8	18.0	.07	.04			.041	.022	.064	.017
65	2.997	45.96	20.10	1.50	.50	3.00	11.88	.0165	.0112	.0214	.0016	.0036	7.50	.6812	1.3015	6.712	3.967	.3093
								-9.4	-.4	7.4	.07	.04			.016	.010	.023	.006
66	1.957	19.61	12.50	1.50	.50	3.00	7.64	.0311	.0126	.0487	.0050	.0128	12.06	.4053	1.5643	6.391	3.905	.3403
								-16.5	-1.0	21.7	.12	.07			.034	.018	.050	.008
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
67	1.891	18.30	12.00	1.50	.50	3.00	7.36	.0208	.0089	.0319	.0035	.0087	12.57	.4288	1.5365	6.351	3.897	.3444
								-13.6	-.0	15.9	.13	.07			.020	.006	.033	.006
68	1.957	19.61	12.50	1.50	.50	3.00	7.64	.0510	.0239	.0764	.0082	.0200	12.06	.4677	1.4976	6.391	3.905	.3403
								-13.1	.4	9.7	.12	.07			.049	.018	.078	.011
69	1.957	19.61	12.50	1.50	.50	3.00	7.64	.0625	.0178	.1005	.0100	.0263	12.06	.2855	1.6075	6.391	3.905	.3403
								-13.8	1.6	7.7	.12	.07			.063	.017	.103	.016
70	2.430	30.23	16.00	1.50	.50	3.00	9.58	.0267	.0168	.0359	.0033	.0075	9.42	.6304	1.3421	6.587	3.943	.3209
								-11.1	-1.2	12.0	.09	.05			.025	.013	.038	.009
71	2.430	30.23	16.00	1.50	.50	3.00	9.58	.0409	.0207	.0614	.0051	.0128	9.42	.5066	1.5013	6.587	3.943	.3209
								-11.2	-1.5	24.1	.09	.05			.040	.018	.063	.010
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
72	2.430	30.23	16.00	1.50	.50	3.00	9.58	.0559	.0178	.0921	.0070	.0192	9.42	.3175	1.6469	6.587	3.943	.3209
								-11.8	-.8	9.7	.09	.05			.057	.016	.098	.024
73	1.431	10.48	8.45	1.50	.50	3.00	5.45	.0335	.0100	.0494	.0079	.0181	17.85	.2993	1.4740	5.908	3.811	.3929
								-21.9	-.9	7.2	.18	.09			.034	.010	.051	.009
74	1.431	10.48	8.45	1.50	.50	3.00	5.45	.0509	.0187	.0731	.0120	.0268	17.85	.3682	1.4365	5.908	3.811	.3929

Va-δ

RLN	T	XLO	XL1	H1	H3	FH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
								-20.0	-.6	.3	.18	.09			.055	.027	.075	.012
75	1.431	10.48	8.45	1.50	.50	3.00	5.45	.0743	.0310	.1040	.0176	.0382	17.85	.4171	1.4005	5.908	3.811	.3929
								-19.1	-1.3	31.7	.18	.09			.075	.032	.105	.010
76	1.077	5.93	5.55	1.50	.50	3.00	3.94	.1187	.0137	.1564	.0428	.0795	27.17	.1150	1.3173	5.157	3.657	.4835
								-28.7	-.4	40.7	.27	.13			.120	.021	.157	.010
RLN	T	XLO	XL1	H1	H3	FH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
77	1.077	5.93	5.55	1.50	.50	3.00	3.94	.0836	.0140	.1117	.0301	.0568	27.17	.1675	1.3368	5.157	3.657	.4835
								-37.6	1.2	13.5	.27	.13			.086	.023	.113	.012
78	1.077	5.93	5.55	1.50	.50	3.00	3.94	.0690	.0202	.0901	.0249	.0458	27.17	.2930	1.3052	5.157	3.657	.4835
								-30.3	1.2	11.1	.27	.13			.069	.019	.091	.009
79	1.077	5.93	5.55	1.50	.50	3.00	3.94	.0430	.0182	.0541	.0155	.0275	27.17	.4240	1.2594	5.157	3.657	.4835
								-29.0	-.7	5.2	.27	.13			.042	.015	.055	.007
80	3.438	60.49	28.10	2.25	1.25	1.80	21.32	.0663	.0364	.0874	.0047	.0082	8.05	.5482	1.3171	8.179	6.207	.3929
								-9.1	-.9	1.1	.08	.06			.070	.042	.089	.012
81	3.438	60.49	28.10	2.25	1.25	1.80	21.32	.0566	.0267	.0772	.0040	.0072	8.05	.4713	1.3631	8.179	6.207	.3929
								-11.4	.9	.5	.08	.06			.059	.031	.078	.008
RLN	T	XLO	XL1	H1	H3	FH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
82	3.438	60.49	28.10	2.25	1.25	1.80	21.32	.0928	.0419	.1279	.0066	.0120	8.05	.4521	1.3783	8.179	6.207	.3929
								-12.5	1.5	6.7	.08	.06			.095	.046	.129	.012

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                    *M4C13-3529,FMS,RESULT,1,5,50C,500,    RCUROCIMOS                                     Vb-1

LIBRARY ENTRY PCINTS,
     .SETUP       (CSHM)       (RTN)        (SPHM)        (FIL)        SQRT        COS         SIN         ATAN        EXP(2


      NAME ORIGIN ENTRY        NAME ORIGIN ENTRY        NAME ORIGIN ENTRY        NAME ORIGIN ENTRY        NAME ORIGIN ENTRY
      MAIN   CC144 00156       AKEFQ   01622 01627      .SETUP 02101 C2106       (RCPM) 02117 03416       FTNBP   02117 02144
     (F2EF) 02117 02307        FTNPM   02117 02145      (F2PM) 02117 02134       (FPT)  07611 07620       (FRM7) 10107 10414
     RSTRTN 1C107 1C400        TIMLFT  1C107 10164      KILLTR 1C107 10346       STOPCL 1C107 10206       RSCLCK 10107 10201
     JOBTM  1C107 10146        TIMER   1C107 10224      (TIME) 10107 10112       FNDJOB 10503 10567       CLKOUT 10503 10535
     EXITM  10503 10511        EXIT    10503 10535      .LOOK  10622 11035       .SCRDS 10622 11037       .READL 10622 10675
     .READ  10622 10675        .TAPRC  10622 10672      (TSHM) 10622 10641       (CSHM) 10622 10640       (TSH)  10622 10651
     (CSH)  10622 10664        ICHSIZ  11345 14707      (RTN)  11345 14557       (FIL)  11345 14542       STQUO  11345 11605
     (IOH)  11345 11610        .03311  16220 16222      .C3310 16220 16222       SFDP   16237 16322       DFDP   16237 16326
     DCEXIT 16237 16402        DFMP    16237 16274      DFSB   16237 16257       DFAD   16237 16242       (TCO)  16412 16516
     (TEF)  16412 16515        (RCH)   16412 16514      (ETT)  16412 16513       (REW)  16412 16512       (WEF)  16412 16511
     (BSR)  16412 16510        (WRS)   16412 16507      (RDS)  16412 16506       (IOS)  16412 16417       (TRC)  16412 16517
     (EXE)  16555 16564        (IOU)   17415 17422      (TES)  17437 17441       RECOUP 17442 17445       .SPRNT 17450 17674
     .PRINT 17450 17554        .TAPWR  17450 17547      .PUNCH 17450 17530       (SCH)  17450 17475       (STHD) 17450 17513
     (STHM) 17450 17463        (STH)   17450 17464      (SPHM) 17450 17462       (SPH)  17450 17521       (PRNT) 17450 20073
     (SCHM) 17450 17472        .FOUT   17450 20410      .CLOUT 17450 20405       .COMNT 17450 17554       .PNCHL 17450 17530
     ERRCR  20650 20654        (WTC)   21044 21133      (WER)  21044 21060       (BST)  21170 21201       (RDPM) 21230 21324
     (RDC)  21230 21307        (RER)   21230 21243      SQR    21333 21337       SQRT   21333 21337       EXP(2  21443 21447
     LDUMP  21577 21602        ATN     21606 2161C      ATAN   21606 21610       SIN    21735 21750       COS    21735 21737
     MOVIE) 22127 22127
PRCGRAM LENGTH =  22504. LOWEST COMMON =  77461

       .87 MINUTES ELAPSED SINCE START OF JOB

      EXECUTICN
THE FCLLCWING RUNS HAVE BED SLCPE =  .125000 AND SIDEWALL SLOPE =  .052000
RLN    T      XLO     XL1     H1     H3    HH13    XL3    A1P     A2P     A3P    STEPU  STEPD    Z1      XKR     XKT    CELAU  CELAD  CGRAT
                                                              DELT1  DELT2  DELT4   HL11   HL33                     A1     A2     A3     A4
 83  4.194   9C.00   34.73   2.25   1.25   1.80   26.2C  .0818  .0126  .1280  .0047  .0098   6.51    .1537  1.5643  8.288  6.252  .3861

                                                               -8.6    -.4    1.3    .06    .05                     .085   .022   .132   .023

 84  4.194   9C.00   34.73   2.25   1.25   1.80   26.20  .0559  .0160  .0834  .0032  .0064   6.51    .2864  1.4910  8.288  6.252  .3861

                                                               -9.2     .C     .6    .06    .05                     .055   .008   .086   .015

 85  4.194   9C.00   34.73   2.25   1.25   1.80   26.20  .0386  .0087  .0604  .0022  .0046   6.51    .2244  1.5627  8.288  6.252  .3861

                                                               -8.6    -.5    1.1    .06    .05                     .040   .012   .062   .010

 86  5.C90  132.58   42.52   2.25   1.25   1.80   31.95  .0363  .0112  .0552  .0017  .0035   5.32    .3086  1.5207  8.360  6.282  .3817

                                                               -6.0   -1.1    4.4    .05    .04                     .039   .017   .057   .010

 87  5.C90  132.58   42.52   2.25   1.25   1.80   31.95  .0326  .0122  .0467  .0015  .0029   5.32    .3757  1.4336  8.360  6.282  .3817

                                                               -5.9   -1.1    4.2    .05    .04                     .035   .017   .048   .008

RLN    T      XLO     XL1     H1     H3    HH13    XL3    A1P     A2P     A3P    STEPU  STEPD    Z1      XKR     XKT    CELAU  CELAD  CGRAT
```

V-2

RUN	T	XL0	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
88	5.090	132.58	42.52	2.25	1.25	1.80	31.95	.0334	.0142	.0484	.0016	.0030	5.32	.4257	1.4492	8.360	6.282	.3817
									-5.9	-1.0	7.5	.05	.04		.032	.009	.050	.009
89	5.090	132.58	42.52	2.25	1.25	1.80	31.95	.0541	.0121	.0837	.0025	.0052	5.32	.2238	1.5475	8.360	6.282	.3817
									-6.1	-.8	4.1	.05	.04		.055	.013	.086	.014
90	5.566	158.58	46.64	2.25	1.25	1.80	35.00	.0420	.0149	.0635	.0018	.0036	4.85	.3539	1.5108	9.385	6.292	.3802
									-6.2	-1.1	2.8	.05	.04		.041	.010	.065	.010
91	5.566	158.58	46.64	2.25	1.25	1.80	35.00	.0530	.0205	.0772	.0023	.0044	4.85	.3867	1.4552	8.385	6.292	.3802
									-6.5	-.5	2.7	.05	.04		.051	.011	.080	.015
92	5.566	158.58	46.64	2.25	1.25	1.80	35.00	.0339	.0162	.0470	.0015	.0027	4.85	.4788	1.3854	3.385	6.292	.3802
									-5.7	-.8	2.2	.05	.04		.032	.010	.049	.010
RUN	T	XL0	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
93	5.566	158.58	46.64	2.25	1.25	1.80	35.00	.0612	.0200	.0915	.0026	.0052	4.85	.327[illegible]	1.4938	8.385	6.292	.3802
									-5.7	-1.0	2.4	.05	.04		.060	.014	.093	.012
94	5.679	165.06	44.97	2.00	1.00	2.00	32.00	.0365	.0144	.0542	.0016	.0034	4.47	.3940	1.4869	7.923	5.638	.3604
									-6.2	-.5	5.0	.04	.03		.039	.019	.056	.010
95	5.679	165.06	44.97	2.00	1.00	2.00	32.00	.0281	.0141	.0398	.0013	.0025	4.47	.5001	1.4153	7.923	5.638	.3604
									-5.7	-.2	4.6	.04	.03		.030	.017	.041	.007
96	5.679	165.06	44.97	2.00	1.00	2.00	32.00	.0598	.0167	.0945	.0027	.0059	4.47	.2795	1.5801	7.923	5.638	.3604
									-5.6	-.1	7.1	.04	.03		.060	.012	.099	.021
97	4.979	126.86	39.27	2.00	1.00	2.00	28.00	.0431	.0013	.0693	.0022	.0049	5.12	.0291	1.6090	7.392	5.628	.3525
									-3.5	1.5	3.7	.05	.04		.045	.010	.072	.014
RUN	T	XL0	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
98	4.979	126.86	39.27	2.00	1.00	2.00	28.00	.0297	.0126	.0438	.0015	.0031	5.12	.4254	1.4779	7.392	5.628	.3625
									-5.5	-1.3	4.3	.05	.04		.034	.020	.046	.010

VI-3

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
99	4.979	126.86	39.27	2.00	1.00	2.00	28.00	.0476	.0108	.0759	.0024	.0054	5.12	.2276	1.5917	7.892	5.628	.3625
								-5.5	-1.5	3.5	.05	.04			.031	.019	.079	.016
10	4.979	126.86	39.27	2.00	1.00	2.00	28.00	.0586	.0131	.0910	.0030	.0065	5.12	.2237	1.5526	7.892	5.623	.3625
								-8.6	1.2	3.5	.05	.04			.065	.028	.096	.022
101	4.263	93.00	33.41	2.00	1.00	2.00	23.90	.0594	.0183	.0899	.0036	.0075	6.02	.3088	1.5139	7.844	5.611	.3658
								-5.2	-1.6	6.4	.06	.04			.058	.008	.093	.017
102	4.263	93.00	33.41	2.00	1.00	2.00	23.90	.0363	.0099	.0566	.0022	.0047	6.02	.2715	1.5564	7.844	5.611	.3658
								-12.4	.5	6.6	.06	.04			.040	.018	.059	.012
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
103	4.263	93.00	33.41	2.00	1.00	2.00	23.90	.3147	-.2137	.1301	.0188	.0109	6.02	-.6791	.4133	7.844	5.611	.3658
								-5.6	-1.5	6.4	.06	.04			.332	-.237	.132	.016
104	6.573	221.15	52.22	2.00	1.00	2.00	37.10	.0304	.0118	.0461	.0012	.0025	3.85	.3888	1.5138	7.949	5.648	.3586
								-5.9	-.6	7.0	.04	.03			.030	.008	.049	.012
105	6.573	221.15	52.22	2.00	1.00	2.00	37.10	.0253	.0176	.0337	.0010	.0018	3.65	.6968	1.3345	7.949	5.648	.3586
								-6.2	-.3	4.1	.04	.03			.024	.015	.036	.009
106	6.573	221.15	52.22	2.00	1.00	2.00	37.10	.0612	.0156	.0960	.0023	.0052	3.65	.2549	1.5686	7.949	5.648	.3586
								-4.8	-.5	4.2	.04	.03			.067	.029	.100	.020
107	6.216	197.75	45.15	1.67	.67	2.49	28.75	.0371	.0147	.0590	.0016	.0041	3.72	.3971	1.5923	7.268	4.628	.3218
								-5.2	.5	7.9	.04	.02			.036	.010	.061	.011
RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
108	6.216	197.75	45.15	1.67	.67	2.49	28.75	.0211	.0092	.0332	.0009	.0023	3.72	.4369	1.5712	7.263	4.628	.3218
								-4.8	.0	6.4	.04	.02			.021	.007	.036	.010
109	6.216	197.75	45.15	1.67	.67	2.49	28.75	.0572	.0198	.0922	.0025	.0064	3.72	.3458	1.6119	7.268	4.628	.3218
								-4.6	-.3	4.5	.04	.02			.061	.028	.094	.013
110	5.497	154.65	39.83	1.67	.67	2.49	25.40	.0274	.0121	.0439	.0014	.0035	4.22	.4432	1.6055	7.250	4.624	.3232

								-7.0	1.4	7.6	.04	.03			.029	.015	.045	.007
111	5.497	154.65	39.83	1.67	.67	2.49	25.40	.0339	.0148	.0254	.0017	.0020	4.22	.4374	.7496	7.250	4.624	.3232
								-4.5	-1.3	2.0	.04	.03			.034	.015	.026	.004
112	5.497	154.65	34.83	1.67	.67	2.49	25.40	.0497	.0122	.0820	.0025	.0065	4.22	.2459	1.6491	7.250	4.624	.3232
								-5.5	-.6	2.0	.04	.03			.049	.007	.083	.009

RUN	T	XLO	XL1	H1	H3	FH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
113	7.236	267.99	52.68	1.67	.67	2.49	33.50	.0347	.0119	.0567	.0013	.0034	3.19	.3421	1.6345	7.285	4.633	.3204
								-2.3	-1.2	6.0	.03	.02			.035	.010	.060	.014
114	7.236	267.99	52.68	1.67	.67	2.49	33.50	.0618	.0228	.0968	.0023	.0058	3.19	.3681	1.5665	7.285	4.633	.3204
								-2.7	.8	5.9	.03	.02			.060	.011	.102	.023
115	7.236	267.99	52.68	1.67	.67	2.49	33.50	.0501	.0143	.0837	.0019	.0050	3.19	.2844	1.6697	7.285	4.633	.3204
								-4.9	1.2	6.1	.03	.02			.055	.025	.087	.017
116	6.997	250.60	48.29	1.50	.50	3.00	28.00	.0293	.0136	.0470	.0012	.0034	3.12	.4635	1.6031	6.906	4.004	.2923
								-5.2	-.2	2.7	.03	.02			.028	.009	.049	.010
117	6.997	250.60	48.29	1.50	.50	3.00	28.00	.0192	.0076	.0319	.0008	.0023	3.12	.3984	1.6638	6.906	4.004	.2923
								-4.0	-1.2	8.6	.03	.02			.021	.011	.033	.006

RUN	T	XLO	XL1	H1	H3	FH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XKR	XKT	CELAU	CELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
118	6.997	250.60	48.29	1.50	.50	3.00	28.00	.0310	.0170	.0473	.0013	.0034	3.12	.5469	1.5274	6.906	4.004	.2923
								-5.4	.5	3.2	.03	.02			.029	.012	.049	.009
119	8.616	379.91	59.59	1.50	.50	3.00	34.50	.0076	.0047	.0107	.0003	.0006	2.53	.6214	1.4015	6.921	4.007	.2911
								-6.1	.9	4.8	.03	.01			.007	.003	.012	.004
120	8.616	379.91	59.59	1.50	.50	3.00	34.50	.0116	.0092	.0133	.0004	.0008	2.53	.7916	1.1447	6.921	4.007	.2911
								-.1	.3	5.0	.03	.01			.011	.008	.015	.005
121	8.616	379.91	59.59	1.50	.50	3.00	34.50	.0252	.0159	.0354	.0008	.0021	2.53	.6326	1.4057	6.921	4.007	.2911
								-	1.0	4.6	.03	.01			.023	.011	.038	.010

RLN	T	XLO	XL1	H1	H3	FH13	XL3	A1P DELT1	A2P DELT2	A3P DELT4	STEPU HL11	STEPD HL33	Z1	XKR	XKT A1	CELAU A2	CELAD A3	CGRAT A4
122	6.065	188.23	41.77	1.50	.50	3.00	24.25	.0167	.0123	.C202	.0008	.0017	3.61	.7322	1.2085	6.892	4.001	.2936
								-5.8	.8	2.2	.04	.02			.016	.011	.021	.004
123	6.065	188.23	41.77	1.50	.50	3.00	24.25	.0206	.0071	.0350	.0010	.0029	3.61	.3455	1.6999	6.892	4.001	.2936
								-3.8	-1.2	2.1	.04	.02			.022	.010	.036	.006
124	6.065	188.23	41.77	1.50	.50	3.00	24.25	.0356	.0159	.0576	.0017	.0048	3.61	.4481	1.6203	6.392	4.001	.2936
								-3.7	.5	7.7	.04	.02			.034	.011	.059	.009

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•M4013-3529,FMS,RESULT,1,1,500,500,    BCLRCCIMCS
•      XEQ                                                                .C4

JCB TIME =     .C6 MIN.

LIBRARY ENTRY POINTS,
     .SETLF      (CSHM)       (RTN)        (SFHM)       (FIL)        SCRT         COS          SIN          ATAN         EXP(2

     NAME ORIGIN ENTRY      NAME ORIGIN ENTRY      NAME ORIGIN ENTRY      NAME ORIGIN ENTRY      NAME ORIGIN ENTRY
     MAIN   00144 C0156     AKEFQ  C1616 C1623     .SETLP C2075 C21C2     (RORM) 02113 C3412     FTNBP  C2113 C2140
     (F2EF) 02113 C2303     FTNPM  C2113 C2141     (F2PM) 02113 C2130     (FPT)  07605 C7614     (FRW7) 1C1C3 1C410
     RSTRTN 10103 1C274     TIMLFT 1C1C3 1C16C     KILLTR 1C1C3 10342     STORCL 10103 1C2C2     RSCUCK 1C1C3 1C175
     JCBTM  101C3 1C142     TIMER  1C1C3 1C22C     (TIME) 10103 10106     ENDJOB 10477 1C563     CLHOLT 1C477 1C531
     EXITM  10477 1C5C5     EXIT   1C477 1C531     .LCCK  10616 11031     .SORDS 10616 11C33     .REACL 1C616 1C671
     .REAC  10616 1C671     .TAPRD 1C616 1C666     (TSHM) 1C616 1C635     (CSHM) 10616 1C634     (TSH)  1C616 1C645
     (CSH)  10616 1066C     IOHSIZ 11341 147C3     (RTN)  11341 14553     (FIU)  11341 14536     ETQCC  11341 11601
     (ICH)  11341 11604     .C3311 16214 16216     .0331C 16214 16216     SFDR   16233 16316     CFER   16233 16322
     CCEXIT 16233 16376     CFMP   16233 1627C     DFSB   16233 16253     DFAD   16233 16236     (TOO)  164C6 16512
     (TEF)  16406 16511     (RCH)  164C6 1651C     (ETT)  16406 16507     (RBW)  16406 165C6     (WBR)  164C6 16505
     (BSR)  164C6 16504     (WRS)  164C6 165C3     (RCS)  164C6 16502     (IOS)  16406 16413     (TRO)  164C6 16513
     (EXE)  16551 1656C     (ICL)  17411 17416     (TES)  17433 17435     REODUP 17436 17441     .SPRNT 17444 1767C
     .PRINT 17444 1755C     .TAPWR 17444 17543     .FLNCH 17444 17524     (SOH)  17444 17471     (STHC) 17444 17507
     (STHM) 17444 17457     (STH)  17444 1746C     (SFHM) 17444 17456     (SRH)  17444 17515     (FRNT) 17444 2CC67
     (SCHM) 17444 17466     .FCLT  17444 2C4C4     .CLCLT 17444 20401     .COMNT 17444 1755C     .PNOHL 17444 17524
     ERRCR  20644 2065C     (WTC)  21C4C 21127     (WER)  21040 21054     (BST)  21164 21175     (RCRM) 21224 2132C
     (RCC)  21224 21303     (RER)  21224 21237     SCR    21327 21333     SQRT   21327 21333     EXP(2  21437 21443
     LCUMP  21573 21576     ATN    216C2 216C4     ATAN   21602 21604     SIN    21731 21744     CCS    21731 21733
     MOVIE) 22123 22123
PROGRAM LENGTH =  225CC. LOWEST COMMON =  77461

        .27 MINUTES ELAPSED SINCE START OF JOB

        EXECUTION
THE FOLLOWING RUNS HAVE BED SLOPE = C.        AND SIDEWALL SLOPE = 1.25CCGO
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RUN	T	XLO	XL1	F1	H3	HH13	XL3	A1P	A2P	A3R	STEPU	STEPD	Z1	XMR	XMT	CELAU	CELAD	CGRAT
									DELT1	DELT2	DEUT4	HL11	HL33		A1	A2	A3	A4
1CC	3.C89	48.82	25.CC	2.25	2.25	1.CC	25.CC	.C642	.0117	.C864	.0051	.0069	0.	.1817	1.3449	8.1CC	8.099	.4999
									-18.5	-.4	-.9	.09	.09		.C84	.046	.1C7	.047
2CC	3.C89	48.82	25.CC	2.25	2.25	1.CC	25.CC	.C249	.C039	.C344	.C020	.0028	0.	.1548	1.3817	8.1CC	8.C99	.4999
									-18.7	.0	-.5	.09	.09		.028	.0C9	.04C	.016
3CC	3.C89	48.82	25.CC	2.25	2.25	1.CC	25.CC	.C864	.CC60	.1203	.C069	.0096	0.	.C658	1.3927	8.1C0	8.C99	.4999
									-18.2	-.4	-.9	.09	.09		.1CC	.C36	.141	.054
4CC	2.534	32.86	20.CC	2.25	2.25	1.CC	2C.CC	.C716	.C255	.0966	.C072	.0097	0.	.3562	1.3489	7.859	7.859	.5000
									-2C.6	.5	-3.0	.11	.11		.C77	.028	.111	.040
5CC	2.534	32.86	2C.CC	2.25	2.25	1.CC	2C.CC	.C481	.C012	.C660	.C048	.0066	0.	.C255	1.3726	7.899	7.899	.5000
									-27.5	1.5	-2.4	.11	.11		.C59	.C2C	.075	.026

RUN	T	XL0	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XNR	XNT	DELAU	DELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
600	2.534	32.86	20.00	2.25	2.25	1.00	20.00	.0218	.0021	.0302	.0022	.0030	0.	.0961	1.2868	7.859	7.859	.5000
								-18.6	-1.5	-1.9	.11	.11			.028	.009	.035	.013
700	1.880	18.08	13.90	2.25	2.25	1.00	13.90	.0445	.0194	.0554	.0064	.0080	0.	.4357	1.2452	7.400	7.400	.5000
								-37.2	-.1	-8.2	.16	.16			.048	.012	.065	.025
800	1.880	18.08	13.90	2.25	2.25	1.00	13.90	.0977	.0419	.1229	.0141	.0177	0.	.4290	1.2570	7.400	7.400	.5000
								-37.2	-.1	2.2	.16	.16			.054	.012	.137	.044
900	1.880	18.08	13.90	2.25	2.25	1.00	13.90	.1294	.0441	.1687	.0186	.0243	0.	.3411	1.2626	7.400	7.400	.5000
								-36.8	-.7	-3.1	.16	.16			.130	.018	.189	.062
1000	1.478	11.18	9.95	2.25	2.25	1.00	9.95	.1439	.0527	.1826	.0289	.0367	0.	.3659	1.2692	6.736	6.736	.5000
								-47.4	1.4	-3.3	.23	.23			.143	.011	.207	.071
RUN	T	XL0	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XNR	XNT	DELAU	DELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
1100	1.478	11.18	9.95	2.25	2.25	1.00	9.95	.1103	.0328	.1452	.0222	.0292	0.	.2977	1.3159	6.736	6.736	.5000
								-47.0	1.5	2.9	.23	.23			.110	.016	.155	.039
1200	1.478	11.18	9.95	2.25	2.25	1.00	9.95	.0389	.0032	.0538	.0078	.0108	0.	.0834	1.3822	6.736	6.736	.5000
								-56.0	-1.4	-14.4	.23	.23			.043	.012	.081	.021
1300	1.053	5.67	5.60	2.25	2.25	1.00	5.60	.0871	.0139	.1179	.0311	.0421	0.	.1591	1.3533	5.323	5.323	.5000
								-91.9	.3	-7.7	.40	.40			.090	.014	.126	.032
1400	1.053	5.67	5.60	2.25	2.25	1.00	5.60	.1453	.0251	.1929	.0519	.0689	0.	.1726	1.3278	5.323	5.323	.5000
								-79.8	.6	-1.6	.40	.40			.149	.015	.207	.054
1500	3.044	47.42	24.60	2.25	2.25	1.00	24.60	.0135	.0065	.0175	.0011	.0014	0.	.4827	1.2972	8.087	8.087	.4999
								-18.2	.9	1.5	.09	.09			.014	.006	.020	.007
RUN	T	XL0	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XNR	XNT	DELAU	DELAD	CGRAT
								DELT1	DELT2	DELT4	HL11	HL33			A1	A2	A3	A4
1600	3.044	47.42	24.60	2.25	2.25	1.00	24.60	.0250	.0069	.0332	.0020	.0027	0.	.2767	1.3315	8.087	8.087	.4999
								-15.4	-1.3	1.1	.09	.09			.030	.015	.039	.015

17C	3.044	47.42	24.6C	2.25	2.25	1.CC	24.6C	.C49C	.C181	.064C	.C040	.0052	0.	.3696	1.3064	8.087	8.087	.4999
								-18.4	.6	1.4	.09	.09			.C56	.C13	.072	.024
18C	2.55C	33.28	2C.15	2.25	2.25	1.CC	2C.15	.C636	.0260	.0810	.0C63	.008C	0.	.4C87	1.2722	7.9C7	7.907	.5000
								-24.8	-.0	-1.3	.11	.11			.C7C	.C30	.096	.038
19C	2.55C	33.28	2C.15	2.25	2.25	1.CC	2C.15	.C45C	.0087	.0622	.0045	.0062	0.	.1928	1.3826	7.987	7.907	.5000
								-17.9	-.9	-1.4	.11	.11			.C59	.C26	.075	.031
20C	2.55C	33.28	2C.15	2.25	2.25	1.CC	2C.15	.C172	.0037	.0227	.C017	.0023	0.	.2165	1.3228	7.9C7	7.9C7	.5000
								-24.2	-.8	4.2	.11	.11			.C22	.C1C	.031	.016

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XMR	XMT	CETAU	CETAD	CGRAT
								DELT1	DELT2	DEUT4	HL11	HL33			A1	A2	A3	A4
21C	1.984	20.16	14.9C	2.25	2.25	1.CC	14.9C	.C215	.C029	.C301	.0029	.0040	0.	.1347	1.3982	7.513	7.513	.5000
								-28.1	-1.0	-4.3	.15	.15			.C28	.C15	.037	.016
22C	1.984	20.16	14.9C	2.25	2.25	1.CC	14.9C	.C694	.C212	.0929	.0C93	.0125	0.	.3C59	1.3375	7.513	7.513	.5000
								-30.8	.7	-4.5	.15	.15			.C69	.C14	.097	.020
23C	1.984	20.16	14.9C	2.25	2.25	1.CC	14.9C	.1011	.C415	.1273	.0136	.0171	0.	.41C4	1.2594	7.513	7.513	.5000
								-3C.7	.6	-4.1	.15	.15			.C98	.C14	.141	.044
24C	1.633	13.65	11.5C	2.25	2.25	1.CC	11.5C	.C863	.0077	.1193	.0150	.0208	0.	.C89C	1.383C	7.C46	7.046	.5000
								-37.4	-.5	-7.1	.2C	.2C			.1C1	.C35	.145	.061
25C	1.633	13.65	11.5C	2.25	2.25	1.CC	11.5C	.C720	.C294	.0916	.0125	.0159	0.	.4C85	1.2729	7.C46	7.046	.5000
								-38.1	.C	-2.5	.20	.20			.C69	.C12	.098	.025

RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XMR	XMT	CETAU	CETAD	CGRAT
								DELT1	DELT2	DEUT4	HL11	HL33			A1	A2	A3	A4
26C	1.633	13.65	11.5C	2.25	2.25	1.CC	11.50	.C345	.C183	.0411	.C060	.0071	0.	.5297	1.19C4	7.C46	7.C46	.5000
								-46.7	1.C	-5.4	.20	.20			.C32	.008	.046	.015
27C	1.375	9.67	8.9C	2.25	2.25	1.CC	8.9C	.0525	.C155	.0699	.0118	.0157	C.	.2957	1.3326	6.477	6.477	.5000
								-52.8	-1.5	-7.0	.25	.25			.C52	.C09	.073	.015
28C	1.375	9.67	8.9C	2.25	2.25	1.CC	8.9C	.1027	.C454	.1273	.0231	.0286	0.	.4418	1.2395	6.477	6.477	.5000

								-47.1	-.8	-14.1	.25	.25			.095	.014	.144	.049
290	1.375	9.67	8.90	2.25	2.25	1.00	8.90	.1273	.0259	.1703	.0286	.0383	0.	.2036	1.3377	6.477	6.477	.5000
								-48.1	-.4	-14.6	.25	.25			.135	.038	.196	.071
300	1.964	19.74	13.10	1.67	1.67	1.00	13.10	.0699	.0163	.0939	.0107	.0143	0.	.2325	1.3435	6.676	6.676	.5000
								-40.7	.3	-2.5	.13	.13			.076	.027	.112	.045
RUN	T	XL0	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	I1	XNR	XNT	DELAU	DELAD	CGRAT
								DELT1	DELT2	DEUT4	HL11	HL33			A1	A2	A3	A4
310	1.964	19.74	13.10	1.67	1.67	1.00	13.10	.0596	.0172	.0795	.0091	.0121	0.	.2883	1.3333	6.676	6.676	.5000
								-44.3	-1.5	-8.9	.13	.13			.060	.014	.083	.017
320	1.964	19.74	13.10	1.67	1.67	1.00	13.10	.0285	.0097	.0370	.0044	.0057	0.	.3402	1.2981	6.676	6.676	.5000
								-34.4	-.1	-2.4	.13	.13			.030	.008	.043	.016
330	1.486	11.30	9.20	1.67	1.67	1.00	9.20	.0390	.0098	.0515	.0085	.0112	0.	.2527	1.3222	6.195	6.195	.5000
								-47.6	1.3	-13.7	.18	.18			.048	.026	.059	.021
340	1.486	11.30	9.20	1.67	1.67	1.00	9.20	.0605	.0174	.0798	.0132	.0173	0.	.2868	1.3190	6.195	6.195	.5000
								-44.5	-1.6	-12.9	.18	.18			.082	.025	.095	.038
350	1.486	11.30	9.20	1.67	1.67	1.00	9.20	.1000	.0306	.1310	.0217	.0285	0.	.3060	1.3104	6.195	6.195	.5000
								-47.1	.1	-13.9	.18	.18			.059	.016	.138	.031
RUN	T	XL0	XL1	H1	H3.	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	I1	XNR	XNT	DELAU	DELAD	CGRAT
								DELT1	DELT2	DEUT4	HL11	HL33			A1	A2	A3	A4
360	1.082	6.00	5.70	1.67	1.67	1.00	5.70	.0470	.0194	.0594	.0165	.0209	0.	.4132	1.2638	5.269	5.269	.5000
								-90.3	.4	-13.4	.29	.29			.045	.009	.063	.015
370	1.082	6.00	5.70	1.67	1.67	1.00	5.70	.0920	.0155	.1243	.0323	.0436	0.	.1689	1.3514	6.269	6.269	.5000
								-83.9	-.5	-11.4	.29	.29			.056	.017	.135	.038
380	2.985	45.74	21.05	1.67	1.67	1.00	21.05	.0435	.0123	.0581	.0041	.0055	0.	.2837	1.3356	7.046	7.046	.4999
								-22.2	-1.4	-3.6	.08	.08			.046	.009	.068	.026
390	2.985	45.74	21.05	1.67	1.67	1.00	21.05	.0279	.0056	.0382	.0027	.0036	0.	.1994	1.3688	7.046	7.046	.4999
								-22.2	-1.3	-3.6	.08	.08			.031	.010	.044	.016

4CC	2.55E	33.45	17.75	1.67	1.67	1.CC	17.75	.C465	.C240	.0546	.0052	.0062	C.	.516E	1.1741	6.943	6.943	.5000
									-27.1	-.4	-1.2	.09	.09		.C44	.CC7	.065	.026
RUN	T	XLC	XL1	H1	H3	HH13	XL3	A1P	A2P	A3R	STEPU	STEPD	Z1	XHR	XHT	CEDAU	DELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
41C	2.55E	33.45	17.75	1.67	1.67	1.CC	17.75	.0486	.C077	.0665	.C055	.0075	0.	.15EE	1.36EE	6.943	6.943	.5000
									-28.3	1.1	-1.8	.09	.09		.C55	.C28	.078	.030
42C	3.CC5	46.33	15.65	1.42	1.42	1.CC	15.64	.C292	.C054	.C408	.0030	.0041	0.	.1862	1.3701	6.536	6.536	.4999
									-22.2	-.0	-2.7	.07	.07		.C31	.C19	.045	.021
43C	3.CC5	46.33	15.65	1.42	1.42	1.CC	15.64	.0285	.0133	.0348	.0029	.0035	0.	.4671	1.22C4	6.536	6.536	.4999
									-21.5	-.5	-2.0	.07	.07		.C2E	.C06	.041	.018
44C	2.277	26.53	14.5C	1.42	1.42	1.CC	14.5C	.C355	.C264	.C416	.0055	.0057	0.	.66C5	1.C431	6.373	6.373	.5C00
									-25.9	.6	-4.3	.10	.10		.C35	.013	.049	.019
45C	2.277	26.53	14.5C	1.42	1.42	1.CC	14.5C	.C5C9	.C291	.C576	.0070	.0080	0.	.5722	1.131E	6.373	6.373	.5000
									-31.0	1.5	-8.9	.10	.10		.C47	.C11	.069	.028
RUN	T	XLC	XL1	H1	H3	HH13	XL3	A1P	A2P	A3R	STEPU	STEPD	Z1	XHR	XHT	CEDAU	DELAD	CGRAT
									DELT1	DELT2	DELT4	HL11	HL33		A1	A2	A3	A4
46C	1.73E	15.46	1C.6C	1.42	1.42	1.CC	1C.6C	.C855	.0311	.1096	.0161	.0207	0.	.2641	1.2822	6.102	6.1C2	.5000
									-4C.2	-1.4	-10.3	.13	.13		.C8E	.C20	.127	.047
47C	1.73E	15.46	1C.6C	1.42	1.42	1.CC	1C.6C	.C509	.0144	.0675	.CC56	.0127	0.	.282E	1.32E5	6.102	6.1C2	.5000
									-4C.9	-1.0	-11.1	.13	.13		.C55	.C22	.072	.018
48C	1.73E	15.46	1C.6C	1.42	1.42	1.CC	1C.6C	.0366	.CC71	.0503	.0069	.0095	0.	.1935	1.3759	6.102	6.102	.5000
									-43.5	1.2	-11.2	.13	.13		.C3E	.E06	.055	.016
49C	1.14C	6.65	6.CC	1.42	1.42	1.CC	6.CC	.0547	.0154	.0729	.0182	.0243	0.	.2814	1.3334	5.266	5.266	.5000
									-76.6	-.2	-10.2	.24	.24		.C55	.C09	.078	.020
50C	1.14C	6.65	6.CC	1.42	1.42	1.CC	6.CC	.C680	.0110	.0902	.0227	.0301	0.	.1616	1.3276	5.266	5.266	.5000
									-82.C	-.1	-18.4	.24	.24		.C84	.C39	.111	.048

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RUN	T	XLO	XL1	H1	H3	HH13	XL3	A1P	A2P	A3P	STEPU	STEPD	Z1	XMR	XMT	DELAL	DELAD	CGRAT
									DELT1	DELT2	DEUT4	HL11	HL33		A1	A2	A3	A4
510	1.998	20.43	12.50	1.42	1.42	1.00	12.50	.0703	.0224	.0921	.0112	.0147	0.	.3186	1.3113	6.260	6.260	.5000
									-36.2	-.0	-6.4	.11	.11		.071	.018	.098	.024
520	1.998	20.43	12.50	1.42	1.42	1.00	12.50	.0510	.0329	.0539	.0082	.0086	0.	.6464	1.0547	6.260	6.260	.5000
									-35.7	-.7	-6.1	.11	.11		.045	.016	.063	.024

APPENDIX D

THE COMPUTER PROGRAM P_I

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      A1      = UPSTREAM INCIDENT WAVE AMPLITUDE,FT                    03/13 2108.3        PAGE 1
C     A2      = UPSTREAM REFLECTED WAVE AMPLITUDE , FT
C     A3      = DOWNSTREAM TRANSMITTED WAVE AMPLITUDE , FT
C     A4      = DOWNSTREAM REFLECTED WAVE AMPLITUDE FROM FAR END , FT
C     A1P     = TRANSFORMED INCIDENT WAVE AMPLITUDE , FT
C     A2P     = TRANSFORMED REFLECTED WAVE AMPLITUDE , FT
C     A3P     = TRANSFORMED TRANSMITTED WAVE AMPLITUDE , FT
C     XL1     = UPSTREAM WAVE LENGTH , FT
C     XL3     = DOWNSTREAM WAVE LENGTH , FT
C     DELTA1 = UPSTREAM INCIDENT WAVE PHASE ANGLE , RADIANS
C     DELTA2 = UPSTREAM REFLECTED WAVE PHASE ANGLE , RADIANS
C     DELTA3 = DOWNSTREAM TRANSMITTED WAVE PHASE ANGLE , RADIANS
C     DELTA4 = DOWNSTREAM REFLECTED WAVE PHASE ANGLE  , RADIANS
C     XMAXU  = UPSTREAM DISTANCE FROM ORIGIN AT WHICH MAXIMA OF WAVE ENVELOPE
C     OCCUR , FT
C     XMAXD  = DOWNSTREAM DISTANCE FROM ORIGIN AT WHICH MAXIMA OF WAVE ENVELOPE
C     OCCUR , FT
C     XAU     = UPSTREAM DISTANCE FROM ORIGIN AT WHICH SIMULTANEOUS MAXIMA OCCUR
C     XBD     = DOWNSTREAM DISTANCE FROM ORIGIN WHERE SIMULTANEOUS MAXIMA OCCUR
C     PAI     = 3.1416
C     XK1     = UPSTREAM WAVE NUMBER, (2.0*PAI)/XL1
C     XK3     = DOWNSTREAM WAVE NUMBER , (2.0*PAI)/XL3
C     SUM12  = SUM OF AMPLITUDE A1 AND A2 , FT
C     SUM34  = SUM OF AMPLITUDES A3 AND A4 , FT
C     DIF12  = DIFFERENCE OF AMPLITUDES A1 AND A2 , FT
C     DIF34  = DIFFERENCE OF AMPLITUDES A3 AND A4 , FT
C     H1      = UPSTREAM WATER DEPTH , FT
C     H3      = DOWNSTREAM WATER DEPTH , FT
C     HL11   = UPSTREAM DEPTH WAVE LENGTH RATIO
C     HL33   = DOWNSTREAM DEPTH WAVE LENGTH RATIO
C     HL10   = UPSTREAM DEPTH TO DEEP WATER WAVE LENGTH RATIO
C     HL30   = DOWNSTREAM DEPTH TO DEEP WATER WAVE LENGTH RATIO
C     BSLOPE = CHANNEL BED SLOPE IN TRANSITION
C     SSLOPE = CHANNEL SIDEWALL SLOPE IN TRANSITION
C     XLO     = DEEP WATER WAVE LENGTH , FT
C     DEL12  = SUM OF PHASE ANGLES DELTA1 AND DELTA2
C     HH13   = UPSTREAM TO DOWNSTREAM DEPTH RATIO
C     XKR     = REFLECTION COEFFICIENT
C     XKT     = TRANSMISSION COEFFICIENT
C     STEEPU = UPSTREAM WAVE STEEPNESS
C     STEEPD = DOWNSTREAM WAVE STEEPNESS
C     Z1      = DEANS PARAMETER
C     CELAU  = UPSTREAM WAVE CELERITY , FT/SEC
C     CELAD  = DOWNSTREAM WAVE CELERITY , FT/SEC
C     CGRAT  = GROUP VELOCITY RATIO
      READ 1 , BSLOPE , SSLOPE ,B1  ,B3
      N=0
    1 FORMAT ( 4F10.6 )
      PRINT 2,BSLOPE , SSLOPE
    2 FORMAT ( 37H THE FOLLOWING RUNS HAVE BED SLOPE = , F8.6 , 22H AND
     1SIDEWALL SLOPE =   , F8.6 )
      PRINT 1001
 1001 FORMAT(130H RUN     T      XLO     XL1     H1     H3   HH13   XL3     A1
     1P     A2P     A3P    STEPU  STEPD    Z1      XKR      XKT     CELAU  CELA
     2D    CGRAT)
      PRINT 1006
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P_I-2

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1C06  FCRMAT(129H
     1 DELT1  DELT2  DELT4   HL11   HL33              A1    A2    A3
     2  A4    )
 2CC2 READ 2001 , H1 , H3
 2CC1 FCRMAT ( 2F12.5 )
 1CC4 READ 3,RUN , ID , XL1 , SUM12 , DIF12 , XMAXU , XAU , SUM34 ,
     1 DIF34 , XMAXD , XBD
    3 FCRMAT ( A3 , I3 , 3X , F7.3 , 3X , F8.5 , 3X , F8.5 , 3X , F7.3 ,
     1 3X , F7.3 , 3X , F8.5 , 3X , F8.5 / F7.3 ,F7.3)
      IF ( ID ) 2002 , 2002, 2003
 2CC3 CALL  AKEFQ ( HTAN , HSEC2 , H1 , XL1 , HL11 , HL10 , HSIN2A )
      HL30 = ((HL10 ) *H3 ) / H1
      XLO = (1.0 / HL30 ) *H3
      T = SQRTF (( XLO ) / ( 5.118 ))
      HH13 = H1 / H3
      XL3=XL1-2.0
   22 CALL  AKEFQ ( HTAN , HSEC2 , H3 , XL3 , HL33 , HLT , HSIN2A  )
      FUNCL = (( HL33 ) * (HTAN )) - HL30
      FUNCLP = -(HL33 * ((1.0 / XL3 ) * HTAN + (( 2.0 * 3.1416 * H3 )
     1 / ( XL3 ** 2 )) * HSEC2  ))
      XL3 = XL3 - (FUNCL)/FUNCLP
      IF(ABSF(FUNCL)-0.001)20,20,22
   20 CCNTINUE
C     CALCULATICN OF REFLECTION AND TRANSMISSION COEFFICIENTS
      PAI = 3.1416
      A1 = (SUM12 + DIF12 )/ 2.0
      A2 = ( SUM12 - DIF12 ) / 2.0
      DEL12 = 1.0 * PAI - 2.0 * (2.0 * PAI / XL1 ) * XMAXU
      A3 = (SUM34 + DIF34 ) / 2.0
      A4 = ( SUM34 - DIF34 ) / 2.0
      DELTA4 = ( 1.0 * PAI ) - 2.0 * (2.0 * PAI / XL3 ) * XMAXD
      R1 = ( 2.0 * PAI / XL3 ) * XBD
      R2 = R1 + DELTA4
      R3 = -COSF( R1 ) + ( A4 / A3 ) * COSF ( R2 )
      R4 = SINF ( R1 ) + ( A4 / A3 ) * SINF( R2 )
      R = ( R3 / R4 )
      ARG1 = ((( 2.0 * PAI ) / XL1 ) * XAU + DEL12 )
      ARG2 = (( 2.0 * PAI ) / XL1 ) * XAU
      XNUM = ( COSF ( ARG1 )) + ( R * SINF ( ARG1 )) -  ( A2 / A1 ) *
     1 ( COSF(ARG2) - R * SINF ( ARG2 ))
      XDENM =  - ( SINF ( ARG1 )) + ( R * CCSF ( ARG1 )) - ( A2 / A1 ) *
     1  ( SINF ( ARG2 ) + R* COSF ( ARG2 ))
      DELTA2 = ATANF (( XNUM) / ( XDENM ))
      DELTA1 = DEL12 - DELTA2
      ARG3 = CCSF ( DELTA1 - DELTA2 + DELTA4 )
      RAD1 =(((A2/A1)**2)*((A4/A3)**2)- 2.0*(A2/A1)*(A4/A3)*ARG3) + 1.0
      RAD2 = (((A1/A2)**2)*((A4/A3)**2) - 2.0*(A1/A2)*(A4/A3)*ARG3) +1.0
      A1P = A1 * SQRTF(ABSF(RAD1))
      A2P = A2 * SQRTF(ABSF(RAD2))
      A3P = A3 * ( 1.0 - (A4/A3)**2)
      XKR  =  (A2P / A1P )
      XKT  =  ( A3P / A1P )
      STEEPU  = ( 2.0 * A1P ) / XL1
      STEEPD = (2.0 * A3P ) / XL3
      Z1 = (4.0 * PAI * H1 ) / (( XL1 ) * BSLOPE )
C     CALCULATICN OF GROUP VELOCITY RATIO
```

```
      CALL  AKEFQ ( HTAN, HSEC2,H1,XL1,HL11,HL10,HSIN2A )
      CELAU2 = (( 32.2 * XL1 ) / ( 2.0 * PAI )) * HTAN
      CELAU  = SQRTF ( CELAU2 )
      XN1 = ( 1.0 + ((( 2.0*2.0*PAI*H1)/XL1)/(HSIN2A ))) / 2.0
      CALL  AKEFQ ( HTAN, HSEC2,H3,XL3,HL33,HL30,HSIN2A)
      CELAD2  = (( 32.2*XL3)/(2.0*PAI))*HTAN
      CELAD  = SQRTF ( CELAD2 )
      XN3  =( 1.0 + (((2.0*2.0*PAI*H3) / XL3)/(HSIN2A))) / 2.0
      CGRAT  = (XN3 * XL3 * B3 )/( XN1 * XL1 * B1 )
      HL11=H1/XL1
      HL33=H3/XL3
      PRINT 101, ID,T,XLO,XL1,H1,H3,HH13,XL3,A1P,A2P,A3P,STEEPU,STEEPD,Z
     11,XKR,XKT,CELAU,CELAD,CGRAT
  101 FCRMAT(1X,I3,1X,F6.3,1X,F6.2,2X,F5.2,2X,F4.2,2X,F4.2,2X,F4.2,2X,F5
     1.2,F7.4,F7.4,F7.4,F7.4,F7.4,2X,F5.2,2X,F6.4,2X,F6.4,2X,F5.3,2X,F5.
     23,F7.4/)
      PRINT 1003,DELTA1,DELTA2,CELTA4,HL11,HL33,A1,A2,A3,A4
      N=N+1
      IF(N-5)1004,210,210
  210 PRINT 1001
      PRINT 1006
      N=0
 1003 FCRMAT(55X,F6.1,2X,F5.1,2X,F5.1,3X,F4.2,3X,F4.2,15X,F5.3,1X,F5.3,1
     1X,F5.3,1X,F5.3//)
      GC TO 1004
      END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)

JCB TIME =   0.11 MIN.
```

P$_I$-4

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
810	01452	32561	77461

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN COMMON, DIMENSION, OR EQUIVALENCE STATEMENT

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
A1P	809	01451	A1	808	01450	A2P	807	01447	A2	806	01446	A3P	805	01445
A3	804	01444	A4	803	01443	ARG1	802	01442	ARG2	801	01441	ARG3	800	01440
B1	799	01437	B3	798	01436	BSLOPE	797	01435	CELAD2	796	01434	CELAD	795	01433
CELAU2	794	01432	CELAU	793	01431	CGRAT	792	01430	DEL12	791	01427	DELTA1	790	01426
DELTA2	789	01425	DELTA4	788	01424	DIF12	787	01423	DIF34	786	01422	FUNCLP	785	01421
FUNCL	784	01420	H1	783	01417	H3	782	C1416	HH13	781	01415	HL10	780	01414
HL11	779	01413	HL30	778	01412	HL33	777	01411	HLT	776	01410	HSEC2	775	01407
HSIN2A	774	01406	HTAN	773	01405	ID	772	01404	N	771	01403	PAI	770	01402
R1	769	01401	R2	768	01400	R3	767	01377	R4	766	01376	RAD1	765	01375
RAD2	764	01374	R	763	01373	RUN	762	01372	SSLOPE	761	01371	STEEPD	760	01370
STEEPU	759	01367	SUM12	758	01366	SUM34	757	01365	T	756	01364	XAU	755	01363
XBD	754	01362	XDENM	753	01361	XKR	752	01360	XKT	751	01357	XLO	750	01356
XL1	749	01355	XL3	748	01354	XMAXD	747	01353	XMAXU	746	01352	XN1	745	01351
XN3	744	01350	XNUM	743	01347	Z1	742	01346						

SYMBOLS AND LOCATIONS FOR SOURCE PROGRAM FORMAT STATEMENTS

	EFN	LOC		EFN	LOC		EFN	LOC		EFN	LOC		EFN	LOC
8)1	1	01336	8)2	2	01334	8)3	3	01235	8)35	101	01213	8)V9	1001	01315
8)VB	1C03	01164	8)VE	1006	01266	8)1UH	2001	01237						

LOCATIONS FOR OTHER SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
1)	735	01337	2)	599	01127	3)	603	01133	6)	610	01142			

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
AKEFC	5	0C005	ATAN	9	00011	COS	7	0C007	.SETUP	0	00000	SIN	8	00010
SCRT	6	0C006	(CSH)	1	C0001	(FIL)	4	CC004	(RTN)	2	00002	(SPH)	3	00003

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY

AKEFC ATAN COS .SETUP SIN SQRT (CSH) (FIL) (RTN) (SPH)

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
2C02	17	C0047	1004	19	00056	2003	22	C0113	22	28	00153	20	33	00227
210	80	01116												

TIME SPENT IN FORTRAN.. .52 MINUTES.

```
          SUBROUTINE     AKEFQ(HTAN,HSEC2,A,B,C,D,E )
          C = A/B
          PAI = 3.1416
          ARG = (2.*PAI)*C
          HSIN = ARG
          ZI=1.0
          ZK=ZI
    7     ZN=ZI+2.0
    6     ZJ=ZI+1.0
          IF(ZN-ZJ)4,5,5
    5     ZK=ZK*ZJ
          ZI=ZJ
          GC TO 6
    4     AK=ZK
          N=ZN
          ADD1 = ((ARG)**N)/AK
          HSIN = HSIN + ADD1
          IF ( ADD1 - 0.0001 ) 8 , 8 , 7
        8 HCCS = 1.0
          ZI=0.0
          ZK=ZI+1.0
   13     ZN=ZI+2.0
   11     ZJ=ZI+1.0
          IF(ZN-ZJ)9,10,10
   10     ZK=ZK*ZJ
          ZI=ZJ
          GC TO 11
    9     AK=ZK
          N=ZN
          ADD2 = ((ARG)**N)/AK
          HCCS = HCCS + ADD2
          IF ( ADD2 - 0.0001 ) 12 , 12 , 13
       12 HTAN = HSIN/HCOS
          HSEC2 = (1.0/HCOS)**2
          D = C*HTAN
          E = 2.0 * HSIN * HCOS
          RETURN
          END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)

JCB TIME =   0.61 MIN.
```

P_I-6

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
175	00257	32561	77461

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN COMMON, DIMENSION, OR EQUIVALENCE STATEMENT

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
ADD1	174	00256	ADD2	173	00255	AK	172	C0254	ARG	171	00253	HCOS	170	00252
HSIN	169	00251	N	168	00250	PAI	167	00247	ZI	166	00246	ZJ	165	00245
ZK	164	00244	ZN	163	00243									

LOCATIONS FOR OTHER SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
1)	162	00242	2)	150	00226	3)	151	00227	6)	156	00234			

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
EXP(2	C	0C000												

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY

EXP(2

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
7	9	C0053	6	10	00C56	5	12	00066	4	15	00074	8	20	00123
13	23	00132	11	24	00135	10	26	00145	9	29	00153	12	34	00202

TIME SPENT IN FORTRAN.. .26 MINUTES.

THE COMPUTER PROGRAM P_{II}


```
C      A1      = UPSTREAM INCIDENT WAVE AMPLITUDE,FT
C      A2      = UPSTREAM REFLECTED WAVE AMPLITUDE , FT
C      A3      = DOWNSTREAM TRANSMITTED WAVE AMPLITUDE , FT
C      A4      = DOWNSTREAM REFLECTED WAVE AMPLITUDE FROM FAR END , FT
C      A1P     = TRANSFORMED INCIDENT WAVE AMPLITUDE , FT
C      A2P     = TRANSFORMED REFLECTED WAVE AMPLITUDE , FT
C      A3P     = TRANSFORMED TRANSMITTED WAVE AMPLITUDE , FT
C      XL1     = UPSTREAM WAVE LENGTH , FT
C      XL3     = DOWNSTREAM WAVE LENGTH , FT
C      DELTA1 = UPSTREAM INCIDENT WAVE PHASE ANGLE , RADIANS
C      DELTA2 = UPSTREAM REFLECTED WAVE PHASE ANGLE , RADIANS
C      DELTA3 = DOWNSTREAM TRANSMITTED WAVE PHASE ANGLE , RADIANS
C      DELTA4 = DOWNSTREAM REFLECTED WAVE PHASE ANGLE  , RADIANS
C      XMAXU  = UPSTREAM DISTANCE FROM ORIGIN AT WHICH MAXIMA OF WAVE ENVELOPE
C      OCCUR , FT
C      XMAXD  = DOWNSTREAM DISTANCE FROM ORIGIN AT WHICH MAXIMA OF WAVE ENVELOPE
C      OCCUR , FT
C      XAU     = UPSTREAM DISTANCE FROM ORIGIN AT WHICH SIMULTANEOUS MAXIMA OCCUR
C      XBD     = DOWNSTREAM DISTANCE FROM ORIGIN WHERE SIMULTANEOUS MAXIMA OCCUR
C      PAI     = 3.1416
C      XK1     = UPSTREAM WAVE NUMBER, (2.0*PAI)/XL1
C      XK3     = DOWNSTREAM WAVE NUMBER , (2.0*PAI)/XL3
C      SUM12   = SUM OF AMPLITUDE A1 AND A2 , FT
C      SUM34   = SUM OF AMPLITUDES A3 AND A4 , FT
C      DIF12   = DIFFERENCE OF AMPLITUDES A1 AND A2 , FT
C      DIF34   = DIFFERENCE OF AMPLITUDES A3 AND A4 , FT
C      H1      = UPSTREAM WATER DEPTH , FT
C      H3      = DOWNSTREAM WATER DEPTH , FT
C      HL11    = UPSTREAM DEPTH WAVE LENGTH RATIO
C      HL33    = DOWNSTREAM DEPTH WAVE LENGTH RATIO
C      HL10    = UPSTREAM DEPTH TO DEEP WATER WAVE LENGTH RATIO
C      HL30    = DOWNSTREAM DEPTH TO DEEP WATER WAVE LENGTH RATIO
C      BSLOPE = CHANNEL BED SLOPE IN TRANSITION
C      SSLOPE = CHANNEL SIDEWALL SLOPE IN TRANSITION
C      XLO     = DEEP WATER WAVE LENGTH , FT
C      DEL12   = SUM OF PHASE ANGLES DELTA1 AND DELTA2
C      HH13    = UPSTREAM TO DOWNSTREAM DEPTH RATIO
C      XKR     = REFLECTION COEFFICIENT
C      XKT     = TRANSMISSION COEFFICIENT
C      STEEPU = UPSTREAM WAVE STEEPNESS
C      STEEPD = DOWNSTREAM WAVE STEEPNESS
C      Z1      = DEANS PARAMETER
C      CELAU   = UPSTREAM WAVE CELERITY , FT/SEC
C      CELAD   = DOWNSTREAM WAVE CELERITY , FT/SEC
C      CGRAT   = GROUP VELOCITY RATIO
       READ 1 , BSLOPE , SSLOPE ,B1  ,B3
       N=0
     1 FORMAT ( 4F10.6 )
       PRINT 2,BSLOPE , SSLOPE
     2 FORMAT ( 37H THE FOLLOWING RUNS HAVE BED SLOPE = , F8.6 , 22H AND
      1SIDEWALL SLOPE =   , F8.6 )
       PRINT  1001
  1001 FORMAT(130H RUN     T      XLO     XL1     H1     H3    HH13    XL3     A1
      1P     A2P      A3P     STEPU   STEPD    Z1       XKR      XKT     CELAU   CELA
      2D    CGRAT)
```

```
      PRINT 1006
1C06  FCRMAT(129H
     1 DELT1  DELT2  DELT4   HL11   HL33              A1    A2    A3
     2  A4     )
 2CC2 READ 2001 , H1 , H3
 2CC1 FCRMAT ( 2F12.5 )
 1C04 READ 3,RUN , ID , XL3 , SUM12 , DIF12 , XMAXU , XAU , SUM34 .
     1 DIF34 , XMAXC , XBD
    3 FCRMAT ( A3 , I3 , 3X , F7.3 , 3X , F8.5 , 3X , F8.5 , 3X , F7.3 ,
     1 3X , F7.3 , 3X , F8.5 , 3X , F8.5 / F7.3 ,F7.3'
      IF ( ID ) 2002 , 2002, 2003
2C03  CALL AKEFQ(HTAN,HSEC2,H3,XL3,HL33,HL30,HSIN2A)
      HL10=HL30*H1/H3
      XL0 = (1.0 / HL30 ) *H3
      T = SQRTF (( XL0 ) / ( 5.118 ))
      HH13 = H1 / H3
      XL1=XL3+2.0
   22 A = XL1
      CALL AKEFQ(HTAN,HSEC2,H1,XL1,HL11,HLT ,HSIN2A)
      FUNCL=HL11*HTAN-HL10
      FUNCLP=-HL11*(1.0/XL1*HTAN+2.0*3.1416*H1/XL1**2*HSEC2)
      XL1=XL1-FUNCL/FUNCLP
      IF(ABSF(A-XL1)-0.001 )20,20,22
   2C CONTINUE
C     CALCULATICN OF REFLECTION AND TRANSMISSION COEFFICIENTS
      PAI = 3.1416
      A1 = (SUM12 + DIF12 )/ 2.0
      A2 = ( SUM12 - DIF12 ) / 2.0
      DEL12 = 1.0 * PAI - 2.0 * (2.0 * PAI / XL1 ) * XMAXU
      A3 = (SUM34 + DIF34 ) / 2.0
      A4 = ( SUM34 - DIF34 ) / 2.0
      DELTA4 = ( 1.0 * PAI ) - 2.0 * (2.0 * PAI / XL3 ) * XMAXD
      R1 = ( 2.0 * PAI / XL3 ) * XBC
      R2 = R1 + DELTA4
      R3 = -COSF( R1 ) + ( A4 / A3 ) * COSF ( R2 )
      R4 = SINF ( R1 ) + ( A4 / A3 ) * SINF( R2 )
      R = ( R3 / R4 )
      ARG1 = ((( 2.0 * PAI ) / XL1 ) * XAU + DEL12 )
      ARG2 = (( 2.0 * PAI ) / XL1 ) * XAU
      XNUM = ( COSF ( ARG1 )) + ( R * SINF ( ARG1 )) -  ( A2 / A1 ) *
     1 ( COSF(ARG2) - R * SINF ( ARG2 ))
      XDENM =  - ( SINF ( ARG1 )) + ( R * COSF ( ARG1 )) - ( A2 / A1 ) *
     1  ( SINF ( ARG2 ) + R* COSF ( ARG2 ))
      DELTA2 = ATANF (( XNUM) / ( XDENM ))
      DELTA1 = DEL12 - DELTA2
      ARG3 = CCSF ( DELTA1 - CELTA2 + DELTA4 )
      RAD1 =(((A2/A1)**2)*((A4/A3)**2)- 2.0*(A2/A1)*(A4/A3)*ARG3) + 1.0
      RAD2 = (((A1/A2)**2)*((A4/A3)**2) - 2.0*(A1/A2)*(A4/A3)*ARG3) +1.0
      A1P = A1 * SQRTF(ABSF(RAD1)).
      A2P = A2 * SORTF(ABSF(RAD2))
      A3P = A3 * ( 1.0 - (A4/A3)**2)
      XKR  =  (A2P / A1P )
      XKT  =  ( A3P / A1P )
      STEEPU  = ( 2.0 * A1P ) / XL1
      STEEPD = (2.0 * A3P ) / XL3
      Z1 = (4.0 * PAI * H1 ) / (( XL1 ) * BSLOPE )
```

```
C     CALCULATION OF GROUP VELOCITY RATIO
      CALL  AKEFQ ( HTAN, HSEC2,H1,XL1,HL11,HL10,HSIN2A )
      CELAU2 = (( 32.2 * XL1 ) / ( 2.0 * PAI )) * HTAN
      CELAU  = SQRTF ( CELAU2 )
      XN1 = ( 1.0 + ((( 2.0*2.0*PAI*H1)/XL1)/(HSIN2A ))) / 2.0
      CALL  AKEFQ ( HTAN, HSEC2,H3,XL3,HL33,HL30,HSIN2A)
      CELAD2  = (( 32.2*XL3)/(2.0*PAI))*HTAN
      CELAD  = SQRTF ( CELAD2 )
      XN3  =( 1.0 + (((2.0*2.0*PAI*H3) / XL3)/(HSIN2A))) / 2.0
      CGRAT  = (XN3 * XL3 * B3 )/( XN1 * XL1 * B1 )
      HL11=H1/XL1
      HL33=H3/XL3
      PRINT 101, ID,T,XL0,XL1,H1,H3,HH13,XL3,A1P,A2P,A3P,STEEPU,STEEPD,Z
     11,XKR,XKT,CELAU,CELAD,CGRAT
  101 FORMAT(1X,I3,1X,F6.3,1X,F6.2,2X,F5.2,2X,F4.2,2X,F4.2,2X,F4.2,2X,F5
     1.2,F7.4,F7.4,F7.4,F7.4,F7.4,2X,F5.2,2X,F6.4,2X,F6.4,2X,F5.3,2X,F5.
     23,F7.4/)
      PRINT 1003,DELTA1,DELTA2,DELTA4,HL11,HL33,A1,A2,A3,A4
      N=N+1
      IF(N-5)1004,210,210
  210 PRINT 1001
      PRINT 1006
      N=0
 1003 FORMAT(55X,F6.1,2X,F5.1,2X,F5.1,3X,F4.2,3X,F4.2,15X,F5.3,1X,F5.3,1
     1X,F5.3,1X,F5.3//)
      GO TO 1004
      END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)

CB TIME =   0.11 MIN.
```

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
814	01456	32561	77461

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN COMMON, DIMENSION, OR EQUIVALENCE STATEMENT

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
A1P	813	01455	A1	812	01454	A2P	811	01453	A2	810	01452	A3P	809	01451
A3	808	01450	A4	807	01447	ARG1	806	01446	ARG2	805	01445	ARG3	804	01444
A	803	01443	B1	802	01442	B3	801	01441	BSLOPE	800	01440	CELAD2	799	01437
CELAD	798	01436	CELAU2	797	01435	CELAU	796	01434	CGRAT	795	01433	DEL12	794	01432
DELTA1	793	01431	DELTA2	792	01430	DELTA4	791	01427	DIF12	790	01426	DIF34	789	01425
FUNCLP	788	01424	FUNCL	787	01423	H1	786	01422	H3	785	01421	HH13	784	01420
HL10	783	01417	HL11	782	01416	HL30	781	01415	HL33	780	01414	HLT	779	01413
HSEC2	778	01412	HSIN2A	777	01411	HTAN	776	01410	ID	775	01407	N	774	01406
PAI	773	01405	R1	772	01404	R2	771	01403	R3	770	01402	R4	769	01401
RAD1	768	01400	RAD2	767	01377	R	766	01376	RUN	765	01375	SSLOPE	764	01374
STEEPD	763	01373	STEEPU	762	01372	SUM12	761	01371	SUM34	760	01370	T	759	01367
XAU	758	01366	XBD	757	01365	XDENM	756	01364	XKR	755	01363	XKT	754	01362
XLO	753	01361	XL1	752	01360	XL3	751	01357	XMAXD	750	01356	XMAXU	749	01355
XN1	748	01354	XN3	747	01353	XNUM	746	01352	Z1	745	01351			

SYMBOLS AND LOCATIONS FOR SOURCE PROGRAM FORMAT STATEMENTS

	EFN	LOC		EFN	LOC		EFN	LOC		EFN	LOC		EFN	LOC
8)1	1	01341	8)2	2	01337	8)3	3	01240	8)35	101	01216	8)V9	1001	01320
8)VB	1003	01167	8)VE	1006	01271	8)1UH	2001	01242						

LOCATIONS FOR OTHER SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
1)	738	01342	2)	602	01132	3)	606	01136	6)	613	01145			

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
AKEFC	5	00005	ATAN	9	00011	COS	7	00007	.SETUP	0	00000	SIN	8	00010
SQRT	6	00006	(CSH)	1	00001	(FIL)	4	00004	(RTN)	2	00002	(SPH)	3	00003

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY

AKEFQ   ATAN   COS   .SETUP   SIN   SQRT   (CSH)   (FIL)   (RTN)   (SPH)

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
2002	17	00047	1004	19	00056	2003	22	00113	22	28	00153	20	34	00232
210	81	01121												

TIME SPENT IN FORTRAN.. .39 MINUTES.

```
      SUBROUTINE    AKEFQ(HTAN,HSEC2,A,B,C,D,E )
      C = A/B
      PAI = 3.1416
      ARG = (2.*PAI)*C
      HSIN = ARG
      ZI=1.0
      ZK=ZI
    7 ZN=ZI+2.0
    6 ZJ=ZI+1.0
      IF(ZN-ZJ)4,5,5
    5 ZK=ZK*ZJ
      ZI=ZJ
      GO TO 6
    4 AK=ZK
      N=ZN
      ADD1 = ((ARG)**N)/AK
      HSIN = HSIN + ADD1
      IF ( ADD1 - 0.0001 ) 8 , 8 , 7
    8 HCOS = 1.0
      ZI=0.0
      ZK=ZI+1.0
   13 ZN=ZI+2.0
   11 ZJ=ZI+1.0
      IF(ZN-ZJ)9,10,10
   10 ZK=ZK*ZJ
      ZI=ZJ
      GO TO 11
    9 AK=ZK
      N=ZN
      ADD2 = ((ARG)**N)/AK
      HCOS = HCOS + ADD2
      IF ( ADD2 - 0.0001 ) 12 , 12 , 13
   12 HTAN = HSIN/HCOS
      HSEC2 = (1.0/HCOS)**2
      D = C*HTAN
      E = 2.0 * HSIN * HCOS
      RETURN
      END(1,0,0,0,0,0,0,0,0,1,0,0,0,0,0)

JOB TIME =   0.50 MIN.
```

PII-6

STORAGE NOT USED BY PROGRAM

DEC	OCT	DEC	OCT
175	00257	32561	77461

STORAGE LOCATIONS FOR VARIABLES NOT APPEARING IN COMMON, DIMENSION, OR EQUIVALENCE STATEMENT

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
ADD1	174	00256	ADD2	173	00255	AK	172	00254	ARG	171	00253	HCOS	170	00252
HSIN	169	00251	N	168	00250	PAI	167	00247	ZI	166	00246	ZJ	165	00245
ZK	164	00244	ZN	163	00243									

LOCATIONS FOR OTHER SYMBOLS NOT APPEARING IN SOURCE PROGRAM

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
1)	162	00242	2)	150	00226	3)	151	00227	6)	156	00234			

LOCATIONS OF NAMES IN TRANSFER VECTOR

	DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT		DEC	OCT
EXP(2	0	00000												

ENTRY POINTS TO SUBROUTINES NOT OUTPUT FROM LIBRARY

EXP(2

EXTERNAL FORMULA NUMBERS WITH CORRESPONDING INTERNAL FORMULA NUMBERS AND OCTAL LOCATIONS

EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC	EFN	IFN	LOC
7	9	00053	6	10	00056	5	12	00066	4	15	00074	8	20	00123
13	23	00132	11	24	00135	10	26	00145	9	29	00153	12	34	00202

TIME SPENT IN FORTRAN.. .21 MINUTES.

BIOGRAPHICAL NOTE

of

Efstathios Lampros Bourodimos

The author was born in Gialtra-Aedipsos, Greece on March 1, 1925, and attended elementary school in that village and high school in Istiea-Euboea. He entered the Greek National Technical University of Athens in 1945 and received the Civil Engineering Degree in June, 1953.

From October, 1947 through April, 1950 the author served on active duty in the Greek National Army. From July, 1953 through August, 1961 he worked as a professional structural and hydraulic engineer in Athens.

In 1959 he received a three-year scholarship from the Greek State Scholarship Foundation to pursue advance studies in Water Resources and Hydrodynamics. He entered M.I.T. in September, 1961 and was employed as a Research Assistant at the Hydrodynamics Laboratory of M.I.T. from June, 1962 until June,1966.

In January, 1963, after receiving his degree of Master of Science, he continued as a Candidate for the degree of Doctor of Philosophy.

The author is a member of the Greek Society of Civil Engineers, of the American Society of Civil Engineers and a member of the Society of Signa Xi.