BREAKEVEN ANALYSIS OF THE PROFIT-VOLUME RELATIONSHIP

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Professor Joseph S. Newell
Secretary of the Faculty
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Dear Sir:

In accordance with the requirements for graduation, I herewith submit a thesis entitled "Breakeven Analysis of the Profit-Volume Relationship."

I wish to express my appreciation for assistance rendered in the preparation of this paper to my thesis adviser, Mr. John H. Kempster, of the Department of Business and Engineering Administration.

Sincerely yours,

William S. Edgerly

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I. Background of breakeven analysis

The breakeven point is the subject of frequent discussion in business circles today. Companies are making free use of the term in explaining corporate finance to their stockholders. A dividend notice issued to the stockholders of the Willys-Overland Motor Company on May 26, 1948 said in part "Management recognizes the importance of lowering the breakeven point in anticipation of the filling of the pipelines at some future time." In other words, costs were going to have to be cut if the expected reductions in sales volume were not to have an adverse effect on profits. A great deal of emphasis is being laid on the importance of increased productivity if a company is to stay in business. The difficulty lies in the fact that increased productivity usually means nothing to labor but harder work. Management has found that the breakeven point is a useful concept to bring out in its negotiations with the unions. The theory of the breakeven point is a convenient device in the preparation of publicity releases. It lends itself readily to the kind of clear, forceful language that public relations men find most effective. But after reading some of the gloomy descriptions of the present business outlook painted in such broad strokes with the help of the magical breakeven point,

one begins to wonder whether the breakeven chart is not just a useful gadget which cannot really prove anything. Does a technique which appears on the surface to be so unscientific have any real basis in fact?

Breakeven analysis has its weaknesses, but it does have a contribution to make to business knowledge. Its wide use in a variety of situations is due to the clarity of the technique, not to a lack of significant meaning. The accounting profession testifies to the significance of the concept by the attention given it in recent years. Accountants attempt to present the information management wants to management in a form which management can understand. They are interested in accurate pictures of the situation because that is what management expects from them. The breakeven chart has in recent years become an accepted, if not an indispensable, accounting tool.[1] Articles have appeared in the Bulletin of the National Association of Cost Accountants from time to time describing various applications of the technique to the problems of the cost accountant. And the Association devoted an entire session of its twenty-ninth International Cost Conference on June 22, 1948 to the subject, "Your Breakeven Point -- Today and Tomorrow." The chairman of that session, Arthur C. Chubbuck, a partner in the firm of Patterson, Teale, and Dennis of Boston, said

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[1] "Breakeven Analysis -- Common Ground for the Economist and the Cost Accountant," J.H. Kempster, NACA Bulletin, Feb. 15, 1949.

in his opening address, "Probably at no previous time has there existed a more perplexing uncertainty as to the immediate business future. Whatever is ahead, however, a knowledge of breakeven points and all surrounding circumstances is vital to the obviously desirable flexibility of business policy."

The current emphasis on breakeven analysis is largely due to uneasiness about the possibility of a decline in sales volume and the effect such a decline would have on profits. Ever since the end of the war the business world has been looking ahead to what is believed an inevitable recession. "A business recession can quickly turn into a major depression if business generally is found to be operating at too high a breakeven volume," according to Charles H. Gleason of the Sylvania Electric Company.[1] Breakeven points are generally higher in the postwar period than they were in 1939. Mr. Gleason quotes an article in the September 27, 1947 issue of Business Week entitled, "Breakeven Points Rise Dangerously" to show a comparison. A survey among a carefully selected group of manufacturers showed that in 1939 58% of the companies questioned had a breakeven point of 60% of capacity or below; while today only 38% of the same companies would break even or show a profit if their operations dropped to 60% of capacity. Mr. Gleason goes on to say, "Many companies have grown substantially during the

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[1] "What is Your Breakeven Point," Charles H. Gleason, Address before New England Regional Cost Conference, Nat'l Assoc. of Cost Acct., Boston, April 9, 1948.

past seven years. In some cases, sales volume has more than tripled. Therefore, in terms of the actual volume of goods and services manufacturers must sell to meet expenses, the breakeven point has increased far more since 1939 than the percentage of capacity production required to make a profit would indicate." Responsibility for the current high breakeven point of industrial operations has not been fixed. But that the point is high and that management is concerned about it is undisputed.

Although current business conditions have caused great interest in breakeven points, the breakeven chart technique itself is an old one. C.E. Knoeppel, a Philadelphia management engineer, claims to have originated the idea in the form of a "Profitgraph" in 1909. Mr. Knoeppel had great faith in graphical representation of business situations. He also believed that if management would concentrate all its energies on the single objective of profit, all other considerations would take care of themselves. Combining these two beliefs he brought forth his Profitgraph, a graphical picture of the relationship between profit and volume. Since that time a great deal of work by many people has brought the technique to its present stage of development.

Persons who have contributed to the theory and application of breakeven analysis may be divided into three general groups. They include the management engineer, the economist, and the cost accountant. The specific analytical technique of the breakeven chart was given its major impetus by management engineers. It has been shown that the

originator of the method was probably a member of that profession. A Boston firm of management consultants which had close connections with Mr. Knoeppel during his lifetime provides an illustration of the sort of interest the group as a whole has in the technique. Bigelow, Kent, and Willard and Company use breakeven analysis as a regular part of their service to management. When requested to make a study of the general health of a company, the firm lists a breakeven analysis as one of seven steps in its procedure, the other items including studies of company balance sheets and profit-and-loss statements, calculation of financial ratios, an analysis of product profitability, a summary of financial progress, and an economic conclusion. The breakeven chart prepared by Bigelow, Kent, and Willard is arrived at simply and quickly in a maximum time of two and one-half weeks. Simplicity and practicality are stressed above theoretical exactness. The firm knows that the results obtained are not perfectly accurate, but believes that they are close enough for practical purposes. This attitude illustrates the view taken by management engineers in general and may be one reason why the group has contributed so much to the technique of breakeven analysis.

The major contribution of the economist to breakeven theory has been his basic interest in the relationship between cost and volume. The theoretical aspects of this relationship have been dealt with in considerable detail. The possibilities that total cost advances with volume in a

stepwise fashion or along a curve have been studied. If cost did behave in either of these fashions, breakeven analysis as used in practice might have no basis in fact, for the analysis depends for its simplicity on the assumption that total cost is a straight-line function of production volume. Fortunately, the economists have found that a straight line is a fair approximation of the cost relationship within the relevant ranges of volume. However, neither costs nor revenue are directly related to volume, and as a result the basic premise that profit is a direct function of output is not correct. But the studies of the economist have indicated that there is some basis for confidence in the breakeven technique. The limitations of the technique will be discussed more fully in a later section.

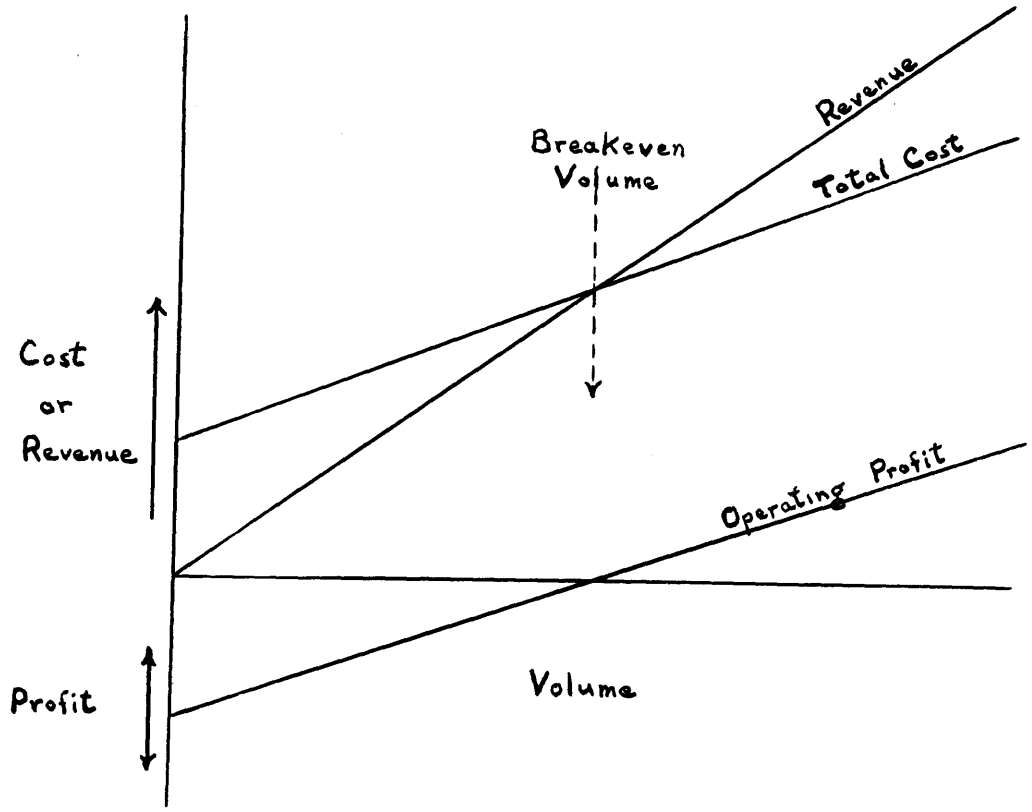
The fundamental data for the preparation of a breakeven chart are provided by the cost accountant. The interest of this group in the technique has been demonstrated. The use of profit-volume analysis has grown naturally in cost accounting on the broad basis of budgeting. Flexible budgeting ties in closely with breakeven analysis in that it provides a method for estimating costs at various levels of production. Standard cost data simplify the task of calculating the cost-output function. Thus the methods of the cost accountant and his primary interest in providing information to management are adapted to the development of knowledge about the rela-

tionship between profit and volume.

II. Basic theory of breakeven analysis

The basic theory employed in the preparation of a breakeven chart is easily understood. Since part of the cost of operating a plant does not vary with volume, it is obvious that some volume of operation must exist above which a profit will be realized and below which a loss will be taken. The problem is to determine how cost and revenue vary with volume; the point at which cost equals revenue is the breakeven point of operations. In drawing a breakeven chart the assumption is made that costs and revenue are linear functions of volume. Thus if cost is known at two specific outputs and a standard unit selling price is established, the complete chart can be drawn. The revenue line will pass through the origin since operating income is zero at zero output. The cost line will intersect the ordinate at some positive value representing the costs incurred at zero volume. These costs are normally taken equal to the fixed costs incurred when the plant is in operation. The assumption is accurate to the extent that depreciation, taxes, and other usual fixed charges continue when the plant is shut down. Actual shutdown costs will be less than normal fixed costs because of the elimination of much of the indirect labor, supervision, and other charges which are usually wholly or partly fixed regardless of the volume of operations. In the relevant volume range, however, failure to observe

this refinement is not serious. The essential form of the breakeven chart is as follows:



Ordinarily only one section of the chart is used, for both the upper section showing revenue and total cost, and the lower section showing operating profit give essentially the same information.

Although theoretically it would be possible to construct a breakeven chart knowing only two points on the cost line and the standard unit selling-price, the charts are not constructed this way in practice. There are two ways in which the cost at a particular volume may be determined. The first method is to examine past records of the company to find what costs were when production

reached that volume during some prior period. The obvious objection to this method is that present conditions are not likely to be the same as those in the past. The price paid for a certain amount of materials might be higher or lower, the production methods might have been different, or the kind of machinery used might have been different. Thus a cost figure obtained in this way would require adjustment if it were to have any significance. To plot only two points obtained from past operating records would indicate more confidence in the accuracy of the adjusted figures than would normally be warranted. The procedure which is followed is to plot a series of points and draw the best straight line through them, the best straight line being determined by one of the statistical methods for the correlation of data or by visual inspection. There are several variations of this general approach. One method is to make rough scatter diagrams of a few uncorrected observances at the extreme ends of the output range and to fit a line visually through the several points. Another approach is to perform a simple correlation analysis of annual cost data, uncorrected for dynamic changes, and of output measured in current dollar sales. Multiple-correlation analysis of cost and output data is a third variation. Finally, lines may be visually fitted to carefully selected samples of roughly adjusted cost observations. The essential feature of all these methods is the fact that each deals with the corre-

lation of total cost or overall profit figures with volume over a period of time. Some one of the variations on the general technique will often give results which will be suitable for the purpose at hand.

Instead of dealing with total cost figures, the second general approach to the problem of relating cost to output breaks total cost down into its components. Each account is analyzed to determine whether it is a fixed cost, a variable cost, or a partially variable cost. All costs fall into one or another of these classifications. An example of a fixed cost would be property taxes, the size of which has no relation whatsoever to the amount of production. Variable costs include such items as material and direct labor which vary in amount directly with the number of units produced. Supervisory charges have both a fixed and a variable component and are typical of the third class of costs. They are not in direct proportion to the amount of production, but can be separated into a portion which is constant at any output and a portion which varies directly with output. The past history of an account can be analyzed by means of scatter diagrams in order to determine in what classification it should fall. Or if past operating conditions provide no parallel the classification can be done mathematically by means of estimates. The procedures followed are described in a later section. The end result of the analysis is a formula showing how the cost in each account , broken

down into a fixed and a variable component, varies with output. The sum of the individual account formulae is the formula for total cost.

The determination of the revenue function is a less difficult part of breakeven analysis than that of the cost function. It is generally felt that the effect of price on demand does not fall within the province of breakeven analysis. The normal procedure is to fix a standard selling-price and consider that price to remain constant over the range of output covered.

If all costs varied directly with output and unit selling-price was constant, the breakeven point of operations would be at zero output. Above zero volume a manufacturer would always either make a profit or take a loss. In preparing a breakeven chart selling price is normally assumed to be constant. Therefore the separation of costs into fixed and variable components is the most important feature of the breakeven chart technique. Both of the general methods for determining the cost function are in essence the performance of this separation. The evidence for this statement lies in the end result of both methods -- a cost line having a slope equal to unit variable cost and intersecting the ordinate at a value equal to total fixed costs.

A final problem should be covered in a discussion of the basic theory of breakeven analysis. This is the selection of a suitable measure of volume. The major difficulty is presented by the fact that most plants turn out a

variety of products in variable proportions. A breakeven chart covering the entire plant operation must assume a standard product mix. Otherwise there would be any number of values of total cost and operating revenue at each given value of output. The goal in selecting an output index is to find one which would cause the smallest spread of cost and revenue values at any given volume if product mix were allowed to vary in all proportions. There are four general measures of volume used in breakeven analysis. They include physical indices of output, sales value of output, percentage of capacity utilized, and various measures of input.

Any physical index of output must be chosen so that total unit cost will be roughly proportional to the unit physical characteristic used. For example, a glass manufacturer bases his breakeven charts on pounds of glass produced. The total cost for materials, labor, and overhead is approximately the same for every pound of glass made. Therefore the physical index is an appropriate one. This sort of index would not be suitable for a manufacturer of silver and brass candlesticks. A certain volume of production expressed in pounds when 80% of the output consisted of silver candlesticks would have a much higher total manufacturing cost than the same volume of output when 80% of the candlesticks were made of brass. The revenue lines in the two cases would be considerably different also, the average selling price per pound being

much higher in the former case. It is unwise to attempt to combine two products on a breakeven chart using a physical index of output when the physical characteristic measured does not have a similar relationship to total cost in each case.

Even when a physical index is available which is roughly proportional to total cost, the relationship between cost and revenue will provide further difficulty. Product A and Product B may have approximately the same total cost per unit, so that we may use units produced as an index of activity. Suppose the unit selling price for A is quite a bit higher than that for B. The profit-volume relationship will not be the same for the two products. At a given volume the amount of profit realized will depend on what proportion of the total output is made up of A and what proportion consists of B. In order to draw a breakeven chart we have to assume a standard product-mix. In using the chart thereafter the profit figure we anticipate at a given volume will be in error to the extent to which the actual product mix deviates from standard.

When a sales value index is substituted for a physical index the same inaccuracy is involved where marginal contributions to profit are different for the different products. At a particular sales value of production total cost will then vary as product mix varies. But this output index is quite commonly used largely because it

provides a convenient common denominator among widely different products. In many plants the list of products includes items completely dissimilar in physical characteristics such as weight and widely differentiated in unit production costs. In such a case the best alternative is to measure production in terms of dollar sales value. This index will be erroneous if selling prices change during the analysis period. With a change in selling price the entire relationship between cost, profit, and volume will be altered. The sales value index is therefore not well suited to a situation in which prices fluctuate rapidly.

With both measurements of output described above, it is usually preferable to use volume produced rather than volume sold, even though sales figures for a given period may be more readily accessible than production figures. The exception to this rule would be where there is very little time lag between production and sales so that the volume of production would roughly equal sales volume. If the time lag between production and sales were considerable, such as when a company is building up its finished goods inventory, and the volume of sales were used as an index, production costs might be considerably out of line with volume as shown on the breakeven chart.

Output is often measured as a percentage of capacity. Since capacity must be expressed in some such units as sales value of production, this index presents no essentially new information. It does, however, facilitate

visualization of a company's position. To say that the company is operating at 90% of capacity has more meaning to most people than to say what the actual volume is, for a basis of comparison is provided. The major difficulty is that capacity cannot normally be measured accurately. It is affected by such things as the availability of materials and labor, product mix, and how long one is able and willing to defer maintenance, and it can often be changed by relatively minor expenditures on bottleneck operations. Thus the expression of output in terms of capacity alone is a sacrifice of accuracy for clarity.

Specification of output as a percentage of capacity naturally leads to the attempt to determine a standard safe breakeven percentage. The management-consultant firm of Bigelow, Kent, and Willard mentioned previously considers a breakeven point at less than 30% or more than 60% of capacity to be unhealthy. Many of the current articles on profit-volume relationships stress the dangers of breaking even at a high percentage of capacity. Actually the peril of a particular breakeven point depends wholly upon the probability that output will fall below that point. The probability that the demand for a firm's products will fall below the breakeven volume depends on factors other than the location of that firm's breakeven point. Therefore, care should be taken to consider the demand picture before placing too much confidence in statements concerning safe breakeven percentages.

The final alternative is to base the measure of acti-

vity on some unit of input, such as materials or direct labor hours. In a refinery, for example, where the output is spread over a large number of products whose proportions can be varied over broad ranges, the throughput of crude oil is a good index. Direct labor hours are a satisfactory index when the input of other factors, notably materials and equipment hours, stand in about the same ratio to labor hours for each of the various products. Determination of the revenue function is somewhat more difficult when this type of index is used since the relation of selling price to such a base as labor hours involves an additional calculation.

III. Usefulness of breakeven analysis

Some of the general uses of the breakeven chart have been indicated in discussing the current interest in breakeven analysis. The popularity of the technique as a method of illustrating basic business problems has been pointed out. Two fields in which this application of the breakeven chart has proved useful are public relations and labor relations. Two questions often asked by labor and the general public, "Why are prices so high?" and "Why must costs be cut?" can sometimes be answered with the help of the breakeven chart. A picture of the relationship between costs, revenue, and operating volume can often give a clearer idea of the situation than many words. Another question which arises relates to productivity and is also explainable in terms of the breakeven point.

Management's attempts to increase productivity are often greeted with cries of "Speed-up!" from the workers. Some clear illustration of the relationship between volume and profit -- or loss -- is needed in order to convince the workers that increased productivity is important.

Breakeven analysis has its greatest potential usefulness as an aid to management in the planning of operations. Although the word "breakeven" seems to indicate that finding the profitless point of operations is the ultimate end of the analysis, that is in reality merely a starting point. Considered broadly, the breakeven chart provides a picture for management of what can be expected under future conditions and under alternative management programs. It indicates what results will follow a change in product mix, a change in pricing policy, or a reduction in costs. It shows whether a reduction in fixed costs will have a greater effect on profits than a reduction in variable costs. It is an aid in the planning of capital expenditures. It provides a framework for the setting of output levels and profit goals.

The usefulness of the breakeven chart as a management-engineering tool is derived from its ability to forecast the results of five specific changes frequently experienced by any business -- changes in fixed cost, in variable cost, in price, in volume, or in product mix. To understand how the technique can be of use to management it is helpful to know how these five predictions are made with the aid of the breakeven chart.

Exhibit I is the breakeven chart of an imaginary firm. Total variable costs in the company amount to 75% of the sales value of production. The so-called Margin of Income ratio is therefore 25%. This figure represents the fraction of each sales dollar which is available for fixed costs and profits. It is a very useful figure in breakeven analysis. The ratio is so named because it can be used to calculate the amount of additional profit resulting from an increased volume of sales. At the breakeven point of operations the company will just recover its fixed and variable costs without making any profit. As the volume of sales increases, each extra dollar of revenue will be divided between variable cost and profit since there is no change in the amount of fixed costs. In this case 75¢ will be required to cover variable costs while 25¢ will be profit. Thus additional sales multiplied by the Margin of Income ratio equals additional profit.

The Margin of Income ratio is used to calculate the breakeven volume of operations. At the breakeven point 25¢ out of each dollar is required to cover fixed costs and there are no additional dollars to contribute to profit. Therefore 25¢ multiplied by the sales volume at the breakeven point is equal to fixed cost. To find the breakeven volume we simply divide total fixed cost by the Margin of Income ratio. If a volume index other than sales value of production were used, the Margin of Income ratio would still represent the additional profit contri-

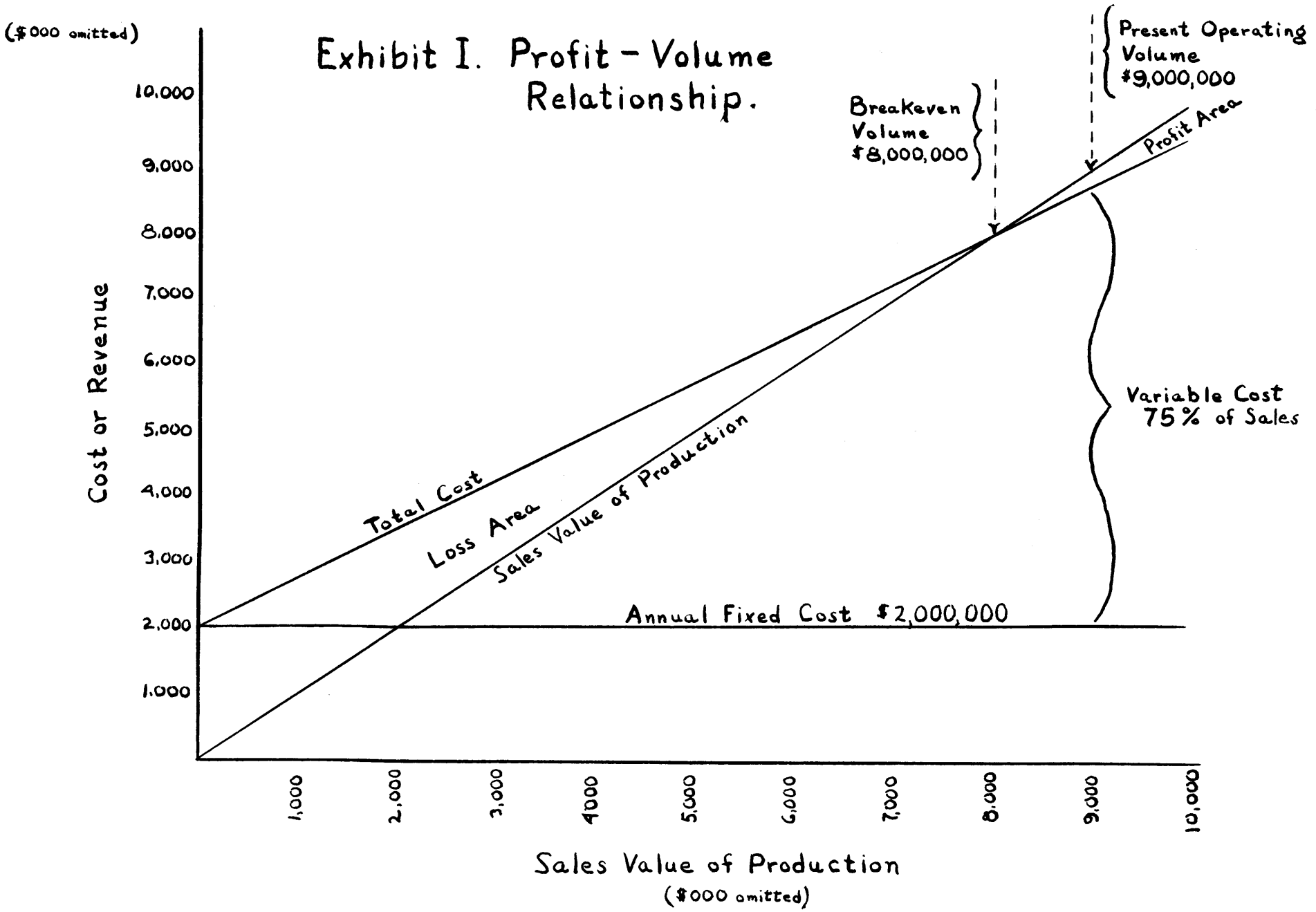


Exhibit I. Profit - Volume Relationship.
($000 omitted)
Cost or Revenue
10,000
9,000
8,000
7,000
6,000
5,000
4,000
3,000
2,000
1,000
Breakeven Volume $8,000,000
Present Operating Volume $9,000,000
Profit Area
Total Cost
Loss Area
Sales Value of Production
Variable Cost 75% of Sales
Annual Fixed Cost $2,000,000
1,000
2,000
3,000
4000
5,000
6,000
7,000
8,000
9,000
10,000
Sales Value of Production
($000 omitted)

buted by each additional unit of volume above the breakeven output. The use and meaning of the ratio would not be changed.

The most satisfactory range for the Margin of Income ratio is between 15% and 40%, preferably at about 25%. When the ratio is low, large changes in volume are required to produce any material change in profit or loss. If large increases in volume are attained at a low Margin of Income ratio, additional working capital may be required faster than it is made available by the marginal income. In such a case a business with inadequate working capital is apt to encounter financial difficulties. When the ratio is high, large profits and an easy cash position result from comparatively small increases in volume above the breakeven point. On the other hand, heavy losses will result from relatively small decreases in volume below the breakeven point. Under the present circumstances the firm we are considering has a favorable Margin of Income ratio.

The company is operating at a sales volume of $9,000,000. The breakeven point of operations is $8,000,000. Since a reduction in sales volume of 11.1% would bring the company to the breakeven point, this figure is known as the Margin of Safety. At the present sales volume profit before taxes is 25% of $1,000,000 or $250,000 -- only 2.77% of sales. Considering this low level of profit and the correspondingly small Margin of Safety, it is evident that there is room for improvement. Not

only is current profit low, but only a slight reduction in sales volume or prices would cause the company to operate at a loss. Nothing has been said about how close to capacity the $8,000,000 sales volume lies. If the possibility of increasing volume is limited by the plant's capacity, the situation is, of course, worse. How would the situation be altered by changes in volume, fixed or variable costs, selling price, or product mix? Breakeven analysis provides the answers to these questions.

The effect of an increase in volume can be seen from the chart. If sales volume were to rise to $10,000,000, profit would become $500,000 or 5% of sales. The breakeven chart gives management a clear picture of how a changing level of output will affect profits.

Suppose management were able to decrease the amount of fixed costs to $1,500,000. If other factors remained the same, the new breakeven point of operations would be $6,000,000. The change can be visualized by mentally lowering the fixed cost line in Exhibit I. The Margin of Safety would become 33.3%, while profit would be increased to $750,000 or 8.31% of sales.

A reduction in variable cost to 65% of sales would have a similar effect on the Margin of Safety and profit. The Margin of Income would now be 35% instead of 25% and the breakeven point would fall to $5,700,000. If sales remained at $9.000,000, the Margin of Safety would become 36.6%. Profit would rise to $1,160,000 or 12.8% of sales.

What would be the effect of a reduction in sales price upon profits? In order to know the complete story we would need to have information on the firm's demand. Without that information we do not know how much extra volume will result from a drop in price. Assume for the present that a 2% reduction in sales price would cause no change in physical volume of sales. Variable costs would now be 76.5% of sales and Marginal Income would drop to 23.5%. The breakeven volume would rise to $8,500,000 while revenue would fall to $8,820,000. The resultant Margin of Safety would be 2.76% for a profit before taxes of $75,200 or 0.85% of sales. Before adopting such a price cut management should have some indication that volume will increase enough to offset the lower selling price. Breakeven analysis tells management what the new volume of sales will have to be if no reduction in profit is to be taken. In this case, if management expected to continue to realize a profit of 2.77% of sales, the new sales volume would have to be $9,650,000.

The final management decision for which breakeven analysis can forecast the results is a change in the product mix. Suppose the company manufactures two different products. Product A has a ratio of variable cost to sales of 80%, while the corresponding ratio for Product B is only 70%. The Margins of Income will be 20% and 30% respectively. At the present time each product comprises one half of the total output. The overall Mar-

gin of Income for the firm is therefore the average, or 25%. Assuming fixed cost and current total sales volume to be the same as in Exhibit I, we find the same Margin of Safety of 11.1% and the same profit at 2.77% of sales. Management now wishes to know what the effect on profits would be if more emphasis were placed on the sale of Product B and less on that of Product A. Breakeven analysis would show that a reduction of Product A to 25% of output with a corresponding increase of Product B to 75%, provided overall sales volume remained the same, would improve the company's situation considerably. The new average Margin of Income would be raised to 27.5%. The breakeven volume would become $7,270,000. Profits would equal $476,000 or 5.28% of sales with a Margin of Safety of 19.2%.

The foregoing examples have been intentionally simplified for purposes of illustration. Only one factor was varied at a time whereas in an actual situation several factors would change at once. The gathering of data on costs, one of the most difficult phases of the analysis, was not discussed. The data used were stripped of complications such as would enter where a large number of different products were manufactured. And the theoretical limitations of the technique were ignored. Both the limitations of the technique and a practical application in an industrial situation will be discussed.

The point here is to illustrate the kinds of questions which breakeven analysis is equipped to answer. It has

been shown that they are questions with which management is vitally concerned. Provided the data are obtainable and the limitations can be overcome, breakeven analysis can be of considerable value to management.

IV. Limitations of the technique

The accuracy of breakeven analysis of the profit-volume relationship is limited by the approximations which must be made in order to obtain a linear cost function and a linear revenue function. Two general methods of determining the cost function have been described. The first consisted of an analysis of the past relationship between overall cost and volume while the second depended on an analysis of individual components of cost. Both methods rely on cost data covering past operations as a basis for prediction of how costs will vary with volume in the future. This would not be true in the case of a firm entering a new field of production in which it had no experience. Breakeven analysis in such a case would depend wholly on engineering cost estimates without the benefit of past experience. The cost function so determined would have less probability of accuracy than one based on past records. In considering the obstacles standing in the way of an accurate determination of the cost function we shall discuss only the common situation where past records are available.

No attempt to link cost with output can be precisely accurate for the reason that no direct relationship exists

between the two variables. The separation of all costs into a fixed and a variable component is a practical expedient which has a theoretical limitation. The relative variability of a particular cost depends on whether it is viewed in the long run or in the short run. In the short run a firm is limited by the machinery and equipment available at the moment. Depreciation charges are fixed in the short run. But in the long run a firm can invest in more machinery as the volume of production increases. Depreciation might be considered a variable cost in the long run. For practical purposes we make a clear distinction between the two types of costs. Actually, there are any number of graduations between complete fixity and complete variability in any one account.

Assuming that a clear distinction between fixed and variable costs does exist, we proceed in our attempt to find the relationship between cost and volume in past records in order to determine what the fixed and variable components are. In the records a particular cost will be found to correspond to a given output. There is, however, little assurance that at some future period the same cost will match the same volume of production. The outlay required to produce a certain amount of goods varies with the price paid for the factors of production and with changes in technology, in plant size, and in efficiency. It might be said that an increased cost for the factors of production would probably correspond to a generally higher price level. If the sales value of production were

used as a volume index and selling price changed in the same ratio as factor prices, there would be no change in the relationship between cost and output. The assumption that selling price and factor prices change in the same ratio is not likely to be correct, but this point is one argument in favor of the sales value index of volume. A more complete discussion of the choice of an output index has been presented previously. The fact that no output index can provide perfect correlation between cost and volume is in itself a limitation which should be considered. Regardless of the index chosen, however, changes in factor prices, technology, plant size, and efficiency disturb the relationship between cost and output.

Another limiting factor is the frequent inability of cost accounting to determine the incidence of costs. Maintenance expense is generally charged to the period in which the repair work was performed. But the equipment wear involved was due to the production of some prior period. Similarly, amortization of the undepreciated portion of equipment assets when they become obsolescent sooner than originally predicted is not related to production volume in the accounting records. A third group of costs in this category are selling costs. An outlay for advertising or salesmen's salaries may have no effect on volume for the period in which the expense is recorded, but may affect volume in some future period.

Another factor which interferes with the relation-

ship between costs and output is management's ability to exercise discretion in the timing of some expenditures. In a period of declining volume management will postpone necessary outlays for new equipment or new facilities. In a boom period management tends to be less conscious of production economies. When volume begins to fall off an effort is made to cut costs wherever possible and many savings are realized which might have been effected at a higher volume. An example would be the more effective utilization of the labor force in slack periods. There is no inflexible ratio of cost to output over which management has no control.

Inaccurate valuation of assets inserts a non-recurring element into profits. Valuation errors can have an important effect in a period of rapidly changing price levels. Inventory gains and losses arising from price variations tend to distort both the cost and profit functions. Rising prices cause understatement of depreciation when replacement cost at the end of the life of the equipment is greater than original cost.

In determining the cost function of an enterprise these limitations should be considered. An effort should be made to allow for changes in factor prices, technology, plant size and efficiency when examining past records. Care should be taken to select the most suitable output index. And the assumptions implicit in the cost accounting procedure should be borne in mind. Exact precision in the cost function cannot be achieved. With a reasonable

amount of effort a relationship can be found which will be suitable for the purposes of breakeven analysis, since the identification of major shifts in the cost function will often be more significant than perfect accuracy. Accuracy can be improved by the use of more complicated and expensive methods of analysis. It is up to the analyst to decide how much economy is to be sacrificed for additional accuracy.

Further limitations of breakeven analysis are linked with the determination of the revenue function. According to economic theory, sales volume in most cases changes continuously with price, the exceptions occurring in price competition or price monopoly. Breakeven analysis follows a practice contrary to the theory in assuming a constant selling price at every output. The principle of the demand curve is considered beyond the scope of breakeven technique. Although the assumption of a constant selling price may be contray to actual conditions, it greatly simplifies the analysis and is not a serious limitation. In cases where selling price changes a family of revenue lines can be drawn. The way in which the breakeven chart is altered when a new selling price is assumed was discussed in the section on uses of breakeven analysis.

The only stable revenue line is achieved when the sales value of production is used as an output index. When a physical index is used the revenue at a given volume will change as product mix is varied. But a change in product mix will alter the profit-volume relationship

with either type of output index if the contribution to profit varies between products. The best solution in such a case is to draw a family of cost and revenue lines for representative product mixes.

The limitations of the breakeven chart technique impair but do not destroy its usefulness. Its major weakness lies in oversimplification of the correlation between profit and volume. Greater accuracy can be achieved by greater attention to detail. For the purpose of providing a profit-making tool for management perfect accuracy can safely be sacrificed to economy and convenience. The limitations of the technique should be considered in planning a breakeven analysis and in the uses to which it is put. They need not destroy confidence in it.

V. Case study -- Murray Radio Company

The Murray Radio Company is a small manufacturer of low-priced radios. It operates a single plant in upstate New York employing about one thousand men and women. The company limits itself to the production of low-priced radios, but turns out a wide variety of models within that price range. In spite of its small size the company is modern and progressive. It is operated by a skilled management which is constantly looking for new ways of improving the profit-making potentialities of the business. By constant attention to sound business and managerial practices the company has established itself over a period of years as one of the permanent contenders in its field.

Murray Radio operates under a flexible budget based on a six-month budgeting period. The significant characteristic of a flexible budget is the development of some sort of formula to show how costs vary with output. A fixed budget shows estimated costs at one volume only. If sales forecasts are in error the fixed budget provides no way of revising cost estimates to correspond with the new level of production. The flexible budget has the advantage that, if sales forecasts are wrong, the company will still have a standard by which to judge the effectiveness of its operations. The Murray Radio Company develops cost formulae as a part of its regular accounting routine. We shall see how these formulae facilitate the breakeven analysis of operations in this company.

The form in which the budget is prepared is illustrated in Exhibit II. The year is divided into two equal budgeting periods. At the end of each period the current estimates for the ensuing six months are devised in line with recent developments. At the same time a forecast is prepared for the period starting a year and a half hence which has not yet been covered. In this way the budget is constantly being brought up to date and extended to cover a span of two years in the future.

The sales forecast upon which the budget is based is derived from estimates received from sales outlets in various parts of the country. Deductions from sales such as returns and discounts are budgeted at current going rates based upon past experience and current trends.

Exhibit II

Profit & Loss Statement -- Budget 1949

	1949 1st Half	1949 2nd Half	1949 Total Year
Gross Sales	5,303,000	3,931,200	9,233,200
Deductions from Sales	426,930	302,006	728,936
Net Sales	4,876,070	3,629,194	8,505,264
Cost of Goods Sold at Standard	3,643,210	2,718,240	6,361,450
% of Net Sales	74.7	74.9	74.8
Gross profit at Standard	1,232,860	910,954	2,143,814
Variances from standard			
Volume		183,758	183,758
Operating		48,216	48,216
Price			
Total		231,974	231,974
Gross Profit at Actual	1,232,860	678,980	1,911,840
Operating & Other Expenses			
Warehousing	137,318	141,560	278,878
Selling	491,730	498,424	990,154
Advertising	139,824	151,624	291,448
Administration	162,544	148,248	310,792
Other Charges less Other Income -- Net	-21,000	-21,000	-42,000
Total	910,416	918,856	1,829,272
Income before Federal Income Tax	322,444	-239,876	82,568
% of Net Sales	6.6	-6.6	0.97

Cost of Goods Sold is composed of estimated costs for materials, direct labor, and overhead at the forecasted volume of production. Usage and price standards are used to predict the probable outlays for direct labor and materials. Overhead costs are computed from the current budget formula. It will be noted that a variance from standard of $231,974 is included in the budget for the second half of the year. A contemplated rise in overhead expenses is responsible for this variance. It is believed that the current budget formulae predict overhead expenses at a lower rate than will actually be realized during the second half of the year. Finally, operating expenses, like deductions from sales, are based on past experience and current trends.

The total cost of sales is built up from allowances for each individual account. The person in charge of each account is expected to operate within his allowance if it is possible to do so. The company does not expect that performance will coincide perfectly with predictions. The budget does, however, provide a standard for gauging individual and overall performance. When variations occur management has a good chance of locating the trouble.

The development of the budget formulae is an important part of the accounting procedure as it applies to breakeven analysis. Each overhead account contains both a fixed and a variable component. The separation of fixed and variable costs is a basic part of any breakeven analysis. The budget formulae provide this separation. They are

derived from analyses of the past history of each overhead account by means of scatter diagrams. These scatter diagrams, therefore, are the foundation of breakeven analysis at Murray Radio.

The first step in constructing such a scatter diagram is to select an appropriate index of production activity. For the analysis of individual overhead accounts the number of direct labor hours is a suitable index. The Murray Radio Company bases all of its variable budget formulae on direct labor hours. For the purposes of breakeven analysis the index is later changed to the sales value of production.

Having selected a base line for the scatter chart, the next step is to plot a series of values of cost versus production activity. Take, for example, indirect labor cost. Approximately twelve monthly values of indirect labor cost are plotted against the corresponding volume expressed in direct labor hours. The number of points required for the plot depends on how consistent the values appear to be. When a given value for cost appears to be far out of line with the corresponding number of direct labor hours, that point is ignored. Additional points are plotted until a definite trend of cost versus production volume appears.

A straight line is now drawn through the points on the graph and extended to intersect the ordinate. This cost line is located by visual inspection or by the rule of least squares, a statistical method for correlating

two sets of data. The analyst does not attempt a perfectly accurate placement of the line. Since no method can give absolute accuracy in the field of budgeting, cost analysis, and profit determination, it is felt that this technique is sufficiently precise without being unduly complicated.

If the account is completely variable, the line will intersect the vertical axis at the zero point indicating zero fixed cost. A completely fixed account would be represented by a horizontal line. Finally, a semi-variable account appears as a line sloping upward to the right and intersecting the vertical axis at a value greater than zero. A typical graphical analysis of an overhead account is pictured in Exhibit III.

Exhibit III

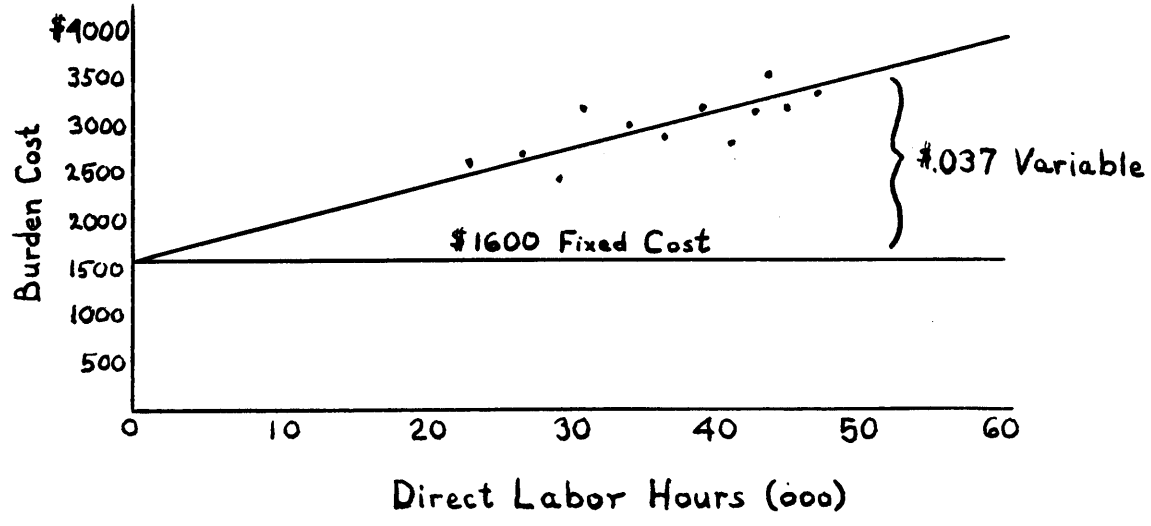
Graphic Analysis of Past Operations

Separation of Variable and Fixed Costs
to Establish Standards for the Control of Burden

Actual Burden

Variable Cost -- $.037 per Direct Labor Hour

Fixed Cost -- $1600 Per Month



In the case where past operating conditions are substantially different from those planned for the future, a graphic analysis of past operations cannot be used. Here the separation is done mathematically. A high and a low volume of production are selected to represent the probable maximum range of future operations. The costs at both the high and the low operating levels are estimated, based on known or expected conditions. Calculation of fixed and variable costs is then accomplished as in Exhibit IV.

Exhibit IV

Mathematical Formula for Separation of Variable and Fixed Costs to Establish Standards for Control of Burden.

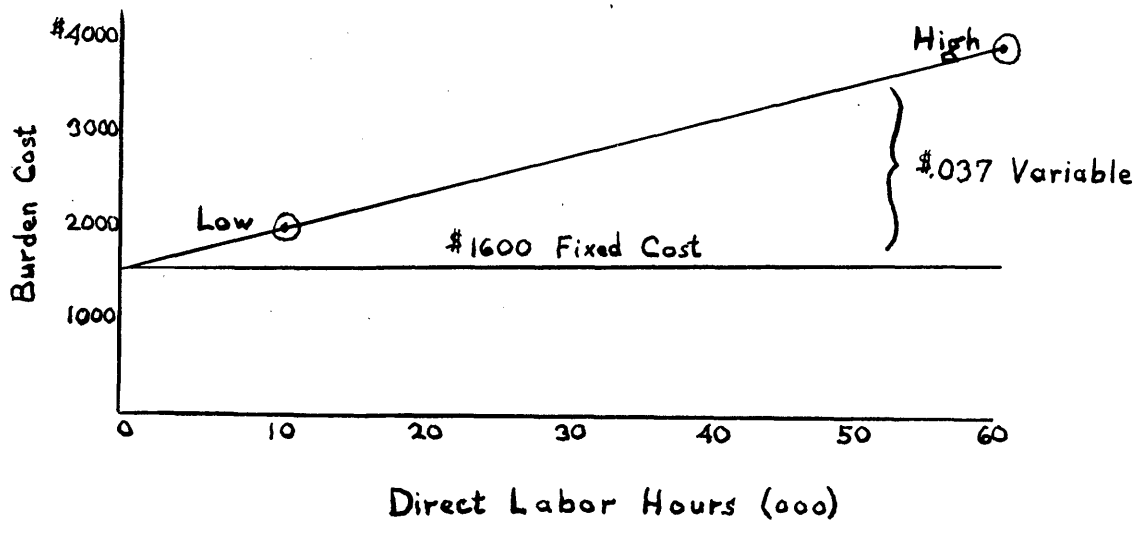
(1) To determine the ratio of variable costs to direct labor hours:

Monthly Direct Labor Hours		Indirect Labor Cost
High Volume	60,000 hrs	$3,820
Low Volume	10,000 hrs	1,970
Variance	50,000 hrs @ $.037	$1,850

(2) To determine the monthly fixed cost:

	Low Volume	High Volume
Total Indirect Labor Costs	$1,970	$3,820
Variable Cost @ $.037 per Labor Hr.	370	2,220
Monthly Fixed Cost	$1,600	$1,600

Graphical representation:



An analysis of the sort described is performed on each individual overhead account. The results of this series of analyses are presented in Exhibit V* under the section entitled "Budget Formula." The figure for "variable cost per standard direct labor hour" is drawn directly from the graphic or mathematical analysis of the account, as is the amount listed under "monthly fixed cost." In order to determine the "percent of the standard direct labor dollar" the following calculation is performed:

Calculation of "% of Standard Direct Labor Dollar"

Standard monthly direct labor hours	55,000 (see Exhibit VI)
Indirect labor cost per standard direct labor hour	$0.1240 (see Exhibit V)
Total indirect labor cost	55,000 x 0.1240 = $6,820
Standard monthly direct labor payroll	$61,596 (see Exhibit VI)
Percent of standard direct labor dollar	$\frac{6,820}{61,596}$ x 100 = 11.09%

The budget formula thus derived is used to calculate the monthly allowable overhead for each account. Three different allowances are determined corresponding to three different production volumes -- low, high, and standard. The following calculations illustrate how the monthly allowable costs are obtained:

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* See p. 39

Calculation of "Monthly Allowable Cost"

Monthly Variable Indirect Labor		
at Standard Volume	55,000 × 0.1240 =	$6,820
at High Volume	80,000 × 0.1240 =	$9,920
at Low Volume	40,000 × 0.1240 =	$4,960
Monthly Fixed Indirect Labor		$5,427
Total Monthly Indirect Labor		
at Standard Volume		$12,247
at High Volume		$15,347
at Low Volume		$10,387

Costs collected in the various overhead accounts are distributed among the several departments according to four different bases. These are Direct Labor Hours, Direct Labor Dollars, Floor Space, and Analysis. The budget control bases are indicated in the upper left-hand corner of Exhibit V together with the proportion of total overhead which is distributed according to each basis. The code letters correspond to those appearing in the column entitled "Cost Center Distribution." To illustrate the procedure, take the allowable cost for Supervision at high volume. This charge of $1450 must be distributed among the several cost centers. The actual proportion of this time which each supervisor devotes to directing the activities of a particular cost center is not known. Since the amount of supervision required stands in a direct relationship to the amount of labor employed, a suitable approximation is to allocate charges for supervision according to the number of direct labor hours used in each cost center. This data is obtainable from Exhibit VI.* Exhibit VII** shows the final result of the allocation of

* See p 43.
** See p 44.

overhead and the calculation of the corresponding burden rates.

Exhibits VIII* and IX** illustrate the meaning of the budget allowances by individual accounts. The cash amounts allowed for Indirect Labor and Expense Labor at various production volumes are interpreted in terms of specific manpower requirements.

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* See p 45.

** See p 47.

Exhibit V

Variable Budget Allowances by Individual Account

Budget Control Base	Distribution of Monthly Allowable at Standard		Monthly Volume	
A. Direct Labor Hours	$17,711	17.8		
B. Direct Labor Dollars	13,021	13.1	Low 40,000 DL hrs	Total Plant
C. Floor Space	12,162	12.3	High 80,000 DL hrs	"
D. Analysis	56,354	56.8	Std 55,000 DL hrs	"
	$99,248	100.0%		

Overhead Accounts	Budget Formula			Monthly Allowable Cost		Cost Center Distribution
	Variable Cost Per Std DL Hour	Variable Cost %of Std DL $	Monthly Fixed Costs	Low	High	
Indirect Labor						
Dept A	$.0906*	8.50%*	$1,145	$2,595	$4,045	
Dept B	.2731*	22.51*	1,556	3,468	5,380	
Dept C	.0700*	5.33*	1,060	1,410	1,760	
Dept D	.0912*	8.60*	1,372	2,102	2,832	
Dept E	.1292	12.48*	294	811	1,328	
Total Indirect Labor	$.1240	11.09%	$5,427	$10,386	$15,345	D
Gen'l Plant Costs						
51 Supervision	$.0075	.67%	$ 850	$ 1,150	$ 1,450	A
56 Indust. Engg.	.0267	2.39	2,171	3,240	4,309	A
57 Purchasing	.0094	.84	985	1,360	1,735	A
88 Plant Acct.	.0105	.94	2,640	3,060	3,480	A
59 Quality	.0166	1.49	1,454	2,118	2,782	A
61 Shipping & Finished Stock	.0375	3.36	1,500	3,000	4,500	D
62 Personnel	.0120	1.07	465	946	1,427	A
63 Product. Control	.0754	6.74	3,264	6,278	9,292	D
64 Material Inspec.	.0109	.97	620	1,054	1,488	D
67 Salvage Labor	.0050	.45	216	425	634	D
68 Miscellaneous	.0050	.45	250	450	650	A
16 Travel	.0050	.45	400	600	800	A
30 Periodicals & Membership	----	---	25	25	25	A
38 Raw Mat'l Losses	.0250	2.23	500	1,500	2,500	D
43 Scrap & Salvage	-.0050	-.45	-100	- 300	- 500	D
52 Moving & Install.	.0075	.67	200	500	800	A
57 Auto & Truck Exp.	.0013	.12	100	150	200	A
Total Genl Plant Costs	$.2503	22.39%	$15,540	$25,556	$35,572	

Exhibit V (cont.)

Overhead Accounts	Budget Formula			Monthly Allowable Cost		Cost Center Distribution
	Variable Cost Per Std DL Hour	% of Std DL $	Monthly Fixed Costs	Low	High	
Mechanical Maintenance						
52 Labor	$.0066	.59%	$ 961	$ 1,223	$ 1,485	D
02 Material	.0125	1.12	400	900	1,400	D
Total Mech Maint	$.0191	1.71%	$1,361	$ 2,123	$ 2,885	
Tools & Supplies						
04 Small Tools	$.0050	.45%	$ 300	$ 500	$ 700	A
06 Parts Packing	.0056	.50	25	250	475	D
19 Stationery	.0050	.45	100	300	500	A
20 Factory Supplies	.0150	1.34	500	1,100	1,700	D
47 Degreasing & Treating Solvents	.0200	1.79	300	1,100	1,900	D
Total Tools & Supplies	$.0506	4.53%	$1,225	$ 3,250	$ 5,275	
Utilities						
07 Telephone & Tel.	$.0075	.67%	$ 400	$ 700	$ 1,000	A
10 Manufacturing Gas	.0250	2.24	400	1,400	2,400	D
14 Power	.0163	1.45	100	750	1,400	D
21 Water	.0013	.12	150	200	250	D
Total Utilities	$.0501	4.48%	$1,050	$ 3,050	$ 5,050	
Payroll Taxes, etc.						
25 Ins. (Exc. Bldg)	$.0075	.67%	$ 240	$ 540	$ 840	B
28 Employees Service	.0056	.50	105	330	555	A
29 Group Insurance	.0038	.34	110	260	410	A
73 Vacation Accrual	.0575	5.14	900	3,200	5,500	B
78 Unemploy. Ins.	.0400	3.59	450	2,050	3,650	B
79 FOAC	.0143	1.28	160	730	1,300	B
80 Pension Expense	.0350	3.13	600	2,000	3,400	B
81 Awards, Prizes, Tuition	.0006	.05	25	50	75	A
84 Paid Holiday	.0310	2.77	220	1,460	2,700	B
87 Vacation Payroll Tax	.0038	.34	50	200	350	B
Total Payr'l Taxes	$.1991	17.81%	$2,860	$10,820	$18,780	
Overtime & Premium						
Direct Labor	$.0874	7.82%	-$3,494	----	$ 3,494	D
Indirect Labor	0263	2.35	- 1,050	----	1,050	D
Expense Labor	----	----	1,000	1,000	1,000	D
Total OT & Premium	$.1137	10.17%	-$3,544#	$1,000	$ 5,544	

Exhibit V (cont.)

Overhead Accounts	Budget Formula			Monthly Allowable Cost		Cost Center Distribution
	Variable Cost Per Std DL Hour	% of Std DL $	Monthly Fixed Costs	Low	High	
Customer's Service						
66 Labor	$----	----	$ 512	$ 512	$ 512	D
93 Material	.0025	.22	100	200	300	D
Tot.Customer's Serv.	$.0025	.22%	$ 612	$ 712	$ 812	
Cafeteria						
55 Labor 77 Other Costs	Because Cafeteria is supposed to operate on a breakeven basis, no budget allowable was set up for this account.					
Occupancy	$.0116	1.04%	$11,524	$11,987	$12,450	C
Depreciation	$ ----	----	$ 3,725	$ 3,725	$ 3,725	D
Special Burden 41 Dies & Molds	$-----	----	$ 1,800	$ 1,800	$ 1,800	D
Engineering	$ ----	----	$13,000	$13,000	$13,000	D
Tot. Monthly O'head	$.8210	73.44%	$54,580	$87,409	$120,238	
Total Monthly Overhead less OT	$.7073	63.27%	$58,124	$86,409	$114,694	

Note: #For simplicity the variable std for OT & Premium is computed on a straight line basis. This assumes that the relative amount of OT & Premium is as great at low volume as at high. The red adjusting figures automatically correct this when calculating allowables.

*These are variable rates to be applied on the basis of Cost Center Activity. All others are based on total plant activity.

Exhibit V (cont.)

Summary by classes of overhead

Overhead Accounts	Budget Formula			Monthly Allowable Cost	
	Variable Cost Per Std DL Hour	Variable Cost % of Std DL $	Monthly Fixed Costs	Low	High
Indirect Labor	$.1240	11.09%	$ 5,427	$10,386	$ 15,345
Expense Labor	.2181	19.51	15,638	24,366	33,094
OT & Premium	.1137	10.17	- 3,544	1,000	5,544
Mfg Expenses	.3652	32.67	22,259	36,857	51,455
Special Burden	----	----	1,800	1,800	1,800
Engineering	----	----	13,000	13,000	13,000
Total Monthly OH	$.8210	73.44%	$54,580	$87,409	$120,238
Total Monthly OH Less OT	$.7073	63.27%	$58,124	$86,409	$114,694

Exhibit VI

Estimated Direct Labor Activity Used as a Basis for Calculating Variable Budget Allowances

Cost Centers	Average Hourly Rate	Direct Labor Hours Per Month		Monthly Direct Labor Payroll	
		Low	High	Low	High
Dept A	$1.066	16,000	32,000	$17,056	$34,112
Dept B	1.213	7,000	14,000	8,491	16,982
Dept C	1.313	5,000	10,000	6,565	13,130
Dept D	1.060	8,000	16,000	8,480	16,960
Dept E	1.035	4,000	8,000	4,140	8,280
Total Plant	$1.1183	40,000	80,000	$44,732	$89,464

Exhibit VII

Variable Budget Allowances
and Burden Rates by Cost Centers

Cost Centers	Budget Formula			Monthly Allowable Overhead		
	Variable Cost Per Std DL Hour	% of Std DL $	Monthly Fixed Costs	Low	High	Std
Dept A	$.7147	67.05	$29,971	$40,447	$53,594	$43,734
Dept B	1.7252	142.23	5,218	16,227	28,421	21,253
Dept C	.4916	37.44	5,138	7,554	10,091	8,902
Dept D	.5397	50.92	8,769	12,913	17,566	15,821
Dept E	.6352	61.37	6,484	8,828	11,666	9,538
Total Plant	$.8210	73.44%	$54,580	$85,969	$121,338	$99,248

	Burden Rate Per Standard Direct Labor Hour			Burden Rate as a % of Std DL %		
	Low	High	Std	Low	High	Std
Dept A	$2.53	$1.67	$2.24	237%	157%	209.8%
Dept B	2.32	2.03	2.12	191	167	174.8
Dept C	1.51	1.01	1.27	115	77	96.4
Dept D	1.61	1.10	1.16	152	104	109.5
Dept E	2.21	1.46	1.90	213	141	183.7
Total Plant	$2.15	$1.52	$1.81	192%	136%	161.5%

Exhibit VIII

Analysis of Indirect Labor Requirements

	No.of Employees Low	Ave. Mo. Payroll	No.of Employees High	Ave. Mo. Payroll
Dept A	17,000 hrs		34,000 hrs	
Foreman	1		1	
Prod. Supvr I	1		3	
Floor man	2		4	
Group leader	2		2	
Service Man	1		1	
Packer-Special Work	2		4	
Tool & Fixture Attendant	1		1	
Utility Operator	1		1	
Total	11	$2,595	17	$4,045
Dept B	7,000 hrs		14,000 hrs	
Foreman	1		1	
Prod. Supvr I	1		2	
Group Leader	3		6	
Booth Cleaners	4½		5	
Paint Mixer	1		1	
Paint Mix Helper	1		2	
Floor Man Sludge Handler	1		2	
Salvage	2		4	
Total	14½	$3,468	23	$5,380
Dept C	5,000 hrs		10,000 hrs	
Foreman	1		1	
Prod. Supvr I	1		2	
Die & Tool Maint.	2		2	
Set Up	1		1	
Total	5	$1,410	6	$1,760
Dept D	8,000 hrs		16,000 hrs	
Foreman	1		1	
Prod. Supvr I	1		1	
Coil Winder Set-up Maintenance	1		1	
Floor Man	3		6	
Utility Operator	1		1	
Salvage	2		3	
Total	9	$2,102	13	$2,832

Exhibit VIII (cont.)

	No.of Employees Low	Ave. Mo. Payroll	No.of Employees High	Ave. Mo. Payroll
Dept E	4,000 hrs		8,000 hrs	
Foreman	1		1	
Prod. Supvr			1	
Floor Man & Mech Helper	1		1	
Stock Handler - Clerk & Shipper	1		2	
Total	3	$ 811	5	$ 1,328
Total Indirect Labor	42½	$10,386	64	$15,345

Summary of Direct and Overhead Labor

	Total Personnel Low	Total Personnel High		Total Salaries & Wages Low	Total Salaries & Wages High
Direct Labor	244	488	Direct Labor	$44,732	$89,464
Indirect Labor	42½	64	Indirect Labor	10,386	15,345
Expense Labor	89	124	Expense Labor	24,366	33,094
Total	375½	676		$79,484	$137,903

Exhibit IX

Analysis of Expense Labor Requirements

	No. of Employees Low 40,000 Hrs	Average Monthly Payroll	No. of Employees High 80,000 Hrs	Average Monthly Payroll
51 Supervision				
Plant Mgr	1		1	
Secretary			1	
Supt			1	
Genl Foreman	1			
Total	2	$1,150	3	$1,450
52 Maintenance -- Mech & Elec				
Mechanics B	4		5	
Gas & Elec Equip	1		1	
Total	5	$1,223	6	$1,485
56 Indust. Engineer.				
Supervisor	1		1	
Engineers Sr.	4		6	
Clerk Sr.	1		1	
Secy-Stenog.			1	
Mastercraftsman	1		1	
Machinist 1st class	1		1	
Total	8	$3,240	11	$4,309
61 Shipping				
Supervisor	1		1	
Foreman II	1		1	
Product. Supvr II	1		1	
Prod.Supply-Hourly			1	
Clerk			1	
Handler-Clerk	1		3	
Stock Hander A	7		9	
Stock Handler & Elec Trk Oper.	1		2	
Total	12	$3,000	19	$4,500
62 Personnel				
Supervisor	1		1	
Interview & Counselor			1	
Clerk Sr.	1		2	
Nurse	1		1	
Total	3	$ 946	5	$1,427

Exhibit IX (cont.)

	No. of Employees Low 40,000 Hrs	Average Monthly Payroll	No. of Employees High 80,000 Hrs	Average Monthly Payroll
63 Prod. Control				
Supervisor			1	
Prod. Supvr I	4		5	
Foreman I	3		4	
Secy - Stenog	1		1	
Stock Handlers	9		15	
Elec Trk Oper	2		3	
Group Leaders	4		6	
Receiving Clerk	1		1	
Salvage	2		2	
Total	26	$6,278	38	$9,292
66 Customer Service				
Prod. Supvr I	1		1	
Utility Operator	1		1	
Total	2	$ 512	2	$ 512
57 Purchasing				
Chief Pur. Agent	1		1	
Purchasing Agents	1		2	
Secretary	1		1	
Total	3	$1,360	4	$1,735
58 Plant Accounting				
Supvr - Cost Sec.	1		1	
Clerks - Cost	4		5	
Supvr - Genl Acct.	1		1	
Clerks - General	4		5	
Functional Clerk	1		1	
Payroll Clerks	2		2	
Total	13	$3,060	15	$3,480
59 Quality				
Dept Supvr	1		1	
Asst to Supvr	1		1	
Secy - Stenog	1		1	
Prod. Supvr II	2		3	
Floor Man	1		2	
Inspectors	2		3	
Total	8	$2,118	11	$2,782
64 Materials Insp Dept				
Prod. Supvr II	1		1	
Inspectors	2		2	
Inspectors (Parts Check & Handlers)	2		4	
Total	5	$1,054	7	$1,488
67 Salvage	2	425	3	634
Total Expense Labor	89	$24,366	124	$33,094

The Murray Radio Company is at present faced with a substantial decline in sales volume. This decline is attributed by the management to two factors. The first cause is believed to be the business conditions prevailing in the country as a whole at the present time. It is felt that part of the reduction in sales volume is symptomatic of the return to more normal levels of production which many industries are experiencing as the postwar boom levels off. The second reason for declining sales is thought to be the entrance of low-cost producers into the industry. The nature of the business makes it possible to go into the production of low-priced radios without a very substantial capital investment. As a result of the postwar boom many small shops have entered the field. Because of the smallness of their operations and their concentration on one or two models they have been able to produce more cheaply than Murray Radio. The price-cutting tactics of these small producers are believed by the Murray management to be an important cause of the reduction in sales volume.

The adverse effect of the declining sales volume on profits will be exaggerated by a contemplated rise in the ratio of cost of goods sold to sales. In order to assist its jobbers in meeting the low-priced competition the company has had to make available a special low-priced model. The introduction of this model is expected to improve sales volume somewhat, but since the model is very low-priced in relation to cost, its contribution against overhead and

profit will be small. The model will constitute a large percentage of sales and will therefore affect the overall cost-price relationship of the company to a considerable extent.

A second factor contributing to the adverse relationship of cost of goods to sales is the rise in the ratio of overhead charges to sales in comparison with prior periods. This rise is characteristic of administrative and selling expenses as well as of manufacturing.

As a result of the declining sales volume, budgeted sales for the first half of this year are far above actual sales. The profit forecast for the six-month period has proved far too optimistic. And this adverse sales trend, together with the contemplated rise in the ratio of cost of goods to sales, has created a gloomy outlook for the second half of the year. It is expected that a loss will be taken for that period.

Management at Murray Radio has become seriously interested in the profit-volume relationship. It wants to know at what point in the declining volume of sales the company will begin to lose money. It wants to know what profits or losses can be expected at various levels of production. Finally, it wants to know what can be done to minimize its losses.

In April management instructed the accounting department to develop a preliminary analysis of the profit-volume relationship in the form of a breakeven chart.

The analysis was to cover the period from the first of the year through June. The purpose of the work was largely to determine whether this form of analysis could be of any value to management. If it appeared that the work could be performed in a reasonable amount of time and that the results obtained were of some significance, management would continue the project as a regular part of its forecasting procedure. The cost formulae prepared in connection with the flexible budget were of great help in the breakeven analysis. All of the information needed for determination of the cost function was readily available. The necessary figures were taken from the flexible budget records and compiled in the form shown in Exhibit X.

Since overhead expenses are analyzed on a monthly basis, the breakeven analysis was performed on that basis also. The budgeted figure for gross sales had to be converted into an average monthly volume. The half-year estimate of gross sales was $5,303,000. The budgeted amount of okay returns was $49,760. Subtracting this amount we obtain a value of gross sales less okay returns of $5,253,240. On a monthly basis the average value was $875,540. This figure was the basis for the remaining calculations.

Budgeted amounts for freight, discounts, and defectives were used to determine the percentage figure shown on the calculation sheet. For the first half of 1949 the calculation was as follows:

Exhibit X. Analysis of Costs and Profit at Various Levels of Production.

	Variable %	Fixed Cost	Breakeven Volume	Jan.-June '49 Average Budget Volume	Jan.-June '49 Budget + 25% Increase	Capacity*
Gross Sales (less ok returns)	100.00		$585081	$875540	$1094425	$1491067
Freight, Discounts, Defectives	7.17	—	41950	62776	78470	106910
Manufacturing Cost						
Materials	51.60	—	301902	451779	564723	769391
Direct Labor	6.00	—	35105	52532	65666	89464
Indirect Labor	.67	$5427	9347	11293	12760	15417
Expense Labor	1.17	15638	22483	25882	28444	33083
Overtime and Premium	.61	−3544	25	1797	3132	5552
Manufacturing Expense	1.96	24059	35527	41220	45510	53284
Engineering	—	13000	13000	13000	13000	13000
Total Mfg. Cost	62.01	$54580	$417389	$597503	$733234	$979191
Operating Expenses	9.19	71973	125742	152435	172550	209002
Grand Total	78.37	$126553	$585081	$812714	$984254	$1295103
Profit			—	62826	110171	195964
% Profit			—	7.18	10.07	13.14

Marginal Income	21.63%
Breakeven Volume	$585,081
Margin of Safety	33.18%

* Based on "High" of Variable Budget

Freight Out	$224,620
Cash Discount	127,015
Defectives	19,755
	$371,390
Gross Sales less ok Returns	$5,303,000
$371,390/$5,303,000 =	7.17%

To determine the materials and direct labor percentages to sales the budgeted amounts for these costs were taken from the budget for the first half of the year. It will be recalled that standard cost data were used in estimating the budget requirements.

The source of the figures for indirect labor, expense labor, overtime and premium, manufacturing expense, and engineering can be found in Exhibit V under "Summary by Classes of Overhead." The index of volume used in Exhibit V is the amount of direct labor expended during the month. The index used in the breakeven analysis was the value of sales for the month. Therefore the variable cost figures of Exhibit V had to be converted to the new basis. Variable overhead costs as a percentage of the direct labor dollar were multiplied by the direct labor percent to sales in order to obtain the percentage of variable overhead costs to sales. The fixed component of overhead was transferred to the breakeven calculation sheet unchanged except that, for simplicity, the fixed charge for special burden was combined with the fixed component of manufacturing expense.

The charge for operating expenses, which included selling, advertising, and administrative expenses, was

taken from the budget. It was broken down into a fixed and a variable component in line with past experience.

The sum of the individual variable cost percentages represents the fraction of each sales dollar which goes toward the recovery of variable costs. In this case the fraction was 78.37%, which left 21.63% of the sales dollar for recovery of fixed costs and for profit. The Margin of Income ratio was therefore 21.63%. If the company were just breaking even, sales volume multiplied by the Margin of Income ratio would equal fixed costs. Therefore to calculate the breakeven point of operations we simply divide the total estimated fixed costs for the month by the Margin of Income ratio. The breakeven volume was $585,081. This was 33.18% less than the estimated sales volume of $875,540. The Margin of Safety was therefore 33.18%.

The calculation of profit, using the Margin of Income ratio, is shown below for several levels of volume:

	Sales		Profit
Jan.-June '49 Ave. Budget	$825,540		
Breakeven Volume	585,081		
	$290,459	@21.63% Margin of Income	$62,826
Jan.-June '49 Budget	$1,094,425		
+ 25% Increase			
Jan.-June '49 Ave. Budget	875,540		
	$218,885	@21.63% Margin of Income	47,345
			$110,171
Capacity Volume	$1,491,067		
Jan.-June '49 Budget	1,094,425		
+ 25% Increase			
	$396,642	@21.63% Margin of Income	85,793
			$195,964

The final step in the analysis was the construction of the breakeven chart shown in Exhibit XI. This merely involved graphing the information compiled on the calculation sheet. It added nothing to the data except an easily-understood presentation.

After examination of the breakeven chart submitted by the accounting department, management decided that a breakeven analysis of operations should be a regular part of the forecasting procedure. A breakeven chart would be prepared on the basis of the budget for the second half of the year in order that a comparison might be made between the two charts. It was expected that the comparison would clearly point out the effect on profits of declining sales volume and increasing ratio of cost of goods sold to sales. At the end of the first budgeting period in June a revised budget would, as usual, be prepared for the next half-year. A breakeven chart for the second half of the year would be prepared using the revised figures. It was hoped that an improvement might be shown. Meanwhile, management planned definite action to make sure that the picture would be improved. The first chart prepared showed a Margin of Safety of 33.18%. In other words, a 33.18% reduction in sales volume could be sustained without taking a loss. It was expected that final figures for the period from January to June would reveal a 40% reduction in volume from that originally budgeted. Management thought reductions in both fixed and variable costs

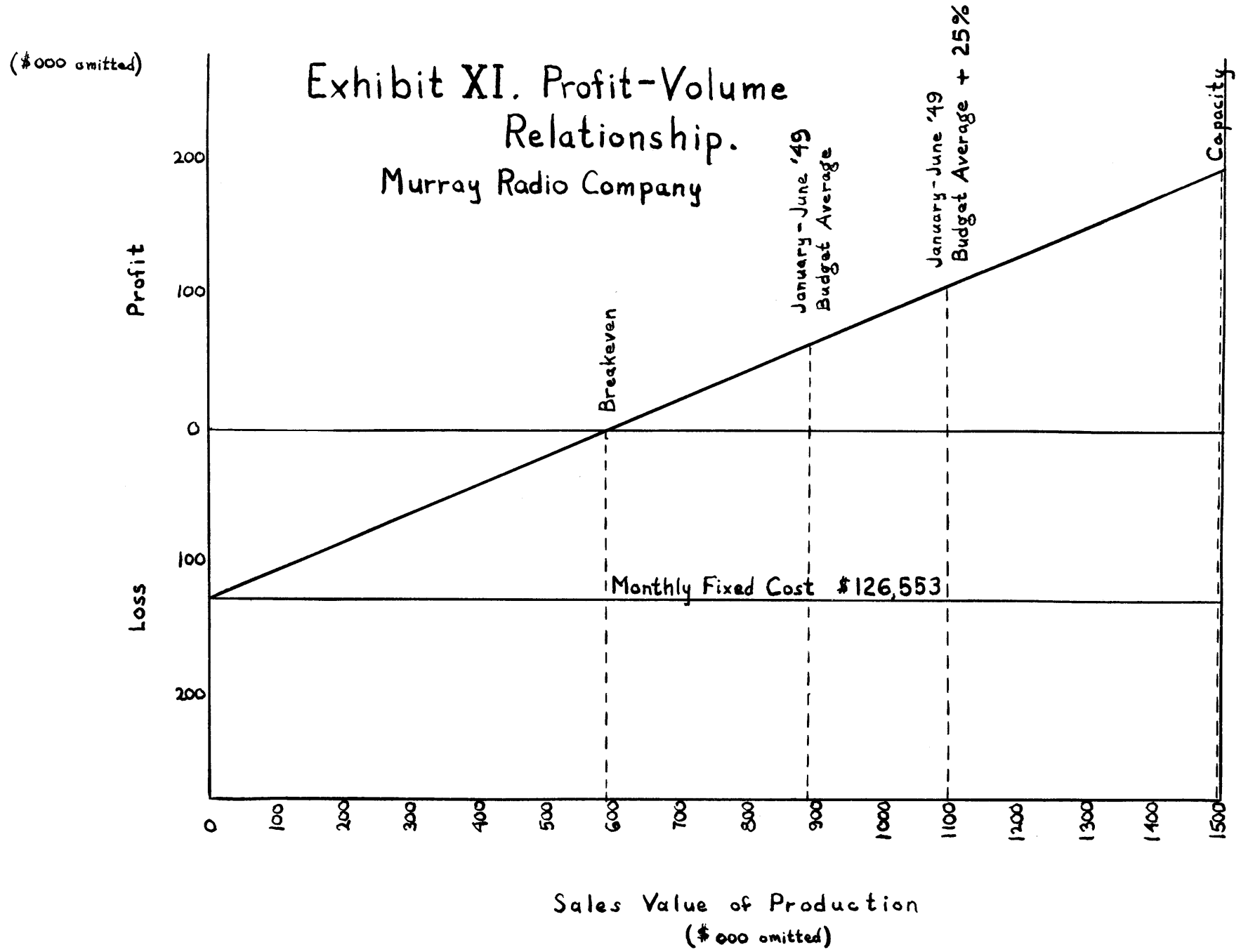
Exhibit XI. Profit-Volume Relationship.
Murray Radio Company
($000 omitted)
Profit
Loss
200
100
0
100
200
Breakeven
January-June '49 Budget Average
January-June '49 Budget Average + 25%
Capacity
Monthly Fixed Cost $126,553
0
100
200
300
400
500
600
700
800
900
1000
1100
1200
1300
1400
1500
Sales Value of Production
($000 omitted)

would be advisable. If both types of costs were reduced, the breakeven point might be lowered considerably. Management was already conducting a study to determine where costs might be cut. In an attempt to reduce variable costs an analysis of materials handling procedures was being made. The various departments at Murray Radio are separated from one another to a considerable extent and there is a great deal of handling of material as it passes from one department to the next. Management hoped to reduce the costs of labor used in handling materials by possibly changing the plant layout in some way and by streamlining procedures. It was also hoped that fixed costs might be cut by reducing the amount of indirect and expense labor employed. An analysis of the office force was then in progress to determine whether some of the production control, accounting, and other personnel might be eliminated from the payroll. Finally, management hoped to further reduce variable costs by obtaining lower material costs and improving labor efficiency. By continual concentration on costs management intended to make the best of the situation. It was hoped that the revised breakeven chart to be prepared in July would picture a somewhat more promising situation.

A final step now under way in the program aimed at lowering the breakeven point of operations is a series of individual Margin of Income studies on the various radio models in the line. If the products having the largest

margin of income can be determined, greater sales emphasis can be laid on them. Thus they will constitute a larger proportion of the total output and will affect the over-all Margin of Income accordingly. For example, one radio may sell at a list price of $31.54 while the price on another is $28.10. The variable cost of producing the first is $25.87 and is $25.69 on the second. The contribution of the first model toward fixed cost and profit is $5.67 or 17.97%. From the sales price of the second model only $2.41 or 8.56% is available for fixed costs and profit. It is obvious that if the first model forms the major part of the total output a smaller dollar volume of sales will be required in order to recover fixed costs. In other words, sales emphasis on that model will tend to lower the breakeven point. Of course it is not always possible to increase the sales of the more profitable models. The management knows, for example, that the cost-price relationship on the special low-priced model now being introduced is not as good as it should be. But the nature of consumer demand and the competition of low-cost producers have forced the company to introduce this model in order to bolster sales volume, and in spite of management's wishes this model is expected to constitute a major portion of total output. Thus consumer demand is a factor which must be taken into consideration when planning any program of selective selling.

In the present period of falling prices the Murray Radio Company has another reason for special interest in

the cost breakdown of each individual model. As competition becomes more severe the company is forced to cut prices in order to maintain volume. How far should the price be allowed to fall before production of a given model is discontinued? One group of accountants would say that production should not be continued when the books show that a loss is being taken. If total fixed and variable costs allocated to a product cannot be recovered, the product should be removed from the market. The management at Murray Radio operates under a different philosophy. Management believes the recovery of variable costs alone to be the important consideration. If the out-of-pocket costs to produce a given model are greater than the cash receipts from the sale of that model, production should be discontinued. But if receipts recover out-of-pocket costs and in addition make some contribution toward fixed costs, the model should be kept in the line. For each contribution toward fixed costs brings the company closer to the breakeven point of operations. Thus where regular accounting procedures show a loss, the differential cost approach shows management how to minimize that loss.

The Margin of Income studies on the individual models determine the percentage relationship of variable cost to list price and the minimum allowable selling price. The information needed for these calculations, the variable cost figures on each model, are provided by cost analysis based on standard costs and past experience. Thus material, direct labor, and overhead charges are derived

from standards. The variable component of overhead for each department is calculated by taking the product of the departmental direct labor charge and the variable overhead percentage of the direct labor dollar for that department as shown in Exhibit IX. To illustrate, the direct labor charge for Model X in Department A is found to be $.784. From Exhibit IX we find the variable overhead percentage of the direct labor dollar to be 67.05%. The product, $.526, is the variable overhead charge for Department A. Discounts and allowances as a percentage of the selling price are based on the experience of the previous year. Freight charges are calculated from the known shipping weight and a standard freight rate to a central point in the Middle West. And finally, the same figure is used for each product to indicate the variable component of operating expenses. This figure is derived from estimates based on past experience. Cost analyses of the two models mentioned above are included in Exhibit XII. The corresponding Margin of Income studies follow in Exhibit XIII.

Exhibit XII

Cost Analyses -- Model X and Model Y

Cost Analysis -- Model X

		Material	Labor	Overhead	Total Mfg Cost
Component 1	Dept C	.670			
	B	.121	.062	.108	.291
2	C	.610			.610
	B	.042	.050	.088	.180
3	C	.455			.455
	B	.121	.086	.150	.357
4	C	.200			.200
	B	.017	.044	.077	.138
5	C	.023	.020	.019	.062
	B	.005	.014	.024	.043
6	C	.945	.482	.448	1.875
	B	.320	.083	.144	.547
7	C	.004	.002	.002	.008
	B				
8	C	.004	.028	.026	.058
	B	.002	.012	.020	.034
9	C	.062	.011	.010	.083
	B	.005	.014	.024	.043
10	A	1.760			1.760
		.098)			
11	A	.040)			
		.322)			
12	A	.105)			.565
		5.890			5.890
13		1.568			1.568
14		.052			.052
15	A	1.700			1.700
16	A	.131			.131
Miscellaneous		.062			.062
Packing		1.056	----	----	1.056
Sub Total		16.390	.908	1.140	18.438
Material Loss		.164			.164
Freight		.206			.206
Dept A		----	.784	1.646	2.430
Tools		----	----	.005	.005
		16.760	1.692	2.791	21,243

Recapitulation by departments

	Material	Labor	Overhead	Total	Variable Overhead
Dept A	13.154	.784	1.651	15.589	.526
Dept B	.633	.365	.635	1.633	.519
Dept C	2.973	.543	.505	4.021	.204
Total	16.760	1.692	2.791	21.243	1.249

Exhibit XII (cont.)

Cost Analysis -- Model Y

		Material	Labor	Overhead	Total Mfg Cost
Component 1	Dept C	.775			.775
	B	.104	.068	.118	.290
2	C	.425			.425
	B	.097	.059	.103	.259
3	C	.580			.580
	B	.130	.072	.126	.328
4	C	.029			.029
	B	.004	.012	.020	.036
5	C	.602	.108	.100	.810
	B	.117	.184	.320	.621
6	C	.202	.009	.008	.219
7	C	.202	.009	.008	.219
8	C	.589			.589
	B	.060	.050	.088	.198
9	C	.190			.190
	B	.014	.020	.035	.069
10		1.760			1.760
11		.590			.590
12		5.890			5.890
13		1.568			1.568
14		.092			.092
15		2.424			2.424
16		.114			.114
Miscellaneous		.035			.035
Packing		.582	----	----	.582
Sub Total		17.175	.591	.926	18.692
Material Loss		.172			.172
Freight		.149			.149
Dept A		----	.835	1.754	2.589
Tools					
Total		17.496	1.426	2.680	21.602

Recapitulation by departments

	Material	Labor	Overhead	Total	Variable Overhead
Dept A	13.376	.835	1.754	15.965	.560
Dept B	3.594	.126	.116	3.836	.047
Dept C	.526	.465	.810	1.801	.661
Total	17.496	1.426	2.680	21.602	1.268

Exhibit XIII

Margin-of-Income Analyses -- Model X and Model Y

	Model X		Model Y	
	$	Variable %	$	Variable %
Selling price	$31.54	100.00	$28.10	100.00
Freight	1.38	4.38	1.00	3.56
Discounts & Allowances		2.30		2.30
		6.68		5.86
Mfg Cost				
Materials	16.760	53.14	17.496	62.26
Direct Labor	1.692	5.36	1.426	5.07
Overhead	1.249	3.96	1.268	4.51
Total Mfg Cost		62.46		71.84
Operating Expense		5.43		5.43
Total Variable Cost		74.57		83.13
Add 10% Variance Factor		7.46		8.31
Total Variable % of Selling Price		82.03		91.44
Minimum Price		$25.87		$25.69

VI. Discussion of the case study

Breakeven analysis ties in with the regular accounting records conveniently where a flexible budget system is used as at Murray Radio. The purpose of a flexible budget is to provide an indication of what costs should be if sales forecasts prove to be in error. The advantage over a fixed budget is that the estimates are not useless if sales volume is greater or less than predicted. The breakeven chart provides a convenient form in which to summarize the material contained in the flexible budget. By bringing the figures together it clearly shows what the effects on profit will be if there is a change in sales volume. What would be a difficult calculation from the accounting records themselves becomes a simple problem through the aid of the breakeven chart. The clear picture which the breakeven chart presents is a helpful aid to management.

The company uses gross sales less okay returns as an output index. Gross sales as estimated in the budget are the equivalent of the estimated sales value of production. Budgeted production costs are based on that volume. Therefore an output index based on gross sales is the same thing as one based on the sales value of production. There is, however, no basis for the subtraction of returns from gross sales. The sets returned were responsible for part of the production costs of the period and should be included as part of the volume produced.

The reason for selection of the sales value of production as an output index is the fact that it provides a

convenient common denominator for tying together the data on the variety of different models produced. Physical characteristics of the various models differ widely, so that a physical index would not be well suited to the situation. Since the analysis of overhead is done on a basis of direct labor input, it might be expected that the same index would be carried into the breakeven analysis. Again the factor of ease of calculation enters into the decision. It is simple to convert the costs based on direct labor into a fraction of sales. It would be difficult to estimate an average selling price per direct labor hour. Sales value of production is probably the best choice of an output index in this particular instance.

Management has recently been discussing the advisability of a 5% reduction in selling prices. The breakeven chart will be helpful in predicting the effect of changes in pricing policy. Management should know how much extra volume would be required to offset the price cut. Breakeven analysis can furnish this information.

It is hoped that the breakeven chart for the revised budget covering the second half of the year will show an improvement in the relationship of cost to revenue. The budget is based on reliable estimates in so far as possible and should indicate some improvement due to cost reduction during the current period. The breakeven chart might be expected to point the way to further decreases in fixed or variable costs.

A great deal might be done with the question of pro-

duct mix. A "standard" product mix is used at the present time in preparing the flexible budget and the breakeven chart. Comparative costs and Margins of Income are known on all models. Models having similar demand characteristics might be grouped together and the average Margin of Income for each group found. For various mixes of these groups new breakeven charts could be drawn to show the possible effect of a program of selective selling. The low-priced model now being introduced by the company in order to increase volume has a low Margin of Income. It is expected to comprise about 30% of total sales in the second half of the year. Management would, of course, like to replace it with a model or group of models which would contribute more toward cost. Marketing costs of another model less tailored to demand would probably be higher. Breakeven analysis of the sort suggested might provide management with useful information as to the effects on volume and profit of changing the product mix. It could not tell what sales effort would be required to meet the required volume, but it could show what volume would be required to make an adequate return. Coupled with market analysis it could conceivably be of considerable help in the planning of selling programs.

The limitations of breakeven analysis which have been discussed are observable in its application to the problem at Murray Radio. A sharp distinction has been drawn between fixed and variable costs. It has been shown that

this procedure, although essential in breakeven analysis, is an approximation to the actual facts. Thus depreciation is considered to be perfectly fixed independent of volume whereas if viewed in the long run it might be considered to vary with output.

In determining the budget formula for overhead costs reliance is placed on past records. Other factors than output have been shown to influence the cost level in any prior period. To allow for all of these factors would require a complex analysis. No attempt is made to remove completely the influence of changing factor prices, technology, and efficiency in this case. Where a value of cost appears to be out of line with the corresponding volume, additional points are plotted until a definite trend appears. Thus the budget formulae lack some of the accuracy which they might otherwise have. The company feels that the formulae obtained in this manner are accurate enough for the use to which they are put. Perhaps the breakeven analysis would have more validity without a prohibitive increase in complexity if a more concerted effort were made to remove the influence of outside factors. Cost data might be adjusted roughly to current price levels, and allowances might be made for major changes in efficiency and technology.

The difficulty in determining the incidence of some expenses is as real at Murray Radio as in other companies. Use of the best possible estimate is the only available course of action. The budget formula for maintenance

expense, like that of other overhead costs, is based on an analysis of past records. The assumption is that outlays for maintenance will be in roughly the same relation to volume during any given period and that over a long period the discrepancies will even out. This is probably the best solution to the problem. The same assumption is made in the case of other questionable items such as selling expense.

The accuracy of breakeven analysis of the operations of this company is limited by the fact that product mix can vary to a considerable extent. On the other hand the products made are fairly homogeneous in price range and in marginal contribution to profits. The number of different items made is not large in comparison to such an organization as Dennison's, where an attempted breakeven analysis was unsuccessful because of the wide variety of articles produced. It is possible at Murray Radio to predict with some degree of accuracy the mix of products that will be sold in the next six months. In such a situation breakeven analysis can and has been applied successfully. Suggestions have been presented as to how the technique might be of more value to management in the analysis of varying product mixes.

In an industrial situation we have seen how breakeven theory can be applied to the analysis of an actual production operation. The technique is not devoid of theoretical and practical limitations. However, provided these limitations are properly taken into account, the breakeven chart can be a useful tool of management.

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