PRESTRESSED CONCRETE PENSTOCK DESIGN FOR WATER HAMMER

by

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To the Secretary of the Faculty
Massachusetts Institute of Technology
Cambridge, Massachusetts

Dear Sir:

In partial fulfillment of the requirements for the degree of Civil Engineer from the Massachusetts Institute of Technology, I submit this thesis entitled, "Prestressed Concrete Penstock Design for Water Hammer."

Respectfully submitted,

Rajnikant Jayachand Kumbhani

Eng'g (C.E.) Nov. 1953

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ABSTRACT

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Prestressed concrete pipe has successfully competed with conventional reinforced concrete pipe as well as steel and cast-iron pipes for high pressure supply lines. This thesis explores its suitability for penstocks, with particular reference to the comparative magnitudes of water hammer, materials-wise economy and weights of the alternatively-designed pipe sections.

An ultimate design procedure allowing for the maximum possible accidental water hammer has been outlined.

An analytical comparison of the maximum water hammer pressures that may occur in penstocks of prestressed concrete and reinforced concrete shows significant reductions amounting to 20 to 50% in favor of the former for design heads of 150 to 500 ft respectively. The advantage of steel penstocks over prestressed concrete penstocks in this respect, however, amounts to naught at 50% joint efficiency, and only 10 to 20% at 90% joint efficiency for design heads of 600 to 150 ft respectively.

A comparison of materials shows that the ratio of the quantities of mild steel required in penstocks of reinforced concrete or steel, to the quantities of high tensile steel required in prestressed concrete penstocks ranges between 6 and 10. The ratio of the quantities of concrete required in reinforced concrete penstocks to quantities required in prestressed concrete penstocks varies between 4 and 7, the same ratio holding also for the weights of the respective pipe sections. Yet prestressed concrete pipe sections weigh only about twice to three times as much as the alternative steel pipe sections.

It is concluded that for sizes up to 12½ ft, prestressed concrete penstocks may take the place of reinforced concrete penstocks for 50 to 150 ft heads. And with the same size restrictions, prestressed concrete penstocks may also compete with steel penstocks for medium and medium-high heads.

Thesis Supervisor: Prof. Myle J. Holley, Jr.
Title: Associate Professor of Structural Engineering

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NOMENCLATURE

D	Internal diameter of the pipe (inches).
t	Thickness of the concrete-lining of a prestressed concrete pipe, or thickness of the concrete pipe-wall of a reinforced concrete pipe (inches).
H	Design head (ft).
A_s	Cross-sectional area of the high tensile steel of a prestressed concrete pipe, or of the mild steel of a reinforced concrete or plate steel pipe. (sq. in. per lineal inch of the pipe)
A_b	Cross-sectional area of the mild-steel cylinder of a prestressed concrete pipe, i.e., thickness of the cylinder wall (sq. in. per lineal inch of the pipe).
p_w	Working pressure corresponding to the design head (psi).
p_m	Pressure corresponding to the "ultimate" or elastic limit of the pipe (psi).
f_{co}	Initial compressive prestress in the concrete (psi).
f_{cf}	Residual compressive stress in the concrete at the design head (psi).
f_{so}	Initial tensile stress in the prestressing wire (psi).
η	Ratio of the tensile stress in the wire after shrinkage and plastic flow to its initial tensile stress.
T_s	Tensile strength of the prestressing wire (psi).
Y_b	Yield strength of the mild-steel cylinder (psi).
f'_s	Stress in the wire when the stress in the cylinder is equal to Y_b (psi).
n	Ratio E_s/E_c
H_o	Initial steady head near the gate prior to start of the gate closure (ft).
V_o	Initial velocity in the penstock prior to start of the gate closure (fps).
a	Celerity of the pressure-wave inside the pipe (fps).

w	Unit weight of water (lb/cft).
K'	Bulk modulus of water (psi).
E	Modulus of elasticity of the pipe (psi).
h_{max}	Maximum pressure rise due to instantaneous closure = aV_o/g (ft).
T	Time of gate closure (sec).
L	Length of the penstock pipe from the gate to the forebay or other point of relief (ft).
h	Pressure rise due to slow closure (ft).
K	Pipe line constant = $aV_o/2gH_o$
N	Time constant = $aT/2L$
e	Joint efficiency of the steel penstock.
C	Initial cost of a linear foot of the penstock pipe.
i	Yearly fixed charges on the pipeline expressed as a ratio.
f	Friction factor of the pipe.
P	Power lost in friction per foot of the pipe (hp).
b	Profits on the sale of 1 hp-year of power.
α	Ratio A_b/D
c_1	Cost of concrete per cubic yard.
c_2	Cost of high tensile steel wire per pound.
c_3	Cost of mild steel cylinder per pound.
ϕ'	A stress function
ϕ''	A stress function
K''	Constant C/D^2

Subscripts p, r, s refer to prestressed concrete, conventional reinforced concrete, and steel penstocks respectively.

I

INTRODUCTION

The penstock of a hydro-electric plant is essentially a pressure pipe line. Therefore, in order to appraise the prospects of utilizing the prestressed concrete pipe for this purpose, it will be helpful to take a glance at its short but eventful record in the field of concrete pressure pipes of water-supply lines.

History of the Concrete Pressure Pipe

The first use of the reinforced concrete pipe in pressure supply lines within the U. S. A. and Canada can be traced back to the year 1909 when the city of Toronto built 72-, 54-, and 36-inch diameter, light-headed reinforced concrete supply-lines. Salem, Massachusetts, followed a few years later with 48- and 36-inch diameter reinforced concrete supply-lines under an operating head of 20 feet at the pumping station.[1] In the early part of its history, the reinforced concrete pipe suffered from two main handicaps: (1) the difficulty of obtaining dense concrete under adverse placing conditions; and (2) the unreliability of the mortar joint then being used. These handicaps in turn caused serious limitations of working pressures and carrying capacity.

Development of precasting techniques and good water-tight pressure-joints helped to overcome the above limitations. However, it was the insertion of a continuous sheet-steel cylinder into the concrete pipe that caused a phenomenal boost in the working pressure range. Lastly, these working pressures of the noncylinder and cylinder precast pipes were almost doubled with the advent of prestressing, which was consequent upon two major developments: (1) advances in the production of low W/C-ratio

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[1] Longley, F.F. "Prestressed Reinforced Concrete Pipe," Waterworks and Sewerage, December, 1954

concrete; and (2) production of very high strength steel wire. The first non-cylinder prestressed concrete pipe-line in the U. S. A. was reportedly constructed in a suburb of Chicago in 1941, followed by the city of Chicago in 1943 when similar pipe-lines of 36-inch diameter were laid to operate under a pressure of 37 psi, but previously subjected to a factory-test at 200 psi.[2] Subsequently, the Lock Joint Pipe Company incorporated the principle of prestressing into their cylinder precast pipes, and have since then, (along with Price Brothers Company and others) laid miles of this type of pipe for high pressures ranging up to 500 psi and more. Table 1 shows the pressure ranges applicable to the different types of concrete pipe, as compiled from several sources.

Table 1. Pressure Ranges of Concrete Pressure Pipes

Type	Maximum Operating Head in Feet		
	F.F. Longley[1]	C.V. Davis[3]	Creager and Justin[4]
Cast in place (ord. Reinf.)	Low	100-150	100
Non-cyl.--Unprestressed-Precast	75	80-100	-
Non-cyl.--Prestressed-Precast	150+	-	-
Steel-cyl.--Unprestressed-Precast	575+	500	500-600
Steel-cyl.--Prestressed-Precast	1150+	-	-

<u>Early Objections to the Prestressed Concrete Pipe</u>

Misinformed quarters raised the following objections to the prestressed concrete pipe in its infancy:

(1) Deformations due to shrinkage and plastic flow would cause the wires to slacken and in so doing endanger the pipe; and

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[2] DeBerard, W.W., and Weldon, W.B. "Experiences with Wire-Wound Prestressed Concrete Pipe, Journal, American Waterworks Association, Oct. 1943

[3] Davis, C.V. Handbook of Applied Hydraulics, 1952 ed.

[4] Creager, W.P. and Justin, J.D. Hydro-Electirc Handbook, 1950 ed.

(2) Prestressing would not change the ultimate strength of the pipe against bursting pressure and as such serves no real purpose.

Obviously, these are only half truths, because

(1) The loss of prestress due to shrinkage and flow, although it does occur, amounts to about 20% only when high-tensile wires are used; and an allowance can be made for the same while winding the wire with a predesigned tension; and

(2) Even though prestressing does not enhance the bursting pressure of the pipe, it improves the elastic qualities of the pipe considerably. Tests[5] have proved the superiority of the prestressed concrete pipe in beam-strength, in strength against external loading, and in cylinder compression strength. It is further shown in this thesis that prestressing actually increases the ultimate hydrostatic pressure of the pipe by decreasing the magnitude of maximum water-hammer pressure, for a given total ultimate strength.

Merits of the Prestressed Concrete Pipe

Numerous advantages offered by the prestressed concrete pipe may be summarized as follows:

(1) Elastic performance of this pipe is far superior to that of the ordinary reinforced concrete pipe. Due to prestressing, concrete is under compression (of 1200 to 2000 psi as per design) at zero pressure. Over the range of pressures corresponding to the stress in concrete from about 2000 psi initial compression up to cracking at 350 to 500 psi tension, the concrete acts in conjunction with the steel in a homogeneous elastic manner. Crackless concrete within working pressures can be ensured by so designing the pipe as to leave a residual compression in the concrete at the design head.

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[5] Ross, C.W. "Tests of Prestressed Concrete Pipes Containing a Steel Cylinder," Journal, ACI, Sept. 1945.

(2) Cracks opened by overloads close up completely on return to the design head.

(3) Use of dense concrete which is also in compression at working pressures makes the pipe less pervious than the reinforced concrete pipe.

(4) Prestressing results in a much thinner pipe of lighter weight. Consequently not only are materials economized, but also the handling and laying of the pipe-sections become much easier.

(5) The prestressed concrete pipe is considerably stronger as a beam, as a column, and under external loadings such as earth pressures.[5]

(6) The relative thinness of the prestressed concrete pipe causes significant reductions in maximum water-hammer pressures (see Section III) and thus increases the safety factor of the pipe considerably.

(7) Due to imperviousness, good resilience, and the absence of cracks, the prestressed concrete pipe may be expected to have a much longer life than the reinforced concrete pipe.*

(8) Applicability of higher pressures (about twice) at per Table 1.

(9) Lower initial costs, as widely publicized by all manufacturers.

Application of the Prestressed Concrete Pipe to Penstocks

Due to the above advantages, prestressed concrete pipes have been extensively used for water-supply lines, culverts, and sewers over the last ten years; and in the case of pressure supply lines, they have successfully withstood competition from even steel and cast-iron pipes. But their application to hydro-electric projects is still awaited, although the principle of partial prestressing has been used in the case of the

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*Informative literature from Price Brothers Company places the life of its cylinder prestressed concrete pipes at more than 100 years, which is about twice that of reinforced concrete pipes.

steel penstocks at Boulder Dam[6], and in the tightening of the steel bands of wood-stave penstocks. The earliest use of reinforced concrete in penstock construction in the U. S. A. can be traced back to 1914 at Plant No. 5 of the New England Power Company at Deerfield River, Vermont.[7] Reinforced concrete penstocks are now quite common, limited in general, however, to heads less than about 150 feet. As a rule, they have been cast-in-place, due to the size limitations (now about 12½ feet maximum) of precast pipe, and due to the difficulty of transportation and erection of heavy precast sections over mountainous terrain. The present day practices in penstock design are summarized in Table 2.

Table 2. Head, Size, and Life Limitations of Penstock Pipes

Penstock Type	Source	Max. Head	Min. Head	Max. Diam.	Life
Reinforced Concrete	Barrows[7]	60 ft*	-	12 ft	25-50 yrs
	Davis[3]	100-150 ft	-	12 ft	-
	Cr. & Justin[4]	100 ft	-	-	50 yrs
Wood-stave	Barrows	150 ft	-	16 ft	20-30 yrs
	Davis	-	-	17 ft	10-40 yrs
	Cr. & Justin	-	-	22 ft	20-30 yrs
Steel	Barrows	No limit	150 ft	30 ft	40-50 yrs
	Davis	"	-	30 ft	-
	Cr. & Justin	"	-	30 ft	20-30 yrs

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[6] U.S. Bureau of Reclamation. "Penstocks and Outlet Pipes," Part IV, Boulder Dam Project Reports, 1940.

[7] Barrows, H.K. Water Power Engineering, 1943 ed.

*Stated with the qualification: "....this limitation particularly applying in colder climates, where alternate freezing and thawing tend to cause concrete under high water pressure to deteriorate rapidly."

For all heads below about 150 feet, a steel penstock as practically built is wasteful of metal because it must usually be of some minimum thickness for practical reasons. The steel reinforcement of the concrete pipe and the band-steel of the wood-stave pipe, however, can be exactly proportioned to fit the given conditions of pressure. But wood-stave penstock would be generally used in the rather special circumstances of abundant good wood, location in rugged forests, and severe cold climate. Under normal circumstances, reinforced concrete penstock, having almost twice the life of wood-stave penstock, may be preferred to the latter. With this in mind, and judging from its record in high-pressure supply lines, one may confidently expect prestressed concrete penstock to replace reinforced concrete penstock wherever the size requirements are smaller than about 150 inches, and wherever the laying of precast sections is not too hazardous. Whether prestressed concrete penstock can compete with steel penstock for heads higher than 150 feet will, however, depend on the relative weights of the pipe sections, the prevailing site characteristics, and other construction features.

Statement of the Purpose of this Thesis

With a view to arriving at an understanding of the relative suitability of prestressed concrete penstocks as compared with reinforced concrete and steel penstocks, the author has endeavored in this thesis to investigate the following salient points of consideration:

(1) Derivation of expressions suitable for working-stress design, and ultimate design of prestressed concrete penstocks.

(2) Determination of the economic size of prestressed concrete penstock, and its comparison with the economic sizes of reinforced concrete and

steel penstocks designed for identical head and flow.

(3) Investigation of the relative magnitude of water-hammer pressures in the three types of penstock, occuring under identical conditions.

(4) Comparison of the quantities of different materials used in the three types of penstock, and comparison of the relative weights of pipe-sections.

II

STRUCTURAL DESIGN OF THE PRESTRESSED CONCRETE PENSTOCK

As in the case of all other structures, the penstock-pipe is also subjected to a controversy over whether the working-stress design or the ultimate-load design should be preferred. In either case, however, it is necessary to have a perfect understanding of: (1) performance of the pipe right from zero pressure up to the bursting pressure, and (2) nature and composition of the loading to which the pipe may be subjected while in service.

Performance of the Steel-Cylinder Prestressed Concrete Pipe

Stage 1. (From zero pressure up to the cracking limit of the concrete) Prior to cracking of the concrete (which may not occur at less than 350 to 500 psi tension), the circumferential elongation of the pipe-wall under increments of internal pressure is governed by the combined characteristics of the steel and the concrete which are firmly and intimately bound together by the prestressing wire. The three elements--high tenile wire, mild-steel cylinder, and concrete lining--having been fabricated into a single unit in stress-equilibrium, are forced to change their dimensions in identical amounts when subjected to load. All three act elastically and homogeneously, and take their proportionate shares of the load.

Stage 2. (From cracking of the concrete to the elastic limit of the steel cylinder) After the concrete is cracked, it will have lost its capacity to carry any stress whatsoever, and will presumably ride along inertly with the steel-cylinder through mutual bond, while the steel-cylinder and the wire are resisting the load by undergoing identical elongations governed by the common Elastic Modulus of the two types of steel alone. Since they have identical lengths, their unit strains and hence their unit stress increments are also identical. It may be noted that before the conclusion of this stage, concrete cracks are capable of closing completely on the removal of the overload, there is no permanent set, and hence no loss of prestress; consequently, the strength and quality of the pipe are not impaired by the overload. End of this stage may also be called the elastic limit of the whole pipe.

Stage 3. (From the elastic limit of the cylinder to the ultimate tensile strength of the high-tensile wire) In this stage, the cylinder and the wire undergo identical strains because of the restraint exerted by the wire on the cylinder, but now the cylinder has a very small modulus of elasticity, and so its stress increments are very small; a major share of the load is now thrown onto the wire which has no definite yield point but a continuously decreasing modulus. This goes on until the wires break, and the cylinder bellies out. This pressure may be called the bursting pressure of the pipe. Typical performance of such a pipe is shown in Fig. 1. on the following page.

Performance of the Non-Cylinder Prestressed Concrete Pipe

The performance in this case is similar to that in the previous case except for the omission of Stage 2. Elastic limit of the pipe in this case however is the elastic limit of the wire. And now the pipe begins to leak at the end of the first stage, whereas in the previous case, the pipe does not leak right up to the bursting pressure.

Nature of the Loading Acting on the Penstock-Pipe

The penstock-pipe is subjected to the following components of loading:

(1) Internal hydro-static pressure - its maximum value at any point

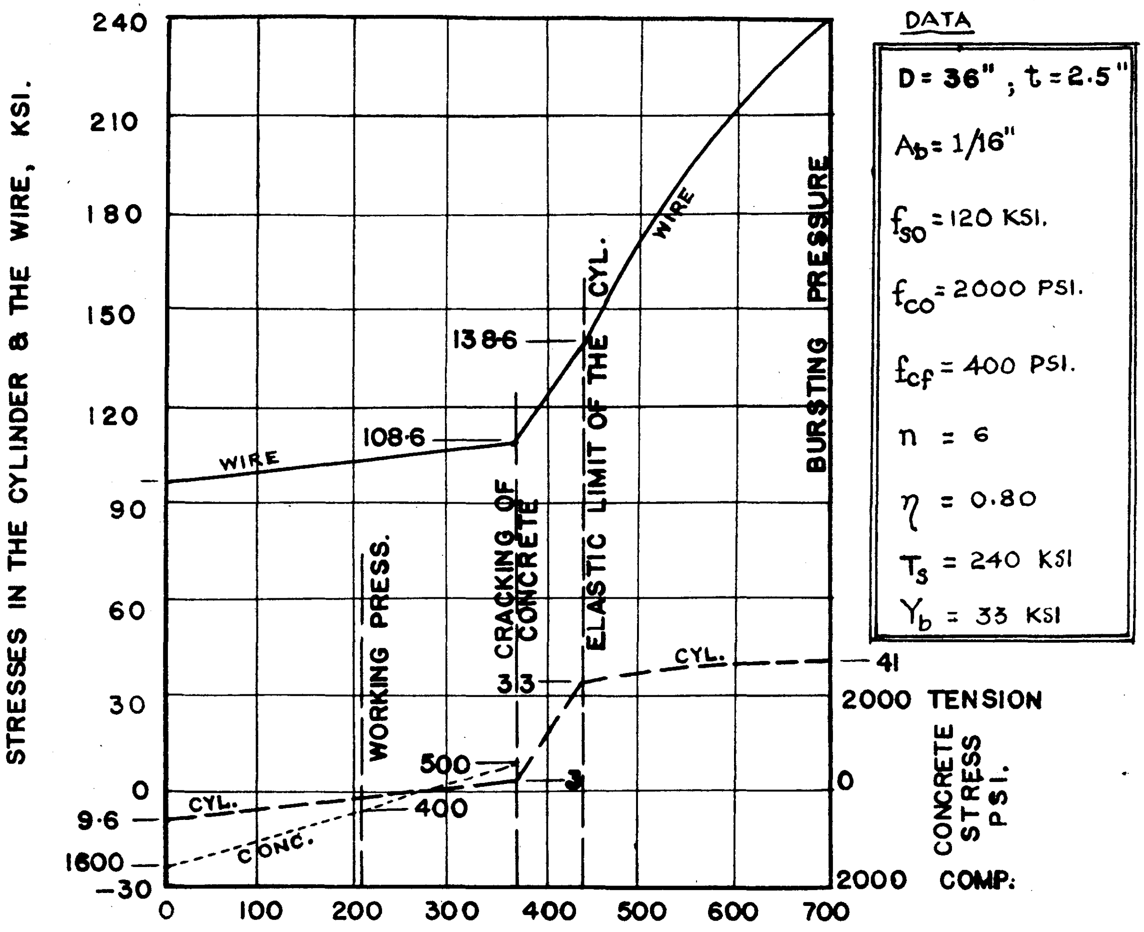

FIG. I. TYPICAL PERFORMANCE OF A PRESTRESSED CONCRETE PIPE

along the penstock is that corresponding to the HFL in the reservoir and can never be exceeded.

(2) Weight of the water flowing through the pipe.

(3) Self-weight of the pipe distributed over the circumference.

(4) External loading - In the case of buried pipes continuously bedded on the ground, external loading will consist of peripheral earth pressures and a bulb-shaped reaction from the bedding roughly confined to the bottom 90 degrees.[8] In the case of penstocks supported on intermittent

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[8] U.S. Bureau of Reclamation. Stress Analysis of Concrete Pipe, Eng. Monographs No. 6, Oct. 1950

cradle-supports, there will be reactions at the cradles causing a beam-action.

(5) Temperature stresses - They may be neglected in the case of buried pipes.

(6) Water-hammer pressures - These are of the most uncertain nature, and may constitute a substantial portion of the total design-head. The designer has to investigate not only the surges resutling from day-to-day governing, but also the possibility of extreme surges arising from probable accidents and emergency load-changes. Evidently, the selection of the severity of the extreme surges will have a complimentary effect on the choice of the "factor of safety." The water-hammer problem is more extensively treated in the next section.

Items (2), (3), (4), and (5) of the loading deserve serious attention only in the case of large thin steel penstocks which are generally exposed and large concrete pipes subjected to light pressure heads as in the case of some supply lines and aqueducts. It is common design practice, so far as penstocks are concerned, to ignore these items, at least in the first design.

Structural Design of the Steel-Cylinder Prestressed Concrete Pipe

Nomenclature to be followed is given on page i.

(1) Working Head Design

According to normal practice, the working head, H, will comprise the static head and the maximum water-hammer head as determined from the governor guarantees and relief-valve characteristics. Working pressure, $p_w = 0.434H$ psi where H is in feet. Concrete stresses must be guarded against two loading conditions: (a) the maximum compressive stress occuring at the time of wire-winding must be equal to a safe stress, f_{co}; and (b) a certain residual compressive stress, f_{cf}, must be ensured at the working head. Stresses in the mild steel cylinder will be automatically safe for the usual values of f_{co}, f_{cf}, and n. The stress, f_{so}, must, however, be properly chosen from the stress-strain characteristics of the wire with the ultimate load condition in mind (see Appendix A).

Equilibrium of one inch length of the pipe under the above two conditions is shown in Fig. 2.

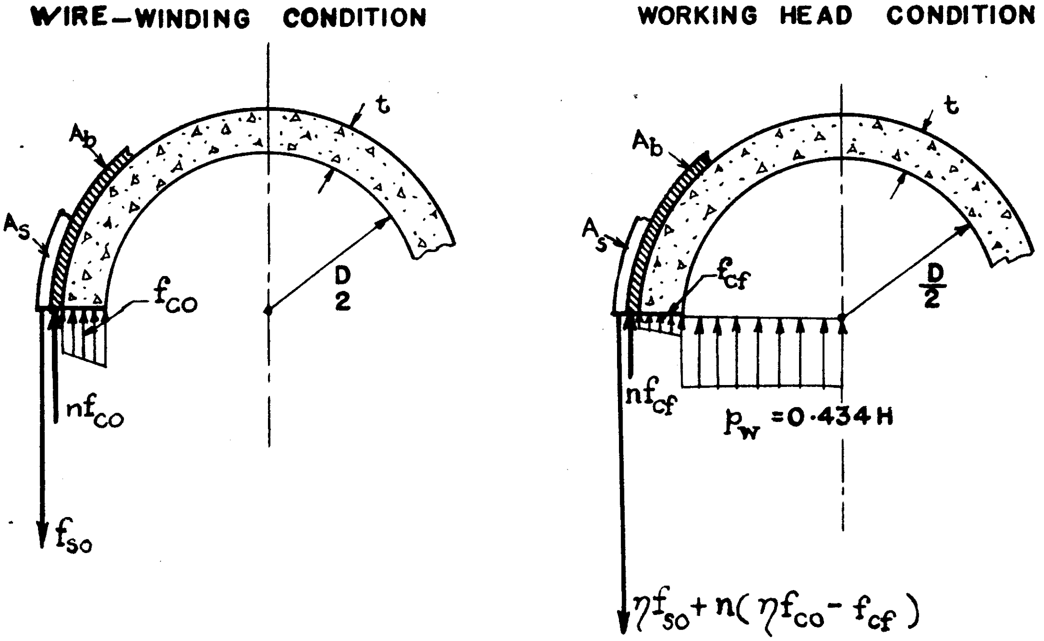

FIG.2. WORKING HEAD DESIGN CONDITIONS

Equations of equilibrium are:

$$A_s = (t + nA_b)\frac{f_{co}}{f_{so}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (1)$$

$$0.217HD = (\eta f_{co} - f_{cf})(t + nA_b + nA_s) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (2)$$

Eliminating A_s between (1) and (2), we obtain:

$$\frac{t + nA_b}{D} = \frac{0.217H}{(\eta f_{co} - f_{cf})(1 + \frac{nf_{co}}{f_{so}})} \quad \ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Design Procedure

It may be noted that there are three unknowns, t, A_b, and A_s, with only two relations among them. But the function of A_b is not mainly structural, and so it can be selected arbitrarily for watertightness, say 16 gage sheet-steel.[9] Then t and A_s can be calculated from equations (3) and (1) respectively. If t thus calculated is less than a desired minimum*, then t must be chosen accordingly, and A_s in this case will have to be obtained from:

$$0.217HD = (\frac{\eta f_{so} A_s}{t + nA_b} - f_{cf})(t + nA_b + nA_s) \quad \ldots\ldots\ldots\ldots\ldots (4)$$

(2) Ultimate or Elastic-Limit Design

As mentioned previously, prestressed concrete pipe should never be loaded beyond its elastic limit because any permanent set would critically impair the strength of the pipe. Hence the load corresponding to the elastic limit of the pipe may be called the ultimate load of the pipe. Even in the case of steel penstocks, a good case has been made out for linking the factor of safety to the elastic limit, and not the ultimate tensile strength because the "fatigue limit" closely approximates the elastic limit. Specifications of the Societe Hydro-technique de France have adopted this principle for many years.[10]

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[9] American Water Works Association. Tentative Emergency Specifications for Reinforced Concrete Pressure Pipe, (Designation 7B-T) specifies a minimum cylinder thickness of No. 16 Gage U.S. Std for unprestressed cylinder concrete pipes.

*Reference 9 also specifies a minimum concrete lining of 1 inch within the steel cylinder.

[10] Ferrand, G. "Pressure Pipe Lines," Water Power (Lond.) March-April 1950.

At the elastic limit of the prestressed concrete pipe, the mild steel cylinder has reached the elastic limit, Y_b, and the stress in the wire is given by:

$$f'_s = \eta f_{so} + (Y_b + \eta n f_{co}) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

The corresponding "ultimate" pressure, p_m, is given by:

$$0.217 H_m D = p_m \frac{D}{2} = A_s f'_s + A_b Y_b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (6)$$

It may be noted that f'_s must be smaller than, or equal to, the proportionality limit or elastic limit of the high tensile wire.

Design Procedure

It is proposed hereby that instead of selecting A_b arbitrarily as in the working-head design, equation (6) should be used in conjunction with equations (1) and (3) to determine the three unknowns, t, A_b, and A_s. The procedure would be: (a) to obtain A_s by eliminating $(t + nA_b)$ between (1) and (2), that is, from:

$$0.217HD = (\eta f_{co} - f_{cf})(\frac{f_{so}}{f_{co}} + n)A_s \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (7)$$

and (b) to obtain A_b next from equation (6); and (c) to obtain t from equation (1).

If A_b thus calculated is smaller than the minimum specified (e.g. No. 16 gage), then the latter may be adopted with an assured increase in the factor of safety, i.e., $\frac{H_m}{H}$.

(3) Determination of the Bursting Pressure

The bursting pressure of prestressed pipes is much higher than the "ultimate" or elastic-limit pressure as defined before. At the bursting pressure, the wires will have attained their ultimate tensile strength, while the steel cylinder, still restricted to finite strains by the wire, will be only somewhat above its yield-point stress due to a very low modulus. A close approximation[5] of the bursting pressure can be obtained from:

$$p_{bursting} \times \frac{D}{2} = T_s A_s + Y_b A_b \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (8)$$

Structural Design of the Non-Cyl. Prestressed Concrete Pipe

All the relations derived above are also applicable to the non-cylinder prestressed concrete pipe by equating A_b to zero. Elastic limit of the pipe can be obtained by equating f'_s to the elastic limit of the wire. Leaking of the pipe will occur with the cracking of the concrete at a pressure that may be calculated from equation (2) by equating f_{cf} to the cracking stress of the concrete with the proper algebraic sign. In this case, because of the absence of the cylinder, longitudinal reinforcement must be provided to take care of the temperature and beam stresses.

Selection of Stress-Values, n, and η

A review of the present practices and some suggestions of the author, particularly for f_{so} and η, are given in Appendix A.

III

COMPARISON OF WATER-HAMMER IN PRESTRESSED CONCRETE PENSTOCK WITH THAT IN REINFORCED CONCRETE AND STEEL PENSTOCKS

Computation of Water-Hammer Pressures for Penstock Design

The problem of the manner in which the water-hammer head should be incorporated into penstock design is still unsettled. A survey of the literature points to the following two methods being primarily discussed:

(1) Working Head Design: The general practice herein is to determine the maximum water-hammer pressure due to governing from the governor guarantees and relief valve characteristics, expressed ordinarily as a percentage of the static head; then to design for somewhat more than this percentage above static pressure, adopting the usual working stresses corresponding to the conventional range of the so-called "factor of safety" based on the yield point or on the ultimate strength. In some practices, this percentage is arbitrarily fixed at from 50% for low head plants[3] to 15% for high head plants [11, 12], and then this limit is scrupulously safeguarded by means of relief, bypass, and air valves and proper governing.

(2) Ultimate Design: The previous method of design disregards the fact that penstock breaks result very rarely or never from governing conditions, even abnormal ones, but rather from accidental conditions producing surges of instantaneous or rapid type[13]. Causes of severe

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[11] Schoklitsch, A. Hydraulic Structures, trans. by L. G. Straub, 1937.

[12] Ferrand, Georges. "Overpressured and Self-Hooped Penstocks," (Reference to the Cestrede Penstock), Water Power, (Lond.) Oct. 1952. See also the illustrative example in Reference 10.

[13] Billings, A.W.K, and others. "High-Head Penstock Design," ASCE-ASME Symposium on Water Hammer, 1933.

accidental surges are numerous, for example: failure of the mechanism, error of the gate-man, resonance caused by rhythmic gate movement, hasty priming of the penstock at the head gate, parting and rejoining of the water column due to a negative wave, parting and rejoining of the water column in the draft tube, and so on. Recognizing this situation, and the cost of penstock installations, it has been suggested[13] that reasonably probable accidental surges be carefully estimated and then the penstock be proportioned to resist these extreme conditions without perceptible deformation.

Basis of Comparison Among Penstocks of Different Materials

For realistic comparison among penstocks made of prestressed concrete, reinforced concrete and steel, it must be postulated that they all should be designed for identical design data; that is, for identical heads and discharges.

Comparison of Sizes

It is shown in Appendix B that if the cost of penstock-pipe per foot-length can be expressed in terms of D^2 as in these three cases, then the economic diameter (namely, the one with minimum annual cost) varies only as the seventh root of all the variables involved except the discharge, which affects the diameter to the extent of three-sevenths root. Since the discharge in all cases is identical, it follows that the alternative designs of the penstock for the same design data but different materials, will result in approximately equal diameters. It may be observed that a 10% difference in cost will change the diameter by only 1.36%. Values of the friction factor - f for the three materials are also not likely to vary from one another by more than 10 to 15%[*], and therefore its effect on the size will also be negligible. In this discussion, therefore, alternative penstock designs will be assumed to have equal diameters.

Comparison of Water-Hammer

Water-hammer pressures occuring in alternatively designed penstocks will be compared on the basis of the following conditions:

(1) Identical heads, discharges, and sizes will give rise to identical velocities for a given gate-opening, that is for a given load.

(2) Load characteristics and hence gate operations will have to be identical in all designs. Consequently, equal velocities, V_o, will be destroyed in equal time intervals, T, in all the three penstocks.

(3) For the comparison to be of a generally applicable nature, it is not possible to take into consideration special plant characteristics, like branched pipes, surge tank complexities, and non-uniform gate motions. Quantitative comparison will be possible with the common assumptions of

(a) Simple conduit

(b) Uniform gate motion

(4) Lastly, equal penstock lengths, L, must be assumed in all the cases.

Comparison of Celerities of the Pressure-Wave

(1) Wave-Celerity in Reinforced Concrete Penstock: It can be checked with available designs and can be generally demonstrated[**] that, according

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[*] See page 406 of Reference 7.

[**] $(t + nA_s)f_c = 0.217HD$. Substituting $nA_s = 0.05t$ and $t/D = 1/12$ (approximately correct values, documented later on), we obtain $f_c = 2.48H$. This gives $f_c = 248$ psi at 100 ft head and $f_c = 124$ psi at 50 ft head. Actually, t/D will be greater than 1/12 for heads greater than about 40 ft, so that f_c for heads greater than about 40 ft will be somewhat smaller than demonstrated here.

to the conventional design of reinforced concrete penstock, the stress in the concrete at the design head is of the order of only 150-200 psi tension. Therefore, the concrete can be safely counted on to do useful work during the passage of the pressure wave.

From the work-energy* principle, following expression is obtained for the celerity of the pressure wave:

$$a_r = \frac{1}{\sqrt{\frac{w}{g}\left[\frac{1}{K'} + \frac{D}{E_c(t + nA_s)}\right]}} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (9)$$

A_s in most cases amounts to 0.5% of the pipe-wall area[11] so that nA_s is approximately equal to 0.05t. Therefore, the contribution of the steel to the rigidity of the pipe may be neglected.

Design practices and specifications** require t in inches equal to D in feet; i.e., $t = D/12$ (both being in inches). The U. S. Bureau of Reclamation[14] specifies: $t = D/12$ (6 inch minimum) for $H = 0 - 40$ ft, $t = D/12 + 1''$ for $H = 40$-80 ft, and then an extra inch for every additional 20 feet. Ignoring these slight increases, however, essentially $t = D/12$ for reinforced concrete penstocks irrespective of the head. According to equation (9), this will make a_r approximately constant for all reinforced concrete penstocks.

Substituting $w = 62.5$ lb/cft, $g = 32.2$ ft/sec^2, $K' = 42.4 \times 10^6$ psf, $E_c = 432 \times 10^6$ psf, and $D/t = 12$, we obtain from equation (9):

$$a_r = 3150 \text{ fps approximately} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \; (10)$$

(2) <u>Wave-Celerity in Prestressed Concrete Penstock</u> The formula for wave-celerity in this case will be

---

*Effect of Poisson's Ratio has been neglected.

**References 2, 3, 7, and 9.

[14] "Soap Lake Siphon," Eng. Monograph No. 5, U.S. Bureau of Reclamation, 1950.

$$a_p = \frac{1}{\sqrt{\frac{w}{g}\left[\frac{1}{K'} + \frac{D}{E_c(t + nA_b + nA_s)}\right]}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (11)$$

Neglecting nA_s on a comparative basis, and substituting previous values of w, g, K', and $E_c = 720 \times 10^6$ psf ($= 5 \times 10^6$ psi) we obtain

$$a_p = \frac{4660}{\sqrt{1 + 0.059\frac{D}{(t + nA_b)}}} \quad \ldots\ldots\ldots\ldots\ldots\ldots (12)$$

This relation has been plotted in Fig. 3.

For any design head, H, $\frac{D}{(t + nA_b)}$ can be obtained from the structural design of the pipe as represented by equation (3). This relation between $\frac{D}{(t + nA_b)}$ and H is plotted in Fig. 4, with common assumed values of $(\eta f_{co} - f_{cf}) = 1200$, and $\frac{nf_{co}}{f_{so}} = 0.10$. To get a_p corresponding to any value of H, one can read off $\frac{D}{(t + nA_b)}$ against this value of H from Fig. 4 and then enter Fig. 3 to read off the corresponding a_p. A plot of a_p versus H is given in Fig. 5.

(3) Wave Celerity in Steel Penstock For steel penstock,

$$a_s = \frac{4660}{\sqrt{1 + \frac{K'}{E_s}\frac{D}{t}}} = \frac{4660}{\sqrt{1 + \frac{D}{100t}}} \text{ fps} \quad \ldots\ldots\ldots\ldots\ldots\ldots (13)$$

$$\frac{D}{t} = \frac{f_s}{0.217H} = \frac{15000e}{0.217H} \quad \ldots\ldots\ldots\ldots\ldots\ldots (14)$$

where e = the joint efficiency

Generally e varies from 50% for single riveted lap joints or single-strap butt joints, uptto 90% for welded joints. For these two extreme values of e, by eliminating D/t between equations (13) and (14), the resulting values of a_s are plotted against H in Fig. 5.

Comparison of Water Hammer Due to Instantaneous and Rapid Surges

Instantaneous surges are those caused by a very rapid velocity change in much less than the critical period, 2L/a, of the pipe, producing a

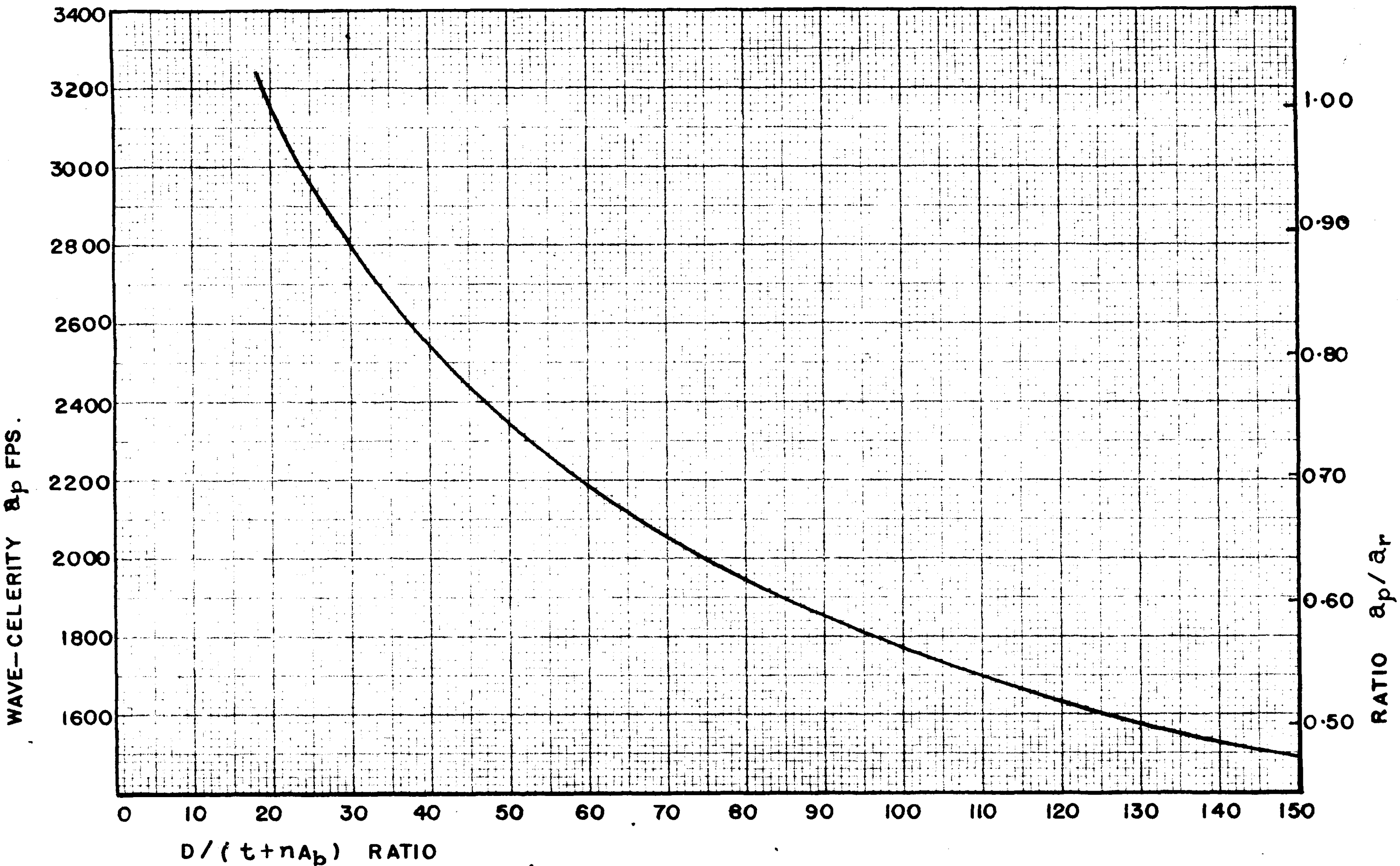

FIG.3. CURVE a_p VS. $D/(t+nA_b)$ FOR PRESTRESSED CONCRETE PIPES

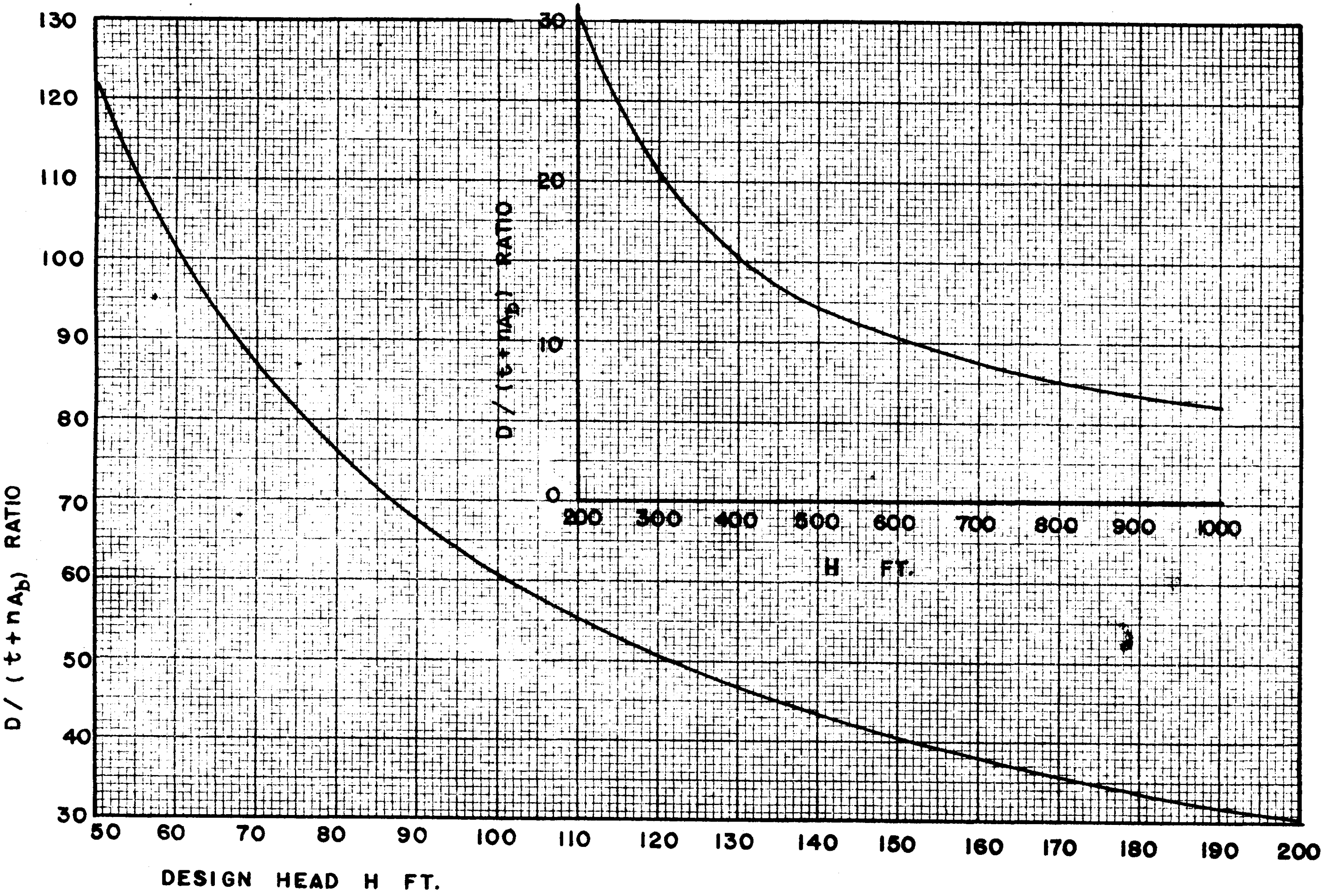

FIG. 4. CURVE $D/(t + nA_b)$ VS. H FOR PRESTRESSED CONC. PIPES

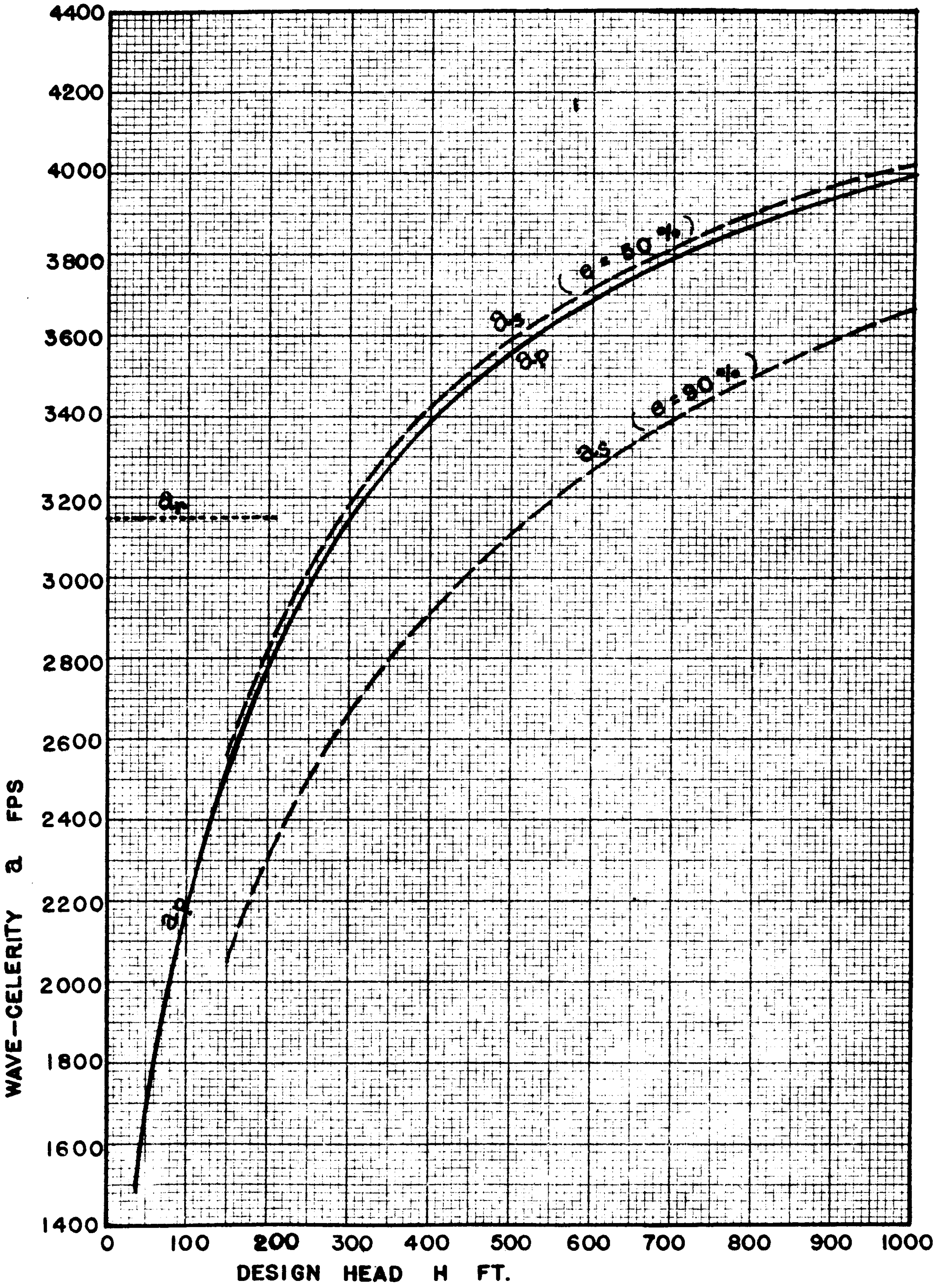

FIG. 5. CURVES OF WAVE-CELERITY VS. DESIGN HEAD FOR PRESTRESSED CONCRETE, REINFORCED CONCRETE, & STEEL PIPES

pressure fluctuation which is transmitted with no diminution or change right up to the relief point. The water-hammer pressure is identical at all points and is the maximum that can ever occur. Its magnitude is given by Joukowsky's formula

$$h_{max} = \frac{aV_o}{g} \quad \ldots \quad (15)$$

Since the three penstock types have equal V_o,

$$h_{pmax} : h_{rmax} : h_{smax} = a_p : a_r : a_s \quad \ldots \quad (16)$$

Rapid type of surges[13] are caused by relatively slower closures which take place in less than 2L/a seconds; and the maximum pressure rise which is of the same magnitude as given by equation (15) is transmitted undiminished along the pipe up to a point where the distance to the intake is equal to Ta/2. From that point to the intake, the pressure reduces uniformly to zero. Consequently, comparative magnitudes of pressure at any point on the pipe are given by

$$h_p : h_r : h_s = a_p : a_r : a_s \quad \ldots \quad (17)$$

The values of a_p/a_r and a_p/a_s can be obtained from Fig. 5. Inasmuch as these ratios depend on the design head, the values of h_p/h_r and h_p/h_s for instantaneous and rapid surges are also dependent on the design head.

The following values are obtained from Fig. 5:

H in ft =	45	75	120	150	225	300
$\frac{h_p}{h_r} = \frac{a_p}{a_r}$ =	0.494	0.615	0.739	0.803	0.915	0.989

Comparable values for prestressed concrete and steel penstocks are as follows:

H in ft =	150	200	300	400	500	600
$\frac{h_p}{h_s} = \frac{a_p}{a_s}$ (for e = 50%) =	0.98	0.989	0.990	0.991	0.992	0.992
$\frac{h_p}{h_s} = \frac{a_p}{a_s}$ (for e = 90%)[*] =	1.231	1.212	1.180	1.163	1.143	1.128

Comparison of Water Hammer Due to Slow Uniform Closure

For uniform valve closure in more than 2L/a seconds, Allievi's and Quick's charts, and R. D. Johnson's formula give extremely close results to those computed by arithmetic integration or graphical procedures[15]. Therefore, for all practical purposes, R. D. Johnson's formula may be used to compare the water-hammer in the three penstock types for slow uniform closure. According to this formula,

$$\frac{h}{h_{max}} = \frac{K+\sqrt{K^2+4N^2}}{4N^2} \ldots\ldots\ldots\ldots\ldots\ldots (18)$$

Where h_{max} is given by equation (15), K is pipe-line constant = $aV_o/2gH_o$, and N is time-constant = $aT/2L$. Since V_o, H_o, T, and L are identical for the three penstock-types, the following relations hold:

$$K_p : K_r : K_s = a_p : a_r : a_s \ldots\ldots\ldots\ldots\ldots\ldots (19)$$

$$N_p : N_r : N_s = a_p : a_r : a_s \ldots\ldots\ldots\ldots\ldots\ldots (20)$$

Now, by equation (15), we have

$$\frac{h_p}{h_r} = \frac{h_p}{h_{pmax}} \times \frac{h_{rmax}}{h_r} \times \frac{a_p}{a_r}$$

Therefore, substituting from (18), (19), and (20)

$$\frac{h_p}{h_r} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (21)$$

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[*] This ratio will in general be closer to unity than stated here because certain minimum plate-thicknesses must be used (see pp. 383-385, Ref. 7) and because practical thickness will have to be always larger than the calculated decimal value, especially for low-medium heads up to 300 ft.

[15] Quick, R.S. "Comparison and Limitation of Various Water-Hammer Theories," Mechanical Engineering (ASME) May, 1927.

Similarly,

$$\frac{h_p}{h_s} = 1 \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (22)$$

These results are also applicable to penstocks of varying diameter and thickness[4] (especially if N is greater than 5), and to such non-uniform gate motions where it is possible to obtain "net equivalent time." [3,4]

Summary of Water Hammer Comparison

(1) Prestressed concrete penstock has a significant advantage over reinforced concrete penstock for instantaneous and rapid surges that may arise from accidental or emergency conditions. For the range of use of the reinforced concrete penstock, i. e., 50 to 150 ft head, prestressing would result in a reduction of water-hammer pressures by approximately 50% to 20%. Therefore if the penstock is designed according to working head design, then prestressing will increase the factor of safety as applied to the static pressure by curtailing the maximum possible water-hammer pressure. But if the basis of design is ultimate load, then prestressing will result in a smaller total ultimate design head (due to smaller maximum water-hammer head) and thus economize materials.

(2) Water-hammer due to sudden closure is of approximately the same magnitude in a steel penstock designed with 50% joint efficiency, as in an alternatively designed prestressed concrete penstock. For 90% joint efficiency, however, the latter is subjected to pressure fluctuations higher than the former by about 13% for 600 ft design up to a maximum of 23% for 150 ft design. This margin will, however, be much smaller for practical steel penstock designs.

(3) For slow uniform closures, no penstock type has an advantage over the others with respect to water-hammer pressures.

IV

COMPARISON OF MATERIALS AND WEIGHTS OF PENSTOCK PIPES

There are many variables involved in the costs of production and erection of a penstock pipe-line. The most fundamental variables contributing to these two items of cost are: the quantities of different materials used, and the weight of the pipe-section respectively. They may well form the basis of estimating the relative economy of different types of penstock pipes. Comparative knowledge of materials is also necessary for nation-wide planning of materials-resources, especially in countries like India where the hydro-power developed is only a few percent of the potential available.

The following comparison of materials and weights is based on alternative designs for the same design head and diameter.

Comparison of Materials

Quantities of materials (in cubic inches) as required in terms of head and diameter per inch length of the pipe (as per its structural design) are listed in Table 3 for the three penstock types under discussion.

Table 4 gives numerical values of the quantities of materials at certain design heads and diameters.

Table 3. Comparison of Penstock Materials

Item	Non-cyl. P/C Penstock	Steel-cyl. P/C Penstock	R/C Penstock	Steel Penstock
1 Cement Concrete	$\frac{\pi D^2 H}{6080}$	$\frac{\pi D^2 H}{6080} - 0.375\pi D$	$\frac{\pi D^2}{12} +$	-
2 Cement Mortar	$0.75\pi D$	$0.75\pi D$	-	-
3 High Tensile Steel	$\frac{\pi D^2 H}{364800}$	$\frac{\pi D^2 H}{364800}$	-	-
4 Circumferential Mild Steel	-	$0.0625\pi D$	$\frac{0.217\pi D^2 H}{16000 - 40H}$	$\frac{0.217\pi D^2 H}{15000\ e}$
5 Longitudinal Mild Steel	0.25%-0.50% of concrete	-	0.25%-0.50% Of concrete	-

Note: (1) A steel cylinder of 16 gage U. S. Std has been assumed for Steel-cyl. P/C Penstock; (2) Design stresses for P/C Penstocks are as used for Fig. 3: (3) Circumferential steel stress for R/C Penstock is according to the U. S. Bureau of Reclamation, and that for steel penstock, according to Reference 7; (4) These quantities do not include the materials required in pipe-joints; and (5) D is in inches, and H in feet.

Table 4. Comparison of Penstock Materials

H ft.	D in.	Penstock Type	Circum. M.S. in.^{3}	Long M.S. in.^{3}	High Tensile Steel in.^{3}	Concrete in.^{3}	Cement Mortar in.^{3}
50	144	Non-cyl.P/C	-	2.67	8.91	535	338
		R/C	50.60	27.15	-	5430	-
100	72	Non-cyl.P/C	-	1.34	4.45	268	169
		Cyl.P/C	14.10	-	4.45	183	169
		R/C	29.40	9.05	-	1810	-
150	72	Non-cyl.P/C	-	2.05	6.67	410	169
		Cyl.P/C	14.10	-	6.67	318	169
		R/C	52.90	12.40	-	2480	-
		1 RL Steel	76.90	-	-	-	-
		2 RL Steel	61.50	-	-	-	-
		W Steel	113.00	-	-	-	-
300	72	Cyl. P/C	14.10	-	13.35	735	169
		1 RL Steel	141.50	-	-	-	-
		2 RL Steel	113.00	-	-	-	-
		W Steel	113.00	-	-	-	-
600	72	Cyl. P/C	14.10	-	26.70	1470	169
		2 RB Steel	169.50	-	-	-	-
		W Steel	169.50	-	-	-	-

Note: (1) 1 RL means single-riveted lap joint; 2 RL means double-riveted lap joint; W means welded; 2 RB means double-riveted double-butt joint.

(2) Joint efficiencies and minimum thickness of steel penstocks are as given on pp. 385, and 387 of Ref. 7.

(3) Minimum thickness of R/C Penstock according to the U. S. Bureau of Reclamation.

(4) Longitudinal reinforcement at 0.5% of the concrete section.

The following approximate expressions for the ratios of quantities are obtainable from Table 3, and may be checked roughly with Table 4.

Materials	Ratio: Penstocks R/C to P/C	Ratio Penstock Steel to P/C
Mild Steel / High Tensile Steel =	$\frac{5}{1 - 0.0025H}$	$\frac{5.3}{e}$
Concrete / (Concrete & Mortar) =	$\frac{510}{H + \frac{4550}{D}}$	-

Note: D is in inches and H in feet.

It is noteworthy that the ratio of mild steel to high tensile steel varies from about 6 to 10 in both comparisons, which is very much more than the inverse ratio of the costs of these materials. The ratio of concrete in reinforced concrete penstock to that in prestressed concrete penstock varies from roughly 4 to 6. This shows the possibilities of tremendous savings in materials accruing from the use of prestressed concrete pipe.

Comparison of Weights

Weights of pipe-sections of equal diameters and designed for equal heads can be obtained from Table 3. Approximate expressions for ratios of weights and their numerical values at certain values of H and D are given in Table 5.

Table 5. Comparison of Penstock Weights

H ft.	D in.	Ratio of wts. for P/C to R/C $= \frac{0.073H + \frac{180}{D}}{0.128H + 41}$	Ratio of wts. for P/C to Steel $= 3.29e + \frac{8100e}{DH}$	
			e = 90%	e = 50%
50	72	1/6.63	-	-
50	144	1/ 5.48	-	-
100	72	1/5.50	-	-
150	72	1/4.33	3.12	1.73
300	72	-	3.28	1.81
450	72	-	3.18	1.76
600	72	-	3.12	1.73

Note: These weights do not include the end-connections. The weights of mortar and reinforcements are included in the weights of the concrete pipes. Densities of 490 lb/cft and 150 lb/cft are used for steel and concrete respectively.

Table 5 shows that prestressed concrete pipe-sections are about 4 to 7 times lighter than conventional reinforced concrete pipe-sections, but only about 2 to 3 times heavier than steel pipe-sections. If steel penstocks of plants with 1500-1800 ft design head can be safely handled, it follows that prestressed concrete penstocks with 500-900 ft design heads (being of equal diameters also) can be handled with the same equipment and safety.

Relative Economics of Pressure Pipes

Production costs of different types of pipes are closely guarded secrets of the manufacturers. Competitive bids with an alternative for materials have been rarely invited. Prestressed concrete pipe has not as yet entered the competition in the field of penstocks, but it has already beaten the conventional reinforced concrete pipe and the steel and cast-iron pipe in many high-pressure supply projects. At the same time, reinforced concrete has successfully competed with steel in penstock projects of low heads, and its prospects (especially with cylinder lining) have been strongly indicated for medium heads. The following instances along these trends have been reported:

(1) Non-cylinder prestressed concrete pipe versus non-prestressed steel-cylinder concrete pipe, and steel & cast iron pipe : Reference 2 reports that on competitive bids for 11244 linear feet of 36 in. pipe (37 psi working pressure), $9.44 per foot was quoted for non-cylinder prestressed concrete pipe (even though a novel experience) as compared with $10.25 per foot for steel-shell-encased concrete pipe. And it was "expected that the cost of the pipe laid will be $5.00 per foot less than 36 in. pipe has usually cost the city..." (meaning thereby a comparison with the steel and cast-iron pipes in use before). These figures show that prestressed concrete pipe was about 33% cheaper than steel and cast-iron pipe.

(2) Steel-lined concrete pipe (non-prestressed) versus plate-steel pipe : Reference 14 reports that for 8264 lineal feet of 22 1/3 feet diameter siphon pipes (maximum hydrostatic head 225 feet), the bid on behalf of the former amounted to $5,398,000 as compared with $7,761,000 on behalf of the latter---a saving of about 30.5%. The report further suggests that "in view of this impressively large saving, the use of steel-lined concrete pipe is being studied by the Bureau of Reclamation for use not only as high-pressure pipe for canal siphons, but as an alternative design for plate-steel penstocks and outlet pipe for dam and power plant work."

V

CONCLUSION

(1) An ultimate design of the penstock pipe allowing for the maximum possible water hammer pressures that may occur during emergency or accidental circumstances, seems to be a realistic approach to the problem in view of the history of penstock failures. Such a design procedure for the prestressed concrete pipe, with a suggested definition of the "ultimate" as the elastic limit of the pipe, has been proposed in this thesis and may deserve attention.

(2) An analytical comparison between alternative designs of the reinforced concrete pipe and the prestressed concrete pipe shows the following advantages in favor of the latter:

(a) Reductions of 20-50% in the maximum water hammer pressures;

(b) Savings of 75-85% in the quantity of concrete;

(c) Savings of 80-90% in the quantity of steel; even though not equally in the cost of steel;

(d) Reductions of 75-85% in the weight of precast pipe sections.

These figures are impressive enough to warrant the use of the prestressed concrete pipe in the place of the reinforced concrete pipe for penstocks of 50-150 feet heads, provided the size required is not larger than approximately $12\frac{1}{2}$ feet and provided the topography does not necessitate an in-situ monolithic construction.

(3) Comparison between alternative designs of the steel pipe and the prestressed concrete pipe shows that the latter

(a) is subjected to maximum water hammer pressures which are higher than those in the former by zero to 20% only, for joint efficiencies of 50-90% of the steel pipe;

(b) causes savings of 80-90% in the quantity of steel;

(c) weighs about 2 to 3 times as much as the steel pipe.
These figures and the available information on competitive bids for high-pressure supply lines indicate that the prestressed concrete pipe may also compete with the steel pipe for medium to medium-high head penstocks. Production costs of the former will almost certainly be smaller than those of the latter, as proved in the field of supply lines where the handling and laying operations are equally simple for both the types of pipes. For penstock construction, however, the weight handicap of the prestressed concrete pipe coupled with difficult topography would raise the construction costs by a variable amount. As such, it may not be possible for prestressed concrete penstocks to compete with steel penstocks all the way up to very high heads. But since the handling equipment now being used for such high head steel penstocks can also handle medium or medium-high head prestressed concrete penstocks, the prospects of a keen competition in this range of heads are bright, particularly in countries suffering from a short supply of steel. Another situation favorable to prestressed concrete penstocks is a dearth of expert welders and heavy riveters, necessary for high-head, thick-plate steel penstocks.

(4) It may be pointed out that while the high tensile steel used for prestressing concrete would save as much as 80-90% of the structural steel, it would at the same time entail extra processing and extra labor. Actually these are ideal solutions for the twin problems of unemployment and short supply. A country like India, therefore, as she stands on the threshold of a large hydro-electric development program, may explore the possibility of manufacturing high tensile steel wires at her existing steel plants.

APPENDIX A

SELECTION OF STRESSES, n, and η FOR THE PRESTRESSED CONCRETE PIPE DESIGN

(1) f_{co} may be selected in the same manner as safe compressive stress for linear prestressed concrete members. Specifications[11] for this stress range from $0.3f'_c$ to $0.4f'_c$, where f'_c is the cylinder strength. f'_c for concretes used in prestressed structures may vary from about 4800 psi to 7000 psi.

(2) f_{cf} desired in concrete pressure pipes will depend on the margin of pressure fluctuations about the design head. Quality of the pipe may be substantially improved by leaving f_{cf} of 200 to 400 psi in the concrete. However, most of the current practices* make $f_{cf} = 0$.

(3) E_c varies from 4×10^6 psi to 5×10^6 psi. E_s of the high tensile wire is practically the same as that for mild steel, and may be taken as 30×10^6 psi. This would make n equal to 6 to 7.5.

(4) Y_b of the mild steel cylinder is more or less standard and may be taken as 33000 psi.

(5) f_{so} should be selected in such a way that the elastic limit of the wire should be reached at the same time as, or after, the elastic limit of the cylinder. Relation (5) would be helpful in this connection. Maximum value of f_{so} for any given wire is that obtained from this relation by making f'_s equal to the elastic limit of the wire. For example, corresponding to the elastic limits of 160,000 psi and 120,000 psi for strain-relieved and galvanized wires respectively[16], maximum values of

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*References 1, 3, and 5

[16] J. A. Roebling's Sons Co. <u>Roebling Strand and Fittings for Prestressed Concrete</u>, Catalog T-918.

f_{so} obtained are 145,000 psi and 97.000 psi respectively (using $f_{co} = 2000$, $n = 6$, $\eta = 0.80$).

(6) η is subject to controversy. Some authors suggest that it is so close to unity that it may be assumed to be equal to unity. Crepps[17] is doubtful if the loss in prestress would exceed 3000 to 5000 psi, and suggests that this will be offset by a circumferential expansion due to water absorption and by an increase in the wire tension due to internal pressure. The latter argument is, however, invalid because this increase in wire tension does not counter the effect of the loss of prestress so far as concrete is concerned. The role of expansion due to absorption is qualitatively correct, but its approximate magnitude[18] is probably about 0.0001 as compared with the combined shrinkage and flow value of 0.0003 to 0.0008. Ross[8] observed in his tests that the wire prestress had reduced to 65,000 psi from 86,000 psi, the pipe however having been kept dry into the test. Longley[1] acknowledges that the loss of prestress may amount to 20%. The part played by crrep of the steel is probably negligible. Roebling Sons Company[16] assures that if the wire is stressed no higher than 55% of its ultimate strength, there will be virtually no creep. The discussion leads up to the conclusion that a value of η between 0.85 and 0.80 will be on the safe side. These values of η are widely used in linear prestressing.

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[17] Crepps, R.B. "Wire-Wound Prestressed Concrete Pressure Pipe," Journal, ACI, June, 1943.

[18] Terzaghi, R.D. "Differences in Characteristics of Concrete in Wet and Dry State," Journal, ACI, Nov. 1946.

APPENDIX B

ECONOMIC DIAMETER OF PRESTRESSED CONCRETE PENSTOCK

Using the nomenclature given in pp. i-ii, the cost per lineal foot of prestressed concrete pipe will be given by

$$C = \frac{c_1}{27}\,\frac{\pi D}{12}\,\frac{t}{12} + c_2\left(\frac{12A_s}{144}\right)\frac{\pi D}{12}(490) + c_3\left(\frac{12\alpha D}{144}\right)\frac{\pi D}{12}(490)$$

But from structural design we have

$$t + n\alpha D = \frac{0.217HD}{(\eta f_{co} - f_{cf})\left(1 + \frac{nf_{co}}{f_{so}}\right)} = \phi' HD, \text{ say}$$

$$A_s = \frac{0.217\ HD}{\frac{f_{so}}{f_{co}}(\eta f_{co} - f_{cf})\left(1 + \frac{nf_{co}}{f_{so}}\right)} = \phi'' HD, \text{ say}$$

Substituting for t, αD, and A_s in the expression for C:

$$C = \frac{\pi D^2}{144}\left\{ H\left(\frac{c_1\phi'}{27} + 490\ c_2\phi''\right) + \alpha\left(490\ c_3 - \frac{c_1 n}{27}\right)\right\} = K''D^2, \text{ say}$$

Now power lost (in terms of horse power) per lineal foot of pipe is given by

$$P = \frac{Q}{8.8}\left(\frac{f}{D/12}\,\frac{V^2}{2g}\right) = \frac{714fQ^3}{D^5} \qquad \text{since } V = \frac{Q}{\frac{\pi}{4}\left(\frac{D}{12}\right)^2}$$

Yearly cost per foot of the pipe line

$$= Ci + Pb$$

$$= K''iD^2 + \frac{714fbQ^3}{D^5}$$

Differentiating with respect to D for minimizing the yearly cost,

$$2K''iD - 3570\,\frac{fbQ^3}{D^6} = 0$$

$$D = \sqrt[7]{\frac{1785fbQ^3}{K''i}}$$ where K" is a linear function of H,   , and cost-rate constants.

It is to be noted that $\alpha = A_b/D$ will have to be guessed for the first approximation, but the effect of the accuracy of this guess on the value of D will be almost negligible. Furthermore, f, b, and H also affect the value of D only to a seventh root; Q is the only variable exercising considerable influence on the value of D.

Similar expressions for steel and reinforced concrete penstocks[7] show that in those cases also the initial cost of the pipe affects the value of economic diameter, D, only to a seventh root.

It follows that, even though the initial costs of alternative pipes may be considerably different, they all will require the same diameter size provided that they are designed for identical heads and flow.

BIBLIOGRAPHY

1. Longley, F.F. "Prestressed Reinforced Concrete Pipe," Waterworks and Sewerage, Dec. 1945.

2. DeBerard, W.W. and W.B. Weldon. "Experiences with Wire-Wound Prestressed Concrete Pipe," Journal, Am. W.W. Assn, Oct. 1943.

3. Davis, C.P. Handbook of Applied Hydraulics, 1952 ed.

4. Creager, W.P. and J.D. Justin. Hydroelectric Handbook, 1950 ed.

5. Ross, C.W. "Tests of Prestressed Concrete Pipes Containing a Steel Cylinder," Journal, ACI, Sept. 1945.

6. U. S. Bureau of Reclamation. Penstocks and Outlet Pipes, Part IV, Boulder Canyon Project Reports, 1940.

7. Barrows, H.K. Water Power Engineering, 1943 ed.
8. U.S. Bur. of Recl. Stress Analysis of Concrete Pipes, Eng. Mon. No.6,Oct.1950
9. American W.W. Assn. Tentative Emergency Specifications for Reinforced Concrete Pressure Pipe (Designation 7B-T)

10. Ferrand, Georges. "Pressure Pipe Lines," Water Power (Lond.), March-April, 1950.

11. Schoklitsch, A. Hydraulic Structures, trans. by L. G. Straub, 1937.

12. Ferrand, Georges. "Overpressures and Self-Hooped Penstocks," Water Power (Lond.), Oct. 1952.

13. Billings, A.W.K. and others. "High-Head Penstock Design," ASCE-ASME Symposium on Water Hammer, 1933.

14. U.S. Bur. of Recl. Soap Lake Siphon, Eng. Monographs No. 5, Aug. 1950.

15. Quick, R.S. "Comparison and Limitation of Various Water Hammer Theories," Mechanical Engineering (ASME), May 1927.

16. Roebling, J.A. Sons Co. "Roebling Strand and Fittings for Prestressed Concrete," Catalog T-918.

17. Crepps, R.B. "Wire-Wound Prestressed Concrete Pressure Pipe," Journal (ACI), June 1943.

18. Terzaghi, R.D. "Differences in Characteristics of Concrete in Wet and Dry State," Journal (ACI), Nov. 1946.