Lateral Vibration of Pedestrian Bridges

by

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Abstract

Engineering is taking the art of pedestrian bridge design to new limits as bridges become longer and more slender. The current trend to produce sleek structures is resulting in the appearance of behavioral modes that were not known to exist. One such mode is the human synchronization with the lateral motion of the bridge. For a number of years vertical deflections of footbridges have been known to cause discomfort in pedestrians and have therefore been kept under certain predetermined levels. However, the lateral sway of bridges, which occurs in long and slender structures, is a relatively new phenomenon. Very little research has been carried out to study this problem, nor is it addressed in bridge design codes.

In order to predict the behavior of a bridge under synchronized human forces, a force model must be developed. The dynamic force models that may be used for analysis purposes are described in the following sections. In addition to this, methods available to either avoid or mitigate this problem in pedestrian bridges such as frequency tuning and damping – both active and passive systems, are also discussed.

The final sections of this thesis discuss the characteristics of some bridges that have displayed large lateral vibrations due to pedestrian induced forces, and the various steps taken to mitigate the vibration problem. Results for a very preliminary finite element analysis on a pedestrian bridge modeled first as a beam and then a plate to study its natural frequencies of vibration are also presented.

Thesis Supervisor: Jerome J. Connor
Title: Professor of Civil and Environmental Engineering

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CONTENTS

LIST OF FIGURES

LIST OF TABLES

1. Introduction

Dynamic forces are induced due to human body motions on pedestrian bridges. The vertical motion of a bridge due to forces induced by human motion has been the subject of much analysis and research. Since bridges are usually stiff enough in the lateral direction, lateral motion is generally not a problem and it has not been well-studied nor well-analyzed. However, in the year 2000 when a large crowd of people crossing the newly opened Millennium Bridge set the bridge moving laterally with deflections that were far greater than those permitted, lateral vibration of bridges was brought into the public eye, and into the forefront of concerns within the engineering community.

The problem addressed in this paper, although similar to that of soldiers needing to break step before crossing bridges, is not the same. A large crowd of people causes both these phenomena, but through different ways. The soldiers are initially walking in step and need to break this rhythm before crossing the bridge to prevent setting up a resonance within the structure. However, with the problem of lateral vibration in pedestrian bridges, the crowd is initially walking randomly. After the bridge begins some movement, they start becoming more synchronized with each other and the bridge. This is due to the natural human response in trying to synchronize with the motion of the bridge.

In order to predict the behavior of a bridge under the synchronized human forces, a force model must be developed. In addition to this it is necessary to investigate methods to either avoid or mitigate this problem in pedestrian bridges.

2. Dynamic Force Model

Vibration problems arise when the natural frequencies of the structure coincide with the frequency of the force imposed on the structure. This causes a resonance problem. In the case of pedestrian bridges, the forcing frequencies under consideration are that of walking and running. Many of the codes such as the British Standard and Ontario Highway Bridge Design Code provide design requirements and guidelines. However, these codes do not address the issue of lateral motion of bridges due to pedestrian motion. It is therefore necessary to devise methods to model these forces. As a first approximation the models used to apply pedestrian loading in the vertical direction can also be used to apply loads in the lateral direction.

Typical pedestrian walking frequencies range from 1.6 to 2.4 Hz while running frequencies range from 2.0 to 3.5 Hz. It should be noted that the frequencies of the second and third harmonic of the normal walking rate are 4 Hz and 6Hz, and these could also be important in some instances. In addition to walking and running, vandal loading must also be taken into consideration. Vandal loading is when people purposely try to cause bridge vibration by exerting forces that are known to cause oscillation. It is suggested that jumping also be included in the loading models for footbridges [12].

Statistical data on bridges show that typically concrete bridges are more susceptible to vibrations than steel bridges. Also, steel footbridges tend to have less vibration damping than do concrete bridges. Some approximate measures of the fundamental frequency of vibration are given by the following equations [1]:

Let L – span (m)

f – fundamental natural frequency (Hz)

$$f = 39L^{-0.77} \text{ (for Concrete)} \quad (2.1)$$

$$f = 35L^{-0.73} \text{ (for Steel)} \quad (2.2)$$

The following section describes more accurate models that can be used to predict the bridge response to human loading.

Fourier Series Model

There are several possible ways to represent pedestrian induced forces. One accurate method of doing so is as normalized dynamic forces that are described by a Fourier series of the form [1]

$$F_p(t) = G + \sum_{i=1}^{n} G.\alpha_i.\sin(2\pi_i f_p t - \phi_i) \qquad (2.3)$$

where G – weight of the person (=800N)

α_i – Fourier coefficient of the i-th harmonic

$G.\ \alpha_i$ – force amplitude of the i-th harmonic

f_p – activity rate (Hz)

ϕ_i – phase lag of the i-th harmonic relative to the 1st harmonic

i – number of the i-th harmonic

n – total number of contributing harmonics

In the case of lateral forces induced due to walking, Equation (2.3) results in

$$F_p(t) = 400.(\sin(2\pi..t) + 3.\sin(6\pi..t)) \qquad (2.4)$$

with $\alpha_{1/2}$ = 0.1

$\alpha_{3/2}$ = 0.1

f_p = 2 Hz

The values of α and ϕ are obtained from Tables A.1 and A.2 in the Appendix (A.1).

Equation (2.3) expressed for running is

$$F_p(t) = 1280.\sin(4\pi t) + 560.\sin(8\pi t) + 160\sin(12\pi t) \qquad (2.5)$$

and

$$F_p(t) = 1280.\sin(3\pi.t) + 560.\sin(12\pi t) + 160.\sin(36\pi t) \qquad (2.6)$$

with α_1 = 1.6

α_2 = 0.7

α_3 = 0.2

f_p = 2 Hz (Equation 2.5) and 3 Hz (Equation 2.6)

This suggests that bridges showing natural frequencies that are in the proximity of the frequency range 2-3 Hz must be carefully analyzed for pedestrian induced motion. It should be noted however, that these methods have been shown to result in an overestimation of the peak acceleration for the frequency ranges of the second harmonic [11]. This may be due to the varying phase between the structural vibrations and the actual load applied by a pedestrian.

Model Refined for Lateral Motion

In the case of lateral forces induced by pedestrian walking, the lateral sway of a person walking will usually be half the walking frequency. Therefore for lateral loading, frequencies as low as 1Hz should also be considered critical. Figure 2.1 illustrates this.

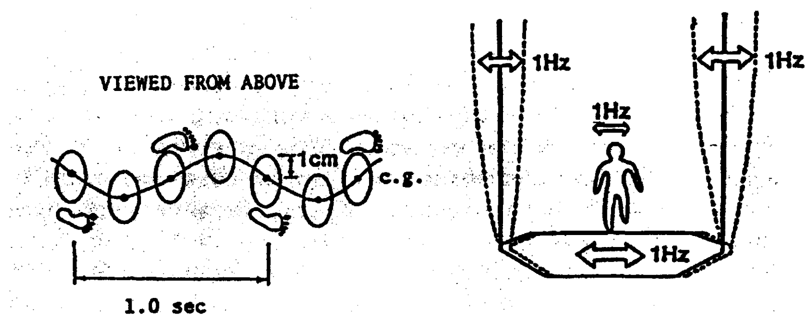


Figure 2.1: Resonance due to lateral pedestrian motion [7]

In order to consider loading due to a large number of pedestrians, n, where ϕ_i is a mutually independent random variable, the lateral force on the girder can be given by

$$F_p(t) = \sum_{i=1}^{n} G.\alpha_i.\sin(2\pi_i f_p t - \phi_i) \tag{2.7}$$

This equation can be re-written as

$$F_p(t) = \sqrt{n} G.\alpha_i.\sin(2\pi_i f_p t - \phi) \tag{2.8}$$

Therefore, the total effective force from n persons' walk is equivalent to that from $n^{1/2}$ persons' completely synchronized walk [7]. Then assuming the lateral girder to be a single-degree-of-freedom system, the lateral response can be computed by [7]:

$$m(\ddot{X} + 4\pi\xi f_0 \dot{X} + 4\pi^2 f_0^2 X) = \frac{1}{2}\sqrt{n}F \sin(2\pi f t + \phi) \quad (2.9)$$

where X – is the amplitude of the first lateral mode of the girder

ξ – is the damping ratio

f_0 – is the natural frequency of the first lateral mode

n – is the number of persons

f – is the frequency of lateral force from human walk

F – the lateral force from human walk (23 N normally)

m – the effective mass of the first lateral girder mode

However, this approach underestimates the lateral loading due to pedestrians, as it does not account for the possibility of feedback and synchronization. Fujino et al [7] conducted a detailed study on human walk on a cable-stayed bridge. This study provides a great deal of insight into the human response to lateral vibration of bridges. The frequency of walking was seen to change with time, was different for different people, and depended on the congestion levels.

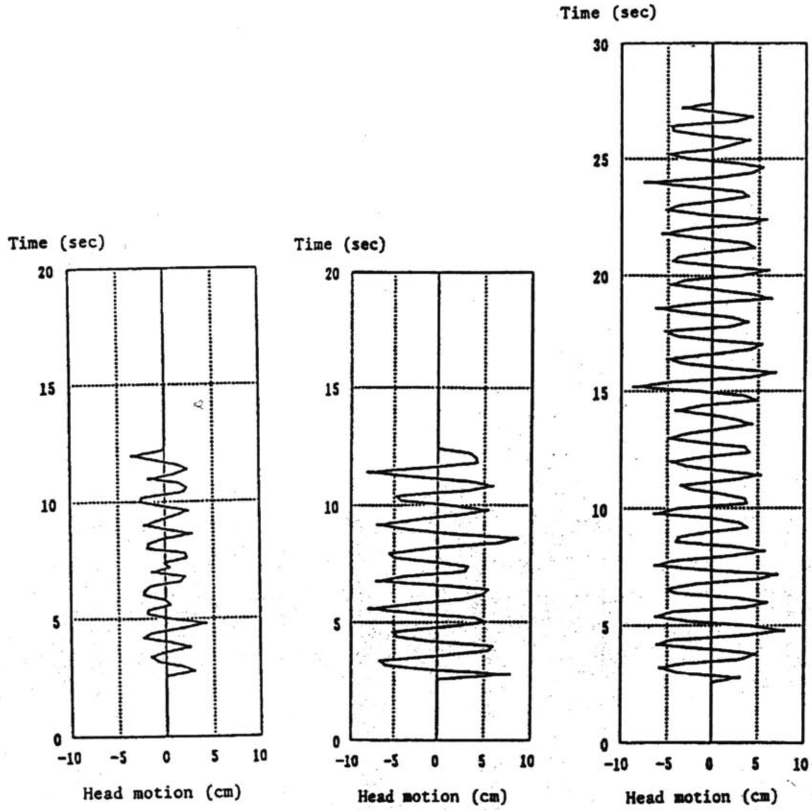
Table 2.1: Frequencies of human walk and bridge vibration [7]

Time	Stage	N	Walking Frequency (Hz)		Pedestrian Condition
			Mean	Std. Dev.	
4.27 (May 5, 1989)	1	8	0.984	0.113	Not congested
4.30 (May 5, 1989)	2	15	0.830	0.086	Congested
4.35 (May 5, 1989)	3	18	0.848	0.082	Extremely Congested
4.40 (May 5, 1989)	4	20	0.864	0.053	(2000 persons)
4.50 (May 5, 1989)	5	25	0.658	0.087	Decreasing
4.30 (Dec. 31, 1989)	6	25	0.883	0.078	Large

Table 2.1 shows the walking frequencies of pedestrians at different times and in different crowd conditions. N is the number of people whose head motions are analyzed. It is clear from this table, that in uncrowded situations, the walking frequencies are close to 1 Hz,

which is the frequency that is expected for lateral swaying. However, as the bridge gets more crowded, the walking frequency becomes smaller and gets closer to the natural frequency of the bridge, which is approximately 0.9 Hz in this particular example. This could be because, when the bridge is not congested with people, they are able to walk normally at a normal pace. However, in congested situations, people are not able to pace normally.

In this experiment the lateral head movement of a number of pedestrians was recorded. The head movement is used as a representation of the lateral body movement of the pedestrian. The results show that pedestrians tend to display larger lateral movements in crowded situations than they do in uncrowded situations. This is illustrated in Figure 2.2. It should be noted that in this example, the lateral girder motion has not been subtracted from the total lateral head motion. However, the girder motion has a magnitude of only about 1-cm, which is extremely small compared to the lateral pedestrian amplitude of 5-cm. This increased magnitude of lateral motion means the lateral force induced will be greater than the 23 N normally used. A possible reason for the increased magnitude of lateral motion could be that the pedestrians cannot freely step forward in crowded situations and hence tend to step sideways instead [7]. Another explanation could be that the pedestrians try to stabilize themselves while walking on a vibrating bridge by stepping sideways. A study carried out by Okamato et al confirms this, and also states that the walking frequency becomes more synchronized to the floor frequency when the lateral amplitude reaches 1-2 cm [7]. This synchronization of pedestrian motion to the motion of the bridge is a subconscious effort to stabilize oneself.



a) Not congestion and small lateral girder vibration (Stage 1 in Table 2.1)

b) Extreme congestion and large lateral girder vibration (Stage 4 in Table 2.1)

c) Extreme congestion and large lateral girder vibration (Stage 6 in Table 2.1)

Figure 2.2: Examples of head motions [7]

Figure 2.3 shows two examples of human head motion corresponding to Stages 1 and 2 in Table 2.1. These are examples of fully synchronized head motions. Since these people were not standing next to each other the identical lateral motion (even though different in amplitude) is attributed to synchronization with bridge motion. Figure 2.4 shows three examples of partially synchronized motion. In this study, phase difference between a reference sinusoidal motion (taken as 0.92 Hz) and the head motions was also calculated. This phase difference was calculated to maximize

$$\int_{\tau}^{\tau+1} h_i(t)\sin(2\pi ft + \phi_i(t))dt \qquad (2.10)$$

where $h_i(t)$ – head motion of the ith person

t and τ – time in seconds

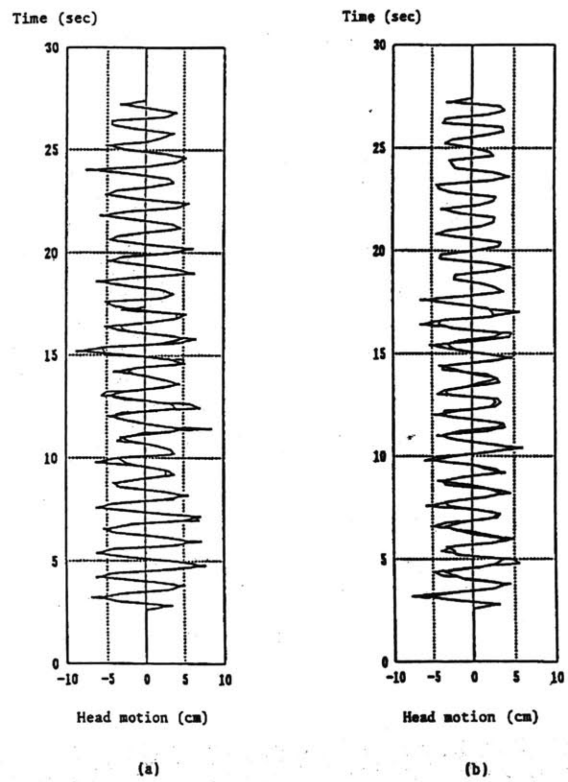


Figure 2.3: Examples of head motions of two completely synchronized persons [7]

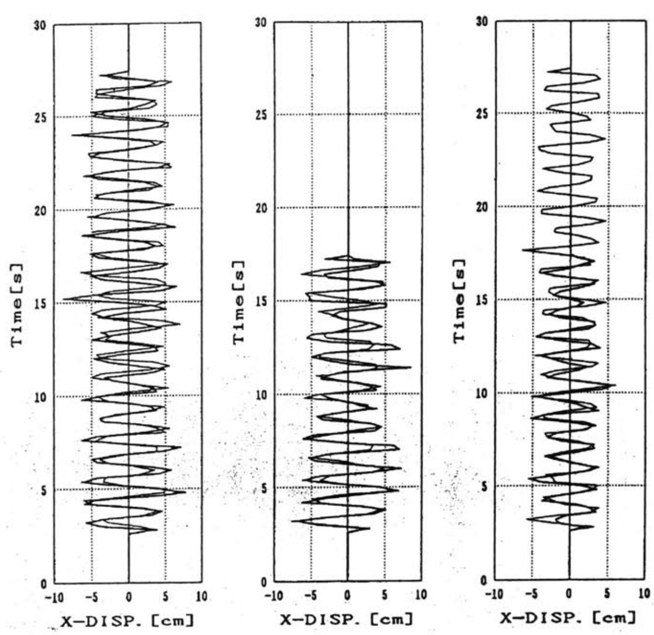


Figure 2.4: Examples of head motions of two partially synchronized persons [7]

Figure 2.5 shows the phase angles plotted against time. Figure 2.5a shows that the five people are almost in phase with each other and Figure 2.5b shows that another five people are well synchronized at another phase.

Therefore, out of the twenty-five people whose motion was recorded, twenty per cent or more show synchronization to the lateral girder vibration. This synchronized force acts as a resonant force on the bridge. It was suggested earlier on that the equivalent number of persons whose frequency is perfectly in tune with the bridge is $n^{1/2}$. However, with the new evidence, it can be stated that the number of people perfectly in tune with the bridge is given by $0.2n$ [7]. Furthermore, it was noticed that the amplitudes of lateral motion in congested bridges are much larger than normal. Hence the lateral force due to walking should also be greater than the normally accepted 23 N. 35 N is considered a reasonable value to use instead [7]. These redefined parameters provide a far more accurate estimate of lateral deflection due to pedestrian motion in congested situations.

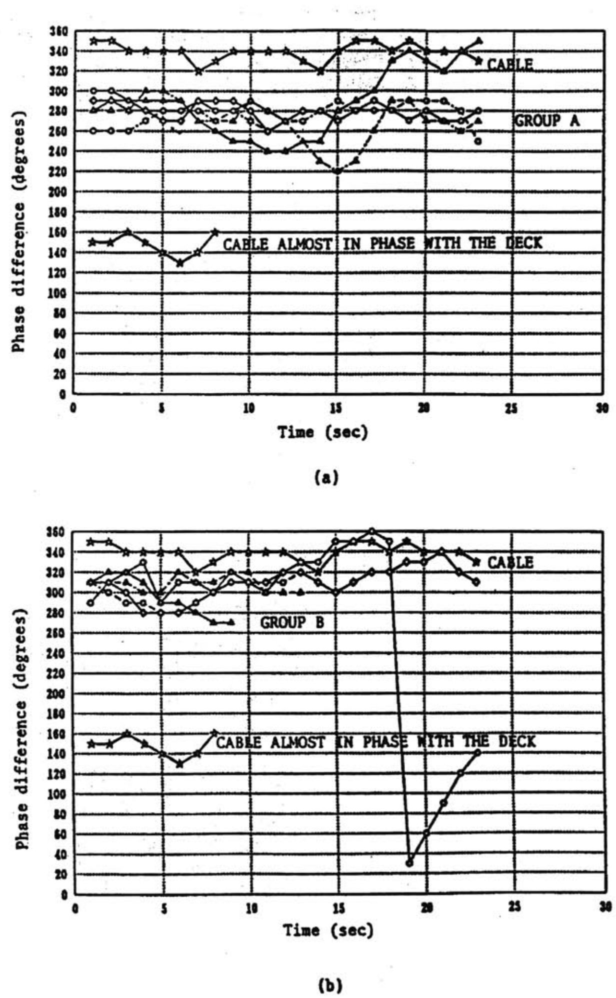


Figure 2.5: Nonstationary phase difference between human head motions and the reference sinusoidal motion [7]

3. Dynamic Analysis

The stiffness of a bridge is governed by all the elements that make up the structure. In the case of lateral stiffness, the bridge deck usually provides a large portion of this stiffness. In addition to this, other structural elements such as cables and pylons in cable-supported bridges, the arch structure in arch bridges, truss members in truss bridges also provide additional stiffness.

Dynamic Behavior of Deck under Lateral Loading

The deck is one of the major contributors to the lateral stiffness of a bridge. In most bridges, the span to width ratio is low enough for the lateral loading due to pedestrians not to be governing factors in terms of serviceability requirements. Appendix A.2 shows frequencies and mode shapes of a hybrid suspension bridge 750-feet long and 132-feet wide. The frequency of vibration of the lowest lateral mode for this bridge is 1.41 Hz. This is larger than the lateral frequency of walking for pedestrians, and will therefore not be excited by pedestrian motion. However, as pedestrian bridges tend to get longer and more slender, we see a trend towards very high span to width ratios. This results in minimal deck stiffness, which requires the bridge to possess other means of providing the necessary stiffness for lateral resistance, or methods to oppose lateral motions such as damping.

Dynamic One-Dimensional Analysis

As a first approximation a pedestrian bridge deck may be modeled as a beam clamped at both ends. This will be a 1-D analysis taking into consideration only the beam bending. This approach will obviously neglect the lateral stiffness provided by the suspension cables, truss members or any other lateral stiffness contributors. However, the results will not be too far from the real frequencies with regard to cable-supported bridges since the cables provide little stiffness in the lateral direction.

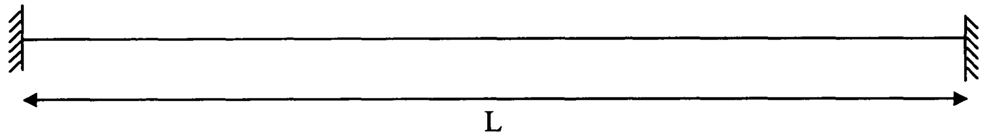


Figure 3.1: Beam Model

The governing equation for a bending beam is given by

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 y}{\partial x^2}\right)+\mu\frac{\partial^2 y}{\partial t^2}=0 \tag{3.1}$$

where $y(x,t)$ represents the transverse displacement of the beam.

Assuming the beam is homogeneous throughout its length, Equation (3.1) becomes

$$EI\frac{\partial^4 y}{\partial x^4}+\mu\frac{\partial^2 y}{\partial t^2}=0 \tag{3.2}$$

When the beam cross-sectional dimensions are small compared to its length, we can assume a harmonic time-dependent solution $y(x,t) = u(x)e^{i\omega t}$. Substituting this into Equation (3.2) we arrive at the fourth-order ordinary differential equation

$$\frac{\partial^4 u}{\partial x^4}+\gamma^4 u=0 \tag{3.3}$$

where we define

$$\gamma^4=\frac{\omega^2\mu}{EI} \tag{3.4}$$

Trying a solution of the form $u(x) = e^{\alpha x}$ and solving the characteristic equation $\alpha^4 - \gamma^4 = 0$ gives $\alpha = \gamma$ and $\alpha = i\gamma$ so that

$$u(x)=A_1e^{\gamma x}+A_2e^{\gamma x}+A_3e^{\gamma x}+A_4e^{\gamma x} \tag{3.5}$$

$$u(x)=C_1\cosh(\gamma x)+C_2\sinh(\gamma x)+C_3\cos(\gamma x)+C_4\sin(\gamma x) \tag{3.6}$$

where A_i or C_i (i=1, 2, 3, 4) are determined from the boundary and initial conditions. For this problem, the boundary conditions are given by

$$u(0) = u(L) = 0 \tag{3.7}$$

$$\left.\frac{du}{dx}\right|_{x=0} = \left.\frac{du}{dx}\right|_{x=L} = 0 \tag{3.8}$$

$$\frac{du}{dx} = \gamma C_1 \sinh(\gamma x) + \gamma C_2 \cosh(\gamma x) - \gamma C_3 \sin(\gamma x) + \gamma C_4 \cos(\gamma x) \tag{3.9}$$

$$u(0) = C_2 + \gamma C_4 = 0 \tag{3.10}$$

$$\left.\frac{du}{dx}\right|_{x=0} = C_1 + C_3 = 0 \tag{3.11}$$

$$u(L) = C_1 \cosh(\gamma L) + C_2 \sinh(\gamma L) + C_3 \cos(\gamma L) + C_4 \sin(\gamma L) = 0 \tag{3.12}$$

$$\left.\frac{du}{dx}\right|_{x=L} = \gamma C_1 \sinh(\gamma L) + \gamma C_2 \cosh(\gamma L) - \gamma C_3 \sin(\gamma L) + \gamma C_4 \cos(\gamma L) = 0 \tag{3.13}$$

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ \cosh \gamma L & \sinh \gamma L & \cos \gamma L & \sin \gamma L \\ \sinh \gamma L & \cosh \gamma L & \sin \gamma L & \cos \gamma L \end{bmatrix} \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{3.14}$$

The determinant of this 4X4 matrix must be equal to zero in order to obtain nontrivial solutions. We therefore get

$$1 - \cosh(\gamma L)\cos(\gamma L) = 0 \tag{3.15}$$

The first eight roots of this equation are given by

$\gamma_1 L = 4.730$

$\gamma_2 L = 7.853$

$\gamma_3 L = 10.996$

$\gamma_4 L = 14.137$

$\gamma_5 L = 17.279$

$\gamma_6 L = 20.420$

$\gamma_7 L = 23.562$

$\gamma_8 L = 26.704$

Therefore, knowing L, E, I, and μ for a bridge deck, an estimate can be obtained for the natural frequencies. This method is used to perform an analysis of a suspension bridge in Chapter 6. In addition to this analytical method, finite element procedure may also be used to find the natural frequencies and mode shapes of a bridge deck. This is also illustrated in Chapter 6.

Dynamic Two-Dimensional Analysis

A more refined model of the bridge deck would be as a 2-D plate element with dimensions equal to that of the bridge deck (Figure 4). This system would be clamped along both the short sides allowing for no translations or rotations.

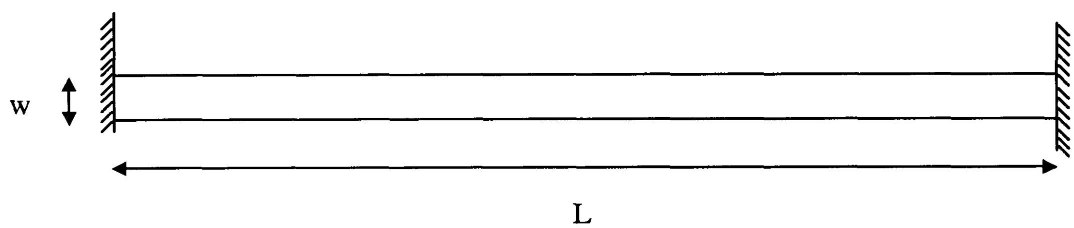


Figure 3.2: Plate Model

Once again finite element analysis may be carried out to assess the natural frequencies and mode shapes of the plate element. This is carried out for a suspension bridge in Chapter 6.

Dynamic Behavior of Cables under Lateral Loading in Cable-Supported Bridges

It is quite clear that cable systems are able to provide resistance against vertical loading. However, it is not as clear if and how cable systems provide lateral stiffness to a bridge structure. The ability of the cable system to provide stiffness in the horizontal direction is a result of second order effects that are determined by the support conditions (Figures 3.3 and 3.4)

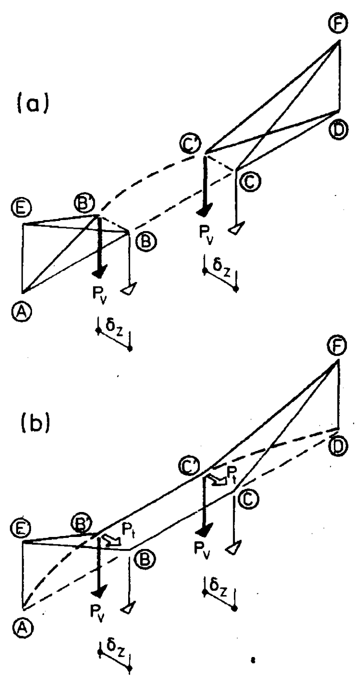


Figure 3.3: (a) Self-anchored and (b) Earth-anchored cable systems under lateral loading [9]

In these figures ABCD represents the deck, AE and DF represent the pylons, and EB and FC represent the cables in the undeformed configuration; AB'C'D represents the deck and EB' and FC' represent the cables under lateral loading. In the self-anchored system, under lateral loading, the deck and cables rotate about the pylons remaining in the same vertical plane i.e. the triangles formed by AB'E and DC'F are vertical. This means that there will be no lateral component of the cable force. However, in the earth-anchored system, the cable forces will have lateral components as a result of the cables moving into an inclined plane with the pylon i.e. AB'E and DC'F are no longer vertical but are slanted. Therefore earth-anchored systems offer lateral resistance while self-anchored systems do not. This is further illustrated in Figure 3.4

The lateral load P_t is given by equation (3.16):

$$P_t = \frac{\delta_z}{h} P_v \tag{3.16}$$

where δ_z – lateral displacement i.e. distance between B and B'

h – height of pylon above deck i.e distance between B and E

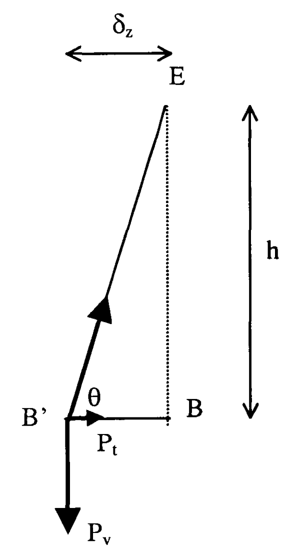


Figure 3.4: Lateral load P_t resulting from rotation of an initially vertical cable plane [9]

In order to analyze a cable system, the cables can be modeled as springs with stiffness k in the lateral direction where

$$k = \frac{P_t}{\delta_z} = \frac{P_v}{\delta_z \sin\theta}\cos\theta = \frac{w}{\delta_z \tan\theta} = \frac{w\delta_z}{\delta_z h} = \frac{w}{h} \qquad (3.17)$$

This lateral stiffness is considered to be distributed uniformly along the deck. This type of modeling is possible only for symmetric cable systems. The pylons and end piers may be considered as rigid supports. Now, with these assumptions the lateral displacement δ_{zc} due to a horizontal loading, u, is given by

$$\delta_{zc} = \frac{uh}{w} \qquad (3.18)$$

Vibration in cable systems may occur due to local or global oscillations. Local cable oscillations are primarily caused by vortex shedding occurring around the cables in the wind. Long cables acting as hangers or stays may undergo vibrations. Sometimes local oscillations may occur when the natural circular frequencies of the cable are equal to the frequency of walking or running of pedestrians. Local oscillations do not generally influence the immediate safety of the bridge. However, the cables undergo cyclic changes during these types of oscillations and hence may eventually result in failure as a result of fatigue.

Global oscillations comprising of the cables, the deck, the pylons and any other supporting structure are usually analyzed using computers. The mode shapes for typical three-span bridges with a symmetric layout will be either symmetric or antisymmetric. A typical first mode displacement form is shown in Figure 3.5

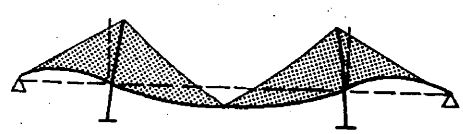


Figure 3.5: Typical displacement form of first symmetric mode [9]

In general the bridge deck model will give very approximate models of the natural frequencies of the structure. It is therefore essential to model the entire bridge system and obtain the natural frequencies of the structure as a whole since all other structural elements contribute to the overall stiffness.

4. Vibration Control

Pedestrian bridges are being designed to be lighter and more flexible. This evidently may result in the deflections being too large, and even though there may not be structural problems as a result of these vibrations, accelerations may be higher than those considered tolerable by users of the structure. Therefore in design, three different procedures may be used to control the excitation levels of a pedestrian bridge:

1. Frequency tuning of the structure. This is based on the critical natural frequencies of the bridge and the frequency of the forcing function i.e. pedestrian footfall frequency in this case.
2. Calculating the forced vibration response of the structure and comparing the amplitudes with serviceability criteria.
3. Several measures can be taken to mitigate vibration problems: increasing structural stiffness, increasing damping with the use of viscous dampers or tuned vibration absorbers, or even restricting use of the structure.

Frequency Tuning

This is simply a method by which critical frequency ranges are avoided during the design stage and hence the frequencies of the dominant structural modes are kept out of a previously determined frequency range. In the case of footbridges, since dynamic forces are induced due to pedestrian footfalls the frequency ranges for walking and running are avoided, i.e. 1.5 Hz to 3.5 Hz. If the bridge has very low damping values (~1%) then the second harmonic of the dynamic force from walking must also be avoided (3.5 Hz to 4.5 Hz) [2]. When dealing with lateral vibrations however, frequencies in the range 0.5 to about 1.5 Hz should be avoided, since pedestrians have lateral motions in this frequency range, and in crowded situations tend to fall into step with bridge motion thus causing resonance.

Satisfying Serviceability Criteria

If it is not possible to keep the vibration frequencies out of the critical range, then the response to pedestrian loading must be analyzed to ensure that the accelerations and amplitudes are within tolerable limits. Unfortunately design codes do not provide specifications for lateral vibration. Therefore the vertical serviceability requirements can be assumed to be applicable to lateral vibration control. The British design code [BS 5400] gives a vibrational acceleration limit of

$$0.5 \cdot f_1^{0.5} [m/s^2] \qquad (4.1)$$

for fundamental frequencies f_1 Hz less than 5 Hz. Unfortunately this will just give an extremely rough estimate of the serviceability of the bridge. As described in Chapter 2, lateral vibration is made very complicated due to pedestrian synchronization with the bridge movements. Due to this, detailed analysis using the dynamic force models developed in Chapter 2 will have to be carried out to ensure that even if a large number of pedestrians fall into step with the bridge vibration, accelerations and amplitudes will remain at acceptable levels.

Vibration Control

A bridge that does not satisfy serviceability requirements will have to be modified in order to do so. There are several options available to do this as discussed below.

Passive Control by Increasing Stiffness

Since excessive lateral vibration occurs due to the low bridge stiffness in the lateral direction. The most logical thought process would be to increase stiffness in this direction. However, this would involve adding external elements such as bracing in the bridge deck that might detract from the bridge aesthetics. Additional structural elements would also increase the total bridge mass, which in turn may result in the superstructure and/or the substructure having to be redesigned. Increasing bridge stiffness could therefore be an extremely expensive, inconvenient and intrusive remedial measure.

Passive Control by increasing Damping

Damping refers to the dissipation of vibrational energy. The energy dissipation equals work done by the damping force. Damping suppresses system responses, especially near resonance conditions where damping governs the response [5]. This energy dissipation can take place in several ways:

- Material damping whereby energy dissipation occurs due to the viscosity of the material.
- Hysteretic damping whereby the material undergoes cyclical energy dissipation and absorption due to inelastic deformations.
- Friction damping whereby energy is dissipated due to friction between moving bodies in contact.
- Energy radiation into the soil

All physical systems have some inherent damping, which are due to contributions from the structure and by energy radiation into the soil. Table 4.1 shows damping ratios for different types of materials. Damping by means of radiation into the soil can be high in medium stiff or soft soil. However stiff soil or rock provides low damping by this method.

Table 4.1: Material damping of different materials [1]

Material		ζ
Reinforced Concrete	small stress intensity (uncracked)	0.007 – 0.010
	medium stress intensity (fully cracked)	0.010 – 0.040
	high stress intensity (fully cracked), but no yielding of reinforcement	0.005 – 0.008
Prestressed concrete (uncracked)		0.004 – 0.007
Partially prestressed concrete (slightly cracked)		0.008 – 0.012
Composite		0.002 – 0.003
Steel		0.001 – 0.002

In the case of footbridges, if the internal structural damping is not sufficient, additional damping must be provided by external means. This could be done using devices installed at various locations along the structure to dissipate energy in particular vibration modes. In this way, the response of the bridge driven at the resonant frequency can be greatly decreased. This in turn can significantly reduce overall motion or acceleration of the structure.

Tuned mass dampers (TMDs) can be an effective and practical means for reducing resonant vibration in a bridge. A TMD is a device composed of a spring, mass, and dashpot (Figure 4.1). These three components can each be implemented in a number of different ways.

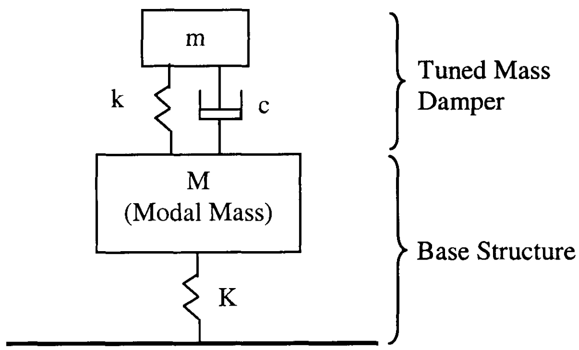


Figure 4.1: Schematic of a tuned-mass damper (TMD)

A TMD is itself a single degree-of-freedom resonant system. It adds a mode of vibration to the base structure. The equations of motion for the system and with the TMD would be [1]

$$m_s \cdot \ddot{x}_s + c_s \cdot \dot{x}_s - c_t(\dot{x}_t - \dot{x}_s) + k_s \cdot x_s - k_t \cdot (x_t - x_s) = \hat{F}\cos\Omega t \qquad (4.2)$$

$$m_t \cdot \ddot{x}_t + c_t(\dot{x}_t - \dot{x}_s) + k_t \cdot (x_t - x_s) = 0 \qquad (4.3)$$

where s – parameter subscript for the primary system

t – parameter subscript for the tuned vibration absorber

x – total displacement

m – mass

c – damping coefficient

k – spring constant

$\hat{F}\cos\Omega t$ - harmonic excitation of the primary system

These equations can be solved to obtain the optimum frequency of the damper:

$$f_t = \frac{f_s}{1 + \dfrac{m_t}{m_s}} \qquad (4.4)$$

where

$$f_t = \frac{1}{2\pi}\sqrt{\frac{k_t}{m_t}} \qquad \text{and} \qquad f_s = \frac{1}{2\pi}\sqrt{\frac{k_s}{m_s}}$$

From these equations we can see that the frequency of the tuned mass damper f_t is slightly smaller than the frequency of the bridge, f_s. The difference between the two frequencies depends on the ratio of m_t to m_s. Therefore the stiffness and mass of the TMD are chosen to put the TMD natural frequency just below the frequency of the "target" mode of the bridge structure, i.e. the mode to be damped. This causes a strong dynamic interaction between the TMD and the target mode. The target mode is replaced by two modes, one slightly above and one slightly below the original frequency. Most important, both of these "split modes" will be damped by the dashpot of the TMD. In essence, the TMD attracts to itself the vibrational energy of the target mode and dissipates it into heat through the action of its dashpot. The dashpot is the damping element added to the absorber. The optimum damping ratio of the absorber is given by:

$$\zeta_{opt} = \sqrt{\frac{3\left(m_t/m_s\right)}{8(1+m_t/m_s)^3}} \tag{4.5}$$

This formula gives a good approximation for the damping required in the bridge and TMD combined structure. In some cases damping by means of a dashpot is not essential if the excitation frequency is certain to remain constant. However, with pedestrian motion this is not the case and therefore a dashpot should be used.

As a solution to a vibration problem, TMDs have a number of very attractive features:

- They are compact, modular devices that can have a simple interface to the base structure.
- They can be added to the bridge structure that is already designed or even built.
- A well-designed TMD can add high damping with minimal weight. Figures 4.2 and 4.3 shows the relationship. Figure 4.3 shows that a TMD having even 1% of the base structure weight can produce high damping: over 5% of critical when tuned correctly.
- The TMD does not impact the static strength or stiffness of the base structure.

- For designing the TMD, it is often possible to characterize the base structure by inexpensive test or analysis. In effect, the base structure is modeled simply in terms of a single mode: the target mode.

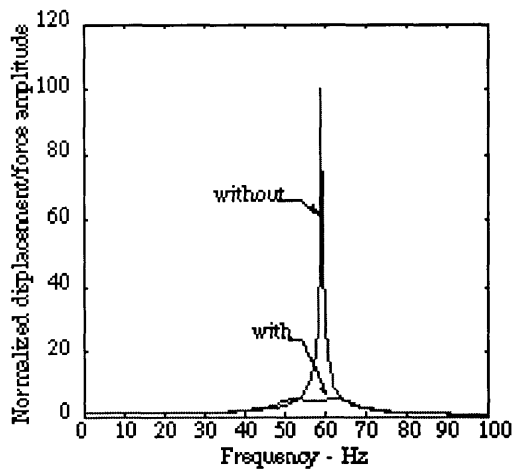


Figure 4.2: Typical frequency response with and without a TMD

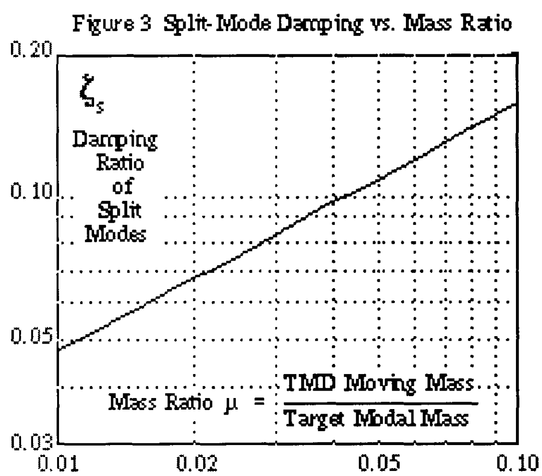


Figure 4.3: Split Mode Damping vs. Mass Ratio

The TMD is considered a narrow-band device. It adds high damping only to modes with natural frequencies close to its own. This means that multiple TMDs may be needed to obtain high damping of modes if the frequency range is too large. However, this suits the vibration control problem of a pedestrian bridge ideally since only frequencies in the critical pedestrian swaying motion need to be damped. A TMD is most effective when its mass is comparable to the modal mass of the bridge, and when the inherent damping of the bridge is very low. One of the primary design aspects to be noted with a TMD is that its ability to operate effectively is highly dependent on accurate tuning of the damper. Therefore detuning over a period of time or inaccurate installation may result in poor performance of the damper. In addition to this TMDs show large amplitude motions when in operation. Hence, they must be installed in open spaces that allow for their motion. In practice is advisable to measure the frequencies of the bridge structure in situ, and the TMD installation process must leave allowance for fine tuning [1]. This is usually easier to do by varying the mass of the damper rather than its stiffness.

Active Control

Passive control mechanisms do not require any external energy whereas active mechanisms operate through input of external energy. Active damping is sometimes necessary to achieve greater performance, or to produce system properties that are controllable electronically. A hybrid active/passive approach may also be used if necessary. Active systems use actuators, sensors, electronics and software to control physical systems.

Figure 4.4 shows a typical architecture of active systems. The usual goal is to achieve some desired response that can be quantified in a set of performance objectives, in response to commands, or in spite of disturbances.

A number of active control strategies exist. One possible method that can be used in vibration control of pedestrian bridges is the use of the predictive control strategy. Cunha and Moutinho [6] have used this approach successfully to study a real pedestrian bridge at Martorell, near Barcelona. This strategy seemed to be rather general, versatile and fairly easily implemented. It also achieved a significant reduction in the levels of vibration in the numerical simulations.

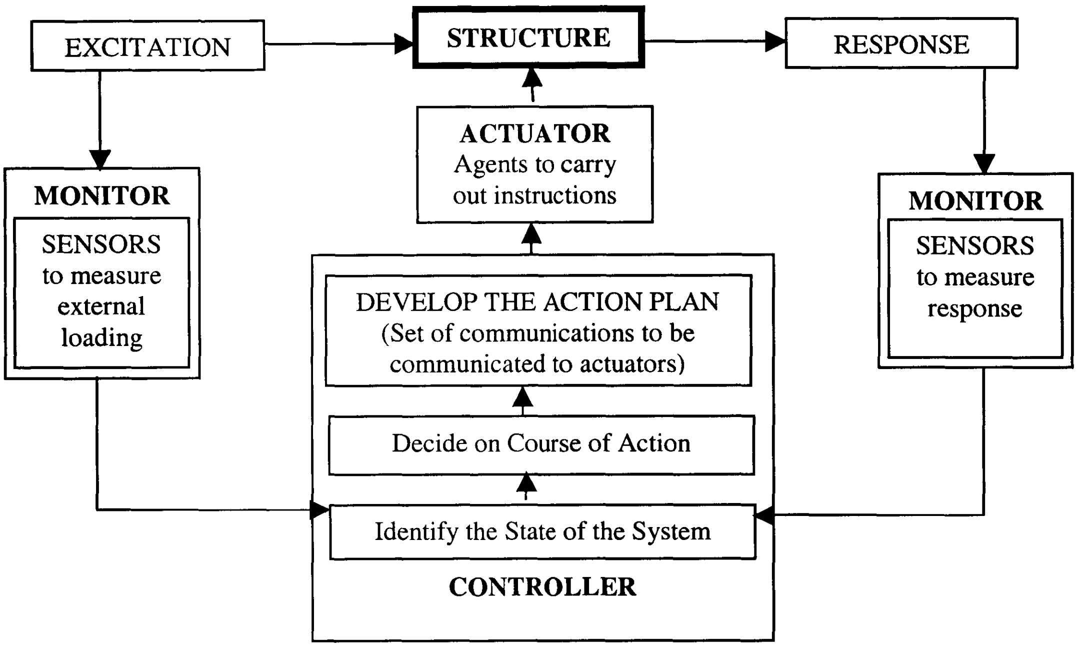


Figure 4.4: Typical layout of an active system [5]

The primary advantage of active systems is that control is carried out dynamically i.e. a monitoring system is able to detect the state of the system at a given time and respond by means of various devices appropriately. This prevents overdesign, which is inherent in the design of passive systems. An active system is also able to respond to different types of forces (e.g different frequencies of vibrations) unlike a passive system which is designed to specifically mitigate certain responses in structures and will not be able to act against a sudden unpredicted stimulation that it was not designed to withstand. However, great care must be taken when using active control systems. Since they operate by input of energy into the system, if designed incorrectly, instability is a very likely outcome.

If the only aspect of the bridge that needs to be controlled is its response to lateral pedestrian swaying, then a passive system may be the most appropriate. An active system that would be constantly monitoring the bridge might be too expensive an option for just this problem. However, if the bridge has other problems along with the lateral vibration issue, then active control may be implemented in order to mitigate all problems. The cost-benefit to using such a system could be realized in this case.

5. Case Studies

A large number of footbridges have undergone lateral vibration problems induced due to pedestrian motion. In this section, some of the bridges around the world that have displayed excessive lateral motion are discussed.

A Cable-Stayed Bridge

The bridge discussed in this section is a cable-stayed bridge with a total span of 180-meters and a width of 5.25-meters (Figure 5.1). The bridge connects a boat race stadium and a bus terminal. When large crowds of people cross the bridge, it has shown both vertical and lateral vibrations. Lateral vibrations with amplitudes up to 1-cm were observed, and the horizontal amplitudes of some of the cables were found to be up to 30-cm. An interesting point to note is that the vibration of the bridge did not occur instantaneously. It took some time for the vibration to build up to a steady state after which it lasted a few minutes.

It was noted that different cables vibrated at different times at different congestion levels. This was a result of changes in cable tensions with pedestrian loading so that different cables were tuned to the lateral girder motion at different loading levels. The first three natural frequencies of the vertical modes of the bridge were 0.7 Hz, 1.4 Hz and 2.0 Hz; and the first lateral mode had a natural frequency of 0.9 Hz.

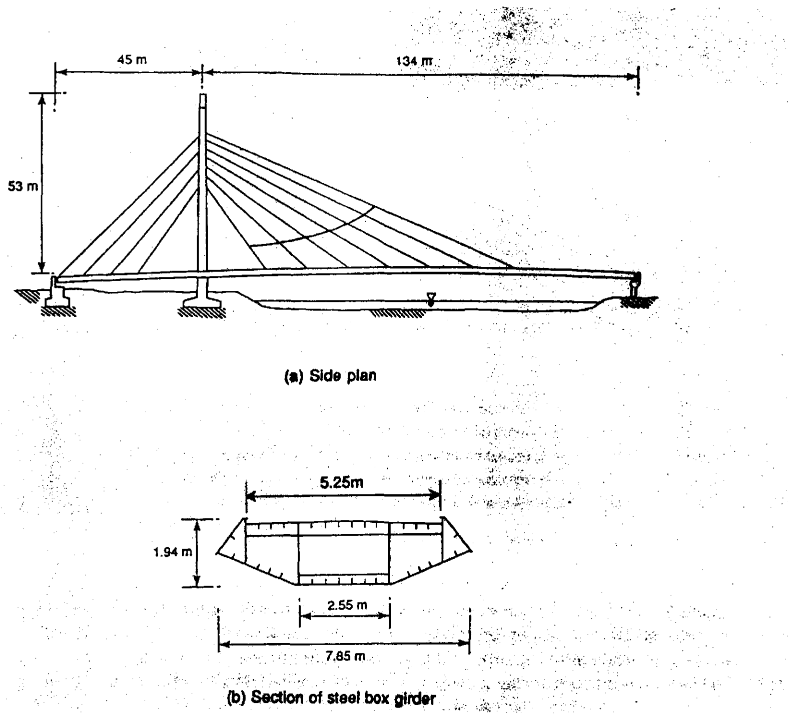


Figure 5.1: A pedestrian cable-stayed bridge (a) side plan; (b) section of steel box girder [7]

A model using forced excitation due to human walking explains why the lateral motion of the bridge occurred specially since its first lateral frequency of vibration is close to the lateral swaying frequency of human walking. However, this simple model is unable to predict the large amplitudes of motion that were observed on this bridge. The model described in Chapter 2, which includes human synchronization with the bridge vibration causing a resonance effect, is able to explain the 1-cm vibration amplitude of the girder.

The gradual increase in amplitude can be explained as follows. Initially a small lateral motion is induced due to the lateral motion of pedestrians in a random manner. This small bridge motion causes some of the pedestrians to start walking in step with the bridge motion. This results in an increase in the lateral force induced by pedestrians and hence increases the amplitude of bridge vibration which in turn causes more pedestrians to get in step with the bridge motion. This way the amplitude gradually builds up. However, it does not keep increasing to infinity but reaches a steady state because of the adaptive nature of humans [7].

The vibration problem was controlled by the installation of a large number of small tuned liquid dampers inside the bridge box girder.

An Angular Arch Bridge

A footbridge consisting of an S-shaped continuous girder suspended from an angular arch is shown in Figure 5.2. This is a steel footbridge, which was excited to strong lateral vibrations with a frequency of approximately 1.1 Hz [2]. This structure consists of several spans with the intermediate supports of the girder being hinged bearings. The first three mode shapes of the vibrations can be seen in Figure 5.3(a). The natural frequency of the lowest lateral mode of this bridge was 1.1 Hz. Since a person sways laterally at half the stepping rate, this mode was excited and resonance took place. In addition to this, pedestrian synchronization to the bridge vibration also occurred amplifying the magnitude of vibrations. In this case the bridge was retrofitted with tuned vibration absorbers that opposed the lateral motion.

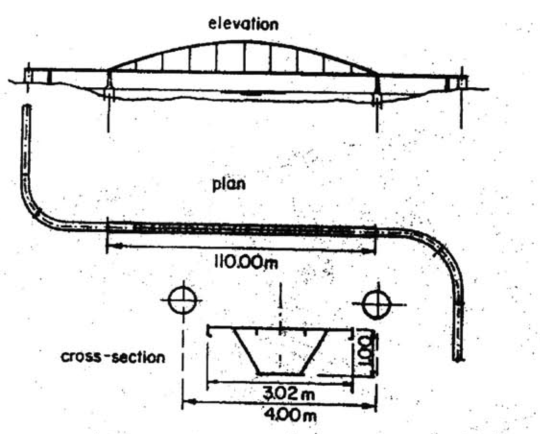


Figure 5.2: Footbridge with Lateral Vibrations at 1.1 Hz: Elevation, Plan, and Cross Section

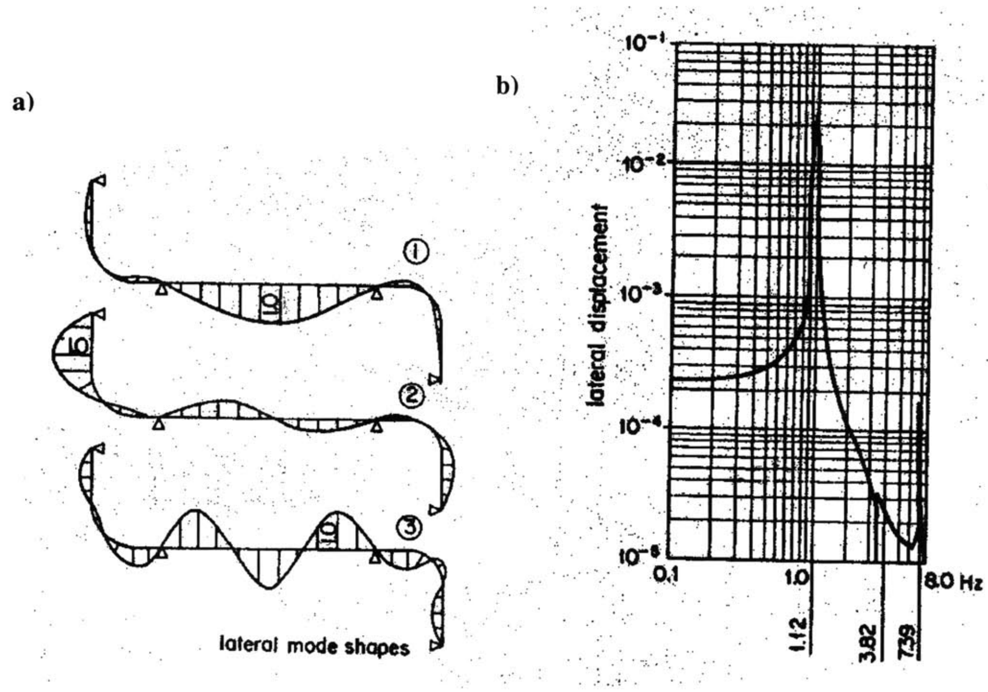


Figure 5.3 (a) First three Mode Shapes of Lateral Vibration; (b) Transfer Function for Lateral Displacement at Midspan [2]

The Millenium Bridge

Vibration Problem

Movements of unexpected amplitudes were experienced on London's Millennium Bridge when it opened on 10 June 2000. The bridge was subsequently closed because of the small perceived risk to pedestrians of losing balance and coming to harm in crowded situations. The Millennium Bridge links the City of London at St. Paul's Cathedral with the new Tate Gallery at Bankside. It is a 330-m long, 4-m wide suspension bridge with a center span of 144-m. The behavior of the Millennium Bridge is dominated by the behavior of the cables that carry a total tension force of some 2000 tons. The lateral vibration experienced on the Millennium Bridge occurs because some of the lateral natural modes of vibration are similar in frequency to the sidesway component of pedestrian footsteps on the bridge (between 0.5 Hz and 1.0 Hz).

Figure 5.4: The Millennium Bridge in London

The bridge was open from Saturday 10 June to Monday 12 June. An estimated 80 000 - 100 000 people crossed on the opening Saturday with about 2000 people at some points during this time period. When the bridge was crowded, the south and center spans underwent lateral vibrations large enough to cause pedestrians to stop walking or to hold onto the balustrades to regain their balance. The south span showed a combination of horizontal and torsional oscillations while the center span showed mainly horizontal oscillations. The south span moved about 50-mm and the center span moved up to 70-mm with frequencies of 0.77-Hz and 0.95-Hz respectively. A loading of 200 people on the bridge was sufficient to cause motion in the center span. The north span however, showed no substantial motion.

Due to concern for the safety of individuals the number of people present on the bridge was controlled from noon onwards on the day of its opening. This was only a serviceability problem. There was never any danger of structural damage.

Investigation into Cause of Vibrations

Once the bridge was closed on June 12, the design engineers –Arup – began to investigate various different options to mitigate the vibration problem. Extensive research, testing and computer simulation was carried out by the engineers. The investigation took into consideration practicality, appearance, cost, and maintenance requirements when proposing the final solution.

The first step in the investigation was to check the assumptions made in the original design. In order to do this, bridge behavior had to be studied by monitoring its dynamic characteristics. These tests were carried out between 15 June and 7 July. The instruments placed on the bridge were used to record bridge motion resulting from forces exerted by Mechanical shakers. The effect of weather also needed to be checked and hence a small weather station was set up to record wind speed and changes in temperature and their effects on the bridge behavior. The test results were compared to the bridge behavior predicted by the original model. This proved that the bridge behaved as expected under loadings that had been taken into consideration during the design process. Hence it was concluded that forces that had not been considered in the original design caused the lateral vibrations of the bridge.

The dynamic forces identified as needing further research were wind effects and pedestrian loading. Investigation into the dynamic effect of wind loading during opening weekend confirmed that it could not have been the cause for the excessive deflections and vibrations. This left the investigators with only one more cause to look into – pedestrian loading.

It had been noticed that the crowd of pedestrians was walking in time with the motion of the bridge during the opening weekend. Once again the original design assumptions were checked and it was concluded that the bridge had been designed following all the requirements specified by the UK bridge design code as well as other overseas codes and non-statutory documents. Since the UK Bridge codes only required vertical excitation to be considered in bridge design, all the critical vertical loads were studied. In addition to this the recommended loading was increased by 33% as an additional safety factor. The design also took into consideration vandal loading.

Hence research was conducted into the design loadings used for bridge design and their adequacy. As mentioned earlier, it was noticed during opening day that the crowd tended to walk in step with the motion of the bridge. This was noticed from video footage and confirmed later on with experimental evidence. In crowds, individuals started walking in step with each other. The summation and synchronization of the forces induced by each individual causes the bridge to also sway in rhythm with the crowd when the frequencies of the crowd imposed lateral loading are equal to any of the natural lateral frequencies of the bridge. Hence a build up situation takes place, where a small group of people begin to walk in step and cause small motions of the bridge. This results in more people falling into step with these motions hence increasing the applied forces which in turn increases the swaying motion of the bridge.

Redefining Bridge Design Loading

Next, an extensive research program was carried out to study how people walk in crowded situations i.e. the magnitude of force exerted by pedestrian footfalls in the lateral direction as well as typical frequencies. The research included a series of laboratory tests at the Institute of Sound and Vibration Research (ISVR) at the University of Southampton, and Imperial College, London. All these places conducted experiments on how individuals walk on vibrating platforms. By controlling the motion of the platform it is possible to monitor human response to different types and magnitudes of platform vibrations.

A field test was also carried out on a small footbridge in Scotland. This bridge was used to test if walking in the open air (i.e outside the confines of a laboratory) resulted in any changes in human response to vibration or "lock-in" with bridge motion. All these tests were used to determine input forces that the Millennium Bridge had to be retrofitted to carry. These tests led to the conclusion that crowd responses are affected by surroundings.

Hence it was decided that the Millennium Bridge itself would be tested with crowd loading. This type of testing would yield very accurate input loadings for the bridge. The experiment involved 100 people walking along the bridge at different speeds. The crowd density was also varied. In addition to this, crowd reaction to bridge motion induced by means of mechanical shakers was also recorded. Video cameras were used to record the human responses. In addition to this foot switches were also used. The response of the bridge was measured as well during these tests. Hence a highly accurate "crowd loading" model was developed. This model was used to test if the retrofitting suggested would restrict bridge motions to acceptable levels.

Bridge Retrofit Options

Three Options were studied:

1. Increase bridge stiffness

The frequency of the bridge is dependent on the square root of its stiffness i.e.

$$f = \sqrt{\frac{k}{m}} \qquad (12)$$

where f – frequency

k – stiffness

m – mass

In order to avoid the bridge and pedestrian frequencies matching, the natural frequencies of the bridge would need to be increased from 0.5 Hz to around 1.5 Hz (i.e. tripled). To do this, the lateral stiffness of the bridge would need to be increased by a factor of nine. Only adding large structural elements that would completely change the unique and slender nature of the bridge could provide such a large increase in stiffness. Adding bracing in the bridge deck could do this. However this type of retrofit resulted in the torsional vibration modes becoming controlling factors. Adding tuned mass dampers, which would further increase the bridge mass, could control these torsional modes. Not only would the increase in mass affect the aesthetics of the bridge but could also affect the structural integrity of the bridge since the superstructure and substructure were designed for the original lightweight bridge. In addition to these two major disadvantages, stiffening the bridge would also be an expensive operation. Due to these factors, Arup concluded that this type of retrofitting would not be the optimal solution.

2. Bridge damping

   Reduce dynamic motion by increasing the energy absorbed by the bridge in each cycle of vibration. A number of active and passive damping systems were studied. Active dampers act by 'reacting' to the motion of the bridge and 'adjusting' the level of damping induced accordingly. This involved monitoring the bridge motion and computerizing this information in order to form a feedback system that would control the amount of damping provided. Even though extremely efficient, active control would be an extremely expensive option and was thus eliminated. Hence Arup concentrated on Passive systems which do not involve feedback systems but react directly to bridge motion. It was decided that a combination of viscous dampers and tuned mass dampers would provide the optimal cost-efficient solution. Viscous dampers act by absorbing energy. The damping provided by viscous dampers is depended on the speed of motion since they act by compressing and decompressing fluids. The resistance of the fluids provides the damping mechanism. Tuned mass dampers use their inertia to resist certain motions. The tmd's are used to stop the structure from vibrating at certain predetermined frequencies.

3. Restrict bridge usage

   The last option considered was to either restrict the number of pedestrians on the bridge at a time or to interrupt their walking patterns by placing various obstructions along the bridge such as street furniture. This option was not considered feasible since it would defeat the entire purpose of the bridge.

The final design for retrofitting included a combination of stiffening, viscous dampers and tuned mass dampers. Additional stiffening would be provided by means of steel bracing. The viscous dampers would be attached between the existing structure and the steel bracing. The steel bracing acted also to transmit bridge motion to the dampers from the deck. Most of the viscous dampers were placed under the deck while some of them were placed between the deck and the river piers. Two pairs of struts and dampers under the south span extended significantly below the bridge to be supported by concrete bases next to the central pedestrian ramp. The viscous dampers were designed to provide mostly lateral vibration control. However some of the dampers were also designed to control vertical vibration and vibrations at inclinations.

The tuned mass dampers were primarily added to provide further vertical vibration control. Four tuned mass dampers will be placed in the center span to control the first lateral mode of vibration. These tmd's will also be fitted under the deck.

6. An Analysis

An analytical and finite element analysis was carried out on a beam model of a bridge. The dimensions used are those of the deck of the Pforzheim ll bridge across the River Enz at Pforzheim built for the city's 1992 Landesgartenschuau (LGS). This is also a suspension bridge with a span of approximately 270-feet and deck width of 8-feet (Figure 6.1). The Pforzheim ll has masts that lean outwards. This was done for the visual effect and to simplify the anchorage of the hanger cables to the deck. The bridge was tested with a group of pedestrians and showed build-up of lateral vibrations. This vibration was observed again when the bridge was opened.

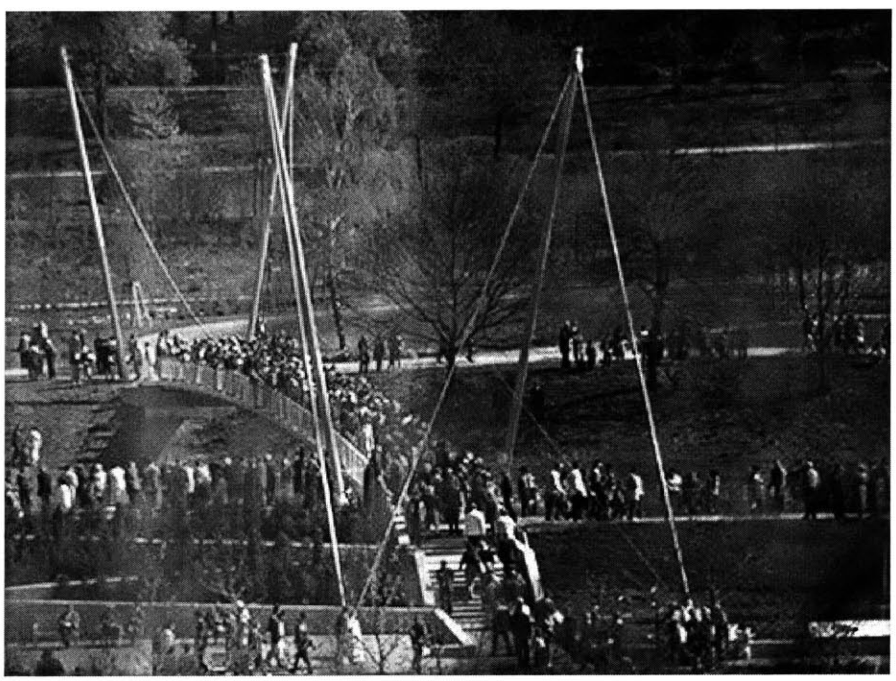


Figure 6.1: The Pforzheim ll Bridge over the River Enz. A back-anchored suspension bridge

One Dimensional Analysis

Analytical Results

From the first eight roots of Equation 3.15 for the beam under consideration,

$L = 270\text{-}feet$

$E = 4176000\text{-}kips/ft^2$

$I = bh^3/12 = 0.525X8^3/12 = 22.4\, ft^4$

$\mu = \rho bh = 0.49X0.525\ X\ 8 = 2.058\ kips/ft^3$

$\omega_1 = 2.069\ rad/s = 0.329\ Hz$

$\omega_2 = 5.703\ rad/s = 0.908\ Hz$

$\omega_3 = 11.182\ rad/s = 1.78\ Hz$

$\omega_4 = 18.483\ rad/s = 2.942\ Hz$

$\omega_5 = 27.612\ rad/s = 4.395\ Hz$

$\omega_6 = 38.564\ rad/s = 6.138\ Hz$

$\omega_7 = 51.342\ rad/s = 16.343\ Hz$

$\omega_8 = 65.946\ rad/s = 20.991\ Hz$

Finite Element Results

The bridge was modeled as a beam with a width of 8-feet and height of 0.525-feet. During the analysis the degrees of freedom perpendicular to the lateral vibration of the beam were taken out so that only in-plane vibration could be studied. The mode superposition method was used in ADINA (finite element analysis software) since only a certain number of modes and their corresponding frequencies were required.

In the first step, the beam was divided into two subdivisions with two nodes per division. This produced a three-node model that was able to predict only the first three eigenvalues and eigenvectors. The results as shown in Table 6.1 were quite inaccurate.

Table 6.1: Modal Frequencies for finite element analysis of beam with 3 nodes

Mode Number	Frequency (rad/s)	
	Exact	FEM
1	2.069	0.2659
2	5.703	0.4975
3	11.1813	1.2186

This three-node model is not able to display the second or third mode shape since there are not enough nodes to model this vector.

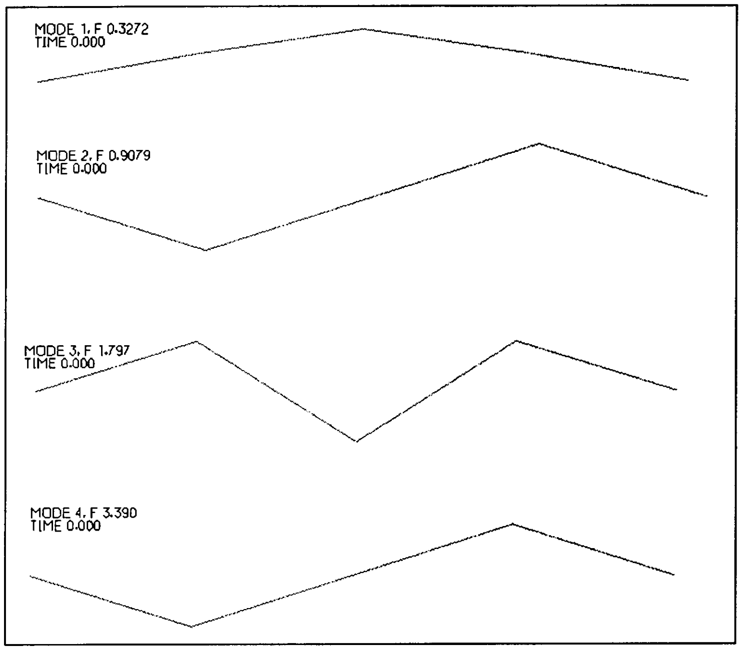


Figure 6.2: Mode shapes for finite element analysis of beam with 5 nodes

Table 6.2: Results for finite element analysis of beam

Mode Number	Exact Solution	Mesh Density					
		11 nodes	21 nodes	31 nodes	41 nodes	51 nodes	101 nodes
1	2.0691	2.0530	2.0530	2.0530	2.0530	2.0530	2.0529
2	5.7036	5.6536	5.6522	5.6521	5.6521	5.6521	5.6521
3	11.1813	11.0702	11.0600	11.0594	11.0593	11.0593	11.0593
4	18.4833	18.2819	18.2369	18.2345	18.2341	18.2339	18.2339
5	27.6108	27.3033	27.1587	27.1505	27.1491	27.1487	27.1484
6		33.9819	33.8773	33.8580	33.8512	33.8481	33.8439
7	38.5639	38.1702	37.7951	37.7729	37.7692	37.7681	37.7675
8	51.3424	50.9334	50.1137	50.0623	50.0534	50.0510	50.0494
9	65.9465	65.5682	64.0810	63.9741	63.9553	63.9503	63.9471
10		68.8034	67.9640	67.8088	67.7547	67.7296	67.6963

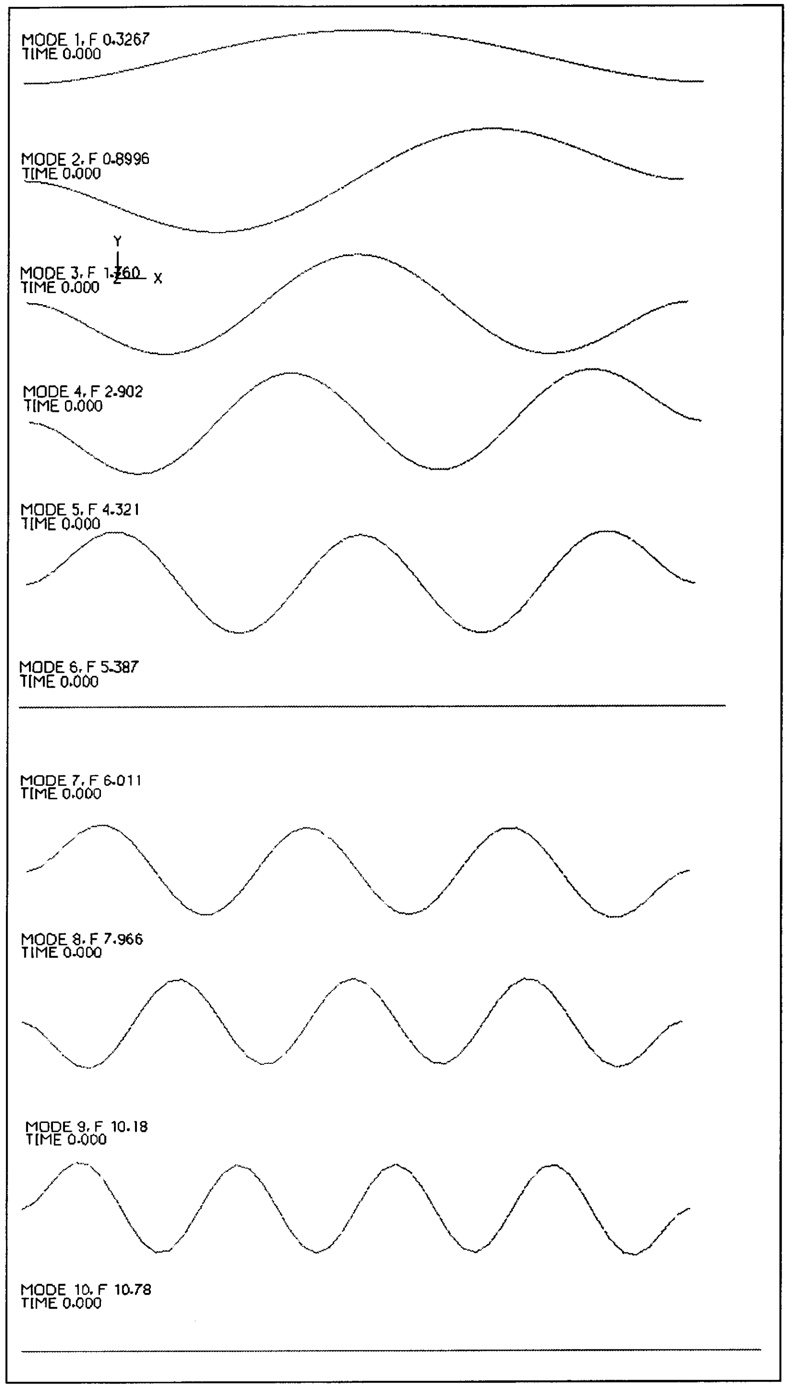


Figure 6.3: Mode shapes for finite element analysis of beam with 51 nodes

Similarly the 5-node model yields 5 modal frequencies and mode shapes (Figure 6.2).

Increasing the number of nodes in the beam results in more accurate results with a larger number of modes included in the solution. Table 6.2 summarizes the results for all the different mesh densities used. One thing to be noted from the FEM results is that the frequencies obtained for Modes 6 and 10 were not part of the exact solution. This point will be addressed and explained in the section that discusses FEM analysis in 2-D.

The first ten mode shapes are obtained using higher mesh densities (Figure 6.3). All these shapes are formed as expected. Once again note that Mode Shape 6 and 10 do not display any variation in lateral displacement along the length of the beam.

Two-Dimensional Analysis

Finite Element Results

Analysis was started out with a small number of elements after which the mesh density was gradually increased to obtain more accurate results. The mode superposition method was used to find the natural frequencies of vibrations and corresponding mode shapes for the plate. The first run was carried out on a model with 2 subdivisions of 8-node elements. The resulting frequencies were nowhere near the exact solution obtained for the beam. The mode shapes could not be represented accurately either.

With higher mesh densities a more accurate representation of the mode shapes was obtained (Figure 6.4). It is seen that for the higher modes lateral motion of the edge nodes is different from that of the central nodes. Hence the deck goes through contraction and expansion cycles at these higher modes. The Mode 6 and Mode 10 motion also becomes clearer with the plate element. These modes represent motion in the longitudinal directions – not the transverse direction. Hence this motion was not captured by the analytical

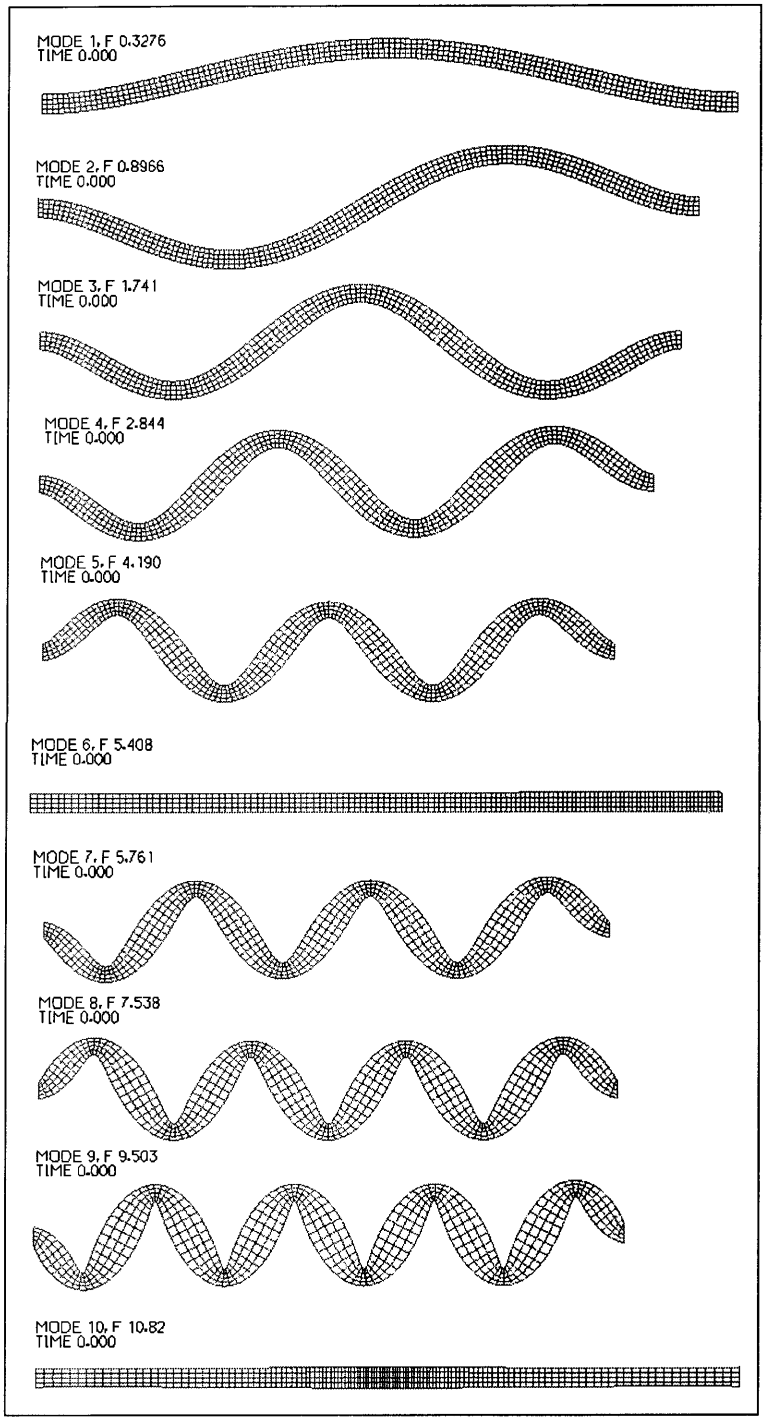


Figure 6.4: Mode shapes for finite element analysis of plate (128 by 4 subdivisions, 8-node elements)

calculations performed for the beam element since they looked at motion only in one direction.

Table 6.3: Results for finite element analysis of plate

Mode Number	Mesh Density - 8 Node Elements						
	2 by 2	4 by 2	8 by 2	16 by 2	32 by 2	64 by 2	128 by 2
1	9.8789	2.3321	2.1040	2.0657	2.0601	2.0591	2.0587
2	25.9026	7.5579	5.8973	5.6685	5.6396	5.6357	5.6346
3	34.3196	28.1165	11.8287	11.0467	10.9554	10.9447	10.9424
4	68.8555	34.0662	20.1534	18.1291	17.9054	17.8817	17.8773
5	73.2148	38.1159	31.2715	26.8601	26.3937	26.3473	26.3397
6	124.8111	68.4025	34.0115	33.9914	33.9837	33.9814	33.9803
7	743.9480	78.5775	45.6301	37.1836	36.3132	36.2302	36.2180
8	819.6729	104.0043	68.0419	49.0514	47.5530	47.4144	47.3958
9	825.8367	129.8910	73.8620	62.4295	60.0028	59.7836	59.7561
10	1174.2834	137.2468	81.2190	67.9816	67.9646	67.9596	67.9577

The results for the plate model are summarized in Table 6.3. This method yields slightly more accurate results when compared to the beam model. It certainly provides better understanding of the mode shapes when compared to the one-dimensional beam analysis.

Discussion of Results

The beam model is an accurate enough representation of the bridge deck and can be used effectively to study the various dynamics of the bridge. However, the plate model gives a more accurate representation of mode shapes. Mesh densities of 21 nodes will yield sufficiently accurate results for the beam model and a 32 by 2 mesh using 8 node elements yields accurate results for the plate model. The frequency of vibration of the first two modes are 0.329 and 0.908 Hz which are close to or within the frequency range that may be excited by the lateral motion of walking pedestrians. However, this model has not accounted for the contribution of the other structural element of the bridge to its stiffness. Hence, it obviously underestimates the bridge stiffness and provides a low estimate of the natural bridge frequencies. Including the cables and the pylons into the bridge model will increase the stiffness in the lateral direction, and hence increase the natural frequencies. Taking into consideration the results obtained for the beam and plate models, it is highly unlikely that the frequencies will increase sufficiently to fall out of the frequency range that can be excited by humans. Most likely the first mode will stay within a range that could be excited by pedestrian motion.

7. Conclusion

Engineering is taking the art of pedestrian bridge design to new limits, as bridges become longer and more slender. The current trend to produce sleek structures is resulting in the appearance of bridge characteristics that were not known to exist before. One such property is the human synchronization with the lateral motion of the bridge. For a number of years vertical deflections of footbridges have been known to cause discomfort in pedestrians and have therefore been kept under certain predetermined levels. However, the lateral sway of bridges, which occurs in long and slender structures, is a relatively new phenomenon. Very little research has been carried out to study this problem, nor is it addressed in bridge design codes.

Quite a few bridges have shown this type of motion over the past few years. This problem does not seem to be dependent upon the type of bridge under consideration, and hence has been observed in arch bridges, cable-stayed bridges, and suspension bridges. The primary characteristic of a bridge that makes it susceptible to this problem is the natural frequency of the bridge being close to 1-Hz.

Pedestrians have an average walking rate of 2-Hz. While walking, people tend to sway at half the walking rate laterally i.e. 1-Hz. If a large enough crowd of people is using the bridge at the same time, it is quite likely that some of the pedestrians will be walking in phase with each other at the natural frequency of the bridge. This exerts a force on the structure, which causes it to vibrate. The next stage in this excitation process is what makes it different from vertical vibration. Once the bridge is set in motion, more pedestrians will begin to walk in step with the vibration of the bridge – this is a normal human reaction in an attempt to steady oneself. This results in the pedestrian force on the bridge becoming larger and hence causing resonance. This is how the large amplitude vibrations result.

Now that engineers are aware of this possible problem with footbridges, the easiest method of avoiding it is to try and keep the natural frequencies of the bridge at levels greater than 1-Hz. Designing structures that are stiff both laterally and vertically does this. If this is not possible, and the bridge does end up having a critical frequency, it is also possible to add external damping to the structure by means of viscous or tuned mass dampers, which will prevent the build up in amplitude of the vibrations. Several bridges have been retrofitted using this method successfully. Active damping systems may also be implemented to provide resistance against the lateral swaying. However, it has not been implemented yet since it happens to be the expensive option. It certainly seems likely that this may turn out to be the most optimum solution if the active system is implemented into the bridge design from the initial stages itself and is used to mitigate not only the lateral vibration problems but is also used to fulfill other serviceability requirements.

The large lateral vibrations are caused by human 'reaction', which could be considered highly subjective in nature. Some studies have shown that surroundings influence this human reaction to vibration. This necessitates adequate testing of a pedestrian bridge after construction. Testing is especially necessary if TMDs are being used since their performance is highly dependent on accurate tuning.

Finally, it is absolutely essential that further in-depth research be carried out on this topic to provide basic design guidelines for engineers. It is also necessary to include serviceability requirements that address the lateral stability of a pedestrian bridge in bridge design codes.

Appendix

A.1 Dynamic Forces

Table A.1: Representative types of activities and their applicability to different actual activities [1]

Representative types of activity			Range of Applicability	
Designation	Definition	Design activity rate (Hz)	Actual activities	Activity rate (Hz)
"walking"	walking with continuous ground contact	1.6 to 2.4	. slow walking (ambling) . normal walking . fast, brisk walking	~1.7 ~2.0 ~2.3
"running"	running with discontinuous ground contact	2.0 to 3.5	. slow running (jog) . normal running . fast running	~2.1 ~2.5 >3.0

Table A.2: Normalized dynamic forces assigned to the representative types of activity defined in Table 1 [1]

Representative type of activity	Activity rate (Hz)		Fourier coefficient and phase lag					Design density (person/m²)
			α_1	α_2	ϕ_2	α_3	ϕ_3	
"walking"	vertical	2.0	0.4	0.1	$\pi/2$	0.1	$\pi/2$	~1
		2.4	0.5					
	forward	2.0	0.2 $\alpha_{1/2} = 0.1$	0.1				
	lateral	2.0	$\alpha_{1/2} = 0.1$	$\alpha_{3/2} = 0.1$				
"running"		2 to 3	1.6	0.7			0.2	-

A.2 Mode Shapes and Frequencies of a Hybrid Arch-Suspension Bridge

Table A.3: Characteristic modal frequencies of the hybrid arch-suspension bridge

Mode	Period (s)	Frequency (Hz)
1	1.68	0.59
2	1.49	0.67
3	1.22	0.82
4	1.05	0.95
5	0.71	1.41
6	0.70	1.43
7	0.67	1.49
8	0.63	1.59
9	0.61	1.61

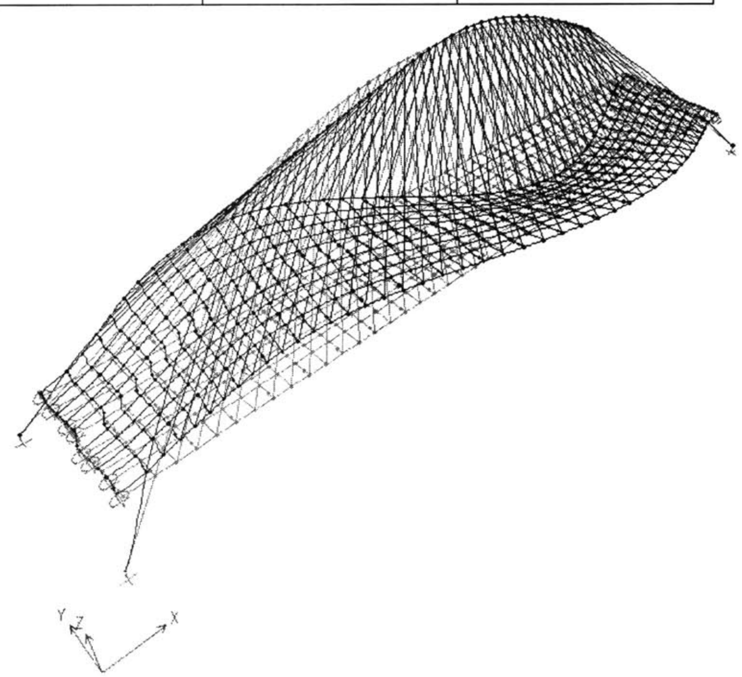


Figure A.1: 5^{th} Mode, T = .71s

References

1. Bachmann, Hugo et al. Vibration Problems in Structures: Practical Guidelines. Berlin, Germany: Birkhauser Verlag Basel, 1995.

2. Bachmann, Hugo "Case Studies of Structures with Man-Induced Vibrations." Journal of Structural Engineering 118 (1992): 631-647.

3. Bathe, Klaus-Jürgen. Finite Element Procedures. New Jersey: Prentice Hall, 1996.

4. Chopra, Anil K. Dynamics of Structures: A Primer. California: Earthquake Engineering Research Institute, 1995.

5. Connor, Jerome J. Introduction to Structural Motion Control. Massachusetts Institute of Technology. May 2001. http://moment.mit.edu.

6. Cunha, Alvaro & Moutinho, Carlos "Active Control of Vibrations in Pedestrian Bridges." Structural Dynamics – EURODYN `99 2 (1999): 783-788.

7. Fujino, Y. et al. "Human-Induced Lateral Vibration of the Pedestrian Bridge." Earthquake Engineering & Structural Dynamics 22 (1993): 743-758.

8. Gatti, Paolo L. & Ferrari, Vittorio. Applied Structural and Mechanical Vibrations: Theory, methods and measuring instrumentation. Padstow, Cornwall: TJ International, 1999.

9. Gimsing, Neils J. Cable Supported Bridges: Concept and Design. Chichester: John Wiley & Sons, 1983.

10. Introducing the Millennium Bridge. ARUP. 4 April 2001. <http://www.arup.com/MillenniumBridge/frameset.htm>.

11. Pimentel, R.L., Waldron, P. "Validation of the Pedestrian Load Model through the Modal Testing of a Composite Footbridge." Proceedings of the 15th International Modal Analysis Conference 1 (1997): 286-292.

12. Rainer, J.H., Pernica, G., Allen, D.E. "Dynamic Loading and Response of Footbridges." Canadian Journal of Civil Engineering 15 (1988): 66-71.