Structural Applications and Feasibility of Prestressed Steel Members

by

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B.S., Architectural Engineering (2000)

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Submitted to the Department of Civil and Environmental Engineering
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ABSTRACT

Prestress mechanism is an effective means to increase strength and control the deflection of a structural member. In structures today, prestress mechanism is primarily associated with concrete members and prestressed steel members are seldom used or even considered for implementation. However, prestressed steel members have been effectively used in rehabilitation or post-strengthening of bridge structures in the past.

Prestressing by tendons, whether it is made of high strength steel or other materials such as FRP (Fiber Reinforced Polymer), is the most versatile system available and is used frequently in the industry. There are other prestressing systems, however, such as prestressing by predeflection, prestressing by bending, and redistribution of moments by support level manipulation. All these are highly effective means to apply prestress on steel members.

Using the method of prestressing by tendons, feasibility of a bridge design with a unique structural orientation was investigated. Due to its orientation, steel was used as a primary structural member to reduce the stresses induced by the structure's own weight. The static analysis of the design showed that the stresses created within such structures can be resolved effectively using the prestressing mechanism described.

Thesis Advisor: Jerome J. Connor
Title: Professor of Civil and Environmental Engineering

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1 Introduction

During an engineering or design process, many challenges in problem solving maybe encountered. Sometimes such challenges are resolved using conventional approaches or means that have been tried before, simply for the sake of solving the problem. On the other hand, many designers' that strive to implement creative and innovative methods for those challenges. These methods may have never been tried and tested, they maybe considered unconventional and seldom used, or they maybe new applications or modifications to existing methods. Whether these new means are proven to be successful or are completely inappropriate for those specific situations, they often give new insights and contribute to technological advancements.

In this paper, it is the author's intention to consider an unconventional solution for a specific problem encountered during an engineering design project. This method, "prestressing," has been used in ways of trial and error long before the mathematical tools in structural engineering were developed. Even today, the idea of prestressing is widely used in many different industries utilizing variety of materials. Among these materials, however, most common material for prestressing in structural engineering today is concrete. Concrete, although versatile in nature and inexpensive in cost, cannot be utilized in all aspects due to its heavy weight and relative strength. For this reason, structural steel is more favorable in certain applications where weight reduction is a key issue in making the structure work.

The idea of prestressing steel is not new. In fact, many studies have been done that have shown considerable benefits in economy and strength capacity using prestressed steel members. Mysteriously, however, prestressed steel never thrived like its concrete counterpart. The method is seldom used and even in academic context it is rarely studied.

In the process of researching information about prestressing means for steel elements, it became evident that there were extensive studies that were carried out regarding the subject up to about the 1970's. In fact, numerous international conventions specifically addressing this topic were held around the world, with participants from well known industrialized countries such as the USSR, England, Belgium, Germany, and the US[14]. However, such studies and researches related to the subject of prestressing steel became scarce around the 1970's and all the attention was focused on prestressed concrete. Much of the existing information on this subject has only been available through engineering journals and publications, most of them foreign[14].

Today, most popular use for prestressing mechanisms on steel structural members is for rehabilitating or retrofitting of existing bridges, generally associated with older or well-traveled highways and railroads. It is the author's understanding that new structures extensively implementing the use of prestressed steel has not been built recently.

2 Concept

The basic idea behind prestressing is not a difficult concept to grasp. It merely is a form of providing additional strength and deflection control to a member to counter-act the applied loading. In ideal cases, due to the static nature of the applied loading, the prestressing mechanisms can be implemented in a customizable fashion to mimic the applied loading pattern, and therefore, directly counter-acting the applied load. Theoretically, the prestressing, then, could completely negate the applied loading, given that the member is able to withstand the stresses induced as a byproduct of the prestressing (i.e. compression). The figures below show a basic prestressing idea at work.

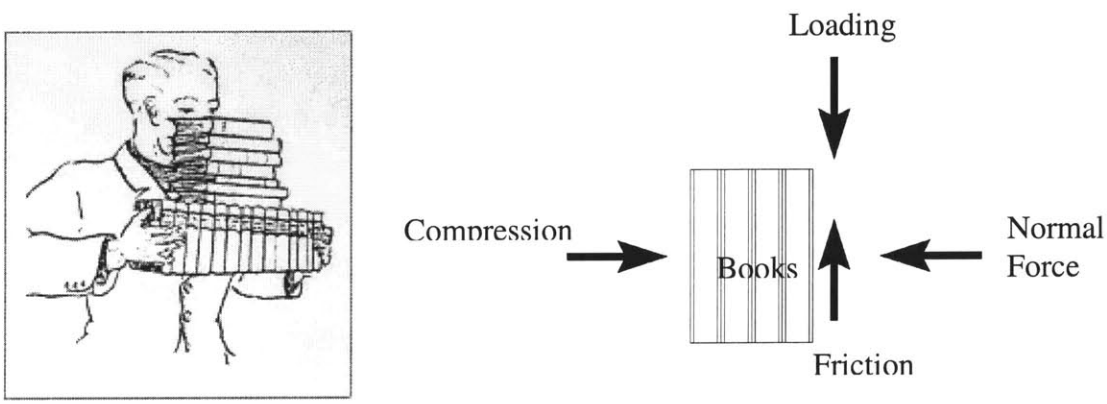


Figure 1: Basic Concept of Prestressing[1]

In this simple case, the friction induced by the compression force opposes the loading in directly. If the loading increased, applying larger compression force can increase the opposing force. Since the compression force and the normal force are equal in magnitude, increasing the compression would increase the friction. This specific case works because the books can withstand the compression forces applied by the person. In other cases, buckling of the member due to the compression must be taken into account.

2.1 Steel vs. Concrete

There are numerous advantages to using prestressing mechanism for steel rather than its concrete counter part. Although steel has a higher material cost by weight compared to concrete, the construction, in most cases, requires less labor and provides faster erection time than that of concrete structures. Concrete structures require formwork and layout of cumbersome reinforcements, entailing greater labor, coordination, cost, and time. In addition, concrete also requires a specified period to reach a workable strength - 28 days to achieve its full strength for normal concrete. Steel is also stronger than concrete of equal weight and has compressive strength equal in magnitude of its tensile strength, compared to concrete, which has virtually no tensile capacity.

In concrete, due to its inability to take tension, prestressing is applied to achieve distributed compression with little or no tension. On the other hand, because there exists load capacity for both compression and tension, equivalent steel section can utilize a greater cross-sectional area for stress distribution. This ability, therefore, provides steel with economical advantage over concrete for resolving stresses within a member, as it is more efficiently used.

There are also many disadvantages in using steel instead of concrete as a structural material. Almost all steel sections are standardized and to deviate from the standards requires additional cost related to fabrication. Also to provide the same support conditions for prestressing in the concrete members, steel sections require additional materials, such as anchorages and stiffeners. And, finally, concrete can be formed freely to take just about any desired shape while it is extremely difficult and costly to achieve the same effect using steel.

Although prestressed concrete is far more popular in structures than steel, it is the author's belief that prestressed concrete cannot possibly be utilized in every situation due to its weight. As it will be discussed later, a conceptual project has used prestressed steel pylon for cable-stay bridge instead of prestressed concrete due to the strength and the weight reduction required.

2.2 Challenges

As simple as this concept may seem, the challenges of prestressing occur when other variables come in to play. For example, in many situations, the applied loading is not static but dynamic in nature. This poses difficulty in trying to match the prestressing pattern to counter the applied loading since the location or the magnitude of the applied loading is not constant.

Perhaps, one of the most challenging aspects of the prestressing mechanism is the fact that the problems are statically indeterminate in nature. Although the analysis for prestressing, concrete or steel, are often done using statically determinate formulations, they do not always yield the most accurate solutions. At the same time geometrically nonlinear behavior is observed due to P-δ effect when extreme prestressing with tendons is installed before the total design load is applied[16]. In such cases, prestressing and the design load must be applied incrementally to reduce the effects of the axial compression. One must keep in mind, though, that in general, linear approximation of prestressing mechanisms, in addition to the code limitations and added safety factors, provide sufficient approximations for practicality.

3 History

As mentioned above, the idea behind prestressing is far from being new. Even before the term "prestressing" was invented the principle was used effectively in different applications. The oldest known application of prestressing in structures was in Egyptian shipbuilding, around 2700 B.C. They had used wire ropes and turnbuckles to hold the sides of the ship together (Fig. 2)[14]. Other applications include wheels, barrels and cannons where iron rings or wires were drawn out and tightly wound to give added stiffness and strength. For example, cartwheels were assembled using heated iron rings that wrapped the rim of the wheel. As the heated metal cooled, the ring contracted, inducing compressive force in the spokes and tightening the joints (Fig. 3). Another example of prestressing, which is still in use today mainly in the wine industry, is the method of barrel making. Wooden barrel sides are held in place tightly by forcing a metal hoops, smaller in diameter, around the barrel, creating a watertight seal between the staves (Fig. 3)[11].

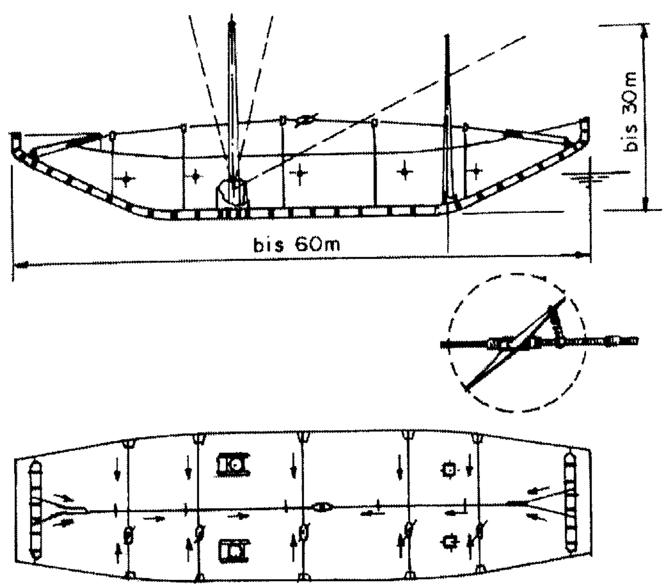


Figure 2: Prestressing Used in Shipbuilding[14]

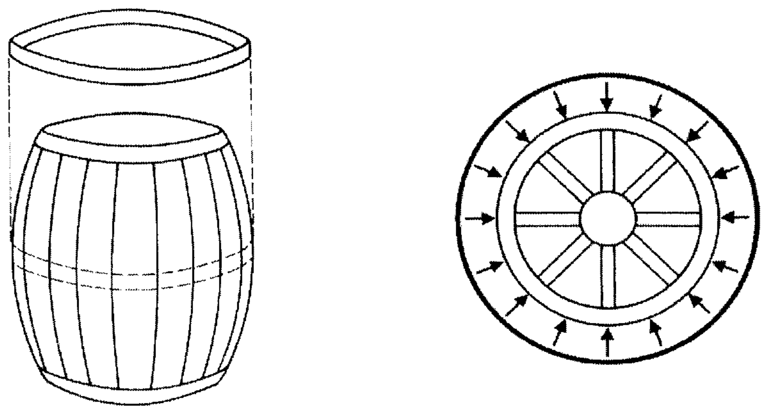


Figure 3: Prestressing Used for Barrels and Wheels[11]

It wasn't until the middle of 1800's when the prestressing schemes were employed in bridge structures. In U.S., Howe trusses, utilizing timber chord members and cast iron diagonal and vertical ties, were patented in 1840. From 1847 to 1850, H. Rider designed prestressed trusses, consisting of cast iron chords and wrought iron diagonals, which were prestressed in similar manner as the Howe trusses, by tightening of nuts at both ends.[14]

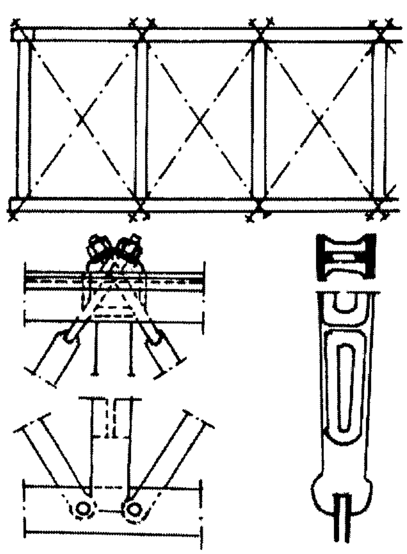


Figure 4: Rider's Prestressed Trusses[14]

One of the more famous prestressed steel structures is the Britannia Bridge over the Menai Straits. Designed and built in 1850 by Robert Stephenson, this 1380 ft bridge, with two 460 ft main spans and two 230 ft side spans, utilizes rectangular tube girders to carry the railway tracks within the girders[7]. The Britannia Bridge used a form of prestressing method called the "pre-deflection" method, which forces a deflection in the direction that will produce a moment countering the applied loading[14]. Details of this prestressing method will be discussed in the later chapters.

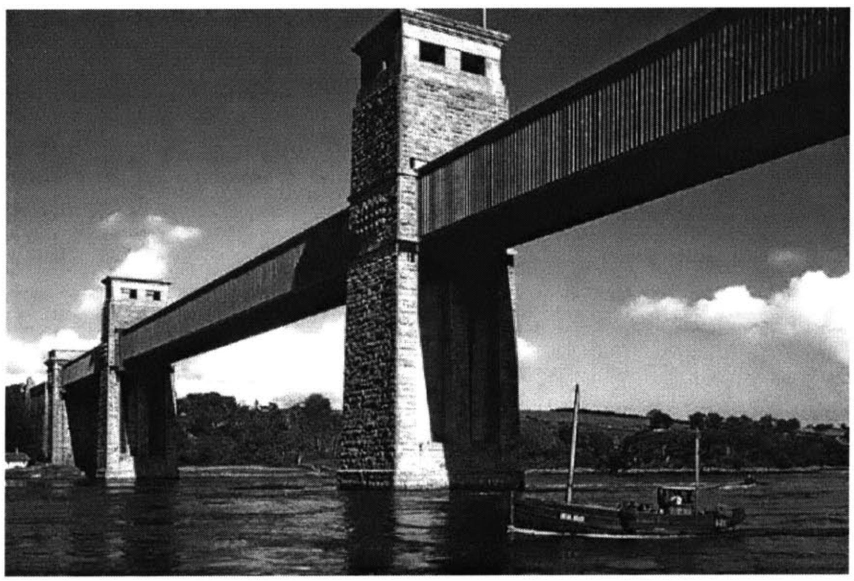


Figure 5: Britannia Bridge over Menai Straits[5]

Many other bridges incorporating the method of prestressing steel were built in the early 19th century but nearly all of them have either been destroyed or replaced. At the same time with the onset of the prestressed concrete developments, structures using prestressed steel members diminished rapidly. However, this development did not completely eliminate the use of prestressing mechanisms for steel members in structures. Prestressing of steel members is, and has been, continuously utilized for rehabilitating or strengthening of existing highway bridges. As an example, recently, two continuous-span steel stringer bridges were strengthened in Iowa, using post-tensioning mechanisms. The bridges in discussion were overstressed in both positive and negative moment regions when service live loads were applied. After, the installations of the prestressing mechanisms, the procedures were determined to be viable, economical strengthening techniques that should significantly extend the useful life of a given bridge[15]. The following figure illustrates the mechanism that was installed (Fig. 6).

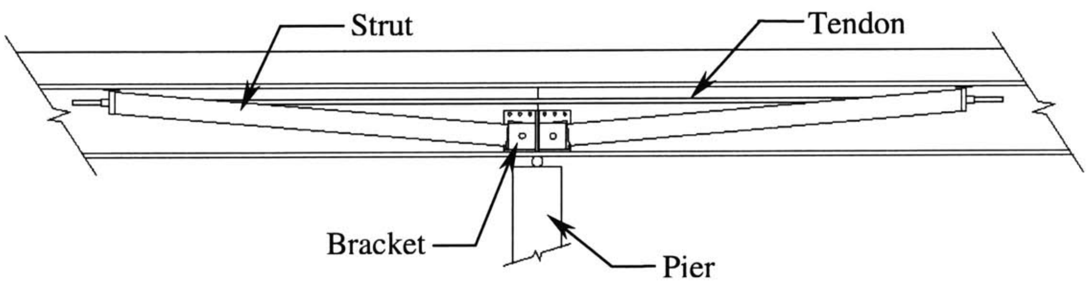


Figure 6: Bridge Rehabilitation Using Prestressing[15]

In 1996, some of the most spectacular structures, which some consider would have been impossible in the previous years, were engineered in Madrid, Spain, by Leslie E. Robertson Associates. The building structures known as "Puerta de Europa", or "Torres Kio," was one of the break-through structures that surpassed the barriers of rigid, verticality of high-rise buildings. The two 26 floor buildings lean 15° from the vertical relying on the lateral support provided by a combination of prestressing mechanisms, post-tensioned in this case, that are implemented on concrete core elements and steel framing elements (Fig. 7)[13]. The prestressing elements in this case provided the necessary deflection control, counter-balance moment, and increased stiffness for the overall system.

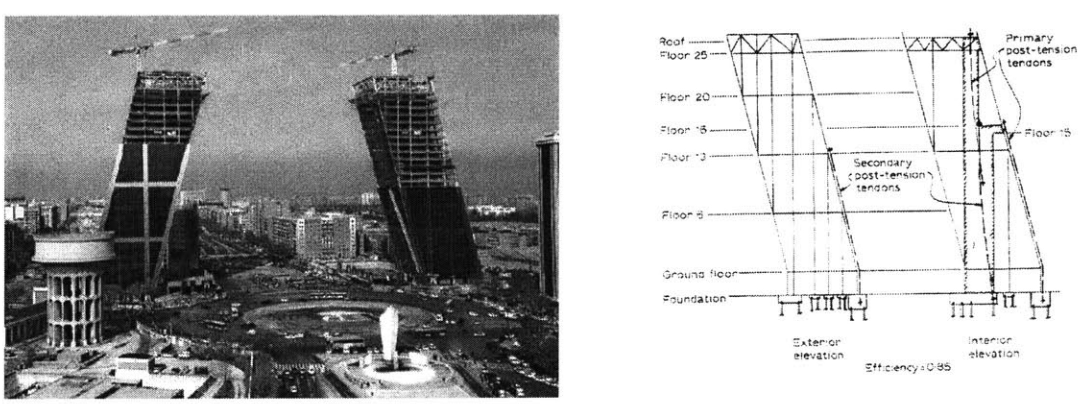


Figure 7: Puerta de Europa[3, 13]

Leslie E. Robertson Associates is currently involved in engineering of several leaning high-rise buildings, a total of five, the Puerta de Europa being one of them, and the other currently in construction is the new Domino's Pizza World Headquarters in Ann Arbor, Michigan (Fig. 8). These examples distinctively rely on the prestressing mechanisms for stability and strengthening of the structures. With further developments in this field, the possibilities for future structures could be limitless.

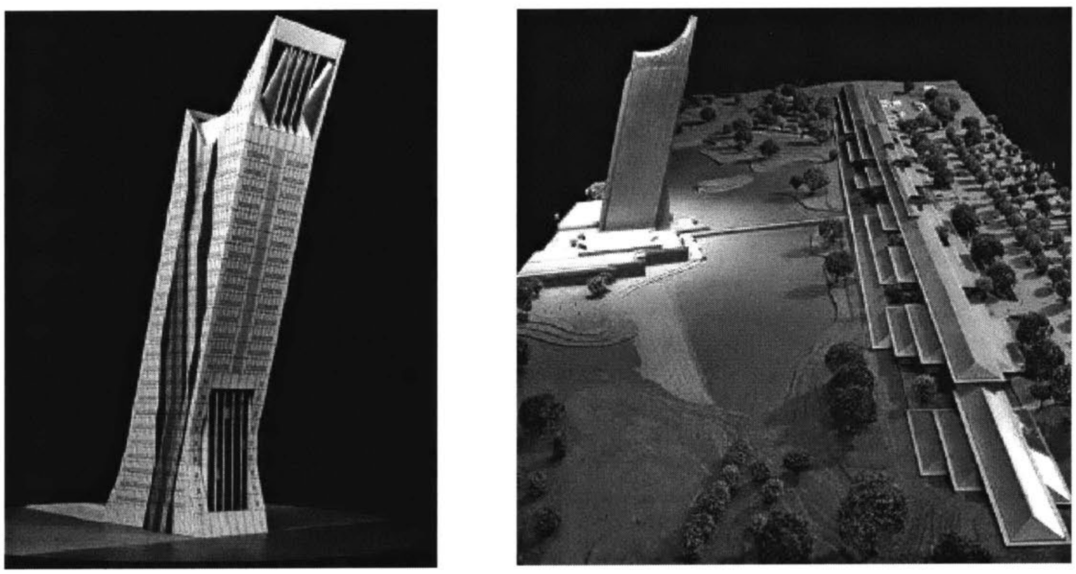


Figure 8: Domino's Pizza World Headquarters[2]

4 Prestressing Methods

There are many variations of applying prestress to steel members. These variations can be grouped into two distinct categories. One category is considered the most versatile method of prestressing mechanism and can be applied to range of structural systems. This particular method utilizes what is known as tendons to induce the necessary prestressing. Because the mechanisms can be installed externally after the member has been constructed, it is often used for rehabilitation or post-strengthening of existing structures. The tendons are generally made of high-strength steel, but other materials such as FRPs (Fiber Reinforced Polymers) are currently being used in the industry.

The other category of prestressing mechanism includes all other means of inducing prestress in the member. These mechanisms include bending of rolled sections reinforced with cover plates, predeflection technique, and redistribution of bending moments using differential support level regulations.

4.1 Prestressing by Tendons

Its simple concept and ease of installation, perhaps, made the use of tendons a popular method of applying prestress to both steel and concrete structural members. In steel members, prestressing with tendons provide a specific advantages in that they can be applied after the structural system is in place, or in many cases, even after the design loads have been applied to the system. This specific advantage makes the tendon prestressing mechanism ideal for post-strengthening of structures that have deteriorated or are in need of rehabilitation. Often this mechanism is used in older highway structures, such as overpasses and bridges, to increase the life span of the structures.

There are a few different orientation of prestressing with tendons that can be applied to the structure depending on the desired stress distribution in the member. One of these orientations requires little effort in installation and provides an excellent result for a member with constant distributed load as its applied loading. In this case, the prestressing tendon is placed under the bottom flange of a beam (i.e. wide flange steel section) and a eccentric stress is induced in the cross section of the member (Fig. 9).

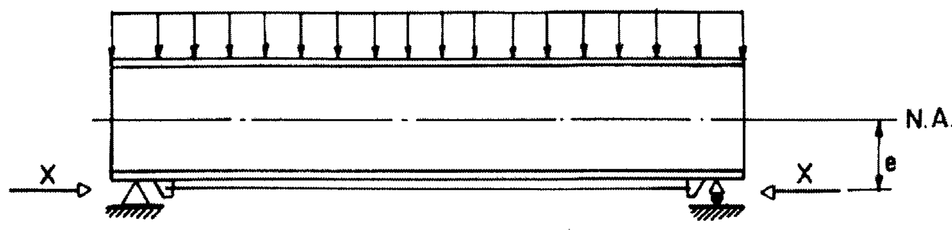


Figure 9: Eccentric Tendon Placement[14]

Since the tendon is placed eccentrically with a distance, *e*, from the neutral axis of the member, the stress induced will be consisted of an axial compression and bending. If *X* is the amount of prestressing force in the tendon, the total stress induced in the member due to pressing, f_p, is

$$f_p = -\frac{X}{A} \pm \frac{Xec}{I}$$

where *e* is the eccentricity, *c* is the distance to the exterior edge of the flange from the neutral axis, and *I* is the moment of inertia for a given section.[14]

If the member is subjected to a loading that generated moment *M*, then the total stress in the member is given by[14]

$$f_{total} = -\frac{X}{A} \pm \frac{Xec}{I} \mp \frac{Mc}{I}$$

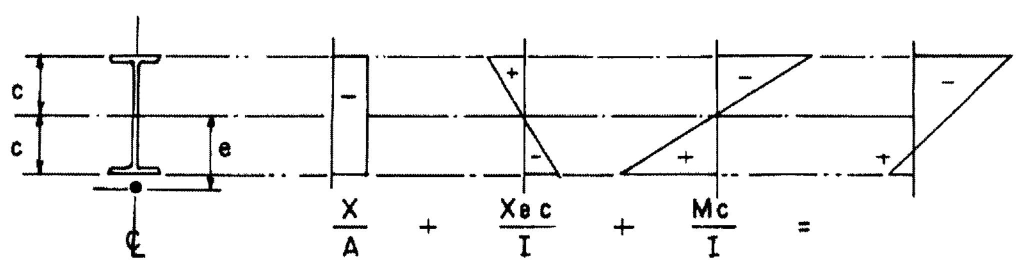


Figure 10: Stress Distribution for a Symmetric Member[14]

In this case, the prestressing reduces the tensile stress in the bottom flange while increasing the compressive stress in the upper flange. The added compressive stress results in decrease of tensile area and therefore increase the overall capacity of the beam.

Another variation of the prestressing mechanism using tendons is the draped orientation of the prestressing cable. This method is preferred in specific situations where the applied loading is a constant distributed loading or point loads of equal magnitudes spaced evenly along the length of the member, and the benefits of using the prestressing will be optimal. The benefits of the draped tendon system draw from the fact that the prestressing is able to provide similar but opposite distribution of stress to those induced by the applied loading[14]. For example, if the prestressing tendons are draped in a predetermined polygonal shape with tensile force of *P*, as shown in Fig. 11,

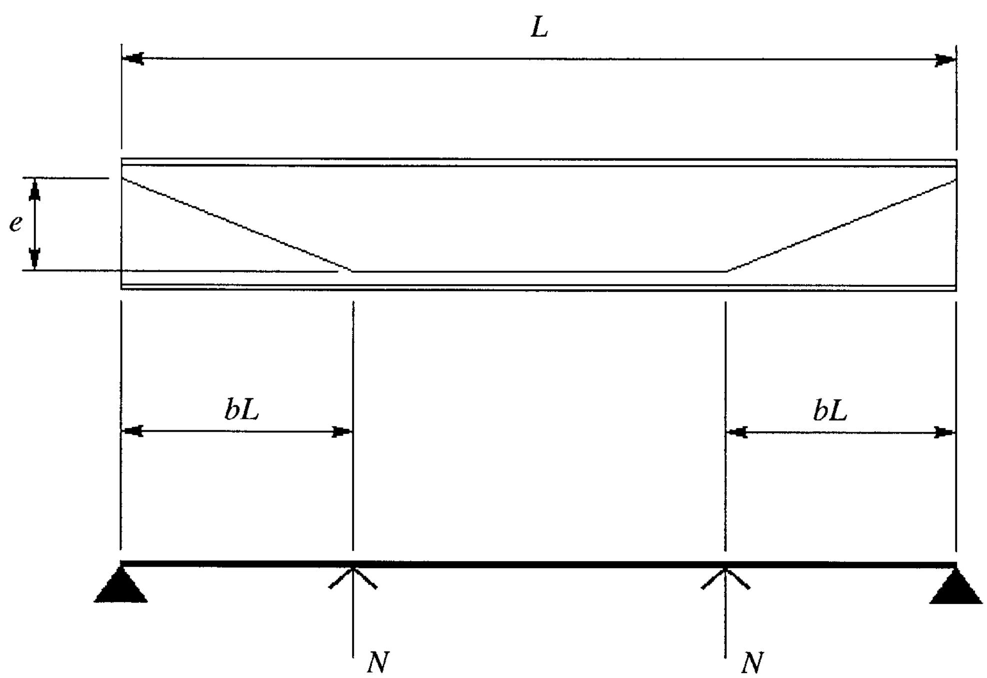


Figure 11: Polygonal Shape Tendon Placement[12]

the resulting vertical components of the tensile force in the tendon will be in the exact opposite direction as the applied loading with a magnitude of[12]

$$N = \frac{Pe}{bL}$$

A similar concept can be applied to account for a constant distributed loading. If the prestressing cable is draped in a parabolic shape, it provides a counter stress distribution that is similar but in opposite direction of the distributed loading (Fig. 12). The magnitude of the equivalent loading, w, due to the parabolic draping of the tendon will be[12]

$$w = \frac{8Pe}{L^2}$$

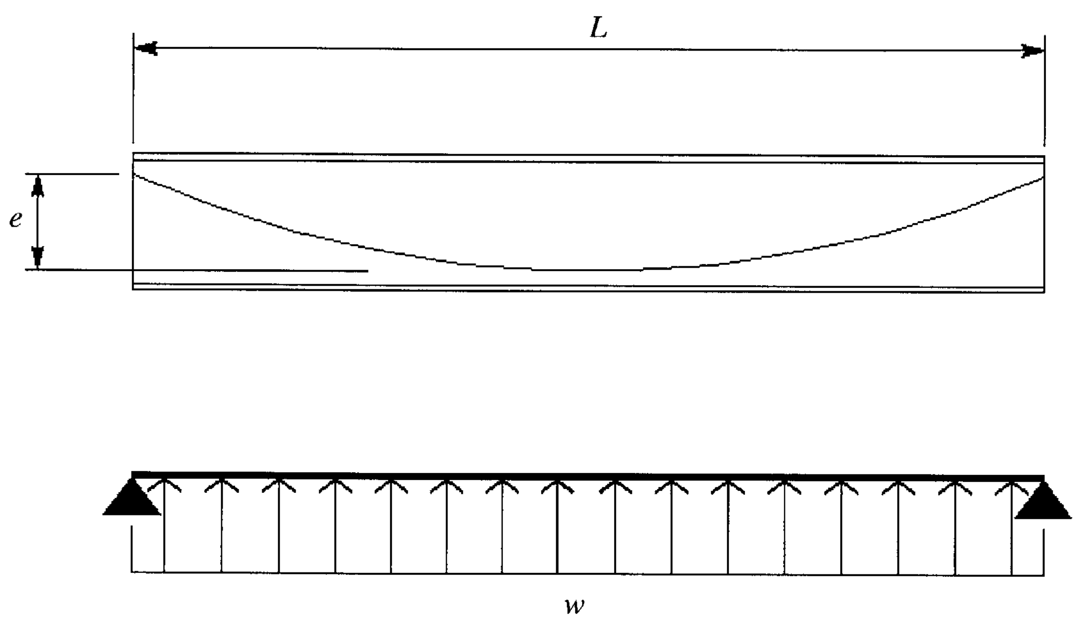


Figure 12: Parabolic Shape Tendon Placement[12]

This particular draping method, however, is not easily implemented on a steel structural member since it requires extra support materials to provide the needed shape of the draping. In prestressed concrete members, the formed concrete can provide the support for the parabolic shape of draping.

In theory, given that the compressive stress induced by the prestressing tendon can be resolved to avoid buckling or compressive yielding, specific cases of applied loading can be completely negated by the use of prestressing mechanism. However, the cost saved by negating the applied loading generally does not compensate for the cost generated by additional materials to support the compressive stress (i.e. increased member size, added web stiffeners, reinforced anchoring for tendons, etc.). Therefore, costs and benefits of prestressing must be investigated before simply applying the prestress to negate the total loading.

So far, the stress formulation has been based on the statically determinate formulations of the system. However, these formulations must be modified for indeterminacies and nonlinear behaviors in specific situations. For example, the modulus of elasticity for the prestressing tendon changes nonlinearly according to the change in the tension[16].

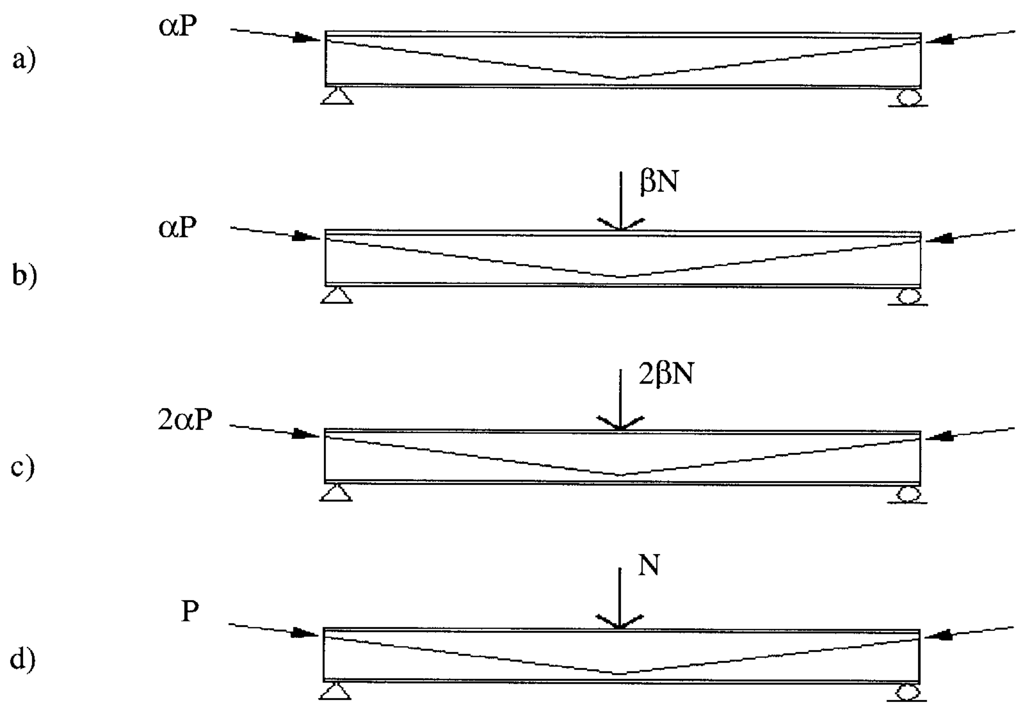
$$E_{eff} = \frac{E}{1 + \frac{AE}{T}\left(\frac{w_n L}{T}\right)^2}$$

where

E_{eff}	=	modified modulus of elasticity (effective modulus)
A	=	cross-sectional area of the prestressing tendon
E	=	original modulus of elasticity
T	=	tension in the tendon
L	=	unsupported length of the tendon
w_n	=	vertical component of the weight per unit length of the tendon

It can be seen in the above equation that the nonlinear material behavior of the cable often does not play a significant role in prestressing systems unless the weight and the length of the tendons are significantly large. Such is the case in cable-stay bridges where the cables' unsupported lengths and the weight of the cables have large affect on the engineering of the structure.

The complication of nonlinear analysis becomes significant when the magnitude of the prestressing becomes large and P-δ effect cannot be ignored. Since the prestressing tendons induce a large component of its stress through compression, the member could buckle if the desired prestressing is applied all at once in a single step without the design load countering the stress. Increasing the member size to accommodate for the prestressing can prevent this buckling but this would defeat the purpose of installing the prestressing mechanism to begin with. A better solution is to incrementally apply the prestressing and the design loading, so that the member will only see a fraction of either the prestressing or the design loading (Fig. 13) at each increment.



α = a fraction of the total prestressing
β = a fraction of the applied loading

Figure 13: Incremental Prestressing

Incremental prestressing is often desired not just to reduce the P-δ effect or to reduce the member size, but also for reasons of constructability, where single step installation of the prestressing mechanism would be limited due to the length and the support condition of the member. However, if the prestressing is done incrementally, the resolution of stresses become statically indeterminate and an additional equation must be established.[14] This is also the case where the prestressed member is subjected to a live load and an additional deflection is induced to the system. Under such conditions, the length of the tendon is increased by an increment of length, ΔS, and accordingly the force in the tendon, X, is also increased by ΔX. To determine ΔX, three equations of

$$\sum M = 0 \qquad \sum V = 0 \qquad \sum H = 0$$

equilibrium, are not sufficient and an additional equation is needed.

An additional equation for the above case can be derived from the compatibility criteria when the cable is cut vertically. Here, the horizontal displacement in the cable due to live load (δ_{ip}) and the force increment ΔX must equal 0.[14]

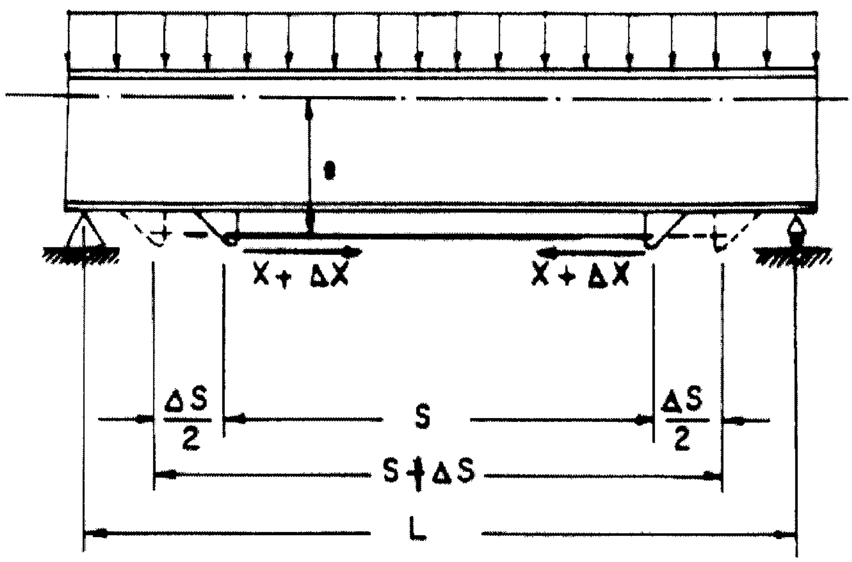


Figure 14: Indeterminacy Due to Stretching of Tendon[14]

Therefore,

$$\Delta X = -\frac{\delta_{ip}}{\delta_{11}}$$

or

where δ_{11} is equal to the displacement due to unit prestressing force.[14]

Solving the equations utilizing the virtual work method, the displacements are:

$$\delta_{ip} = \int_0^l \frac{Mxm}{EI} dx$$

$$\delta_{11} = \int_0^l \frac{m^2}{EI} dx + \frac{l}{E_t A_t} + \frac{l}{EA}$$

and therefore,

$$\Delta X = -\frac{\int_0^l \frac{Mxm}{EI} dx}{\int_0^l \frac{m^2}{EI} dx + \frac{l}{E_t A_t} + \frac{l}{EA}}$$

where[14]

M	=	the bending moment due to external loading
M	=	the bending moment due to unit load
A	=	cross-sectional area of the beam
A_t	=	cross-sectional area of the tendon
E	=	modulus of elasticity of the beam
E_t	=	modulus of elasticity of the tendon
I	=	moment of inertial of the beam
L	=	length of the tendon

As was mentioned earlier, in theory, with the addition of necessary supports for compression, the prestressing limitation would only be bounded by the tensile strength of the tendons, which can increase with addition of extra tendons. Implication of this theory is that with proper engineering, relatively thin, slender structures, generally architecturally more pleasing, can be built using prestressing mechanisms to negate the applied loading. However, installations of prestressing mechanisms are generally associated with cost reduction, and using these mechanisms may not be the most economical means to construct slender structures.

4.2 Prestressing by Bending

Steel members can be prestressed also by bending the members using jacks and then fixing their positions by means of welding (Fig. 15). The prestress created will have a uniform distribution throughout the cross-section of the beam in the opposite direction to that of the stresses induced by the applied loading. Since the final cross-section of the prestressed member will be symmetric, the magnitudes of the compressive stress in the bottom flange will be equal to that of the tensile stress in top flange. This ensures that the negating bending stress will be equally distributed between the top and the bottom flanges, effectively utilizing the material.[14]

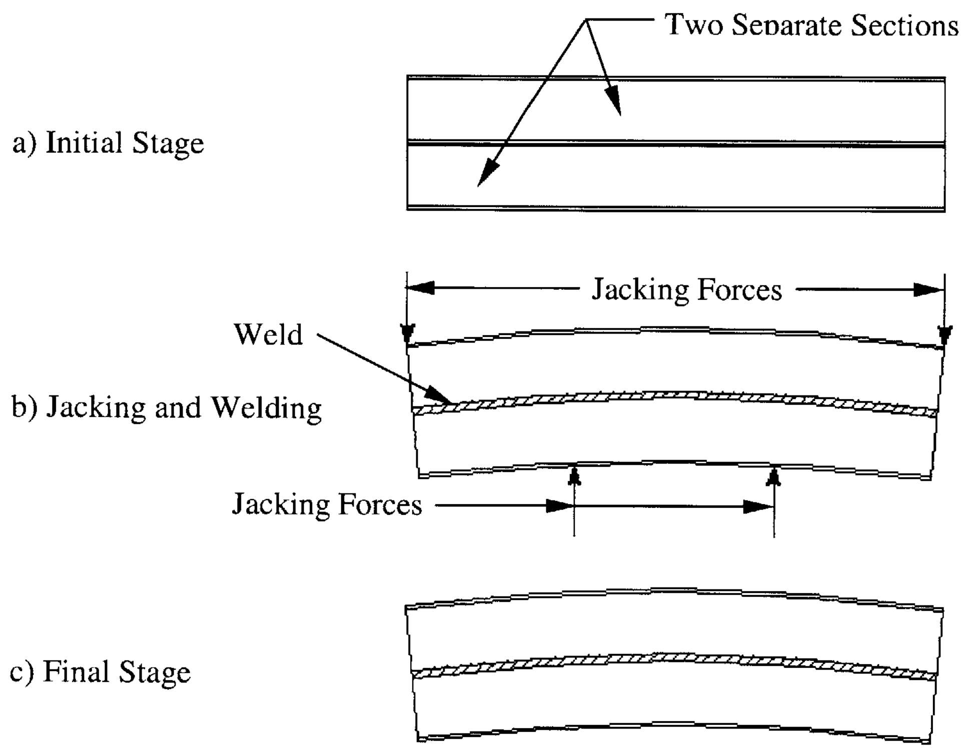


Figure 15: Prestressing by Bending[14]

4.2.1 Combining Two Symmetrical Members

The stresses in one symmetric member at the exterior edges of the flanges due to bending will be

$$\sigma_0^1 = \pm \frac{M_0 c}{I_0} = \pm \frac{M_0}{S_0}$$

Where[14]

M_0	=	the bending moment of one member
I_0	=	the moment of inertia of one member
S_0	=	the section modulus of one member
c	=	distance to the exterior edge of the flange from the neutral axis

After the two members are welded and the jacks are released, the prestressing moment in the combined member will be equal to $2M_0$. Therefore the stresses in the extreme edges of the top and bottom flanges of the combined member are

$$\sigma_{01}^{1} = \pm \frac{2M_0}{S_w} = \pm \frac{2\sigma_0^1 S_0}{S_w}$$

where S_w is the section modulus of the combined member.[14]

Finally, the stresses induced in the combined member due the applied loading will be

$$\sigma_A^1 = \pm \frac{M_A}{S_w}$$

Therefore, the total resulting stresses in the combined member will be

$$\sigma^1 = \pm \frac{M_0}{S_0} \mp \frac{2\sigma_0^1 S_0}{S_w} \mp \frac{M_A}{S_w}$$

As it can be seen, the magnitude of the stresses in the top and the bottom flange of the combined member are equal in magnitude.[14]

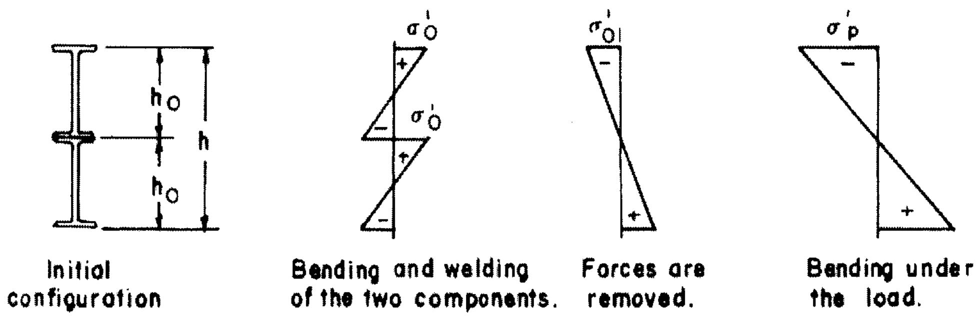
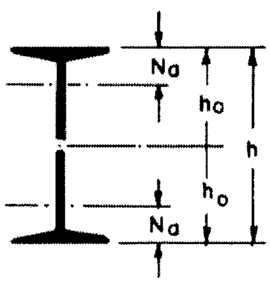


Figure 16: Stress Distribution for Combined Member[14]

4.2.2 Combining Two Asymmetrical Members

In general, if two asymmetric sections are combined to form the prestressed member, two T-sections or two halves of the wide flange sections are used to form a symmetric prestressed I-beam (Fig. 17). The resulting member prestress distribution is similar to that of the prestressed member using two symmetrical members.



$$\sigma^1 = \pm\frac{M_0}{S_t} \mp \frac{2M_0}{S_w} \mp \frac{M_A}{S_w}$$

Figure 17: Two Asymmetrical Sections Combined[14]

where S_t is the sectional modulus of a single asymmetric member.[14]

4.3 Prestressing by Predeflection

Also known as Preflex, the predeflection technique was first developed by Lipski in 1949, utilizing both high strength steel members and high strength concrete.[14] Unlike prestressed concrete members, the high strength concrete in this case is used to maintain the prestressing in the structural steel member. Although the method is seldom used in

US today, it is a popular method for construction of bridges and railroad structures in certain Asian and European countries.

The predeflection technique requires a rolled beam pre-cambered to a specified design shape with shear connectors to transfer forces between the steel member and the concrete encasing (Fig. 18a). The pre-cambered member is prestressed using jacking forces similar to the process used for the prestressing by bending (Fig. 18b). With the jacking forces still in place, the flange in tension is encased in high strength concrete (Fig. 18c). The concrete encasing can be reinforced for additional stiffness and ductility. After the concrete sets and desired strength is reached, the jacking forces are removed and the prestressing forces are maintained by the compression in the concrete encasing (Fig. d). Loss of prestress is expected due to the shrinkage and creep of the concrete.

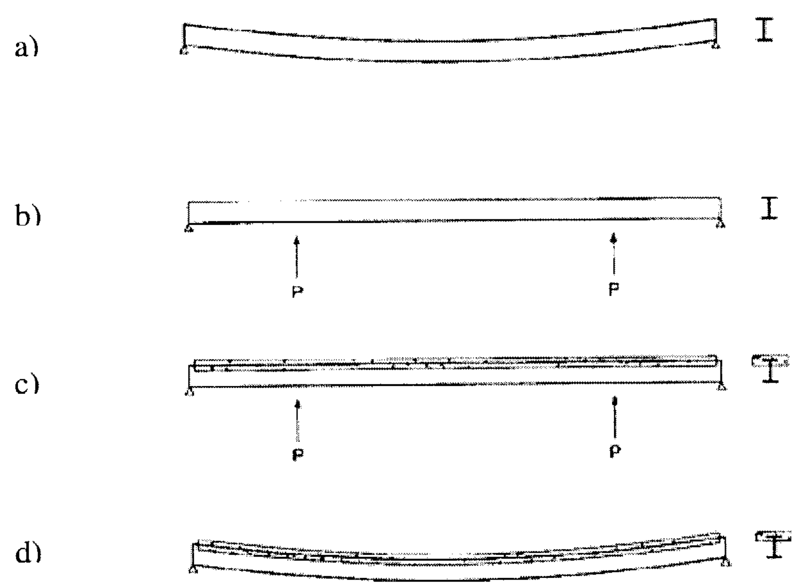
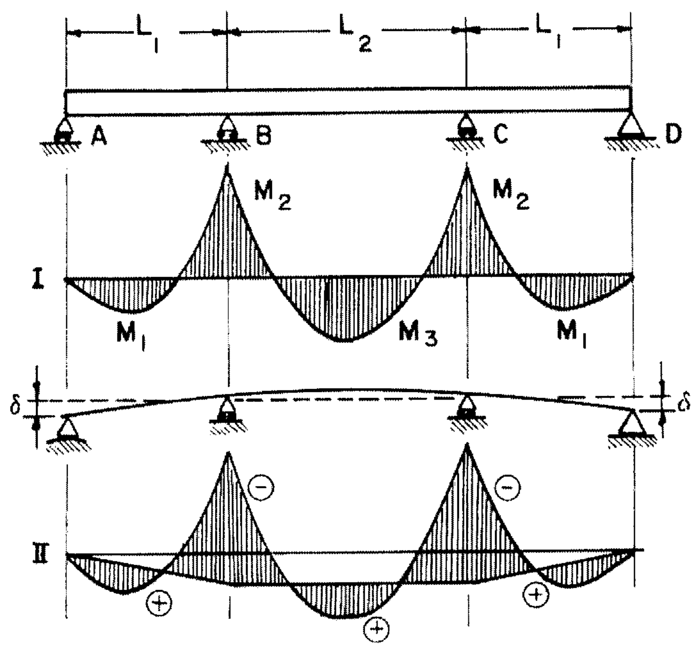


Figure 18: Prestressing by Predeflection[4]

4.4 Prestressing by Moment Redistribution

The concept of prestressing by moment redistribution arises from the manipulation of the support levels to induce bending stress within the members. This type of stress is generally seen in unequal support settlement where the difference in the support settlement results in stresses in the member due to bending. Utilizing this idea, predetermined settlement, or deliberate displacement, can be applied to the structure, producing desired moment in the system. The direction and the magnitude of the support displacements can be manipulated to achieve desired moment distribution.[14]



5 Economic Benefits and Feasibility

When prestressing mechanism is designed and engineered for a new structure, the goal is to achieve stronger system with smaller deflection using least material possible. In concrete, the benefits of prestressing are two fold. One benefit is the obvious increase in the load capacity and the other is that prestressing by tendon primarily increases axial compression, which can help to prevent formation of tensile cracks.

The benefits of prestressing for steel is similar in that less steel can be utilized to provide the needed load capacity or even greater capacity than the member without prestressing. Savings in material due to prestressing mechanism has been estimated to range between 10 and 30 percent[14]. Since the cost of steel is greater than concrete by weight, benefits of material reduction due to prestressing will be even more favorable than that of the concrete. However, there are two cost aspects that must be considered before realizing the economical benefits of prestressing steel members. First, installation of prestressing mechanism requires additional equipment and labor, which is not necessary in steel construction without prestressing. Second, the extent of the economical benefit is limited by the cost associated with additional material needed for anchorage and increase in member size to prevent compression failures. If extreme magnitude of prestressing is desired, for example to negate the entire design loading, the compression stress applied by prestressing can become significant. In such case, the cost to accommodate the additional increase in axial stress, to reinforce against compression failures, must also be considered.

Even with these factors in mind, the designer may wish to apply the necessary prestressing solely for the aesthetic benefits in constructing slender structures. In terms of architectural popularity, slender structures have become increasingly desirable in the past. Since the prestressing mechanism for steel can be applied on just about any structural system, it could be utilized to construct significantly more slender structures. For example, depth of a steel girder could be reduced up to 30 percent if prestressing using tendon was applied. Such reduction, given that the deflection criteria have been met, would impose a significant visual impact in an aesthetical sense.

In new structures, prestressing would be ideal for specific members that require both reduced section size and increased capacity. Application of the prestressing mechanism for repetitive members throughout the structure is not recommended since the construction time required for application of prestressing will rapidly decrease in its efficiency. However, such structures as bridge deck segments that can be fabricated at a plant, using template like setup, prestressing of repetitive members is highly feasible.

6 Case Study

While working on a cable-stayed bridge design, a problem was encountered due to the unique orientation of the main structural element, the masts. The design of the bridge consisted of two masts, on both sides of river, which held up the decks using cables. What made this bridge unique was that the masts lack the forces that counteract the weight and the forces on the decks (Fig. 20).

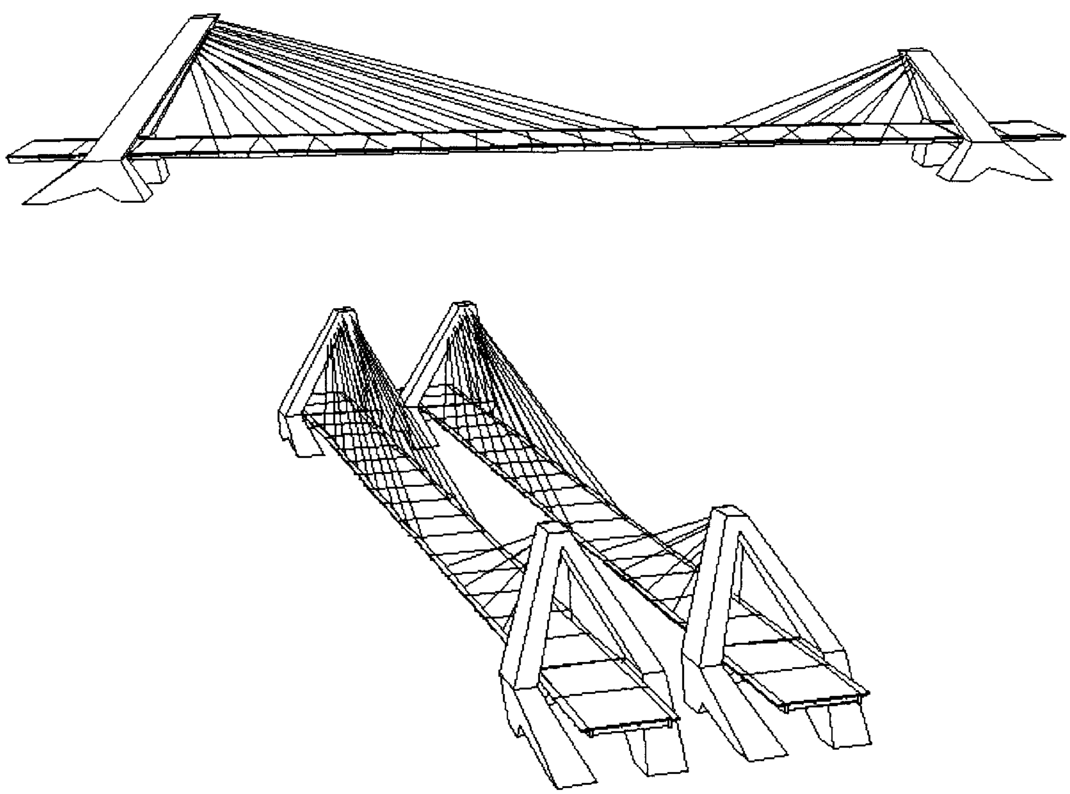


Figure 20: Proposed Bridge Design

In general, the masts on cable-stayed bridges have what they call "back spans," spans which provide balancing forces to the main span of the bridge. In this particular bridge there are no back spans and to make the matters worst, the masts lean in towards the main span approximately 55° from the horizontal. This means that the self-weights of the masts also add to the loading. One of the great architects, Santiago Calatrava, used the similar but opposite principles in his acclaimed El Alamillo Bridge of Seville, Spain.

It also does not have back spans but rather uses the weight of the massive mast made of concrete to counter balance the deck loading (Fig. 21).

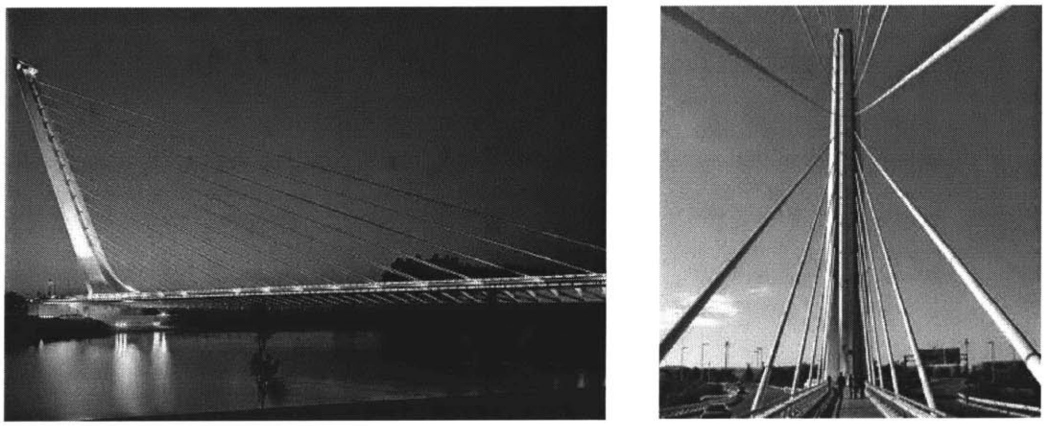


Figure 21: Calatrava's Alamillo Bridge[8]

Therefore, the masts on this design act as large cantilever beams that have concentrated loading near the tip. The larger mast has a length of 140 ft and the shorter one of 95 ft. According to the preliminary calculations that were carried out, required concrete section to withstand the stress would be enormous, and near impossible within the design restriction. At the same time, adding more concrete area to bear the stress would only increase the weight and therefore would increase the induced stress. Composite sections were also considered but in the end it was concluded that the use of steel for the primary structural member would be the optimal solution for this design.

Simply replacing the concrete with steel, however, did not solve the main problem in hand. The structure still lacked the necessary back span or other counteracting forces. Analyzing only in terms of structural strength, the bridge can be built to withstand the design loads. This means that the bridge will not collapse if it was built. The governing factor in this design, though it seemed at first, was not the issue of strength but rather was an issue of stiffness.

A cantilever mast with a length of 140 ft would deflect significantly, even to the point of discomfort for those traveling on the bridge. Again, in a conventional cable-stayed bridge, the combination of the main span loading and back span loading creates a large axial load in the mast, stiffening up the structure. Without the back span, a large moment is induced causing tension on one side and compression on the other. It is this large moment that must be equalized in the structure (Fig. 22).

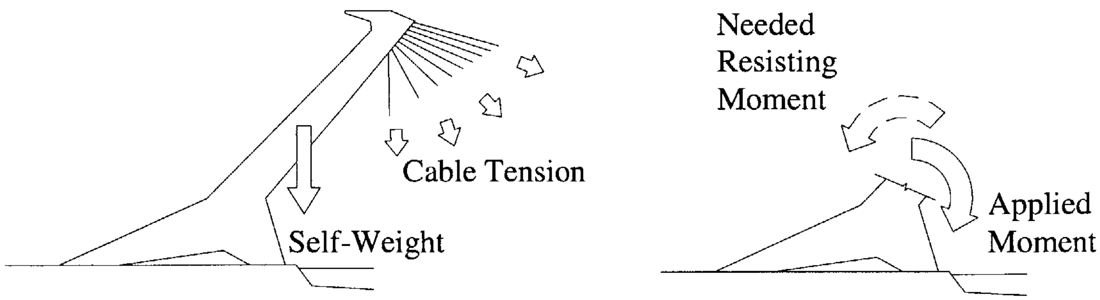


Figure 22: Balancing of Forces

Application of prestressing to this structure, therefore, may help to reduce the effect of induced moment and stiffen up the structure as a whole.

6.1 Prestressing Considerations

As discussed, there are a variety of prestressing methods possible for steel members. However, for this specific case, only one of the options was considered, which utilize the steel cables for prestressing. For the length and the scale of prestressing needed for this project, only prestress using tendons deemed feasible. Although both draping and eccentric tendon orientations were considered, in the end, eccentric orientation was chosen for the structure due to the structure's constructability.

Since applying the dead loads of the full length of the deck in a single increment would not be feasible, the prestressing system was divided up into segments, each set of two prestressing tendons corresponding to a deck section. The design connected nine deck segments to the taller mast and, therefore, nine incremental applications of the prestressing tendons were considered. Simply, two prestressing tendons will counter-balance each deck section.

The eccentric tendon orientation would provide a type of resisting load and stiffening mechanism for the structure. Two schemes were considered for installing the prestressing system for the structure using eccentric system (Fig. 23). In the end, installation of the tendons within the core of the mast was selected for architectural reasons.

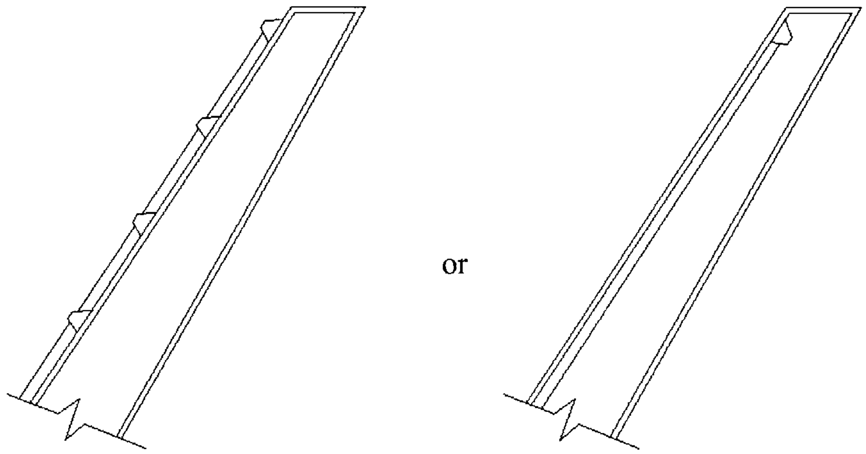


Figure 23: Prestressing Orientation Options

Both of these orientation are sometimes referred to as an outrigger system in building structures, and it uses forces and moment arms to provide the necessary resistance (Fig. 24).

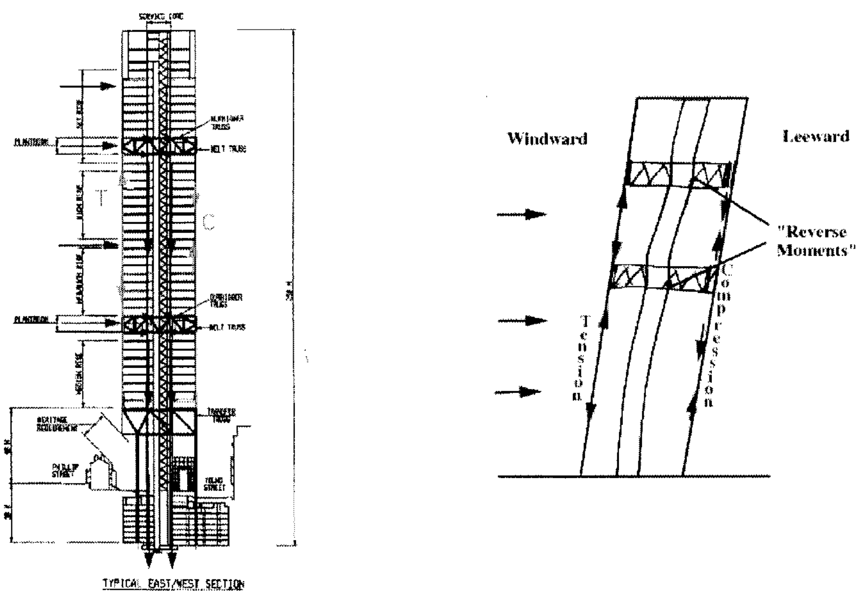


Figure 24: Outriggers in Building Structures[9]

The prestressing tendons will be stressed incrementally to minimize the stress and deflection imposed on the mast section during the construction. Each prestressing increments will correspond to an installation of one deck section. The stresses in the strands will be gradually increased accordingly as the cables supporting the decks are gradually loaded. This method of installation will ensure that the mast section will only experience a maximum moment when the last set of prestressing strands is installed. During the installation of the preceding sets of prestressing strands and the decks, the moments applied to the structure will be less than that of the last set.

It is clear that the magnitude of prestressing is mainly determined by the load on the mast, which is directly related to the dead load of the deck. To reduce the need of applying extreme prestressing loads on the mast, prestress was also considered for the transverse girders in the deck for deck weigh reduction. The transverse girders were designed to carry the four-point loads transferred from the intermediate stringers. If prestressing cables are to be installed for size reduction, the layout and positioning of the cables must be such that the forces induced by prestressing will directly counter the point loads for the optimal benefit. To do so, the cables should be draped and the anchors placed at calculated points. For this case where the point loads are applied symmetrically, equidistance from each other by L/5, two anchors should be placed at 2/3 the depth of the draping in line with the point loads and two at the depth of the draping also in line with the loads for the optimal configuration (Fig. 25).

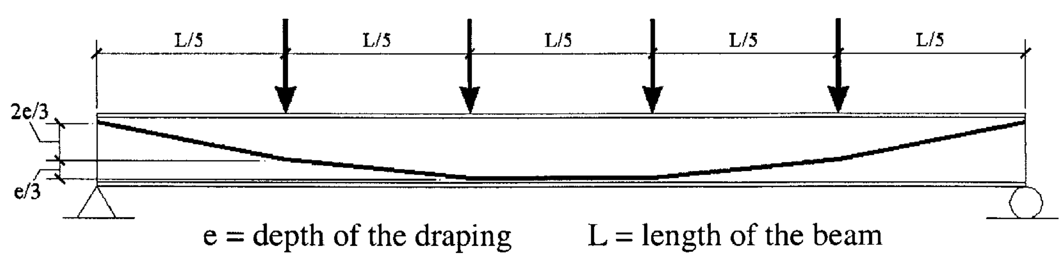


Figure 25: Prestressing Scheme for Transverse Girders

The prestressing of the transverse girders weren't included in the final design of the structure due to uncertainties associated with the weight reduction in the deck. Since the deck will be subjected to a lateral wind loading, significant weight reduction may have lead to unexpected circumstances such as loss of rigidity to resist vortex shedding of wind from the adjacent bridge.

6.2 Design and Analysis

The design and analysis of the masts in discussion were carried out as a part of semester bridge project as required in Master of Engineering program at MIT. For the purpose of this report, only the calculations pertaining to the taller mast, 140 ft in length, will be presented. For numerical reference and procedure, refer to the appendix at the end of the report.

The design of the bridge requires that the taller mast carry the load of nine bridge deck sections, transferred through sets of cables attached within the top sixty feet of the mast (Fig. 26). A set of two cables will carry the load of each deck, which will be counter-balanced by a set of two prestressing cables attached on the opposite side of the mast. Corresponding to nine deck sections, therefore, nine pairs of prestressing cables, eighteen in total will be installed. The prestressing cables will be installed within the core of the mast, which will be constructed of steel box girders for reasons of weight reduction (Fig. 26).

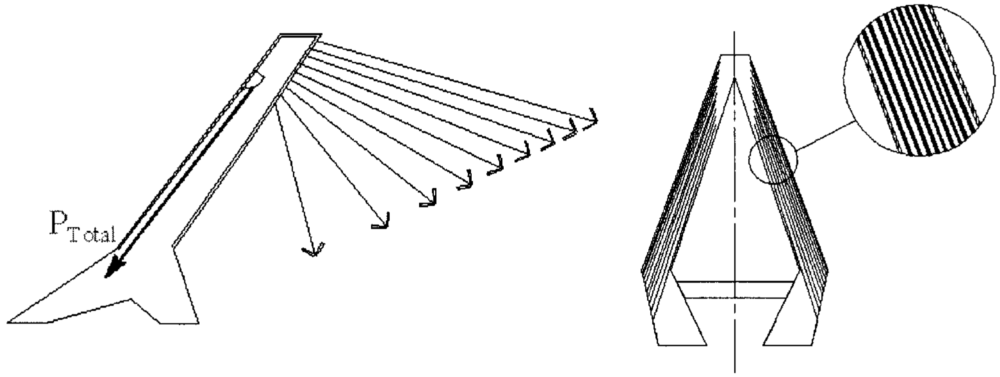


Figure 26: Prestressing Placement

As discussed earlier in the section, the prestressing tendons will be loaded incrementally, a set at a time, as each deck sections are hung from the cables. According to the calculations, the maximum stress that a prestressed tendon will experience will be 9365 kips. Depending on the strength of the tendons used, the number of strands and cross-sectional area required to withstand the force will vary. For example, using 270 ksi steel tendons[6], approximately 7" prestressing tendon diameter will be required. Since this corresponds to the maximum tensile value that the prestressing tendon will experience, all other sets of prestressing will be of smaller diameter.

To calculate the stresses developed at the base of the mast, where the stresses will be highest due to the total moment and axial forces created, moment arm distances, cable angles, and the varying eccentricity of the prestressing tendons were considered. Again, since the prestressing and the deck loading will be applied incrementally, only the maximum moment induced per increment was considered for the design. However, when considering the stress development due to the axial forces, total axial stress was used since axial stress in each increment will be additive.

For the project, the indeterminacy related to the incremental prestressing method was not considered. When the usage of prestressing was considered to resolve the back-span issue, the goal of the project became a challenge simply to see if the structure would be feasible rather than to produce a detailed, working set of calculations. When it was shown that the strength capacity and load demands could be met using static, dead and live load analysis, detailed calculations involving energy methods became unnecessary. In addition, the limitations imposed on the bridge design was mainly based on the requirements to meet the deflection criteria, which led to an over-design of the structure assuring that the failure of the bridge will not be due to its load capacity.

7 Conclusion

Prestressed steel mechanism has been recommended in the past as one of the more effective means to increase the capacity of a structural member and to economically benefit from the reduction in material used. It has also been proven effective when the method was used for retrofitting of highway bridges and structural elements in rehabilitation of buildings. Unlike its concrete counterpart, however, prestressed steel mechanisms have not been deeply rooted in the industry as expected. Rather, the method is rarely used and seldom even suggested as a viable solution.

Speculation to this trend would be that the prestressed steel mechanism might be just too good for its own sake. The mechanism improves the system's strength capacity and reduces the amount of material needed for the structure. For the engineering industry, this would be a practical solution for majority of steel structures. From a commercial point of view, however, this wouldn't be a favorable option for the steel industry. Unlike prestressed concrete, the usage of steel prestressing tendons does not promote the usage of more steel, but rather reduces it.

Another aspect of prestressed steel that may seem to be a disadvantage is that all steel structural elements must be protected and maintained to prevent corrosion. Corrosion drastically decreases the effectiveness and the service life of prestressing tendons, especially. Specifically to prevent this, tendons are generally encased in a water proof tubes or sleeves to isolate them from the external environment.

Whether the prestressing steel is favorable commercially or not, the mechanism remains as one of the most valuable means to strengthen and add stiffness to an existing steel structure. The effect cannot be ignored due to its ease of installation and cost benefits that can result from its use. It is the author's belief that the prestressing mechanism, in the future, will be implemented in the areas where the concrete counterpart currently dominate the industry; Cable-stayed bridges, structural beams and girders, and even foundations. As recent designs have shown, with careful engineering and design process utilizing prestressing systems could push structures beyond their conventional boundaries.

Appendix

Cable Angles

$TL := 5.37 \cdot 10^5$

Geometric angles formed between the cables and the horizontal axis

$\theta_1 := 16.2$ $\quad \theta_6 := 38.4$

$\theta_2 := 18.4$ $\quad \theta_7 := 50.5$

$\theta_3 := 21.27$ $\quad \theta_8 := 68.71$

$\theta_4 := 25.11$ $\quad \theta_9 := 92.65$

$\theta_5 := 30.5$

Geometric angles formed between the vertical axis and the cables

$\phi_1 := 73.8$ $\quad \phi_6 := 51.6$

$\phi_2 := 71.6$ $\quad \phi_7 := 39.5$

$\phi_3 := 68.73$ $\quad \phi_8 := 21.29$

$\phi_4 := 64.89$ $\quad \phi_9 := 2.65$

$\phi_5 := 59.5$

Tension in Cables

$$T1 := \frac{TL}{\sin\left(\theta_1 \cdot \frac{\pi}{180}\right)} \qquad T1 = 1.925 \cdot 10^6$$

$$T2 := \frac{TL}{\sin\left(\theta_2 \cdot \frac{\pi}{180}\right)} \qquad T2 = 1.701 \cdot 10^6$$

$$T3 := \frac{TL}{\sin\left(\theta_3 \cdot \frac{\pi}{180}\right)} \qquad T3 = 1.48 \cdot 10^6$$

$$T4 := \frac{TL}{\sin\left(\theta_4 \cdot \frac{\pi}{180}\right)} \qquad T4 = 1.265 \cdot 10^6$$

$$T5 := \frac{TL}{\sin\left(\theta_5 \cdot \frac{\pi}{180}\right)} \qquad T5 = 1.058 \cdot 10^6$$

$$T6 := \frac{TL}{\sin\left(\theta_6 \cdot \frac{\pi}{180}\right)} \qquad T6 = 8.645 \cdot 10^5$$

$$T7 := \frac{TL}{\sin\left(\theta_7 \cdot \frac{\pi}{180}\right)} \qquad T7 = 6.959 \cdot 10^5$$

$$T8 := \frac{TL}{\sin\left(\theta_8 \cdot \frac{\pi}{180}\right)} \qquad T8 = 5.763 \cdot 10^5$$

$$T9 := \frac{TL}{\sin\left(\theta_9 \cdot \frac{\pi}{180}\right)} \qquad T9 = 5.376 \cdot 10^5$$

Cable Design

Max. Tension in Cable, $T_{max} := T1$

Using 150 ksi strands,

$$\frac{T1}{150000} = 12.832 \text{ in}^2$$ For tension, Safety Factor = 2.0

Area required,

$$A_{req} := 2 \cdot 12.831 \rightarrow 25.662$$

Using 1/2" ϕ strands

$$\frac{25.662}{\pi \frac{(0.5)^2}{4}} = 130.695$$ ==> **132 strands/deck = 66 strands/side**

Pylon Design

Axial forces at the base of the cantilever due to the loading

$$F_1 := \frac{T1}{1000} \cdot \sin\left[\left(\phi_1 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_1 = 620.295 \quad K$$

$$F_2 := \frac{T2}{1000} \cdot \sin\left[\left(\phi_2 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_2 = 486.029 \quad K$$

$$F_3 := \frac{T3}{1000} \cdot \sin\left[\left(\phi_3 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_3 = 351.345 \quad K$$

$$F_4 := \frac{T4}{1000} \cdot \sin\left[\left(\phi_4 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_4 = 217.349 \quad K$$

$$F_5 := \frac{T5}{1000} \cdot \sin\left[\left(\phi_5 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_5 = 83.013 \quad K$$

$$F_6 := \frac{T6}{1000} \cdot \sin\left[\left(\phi_6 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_6 = -51.272 \quad K$$

$$F_7 := \frac{T7}{1000} \cdot \sin\left[\left(\phi_7 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_7 = -185.98 \quad K$$

$$F_8 := \frac{T8}{1000} \cdot \sin\left[\left(\phi_8 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_8 = -319.858 \quad K$$

$$F_9 := \frac{T9}{1000} \cdot \sin\left[\left(\phi_9 + 35 - 90\right) \cdot \frac{\pi}{180}\right] \qquad F_9 = -425.629 \quad K$$

Shear forces at the base of the cantilever due to the loading

$$V_1 := \frac{T1}{1000} \cdot \cos\left[(\phi_1 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_1 = 1.822 \cdot 10^3 \quad K$$

$$V_2 := \frac{T2}{1000} \cdot \cos\left[(\phi_2 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_2 = 1.63 \cdot 10^3 \quad K$$

$$V_3 := \frac{T3}{1000} \cdot \cos\left[(\phi_3 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_3 = 1.438 \cdot 10^3 \quad K$$

$$V_4 := \frac{T4}{1000} \cdot \cos\left[(\phi_4 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_4 = 1.247 \cdot 10^3 \quad K$$

$$V_5 := \frac{T5}{1000} \cdot \cos\left[(\phi_5 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_5 = 1.055 \cdot 10^3 \quad K$$

$$V_6 := \frac{T6}{1000} \cdot \cos\left[(\phi_6 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_6 = 863.007 \quad K$$

$$V_7 := \frac{T7}{1000} \cdot \cos\left[(\phi_7 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_7 = 670.623 \quad K$$

$$V_8 := \frac{T8}{1000} \cdot \cos\left[(\phi_8 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_8 = 479.426 \quad K$$

$$V_9 := \frac{T9}{1000} \cdot \cos\left[(\phi_9 + 35 - 90) \cdot \frac{\pi}{180}\right] \qquad V_9 = 328.37 \quad K$$

Moment forces at the base of the cantilever due to the loading

$$M1 := \left[V_1 \cdot (140 - 6)\right] \qquad M1 = 2.442 \cdot 10^5 \quad K$$

$$M2 := \left[V_2 \cdot (140 - 12)\right] \qquad M2 = 2.087 \cdot 10^5 \quad K$$

$$M3 := \left[V_3 \cdot (140 - 18)\right] \qquad M3 = 1.754 \cdot 10^5 \quad K$$

$$M4 := \left[V_4 \cdot (140 - 24)\right] \qquad M4 = 1.446 \cdot 10^5 \quad K$$

$$M5 := \left[V_5 \cdot (140 - 30)\right] \qquad M5 = 1.16 \cdot 10^5 \quad K$$

$$M6 := \left[V_6 \cdot (140 - 36)\right] \qquad M6 = 8.975 \cdot 10^4 \quad K$$

$$M7 := \left[V_7 \cdot (140 - 42)\right] \qquad M7 = 6.572 \cdot 10^4 \quad K$$

$$M8 := \left[V_8 \cdot (140 - 48)\right] \qquad M8 = 4.411 \cdot 10^4 \quad K$$

$$M9 := \left[V_9 \cdot (140 - 54)\right] \qquad M9 = 2.824 \cdot 10^4 \quad K$$

Prestressing cable eccentricity

$e1 := \left(5 \cdot \frac{6}{140}\right) + 7.5 + 0.5 \quad e1 = 8.214 \quad ft$

$e2 := \left(5 \cdot \frac{12}{140}\right) + 7.5 + 0.5 \quad e2 = 8.429 \quad ft$

$e3 := \left(5 \cdot \frac{18}{140}\right) + 7.5 + 0.5 \quad e3 = 8.643 \quad ft$

$e4 := \left(5 \cdot \frac{24}{140}\right) + 7.5 + 0.5 \quad e4 = 8.857 \quad ft$

$e5 := \left(5 \cdot \frac{30}{140}\right) + 7.5 + 0.5 \quad e5 = 9.071 \quad ft$

$e6 := \left(5 \cdot \frac{36}{140}\right) + 7.5 + 0.5 \quad e6 = 9.286 \quad ft$

$e7 := \left(5 \cdot \frac{42}{140}\right) + 7.5 + 0.5 \quad e7 = 9.5 \quad ft$

$e8 := \left(5 \cdot \frac{48}{140}\right) + 7.5 + 0.5 \quad e8 = 9.714 \quad ft$

$e9 := \left(5 \cdot \frac{54}{140}\right) + 7.5 + 0.5 \quad e9 = 9.929 \quad ft$

Forces in prestressing cables

$P1 := \left[\frac{M1}{e1 + (0.036 \cdot 134)}\right] \quad P1 = 1.873 \cdot 10^4$

$P2 := \left[\frac{M2}{e2 + (0.036 \cdot 128)}\right] \quad P2 = 1.601 \cdot 10^4$

$P3 := \left[\frac{M3}{e3 + (0.036 \cdot 122)}\right] \quad P3 = 1.346 \cdot 10^4$

$P4 := \left[\frac{M4}{e4 + (0.036 \cdot 116)}\right] \quad P4 = 1.11 \cdot 10^4$

$P5 := \left[\frac{M5}{e5 + (0.036 \cdot 110)}\right] \quad P5 = 8.904 \cdot 10^3$

$P6 := \left[\frac{M6}{e6 + (0.036 \cdot 104)}\right] \quad P6 = 6.888 \cdot 10^3$

$P7 := \left[\frac{M7}{e7 + (0.036 \cdot 98)}\right] \quad P7 = 5.045 \cdot 10^3$

$P8 := \left[\frac{M8}{e8 + (0.036 \cdot 92)}\right] \quad P8 = 3.386 \cdot 10^3$

$P9 := \left[\frac{M9}{e9 + (0.036 \cdot 86)}\right] \quad P9 = 2.168 \cdot 10^3$

Moment at the base of the cantilever due to prestressing

$Mp1 := P1 \cdot (e1 + (0.036 \cdot 134)) \quad Mp1 = 2.442 \cdot 10^5 \quad Kft$ <== Maximum

$Mp2 := P2 \cdot (e2 + (0.036 \cdot 128)) \quad Mp2 = 2.087 \cdot 10^5 \quad Kft$

$Mp3 := P3 \cdot (e3 + (0.036 \cdot 122)) \quad Mp3 = 1.754 \cdot 10^5 \quad Kft$

$Mp4 := P4 \cdot (e4 + (0.036 \cdot 116)) \quad Mp4 = 1.446 \cdot 10^5 \quad Kft$

$Mp5 := P5 \cdot (e5 + (0.036 \cdot 110)) \quad Mp5 = 1.16 \cdot 10^5 \quad Kft$

$Mp6 := P6 \cdot (e6 + (0.036 \cdot 104)) \quad Mp6 = 8.975 \cdot 10^4 \quad Kft$

$Mp7 := P7 \cdot (e7 + (0.036 \cdot 98)) \quad Mp7 = 6.572 \cdot 10^4 \quad Kft$

$Mp8 := P8 \cdot (e8 + (0.036 \cdot 92)) \quad Mp8 = 4.411 \cdot 10^4 \quad Kft$

$Mp9 := P9 \cdot (e9 + (0.036 \cdot 86)) \quad Mp9 = 2.824 \cdot 10^4 \quad Kft$

*Note: The mast should be designed for the maximum shear, maximum axial, and maximum moment that will be present only during construction; Maximum moment induced is when the last pre-stressing cable (cable 1) is installed. After the construction, the stresses induced by the prestressing will be counter-balanced by the dead and live load of the deck.

Axial forces at the base of the cantilever due to prestressing and the loading

$F_{b1} := P1 - F_1 \qquad F_{b1} = 1.811 \cdot 10^4 \quad K$

$F_{b2} := P2 - F_2 \qquad F_{b2} = 1.552 \cdot 10^4 \quad K$

$F_{b3} := P3 - F_3 \qquad F_{b3} = 1.311 \cdot 10^4 \quad K$

$F_{b4} := P4 - F_4 \qquad F_{b4} = 1.088 \cdot 10^4 \quad K$

$F_{b5} := P5 - F_5 \qquad F_{b5} = 8.821 \cdot 10^3 \quad K$

$F_{b6} := P6 - F_6 \qquad F_{b6} = 6.94 \cdot 10^3 \quad K$

$F_{b7} := P7 - F_7 \qquad F_{b7} = 5.231 \cdot 10^3 \quad K$

$F_{b8} := P8 - F_8 \qquad F_{b8} = 3.706 \cdot 10^3 \quad K$

$F_{b9} := P9 - F_9 \qquad F_{b9} = 2.594 \cdot 10^3 \quad K$

$$F_{btot} := F_{b1} + F_{b2} + F_{b3} + F_{b4} + F_{b5} + F_{b6} + F_{b7} + F_{b8} + F_{b9}$$

$$F_{btot} = 8.49 \cdot 10^4 \quad K$$

try 15' W x 25' D x 6" thk tapered box girder

$A := (15 \cdot 12 \cdot 25 \cdot 12) - (14 \cdot 12 \cdot 24 \cdot 12) \qquad A = 5.616 \cdot 10^3 \quad in^2$

$$I_{xx} := \frac{\left[(15 \cdot 12) \cdot (25 \cdot 12)^3 - (14 \cdot 12) \cdot (24 \cdot 12)^3\right]}{12} \qquad I_{xx} = 7.057 \cdot 10^7 \quad in^4$$

*Note: Cantilever is overdesigned to account for the deflection criteria for the deck. The full span of the deck must also meet the criteria of L/1000 = 750'/1000 = 9 in.

Self-weight

$((20 \cdot 15 \cdot 140) - (14 \cdot 19 \cdot 140)) \cdot 0.49 = 2.332 \cdot 10^3 \quad K$

$Moment_{sw} := 2332 \cdot 70 \qquad Moment_{sw} = 1.632 \cdot 10^5 \quad Kft$

Total stresses

$$\delta_{axial} := \frac{F_{btot}}{A} \qquad \delta_{axial} = 15.118 \quad ksi$$

$$\delta_{moment} := \left[(Mp1 + Moment_{sw}) \cdot \frac{(12.5 \cdot 12)}{I_{xx}} \right] \qquad \delta_{moment} = 0.866 \quad ksi$$

$$\delta_{total} := \delta_{axial} + \delta_{moment} \qquad \delta_{total} = 15.984 \quad ksi \quad << \quad 0.5 \cdot 36 \quad ksi$$

Checking for the worst case condition deflection (i.e. cantilever w/o back-span)

$$V_{tot} := V_1 + V_2 + V_3 + V_4 + V_5 + V_6 + V_7 + V_8 + V_9 \qquad V_{tot} = 9.533 \cdot 10^3 \quad K$$

Cantilever deflection,

$$\upsilon := V_{tot} \cdot \frac{(110 \cdot 12)^2 \cdot (3 \cdot 140 \cdot 12 - 110 \cdot 12)}{6 \cdot 29000 \cdot I_{xx}} \qquad \upsilon = 5.032 \quad in$$

$$\upsilon_{sw} := (2332) \cdot (70 \cdot 12)^2 \cdot \frac{(3 \cdot 140 \cdot 12 - 70 \cdot 12)}{6 \cdot 29000 \cdot I_{xx}} \qquad \upsilon_{sw} = 0.563 \quad in$$

$$\upsilon_{tot} := \upsilon + \upsilon_{sw} \qquad \upsilon_{tot} = 5.595 \quad in$$

Checking for L/300 for cantilever deflection criteria (AASHTO)

$$\frac{L}{300} = 140 \cdot \frac{12}{300} = 5.6 \quad in \quad = \quad 5.595 \quad in \qquad ok$$

Reference

1. Canadian Precast/Prestressed Concrete Institute. "Outstanding Engineering Achievements: Prestressed Concrete in Canada." History. http://www.cpci.ca/history.html (6 May 2001).

2. Leslie E. Robertson Associates. "Project Gallery." Project Gallery. Domino's Headquarters. http://www.lera.com/projects.html. (2 May 2001)

3. Leslie E. Robertson Associates. "Project Gallery." Project Gallery. Puerta de Europa. http://www.lera.com/projects.html. (2 May 2001)

4. Manse Corea. "Method of Manufacturing Preflex Beams." Descriptions and Drawings. http://www.mansecorea.com/us2/us2.htm. (2 May 2001)

5. National Information Service for Earthquake Engineering. "Golden Collection: Structural Engineering Slide Library." Index by Country, City, Structure. 1998. http://www.anglesey-history.co.uk/places/bridges/brittube.html. (2 May 2001)

6. Post Tension & Steel, Webmaster. "Technical Information about Post Tension Cables." Information Page. 2001. http://www.posttension-steel.com/strand_info.htm. (15 May 2001)

7. Ryall, M. J. "Britannia Bridge: from concept to construction." May/August 1999. http://www.icenet.org.uk/club/ce/ce320209.html. (4 May 2001).

8. Sevilla5.com. "The Seville City Guide." Alamillo Bridge. http://www.sevilla5.com/monuments/alamillo.html. (2 May 2001)

9. Structures And Materials. "SAM's Cases by Building Name." Steel Outrigger Truss System. http://www.arch.su.edu.au/kcdc/caut/html/GPT/outrig.htm. (3 May 2001)

10. American Association of State Highway and Transportation Officials. Standard Specifications for Highway Bridges. 14th ed. Washington, D.C.: AASHTO, 1989

11. Hurst, M. K. Prestressed Concrete Design. 2nd ed. London: E & FN Spon, an Imprint of Routledge, 1998

12. Raths, Raths and Johnson, Inc., ed. PCI Design Handbook: Precast and Prestressed Concrete. 5th ed. Chicago: Precast/Prestressed Concrete Institute, 1999

13. Robertson, Leslie E. On the Design of Leaning High-Rise Buildings. Journal of Constructional Steel Research, Vol. 17 (1990): 163-191.

14. Troitsky, M. S. Prestressed Steel Bridges: Theory and Design. New York: Van Nostrand Reinhold, 1990

15. Klaiber, F.W., Wipf, and Fanous. Strengthening of Two Continuous-span Steel Stringer Bridges. Construction and Building Materials, Vol. 12 (1998): 151-159.

16. Viteri, Victor. 1.571: Structural Analysis and Control. Notes on Non-linear Analysis of Cables. Spring 2001. Massachusetts Institute of Technology, Department of Civil and Environmental Engineering.