An Alternative Method of Long Lead-Time Tool Purchases

By

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Submitted to the Sloan School of Management and the Department of Mechanical Engineering in partial fulfillment of the requirements for the degrees of

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ABSTRACT

The intent of the thesis is to examine the problem of a supplier with a long-lead time product and a buyer that wants the product as soon as possible. The products examined are build-to-order in a rapidly changing industry, so the reduction of the difference between the two times is important. This paper attempts to lay out two primary ways of thinking about this problem, both of which involve real options thinking. The first is the construction of a management decision tree in which there is an upfront payment that is determined by the discounting of the future cash flows based on some discrete distribution of the outcomes. The second is through the use of the Black-Scholes equation most commonly used in pricing financial options.

The primary motivation for the project is to try and reduce the lead-time that a buyer faces when a company is purchasing a long lead-time product. The microprocessor industry is moving quickly with chip speeds doubling every year to year and a half. This means that it is difficult to forecast the number of tools that will be necessary if the tool has a lead-time that is as long as the actual product cycle. By reducing the lead-time, it may be possible to better time the delivery of the tools to capture the demand as it appears rather than having the tool arrive late, which means missing revenue, or early, which means having unused capacity on hand.

While this thesis will not answer all of the questions regarding options theory as applied to long lead-time tools, the hope is that this work can be used to lay the future for a more intense study of the subject. The process of thinking about these projects as options, having the right at a future point in time to continue with the project or to cancel the project, is important. This line of thinking has the potential to add an immense amount of value to corporate decisions and should not be neglected.

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1 INTRODUCTION AND OVERVIEW

1.1 INTRODUCTION

The purpose of any firm is to create value for the stakeholders and the decisions made by management in any entity should be made in recognition of that. Business managers are now faced with need to make decisions at a more rapid pace than at any time in the past. These decisions have consequences that can ripple through the supply chain with surprising speed and ferocity. The semiconductor industry is, in particular, faced with the need to make quick decisions due to shortened product life-cycles, increasing competition, and hazy demand forecasts.

This project was conducted at the Intel Corporation in Chandler, Arizona with the purpose of examining ways to reduce the lead-time of tools used in the semiconductor manufacturing process. However, the methods discussed in this paper can be used for a variety of purchased products. One can simply change the time frame and the values that were utilized if other products are being examined.

Intel has had the ability to make exceptional profits for most of its history for two reasons. The first reason is the amount of research and development that the company conducts. The company has always felt it was important to invest in the next generation of products. In fact, Gordon Moore predicted that the processing power of semiconductors would double approximately every 18 months. This law, for the most part, has held true in the industry. For each generation of chip that Intel produced, a significant portion of the profits were reinvested in developing the next generation of
chips. This allowed Intel to introduce next generation processors faster than any other company on the market, allowing market domination and high profit levels.

The second reason is the excellence of their manufacturing function. Intel is able to quickly ramp-up production of a new project and bring the price of the product down to a reasonable level. The demand for their products usually skyrockets soon after introduction while, at the same time, the company is faced with a product life cycle that lasts no more than 18 months. For this reason, the company needs a massive amount of capacity to quickly produce their product and meet their demand. They copy their manufacturing exactly from one site to the next to help make sure that they have the necessary capacity in place.

Through this combination of large investments in research and manufacturing, the company has come to dominate the semiconductor market. However, this domination has led the company to be lax in a number of important areas. The most important to this project is their supply chain design. Up until the mid 1990’s, Intel was able to tell each user of their product exactly how much of their demand they would actually receive. For example, Intel may have been able to produce one million chips in a given time period, while demand was 1.2 Million chips. Intel could tell the computer manufacturers that they would only be getting 83% of their requested demand and the computer manufacturers would have to live within this limit. This allowed Intel to concentrate on manufacturing while leaving their supply chain to be examined later because it just did not matter. If production was delayed for some reason, the product delivery was pushed out and there were no negative repercussions to Intel.
In the mid-1990's, greater competition for Intel started to emerge. All of a sudden there were other companies entering the market (AMD, 3rd party manufacturers, etc.) that were able to compete with Intel. No longer could Intel tell a manufacturer that they would only receive 83% of their demand. The manufacturers now had other suppliers that they purchase chips from if Intel could not meet the necessary demand. Intel's supply chain became significantly more important as their competitors started to eat away at their market share. It was no longer a non-issue if a tool was not delivered when promised. It meant that Intel had missed out on a significant portion of demand, revenue, and profit. More importantly, Intel could begin to lose customers, as they could not meet supply. Intel's competitors were able to take a small portion of demand, reinvest the profit, and slowly build their capacity until there was viable competition to Intel’s domination of the market.

1.2 Overview

This thesis will begin with an overview and background of the problem in Section 2. Section 3 introduces real options and the current state of real options literature and applications. Section 4 will show a management decision tree model that was created and the variables and formulas in that tree. Section 5 will discuss the results of a number of these scenarios including the shortcomings of the method. In addition, several other potential areas of option pricing are examined. This section will discuss when it makes sense, and when it does not, to examine alternative purchasing arrangements. Section 6 is a conclusion with some potential future areas of research. Section 7 includes an annotated bibliography of papers that the reader may find of interest.
2 Problem Description

2.1 Background

This project was defined as looking at ways to decrease the lead-time on long lead-time equipment purchases. The ultimate goal of the project was to examine ways in which we could decrease time between placing the order for a photolithography tool and receiving the tool. A photolithography tool has amongst the longest lead-time of any tool in a fabrication facility, with a time of nearly 18 months. It is important to note that a new generation of photolithography equipment is necessary for each new generation of processor. Given that the typical life-cycle for a generation of semi-conductor chips is 18-months, it is apparent that as soon as a new chip is introduced, orders must be placed for the next generation of equipment to produce the next generation of chips. Figure 1 shows the problem that was being examined.

Figure 1. Time Flow

The supplier wants the order placed as early as possible for a number of reasons. They need to plan for their headcount, their material flows, and their production space.
The sooner a firm order is placed, the sooner the supplier can begin to plan for the manufacturing. At the other end, the buyer of the tool wants to place the order as late as possible. The primary reason for this is because as time goes forward, the buyer has better visibility as to what the realized demand will be. If the order is placed 18-months before the demand will be realized, it is difficult to forecast what the actual demand will be. If the tool can be ordered nine-months before the demand is realized, the demand visibility is not as cloudy. The solution has been to order more tools than are actually necessary. These tools cost nearly $10 Million each, so buying excess tools can quickly eat into a company's profit. Figure 2 gives a second view of the problem that was being examined. The x-axis represents time in Figure 2 while the y-direction is representing demand. The demand can be assumed to follow a normal distribution in which demand ramps-up, reaches a peak, and ramps-down. Each of the early tools (tool 1, 2, …, n) are tools that will be used throughout the production of the semiconductor generation. The tool capacity that is necessary at the lower level of demand is easy to forecast as this capacity will almost certainly need to be utilized. However, the tools that will be used as demand reaches a peak are much more difficult to forecast. These tools will have a limited life in that they will be producing product for a short period of time. The tools are brought on-line as the production begins to peak and can be called spike demand tools. Since these tools are used for such a short period of time, there overall value is reduced as compared to other tools as not as much product will flow through these spike tools. However, to meet demand and ensure profits, they are still necessary.
The demand is not known but an attempt can be made to forecast the demand. Demand is assumed to follow a somewhat normal distribution, so tools 1 through n can be ordered to cover some certain percentage of the demand that we is sure to be realized in extreme downturns. For example, a company may be 100% confident that 50 Million chips in a particular generation will be sold. Therefore, enough tools can be ordered 18-months from the needed date to produce those 50 Million chips. However, the company may think that demand can range up to 80 Million chips even though the expected demand is 70 Million chips. So the expected possible demand on the upside is 80 Million chips, with some uncertainty, while the lowest demand forecast possible is 50 Million chips. The questions is how can the company minimize their exposure to paying for tools that they might not be able to use. That was the central point of the project.

The expectations for the internship were three-fold. The first was to examine ways in which purchasing agreements could be constructed to minimize the overexposure of Intel to excess tool purchases. Different methods of doing this were examined and narrowed down. The second expectation was to examine the method chosen with regards
to what had already been done in the real world. If the project had already been completed then there was no reason to reinvent the wheel. The third purpose of the project was to attempt to build a model that could eventually be used in Intel to assist with purchasing. It was known going into the internship that there was not enough time to complete a fully functioning model that could be used by Intel. However, the purpose was to start a model that could be handed over to Intel for eventual integration in purchasing.
3 WHAT ARE OPTIONS?

3.1 WHAT IS AN OPTION

An option is the right, but not the obligation, to take some action in future. An option is valuable when there is uncertainty in the final distribution of the value of the asset. As uncertainty increases, the value of an option increases. The central tenant of options theory is that there is a probability distribution for the final price of an asset. There is an expected value but there is a chance that the value will be higher and there is a chance that the value will be lower. This distribution is characterized by the expected return and the variance of those returns.

There are two basic types of options. The first is a call option. This gives the holder of the option the right to buy the underlying asset by a specified date at a specified price. The specified date of the contract is known as the time-to-maturity and the specified price is called the strike price. An American call option gives the owner the right to exercise the option at any time up to and including the expiration date. A European call option allows the owner to exercise the option only at the date of expiration. In the case of a call option, the owner believes that the value of the underlying asset will increase with time. Upon exercise, the owner can buy the asset at the lower price and sell it at the higher price. Figure 3 shows the payoff on a typical call option.
Figure 3. Call Option Payoff

A put option gives the owner the right to sell the asset by a specified time at a specified price. In this case, the owner believes that the price of the asset will decrease. This will give the owner the right to buy the asset at the lower price and sell it at the higher price. Figure 4 shows the payoff for a typical put option.

Figure 4. Put Option Payoff

Notice that in the payoff for both the call and the put options, there is a portion of the payoff that has a negative value. This is because the purchaser of an option is required to pay a set price to purchase the option. This is the option purchase price. As Figure 3
shows, before the asset hits the dictated strike price there is a loss of the amount of the premium. If the premium for a call option is $3 with a strike price of $10 and the asset never crosses the $10 value, the holder of the option has lost the $3. When the strike price is hit, the holder begins to make money with the increase in price. When the asset is worth the strike price plus the premium the holder makes money. This is where the line begins to cross the x-axis in Figure 3. Similarly, the owner of a put option will not make money until the price of the asset has dropped below the total strike price less the purchase price of the option.

An investor in an option can take advantage of the upside of a transaction while avoiding the downside. The payoff is truncated in that the downside of the asset movement is cut off while the upside is not. Due to this truncation, it was difficult for people to determine what the proper price to charge for an option. However, academics developed a means of duplicating the cash flows from an option investment by building a basket of publicly traded financial instruments. When this basket can be built, the option price can be determined exactly. This led to three primary means of valuing an option. They are the Black-Scholes method, the binary tree method, and through Monte Carlo simulation. While there are a number of alternative methods, these are the three most used in practice.

3.2 The Binary Tree

Consider a stock with an initial value of $S_0$ and a single option on that stock with a current price of $V$ (Hull, 2000: The reader is encouraged to examine this reference for a more detailed description of binary tree option pricing). The option has a time to maturity of $T$ and the option can either move up to a price of $S_0 \mu$ or down to a price of
$S_0d$ where $u$ is greater than 1 and $d$ is less than 1. The proportional increase when there is an up movement in the stock is $u-1$ and the proportional decrease when there is a down movement in the stock is $1-d$. If the stock moves to the up position then the option has a value of $V_u$ and if the stock moves to the down position the option has a value of $V_d$. This situation is shown in the following Figure 5.

\[ S_u \quad \text{or} \quad S_v \]
\[ S_0 \quad V \]
\[ S_d \quad V_d \]

Figure 5. Stock and Option Prices in a Binary Tree

A fairly simple assumption can be made in pricing the option that only relies on the principle of no arbitrage, or the law of one price. A portfolio of the stock and option can be set up in such a way that there is no uncertainty about the value of the portfolio at the end of the three months. Since the value of such a portfolio is known, there is no risk and it must earn the risk-free rate of interest.

Imagine a portfolio that consists of $\Delta$ shares of the asset and that is short one option. The value of $\Delta$ that makes the portfolio risk-free can then be calculated. If the asset moves up in value, the value of the portfolio at the end of the life of the option will be $S_0u\Delta - V_u$. If there is a down movement in the stock then the value becomes $S_0d\Delta - V_d$. The two are equal when
\[ S_0u\Delta - V_u = S_0d\Delta - V_d \]

Or:

\[ \Delta = \frac{V_u - V_d}{S_0u - S_0d} \]

If the risk free rate of interest is \( r \), then the present value of the portfolio is:

\[ (S_0u\Delta - V_u)e^{-rT} \]

Note that the exponent is used to recognize that the interest is continually compounded. It then follows that

\[ (S_0u\Delta - V_u)e^{-rT} = S_0\Delta - V \]

This last statement is saying that the cost to set-up the portfolio is equal to the present value given the law of one price. So:

\[ V = S_0\Delta - (S_0u\Delta - V_u)e^{-rT} \]

Substituting the equation for delta and simplifying the equation yields:

\[ V = e^{-rT}[pV_u + (1-p)V_d] \]

where:

\[ p = \frac{e^{rT} - d}{u - d} \]

These last two equations allow an option to be priced using a one step binomial model. This means looking out one period of time in the future and calculating the value of the option at time zero. For example, consider a stock currently priced at $20. The stock will be either $22 or $18 in three months. We would like to value a European call option that expires in three months with a strike price of $21. If the stock ends up worth $22, the option will have a value of $1. If the stock ends up at $18, then the option will be worthless.
The value for $u$ is 1.1 as $22/20 = 1.1$. The value for $d$ is .9 as $18/20 = .9$. If the risk-free interest rate is 12% per year, then the value for $p$ and $f$ is

$$p = \frac{e^{12* .25} - .9}{1.1 - .9} = .6523$$

$$V = e^{-12* .25} [.6523 \times I + (1 - .6523)O] = .633$$

In the absence of arbitrage opportunities, the current value of the option is $.633.

Although binomial trees are used in practice, using a one-step binomial tree is an extremely simplistic approach. The life of a stock option is usually broken into $n$ time steps of $\Delta t$ which leads to $2^n$ possible stock paths. So if an option expires in 30 days and each day is considered a time step, there are $2^{30}$ possible final stock values. Typically, the up and down movements are chosen to represent the volatility of the stock. That is done by setting:

$$u = e^{\sigma \sqrt{t}} \text{ and } d = e^{-\sigma \sqrt{t}}$$

where $\sigma$ is the volatility of the asset and $t$ is the time to expiration, or the time-frame being examined. These values can be used instead of the set end values of the stock. If one knows the upward and downward movements of the stock then the value for the volatility (sigma) can be estimated. This relationship will be used later in the paper.

3.3 **The Black-Scholes Method**

A second method of option valuation is the Black-Scholes Method. The equation that was derived is a closed form solution for a European option. This is a continuous time domain as compared to binary trees that work in discrete time. For the binary trees, a value has to be assigned at the end of each branch of the tree and then the preceding value of the option is calculated. Imagine breaking the time down into infinitely small
buckets and one can see how this would collapse into a differential equation. There are a number of assumptions used in this equation which are detailed in the bibliography (Black, 1973).

Various equations have been derived that work around one or more of these constraints. For example, there is an equation for valuing stock options that allows for dividends to be paid on stocks, as long as they are predictable. It is also possible to have $\sigma$ as a function of $t$ or interest rates can be allowed to be stochastic. However, there are no equations that can work around all of the assumptions at once. Again, in the bibliography, there are sources listed that detail a work around for each assumption (Black, 1989).

The equation for pricing a European call option is shown below.

$$C_o = S_o N(d_1) - X e^{-r t} N(d_2)$$

where:

$$d_1 = \frac{\ln \left( \frac{S_0}{X} \right) + (r + \sigma^2/2) t}{\sigma \sqrt{t}}$$

and:

$$d_2 = d_1 - \sigma \sqrt{t}$$

The variables are:

- $S_0$ = The current price of the stock
- $X$ = The exercise price of the stock
- $r$ = The continuously compounded risk-free interest rate per annum
- $\sigma$ = Measure of uncertainty of the returns (volatility = standard deviation of the stock price)
- $t$ = Time to expiration of the option
N( ) = Cumulative probability distribution function for a standard normal variable

An excellent way to think about the breakdown of the overall formula is:

\( S_0 N(d_1) = \) The expected value of \( S_0 \) if \( S_0 \) is greater than \( X \) at expiration. The expectations are taken using risk-neutral probabilities.

\( X e^{-r} = \) Present Value of the cost of the investment.

\( N(d_2) = \) The risk-neutral probability of \( S_0 \) being greater than \( X \) at expiration.

These values can all transfer to the domain of real options. \( S_0 \) becomes the present value of all of the future cashflows that are expected from an investment opportunity. \( X \) is the present value of the cost associated with the investment opportunity. The volatility is the standard deviation of the growth rate of the future cashflows associated with the asset. The time to expiration becomes the period for which the investment opportunity is valid. This can be the length of a lease or a product life cycle. The risk-free rate of interest remains the same.

### 3.4 Real Options

Real options are a means of looking at decisions, strategic and financial, that take into account ideas that a normal valuation or decision-making process would not. In particular, real options recognize that options are contingent decisions. They give a person the opportunity to make a decision after the events have unfolded. If events have turned out well on the decision day, then one decision can be made. If they have not, then another decision is made. This process allows one to make an attempt to reduce the exposure to the downside of a situation while increasing the payoff from the upside.
Aristotle (Copeland, 1998) writes of how Thales the Melesian, a philosopher, divined from tea leaves that in six-months there would be an excellent olive harvest. Thales spoke with the owners of the olive presses and told them that he would give them money today to be given the right to use the presses in six-months time. In six months there was a bumper harvest and the olive growers were in need of all available capacity to press their harvest. Thales rented the presses to the olive growers at a high rate, paid the predetermined rate to the press owners, and kept the rest. He had paid a small fee up front for the right, but not the obligation, to use the olive presses in the future. This is an example of a real option.

In options theory, the predetermined rate that is paid upon exercise is called the exercise price. When the exercise price is lower than the market price, the option is called “in the money”. When the exercise price is higher than the market price, the option is called “out of the money. When the olive harvest came about, Thales had an exercise price of the normal cost of using the press while the market price was higher. So his option was in the money.

The value of the option increases with an increase in uncertainty. Imagine the situation where the size of the harvest is known exactly six-months in advance. The owners of the press are then able to plan their capacity precisely meaning that the price charged for the use of the presses will also be normal. If this occurs, then the option that is held by Thales is worthless. However, as the uncertainty increases, the chance that the option will finish somewhere other than the expected value increases. Therefore, the value of the option increases. The source of uncertainty in this story is the size of the olive harvest. This affected the value of the olive presses. As the size of the harvest
increased, there was less excess capacity so the value of press time increased. As the value of the underlying asset increases, so does the option value.

In addition, the time to expiration of the option also affects the value. If Thales had signed a contract with the press owners one-day before the expiration of the option then there would have been very little chance that the harvest would change by a great amount over that one day. However, if the contract had been signed one-year before the harvest, the value would have been larger. This is because the uncertainty has a greater effect as time increases. As the time increases, the chance that the value of the underlying asset will move up increases, driving an increase in the value of the option.

With this example, the major sources of value in an option are shown. They are the value of the underlying asset, the uncertainty in the option, the exercise price of the option, and the time to expiration of the option. The final variable is the risk-free interest rate. As the interest rate increases, the value of the option decreases because the future cash flows have to be discounted at a higher rate.

Real options are often difficult to recognize when managers are making decisions. Consider the example of life insurance companies in the 1960s that were trying to sign people up for whole life policies. A feature that was often included in the policies was the right of the policy holder to borrow money against the cash value of the policy at a certain fixed interest rate. The interest rates at the time were low, 3% or 4%, so the companies did not think that this feature was important. However, in the late 1970’s and early 1980’s, interest rates jumped in the double digits. All of a sudden the policy holders were able to borrow against their policies at 8% and the insurance companies had to borrow at a much higher rate to finance their policy holders loans. The life insurance
companies were giving away a call option on borrowing money and not charging a premium. While the life insurance companies almost certainly prices these policies with a normal discounted cash flow analysis, the companies did not take into account the volatility of interest rates.

Real options involve uncertainty about two separate items. The first is the future and the other is the ability of management to respond to what it learns as that future becomes clearer. That is the primary way to think about real options. Figure 6 shows an asset or project with a present value of $1M.

![Figure 6. Range of Future Values](Image)

As shown in Figures 6 and 7 (Amram, 1998), that project has some future range of values, although it is not known with certainty. The distribution is typically skewed so that the range of positive outcomes is larger than the range of negative outcomes as we expect, on the average, that companies will increase value over time instead of destroy value.
Figure 7. Range of Future Values and the Distribution

A company needs to take that uncertainty and try to take advantage of it. In other words, increase the range of positive outcomes, while trying to reduce the possibility of negative outcomes. In options terminology, purchase a call to take advantage of movement up and purchase a put to protect against any movement down. Figure 8 (Amram, 1998) attempts to show this rotation through the company's assets.

Figure 8. Uncertainty and Value
Options are most valuable when there is a large amount of uncertainty in the future and when management has the flexibility to take advantage of the uncertainty. When it is likely that more information about the demand will be received over time, the value increases. This is shown in Figure 9 (Copeland, 1998).

<table>
<thead>
<tr>
<th>Uncertainty</th>
</tr>
</thead>
<tbody>
<tr>
<td>Likelihood of receiving new information</td>
</tr>
<tr>
<td>Low</td>
</tr>
<tr>
<td>Room for managerial flexibility</td>
</tr>
<tr>
<td>Ability to Respond</td>
</tr>
</tbody>
</table>

**Flexibility value is greatest when:**

1. There is high uncertainty about the future and it is likely that new information will be received over time.
2. There is room for managerial flexibility which gives management the chance to respond.
3. The Net Present Value without considering flexibility is near zero, meaning that the project is not obviously good or obviously bad so management might want to change direction while the project is in work.

Figure 9. The Value of Uncertainty and Flexibility

It is important to recognize the difference between an option and a bet. Imagine that a company is building a factory for $1M dollars that must be paid entirely up front. If the company does not know what the final demand will be, then this is a bet. The company must either invest the money or not invest the money. Now imagine that this company can go ahead with a $100,000 pilot project and then spend $1.1M in one-years time. They are using the pilot project to see if all will go according to plan or not. If it does, the company can spend the $1.1M to build the full-scale project. If not, for a cost of
$100,000, the company has saved itself $1M in present value terms (assuming a 10% interest rate). The company has given itself the option to expand. If the project is successful, then the company will have spent $100,000 more than they otherwise would have. However, some managers would think it well worthwhile to spend a little bit more money to garner the information that will help adjust the probability of outcomes favorably.

3.5 Types of Real Options

There are a variety of types of real options but, for the most part, they can be classified as one of the following six types of options.

3.5.1 The Option to Grow

The first type of option is the option to grow. The option to grow usually has the greatest potential value in investment decisions. When a company invests in its infrastructure, its people, or its technological base it is buying a growth option. The company now has the ability to make future choices that it was not previously able to make. Research and Development projects are an example of purchasing an option to grow. If the projects are successful then the company has the potential to open a whole new market. If a company invests small amounts of money in 100 different R&D projects to see which projects have early signs of success, they can then exercise their options on the successful pilots.
3.5.2 The Option to Expand

Closely tied to the growth option is the option to expand. In a manufacturing facility, this option might take the form of purchasing a machine that has a negative net present value but sets the stage for future expansion. For example, a firm may purchase a new machine that generates $300,000 of revenue per year but costs $400,000 per year to operate. If they do not purchase the machine now then they are not allowed to purchase the machine in the future. However, in another year a new type of material may, or may not, be developed which will cause this machine to generate $500,000 of revenue per year instead of $300,000. The paper company has purchased a call option that will allow them to expand into this new product. It is important to note that the option for growth and the option to expand are similar in nature.

3.5.3 The Option to Abandon

When a project commences, the company still may have the ability to cancel a project and receive an inflow of cash in exchange. This can be thought of as the salvage value of a project. For example, a real estate company may begin construction of an office building at a cost of $100 Million. The real estate company has a contract from a client to rent the building for $120 Million (in present value terms) over the life of the building. At some future date, during construction, that client has to back out of the agreement for some reason. The real estate company is able to find a new renter but they are only willing to pay $90 Million over the life of the building. If the company can terminate the construction project and sell the partially completed building to another company with a loss of less than $10 Million, the real estate company should do it. This is considered an option to abandon.
This option is equivalent to an American put option. The company can exercise this option whenever they want, which means stopping the construction, and the option allows them to protect against downside risk.

3.5.4 The Option to Contract

Closely tied to the option to abandon is the option to contract. This allows a company to change the scope of the project while the project is in process. For example, a manufacturing firm is building a new factory that has several financial milestones. The company is planning a factory that can produce 10,000 boats and it will cost $5 Million per year over the next five years to construct the factory. If in year two the company decides that they want to scale down the factory so that it only produces 5,000 boats, they will pay only $3 Million. The company has an option to contract because they can decrease the scale of their project. As with the option to abandon, the company has a put option to protect against a sudden decrease in demand.

Like the option to grow and the option to expand, the option to abandon and the option to contract are very similar. Again, they are on the same continuum but at different extremes. The option to abandon is, usually, considerably more valuable than the option to contract because the scope is typically much larger.

3.5.5 The Option to Switch

A fifth type of option is known as the option to switch. This option usually refers to the ability to accommodate different inputs or different outputs. For example, a manufacturing company is deciding whether to install new furnaces for a new production line that will run off of natural gas, off of electricity, or off of both. The natural gas
furnaces cost $1 per furnace, the electrical furnaces are $1.25 each, and the dual furnaces are $1.5 each. Each type of furnace has the same capacity and produces 1 unit per $ of energy input. Currently, the cost to run the electrical furnace is half the cost to run the natural gas furnace.

In the current state, the company will choose to install the electrical furnaces because the cost of using the furnace is less than the other two furnaces. However, the inputs to these furnaces are volatile and it is possible that in the future the cost of electricity will skyrocket while the cost of natural gas stays the same. If this happens, the company will not be able to produce products at a reasonable price. If the company purchases the furnaces that run off of both inputs, they can switch back and forth depending on which input is less expensive. This is an example of the option to switch. Typically, the less correlated the inputs, or outputs depending on the option, the more valuable the option will be.

3.5.6 The Option to Wait

Every investment decision involves an option to wait. Common sense says that the longer a company or an individual waits to make a decision, the more information there will be available to make that decision with. The emergence of information over time effects an investment and the decision that will be made. There is, however, a trade-off to the waiting in most situations. The company runs the risk of missing out on some of the return from the investment by not investing up front. This is the case when companies do not have monopoly power and face competition.

There are situations where this option has value. For example, a orchard may own land that produces apples on the outskirts of a city that has a quickly developing
technology industry. The company can develop this land for $2 Million and sell it for $2.2 Million. The project currently has a positive net present value but, at the same time, the company recognizes that the real estate market is volatile and that the technology industry is volatile. Even though the net present value is positive, the company may calculate that there is addition value added by waiting a year to see if the real estate and technology industries are going to hold up and continue to grow. The company has a valuable option to wait because the information that they will gather over the next year may be worth more than the present value of the project.

A second example might take the form of a sleeping patent. A sleeping patent is a patent on a product or a process that a company holds but is not currently using. For example, an electronics processor may hold a sleeping patent on a new type of digital recording device that it is not producing. The company is not sure how large the market is for the product and is not willing to invest in the capacity to produce the product without knowing. The company can wait for competitors to introduce similar products that define the market size. The company can then begin production of its superior product and steal market share from its competitors. The company gained value by being able to wait to begin production of its product.

3.6 **When are Real Options Valuable**

Real Options thinking is not always a necessity. There are some decisions that require very little in the way of financial analysis and are certain to generate a profit no matter the outcome. In the oil industry, if there is excess demand of two-million barrels per day at $40/barrel (with the price expected to be constant for a certain time period) and we can produce oil at $10/barrel, it is obviously advantageous to start producing extra oil.
On the other hand, there are times when options are not necessary because there is no value added from a certain decision. An example of this would be where the price of oil is $10/barrel and we can produce it at $40/barrel. The decision to produce would be incorrect.

Thinking about options is most advantageous when these decisions are in a gray area. The areas where a more traditional net present value decision is hovering around $0 or the times when there are strategic reasons for making an investment. Traditional tools work very well when there are no options to consider in a decision or where there is little uncertainty within a decision. An excellent example of this is the company that consistently generates a specified cash flow that is declining over the years. This company will not have any follow on opportunities and is more than likely in the declining phase of the company or product life-cycle. A company that supplies a certain piece of equipment for a product that is being phased out might be an example of this. This type of company is known as a “cash-cow”.

So real options are needed in situations where there is enough uncertainty that is sensible to wait for more information. The goal is to avoid the regret of making irreversible investments that could harm the value of the company. One can imagine a situation where demand forecasting is less precise the further in time it is from the time that the demand will come due.
As time moves forward, the accuracy of the demand forecast should be increased. This means that there is some value in waiting for more information to appear so that the forecast is more precise if the risk of losing market share is not large.

3.7 **PROJECT FIT**

The purchase of long-lead time tools was examined through use of a decision tree model. Decision tree models and real options are similar in nature but there are several key differences. Decision tree analysis involves building a tree that represents all possible decisions and how management can and should respond to various outcomes. The valuation of a decision tree is completed by calculating the expected future cash flows based on some objective probability and discounting those cash flows at the weighted average cost of capital.

Option valuation is different in that it calculates values based on the law of one price. In other words, an entity should not be able to profit by shorting one bundle of assets that are trading and buying another bundle of assets if those assets have the same price. Two
different assets that produce the same cash flows must be worth exactly the same amount or people will be able to profit with no risk.

The option valuation approach can be thought of as modifying the discount rate in a decision tree to reflect the actual riskiness of the cash flows. A call option, for example, is the same as a leveraged position in the asset. If a stock has a price of $50 and a call option has an exercise price of $45, the option should have a value of approximately $5. If the asset goes down by $1, the value has decreased by 2%. However, the value of the option has gone from $5 to $4 and had decreased by 20%. So the option is riskier than the stock. So in valuing an option, the discount rate must be higher than the weighted average cost of capital to reflect the additional risk. In addition, this discount rate can change throughout the decision tree depending on if the option is in or out of the money.

Traditionally, the law of one price can be satisfied in real options models by examining the market. For example, in the case of an oil company pricing an option on developing an oil field, the company can look at the futures market for oil prices and use that as a proxy for their volatility. The company can then build a basket of publicly traded securities that matches the cash flows of the oil field development and compare the prices. If the price of the oil development field is higher than the basket of market securities the company can short sell the project, buy the market basket, and buy back the project at termination. The cash flows would have been the same so the company would have made arbitrage profits.

There are two problems with trying to price real options in the semiconductor industry. The first is the amount of correlation that is present in the industry. If one tool buyer purchases an option on a tool that is used to produce a specific microprocessor and
demand on that microprocessor quickly drops there is no other party that will want to purchase this tool. The tools are specific to the processes on the microprocessors and if one company is not going to need the excess capacity, it is extremely unlikely that a second company will want to purchase the option. In the financial markets, parties are purchasing options for a variety of reasons such as to hedge a portfolio from decreasing beyond a specified limit or to lock in currency or commodities prices. When oil prices are dropping or oil prices are falling, parties are still going to trade options on oil due to their separate needs. In the case of purchasing tools, nobody is gaining if the demand falls. The options will simply go unexercised. So there is a break down in the sale and purchase of this type of option.

The second reason is that there has not been enough research at the academic level on how to work around the law of one pricing when there is nothing to compare the price of a real option to. At the present time, there is ongoing research at the Massachusetts Institute of Technology trying to find ways around this type of problem. One of the primary thrusts of this research is the examination of whether or not a company’s stock price can be used as a proxy for the volatility in a real options problem that does not have a common forecast and pricing curve from which the volatility can be garnered. If this research is successful then it can be incorporated in the decision tree model to give the proper discount rate. This model would then be a fully functioning real option.
4  DECISION TREE MODEL

The following model was created with Decision Processing Language. A decision tree allows the user to lay out the decisions in a rational process, run a scenario, and calculate the expected value of the decision process. In addition, the optimal decision strategy is displayed based on this expected value. The user can vary the inputs as necessary and can run simulations to see possible results. This section will show the full decision tree model that was constructed and discuss each of the pieces of that model in detail.

4.1  THE FULL MODEL

The full decision tree model is shown in Figure 11. This display is called an influence diagram. With an influence diagram the user can lay out each of the important variables in a model and set-up the flow between each of those variables. In addition, the major decisions of interest can be included in the model as well as any uncertainties that are tied to the model.

The flow of information is determined by the way in which the model is set-up. Constants are the first variables to have their information. The tree then works back from the final value, Overall Profit in this case, and determines which values need to be calculated first. The flow of the data and time is discussed in this section following the description of the overall model.

The full model shown below can be broken up into three separate sections. There is a section that deals with the immediate needs of the capacity. This is the six-month demand forecast that represents the actual expected demand six-months from today.
There is then a section that has a nine-month demand forecast and an eighteen-month demand forecast. These are the actual nine-month and eighteen-month expected demands.
Figure 11. Full Decision Tree
Figure 12 shows the actual time flow for the purchase of a photolithography machine. The tool takes approximately 18-months, from the beginning of the manufacturing process to the delivery of the tool, to be completed. There are primarily two stages in the production of a photolithography machine. The first is the growing and polishing of the crystal that is used in the tool. This is the majority of the lead-time and is where most of the time is used. The second portion of the lead-time is the completion of the tool. This involves the assembly of the remaining components. There is a natural break 12-months into the process. The crystal is not specific to any one buyer and can be used in any tool. The remainder of the tool, however, may be specific to a certain buyer. So given these time constraints, any option would have to be exercised at a time of six-months.

![Diagram of Time Flow](image)

**Figure 12. Critical Times**

The portion of the tree that is in the six-month time frame is shown in Figure 13. This portion of the tree models the decisions and activities based on the forecast for demand six-months from now. Each of the variables in the tree is discussed in Section 4.2. In fact, each of the time frames looks similar with the differences being the time frame that each of the decisions are made in. In the six-month time frame, there are two decisions that are made. The first is the decision to exercise any options that are currently due. The second is, if these options are not exercised, should they be allowed to expire or
should they be delayed for a period. If the option is delayed, it is then active for the next six-month period.

This model would need to be run every three months. In other words, at time zero the user will enter the six-month forecast, the number of options that were exercised in the past, and the number of options that come due in this time period.

Figure 13. The Decision Tree for the Six-Month Time Frame

In addition, the user must enter the data for the nine-month time frame, which represents the decisions and activities based on the nine-month demand forecast. This is shown in Figure 14. The only data that needs to be entered at this time are the forecast for the nine-month time frame along with the various costs and prices.
The user then enters the data for the 18-month time frame, which is shown in Figure 15. Again, this models the decisions and activities made in the current time period in response to the 18-month demand forecast. The user needs to enter a long-term forecast and the number of options on tools that are currently outstanding. For example, if an option is bought in the current time period then that option will be outstanding in the next time period. In fact, an option is outstanding for a total of 12-months before it becomes active.
When the user has entered data for each of the time frames, the model can be run. Three-months later, the updated data will then be inputted and the model is run again.

4.1.1 Real-Time Flow

It is also important to have an understanding of what is happening in real-time before the model is described in detail. Figure 16 shows an 18-month delivery time for the options. The description for each time period is given below.
Figure 16. Real-Time

a) At time = 0 months, a decision is made to purchase options that can be exercised in 12 months. This decision is made based on the demand forecast for 18-months in the future and the amount of capacity that is currently available.

b) At time = 12 months, the 18-month forecast that was generated in step a) can now be updated to provide a new six-month forecast. This six-month forecast should be more accurate given that new information should have appeared over the past year. With this new six-month forecast, decisions are made as to how many of the options purchased in part a) should be exercised, how many should be delayed, and how many should expire.

c) At time = 15 months, the options that were delayed in part b) must either be exercised, delayed again, or left to expire. If the options are exercised then they will be delivered at time = 21 months.

d) At time = 18 months, the tools are delivered if there were options exercised in part b).

The uncertainty in the tool production rate is realized as is the actual demand.

While these steps lay out what is happening in real time, it is important to realize that this is an ongoing process. In other words, at time = 0 above, a decision is made to purchase a certain number of options given the 18-month forecast. Three months later,
there is a new step a). This means that a new 18-month forecast is needed (21 months from the first forecast) to determine the number of options that are necessary, given the options already purchased, to make sure that there is enough capacity to meet demand. This is shown in Figure 17.

Figure 17. Ongoing Real-Time

Figure 17 shows how the model flows in real-time when the model is run over and over again. At time = 0 months the data is put in the model and run. The decision is made as to the number of options that should be purchased. At time = 3 months, the model is rerun with the new 18-month forecast to see how many additional options should be purchased. When the time is 12-months, there are decisions to purchase new options for
the new 18-month forecast, decisions to exercise options that were purchased at time = 0 (based on the original 18-month forecast that is now updated to a six-month forecast), and decisions to delay options that are not exercised. When the time is 18-months, the options that were exercised result in tool delivery and demand realization.

In summary, the model is used to make decisions in a certain time period based on the forecast information that is placed in the model. Then, moving forward in time, the tree is resolved in the next time period based on new information received over the course of the time period. As time moves forward, the decisions are contingent on the decisions that were made in the prior time period. The goal is to make decisions that are optimal given the information at hand and to try and protect against sudden downturns in demand.

4.1.2 Data Flow

All of the variables in the model are discussed in this section but it is important to have an understanding of the time and data flow in the model before the variables are discussed. Figure 18 shows the decision flow through the model. The information moves from left to right.

The model is first run when the user is at time zero. The variables that are fixed (cost, price, etc.) are specified by the user and the model is run. The model creates a decision tree that shows the value of all possible end nodes in the tree. The model also calculates an expected value which is the probability weighted value from each end node.

The first decision made in the model is the number of options that are currently held that will be exercised. If the user is at time zero and the company currently holds four options that are due to be exercised, the tree will calculate all possible values if zero, one,
two, three, or four of the options are exercised. The model then runs through two separate chance nodes. The first is the uncertainty in the tool and the second is the six-month forecast of demand. This completes the six-month portion of the model.

The next step in the process is the nine-month section of the model. It runs through the number of options to pay to delay and that are exercised nine-months out. If the company owns five options and only exercises three, it may make more sense to delay those options for three-months instead of letting them expire. This will be dependent on the nine-month demand forecast. If the six-month forecast is low and the nine-month forecast is high, then the options will be delayed. The tree will run through every possible scenario. If the company owns five options and only one is exercised then the tree will give a value for delaying zero options through four options. When the tree runs through the nine-month demand forecast the nine-month time frame is complete.

The final decision is to look at the eighteen-month forecast and decide how many options should be purchased so that tools can be available in eighteen months. Again, the tree runs through every scenario so that zero through five options can be purchased and exercised. When this is complete the model has run through all scenarios.
Figure 18. Time Flow of the Decision Tree

Figure 19 provides a second way to think about the decision flow in the model. The company is at time zero with a certain number of options that are due and there is a six-month demand forecast. A decision is made as to whether to exercise or pay to delay an option that expires in the current time period. Any option that is not exercised or delayed automatically expires. There is uncertainty in the tool when it is delivered so it may or may not produce as much as expected. The options that are delayed when looking at the six month forecast are then exercised for the nine-month forecast. The company then has to make a decision based on their long-term eighteen-month forecast on whether or not to purchase options on tools that will be needed for that eighteen month forecast. The profit is then calculated for each node and the optimal decision is shown based on the data placed in the model.
4.1.3 Data Flow Description

The summary of the flow of data in the model is shown in the Figure 20. The data flows in steps running from one to 11. The nodes that are marked with a one are the nodes that need no information to go to the next step. This includes all of the cost and pricing data as well as the number of options that are held from previous time periods. The following steps all require information from the prior steps to run the calculations or to show the decisions. The final step, number 11, is the calculation of the overall profit.
4.2 **Model Description**

Section 4.2 includes a description of each of the variables in the model and diagrams of several of those variables.

4.2.1 **Fixed and Calculated Variables**

**Number of Options:** This is the number of options that the buyer currently has that are coming due in the current period.

**Options Remaining:** This is the number of options, after exercising this period, that the company has that will expire this period. For example, if the buyer has three options that will expire this period and the buyer exercises two of those options, there is one option remaining. The formula is:

\[
\text{Number of Options} - \text{Exercise Option}
\]

**Tool Capacity:** The tool capacity represents the output that a tool is capable of producing. This is tied to the tool uncertainty in that there is some expected value that the tool will be able to produce but there is also some probability that the tool will be capable of producing more or less than is expected. The value is not known until the tool is installed and operating. The figure below shows the dependence of tool capacity on tool uncertainty. For example, if the value is low then the machine can produce 900,000 wafers per time period and if the value is high the machine can produce 1.1 Million wafers per time period.
Figure 21. Tool Uncertainty

**Current Capacity:** The current capacity is a function of the previous options that have been exercised, the capacity of a tool, and the beginning capacity. Suppose a tool has a capacity of one unit, if there have been four options exercised in the past, and the beginning capacity was three units, then the current capacity is seven units. The formula is:

$$\text{Beginning Capacity} + \text{Previous Options Exercised} \times \text{Tool Capacity}$$

**Previous Options Exercised:** This is the number of options that have been exercised in the past. It is used to calculate the current capacity.

**End Capacity:** This variable represents the capacity at the end of the six-month period. It is a function of the current capacity and the number of options that are exercised in the current period. The formula is:

$$\text{Current Capacity} + \text{Tool Capacity} \times \text{Exercise Option}$$

**Excess 6 Month Capacity:** This is capacity that the company has that is not being utilized. If the demand for product is eight units and there is nine units of demand then the company has one excess unit of capacity. This represents capital that is tied up in equipment that is not being used. The formula is:
Tool Value: The tool value is the value of the tool being held by the company. This means that if the company purchased a tool for $10 Million and the tool is not being used, the tool has a tool value of $10 Million (this can be adjusted for depreciation).

Low Confidence Interval Six: This variable is used in the calculation of six-month demand. If the expected demand is 100 units but the company believes that there is a lower band on the demand of 95 units, this variable will hold a value of .95.

High Confidence Interval Six: This variable is used in the calculation of six-month demand. If the expected demand is 100 units but the company believes that there is an upper band on the demand of 103 units, this variable will hold a value of 1.03.

Nominal Forecast: This variable is the expected demand at the six-month mark.

Wafer Cost: The wafer cost is the average cost of the wafers being sold. This can include labor, materials, machine time, etc.

Wafers Sold: This is the projected demand for wafers that will be experienced in six-months. It is a function of the six-month demand forecast and the capacity that is installed at that time. If the capacity is not enough to meet the demand then the wafers sold will just be the end capacity at six-months. If the capacity is enough to meet demand then the wafers sold will be the demand. The formula is:

\[ \text{min}(\text{End\_Capacity}, \text{Forecast\_Six\_Month\_Demand}) \]

Missed Rev: This variable is meant to capture the fact that some revenue will be missed and that there should be a charge for missing this revenue. For example, if the demand is 20 units but the company can only produce 15 units, then the company has missed out on 5 units worth of revenue. The goal, as stated previously, of the company is to maximize
shareholder value so there needs to be a charge for missing out on revenue. The missed revenue is a function of the six-month demand forecast, the ending capacity, the chip price, and the wafer price. The formula is:

\[
\text{if}(\text{Forecast\_Six\_Month\_Demand-End\_Capacity}>0, (\text{Forecast\_Six\_Month\_Demand-End\_Capacity}) \times \text{Chip\_Price-Wafer\_Cost}, 0)
\]

This formula is saying that if there is excess demand, the missed revenue will be the amount of missed demand multiplied by the gross margin on the demand.

**WACC:** This is the weighted average cost of capital of the company. The weighted average cost of capital is calculated as follows:

\[
WACC = \frac{D}{D+E} \times (1-T)k_d + \frac{E}{D+E}k_e
\]

where:

- \(D\) = The market value of the debt that a company holds
- \(E\) = The market value of the equity for a company
- \(T\) = The tax-rate for a company
- \(k_d\) = The cost of debt for a company. This is usually defined as the interest rate on the company’s debt
- \(k_e\) = The cost of equity for a company. This is the return that a shareholder expects for holding a share of stock. This is often the company’s return on equity.

The weighted average cost of capital is usually given in a company’s financial statements or, if not, can be garnered from independent financial sources.

**Revenue:** The revenue is the number of chips sold multiplied by the price of each of the chips. The formula is:
\[ \frac{Wafers\_Sold \times Chip\_Price}{(1 + WACC)^{0.5}} \]

The total has to be brought back one-half of a year because the cash flows occur six-months in the future.

**Profit Six Month:** The six-month profit represents the revenue from the six-month demand forecast minus all of the costs involved in producing the product. This is a function of the revenue, the cost, and the missed revenue. The formula is:

\[ \text{Revenue} - \text{Cost} - \text{Missed Rev} \]

Notice that this is not the same as the accounting profit because there is a penalty for missing revenue. Accounting profit will not include a penalty for missing out on any demand even though it may hurt the company in the future.

**Exercise Price:** This is the price charged to the company for exercising an option that they hold.

**Cost:** The cost is the total cost to the company in the six-month demand frame. There are a variety of costs involved including the exercise price of any options that are exercised, the cost of the chips that are sold, and the holding cost charged to the company. The cost is the sum of these three variables. The formula is:

\[ \frac{(Wafer\_Cost \times Wafers\_Sold)}{(1 + WACC)^{0.5}} + Exercise\_Price \times Exercise\_Option + \frac{(Holding\_Cost)}{(1 + WACC)^{0.5}}. \]

The cost of the wafers sold and the holding cost must be brought back one-half of a year because those cash flows occur in the future. The exercise price occurs at the time of execution.

**Chip Price:** The chip price is the average price of the chips being sold. In terms of a particular semiconductor production line, it is the projected price point that the chips will
be sold at. If a line produces more than one type of product then this would be the average price that they will be sold for.

**Holding Cost:** The holding cost represents the opportunity cost to the company for holding too much tool capacity. There is a cost because the company, if they did not have this excess capacity, would be able to invest the capital that is tied up in another venture. For example, the company could invest $1 Million that is used to purchase a tool in government bonds. They would then gain a certain return from the bonds vs. having the capital tied up in a tool that is idle. The formula used is:

\[
\text{if(Excess\_6\_Month\_Capacity/Tool\_Capacity>1,(Excess\_6\_Month\_Capacity/Tool\_Capacity)\times Tool\_Value\times .25\times WACC,0)}
\]

This formula is worth discussing. The first term involves the excess capacity divided by the tool capacity. This was used to determine the amount of capacity that there is in excess in terms of tools. For example, if there is excess capacity of 12 units and the tool capacity is 10 units, this division will yield an answer of 1.2. This means that there is just over one tool that is held that does not have to be held. This first expression is surrounded by an if-then statement. The if statement says that if this excess capacity in terms of tools is greater then zero, then there is a penalty. If there is no excess capacity, then there is not a holding cost. The second part of the equation is the actual calculation of the holding cost. It involves the excess capacity in terms of tools multiplied by the length of time that the tools will be held in inventory. For example, if the tool is worth $1 Million and the tool will not be used for the next three months, the penalty is $1 Million multiplied by .25. This then yields a value of $250,000. If the WACC is 10%, the total value is $25,000. This means that if the company had invested that $1 Million in a project of comparable risk to the company as a whole, the company could have
generated $25,000 in profit over the course of those three-months instead of having the money tied up in the excess tool.

**Wafer Cost Nine Month:** This is the same as the six-month wafer cost. The cost can be varied if the cost of production is expected to increase.

**Profit Nine Month:** This is the profit generated at the nine-month mark.

**End Capacity Nine Month:** This is the end capacity at the nine-month time period. The formula is:

\[
\text{End Capacity} + \text{Exercise Nine Month} \times \text{Tool Capacity}
\]

The formula is taking the capacity that was available at the end of the six-month period and adding in any options that are exercised in the nine-month period.

**Growth Rate:** The growth rate is used as a proxy for an increase in the demand for product that will take place between the different time periods. For example, if the demand is expected to be 15% greater at the nine-month mark then at the six-month mark, the growth rate will be 1.15. This is used to simplify data entry. Specific numbers can be entered instead of using this growth rate. If growth will be 1% from the six-month to the nine-month time period and then 10% from the nine-month to the eighteen-month period, these values can be entered separately.

**Low Confidence Interval Nine:** This is the same as the confidence interval on the six-month demand.

**High Confidence Interval Nine:** This is the same as the confidence interval on the six-month demand.

**Missed Revenue Ninth Month:** This is equivalent to the missed revenue for the six-month demand. The formula is:
\[ @\text{if}(\text{Forecast\_Nine\_Month\_Demand} - \text{End\_Capacity\_Nine\_Month} > 0, (\text{Forecast\_Nine\_Month\_Demand} - \text{End\_Capacity\_Nine\_Month}) \times (\text{Chip\_Price\_Nine\_Month} - \text{Wafer\_Cost\_Nine\_Month}), 0) \].

**Wafers Sold Nine Month:** This is the same as the wafers sold for the six-month except that it uses the nine-month projected capacity and demand figures. The formula is:

\[ \text{min}(\text{End\_Capacity\_Nine\_Month}, \text{Forecast\_Nine\_Month\_Demand}) \]

**Revenue Nine Month:** The formula for this variable is:

\[ (\text{Wafers\_Sold\_Nine\_Month} \times \text{Chip\_Price\_Nine\_Month}) / (1+WACC)^{.75} \]

This value is discounted by three-quarters of a year to recognize that the revenue is nine-months in the future.

**Chip Price Nine Month:** This is the projected price that will be charged nine-months in the future. The price can be the same, or different, as the six-month price.

**Pay to Delay Cost:** This is the cost to the company to delay exercising an option until the next period. When the option is at its expiration date, the company holding the option has three choices. The first is to exercise the option and the second is to let the option expire. The third is to pay a fee to the company that has issued the option to allow the holding company to extend the deadline until the next time-period. In this model, paying to delay will allow the company to exercise the option in the nine-month time frame instead of letting the option expire in the six-month time period.

**Cost Nine Month:** This is the total cost in the nine-month time frame. It is similar to the six-month frame except that the pay to delay cost has been added in. This is the number of options that the company decides to delay multiplied by the cost per option. The formula is:

\[ (\text{Wafer\_Cost\_Nine\_Month} \times \text{Wafers\_Sold\_Nine\_Month}) / (1+WACC)^{.75} + \text{Exercise\_Price} \times \text{Exercise\_Nine\_Month} + \text{Pay\_to\_Delay\_Cost} \times \text{Pay\_to\_Delay}. \]
**Overall Profit:** The overall profit is the profit generated by the company over the 18-month time frame. The formula is:

\[ \text{Profit}_{\text{Six\_Month}} + \text{Profit}_{\text{Nine\_Month}} + \text{Profit}_{\text{Eighteen\_Month}} \]

The formula is made up of three parts. None of these are discounted because the values were discounted in the cost and revenue equations.

**Wafers Sold Eighteen Month:** This is the number of wafers that are sold in the eighteen-month time period. The formula is:

\[ \min(\text{End\_Capacity\_Eighteen\_Month}, \text{Forecast\_Eighteen\_Month\_Demand}) \]

**Wafer Cost Eighteen Month:** This is the cost of the chips that are sold in the eighteen-month time period. This can be the same as the other wafer costs of different if the cost is expected to rise or fall.

**Missed Revenue Eighteen Month:** This variable is the missed revenue in the eighteen month period. The formula is:

\[ @\text{if}(\text{Forecast\_Eighteen\_Month\_Demand}-\text{End\_Capacity\_Eighteen\_Month}>0,(\text{Forecast\_Eighteen\_Month\_Demand}-\text{End\_Capacity\_Eighteen\_Month})*(\text{Chip\_Price\_Eighteen\_Month}-\text{Wafer\_Cost\_Eighteen\_Month}),0) \]

**Chip Price Eighteen Month:** This is the price of the chips that are sold in the eighteen-month time period. This can be the same or different as the previous prices.

**End Capacity Eighteen Month:** This is the end capacity at eighteen-months. The formula is:

\[ \text{End\_Capacity\_Nine\_Month} + \text{Exercise\_Eighteen\_Month}^{*}\text{Tool\_Capacity}^{*}\text{Options\_Outstanding}^{*}\text{Tool\_Capacity} \]
This is taking the capacity from the nine-month period and adding in the options that are outstanding in the period. In addition, it is adding in any options that are exercised in this period.

**Profit Eighteen Month:** This is the profit generated in the eighteen-month time period. The formula is:

\[ \text{Revenue Eighteen Month} - \text{Cost Eighteen Month} - \text{Missed Revenue Eighteen Month} \]

**Revenue Eighteen Month:** This is the revenue generated in the eighteen-month time period. The formula is:

\[ \frac{\text{(Wafers Sold Eighteen Month} \times \text{Chip Price Eighteen Month})}{(1+WACC)^{1.5}} \]

**Cost Eighteen Month:** This is the cost of all activities in the eighteen-month period. The formula is:

\[ \text{Purchase Options} \times \text{Option Purchase Price} + \frac{\text{(Exercise Eighteen Month} \times \text{Exercise Price 18 Month})}{(1+WACC)} + \frac{\text{Wafer Cost Eighteen Month} \times \text{Wafers Sold Eighteen Month}}{(1+WACC)^{1.5}}. \]

The cost is a function of the number of options that are purchased, the number that are exercised, and the number of wafers that are sold in this period.

**Exercise Price 18 Month:** This is the cost to exercise an option in this period. This cost will come due twelve months later when the company makes the decision as to whether or not they want to exercised the option. However, to estimate the profit for the eighteen month period the charge for exercising the option must be included in the cost.

**Options Outstanding:** This is the number of options that the company has that are not at the expiration stage. For example, the company might purchase two options. They then have twelve months before the decision has to be made as to whether the option will be
exercised or not exercised. These options are counted here. These can be considered as capacity for the eighteen-month projection because these can be exercised before the eighteen-month projection is actually realized.

**Option Purchase Price:** This is the cost to purchase a new option that will expire in twelve months.

### 4.2.2 Decision Variables

Note: The decision and chance variables in the following sections only show five branches. All have eight branches in the actual model, with the exception of the forecasting nodes. The figures show five branches to ensure readability. The formulas from the branches carry over to the eight branch nodes.

**Exercise Option:** This variable represents the decision to exercise options that have reached their expiration date or to not exercise them. The decision is shown in Figure 22.

```
Exercise_Option
  zero
  one @if(Number_of_Options>0,1,0)
  two @if(Number_of_Options>1,2,0)
  three @if(Number_of_Options>2,3,0)
  four @if(Number_of_Options>3,4,0)
```

**Figure 22. Exercise Option**

In this model, a maximum of seven options can be exercised at any one time and there are eight decisions that can be made. The company can exercise zero, one, two, three, four, five, six, or seven options. A seven-option limit was used for two reasons.
The first is due to computer computation speed. As the number of branches increases at each node there is an increase in the number of end nodes. For example, if there are three nodes with five branches on each, there are a total of $3^5 = 243$ end nodes. When there are three nodes with 12 branches each, there are a total of $3^{12}$ end nodes, or over 500,000. The speed with which the end node values were calculated dropped dramatically when there were more than eight branches per decision node. The second reason is that there should be no reason that the company purchases an excessive number of options. As will be discussed in the conclusion section, if this is integrated with the purchasing decision a certain number of tools will be full purchased upfront and a certain number will be optioned. An average production line may have 25 tools total, so there is no reason to option more than eight tools.

The decision is contingent on the number of options that the company currently holds in their possession. For example, if the company holds four options then five decisions are possible. However, if the company holds two options then they can only exercise zero, one, or two options. The decision tree above works within this constraint by saying that the company has to hold a certain number of options to gain a value of a branch. In the case where the company holds two options, the fourth and fifth branches will have a value of zero. The first branch will also have a value of zero which says that the company has decided to exercise zero options. The second branch will have a value of one and the third branch will have a value of two. The formulas say that if the company has above $X$ number of options, then $X+1$ will be the value placed on the branch, otherwise the value will be zero.
This node flows into the End Capacity node which, as discussed in the previous section, calculates the amount of capacity available after the options are exercised. There will be eight values in the end capacity node depending on how many options are exercised.

**Pay-To-Delay:**

The Pay-To-Delay node shows how many options will be delayed until the next period. The decision is shown in the following figure.

![Pay-To-Delay Diagram](image)

Figure 23. Pay-To-Delay

The decision is contingent on the number of options that were exercised in the previous period. This variable was discussed in the previous section and represents the number of options that are left after the exercise decision is made.

There can be a maximum number of four and a minimum of zero options that are delayed. For example, if a company has three options remaining, then the company can delay zero, one, two, or three of those options.

This variable flows into the decision to exercise at nine-months and the nine-month cost. The cost will depend on how many of the options are delayed while the decision to
exercise at nine-month is conditional on the number of options that are delayed. If the company delays two options then they can exercise both of them. If the company is not going to exercise the option then there is no reason for the company to pay-to-delay this option. The company will let the option expire. If a company has a total option count of four in the six-month period, they may exercise one, pay-to-delay one, and let one expire. The decision to pay-to-delay will have effects on the total capacity at nine-months as well, although this is controlled by the nine-month exercise as discussed below.

**Exercise Nine-Month:** This variable controls the number of options that are exercised in the nine-month period. The decision is shown in Figure 24.

![Figure 24. Nine-Month Exercise](image)

This is dependent on the number of options that were delayed in the previous decision. For example, if the company delays zero options, then the company can exercise no options. If the company delays two options, then the company can exercise zero, one, or two options. However, it is important to recognize that a company will never exercise fewer options in the nine-month scenario than are delayed. There is no reason for a company to do this. If the company pays to delay an option and does not exercise it, then the company has wasted the delay price.
At the same time, this decision is not happening in “real-life”. In other words, the option is being delayed so that the company has an additional three-months to consider making the decision. Over the course of those three-months, the forecast that the company is using should become closer to the actual demand. Therefore, a company might pay-to-delay an option when they are looking at the options nine-months out and then decide, when they have moved three months in the future, that the additional tool will lead to overcapacity. The company might then decide that it does not make sense to exercise the option. So even though the company paid the addition fee to delay the option, they will let the option expire if the new forecast shows that the tool is not necessary.

This variable is used in the calculation of the nine-month cost as well as the final nine-month capacity. Both of these are dependent on the decision to pay-to-delay in terms of dollars for the cost and in terms of additional capacity for the final capacity.

**Purchase Options:** This variable represents the company’s decision to purchase new options on tools at 18-months. The decision tree is shown in Figure 25.

![Figure 25. Option Purchase](image-url)
In this model, the company can decide to purchase up to four new options in each time period. This, as with all variables, can be expanded for more options by adding branches to the tree.

In the 18-month time frame, the company needs to decide how many options it will need to be purchased that can be exercised in 12-months. The company is forecasting their demand 18-months in the future and trying to decide on the capacity that will be necessary to meet this demand.

**Exercise Eighteen-Month:** This variable represents the number of options purchased eighteen months in advance that the company expects to exercise. The tree is shown in Figure 26.
The option exercise is contingent on the number of options that are purchased. For example, if three options are purchased the company can exercise zero, one, two, or three of the options. This variable is used both in the calculation of the 18-month cost and the calculation of the capacity at 18-months.
4.2.3 Uncertainties

This section contains the major uncertainties in the model. Although the values shown are baseline, they can be varied in any number of ways.

**Tool Uncertainty:** The tool uncertainty represents the uncertainty in the amount of output that the tool will actually provide. The tree is shown in the figure that follows.

![Tool Uncertainty Diagram]

Figure 27. Tool Uncertainty

In this model, there is an expected production from each tool. This represents the nominal branch for the tool uncertainty. The tool can also produce more or less than is expected. This variable is tied to the tool capacity which was discussed in the section on fixed and calculated values.

**Forecast Six Month Demand:** The six-month demand forecast uncertainty is the company's demand forecast for six-months out. When the company is at time zero, marketing will create a forecast for the expected demand in six-months. In addition, they may or may not set upper and lower limits on the demand. This node attempts to capture the forecast from marketing (or the relevant group). The tree is shown in Figure 28.
In this model, there are three possible levels of demand. There is a nominal forecast and there is a forecast that depends on the confidence interval levels previously discussed. This variable is one of the more important variables in the model. The six-month demand forecast determines the number of options that are coming due that will be exercised. If the demand cannot be met with the current capacity and there are options available, then the company should exercise the options.

Each of the demand variables can be calculated in a number of ways. The first is to input demand forecasts that are turned over by the marketing or forecasting groups. If these groups give point forecasts for expected, low, and high level of demands then these can be put directly in the model. If marketing provides six different point forecasts with discrete probabilities for each forecast, this can be implemented in the model by increasing the number of branches in the model and changing the probabilities.

The second way to use demand data in this model is through Monte-Carlo simulation. A continuous distribution can be set for each of the demand nodes (not necessarily normal) and run a number of times to see what the expected demand and optimal choices are to maximize profits under the given distributions. To conduct this analysis, marketing needs to provide a description of the demand data. In other words, marketing needs to provide the data that will describe the distribution. If it is a normal distribution, they can
provide an estimate of the mean and an estimate of the standard deviation (or demand volatility). With this data, a number of scenarios can be run.

**Forecast Nine Month Demand:**

The nine-month demand forecast is the expected demand nine-months out. Much like the six-month demand forecast, marketing will provide a forecast of the demand for nine-months out. If the company is in a product growth stage then the nine-month forecast demand should be greater than the six-month forecast. The tree is shown in Figure 29.

![Figure 29. Nine-Month Demand Forecast](image)

The primary difference between the nine-month forecast and the six-month forecast is the use of the growth variable. As previously stated, it is not necessary to use the growth variable. A separate variable can be set up called “Nominal Forecast Nine-Month” and used instead. In addition, the growth rate can be negative if demand is ramping down.

This forecast is used to determine the number of options that will be delayed in the nine-month period. If the company has an option that is at the expiration date but does not need the extra capacity, they can decide to delay the option and exercise it in the next time period. That next time period is the nine-month time frame.

**Forecast Eighteen Month Demand:**

The 18-month forecast is used in the decision as to whether new options should be purchased or not. The tree is shown in the following figure.
As with the nine-month forecast, it is not necessary to use the growth rate variable for the 18-month demand. It is used for ease of data entry. In addition, the growth rate is raised to the fourth power in this tree. The time between the six-month forecast and the 18-month forecast is made up of four time period of three-months each. This is the reason for the fourth power.

The decision to purchase new options is made in the 18-month time period because it takes 18-months to produce the tool. The forecast is used as a proxy for the actual demand in the profit calculations.

### 4.3 Supplier Model

This section shows a supplier model that was created to match with the management decision tree for the purchasing company. An overview of the supplier's decisions are shown in Figure 31.
Figure 31. Supplier Decision Overview

The model for the supplier is much simpler than the model for the buyer. This is because the buyer dictates to the supplier the number of options that will be exercised, purchased, or delayed. There are only two items that the supplier needs to think about. The first is whether or not there is manufacturing space to process orders. The second is the supplier’s view of the chance that any options sold will be exercised. In the semiconductor industry, the tool suppliers typically create forecasts that are different than the forecasts created by the chip manufacturers. It is entirely possible that the supplier will forecast a much higher chance of exercise than the buyer in which case the supplier will be happy to sell the option because they will receive a premium. However, if the supplier sees a very low chance of exercise they may try and lock the buyer into purchasing the full tool upfront. This way, the supplier receives profit from the
production of the entire tool instead of just the option price. However, game theory is beyond the discussion of this paper.

The full supplier model is shown in Figure 32 and discussion of each of the variables follows.

![Overall Supplier Model](image)

**Figure 32. Overall Supplier Model**

### 4.3.1 Fixed and Calculated Variables

**Number of Pay to Delay:** This variable represents the number of options that the buyer has decided to pay to delay.

**Number to Exercise:** This is the number of options that the buyer has decided to exercise in the current period.

**Max Capacity:** This is the maximum capacity that the supplier has. This is represented by the maximum number of tools that the supplier is capable of producing at one time.
**Current WIP:** This is the current number of tools that the supplier has in currently in production.

**Options Orders:** This is the number of options on tools that the buyer is purchasing.

**Option Sale Price:** This is the up-front payment that the manufacturer receives from the buyer for the purchase of an option.

**Option Exercise Price:** This is the fee charged to the buyer for the exercise of the option.

**Pay to Delay Price:** This is the fee charged to the buyer to allow the time to expiration for the option to be extended for one additional time period.

**Decisions to be Made:** This is the number of options that are coming due in the current time period. If the buyer has three options outstanding that must be exercised in the current period, then there are three decisions that must be made.

**Number Expired:** This is the number of options that expired in the current period. It is a function of the number of options that are outstanding, the number that have been delayed, and the number that have been exercised. The formula is:

\[ \text{Decisions To Be Made - Number of Pay To Delay - Number To Exercise Remaining Capacity:} \]

The remaining capacity represents the amount of room that the supplier has to begin additional construction. It is a function of the maximum capacity and the work-in-progress. If the company has a maximum capacity of eight, they have five that are already being produced, and the buyer wants to purchase an additional five orders, the supplier will not be able to meet all of the demand. The formula is:

\[ \text{Max Capacity - Current WIP} \]
Manufacturing Cost Stage I: This is the cost to the supplier of manufacturing the tool from the initial order to the date where the buyer must make the decision to exercise the option or let the option expire.

Manufacturing Cost Stage II: This is the cost to the supplier of manufacturing the tool from the option exercise date to the tool completion date.

Orders Not Filled: This is the number of options ordered that the company cannot provide because they are past their manufacturing capacity. The formula is:

\[ \text{Option} - \text{Orders} - \text{Orders Manufactured} \]

Holding Cost Time Period: This variable is used in the calculation of holding costs for the supplier. The supplier realizes a holding cost when the buyer of an option decides to pay-to-delay an option. The supplier has a tool that is partially finished in inventory. This tool represents capital that is tied up and can be used elsewhere. In this model, this variable is always a three to represent a time period of three months. The cost equation turns this variable into a time period.

Cost: The cost variable represents the total cost to the supplier in the current time period. The equation is:

\[ \text{Manufacturing Cost Stage I} \times \text{Orders Manufactured} + \text{Manufacturing Cost Stage II} \times \text{Option Exercise} + \text{WACC} \times \text{Holding Cost Time Period}/12 \times \text{Manufacturing Cost Stage I} \]

The first portion of the equation is the cost of completing Stage I of the orders. The second part of the equation is the cost of completing the orders from the option exercise date to completion. The third portion represents the holding cost to the supplier. The holding cost is for a three month time period.

WACC: Weighted Average Cost of Capital.

Profit: The profit is the revenue – cost.
Revenue: This is the amount of revenue that the company receives in the current time period. It is the revenue from option sales, option exercises, and pay-to-delay fees. The formula is:

\[
\text{Revenue} = \text{Option Sale Price} \times \text{Orders Manufactured} \times \text{Forecast Chance Of Exercise} + \text{Option Exercise Price} \times \text{Option Exercise} + \text{Pay To Delay Price} \times \text{Pay To Delay}.
\]

The revenue made up of option sales has a variable called the forecast chance of exercise. This is a forecast, by the supplier, of the chance that the buyer of the option will actually exercise the option that they are being sold.

4.3.2 Decision Variables

Forecast Chance of Exercise: This is the probability that the supplier assigns to an option as to whether it will actually be exercised or not.

Pay to Delay: The pay-to-delay variable is the number of options that the buyer is going to delay in the current period. The tree is shown in the Figure 33.

![Figure 33. Pay-to-Delay on the Supplier Side](image)

Each branch is an if-then statement saying that if the number of options that will be delayed is equal to the branch, the branch will hold that number. If not, then the branch is
given a value of zero. This node is not actually a decision by the supplier but it is shown as a decision calculation to aid the flow into the revenue and option exercise nodes. The revenue equation uses this decision node to calculate the fees that will be paid in delaying the option.

**Option Exercise:** The Option exercise is the number of options that the buyer has told the supplier they will exercise in the coming period. The tree is shown in Figure 34.

![Option Exercise Tree]

Figure 34. Option Exercise on the Supplier Side

Again, each branch is an if statement that states if the number of options to be exercised is equal to the branch, the branch will contain that number. Otherwise, the branch will contain a zero.

**Orders Manufactured:** This variable is the number of orders that need to be manufactured by the company. The tree is shown in the figure below.
Figure 35. Orders Manufactured

The branches show that if the remaining capacity is enough to handle the option orders, then the full number of orders can be fulfilled.
The purpose of this section is to show some of the various scenarios through which the model was run. The variables in the model were discussed in Section 4 of the paper. For the initial scenario, the variables had the following base values:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Starting Capacity</td>
<td>400,000 units</td>
</tr>
<tr>
<td>Tool Capacity</td>
<td>100,000 units</td>
</tr>
<tr>
<td>Tool Uncertainty</td>
<td>80% chance of 100,000 units, 10% chance of 90,000 units, 10% chance of 110,000 units</td>
</tr>
<tr>
<td>Wafer Cost</td>
<td>$80/unit</td>
</tr>
<tr>
<td>Wafer Price</td>
<td>$100/unit</td>
</tr>
<tr>
<td>Option Purchase Price</td>
<td>$6,000,000</td>
</tr>
<tr>
<td>Option Exercise Price</td>
<td>$6,000,000</td>
</tr>
<tr>
<td>Pay to Delay Cost</td>
<td>$300,000</td>
</tr>
<tr>
<td>Six-Month Low Confidence Interval</td>
<td>.97</td>
</tr>
<tr>
<td>Six-Month high Confidence Interval</td>
<td>1.04</td>
</tr>
<tr>
<td>Nominal Six Month Demand at time zero</td>
<td>400,000 units</td>
</tr>
<tr>
<td>Nine-Month Low Confidence Interval</td>
<td>.95</td>
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<td>Nine-Month High Confidence Interval</td>
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<td>Eighteen-Month Low Confidence Interval</td>
<td>.85</td>
</tr>
<tr>
<td>Eighteen-Month High Confidence Interval</td>
<td>1.14</td>
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<tr>
<td>WACC</td>
<td>17%</td>
</tr>
<tr>
<td>Growth Rate</td>
<td>15%</td>
</tr>
<tr>
<td>Demand Uncertainties</td>
<td>80% Nominal, 10% Low, 10% High</td>
</tr>
<tr>
<td>Cost to Purchase a Tool</td>
<td>$10,000,000</td>
</tr>
</tbody>
</table>
These values were used in all of the scenarios unless otherwise indicated. If there are changes in the scenarios then the new values for the variables are shown. There are a total of six scenarios discussed. They are:

1. A Steady Demand Scenario
2. A slight drop in forecasted demand in one period
3. A sharp drop in forecasted demand in one period
4. A very large drop in demand
5. Demand forecasts that are volatile over time
6. A more realistic volatile situation

Discussion is given for each of the scenarios as to whether the model is helpful in the scenario or if it is unnecessary. Some of these numbers are realistic while some are not. The demand numbers, for example, are not actual demand numbers from Intel and the tool capacity numbers are not real. However, the real numbers can be substituted easily. One would just change the demand and the tool capacity numbers in the model to create realistic scenarios. In addition, the margin on the product ($20 profit per chip sold) is not the actual margin. The WACC is realistic as it is publicly available information. Most of the data is not realistic for confidentiality reasons.
5.1 Steady Demand Scenario

The first scenario examined was a steady demand scenario. In this scenario, the forecasts for the six, nine, and eighteen-month demand all match exactly. This can be thought of as a time in a product life when demand is growing steadily with no unforeseen up or downs in the market. The first nine-month forecast matches the six-month forecast that is made three months later. So demand forecasting is exact. The graph for the demand is shown in Figure 36 and the data are shown in Table 9 in the Appendix. Note that the forecasts match exactly as previously described.
Figure 36. Demand for the Steady Demand Scenario
Table 1 shows the decisions that were made at each time period. The first row shows the time period being examined. The time frame is three months because the decision tree is run every three months. The second row shows the number of options that are available to be exercised in the current time period. For example, in time period zero there are no options available for exercise because none had been purchased previously. So the assumption in this model is that at time zero, there are no options available.

The third row, Options Outstanding this Period, is the number of options that have been previously purchased but are not available for exercise. The options are purchased 18-months in advance of projected need. This means that the decision to exercise will be made 12-months later. So if an option is purchased at month zero then the decision for that option will be made in month 12. This is a running total of options that have been purchased but are not yet at the decision stage. Since this model is run in three month time-periods, this will be the summation of the options purchased at time zero, three, six, and nine. When the model is moved to month twelve, the options from month zero become available to exercise. These options are lost from the running total but any options purchased in month twelve are added.

The fourth row is the running total of the options that had previously been exercised. The fifth row is the starting capacity for the time period. This is the sum of any initial capacity and all of the options exercised to date. In this model, and in the future scenarios, the capacity was assumed to start at 400,000 units per time period. The sixth through ninth rows reflect the output from the model for the current time period. This is the number of options that are purchased, delayed, exercised, or expired in each time period.
Table 1.

The Figure 37 shows the cost of purchasing the tools outright versus using options to purchase the tools (Note: For the remaining scenarios, the discounted costs are not graphically shown). When purchasing the tools outright, the buyer is forced to pay the entire cost of the tool. To compute the difference in the cost to purchase the overall capacity, it is important to recognize the difference in timing that exists. For example, in period zero there were three options that were purchased on three tools. This means that a total cost of $18M is spent on the options (3 options * $6M/option). If a regular purchasing agreement were used instead of the option purchases, then the total cost would have been $30M at time zero. The additional cost of the option is added in when the options are exercised in time period 12. In this time period, there will be a second charge of $18M (3 options * $6M/option) while there will be no cost in that time period if the tools had been purchased. They would have already been paid for.
This leads to an important point regarding the final periods of the scenario. The last four time periods will not have a charge placed if there is an option purchase. This means that options purchased in periods 30, 33, 36, and 39 are not charged. The capacity will not come into being until time period 42, which is outside of the scenario. Correspondingly, there is no purchase charge in this time period. However, there is a charge if an option is exercised in this time period. In previous time periods the purchase of an option corresponds to the purchase of a full tool, so in the later time periods the option exercise price must be charged to match the purchase price of the tool.

In looking at the cost of using options versus full purchasing, one would expect the charges to be much higher in the initial stages for a purchase arrangement since the full payments are made in the beginning. In the latter time periods, the charge for purchasing would be zero as compared to the options because half of the option price is paid at a later date.

There are two other charges as well. The first is the pay-to-delay charge that the option holder incurs if the option is not exercised and not expired. When the holder decides to pay-to-delay, he is charged for that. At the same time, if the person had purchased the tool outright, that tool would be sitting idle in the factory. So the purchasing party would incur a holding cost for that extra tool sitting idle.

In summary, the discounted tool purchases is the sum of the up-front purchase of the tool in the same time period that options are purchased and holding costs that occur in time periods where there is a pay-to-delay decision. Each time period’s cost is discounted with the WACC and plotted in the following figure. The discounted option total is the sum of option purchases, exercises, and pay-to-delay decisions in each time period. Each of these are discounted and plotted in the figure as well.
The final variable in the following figure is the discounted cumulative cost. This is equal to the discounted tool purchases – the discounted option total in each time period and then summed up. So the final point on this line in time period 39 is, in effect, the cost of having used a regular purchase agreement versus using an options agreement. If the final number is positive, then it costs the company more money to purchase the tools outright then it does to purchase the tools using options. If the final number is negative, then the total for the options purchases was greater which means that the purchasing total was smaller. So the company would have been better off using regular purchase agreements to purchase the tools.
Figure 37. Cost Comparison for the Steady Demand Scenario
In this case, the final cumulative cost value is negative. This is what one would expect to happen in the case of steadily rising demand that is easily forecasted. There is no need to use an arrangement that includes the right to cancel a tool order because the order will never be cancelled. Using contracts that include the option to cancel an order in this type of demand scenario will only add an unnecessary cost to the total. Looking at the chart, the purchases cost more in the early months than the options do which causes the cumulative cost to increase. However, eventually the cost of exercising the options begins to add to the option total cost, which causes the cumulative cost to go negative.

5.2 One-Period Demand Drop Scenario

In this scenario, a slight drop in demand in one period followed by a quick recovery was considered. In month 24, the demand forecast drops by 10% as compared to month 21. So the six-month, nine-month, and 18-month forecasts are all 90% of the previous time months forecast. After this quick drop, the demand recovers and returns to 15% growth. The forecasts are shown in the Figure 38 and the data used are given in Table 10 in the Appendix. The demand forecasts are all the same until there is a drop in demand. When this occurs, the forecasts all collapse, and then they are all the same when demand recovers. For example, when the model is being run at month zero, the nine-month forecast is 15% greater than the six-month forecast. When the model is being run at month 24 the six-month forecast has dropped by 10%. The nine-month forecast has also dropped by 10% although it is still 15% greater than the six-month forecast. So with the exception of the drop in demand, the forecasts are exact.
Figure 38. Demand for the Slight Demand Drop
The table below shows the action for each time period. Beginning in month 24, there is a string of pay-to-delay decisions that are made. This means that if the options were exercised in month 24, there would be excess capacity leading to a holding cost. So it makes more sense to pay the delay fee and make a new decision in the following time period.

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>3</th>
<th>6</th>
<th>9</th>
<th>12</th>
<th>15</th>
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<tbody>
<tr>
<td>Options Ready this Period</td>
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<td>0</td>
<td>0</td>
<td>3</td>
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<td>Options Outstanding this Period</td>
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<tr>
<td>6-Month Options Exercised to Date</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
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<td>7</td>
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<td>10</td>
<td>12</td>
<td>15</td>
</tr>
<tr>
<td>Starting Capacity</td>
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<td>4.00E+06</td>
<td>4.00E+06</td>
<td>4.00E+06</td>
<td>7.00E+06</td>
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<td>1.60E+07</td>
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<td>Options Purchased</td>
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<td>2</td>
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<td>3</td>
<td>4</td>
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</tr>
<tr>
<td>Pay-To-Delay</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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</tr>
<tr>
<td>Options Exercised</td>
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<td>0</td>
<td>0</td>
<td>0</td>
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<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
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<td>Options Expired</td>
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<td>0</td>
<td>0</td>
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<td>0</td>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 2.

To maximize the profit in this scenario, the discounted cost to purchase using the option method is approximately $155 Million while using the regular purchase method the discounted cost is $142 Million. As shown in the table, in month 24 the number of options delayed begins to increase. The model still shows that purchasing the tools in a normal fashion is the preferred method when there is a quick drop in demand followed by an immediate recovery. This makes sense for the following reason. In this model, the price to purchase and exercise an option is approximately 20% more than purchasing a tool outright. So the cash saved by making the
decision to delay the options in the later time periods rather than pay the holding cost of having the unused tool is not enough to make up for the extra cash that is paid for that right. So again, with fairly steady demand and a small drop in demand it might be better to purchase the tools fully in the beginning. Note that the overall benefit to purchasing the tools outright has dropped significantly from the steady demand scenario. In the first scenario, the benefit was approximately $20M while in this scenario it is about $10M. This means that the option versus purchase method is highly dependent on the price of the option in that if the total to purchase and exercise an option was $11M instead of $12M, there would be a benefit to using the option method instead of the purchase method.

5.3 One Period Sharp Demand Drop Scenario

In this scenario, a pronounced drop in demand in one period followed by a quick recovery was considered. In month 21, the demand forecast drops by 30% as compared to month 18. So the six-month, nine-month, and 18-month forecasts are all 70% of the previous time months forecast. After this quick drop, the demand recovers and returns to 15% growth. The forecasts are shown in Figure 39 and the demand data are given in Table 11 in the Appendix. As with the previous scenario, the demand forecasts all track exactly. For example, when the model is run at month 21 the six-month forecast is shown as month 27 (1.15% - 1 = 32.25% greater than month 21). The nine-month forecast is 15% greater than the 27-month forecast. When the model is run in month 24, the six-
The month forecast has collapsed. So the nine-month and 18-month forecasts have also collapsed, but they are growing at 15% per three-month time period based off of the six-month forecast.

Figure 39. Demand for the Sharp Demand Drop
The table below shows the action for each time period. Beginning in month 24, there are options that are delayed. In addition, in this scenario there are actually options that are cancelled. In months 21 and 24, there is one option cancelled in each time period. Also, in months 27 through 36, there are options being delayed even while new options are being purchased. The reason that this is happening is because of the long-term forecast versus the short-term forecast.

In month 21, the demand has dropped so much that even the 18-month forecast (for month 37) is below the capacity that is currently in place. So given the demand forecast, the model has no reason to keep options that are currently on the books. However, demand recovers quickly, due to the 15% growth rate, so that in period 27 there is an option purchase even though an option expired six-months previously. If the demand were at a lower growth rate after the drop, no options would be bought in the later time periods. If demand grows at a faster rate then the options would not be allowed to expire.

<table>
<thead>
<tr>
<th>Time</th>
<th>0</th>
<th>3</th>
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<tbody>
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<td>Options Outstanding this Period</td>
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<tr>
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<td>0</td>
</tr>
</tbody>
</table>

Table 3.
To maximize the profit and minimize the purchasing cost in this scenario, the discounted cost to purchase the tools using the options method is $144.6 Million. The discounted cost to purchase the tools using the full up-front purchase method is approximately $140 Million. The purchasing method still shows an overall lower cost than the option method. However, the difference has once again narrowed. The purchasing method saves approximately $3.9M over the options purchasing method. This represents approximately $250,000 per option. So if the option price and the exercise price were $5.75M instead of $6M (giving the supplier a premium of $1.5M instead of $2M per tool) it would be less expensive to purchase with the options. The reason that this occurs is because of the expiring options. When the options expire and more options are purchased, the premium is being paid a second time.

5.4 LARGE DEMAND DROP SCENARIO

In the next scenario, there is a very large drop in demand followed by a slow recovery. In month 21, the forecasted six-month demand drops my 50%. The six-month, nine-month, and 18-month demand projections are all 50% of the previous months total. After the drop, demand begins to grow at 10% and continues to grow at that rate. The forecasts are shown in Figure 40 are the data are given in Table 12 in the Appendix. As the plot shows, at month 21 the six-month forecast drops to near the original level of demand.
Large Drop in Demand

Figure 40. Demand for the Large Demand Drop
There were several assumptions that differed from the previous scenarios in this scenario. The variables that changed are:

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Six-Month high Confidence Interval</td>
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</tr>
<tr>
<td>Nine-Month Low Confidence Interval</td>
<td>.95</td>
</tr>
<tr>
<td>Nine-Month High Confidence Interval</td>
<td>1.4</td>
</tr>
<tr>
<td>Eighteen-Month Low Confidence Interval</td>
<td>.85</td>
</tr>
<tr>
<td>Eighteen-Month High Confidence Interval</td>
<td>1.5</td>
</tr>
<tr>
<td>Growth Rate Before Month 21</td>
<td>15%</td>
</tr>
<tr>
<td>Growth Rate After Month 24</td>
<td>10%</td>
</tr>
</tbody>
</table>

These variables were changed to make the forecasting scenarios more realistic. In a typical forecast, the long-term forecasts are significantly more uncertain than the near-term forecast. Referring back to Figure 9, this was called the cone of uncertainty. As time moves forward, future demand should become more visible. In summary, this type of scenario is saying that the forecasts are more variable on the upside in the long-term forecasts. The company is taking a consistently optimistic view of the long-term demand.

Table 4 shows the results from running this kind of scenario. Six options were purchased in each of the first two periods due to the rapid buildup and the current shortage in capacity, and three options were purchased in months six and nine. However, when the first chance to exercise an option comes along, in month 12, only three of the options are exercised even though there are six options available for exercise. This is due to the funnelling down of the demand to the nominal six-month forecast from the highly uncertain
18-month forecast. Although the capacity was forecasted as necessary when the eighteen-month forecast was being used, they are not necessary when the time comes around.

The options on the tools that are not exercised are delayed into the next period. The eighteen-month forecasts are still showing an eventual necessity for the tool capacity and it is less expensive to defer the options than it is to let the options expire and purchase a new option in a later time period.

As discussed before, there is a large drop in demand in period 21. Looking at the table below all options are allowed to expire from month 21 to month 27. The eighteen-month forecast does not begin to recover until month 30. The model still lets four of the five options expire that come due in this time period but it also recognizes that demand is beginning to grow and elects to delay one of the options. In the final period, the option is delayed a second time. After the final period, the model begins to behave like it previously did, slowly building up tool capacity in order to prepare for the long-term forecasts.
<table>
<thead>
<tr>
<th>Time</th>
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<tbody>
<tr>
<td>Options Ready this Period</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>6</td>
<td>8</td>
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Table 4.

In this scenario, to maximize the profit and to minimize the purchasing costs, the discounted cost for purchasing the tools outright is $275.5 Million. The discounted cost to purchase the tools using options is $210.5 Million. In the scenario that has a large drop in demand, the purchasing method ends up costing significantly more than the options method. The reason is that a large proportion of the options are not exercised and left to expire. So the second half of the tool purchase is never made. When the demand collapses this means that a large amount of cash is not paid. In the full purchasing method, all of the cash is paid up front so the tools have already been purchased, even though they will not be used. This is one of the ideal scenarios for using option purchases versus a straight purchase. The option to cancel the orders becomes valuable.
5.5 Volatile Demand Scenario

In the next scenario, the demand is volatile. As shown in Figure 41, the forecasts are changing on a regular basis after a steady growth period for the first few months. When demand is growing, it is growing at 15%. When the demand collapses, it collapses by 50%. The data are given in Table 13 in the Appendix.
Figure 41. Demand for the Volatile Demand
Table 5 shows the result from this scenario. This scenario shows periods of option purchases, option delays, exercises and expirations. Interestingly, month 21 shows a severe drop in the forecasted demand in which all six options are allowed to expire. Then in the very next period six new options are purchased. This would be the equivalent of removing six tools that are already owned and then replacing those tools in the next period. This, of course, should not be allowed to happen. However, the forecast dictates that it should happen. The six-month, nine-month, and eighteen-month demand have all collapsed which means that the company sees no need for extra capacity in the immediate future. In the next scenario, the fact that the options should be delayed instead of cancelled will be examined. The 18-month forecast needs to be held fairly steady to account for the volatility.

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Table 5.

In this scenario, to maximize the profit and minimize the cost, the options purchasing method yields a total cost of $207.4 Million and the full-purchase method has a cost of $280 Million. The cost to purchase capacity shows that it is much less expensive to
purchase the capacity with the options method. Some of the options that are purchased earlier in the time period are not even exercised which leads to a significant cost savings over the full purchases. When the options are left to expire, as discussed before, it is assumed that the tools that would have been purchased under the purchasing method are disposed of. This means that if options are purchased after other options expire, the company is again charged for a full tool purchase, even if those new options again expire. Again, this is not a realistic assumption as a company that realizes demand may have collapsed for only a period or two is going to keep their existing capacity or, in the case of options, delay them.

5.6 Volatility with a Non-Volatile Long Lead-Time Forecast

In this case, the six-month and nine-month demands were allowed to collapse and move back up as they were in the previous scenario. However, the 18-month demand is held steady at 15% growth. This shows we are forecasting a steady demand rise over the long-term but, in the short-term demands fluctuates around the 15% growth line. This scenario represents the more common forecast in which the long-term forecast is steady but there is significant fluctuation in the short term forecast and realized demand. The plotted demand graph is shown in Figure 42 and the data are shown in the Appendix in Table 14.
Figure 42. Demand for the Realistic Volatile Forecast
Table 6 shows the result from this analysis. As expected, there are a number of situations in which the options are delayed by a period. This occurs because the long-term demand is forecasting a steady demand rise. When the short-term demand collapses, the options are delayed because it is less expensive to delay a tool than it is to remove the tool and purchase a new tool.

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Table 6

In this scenario, to maximize the profit, purchasing under the options method gives a total discounted cost of $212.8 Million. Purchasing in the normal manner gives a cost of $303.9 Million. As expected, purchasing via options is less costly than purchasing the full tool up front.
5.7 Summary

It seems that purchasing options makes more sense in periods of high volatility in the short-term forecast. The company is given the right, but not the obligation, to cancel or delay the order instead of receiving the tool when it is not necessary. When forecasts are rising steadily or the firm is faced with the occasional hiccup in demand, there is no need for this type of purchasing investment. It will cost the company more money in the long-term than is necessary to spend. If demand is steady and can be well forecasted there is no reason to spend the extra capital on purchasing the options. In this case they are rarely, if ever, delayed or expired.

There are benefits to the purchasing company and there are also benefits to the company that is making the sale. The selling company stands to receive a premium from the purchasing company if the option is exercised. The selling company is making more if they sell a tool with an option than if they do a straight sale. When the exercise price is discounted back to the present, the company makes a premium of approximately $1M over the sale of the tool.

In addition, the company can make option sales in environments that the purchasing company might not typically purchase a tool. For example, if the purchasing company believes that the final tool purchase is on the margin, meaning that the present value of the production from that tool will be close to zero, the company might not normally buy that tool to avoid the risk. However, if the seller can offer the option to cancel, it might persuade the purchasing company to purchase the tool.

Also, the selling company has the option to resell the tool if the purchasing company decides to let an option expire. While this might not happen very often, due to correlation in the industry, there may be situations where companies are showing a shift in
market share and one company is gaining. The company that is gaining may decide to pick up the tool that the option was not exercised on.

Finally, in the option purchasing method, the selling company is given the fee for the first manufacturing stage up-front. The current purchasing method is that there is no payment made by the purchasing company to the selling company for three-months. While this is extremely advantageous to the buyer, the supplier bears an unfair amount of risk in this situation. This is a way for the supplier to mitigate that risk.

So there is potential for a win-win in this arrangement. For the buyer:

1. Protection against paying out more in purchasing fees than is necessary by canceling tools half-way through production
2. A reduction in holding cost by delaying a tool that is half-produced

For the supplier:

1. An increase in price due to the cancellation risk at the half-way point
2. Up-Front payment of manufacturing costs mitigates the cancellation risk in the beginning
3. A potential increase in tool purchases
4. The chance to resell the option to another party

As for the model, the primary drivers are the demand forecasts, the price of the option to purchase, the price of the option to delay, and the holding cost. The option to delay must be less than the holding cost of the tool. If the tool is worth $12M when it is
delivered, the holding cost is $510,000 for a three-month period (assuming a 17% WACC). On the supplier side, the price to delay the option must be more than their holding cost. However, they are working off of a separate value. Because the tool has only finished the first phase of production, the value of the tool is less to the supplier than it is to the buyer. If the value of the first manufacturing steps is $4M, the holding cost is $170,000 to the supplier for a three-month period (assuming a 17% WACC which is high for a tool supplier). So the pay to delay must be bounded by these two conditions.

The forecasts are obviously important as they will dictate the actions that are taken during the course of the period. If the forecasts are volatile then being allowed to delay or let options expire is more valuable to the purchasing company than if the forecasts are steady.
5.8 **Black-Scholes Discussion**

As discussed in section 3, the Black-Scholes formula provides another means of looking at option pricing.

Imagine that a semiconductor company can purchase a tool today for $10M. The expected demand forecast shows that the company has a 60% chance of selling 700,000 units and a 40% chance of selling 412,500 units, all at $20/unit and a WACC of 17%. The expected value is:

\[
NPV = -S10M + \left(\frac{700,000\text{units} \times $20/\text{unit}}{1.17}\right) \times 0.6 + \left(\frac{412,500\text{units} \times 20/\text{unit}}{1.17}\right) \times 0.4 = -S10M + S10M = 0
\]

In other words, the project is of marginal value to the company. The NPV of the project is such that the company may take the project and it may not. The company should be indifferent. However, this is not taking into account the volatility of the project and, therefore, it is not taking full account the chance that the tool will produce more cash than the tool costs.

Recall the definition of the up movements and the down movements in a stock price from section 3.2. The up movement \(u\) was defined as the end price of the stock divided by the original price of the stock. In real options, this can be thought of as the optimistic cash flows divided by the expected cash flows. In this case, the optimistic cash flows occur when the 700,000 units are sold which leads to a cash flow of $14M. This gives an up movement of 1.4 ($14M/$10M). The down movement occurs when 412,500 units are sold and is equal to .825 ($8.25M/$10M).
Often, these up and down movements are used to calculate the volatility for a real options problem. The upside volatility and the downside volatility are calculated and the two are averaged to provide a proxy for the overall volatility.

\[
\text{upside volatility} = \sigma_u = \frac{\ln(u)}{\sqrt{t}}
\]
\[
\text{downside volatility} = \sigma_d = \frac{-\ln(d)}{\sqrt{t}}
\]
\[
\text{volatility} \approx \sigma = \frac{\sigma_u + \sigma_d}{2}
\]

In this case, the estimated volatility is .264. Our exercise price, X, is the expenditure for the tool, $10M. The value for the stock price, S, is the present value of the expected cash flows from the tool, $10M. In the example above we assumed the tool can produce product for one year before the tool is outdated, so our time to expiration is one year. The risk-free rate is estimated to be 4%. Recall the formula for the Black-Scholes equation in this case:

\[
d_1 = \frac{\ln\left(\frac{10,000,000}{10,000,000}\right) + (0.04 + 0.264^2/2)}{0.264\sqrt{1}} = 0.2835
\]
\[
d_2 = 0.2835 - 0.264\sqrt{1} = 0.0191
\]

and:

\[
C_0 = 10,000,000N(0.2835) - 10,000,000e^{-0.04*1}N(0.0191) = 1,238,962
\]

This means that this type of call option is worth over $1M according to the Black-Scholes formula. In other words, the overall NPV of this project is the $0 calculated previously plus this option value, for a total NPV of $1,238,962. The additional NPV is the option determined NPV.
This additional NPV represents the value of waiting to see what happens with the demand. In effect, this is saying that the buyer of the tool is willing to pay up to $1.238M to be given the option, for the next year, to receive delivery of a tool at anytime within the next year.

In the scenario of a semiconductor company purchasing a long-lead time tool, the tool manufacturer is not going to be willing to provide this option to the buyer. The costs of manufacturing the tool are well above this upfront fee. The supplier will not be willing to begin the manufacture of the product with just this upfront fee.

However, imagine that the volatility and the time-frame of the option are increased. It is not at all uncommon for volatility in the semiconductor industry to be very large. Imagine a volatility of 50% around the growth rate in semiconductor demand and an option time of five-years. The exercise price, estimated future cash flows, and risk-free interest rate remain the same. In this case, the value of a call option on the tool is $4.8M. This means that they buyer is willing to pay up to $4.8M for the right to exercise that option within five-years that includes the exercise fee of the full tool amount.

In this case, the supplier can complete the upfront construction of the tool for less than the cost of the option premium. Since the semiconductor company closes its forecast six-months prior to the necessary delivery time of the tool, the supplier will be able to finish the tool within that six-months. In this case, the value of waiting to see what happens with demand is worth $4.8M to the purchasing company, the value of the flexibility.

Thinking about it in terms of a financial option, imagine that a call option with a strike price of $30 is selling at $14 and the stock is currently at $42. If that option is
exercised today, the payoff is $12 and the buyer would lose $2. That $2 represents the flexibility in waiting to see what the stock price will be at the end of that year. That is equivalent to the $4.8M in the tool scenario.

Now suppose that a semiconductor company can purchase a tool today for $10M. This tool is considered a marginal tool in that the company believes that the expected demand can be covered with the tools that the company already owns. The tool is capable of producing 1M units per quarter and the margins are $20/unit sold (consistent with the previous section). The company believes that there is a 10% chance that the tool will be able to produce at full capacity for six quarters and a 90% chance that the tool will only produce 1000 units in each of six quarters. In other words, there is a small chance that demand is going to shoot up for some reason, although it is not very likely.

The net present value for this tool is:

\[ NPV = -10M + \left( \frac{3M \times $20}{1.17} + \frac{3M \times $20}{1.17^2} \right) \times 0.1 + \left( \frac{3000 \times $20}{1.17} + \frac{3000 \times $20}{1.17^2} \right) \times 0.9 = -$403,000 \]

In the above formula, the -$10M is the upfront cost of the tool. The first quantity represents the potential upside of the scenario. The first figures in the first quantity represent three million chips sold at a profit per chip of $20. This value is then discounted by the WACC of 17%. The second figure in the first quantity represents the same sales level in the second year so it is discounted by the WACC-squared. These two values are summed and multiplied by the 10% chance of this happening. The second quantity is the chance of selling 3000 units in the first-year discounted by the WACC plus 3000 units in the second-year discounted by the WACC-squared. This is then multiplied by the 90% chance of this happening.
This project has a negative net present value caused by the expected demand. The present value is about $9.6M, which is below the initial cost of the tool. So management should pass on purchasing the tool.

Now the same tool will be priced using the Black-Scholes equation. The data remains the same but it is assumed that there is an exercise price of $200,000. This represents the cost of putting the tool on the factor floor. In calculating the upward movement, the expected cash flow is $9.6M while the maximum cash flow is $120M. This gives a $u$ value of approximately 12.504. Since the time-frame is two years, the estimated volatility is 1.79. For the downward movement, the expected cash flow is about $85,600. This gives a $d$ of about .0089 which leads to a volatility of approximately 3.34. The average volatility used is then 2.56. In this example, the equations are:

\[
\begin{align*}
    d_1 &= \frac{\ln(\frac{9596888}{200000}) + (0.04 + \frac{2.56^2}{2})}{2.56\sqrt{2}} = 2.902 \\
    d_2 &= 2.902 - 3.62\sqrt{2} = -0.72
\end{align*}
\]

and:

\[
C_0 = 9596888N(2.902) - 200000e^{-0.04 \times 2}N(-0.72) = 9,535,612
\]

In this case, the company would be willing to pay up to $9.54M for a tool that the company can put in storage and that can be brought on to the factory floor and set up for $200,000. The $9.54M does not represent the value of the additional capacity. It represents the break-even point regarding the amount that the company is willing to pay. In other words, the company is only willing to pay up to $9.54M for the tool. The potential upside if this option is in place is $120M as previously discussed.
Notice how close the value of the option and the expected cash flows are. In fact, as the strike price (the price to bring the tool on-line) approaches zero, the value of the option will approach the expected cash flows. This is because the value of the option is nearly as far in-the-money as it can possibly get.

In summary, these results show that there is value inherent in being allowed to wait before making the decision whether to bring a tool online or not. However, this assumes that the tool is instantly deliverable which is not the case. In the first Black-Scholes example, the exercise price of the option was the full tool purchase price and the value for waiting a year before making a decision was over $1M. Yet this assumes that when the exercise price is paid the tool is there and can be used. This obviously cannot occur given the constraints of the supplier. The cost to the supplier to prepare the tool for the purchasing company’s exercise is the full cost of the tool, or $10 Million. The supplier will not be willing to except $1M to manufacture the tool when it is going to cost them the full amount. There is too much risk associated with that strategy.

Both of these examples are, however, applicable if the supplier has an unsold tool in inventory that is not dedicated. In the first case, the purchasing company will be willing to pay up to $1.2M at time zero if they are allowed to exercise that option anytime within the next year, with an exercise price of $10M. This is an interesting scenario because the net present value showed a value of zero meaning that the company would not be willing to purchase the tool. However, the option value is higher than that. The company is willing to pay $1.2M now for the chance to make $14M in the future. In the next section, a way to think about working with the Black-Scholes equation is discussed that holds more merit for purchasing long lead-time tools.
5.8.1 Working with Black-Scholes

In this discussion, the data is the same as the second example in section 5.8 with the following exceptions. The time to exercise in this case will be 12-months, the time that the buying company is required to make a decision as to whether the option will be exercised or not. The exercise price is $6,000,000, the amount of money that the buying company is required to pay to exercise the option.

The time-frame for this example is shown in Figure 43. The option can be purchased at time 0 for $6M. There is then one-year until the option has to be exercised or expired. When the option is exercised, there is six-months until the tool is delivered. One-year after tool delivery the first cash flows are realized. Note that this assumes that the cash-flows in year one and year two occur at the end of the year. In reality, they are spread out over the course of the year. When the cash flows occur at the end of the year, the overall value is driven down. So this can be considered the worst case scenario.

![Diagram of time frame for Black-Scholes example](image)

Figure 43. Time Frame for the Black-Scholes Example

The net present value of the cash flows is shown below. Again, notice the time used
in the denominator to bring the cash flows back. They take place at year 2.5 and year 3.5.

\[
NPV = -10M + \left(\frac{3.5M \times 20}{1.17^{2.5}} - \frac{3.5M \times 220}{1.17^{3.5}}\right) + \left(\frac{50000 \times 20}{1.17^{2.5}} + \frac{50000 \times 220}{1.17^{3.5}}\right) \times 0.9 = -804,533
\]

The net present value in this scenario is negative meaning that management is likely to pass on project. However, looking back at the time frame in Figure 45, it is possible to think of this as an option decision. The calculation of the call value is shown below:

\[
d_1 = \frac{\ln\left(\frac{9,895,467}{6,000,000}\right) + \left(0.04 + \frac{2.124^2}{2}\right)}{2.124\sqrt{1}} = 1.3165
\]

\[
d_2 = 1.3165 - 2.124\sqrt{1} = -0.8078
\]

and:

\[
C_0 = 7,583.185N(1.3165) - 6,000,000e^{-0.04\times1}N(-0.8078) = 7,756,870
\]

The call option in this scenario is worth $7,756,870. In this case, the company is willing to pay up to $7.756M in time zero to be given the right to exercise this option at the one-year time frame. Since this calculates the value of a European call option, there is an assumption that the option cannot be exercised until the exercise date. In this situation, it fits perfectly given that there is no reason for early exercise. The tool cannot be delivered any earlier if the option is exercised early, so there is no reason to do so.

In this last example, the implied volatility was used instead of the volatilities that were calculated above. The implied volatility was calculated from the stock market by reversing the Black-Scholes equation. A call option was used with a strike price of $22.5/share and a stock price of $28.6/share. The option was trading at $6.60/share on the day that the values were examined. The implied volatility was calculated to be 1.71 by adjusting the volatility in a spreadsheet until the price of the call option matched the real-
world price of the option. Note that a risk-free rate of 4% was used, although the risk-free rate is probably at a lower level at the current time given the recent interest rate cuts.

Using the same data from above, the value of the call option is $6,999,829.

\[
d_1 = \frac{\ln(\frac{9,895,467}{6,000,000}) + (0.04 + \frac{1.71^2}{2})t}{1.71\sqrt{t}} = 1.171
\]

\[
d_2 = 1.171 - 1.71\sqrt{t} = -0.539
\]

and:

\[
C_0 = 7,583,185N(1.171) - 6,000,000e^{-0.04t}N(-0.539) = 6,999,829
\]

Table 7 shows how the value of the call option varies with the expected cash flows and with the strike price. As the expected value increases, the value of the call option increases regardless of the strike price. In addition, the call option is worth more when the value of the strike price is lower. For this example, the break-even expected value is $8.75M when the strike price is $6M. In other words, the expected value from the cash flows has to be greater than $8.75M to make it worth purchasing the option. If the expected value drops below this then the value of the call option is worth less than $6M, which means that the purchasing company is not willing to purchase the option. Figure 44 shows this as a graph. As the expected value increases above $8.75M, the option is in the money if it has a strike price of $6M.

<table>
<thead>
<tr>
<th>Expected Value</th>
<th>$7,000,000</th>
<th>$8,000,000</th>
<th>$9,000,000</th>
<th>$10,000,000</th>
<th>$11,000,000</th>
<th>$12,000,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>$6M Strike</td>
<td>$4,513,969</td>
<td>$5,357,289</td>
<td>$6,217,557</td>
<td>$7,091,798</td>
<td>$7,977,750</td>
<td>$8,873,650</td>
</tr>
<tr>
<td>$5M Strike</td>
<td>$4,749,623</td>
<td>$5,617,153</td>
<td>$6,499,761</td>
<td>$7,394,708</td>
<td>$8,299,928</td>
<td>$9,213,824</td>
</tr>
</tbody>
</table>

Table 7.
Figure 44. Option Value and Expected Value

Table 8 shows the sensitivity of the option value to the volatility. As expected, the value of the option drops quickly with volatility. Figure 45 shows the value of the option as volatility increases. It also shows the breakeven volatility of 1.245. When the volatility is greater than this, the option is in the money.

<table>
<thead>
<tr>
<th>Volatility</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call Option Value</td>
<td>$4,252,362</td>
<td>$4,573,059</td>
<td>$4,991,422</td>
<td>$5,445,025</td>
<td>$5,903,540</td>
<td>$6,350,649</td>
<td>$6,776,719</td>
</tr>
</tbody>
</table>

Table 8.
Figure 45.  Option Value and Volatility

This last example shows how a real option value on a tool can be calculated using the expected cash flow along with the implied volatility that is being traded on the stock market. The assumption inherent in this is that the volatility on the stock option, meaning the volatility of the entire company, is equivalent to the volatility of the individual project. This may or may not be the case. As discussed earlier, there is currently research being conducted to determine if this is a valid assumption or not.

The decision process in this case is simple. When it comes time to exercise the option or let it expire a standard net present value analysis is run. The option is exercised if the net present value is greater than the exercise price. There is no flexibility, with two exceptions, from this point forward so the net present value is the decision criteria. The forecast freezes at six-months.

The first exception might be if the supplier allows a pay to delay option at this point. If so, this can be priced as another option where the pay to delay price is used as the exercise price. The second exception might be if there is a salvage value to the tool. For
example, the buying company might be able to donate the tool to a university and take a tax deduction. If this is possible, a put option can be priced with the Black-Scholes equation where the exercise price is the tax deduction to the donating company.
6.1 Summary

This paper attempted to lay out a framework of thinking about the act of forecasting demand and the ways in which decisions can be made to minimize the risk if the forecasts are high or maximizing the gain if forecasts are low. The decision tree/options can be an excellent way of making decisions if a company is trying to protect the downside. If demand falls significantly, the tool orders can be canceled and a significant amount of money can be saved.

This model is a baseline model for thinking about purchasing long lead-time tools in an uncertain environment. The model attempts to lay out a rational methodology by which decisions can be made as to whether a tool should be bought which includes an option to cancel an order. This model lays the groundwork for future work or research that can be conducted into these types of purchasing methodologies.

A number of scenarios were examined to see if any value was added to the process by purchasing with a cancellation option imbedded. As expected, this can be valuable when demand is volatile and can drop rapidly. This is especially true in the case of tools that are only going to be used for a short period of time and will not be reused in the future. In this case, the option to cancel can save a substantial sum of money if demand does not materialize.

In addition, the Black-Scholes model was used to examine the possibility of purchasing an option on a tool using the European call option pricing formula. While the option was shown to have value, the scenario in which it is used is unrealistic. There was
an implicit assumption that, upon exercise, the tool could be delivered instantaneously. In
the case of long lead-time tools, this is not the case.

However, another way of thinking about the Black-Scholes example was shown in
which the expiration of the option occurs at the end of the first manufacturing phase of
the tool. This is then a viable way of thinking about the purchase of an option on a
marginal tool or tools. The implied volatility can be garnered from the market, albeit this
is not an exact method for calculating volatility on individual projects.

The more complicated part of the process is trying to catch the upside of the
opportunity. This is entirely due to the production process that the tool must go through.
The tool has an extremely long lead-time and a substantial amount of capital that must be
invested before the tool can be ready for use. Traditional option pricing theories cannot
capture this facet of the process. With a stock option, the option can be delivered at any
time. There is no need to build the option. With a tool that requires a substantial amount
of time to build, this is not the case.

The type of option discussed in this paper has enough cash paid for the option right
up front to cover the manufacturing costs and the rest is paid as an exercise price at a later
date. As stated previously, this protects the downside. To attempt to capture the upside,
the forecasted demand that is furthest from the current point in time (18 months) must be
forced to the high level. This is the only way that this upside can be captured. However,
this means holding a tool in inventory which may not be a viable candidate for future
exercise. However, the tools held would be minimal and the cost is lowered because an
option is being held instead of a tool. In addition, the payoff is large. The overriding
constraint is the time frame involved in the production of the tool.
There are three additions that will be necessary to the model before it can be utilized in practice. The first is that the purchase of tools needs to be added to the model. This means that the entire tool purchase amount is paid at the beginning of the manufacturing process versus an option purchasing methodology. There is no reason that the tools purchased to satisfy the lower levels of demand should be purchased via options. For example, if the company is 100% sure that a tool will be used, then that tool should be purchased up-front which will save the cost of an option. If the purchases are added to the model then it may be possible to have a model that covers purchases in addition to options. It will give the decisions to purchase tools and options that gives the maximum expected value.

The second necessary additions are the realistic confidence limits that will be used in the forecasting. In this model, confidence limits are set arbitrarily because forecasts are based on a point forecast. However, the model was built to allow forecasting via confidence limits. This can be used in any number of ways. For example, the forecasting uncertainties can be adjusted to allow any number forecasting scenarios. The model was built with a low, nominal, and high confidence settings. There is no reason that there cannot be more branches than that.

Marketing could provide a forecast with a range of probabilities. For example, marketing might say that there is a 40% chance of nominal demand, a 20% chance of slightly higher demand, a 10% chance of high demand, a 10% chance of extreme demand, and a 20% chance of low demand. These forecasts can be entered in the model and it will produce a result that says purchase a certain number of tools outright and purchase options on the remainder of the tools. In addition, a continuous rather than a
discrete probability distribution can be entered in the demand nodes and Monte Carlo simulation can be run to determine the potential end values.

In capital budgeting, when the payoff characteristics change, the discount rate needs to change as well. In financial theory, a financial option is more risky than owning the underlying stock. For a call option, the discount rate increases while for a put option the discount rate decreases. In Black-Scholes, that volatility can either be calculated by looking at past movements of the stock or looking at the implied volatility. The implied volatility can be calculated by taking an option that is traded on the market, holding all values fixed, and calculating the volatility that is necessary to price that option as it stands. In a real option, a tracking portfolio is built to mimic the option. In other words, a portfolio of publicly traded assets is built that matches the cash flow payoffs of the real option. From this, the volatility of the real option can be calculated since it must be equal to the publicly traded securities on the market. If not, the law of one price has been violated.

In a binary tree, the volatility is accounted for by conducting a risk-neutral valuation of the option and discounting the cash flows at the risk-free rate. The relative riskiness of the outcomes is then accounted for by modifying the probability of each outcome in the tree. This is completed in the same manner as estimating the volatility for the Black-Scholes model. A replicating portfolio is built with the same cash flows as the real option.

To date, no research has been successfully completed that will allow an option to be built with a decision tree that does not include a replicating portfolio. However, there is ongoing research at a number of management schools that are examining this problem.
and trying to figure out ways to work around the necessity of the replicating portfolio. One of the primary means being examined is looking at the impact of a company's project on the company's stock price. In other words, using the implied volatility from the company's stock as the volatility on individual projects within that company. To date, the research has not been successful although it is expected that results will be successful in a short period of time.

When this occurs, the decision tree used in this paper can be adjusted to use the risk-neutral probabilities to calculate the present cash flows rather than the weighted average cost of capital. These tree-building programs will allow the calculation of risk-neutral probabilities at each branch of the tree. So the tree can be adjusted to account for this when it is possible to do so. The WACC would then be replaced by the risk-free rate of interest. In addition, the Black-Scholes method of calculating the option value can be utilized when the volatility research is complete.

This type of model is only going to be as good as the forecast that is put in. As show in the results section, some of the decisions that lead to short-term optimality at the expense of the long-term can be eliminated. However, it is not possible to make tools appear if that was not prepared for. People purchase stock options because they believe that the underlying derivative has some probability of ending well above, or below, the price that is expected. If the stock ends at the expected price then the strike price will equal the stock price. The holder will lose the premium. If the stock ends well above the consensus expected price then the holder stands to reap a large benefit. This only occurs because the holder has recognized that while there is an expected value, there is a
probability distribution attached to that value that allows a stock to move around its mean forecast.

This type of option pricing is limited and it faces a number of constraints. The primary constraint is the fact that the supplier has ongoing costs if an option is purchased. The lead-times are fixed and cannot be reduced. This means that each time a company purchases an option the supplier is forced to incur the cost of building at least half of the tool. If the option to the purchasing company is not at least as much as the cost incurred by the supplier, then the supplier will not be willing to sell the option on the tool.

It is also important to recognize that these types of options may need to be tied to options on other tools or items. If a certain tool is the bottleneck that will allow capacity to be raised then the stand-alone option is valid. However, if there are four tools on an assembly line and a capacity expansion option is being considered, an option must be placed on each of those four tools. The cash flows can be broken down and assigned to each of the available tools based on their cost relative to the overall cost. Or, in one supplier manufactures all of the tools, an option can be priced on a package delivery in which each of the tools is delivered as part of the option exercise.

Greater uncertainty increases the value of an option. This is one of the crucial differences between an option and a traditional net present value analysis. When a company is fully invested uncertainty has a negative effect because the returns from the investment are symmetrical. Losing the entire investment is often as much a possibility as doubling the investment. When a company buys an option, the company is either exposed to the upside or it is protecting against the downside. This means that the company wants
to see an increase in the uncertainty of the expected returns and then exercise the option to back out or exercise at the top or bottom.

6.2 Future Work

There are two areas that would be worth investigating in the future regarding options in the manufacturing environment. Each of these options areas has the potential for a large payoff if successful.

The first involves pricing the option of having an additional tool sitting in inventory that can be installed and used at any time. The pricing of this type of an option would be valuable to know. This type of option was shown using the Black-Scholes method. There is a small setup cost but the potential cash flows associated with the tool can be large. The option value must be worth more than the cost of purchasing the tool and sticking the tool in inventory, which is useful only in limited cases. One such case is discussed further in this section and involves expanding the capacity of a particular tool. In other words, there is a base tool but there is also an option to add capacity to that particular tool.

The second involves pricing the option on “owning” a certain amount of a manufacturers capacity. For example, as discussed in section 3.3, imagine that the tool supplier and the purchasing company have different forecasts of the semiconductor for the next three years. Or, imagine that the purchasing company has an expected forecast that is equal to the suppliers forecast, but the purchasing company has an upper limit on the forecast that is above the upper limit of the forecast for the selling company. In this case, the supplier owns the olive presses (tool production lines) and the purchasing company is looking to purchase the right to use those presses in advance. If the
semiconductor manufacturing company's upper forecast is significantly higher than the tool suppliers forecast, it is likely that it will be valuable to purchase a certain percentage of the tool company’s capacity in advance.

This is valuable for two reasons. The first is that the purchasing company has locked in the capacity that may be necessary well in advance. If the upper forecasts are actually realized, then the purchasing company is not as likely to be capacity constrained because they will have access to the extra capacity. The second advantage is that the cost will more than likely be lower. The company that purchases the option on the capacity is less likely to have the price of a tool increased when they are purchasing a tool. If demand in the industry is high, the tool supplier is more likely to increase the purchasing price of the tools because demand is higher. However, the company that purchased the option has already locked in a certain amount of capacity and is less likely to face these price hikes.

There is value in options when looking at an expansion of capacity that is already in place. For example, if a standard photolithography tool is capable of producing 1000 units per day but the supplier can provide a an expansion on this tool that provides an additional 50 units per day of production, for a total of 1050 units per day, the option on that capacity expansion can be priced. The exercise on this option is instantaneous in that a switch needs to be thrown to increase the production rate. The tool is going to be bought whether the 1000 unit or the 1050 unit machine is bought, so this cost is not considered in the option pricing. It is the additional potential cash flows from the additional capacity, the volatility of the cash flows, the additional price for the 1050 unit machine (strike price) and the length of time that the machine will be used. This option will have value. While a traditional NPV analysis might say that there is marginal or
negative value to the additional capacity, if the volatility is high then the additional capacity will have value.

However, one method that should be investigated is by breaking the tool down into distinct time periods. Imagine that a $10M tool can be broken into five distinct payments of $2M. As Dixit and Pindyck (Dixit, 1987) discuss, if the project is broken into distinct phases, and there is a known volatility, the option to expand at each phase can be calculated through the use of dynamic programming. In this case, when it is time to pay the next $2.5M, a decision can be made as to whether the option value is above or below a critical value depending on the future cash flows. If the option is above the value, then the next installment is paid. If the option is below the value, then the project should be abandoned. In fact, a value can be calculated for each time period above which the next payment is made and below which the project is terminated. In their paper, they use dynamic pricing to solve the problem. However, the problem still stands that there is a lack of volatility data in semiconductor applications.

This is the purpose of trying to think in an options state of mind. While trying to assign an actual value to all decisions is necessary, making decisions by framing them properly is just as important. Companies that think of uncertainty as a potential source of value when making decisions stand to reap large rewards. Returning to the example of the grape press, the future harvest was uncertain but the subject locked up the capacity in advance. When the harvest was large, Thales was able to make a substantial return on his investment. Without taking the risk to pay for that capacity in advance he would have not made any money.
6.3 Conclusion

The primary purpose of this paper was twofold. The first was to build a model and the second was to think about real options in terms of manufacturing companies. It is important to think about the places that real options thinking can be implemented to try to increase shareholder value. The projects that are best examined with options methodology are projects that can be instantly exercised. These include items such as licenses and research and development. For example, a company can purchase a license for a new technology with the understanding that they will be the sole owner of that license. The company can then wait to see if there is demand for this in the future and, if there is, they can exercise their option. If a research and development project can be broken into distinct phases then it is possible to price an option on this.

Options can be priced on owning a certain amount of capacity. A semiconductor company can purchase an option for an upfront fee on the manufacturing capacity of a supplier. The company can then exercise this option at any point in time and the capacity belongs to them. If there is a queue and a manufacturing company is forced to wait in line before production of a tool can start, an option can be priced to allow the company to cut in line in the queue.

A European call option can be priced on delivery of a tool if the exercise price is assumed to be the amount of cash that must be spent to bring the tool online. If the expiration time is 18 months, this matches the lead-time of the tool. However, the value of the option becomes nearly equivalent to the net present value analysis, which makes the option analysis unnecessary.
On the other hand, thinking of the call option as expiring in 12 months presents a viable way of examining the problem. The forecast is frozen six-months prior to delivery and no changes are allowed after this point. Therefore, the option can expire at this point and the Black-Scholes formula can be utilized.

It is possible to show that the option to cancel, or the put, has value in this type of scenario. This was shown through the analysis of the decision tree and the scenarios that were presented.
Tagir Agmon\(^1\) examines the flow of information and the effect of that option in a particular options case. Agmon examines a joint venture in which both parties had options to purchase at the end of the third year. After two-years of poor returns, one of the partners sells out at a low price. In the third year, the profitability of the venture increases. Agmon examines how the flow of information should have been evaluated prior to the first party selling. The project was still within the standard deviation of the potential outcomes so the option was still worth holding.

Amram\(^2\) and Kulatilaka wrote a book that introduces real options thinking. The book begins with an introduction to options theory, how this options theory maps onto real options, and provides a number of examples of real options in industry. The biggest contribution of this book is the number of real options examples that are utilized.

Barnes-Schuster, Bassok, and Anupindi\(^3\) investigate the role of contingent claims in a buyer-supplier system. The main contribution of this paper is examining a two-period model with correlated demand and how options provide flexibility for a supplier to respond to market changes. The authors demonstrate that channel coordination can only be achieved if the exercise price for the options is piecewise linear and they develop cost parameters such that linear prices coordinate the channel. However, the prices end up being non-rational for the supplier so they use returns policies to allow the supplier to achieve profitability. They demonstrate the benefits of options in channel coordination and show the loss due to non-coordination.

Bassok and Anupindi\(^4\) examine a class of supply contracts known as Rolling Horizon Flexibility (RHF) contracts. A buyer is required to commit to a certain number of components at the beginning of the time period. The supplier then provides the buyer with a certain amount of flexibility to adjust the order. The amount of flexibility is stepped down as the delivery time approaches zero. The authors demonstrate that this type of contract has decreasing order process variability as flexibilities decrease without increasing the buyer’s costs. The authors provide guidance as to how much flexibility is sufficient and what the value of adding additional flexibility is.


\(^3\) Barnes-Schuster, Dawn, Bassok, Yehuda, and Ravi Anupindi, “Coordination and Flexibility in Supply Contracts with Options,” Working Paper – University of Chicago, University of Southern California, and Northwestern University, March 2000

In his graduate thesis, Craig Belnap wrote a paper with an introduction to real options for manufacturing firms. He walks through financial options, explains real options, and offers several examples of problems that a manufacturing firm might have that can be priced as real options.

Fischer Black discusses the assumptions underlying the Black-Scholes formula and discusses ways in which adjustments can be made to the formula to account for violations. Black walks through each of the assumptions and adjusts the formulas as necessary.

In 1973, Fischer Black and Myron Scholes introduced the Black-Scholes equation for the pricing of European call options. The paper discusses the fact that the bond-holders are the actual owners of a corporation while the stockholders possess an option to purchase the assets of a corporation by paying down the debt. So the equity holders have, in effect, a call option on the assets of a company.

The paper presents the underlying assumptions of the Black-Scholes equation, which include the following:

There is a constant interest rate
The stock-price is a random walk with a lognormal distribution
There are no dividends paid on the stock
There is no early exercise of the option (so it is European)
There are no transaction costs (i.e. no costs to buying or selling the option)
There are no borrowing costs
There are no penalties for short selling (no interest rate charges)
There is an infinite market (ones actions will not affect the market price)

Fischer Black describes the assumptions underlying the Black-Scholes equation in this paper. He has written it in a way that is easily readable yet useful. He describes how investors can use the formula when the underlying assumptions are not satisfied. Such as when the volatility will not remain constant, when interest rates are changing, or when there are transaction costs.

Nicolas Bollen, in 1999, develops an options valuation that accounts for a products lifecycle. Typically, real option valuation techniques ignore the life cycle and assumes a constant expected growth rate in either demand or price. Bollen shows that not accounting for the life cycle tends to undervalue the option to contract capacity and

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overvalue the option to expand capacity. He values the option to change a project’s capacity.

Michael Brennan\textsuperscript{10} presents an example of options analysis in a copper mine. The authors determine the value of opening and closing the mine based on copper prices and the variance of the prices.

Alexander Brown\textsuperscript{11} and Hau Lee examined the possibility of integrated circuit manufacturers optioning out capacity from subcontract foundries for wafer fabrication. The paper compares this option method, which is more expensive, to the less expensive firm commitment. The work examines the effect of information quality, reservation costs, and demand parameters on the reservation decisions and the expected profit. The work attempts to arrive at a solution as to how much capacity reservation should take the form of options and how much should be firmly committed.

John Charnes\textsuperscript{12} discusses Monte Carlo simulation and its use in pricing financial options. He introduces options, discusses options theory, and presents several examples of option pricing and variance reduction using Crystal Ball (a simulation program for Excel).”

Thomas Copeland\textsuperscript{13} and Philip Keenan provide an introduction to compound options. The paper lists a number of options in various manufacturing and retail sectors. The paper discusses compound and learning options and provides an example of a situation that results in a compound option. The paper steps through the real option and how it functions.

John Cox\textsuperscript{14} and Stephen Ross demonstrate the binomial option pricing approach. The authors show that as the time period of the tree is decreased towards zero, the tree approaches the Black-Scholes formula. The tree is essentially a decision tree that determines risk-neutral probabilities and works backwards through to the tree to calculate the option value.

Peter Coy\textsuperscript{15} discusses the basics of real options and provides several examples of real options that were used in industry.

\textsuperscript{11} Brown, Alexander O. and Hau L. Lee, “Optimal “Pay to Delay” Capacity Reservation with Application to the Semiconductor Industry”, Owen Graduate School of Management at Vanderbilt University (Brown), Department of Industrial Engineering and Engineering Management, and the Graduate School of Business (Lee), White Paper
\textsuperscript{15} Coy, Peter “Exploiting Uncertainty”, \textit{Business Week}, June 7, 1999 pp. 118-124
Avinash Dixit and Robert Pindyck prepared this paper as an introduction to real options for managers in industry. They point out that the net present value means of valuing a project makes two false assumptions that real options does not. The first is that the project is reversible and the second is that the project cannot be delayed. The primary contribution of this paper is the discussion of the differences between NPV and options.

Dixit and Pindyck also wrote a book on investments under uncertainty. They discuss sequential investments (tied to the paper discussed in footnote 23) via dynamic programming and spanning portfolios.

Hemantha Hebrath and Chan Park discuss using options pricing as a means of analyzing Research and Development (R&D) projects. Their main argument is that a traditional discounted cash-flow method does not adequately capture all of the future growth opportunities. They view the R&D process as an initial investment that creates future follow-on opportunities only if the initial project is successful. The authors create a valuation model based on a binomial option pricing model in the form of a management decision tree. The model incorporates the risk-free arbitrage features of the binomial model. The paper links traditional economic analysis and stock prices and uses the MACH3 project by Gillette as an example.

John Hull wrote a book on options and futures. It includes the description of how to price options with binary trees as well as with the Black-Scholes equation. The assumptions of underlying both methods are detailed.

Robert Jarrow and Andrew Rudd wrote a book on the pricing of options. The book provides a number of methods for estimating the volatility of a stock. In addition, alternative methods of options are priced including Monte Carlo simulation.

Angelienn Kemna discusses real options through several examples including an oil field development, a distiller abandonment option, and a venture growth option. The paper works through the equations and estimates the variables to price the real options.

Nalin Kulatilaka and Enrico C. Perotti wrote a paper that examines the strategic rationale for growth options under uncertainty and imperfect competition. Real options analysis is typically based on the assumptions that a firm has monopoly power over an
investment opportunity and that the product market is perfectly competitive. This paper looks at the possibilities of real options when this is not the case.

Keith J. Leslie and Max P. Michaels22 wrote a paper with the intention of introducing real options to people working in corporations where real options might be applicable. The article is analytically light on purpose. The article introduces real options in a strategic sense and examines the levers that can decrease or increase the value of a real option. The paper walks through several examples of real options in industry and how they were priced.

Timothy Luehrman23 wrote a paper on the analogy between financial options and corporate investments. He writes this as an introduction for corporate managers and introduces a framework for thinking about real options and potential uses in corporations. The author walks through a capital investment example and describes the similarities to a call option. He compares this with the discounted cash flow technique and critiques the similarities and differences between the two.

Saman Majd24 and Robert S. Pindyck wrote a paper on viewing sequential cash outlays over time as a contingent claim. They use, as an example, a project that requires an investment of $6M invested at $1M per year. They show how this analysis can be used to decide at what level purchasing the option on the next phase of the project will break even.

Stewart Myers25, in 1977, wrote the first paper to suggest that investments in real assets could be viewed as 'real options'. He discusses the debt/equity relationship on real investment decisions and how the relationship will change the way in which choices are made. He uses the term growth options to describe the purchase of potential future growth opportunities and notes that firms with growth options are often penalized with risky debt. In other words, firms with real growth options are required to issue debt at a premium. He notes that corporate borrowing is inversely related to the proportion of market value accounted for by real options. So the higher the growth opportunities that a particular company has, the lower the level of debt they will use to finance those opportunities.

Steward Myers26 and Saman Majd discuss the valuation of an option to abandon a project for its salvage value. The paper points out that in capital budgeting, managers will often

22 Leslie, Keith J. and Max P. Michaels, "The real power of real options," The Mckinsey Quarterly, 1997, Number 3, pp. 4-22
26 Myers, Stewart and Saman Majd, "Abandonment Value And Project Life," Advances in Futures and Options Research. Volume 4, 1990, pp. 1-21
depart from the operating and investment plans that underlay the original forecasts. This paper tries to account for part of this through the value of the option to abandon.

Stewart Myers and Saman Majd\textsuperscript{27}, in 1983, discuss the potential for abandoning an asset or a project early. This creates an ‘abandonment option’ which functions much like a put option. They use option pricing theory to calculate this option to abandon as an American put option on a dividend paying stock. This is counter to the traditional approach of estimating the life of an asset and assigning a salvage value. This work offers a fairly simple description of the impact of an abandonment option. It includes two qualitative examples regarding two different manufacturing plants for production of a product and the value of abandoning the construction.

In 1988, Robert Pindyck\textsuperscript{28} wrote a paper that provides a middle ground between conceptual and analytical introductions to the applications of options theory to real assets. The intent of the paper is to provide the reader with a method to determine optimal investment rules, to survey the literature outstanding, and to discuss policy implications of real options.

In addition, in 1993, Pindyck\textsuperscript{29} discusses irreversible investment decisions when projects have a certain lead-time and are subject to two different types of uncertainty. The first is technical uncertainty and the second is the input cost uncertainty. Technical uncertainty is uncertainty covering the actual completion of the project. Input cost uncertainty is uncertainty regarding costs that are external to the firm. These would include raw material costs, government regulations, etc. He derives a rule to maximize firm value using the decision to start or continue building a nuclear power plant as an example. The author states that technical uncertainty makes the investment more valuable while input cost uncertainty lowers the value.

Also, in 1987, Pindyck\textsuperscript{30} discusses the use of contingent claims analysis (options) to value investment decisions and outlays that are made sequentially. Traditional discounted cash-flow analysis treats sequential cash outlays as fixed in time and ignores the flexibility of timing and can understated the value of the project. The effects of time to build, opportunity cost, and uncertainty are all examined to derive optimal decision rules to value these investments. The authors show that for reasonable values, NPV analysis can lead to incorrect conclusions.

\textsuperscript{27} Myers, Stewart and Saman Majd, “Calculating Abandonment Value using Option Pricing Theory,” MIT Sloan School of Management Working Paper, No. 1462-83
\textsuperscript{28} Pindyck, Robert S., “Irreversibility, Uncertainty, and Investment,” Journal of Economic Literature (September 1991), pp. 1110-1148
\textsuperscript{29} Pindyck, Robert S., “Investments of uncertain cost,” Journal of Financial Economics (Volume 34, 1993), pp. 53-76
Robichek and Home\textsuperscript{31} wrote a paper that discusses the weakness of the Net Present Value Analysis in valuation. The primary weakness, according to the authors, is the fact that NPV analysis assumes that decisions cannot be made once the project has started. They use simulation and a binomial tree to value abandonment options of an asset. They show that the abandonment option can significantly increase the value of projects with an NPV that is close to zero. They show that including the value of the option can increase the project NPV and decrease the standard deviation of the project.

Sarah Ryan\textsuperscript{32}, in 2000, discusses the options values and the choice of a capacity expansion policy. Ryan states that a combination of demand uncertainty and lead-time for adding capacity create the risk of a shortage in production. The author creates a model for geometric demand growth and deterministic expansion lead-times. The author estimates the shortages that will result from different expansion timing policies with option valuation methods. The main thrust of the paper is to determine how to time expansions according to realized and forecast demand. The author also explores economies of scale and its impact on the model.

Siegal, Smith, and Paddock\textsuperscript{33} wrote a paper on how to value the real option of offshore oil properties utilizing financial options. The authors look at the similarities and differences between real and financial options. The authors conclude that the advantage of valuing with options vs. discounted cash flow analysis is most useful when the DCF analysis is very close to zero. Otherwise, the option valuation makes no difference.

Smith and Nau\textsuperscript{34} compare three different methods of valuing risky projects. They look at risk-adjusted discount-rate analysis, option pricing analysis, and decision analysis. The focus is on the latter two methods. The main contribution of this paper is that the authors show that when options pricing methods and when decision analysis methods are used, they must yield similar results. In addition, the authors demonstrate that the option pricing techniques can be used to simplify decision analysis techniques when some of the risks can be hedged through financial trading. In addition, they demonstrate how decision analysis techniques can be used to extend option pricing techniques in problems where there are incomplete security markets.

John E. Stonier\textsuperscript{35} discusses the pricing of options granted to airlines to reduce the lead-time of their airframe delivery. This is not an option to reduce manufacturing lead-time, but rather the option to "cut" the queue. He discusses basic option pricing.

\textsuperscript{34} Smith, James E. and Robert F. Nau, “Valuing Risky Projects: Option Pricing Theory and Decision Analysis,” \textit{Management Science}, Volume 41, Number 5, pp. 795-815
mechanisms including the Black-Scholes equation, decision tree analysis, and risk-adjusted binary trees and their application to this problem.

Alexander Triantis and James Hodder\textsuperscript{36} discuss the pricing of flexible production systems using contingent claims. Their approach allows a downward sloping demand curve for the underlying asset, which means that it is not a competitive market. They also allow for increasing marginal production costs, which is their exercise price.

Lenos Trigeorgis and Eero Kasanen\textsuperscript{37} wrote a paper on the necessity of investigating and accounting for options in strategic planning. The papers introduces the different types of options and describes their place in strategic planning. The authors also present an example of growth options as sequential projects over time, or staircase options. The authors state that capital budgeting cannot be thought of as a static activity but must, instead, be revisited constantly while paying particular attention to potential imbedded options.

Lenos Trigeorgis\textsuperscript{38} creates a new numerical method of pricing options through a transformed binomial method (a diffusion process). He compares his method with the Black-Scholes formula, the Compound Analytic method, the Quadratic method, the Johnson Method, the Finite-Difference Method, Numerical Integration, and the Cox-Ross-Rubenstein binomial method for pricing European Puts. He provides an example of a chemical company and examines five different types of methods: defer, abandon, contract, expand, and switch. He shows that the value of the combined options is not the sum but is rather less than the sum, about 70\%, due to the interaction amongst the options.

Lenos Trigeorgis\textsuperscript{39} discusses the impact of competitive analysis on the value of an investment when that investment can be priced as an American Call. He examines five different scenarios: no competition, preempt the competition by exercising immediately, exercise right before the competitor enters the market, exercise at the same time as the competitor enters the market, and deferring while the competition enters the market. He examines the various scenarios to better determine the optimal exercise price and concludes with a discussion of competitors arriving randomly.

Trigeorgis\textsuperscript{40} wrote a literature review of outstanding real options literature. He suggests possibilities for future research and provides an exhibit that shows most of the

\textsuperscript{36}Triantis, Alexander J. and James E. Hodder, "Valuing Flexibility as a Complex Option,"\textit{The Journal of Finance,} (Vol XLV, No 2), June 1990, pp. 549-565


\textsuperscript{40}Trigeorgis, Lenos, “Real Options and Interactions with Financial Flexibility,”\textit{Financial Management,} August 1993, pp. 202-224
analytical papers with the type of real option that they are discussing. The author also uses a company example to show the valuation of the option to defer, the option to stop, the option to default, the option to abandon, the option to switch, the option to contract, and how the growth options interact. He uses the binomial technique to calculate the value of the options to abandon, default, and defer.

Trigeorgis wrote a paper examining the nature of option interactions and capital budgeting projects that possess flexibility in the form of multiple real options. The option illustrates the importance of accounting for the various types of options and the order in which the options can be exercised. The paper shows that the incremental value of an option is worth less than the value of the option in isolation and the overall value declines as additional options are added.

## APPENDIX

<table>
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Table 9. Demand data for the steady demand scenario.

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Table 10. Demand data for the one period demand drop.
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Table 11. Demand data for the one period sharp demand drop.

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Table 12. Demand data for the huge drop.
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Table 13. Demand data for volatile.

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Table 14. Demand data for realistic volatile.