MASSACHUSETTS INSTITUTE OF TECHNOLOGY

15.053 – Optimization Methods in Management Science (Spring 2007)

Recitation 1, February 8th and February 9th, 2007

Problem 1: Production Planning for PepsiCo

The Wall Street Journal announced on February 1st that at PepsiCo, the bottling company for Pepsi, profits were flat over the past few quarters. They call on you, a top OR student, to help change this stagnation.

PepsiCo Company owns two different bottling plants that output different amounts of product for each hour run. If they run Plant 1 for a full hour, they will produce a certain number of cases of Pepsi, Sierra Mist, and Strawberry Fanta Light. PepsiCo must produce at least 12 tons of Pepsi, 8 tons of Sierra Mist, and 24 tons of Strawberry Fanta Light per day. It costs $20,000 per hour to run Plant 1, and costs and $16,000 per hour to run Plant 2. Each plant can be run for any amount between 0 and 8 hours per day. In one hour each plant produces the following tonnage:

	Pepsi	Sierra Mist	Diet Strawberry Fanta
Plant 1	3	1	2
Plant 2	1	1	6

Part A:

How can PepsiCo meet the requirements of the parent company at the lowest cost?

Part B:

Say there is an added constraint that Diet Strawberry Fanta can not make up more then 40% of the output, can we still model this as an LP?

Part C:

Suppose plant two must be run between 3 and 6 hours, incorporate this restriction into your LP.

Part D:

Suppose we can run Plant one for up to an additional two hours for an added cost of $30,000 for each additional hour we run the plant. Incorporate this option into your linear program

Problem 2: Conversion and Convexity

Consider the following Mathematical Program:

$$z = \min(\max(2a - 3b, -7a + 8b))$$
$$SubjectTo:$$
$$a - 3b \geq 8$$
$$|\, 9a - b \,| \leq 15.053$$
$$a, b \geq 0$$

Part A:

Is the objective function linear?

Part B:

Is it a convex function? A friendly function?

Part C:

Convert to a Linear Program

Problem 3: Automobile Manufacturing (Team Problem)

An automobile manufacturer produces three types of car known as T, C, and L. Each kind of car requires a certain amount of factory time to produce, and yields a certain profit. The factory has allocated 120 hours for next week. They want to produce cars to maximize their profits, while satisfy existing orders for the week (For example, they need to produce at least 15 T-cars.)

Part A:

Suppose there are three kinds of cars, known as T, C, and L with the following times to produce per car, profit per car, and dealer orders for each car.

Car	Time	Profit	Orders
T	1	$200	10
C	2	$500	20
L	3	$700	15

Write this problem as a linear program that maximizes profit.

Part B:

How could you generalize this model for a problem with N types of cars? Describe first a set of variables and notation you will need to do this.

Problem 4: School District Assignments (Team Problem)

Consider a school district with I neighborhoods, J schools and G grades at each school. Each school j has a capacity of C_{jg} for grade g. In each neighborhood i, the student populations of grade g is S_{ig}. The distance of School j from Neighborhood i is d_{ij}. Formulate a linear programming problem whose objective is to assign all students to schools, while minimizing the total distance traveled by all students. (You may ignore the fact that numbers of students must be integer.)

Problem 5: True or False(Optional)

The following is a linear program.

$$\begin{aligned} \text{Min} \quad & x_1 + x_2 \\ \text{st.} \quad & x_1 < 1 \\ & x_2 = 1 \\ & x_1 \geq 0 \; x_2 \geq 0 \end{aligned}$$

Problem 6: Britney and Kevin's Divorce Proceedings(Optional)

According to recent reports in US Weekly, after realizing Kevin trailer trash(Unlike Britney of course ☺) Britney has decided to leave Kevin. Currently they are trying to determine how to optimally divide some of their possessions. They hire "you" to try and help them accomplish this task. To start, you ask them to assign "value points" to all of the items under consideration such that they total 100. Their allocation is shown in the table below.

Item	Kevin's Allocation of Points	Britney's Allocation of Points
House	35	14
Children	10	43
Kevin's CDs	35	0
Wedding Jump Suits	10	29
Trailer Full of Garbage	10	14

Let us assume that the assets can be divided into fractional parts. For example, Kevin can get 47% of the Wedding Jump Suits while Britney can get the remaining 53%. The allocation should meet the following two criteria:

- Kevin's total point allocation for the items he receives should be the same as Britney's total point allocation for the assets she receives. This is the "jealousy prevention clause."
- The sum of the number of points they receive should be maximized.

Part A:

Develop an LP such that solving it will accomplish the task above.

Part B:

Modify the 100 point allocation for Britney while keeping Kevin's point allocation fixed so that there will be multiple optimal solutions for the LP from part A. Give two alternative optima for this revised problem.

Part C:

Does the LP (In Part A) have a feasible solution if they cannot subdivide items? Give a short two or three sentence explanation.

Part D:

Suppose a point is worth $100,000 to Britney. Suppose that Britney can spend $x to change the first clause to:

- Kevin's total point allocation for the items he receives should 10 less then Britney's total point allocation for the assets she receives. This is the "revised jealousy prevention clause."

Using Linear Programming determine how to find 'x'