13 Experimental attractors

In this brief lecture we show examples of strange attractors found in experiments.

13.1 Rayleigh-Bénard convection

Two dynamical variables, $\Delta T$ and $(\Delta T)'$, represent time-dependent thermal gradients measured by the refraction of light, inside the convecting system.

A 3-D phase space is defined by the coordinates

$$(\Delta T), (\dot{\Delta}T), \text{ and } (\Delta T)'$$.

The Poincaré section in the plane $(\Delta T), (\dot{\Delta}T)$ is obtained by strob ing the system at the dominant frequency of the fluctuations of one of the dynamical variables.

Like the Hénon attractor, the points are arranged in a “complex but well-defined structure” (BPV, p. 137).

13.2 Belousov-Zhabotinsky reaction

See Strogatz, Section 8.3.

The B-Z reaction is a particularly well studied chemical reaction that is of interest for

- dynamics, because it is oscillatory; and
- pattern formation and nonlinear waves.
An example of the pattern formation is

Strogatz, Plate 1, Section 8.3

For us the dynamical aspects are of greater interest. This chemical reaction, like many others, can be envisioned as a flux of reactants into a reactor, and a flux of products out of the reactor:

Nonlinearities arise from chemical reactions like

\[ A + B \rightarrow C \]

which yield terms like

\[ \frac{dC}{dt} = AB \]

The control parameter is typically the reactant flux.

The dynamical variable \( X(t) \) measures the concentration of a particular chemical species within the reactor.

Phase portraits are obtained in the 3-D phase space formed by

\[ X(t), X(t + \tau), X(t + 2\tau). \]

The periodic regime (limit cycle) is evident in the 2-D projection given by

BPV, Figure VI.26

The chaotic regime is evident in the 2-D projection given by

BPV, Figure VI.27
A Poincaré section reveals evidence of strong dissipation, as seen in a plot of intersections with a plane perpendicular to \(X(t), X(t + \tau)\):

**BPV, Figure VI.28**

The straightness of the line is insignificant, but its thinness indicates strong dissipation.

Due to dissipation, a nice 1-D first-return map can be formed by plotting \(X_{k+1}\) vs. \(X_k\), where \(X_k\) is the \(k\)th observation of \(X(t)\):

**BPV, Figure VI.29a**

That all the points essentially lie on a simple curve implies *deterministic order* in the system.

*Attraction* is demonstrated by inducing a small perturbation and then observing the re-establishment of the first return map:

**BPV, Figure VI.29b**

Sensitivity to initial conditions is observed by plotting the distribution of points that results after passage close to the same point:

**BPV, Figure VI.30**

Stretching and folding may also be observed. Plotting the data with a different choice of \(\tau\):

**BPV, Figure VI.31**

Nine different slices through the system reveal stretching (from 9 to 1) and folding (between 2 and 8):

**BPV, Figure VI.32**