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Quantum Cherenkov radiation and noncontact friction

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We present a number of arguments to demonstrate that a quantum analog of the Cherenkov effect occurs when two nondispersive half spaces are in relative motion. We show that they experience friction beyond a threshold velocity which, in their center-of-mass frame, is the phase speed of light within their medium, and the loss in mechanical energy is radiated through the medium before getting fully absorbed in the form of heat. By deriving various correlation functions inside and outside the two half spaces, we explicitly compute this radiation and discuss its dependence on the reference frame.

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I. INTRODUCTION

An intriguing manifestation of quantum theory in macroscopic bodies is the noncontact friction between objects in relative motion. For example, two surfaces (or half spaces) moving in parallel experience a frictional force if the objects’ material is lossy [1–3]. The origin of this force is the quantum fluctuations of the electromagnetic field within and between the objects; the same fluctuations also give rise to Casimir and van der Waals forces. In brief, quantum fluctuations induce currents in each object, which then couple to result in the interaction between them. For moving objects, a phase lag between currents leads to a frictional force between them.

For parallel plates (half spaces), the friction force is related to the amplitude of the reflected wave upon scattering of an incident wave from each surface (formalized into a reflection matrix below) [1,2]. Due to its quantum origin, friction persists even at zero temperature, where it is related to the imaginary part of the reflection matrix corresponding to evanescent waves. It is usually assumed that the dielectric function itself has an imaginary part due to dissipative properties of the material; this then leads to an imaginary reflection matrix and hence friction [4]. However, this is not necessary as, even for a vanishingly small loss, evanescent waves lead to an imaginary reflection matrix.

We consider nondispersive half spaces described by a real constant dielectric function, such that light propagates in the medium with a constant (reduced) speed. Note that the frequency independence of the dielectric function follows from a vanishing imaginary part due to Kramers-Kronig relations. We show that when the velocity of moving half spaces, in their center-of-mass frame, is larger than the phase speed of light in the medium, a frictional force arises between them. This is in fact a quantum analog of the well-known classical Cherenkov radiation. We elaborate on the relation between the friction and radiation in the gap as well as within the half spaces. We emphasize, however, that dispersive half spaces can experience friction at any velocity. Even nondispersive bodies moving nonuniformly experience vacuum friction at arbitrarily low speeds.

Quantum Cherenkov radiation was first discovered by Frank and Ginzburg [5] in a rather different setup. They argued that when an object (an atom, for example) moves inertially and superluminally, i.e., larger than the phase speed of light in a medium, it spontaneously emits photons; see Refs. [6,7] for subsequent reviews by Ginzburg. This phenomenon is intimately related to super-radiance, first discovered by Zel’dovich [8] in the context of rotating objects and black holes: A rotating body amplifies certain incident waves even if it is lossy. The underlying physics is that a moving object (atom) can lose energy by getting excited. This is because at superluminal velocities an excitation in the rest frame of the object corresponds to a loss of energy in the lab frame. Frank and Ginzburg refer to this eventuality as the anomalous Doppler effect [5] (see also Ref. [9]).

Since these unusual observations span several subfields of physics, we find it useful to demonstrate the results by a number of different formalisms. We first generalize the arguments by Ginzburg and Frank to prove dissipation effects associated with the relative motion of two parallel plates. We then use the input-output formalism of quantum optics to derive and compute the friction force based on scattering matrices. An alternative proof follows approaches introduced in the context of quantum field theory in curved space-time, making use of an inner product to identify the wave functions and their (quantum) character. Application of the latter formalism to vacuum friction is particularly suited to a real dielectric function. Finally, we employ the Rytov formalism [10], which is grounded in the fluctuation-dissipation theorem for electrodynamics and well known to practitioners of noncontact friction. We thereby extend previous results on friction and radiation in the gap between the half spaces to those within their medium, which is desired to establish a connection to the Cherenkov radiation.

To ease computations, however, we consider a scalar field theory as a simpler substitute for electromagnetism. The former shares the same conceptual complexity while being more tractable analytically. This is particularly useful in expressing complicated Green’s functions with points both inside and outside each half space, or within the gap between

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them. The generalization to vector and dyadic electromagnetic expressions should be straightforward but laborious. Finally, to avoid complications of the full Lorentz transformations, we limit ourselves to small velocities—both the relative velocity of the objects and the speed of light in their media.

The remainder of the paper is organized as follows. In Sec. II, existing formulas are used without proof to compute friction and to discuss the similarities with the classical Cherenkov effect. In this section, we elaborate on friction in a specific example. In Sec. III, we consider a general setup, argue for and derive the friction force, as well as emitted radiation, in great detail. This section comprises four subsections each devoted to one particular formalism. Specifically, we discuss how the radiation within the half spaces, and in the gap, depend on the reference frame.

II. FRICTION

We start with a scalar model that is described by a free field theory in empty space, while inside the medium a “dielectric” (or, a response) function ε is assumed which characterizes the object’s dispersive properties. The field equation for this model reads

$$\left(\nabla^2 + \epsilon(\omega, \mathbf{x}) \frac{\omega^2}{c^2}\right) \Phi(\omega, \mathbf{x}) = 0,$$

with ε = 1 in the vacuum, and a frequency-dependent constant inside the medium.

We consider the configuration of two parallel half spaces in D spatial dimensions, separated by a vacuum gap of size d. For each half space in its rest frame, a plane wave of frequency ω and wave vector k is reflected with amplitude (“reflection matrix”)

$$R_{\omega k} = \frac{-\sqrt{\frac{\epsilon(\omega, \mathbf{x})}{\epsilon(\omega, \mathbf{x})}} - \sqrt{\frac{\omega^2/c^2 - k_\perp^2}{\omega^2/c^2 - k_\parallel^2}}}{\sqrt{\frac{\epsilon(\omega, \mathbf{x})}{\epsilon(\omega, \mathbf{x})}} + \sqrt{\frac{\omega^2/c^2 - k_\perp^2}{\omega^2/c^2 - k_\parallel^2}}},$$

where k_0 is the component of the wave vector parallel to the surface. This result is easily obtained by solving the field equations inside and outside the half space, and matching the reflection amplitude to satisfy the continuity of the field and its first derivative along the boundary. We are particularly interested in friction at zero temperature, which is mediated solely by evanescent waves [1–3]; for further discussion see Ref. [11]. Such waves contribute to friction through the imaginary part of the reflection matrices. If one half space moves laterally with velocity v along the x axis while the other is at rest, the friction force is given by (introducing the notation dx = dx/2π)

$$f = \int_0^\infty d\omega L_{D-1} \int d\omega k_\perp \hbar k_\parallel e^{-2|\omega|/c} \left(2 \text{Im } R_1 \right) \left(2 \text{Im } R_2 \right) \frac{e^{-2|\omega|/c} \left(1 - e^{-2|\omega|/c} R_1 R_2 \right)^2}{\left(1 - e^{-2|\omega|/c} R_1 R_2 \right)^2} \times \Theta(-\omega + v k_\perp),$$

where Θ is the Heaviside step function, k_⊥ = √ω^2/c^2 - k_∥^2, and L_{D-1} is the area. Note that the reflection matrix of the static half space is given by Eq. (2) but that of the moving half space is obtained after Lorentz transformation to the laboratory frame.

We leave the derivation and extension of Eq. (3) to the next section but discuss its implications here. While this equation has been studied extensively in the literature, it is usually assumed that the dielectric medium is lossy, with a nonzero imaginary part of ε. However, even when Im ε is vanishingly small, a frictional force can be obtained as follows. With Im ε ≈ 0, the medium can be characterized by the modified speed of light v_0 = c/√ε. The only relevant length scale in the problem (aside from the overall area L_{D-1}) is the separation d. We can then construct the frictional force on purely dimensional grounds as

$$f = \frac{\hbar v_0 L_{D-1}^{-1}}{d^{D+2}} g \left(\frac{v}{v_0}, \frac{v}{c}\right),$$

where g is a function of two dimensionless velocity ratios. Any velocity could have appeared as prefactor (with a correspondingly modified function g); we have chosen v_0 for convenience. For small velocities, the dependence on vacuum light velocity c drops out^1 and

$$f \approx \frac{\hbar v_0 L_{D-1}^{-1}}{d^{D+2}} g \left(\frac{v}{v_0}\right),$$

with g depending only on the ratio of the velocity v to the light speed in the medium v_0. Interestingly, at small v, only the modified speed within the media is relevant. Our assumption pertains to the nonretarded limit when the speed of light can be formally taken to c → ∞. Barton has also considered the same limit in Ref. [12], where he computes the frictional (drag) force between weakly dissipative media described by the Drude model, hence obtaining different power laws in the limit of zero temperature.

Now note that the Heaviside function in Eq. (3) restricts us to frequencies

$$(0 <) \omega < v k_\perp.$$

Furthermore, the imaginary part of the reflection matrix R_1, given by Eq. (2), is only nonzero when ω^2/c^2 - k_2^2 < 0 and εω^2/c^2 - k_\parallel^2 > 0, which, in turn, implies

$$|\omega| > v_0 |k_\parallel| > v_0 |k_\perp|.$$  

A similar condition holds for the second half space: |ω'| > v_0 |k_\prime_\perp| with primed values defined in the moving reference frame. For simplicity, we assume that v, v_0 ≪ c, and thus neglect the complications of a full Lorentz transformation. Hence, ω' ≈ ω - vk_∥ and k_\prime_\perp ≈ k_\perp - v_0ω/c^2 ≈ k_\perp. Then the analog of Eq. (6) for the second half space reads

$$|\omega - vk_\perp| \geq v_0 |k_\perp|.$$  

The above conditions limit the range of integration to

$$k_\perp > 0, \quad \text{and } v_0 k_\perp < \omega < (v - v_0) k_\perp.$$  

One then finds the minimum velocity where a frictional force arises as

$$v_{\min} = 2v_0 = 2 \frac{c}{\sqrt{\epsilon}}.$$  

---

1One can see this explicitly from Eq. (3).
This threshold velocity is reminiscent of the classical Cherenkov effect, although larger by a factor of two. However, in the center-of-mass frame where the two half spaces move at the same velocity but in opposite directions, we find the same condition as that of the Cherenkov effect: A frictional force arises when, in the center-of-mass frame, the half spaces’ velocity exceeds that of light in the medium. As a specific example, we consider a two-dimensional space, i.e., surfaces represented by straight lines. The dependence of the friction force on relative velocity is then plotted in Fig. 1.

III. FORMALISM AND DERIVATION

In the previous section we argued for the appearance of friction between moving parallel plates, which is reminiscent of the Cherenkov effect. Establishing a complete correspondence requires a full analysis of the radiation within each object. In this section, we provide several arguments to demonstrate why and how a fluctuation-induced friction arises in the context of macroscopic objects in relative motion. Our presentation is not a repetition of the existing literature. Friction, as well as radiation within the gap, are obtained through our methodologies, and further extended to compute radiation inside the media. We start with a heuristic argument, making a connection with the Frank-Ginzburg condition. We then present three distinct derivations of the friction force using techniques developed in different fields. The first method relies on the input-output formalism in a second-quantized picture; the second one appeals to quantum field theory in curved space-time. The last two approaches can be applied to quantum friction between moving half spaces. The last, and the longest, derivation is based on fluctuation-dissipation theorem, or the closely related Rytov formalism. The advantage of the latter approach is in finding correlation functions inside and outside the two half spaces which can be used to compute the radiation within each half space and in the gap between them.

A. Why is there any friction/radiation?

To start with, let us consider a space-filling dielectric medium described by a constant real \( \epsilon \). A wave described by wave vector \( \mathbf{k} \) satisfies the dispersion relation \( \omega = v_0|\mathbf{k}| \), with \( v_0 = c/\sqrt{\epsilon} \) being the speed of light in the medium. This relation describes the spectrum of quantum field excitations. If the medium is set in motion, the new spectrum can be deduced simply by a Lorentz transformation from the static to the moving frame. Assuming again that the speed of light in medium is small (or \( \epsilon \) is large), we find

\[
\omega = v_0|\mathbf{k}| + vk_x ,
\]

with the medium moving with speed \( v \) parallel to the \( x \) axis.

Next consider two (semi-infinite) media one of which moves laterally with velocity \( v \), whereas the other is at rest. Although the boundaries modify the dispersion relation, we may assume that Eq. (10) approximately describes each medium (with \( v = 0 \) for the stationary body). This is justified by considering wave packets away from boundaries. We thus have two distinct spectra: The spectrum for the half space at rest is akin to a cone while that of the moving half space is tilted towards the positive \( x \) axis, as in Fig. 2. For the sake of simplicity, we limit ourselves to \( \mathbf{k} = k_x \hat{x} \). Let us consider the (spontaneous) production of two particles, one in each medium. Since linear momentum is conserved the two particles must have opposite momenta (\( k_x \) and \( -k_x \)). This process is energetically favored if the sum of the energy of the two particles is negative, that is, spontaneous pair production occurs if it lowers the energy of the composite system. This condition is satisfied when

\[
\omega_1 + \omega_2 = 2v_0|k_x| + vk_x < 0 ,
\]

which is possible only if \( v > 2v_0 \). We stress that our argument is not specific to a particular reference frame. If both half spaces are moving with velocities \( v_1 \) and \( v_2 \), the velocity \( v \) in Eq. (11) is replaced by \( v_2 - v_1 \), thus this equation puts a bound on the relative velocity.

The above argument is similar to the Landau criterion for obtaining the critical velocity of a superfluid flowing past a wall [13]. The instability of the quantum state against spontaneous production of elementary excitations (and vortices) breaks the superfluid order beyond a certain velocity. Quantum friction provides a close analog to Landau’s argument in the context of macroscopic bodies. The same line of reasoning is adopted in the work of Frank and Ginzburg [5–7]. While this argument

![Figure 1](https://example.com/fig1.png)

**FIG. 1.** (Color online) Friction depends on velocity \( v \) through the function \( g \). Below a certain velocity, \( v_{\text{min}} = 2v_0 = 2c/\sqrt{\epsilon} \), the friction force is zero; it starts to rise linearly at \( v_{\text{min}} \), achieves a maximum, and then falls off.

![Figure 2](https://example.com/fig2.png)

**FIG. 2.** (Color online) The energy spectra for a medium at rest (solid curve denoted by \( \omega_1 \)) and a moving medium (dashed curve denoted by \( \omega_2 \)), in the \((\omega, k_x)\) plane. The spectrum for the moving medium is merely tilted. The production of a pair of excitations, indicated by solid circles at opposite momenta, is energetically possible for \( v > 2v_0 \).
correctly predicts the threshold velocity for the onset of friction, it does not quantify the magnitude of friction and its dependence on system parameters.

## B. The input-output formalism

The input-output formalism deals with the second quantized operators corresponding to incoming and outgoing wave functions, relating them through the classical scattering matrix [14–17]. From the (known) distribution of the incoming modes, one can then determine the outflux of the outgoing quanta. The input-output formalism has been used to study the dynamical Casimir effect—a consequence of the quantum field theory in the presence of moving boundaries [18]—in theoretical [19,20] as well as experimental [21] contexts, and recently generalized in application to lossy objects by [14–17]. From the (known) distribution of the incoming operators corresponding to incoming and outgoing wave functions, one can then determine the outflux of the outgoing modes, one can then compute the expected flux of the outgoing modes, \( \langle a_{1}^{\text{out}} | a_{1}^{\text{in}} \rangle \). At zero temperature, \( \langle a_{1}^{\text{out}} | a_{1}^{\text{in}} \rangle = 0 \), and the only contribution to the outflux is due to the second term in the right-hand side of Eq. (15), resulting in

\[
\langle a_{1}^{\text{out}} | a_{1}^{\text{in}} \rangle = \Theta(vk_x - \omega) |S_{21}|^2, \tag{16}
\]

with \( \Theta \) being the Heaviside step function. The friction, or the rate of the lateral momentum transfer from one half space to another, is then

\[
f = \int_0^\infty d\omega \int d\mathbf{k}_1 \ h \kappa \Theta(vk_x - \omega) |S_{21}|^2. \tag{17}
\]

Similarly, the energy radiation can be computed by replacing \( h \kappa \) by \( \hbar \omega \) in the last equation.

It is a simple exercise to compute the scattering matrix. By exploiting the continuity equations (of the field and its first derivative) at the interface of the two half spaces and the gap, we find the set of equations

\[
1 + S_{11} = A + B, \quad i\kappa_\perp (1 - S_{11}) = |k_\perp| (A - B), \tag{18}
\]

\[
Ae^{k_\perp |d|} + Be^{-k_\perp |d|} = \sqrt{k_\perp} S_{21} e^{\kappa_\perp d}, \tag{19}
\]

where \( S \) is the 2 \times 2 scattering matrix, and the dependence on \( \omega \) and \( \mathbf{k}_1 \) is implicit. The scattering matrix can be straightforwardly computed by matching the wave function and its first derivatives along the boundaries. Note that a scattering channel relates a wave function labeled by \((\omega, \mathbf{k}_1)\) on one half space to \((\omega', \mathbf{k}_1')\), the frequency and wave vector as seen from the moving frame on the second half space. At small velocities, we have \( \omega' \approx \omega - vk_x \) and \( \mathbf{k}_1' \approx \mathbf{k}_1 \). Therefore, a positive-frequency mode on one half space can be coupled to a mode with negative frequency on the other half space. However, as remarked above, an operator \( a_{\mathbf{k}_1} \) with negative \( \omega \) is, in fact, a creation operator. This mixing between positive and negative frequencies is at the heart of the dynamical Casimir effect [4,22]. In a frequency window where this mixing occurs, i.e., for \( 0 < \omega < vk_x \), the input-output relation is recast as

\[
d^{\text{out}}_{1, \omega \mathbf{k}_1} = S_{11} d^{\text{in}}_{1, \omega \mathbf{k}_1} + S_{21} d^{\text{in}}_{2, \omega \mathbf{k}_1 - \mathbf{k}_1}. \tag{15}
\]
[Note that $R_1 = -\langle \hat{k}_\perp - i k_\perp \rangle / \langle \hat{k}_\perp + i k_\perp \rangle$ and similarly for $R_2$ with $\hat{k}_\perp \rightarrow \hat{k}_\perp'$.] It is immediately clear that Eq. (17) reproduces the friction in Eq. (3). Indeed, the input-output formalism makes the derivation rather trivial.

Equations (14) and (15) relate annihilation and creation operators which satisfy canonical commutation relations,

$$\left[ a_{\text{out/in}}^{\dagger} | k_\perp \rangle, a_{\text{out/in}} | k_\perp \rangle \right] = \text{sgn}(\omega) \delta_{ij} . \quad (20)$$

The function $\text{sgn}(\omega)$ merely indicates that, for negative frequencies, the creation and annihilation operators should be identified correctly. The above canonical relations applied to Eq. (15) yield

$$1 - |S_{11}|^2 = \text{sgn}(\omega - vk_\perp)|S_{21}|^2 , \quad (21)$$

implying that the scattering amplitude corresponding to the backscattering in the first medium is larger than unity for $\omega < vk_\perp$. This is an example of the so-called superscattering due to Zel’dovich [8]. For certain modes, a moving (rotating) object amplifies incoming waves, indicating that energy due to Zel’dovich [8].

Equations (17) and (21) can be combined to yield

$$\mathcal{P} = \int_0^\infty \tilde{d} \omega L^{D-1} \int \tilde{d} k_\perp \hbar \omega \Theta(vk_\perp - \omega) (|S_{11}|^2 - 1) . \quad (22)$$

This expression is indeed very similar to quantum radiation from a rotating object (a rotating black hole in Ref. [26] or a rotating cylinder in Ref. [24]) with the following substitutions: $v \rightarrow \Omega$ (linear to angular velocity), $k_\perp \rightarrow m$ (linear to angular momentum), and $S_{11}$ to be replaced by the scattering matrix of the rotating body.

It is worth noting that the only contribution to the radiation is from modes with $k_\perp > \omega / v > \omega / c$, corresponding to evanescent waves in the gap between the two half spaces. In this respect, the radiation is a quantum tunneling process across a barrier (in this case, the gap). We can recast the wave equation in a fashion similar to the Schrödinger equation as

$$\left( - \frac{\delta^2}{\delta x^2} + V \right) \psi = 0 , \quad (23)$$

with $V_1 = -(\epsilon \omega^2 / c^2 - k_\perp^2)$, $V_2 = -(\epsilon (\omega - vk_\perp) / c^2 - k_\perp^2)$, and $V_{gap} = |\omega^2 / c^2 - k_\perp^2|$; see Fig. 3. The relative motion of the two media results in a steady tunneling of particles of opposite momenta from one half space to another, thus leading to the slowdown of the motion.

C. Inner-product method

In this section, we describe a method which is widely used in application to quantum field theory in curved space-time, or in the presence of moving bodies. However, the following discussion applies this method to the problem of moving half spaces, which is possible only when the dielectric function is taken to be a real constant.

To quantize a field theory, a first step is to decompose the quadratic part of the Hamiltonian into a collection of (infinite) harmonic oscillators, define the corresponding annihilation and creation operators, and impose canonical commutation relations. One can then construct the Fock space with the vacuum state of no particles and excited single- and multi-particle states obtained by applying creation operators. In high-energy physics, the usual starting point is empty space, but the above procedure works equally well in the presence of background matter, as is the case for the Casimir effect. The reason is that canonical quantization only relies upon time translation and time reversal symmetry. The former allows construction of eigenmodes of a definite frequency, which is the basis of the notion of modes/quanta/particles. Time reversal symmetry, on the other hand, is used to identify creation and annihilation operators. The coefficient of a positive (negative) frequency mode is understood as an annihilation (creation) operator. To make this correspondence explicit, consider a quantum field $\Phi(t,x)$, possibly in the presence of a background medium which is static. One can find a basis of eigenmodes $\phi_{\omega\alpha}(x)$ labeled by frequency $\omega$ and quantum number $\alpha$ to expand the field as

$$\Phi(t,x) = \sqrt{\frac{n}{2}} \sum_{\omega>0, \alpha} e^{-i\omega t} \phi_{\omega\alpha}(x) \hat{a}_{\omega\alpha} + e^{i\omega t} \phi_{\omega\alpha}^*(x) \hat{a}_{\omega\alpha}^\dagger , \quad (24)$$

with [defining $\delta(x) = 2\pi \delta(x)$]

$$[a_{\omega\alpha}, \hat{a}_{\omega'\beta}^\dagger] = \delta(\omega_1 - \omega_2) \delta_{\omega\beta} . \quad (25)$$

The latter follows from the canonical commutation relations between the field $\Phi(t,x)$ and its conjugate momentum $\Pi(t,y)$,

$$[\Phi(t,x), \Pi(t,y)] = i\hbar \delta(x - y) . \quad (26)$$

When the object is moving, we lose one or both symmetries in time. The case of two parallel plates in lateral motion
respects time translation symmetry as the relative position does not change. Time reversal symmetry, on the other hand, is broken; in the backward direction of time the half space moves in the opposite direction. In the absence of time reversal symmetry, the correspondence between positive (negative) frequency and the annihilation (creation) operators breaks down. There is, however, a more general way to identify operators as follows. Let us consider two functions \( \phi_1 \) and \( \phi_2 \), which are solutions to the classical field equation, and define an inner product as [27–29]

\[
\langle \phi_1, \phi_2 \rangle = \frac{i}{2} \int dx (\phi_1^* \dot{\phi}_2 - \phi_1^* \dot{\phi}_2),
\]

(27)

where \( \pi_i \) is the corresponding conjugate momentum, and the integral is over the whole space. One can easily see that the inner product defined in Eq. (27) is independent of the choice of the reference frame or the (spacelike) hypersurface as the inner product defined in Eq. (27) is still a good quantum number because of the super)unitary relation in Eq. (21) is essential in deriving the δ functions. Note that only the positive-norm norms are diagonal in frequency to compute the δ functions. The integral over the gap is neglected as it does not contribute to the frequency

\[
\delta = \delta(\omega_1 - \omega_2) \delta(|k_2| - |l_2|), \quad \omega_1 > 0,
\]

(33)

and

\[
\delta = \text{sgn}(\omega_1 - v k_x) \delta(\omega_1 - \omega_2) \delta(k_1 - l_1), \quad \omega_1 > 0.
\]

(34)

To obtain these relations, we have exploited the fact that the integrals are diagonal in frequency to compute the δ functions. In Eqs. (33) and (34) which are then converted to those of \( \omega \). The integral over the gap is neglected as it does not contribute to the frequency \( \delta \) functions. Note that the (super)unitary relation in Eq. (21) is essential in deriving the norms. Functions of type I have positive norm so they serve as the coefficients of annihilation operators. However, type-II functions include negative-norm modes for \( 0 < \omega < v k_x \).

Therefore, despite the positive sign of frequency, the latter should be identified as creation operators. We thus expand the field as

\[
\frac{\hat{\Phi}(t, \mathbf{x})}{\sqrt{\hbar/2}} = \sum_{\omega > 0, \mathbf{k}_1} e^{-i\omega t} \phi_{\omega \mathbf{k}_1}^\dagger(\mathbf{x}) \hat{\alpha}_{\omega \mathbf{k}_1} + e^{i\omega t} \phi_{\omega \mathbf{k}_1}(\mathbf{x}) \hat{\alpha}^\dagger_{\omega \mathbf{k}_1}.
\]

(30)

where the summation is a shorthand for multiple integrals, and \( \hat{\alpha} \) and \( \hat{\alpha}^\dagger \) satisfy the usual commutation relations. The friction is given by the rate of the lateral momentum transfer,

\[
\frac{f}{L^{D-1}} = \langle \partial_t \phi, \partial_t \Phi \rangle = \int_0^\infty \tilde{d} \omega \int \tilde{d} k_x \frac{\hbar}{2} \Theta(\omega v k_x - \omega k_x^2) \langle |S_{12}|^2 \rangle \]

(36)

We have again exploited Eq. (21) and arrived at the same results as in the previous sections. Notice that only the super-radiating modes contribute to the radiation while other modes cancel out in the second line of the last equation.
D. Radiated energy: The Rytov formalism

In this section, we employ the Rytov formalism [10] to study the correspondence between friction and radiation in some detail. This formalism is based on the fluctuation-dissipation theorem and has been extensively used in the context of noncontact friction [2]; see also Ref. [3] and citations therein. This section goes beyond the existing literature by computing various correlation functions and the radiated energy inside the half spaces (as well as in the gap between them), thus making an explicit connection to Cherenkov radiation. Specifically, we discuss the dependence of various quantities on the reference frame. While the radiation in the gap depends on the reference frame (in the center-of-mass frame, the latter is simply zero), the radiation within the two half spaces is invariant and presents a close analog to classical Cherenkov radiation.

We start by relating fluctuations of the field to those of “sources” within each medium by

\[- \left( \Delta + \frac{a^2}{c^2} \varepsilon(\omega, x) \right) \Phi(\omega, x) = -i \frac{\omega}{c} \rho_\omega(x) . \tag{37} \]

The “charge” \( \rho \) fluctuates around zero mean with correlations (covariance)

\[ \langle \rho_\omega(x) \rho_\omega^*(y) \rangle = a(\omega) \text{Im} \varepsilon(\omega, x) \delta(x - y) , \tag{38} \]

where

\[ a(\omega) = 2 \hbar \left[ n(\omega, T) + \frac{1}{2} \right] = \hbar \coth \left( \frac{\hbar \omega}{2 k_B T} \right) . \tag{39} \]

Note that the source term on the right-hand side of Eq. (37) comes with a coefficient linear in frequency reminiscent of a time derivative, the reason being that the source couples to the time derivative of the field just in the same way that the response function \( \varepsilon \) correlates the time derivatives of the field at different times.

The field is related to the sources via the Green’s function \( G_r \), defined by

\[- \left( \Delta + \frac{a^2}{c^2} \varepsilon(\omega, x) \right) G(\omega, x, z) = \delta(x - z) . \tag{40} \]

In equilibrium (uniform temperature and static), this results in the field correlations

\[ \langle \Phi(\omega, x) \Phi^*(\omega, y) \rangle = \frac{a^2}{c^2} \int d^3z \int_1^\infty G(\omega, x, z) G^*(\omega, y, z) \langle \rho_\omega(z) \rho_\omega^*(z) \rangle \]

in agreement with the fluctuation-dissipation condition which relates correlation functions to dissipation through the imaginary part of the response function. Note that the second line in Eq. (41) follows from \( \frac{a^2}{c^2} \text{Im} \varepsilon = - \text{Im} \frac{1}{G_r} \) according to Eq. (40).

For the case of two half spaces, we first compute the correlation function for two points in the gap. The source fluctuations in each half space will be treated separately, starting with those in the static half space (indicated by subindex 1 on the integral):

\[
\langle \Phi(\omega, x) \Phi^*(\omega, y) \rangle_1 = \frac{a^2}{c^2} a_1(\omega) \int d^3z G(\omega, x, z) \text{Im} \varepsilon(\omega, z) G^*(\omega, y, z)
\]

\[
= \frac{a^2}{2c^2} a_1(\omega) \int d^3z \left[ \varepsilon(\omega, z) G(\omega, x, z) \right] G^*(\omega, y, z) - G(\omega, x, z) \left[ \varepsilon(\omega, z) G(\omega, y, z) \right]^*
\]

\[
= \frac{i}{2} a_1(\omega) \int d^3z \left[ \Delta_1 G(\omega, x, z) \right] G^*(\omega, y, z) - G(\omega, x, z) \Delta_1 G^*(\omega, y, z)
\]

\[
= \frac{i}{2} a_1(\omega) \int d^3z \left[ \left[ \nabla_1 G(\omega, x, z) \right] G^*(\omega, y, z) - G(\omega, x, z) \nabla_1 G^*(\omega, y, z) \right] . \tag{42}
\]

Note that we used Eq. (40) in going from the second to the third line above, and then integrated by parts to obtain an integral over the surface adjacent to the gap. The contribution due to the other surface at infinity vanishes since \( \varepsilon \) is assumed to have a vanishingly small imaginary part. This assumption is a rather technical point which also arises for the dielectric response of the vacuum in the context of a single object out of thermal or dynamical equilibrium with the vacuum [24,30].

To compute the surface integral in Eq. (42), one needs the (out-out) Green’s function with both points in the gap. The latter is given by Eq. (A1) and leads to

\[
\langle \Phi(\omega, x) \Phi^*(\omega, y) \rangle_1 = -\sum_{\alpha} a_1(\omega) \frac{|e^{ip_\alpha d}|^2}{4 \rho_\alpha^2} \left[ 1 - e^{2ip_\alpha d} R_\alpha \bar{R}_\alpha \right] |U(R)|^2 \times \left( \Phi_{\alpha, reg}(\omega, \tilde{x}) + \bar{R}_\alpha \Phi_{\alpha, out}(\omega, \tilde{x}) \right) \times \left( \Phi_{\alpha, reg}^*(\omega, \tilde{y}) + \bar{R}_\alpha \Phi_{\alpha, out}^*(\omega, \tilde{y}) \right) . \tag{43}
\]
from a reference point on the surface of the first (second) half space—the reference points on two surfaces have identical parallel components, $x_i = x_i$, but differ in their $z$ component as $z_k = d - z_x$. The regular and outgoing functions are defined with respect to the corresponding half space; see Appendix for more details. Furthermore, the overbar notation implies complex conjugation, and $|U(R_a)|^2$ is defined as

$$
|U(R_a)|^2 = \int d\Sigma \left[ \Phi^*_\alpha \Phi_\alpha - (\nabla \Phi^*_\alpha \cdot \nabla \Phi_\alpha) \right],
$$

with $\Phi_\alpha = \Phi^\text{reg}_\alpha(x, z) + R_\alpha \Phi^\text{out}_\alpha(x, z)$, such that

$$
|U(R_a)|^2 = 1 - |R_\alpha|^2, \quad \alpha \text{ propagating waves},
$$

$$
|U(R_a)|^2 = 2 \text{Im} R_\alpha, \quad \alpha \text{ evanescent waves}.
$$

One can similarly find the correlation function due to source fluctuations in the second half space

$$
\langle \Phi(\omega, x)\Phi^*(\omega, y) \rangle = \langle \Phi(\omega, x)\Phi^*(\omega, y) \rangle_1 + \langle \Phi(\omega, x)\Phi^*(\omega, y) \rangle_2.
$$

The frictional force is then computed as the average of the appropriate component of the stress tensor as

$$
f = \int_{-\infty}^{\infty} d\omega \int d\Sigma \left\{ \partial_\omega \Phi(\omega, x) \partial_\omega \Phi^*(\omega, x) \right\}
$$

$$
= \int_{-\infty}^{\infty} d\omega L^{D-1} \int d\mathbf{k}_\parallel \hbar \omega \left[ e^{i[k_\parallel \cdot \mathbf{r}]} / |1 - e^{2ik_\parallel \cdot \mathbf{R}_{\text{in}}}| \right]
$$

$$
\times |U(R_{\text{in}})|^2 |U(R_{\text{out}})|^2 [n_1(\omega) - n_2(\omega - \mathbf{v} \cdot \mathbf{k})],
$$

where we have restored $k_\parallel$ in place of $\alpha$. Further manipulations lead to Eq. (3). Similarly, the Rytov formalism allows us to calculate the energy flux from one object to the other as

$$
P_{\text{gap}} = \int_{-\infty}^{\infty} d\omega \int d\Sigma \left\{ \partial_\omega \Phi(\omega, x) \partial_\omega \Phi^*(\omega, x) \right\}
$$

$$
= \int_{-\infty}^{\infty} d\omega L^{D-1} \int d\mathbf{k}_\parallel \hbar \omega \left[ e^{i[k_\parallel \cdot \mathbf{r}]} / |1 - e^{2ik_\parallel \cdot \mathbf{R}_{\text{in}}}| \right]
$$

$$
\times |U(R_{\text{in}})|^2 |U(R_{\text{out}})|^2 [n_1(\omega) - n_2(\omega - \mathbf{v} \cdot \mathbf{k})],
$$

i.e., by merely replacing $\hbar k_\parallel$ with $\hbar \omega$ in Eq. (47).

Next we compute the energy flux through each half space. Since the dielectric function is assumed to be a real constant (albeit with a vanishingly small imaginary part), we can circumvent ambiguities in defining the Maxwell stress tensor in a lossy medium [31]. In the following, we find the field correlation function in the first (static) half space due to source fluctuations in the moving half space by using an analog of Eq. (42) but evaluating a surface integral on the second half space. However, in this case, the appropriate Green’s function is the (out-in) type given in Eq. (A4). We then find the latter correlation function as

$$
\langle \Phi(\omega, x)\Phi^*(\omega, y) \rangle_2 = \sum \alpha \frac{\omega^2(\omega - \mathbf{v} \cdot \mathbf{k}_\parallel)}{4 \hbar^2} \frac{\left| e^{i[k_\parallel \cdot \mathbf{r}]} / |1 - e^{2ik_\parallel \cdot \mathbf{R}_{\text{in}}}| \right|}{\left| U(R_{\text{in}}) \right|^2}
$$

$$
\times \left[ V_{\text{in}} \psi^*_\alpha(\omega, \mathbf{x}) + W_{\text{in}} \psi'_\alpha(\omega, \mathbf{x}) \right]
$$

$$
\times \left[ V_{\text{out}} \psi^*_\alpha(\omega, \mathbf{y}) + W_{\text{out}} \psi'_\alpha(\omega, \mathbf{y}) \right].
$$

where $\mathbf{x}$ and $\mathbf{y}$ are both inside the first half space, $V$ and $W$ are coefficients depending on $\omega$ and system parameters, and the functions $\psi$ are defined inside the medium; see Appendix for more details. Henceforth, we assume that the objects are at zero temperature. Anticipating that only evanescent waves contribute, we obtain the energy flux in the first half space due to the fluctuations in the second half space. Noting that the “Poynting vector” is defined as $\partial_t \Phi \partial_\omega \Phi$ even within the dielectric medium, we find

$$
P_{1}^{(1)} = \int_{-\infty}^{\infty} d\omega L^{D-1} \int d\mathbf{k}_\parallel \hbar \omega \left[ e^{i[k_\parallel \cdot \mathbf{r}]} / |1 - e^{2ik_\parallel \cdot \mathbf{R}_{\text{in}}}| \right]
$$

$$
\times 2 \text{Im} R_{\text{in}} \text{Im} \mathbf{R}_{\text{in}} |\mathbf{v} - \mathbf{k}_\parallel|,
$$

where we have used the fact that, for evanescent waves, $p_\alpha \equiv \mathbf{k}_\parallel = \sqrt{\omega^2 c^2 - k_\parallel^2}$ is purely imaginary while $\hat{p}_\alpha \equiv \mathbf{k}_\perp = \sqrt{\omega^2 c^2 - k_\parallel^2}$ is real, leading to

$$
|V_{\text{in}}|^2 - |W_{\text{in}}|^2 = \left| p_\alpha / \hat{p}_\alpha \right|^2 2 \text{Im} R_{\text{in}}.
$$

In order to take into account the source fluctuations in the first half space (where we compute the field correlation function), we need the (in-in) Green’s function in Eq. (A7). The energy flux due to the latter fluctuations $P_{1}^{(1)}$ is computed similarly but there is one subtlety. Unlike the previous cases, the correlation function is evaluated at points where there are also fluctuating sources. However, Eq. (42) contains, beyond the surface integral, a term proportional to $\text{Im} G(\omega, x, y)$ which does not contribute to the radiation. The remaining computation is similar to the previous case, and the overall energy flux is obtained as

$$
P^{(1)} = \int_{-\infty}^{\infty} d\omega L^{D-1} \int d\mathbf{k}_\parallel \hbar \omega \left[ e^{i[k_\parallel \cdot \mathbf{r}]} / |1 - e^{2ik_\parallel \cdot \mathbf{R}_{\text{in}}}| \right]
$$

$$
\times \Theta(\mathbf{v} \cdot \mathbf{k} - \omega).
$$

(52)

This is again in harmony with the results in the previous sections.

Comparing Eqs. (52) and (48), we observe that in the reference frame in which the first half space is at rest,

$$
P^{(1)} = P_{\text{gap}}.
$$

(53)

However, $P_{\text{gap}}$ must vanish in the center of mass (c.m.) frame from symmetry considerations. It can be obtained explicitly by a Lorentz transformation from the laboratory frame, which, to the lowest order in velocity, takes the form

$$
0 = P_{\text{gap}} = P_{\text{gap}} - v f / 2,
$$

(54)
indicating $P^{(1)} = v f / 2$. This conclusion can be verified directly as follows. First note that Eqs. (47) and (52) yield

$$
P^{(1)} - \frac{v f}{2} = \int_{k_i, i = 0} d k_i \int_{0}^{\frac{1}{2} v k_i} d \omega h (\omega - \frac{v k_i}{2})\left(1 + \frac{2 \text{Im} R_{\omega, k_i} 2 \text{Im} R_{\omega, k_i}}{|1 - e^{2 i k_i d} R_{\omega, k_i} R_{\omega, k_i}|^2}\right), \tag{55}
$$

where the $x$ axis is chosen parallel to the velocity $v$. Let us make the following change of variables:

$$\omega' = \omega - \frac{v k_i}{2}, \quad k'_i = k_i - \frac{v \omega}{2c^2} \approx k_i, \quad k'_i = k_i, \quad i \neq x. \tag{56}
$$

It then follows that

$$
P^{(1)} - \frac{v f}{2} = \int_{k'_i, i = 0} d k'_i \int_{-k'_i/2}^{k'_i/2} d \omega h \omega'\left(1 + \frac{2 \text{Im} \bar{R}_{\omega, k_i} 2 \text{Im} \bar{R}_{\omega, k_i}}{|1 - e^{2 i k_i d} \bar{R}_{\omega, k_i} \bar{R}_{\omega, k_i}|^2}\right), \tag{57}
$$

where $R^+$ and $R^-$ are the reflection matrices from half spaces moving at velocities $v/2$ and $-v/2$, respectively, along the $x$ axis. Since $\epsilon$ is real [the real part of the response function is even in frequency, i.e., $\text{Re} \epsilon(\omega) = \text{Re} \epsilon(-\omega)$], we have

$$R_{\omega, k_i} = R_{-\omega, k_i}'. \tag{58}
$$

This implies that the integrand in Eq. (57) is antisymmetric with respect to $\omega'$ so that the integral vanishes.

When there is friction, work must be done to keep the moving half space in steady motion. This work should be equal to the total energy dissipated in the half spaces,

$$v f = P_{\text{tot}}, \tag{59}
$$

where $P_{\text{tot}}$ is the sum of energy flux through each half space. For Eq. (59) to hold, the energy flux through the second (moving) half space should also be equal to $P^{(1)} = v f / 2$. In the center-of-mass frame too, we should have the same condition because of the energy conservation $v f = P^{(1)}_{\text{c.m.}} + P^{(2)}_{\text{c.m.}}$, and the symmetry $P^{(1)}_{\text{c.m.}} = P^{(2)}_{\text{c.m.}}$. (The force in the center-of-mass frame is almost identical to the laboratory frame since the velocity is small compared to the speed of light.) Therefore, we conjecture that $P^{(1)}_S = P^{(2)}_S = v f / 2$ irrespective of the reference frame $S$, while $P_{\text{gap}}$ sensitively depends on the reference frame $S$; it is $v f / 2$ when the first half space is at rest, $-v f / 2$ when the second half space is at rest, and zero in the center-of-mass frame.

IV. DISCUSSION AND SUMMARY

Throughout this paper, we explicitly considered half spaces described by a constant and real dielectric function. However, the underlying physics is rather general and does not depend on the idealizations made for the sake of convenience. For example, rather than a half space, we can consider a thick slab of a material with a complex dielectric function $\epsilon$. The slab will act like an infinite medium provided that the imaginary part of $\epsilon$ while small, is sufficiently large to absorb the emitted energy within the slab, with almost no radiation escaping the far end.

Such conditions can be met for a broad range of thickness and lossyness.

Classically, Cherenkov radiation is emitted when a charged particle passes through a medium. However, even a source without a net charge, or even a multipole moment, may result in Cherenkov radiation due to quantum fluctuations [5]. In the present paper, this is demonstrated for two neutral parallel half spaces in relative motion. By employing an amalgam of techniques, usually applied in different contexts, we are able to make several conceptual and technical observations. These techniques are applicable to a variety of other setups. An interesting situation, closer in spirit to Cherenkov radiation, is when a particle passes through a small channel drilled into a dielectric. Another closely related problem is a particle moving parallel to a surface [22,32,33]. A classical analog of the latter, namely, a charged particle moving above a dielectric half space, is discussed in Ref. [34]. Our approach of utilizing scattering theory in conjunction with a host of other methods, including input-output and Rytov formalism, should be useful in analyzing such situations.

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APPENDIX: GREEN’S FUNCTIONS

In this Appendix we compute a number of Green’s functions where the two spatial arguments lie within or outside each half space. We take the first half space to be at rest while the second one is moving at a velocity $v$ parallel to its surface. We further assume that $|v| \ll c$ for simplicity.

(1) **Green’s function with both points lying within the gap (outside both objects):** In this case, the Green’s function is given by (with $z_k > z_k$)

$$G_{\text{out-out}}(\omega, x, z) = \sum_{\alpha} \frac{1}{2 i p_{\alpha}} \frac{e^{i p_{\alpha} d}}{1 - e^{2 i p_{\alpha} d} R_{\alpha} \bar{R}_{\alpha}} \left[\Phi_{\alpha}^{\text{reg}}(\omega, \bar{x}) + \bar{R}_{\alpha} \Phi_{\alpha}^{\text{out}}(\omega, \bar{x})\right] \left[\Phi_{\alpha}^{\text{reg}}(\omega, z) + R_{\alpha} \Phi_{\alpha}^{\text{out}}(\omega, z)\right], \tag{A1}
$$

where we used a compact notation defined as

$$\alpha = k_\parallel, \quad \bar{\alpha} = -k_\perp, \quad p_{\alpha} = k_\perp = \sqrt{\omega^2 - k^2_\perp},
$$

$$\Phi_{\alpha}^{\text{reg}} = e^{i k_\perp z_\perp} \Phi_{\alpha}^{\text{out}} = e^{i k_\perp z_\perp}, \quad R_{\alpha} = R_{\omega, k_\parallel}, \quad R_{\alpha} = R_{\omega, k_\parallel}, \quad z_\perp = d - z_\perp, \quad \bar{x}_\perp = x_\perp,
$$

$$\sum_{\alpha} = L_{D-1} \int d k_\parallel \equiv L_{D-1} \int \frac{d^{D-1} k_\parallel}{(2\pi)^{D-1}}, \tag{A2}
$$

where $\omega'$ and $k_\parallel'$ are the Lorentz transformation of $\omega$ and $k_\parallel$, respectively. Also $D$ is the number of (spatial) dimensions. According to these definitions, $\Phi_{\alpha}(z)$ is defined with respect to an origin on the surface of the first half space, while $\Phi_{\alpha}(\bar{x})$
is the wave function defined with the origin on the surface of the second half space and the direction of the \( z \)-axis reversed. It is straightforward to check that the expression in Eq. (A1) is indeed the Green’s function. First note that, for \( x \neq x \), it solves the homogenous version of Eq. (40). Furthermore, the coefficients are chosen to produce a \( \delta \) function when \( z \rightarrow x \) upon applying the Helmholtz operator.

(2) Green’s function with one point in the gap and the other inside the first half space: This Green’s function can be obtained from continuity conditions, i.e., by matching the Green’s functions approaching a point on the boundary from inside and outside the object

\[
G_{\text{out-out}}(\omega, x, y)\Big|_{y=-}\;=\;G_{\text{out-in}}(\omega, x, y)\Big|_{y=-}
\]

This leads to

\[
G_{\text{out-in}}(\omega, x, z) = \sum_{\alpha} \frac{1/(2ip_{a})}{1 - e^{ip_{a}d}R_{a}R_{a}} \times \left[ \Phi_{\alpha}^{\text{reg}}(\omega, \hat{x}) + R_{a} \Phi_{\alpha}^{\text{out}}(\omega, \hat{x}) \right] \times \left[ V_{a}\psi_{a}^{+}(\omega, x) + W_{a}\psi_{a}^{-}(\omega, x) \right] \times \left[ V_{a}\psi_{a}^{+}(\omega, z) + W_{a}\psi_{a}^{-}(\omega, z) \right],
\]

with

\[
\psi_{a}^{\pm} = e^{ik_{z}x}e^{\pm ip_{a}d},
\]

where \( k_{z} \equiv \hat{p}_{a} = \sqrt{\omega^2/c^2 - k_{\perp}^2} \). The (diagonal) matrices \( V \) and \( W \) are determined by imposing continuity equations, as \( V_{a} + W_{a} = 1 + R_{a} \), \( \bar{p}_{a}(V_{a} - W_{a}) = \bar{p}_{a}(1 - R_{a}) \).

(3) Green’s function with both points inside the first half space: This Green’s function is given by (\( z_{a} > x_{a} \))

\[
G_{\text{in-in}}(\omega, x, z) = \sum_{\alpha} \frac{1/(2ip_{a})}{1 - e^{ip_{a}d}R_{a}R_{a}} \times \left[ V_{a}\psi_{a}^{+}(\omega, x) + W_{a}\psi_{a}^{-}(\omega, x) \right] \times \left[ V_{a}\psi_{a}^{+}(\omega, z) + W_{a}\psi_{a}^{-}(\omega, z) \right],
\]

where, via continuity relations, we have

\[
\bar{V}_{a} + \bar{W}_{a} = e^{ip_{a}d}(1 + \bar{R}_{a}),
\]

\[
\bar{p}_{a}(\bar{V}_{a} - \bar{W}_{a}) = \bar{p}_{a}e^{ip_{a}d}(1 - \bar{R}_{a}).
\]