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Cosmic Evolution from Phase Transition of Three-Dimensional Flat Space

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Flat space cosmology spacetimes are exact time-dependent solutions of three-dimensional gravity theories, such as Einstein gravity or topologically massive gravity. We exhibit a novel kind of phase transition between these cosmological spacetimes and the Minkowski vacuum. At sufficiently high temperature, (rotating) hot flat space tunnels into a universe described by flat space cosmology.

Introduction.—Phase transitions are ubiquitous in physics, with numerous applications in condensed matter physics, particle physics, and cosmology. Interestingly, phase transitions can occur even between different spacetimes, for instance, between black hole spacetimes and (hot) empty space [1]. In this work, we exhibit a novel type of phase transition between cosmological spacetimes and (hot) flat space in three spacetime dimensions.

The existence of a phase transition is quite surprising, given that flat space in three dimensions ($\varphi \sim \varphi + 2\pi$)

$$ds^2 = -dt^2 + dr^2 + r^2 d\varphi^2 \qquad (1)$$

has few interesting features at first glance. It arises as an exact solution of the vacuum Einstein equations $R_{\mu\nu} = 0$. The Euclidean signature version of flat space allows us to introduce a finite temperature by periodically identifying the Euclidean time (possibly with a rotation in φ). We call this hot flat space (HFS).

The other spacetime we shall be concerned with is flat space cosmology (FSC). FSC spacetimes [2,3] [$\Lambda(\tau): = 1 + (E\tau)^2$, $y \sim y + 2\pi r_0$]

$$ds^2 = -d\tau^2 + \frac{(E\tau)^2 dx^2}{\Lambda(\tau)} + \Lambda(\tau)\left(dy + \frac{(E\tau)^2}{\Lambda(\tau)} dx\right)^2 \qquad (2)$$

are locally flat [4] time-dependent exact solutions of the vacuum Einstein equations [5,6]. For positive (negative) τ, they describe expanding (contracting) universes from (toward) a cosmological horizon at $\tau = 0$. The parameter E has inverse length dimension and corresponds physically to the temperature associated with FSC [7].

The main purpose of the present work is to exhibit a phase transition between the (Euclidean versions of the) spacetimes (1) and (2) within Einstein gravity and more general gravitational theories in three dimensions. Thus, remarkably time-dependent cosmological spacetimes can emerge from flat space by heating up the latter.

Flat space cosmological spacetimes.—FSC spacetimes (2) are shifted-boost orbifolds of $\mathbb{R}^{1,2}$ [2,3] and correspond to flat space analogs of nonextremal rotating Bañados-Teitelboim-Zanelli (BTZ) black holes [8] in anti–de Sitter (AdS) space. In flat space chiral gravity [9], FSC spacetimes are conjectured to be dual to nonperturbative states, again in full analogy to the role played by BTZ black holes in AdS quantum gravity. Their Bekenstein-Hawking entropy can be matched by a formula counting the asymptotic growth of states in the putative dual field theory [5,6]. It is useful for our purposes to represent FSC (2) in terms of different coordinates. We make the coordinate transformation $\hat{r}_+ t = x$, $r_0 \varphi = y + x$, and $(r/r_0)^2 = 1 + (E\tau)^2$ with $E = \hat{r}_+/r_0$ and $\varphi \sim \varphi + 2\pi$,

$$ds^2 = \hat{r}_+^2 dt^2 - \frac{r^2 dr^2}{\hat{r}_+^2 (r^2 - r_0^2)} + r^2 d\varphi^2 - 2\hat{r}_+ r_0 dt d\varphi. \qquad (3)$$

With no loss of generality, we assume $r_0, \hat{r}_+ > 0$. These solutions are compatible with asymptotically flat boundary conditions [9,10]. In the absence of sources, Eq. (3) is the most general zero mode solution of the vacuum Einstein equations [11]. At vanishing r, closed null curves are encountered, so that the locus $r = 0$ corresponds to a singularity in the causal structure.

This singularity is screened by a cosmological horizon at the surface $r = r_0$, so that the region $r \geq r_0$ is regular. The horizon's surface gravity determines its Hawking temperature $T = \beta^{-1}$ as

$$T = \frac{\hat{r}_+^2}{2\pi r_0}. \qquad (4)$$

The angular velocity Ω of the horizon is given by

$$\Omega = \frac{\hat{r}_+}{r_0}. \qquad (5)$$

The result for the Hawking temperature (4) agrees with the corresponding one by Cornalba, Costa, and Kounnas [7], who calculated thermal radiation from cosmological particle production in the time-dependent background (2).

Strategy of the calculation.—Given some values of temperature (4) and angular velocity (5), we pose the question

which of the spacetimes (1) or (3) is preferred thermodynamically. To this end, we continue to Euclidean signature and compare which of the smooth Euclidean solutions has smaller free energy.

Free energy can then be derived from the canonical partition function

$$Z(T, \Omega) = \int \mathcal{D}g e^{-\Gamma[g]} = \sum_{g_c} e^{-\Gamma[g_c(T,\Omega)]} Z_{\text{fluct}}, \quad (6)$$

where the path integral is performed over all continuous Euclidean metrics g compatible with the boundary conditions enforced by the temperature T and angular velocity Ω. In the semiclassical approximation, the leading contribution comes from the Euclidean action Γ evaluated on smooth classical solutions g_c compatible with the boundary conditions. We are not concerned here with subleading contributions from fluctuations encoded in Z_{fluct}.

Smooth Euclidean saddle points.—The Euclidean version of flat space (1) is simple, but we also need the Euclidean continuation of FSC (3). A natural choice is

$$t = i\tau_E, \qquad \hat{r}_+ = -ir_+, \quad (7)$$

which then leads to Euclidean FSC

$$ds_E^2 = r_+^2 \left(1 - \frac{r_0^2}{r^2}\right) d\tau_E^2 + \frac{dr^2}{r_+^2 \left(1 - \frac{r_0^2}{r^2}\right)} + r^2 \left(d\varphi - \frac{r_+ r_0}{r^2} d\tau_E\right)^2. \quad (8)$$

Requiring the absence of conical singularities on the FSC horizon fixes the periodicities of the angular coordinate φ and Euclidean time τ_E. Consider $r^2 = r_0^2 + \epsilon\rho^2$. In the near-horizon ($\epsilon \to 0$) approximation, the Euclidean metric (8) can be written as

$$ds_E^2 \approx \frac{\epsilon}{r_+^2} \left(d\rho^2 + \rho^2 \frac{r_+^4}{r_0^2} d\tau_E^2\right) + r_0^2 \left(d\varphi - \frac{r_+}{r_0} d\tau_E\right)^2. \quad (9)$$

Smoothness requires the identifications

$$\tau_E \sim \tau_E + \frac{2\pi r_0}{r_+^2} = \tau_E + \beta,$$

$$\varphi \sim \varphi + \frac{2\pi}{r_+} = \varphi + \beta\Omega, \quad (10)$$

since the term proportional to ϵ demands a precise τ_E periodicity and the transverse direction $\varphi - \tau_E r_+/r_0$ must stay fixed as one moves around the thermal τ_E circle. The expressions for the Hawking temperature $T = \beta^{-1} = r_+^2/(2\pi r_0)$ and angular velocity $\Omega = r_+/r_0$ agree with their Minkowski counterparts (4) and (5) as well as with the flat limit expressions of the ones for inner horizon BTZ thermodynamics [12,13].

Defining the ensemble.—We declare two Euclidean saddle points to be in the same ensemble if (i) they have the same temperature $T = \beta^{-1}$ and angular velocity Ω given by Eqs. (4) and (5), respectively, (ii) the two metrics

obey flat space boundary conditions [9,10], and (iii) the solutions do not have conical singularities. Note that requirement (ii) is somewhat different from what we would usually assume, namely, that the metrics asymptote to the same one at infinity. The peculiarities of the boundary conditions [9,10] for flat space solutions imply that there are leading terms in the metric that can fluctuate like the g_{tt} term. Note finally that the absence of conical singularities does not automatically imply the absence of asymptotic conical defects. A crucial counterexample is FSC (8), which has an asymptotic conical defect if $r_+^2 \neq 1$, since in the large r limit, $ds_E^2 = r_+^2 d\tau_E^2 + dr^2/r_+^2 + r^2 d\varphi^2 + \cdots$, where φ is 2π periodic [14]. On the other hand, Euclidean HFS

$$ds_{\text{HFS}}^2 = d\tau_E^2 + dr^2 + r^2 d\varphi^2 \quad (11)$$

has no conical defects since it has periodicities $(\tau_E, \varphi) \sim (\tau_E, \varphi + 2\pi) \sim (\tau_E + \beta, \varphi + \Phi)$, where inverse temperature $\beta = T^{-1}$ and angular potential $\Phi = \beta\Omega$ are given by Eq. (4).

Cosmic phase transition in Einstein gravity.—The considerations above are valid for any three-dimensional (3D) gravity theory supporting flat space boundary conditions. From now on, we focus on the simplest such theory, namely, Einstein gravity. Its Euclidean action reads [15]

$$\Gamma = -\frac{1}{16\pi G} \int d^3x \sqrt{g} R - \frac{1}{16\pi G} \lim_{r \to \infty} \int d^2x \sqrt{\gamma} K. \quad (12)$$

Here, G is the Newton constant, γ the determinant of the induced metric at the asymptotic boundary $r \to \infty$, and K the trace of extrinsic curvature.

On shell, the bulk term vanishes in Eq. (12). HFS (11) yields $\sqrt{\gamma} = r$ and $K = 1/r$. FSC (9) yields $\sqrt{\gamma} = r_+ r + O(1/r)$ and $K = r_+/r + O(1/r^3)$. Thus, we obtain on shell

$$\Gamma_{\text{HFS}} = -\frac{\beta}{8G}, \qquad \Gamma_{\text{FSC}} = -\frac{\beta r_+^2}{8G} = -\frac{\pi r_0}{4G}. \quad (13)$$

Plugging the on-shell actions (13) into Eq. (6) establishes the respective canonical partition functions $Z(T, \Omega)$. The free energy is obtained from $F(T, \Omega) = -T \ln Z$, where $T = r_+^2/(2\pi r_0)$ is the Hawking temperature:

$$F_{\text{HFS}} = T\Gamma_{\text{HFS}} = -\frac{1}{8G} \qquad F_{\text{FSC}} = T\Gamma_{\text{FSC}} = -\frac{r_+^2}{8G}. \quad (14)$$

So our main conclusion is that there is a phase transition between HFS and FSC, as summarized below ($r_+ > 0$):

$$\begin{aligned} &r_+ > 1, \quad \text{FSC is the dominant saddle;} \\ &r_+ < 1, \quad \text{HFS is the dominants addle;} \quad (15) \\ &r_+ = 1, \quad \text{FSC and HFS coexist.} \end{aligned}$$

The phase transition occurs at the critical temperature

$$T_c = \frac{1}{2\pi r_0} = \frac{\Omega}{2\pi}. \tag{16}$$

Thus, at sufficiently high temperatures, HFS "melts" and tunnels into FSC. Conversely, increasing rotation leads to higher critical temperatures, making HFS more stable. So, at sufficiently high angular velocity, FSC "falls apart" and tunnels to HFS.

Entropy.—Given the free energy (14), we can derive all thermodynamical variables of interest by standard methods. For HFS, the free energy is constant, and thus all quantities besides temperature and angular rotation are trivial. In particular, the entropy of HFS vanishes. By contrast, FSC has nontrivial thermodynamics [5,6]. As a consistency check on the validity of our result (14), we show now that we recover the correct entropy.

Let us first rewrite the free energy in terms of temperature and angular rotation:

$$F_{\text{FSC}}(T, \Omega) = -\frac{\pi^2 T^2}{2G\Omega^2}. \tag{17}$$

The thermodynamical entropy

$$S = -\frac{\partial F_{\text{FSC}}}{\partial T}\bigg|_{\Omega=\text{const}} = \frac{2\pi r_0}{4G} \tag{18}$$

then coincides precisely with the Bekenstein-Hawking entropy, which in turn coincides with the entropy derived from a Cardy-like formula valid for Galilean conformal algebras [5,6]. Our result (18) not only confirms the analysis of Refs. [5,6], but it also strengthens their conclusions, since we have derived above the Bekenstein-Hawking law from first principles, rather than assuming its validity.

Specific heat $C = T\partial S/\partial T = S = \pi^2 T/(G\Omega^2)$ is positive, which implies that the Gaussian fluctuations that contained in Z_{fluct} in Eq. (6) do not destabilize the system. Note that specific heat vanishes linearly with temperature as T tends to zero, just like a free Fermi gas at low temperature.

First law.—For FSC, another thermodynamical quantity of interest is the angular momentum

$$J = -\frac{\partial F_{\text{FSC}}}{\partial \Omega}\bigg|_{T=\text{const}} = -\frac{r_+ r_0}{4G}, \tag{19}$$

which enters in the first law of thermodynamics

$$dF = -SdT - Jd\Omega. \tag{20}$$

Integrating the first law yields $F = U - TS - \Omega J = U$ with the (nonpositive) internal energy $U = \Omega J/2 = -M$, where

$$M = \frac{r_+^2}{8G} \tag{21}$$

is the mass parameter. The first law is also obeyed by internal energy $dU = TdS + \Omega dJ$. The unusual signs

appearing here are reminiscent of inner horizon thermodynamics [12,13,19–21].

Matching the solutions via S transformation.—We now connect FSC and HFS by the flat space analog of a modular S transformation in a conformal field theory (CFT). This is useful for a field-theoretic interpretation of our results. The flat space S transformation reads [22]

$$S{:}(\beta, \Phi) \to (\beta', \Phi') = \left(\frac{4\pi^2\beta}{\Phi^2}, -\frac{4\pi^2}{\Phi}\right). \tag{22}$$

We start with the FSC metric (8). Changing coordinates $r^2 = r_0^2 + r_+^2 r'^2$, $\tau_E = \tau_E'/r_+ - \varphi' r_0/r_+^2$, and $\varphi = \varphi'/r_+$ yields flat space $ds^2 = d\tau_E'^2 + dr'^2 + r'^2 d\varphi'^2$. In terms of the new coordinates, the periodicities read $(\tau_E', \varphi') \sim (\tau_E' - \beta', \varphi' + \Phi') \sim (\tau_E', \varphi' + 2\pi)$ with $\beta' = 2\pi r_0 = 4\pi^2\beta/\Phi^2$ and $\Phi' = 2\pi r_+ = -4\pi^2/\Phi$. These are precisely the values obtained from the S transformation (22). Therefore, FSC with periodicities (β, Φ) is equivalent to HFS with S-dual periodicities (β', Φ'). This is the flat space analog of the $\text{AdS}_3/\text{CFT}_2$ statement that thermal AdS_3 space with modular parameter τ is equivalent to a BTZ black hole with S-dual modular parameter $-1/\tau$ (see, e.g., Ref. [31]).

Consistency check.—The analysis above lets us resolve a seemingly puzzling conceptual issue. In flat space (1), there appears to be no preferred scale, so how is it possible that there is a critical temperature? The key observation is that we are considering flat space with fixed angular rotation, which does provide a length scale $L = 2\pi r_0$. The critical temperature (16) is reached precisely when the periodicity in Euclidean time is one in units of L. We can interpret this property from a field theory perspective, where L is associated with the twist of one of the cycles of the torus on which the field theory lives. Consistently, the critical temperature (16) coincides with the self-dual point of the S transformation (22).

Beyond Einstein gravity.—We now generalize our results to another interesting 3D theory of gravity, namely, topologically massive gravity (TMG) [32]:

$$\Gamma^{\text{TMG}} = \Gamma - \frac{1}{32\pi G\mu} \int d^3x\, \text{CS}. \tag{23}$$

If the Chern-Simons coupling constant μ tends to infinity, we recover the Einstein gravity action (12). The Chern-Simons term expressed in terms of the Christoffel symbols reads $\text{CS} = \epsilon^{\lambda\mu\nu}\Gamma^\sigma_{\lambda\rho}(\partial_\mu\Gamma^\rho_{\nu\sigma} + \frac{2}{3}\Gamma^\rho_{\mu\tau}\Gamma^\tau_{\nu\sigma})$. TMG has all solutions of Einstein gravity, since any spacetime with a vanishing Ricci tensor also has a vanishing Cotton tensor $C_{\mu\nu} = \varepsilon_\mu^{\lambda\sigma}\nabla_\lambda(R_{\nu\sigma} - \frac{1}{4}g_{\nu\sigma}R) = 0$, and thereby trivially solves the field equations of TMG $R_{\mu\nu} + \frac{1}{\mu}C_{\mu\nu} = 0$. We assume with no loss of generality that μ is positive.

A complication in TMG is that it is not known how to compute free energy from the on-shell action. We proceed by assuming the validity of the first law of thermodynamics (20) and then integrate it. For this, we need the angular

momentum at finite μ, $J(\mu) = J - \frac{1}{\mu}M$, and entropy. The latter can be calculated using Solodukhin's conical deficit method [33] or Tachikawa's generalization of the Wald entropy for theories with a gravitational Chern-Simons term [34]:

$$S^{\mathrm{TMG}} = \frac{2\pi r_0}{4G} + \frac{1}{\mu}\frac{2\pi r_+}{4G}. \tag{24}$$

The first term is compatible with the Einstein result (18) obtained in the limit $\mu \to \infty$. The second term is compatible with the conformal Chern-Simons gravity (CSG) result obtained in the limit $G \to \infty$, $8\mu G = 1/k$, which we now derive. To this end, we exploit the flat space chiral gravity conjecture that the dual field theory is a chiral CFT with central charge $c = 24k$ [9] and use a chiral version of the Cardy formula:

$$S^{\mathrm{CSG}} = 2\pi\sqrt{\frac{ch_L}{6}} = 4\pi kr_+. \tag{25}$$

In the second equality, we used the result for the Virasoro zero mode charge $h_L = kr_+^2$ [9]. Integrating the first law (20) with the results above yields the free energy

$$F^{\mathrm{TMG}}_{\mathrm{FSC}} = -\frac{\pi^2 T^2}{2G\Omega^2}\left(1 + \frac{\Omega}{\mu}\right). \tag{26}$$

Comparing the free energies (26) and $F^{\mathrm{TMG}}_{\mathrm{HFS}} = -(1/8G)$, we see that that there is again a phase transition between HFS and FSC, as summarized below ($\mu, \Omega, r_+ > 0$):

$$r_+^2\left(1 + \frac{\Omega}{\mu}\right) > 1, \quad \text{FSC is the dominant saddle;}$$

$$r_+^2\left(1 + \frac{\Omega}{\mu}\right) < 1, \quad \text{HFS is the dominant saddle;} \tag{27}$$

$$r_+^2\left(1 + \frac{\Omega}{\mu}\right) = 1, \quad \text{FSC and HFS coexist.}$$

Consequently, if r_+ is sufficiently large, HFS is thermodynamically not the preferred spacetime and will tunnel to FSC. Thus, our phase transition is not a unique feature of Einstein gravity and arises also in TMG. The phase transition occurs at the critical temperature

$$T^{\mathrm{TMG}}_c = \frac{\Omega}{2\pi}\frac{1}{\sqrt{1 + \Omega/\mu}}. \tag{28}$$

It could be interesting to extend our results to other 3D models, like new massive gravity [35], pure fourth-order gravity [36], or generalizations thereof.

Concluding remarks.—We conclude with some remarks concerning four dimensions. The Gross-Perry-Yaffe instability of four-dimensional HFS due to nucleation of Schwarzschild black holes [37] is qualitatively different from the instability discussed in the present work, since the former only involves static spacetimes. Our transition from HFS into FSC is also different from the well-known

quantum creation of universes [38–40], since the latter requires the presence of some form of matter, like a scalar field with a nonvanishing self-interaction potential. Given these differences with previous constructions, it would be interesting to generalize our results to four (or higher) dimensions. This could be feasible, since also four- (or higher-) dimensional AdS space allows the construction of BTZ-like quotients; see Ref. [41] for a careful analysis and references therein for the original literature.

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