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FURTHER CONSIDERATIONS OF FIXED-COEFFICIENT MODELS

In the preceding paper, "The Effect of Fixed Coefficients in Overpopulated Areas," (Italy, Economics 7) a number of theoretical examples were constructed to illustrate some of the consequences of widespread fixed technical coefficients of production. Even the simplest of these examples was useful in giving insights as to the implications of limited substitutability of factors. However, the theory requires further development to provide more descriptive conclusions which are, perhaps, better suited to testing. The present paper is intended to take several more steps toward a complete analysis.

Section I will revise the analysis of Section IV of the preceding paper in which a model was introduced and discussed composed of a fixed-coefficients sector and a variable coefficients sector. In order to avoid problems related to a changing composition in output the first approximation used assumed that there was only one good, producible by either the fixed coefficient method or the variable coefficient method. There were inadequacies in the original presentation whose correction will enable a more general treatment and additional insights in this restrictive example.

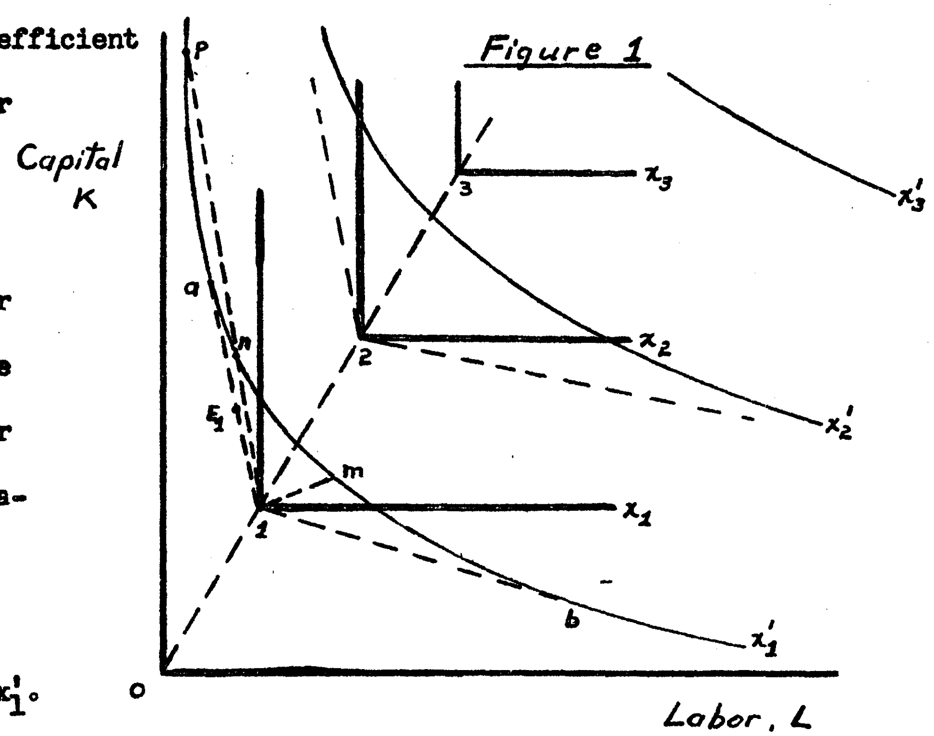
Restricting the two sector hypothesis to the case of a single good, while simplifying the analysis considerably, also rules out problems which should not be ignored. It is, therefore, necessary to extend the treatment to the case where the fixed coefficient and variable coefficient sectors produce different goods. This will be done in Sections II and III of this paper. Section II, though perhaps of some interest in itself, is intended mainly as a simplified version of the problems discussed in Section III.

The geometrical techniques used can be made rigorous and may be of considerable help in appreciating the elements of the situation. They cannot handle, however, all the questions which can be asked about even such a simple system. Thus, this paper does not yet represent an exhaustive analysis.

I. <u>A One Good, Two Factor Case of the Hypothesis For Underdeveloped Areas.</u>

The discussion of the model, in which in some industries only a few alternative processes of production are available while other industries are characterized by variable coefficients, has been initiated under the assumption that all industries produced the same product. It is possible to construct more comprehensive diagrams of this case than those developed in the previous paper (Figures 2 and 5). These can in turn be specialized to illustrate conditions which may be particularly relevant for underdeveloped areas. To begin suppose that the constant product lines of the fixed-coefficient[1] industries are represented by the heavy lines x_1, x_2,... and the constant product lines of the variable coefficient industries by the lighter lines x'_1, x'_2,...

The output x_1 could be produced by the factor combination and technique represented by point 1 or any of the factor combinations using the variable proportions technique represented by the line x'_1.



1. Although the constant product lines for the fixed-coefficients sector are drawn in Figure 1 as if only one process is available, the demonstration is perfectly general and its implications are applicable when more than one process is available for the fixed-coefficient industries.

Moreover, following the reasoning on page 5 of the preceding paper it is also possible to produce x_1 by simultaneously using both the fixed coefficients and variable coefficients techniques. All of the lines which could be drawn from point 1 to line x_1' represent a <u>combination</u> of methods which would produce output x_1; all such lines fall between the lines al and lb which are drawn from point 1 just tangent to line x_1'.[1]

Figure 1 not only contains both of the cases considered separately in Section 4 of the previous paper but several variants of each case. The "efficiency locus" for specified outputs can be traced out by determining, for given amounts of one of the factors, the minimum amount of the other factor necessary to produce the output. If this is done for output x_1, when very little labor is available it is best to produce by use of the variable coefficients process alone; a representative factor use for this case is at point P. As more labor becomes available the minimum amount of capital required to produce x_1 is found by sliding down the variable coefficients equal product line to point a. It was pointed out above that line al represents different combinations of the variable coefficients process located at a and the fixed coefficients process. When the labor available is further increased, the minimum amounts of capital necessary to produce x_1 is found by moving along line al. As labor available is still further increased the line lb is the next segment of the efficiency locus used, for reasons analogous to those given for the use of segment al. Finally, when labor is increased beyond the amount available at point b, again only the variable coefficients method should be used to produce x_1.

Output x_1 could also be produced by process combinations and amounts of factors which do not lie on the efficiency locus, of course. Line pl represents a series of such combinations, using in varying proportions the

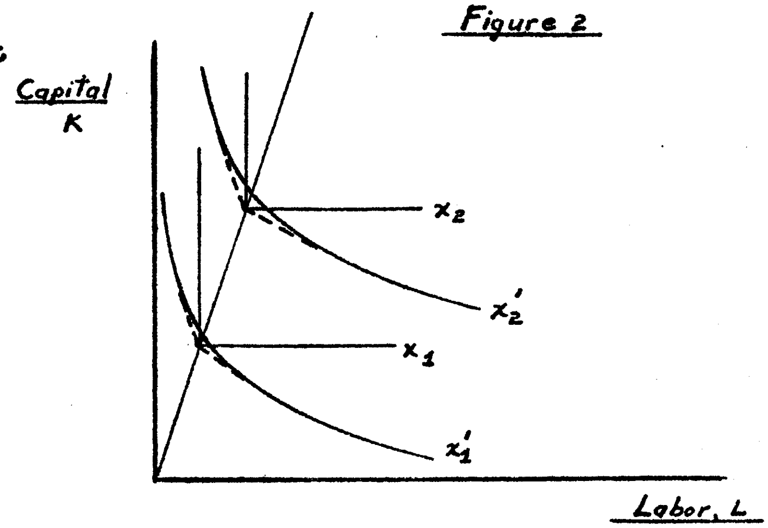
1. I am indebted to Prof. R. Solow for the criticism of the previous paper which led to this reformulation.

variable coefficients processes located at p or n and the fixed coefficient process located at point 1. Any combination of methods along mnl, however, would result in higher costs for x_1 than a method found on the efficiency locus; methods along pnl could also be used to produce larger amounts than x_1. Of course, many lines like pnl could be drawn between al and the vertical portion of the fixed-coefficient x_1 iso-product line and, analogously, between lb and the horizontal portion of the fixed-coefficient iso-product line. Lines like lm, of which many could also be drawn, represent combinations of methods which would also produce x_1, but require more of both labor and capital than points on the efficiency locus; line lm illustrates the case in Figure 5 of the preceding paper. The boundaries of lines such as lm are the vertical and horizontal portions of the fixed coefficients x_1 iso-product line.

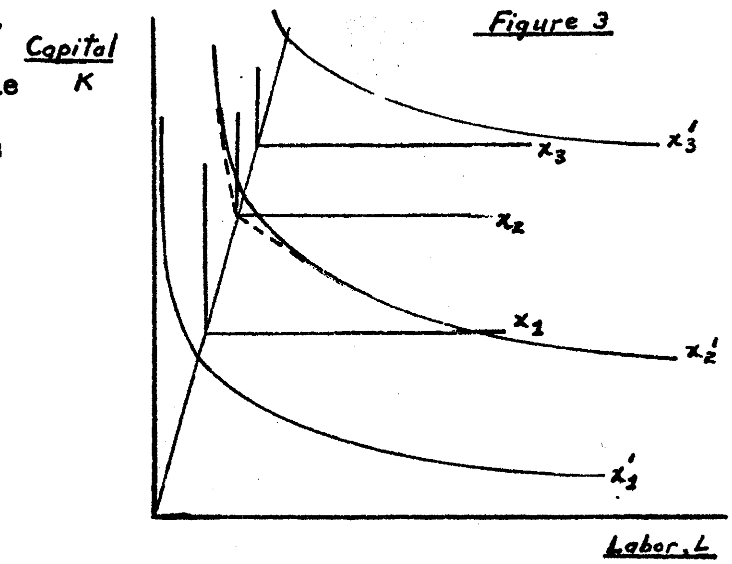
Figure 1 embodies the constant returns to scale assumption for both the fixed coefficients and variable coefficients method. This is not necessarily the most realistic or relevant assumption, however, nor does the relative position of the two types of curves, or the shape of the variable coefficients isoquants necessarily correspond closely to reality. It is useful to recognize other, special cases which may have important empirical significance. In Figure 1, for example, only the extremes of the iso-product curves of the variable coefficients process were a part of the efficiency locus for any particular output, and, as drawn, relatively little substitution was possible at such extremes.

Figure 2 depicts a rather different possible situation from that in Figure 1 in which the "efficient" iso-product ridge lines follow the variable-coefficients lines so as to allow substitution of factors over a considerable range.

The effect of different rates of return to scale on the shape and slope of the equal product ridge lines is illustrated in Figure 3 for one possible set of relations. In the fixed coefficients process it is



assumed that there are increasing returns to scale. (Shown by decreasing distances between x_1, x_2, x_3 along any ray from the origin). In the variable coefficients method constant returns to scale are the rule (Shown by constant distances between x_1', x_2', x_3' along a ray from the origin). In this example, the efficient iso-product lines change their shape as output is increased. For output x_1



only the bariable coefficients equal-product line is relevant. For output x_2, the "efficient" iso-product ridge line involves use of both the fixed and variable coefficient techniques. Finally, for output x_3, only the fixed-coefficient technique is "efficient" and the ridge lines involving the variable coefficient method necessarily have a positive slope.

Figure 3 provides the formal basis for some deductions which would be obvious even without it. Changes in factor prices which might induce shifts in the proportions in which factors are used at one scale of output, at another scale of output may not induce such shifts or may only produce smaller shifts. Likewise techniques of production not feasible at one scale of production may become mandatory for efficiency at another scale.

In both Figures 2 and 3 it is particularly clear that in order for the system to travel along its most efficient production isoquant it is necessary that factor prices be flexible. Factor price rigidities would make parts of the efficient equal product lines unattainable for profit-maximizing businessmen.

II. Two Goods, Two Factors, Two Fixed-Coefficient Sectors.

The assumption that the two hypothesized sectors produce the same commodity has been only an extremely crude first approximation, intended to set out the basic hypothesis in its most stark form. Analysis of the general case in which a number of different commodities are produced by both the fixed-coefficient and variable coefficient sectors will have more practical relevance but, at the same time will be more difficult. As an intermediate step in which the relations involved remain fairly obvious, it will be assumed that only one good is produced by the fixed-coefficients sector and one, different commodity in the variable-coefficients sector. Restricting the analysis in this way will make it possible to continue to rely on graphical demonstrations by extensive use of transformation curves.

Although the transformation curve was used briefly in the preceding paper it may be worthwhile to discuss it briefly in general terms in view of its frequent use in the following analysis. The transformation curve is a relation derived from more basic data and can be used in at least two different senses. The basic data consists of the amounts of the two factors of production which are available, and the production functions for the two goods. Then, subject only to the technological and resource limitations by finding the maximum of one of the goods which can be produced for various specified amounts of the other good, the transformation curve can be derived. This process can be represented graphically in the two good-two factor case by the box diagram in Figure 4a, familiar to economists.[1] The dimensions of each side of the box represent the total amount of factors available. Any point within the box simultaneously represents four quantities: the amount of capital and the amount of labor used in the clothing industry, which is determined by measurement from the lower left hand corner, and the amount of capital and the amount of labor used in the food industry, measured from the upper right hand corner. The heavy curved lines c_1, c_2, c_3, c_4 are only a few of the infinity of the equal product lines which could be drawn for different outputs. Along each of the lines the different combinations of factors result in the same output of clothing. The lighter curved lines f_1, f_2, f_3, f_4 are similarly iso-product lines for the food industry. The curved equal product lines for each industry in Figure 5a represent the assumption, used for illustrative purposes, that it is possible to use the factors in smoothly varying proportions. If, now we specify a particular amount of clothing to be produced, say, c_1, and find the maximum of food which can simultaneously be produced, we must move along curve c_1, crossing food iso-products lines such as f_3, until we reach the highest food line obtainable, in this case f_4. Similar optimum positions will be reached at other tangencies of the food and clothing iso-product lines.

The line tt' is drawn so as to pass through each of the tangency positions of the clothing and food isoquants. tt' will be called the "efficiency locus" for the two goods. If production takes place at any

1. This type of box diagram is an adaptation of the Edgeworth-Bowley box used in the study of consumers' behavior and development of the contract curve. P.A. Samuelson has used this device most frequently, I believe. cf. "Protection and Real Wages," Review of Economic Studies, IX, 1941, 58-74 (with W.F. Stolper).

point off tt', it is possible by recombination of the factors to get more of one good without diminishing output of the other. If along tt' corresponding amounts of the two goods are read off and plotted on a chart as in Figure 4b, the transformation curve between food and clothing is obtained. Under the usual assumptions of diminishing returns the curve will be concave to the origin.

Figure 4a

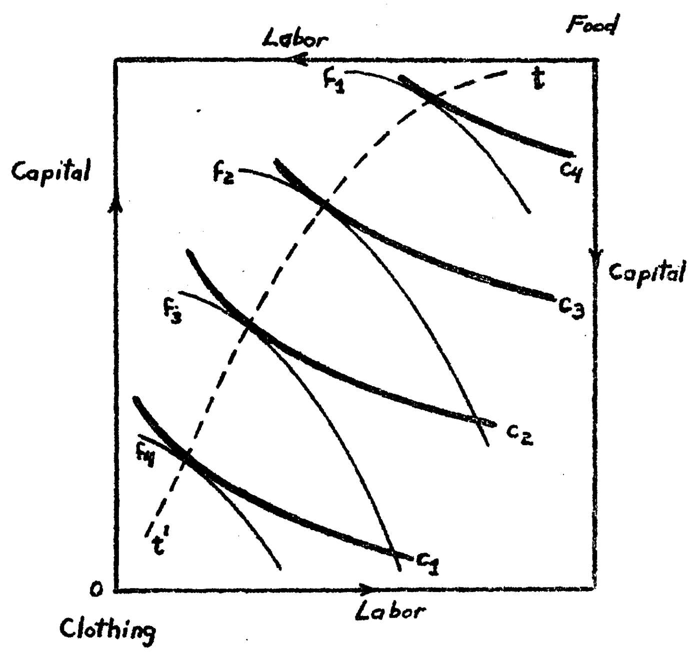
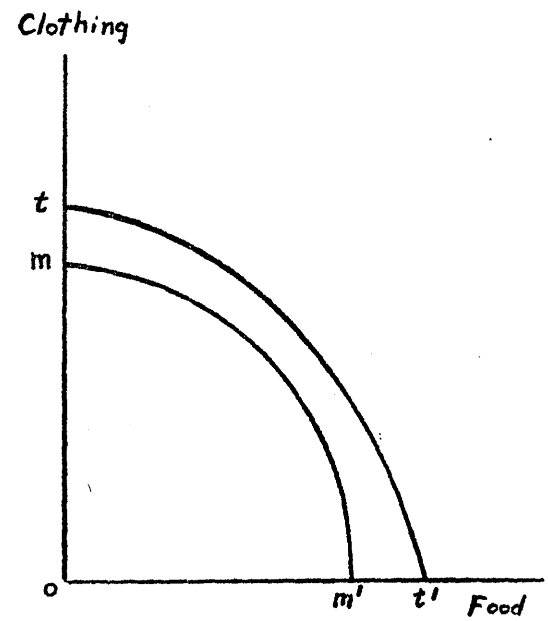


Figure 4b



It should be clear that the transformation curve as derived depends only on the original factor endowments and the technological conditions of production and requires considerable substitution of factors in the production of each good. Commodity demand conditions play no part in the determination of the efficiency locus and transformation curve in Figures 4a and 4b, just as they played no part in tracing out the efficiency locus in Figure 1. Demand conditions will, of course, determine just where on the transformation curve the system settles.

Behind the transformation curve tt' lie many fine adjustments as factors are shifted from one industry to another and recombined in varying proportions to obtain the maximum output from one industry for given outputs of the other industry. It has been assumed that the necessary adjustments would be accomplished as they would be in a perfect factor market. When imperfections and rigidities of various types obstruct the movements of factors and prices, the system will not be able to achieve tt' but will instead do no better than to move along some other curve such as mm'. We shall be interested below in the effect of imperfections in the factor markets and it will be useful to distinguish between transformation curves like tt' and mm'. The curves like tt', dependent on factor endowments and technological limitations and which require perfect adjustments in the factor markets, will be called "technical transformation curves." The transformation curves which take into account market imperfections may or may not be different from the technical transformation curves; the concept is useful to isolate, however, and these latter realtions will be designated "market transformation curves."

In Section 111 of the previous paper a simple case of two goods each produced by two factors which could be combined only in fixed proportions was adduced to illustrate the possibility of discrepancies between a full employment output and a maximum value output. I now wish to extend this analysis but it may again be useful to take several small steps rather than one large one.

Let us now assume that each good, x_1 and x_2 can now be produced by two, fixed proportions processes, and that constant returns to scale prevail;[1] only two factors, capital, K, and labor, L, are used. Figure 5a shows the

1. The assumption of constant returns to scale is, of course, maintained not because it is considered the best description of reality but for its analytical convenience. Some comments on the effects on the analysis of dropping this assumption are made below.

ridge lines indicating the factor proportions which can be used for each product for the two goods. The solid lines refer to product 1, the dashed lines to product 2. In Figure 5b these ridge lines are used to form a box diagram of the type in Figure 4a, the dimensions of which correspond to the given factor endowments of capital and labor. The "efficiency locus" for these two goods is a rather complicated succession of line segments.

Figure 5a

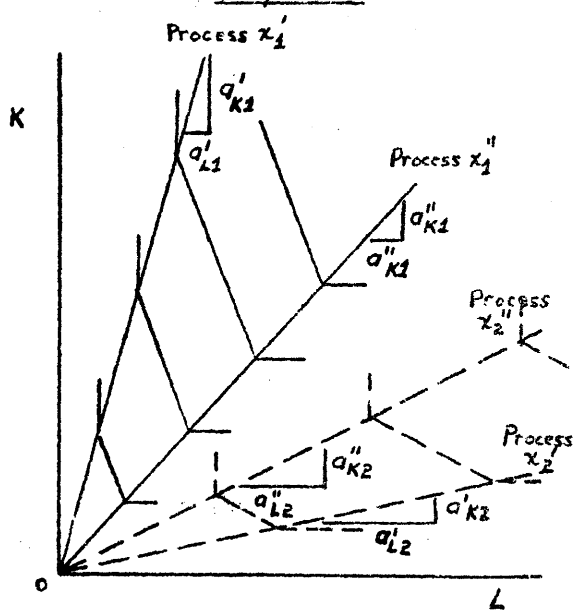


Figure 5b

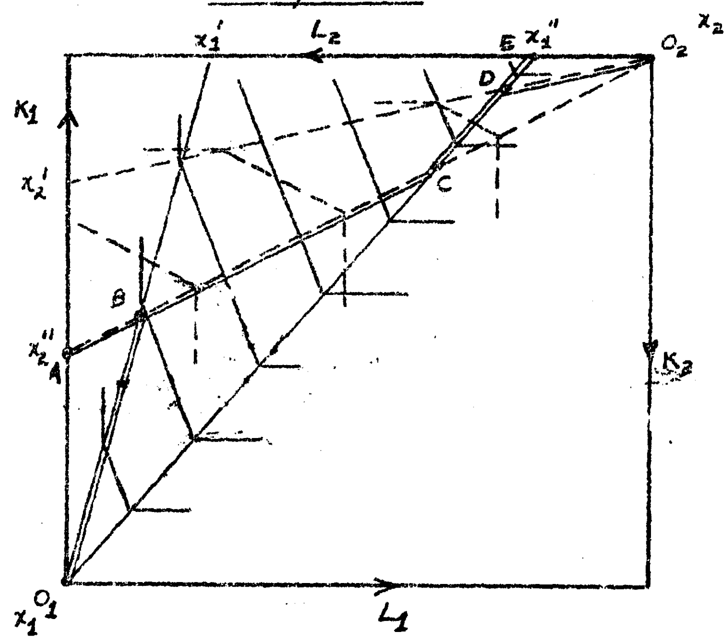
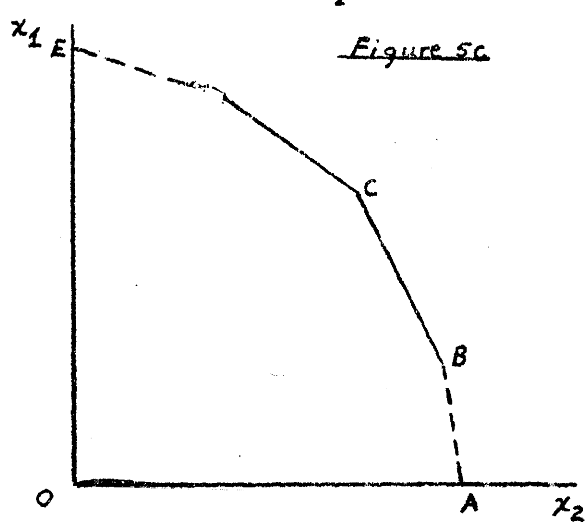


Figure 5c



Starting at o_1, zero output of x_1, the maximum output of x_2 obtainable is indicated at point A and could be computed by dividing O_2A by the scale factor applicable to process x_2''. If output of x_1 is increased relative to x_2, it will be most efficient, at first, to use process x_1' and for x_2 to be produced with process x_2''; production of x_1 would then move along the expansion path O_1B and x_2 should decrease along the expansion path x_2'' to point B.

As production of x_1 is expanded beyond point B it would be best to begin to use both process x_1' and process x_1'' in combinations indicated by the intersection of O_2A with the production possibility lines joining equal outputs on the expansion paths of x_1' and x_1''; production of x_2 should continue to be by means of process x_2'' alone, however. This second stage is indicated by the points on the segment BC which belong to both the combination of x_1' and x_1'' and process x_2''. It can be seen that, as production of x_1 is expanded in this stage and production of x_2 is decreased, and increasing proportion of x_1 is produced by process x_1''.

In the third stage, as output of x_1 is further increased it is most efficient to use only process x_1''. But now for any given output of x_1, the maximum amount of x_2 can be obtained by use of both process x_2' and x_2'' : The third stage on the efficiency locus is indicated by line CD.

Finally, expansion of x_1 still further continues to be best done along the expandion path of process x_1'' until at point E x_1 is being produced to the complete exclusion of x_2. In this fourth and final stage output of x_2 should be produced only by process x_2'. Only in the fourth stage and the first stage of the efficiency locus would optimum allocation imply some unemployment of one of the factors. In the first the unemployment of capital for different outputs of x_1 and x_2 is indicated by the vertical distance between lines AB and O_1B. In the final stage the unemployment [1]

1. This unemployment it should be noted is exactly the same as that described in the previous paper and is "technological" rather than due to lack of effective demand.

of labor is measured by the horizontal distance between lines DE and DO_2.

In actuality the occurrence and significance of any of the stages, depends on both "technology" and "factor endowments." If process x_1'' were relatively more labor intensive its expansion path would pivot to the right and stage 2 in Figure 5b would be prolonged. As common sense would suggest development of a sufficiently labor intensive process x_1'' could cause stages three and four to disappear entirely and with them the possibility that there could be any "optimal" configuration which involved unemployment of labor. A similar effect would result from a decrease in the amount of labor endowment. This could be depicted by moving together the left and right hand sides of the box in Figure 5b. Increasing the labor supply would mean stretching the box horizontally. This would not only increase the range of outputs associated with stage 4 but also, if pushed far enough, eliminate first stage 1, the capital unemployment stage and then stage 2.

The technical transformation curve which is associated with the efficiency locus sketched out in Figure 5b is drawn in Figure 5c as curve ABCDE. The shape of this curve requires rigorous justification although it can be made intuitively reasonably plausible. Its precise derivation will, therefore, be left to the appendix.

The points ABCDE on the technical transformation curve in Figure 5c correspond to the similarly lettered points on the efficiency locus in Figure 5b. At first when only a little x_1 is produced and a lot of x_2, relatively we should move along the segment AB using process x_1' and x_2''. Unemployment of capital associated with this segment on Figure 5b will be reduced as we approach B. Relative labor scarcity is limiting along this segment and the slope of the line segment AB will depend on the ratio of the labor inputs per unit of output of x_2 to x_1. The relative labor intensity of process x_2'' compared to process x_1' as drawn on Figure 5a accounts for the steepness of the segment.

The line segment ED on Figure 5c has an exactly analogous justification to that above for the segment AB. Labor unemployment will be reduced as D is approached from point E. Capital is the only scarce factor and the relative capital intensity of process x_1'' as compared to process x_2' accounts for the flatness of ED.

Point C is located conveniently relative to points B and D. More of x_1 is produced at C than at B, though not so much more as produced at point D. Likewise less x_2 is produced at C than at B though much less than at D.

Referring to Figure 5b it can be seen that at point C only processes x_1'' and x_2'' are used. Before that between points A and B only process x_1' was used with process x_2''. The dashed lines on Figure 5c outline the technical transformation curves to which production would be limited if only one process could be used for the production of each good. On the other hand, between points B and C two processes, x_1' and x_1'', are used to produce x_1 in conjunction with process x_2'' for x_2. And between D and C processes x_2' for x_2'', are used together along with process x_1''. As a result we would expect the sections of the technical transformation curve between B and C and between D and C to be "further out" than any of the transformation curves which involve the use of only a single process for each good. This is in fact the case and, moreover, the segments BC and CD will be straight lines as can be verified by noting in Figure 5b that there must be a constant ratio between changes in output of x_2 along the line O_2A, for example, and changes in output of x_1 on the negatively slanting production possibility lines for x_1.

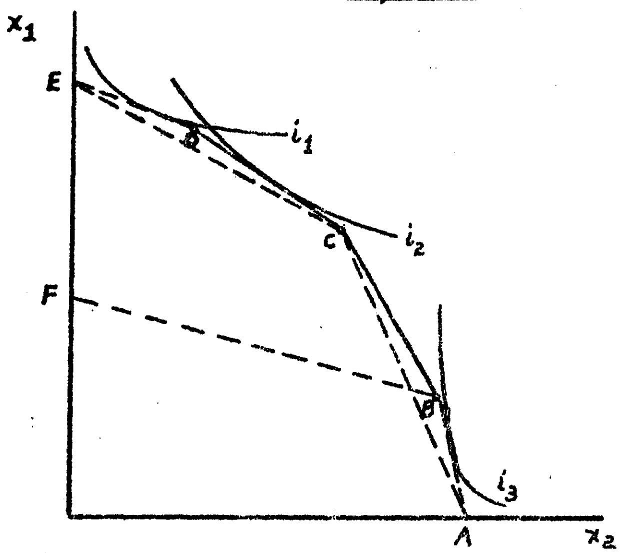
It was pointed out for the efficiency locus on Figure 5b that changes in factor endowment and technology could shorten or extend or even completely eliminate various stages of the efficiency locus. This applies also to the separate segments of the technical transformation curve. The technical transformation curve of Figure 5c illustrates all the possible stages which could be produced by this simple case, from unemployment of capital to unemployment of labor. It should not be presumed that this range of possibilities will actually exist in a particular system at any one time. Rather, it is one of the objectives of empirical research to determine the practical significance of the theoretical possibilities.

It was pointed out in connection with the general introduction to the use of transformation curves that there may be differences between the technical transformation curve and the market transformation curve. In this section I should like to explore this possibility.

The market transformation curve is that produced by the action of profit-maximizing or cost-minimizing entrepreneurs in response to the demand for different combinations of (two) goods. The cost minimizing combination of factors will be determined for any particular output at the point at which the production possibility schedule for that output touches the lowest expenditure line. If the production possibility schedule has any slope at this point of "touching," a condition of equilibrium is that this slope be equal to the factor price ratio which determines the slope of the expenditure line.

In the previous paper the technical transformation curve was drawn for the case of two goods each produced by single, fixed coefficients process. Since no substitution of factors was possible in the production of each good, the efficiency locus and technical transformation curve were independent of movements in factor prices. In the present case, however, since a limited amount of substitution of factors is possible, as there are now two processes available for the production of each good, movements of factor prices may affect the combinations in which processes are used. In fact, achievement of the outer boundary of the technical transformation <u>requires</u> a certain amount of flexibility of market prices. A competitive factor market would satisfy these requirements and make cost-minimizing or profit-maximizing adjustments identical to those on the efficiency-locus or <u>technical</u> transformation curve. On the other hand, factor price rigidities would cause discrepancies between the market adjustments and those necessary to keep the system on its technical transformation curve. For example, in a perfect factor market the wage of the surplus factors, in the AB and CD stages of the technical transformation curve, capital and labor respectively, would fall to zero. Suppose, however, that by means of union pressures or minimum wage legislation real wages of labor were maintained so that the ratio of labor-capital prices was greater than the negative slope of the constant product lines for good x_2 but less than the negatively sloped portion of the x_1 isoquants. This would mean that it would always be more profitable to use process x_2'' alone in producing x_2 than process x_2' or a combination of processes x_2' and x_2''. As a result the efficiency locus would become $ABCO_2$ in Figure 5b and the curve ABCE in Figure 6 (a redrawing of Figure 5c) would be the market transformation curve. If labor wages were raised still further cost-minimizing behavior would rule out the relatively labor-intensive process for x_1, x_1'', as well as the relatively labor-intensive process for x_2, x_2'. As a result stage BC would be no longer achievable by market processes and the effective market transformation curve would be ABF,[1] with full employment of factors only at point C.

Figure 6



On the other hand, if the ratio of the price of labor to the price of capital were kept at low levels by maintenance of high interest rates, the market transformation curve would also be different from the technical transformation curve. A factor price ratio maintained at less than the slope of the negatively slanting portion of the x_1 isoquants would restrict the market transformation curve to ACDE. An even lower ceiling on the factor price ratio would move the market transformation curve into GDE.

It is possible in this case of two processes for each of two goods for a divergence to exist between the full employment output and the output with maximum value just as in the case of one process each for two goods depicted in Figure 4 of the previous paper. This could result even if there were no market imperfections. If, in Figure 6 the community indifference

1. The effect of factor-market imperfections in shifting the market transformation curve inside the technical transformation curve have been pointed out and analysed for international trade by G. Habeler, "Some Problems in the Pure Theory of International Trade, "Economic Journal, LX, June, 1950, pp. 223-240 and by P. Samuelson, "Evaluation of Real National Income," Oxford Economic Papers, II (N.S.) Jan., 1950, pp. 18-19 for welfare economies; others have probably also noted the effect.

curves were like i_1 rather than i_2 so that the tangency occurred in the capital scarcity-labor surplus stage the divergence could exist. On the other hand, community indifference curves shaped like i_2 would mean that it would be possible for full employment output and maximum value output to be identical.

There are several important qualifications to this demonstration. First of all, the shape and position of the community indifference curves might not be independent of the particular processes or combinations of processes which are used. To handle this difficulty it would be necessary to determine the shifts in income distribution which result from changes in factor prices and proportions of factor use and to explore the differentials in tastes of the recipients of the different types of income. Secondly, without actual knowledge of the technology available and factor endowment it is impossible to say anything about whether slopes like that of ED, DC, CB or BA will be most significant, and offer opportunities for tangencies.

Removal of factor price rigidities which limited the market transformation curve to say ABCE or ABF would improve the community's welfare if the community indifference curves are shaped like i_1 or i_2 and did not change their general shape. If the community indifference curves were like i_3, introduction of flexibility in factor prices would not change the community's welfare position, other than through effects on income distribution and thus possibly to shift i_3.[1]

The preceding construction of the transformation curve for this case of two goods is dependent on the assumption of constant returns to scale as well as the postulation of fixed coefficient processes for each product.

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1. P.A. Samuelson, op cit.

If there were decreasing returns to scale in each process, there would still be four possible separate stages of the transformation curve. The segment representing each stage would, however, be curved and concave to the origin and the possibility would arise that the junction of the segments would create a portion of the curve which is convex to the origin. Increasing returns to scale would have opposite effects. The anlaysis above of the effects of rigid factor prices on the transformation curve would not be modified by dropping the assumption of constant returns to scale. The welfare implications of such rigidities would not necessarily hold, however.

Still further observations could be made on the implications of this case but it has, I believe, already served its mainly introductory purpose. While of some interest in itself this section was intended as a step toward the analysis of the hypothesis stated in the previous paper.

III A Model of Underdeveloped Areas.

The primary interest of this paper is to examine the hypothesis that some underdeveloped areas can be characterized by a two sector model: In one sector there are only a few alternative, relatively capital intensive processes while in the other sector widely different and smoothly variable combinations of factors are possible.

The previous extensive treatment of the case in which the two sector model is characterized by having only two alternative processes available for the production of each product will permit the analysis of this section to move rapidly. The diagrammatic representation of the hypothesis being considered is presented in Figure 7a. The assumption for x_1 that only two alternative processes are available is maintained for convenience; the resulting production possibility curves for x_1 are shown by the solid lines.

The assumption of variable coefficients in the production of x_2 is represented by the dashed lines in Figure 7a. As shown in Figure 7a, however, the variability of the coefficients in production of x_2 is limited to the sector between its ridge lines because at these ridge lines the marginal productivity of one of the factors becomes zero and further input this factor would have no effect on output.[1] I really do not intend to commit myself at this time to the view that the ridge lines for x_2 are, in fact, quite close together.

Figure 7a

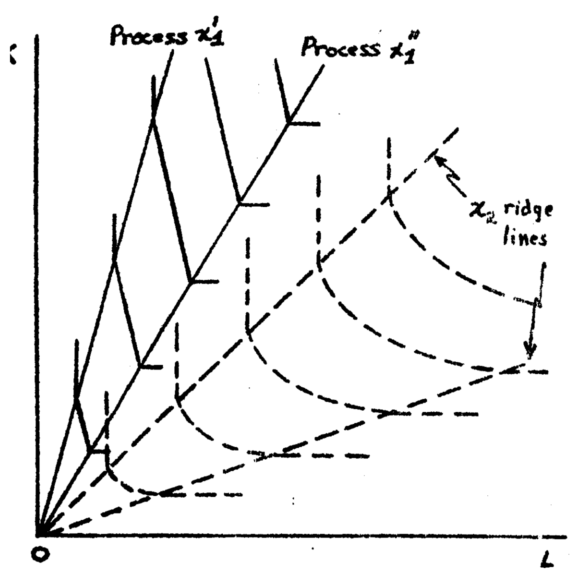
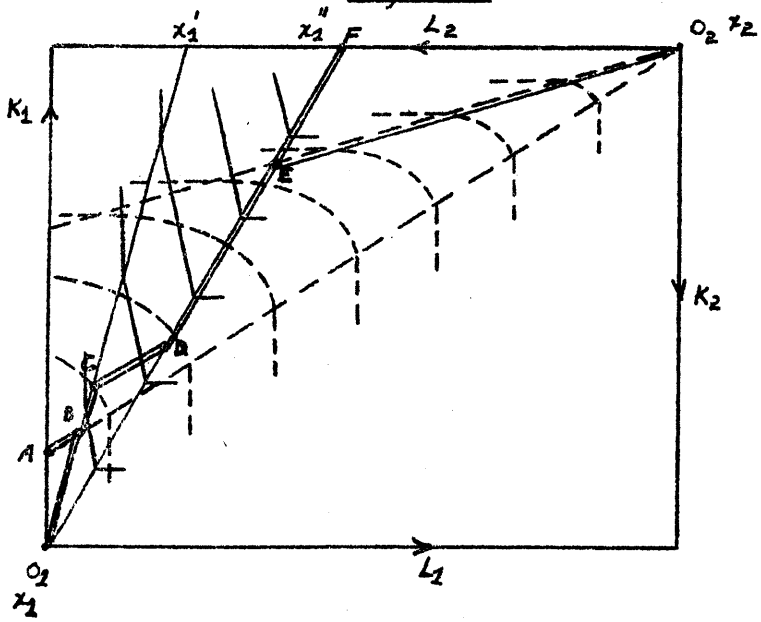


Figure 7b



This may, indeed, be the case but the curves are drawn as shown mainly to make it possible to represent a complete range of possibilities on the transformation curve. It may then be fruitful to speculate as to which of the possibilities correspond most closely to actual conditions. Different factor endowment ratios, differences in the range of factor proportions within which substitutability is possible, will, along with other influences, change the position and shape of the efficiency locus and the corresponding

1. It is assumed that there is no disposal problem and, thus, that the production isoquants do not bend back on themselves.

transformation curve. The box diagram approach helps to clarify the implications of the differences in substitutability in the fixed coefficient and variable coefficient sectors; it may be useful to consider these differences one of degree rather than kind.

The production functions for x_1 and x_2 are reproduced in the box diagram in Figure 7b: the dimensions of the box are determined by the factor endowments.[1] Using this box diagram we can trace out the efficiency locus for the two products by repeatedly asking the question, "For a given amount of x_1 what is the maximum amount of x_2 which can be produced?" In the process of tracing out the efficiency locus, the transformation curve can be drawn for the two goods. Again it will be useful to proceed in steps.

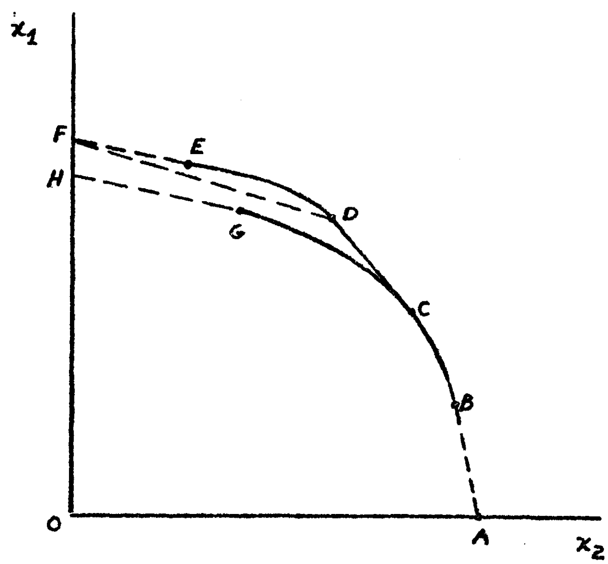
Starting at O_1, zero output of x_1 the maximum amount of x_2 producible is given by O_2A. If the output of x_1 is increased relative to x_2, it would be most efficient at first to use process x_1' for x_1 and to produce x_2 by traveling along its most capital intensive ridge line, O_2A. This represents optimal behavior up to point B. It will be noticed that in this stage both products are being produced with factors used in fixed proportions and capital is a redundant factor. This results in spite of the variability of coefficients in production of x_2 because outside the upper ridge line of x_2 capital has zero marginal productivity. Stage 1 is represented on the transformation curve in Figure 7c as segment AB.

The efficiency locus from B to C is traced out by finding the succession of points at which the x_1 equal product curves touch the highest x_2 equal product curves. In this stage process x_1' will be used for x_1 and a varying

1. In fact only the ratio of the factor endowments is important for this case as long as constant returns to scale is assumed. This is also true, of course, of the previous case.

combination of factors for x_2. In step 2 x_1 is increased and x_2 decreased, the capital labor ratio used in x_2 will decrease. The segment of the transformation curve for this stage BC will be curved as equal increases

Figure 7c



in production of x_1 along x_1' will not result in constant changes in output of x_2. Only along rays from the origin O_2 will equal distances imply equal differences in output of x_2. BC will be concave to the origin as is "normal" for transformation curves; graphically it can be seen that smaller and smaller changes along x_1' are needed in order to move across equal changes in output of x_2. As production moves from B to C the points at which the x_1' corners touch the x_2 isoquants will be characterized by smaller and smaller slopes on the x_2 isoquants, corresponding to the decreasing capital-labor ratio used in the production of x_2. At some point, C, the capital-labor ratio in x_2 will become equal to the capital-labor ratio represented by the negatively slanting portion of the x_1 isoquants. A ray from the origin O_2 to point C will intersect every x_2 isoquant at a point with identical slopes. Thus the third stage of the efficiency locus and the transformation curve will be the series of tangencies of the negatively slanting portion of the x_1 isoquants, representing combinations of processes x_1' and x_1'' and the x_2 isoquants along the ray O_2C. As production moves from C to D there is a shift in the proportion of output produced processes by x_1' and x_1'' until at point D all of x_1 is produced by process x_1'. The segment of the transformation curve corresponding to CD on the efficiency locus will be a straight line as CD lies on a ray from O_2 and thus the equal jumps across the x_1 isoquants will mark out constant changes in production of x_2.

At point D and for further increases in output of x_1 relative to x_2 it would be best to use only process x_1'' for production of x_1. In stage 4 as in stage 2 the efficiency locus, DE, is traced out as the series of points at which the x_1'' process isoquants touch the "highest" isoquants of x_2 in the curved portions. The segment of the transformation curve, DE, corresponding to DE on the efficiency locus is curved, for reasons similar to those which created the curvature of segment BC.

The final stage of the efficiency locus is the labor unemployment stage. Output of x_1 is exclusively by use of process x_1'' and output of x_2 falls along the labor intensive ridge line, O_2E. Unemployment of labor is measured as the horizontal distances between these lines in this stage. The marginal productivity of labor has fallen to zero in the x_2 sector and in a perfect factor market wages would fall to zero. EF represents this final stage on the transformation curve in Figure 6c.

ABCDEF in Figure 7c is the full transformation curve for this case. There are now curved as well as straight line segments and the kinks, characteristic of the previous transformation curves have disappeared.[1]

1. Being off the efficiency curve, it may be noted is like being on a isoquant lm in the single good case. Cf. Figure 1 above.

In Figure 7b it is also possible to visualize the effects of innovations which change the shape and position of the production isoquants for each product and the relative factor endowments.[1] Stretching the box vertically, for example, would represent an increase in the amount of capital available; it would, if carried far enough, eliminate all or part of the relatively labor intensive segments EF and DE and would stretch the capital unemployment sector AB. Developing more capital intensive processes for either or both goods would swing the capital intensive ridge lines toward the vertical axes. This would reduce the capital unemployment stages and might add stages like CD. Similar effects for labor would result from development of more labor intensive processes.

Without empirical knowledge, it is not possible to evaluate certainly the relative importance of each of the stages. However, according to the hypothesis advanced in the previous paper the transformation curve for underdeveloped areas would consist mainly of high labor intensity and labor unemployment segments such as DE and EF.

In this case as in the previous case for the market mechanism to keep production on the efficiency locus it is necessary that factor prices be flexible. But flexibility within the wides limits is required to achieve every possible position on the transformation locus in Figure 7c. If factor endowments or technology eliminate one or more of the segments of this curve, the range within which factor price flexibility is required is correspondingly reduced. Limited factor price flexibility may be quite serious when at least one good is produced with fixed coefficient processes.

1. It may be noted in passing that in the constant returns to scale case, only relative factor endowments are important in determining the <u>shape</u> of the transformation curve. If the absolute factor endowments were changed while relative factor endowments remain constant, it would amount to sliding the northeast and southwest vertices of the box on the connecting diagonal.

If rigid factor prices render a relatively labor intensive process unprofitable, the only alternative process may represent a big jump to a quite capital intensive process as well as a drastic obstacle to substitution in the variable coefficients process.

It may approximate reality to assume that factor price ratios in the variable coefficients sector are relatively more flexible than those in the fixed coefficients sector. This might result from differential strength of union organization or susceptibility to government wage controls. Suppose, for example, the labor-capital price ratio was set at, or above, that represented the slope of the constant product curve combining processes x_1' and x_1''. Process x_1'' would never be used and the transformation curve would be ABCGH; this is below the optimum transformation curve and has a much longer range of unemployment. If both sectors were characterized by such high, inflexible factor price ratios, the economy's transformation curve would approach ABJ.

Although the effects of rigid wages on the transformation curve of the economy are clear, welfare judgments as to the results of removal of the factor price rigidities are subject to the same qualifications spelled out for the previous case. Much depends on the effects of a change in methods on the income distribution and, <u>via</u> income distribution, on community preferences. One result of assuming variable coefficients for one sector is to provide a continuous range of slopes on the transformation curve at which tangencies with community indifference curves could occur.

As in the previous analysis the existence of straight line segments in the transformation curve depends on the constant returns to scale assumption. Non-constant returns to scale would introduce curvature throughout the transformation curve.

The two good-two factor case contains, I believe, most of the essential characteristics of the many good-many factor case. A number of problems of analysis remain but it is interesting to find that the major impressions of the simpler one good-two factor examples of the previous paper have, on the whole, been confirmed.

APPENDIX

Figure 5a represents the two processes which are available for the production of each of the two goods and on Figure 5b, along the efficiency locus, it is possible to read off the combinations of processes actually used.

From A to B in Figures 5b and 5c processes x_1' and x_2'' are used. The boundaries of the transformation curve for this combination of processes are given by the factor use equations

$$(1.1) \qquad a_{11}'x_1 + a_{12}''x_2 \leq \bar{K} \quad ;$$

$$(1.2) \qquad a_{21}'x_1 + a_{22}''x_2 \leq \bar{L} \quad ;$$

where K and L are the factor endowments. These equations say that the amount of capital and labor, respectively, used in the production of x_1 and x_2 can be no more than the endowed amounts of the factors. (1.1) and (1.2) provide the boundaries of the technical transformation for this set of processes. Rewritten in the form

$$(1.11) \qquad x_1 \leq \frac{\bar{K}}{a_{11}'} - \frac{a_{12}''}{a_{11}'} x_2 \quad ;$$

$$(1.21) \qquad x_1 \leq \frac{\bar{L}}{a_{21}'} - \frac{a_{22}''}{a_{21}'} x_2 \quad .$$

they are somewhat more easily visualized. It has been assumed that in this stage labor is the limiting factor and capital is redundant so that (1.21) provides the equation for the line along the AB segment of the transformation curve in Figure 5c. If capital rather than labor is limiting the equation (1.11) becomes relevant.

The boundary conditions for processes x_1'' and x_2' used along the DE stage of Figures 5b and 5c can be similarly derived as

$$(2.11) \qquad x_1 \leq \frac{\bar{K}}{a_{11}''} - \frac{a_{12}'}{a_{11}''} x_2 \quad ;$$

$$(2.21) \qquad x_1 \leq \frac{\bar{L}}{a_{21}''} - \frac{a_{22}'}{a_{21}''} x_2 \quad .$$

In this stage it was assumed capital was limiting, so that (2.11) provides the only operative boundary condition.

At point C processes x_1'' and x_2'' are used together with full employment of factors. This point can be found by solution of the boundary line equations for the process,

(3.11) $$x_1 \leq \frac{\bar{K}}{a_{11}''} - \frac{a_{12}''}{a_{11}''} x_2 \quad ;$$

(3.21) $$x_1 \leq \frac{K}{a_{21}''} - \frac{a_{22}''}{a_{21}''} x_2 \quad ;$$

to give

(3.3) $$x_1 = \frac{a_{12}'' \bar{L} - a_{22}'' \bar{K}}{a_{12}'' a_{21}'' - a_{11}'' a_{22}''}$$

(3.4) $$x_2 = \frac{\bar{a}_1'' \bar{L} - a_{21}'' \bar{K}}{a_{11}'' a_{22}'' - a_{12}'' a_{21}''}$$

Between B and C and between C and D two processes are used to produce one of the goods along with one process for the other. Working with the BC section which uses processes x_1' and x_1'' for x_1 and x_2'' for x_2, the factor use equations are

(4.1) $$a_{11}' x_1' + a_{11}'' x_1'' + a_{12}'' x_2'' \leq \bar{K} \quad ;$$

(4.2) $$a_{21}' x_1' + a_{21}'' x_1'' + a_{22}'' x_2'' \leq \bar{L} \quad .$$

Note that

(4.3) $$x_1' + x_1'' = x_1$$

(4.4) $$x_2' = x_2$$

We can express the proportion of x_1 produced by x_1' as

(4.31) $$x_1' / x_1 = C_1$$

so that

(4.32) $$x_1'' = (1-C_1)\, x_1$$

where it is recognized that C_1 will vary.

It is now possible to express the boundary equations for this stage in terms of x_1, x_2 and C_1 :

(4.11) $$x_1 \leq \frac{\bar{K}}{C_1\,(a_{11}' - a_{11}'') + a_{11}''} - \frac{a_{12}''}{C_1\,(a_{11}' - a_{11}'') + a_{11}''}\, x_2$$

(4.21) $$x_1 \leq \frac{\bar{K}}{C_1\,(a_{21}' - a_{21}'') + a_{21}''} - \frac{a_{22}''}{C_1\,(a_{21}' - a_{21}'') + a_{21}''}\, x_2$$

C_1 could be eliminated from these equations to find the equation for the segment BC.