**1.021, 3.021, 10.333, 22.00 Introduction to Modeling and Simulation** Spring 2011

**Part I – Continuum and particle methods**

# **Application to modeling brittle materials**

Lecture 7

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#### Content overview



- 3. The Many-Body Problem: From Many-Body to Single-Particle
- 4. Quantum modeling of materials
- 5. From Atoms to Solids
- 6. Basic properties of materials
- 7. Advanced properties of materials
- 8. What else can we do?

#### Overview: Material covered so far…

- П **Lecture 1: Broad introduction to IM/S**
- **Lecture 2**: **Introduction to atomistic and continuum modeling** (multiscale modeling paradigm, difference between continuum and atomistic approach, case study: diffusion)
- **Lecture 3**: **Basic statistical mechanics – property calculation I** (property calculation: microscopic states vs. macroscopic properties, ensembles, probability density and partition function)
- $\blacksquare$  **Lecture 4**: **Property calculation II** (Monte Carlo, advanced property calculation, introduction to chemical interactions)
- **Lecture 5: How to model chemical interactions I** (example: movie of copper deformation/dislocations, etc.)
- $\blacksquare$  **Lecture 6: How to model chemical interactions II** (pair potentials, fracture – introduction)
- П **Lecture 7: Application – MD simulation of materials failure**

#### **Lecture 7: Application to modeling brittle materials**

#### **Outline:**

- 1. Basic deformation mechanism in brittle materials crack extension
- 2. Atomistic modeling of fracture
	- 2.1 Physical properties of atomic lattices
	- 2.2 Application
- 3. Bond order force fields how to model chemical reactions

#### **Goal of today's lecture:**

- П Apply our tools to model a particular material phenomena – brittle fracture (useful for pset #2 )
- П Learn basics in fracture of brittle materials
- ш Learn how to build a model to describe brittle fracture (from scratch)

### 1. Basic deformation mechanism in brittle materials - crack extension

#### Brittle fracture

- $\blacksquare$ Materials: **glass, silicon, many ceramics, rocks**
- $\blacksquare$  At large loads, rather than accommodating a shape change, materials break

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#### Basic fracture process: dissipation of elastic energy



### Continuum description of fracture

- Fracture is a dissipative process in which elastic energy is dissipated to break bonds (and to heat at large crack speeds)
- $\blacksquare$ **E** Energy to break bonds = surface energy  $\gamma_s$  (energy necessary to create new surface, dimensions: energy/area,  $\rm Nm/m^2$ )



# Griffith condition for fracture initiation

 $\blacksquare$ **Energy release rate** G, that is, the elastic energy released per unit crack advance must be equal or larger than the energy necessary to create new surfaces

$$
G := \frac{1}{2} \frac{\sigma^2}{E} \xi = 2 y_s \qquad G = 2 \gamma_s
$$

- $\blacksquare$ Provides **criterion to predict failure initiation**
- $\blacksquare$  $\blacksquare$  Calculation of G can be complex, but straightforward for thin strips as shown above
- $\blacksquare$  Approach to calculate *G* based on "**stress intensity factor**" (see further literature, e.g. Broberg, Anderson, Freund, Tada)

### Brittle fracture mechanisms

- $\blacksquare$  Once nucleated, cracks in brittle materials spread rapidly, on the order of sound speeds
- ٠ Sound speeds in materials (=wave speeds):
	- **•** Rayleigh-wave speed  $c_R$  (speed of surface waves)
	- $\textcolor{red}{\bullet}$  shear wave speed  $c_{\scriptscriptstyle S}$  (speed of shear waves)
	- $\textcolor{red}{\bullet}$  longitudinal wave speed  $c_l$  (speed of longitudinal waves)
- $\blacksquare$  *Maximum speeds of cracks is given by sound speeds, depending on mode of loading (mode I, II, III)*

*Linear elastic continuum theory*

# Brittle fracture loading conditions

 $\blacksquare$  Commonly consider a single crack in a material geometry, under three types of loading: mode I, mode II and mode III



#### Limiting speeds of cracks: linear elastic continuum theory



Image by MIT OpenCourseWare.

• Cracks **can not exceed** the limiting speed given by the corresponding wave speeds **unless material behavior is nonlinear**

• Cracks that exceed limiting speed would produce energy (physically impossible - *linear elastic continuum theory*)

#### Sound speeds in materials: overview



Wave speeds are calculated based on elastic properties of material

$$
c_l = \sqrt{\frac{3\mu}{\rho}} \qquad c_s = \sqrt{\frac{\mu}{\rho}} \qquad c_R \approx \beta c_s \qquad \beta \approx 0.923
$$

 $\mu$  = shear modulus  $E = 8/3 \mu$   $\mu = 3/8E$ 

#### Brittle fracture mechanisms: fracture is a multiscale phenomenon, from nano to macro

Image removed due to copyright restrictions. Fig. 6.2 in Buehler, Markus J. *Atomistic Modeling of Materials Failure*. Springer, 2008.

# Physical reason for crack limiting speed

- ٠ Physical (mathematical) reason for the limiting speed is that it becomes increasingly difficult to increase the speed of the crack by adding a larger load
- $\blacksquare$  When the crack approaches the limiting speed, the **resistance to fracture diverges to infinity (=dynamic fracture toughness )**

*divergence*

Driving force (applied load)

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$$
G = \frac{1}{2} \frac{\sigma^2}{E} \xi = f(y_s, v)
$$

$$
G \sim \frac{\sigma^2}{E}
$$

#### 2. Atomistic modeling of fracture

#### What is a model?

#### **Mike Ashby (Cambridge University):**

■ *A model is an idealization. Its relationship to the real problem is like that of the map of the London tube trains to the real tube systems: a gross simplification, but one that captures certain essentials.* 



#### *"Physical situation" "Model"*

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#### A "simple" atomistic model: geometry



#### Harmonic and harmonic bond snapping potential



$$
\phi = \frac{1}{2}k_0(r - r_0)^2
$$

$$
\phi = \begin{cases} \frac{1}{2}k_0(r - r_0)^2 & r < r_{\text{break}}\\ \frac{1}{2}k_0(r_{\text{break}} - r_0)^2 & r \ge r_{\text{break}} \end{cases}
$$

#### 2.1 Physical properties of atomic lattices

#### *How to calculate elastic properties and fracture surface energy – parameters to link with continuum theory of fracture*

free energy density (energy per unit volume)

Stress 
$$
\sigma = \frac{\partial \psi(\varepsilon)}{\partial \varepsilon}
$$
 Young's modulus  $E = \frac{\partial^2 \psi(\varepsilon)}{\partial \varepsilon^2}$   
 $\sigma = E\varepsilon$ 

#### 1D example – "Cauchy-Born rule"

- ٠ Impose homogeneous strain field on 1D string of atoms
- ٠ **-** Then obtain  $\sigma = E\varepsilon$  from that



# 2D hexagonal lattice





$$
\psi(\varepsilon_{ij}) = \frac{\sqrt{3}}{8} \phi^{i} \left( 3\varepsilon_{11}^{2} + 2\varepsilon_{11}\varepsilon_{22} + 3\varepsilon_{22}^{2} + (\varepsilon_{21} + \varepsilon_{12})^{2} \right)
$$

$$
\sigma_{ij} = \frac{\partial \psi(\varepsilon_{ij})}{\partial \varepsilon_{ij}} \qquad \qquad c_{ijkl} = \frac{\partial^2 \psi(\varepsilon_{ij})}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}}
$$

#### 2D triangular lattice, LJ potential



### 2D triangular lattice, harmonic potential



Elastic properties of the triangular lattice with harmonic interactions, stress versus strain (left) and tangent moduli  $E_x$  and  $E_y$  (right). The stress state is uniaxial tension, that is the stress in the direction orthogonal to the loading is relaxed and zero.

#### Elastic properties

$$
E = \frac{2}{\sqrt{3}}k, \quad \mu = \frac{\sqrt{3}}{4}k \qquad \phi' = k
$$

Enables to calculate wave speeds:

66

 $72\sqrt[3]{2} \approx 57.14$ 





### Surface energy calculation



Image by MIT OpenCourseWare.

Harmonic potential with bond snapping distance  $r_{\operatorname{break}}$ 

$$
\gamma_s^{\text{bs,h}} = \frac{k(r_{\text{break}} - r_0)^2}{\sqrt{3}r_0} \qquad \gamma_s^{\text{bs,l}} = \frac{k(r_{\text{break}} - r_0)^2}{2r_0}
$$

#### **Note:** out-of-plane unity thickness

#### 2.2 Application

#### *Focus: effects of material nonlinearities (reflected in choice of model)*

### Coordinate system and atomistic model



Image by MIT OpenCourseWare.

*Pair potential to describe atomic interactions Confine crack to a 1D path (weak fracture layer)*

#### Linear versus nonlinear elasticity=hyperelasticity



Image by MIT OpenCourseWare.

**Linear elasticity:** Young's modulus (stiffness) does not change with deformation

**Nonlinear elasticity = hyperelasticity:** Young's modulus (stiffness) changes with deformation

### Subsonic and supersonic fracture

- $\blacksquare$  Under certain conditions, material nonlinearities (that is, the behavior of materials under large deformation = hyperelasticity) becomes important
- ٠ This can lead to different limiting speeds than described by the model introduced above



#### *Deformation field near a crack*





Image by MIT OpenCourseWare.

# Limiting speeds of cracks



Image by MIT OpenCourseWare.

- Under presence of hyperelastic effects, cracks can exceed the conventional barrier given by the wave speeds
- This is a "local" effect due to enhancement of energy flux
- Subsonic fracture due to local softening, that is, reduction of energy flux

### Stiffening vs. softening behavior



Image by MIT OpenCourseWare.

*Increased/decreased wave speed*

$$
c_l = \sqrt{\frac{3\mu}{\rho}} \qquad c_s = \sqrt{\frac{\mu}{\rho}}
$$

$$
c_R \approx \beta c_s
$$

 $\beta \approx 0.923$ 

# MD model development: biharmonic potential



- **Stiffness change under deformation**, with different strength
- Atomic bonds break at critical atomic separation
- **Want:** simple set of parameters that control these properties (as few as possible, to gain generic insight)

#### Biharmonic potential – control parameters



#### Biharmonic potential definition

The biharmonic potential is defined as:

$$
\phi \quad (r_{ij}) = \begin{cases} a_0 + \frac{1}{2}k_0(r_{ij} - r_0)^2 & \text{if } r_{ij} < r_{\text{on}}, \\ a_1 + \frac{1}{2}k_1(r_{ij} - r_1)^2 & \text{if } r_{ij} \ge r_{\text{on}} \end{cases}
$$

where  $r_{\rm on}$  is the critical atomic separation for the onset of the hyperelastic effect, and

$$
a_1 = a_0 + \frac{1}{2}k_0(r_{\text{on}} - r_0)^2 - \frac{1}{2}k_1(r_{\text{on}} - r_1)^2
$$

and

$$
r_1 = \frac{1}{2}(r_{\text{on}} + r_0)
$$

are found by continuity conditions of the potential at  $r = r_{on}$ . The values  $k_0$  and  $k_1$  refer to the small- and large-strain spring constants.

# Energy filtering: visualization approach

- Only plot atoms associated with higher energy
- $\blacksquare$  Enables determination of crack tip (atoms at surface have higher energy, as they have fewer neighbors than atoms in the bulk)

Energy of atom 
$$
i
$$
  $U_i = \sum_{j=1}^N \phi(r_{ij})$ 



Image by MIT OpenCourseWare.

### MD simulation results: confirms linear continuum theory



*a*  ( *t* )

#### crack speed =time derivative of *a*

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*v* ( *t* )*=da/dt*

Image by MIT OpenCourseWare.

	Spring constant k Young's modulus E Shear modulus $\mu$ Poisson's ratio $\nu   c_i   c_s$ $c_R$				
$36\sqrt[3]{2} \approx 28.57$		12.4	0.33	6.36 3.67 3.39	
$72\sqrt[3]{2}\approx 57.14$	66	24.8	0.33	$9 \mid 5.2 \mid 4.8$	

#### Virial stress field around a crack

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large stresses at crack tip *- induce bond failure*

#### Biharmonic potential – bilinear elasticity



Image by MIT OpenCourseWare.

#### Harmonic system



#### Subsonic fracture

- $\blacksquare$  Softening material behavior leads to subsonic fracture, that is, the crack can never attain its theoretical limiting speed
- $\blacksquare$ Materials: metals, ceramics



#### Supersonic fracture

- $\blacksquare$  Stiffening material behavior leads to subsonic fracture, that is, the crack can exceed its theoretical limiting speed
- $\blacksquare$ Materials: polymers



### Different ratios of spring constants



Stiffening and softening effect: Increase or reduction of crack speed

# Physical basis for subsonic/supersonic fracture

- $\blacksquare$  Changes in energy flow at the crack tip due to changes in local wave speed (energy flux higher in materials with higher wave speed)
- $\blacksquare$ **•** Controlled by a characteristic length scale  $\chi$



#### **LI** Hyperelastic region

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 $AP$ 

#### Supersonic fracture: mode II (shear)



Please see: Buehler, Markus J., Farid F. Abraham, and Huajian Gao. "Hyperelasticity Governs Dynamic Fracture at a Critical Length Scale." *Nature* 426 (November 13, 2003): 141-6.

# Theoretical concept: energy flux reduction/enhancement



Image by MIT OpenCourseWare.

**Energy flux related to wave speed:** high local wave speed, high energy flux, crack can move faster (and reverse for low local wave speed)

# 3. Bond order force fields - how to model chemical reactions

#### Challenge: chemical reactions



*CHARMM-type potential can not describe chemical reactions*

#### Why can not model chemical reactions with |spring-like potentials?

$$
\phi_{\text{stretch}} = \frac{1}{2} k_{\text{stretch}} (r - r_0)^2
$$

$$
\phi_{\text{bend}} = \frac{1}{2} k_{\text{bend}} (\theta - \theta_0)^2
$$

Set of parameters only valid for particular molecule type / type of chemical bond

$$
k_{\text{stretch},sp^2} \neq k_{\text{stretch},sp^3}
$$

*Reactive potentials or reactive force fields overcome these limitations*

# Key features of reactive potentials

- $\blacksquare$  How can one accurately describe the transition energies during chemical reactions?
- п Use computationally more efficient descriptions than relying on purely quantum mechanical (QM) methods (see part II, **methods limited to 100 atoms**)



## Key features of reactive potentials

- $\blacksquare$ Molecular model that is capable of describing **chemical reactions**
- $\blacksquare$  **Continuous energy landscape during reactions** (key to enable integration of equations)
- $\blacksquare$  No typing necessary, that is, atoms can be sp, sp2, sp3… w/o further "tags" – **only element types**
- $\blacksquare$  **Computationally efficient** (that is, should involve finite range interactions), so that large systems can be treated (> 10,000 atoms)
- $\blacksquare$ **Parameters with physical meaning (such as for the LJ potential)**

#### Theoretical basis: bond order potential

**Concept:** Use pair potential that depends on atomic environment (similar to EAM, here applied to covalent bonds)

$$
\phi(r_{ij}) = \phi_{\mathbf{R}}(r_{ij}) - M_{ij}\phi_{A}(r_{ij})
$$
\nModulate strength of  
\n
$$
M_{ij} \sim Z^{-\delta}
$$
\nModulate strength of  
\n
$$
\begin{array}{c}\n\text{intactive part} \\
\text{(e.g. by coordination,}\\
\text{for "bond order")} \\
\text{Abell, Tersoff}\n\end{array}
$$
\nTherefore, pair-interactions for various C-C (Carbon) bonds

Image by MIT OpenCourseWare.

 $k(r) \sim k_0 M_{ij}(Z,\delta)$ 

Changes in spring constant as function of bond order Continuous change possible <sup>=</sup>**continuous energy landscape during chemical reactions**

#### Theoretical basis: bond order potential



Image by MIT OpenCourseWare.

#### Concept of bond order (BO)



#### Bond order based energy landscape



# Historical perspective of reactive bond order potentials

- $\mathcal{L}^{\mathcal{L}}$  1985: Abell: General expression for binding energy as a sum of near nieghbor pair interactions moderated by local atomic environment
- $\mathbb{R}^3$  1990s: Tersoff, Brenner: Use Abell formalism applied to silicon (successful for various solid state structures)
- $\mathcal{L}_{\mathcal{A}}$ 2000: Stuart et al.: Reactive potential for hydrocarbons
- $\mathcal{L}_{\mathcal{A}}$  2001: Duin, Godddard et al.: Reactive potential for hydrocarbons "ReaxFF"
- $\mathcal{L}^{\mathcal{L}}$  2002: Brenner et al.: Second generation "REBO" potential for hydrocarbons
- $\mathcal{L}_{\mathcal{A}}$  2003-2005: Extension of ReaxFF to various materials including metals, ceramics, silicon, polymers and more in Goddard's group

# Example: ReaxFF reactive force field



William A. Goddard III California Institute of Technology



Courtesy of Bill Goddard. Used with permission.

Adri C.T. v. DuinCalifornia Institute of Technology

#### ReaxFF: A reactive force field

$$
E_{system} = E_{bond} + E_{vdWaals} + E_{Coulomb} + E_{val,angle} + E_{tors}
$$
  
2-body  
3-body  
4-body  
multi-body

*Total energy is expressed as the sum of various terms describing individual chemical bonds*

#### *All expressions in terms of bond order*

*All interactions calculated between ALL atoms in system…*

#### *No more atom typing: Atom type = chemical element*

#### Example: Calculation of bond energy

$$
E_{system} = E_{bond} + E_{vdWaals} + E_{Coulomb} + E_{val,angle} + E_{tors} + E_{over} + E_{under}
$$

$$
E_{\text{bond}} = -D_{\text{e}} \cdot \text{BO}_{ij} \cdot \exp\left[p_{\text{be},1}\left(1 - \text{BO}_{ij}^{p_{\text{be},1}}\right)\right]
$$

Bond energy between atoms *i* and *j* does not depend on bond distance

#### **Instead, it depends on bond order**

#### Bond order functions

BO goes smoothly from 3-2- 1-0









*All energy terms are expressed as a function of bond orders*

#### Illustration: Bond energy

$$
E_{bond} = -D_e^{\sigma} \cdot BO_{ij}^{\sigma} \cdot f(BO_{ij}^{\sigma}) - D_e^{\pi} \cdot BO_{ij}^{\pi} - D_e^{\pi\pi} \cdot BO_{ij}^{\pi\pi}
$$

Image removed due to copyright restrictions. Please see slide 10 in van Duin, Adri. "Dishing Out the Dirt on ReaxFF." <http://www.wag.caltech.edu/home/duin/FFgroup/Dirt.ppt>.

#### vdW interactions

$$
E_{system} = E_{bond} + E_{vdWaal} + E_{Coulomb} + E_{val,angle} + E_{tors} + E_{over} + E_{under}
$$

- Accounts for short distance repulsion (Pauli principle orthogonalization) and attraction energies at large distances (dispersion)
- $\blacksquare$ Included for all atoms with shielding at small distances

$$
E_{\text{vdWaals}} = D_{ij} \cdot \left\{ \exp\left[\alpha_{ij} \cdot \left(1 - \frac{f_{13}(r_{ij})}{r_{\text{vdW}}}\right)\right] - 2 \cdot \exp\left[\frac{1}{2} \cdot \alpha_{ij} \cdot \left(1 - \frac{f_{13}(r_{ij})}{r_{\text{vdW}}}\right)\right] \right\}
$$

$$
f_{13}(r_{ij}) = \left[r_{ij}^{\lambda_{29}} + \left(\frac{1}{\lambda_w}\right)^{\lambda_{28}}\right]^{1/\lambda_{28}}
$$

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Please see slide 11 in van Duin, Adri. "Dishing Out the Dirt on ReaxFF." <http://www.wag.caltech.edu/home/duin/FFgroup/Dirt.ppt>.

#### Resulting energy landscape



Contribution of *E*bond and vdW energy

Source: van Duin, C. T. Adri, et al. "ReaxFF: A Reactive Force Field for Hydrocarbons." *Journal of Physical Chemistry A* 105 (2001). © American Chemical Society. All rights reserved. This content is excluded from our Creative Commons license. For more information, see<http://ocw.mit.edu/fairuse>.

#### Current development status of ReaxFF



: not currently described by ReaxFF

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B

A

A--B

Allows to interface metals, ceramics with organic chemistry: Key for complex materials, specifically biological materials

Periodic table courtesy of Wikimedia Commons.

#### Mg-water interaction: How to make fire with water

Video stills removed due to copyright restrictions; watch the video now: <http://www.youtube.com/watch?v=QTKivMVUcqE>.



[http://video.google.com/videoplay?docid=4697996292949045921&q=magnesium+water&total=46&start=0&num=50&so=0&type=search&plindex=](http://video.google.com/videoplay?docid=4697996292949045921&q=magnesium+water&total=46&start=0&num=50&so=0&type=search&plindex=0) 0

#### Mg – water interaction – ReaxFF MD simulation



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