1.021, 3.021, 10.333, 22.00 Introduction to Modeling and Simulation Spring 2011

Part I – Continuum and particle methods

Application to modeling brittle materials

Lecture 7

Markus J. Buehler

Laboratory for Atomistic and Molecular Mechanics Department of Civil and Environmental Engineering Massachusetts Institute of Technology



Massachusetts Institute of Technology

Content overview

		2

I. Particle and continuum methods

- 1. Atoms, molecules, chemistry
- 2. Continuum modeling approaches and solution approaches
- 3. Statistical mechanics
- 4. Molecular dynamics, Monte Carlo
- 5. Visualization and data analysis
- 6. Mechanical properties application: how things fail (and how to prevent it)
- 7. Multi-scale modeling paradigm
- 8. Biological systems (simulation in biophysics) how proteins work and how to model them

II. Quantum mechanical methods

- 1. It's A Quantum World: The Theory of Quantum Mechanics
- 2. Quantum Mechanics: Practice Makes Perfect
- 3. The Many-Body Problem: From Many-Body to Single-Particle
- 4. Quantum modeling of materials
- 5. From Atoms to Solids
- 6. Basic properties of materials
- 7. Advanced properties of materials
- 8. What else can we do?

Lectures 14-26

Lectures 2-13

Overview: Material covered so far...

- Lecture 1: Broad introduction to IM/S
- Lecture 2: Introduction to atomistic and continuum modeling (multiscale modeling paradigm, difference between continuum and atomistic approach, case study: diffusion)
- Lecture 3: Basic statistical mechanics property calculation I (property calculation: microscopic states vs. macroscopic properties, ensembles, probability density and partition function)
- Lecture 4: Property calculation II (Monte Carlo, advanced property calculation, introduction to chemical interactions)
- Lecture 5: How to model chemical interactions I (example: movie of copper deformation/dislocations, etc.)
- Lecture 6: How to model chemical interactions II (pair potentials, fracture introduction)
- Lecture 7: Application MD simulation of materials failure

Lecture 7: Application to modeling brittle materials

Outline:

- 1. Basic deformation mechanism in brittle materials crack extension
- 2. Atomistic modeling of fracture
 - 2.1 Physical properties of atomic lattices
 - 2.2 Application
- 3. Bond order force fields how to model chemical reactions

Goal of today's lecture:

- Apply our tools to model a particular material phenomena brittle fracture (useful for pset #2)
- Learn basics in fracture of brittle materials
- Learn how to build a model to describe brittle fracture (from scratch)

1. Basic deformation mechanism in brittle materials - crack extension

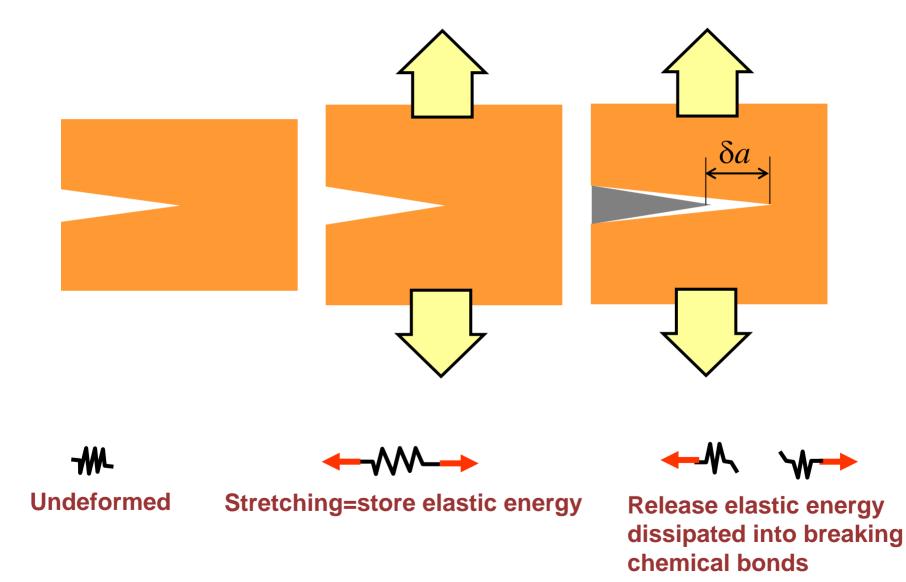
Brittle fracture

- Materials: glass, silicon, many ceramics, rocks
- At large loads, rather than accommodating a shape change, materials break

Image courtesy of quinn.anya. License: CC-BY.

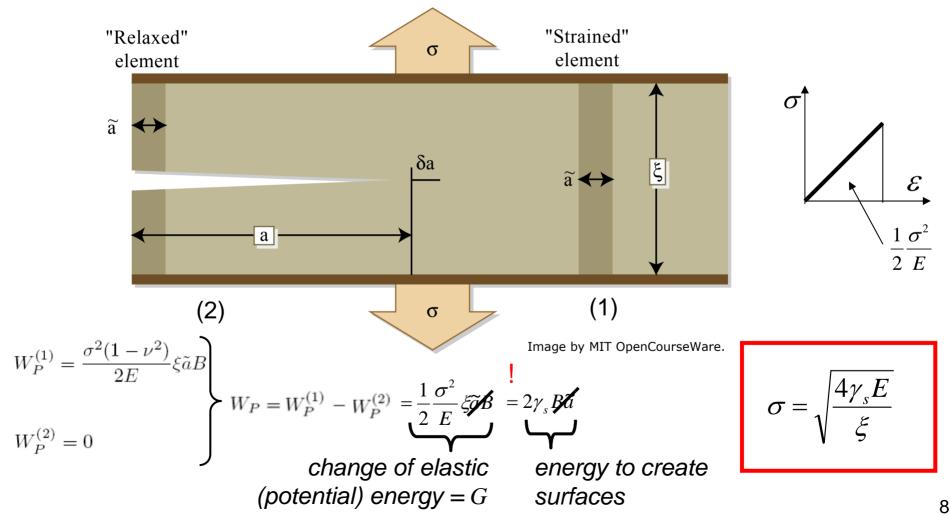


Basic fracture process: dissipation of elastic energy



Continuum description of fracture

- Fracture is a dissipative process in which elastic energy is dissipated to break bonds (and to heat at large crack speeds)
- Energy to break bonds = surface energy γ_s (energy necessary to create new surface, dimensions: energy/area, Nm/m²)



Griffith condition for fracture initiation

 Energy release rate G, that is, the elastic energy released per unit crack advance must be equal or larger than the energy necessary to create new surfaces

$$G := \frac{1}{2} \frac{\sigma^2}{E} \xi = 2y_s \qquad \qquad G = 2\gamma_s$$

- Provides criterion to predict failure initiation
- Calculation of G can be complex, but straightforward for thin strips as shown above
- Approach to calculate G based on "stress intensity factor" (see further literature, e.g. Broberg, Anderson, Freund, Tada)

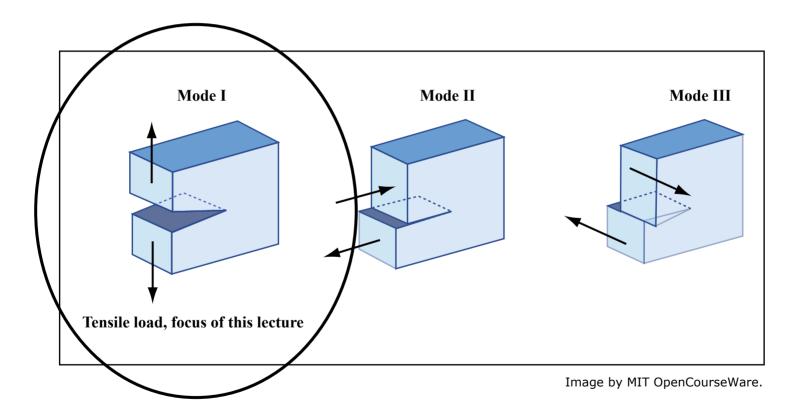
Brittle fracture mechanisms

- Once nucleated, cracks in brittle materials spread rapidly, on the order of sound speeds
- Sound speeds in materials (=wave speeds):
 - Rayleigh-wave speed c_R (speed of surface waves)
 - shear wave speed c_s (speed of shear waves)
 - longitudinal wave speed c_l (speed of longitudinal waves)
- Maximum speeds of cracks is given by sound speeds, depending on mode of loading (mode I, II, III)

Linear elastic continuum theory

Brittle fracture loading conditions

 Commonly consider a single crack in a material geometry, under three types of loading: mode I, mode II and mode III



Limiting speeds of cracks: linear elastic continuum theory

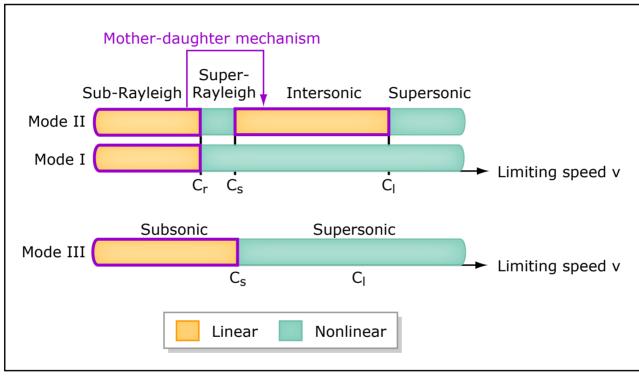


Image by MIT OpenCourseWare.

• Cracks can not exceed the limiting speed given by the corresponding wave speeds unless material behavior is nonlinear

 Cracks that exceed limiting speed would produce energy (physically impossible - *linear elastic continuum theory*)

Sound speeds in materials: overview

Material	$c_R \ (in m/s)$	$c_s \ (\text{in m/s})$	$c_l ~({ m in}~{ m m/s})$
Steel	2,940	3,200	6,000
Al	2,850	$3,\!100$	$6,\!300$
Glass	3,030	3,300	$5,\!800$
PMMA	920	1,000	$2,\!400$

Wave speeds are calculated based on elastic properties of material

$$c_l = \sqrt{\frac{3\mu}{\rho}}$$
 $c_s = \sqrt{\frac{\mu}{\rho}}$ $c_R \approx \beta c_s$ $\beta \approx 0.923$

 μ = shear modulus $E = 8/3\mu$ $\mu = 3/8E$

Brittle fracture mechanisms: fracture is a multiscale phenomenon, from nano to macro

Image removed due to copyright restrictions. Fig. 6.2 in Buehler, Markus J. *Atomistic Modeling of Materials Failure*. Springer, 2008.

Physical reason for crack limiting speed

- Physical (mathematical) reason for the limiting speed is that it becomes increasingly difficult to increase the speed of the crack by adding a larger load
- When the crack approaches the limiting speed, the resistance to fracture diverges to infinity (=dynamic fracture toughness)

divergence

Driving force (applied load)

Image removed due to copyright restrictions. Please see: Fig. 6.15 in Buehler, Markus J. *Atomistic Modeling of Materials Failure*. Springer, 2008.

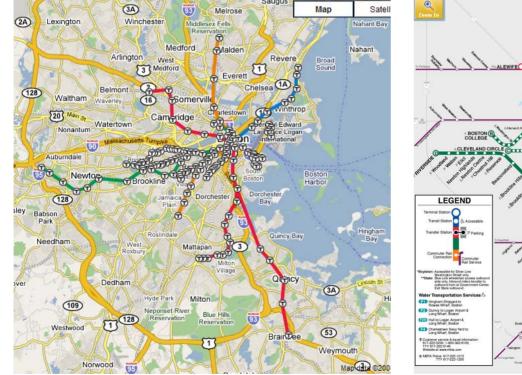
$$G = \frac{1}{2} \frac{\sigma^2}{E} \xi = f(y_s, v)$$
$$G \sim \frac{\sigma^2}{E}$$

2. Atomistic modeling of fracture

What is a model?

Mike Ashby (Cambridge University):

 A model is an idealization. Its relationship to the real problem is like that of the map of the London tube trains to the real tube systems: a gross simplification, but one that captures certain essentials.





A DAK GRO

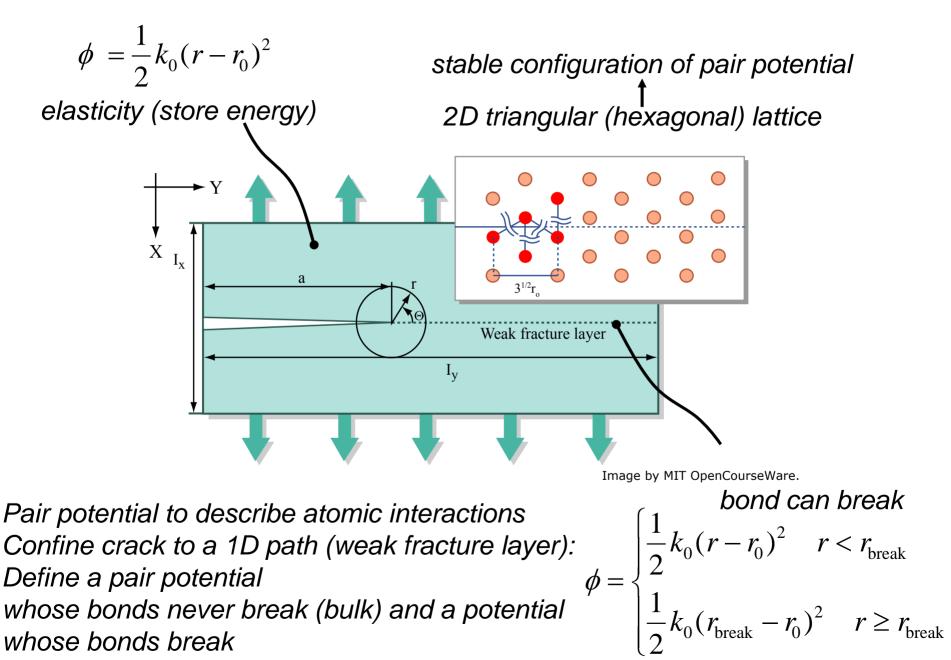
"Physical situation"

© Google, Inc. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/fairuse.

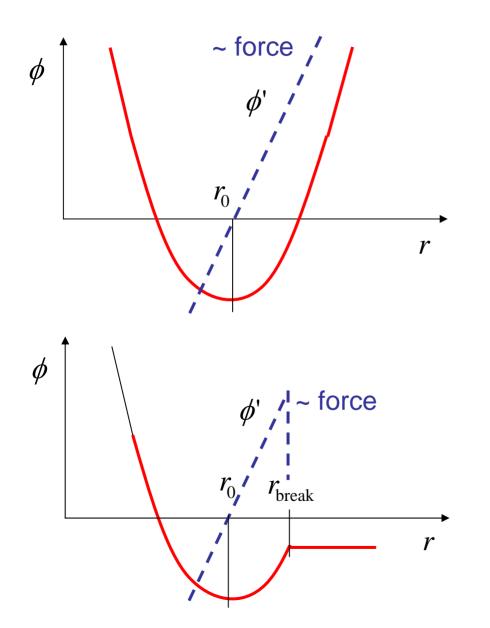
"Model"

© Massachusetts Bay Transportation Authority. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/fairuse. 17

A "simple" atomistic model: geometry



Harmonic and harmonic bond snapping potential



$$\phi = \frac{1}{2}k_0(r - r_0)^2$$

$$\phi = \begin{cases} \frac{1}{2}k_0(r - r_0)^2 & r < r_{\text{break}} \\ \frac{1}{2}k_0(r_{\text{break}} - r_0)^2 & r \ge r_{\text{break}} \end{cases}$$

19

2.1 Physical properties of atomic lattices

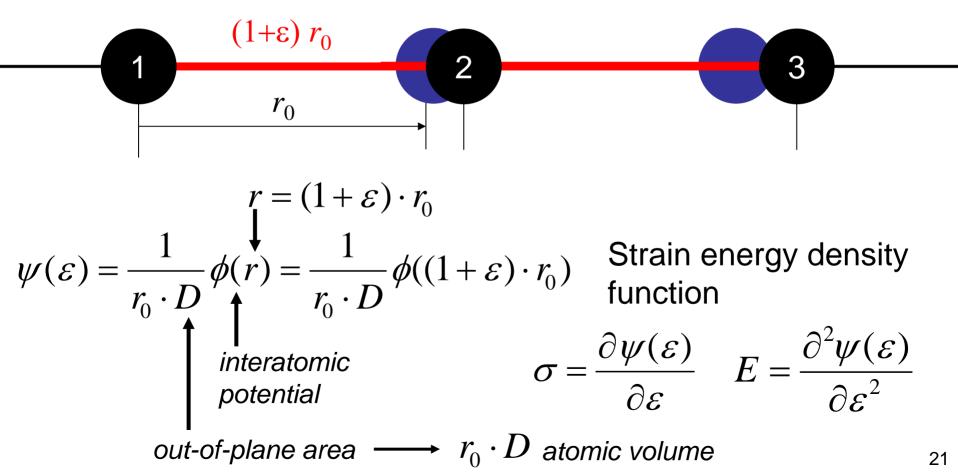
How to calculate elastic properties and fracture surface energy – parameters to link with continuum theory of fracture

_ free energy density (energy per unit volume)

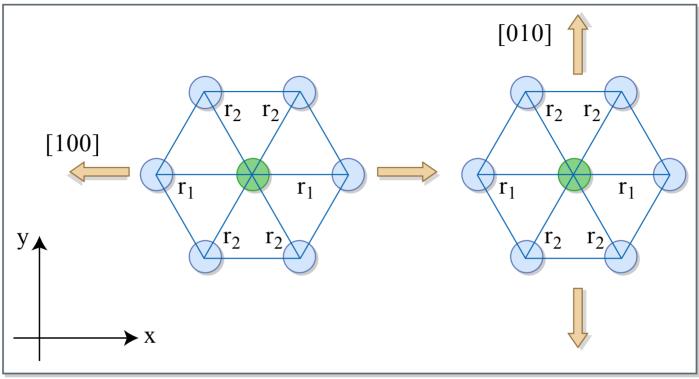
Stress
$$\sigma = \frac{\partial \psi(\varepsilon)}{\partial \varepsilon}$$
 Young's modulus $E = \frac{\partial^2 \psi(\varepsilon)}{\partial \varepsilon^2}$
 $\sigma = E\varepsilon$

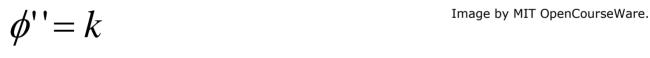
1D example – "Cauchy-Born rule"

- Impose homogeneous strain field on 1D string of atoms
- Then obtain $\sigma = E\varepsilon$ from that



2D hexagonal lattice

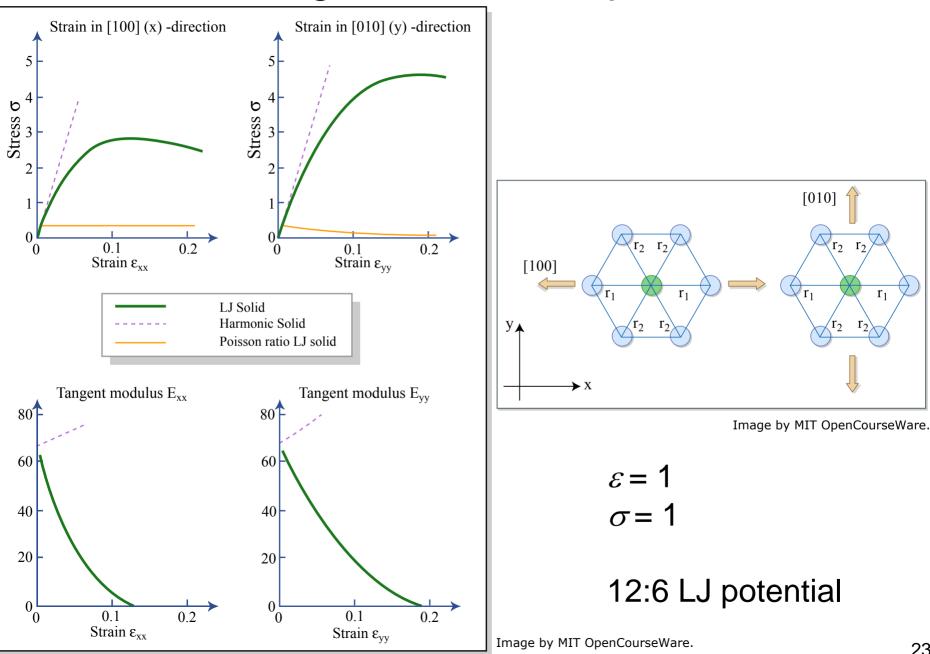




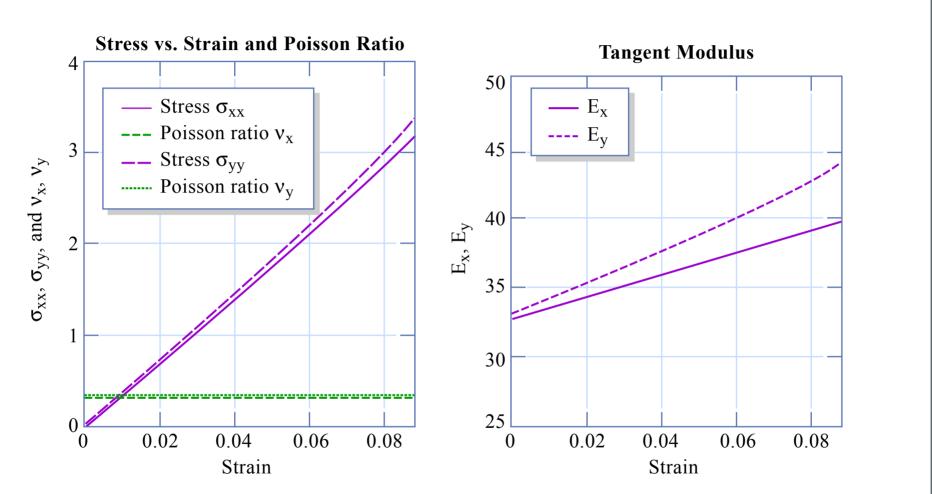
$$\psi(\varepsilon_{ij}) = \frac{\sqrt{3}}{8}\phi'' \left(3\varepsilon_{11}^2 + 2\varepsilon_{11}\varepsilon_{22} + 3\varepsilon_{22}^2 + (\varepsilon_{21} + \varepsilon_{12})^2\right)$$

$$\sigma_{ij} = \frac{\partial \psi(\varepsilon_{ij})}{\partial \varepsilon_{ij}} \qquad c_{ijkl} = \frac{\partial^2 \psi(\varepsilon_{ij})}{\partial \varepsilon_{ij} \partial \varepsilon_{kl}}$$

2D triangular lattice, LJ potential



2D triangular lattice, harmonic potential



Elastic properties of the triangular lattice with harmonic interactions, stress versus strain (left) and tangent moduli E_x and E_y (right). The stress state is uniaxial tension, that is the stress in the direction orthogonal to the loading is relaxed and zero.

Elastic properties

$$E = \frac{2}{\sqrt{3}}k, \quad \mu = \frac{\sqrt{3}}{4}k \qquad \qquad \phi'' = k$$

Enables to calculate wave speeds:

Spring constant k	Young's modulus E	Shear modulus μ	Poisson's ratio ν	c_l	C_s	c_R
$36\sqrt[3]{2} \approx 28.57$	33	12.4	0.33	6.36	3.67	3.39
$72\sqrt[3]{2} \approx 57.14$	66	24.8	0.33	9	5.2	4.8

Surface energy calculation

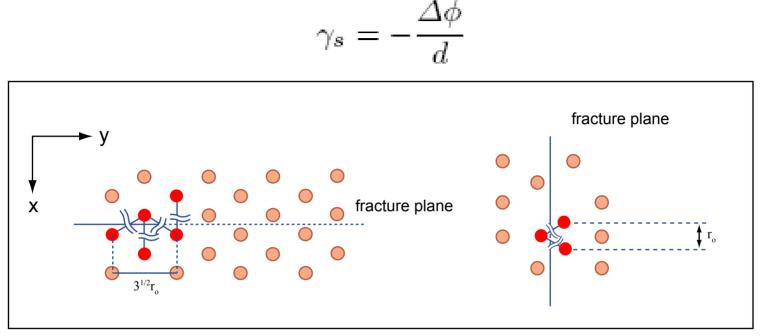


Image by MIT OpenCourseWare.

Harmonic potential with bond snapping distance r_{break}

$$\gamma_s^{\rm bs,h} = \frac{k(r_{\rm break} - r_0)^2}{\sqrt{3}r_0} \qquad \qquad \gamma_s^{\rm bs,l} = \frac{k(r_{\rm break} - r_0)^2}{2r_0}$$

Note: out-of-plane unity thickness

2.2 Application

Focus: effects of material nonlinearities (reflected in choice of model)

Coordinate system and atomistic model

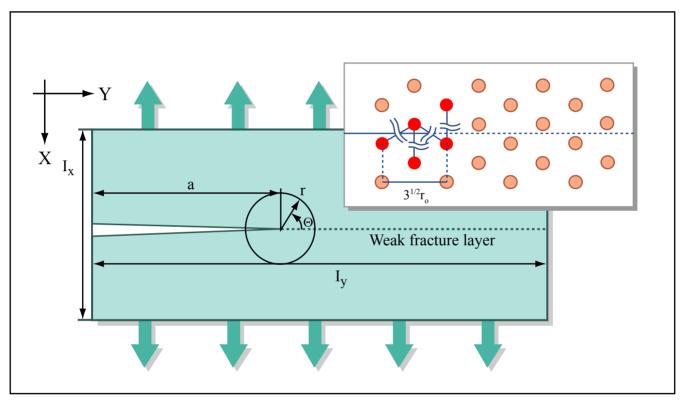


Image by MIT OpenCourseWare.

Pair potential to describe atomic interactions Confine crack to a 1D path (weak fracture layer)

Linear versus nonlinear elasticity=hyperelasticity

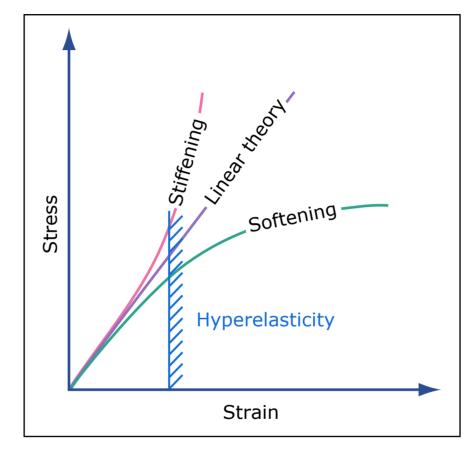


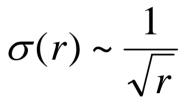
Image by MIT OpenCourseWare.

Linear elasticity: Young's modulus (stiffness) does not change with deformation

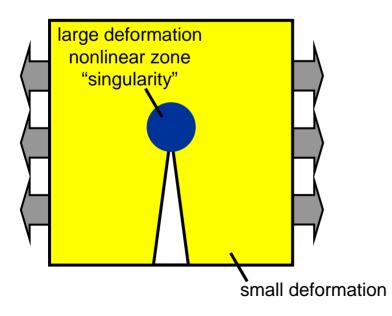
Nonlinear elasticity = hyperelasticity: Young's modulus (stiffness) changes with deformation

Subsonic and supersonic fracture

- Under certain conditions, material nonlinearities (that is, the behavior of materials under large deformation = hyperelasticity) becomes important
- This can lead to different limiting speeds than described by the model introduced above



Deformation field near a crack



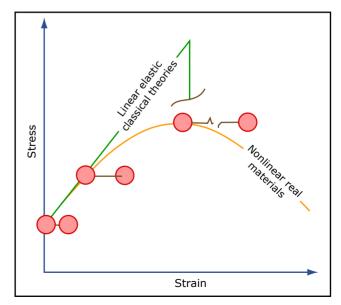


Image by MIT OpenCourseWare.

Limiting speeds of cracks

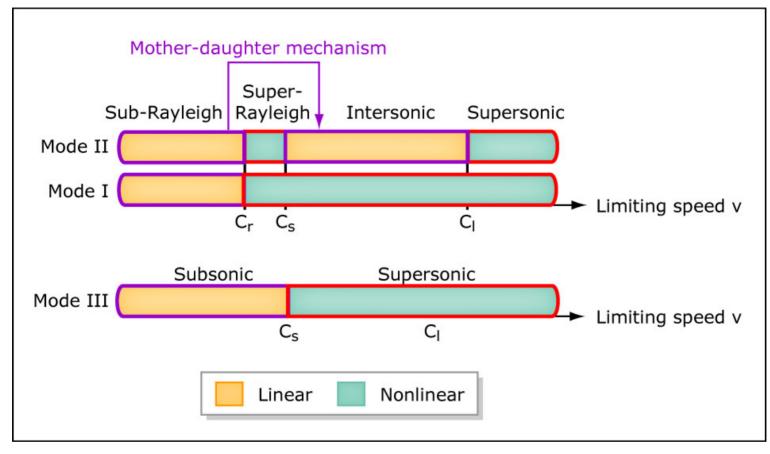


Image by MIT OpenCourseWare.

- Under presence of hyperelastic effects, cracks can exceed the conventional barrier given by the wave speeds
- This is a "local" effect due to enhancement of energy flux
- Subsonic fracture due to local softening, that is, reduction of energy flux

Stiffening vs. softening behavior

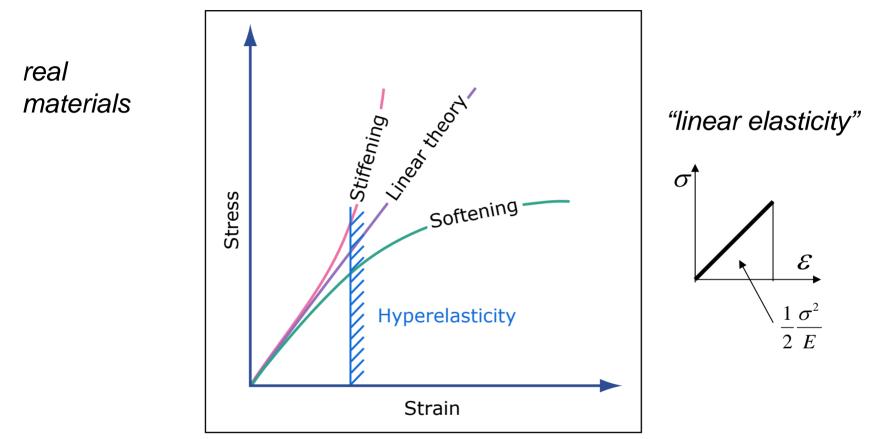


Image by MIT OpenCourseWare.

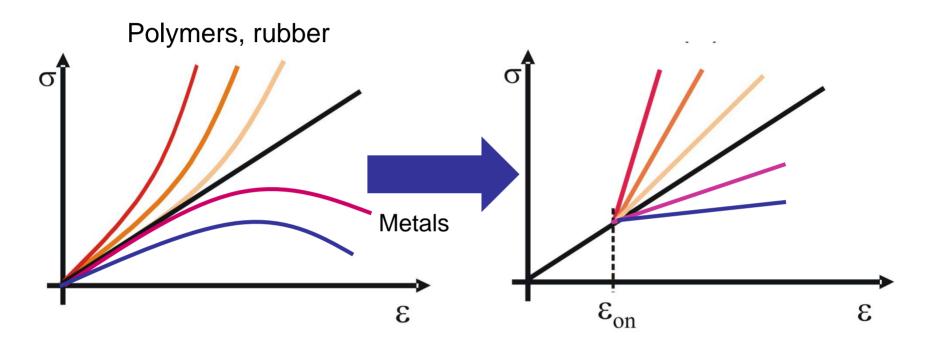
Increased/decreased wave speed

$$c_l = \sqrt{\frac{3\mu}{\rho}} \qquad c_s = \sqrt{\frac{\mu}{\rho}}$$

$$c_R \approx \beta c_s$$

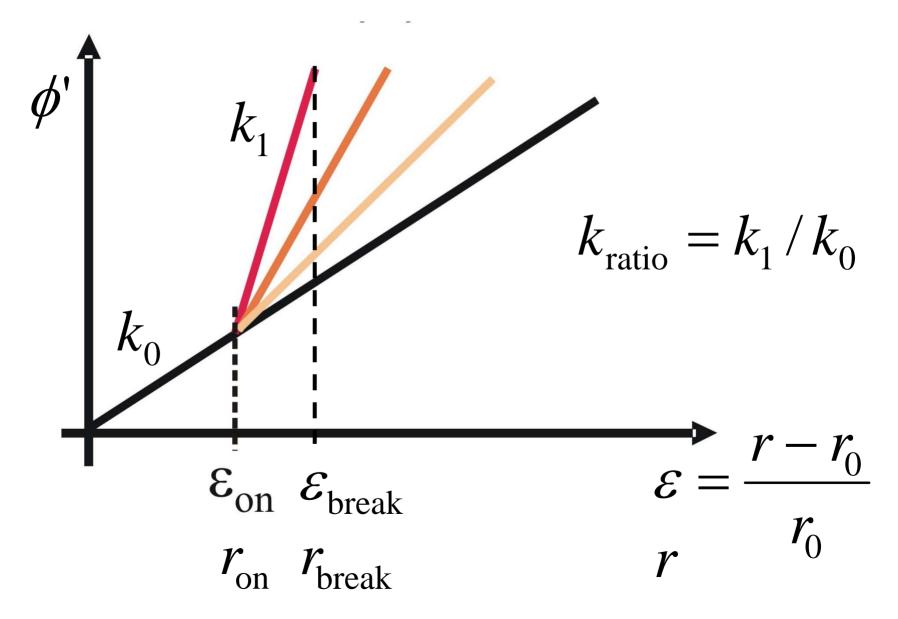
 $\beta \approx 0.923$

MD model development: biharmonic potential



- Stiffness change under deformation, with different strength
- Atomic bonds break at critical atomic separation
- Want: simple set of parameters that control these properties (as few as possible, to gain generic insight)

Biharmonic potential – control parameters



Biharmonic potential definition

The biharmonic potential is defined as:

$$\phi \quad (r_{ij}) = \begin{cases} a_0 + \frac{1}{2}k_0(r_{ij} - r_0)^2 & \text{if } r_{ij} < r_{\text{on}}, \\ a_1 + \frac{1}{2}k_1(r_{ij} - r_1)^2 & \text{if } r_{ij} \ge r_{\text{on}} \end{cases}$$

where r_{on} is the critical atomic separation for the onset of the hyperelastic effect, and

$$a_1 = a_0 + \frac{1}{2}k_0(r_{\rm on} - r_0)^2 - \frac{1}{2}k_1(r_{\rm on} - r_1)^2$$

and

$$r_1 = \frac{1}{2}(r_{\rm on} + r_0)$$

are found by continuity conditions of the potential at $r = r_{on}$. The values k_0 and k_1 refer to the small- and large-strain spring constants.

Energy filtering: visualization approach

- Only plot atoms associated with higher energy
- Enables determination of crack tip (atoms at surface have higher energy, as they have fewer neighbors than atoms in the bulk)

Energy of atom
$$i$$
 $U_i = \sum_{j=1}^N \phi(r_{ij})$

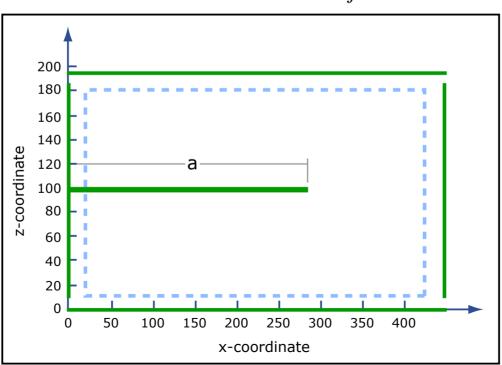
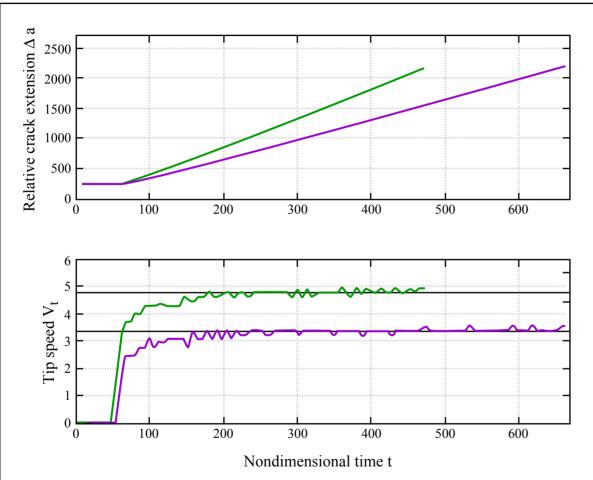


Image by MIT OpenCourseWare.

MD simulation results: confirms linear continuum theory



 $a\left(t
ight)$

crack speed =time derivative of *a*

37

v(t) = da/dt

Image by MIT OpenCourseWare.

Spring constan	it k	Young's modulus E	Shear modulus μ		-		
$36\sqrt[3]{2} \approx 28.5$	7	33	12.4	0.33			3.39
$72\sqrt[3]{2} \approx 57.1$	4	66	24.8	0.33	9	5.2	4.8

Virial stress field around a crack

Images removed due to copyright restrictions. Please see: Fig. 6.30 in Buehler, Markus J. *Atomistic Modeling of Materials Failure*. Springer, 2008.

large stresses at crack tip - induce bond failure

Biharmonic potential – bilinear elasticity

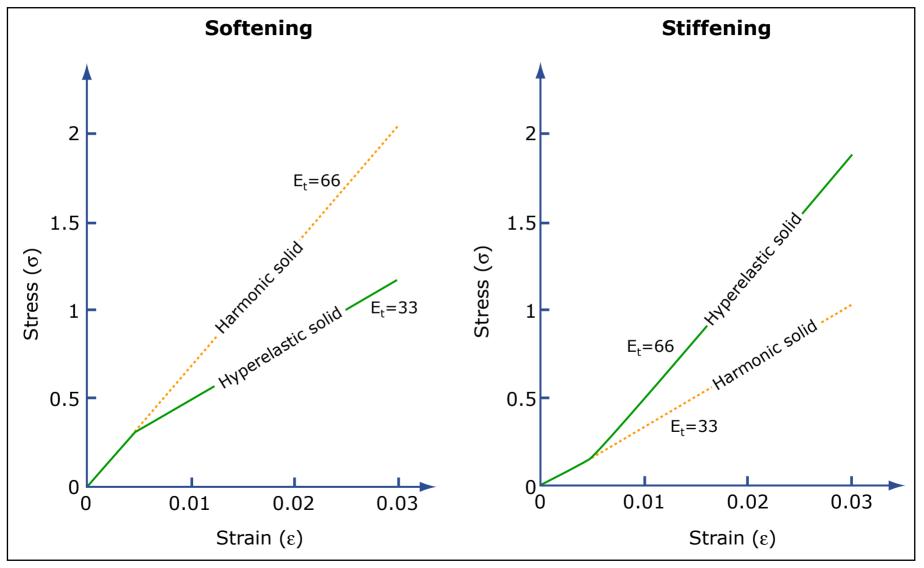
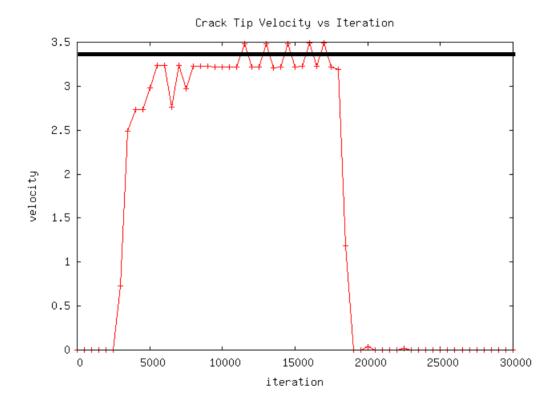


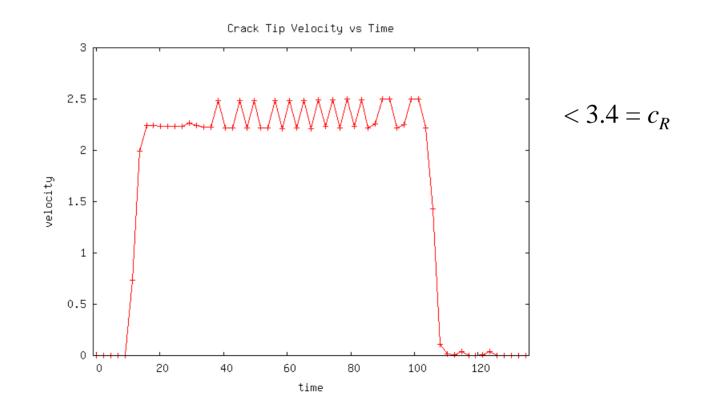
Image by MIT OpenCourseWare.

Harmonic system



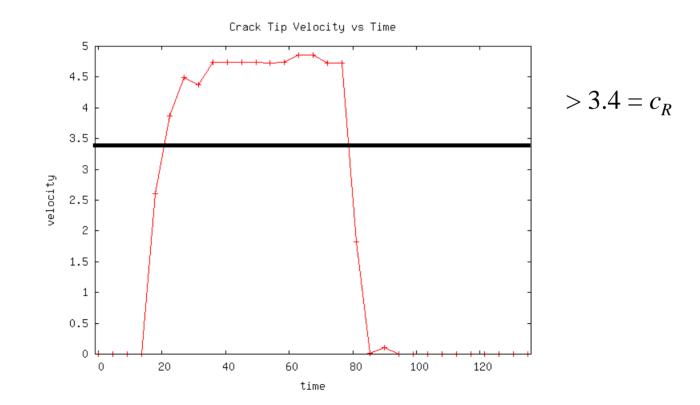
Subsonic fracture

- Softening material behavior leads to subsonic fracture, that is, the crack can never attain its theoretical limiting speed
- Materials: metals, ceramics

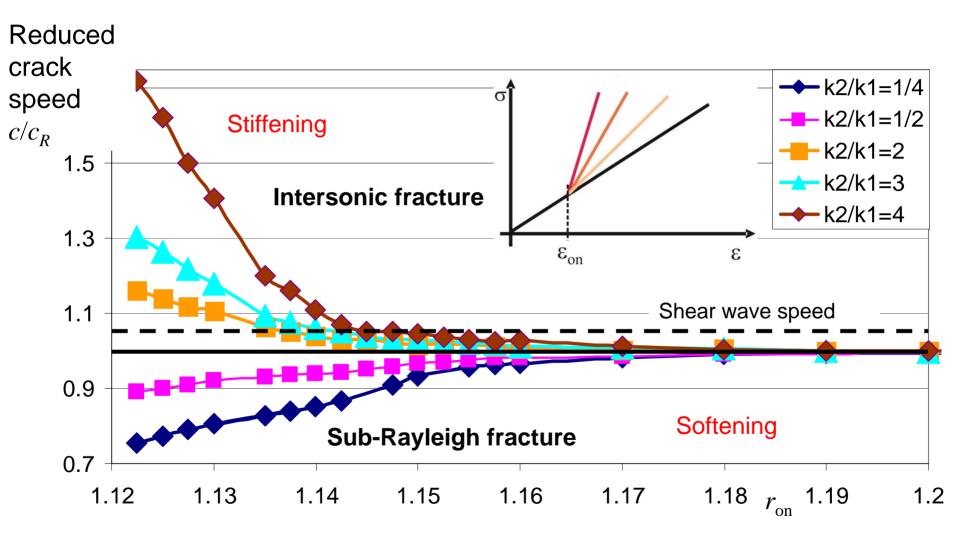


Supersonic fracture

- Stiffening material behavior leads to subsonic fracture, that is, the crack can exceed its theoretical limiting speed
- Materials: polymers



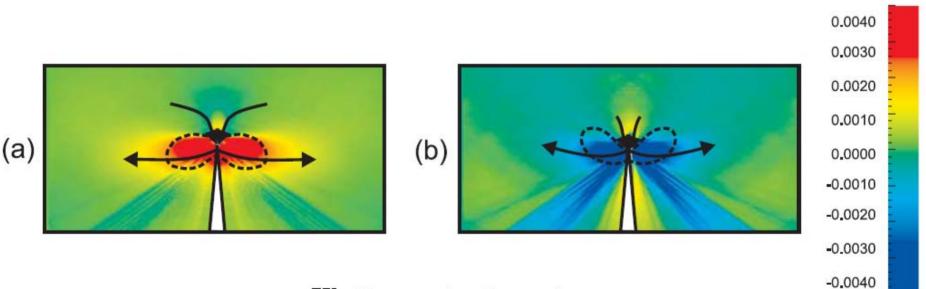
Different ratios of spring constants



Stiffening and softening effect: Increase or reduction of crack speed

Physical basis for subsonic/supersonic fracture

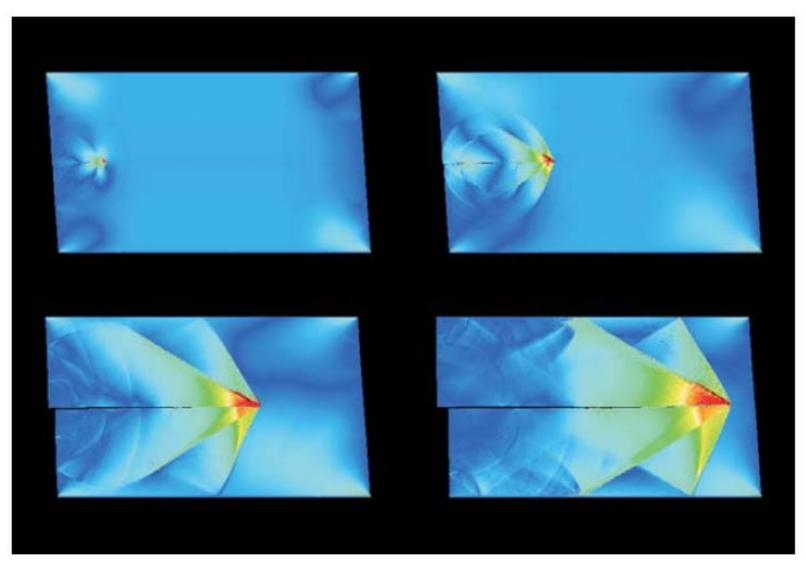
- Changes in energy flow at the crack tip due to changes in local wave speed (energy flux higher in materials with higher wave speed)
- Controlled by a characteristic length scale χ



Hyperelastic region

Reprinted by permission from Macmillan Publishers Ltd: Nature. Source: Buehler, M., F. Abraham, and H. Gao. "Hyperelasticity Governs Dynamic Fracture at a Critical Length Scale." *Nature* 426 (2003): 141-6. © 2003. ΔP

Supersonic fracture: mode II (shear)



Please see: Buehler, Markus J., Farid F. Abraham, and Huajian Gao. "Hyperelasticity Governs Dynamic Fracture at a Critical Length Scale." *Nature* 426 (November 13, 2003): 141-6.

Theoretical concept: energy flux reduction/enhancement

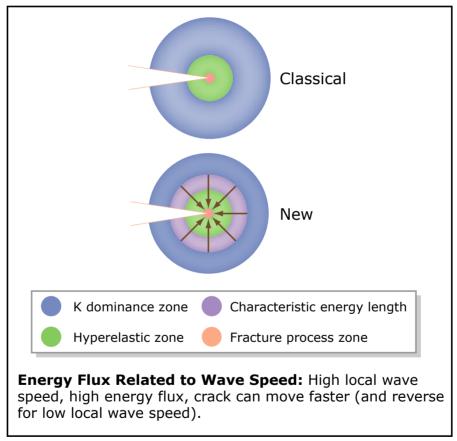
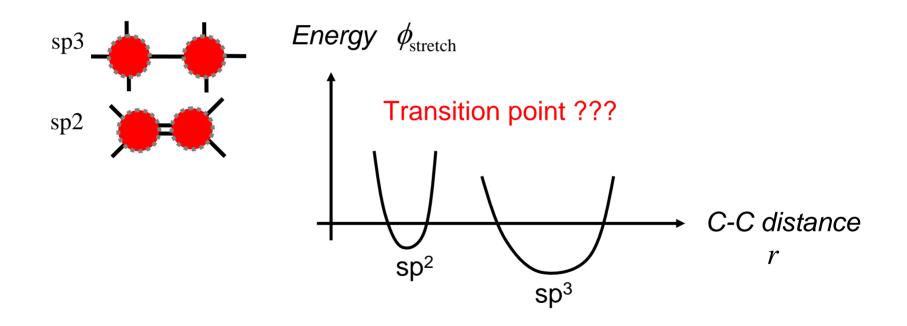


Image by MIT OpenCourseWare.

Energy flux related to wave speed: high local wave speed, high energy flux, crack can move faster (and reverse for low local wave speed)

3. Bond order force fields - how to model chemical reactions

Challenge: chemical reactions



CHARMM-type potential can not describe chemical reactions

Why can not model chemical reactions with spring-like potentials?

$$\phi_{\text{stretch}} = \frac{1}{2} k_{\text{stretch}} (r - r_0)^2$$
$$\phi_{\text{bend}} = \frac{1}{2} k_{\text{bend}} (\theta - \theta_0)^2$$

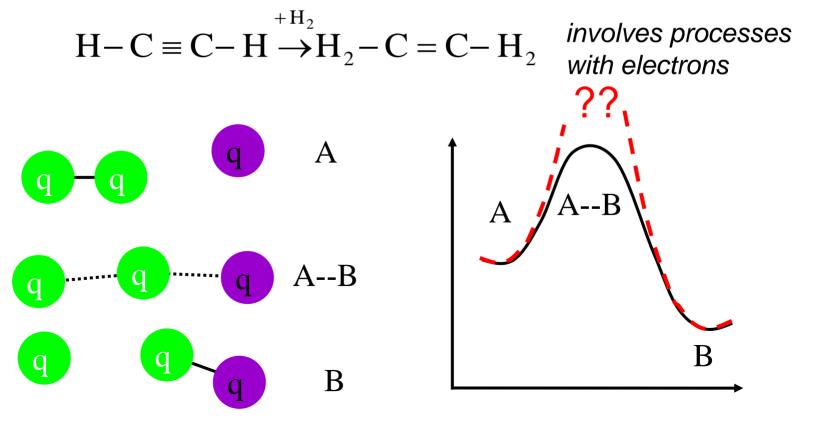
Set of parameters only valid for particular molecule type / type of chemical bond

$$k_{\text{stretch},sp^2} \neq k_{\text{stretch},sp^3}$$

Reactive potentials or reactive force fields overcome these limitations

Key features of reactive potentials

- How can one accurately describe the transition energies during chemical reactions?
- Use computationally more efficient descriptions than relying on purely quantum mechanical (QM) methods (see part II, methods limited to 100 atoms)



Key features of reactive potentials

- Molecular model that is capable of describing **chemical reactions**
- Continuous energy landscape during reactions (key to enable integration of equations)
- No typing necessary, that is, atoms can be sp, sp2, sp3... w/o further "tags" – only element types
- Computationally efficient (that is, should involve finite range interactions), so that large systems can be treated (> 10,000 atoms)
- Parameters with physical meaning (such as for the LJ potential)

Theoretical basis: bond order potential

Concept: Use pair potential that depends on atomic environment (similar to EAM, here applied to covalent bonds)

$$\phi(r_{ij}) = \phi_{R}(r_{ij}) - M_{ij}\phi_{A}(r_{ij})$$

$$\downarrow \qquad Modulate strength of attractive part (e.g. by coordination, or "bond order")$$

$$M_{ij} \sim Z^{-\delta} \qquad Abell, Tersoff$$

Image by MIT OpenCourseWare.

 $k(r) \sim k_0 M_{ij}(Z, \delta)$

Changes in spring constant as function of bond order Continuous change possible = continuous energy landscape during chemical reactions

Theoretical basis: bond order potential

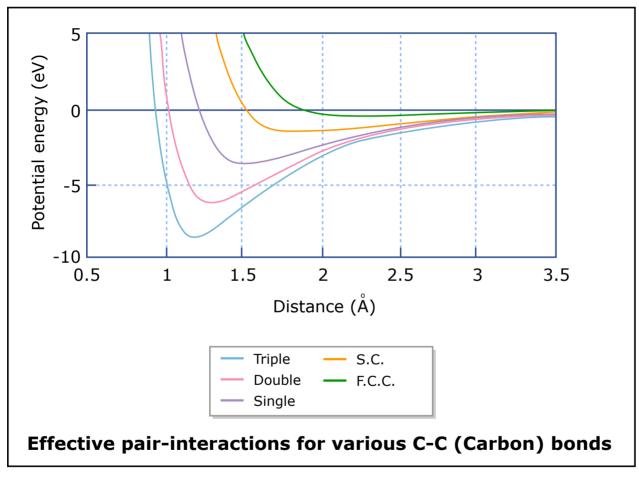
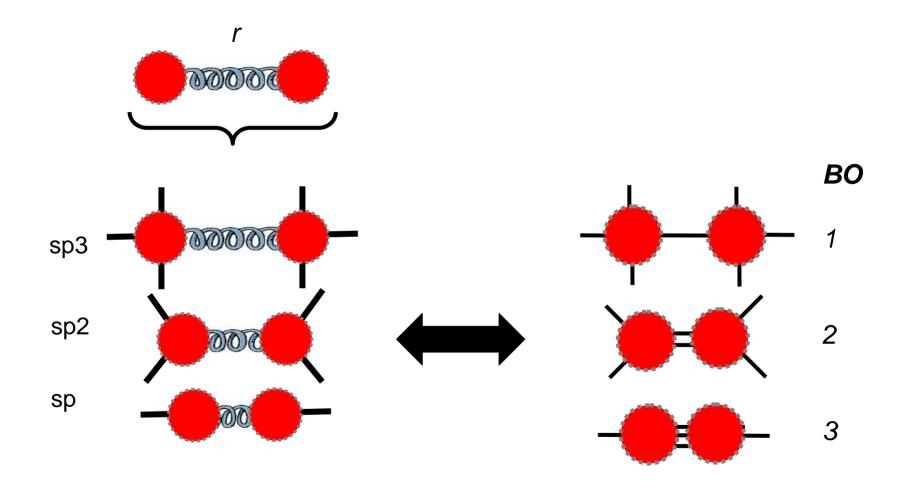
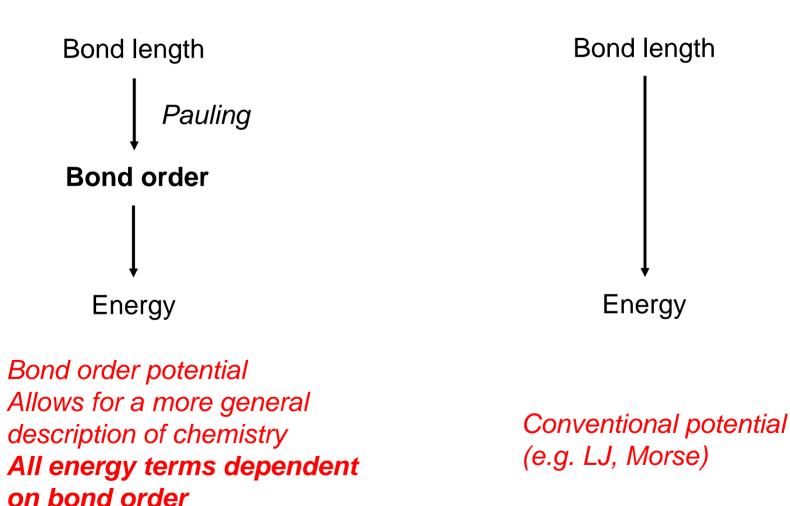


Image by MIT OpenCourseWare.

Concept of bond order (BO)



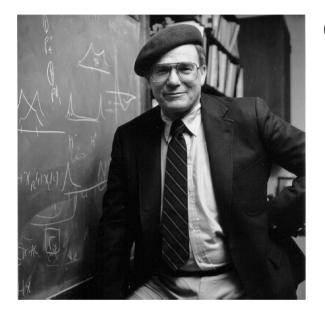
Bond order based energy landscape



Historical perspective of reactive bond order potentials

- 1985: Abell: General expression for binding energy as a sum of near nieghbor pair interactions moderated by local atomic environment
- 1990s: Tersoff, Brenner: Use Abell formalism applied to silicon (successful for various solid state structures)
- 2000: Stuart et al.: Reactive potential for hydrocarbons
- 2001: Duin, Godddard et al.: Reactive potential for hydrocarbons "ReaxFF"
- 2002: Brenner et al.: Second generation "REBO" potential for hydrocarbons
- 2003-2005: Extension of ReaxFF to various materials including metals, ceramics, silicon, polymers and more in Goddard's group

Example: ReaxFF reactive force field



William A. Goddard III California Institute of Technology



Courtesy of Bill Goddard. Used with permission.

Adri C.T. v. Duin California Institute of Technology

ReaxFF: A reactive force field

$$E_{system} = \underbrace{E_{bond} + E_{vdWaals} + E_{Coulomb}}_{2\text{-body}} + \underbrace{E_{val,angle} + E_{tors}}_{3\text{-body}} + \underbrace{E_{over} + E_{under}}_{\text{multi-body}}$$

Total energy is expressed as the sum of various terms describing individual chemical bonds

All expressions in terms of bond order

All interactions calculated between ALL atoms in system...

No more atom typing: Atom type = chemical element

Example: Calculation of bond energy

$$E_{system} = E_{bond} + E_{vdWaals} + E_{Coulomb} + E_{val,angle} + E_{tors} + E_{over} + E_{under}$$

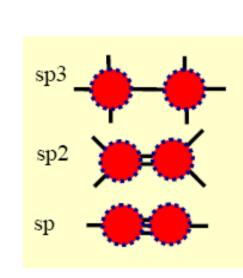
$$E_{\text{bond}} = -D_{\text{e}} \cdot \text{BO}_{ij} \cdot \exp\left[p_{\text{be},1}\left(1 - \text{BO}_{ij}^{p_{\text{be},1}}\right)\right]$$

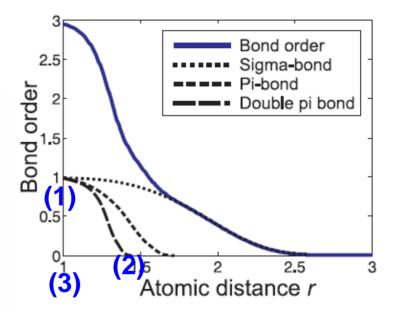
Bond energy between atoms *i* and *j* does not depend on bond distance

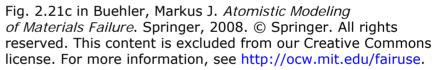
Instead, it depends on bond order

Bond order functions

BO goes smoothly from 3-2-1-0







$$BO_{ij} = \exp\left[\alpha_{\sigma} \cdot \left(\frac{r_{ij}}{r_{0}}\right)^{\beta_{\sigma}}\right] + \exp\left[\alpha_{\pi} \cdot \left(\frac{r_{ij}}{r_{0}}\right)^{\beta_{\pi}}\right] + \exp\left[\alpha_{\pi\pi} \cdot \left(\frac{r_{ij}}{r_{0}}\right)^{\beta_{\pi\pi}}\right] + \exp\left[\alpha_{\pi\pi} \cdot \left($$

All energy terms are expressed as a function of bond orders

Illustration: Bond energy

$$E_{bond} = -D_e^{\sigma} \cdot BO_{ij}^{\sigma} \cdot f(BO_{ij}^{\sigma}) - D_e^{\pi} \cdot BO_{ij}^{\pi} - D_e^{\pi\pi} \cdot BO_{ij}^{\pi\pi}$$

Image removed due to copyright restrictions. Please see slide 10 in van Duin, Adri. "Dishing Out the Dirt on ReaxFF." http://www.wag.caltech.edu/home/duin/FFgroup/Dirt.ppt.

vdW interactions

$$E_{system} = E_{bond} + E_{vdWaal} + E_{Coulomb} + E_{val,angle} + E_{tors} + E_{over} + E_{under}$$

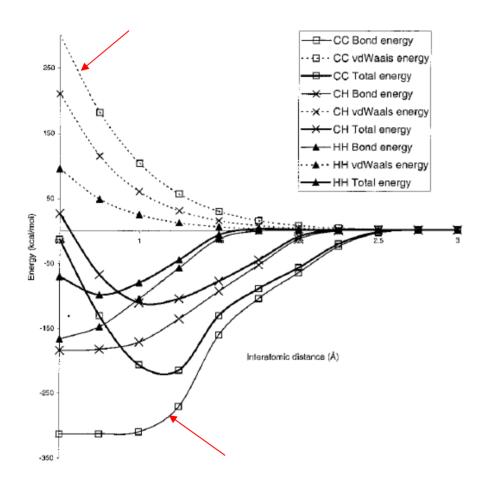
- Accounts for short distance repulsion (Pauli principle orthogonalization) and attraction energies at large distances (dispersion)
- Included for all atoms with shielding at small distances

$$E_{\text{vdWaals}} = D_{ij} \cdot \left\{ \exp\left[\alpha_{ij} \cdot \left(1 - \frac{f_{13}(r_{ij})}{r_{\text{vdW}}}\right)\right] - 2 \cdot \exp\left[\frac{1}{2} \cdot \alpha_{ij} \cdot \left(1 - \frac{f_{13}(r_{ij})}{r_{\text{vdW}}}\right)\right] \right\}$$
$$f_{13}(r_{ij}) = \left[r_{ij}^{\lambda_{29}} + \left(\frac{1}{\lambda_{w}}\right)^{\lambda_{28}}\right]^{1/\lambda_{28}}$$

Image removed due to copyright restrictions.

Please see slide 11 in van Duin, Adri. "Dishing Out the Dirt on ReaxFF." http://www.wag.caltech.edu/home/duin/FFgroup/Dirt.ppt.

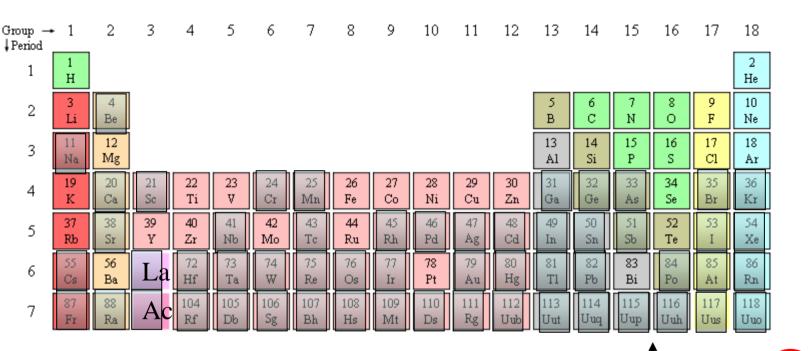
Resulting energy landscape



Contribution of E_{bond} and vdW energy

Source: van Duin, C. T. Adri, et al. "ReaxFF: A Reactive Force Field for Hydrocarbons." *Journal of Physical Chemistry A* 105 (2001). © American Chemical Society. All rights reserved. This content is excluded from our Creative Commons license. For more information, see http://ocw.mit.edu/fairuse.

Current development status of ReaxFF



: not currently described by ReaxFF

A--R

Β

64

Α

Allows to interface metals, ceramics with organic chemistry: Key for complex materials, specifically biological materials

Periodic table courtesy of Wikimedia Commons.

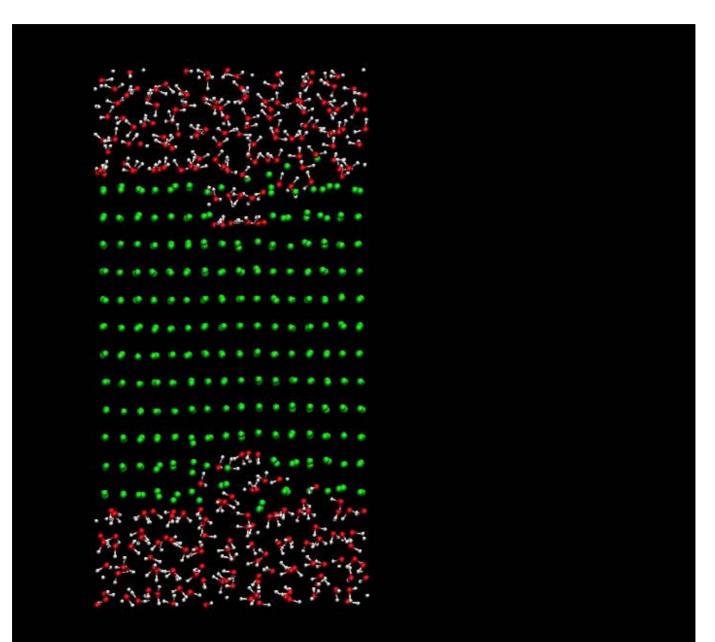
Mg-water interaction: How to make fire with water

Video stills removed due to copyright restrictions; watch the video now: http://www.youtube.com/watch?v=QTKivMVUcqE.



http://video.google.com/videoplay?docid=4697996292949045921&q=magnesium+water&total=46&start=0&num=50&so=0&type=search&plindex=0

Mg – water interaction – ReaxFF MD simulation



3.021J / 1.021J / 10.333J / 18.361J / 22.00J Introduction to Modeling and Simulation Spring 2011

For information about citing these materials or our Terms of use, visit http://ocw.mit.edu/terms.