PRELIMINARY INVESTIGATION FOR DESIGN OF VIBRATION MODEL

by

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PURPOSE

This thesis is a preliminary investigation for the design of a vibration model to be used by the Mechanical Engineering Department at the Massachusetts Institute of Technology for a proposed museum of dynamic models as well as for classroom instruction. The results of the investigation are in the form of suggestions for operation and construction of one such described model. Considerations leading to this particular design are to be fully discussed as an aid to future work on this model as well as to establish reasons for selection of various features of the suggested design in preference to other possible systems.

INTRODUCTION

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The vibration model under consideration consists of a mass, M, suspended by a spring, K, from a support in sinusoidal motion of maximum amplitude, x_{af}, having a damping force, C, as shown by Fig. 1.

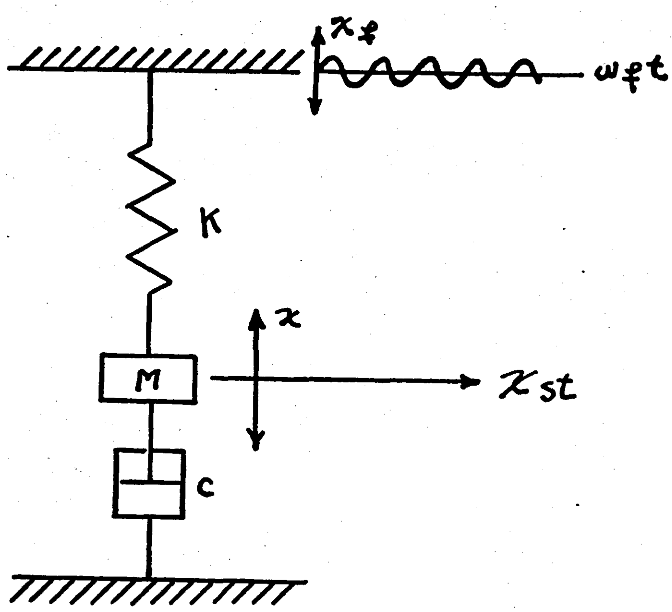
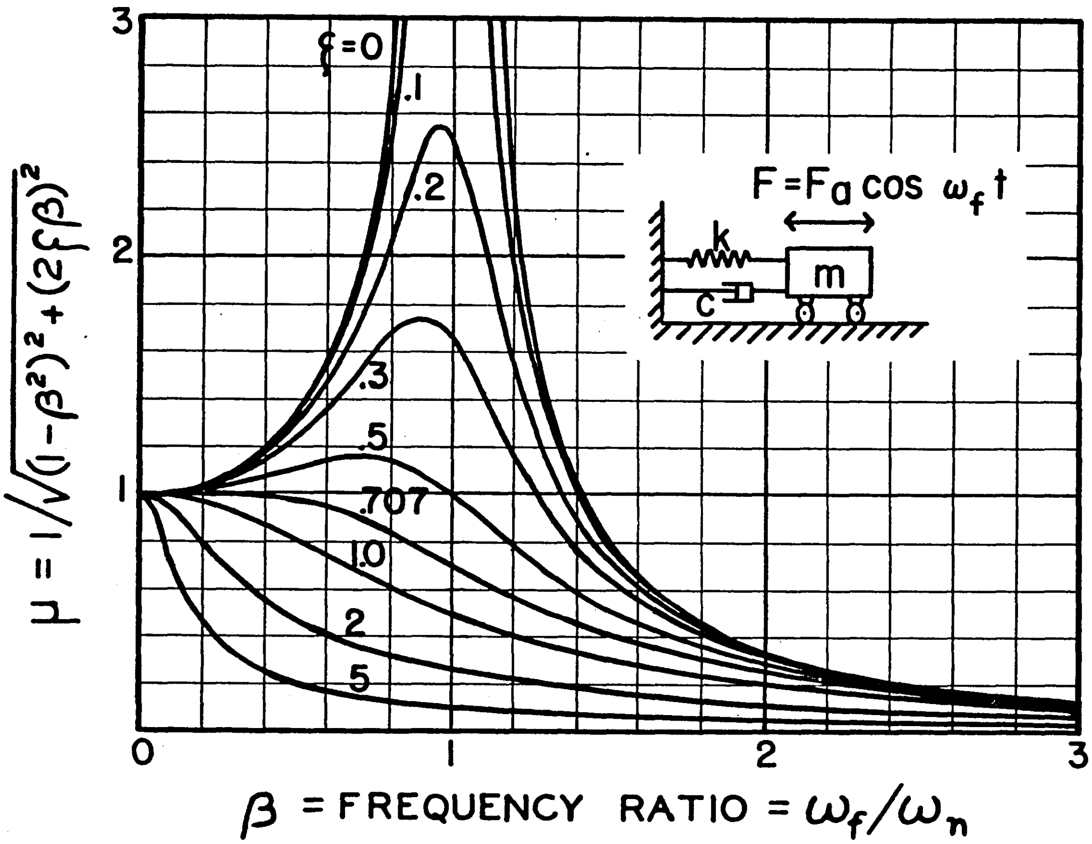
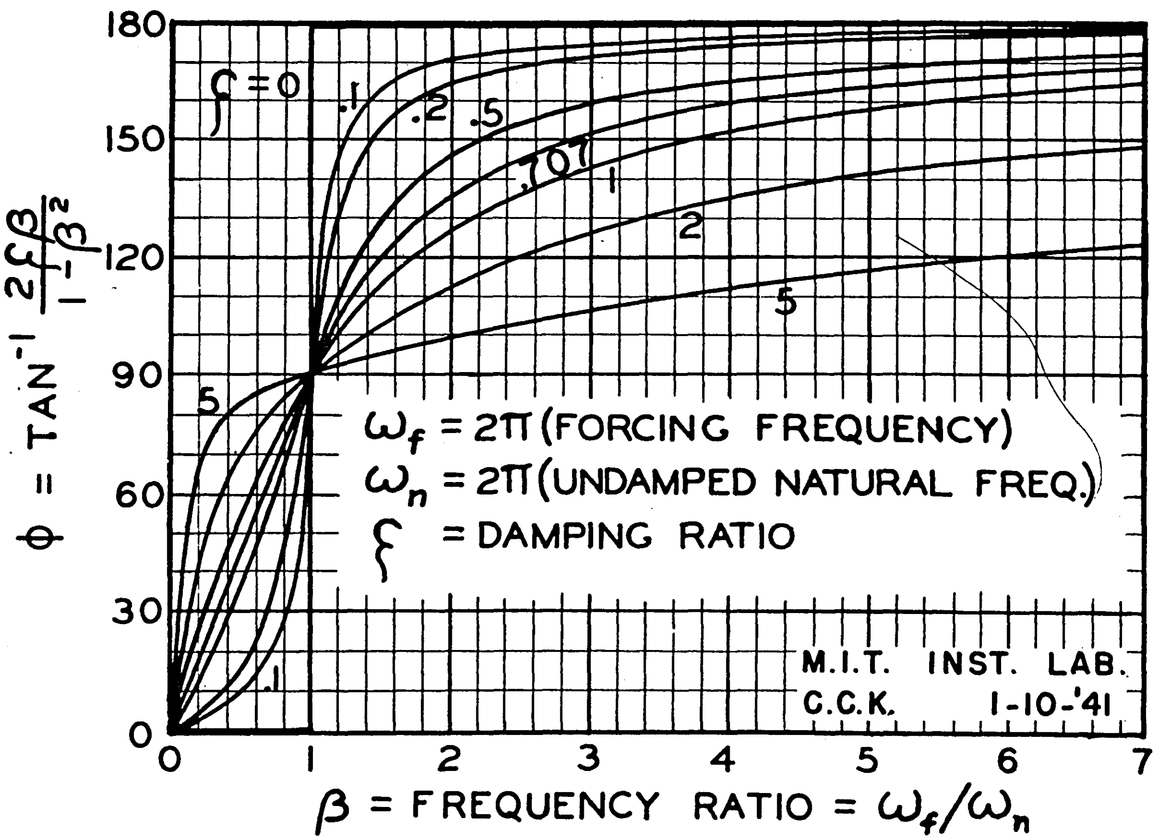


Fig. 1

With variable imputs of mass, spring constant, damping, and amplitude of forcing motion, two curves are plotted to describe the motion of the vibrating mass. First is the plot of forced frequency against μ, the ratio of the maximum displacement of the mass from its position of static equilibrium to the maximum displacement of the support. Second, is the plot of the phase angle, φ, between the sinusoidal motion of the mass and the support. Thus, means of measuring the phase angle and the maximum displacement of the support and mass from their equilibrium position must be considered in the design.

The condition to be fulfilled in this design is that the


$\mu = 1/\sqrt{(1-\beta^2)^2 + (2\zeta\beta)^2}$
β = FREQUENCY RATIO = ω_f/ω_n
$F = F_a \cos \omega_f t$
k
c
m
ζ=0
.1
.2
.3
.5
.707
1.0
2
5


$\phi = \tan^{-1} \frac{2\zeta\beta}{1-\beta^2}$
β = FREQUENCY RATIO = ω_f/ω_n
ω_f = 2π (FORCING FREQUENCY)
ω_n = 2π (UNDAMPED NATURAL FREQ.)
ζ = DAMPING RATIO
M.I.T. INST. LAB.
C.C.K. 1-10-'41
ζ=0
.1
.2
.5
.707
1
2
5

motion of the mass as well as that of the support be sinusoidal since the theoretical derivation of the plots we shall attempt to duplicate are based on the assumption that:

$$x_f = x_{af} \sin \omega_f t$$
$$x = x_a \sin \omega_f t$$

In order to meet this condition there must be no friction other than a force proportional to velocity, spring force must be proportional to displacement, and x_f must be sinusoidal since the differential equation given for this motion is:

$$M\ddot{x} + C\dot{x} + Kx = P_{af} \sin \omega_f t$$

where P_{af} is the sinusoidal force transmitted by the support.

With this condition as the criterion, investigation proceeded in designing a system that would fulfill the following conditions:

1. Relatively simple construction.
2. Relatively simple operation.
3. Indicating system to read x_{af}, x_a, and φ accurately.
4. Unit of shape and weight that could easily be moved between classrooms.
5. Measurement of forced frequency that would be rapid as well as accurate.
6. Electric motor to operate smoothly over the frequency range to be investigated.
7. Possibility of operation for large groups so that the dynamic phenomenon can be seen by all.

The result of the investigation is the unit, as described in Chapter I, that is considered by the author to best fulfill the above conditions. This design was established only after a thorough examination of the other possible systems under consideration; a number of which are discussed in Chapter II.

CHAPTER I

SUMMARY OF RESULTS

CHAPTER I - SUMMARY OF RESULTS

The essential features of the model to be described in this chapter are twofold. First, a thin plate viscous shear dash pot will produce the damping force. Its theoretical development and merits as contrasted with other dash pots available are discussed in Chapter II, Section 1. Second, an optical method using stroboscopic technique is employed for an indicating system. It was chosen for its simplicity, accuracy, and flexibility; and also because of the difficulties encountered using other systems that are not found here.

Section 1

DESCRIPTION OF APPARATUS

A. The Indicating System

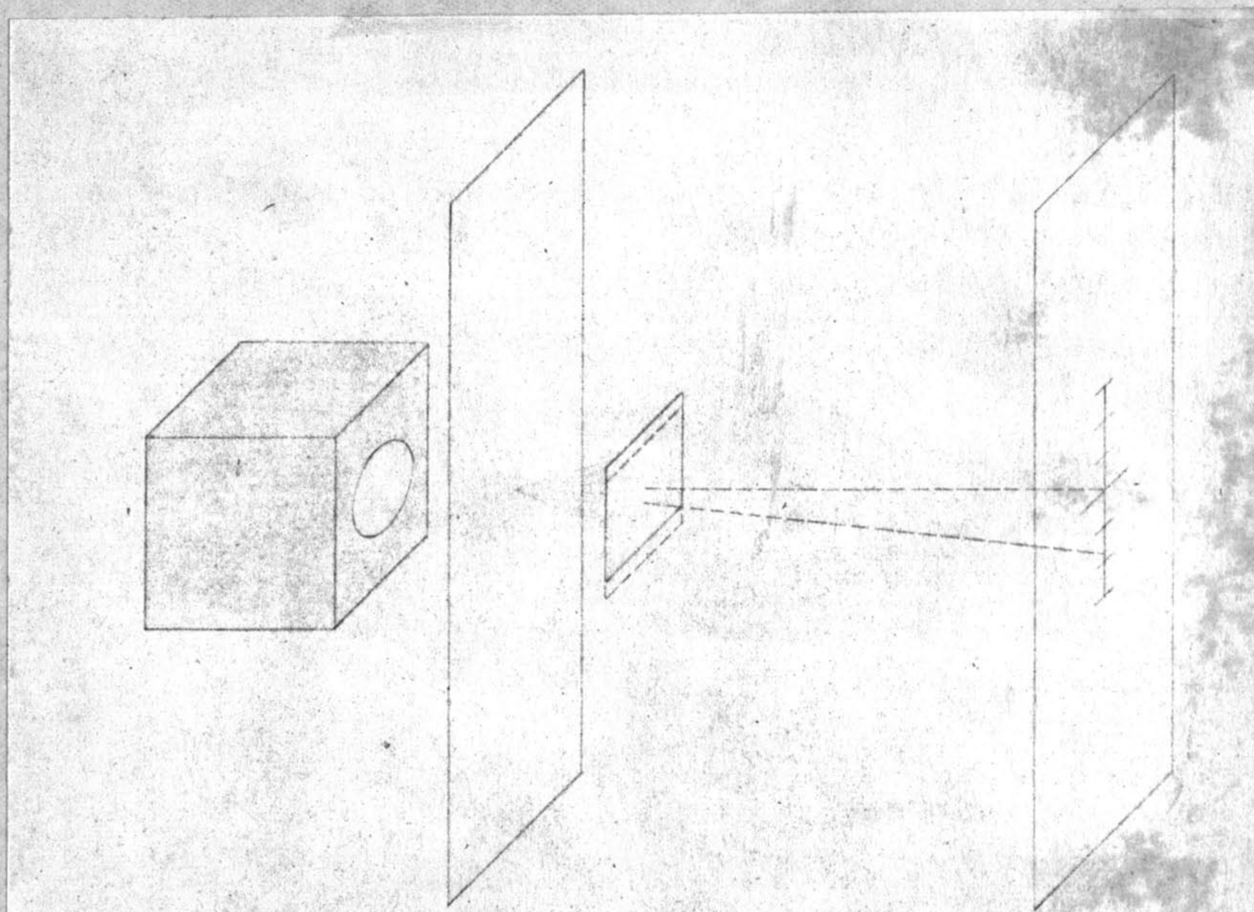


Fig. 2

The sketch above shows a stroboscope on the left flashing light through a pin point in a thin screen so light flashing on the portion of the mass shown will be essentially that from a point source. The light passes through a pin point opening in the mass (actually this will be a narrow slit) to the calibrated frosted glass screen shown on the right. With the mass geometrically nearer to the pin point screen, vertical motion of the mass will be amplified on the frosted glass screen. A contact on the eccentric drive mechanism (to be discussed later in this chapter) will produce light at one position on the glass screen, and by moving this contactor the light can be made to move up or down. At the position of maximum displacement from the zero reading on the screen x_a is read and the phase angle is read from the angular displacement of the contact.

In order to get a slow motion view of the vibrating mass, the stroboscope must flash at slightly higher or lower frequency than the forced frequency or integral multiples thereof. This can be done with another contact on a shaft geared to the motor, or by manual control of the frequency of the stroboscopic light. General Radio Company's type 631-B strobotac would do well for this design since the frequency can be adjusted either by hand or an external contactor.

The frame should be built so that no light enters between the screens other than through the pin hole opening--otherwise difficulties in other than darkroom operation will be encountered.

If the calibrated screen were made removable, the stroboscope may be brought out in front of the model and the same effect can be seen on the mass itself.

B. Description of Apparatus - Plates I, II, III and IV

1. Moving support
2. K'
3. Plate bearing for moving support
4. Adjustable length spring attachment
5. Spring
6. Screen plate
7. Dash pot bearing for mass
8. Mass rod

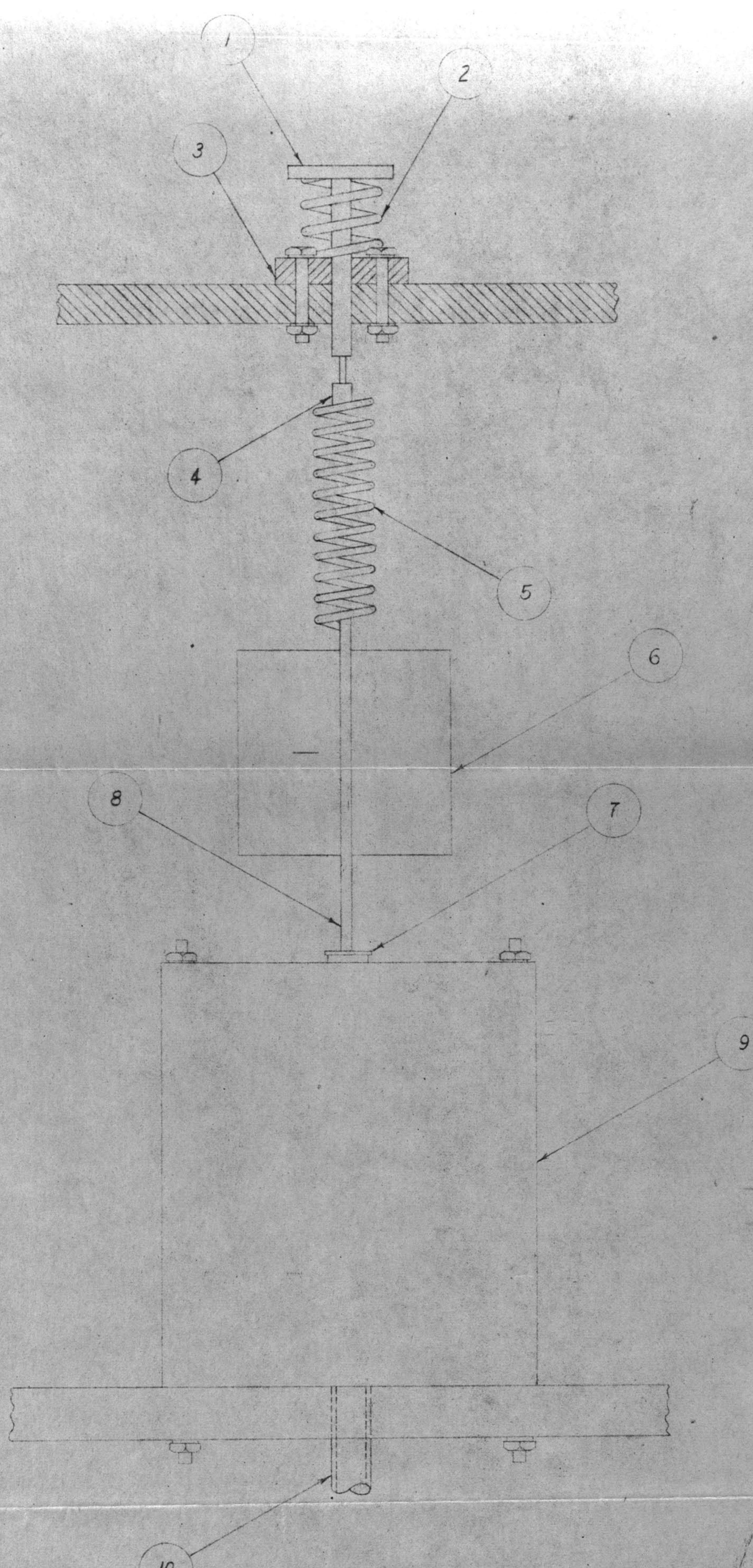
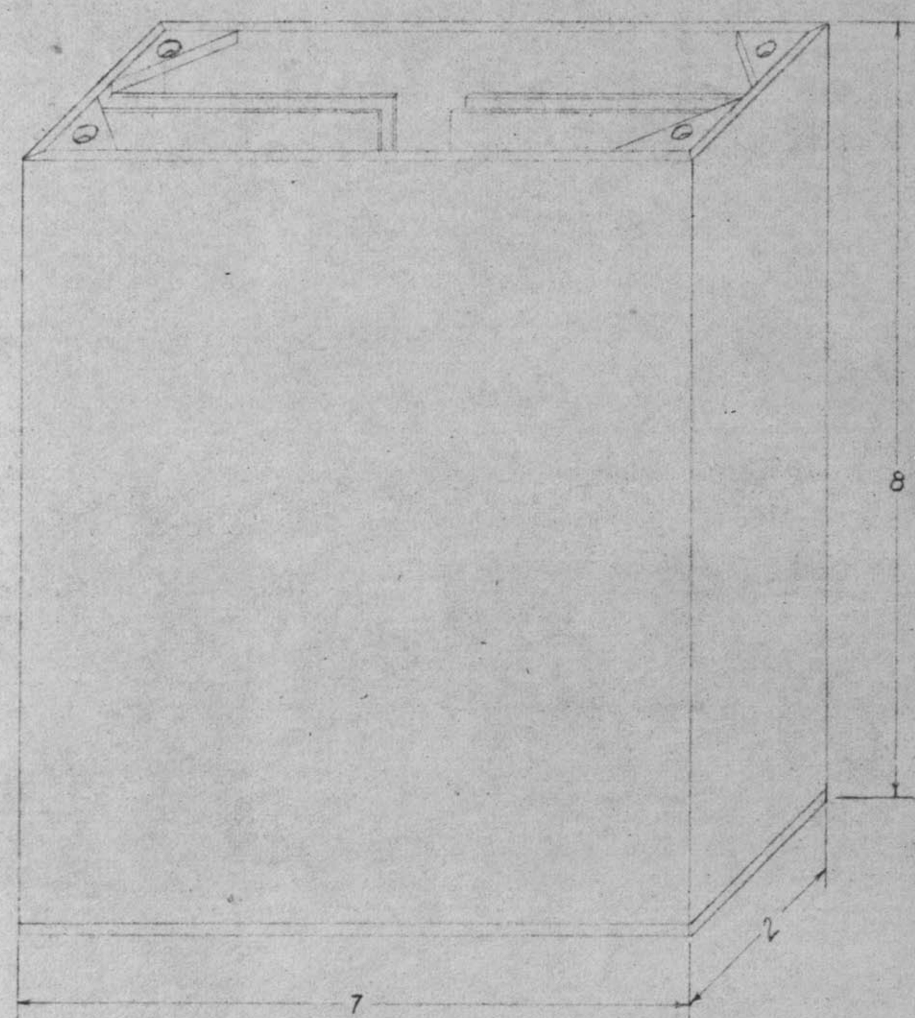


Plate No 1



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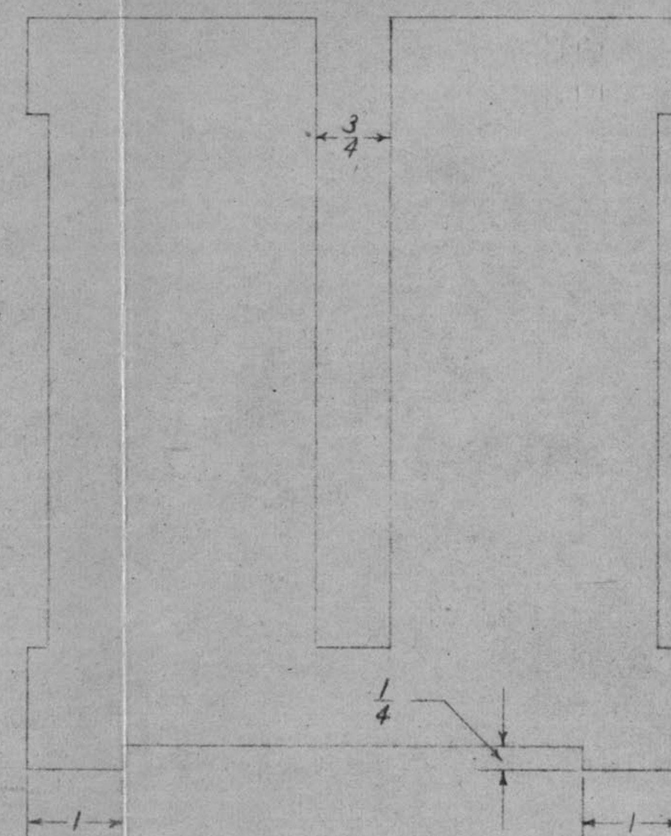

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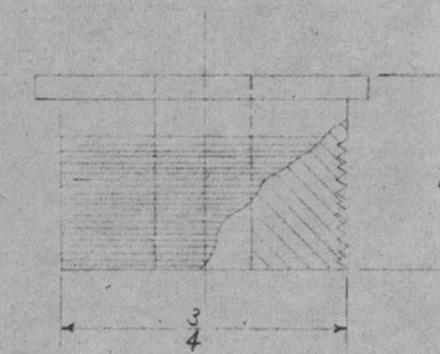

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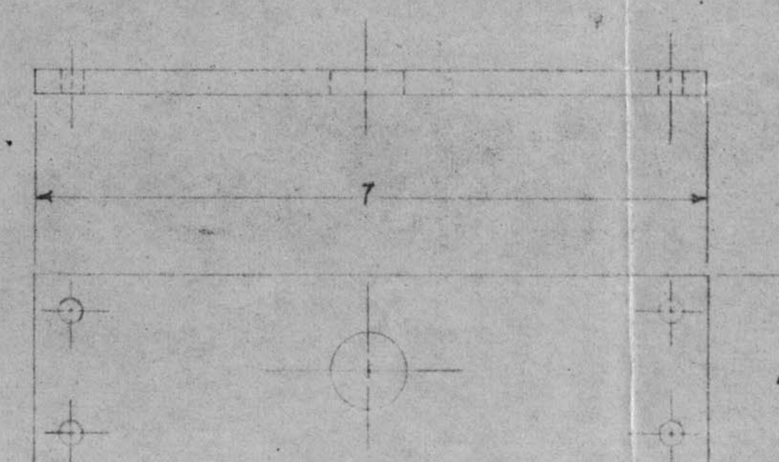

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Plate No 2

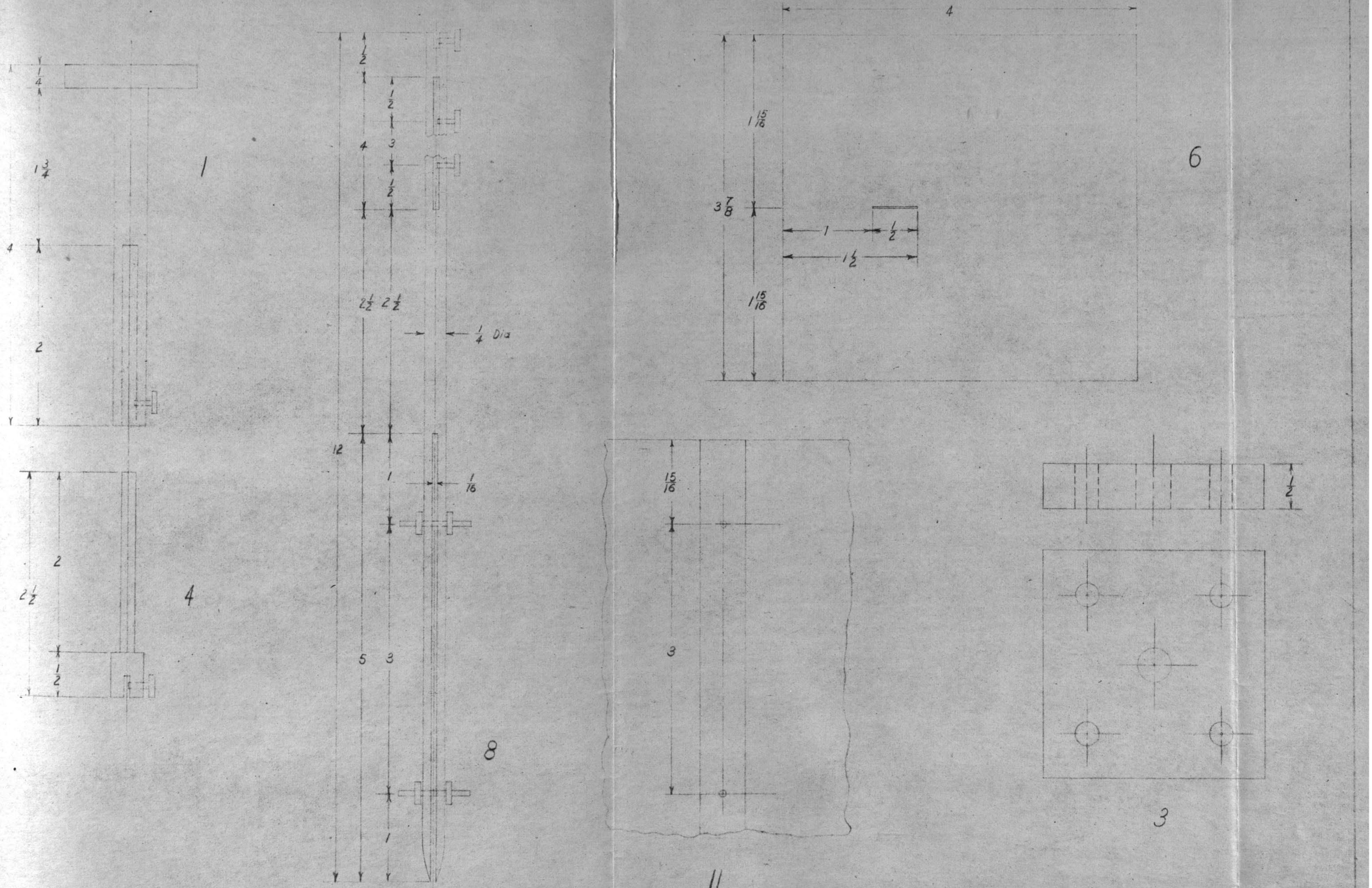


Plate No 3

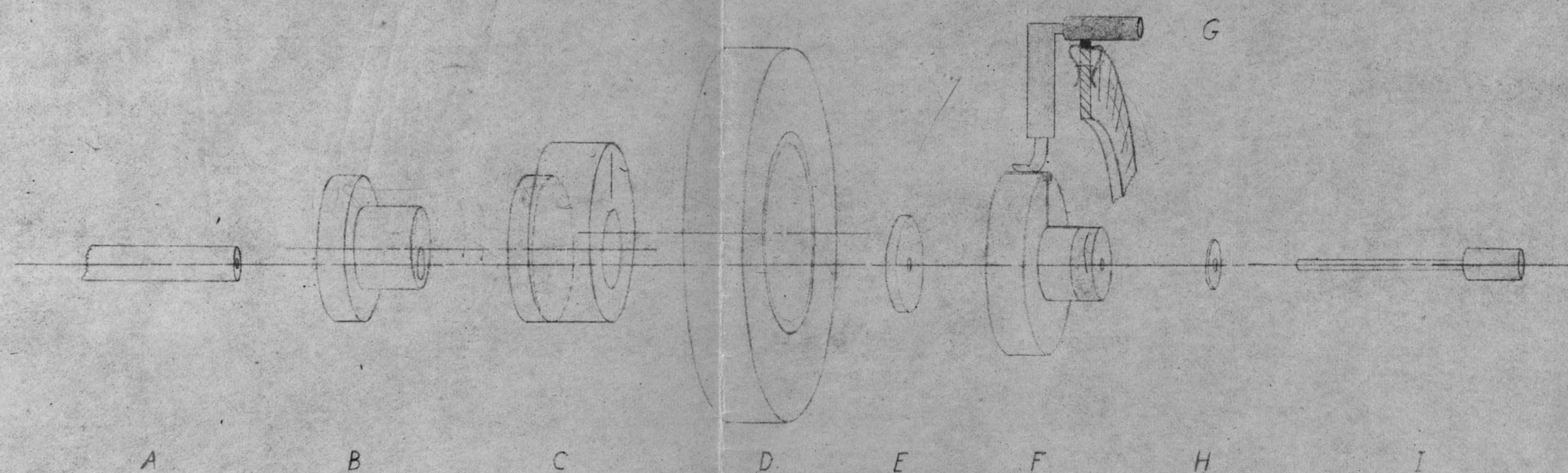


Plate No 4

9. Dash pot
10. Outlet oil orifice
11. Moving plate
12. Stationary plate
13. Dash pot top

C. Operation of Apparatus and Suggestions for Construction

Plate I is an overall picture of the system shown in more detail on Plates II, III and IV.

1. Eccentric driving mechanism — Plate IV.

Part B is eccentric to drive shaft A. For convenience of description this is assumed to be 1/8 inch. Part C is in turn 1/8 inch eccentric to part B and by angular displacement of C relative to B the total eccentricity may be varied from zero to one quarter inch. D is a single row light type ball bearing— see SKF bearing No. 6200 series. Part F is a bakelite disk having a small metallic slug fastened on the large diameter periphery for electrical contact with the copper brush of contactor G. A wire is fixed to the disk between the metal slug and metallic band shown shaded along the small diameter periphery of the disk. One lead to the stroboscope is through the brush on contactor G while the other lead is connected to a metallic brush that is always in contact with the metallic band. Therefore, on every revolution for the instant the brush is in contact with the metal slug the stroboscope ignites. The end portion of I is threaded to screw into drive shaft A to make the mechanism secure after a setting has been made.

There is a force fit between A and B and C and D, snug fit between B and C, and E, F and H are free fits with the unthreaded portion of screw I.

There is a 180° scale calibrated along the periphery of the larger diameter of B. Thus, exact values for x_{af} can be set by turning C relative to B. After this setting has been made it is necessary that the metal slot in F be lined up with the straight line shown on C.

The scale of contactor G need only be 180°. Zero reading is when its brush makes contact with the line on C in a vertical position.

2. Vibrating support — Plates I and III — Part 1.

The yoke shown transmits driving force into sinusoidal motion. The spring (2) which shall be designated as K' forces the piston head to follow the eccentric drive under all operating conditions, thus producing $x_{af} \sin \omega_f t$—a condition that must be fulfilled. A plate (3) of sufficient thickness to function as a sleeve bearing for the vibrating support is bolted to the frame as shown.

In the construction of the model the pin point hole and the calibration mark of zero displacement are of the same elevation. For the indicating system to operate as designed, the narrow slit in the mass screen, 6, must also be at this elevation for the mass in its static equilibrium position. When settings are changed and such constants as M, K, and C are varied, the position of static equilibrium will change somewhat. To bring the mass back to static equilibrium position at the elevation defined above, the eccentricity is set for zero and position of the fastening, 4, to the main spring, 5, is changed with relation to part 1 of the moving support.

3. Mass.

The mass consists of a main rod, 8, to which the indicating screen, 6, and the moving plate of the dash pot, 11, are fastened. Small changes in mass are made by changing the weight of the indicating screen or moving plate, or by the addition of concentric shells that are fastened securely to the main rod. To make a large variation of mass, main rods up to 5/8 inch in diameter can be used as compared with the 1/4 inch rod shown. With this in mind the removable sleeve bearing, 7, was designed. Also rods made of different density may be used to vary mass.

It would be unwise to vary mass in the upper portion of the rod, for example by increasing the weight of the indicating screen since it is far enough above the dash pot bearing for the possibility of a second degree of freedom in the seismic element without an additional bearing. Furthermore, it would be advisable to make the indicating screen thin as possible so that the slit may be narrow and precise readings may then be made on the calibrated screen. The slit is to be a quarter or half an inch in length.

The top and bottom ends of the moving plate should have pointed edges to reduce any pumping effect that may be present as well as to insure equal flow on both sides of the plate.

4. Dash Pot — Plate IV — Part 9.

For a shear plate of heigth, ℓ and width b, the total shear area is 2 ℓb. The shear force is proportional to the total area and inversely proportional to the clearance, h, between it and the stationary plates, 12. The shear force is also proportional to the viscosity, μ, of the working fluid. Thus, damping, C, is given by the equation $C = \frac{2A\mu}{h}$. And herein are four ways of varying the damping constant C.

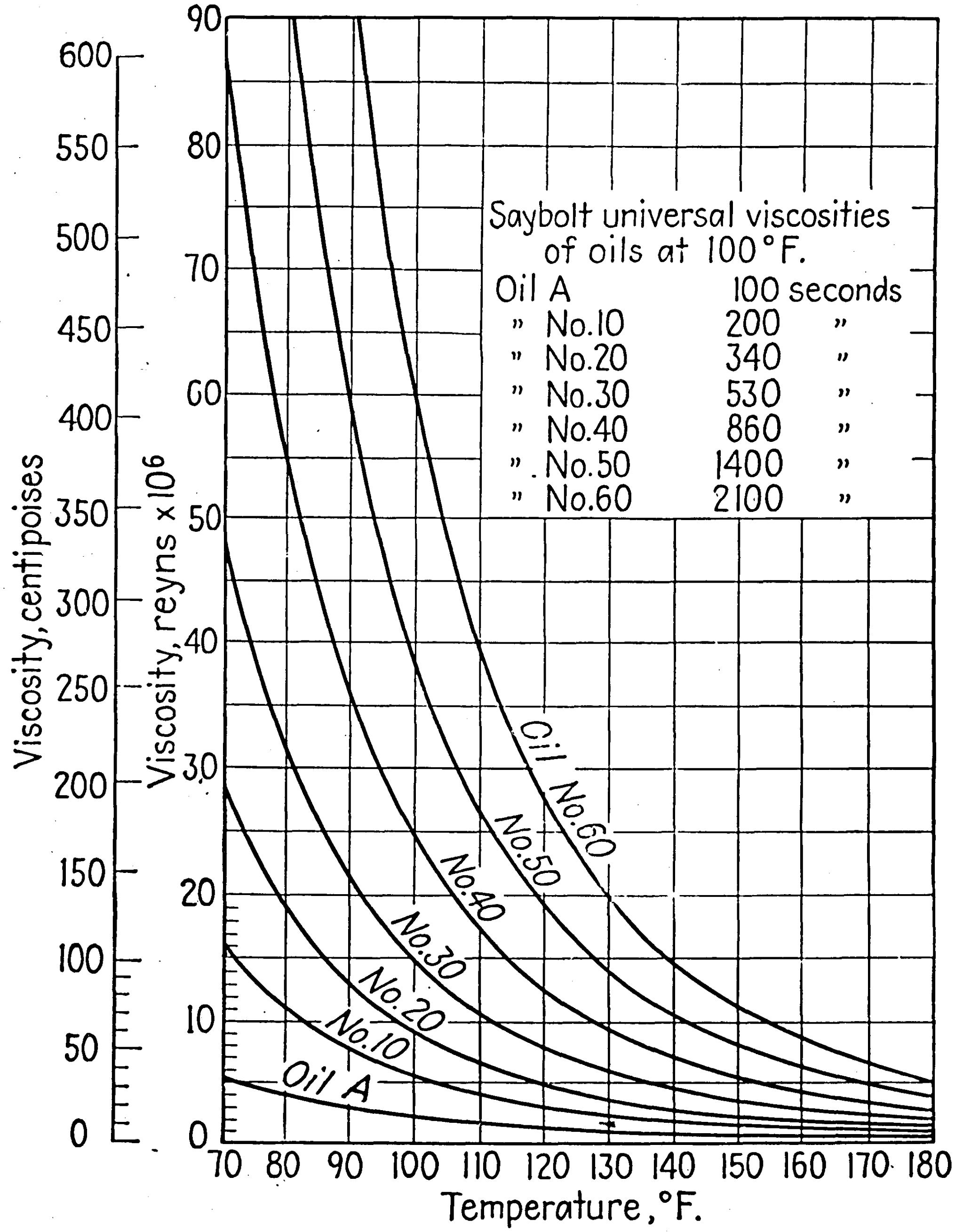
(a) Variation in area of moving plate.

(b) Variation in thickness of moving plate, which in turn varies clearance since the stationary plates are a fixed distance apart.

(c) Changing the working fluid in the system.

(d) Varying the temperature of the working fluid as shown by plot on page 13.

During operation there is a rise in temperature of the working fluid due to the dissipation of energy into heat. For this reason a large reservoir of working fluid is desired so that the temperature change will be kept at a minimum. A dash pot of the overall dimensions given on Plate II will contain enough oil for this purpose if there is good circulation of the oil between the plates to that outside the stationary plates. The shape of the stationary plates was designed with this in mind. Should this design not prove effective, a paddle wheel should be included in the construction to obtain the necessary circulation. The effect of heat rise of the working fluid during operation could be minimized by operation at



Temperature-viscosity chart for S.A.E. classification.

temperatures where the viscosity of the oil will not change appreciably with change in temperature.

Welded construction is the author's suggestion for this dash pot.

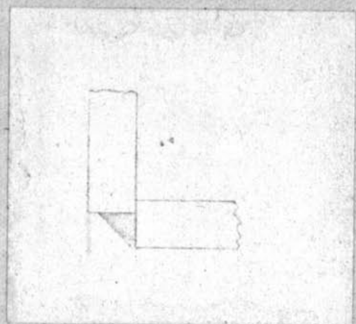


Fig. 3

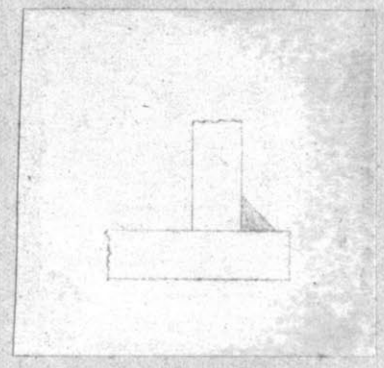


Fig. 4

Plates of 3/16 inch are advised. Thinner plates would warp at the welding temperatures and thicker plates would make the dash pot a heavy combersome unit in the design. The side plates should be designed so that a neat fillet weld as shown in Fig. 3 can be made. The bottom plate can be welded as shown in Fig. 4. Two of the side plates shall be grooved so that the two stationary plates may be slid into position and subsequently welded secure to the dash pot.

The top and bottom plates have center holes drilled for pieces 7 and 10 respectively. Also four holes on each plate are drilled for the bolt construction as shown on Plate I.

D. Discussion of Operating Constants

Values for the maximum displacement of the mass from its equilibrium position are not to exceed 1/2 inch. Therefore, the driving mechanism should be designed to have a maximum eccentricity of about 1/4 inch.

There is but one actual bearing to guide the mass since the plates in the dash pot also guide the motion. Best operation will occur with the moving plate a large portion of the mass. Therefore, it is better to change the plate thickness using moving plates of approximately 5" x 5" instead of varying the plate area to vary damping.

Now for the actual dynamics of the model.

With a working shear area of 50 square inches, clearance of 0.01 inch and working with a heavy oil at room temperature damping close to one pound second per inch may be obtained. Assuming the separation of the two stationary plates is 0.08 inches, the moving plate thickness in this case is 0.06 inches. Using the rod shown on Plate III of one quarter inch diameter, the weight of the seismic element is approximately 0.6 pounds if parts be made of steel. Since the damping here is the maximum we can obtain with this dash pot, it should be the damping used for the highest damping ratio for a series of runs taken with this mass and spring held constant.

$$\text{Let } \zeta = 1.0 = \frac{C}{2\sqrt{KM}} = \frac{1.0}{2\sqrt{KM}}$$

$$KM = (0.5)^2 = 0.25 \ \frac{lb^2 \ sec^2}{in^2}$$

$$M = \frac{0.6}{386} = 1.55 \times 10^{-3} \text{ slug}$$

$$K = 161 \text{ lb/in}$$

$$\omega_n = \sqrt{\frac{K}{M}} = 322 \ \frac{rad}{sec} = 3080 \text{ rpm}$$

Higher frequency may be obtained using a lighter metal for the mass, but of course the forces at these high frequencies are not permissible due to the light construction of the apparatus.

By reducing the thickness of the plate (clearance, h, is increased) and using lighter oils damping may be as low as $.01 \frac{\text{lb sec}}{\text{in}}$ with the same area plate and same separation between stationary plates. This is computed for a clearance of 0.03 inches which leaves a plate thickness of 0.02 inches—and of course the weight of the moving plate has been reduced, and using the same steel rod the weight of the seismic element is approximately 0.3 pounds. In this case, since damping is a minimum the damping ratio will be computed as 0.1.

$$\xi = 0.1 = \frac{C}{2\sqrt{KM}} = \frac{.01}{2\sqrt{KM}}$$

$$KM = (.05)^2 = .0025 \frac{\text{lb}^2\ \text{sec}^2}{\text{in}^2}$$

$$M = \frac{0.3}{386} = 7.77 \times 10^{-4} \text{ slug}$$

$$K = 3.22 \text{ lb/in}$$

$$\omega_n = \sqrt{\frac{K}{M}} = 64.3 \frac{\text{rad}}{\text{sec}} = 615 \text{ rpm}$$

Using this damping and damping ratio, but a 5/8 inch diameter steel rod in place of the quarter inch rod:

$$M \cong \frac{1}{386} \text{ slug}$$

$$KM = .0025 \frac{\text{lb}^2\ \text{sec}^2}{\text{in}^2}$$

$$K \cong 1 \text{ lb/in}$$

$$\omega_n = \sqrt{\frac{K}{M}} \cong 19.65 \frac{\text{rad}}{\text{sec}} \cong 188 \text{ rpm}$$

These three computations show the wide range to which the model may be operated, but it is limited to operation where the transmitted forces are small. Furthermore, operation at extremely low frequency is limited by the performance of the electric motor unless a fractional gear ratio be designed to drive the system, which is unnecessary if constants be chosen as follows, using a shunt wound direct current fractional horsepower motor.

for $\xi = 0.1$

$$C = 0.01 \frac{\text{lb sec}}{\text{in}}$$

and for $\xi = 1.0$

$$C = 0.1 \frac{\text{lb sec}}{\text{in}}$$

$$KM = 0.0025 \frac{\text{lb}^2 \text{ sec}^2}{\text{in}^2}$$

using a mass of $\frac{1}{1000}$ slug

$$K = 2.5 \frac{\text{lb}}{\text{in}}$$

and $\omega_n = 478$ rpm

Section II

FREQUENCY MEASUREMENT AND POWER

Several methods are available for frequency measurement. Those not discussed in this section can be found in Chapter II.

Frequency is controlled by varying the terminal voltage across the armature of a D.C. shunt wound motor. This is best done by varying the voltage drop across a resistance in series with the armature. The discussion to follow will explain how this resistance may be calibrated in rpm.

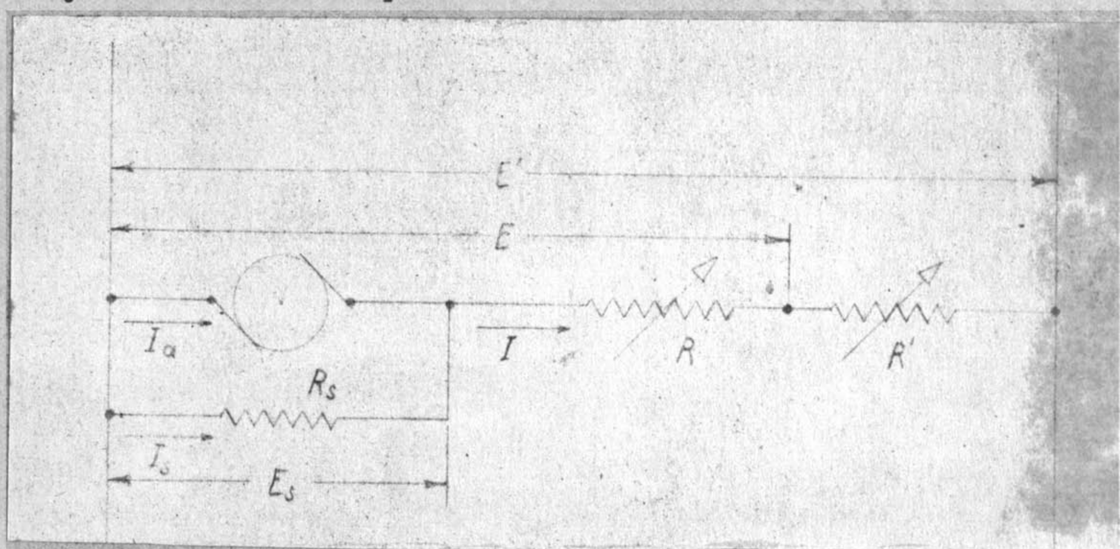


Fig. 5

The circuit shown in Fig. 5 will minimize the variation in speed with changes in load by means of an armature shunting resistor, R_s. The resistance is connected in parallel with the armature, and its purpose is to increase the current, I, flowing through the series resistor, R. If the current flowing through the shunt is high in proportion to that in the armature, changes in armature current will have relatively little effect on the current in the series resistance. Therefore, the voltage across the series resistance will remain more nearly constant. The best value of R_s can only be obtained by trial and error with the electric motor used in the circuit.

A vibration reed is used to check the calibration of the circuit, since the source voltage, E', may vary from day to day. By setting R at the frequency which the reed will vibrate and varying R' until it does vibrate, the voltage E will be that for which R was calibrated.

By having the power rating of the motor large in proportion to the load, the variation of speed will be reduced with changes in load.

If the force due to the spring, K', is large compared with the resultant of the dynamic forces,

$$\text{where } P = \sqrt{(M\omega^2 x - Kx)^2 + \overline{C\omega x}^2}$$

then the load itself will tend to remain constant—that is of course for operation at a calibrated frequency with M, C, and K varied.

The excess power and the spring effect tend to increase the accuracy of the calibrated resistance in rpm—the use of which will simplify operation of the model.

Therefore, in correlation with the results of Section 1 of this chapter, a spring stiffness of 15 lbs. per inch for K' will allow for ample variation of the dynamic constants with power supplied by a tenth horsepower D.C. shunt wound motor rated at 3450 rpm (Westinghouse Type FK, Frame No. 125). Of course, a higher horsepower motor will tend to improve operation of the calibrated resistance.

CHAPTER II

GENERAL NOTES OF INVESTIGATION

Section 1

DAMPING

The first dash pot to be considered will consist of a circular piston of sufficient thickness to insure the formation of laminar flow in the small clearance between it and the cylinder enclosing the piston and working fluid. Damping will be varied by changing the number of drilled holes through which leakage may occur during the piston's harmonic motion. This may be accomplished by a sleeve about the piston shaft that can turn a disk that is in direct contact with the piston face so leakage will pass through "n" holes in accordance with the angular displacement of the sleeve.

Nomenclature

$h \equiv$ clearance between cylinder wall and piston

$\ell \equiv$ height of the piston

$A \equiv$ area of piston of diameter D and radius R

$\dot{x} \equiv$ velocity of the piston

$d \equiv$ diameter of holes drilled in the piston

$r \equiv$ radius of holes drilled in the piston

$n \equiv$ number of holes of radius r

$Q_1 \equiv$ leak through one hole of radius r

$Q_2 \equiv n\, Q_1$

$Q_3 \equiv$ leak through clearance between piston and cylinder wall

$D' \equiv$ diameter of the piston rod

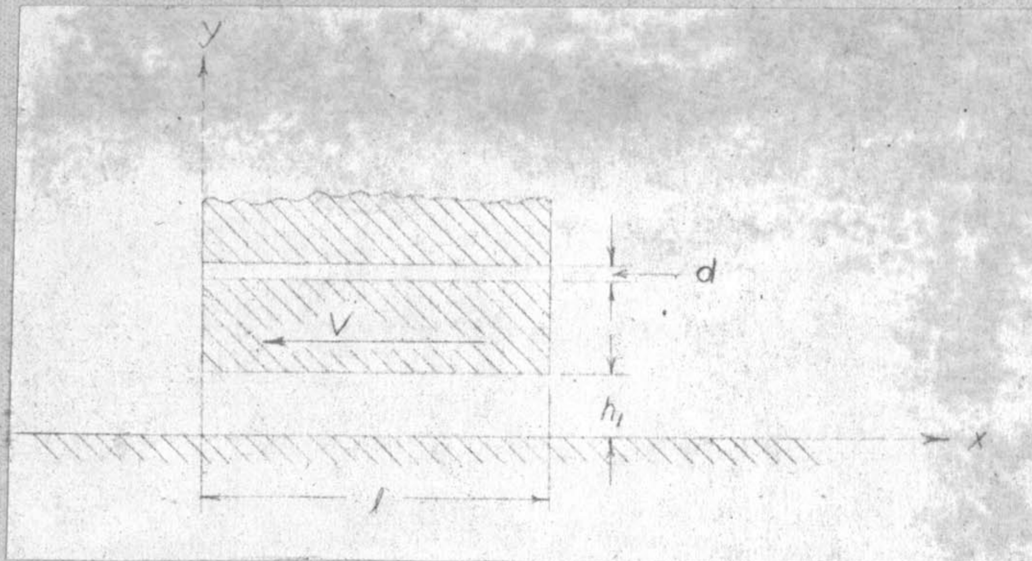


Fig. 6

$$P - P_o = \frac{12\mu}{h_1^3}\left(\frac{\dot{x}\, h\, l}{2} - l Q'\right)$$ [1]

$$Q' = \frac{Q_3}{\pi D}$$ by definition

Therefore

$$-\Delta P = \frac{12\mu l}{h_1^3}\left(\frac{\dot{x}\, h}{2} - \frac{Q_3}{\pi D}\right) \qquad (1)$$

$$\Delta P = \frac{8\, Q_2\, \mu\, l}{n\, \pi\, r^4} \qquad (2)$$ [2]

$$Q_2 + Q_3 = \dot{x}\, A \qquad (3)$$

$$C = \frac{\Delta P\, A}{\dot{x}} \qquad (4)$$

---

[1] Dodge and Thompson, Fluid Mechanics, p. 461.

[2] Hazen-Poiseuille Law.

Adding equations (1) and (2) we find:

$$Q_2 = \frac{3}{2} \frac{n \pi r^4}{h^3} \left(\frac{Q_3}{\pi D} - \frac{\dot{x} h}{2} \right)$$

since $Q_3 = \dot{x} A - Q_2$ (Eq. 3)

$$Q_2 = \frac{3 n \pi r^4 \dot{x} D}{2 h^3 D + 3 n r^4} \left[\frac{A}{D} - \frac{h}{2} \right]$$

The area $A = \dfrac{\pi (D - D')^2}{4}$

The piston rod must extend through the piston as shown to have equal damping on upward and downward strokes.

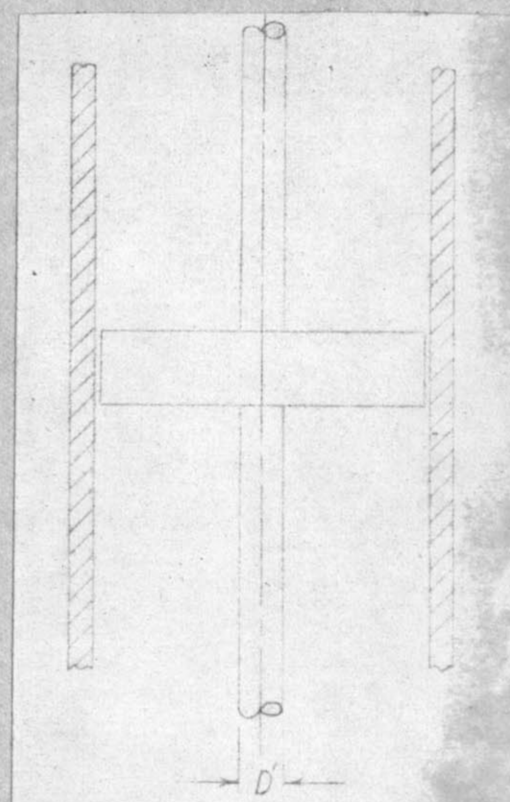


Fig. 7

Since $C = \dfrac{\Delta P \, A}{\dot{x}}$ (Eq. 4)

$$C = \frac{8 Q_2 \mu l A}{n \pi r^4 \dot{x}}$$

$$C = \frac{3 \mu l \pi (D - D')^2 \left[(D - D')^2 - 2 h D\right]}{4 h^3 D + 6 n r^4} \qquad (5)$$

To vary damping with holes drilled directly through the piston will not allow for variation during operation of the equipment. This may be done however with the leakage Q_2 passing through holes external to the cylinder itself in a manner similar to the wedge dash pot (Fig. 8).

Let l_1 = height of the piston

l_2 = length of the drilled holes

In this case,

$$C = \frac{3 \mu l_1 l_2 \pi (D - D')^2 \left[(D - D')^2 - 2 h D\right]}{4 h^3 D\, l_2 + 6 n r^4 l_1} \qquad (6)$$

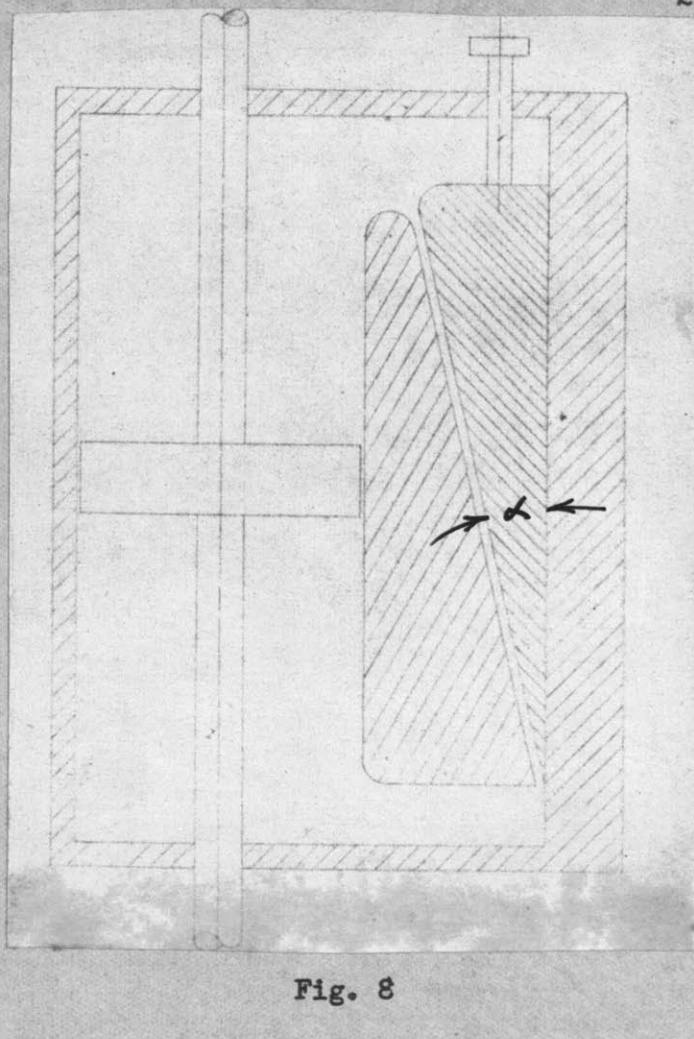


Fig. 8

Instead of having circular holes in the external fluid circuit, a wedge arrangement as shown may be used. This allows for a complete variation in damping having for a maximum, that damping with all leakage through the clearance between the piston and cylinder wall. This of course is the maximum damping for the previously described dash pots, where

$$C = \frac{3\pi\mu\ell_1}{4\,D\,h_1^3}\,(D-D')^2\left[(D_1-D_1)^2 - 2\,h_1\,D\right] \qquad (7)$$

let ℓ_1 = piston height

h_1 = piston clearance

ℓ_2 = wedge length

h_2 = wedge clearance

w = wedge width

in this case,

$$C = \frac{3\mu\ell_1\,\ell_2\,\pi^2\,(D-D')^2\left[(D-D')^2 - 2\,h_1\,D\right]}{4\,\ell_2\,h_1^3\,\pi\,D + 4\,\ell_1\,h_2^3\,w} \qquad (8)$$

The wedge may be designed so that ℓ_2 remains constant for all vertical displacements of the wedge. In that event, the variable damping is a function of h_2 which in turn is a function of α and the pitch of the screw threads—and therein, the angular displacement of the screw. Therefore, the smaller the angle α and the pitch of the screw thread, the greater the precision in calibration—the limit being the loss in accuracy due to changes in viscosity of the working fluid due to temperature rise during operation.

For dash pots with a piston, the clearance h must be small to get sufficient damping. This is especially true in the case of air as the working fluid. In order to prevent a loss of working fluid from the system, the openings through which the piston shaft extends must be of extremely small clearance. With three bearings directly in line, friction other than viscous friction proportional to velocity is very likely to occur. Machining of this type dash pot will be difficult and especially in the case of the unit incorporating the wedge for variations in damping.

There will be a loss of working fluid from the system during operation. This may be of little effect for a dash pot with air as a working fluid; whereas it presents a definite disadvantage for an oil system in that the damping constant will change, and it will be a nuisance replenishing the oil supply after every run.

Damping is directly proportional to the viscosity of the working fluid. The viscosity is a function of temperature. During operation, energy is dissipated in the dash pot in the form of heat; and since the amount of working fluid is limited to a small quantity, it will not take a great deal of heat to raise the temperature of the working fluid a sufficient amount to change the damping constant.

Sample computations for systems working with an air dash pot.

$D = 2$ inches

$D' = 1/4$ inch

$\ell_1 = 1/2$ inch

$T = 75°$ F

$\mu = 2.54 \times 10^{-9} \frac{\text{lb sec}}{\text{in}^2}$

$h = .006$ inch (loose fit for 2 inch dia.)

by solving for the maximum damping (Eq. 7) and applying a value of near unity or slightly greater to be on the safe side, the natural frequency can readily be obtained after selection of mass and spring constant—the product of which is determined by the damping constant "C" and the damping ratio "ζ".

$$C = \frac{3}{4} \frac{\pi \mu \ell_1}{D\, h_1^3} (D - D')^2 \left[(D - D')^2 - 2\, h\, D\right] = 0.1288 \frac{\text{lb sec}}{\text{in}}$$

$$\text{Let } \zeta = 1.2 = \frac{C}{C_c} = \frac{0.1288}{2\sqrt{KM}}$$

$$KM = (.05365)^2 = 0.00288 \frac{\text{lb}^2\ \text{sec}^2}{\text{in}^2}$$

$$\text{for } M = \frac{1.5}{386} \text{ slug}$$

$$K = 0.74 \text{ lb/in}$$

$$\omega_n = \sqrt{\frac{K}{M}} = 13.8 \ \frac{\text{rad}}{\text{sec}} = 132 \text{ rpm}$$

Difficulty will be encountered operating with constants for natural frequencies substantially higher than 132 rpm. Of course, $C = 0.1288 \frac{\text{lb sec}}{\text{in}}$ could be damping of this dash pot for $\zeta = 0.1$. In which case the natural frequency will be 1580 rpm with the 1.5 lb. seismic element, but runs taken with this natural frequency at higher values of damping ratio are impossible because damping itself cannot increase substantially above $0.1288 \frac{\text{lb sec}}{\text{in}}$.

It is well to note here that to increase damping with an air dash pot the piston area must be increased, in which case mass of the system is also increased and the model so designed will be an extremely large unit.

Operation of an air dash pot is also confined to small frequency operation by consideration of turbulent flow on viscous damping. The leakage changes to turbulent flow for Reynolds numbers greater than 2000. The velocity of the flow is greatest at resonance since the amplitude of the seismic element is greatest at this frequency and hence the leakage is a maximum.

The effect of turbulent flow in viscous damping.

To maintain the damping constant, laminar flow must exist. Laminar flow is always present for Re $<$ 2000.

$$Re = \frac{\rho V D}{\mu} = \frac{V D}{\nu}$$

For an extreme case, examine a dash pot of the piston hole type.

Assume,

$$n = 1$$

$$r = \frac{1}{100} \text{ in}$$

Although incorrect, assume all leakage to pass through this hole. Examine air since it is of low viscosity and more possible to produce high Re

$$\text{for} \quad t = 70^\circ \qquad \nu = .0001626 \frac{ft^2}{sec} = \frac{\mu}{\rho}$$

Therein, the problems comes down to, what velocity in this extreme case is permissible.

$$Re = \frac{\rho V D}{\mu}, \text{ where } V \equiv \bar{V}_p/f, \text{ velocity of piston relative to the fluid}$$

$$\bar{V}_p/f = \frac{Re\,\mu}{\rho\, D} = 2000 \times 50 \times 1.626 \times 10^{-4} = 16.26 \text{ ft/sec.}$$

The velocity to be examined is the velocity of the piston relative to the fluid which is the algebraic sum of the absolute piston velocity and the leakage through one hole of $\frac{1}{100}$ in. radius.

To continue with assumptions for the extreme case, find the permissible frequency for this problem for an amplitude of 1" (full swing) and a piston area of 3 square inches.

$$V_p/f = 16.26 \text{ ft/sec} = \bar{V}_p + \bar{V}_f$$

$$\bar{V}_{p_{max}} = \omega x = \frac{\omega}{24} \text{ ft/sec}$$

$$Q_2 = \omega x A$$

$$\bar{V}_f = \frac{Q_2}{\pi r^2} = \frac{\omega x A}{\pi r^2} = 100$$

$$\bar{V}_p/f = 100\,\omega = 16.25$$

$$\omega = .1625 \frac{\text{rad}}{\text{sec}} = 1 \frac{\text{cycle}}{\text{sec}} = 60 \frac{\text{cycles}}{\text{min}}$$

Of course, a portion of the flow will be passing through the clearance and the allowable resonance frequency will be greater than this computed value of 60 rpm—but in any case, for sizable values of x_a at resonance the criterion of laminar flow will not permit high frequency operation.

An oil dash pot however is not restricted to any frequency range, but when operating at high frequency the heat dissipated to the fluid will change viscosity and thus the damping constant very rapidly—especially so when operating at a high damping ratio.

Variable damping by shear action of viscous fluid on the mass.

On to the mass, attach "n" light metallic plates having an area = b x ℓ. Pass these plates into a dash pot consisting of parallel plates so that the clearance between the plates of the mass and those of the dash pot be equal to "h".

In this case, the total damping force

$$F = 2\, b \ell \mu \frac{\dot{x}\, n}{h} = 2\, A \mu \frac{\dot{x}\, n}{h}$$

$$C = \frac{F}{\dot{x}} = \frac{2\, A \mu\, n}{h} \qquad (9)$$

The shearing area (2A) and the clearance h will remain constant.[1] Viscosity, μ of the working fluid will be the varying

[1] A discussion of A and h not held constant, but rather as the varying elements of the dash pot is found in Chapter I.

element of the dash pot. Since μ varies with temperature and especially so with fluids of high viscosity which would be used in this type of dash pot, a rigid temperature control must be maintained. Furthermore, the temperature may not vary while the machine is in operation. This of course is true for all dash pots, although in this particular dash pot, a larger source of working fluid may be maintained. Thus a smaller temperature change when compared with the previously described dash pots for equal amounts of energy being transmitted to the dash pot from the vibrating mass. However, it must be remembered that change in viscosity for changes in temperature is many times greater for heavy oils as compared with light oils.

For SAE oil No. 60 μ at 70° F $= \dfrac{200}{10^6}\dfrac{\#\ \text{sec}}{\text{in}^2}$

let $A = 25\ \text{in}^2$

$n = 1$

$h = \dfrac{1}{32}''$

$$C = 50 \times 32 \times 200 \times 10^{-6} = 0.32\ \frac{\#\ \text{sec}}{\text{in}}$$

for the same geometry and oil, but at an oil temperature of 110° F

$$C = .06\ \frac{\#\ \text{sec}}{\text{in}}$$

and at 130° F

$$C = .03\ \frac{\#\ \text{sec}}{\text{in}}$$

$$\zeta = \frac{C}{C_c} = \frac{C}{2\sqrt{K\,M}}$$

Therefore, with the temperature of SAE oil No. 60 varied from 70° F to 130° F a damping ratio over the range of $\zeta = 1.0 \longrightarrow \zeta = 0.1$ may be obtained by properly chosen values of K and M since the viscosity of the fluid has varied to this extent in the selected temperature range.

At resonance, all the input energy is dissipated in damping. The energy dissipated in damping per cycle is equal to $\pi c \omega x^2$. When operating at low frequencies with low damping, the Btu dissipated in the dash pot per minute is a negligible quantity. But for operation at high frequency with sizable damping, this effect can no longer be neglected. The two pounds of working fluid in the dash pot for Chapter I will rise one degree F. if the model is operated for several minutes at critical condition—that is, at high frequency, large amplitude, and large damping.

Section 2

INDICATING SYSTEMS

A well-known and often used form of recording vibrations is revolving a drum at constant angular velocity with its axis parallel to the motion of the vibrating mass, and thereby sketching the motion of the mass onto paper wound about the drum by a unit on the mass. This could be done equally as well with light sensitive film wound about the drum and a screen with a thin slit in the direction of vibration separating the two. In this case a light source passing through a thin slit cut in a screen on the mass perpendicular to the motion will record the vibration on the film. A third adaptation of the drum could be electric sparks between a point on the mass and the drum recording the vibration by holes cut in paper wound about the drum.

This could just as well be done by translation instead of rotation in which case there will be required a drum to unroll the paper or film as the case may be and a drum on which to wind the record after it has passed the vibrating mass. In the case of light sensitive film, this has been called motion picture photography.

If the film or paper passes the vibrating mass fast enough the sine wave may be recorded. If it were to move slowly with the frequency control mechanism, the envelope of the recorded vibration would be a plot of amplitude vs. frequency. This plot could also be recorded on the film of an ordinary camera by rotating it about its lens as an axis, and the film thus recording the motion of a light source on the moving mass.

Another general method using a light beam reflected from a mirror that is moved mechanically by the vibrating object could be made visible to a large audience, whereas the previously discussed methods would be of no immediate value to a large audience. The sine wave could be shown by reflecting the light beam by a mirror revolving at high angular velocity before passing on to a screen. Use of a light persistant screen and light from a gas filled bulb with electric contact as in Chapter I will improve this method.

A cathode ray oscillograph can be used very effectively to record mechanical vibrations. A number of ways can be used to change the mechanical vibration into an electronic vibration. Some of which are:

1. Changing the resistance of a wire due to its elongation or compression could be used for extremely small vibrations.
2. A light beam controlled by the motion of the vibrating object directed on to a photo cell the output of which is good for large vibrations over a large frequency range.
3. Induction of voltage in a moving coil which moves in the air gap of a permanent magnet.
4. Varying capacitance with one of the plates moving with the mass.
5. Core and coil pickups, with a moving core because that will be more sensitive.

The voltage produced by any one of these methods would be amplified. For frequencies between 15 and 40,000 cycles per second an ordinary A.C. amplifying circuit will operate at constant gain. Below 15 cycles per second gain decreases and therefore this amplifier could not be used for the vibrating model at frequencies well below 1000 rpm. A D.C. amplifier would give constant gain over a wider range. Its use is not advised for this design because it is an unreliable unit that must frequently be recalibrated. Nevertheless, there is one amplifying circuit that can be used over a wide frequency range that will be reliable. That is, the photo-electric recorder. It is a Wheatstone bridge circuit having photo cells sensitive to a light source from the vibrating mass. Their output varies a 500 ~ or 1000 ~ A.C. voltage which records an envelope of the motion of the vibrating light source on the screen of the oscillograph.

A stroboscope may be used to read the amplitude of the displacement on a scale directly adjacent to the vibrating mass. Difficulties arise in that the vibrations are small and will be difficult to read. Amplifying this vibration optically is described in Chapter I. The vibration may be amplified mechanically with a light rod vibrating with the mass at one end and the free end vibrating along a scale that has been magnified since the fulcrum on which the rod rotates is nearer to the end vibrating with the mass. The moment of inertia of this rod must be added to the mass of the dynamic system. This will vary slightly with displacement, and since a light mass is being used for the model, the effect will tend to change the pure sinusoidal motion of the mass and for this reason was not used for the design.

Section 3

FREQUENCY MEASUREMENT

Frequency can be read from indicating systems described in the preceding section that make a record of the sine wave when the paper or film have passed the vibrating mass at a known constant velocity. By recording another wave of known frequency along with the sine wave of the mass, the velocity of the recording film need neither be known or constant. A 60 cps light wave could be used and in the case of the electric sparks all that need be known is the frequency producing these sparks.

Frahm's vibrating reed tachometer cannot be used for this design because it does not cover the wide frequency range desired, but it can be used for checking calibration of an uncertain tachometer.

Electric tachometers accurate to ± 1% can be used with this design. For further information, see Bulletin No. GEA-4324, General Electric Co., Schenectady, N.Y. Three types are listed:

1. 300 - 4000 rpm - using D.C. generator with recording voltmeter plus a properly calibrated dial.
2. 10 - 15000 rpm - using A.C. generator with recording voltmeter and properly calibrated dial.
3. 6000 - 60000 rpm - consists of a phototube, lamp and amplifying equipment to furnish the indicating voltage.[1]

The strobotac used in the design will very easily measure frequency. It has a calibrated dial for values between 600 rpm and 14400 rpm. Speeds below 600 rpm can be read from a disk geared to

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[1] For further information, see Brown Instrument Company's bulletin, Tachometer. (M.I.T. Central Library - 530.81-B88)

the motor. A spot on the disk is stopped by the strobotac, in which case the dial reading divided by the gear ratio is the motor speed. In other instances a number of spots will appear to be stationary, in which case the actual rpm is the dial reading divided by the product of the gear ratio and the number of spots seen. This strobotac uses a vibrating reed for day to day recalibration and is accurate to $\pm$ 1% for dial readings above 900 rpm. (Type 631-B strobotac - General Radio Co., Cambridge, Mass.)

Section 4

ELECTRIC MOTORS

Instead of the D.C. shunt wound motor specified in the design of the vibrations model, a fractional horsepower universal motor could have been used. By armature control its frequency range is 1500 rpm to 15,000 rpm—in which case it must be geared down to frequency of operation of the model. Increased frequency range may be obtained with field control. Nevertheless, the D.C. shunt wound motor is advantageous for this design since it has greater frequency range without field control and gearing is unnecessary.

In mounting the motor on the frame of the vibrating model a rubber pad should be inserted between the motor and frame to absorb extraneous vibrations.

CHAPTER III

SUGGESTIONS FOR FUTURE STUDY

A. High Frequency Operation.

High frequency operation is desirable so that a cathode ray oscillograph may be used to record vibrations, in which case constants would be chosen for a natural frequency near 2000 rpm. Theoretically, the dash pot of Chapter I will provide sufficient damping for operation at this frequency. With this in mind, studies should be made of the dash pot in operation. It may be found that more than one plate is necessary to provide sufficient damping. A design having perpendicular plates may be found desirable in contrast to parallel plate designs.

Certainly, an air dash pot should be considered. To obtain large damping (close to 1 $\frac{\text{lb sec}}{\text{in}}$ will be necessary) a large area piston must be employed. In order to keep the mass small, the piston thickness must be kept at a minimum, but then must be sufficient thickness at the clearance to allow for the formation of laminar flow. Since an oscillograph will record very small vibrations accurately, the leakage and thus the Reynolds number can be held low. Therefore, the design of a wedge type air dash pot having a thin piston with a flange at the cylinder clearance is suggested.

B. External Leakage in Oil Dash Pots.

The disadvantage of using pressure difference type oil dash pot is the loss of working fluid through the lower piston shaft opening. Investigations to determine a material or design to remove this effect could be made.

C. Eddy Current Damping.

D. Design of torsional system in one degree of freedom with variable damping, moment of inertia, and stiffness of connecting shaft.

E. Effect of slight turbulence in viscous damping.

F. Persistent screen indicating method is a good topic for future investigation because these screens are restricted at the present time.

G. As springs K and K' are compressed and elongated they will tend to produce a moment that will start a torsional vibration. This effect may be valuable in the design of Part D of this chapter, but is certainly undesirable in the results of this investigation and its elimination is essential.

APPENDIX A

NOMENCLATURE

NOMENCLATURE

M Mass of vibrating element, $\frac{\text{lb sec}^2}{\text{in}}$

K Main spring stiffness, lb/in

K' Moving support spring stiffness, lb/in

x Displacement of mass from its equilibrium position, in

x_a Maximum displacement of mass from its equilibrium position, in

x_f Displacement of moving support from its equilibrium position, in

x_{af} Maximum displacement of moving support from its equilibrium position, in

$\dot{x}$ Velocity of mass at displacement x, in/sec

$\ddot{x}$ Acceleration of mass at displacement x, in/sec^2

F Viscous damping force, lb

C Damping coefficient, $C = \frac{F}{\dot{x}}$, $\frac{\text{lb sec}}{\text{in}}$

C_c Critical damping $= 2\sqrt{K\,M} = \frac{\text{lb sec}}{\text{in}}$

ξ Damping ratio $= \frac{C}{C_c}$, dimensionless

ω_n Natural frequency $= \sqrt{\frac{K}{M}}$, rad/sec

ω_f Forced frequency, rad/sec

β Ratio of forced frequency to natural frequency, $\frac{\omega_f}{\omega_n}$, dimensionless

φ Phase angle between x and x_f, degrees

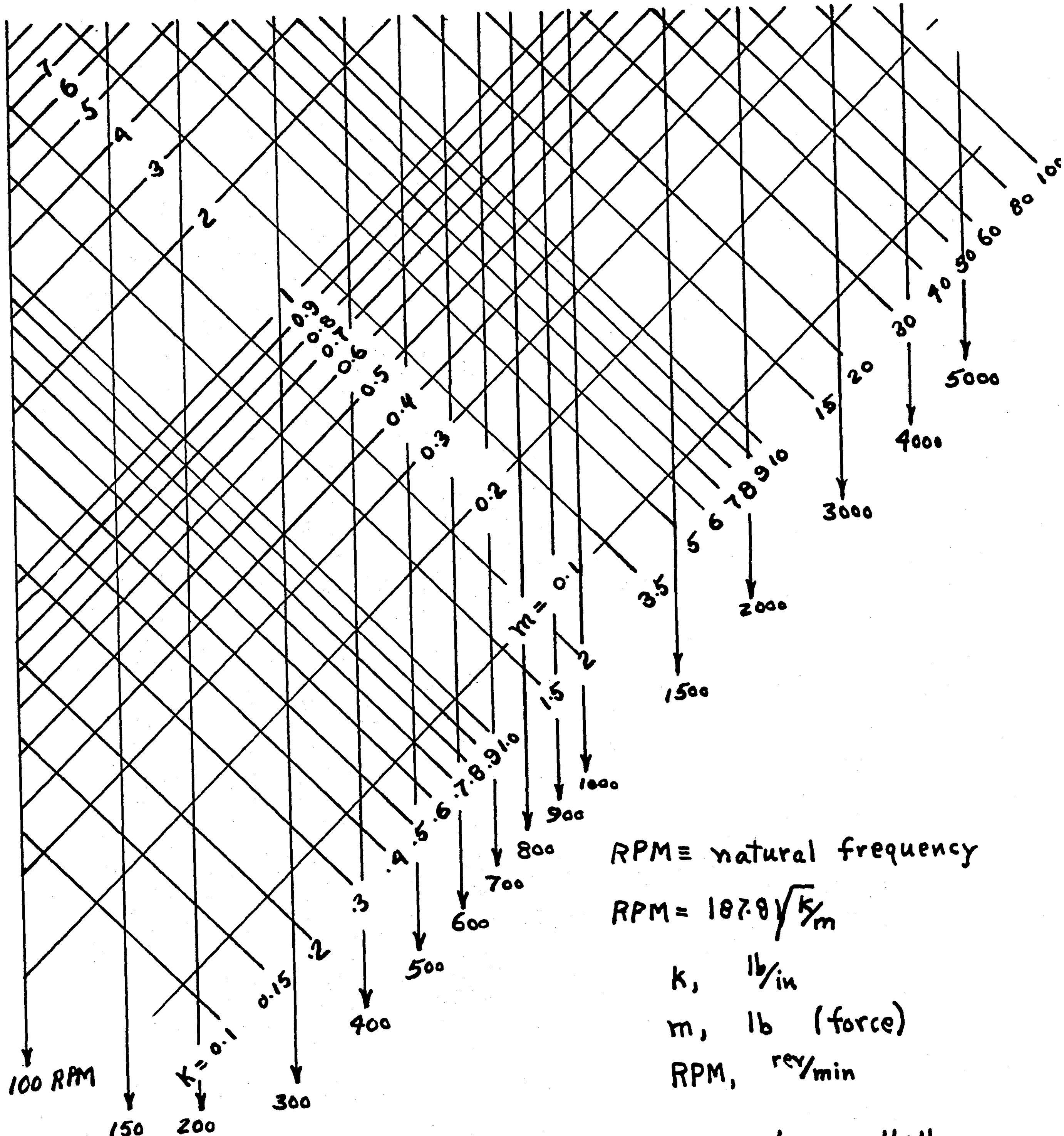
μ Viscosity of fluid, $\frac{\text{lb sec}}{\text{in}^2}$

A Area of viscous plate on the mass, in^2

h Clearance between moving plate and stationary plates of dash pot

APPENDIX B

NATURAL FREQUENCY CHART



RPM ≡ natural frequency

$$RPM = 187.8\sqrt{k/m}$$

k, lb/in

m, lb (force)

RPM, rev/min

James Heller
MIT Thesis
July 1945

APPENDIX C

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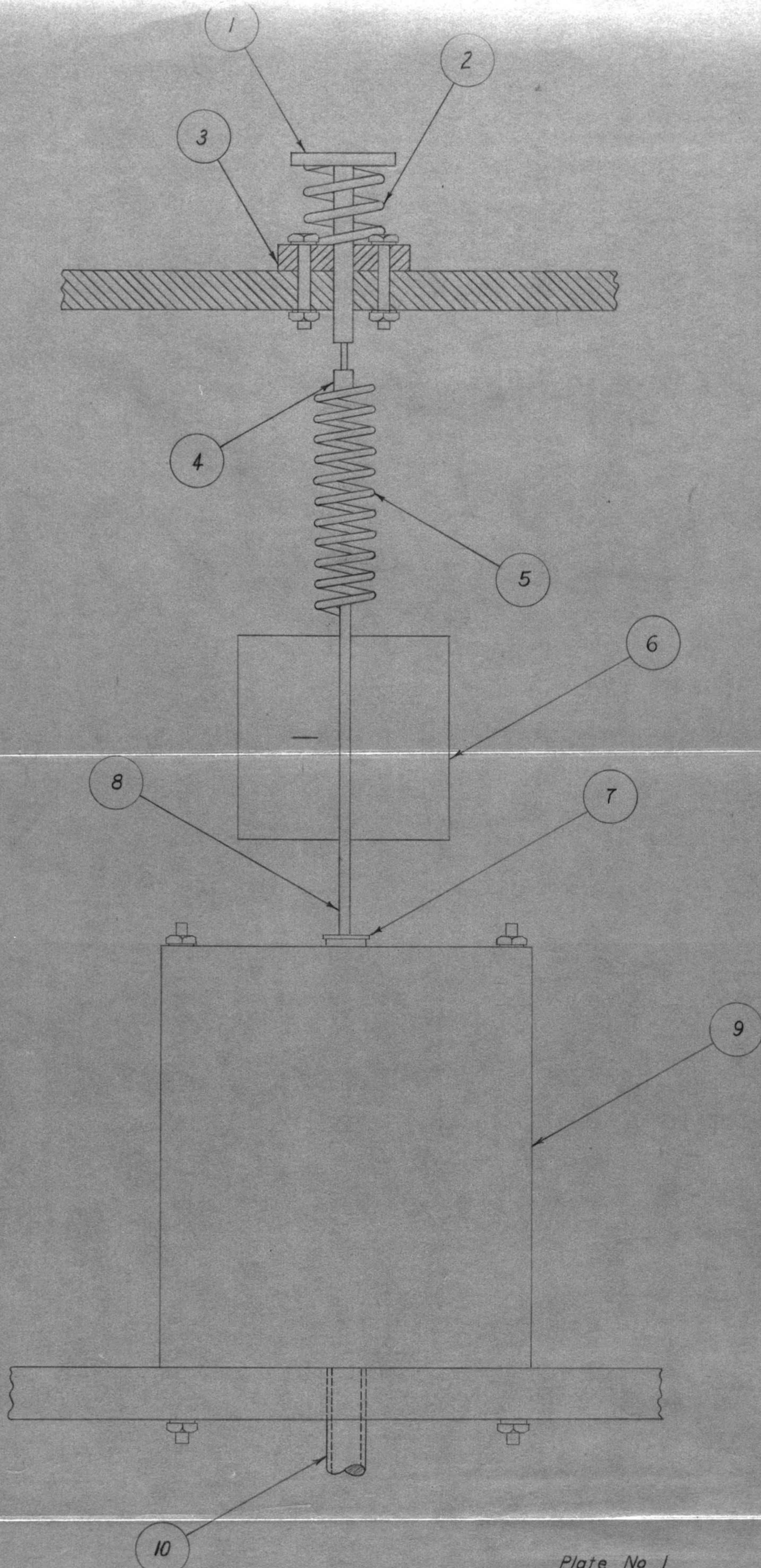

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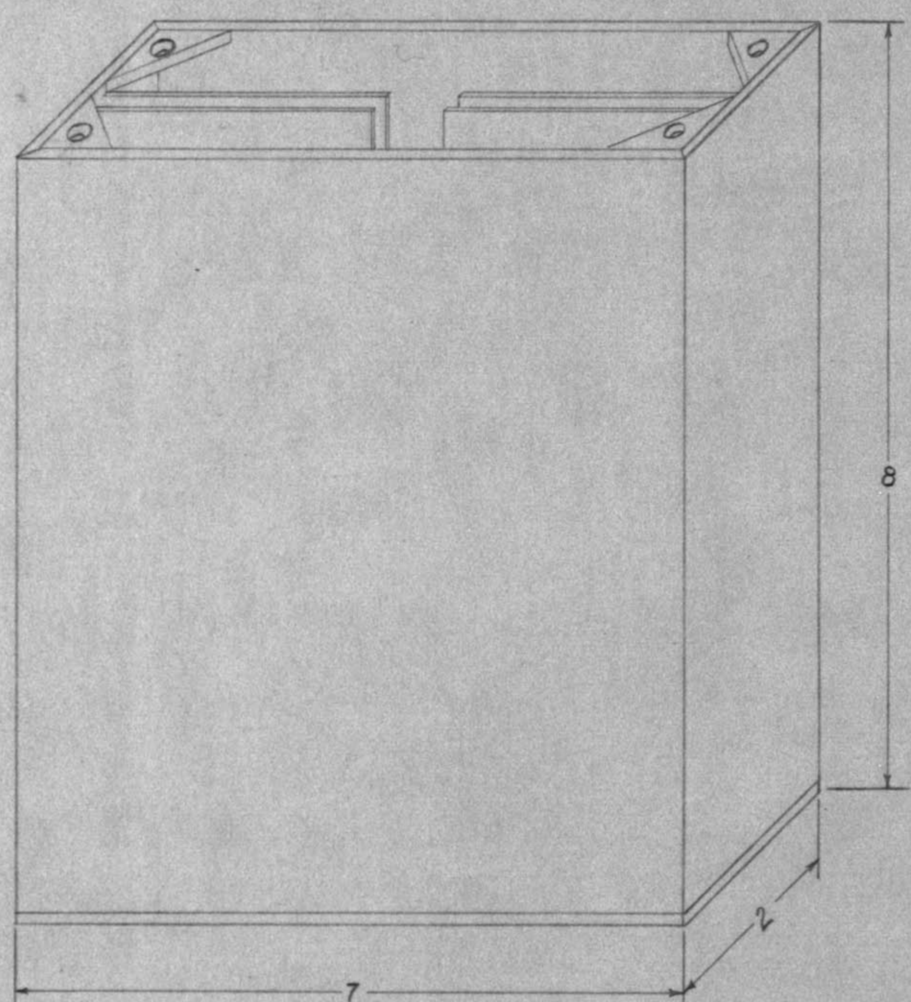

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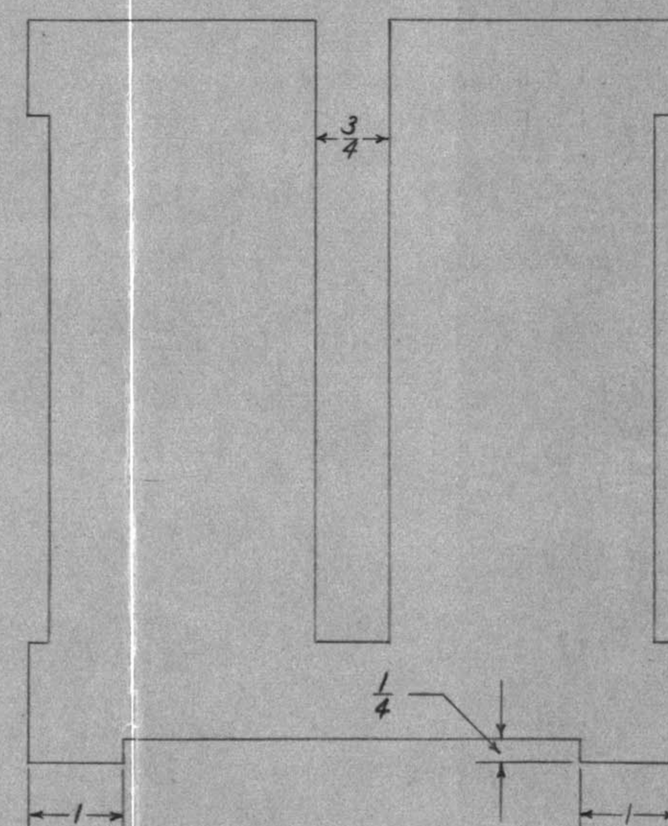

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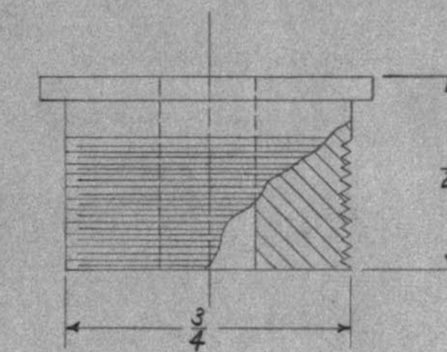

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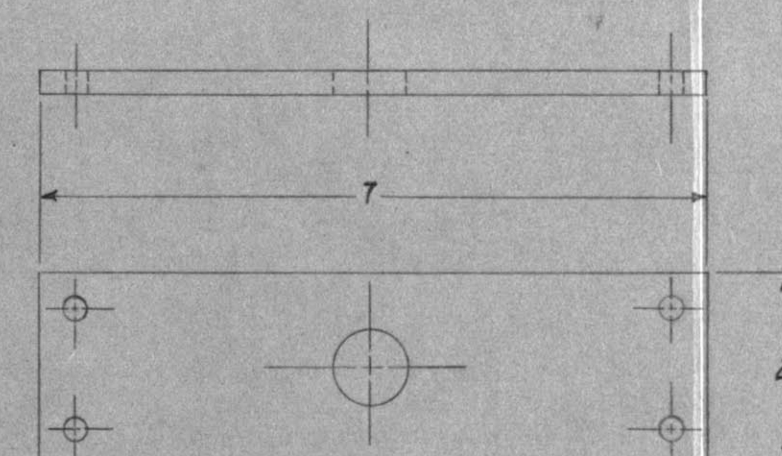

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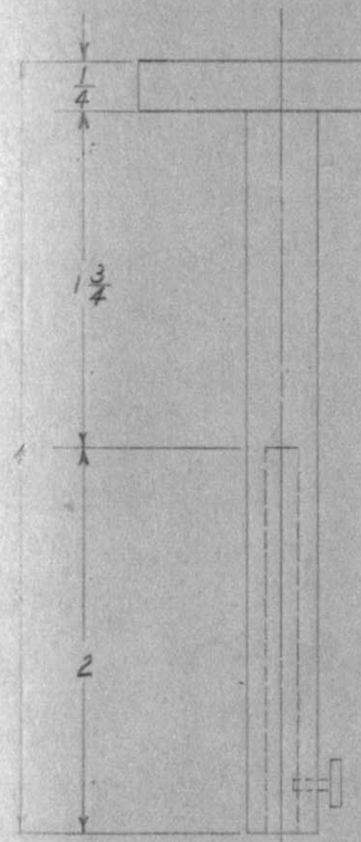

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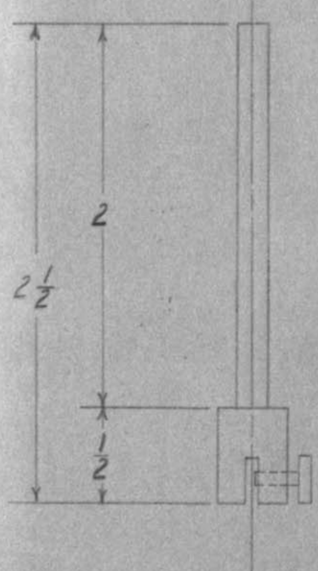

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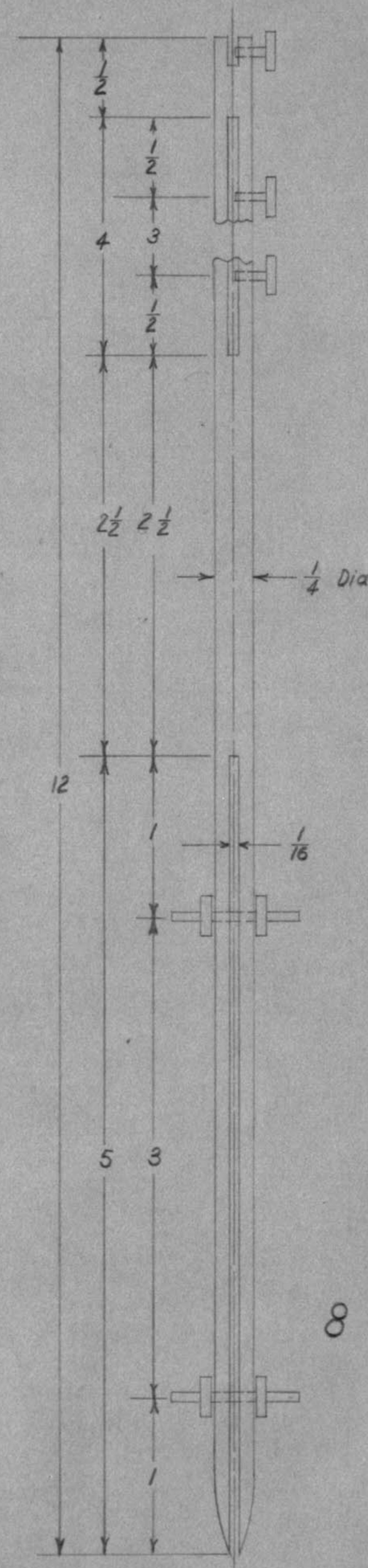

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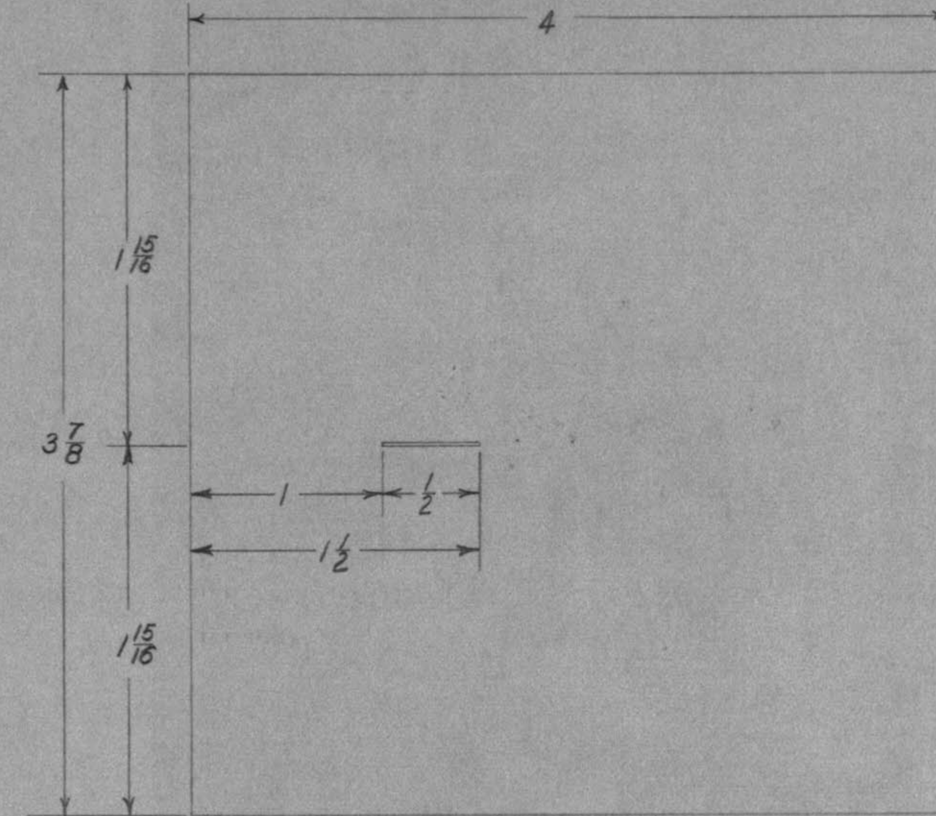

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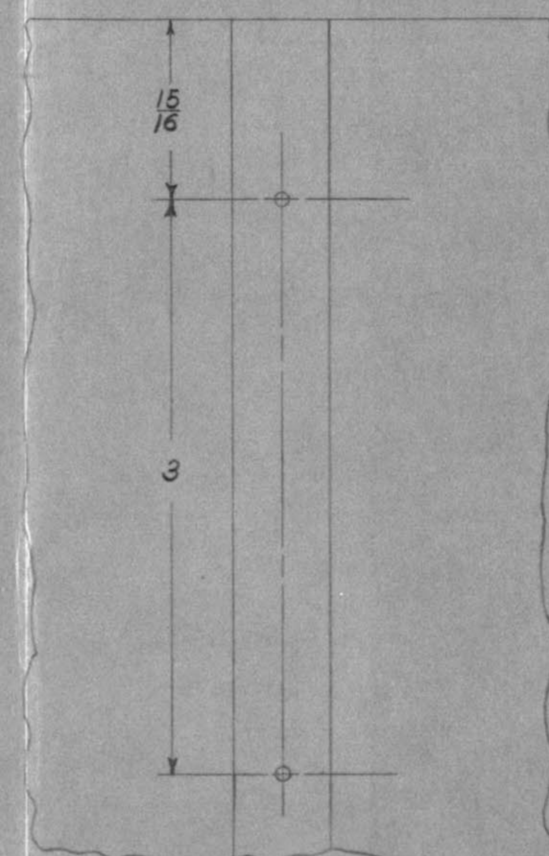

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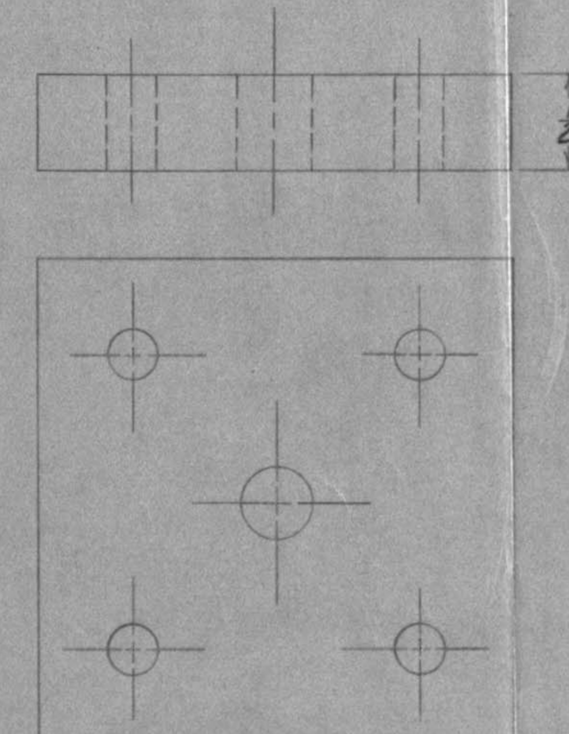

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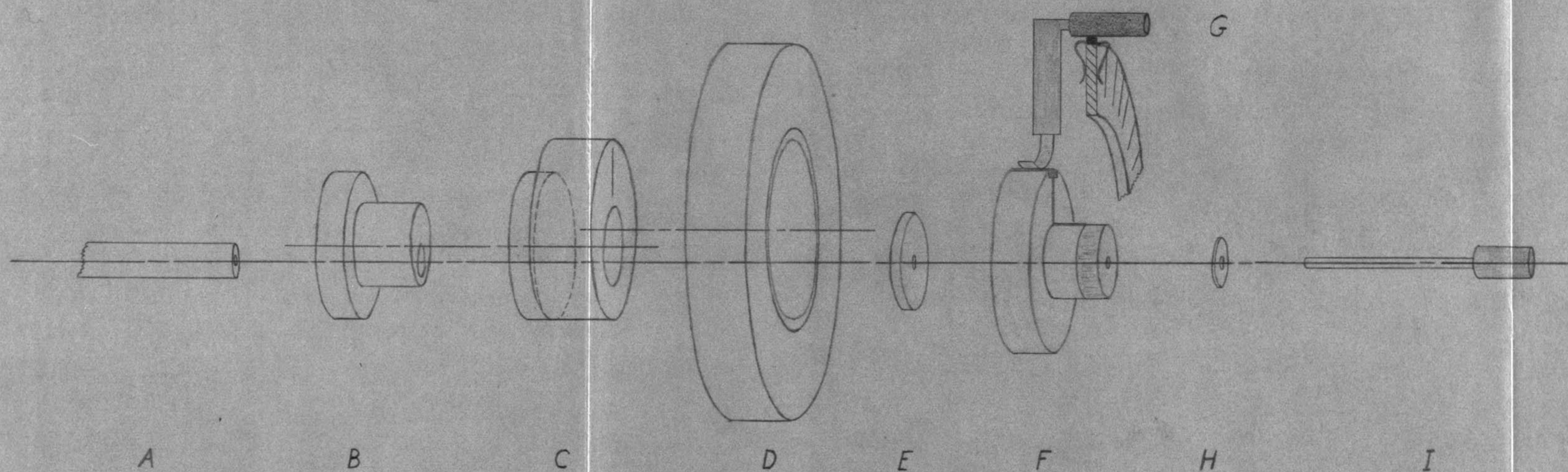

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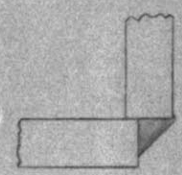

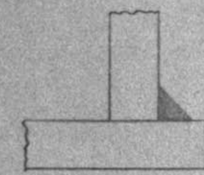

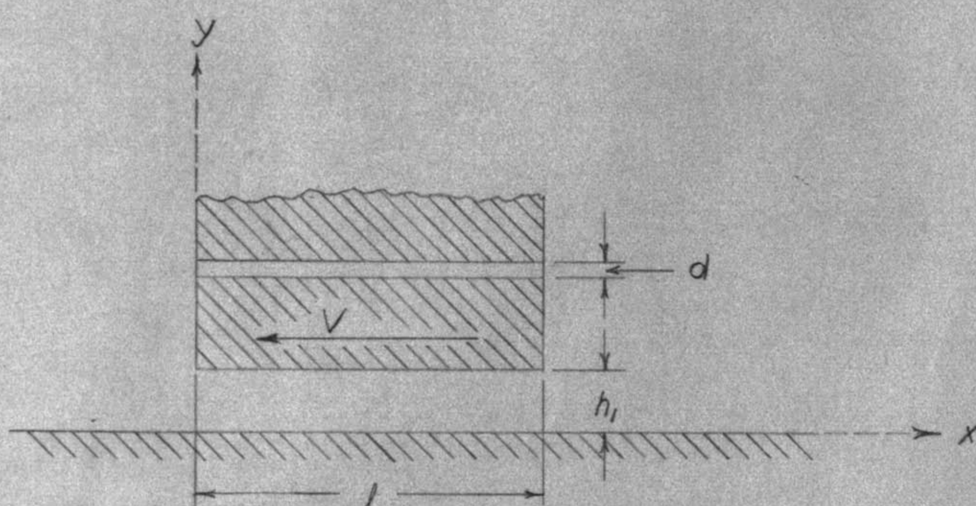
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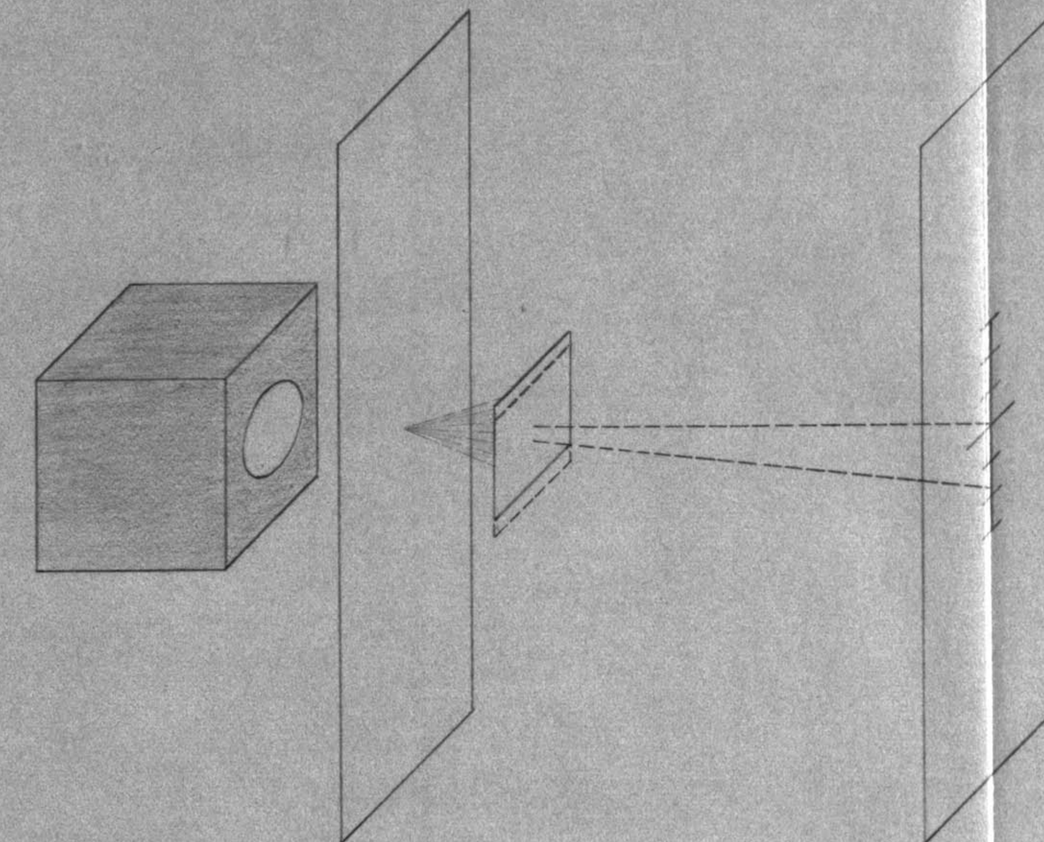

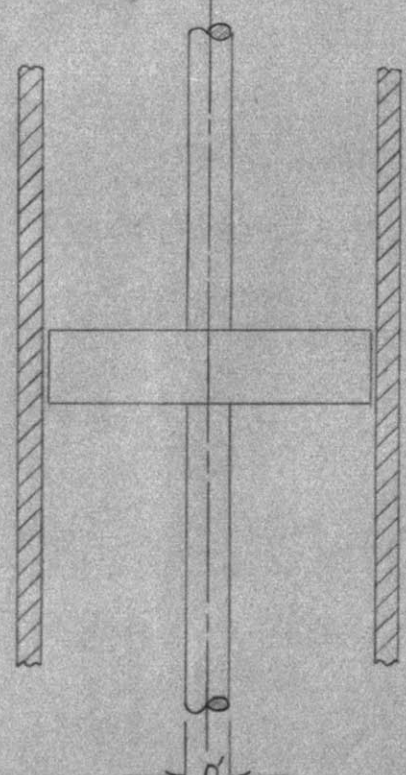
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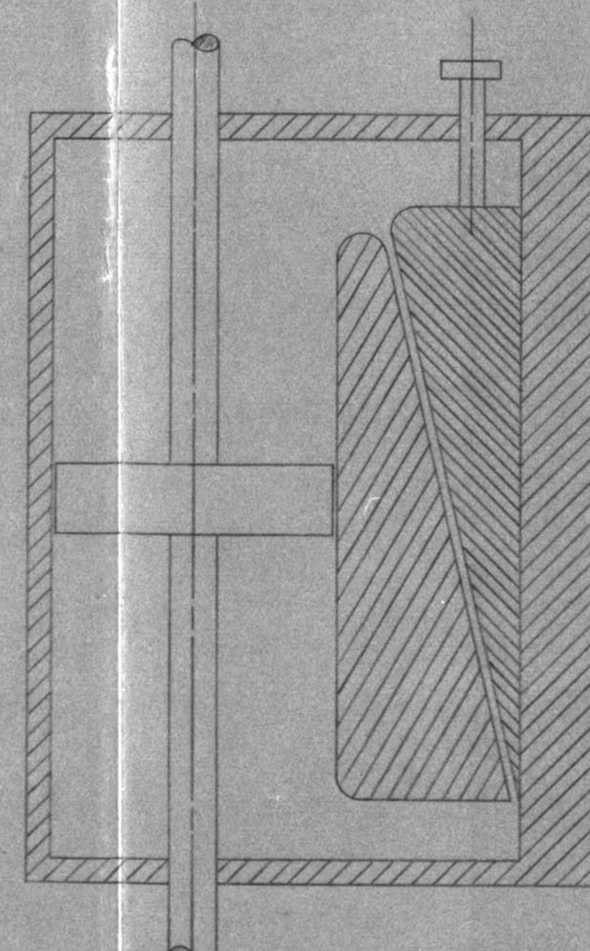

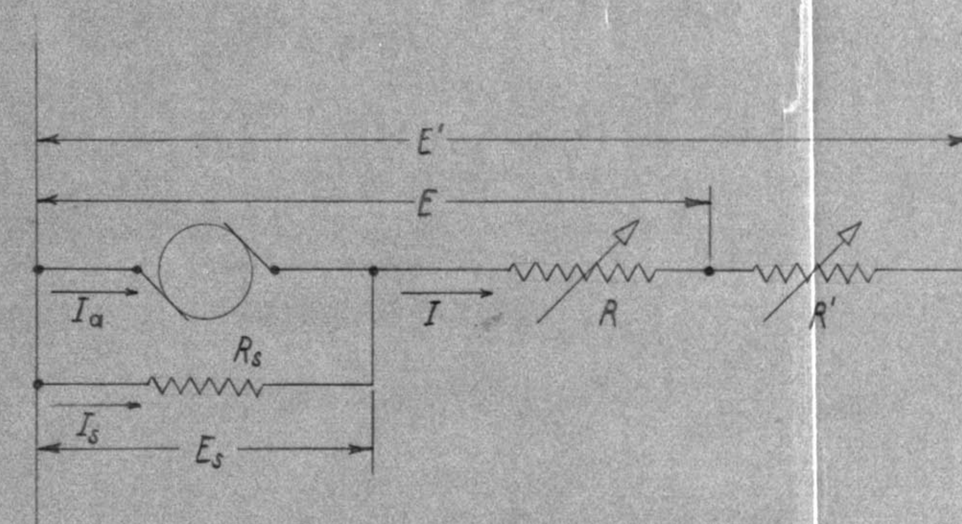
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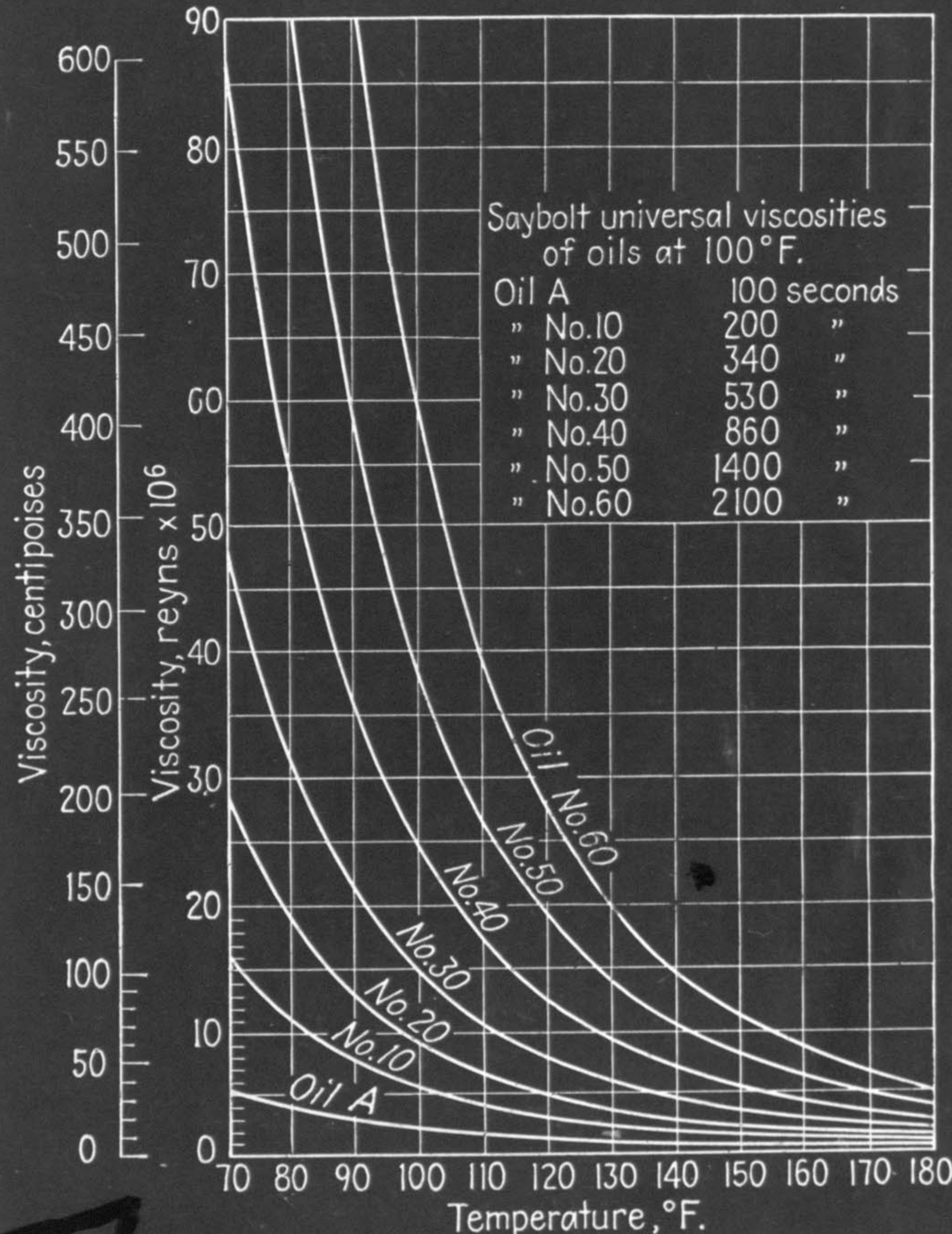

Temperature-viscosity chart for S.A.E. classification.