Charge Separation: How Voltage and Current Are Formed

Lecture 6 – 2.626

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Concept Quiz Results

• Discussion...
General Announcements

• Books
• Print-Outs of Lecture Notes
Homework Assignment

• Read: Martin Green, Chapter 4
• Read: PVCDROM: Chapters 3 and 4
Books: Sticker-shock, anyone?

If not, we’ll call in the order!

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Outline

• Review: pn-junctions
• Minority carrier current
• Ideal diode equation
Review: Diffusion

From PVCDROM

In this animation, 1/4 of the carriers move to the right, 1/4 to the left and the remainder stay put (move up or down).

Carriers in each location at scattering event #1
- Carriers at scattering event #2
- Carriers at scattering event #3
- Carriers at scattering event #4

Process continues until a uniform concentration results.

Courtesy Christiana Honsberg and Stuart Bowden. Used with permission.
Review: Drift Current

From PVCDROM
Courtesy Christiana Honsberg and Stuart Bowden. Used with permission.
Eventually, the accumulation of like charges [(h⁺ + P⁺) or (e⁻ + B⁻)] balances out the diffusion, and steady state condition is reached.
Nicer figure at Wikipedia!

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Pn-junction under bias

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Bias Across a pn-Junction

$$q\psi_o = E_g - E_1 - E_2$$

$$= E_g - kT \ln \left( \frac{N_V}{N_A} \right) - kT \ln \left( \frac{N_C}{N_D} \right)$$

$$= E_g - kT \ln \left( \frac{N_C N_V}{N_A N_D} \right)$$

$$\psi_o = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right)$$

Built-in pn-junction potential a function of dopant concentrations.
Bias Across a pn-Junction

\[ q\psi_o = E_g - E_1 - E_2 \]

\[ = E_g - kT \ln \left( \frac{N_V}{N_A} \right) - kT \ln \left( \frac{N_C}{N_D} \right) \]

\[ = E_g - kT \ln \left( \frac{N_C N_V}{N_A N_D} \right) \]

\[ \psi_o = \frac{kT}{q} \ln \left( \frac{N_A N_D}{n_i^2} \right) \]

Built-in pn-junction potential a function of dopant concentrations.

Bias Across a pn-Junction

The potential across a biased pn-junction device is

\[ \psi_o - V_A = \frac{kT}{q} \ln\left(\frac{N_A N_D}{n_i^2}\right) - V_A \]
Carrier Concentrations Across a pn-Junction

Approximation 1: Device can be split into two types of region: quasi-neutral regions (space-charge density is assumed zero) and the depletion region (where carrier concentrations are small, and ionized dopants contribute to fixed charge).
Width of space charge region

\[ W = l_n + l_p = \sqrt{\frac{2\varepsilon}{q}(\psi_0 - V_a)} \cdot \left( \frac{1}{N_A} + \frac{1}{N_D} \right) \]
Width of space charge region

\[ W = l_n + l_p = \sqrt{\frac{2\varepsilon}{q}(\psi_0 - V_a) \cdot \left(\frac{1}{N_A} + \frac{1}{N_D}\right)} \]

NB: Actually \( \varepsilon \star \varepsilon_o \), where \( \varepsilon_o \), the vacuum permittivity, is 8.85x10^{-12} \text{ F/m} \) or 5.53x10^{7} \text{ e/(V*m)}
Capacitance

\[ n = n_{n0} \approx N_D \]

\[ p = p_{n0} \approx n_i^2/N_D \]

\[ n = n_{p0} \approx n_i^2/N_A \]

\[ p = p_{p0} \approx N_A \]

Device capacitance

Distance, \( x \)

pn-junction area

\[ C = \frac{\varepsilon A}{W} \]
Capacitance

When one side of the pn-junction is heavily doped, the capacitance reduces to this expression:

\[
\frac{C}{A} = \sqrt{\frac{q \varepsilon N}{2(\psi_0 - V_a)}}
\]
Pn-junction under zero bias

\[ n = n_{n0} \approx N_D \]

\[ p = p_{p0} \approx N_A \]

\[ n = n_{p0} \approx n_i^2 / N_A \]

\[ p = p_{n0} \approx n_i^2 / N_D \]
Pn-junction under forward bias

\[ p = p_{p0} \approx N_A \]
\[ n = n_{n0} \approx N_D \]
Pn-junction under **forward bias**

At zero bias:

\[ p_{nb} = p_{n0} = p_{p0} \cdot \exp\left(-\frac{q\psi_0}{kT}\right) \approx \frac{n_i^2}{N_D} \]

\[ n_{pa} = n_{p0} = n_{n0} \cdot \exp\left(-\frac{q\psi_0}{kT}\right) \approx \frac{n_i^2}{N_A} \]
Current flow through the depletion region

For holes:

\[ J_h = q u_h p \xi - q D_h \frac{dp}{dx} \]
Current flow through the depletion region

For holes:

$$\xi \approx \frac{kT}{q} \frac{1}{p} \frac{dp}{dx}$$

Approximation 2: Assume $J_h$ is small!
Current flow through the depletion region

Integrating...

\[ \psi_o - V_a = - \frac{kT}{q} \ln(p) \bigg|_a^b \]

\[ = \frac{kT}{q} \ln \left( \frac{p_{pa}}{p_{nb}} \right) \]
Current flow through the depletion region

Approximation 3: Only cases where minority carriers have a much lower concentration than majority carriers will be considered, i.e., \( p_{pa} \gg n_{pa}, \ n_{na} \gg p_{na} \)

\[
p_{pa} = N_A + n_{pa}
\]
Current densities

Calculate (diffusive) currents in quasi-neutral region:

\[ J_h = -qD_h \frac{dp}{dx} \]

... from previous slide ...

\[ J_h(x) = \frac{qD_h p_{n0}}{L_h} \left( e^{qV/kT} - 1 \right) e^{-x/L_h} \]

\[ J_e(x') = \frac{qD_e n_{n0}}{L_e} \left( e^{qV/kT} - 1 \right) e^{-x'/L_e} \]
Current densities

\[
\frac{1}{q} \frac{dJ_e}{dx} = U - G = -\frac{1}{q} \frac{dJ_h}{dx}
\]

Magnitude of the change in current across the depletion region:

\[
\delta J_e = |\delta J_h| = q \int_{-W}^{0} (U - G) dx \approx 0
\]

Key assumption: \(W\) is small compared to \(L_e\) and \(L_h\). Therefore, integral is negligible. It follows that the current \(J_e\) and \(J_h\) are essentially constant across the depletion region, as shown below.
Ideal Diode Equation

Since $J_e$ and $J_h$ are known at all points in the depletion region, we can calculate the total current:

$$J_{\text{total}} = J_e \bigg|_{x'=0} + J_h \bigg|_{x=0} = \left(\frac{qD_e n_{p0}}{L_e} + \frac{qD_h p_{n0}}{L_h}\right)\left(e^{qV/kT} - 1\right)$$

This leads to the ideal diode law:

$$I = I_o \left(e^{qV/kT} - 1\right), \text{ where}$$

$$I_o = A\left(\frac{qD_e n_i^2}{L_e N_A} + \frac{qD_h n_i^2}{L_h N_D}\right)$$
Next Class

• *Ideal diode equation discussion*
• *Contacts*
• *Review Part 1*