Today’s Topics:

1. Steady state system responses to sinusoidal inputs
2. Second order system example
Consider the following experiment in which a stable linear system is driven by a sinusoidal input

The Laplace Transform of this input is-

If $G(s)$ is the system transfer function then the Laplace Transform of the output is
which can be written as-

\[
N(s) \text{ and } D(s) \text{ are the numerator and denominator polynomials of } G(s).
\]

Then

\[
-z_1, -z_2, \ldots \text{ are the system zeros and } -p_1, -p_2, \ldots \text{ are the system poles}
\]

A partial fraction expansion of \( C(s) \) obtains
The inverse Laplace Transform would then be

Since the system is stable all of the system poles lie in the left half plane. Hence all of the $p's$ have positive real parts so each of the terms of the form $e^{p't}$ will decay to zero as time increases. In particular, if we wait for a long time after startup then

This is the steady state response to the sinusoidal input at the frequency $\omega$. 
The two residuals $\kappa_1$ and $\kappa_2$ can be evaluated in the usual fashion. Recall that

so

and the steady state response is
Now let’s look at the term $G(j\omega)$. It maps the point $j\omega$ in the “$s$” plane to the point $G(j\omega)$ in the “$G$” plane.

The complex number $G(j\omega)$ can be represented either as the sum of real and imaginary parts-

or as a vector of magnitude $M(\omega)$ at the angle $\phi(\omega)$
where

Also, $G(-j\omega)$ is the complex conjugate of $G(j\omega)$

Using these results we can substitute back into the equation for the steady state response
So what do we have?

**Frequency Response:** A steady sinusoidal input, of frequency $\omega$ and magnitude $A$, into a stable linear system, yields, after all transients have died out, a steady sinusoidal output at frequency $\omega$, of magnitude $A \cdot M(\omega)$ and with phase shift $\phi(\omega)$. The amplitude ratio $M(\omega)$ and phase shift $\phi(\omega)$ satisfy the expressions-

Said in another way, the input sinusoid has its amplitude multiplied by $M(\omega)$ and its phase shifted by the angle $\phi(\omega)$, as it passes through the system $G(s)$.
Second Order System Example-

The transfer function of a second order system, with undamped natural frequency $\omega_n$ and damping ratio $\delta$, is

so

which yields the amplitude ratio

and phase shift

At low frequencies, as $\omega$ approaches zero

At high frequencies, as $\omega$ approaches infinity

At $\omega = \omega_n$
**********Freq Rsp plots**********