16.06 Lecture 27

Polar Plots

November 6, 2003

Today’s Topics:

1. First order system polar plot
2. Second order system polar plots
3. Other examples
Recall from last time that if the system $G(s)$ receives a steady sinusoidal input of amplitude $A$, so

then the steady state output will be a sinusoid of magnitude $A \cdot M(\omega)$ with its phase shifted by $\phi(\omega)$

where

Now let’s pause for a bit and think about some interpretations of $G(j\omega)$. 
$G(j\omega)=$ the transfer function $G(s)$ evaluated at the point $s = j\omega$ on the imaginary axis of the “$s$” plane.

In particular, if

then

Now the term $(j\omega - z_i)$ is the vector from the zero at $z_i$ to the point $j\omega$ on the imaginary axis of the “$s$” plane.
Similarly, if the poles $p_1$ and $p_2$ are complex conjugates then

In general

where-

$k_{rl}$ root locus gain

$A_a =$ magnitude of the vector from the zero at $-z_i$ to the point $j\omega$

$A_p =$ magnitude of the vector from the pole at $-p_i$ to the point $j\omega$

$\phi_{z_i}$ angle of the vector from the zero at $-z_i$ to the point $j\omega$

$\phi_{p_i}$ angle of the vector from the pole at $-p_i$ to the point $j\omega$
Example-first order system

As $\omega$ goes from zero to plus infinity $G(j\omega)$ traces a contour in the complex plane
Example: second order system
Some typical second order system polar plots for

\[ G(s) = \frac{k_m}{s^2 + 2\delta\omega_n s + \omega_n^2} \]

Critical damping (\( \delta = 1 \))- 

Medium damping (\( \delta = 0.7 \))
Light damping (\(\delta \ 0.1\))

Zero damping (\(\delta \ 0\))
The Quanser

An integrator (pole at the origin)
A differentiator (zero at the origin)