16.06 Lecture 36
Bode Diagrams

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Today’s Topics

1. Simple Lag

2. Quadratic Lag

3. Bode diagram construction

4. Bode diagram examples

Reading: 7.5
1 Simple Lag

Consider a simple lag term: \[ \frac{1}{S} = \frac{1}{j \omega T + 1} \]

\[ M = \]

\[ M_{dB} = \]

\[ \phi = \]

- If \( \omega \ll 1/T, \omega T \ll 1 \)

\[ M \]

\[ M_{dB} \]

\[ \phi \rightarrow \]

This gives us the

- If \( \omega \gg 1/T, \omega T \gg 1 \)

\[ M \]

\[ M_{dB} \]

\[ \phi \rightarrow \]

This gives us the
Draw the asymptotic approximation:

The asymptotes meet at the **break frequency** or **corner frequency**.

- If $\omega = 1/T$

  $$\omega T =$$

  $$M =$$

  $$M_{dB} =$$

  $$\phi =$$
Insert Bode diagram for a simple lag
- Some details of first-order factors:

1. The true curves depart from the asymptotic approximations by
   \[ \pm 0.15 \text{ log units or } \pm 3 \text{ dB at } \omega = 1/T. \]

2. One octave below the break frequency the angle is \(-26.6^\circ\)

3. One octave above the break frequency the angle is \(-90 + 26.6 = -63.4^\circ\)

4. One decade below the break frequency the angle is \(-5.7^\circ\)

5. One decade above the break frequency the angle is \(-90 + 5.7 = 84.3^\circ\)
2 Quadratic Lag

Consider a quadratic lag term:

\[
\frac{1}{Q} = \frac{1}{(j\omega/\omega_n)^2 + 2\zeta j\omega/\omega_n + 1}
\]

\[
M = \quad M_{dB} = \quad \phi =
\]

- If \(\omega/\omega_n \ll 1\)

\[
M = \quad M_{dB} = \quad \phi
\]

This gives us the

- If \(\omega/\omega_n \gg 1\)

\[
M = \quad M_{dB} = \quad \phi
\]

This gives us the
Draw the asymptotic approximation:

The asymptotes meet at the break frequency or corner frequency.

- If $\omega/\omega_n = 1$

  \[
  M = \frac{1}{2\zeta},
  \]

  \[
  M_{dB} = 20\log \frac{1}{2\zeta},
  \]

  \[
  \phi = -90^\circ
  \]

  Smaller damping ratios cause a larger peak in $M_{dB}$ and a more abrupt change in $\phi$.

- Note that for a quadratic lag, the actual magnitude plot can be very different to the asymptotic approximation.
Insert the Bode diagram for a quadratic lag
3 Examples

General approach:

1. Determine which elementary factors are present and calculate any break frequencies.

2. Plot the low-frequency asymptote.

3. Add in the other elementary factors in order of increasing break frequencies. Note that each factor has a 0dB asymptote below its break frequency, and so does not affect the asymptotic plot below this frequency.

   • A simple lag causes a change in slope of -20 dB/dec
   • A quadratic lag causes a change in slope of -40 dB/dec
Example 1:

\[ G(s) = \frac{10(0.1s + 1)}{s(0.01s + 1)} \]
Example 2:

\[ G(s) = \frac{(-0.1s + 1)}{0.1s + 1} \]
Example 3:

\[ G(s) = \frac{10(s + 1)}{s(10s + 1)} \]