Today’s Topics

1. Poles and zeroes

2. Transient response and inverse Laplace transform

3. Graphical determination of residues

Reading: 1.7 (from the top of pg. 14), 1.8, 1.9
1 The s-plane

We write

\[ C(s) = G(s)R(s) \]

where \( C, G \) and \( R \) are each ratios of polynomials in \( s \), i.e. \( G(s) = \frac{\text{num} \, G}{\text{den} \, G} \).

Consider the following definitions:

- zeroes of \( C, G \) and \( R \) are
- poles of \( C, G \) and \( R \) are
- system zeroes and poles are
- system characteristic polynomial is
- system characteristic equation is

Note that the roots of the C.E. are

Since, the polynomials have real coefficients, the poles and zeros are
We plot the poles and zeros in the \( s(\sigma + j\omega) \) plane.

Example:

Assume \( R(s) = \frac{1}{s} \). Then the pole-zero pattern of \( C(s) = R(s)G(s) \) is the superposition of the patterns of \( R(s) \) and \( G(s) \):

\[
C(s) = \frac{K(s + 2)}{s(s + 4)}
\]
2 Transient response and inverse Laplace transform

Question: Given the pole-zero diagram of $C(s)$, how do you get $c(t)$?

Answer: perform inverse Laplace transform

(a) If $C(s)$ is simple, use a table.

If $C(s)$ is complicated, we can use partial fraction expansion (PFE).

Example:

(b) There is an alternative approach, which will serve us well in the future
3 Graphical determination of residues (real poles)

(a) Typical factor in PFE is

\( a \) is positive and \( b \) is negative

We can write

\[ b - (-a) \]

where \( b - (-a) \) is

So in the s-plane:
(b) The general expression for $K_1$ in the example above is

(c) Using the actual values, we have:

and for $K_2$:

(d) So as before,

\[ c(t) = K \left( \frac{1}{2} + \frac{1}{2}e^{-4t} \right) \]
4 Root locus gain (vdv pg. 20)

Definition: The root locus gain of a transform or a transfer function is that which results if the coefficients of the highest powers of \( s \) in the numerator and denominator polynomials are made equal to unity.

For, say,

\[
C(s) = 2 \frac{0.5s + 1}{(s + 3)(0.1s + 1)} = 2(0.5) \frac{s + 2}{0.1(s + 3)(s + 10)}
\]

the root locus gain is \( 2(0.5/0.1)=10 \).

Reviewing the expressions for \( K_1, K_2 \) and \( K_3 \) reveals the following general rule.

**Graphical Residue Rule:** The residue \( K_i \) at the pole \( -p_i \) of \( C(s) \) equals the root locus gain times the product of the vectors from all zeros of \( C(s) \) to \( -p_i \) divided by the product of the vectors from all other poles of \( C(s) \) to \( -p_i \).