A PHASED ARRAY SONAR FOR AN
UNDERWATER ACOUSTIC COMMUNICATIONS SYSTEM

by
WILLIAM HOWARD HANOT

SUBMITTED IN PARTIAL FULFILLMENT
OF THE REQUIREMENTS FOR THE
COMBINED DEGREES OF
BACHELOR OF SCIENCE
in
NAVAL ARCHITECTURE AND MARINE ENGINEERING
and
MASTER OF SCIENCE
in
OCEAN ENGINEERING
at the
MASSACHUSETTS INSTITUTE OF TECHNOLOGY
August 1980

© Massachusetts Institute of Technology 1980

Signature of Author

Department of Ocean Engineering
21 August 1980

Certified by

Arthur B. Baggeroer
Associate Professor of Ocean and
Electrical Engineering
Thesis Supervisor

Accepted by

A. Douglas Carmichael
Chairman,
Department Graduate Committee
DISCLAIMER OF QUALITY

Due to the condition of the original material, there are unavoidable flaws in this reproduction. We have made every effort possible to provide you with the best copy available. If you are dissatisfied with this product and find it unusable, please contact Document Services as soon as possible.

Thank you.

The images contained in this document are of the best quality available.
A PHASED ARRAY SONAR FOR AN
UNDERWATER ACOUSTIC COMMUNICATIONS SYSTEM

by
William Howard Hanot

Submitted to the Department of Ocean Engineering on
21 August 1980 in partial fulfillment of the
requirements for the Combined Degrees of Bachelor
of Science in Naval Architecture and Marine Engineering
and Master of Science in Ocean Engineering

ABSTRACT

A phase-steerable planar sonar array was designed,
constructed and tested for the acoustic transmission of information over relatively short (< 1 km) near vertical paths.
The array consists of 32 transducers in a 4 x 8 element matrix
and is mounted in an oil-filled housing which forms the end cap
to a pressure case designed for operation at depths up to 2 km.
A theoretical analysis predicted an efficiency of 11 percent,
an acoustical quality factor (Q) of 8.3, and 3 dB beamwidths of
8.0° x 16.0° at resonance in the x-z and y-z planes, respectively.
Measurements taken during preliminary trials approached
theoretical predictions, yielding an efficiency of 20 percent,
a Q of 7.4, and 3 dB beamwidths of 10.0° x 14.0° in the x-z and
y-z planes. In addition, power amplifiers were designed with
40 dB of gain and a lowpass filter (f = 70 kHz) for driving
the transducers, and performed exactly as specified. Overall,
the system advanced the cause of low power, high data rate com-
munication over short path lengths in the challenging ocean
environment.

Thesis Supervisor: Arthur B. Baggeroer

Title: Associate Professor of Ocean Engineering
and Electrical Engineering
ACKNOWLEDGEMENTS

This research was made possible by the support of the Sea Grant research program administered through the National Oceanic and Atmospheric Administration, Department of Commerce.

Most of all I would like to thank my advisor and thesis supervisor, Arthur B. Baggeroer, whose patience, encouragement, and direction made it possible for me to learn the real process of engineering outside of the classroom.

I would also like to thank my colleagues at the Woods Hole Oceanographic Institution, Donald Koelsch and Keith Von der Heydt, for their invaluable insight and advice, and for the frequent excuses for trips to Woods Hole, where work is a vacation.

Thanks are due as well to George Shepard of Bolt, Beranak, and Newman, Inc., for providing some much needed equipment used in conducting the experiments described herein.

Finally, I would like to thank all my friends and all the people that I have met who together have made my five years in Boston and at M.I.T. an experience never to be forgotten.
<table>
<thead>
<tr>
<th>TABLE OF CONTENTS</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Symbols/Coordinate System</td>
<td>6</td>
</tr>
<tr>
<td>CHAPTER 1  INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.0 Scope of the Research</td>
<td>8</td>
</tr>
<tr>
<td>1.1 Summary</td>
<td>8</td>
</tr>
<tr>
<td>CHAPTER 2  BACKGROUND</td>
<td></td>
</tr>
<tr>
<td>2.0 Why Acoustic Communication?</td>
<td>12</td>
</tr>
<tr>
<td>2.1 Implementation of Acoustic Communication Systems</td>
<td>20</td>
</tr>
<tr>
<td>2.2 Selected System Characteristics</td>
<td>24</td>
</tr>
<tr>
<td>CHAPTER 3  SYSTEM OVERVIEW</td>
<td></td>
</tr>
<tr>
<td>3.0 Data Collection Sources</td>
<td>30</td>
</tr>
<tr>
<td>3.1 Data Transmission</td>
<td>30</td>
</tr>
<tr>
<td>3.2 Data Reception</td>
<td>38</td>
</tr>
<tr>
<td>3.3 Steering Command Feedback Link</td>
<td>43</td>
</tr>
<tr>
<td>CHAPTER 4  POWER AMPLIFICATION AND SONAR ARRAY HARDWARE</td>
<td></td>
</tr>
<tr>
<td>4.0 Design Performance Goals</td>
<td>46</td>
</tr>
<tr>
<td>4.1 Power Amplifier Design and Construction</td>
<td>52</td>
</tr>
<tr>
<td>4.2 Sonar Array Design and Construction</td>
<td>56</td>
</tr>
<tr>
<td>4.3 Pressure Case Design</td>
<td>67</td>
</tr>
<tr>
<td>4.4 Design/Experimentation Constraints</td>
<td>71</td>
</tr>
<tr>
<td>CHAPTER 5  SYSTEM THEORETICAL PERFORMANCE</td>
<td></td>
</tr>
<tr>
<td>5.0 Amplifier Characteristics</td>
<td>74</td>
</tr>
<tr>
<td>5.1 Transducer Analysis</td>
<td>76</td>
</tr>
<tr>
<td>5.2 Theoretical Array Performance</td>
<td>101</td>
</tr>
<tr>
<td>5.3 Predicted Signal/Noise Ratio at the Receiver</td>
<td>119</td>
</tr>
<tr>
<td>CHAPTER 6  EXPERIMENTAL RESULTS</td>
<td></td>
</tr>
<tr>
<td>6.0 Scope of Experiments</td>
<td>122</td>
</tr>
<tr>
<td>6.1 Amplifier Tests</td>
<td>123</td>
</tr>
<tr>
<td>6.2 Sonar Array Tests</td>
<td>123</td>
</tr>
<tr>
<td>6.3 Presentation of Measured Data</td>
<td>134</td>
</tr>
<tr>
<td>6.4 Sources of Efficiency Loss</td>
<td>146</td>
</tr>
<tr>
<td>CONCLUSIONS</td>
<td>151</td>
</tr>
<tr>
<td>REFERENCES</td>
<td>152</td>
</tr>
</tbody>
</table>
List of Symbols and Abbreviations

<table>
<thead>
<tr>
<th>Quantity</th>
<th>Description</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>m</td>
<td>length</td>
<td>meters</td>
</tr>
<tr>
<td>kg</td>
<td>mass</td>
<td>kilograms</td>
</tr>
<tr>
<td>s</td>
<td>time</td>
<td>seconds</td>
</tr>
<tr>
<td>a</td>
<td>diameter</td>
<td>meters</td>
</tr>
<tr>
<td>α</td>
<td>attenuation coefficient</td>
<td>dB/km</td>
</tr>
<tr>
<td>c</td>
<td>speed of sound</td>
<td>m/s</td>
</tr>
<tr>
<td>d</td>
<td>interelement spacing</td>
<td>meters</td>
</tr>
<tr>
<td>f</td>
<td>frequency</td>
<td>Hz (cycles/second)</td>
</tr>
<tr>
<td>I</td>
<td>intensity</td>
<td>watts/m^2</td>
</tr>
<tr>
<td>k</td>
<td>wave number</td>
<td>m^-1</td>
</tr>
<tr>
<td>κ</td>
<td>Boltzmann's constant</td>
<td>Joules/Kelvin</td>
</tr>
<tr>
<td>η</td>
<td>efficiency</td>
<td></td>
</tr>
<tr>
<td>ρ</td>
<td>density</td>
<td>kg/m^3</td>
</tr>
<tr>
<td>P</td>
<td>pressure</td>
<td>kg/s^2m</td>
</tr>
<tr>
<td>λ</td>
<td>wavelength</td>
<td>meters</td>
</tr>
<tr>
<td>T</td>
<td>temperature</td>
<td>°Kelvin</td>
</tr>
<tr>
<td>V</td>
<td>voltage</td>
<td>volts</td>
</tr>
<tr>
<td>ω</td>
<td>angular frequency</td>
<td>radians/second</td>
</tr>
<tr>
<td>φ</td>
<td>azimuth angle</td>
<td>radians</td>
</tr>
<tr>
<td>θ</td>
<td>polar angle</td>
<td>radians</td>
</tr>
<tr>
<td>x,y,z</td>
<td>rectangular coordinates</td>
<td></td>
</tr>
</tbody>
</table>
Coordinate System for Sonar Array
CHAPTER 1

Introduction

1.0 Scope of the Research

This research has been part of a larger effort to increase the state of the art in underwater acoustic communication[1]. The subsystem detailed in this thesis comprises a set of 32 identical power amplifiers that drive a 32 element phased sonar array, whose purpose is to transmit the encoded and modulated data from the rest of the communication system to a receiver with a minimum power consumption and a maximum signal/noise ratio at the receiver. The characteristics of the amplifiers and sonar array were derived from the overall requirements of the complete communications system, so the entire system is described in brief to allow a better understanding of the motivation behind some of the design features of equipment that is the fruit of two years of research. The success of this facet of the program paves the way for its integration into the communication system as a whole; then its performance under real and varied operating conditions may be conclusively judged.

1.1 Summary

Chapter 2 provides a brief background of the nature of acoustic communication in the real ocean environment, and a description of the particular communications problem this
project attempts to solve, including some varied approaches to the design of hardware/software that can be implemented as an "optimum" compromise among conflicting theoretical principles and practical limitations.

Chapter 3 describes qualitatively the physical implementation of the entire acoustic communication system, broken down by functional subsystems, and traces the path of data from collection to eventual use by the operator. These functional blocks consist of: data collection sources; the encoding scheme employed; modulation techniques; the array steering electronics; power amplification; the sonar transmitting array; receiving hydrophone; demodulation and decoding; and the steering information feedback link.

Chapter 4 begins the main purpose of this work, relating the desired performance specifications and the mechanical and electrical designs conceived to fulfill those specifications. Part of every design process seeks to optimize the trade-offs that are necessarily involved when time, budget, technical complexity, and more mundane considerations such as purchasing lead times must be balanced to produce a viable product within these and other constraints; some of the effects introduced by this process into the design of the transmitting array are indicated when significant. Power consumption of the system as a whole happened to be one of the most vitally important factors influencing the prototype design without a doubt will continue to be trimmed as much as possible as the system design is refined in the future.
Chapter 5 contains a theoretical analysis of the expected performance of both a single transducer and the entire array. The transducer element is modelled with an equivalent electrical network to provide information about its frequency response, efficiency, and impedance characteristics, and then as a simple piston to calculate its free space beam pattern and directivity index. Linear array theory is employed to extend the beam pattern results and to determine the array gain. The power amplifiers can be described very well with "ideal" models, but differences between their assumed operating characteristics and true performance are enough (particularly from amplifier to amplifier in terms of absolute gain/frequency response) to warrant noting, especially if any subtle array shading is contemplated for the future.

Chapter 6 embodies the actual experimental results obtained from trials both in the laboratory and in field trials, conducted off the docks at the Woods Hole Oceanographic Institution (WHOI). Experiments were performed to obtain frequency response and overall efficiency data for the array, as well as directional characteristics in the x-z and y-z planes at three frequencies (30, 50, and 60 kHz). The test conditions, as described later in some detail, were far from perfect, and the data taken is really of a semi-quantitative nature. More trials under more carefully controlled conditions will have to be made before any final evaluation of the system performance can be made. The power amplifiers were all laboratory tested to verify correct operation, but individual data is not really relevant and so
not included here. A typical response curve is shown, however.
CHAPTER 2

Background

2.0 Why Acoustic Communication?

Before addressing the specific problems inherent in transmitting a coherent acoustic signal through the ocean, we should note that acoustic transmission of information is really the only viable medium available with current technology, and will continue to be so for any sort of long range (more than a few tens of meters) communication under the water's surface (excluding tethered communication, accomplished via a physical link between transmitter and receiver). Electromagnetic radiation, whether in the frequency bands of radio, microwave, or visible light, all suffer severe attenuation of the signal through absorption in the water, and so is an impractical medium for underwater communication.

2.0.1 Some Acoustic Characteristics of the Ocean

The ocean makes a very difficult medium for the coherent transmission of acoustic signals by virtue of its temporal and spatial inhomogeneity, the degree to which acoustic energy is absorbed in the water, and the noisy characteristics of the background through which signals must be propagated. The extent of the problems created by each of these qualities depends on the nature of the acoustic transmission desired, as outlined in the following section. Though specific values are dependent on
immediate ocean parameters such as depth and salinity, Figure 2.0.1 shows how acoustic energy is absorbed in the water as a function inversely proportional to frequency, expressed as the coefficient $\alpha$ in dB/km.\[^{2}\] Figure 2.0.2 relates the sources of additive background noise to their strength over a wide range of frequencies; while noise below 10 kHz has directional properties, in the frequency range of interest to our system (30-60 kHz) it can be assumed as isotropic in nature.\[^{3}\] Figure 2.0.3 illustrates how variations in the speed of sound in water with depth give rise to refraction and reflection, creating sound channels or acoustic waveguides, which can be exploited for long range acoustic communication. While reverberation is generally regarded as an obstacle to overcome in acoustic communication, theoretically the multipath structure can be controlled if very tight tolerances are kept on transmission parameters such as beamwidth, directional of propagation, and so forth. The phenomena of absorption, along with losses in signal level due to spreading (using a spherical model ($\propto 1/r^2$) for short ranges and a cylindrical model ($\propto 1/r$) for long distances) limits the effective range of communication for any given source power output. A noisy background similarly limits range by degrading the signal to noise ratio at the receiver, and can also affect coherence with some modulation schemes, especially with amplitude modulation. The multipath structure of acoustic transmission channels in the ocean, formed by the phenomena of refraction and reflection, causes time spreading of the original signal at the receiver and is generally regard-
Absorption of Acoustic Energy in the Ocean

Figure 2.0.1 Absorption of Acoustic Energy in the Ocean
Figure 2.0.2  Background Noise in the Ocean
Ray diagram for typical Atlantic Ocean sound channel, depicting channeled rays and refracted-surface-reflected (RSR) rays; sound speed profile is at the right. The angles are grazing angles at the axis of the sound channel. (Ewing and Worzel, 1948.)

Figure 2.0.3  Reverberation from Reflected and Refracted Rays
ed as the most difficult obstacle to overcome in underwater acoustic communication, as it severely limits the data rates that can be maintained with coherent reception of data. Another form of distortion, frequency spreading, is produced by the Doppler shift incurred when the transmitter and receiver are in motion relative to one another. This problem is well understood, however, and methods exist to correct for any Doppler shift that may be present in the received signal.

2.0.2 Modes of Acoustic Communication

Geometrically, acoustic communication can be broken down into four separate pathways through the ocean. It can be long range or short range, and may be propagated in either the vertical or horizontal direction. Figure 2.0.4 shows a table outlining the major uses of each mode of data transmission.

The data rates required of any communication system, and its operating environment—particularly whether in shallow or deep water—also have much to do with the determination of the hardware/software selected to implement any given mode of communication. In general, in deep water horizontal communication is thought to be reverberation limited (from refraction and reflection) and vertical transmission to be noise limited. In shallow water, horizontal reception is still reverberation limited, but vertically, surface and bottom reflections can surpass noise as the factor limiting the performance of the communication system. Higher data rates call for increased
APPLICATIONS OF ACOUSTIC TELEMETRY

PATH DIRECTION

<table>
<thead>
<tr>
<th>RANGE</th>
<th>VERTICAL</th>
<th>HORIZONTAL</th>
</tr>
</thead>
</table>
| SHORT (SHALLOW) (HIGH BANDWIDTH) | 1) INSPECTION OF OFFSHORE PLATFORMS  
2) SUBMERSIBLES DEPLOYED FROM TENDERS      | 1) COMMAND/CONTROL FROM WELLHEADS  
2) PIPELINE INSPECTION                     |
| LONG (DEEP) (LOW BANDWIDTH)   | 1) DEEP SEA MINING AND RESOURCE RECOVERY  
2) OCEANOGRAPHIC SENSORS  
3) DEEP SUBMERSIBLES                  | 1) COMMAND/CONTROL COMMUNICATION AMONG NAVAL SUBMARINES |

Figure 2.0.4 Acoustic Transmission Modes
bandwidth in the transmitted signal, which dictates decreased range of more power output capability because of the frequency dependent absorption loss.

2.0.3 Present Day Needs for Underwater Communication

Because of the ever expanding role of the oceans in supporting our increasingly resource-scarce and technologically-based society, new underwater communication systems must be developed as an integral part of other new underwater technologies, such as oil and gas exploration, manganese nodule mining, OTEC development, and large scale aquaculture.[4] In the past most attention has been paid to long range, relatively low data rate communication, particularly by the U.S. Navy. In the future, however, the need for shorter range but much higher data rate communication will become ever more apparent, as remotely controlled vehicles and instrumentation platforms continue to explore the three-quarters of the earth's surface covered by water. This is especially true when video information is necessary to carry out some experimental task, which requires the highest data rate of any form of communication today. Currently available systems do not provide this capability, thus providing the motivation for the research being conducted jointly between M.I.T. and W.H.O.I., which should soon culminate in a prototype of a new generation of underwater acoustic communication systems.
2.1 Implementation of Acoustic Communication Systems

As with most engineering problems, underwater communication systems can be realized in a variety of ways, depending on the nature of the problem, and the resources available for its solution, as well as considerations involving the desired technical complexity and reliability of the system. This section discusses the individual goals desired to be achieved by the system currently under development, and the different approaches available to reach those goals.

2.1.1 Environmental and Operational Characteristics

Our system goals require the transmission of information over relatively short path lengths (< 1 km), oriented in a vertical or near vertical direction. Information to be transmitted will be digitized and sent at data rates exceeding 1 kbit/sec, possibly up to 10 kbit/sec. Since the system is designed for tether-free operation, it must depend on batteries as a power source, and so minimal power consumption becomes a necessity for long term operation. As seen in Figure 2.1.1, there will be two modes for transferring data from the source to a user: in shallow water, the sending array transmits directly to a receiving hydrophone mounted or suspended from the mother ship; in deep water, the free swimming transmitter sends to a receiving hydrophone suspended via a hardwire link from the vessel, where the information is preamplified and repeated up to the surface. This secondary link will be accomplished
Figure 2.1.1
System Operating Modes

DIRECT ACOUSTIC LINK

SURFACE RECEIVER

ACOUSTIC LINK

DATA SOURCE

~1 km

RECEIVING STATION CABLE LINK

SURFACE RECEIVER

RECEIVING/REPEATER STATION

ACOUSTIC LINK

DATA SOURCE

~4 km

~1 km
utilizing newly developed fiber optic cable technology, which is lighter (thus easier to handle and requires less space) and has a wider bandwidth than standard coaxial cable. These general requirements must now be translated into specific methods and equipment for testing in the real ocean environment.

2.1.2 Methods of Information Transmission

In implementing the characteristics described above, three subsystems must be selected from a variety of options: data encoding; modulation; and actual transmission hardware. Encoding the data from the source provides a method of improving the reliability of the data when received. Among the many types of algorithms developed in information theory, two types of codes have been researched extensively—block codes and convolutional codes.\[5\] Block codes operate on successive segments of the data, forming linear combinations of the information bits, while convolutional codes operate continuously on the data stream, also forming linear combinations of the bits. Either method can be performed fairly easily at the source, but decoding tends to require significantly more computational power for convolutional codes, and especially for long block codes, where the capacity of the decoder increases as \(2^k\), where \(k\) is the number of bits in a block. Fortunately, the receiver is generally at the surface where space and power are not at a premium. Since processors have already been built and proven for high-speed satellite communication, the modest rates required for the under-
water acoustic transmission system should pose no problem in the implementation of specific hardware for data encoding.

Modulation methods used in underwater communication have been principally restricted to amplitude and frequency schemes, though recent study has been directed toward phase, differential phase, and pulse code modulation techniques. The particular system employed is to some degree environmentally dependent; for instance, amplitude modulation works best when reverberation is small. In deep water, horizontal communication is limited by reverberation, vertical paths are felt to be limited by additive noise, due mainly to the stratification of the ocean into sound channels. In shallow water, reverberation may dominate both modes of transmission, with refraction continuing to plague horizontal paths, and additive noise in the vertical direction becoming overshadowed by surface/bottom reflections of the transmitted signal.

Transmitters and receivers (interchangeable when used in their linear regions so reciprocity holds) typically consist of a single large transducer, or hydrophone, or an array of smaller hydrophones. There are two parameters that concern an untethered communications system - power requirements and the directivity index. If the system is to operate for an extended period of time, then the required transmitter power can dominate all other subsystems and thus limit the life of the entire system.

The directivity index of a transmitter/receiver indicates its ability to focus acoustic energy in a preferred direction. Significant gains in the signal/noise ratio, or conversely reductions in power (from 10 - 30 dB in either case) may be gained by introducing directionality into the acoustic link. However, problems can result if the directivity index is pushed too high in a system where relative motion exists between the transmitter and receiver.

Pointing errors may occur if the transmitter is extremely directional, so that
the receiver cannot detect its presence when oriented just a few degrees off line-of-sight, thus no link can be established between the source and the target—somewhat akin to the case of searching for a small object in the dark with a narrow beam flashlight. An improvement in performance with directional transmitters and receivers can be made through a technique called adaptive beamforming, where the width and/or direction of the beam is controlled dynamically by the communication system, responding to temporal and spatial changes in the ocean medium.

2.2 Selected System Characteristics

From the list of possible encoding/modulation/transmission schemes available with current technology, the following have been chosen as the most flexible for use in an untethered, short range, vertical path directed underwater communication system that will transmit high data rates in shallow or deep water.

One of many block encoding algorithms was selected to provide reliable data communication through redundancy. Figures 2.2.1 and 2.2.2 show the encoding and decoding logic for a 4 bit Hanning code with a parity check that encodes any 4 bit block (16 possible words) into an 8 bit code, providing a 16-fold redundancy in the data. Given the detection of just one error upon reception, the processor can change the faulty bit to ensure a complete message transmission. If two errors are present, the decoder corrects neither but shows the presence of an error in the message. More than two errors go undetected.
<table>
<thead>
<tr>
<th>SYMBOL</th>
<th>BIT STREAM</th>
<th>ENCODING PATTERN</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0 0 0 0</td>
<td>+ + + + + + +</td>
</tr>
<tr>
<td>1</td>
<td>0 0 0 1</td>
<td>+ + + - - - -</td>
</tr>
<tr>
<td>2</td>
<td>0 0 1 0</td>
<td>+ + - + - + -</td>
</tr>
<tr>
<td>3</td>
<td>0 0 1 1</td>
<td>+ - - + + - -</td>
</tr>
<tr>
<td>4</td>
<td>0 1 0 0</td>
<td>+ - + + - + -</td>
</tr>
<tr>
<td>5</td>
<td>0 1 0 1</td>
<td>+ - - - + - + -</td>
</tr>
<tr>
<td>6</td>
<td>0 1 1 0</td>
<td>+ - - + + - + -</td>
</tr>
<tr>
<td>7</td>
<td>0 1 1 1</td>
<td>+ - - - - + + +</td>
</tr>
<tr>
<td>8</td>
<td>1 0 0 0</td>
<td>- + + + + - -</td>
</tr>
<tr>
<td>9</td>
<td>1 0 0 1</td>
<td>- + + - - + +</td>
</tr>
<tr>
<td>A</td>
<td>1 0 1 0</td>
<td>- + - + - + +</td>
</tr>
<tr>
<td>B</td>
<td>1 0 1 1</td>
<td>- + - - + + +</td>
</tr>
<tr>
<td>C</td>
<td>1 1 0 0</td>
<td>- - + + - + +</td>
</tr>
<tr>
<td>D</td>
<td>1 1 0 1</td>
<td>- - + + + - +</td>
</tr>
<tr>
<td>E</td>
<td>1 1 1 0</td>
<td>- - - + + + +</td>
</tr>
<tr>
<td>F</td>
<td>1 1 1 1</td>
<td>- - - - - - -</td>
</tr>
</tbody>
</table>

Figure 2.2.1  Data Encoding
DECODING LOGIC

1) FREQUENCY SYNTHESIZER OUTPUTS

\[ R_L = \begin{cases} 
1 & \text{IF - FREQUENCY DETECTED} \\
0 & \text{IF + FREQUENCY DETECTED} 
\end{cases} \]

2) COMPUTE DECODING SYNDROME

\[ \lambda_1 = R_2 \oplus R_3 \oplus R_4 \oplus R_5 \]
\[ \lambda_2 = R_1 \oplus R_3 \oplus R_4 \oplus R_6 \]
\[ \lambda_3 = R_1 \oplus R_2 \oplus R_4 \oplus R_7 \]
\[ \lambda_4 = R_1 \oplus R_2 \oplus R_3 \oplus R_8 \]

3) DECODING LOGIC

A) IF \( \lambda_1 \lambda_2 \lambda_3 \lambda_4 \equiv \lambda = 0 \) 0 ERRORS

B) IF \( \lambda \) HAS ODD PARITY 1 ERROR

CORRECT AS FOLLOWS

\( \lambda = 7_8 \) CHANGE SYMBOL 1
\( \lambda = 13_8 \) CHANGE SYMBOL 2
\( \lambda = 11_8 \) CHANGE SYMBOL 3
\( \lambda = 14_8 \) CHANGE SYMBOL 4

C) IF \( \lambda \) HAS EVEN PARITY 2 OR MORE ERRORS

Figure 2.2.2 Decoding Logic in the Receiver
The modulation system to be employed is a hopped, frequency shift keyed (FSK) algorithm. It was selected on the basis of its tremendous flexibility, its performance under high multipath conditions, and relative ease of implementation. The simplest FSK system assigns two frequencies—say \( f_1 \) and \( f_2 \)—to correspond to a transmitted "1" or "0". The two frequencies are usually spaced a given distance apart in the frequency spectrum, centered on the "bit band" frequency. This frequency may be changed periodically in time or "hopped" around the transmittable spectrum, allowing any reverberation in previously used frequency bands to die away before returning to transmit there once again. The FSK algorithm allows the transmitter to independently vary the data rate, number of hopping frequencies, bit sub-bandwidths (width of a "1" or "0"), etc.; conceivably even in real time, to adapt transmission to the instantaneous environment experienced by the system. This type of modulation should be very effective against surface and bottom reflections in shallow water applications. Of course, for a given number of hopping frequencies, the higher the data rate, the less time there remains before one is forced to return to a previously used frequency.

The transmission system chosen utilizes a combination of a highly directional transmitting array, an omnidirectional receiving hydrophone, and adaptive beamforming to attempt to maximize the efficient use of valuable battery power while avoiding as much multipath distortion as possible. Chapters 4 through 6 discuss this subsystem in great detail as indicated in the Introduction.
Figure 2.2.3  Modulation Performance for Two Operating Modes
Figure 2.2.3 illustrates the expected performance of the entire communications system as just described, in terms of two significant parameters: channel capacity versus range, which shows the maximum attainable data rates via an acoustic channel; and error probability versus range, which is a common measure of the performance of digital information transmission systems.\[8\] The curve shows that data rates in excess of 1 kbit/sec are achievable with an error probability per bit of less than $10^{-4}$, or .01 percent.
CHAPTER 3

System Overview

3.0 Data Collection Sources

The possible sources of data for use in the underwater telemetry system are limited only by the maximum data rate that the system will establish[9] Analog sources are run through an A/D converter and transmitted in the same fashion as digital data. Examples of likely users of the system would be fixed oceanographic instrumentation platforms operating on the ocean floor and sending information on temperature, salinity, current, etc., remote or manned submersibles engaged in exploratory or industrial activities, and monitoring units for undersea wellheads. Data rates required for these types of transmission can be grouped roughly as: 100 to 1000 bits/sec for command/control functions; 1 to 10 kbits/sec for ocean bottom instruments such as hydrophones and seismometers, and low frequency sonar acoustic data; and 100 kbit/frame to 10 Mbit/frame for video data, which would dictate realistic frame rates of between 1 frame/sec and 1 frame/10 minutes. In short, a wide range of opportunities exist for potential exploitation of this form of underwater acoustic communication.

3.1 Data Transmission

Figure 3.1.1 traces the path of information from the data
Figure 3.1.1 Block Diagram of Communication System Components
DISCLAIMER

Page has been omitted due to a pagination error by the author.

( Page 32 )
INPUT SHIFT REGISTER BUFFER

8085 MICROPROCESSOR ENCODER & FREQUENCY HOPPING CODE

FREQUENCY SYNTHESIZER

TONES FOR DOPPLER TRACKING AND SYNCHRONIZATION

F_D (60 KHZ)

F_S (30 KHZ)

DOPPLER CONTINUOUS @ 60 KHZ

SYNCHRONIZATION (AT BLOCK LENGTH)

100 µSEC BLOCK @ 30 KHZ

Figure 3.1.2 Encoding and Modulation Components
Eight 1 kHz sub-bands are available for hopping each 8 bit word as it is transmitted. Within each 1 kHz sub-band there exist eight 100 Hz "bit bands" centered on the 100 Hz multiples, and each bit of the encoded word is shifted ±50 Hz from the center of the 100 Hz "bit band", corresponding to its value of "1" or "0" in binary code. As each word is transmitted, a different sub-band is used until eight words later when the original band starts the sequence over again. In this fashion, problems with multipath distortion are reduced by allowing the secondary and later returns to die out before that portion of the frequency spectrum carries "real" data again. The summed signal then enters the next stage of processing where delays are introduced for steering purposes and is finally amplified for transmission by the planar array.

3.1.2 Phased Array Steering and Amplification

Figure 3.1.3 shows a block diagram of the major components in the transmitter phased array steering system. After the received data has been processed, a signal indicating any pointing error is relayed from the ship or operator's station back down to the transmitter. This signal broadcasts at 9.5 ±.25 kHz for North/South steering and at 10.5 ±.25 kHz for East/West steering. The receiving hydrophone passes this input through four narrowband filters, whose output is converted with an A/D to a "steer" or "not steer" command for each of the four directions. Simple logic circuitry translates the commands and controls two voltage-controlled oscillators (VCO's) whose output
FILTERS & PHASE LOCKED LOOPS FOR STEERING

CONTROL LOGIC

COMPUTE STEERING DELAYS

\[ \tau_L \quad \cdots \cdots \quad \tau_{32} \]

FROM FREQUENCY SYNTHESIZER

PHASE SHIFT

STEERING AT 9.5 KHZ \( \pm \) 0.250 KHZ N/S
10.5 KHZ \( \pm \) 0.250 KHZ E/W

PHASED ARRAY OF HYDROPHONES:

4\(\lambda\) x 8\(\lambda\) ARRAY
8° x 16° RESOLUTION @ 50 KHZ

POWER AMP\(_1\)

POWER AMP\(_{32}\)

RECEIVER HYDROPHONE

Figure 3.1.3 Transmitter Phased Array Steering System
controls the phase shift electronics that utilize tapped analog delay (TAD) components that sum orthogonal delay times into a 32 element matrix using only two basic delays and the proper logic control circuitry. This is possible because of the linearity of the array element spacing. Each successive delay along either the x or y-axis is thus a multiple of the initial delay time (element 1 to element 2 delay in the x or y direction). The steering information arriving at the receiver will be updated perhaps once per second, which should be fast enough to accommodate the most severe environmental conditions that can be imagined under reasonable operating conditions.

Figure 3.1.4 shows a typical set of these delays, arranged into a matrix corresponding to each element of the 4 x 8 array. Also shown is the equation governing the required delay per element for any given steering angle in one plane (x-z or y-z plane):[10] The system will steer from 0° to 45° in azimuth, at any polar angle in increments of 3° of azimuth, which ensures that the received signal should always be within 2 dB or better of the maximum possible strength. We could steer in increments of any fraction of a degree, but the steering logic required would become cumbersome in terms of space and power requirements.
Typical Delay Calculations

\[ U_m = \frac{2\pi m d \sin \phi}{\lambda} \]  
(DELAY FOR THE Mth ELEMENT IN RADIANS)

\[ \Delta \tau = \frac{U_m \cdot 1}{2\pi f \omega} = U_m \]  
(DELAY FOR THE Mth ELEMENT IN SECONDS)

<table>
<thead>
<tr>
<th>Y DELAY</th>
<th>(0)</th>
<th>2.33</th>
<th>4.66</th>
<th>6.99</th>
<th>9.32</th>
<th>11.65</th>
<th>13.98</th>
<th>16.31</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1.27</td>
<td>3.60</td>
<td>5.93</td>
<td>8.26</td>
<td>10.59</td>
<td>12.92</td>
<td>15.25</td>
<td>17.58</td>
</tr>
<tr>
<td></td>
<td>2.54</td>
<td>4.87</td>
<td>7.20</td>
<td>9.53</td>
<td>11.86</td>
<td>14.19</td>
<td>16.52</td>
<td>18.85</td>
</tr>
</tbody>
</table>

Figure 3.1.4 Typical Delay Calculations
3.2 Data Reception

3.2.1 Signal Demodulation

The received signal passes through the receiver hydrophone and is preamplified before entering the three sharply cut-off bandpass filters that recover the data, Doppler, and synchronization signals respectively, as shown in Figure 3.2.1. The synchronization waveform is processed through a delay locked loop (DLL) and then routed to the Fast Fourier Transform (FFT) electronics, where it attempts to recover each 8 bit decoded word from the correct 8 encoded bits originally sent by the transmitter. The 60 kHz Doppler tracking tone passes through a phase locked loop (PLL) and acts as a controller for the quadrature demodulator that recovers the original encoded bit stream from the spread spectrum modulated waveform, so that changes in frequency resulting from any relative velocity between the source and receiver will not affect the possibly crowded frequency encoding of the data. The quadrature demodulator preserves phase information in the signal by splitting it into a sin and cos component. The two channels of demodulated data are then sent to the decoder for final processing.

3.2.2 Decoding

The amount of computational power required for decoding far surpasses that needed for encoding; luckily most users will be located on the surface where space, weight, and power are
Figure 3.2.1  Receiver System Front End Block Diagram
relatively minor considerations as compared to the self-contained transmitter below the surface. Figure 3.2.2 shows a block diagram of the decoding hardware used to decipher the messages sent from whatever information source is being employed at the transmitter. First the two demodulated channels are fed through a dual A/D converter, and then each 8 bit block of output is multiplexed into a complex 16 bit word (word formatting). As one of the dual buffers on each channel feeds the HP 2100 FFT via a direct memory access (DMA) line, the other receives the next word and waits its turn to dump data into the 2100, which speeds up the decoding process. This process is controlled from the 2100 through the synchronization pulses received from the 30 kHz input. As an example, for 256 point sampling (just a convenient maximum block rate), the HP 2100 accepts words from the buffer between framing pulses; if no data is present is pads the sample with "0"s. The FFT converts the sample word into a spectrum symbol (see Figure 3.2.3) that is matched against the decoding logic in the array processor. Currently (remember that the various modulation parameters may be altered), the system operates the FFT unit on the received data blocks from the sin and cos channels, which produces a spectrum that should match the original frequency synthesizer's output--each word containing 8 bits with each bit shifted ±50 Hz around the 100 Hz multiple center frequencies contained within each 1 kHz hopping band.

The array processor then matches the data spectrum against each of the 16 (or 32, or 64, etc.) possible encoded words and selects the closes symbol pairing, using the parity check to
Figure 3.2.2 Decoding Hardware Block Diagram
Figure 3.2.3 Signal Flow in the Decoder
correct the data if just one bit cannot be matched correctly. If two errors are present, the condition is detected but not corrected, and three or more errors will pass unnoticed through the decoder. Other codes are available with more redundancy and/or more complexity that may be processed with a greater degree of sophistication, but they also require more computational power in both the encoder and decoder.

The decoded words are sent to the operator over whatever peripheral device is being employed, finishing the last link of communication between the information source and the data user.

3.3 Steering Command Feedback Link

The easiest method of accomplishing the adaptive feedback process is one of manual control, where the operator observes the received signal/noise ratio and directs the feedback transmitter logic with a joystick or similar control device until the S/N ratio is back within acceptable limits. While simply implemented, this requires the presence of a human operator, and if the received data were ever to be collected at a remote location and stored on tape, for instance, an alternative method of steering control would have to be developed.

Figure 3.3.1 illustrates a possible automatic adaptive feedback control loop. Essentially an entirely different signal loop is created, though the original information source/transmitter system may still be utilized for evaluating the accuracy of transmitter/receiver alignment. A small, separate four-element array tuned to the 60 kHz Doppler tracking signal
Figure 3.3.1 Steering System Feedback Loop
with a high value of Q for a good signal/noise ratio mounts in the same housing as the data receiving hydrophone. A phase comparator measures the phase differences between the four elements in the array; then the direction from which the transmitter is sending may be determined, and the feedback command signals may be appropriately broadcast to instruct the source array to steer its acoustic orientation until phase differences at the steering feedback array are minimized. As another, more simply implemented check, the magnitude of the received signal may be compared with the sample obtained from the last steering instruction, and the transmitting array is steered with a simple logic algorithm until the signal is again maximized. The command signals, as previously mentioned, are sent at four discrete frequencies around 10 kHz, each frequency when active denoting a "steer" command that will electronically turn the array in the proper N/S/E/W direction via signal delays introduced before each of the 32 transducers, effecting the focusing of acoustic energy in the desired direction.
CHAPTER 4

Power Amplification and Sonar Array Hardware

4.0 Design Performance Goals

The original conception of the underwater acoustic communication system specified certain parameters to be met in the implementation of system hardware. A few of these have already been introduced, such as the possible 10 kbit/sec (realistically more like 2-4 kbit/sec for the prototype system) data rate that hopefully will be achieved by the system as a whole; here follow the goals dictated for the specific hardware that comprises the last two year's research effort, and the subject of this thesis.

4.0.1 Power Amplifiers

Since a power amplifier merely functions as a "gain block" for any signal that is to be transmitted, ideally its characteristics could be specified by one number—the ratio of voltage output over voltage input. However, in the real world we must live with noise, distortion, and external interference, so additional quantities must be measured to assess the true effectiveness of the amplifier's ability to increase the level of a signal while preserving its fidelity. The communication system requires a minimum of 171 dB SPL (sound pressure level) in the water at the source. Given the transducer's broadband efficiency expectations (see Section 5.1), this translates to an output
power requirement from each amplifier of about 1 watt to each transducer; any power capability beyond that will contribute to an increased operating range. The impedance characteristics of the transducers (see Section 5.1) then require that a 50 V pk-pk output be available from the power amplifiers.

The 60 kHz Doppler tracking signal sets the upper requirement for the frequency response characteristics of the amplifiers. The desired response remains flat from DC to 60 kHz, and then attenuates at 12 dB per octave above that point to avoid wasting power in transmission of the unwanted harmonics (particularly third order harmonics) present in the synthesized data, Doppler, and synchronization signals. The first harmonic to be removed thus occurs at 90 kHz.

The gain through the power amplifiers totals 40 dB from 0 to 60 kHz, plus or minus 1/2 dB.

The phase response of the amplifiers is conservatively designed to be flat throughout the frequency transmission bandwidth, though a significant (but not particularly important) amount of phase shift is introduced by the electroacoustic drivers themselves.

The self-noise characteristics of the amplifiers will be so overshadowed by the additional of ambient background noise from the acoustic channel that they are irrelevant, especially considering the large-signal mode of operation. Harmonic distortion is also inherently low enough in the amplifier design as not to be worth trying to measure; the first odd harmonic lies beyond the transmission bandwidth anyway. Intermodulation
distortion can be a problem with the FSK modulation scheme that relies heavily on coherent transmission of information over closely spaced frequencies, but lacking the equipment to measure it, we must proceed on the assumption that it too will be of an insignificant magnitude in this application.

A summary of the required (measureable) power amplifier parameters is given in Figure 4.0.1.

4.0.2 Sonar Array

The performance of the sonar array, whose task consists of translating the electrical input signal into an acoustical analog, can be characterized by its efficiency, directional characteristics (beam patterns and directivity index), and frequency response. Since it is a passive electroacoustical converter, it introduces no noise of its own (other than thermal noise), though phenomena such as "ringing" can occur when driven with very high slewing signals.

Obviously an efficiency as close to 100% as possible is a desirable figure. Practical transducers have ultimate efficiencies of better than 90% when tuned to one operating frequency; however as bandwidth increases the efficiency must necessarily degrade proportionately. For our operating bandwidth a quality factor Q of 5 was desired; with this figure an efficiency at resonance of 15% is a good goal to work toward.

The FSK modulated data centers on 50 kHz for the prototype sonar array, which becomes the logical design resonant frequency for the electroacoustic transducers. The aforementioned Q fac-
### Amplifier Design Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>FREQUENCY RESPONSE ((V_{out}/V_{in}))</td>
<td>+40 dB ± 1/4 dB, 0 - 60 kHz</td>
</tr>
<tr>
<td>FILTERING</td>
<td>12 dB per OCTAVE CUT ABOVE 70 kHz</td>
</tr>
<tr>
<td>MAXIMUM OUTPUT VOLTAGE</td>
<td>60 VOLTS PEAK-PEAK</td>
</tr>
<tr>
<td>SIGNAL/NOISE RATIO</td>
<td>100 dB</td>
</tr>
</tbody>
</table>

**Figure 4.0.1 Amplifier Design Parameters**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>EFFICIENCY</td>
<td>.15</td>
</tr>
<tr>
<td>QUALITY FACTOR (Q)</td>
<td>5</td>
</tr>
<tr>
<td>3 dB BEAM WIDTH</td>
<td>10°</td>
</tr>
<tr>
<td>MAXIMUM POWER OUTPUT</td>
<td>1.0 (ACOUSTIC WATTS)</td>
</tr>
</tbody>
</table>

**Figure 4.0.2 Sonar Array Design Parameters**
tor relates the half-power or 3 dB down frequencies on each side of a resonant frequency to that resonant frequency, thus determining the general shape of the frequency response. Empirically it is expressed as

\[ Q = \frac{f_r}{(f_u - f_l)} = \frac{f_r}{W} = 5 \quad (4.0.1) \]

where

- \( f_r \) is the resonant frequency (50 kHz),
- \( f_{u,l} \) are the upper and lower 3 dB down frequencies,
- \( W \) is the signal bandwidth (10 kHz).

The last characteristic of the sonar array needing specification is the beam pattern, or directional characteristic of the array. The beam pattern of a line array expresses in two dimensions the output at azimuth angles off-axis relative to the on-axis signal level, usually normalized at 0 dB. The beam pattern is influenced by three independent parameters: the individual element's own beam pattern (the line array is made up of \( n \) elements that may or may not have directional radiation characteristics of their own), which in turn is dependent mainly on it radiating face dimensions compared to the wavelength of the transmitted signal; the spacing between subsequent transducers in the line, which increases directionality when made larger, but introduces other problems if the spacing is made too great; and the number of elements (thus the length) of the line. The longer the line, the narrower the main lobe of the beam pattern. A plane array may be
considered as a line array of transducers which are in fact line arrays themselves (imagine a transducer with an individual beam pattern exactly equivalent to an \( n \) element array of other transducers). Thus a plane array composed of \( m \) line arrays each \( n \) elements long will have a qualitative beam pattern that is directional along the \( n \)-element line array axis proportional to the number of elements \( n \), and directional along the \( m \) array/element axis proportional to the number of arrays/elements \( m \).

The exact beam pattern is the product of the beam patterns resulting from each \((n \text{ length, } m \text{ length})\) line array, so long as all of the elements are linearly spaced (and equally spaced consistently within the \( n \) or \( m \) axes). In fact, a "cross" shaped array of exactly two single line arrays oriented orthogonally to each other has nearly and identical pattern to one whose elements form the same length(s) square or rectangular matrix. The additional elements in a full plane array provide greater control over side lobes and a higher value of array gain, or equivalently for the linear array in isotropic noise, direction index. As a compromise between the ideal of very high efficiency and the possibility of suffering from pointing errors that would be hard to control, an original figure of \( 10^0 \) was proposed as the \( 3 \text{ dB} \) down points of the main lobe. This number subsequently changed as a consequence of some engineering realities, discussed later in this chapter.

Concurrent with the goals of high efficiency and appropriate beamwidth is the objective of side lobe control, which involves minimizing the radiation of acoustic energy in directions other
than the main lobe. This can be accomplished through a variety of techniques, the most common of which is the process of shading the amplitudes of the transducer driving voltages, so that the output of each transducer can be carefully controlled to obtain the desired side lobe levels. The main lobe width and side lobe levels are not completely independent, so that consideration must be given to the desired "optimum" compromise between the two parameters. A number of amplitude shading algorithms have been developed for arrays of any given number of elements, and currently this communication system incorporates an equal, or rectangular shading—that is to say, all of the elements are driven equally. If a different scheme is ever desired, it is only a matter of adjusting the individual power amplifier gains to implement whichever algorithm might be adopted.

Figure 4.0.2 (page 49) summarizes the design parameters chosen for the sonar array.

4.1 Power Amplifier Design and Construction

The power amplifier design was undertaken with the objective of satisfying all of the previously listed performance requirements, while minimizing cost, space, weight, and total power consumption. The choice of operational amplifiers as building blocks in the circuitry was predicated on their ease of application, reliability, and adaptability to changes in circuit function.

Figure 4.1.1 shows a schematic drawing of the present
design. The first of the two 20 dB stages of gain also acts as a lowpass filter with a 3 dB cutoff frequency of 70 kHz. It currently utilizes a high-speed general purpose op-amp, the LM 318, which is also balanced for nulling DC offset voltage. The second stage is one of pure gain, also 20 dB, and uses the high voltage LM 344 op-amp which can deliver output voltage swings of up to 60 V pk-pk at up to 20 mA of current. Like the first stage it is balanced for offset nulling, and has provisions for isolation from the fairly high capacitance seen at the transducer inputs. Overall the circuit maintains the virtues of simplicity and minimal "tweaking" required for the best operation obtainable.

The power amplifiers have been built on specially designed double-sided printed circuit (PC) boards, four channels to a board. The PC boards minimize stray capacitance effects and increase the degree of uniform performance between amplifier channels. Figure 4.1.2 shows a photograph of one of the completed boards.

Two distinct areas for improvement should be noted for future generations of this circuit if it is to be used past the prototype stage. First, the components in the filter section of the initial amplification stage should be replaced with new ones having 1% tolerances. This would assure better uniformity between channels, and thus lead to a tighter degree of control on the beam pattern produced by the sonar array. It might also be wise to replace the source and feedback resistors in the power (2nd) stage as well, especially if nonuniform amplitude
Figure 4.1.2 Photograph of One Four-Channel Amplifier Circuit Board
shading is to be implemented at some future time. The cost of this and the next suggestion would not be trivial, however, as there are 32 separate amplifier channels to consider. Next, the total quiescent power consumption (which accounts for at least 50% of total absolute power consumption, even at the full 60 V pk-pk output) could be reduced by approximately 25% by the replacement of the LM 318, which has a typical 5 mA supply current rating, with a low supply current device such as the LF 355, which normally draws about 2.5 mA from the supply. The LF 355 also happens to be pin-for-pin compatible with the LM 318, so no circuit changes would be necessary. Finally, in the second stage the balancing potentiometer (20K) could probably be replaced with a 250k pot while still retaining sufficient offset nulling capability; this would reduce the total quiescent supply current requirement by an additional 20% over the current circuit load.

4.2 Sonar Array Design and Construction

The overall design of the array required to fulfill the performance goals stated earlier can be divided into three major design tasks: an appropriate transducer must be generated for use in the array, one that individually possesses the efficiency and frequency response characteristics desired for the array as a whole; the geometric design of the array (transducer placement) must be calculated with regard to interelement spacing and the size of the matrix; and finally a suitable housing must be constructed that will allow the array to function without degrading the ideal performance of the separate elements.
4.2.1 Transducer Characteristics

Since the operational characteristics of the sonar array as a whole are dependent on the performance of the individual transducers, they were the first components to be designed. Specifications for the drivers were drawn up consistent with the overall goals of the transmitter system, and the actual construction of the transducers was contracted out to the International Transducer Corporation, located in Goleta, California. Initially the array design called for an interelement spacing of one-half wavelength at resonance (50 kHz), as the one-half wavelength spacing would have guaranteed that no repeated major lobes could occur between \(-\pi/2 < \phi < \pi/2\), and provides a reasonable degree of side lobe control as well. The required physical design of the transducers precluded the use of this spacing, as to achieve the necessary efficiency and frequency response they had to claim a diameter of 2.2 cm, larger than the 1.5 cm wavelength at 50 kHz. As discussed in Section 4.2.2, the final spacing of 2.41 cm was the minimum that could be obtained while still preventing contact between the transducers.

The transducer design is shown in Figure 4.2.1. It comprises a relatively standard longitudinal vibrator design, with piezoelectric ceramic drivers mass-loaded by quarter-wavelength resonators. In this case the head mass located at the top of the transducer is composed of aluminum, and forms the radiating face of the driver. The taper attempts to negate some of the directionality inherent in a piston type sound source whose ka
Figure 4.2.1  Sonar Transducer Element Design
value, where \( k \) is the wave number and \( a \) is the diameter, is much larger than that for an omnidirectionally radiating element (see Section 5.1). While the ultimate objective of the array is to focus the radiated acoustic energy in a very narrow beam, for steering purposes it is desirable that the individual transducers have as close to uniform hemispherical response as possible. Two lead zirconate titanate (PZT) ceramic rings make up the active part of the transducer; they are wired in a center driven mode which places the acoustic center of the transducer at the joint between the cylindrical pieces of ceramic. The heavy tail mass located beneath the PZT rings is mild steel. The entire assembly is cemented together and then precompressed by a stress bolt located through the center of the stack. Piezoelectric ceramic is a very brittle substance that has the property of having a much higher yield stress in compression than in tension, thus the stress bolt ensures that the ceramic will never be subjected to a high state of tension even when being driven with the maximum 60 V pk-pk input. The two small holes shown in the top view allow the oil with which the array is filled to equalize the hydrostatic pressure encountered at depth around the transducer, giving the driver a depth capability limited only by the strength in compression of the steel, aluminum, and ceramic materials of the device; better than 5 km, and easily sufficient for any use contemplated with this system.

In general this type of transducer is modelled fairly effectively with a simple piston analogy. From this model it is apparent that efficiency increases as the radiating area
of the piston at any given frequency, but at the same time the radiation pattern becomes more directional. Thus a compromise must be struck between efficiency and directionality, subject to the importance with which each of these quantities is weighted in the design analysis. Minimum power consumption was the determining factor in this design, necessitating a change in the interelement spacing as indicated previously. As detailed in Chapter 5, the increased directionality of the transducer did in fact remove any potential hazards from the existence of repeated main lobes at $\phi = \pm \pi/2$ as a result of increased inter-element spacing.

The following sections develop the geometric arrangement of the transducers in the array, the design of the array housing, and the incorporation of the array into the entire communications system.

4.2.2 Array Configuration

The initial beam pattern requirements from the 4 x 8 element array were known from well established theory to be 7.5° in the x-z plane by 15.0° in the y-z plane at the resonant frequency of 50 kHz. The 4 x 8 element pattern was selected as the most efficient use of the number of transducers available for use in the array.

Figure 4.2.2 illustrates the geometry of the transmitting elements. The interelement spacing chosen is $d/\lambda = .80$, where $d$ equals the interelement spacing and $\lambda$ is the wavelength of
Figure 4.2.2 Array Geometry
the transmitted signal at resonance. The beam pattern produces an acoustic energy field focused with its narrow cross-section in the x-z plane, and the wider cross-section located in the y-z plane.

4.2.3 Housing Design

Once the proper transducers were procured, and a suitable array geometry established, the task followed of providing a housing that allows—as nearly as possible—ideal realization of the benefits of an array versus a single transducer. Other considerations include maximum depth of operation, life cycle in the water, and the electrical/mechanical connections to the rest of the communications system.

The first stage of design concerned the mounting of the actual array itself, pictured in Figure 4.2.3. The transducers were mounted with noncompliant epoxy under pressure, forming a uniform thin joint to a thin plate of mild steel, with holes provided to channel the wiring down into the lower part of the housing.[13] This plate serves the purpose of decoupling the transducers from one another so that energy from one driver will not be absorbed by others, which would lead to a nonuniform velocity/phase distribution across the face of the array, and a concomitant loss of efficiency, as well as degradation of the beam pattern. The plate is then mounted to the backplane of the upper array housing at a distance of one-quarter wavelength of resonant frequency (0.75 cm). Theoretically any back
Figure 4.2.3  Upper Array Housing
radiation from the transducers is cancelled by the reflected wave, preventing phase incoherence in the radiated signal.

The dimensions of the upper array housing were based on the necessity of a clear path (with no reflections from the inside of the housing) for the acoustic beam at any degree of steering (up to 45° in azimuth at any polar angle). It is constructed of .95 cm thick 6061 aluminum alloy plate. No corrosion protection processes were employed in the exterior finish because its operation as an experimental prototype will not require extended lengths of immersion time in the ocean. Holes drilled in the backplane provide access to the lower array housing, and those located at each end serve as inlet and outlet for oil during the filling process. The top cover is fitted with an acoustic window composed of polyurethane, whose acoustic impedance closely matches that of water, facilitating the transfer of acoustic energy from inside the array out into the ocean with a minimum loss of efficiency. If alternative arrangements for the enclosure of the communication system electronics are ever made, the upper array housing may still be used without modification, except that new pressure-tight connections would have to be made to the new communicator housing: this provided the impetus for the separation of the array housing into upper and lower components; only the lower array housing need by changed to adapt the system.

The upper housing bolts directly to the lower housing, whose purpose is to provide a pressure-resistant electrical connection between the (atmospheric) pressure case containing
the power amplifiers and processing circuitry and the sonar array which must be exposed to ambient pressures. Watertight integrity is maintained using a seal of silicon sealant/adhesive at the upper/lower joint and around the top cover as well. The lower array housing, illustrated in Figure 4.2.4, was machined from a solid piece of 6061 aluminum alloy. It measures 22.86 cm in outside diameter, with outside wall 1.27 cm thick and an interior bulkhead of 3.81 cm. The connections made through the bulkhead must withstand at least 16.1 MN/m² (2000 psi) of pressure without leaking, while providing routing for 32 channels of information (one for each transducer). After abandoning the idea of buying a commercial one-piece 32 contact connector because of its expense and long lead times in ordering, a single contact miniature commercial "mini-Mecca" connector was found that require 32 separate O-ring seals, but leave the option of replacing any that might fail during operation. The housing itself is used for a ground path between the transducers and power amplifiers, cutting in half the required number of pressure-resistant connections.

Co-axial cable was used for wiring between the connectors and the transducer elements, with all the shields tied together and then to the housing at the lower end.

The bottom of the lower array housing fits with a .01 cm clearance into the end of the electronics pressure case, and seals with the aid of both a face-mounted O-ring and an O-ring gland seal fitted with double back-up rings. As hydrostatic pressure increases the pressure case locks down on both O-ring
Figure 4.2.4  Lower Array Housing
seals, promoting a high level of confidence in watertight integrity.

Each bulkhead connector terminates with a short length of 20 gauge hook-up wire ending at a miniature phono plug that mates with female connectors mounted in the last bulkhead of the electronics card cage, providing both a relatively easy disconnection from the communication system electronics and a method of selectively disconnecting any number of individual transducer channels, which allows any array configuration desired within the 4 x 8 element limits. Figures 4.2.5 and 4.2.6 show photographs of the completed sonar array, in unassembled and assembled views, respectively.

4.3 Pressure Case Design

Two very similar pressure cases make up the final components of the underwater acoustic communications system. One serves to hold all the batteries that make up the power source for the system, and the other contains all of the processing electronics and power amplifiers. Connections between the two are made through two 12-contact commercial watertight bulkhead connectors, located through the lower end cap of each pressure case. The transmitting array forms the upper end cap of the electronics housing, and the two cylinders together form an entirely self-contained communication transmitting system, with the exception of the data input which must be provided externally. Figure 4.3.1 illustrates the major details of the pres-
Figure 4.2.5 Photographs of the Array Housing Interior
Figure 4.2.5  Photographs of the Array Housing Assembly
Figure 4.3.1 Pressure Case Design
sure case design. The body of the case is constructed from extruded aluminum tubing (7075 alloy) which has a yield stress about twice that of the more common 6061 alloy. It has an outside diameter of 17.78 cm, a wall thickness of 1.27 cm, and an overall length of 109.22 cm. The two rings mounted at each end of the provide a method of securing the end caps, and were heat shrunk on the main body with an interference of .01 cm, avoiding any compromise in the strength of the case itself. The other three end caps are identical with the exception of the power supply line connectors mounted through two of them, and share the same dual O-ring seals that are provided for the array housing connection. They were machined from 2.54 cm thick 7075 alloy plate, and have an overall diameter of 22.86 cm.

The entire communications system was designed for an initial working depth of 1600 m (2000 psi), although various components in the system would be suitable for very deep operation if the remaining parts were redesigned for the increased pressure.

4.4 Design/Experimentation Constraints

A few words are in order on the subject of the limitations imposed on the design and implementation/experimentation of the communication system. The main obstacles that had to be overcome were time and budget. Shortages of certain parts and long lead ordering times have resulted in at least a three to six month overall delay during the past two years. The transducers, which were promised for a 14 week delivery took 24 weeks. Originally a 6 x 6 square array had been envisioned, but the cost
of the extra four transducers (at $100 each) was prohibitive. Besides the change in the geometry of the sonar array, the other main design concession to budget was the decision not to purchase commercially available connectors that could have extended the range of operating depths to beyond 5 km.

While the above constraints were a continual source of frustration during the design and construction phase of the project, they did not alter the quality of the product significantly. Work was completed far behind schedule, and the scope of application had to be narrowed, but basically the system that eventually took shape met all the desired design criteria, as far as its physical composition is considered.

The one problem that could not be solved in time to affect this thesis concerned the lack of suitable equipment and facilities to accurately measure the parameters that are theoretically predicted in Chapter 5. We did not have access to an accurately calibrated hydrophone with enough bandwidth to cover the necessary spectrum of transmitted acoustic energy, so the validity of all of the data involving absolute sound pressure levels or efficiencies, etc., must be called into question.

Experimental trials were first run at W.H.O.I., where the presence of wind and waves forced the abandonment of the real environment as a test facility as it became obvious that a perfectly stable platform and perfectly calm water were necessary to make any kind of quantitative measurements on the array. The M.I.T. swimming pool offered the last two attributes, but also suffered from two problems of its own. Due to a squeeze
between a heavily scheduled summer swimming program, and an imminent closing of the pool for several weeks for maintenance, only twelve hours of unrestricted time could be scheduled on the short notice available. This time constraint effectively prevented measurements of beam patterns at any angles other than immediately around the main lobe; all of the data in Chapter 6 was taken in that one 12 hour period.

One or two specific problems in taking data in the pool are covered in more detail in Chapter 6, as they have to do with the intensely reverberant nature of the solid concrete-walled body of water that tried to represent the open ocean. Ambient noise in the pool was high enough to pose a significant problem when trying to measure side lobes of the beam patterns as well.

The purpose of this discussion is to caution the reader of the dangers of excessive belief in printed data. Though the data did in fact agree surprisingly well with theory developed in Chapter 5, the possibility of sheer coincidence cannot be ignored, given the range of unknowns and approximations used in the collection of the data. We believe that the data represents at least a fair estimation of the true behavior of the sonar array, and wait eagerly for the opportunity to verify its validity with calibrated equipment under approximately anechoic conditions.
CHAPTER 5

System Theoretical Performance

5.0 Amplifier Characteristics

The operational amplifiers used as building blocks in the design of the power amplifier circuitry reduce the task of circuit analysis to two basic equations, which can be combined to give the overall frequency response characteristics

\[
\frac{V_{\text{out}}}{V_{\text{in}}} = \frac{R_3 \cdot R_8}{R_1 \cdot R_6} \quad (5.0.1)
\]

Equation 5.0.1 shows the passband gain of 40 dB up to the cutoff frequency of the active filter which is expressed as (all component symbols refer to the schematic in Figure 4.1.1)

\[
f_C = \frac{1}{2\pi} \sqrt{\frac{1}{R_2 \cdot R_3 \cdot C_1 \cdot C_2}} \quad (5.0.2)
\]

where \( f_C \) is the cutoff frequency of the filter. Above this frequency (70 kHz) the response rolls off at the rate of 12 dB/octave. Figure 5.0.1 shows a plot of the ideal frequency response curves, along with the expected phase shift.

Phase shift and amplifier self-noise can be quasi-theoretically predicted utilizing information from the manufacturer's data sheets listing typical performance characteristics. The
Figure 5.0.1 Amplifier Theoretical Response
total phase shift determined by this manner is $180^\circ$ from 0 to 60 kHz. The total circuit noise may be approximated as $[14,15]$}

$$E_t = \sqrt{e_n^2 + \left[i_n^2 R_{eq1}^2 + \left(i_n^2 (R_s \parallel R_f) \right)^2 + 4 \kappa T R_{eq2} + 4 \kappa T (R_s \parallel R_f) \right] \cdot \frac{1}{BW}}$$

$\text{(Volts)}$ \hspace{1cm} (5.0.3)

where

$$e_{nt} = \sqrt{e_n^2 + \left[i_n^2 R_{eq1}^2 + \left(i_n^1 (R_s1 \parallel R_f2) \right)^2 + 4 \kappa T R_{eq1} + 4 \kappa T (R_s1 \parallel R_f1) \right] \cdot \frac{1}{B_1}}$$

$\text{(Volts)}$ \hspace{1cm} (5.0.4)

$E_t$ is the total circuit noise (V),
$1/\beta$ is stage 1 total noise plus stage 2 input noise voltage (V$\sqrt{\text{Hz}}$),
$i_{n1,2}$ is the amplifier noise current (A$\sqrt{\text{Hz}}$),
$R_{eq1,2}$ and $R_{s1,2} \parallel R_{f1,2}$ are the source resistances (\Omega),
$\kappa$ is Boltzmann's constant ($1.38 \times 10^{-23}$ J/°K),
$T$ is the absolute temperature in °Kelvin,
$BW$ is the noise bandwidth (Hz).

Equation 5.0.3 yields a total circuit noise figure of .56 mV, and so a maximum $S/N$ ratio of 100 dB is attained at 60 V pk-pk.

5.1 Transducer Analysis

Several methods exist for the analysis of electroacoustic transducers, ranging from solving the partial differential equations describing its distributed electrical and material characteristics to using the common mass-spring-dashpot analogy. Employed here is a similar technique involving equivalent electrical circuits, where the values of the components represent the electrical analog of the transducer's electroacoustic pro-
properties. For the purposes of determining the single transducer beam pattern, the driver is modelled as an ideal piston operating in an infinite baffle.

5.1.1 Equivalent Circuit Analysis

The longitudinal vibrator is shown in its equivalent electrical form in Figure 5.1.1. Capacitor $C_0$ represents the shunt capacitance of the transducer; the rest of the components are mechanical analogs. Inductor $L_m$ represents the mass of the driver, while $C_m$ and $R_m$ represent material compliance and damping respectively. The general impedance term $Z_1$ represents the loading of the medium through which acoustic energy is transmitted against the face of the transducer. It is normally referred to as the radiation impedance, containing both a resistive and reactive term.

The first four circuit elements have values provided by the manufacturer of the transducers; the radiation impedance must be calculated. Modelling the face of the transducer as a piston operating in an infinite baffle, the radiation impedance can be expressed in terms of the real radiation resistance $R_r$ and the imaginary radiation reactance $X_r$ as

$$R_r = \rho_0 c \pi a^2 R_1(2ka) \quad (\Omega) \quad (5.1.1)$$

$$X_r = \rho_0 c \pi a^2 X_1(2ka) \quad (\Omega) \quad (5.1.2)$$
Figure 5.1.1  Transducer Equivalent Circuit
R₁(x) and X₁(x) and called piston resistance and reactance functions respectively, and may be expanded as infinite series:

\[
R₁(x) = \frac{x^2}{2.4} - \frac{x^4}{2.4^2 6} + \frac{x^6}{2.4^2 6^2 8} \ldots (5.1.3)
\]

\[
X₁(x) = \frac{4}{\pi} \left[ \frac{x}{3} - \frac{x^3}{3^2 5} + \frac{x^5}{3^2 5^2 7} \right] \ldots (5.1.4)
\]

where

ρ₀ is the ambient density of the medium (1025 kg/m³),
c is the speed of sound in the medium (1500 m/s),
a is the diameter of the transducer face (.022 m),
k is the acoustic wave number \(\omega/c\), \(\omega\) is frequency in rad/sec.

The shaded region of Figure 5.1.2 shows the range of R₁(x) and X₁(x) included between 30 and 60 kHz[16] For ease in calculation, a value of 1.0 is assumed for R₁(x), and one of 0.2 for X₁(x). The reactive term is not as good an approximation as it varies more widely than R₁(x) over the frequency range of interest, but the constant may still be assumed with little effect because the reactance is such a small part of the total radiation impedance in this region, which is dominated by the real, resistive term. For our transducer this yields a value of 2380 acoustic ohms for the radiation resistance, and 476 acoustic ohms for the radiation reactance. Thus the total radiation impedance seen by the transducer as it is loaded by the fluid medium is

\[
Z_{H₂O} = 2380 + j 476 \hspace{1cm} (\Omega) \hspace{1cm} (5.1.5)
\]
Figure 5.1.2  Radiation Impedance Parameters

\[ R_r = \rho_0 c \pi a^2 R_1(2ka) \quad (\Omega) \]

\[ X_r = \rho_0 c \pi a^2 X_1(2ka) \quad (\Omega) \]
Now that all of the circuit elements have been determined, standard linear network theory may be employed to produce expressions for the transducer's input impedance, voltage transfer function, acoustical Q, and resonant frequency ($f_r$ or $\omega_r$) from the equivalent circuit. This procedure gives for the input impedance

$$Z_{in} = \left[ \frac{R \left[ \frac{1}{C_o} + \frac{1}{C_m} - \omega (\omega L_m + X_r) \right] + \omega R \left[ \frac{1}{C_o} (\omega L_m + X_r) - \frac{1}{\omega C_m} \right]}{\left[ \frac{1}{C_o} + \frac{1}{C_m} - \omega (\omega L_m + X_r) \right]^2 - [\omega R]^2} \right] + j \left[ \frac{\frac{1}{C_o} (\omega L_m + X_r) - \frac{1}{\omega C_m} \left[ \frac{1}{C_o} + \frac{1}{C_m} - \omega (\omega L_m + X_r) \right]^2 - [\omega R]^2}{\left[ \frac{1}{C_o} + \frac{1}{C_m} - \omega (\omega L_m + X_r) \right]^2 - [\omega R]^2} \right] \quad (\Omega) \quad (5.1.6)$$

where $R$ is the sum of $R_m$ and $R_r$.

Equation 5.1.6 does not provide a great deal of insight into the behavior of the input impedance with frequency on mere inspection, so a computer was used to plot the function in several different ways. Shown in Figures 5.1.3 and 5.1.4 and the plots of transducer input $\text{Re}|Z_{in}|$ and $\text{Im}|Z_{in}|$, respectively. The curves are fairly typical for this type of transducer, with a real input impedance value of 1950 ohms at the theoretical resonant frequency of 50 kHz. [It should be strongly emphasized that the equivalent circuit model loses accuracy as one moves
Figure 5.1.3
Real Part of Transducer Input Impedance

FREQUENCY (kHz)

Re [Z]
away from the resonant frequency; thus wideband curves of various parameters plotted from equations developed from the equivalent circuit model may be expanded or compressed, not necessarily linearly, depending on what parameter is being observed.] Using the real part of the impedance, we can calculate the power consumed by the transducer for a constant voltage input (or any voltage input, for that matter) as

\[ P_e = \frac{V^2_{in}}{\text{Re}|Z_{in}|} \quad \text{(Watts)} \quad (5.1.7) \]

Since \( Z_{in} \) is a function of frequency, so is \( P_e \), and a plot of \( P_e \) for a constant 1 Volt input is found in Figure 5.1.5. Examining the curve, we see that the minimal power consumption does take place at resonance, but must remember that the seemingly huge power consumption at 30 and 60 kHz cannot be trusted from the equivalent circuit model. Given the power amplifier's 60 V pk-pk maximum output capacity, we can calculate the maximum available input power to the transducer at resonance to be 1.85 watts peak, or 0.92 watts rms.

The transducer's voltage transfer function may be predicted by the same methods, which is accurate in the far field \((r >> \lambda)\). The equation for the complex transfer function is found to be

\[
H(\omega) = \frac{[-\omega X_r (1 - \omega^2 L_m - \omega X_r) + \omega^2 R R_r] + j[\omega R_r (1 - \omega^2 L_m - \omega X_r) + \omega^2 R X_r]}{(\frac{1}{C_m} - \omega^2 L_m - \omega X_r)^2 + (\omega R)^2} \quad (5.1.8)
\]
Figure 5.1.5
Power Dissipated in Transducer for a 1 Volt Constant Input

$P_{in}$ for a constant 1 V input

$-51.7 \text{ (dB re 1 watt)}$

FREQUENCY (kHz)
The magnitude and phase characteristics of the transfer function may be computed from

\[
H(\omega) = \left[ \text{Re} |H(\omega)|^2 + \text{Im} |H(\omega)|^2 \right]^{1/2} \quad (5.1.9)
\]

\[
\phi(\omega) = \tan^{-1} \frac{|\text{Im} H(\omega)|}{|\text{Re} H(\omega)|} \quad (5.1.10)
\]

The computer plots of these functions are found in Figures 5.1.6 through 5.1.9. The plot of the magnitude of \( H(\omega) \) has no real bearing on this discussion, but is included for completeness. The phase plot in Figure 5.1.7 shows the characteristic 180° phase shift through resonance (the bandwidth shown is not wide enough to accommodate the entire range) that passes through 0° at resonance. The imaginary part of the transfer function is not really of interest either, but with the real part portrayed in Figure 5.1.8 we can calculate the overall efficiency of the transducer as the ratio of power radiated into the water to power dissipated in the transducer

\[
\eta = \frac{P_w}{P_{in}} = \frac{|V_w|^2/\text{Re} Z_w|}{|V_{in}|^2/\text{Re} Z_{in}|^2}
\]

\[
= \text{Re} |H(\omega)|^2 \text{Re} Z_{in}/\text{Re} Z_w \quad (5.1.11)
\]

Equation 5.1.11 shows that the dependence of efficiency on frequency is a function of two variables—\( \text{Re} |H(\omega)| \) and \( \text{Re} |Z_{in}| \)—as the real part of \( Z_w = Z_{H2O} \) may be assumed to be a constant
Figure 5.16
Magnitude of Voltage Transfer Function

\[ \frac{V_{\text{out}}}{V_{\text{in}}} \]

\[ |H(\omega)| \]

FREQUENCY (kHz)
Figure 5.1.7
Phase Characteristics of Transducer Transfer Function

FREQUENCY (kHz)

\( \phi(\omega) \)
Figure 5.1.8
Real Part of the Transducer Transfer Function

$\text{Re } [H(\omega)]$
Figure 5.1.9
Imaginary Part of the Transducer Transfer Function

Im [H(ω)]

FREQUENCY (kHz)
equal to 2380 acoustic ohms over the frequency range of interest. Inserting this relation into the computer yields Figures 5.1.10 and 5.1.11, plotted against a linear and a log scale respectively. The maximum efficiency at resonance is seen to be 11 percent, a good example of the penalty paid when trading efficiency for bandwidth. The quality factor Q, when measured from the 3 dB down points of Figure 5.1.11, comes out to about 8.4, somewhat higher than that specified to the manufacturer.

An examination of Figure 5.1.10 readily yields the correct value for resonant frequency at 50 kHz.

The equivalent circuit analysis of the individual sonar transducer has produced theoretical values for the efficiency, frequency response, and Q of the element. Now a different model is employed to determine the directional characteristics of the transducer.

5.1.2 Beam Pattern and Directivity Index

The transducer may be modelled as a simple piston operating in an infinite baffle for the purposes of determining its beam pattern and directivity index. This model works quite well for forward radiation from the transducer, but neglects entirely any imperfections in the real baffle. Since the housing design leaves the exact degree of baffling an open question, some radiation must be assumed to be present in the rear that will decrease the effective directivity index of the transducer. The magnitude of this effect may only be determined empirically
Figure 5.1.10  Efficiency of the Individual Sonar Transducer

\[ \eta = \frac{P_{\text{out}}}{P_{\text{in}}} \]
Figure 5.1.11
Acoustic Power Radiated for a 1 Watt Constant Input

$P_{out}$ for a 1 watt input.

Frequency (kHz)

-24.5 (dB re 1 watt)
(the geometry of the housing is too complex to permit accurate calculations), so the value of the directivity index predicted by this analysis must be taken as an upper limit.

The complex pressure distribution resulting from the radiation of an infinitely baffled piston into free space is expressed as

\[
\overline{p} = \frac{ip\cdot e^{j(\omega t - kr)}}{2\pi r} \frac{2j_1(k\sin\phi)}{k\sin\phi} \tag{5.1.12}
\]

where

- \(Q_p\) is the source strength of the piston (m³/s),
- \(r\) is the radial distance from the piston face (m),
- \(\phi\) is the azimuth angle

This expression is identical to the equation for a hemispherically radiating simple source except for the presence of a directivity term (enclosed in brackets). It effect can be seen in Figure 5.1.12 where as \(x\) (\(x = k\sin\phi\)) increases, its amplitude diminishes alternating between positive and negative values.\(^{[16]}\)

For a piston of fixed diameter, varying frequencies produce different beam patterns with an increasing number of lobes (resulting from the zero crossings of the directivity term) as frequency gets higher. The transducer radiation patterns are plotted in Figures 5.1.13 through 5.1.15 for 30, 50, and 60 kHz.

Two related quantities can be used to specify the directionality of the transducer. The first is the beam width of the major lobe, which may be defined from an azimuth angle of 0° to any specified drop in intensity, commonly taken at the point 3 dB down from axial intensity. Twice this angle then gives
Figure 5.1.12  Directivity Function for a Fully Baffled Piston
TRANSUDER DIRECTIVITY
(10 DB PER DIVISION)

30 KHZ

Figure 5.1.13 Transducer Directivity at 30 kHz
Figure 5.1.14  Transducer Directivity at 50 kHz
Figure 5.1.15 Transducer Directivity at 60 kHz
the major lobe beamwidth. To find an expression from which the intensity may be plotted or calculated we first find the pressure amplitude $P$ from Equation 5.1.12 to be

$$p = \frac{\rho_0 c k \pi a^2 U_0}{2 \pi r} \left[ \frac{2 J_1(\text{kasin} \phi)}{\text{kasin} \phi} \right] \text{(N/m}^2) \quad (5.1.13)$$

where

$$U_0 = \frac{Q_p}{4\pi a^2} \quad \text{(m/s)} \quad (5.1.14)$$

then the intensity $I$ is

$$I = \frac{p^2}{2\rho_0 c} = \frac{\rho_0 c k^2 U_0^2 (\pi a^2)^2}{8\pi^2 r^2} \left[ \frac{2 J_1(\text{kasin} \phi)}{\text{kasin} \phi} \right] \quad \text{(watts/m}^2) \quad (5.1.15)$$

and the axial intensity $I_0$ reduces to

$$I_0 = \frac{\rho_0 c k^2 U_0^2 S^2}{8\pi^2 r^2} = \frac{\rho_0 c k^2 Q_p^2}{8\pi^2 r^2} \quad (5.1.16)$$

The directivity index relates the axial intensity of the transducer to that of an omnidirectional radiator of the same source strength and is defined as

$$\text{DI} = 10 \log \frac{I_0}{I_{\text{ref}}} \quad \text{(dB)} \quad (5.1.17)$$

where

$$I_{\text{ref}} = \frac{W}{4\pi r^2} \quad (5.1.18)$$

and $W$ is the total radiated acoustic power from the source.
A spherical sound source is generally used as the reference for directionality index as it has a DI of 0 and thus reduces Equation 5.1.17 to that of a single variable. Real world values for DI range from 0 for the spherical source to over 30 dB for highly directional projectors and receivers. The effect of a baffle is seen to introduce a 3 dB increase in DI for any source with a radiation pattern symmetric about the baffling plane, as it cancels all the radiation into one hemisphere, and thus half of whatever pattern the given symmetrical transducer might have.

The directivity index may also be approximated from the 3 dB beamwidth:[3]

\[ DI = -10 \log\left(\sin\frac{\phi_{3\text{dB}}}{2}\right) \]  \hspace{1cm} (5.1.19)

Inserting the proper parameters into Equations 5.1.15 through 5.1.17 we find that for the sonar transducer the beamwidth and directivity index are as listed in the Table below.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>30</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beamwidth (2\phi_{3\text{dB}})</td>
<td>71.6</td>
<td>41.0</td>
<td>34.0</td>
</tr>
<tr>
<td>Directivity index</td>
<td>5.1</td>
<td>7.5</td>
<td>8.3</td>
</tr>
</tbody>
</table>

(DI assumes fully baffled source)
5.2 Theoretical Array Performance

The use of a multi-element array for the sonar transmitter accomplishes two objectives: first, it enhances the directivity of an individual transducer, resulting in a higher on-axis S/N ratio; second, it allows phase control of the signals being fed to each element, so that the effective direction of acoustic radiation may be steered within the limits of the individual transducer response and required S/N ratio at the receiver.

5.2.1 General Array Directionality Characteristics

Before considering the properties of the real sonar array, we examine the effect of operating the same 4 x 8 element array using ideal hemispherically radiating drivers, so that the directional benefits of the array alone may be appreciated.

In the case of linear arrays, a plane array may be considered to be the two-dimensional extension of a line array, so far as its directional characteristics are concerned; a line array of line arrays rather than individual elements. Just as the transducer could be represented as a baffled hemispherical source modified by a directivity term \((2J_1(kasin\phi)/kasin\phi)\), so can the planar array. The directivity function for a line array of \(N,M\) equally spaced elements centered on \(x,y = 0\) is\(^3\)

\[
B_x(\phi) = \frac{\sin\left(\frac{N\pi dsin\phi}{\lambda}\right)}{N\sin\left(\frac{\pi dsin\phi}{\lambda}\right)}
\]

\[
B_y(\phi) = \frac{\sin\left(\frac{M\pi dsin\phi}{\lambda}\right)}{M\sin\left(\frac{\pi dsin\phi}{\lambda}\right)}
\]
Equations 5.2.1 and 5.2.2 express the directivity functions of the sonar array when considered independently in the x-z and y-z planes, respectively. To extend this result for any polar angle \( \phi \) we must include a term that tapers the effect of each direction of line arrays from a factor of 1 when observing \( B_x \) in the x-z plane, for instance, down to 0 when observing \( B_y \) in the y-z plane. This is accomplished by multiplying the arguments of each pseudo-sinc function by \( \cos \theta \) for \( B_x \) and \( \sin \theta \) for \( B_y \) as detailed below: now a single function \( B_p \) can describe the directionality of the 4 x 8 element plane array for any combination of azimuth angle \( \phi \) and polar angle \( \theta \).

\[
B_p(\phi, \theta) = \sin \left[ \frac{N \pi d \sin \phi \cos \theta}{\lambda} \right] \cdot \sin \left[ \frac{M \pi d \sin \phi \sin \theta}{\lambda} \right] \quad (5.2.3)
\]

where

- \( N, M \) are the number of elements in lines parallel to the x-z and y-z planes, respectively, \( (N=8, M=4) \),
- \( d \) is the interelement spacing (here assumed uniform throughout the array) \( (m) \),
- \( \lambda \) is the transmitted wavelength \( (m) \).

The directivity function is normalized so that on the z-axis each function \( B_p(0, \theta) \) is equal to 1. Overall, then, this function serves to trim or taper the response of the array so that instead of looking like a hemisphere (that would have a directivity function \( B(\phi, \theta) = 1 \)), the beam pattern takes the shape of some sort of conglomeration of one main lobe and some number of side lobes, depending on the dimensions of the array, the
size and spacing of the individual elements, and the frequency of operation. Trying to visualize a complicated three-dimensional beam pattern is a very difficult proposition, but one could draw a loose analogy to looking down at the top of an ever-green tree, with the highest part of the trunk corresponding to the main lobe, and the upreaching branches analogous to the smaller but numerous side lobes.

Figures 5.2.1 through 5.2.6 are computer generated polar plots of Equation 5.2.3 with $\theta = 0$ producing the cross-section in the x-z plane, and similarly $\theta = \pi/2$ describing the cross-section in the y-z plane. The plots are made at 30, 50, and 60 kHz. The important point to note is that at any given frequency, the shorter line (y-z plane) produces a beamwidth twice that of the longer (by a factor of 2!) line, and as frequency increases the beamwidths in a given plane grow successively narrower, thus illustrating clearly the role of the parameters of line length and frequency in the action of the directivity function for a linear array.

These figures also illustrate how the beam pattern of a plane array can be described by just two orthogonal cross-sections each lined up with a major or minor axis of the array geometry. The beamwidths and directivity index of a plane array thus may be specified in terms of their axial values; the total DI for the array is then the sum of the x-z and y-z plane figures. The axial directivity functions $B_x$ and $B_y$ may be used to calculate the 3 dB down beamwidths, but since they operate on the pressure distribution, and beamwidths are proportional to levels
Figure 5.2.1 Array Directivity at 30 kHz in the X-Z Plane
Figure 5.2.2  Array Directivity at 30 kHz in the Y-Z Plane
Figure 5.2.3 Array Directivity at 50 kHz in the X-Z Plane
Figure 5.2.4  Array Directivity at 50 kHz in the Y-Z Plane
Figure 5.2.5  Array Directivity at 60 kHz in the X-Z Plane
Figure 5.2.6 Array Directivity at 60 kHz in the Y-Z Plane
of intensity, the value of $\phi$ for which $B_1^2(\phi)$ equals 0.5 must be found. The solutions of Equations 5.2.1 and 5.2.2 for this condition are presented in Table 5.2.1.

<table>
<thead>
<tr>
<th>Table 5.2.1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Directional Parameters for Generalized Array ($^\circ$)</strong></td>
</tr>
<tr>
<td><strong>frequency (kHz)</strong></td>
</tr>
<tr>
<td><strong>beamwidth (2$\phi_{3dB}$)</strong></td>
</tr>
<tr>
<td><strong>x-z plane</strong></td>
</tr>
<tr>
<td><strong>y-z plane</strong></td>
</tr>
</tbody>
</table>

The directivity index for a line array is given by

$$DI = -10 \log (d) \text{ (dB)} \quad (5.2.4)$$

where

$$d = \frac{1}{2\pi} \int_{-\pi/2}^{\pi/2} |B_1(\phi)|^2 d\phi \quad (5.2.5)$$

The directivity factor $d$ is a measure of the average value of $B^2$ integrated over free space; the smaller the value of $d$ compared to unity, the greater the concentration of the acoustic field (and the greater the value of $DI$). The integration required in Equation 5.2.5 cannot be carried out in the case of the segmented line array because of the periodicity of the pseudo-sinc ($\text{sinkx}/\text{ksinx}$) function that makes up $B_x$ and $B_y$, but the approximation given in Equation 5.1.19 should be quite sufficient in this case. Using the values from Table 5.2.1 we find for the directivity index in the x-z and y-z planes:
Table 5.2.2

Directivity Indices for Generalized Array (dB)

<table>
<thead>
<tr>
<th>frequency (kHz)</th>
<th>30</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>DI in x-z plane</td>
<td>13.2</td>
<td>14.9</td>
<td>16.2</td>
</tr>
<tr>
<td>DI in y-z plane</td>
<td>10.2</td>
<td>11.8</td>
<td>13.2</td>
</tr>
<tr>
<td>DI total for array</td>
<td>23.4</td>
<td>26.7</td>
<td>29.4</td>
</tr>
</tbody>
</table>

5.2.2 Predicted DI and Beamwidths for the Sonar Array

The composite values of directivity index and beamwidths are easily determined from the previously calculated figures for the individual transducer elements and the array using hemispherically radiating (non-directional) sources. This fortunate consequence is a result of the linearity of the array. The overall directivity function $B(\phi, \theta)$ can now be expressed as

$$B(\phi, \theta) = B_t(\phi) \cdot B_p(\phi, \theta) \quad (5.2.6)$$

or

$$B(\phi, \theta) = \left[ \frac{2J_1(\text{kasin}\phi)}{\text{kasin}\phi} \right] \left[ \frac{\sin \left( \frac{N \pi d \sin \phi \cos \theta}{\lambda} \right)}{N \pi d \sin \phi \cos \theta} \cdot \frac{\sin \left( \frac{M \pi d \sin \phi \sin \theta}{\lambda} \right)}{M \pi d \sin \phi \sin \theta} \right]$$

$$\quad (5.2.7)$$

The complex pressure distribution radiated by the array is then

$$\bar{p} = \frac{j \rho c k Q_p^' \ e^{j(\omega t - kr)}}{2\pi r} \ B(\phi, \theta) \quad (5.2.8)$$

where $Q_p^'$ is the total source strength of all 32 transducers.
and thus the intensity is given by

\[ I = \frac{p^2}{2\rho_o c} = \frac{\rho_o c k^2 v_0^2 (\pi a^2)^2}{8\pi^2 r^2} B(\phi, \theta) \]  

(5.2.9)

and the axial intensity reduces to

\[ I_o = \frac{\rho_o c k^2}{8\pi^2 r^2} Q_p^2 \]  

(5.2.10)

The beam patterns resulting for the real sonar array when the beam patterns of the transducers and the generalized array are multiplied together may be found in the computer generated polar plots in Figures 5.2.7 through 5.2.12. The most important features to look at in these figures are the differences generated by the introduction of directional transducers into the generalized array. Close to the z-axis the patterns barely show any difference, because the output of the transducers is very uniform for \( \phi < \pm 20^\circ \) or so. At angles farther from the z-axis, the output of the transducers is changing rapidly, which has the effect of drastically reducing the side lobe levels predicted for the array using uniformly radiating elements. Thus we see that the introduction of directional transducers into a line or plane array can provide good improvement in DI without affecting the array's design main lobe beamwidth significantly.

Of course, in the real array beam patterns cannot show the perfect nulling that is predicted by theory, and so measured
Figure 5.2.7  Sonar Directivity at 30 kHz in the X-Z Plane
Figure 5.2.8  Sonar Directivity at 30 kHz in the Y-Z Plane
Figure 5.2.9  Sonar Directivity at 50 kHz in the X-Z Plane
Figure 5.2.10  Sonar Directivity at 50 kHz in the Y-Z Plane
Figure 5.2.11. Sonar Directivity at 60 kHz in the X-Z Plane
Figure 5.2.12  Sonar Directivity at 60 kHz in the Y-Z Plane
The beamwidths of the sonar array are for all intents and purposes identical to that of the generalized 4 x 8 element model, and the total directivity indices are found by summing the nondirectional element array figures with those computed for the real transducers. A summary of all these numbers is given in Table 5.2.3.

Table 5.2.3
Directional Performance of the Real Array

<table>
<thead>
<tr>
<th>frequency (kHz)</th>
<th>30</th>
<th>50</th>
<th>60</th>
</tr>
</thead>
<tbody>
<tr>
<td>beamwidth (x-z) (°)</td>
<td>11.0</td>
<td>7.5</td>
<td>5.5</td>
</tr>
<tr>
<td>beamwidth (y-z) (°)</td>
<td>22.0</td>
<td>15.0</td>
<td>11.0</td>
</tr>
<tr>
<td>DI in x-z plane (dB)</td>
<td>15.8</td>
<td>18.7</td>
<td>20.3</td>
</tr>
<tr>
<td>DI in y-z plane (dB)</td>
<td>12.8</td>
<td>15.6</td>
<td>17.4</td>
</tr>
<tr>
<td>DI total for array (dB)</td>
<td>28.5</td>
<td>34.2</td>
<td>37.7</td>
</tr>
</tbody>
</table>

This concludes the theoretical analysis of the performance of the power amplifiers and sonar transmitting array.

5.3 Predicted Signal/Noise Ratio at the Receiver

Using the data developed thus far for array efficiency, sound absorption in the ocean, sources of broadband noise, etc., we can predict the S/N ratio per bit at the receiver over a 1 km path for the acoustic communication system as a whole. This ratio is expressed from the following chain of terms: first, the source level in terms of electrical power to the array is
specified; that level is diminished by the factor of array transmitting efficiency, and then increased by the directivity index of the transmitter; at the receiver the signal is enhanced by any additional directivity possessed by the receiver. This level is subsequently reduced by subtracting the absorption and spreading loss of the signal in the water, as well as the noise level in the data transmission band. Spreading the bandwidth of the signal reduces the signal further, while some enhancement is provided by the bandwidth expansion ratio (bandwidth/data rate) to arrive at the final signal/noise ratio. Some of these quantities will change as a function of range, local environmental conditions, or input power, but a strongly conservative case is presented in Table 5.3.1, which suggests that accomplishing the design goals under more typical conditions should pose no great problem at all.

The next Chapter presents real data taken during preliminary trials of the power amplifier/sonar array communications subsystem, and provides some interesting comparisons with the theory developed in this Chapter.
Table 5.3.1

SIGNAL TO NOISE RATIO PER BIT ($E_b/N_0$)
FOR A 1 KM PATH

1) SPL for 10 Watts (SOUND PRESSURE LEVEL) +181 dB*
2) n EFFICIENCY -10
3) DI$_T$ of TRANSMITTER +10
4) DI$_R$ of RECEIVER 0
5) αR LOSS (ABSORPTION LOSS) -22
6) Spreading Loss -60
7) N$_O$ LEVEL @ 50 kHZ -45 (NOISE LEVEL)
8) W bandwidth for 10 kHz -40
9) Bandwidth Expansion (W/R) +6

$E_b/N_0$ (SIGNAL NOISE RATIO/BIT) +20 dB
CHAPTER 6

Experimental Results

6.0 Scope of Experiments

At this writing the entire communications system has not yet been assembled, and so could not be incorporated into the testing program designed for the planar array subsystem. The goals of these experiments were directed towards confirming at least qualitatively the predicted frequency response and directional characteristics of the sonar array, as well as to make as accurate a determination as possible of the absolute efficiency of the transmitter. Time was not available to procure sophisticated test equipment and facilities, and so these experiments served not only as a preliminary report on the performance of sonar array, but also allowed some freedom to become acquainted with the idiosyncracies of the system, such as its performance under changing supply voltages, which could be of great benefit in the interpretation of more carefully controlled experiments. In short, these experiments provided an opportunity for a preview of future work, which will help ensure that things run smoothly through to the completion of the project.
6.1 Amplifier Tests

No extensive testing was performed on the power amplifiers beyond careful offset nulling of each channel, and confirmation of the 40 dB gain characteristic with a rolloff above 70 kHz. Instruments were not available to measure noise or distortion, or phase shift. The amplifiers did in general perform exactly as expected, and a typical frequency response curve for the real amplifiers is plotted in Figure 6.1.1.

6.2 Sonar Array Tests

The objectives of the array testing program were to determine the efficiency, frequency response, and directivity patterns of the transmitter. The following sections describe the testing environment, equipment, and procedure; the experimental data are then presented and analyzed.

6.2.1 Test Environment

Strong wind and wave action at the Woods Hole Oceanographic Institution forced the abandonment of the W.H.O.I. docks as the primary test location. To obtain the required degree of accuracy in measuring beam patterns, an absolutely calm body of water was required. The only readily available location meeting that criterion was the M.I.T. swimming pool, which became the main test facility. The M.I.T. pool measures approximately 25 meters long, 15 meters wide, and 2 (shallow)/4 (deep) meters in depth. Its main drawback was its concrete construction which absorbed
Figure 6.1.1 Measured Power Amplifier Response Data
very little acoustic energy, thus the introduction of any sound source generated an extremely reverberant field. Unfortunately, only 12 hours of pool time could be reserved for the entire range of experiments, due to the heavily scheduled summer swimming program and the imminent shutdown of the pool for maintenance. Lack of time notwithstanding, the pool did provide an ideal environment for testing in the respect that the water was clear and entirely undisturbed; the alignment of transmitter and receiver could be checked visually, and the transmitter to receiver range was also easily determined. If given just a reasonable amount of additional time and test equipment, data could be taken in the pool that could only be surpassed by open ocean trials, where reverberation can be effectively eliminated under the right conditions.

6.2.2 Test Equipment

At the transmitting end, the incomplete communications system composed of the 32-element sonar array and corresponding power amplifiers was driven from a specially designed test signal generator that could provide four different signals in four different modes. The signals included fixed frequencies at 30, 50, and 60 kHz, and a band-sweeping signal that generated tones between 30 and 60 kHz in increments of 1 kHz, changing frequencies once every 10 seconds. The four modes of transmission were continuous wave (CW), and three pulsed modes: a 70 ms pulse with a 50% duty cycle; a 35 ms pulse with a 25% duty cycle; and for the confines of the swimming pool, a .5 ms pulse that
repeated once every 140 ms. The latter pulse generated a .75 m wave train that precluded any overlap between the direct arrival and any reflected waves also picked up at the receiver. Output level, offset, and pulse length were all independently variable from the test signal generator, however the output level was itself dependent on the supply voltage which was unregulated and drawn directly from batteries contained temporarily within the instrument case.

The self-contained transmitter was suspended from a jury-rigged directional-indicator/depth-controllable apparatus that may be seen in Figures 6.2.1 and 6.2.2. The device allowed the selection of any mechanical steering angle in $1\frac{1}{4}^0$ increments, and provided for changing the depth of operation via the two lines running down to the transmitter housing. The critical stability requirement demanded that the transmitter be suspended from a perfectly unmoving platform, a task that the device managed to perform quite well.

The highly reverberant environment of the swimming pool necessitated the use of pulsed measurements. In the open ocean the system would have been operated in the CW mode which would have allowed the use of a frequency counter and digital voltmeter—impossible in the pulsed modes, as the instruments would not respond correctly. Therefore, the only equipment on the receiving end consisted of an omnidirectional LC-10 receiving hydrophone, a 20/40 dB preamplifier with a second-order highpass filter ($f_c = 20$ kHz), and a Hewlett-Packard 1720B oscilloscope.
Figure 6.2.1 A View Inside the Communication System
Figure 6.2.2 Test Set-up in the MIT Swimming Pool
6.2.3 Experimental Procedure

Two main types of tests were performed. The first was an on-axis frequency response and efficiency calibration with the transmitter and receiver hydrophone each suspended 2 m below the water's surface. The test signal generator had previously been turned on to the .5 ms pulsed mode, sweeping from 30 to 60 kHz. The array was mechanically steered to an angle of 0°, and the received voltage on the oscilloscope for each frequency step. Since the frequency could not be counted, the cycle was watched until the obvious step from 60 kHz to 30 kHz took place, and then readings were made each time the amplitude of the signal jumped, as the amplitude changes were easier to observe on the 'scope than the gradual crowding of the waveform on the screen. In reality the frequency steps were not exactly 1 kHz, nor did the sweep start exactly at 30 kHz. Supply voltage variations that took place gradually as the tests were conducted (the high voltage supply dropped from +25.4 V dc to +20.1 V dc during the course of the 12 hour test, which was about to be expected) made it impossible to know exactly what frequencies were being transmitted, so it was necessary to assume an even division of the swept bank for the purpose of having something to plot data against. Since the entire curve was taken on-axis, readings were easily made, observing clearly the direct arrival of the signal and the first reflection from the other end of the pool back to the receiver hydrophone. Figure 6.2.3 shows an oscilloscope photograph of one of the better defined waveforms that was observed during the day. Unfortunately the beam
Figure 6.2.3 Observed Waveform for a Direct Arrival
pattern tests did not share the same clarity in the observed waveform.

Before the beam patterns could be tested, and every time the frequency was to be changed, the transmitter was hauled up out of the water, dried off, and opened up so the proper switches could be thrown. New O-ring lubrication was applied each time before the unit was sealed up and again placed back into the pool.** The entire operation took about ten minutes, unless an obstacle such as a leak was discovered.

Six different beam patterns were originally desired; measurements in both the x-z and y-z planes at 30, 50, and 60 kHz. The 1 1/4 increments had been selected over the more usual 1° spacings because it would save about 20% of the time required to go around 360°, and still would provide enough resolution to pick up fairly sharp peaks and dips in the response. When the measurements were being taken it became apparent that even with the absolute stillness of the pool, the pressure case would still oscillate back and forth in the water (rotationally) every time the steering angle was changed. Desiring at least some data at each frequency and in each plane, we decided to cover the main lobe of each pattern first, to about 30°, then fill out the data in descending order of priority with whatever time remained.

** Though the pressures encountered in the swimming pool hardly made a test of the watertight integrity of the transmitter housing, the housing (electronics removed) had been lowered to the bottom at the deepest part of the W.H.O.I. docks (15m) so that some minimal check of the electrical bulkhead connectors and O-ring seals could be performed. It survived.
(the pattern at resonance was of the most interest).

The procedure consisted of turning the array to the next angle, letting the oscillations die down, and reading the "average" peak-peak voltage. The observed pulse was only of uniform height when operating very close to the z-axis, and the weaker it got, the more distorted it became, so an "eyeball integration" was performed to determine the peak-peak voltage that represented the average energy in the pulse (see Figure 6.2.4 for a photo of one of the harder cases). This process was employed as consistently as possible, so that the relative amplitudes of the received pulses could be preserved fairly well.

As the array turned farther off-axis, it became clear that additional time alone would not allow accurate measurements of any beam pattern around 360°, because the signal/noise ratio degraded to below 1 fairly rapidly, at which point the signal was no longer distinguishable on the 'scope. The broadband background noise measured about -26 dB re 1 V, and the signal arriving on-axis at resonance was a maximum of 6 dB re 1 V, giving an absolute maximum S/N ratio of 32 dB (the 30 and 60 kHz patterns were only able to obtain S/N maxima of 16 dB and 22 dB respectively). Most of the side lobes were thus buried in noise, and could never be measured in the swimming pool environment anyway.

The measured data are described and analyzed in the next section. Its interpretation must be basically qualitative, but if the quantitative results are close to being correct, the project must be termed an unqualified success.
Figure 6.2.4  An Example of a Difficult Interpretation...
What Voltage Does One Pick?
6.3 Presentation of Measured Data

Most of the data introduced in this section is plotted in terms of decibel level (dB), and careful attention should be paid to the reference levels, as they were chosen to give the most meaning possible to the various curves, and are not necessarily referred to "standard" reference levels.

6.3.1 Frequency Response Results

The data contained within the frequency response curve shown in Figure 6.3.1 should be reasonable accurate. All of the data were taken on-axis, and the lowest point on the curve still had a S/N ratio of 15.6 dB. The curve shows the observed voltage corrected for three factors: a rolloff present in the test signal generator that is linear from 0 dB at 60 kHz to -3 dB at 30 kHz; the rolloff ending at the highpass filter cutoff in the receiver preamplifier; and corrections for the sensitivity of the receiver hydrophone.

\[
SL(f) = V_{\text{out}}(f) + H_s(f) - A_G(f) + 20 \log(r) + \alpha r
\]

where (dB re 1 m, 1 V) (6.3.1)

SL is the source level from the transmitter,
\(V_{\text{out}}\) is the observed waveform voltage,
\(H_s\) is the hydrophone sensitivity curve,
\(A_G\) is the preamplifier gain,
\(\alpha\) is the acoustic absorption attenuation coefficient (dB/km)
\(r\) is range from the source.
Figure 6.3.1
On-axis Sonar Array Frequency Response (SL)

Source Level (SL)

205 (dB re 1µPa, 1 V, 1 m)

FREQUENCY (kHz) (real) assumed
It should be noted that though the LC-10 hydrophone does come with a calibration curve, a tag dated two years ago on the one we borrowed for the experiments showed a 1 kHz sensitivity 6 dB lower than that listed on the published response curve. Used here is the published curve minus a uniform 6 dB, but its accuracy is questionable.

Figure 6.3.1 is in reasonable agreement with the predicted power output for a constant input shown in Figure 5.1.11. The rise observed after 50 kHz is a consequence of the antiresonant frequency of the transducer, which could not be included in the simple equivalent circuit model described earlier. Calculating the quality factor Q of the sonar array from Equation 4.1.1, we find it to be about 7.4, which is fairly close to the value of 8.3 predicted by Figure 5.1.11 (even though the manufacturer had specified that it had a Q of 5). The frequencies listed below the abcissa in Figure 6.3.1 correspond to the assumed division of the test generator sweeping band into equal 1 kHz segments starting at 30 kHz. Previous tests have confirmed that the true resonant frequency of the transducers is very close to 48 kHz, thus the figures above the abcissa in parenthese are the most likely "true" frequencies corresponding to the real output of the array.

From the source level SL at resonance we can calculate the power output in the water $P_w$, and the maximum efficiency $\eta$.

$$P_w = 10 \left[ \frac{SL - 171.5 - DI}{10} \right] \text{ (watts)} \quad (6.3.2)$$
\[ P_e = \frac{V_{out}^2}{Re|Z_{in}|} \cdot n \quad (6.3.3) \]

\[ n = \frac{P_w}{P_e} \quad (6.3.4) \]

where

\( n \) is the number of driven channels.

Reading the SL from the frequency response curve Figure 6.3.1, we determine that the maximum level is 202.7 dB (re 1V, 1m, 1\( \mu \)Pa). The closest data to determine DI at 48 kHz is the 50 kHz beam pattern which has x-z and y-z beamwidths of 10° x 14°, which yields a total DI for the sonar array at 50 kHz of 25.7 dB, to which we add 3 dB to account for the baffle. Equation 6.3.2 then yields an acoustic output power of 1.78 watts peak, or an rms power of .89 watts.

The electrical input power is calculated from Equation 6.3.3 with \( n = 32 \), \( V_{out}^2 \) (rms) = 264.5, and from Figure 5.1.3 we determine the real part of the impedance to be about 1900 ohms at 48 kHz. With this data Equation 6.3.3 returns an rms input power to the array of 4.45 watts. From Equation 6.3.4 it is easy to deduce that the maximum efficiency of the sonar array at resonance is on the order of 20%.

This cannot be viewed as an exact result, if only for the reason that the figure for DI was taken at a different frequency than the one for which we tried to calculate the efficiency. However, 20% certainly makes a good point from which to fine tune the different parameters that make up the calculations. It is also interesting to note that this value for efficiency
is almost twice that predicted figure of 11%. Of course, either number or both could be in error--more trustworthy data will have to be gathered before any decision on the real value of array efficiency can be made.

6.3.2 Beam Patterns

Beam patterns of highly directional transducers are notoriously difficult to measure accurately, and these experiments were no exception. Having already discussed some problems encountered with the test environment and equipment, here follows what we feel is the most realistic interpretation of the results that can be made under the circumstances.

Figures 6.3.2 through 6.3.7 show the measured beam patterns at 30, 50, and 60 kHz in both the x-z and y-z planes. Figures 6.3.2 and 6.3.3 are consistent within themselves in that the shorter array direction did indeed produce a beamwidth approximately twice that of the long axis, however they are anomalous when compared to the predicted numbers of $11^\circ \times 22^\circ$ for 30 kHz. Whether this phenomena was produced by the array itself, or by the test conditions will have to be verified through further experimentation. The low signal/noise ratio expresses the roll-off in frequency response at 30 kHz, which was to be expected (there was also a fairly high ambient noise level in the pool). No side lobes are found within the range of steering angles included above the noise ceiling, which agrees with the computer generated plots for 30 kHz.

Figures 6.3.4 and 6.3.5 show the most detailed patterns
Figure 6.3.2  Measured Sonar Directivity at 30 kHz in the X-Z Plane
Figure 6.3.3  Measured Sonar Directivity at 30 kHz in the Y-Z Plane
Figure 6.3.4  Measured Sonar Directivity at 50 kHz in the X-Z Plane
Figure 6.3.5  Measured Sonar Directivity at 50 kHz in the Y-Z Plane
Figure 6.3.6  Measured Sonar Directivity at 60 kHz in the X-Z Plane
Figure 6.3.7  Measured Sonar Directivity at 60 kHz in the Y-Z Plane
that could be taken, due to the higher output level of the sonar array at resonance. The 3 dB beamwidths are approximately 10° and 14° in the x-z and y-z planes respectively, which are not too far from the predicted values of 7.5° and 15° from Chapter 5. Side lobes are clearly visible in both plots, with the x-z pattern containing twice as many lobes as the y-z plane, once again to be expected. The relatively high side lobe levels (as compared to the computer plots) are partially unavoidable in any real sonar array because it is impossible to build an array in such a manner that all the transducers are exactly identical, all spacings exactly the correct fraction of a wavelength, etc.

While not shown in the figures because of the high noise ceiling, measurements were taken at all frequencies at 90, 180, and 270 degree steering angles. In every case the signal level was buried in the noise, so aliasing of the major lobes does not appear to be a problem, and the assumption of a fully baffled array in directivity calculations (which adds 3 dB to the DI) seems well justified also.

The final two patterns shown in Figures 6.3.6 and 6.3.7, taken at 60 kHz, show the correct qualitative behavior when compared both to theory and to measurements taken at 50 kHz. The beamwidths are narrower at the higher frequency, and once again the x-z plane exhibits more side lobes than does the y-z data, at least within the included angle of sampling. The on-axis levels are lower than at 50 kHz, but higher than at 30 kHz, which agrees with Figure 6.3.1, the frequency response plot.

An interesting point is that if beamwidths are measured
at the -10 dB down points rather than at the usual -3 dB location, the x-z and y-z patterns at each frequency show a much closer agreement with the theoretical doubling of beamwidth for the direction of the shorter (4 element) line array. This is most likely due to the fact that at the -10 dB points the voltage output from the receiving hydrophone (and thus the source level) was changing much faster with each incremental increase in steering angle, and thus differences in level became more obvious while observing the waveforms on the oscilloscope used to measure the beam patterns.

Table 6.3.1 presents a summary of the experimental results tabulated against the values predicted from theory for parameters such as beamwidth, efficiency, and directivity index. When the conditions under which the tests were performed are taken into account, theory and experiment prove to compare very well.

6.4 Sources of Efficiency Losses

Assuming that a more accurate determination of the array efficiency can be made at some future point with more convivial testing conditions, we might expect the figure to be slightly less than that predicted by the theoretical model. There are three main sources of efficiency loss that may be identified, though only one has any real change of improvement.
Table 6.3.1

Summary of Array Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Theory</th>
<th>Measured</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency, ( n )</td>
<td>.11</td>
<td>.20</td>
</tr>
<tr>
<td>Quality Factor, ( Q )</td>
<td>8.3</td>
<td>7.4</td>
</tr>
<tr>
<td>Beamwidth, 30 kHz, X-Z</td>
<td>11.0</td>
<td>8.0</td>
</tr>
<tr>
<td>Beamwidth, 30 kHz, Y-Z</td>
<td>22.0</td>
<td>12.0</td>
</tr>
<tr>
<td>DI, 30 kHz, X-Z</td>
<td>15.8</td>
<td>17.6</td>
</tr>
<tr>
<td>DI, 30 kHz, Y-Z</td>
<td>12.8</td>
<td>15.8</td>
</tr>
<tr>
<td>Beamwidth, 50 kHz, X-Z</td>
<td>7.5</td>
<td>10.0</td>
</tr>
<tr>
<td>Beamwidth, 50 kHz, Y-Z</td>
<td>15.0</td>
<td>14.0</td>
</tr>
<tr>
<td>DI, 50 kHz, X-Z</td>
<td>18.7</td>
<td>16.6</td>
</tr>
<tr>
<td>DI, 50 kHz, Y-Z</td>
<td>15.6</td>
<td>15.1</td>
</tr>
<tr>
<td>Beamwidth, 60 kHz, X-Z</td>
<td>5.5</td>
<td>7.0</td>
</tr>
<tr>
<td>Beamwidth, 60 kHz, Y-Z</td>
<td>11.0</td>
<td>9.5</td>
</tr>
<tr>
<td>DI, 60 kHz, X-Z</td>
<td>20.3</td>
<td>18.2</td>
</tr>
<tr>
<td>DI, 60 kHz, Y-Z</td>
<td>17.4</td>
<td>16.8</td>
</tr>
</tbody>
</table>
6.4.1 Acoustic Window

The polyurethane acoustic window that forms the barrier between the oil-filled sonar array and the open ocean, and allows the passage of sound to the intended receiver is not a perfect acoustic impedance match from the oil inside to the water exterior to the array. Experiments on high efficiency narrow-band arrays have shown a drop of 1.2 dB in level for the case of a window similar to the one used in the communication system's sonar array housing, as opposed to immersing the transducers directly into the fluid medium. The window cannot be discarded for ocean acoustics applications, and so the loss must be lived with, but it is important to recognize that it exists and that it would account for part of a possible gap between measured efficiency and over the value predicted by theory.

6.4.2 Mutual Radiation Impedance

The phenomena of mutual radiation impedance concerns dynamic changes in the loading experienced by a transducer due to the presence of other transducers also radiating into the acoustic medium. The calculations required to assess the severity of this effect in a 4 x 8 element array are beyond the scope of this thesis, since there is not even a hope for effecting any changes for the better. The desired condition is to make the ratio of transducer self-impedance (the loading into the fluid) to mutual radiation impedance as large as possible. There are several ways to go about this: the elements can be
made physically larger, which increases the transducer's self-impedance; the interelement spacing can be increased, which reduces the effects of mutual radiation impedance (to the detriment of array efficiency and beam pattern); or for narrow-band (single-frequency operation) transducers, the driving circuits may be tuned electrically so that the mutual radiation impedance disappears.\[19\] None of these methods can be applied to the sonar array, but the effect may still account for some loss in expected efficiency, and is mentioned for a more complete understanding of the operation of the array. Figure 6.4.1 shows mutual radiation impedance functions (resistance and reactance) for two equal square pistons mounted in an infinite plane, as an example of the variations of the magnitude of the effect that may be expected.

6.4.3 Mechanical Coupling Effects

Although great effort was made to insure that any coupling between the individual transducers through the mounting into the array housing was minimized, this area remains as the only factor that might still be improved to increase the overall array efficiency. The transducers were mounted as described in Chapter 4, but more sophisticated techniques of adhering the elements to the backplane, as well as a deeper investigation of a backplane material with a better damping factor would at least guarantee that efficiency is not being lost through mechanical coupling, which can produce nonuniform phase in the complex driving velocities, leading to degradation in performance.
Figure 2. Mutual radiation resistance ($R/pcA$) for two equal square pistons of dimension $ka$ as function of separation distance $kd$.

Figure 3. Mutual radiation reactance ($X/pcA$) for two equal square pistons of dimension $ka$ as function of separation distance $kd$.

Figure 6.4.1 Behavior of Mutual Radiation Impedance
Conclusions

A phased array sonar with driving amplifiers was constructed and tested as part of a larger short-range, high data rate underwater acoustic communications system. The maximum acoustic power output of approximately 2 acoustic watts exceeded the design goal by a factor of 2. As far as could be determined, the overall array efficiency was 20%, which also exceeded theoretical expectations. The measured beam patterns showed qualitative agreement with predicted behavior, and the assumption of a fully baffled array seems well justified. The general operation of the sonar and amplifiers met all of the stated design requirements, and the experience gained through the construction of the prototype system should ensure that a future model will perform even better.

In conclusion, the sonar/amplifier/pressure case components of the larger communication system have been proven as effective in transmitting acoustic signals through the water with good efficiency and directionality, and should pose no problem in the incorporation of the remainder of the communications system. The project has accomplished one additional step toward the goal of establishing a new capability in underwater communication and exploration, leading perhaps to the day when the oceans will provide work, living, and recreational space as routinely inhabited as the land today.
References


