14.05 Intermediate Applied Macroeconomics
Problem Set 3 Solutions

Distributed: October 20, 2005 TA: José Tessada
Due: October 27, 2005

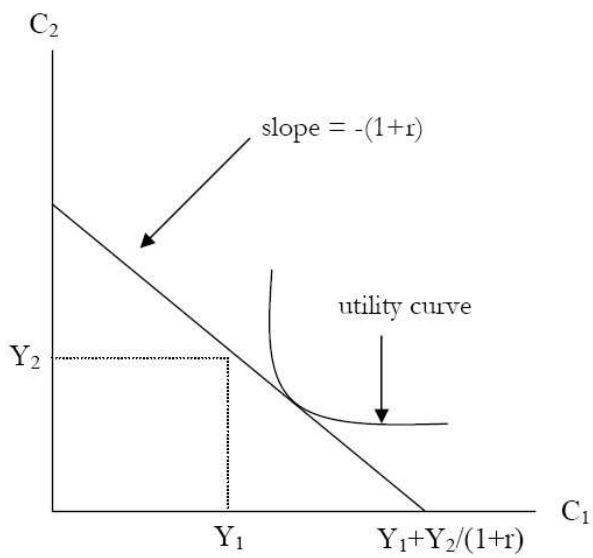
Question 1 Savings and Taxation: Income and Substitution Effects

Assume an individual lives for two periods. Let C_1 and C_2 denote the consumption in the two periods of life. In the first period of life, the individual receives an income of Y_1, and in the second period of life, income is given by Y_2. The interest rate for both savings and borrowing is r. Assume that $Y_1 < Y_2$.

(a) Write out the (intertemporal) budget constraint for this individual.
Answer.

$$C_2 = (1 + r)(Y_1 - C_1) + Y_2$$

(b) Draw the budget constraint from part (a), and draw an example utility curve (indifference curve) tangent to the budget constraint that represents an individual that is borrowing. Be sure to label your axes, the slope of the budget constraint, the (Y_1, Y_2) point and the utility curve.
Answer.
A borrower is when $C_1 > Y_1$, so we just need to make sure the utility curve is tangent to the budget constraint at such a point.



(c) Will an increase in the interest rate, r, increase or decrease savings for this borrower? You must discuss your answer by describing both the income and substitution effects. *[Hint: Remember that borrowing is just negative savings in this case. Thus, a reduction in borrowing is an increase in savings.]*

Answer.

An increase in the interest rate will increase savings (reduce borrowing). The substitution and income effects both work in the exact same direction for borrowers. An increase in the interest rate reduces the price of consumption in the second period of life, and thus the individual will substitute consumption from the first to second period. This substitution effect leads to an increase in savings (or reduction in borrowing). The higher interest rate, however, also reduces the income of the individual because he/she is a borrower and must now pay a higher income. Less income means less consumption in the first period. Thus, the income effect also increases savings.

Now assume that the government decides to implement a tax on financial operations such that the real interest rate is given by $(1-\tau)r$, where τ is the tax rate, and r is the real rate before taxes; this rate applies both to savers and borrowers. Assume that the government simply throws away the tax revenue it collects.

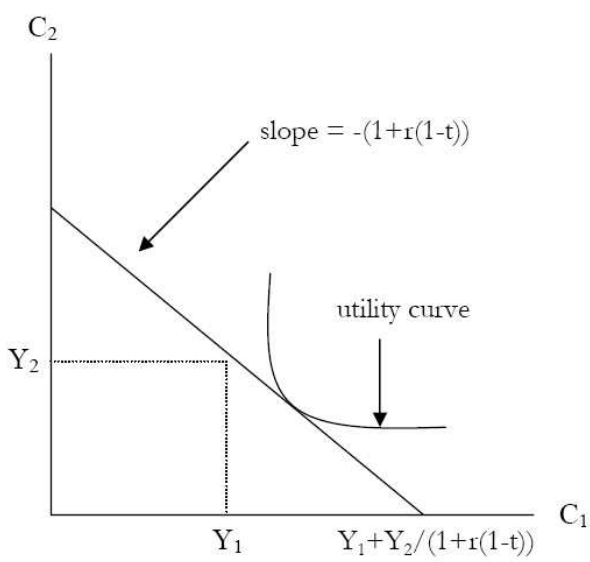
(d) write out the new budget constraint and draw a graph similar to part (b). What is the new slope of the budget constraint?

Answer.

The new budget constraint is given by

$$C_2 = (1 + r(1-\tau))(Y_1 - C_1) + Y_2.$$

The new graph is the same as before, but the budget constraint has a less steep slope now. It will go through the (Y_1, Y_2) point exactly as before.



(e) Again, assume that the individual is borrowing. How will an increase in the tax rate τ affect savings? Explain your answer using both the income and substitution effects.

Answer.

An increase in the tax rate serves to reduce the real interest rate of the borrower. This is the opposite of what we saw in part (c), and the savings will fall. The lower interest rate increases the price of consumption in the second period and leads to lower savings via the substitution effect. The lower interest rate also increases the income of the person since they are a borrower and now have a lower interest rate. The higher income leads to more consumption in the first period and less savings. This is the income effect.

(f) Now assume that the individual is a saver. How will an increase in the tax rate τ affect savings now? Again, explain your answer using both the income and substitution effects.

Answer.

Again, the increase in the tax rate serves to reduce the real interest rate. But now that we are considering a saver, the income and substitution effects will act in opposing directions, and it is uncertain what effect the tax will have on savings. The substitution effect remains the same for the saver as it was for the borrower in part (e). i.e. The lower interest rate increases the price of consumption in the second period and leads to lower savings via the substitution effect. However, the income effect is now different than it was in part (e). Because the person is a saver, the lower interest rate decreases the lifetime income of the person. Thus, the lower lifetime income leads to less consumption in the first period and more savings. This is the income effect. Since the income and substitution effects now work in opposite directions, it is uncertain what effect the tax will have on savings.

Question 2 Consumption "Smoothing"

Consider an individual who lives for $T+1$ periods. Let c_t denote the consumption in period t. Denote by y_t the exogenous income the agent receives in period t. She has the following intertemporal utility function

$$U(c_0, c_1, \ldots, c_T) = \sum_{t=0}^{T} \frac{1}{(1+\delta)^t} u(c_t), \qquad \delta > 0 \tag{1}$$
$$u' > 0, u'' < 0$$

where δ is the subjective discount rate. Assume, for simplicity, that the interest rate $r = 0$ and the subjective discount rate $\delta = 0$. The agents has wealth a_0 to begin with in the first period ($t = 0$).

(a) Write down the intertemporal budget constraint for this agent.
Answer.

$$\sum_{t=0}^{T} c_t = \sum_{t=0}^{T} y_t$$

(b) Write down the agent's optimization problem to obtain the optimal consumption plan. Describe how consumption changes with time, does it increase? decreases? *[No need to determine the actual consumption level, just describe how it evolves on time.]*
Answer.
The agent's problem is

$$\max_{\{c_t\}} \sum_{t=0}^{T} u(c_t) + \lambda \left(a_0 + \sum_{t=0}^{T} y_t - \sum_{t=0}^{T} c_t \right).$$

The first order conditions are

$$u'(c_t) = \lambda \qquad \forall t$$
$$\Rightarrow \quad u'(c_t) = u'(c_s) \qquad \forall t, s.$$

Under the assumption that the instantaneous utility function is the same every period, we obtain that consumption is constant.

(c) How important is the assumption that $r = \delta = 0$ for your answer in (b)? Can we change part of the assumption and still get the same result for consumption?
Answer.
The assumption that $\delta = r = 0$ guarantees that the agent finds optimal to choose a consumption profile that is flat (constant consumption) as in part (b). However, we only need $\delta = r$ for this result to hold, if both are equal but different from 0, the consumption profile would still be flat.

Now assume that the agent's income is constant in each period, $y_t = \overline{y} \quad \forall t$.

(d) What is the optimal consumption level in each period of life now? How much does the individual save in period $t = 0$?

Answer.

From the above assumptions and results, we know that it must be the case that $c_t = c \ \ \forall t$. We can use the budget constraint to write

$$\sum_{t=0}^{T} c = a_0 + \sum_{t=0}^{T} \overline{y}$$

$$\Rightarrow \ c = \overline{y} + \frac{a_0}{T+1}.$$

By definition, $s_0 = y_0 - c_0$. Using the values we just computed we obtain

$$s_0 = \overline{y} - \left(\overline{y} + \frac{a_0}{T+1}\right) = -\frac{a_0}{T+1}$$

Notice that savings are positive if and only if $a_0 < 0$.

(e) Now suppose that the government announces in $t = 0$ that will tax individuals. The government implements a tax on income in each period of life such that the agent receives $(1-\tau)\overline{y}$ each period, where $\tau = 1/(1+T)$. How much does the tax reduces the agent's per-period income? How much does optimal consumption falls at $t = 0$?

Answer.

The individual's per period income is reduced by $\tau\overline{y} = \frac{\overline{y}}{T+1}$.

The individuals lifetime income was given by $a_0 + \sum_{t=0}^{T} y_t = (T+1)\overline{y}$; now it is given by $(T+1)(1-\tau)\overline{y}$. Thus, lifetime income was reduced by $(T+1)\tau\overline{y} = \overline{y}$.

We can use the same formula used in part (d) to obtain

$$c = \frac{a_0}{T+1} + (1-\tau)\overline{y}$$

So, consumption falls by $\tau\overline{y} = \frac{\overline{y}}{T+1}$, and this fall is the same in every period, not just in $t = 0$.

(f) Now assume that instead of a constant tax rate on each period, the government sets a tax rate $\tau = 1$ in $t = T$ and 0 in all the earlier periods. How much does the tax reduces the agent's per-period income? How much does optimal consumption falls at $t = 0$?

Answer.

The per-period income falls by $\overline{y}$ only in the last period of life.

The change in lifetime income is the same as in part (e). Thus, the change in consumption is the same too.

(g) Compare your answers in parts (e) and (f). Explain the intuition.

Answer.

Individuals' consumption only responds to changes in lifetime income (or permanent income). In this particular case, given the assumption that $r = 0$ (the $\delta = 0$ part guarantees that the optimal consumption profile is flat given $r = 0$), all that matters for the consumption level is the sum of all income received during lifetime.

In both tax regimes, the government tax has the same affect in total lifetime income (and thus on permanent income), and therefore, it will have the same effect on consumption in

each period of life. When the tax is actually taken from the individuals doesn't matter. i.e. contemporaneous (or transitory) changes in income matter for consumption only if they change lifetime income!

If you want to see this more clearly, think of the effect on consumption of a tax $\tau = 1$ in $t = s$ followed by a transfer (i.e. the government gives resources to the consumer, or a negative tax) of $\tau = -1$ in $t = s + 1$. What's the effect on consumption today $t = 0$? NO effect!