Brane Worlds, Brane Worlds: It's Party Time: Excellent.

by

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Submitted to the Department of Physics
in partial fulfillment of the requirements for the degree of

Doctor of Philosophy

at the

MASSACHUSETTS INSTITUTE OF TECHNOLOGY

May 2001
[June 2001]

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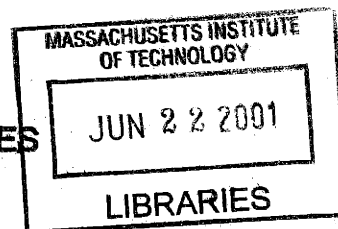
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Abstract

In recent years branes have had significant impact in the construction of higher dimensional theories of SUSY breaking, and of gravitational phenomena. In particular they provide strong motivation for considering anomaly mediation as a mechanism of SUSY breaking communication. Anomaly mediation provides a very predictive spectrum for the superpartner masses, which is quite model independent, and is given in terms of the beta functions and anomalous dimensions of the fields. This spectrum is also largely flavor blind, and thus does not suffer from flavor changing neutral current problems. Anomaly mediation, however, predicts negative slepton mass squares. In this thesis we will try to address this problem by providing positive contributions to the slepton masses, through higher order SUSY breaking effects stemming from having additional heavy fields in the theory. We will also provide natural values for the μ and B parameters of the MSSM. Branes also allow for the surprising possibility that we may be living in a world with infinite extra dimensions with minor observational consequences. In this dissertation we will show that the RS2 and LR models, which have a single infinite dimension, are completely consistent and conform nicely with current experiments. We will also comment on possible holographic interpretations of these models as having 4D gravity coupled to a fully quantum conformal field theory.

Thesis Supervisor: Lisa Randall
Title: Professor

Acknowledgments

There are many people who deserve the credit for my turning out the way I am.

First, let me thank my fun and inventive graduate advisor, Prof. Lisa Randall. Prof. Randall has a wonderful sense of intuition, and has taught me a great deal of physics by asking insightful and often difficult questions. I was also constantly inspired by her knowledge and curiosity concerning a wide variety of different aspects of high energy physics. Regarding the work of this dissertation, few students could claim that their thesis work was entirely based on physics created by their advisor. While on various travels, Prof. Randall always made sure that I was busy with a project, and often took me along with her. Happily, I was hence exposed to various scientific, as well as cultural environments.

I would also like to thank my collaborators for their explanations, patience and help. In particular, O. DeWolfe, M. Gremm, A. Iqbal and especially A. Hanany for teaching me about string theory, branes, and string junctions. Also, I would like to thank Y. Shadmi and Y. Shirman for teaching me quite a bit about SUSY field theory, and the aesthetics of good model building. Finally, I wish to thank S. Giddings for sharing with me his knowledge of gravitation and especially its connection with AdS/CFT.

Let me thank Profs. A. Hanany and F. Wilczek for reading my thesis, for being part of my committee on such short notice, and for making my thesis defense a memorable, and enjoyable experience.

Power to my good friends on the east coast and the west coast, who have endured my insanity and did not fail to remind me that there be life outside of physics.

I would like to thank my entire family for their care, love, and support. Especially, let me thank my mom, Adela for her persistent attempts to guilt me in all the right directions, and my sister, Sarah, who has to constantly face the shock that I am actually her brother. Finally, if there is any family member who is directly guilty of making this thesis possible, it is definitely my dad, Gabriel, who managed to irrevocably warp my mind at a young age towards science.

As a grand finale, I would like to thank my love, companion, and wife, Ilana, who had to put up with my physics lifestyle choice. Throughout my time at MIT, she had to suffer through my chaotic work schedule and my demanding immersion into the physics world, often having to take care of things in the real world for both of us. She also had to endure the challenging months during which we were separated while I was away working in Princeton. Despite all this, she has been a constant lifeforce, helping me to find a sane path, and supporting me in many difficult decisions. For this I am indebted to her, and truly thankful.

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Chapter 1

Introduction

1.1 The challenges of a supersymmetric solution to the hierarchy problem

One of the most fundamental challenges of particle physics is to address the vast hierarchy of scales between the Planck mass and the scale of electroweak symmetry breaking. Assuming, as in the Standard Model (SM), that the Higgs is a fundamental scalar particle, this is two-fold challenge. One must not only explain why the Higgs mass is much smaller than the Planck scale, but also provide a reason for this mass to remain small against radiative corrections. Such corrections would naturally push the Higgs mass to highest available scale, which would be near the Planck scale. Thus, the problem involves two separate types of fine tunings.

Traditionally, these two facets of the so called hierarchy problem were addressed using four dimensional field theory models and techniques. Among these, supersymmetry (SUSY) emerged as a very compelling framework. Models with softly broken supersymmetry naturally protect the mass of the Higgs from radiative corrections due to near cancellation between fermion and partner boson contributions. The simplest supersymmetric extension of the Standard Model was constructed, named the Minimal Standard Supersymmetric Model (MSSM). This model doubles the field content of the Standard Model by introducing a SUSY partner for each SM field. In addition,

a second Higgs must be added in order to cancel gauge anomalies stemming from the spinor partner of the first Higgs. Supersymmetry, however, introduces its own hierarchy problems. These include facing the first fine tuning issue of above, by explaining why it is that mass of the Higgs is so small. This is related to the so called μ problem.

The μ parameter is a supersymmetric Higgs mass which is permitted by the symmetries of the MSSM, and therefore is not a priori related to the mass scale of the soft masses that originate from SUSY breaking. It is in fact a mixing term between the two Higges,

$$\int d^2\theta \mu H_1 H_2, \tag{1.1}$$

which appears in the MSSM Lagrangian. Here, H_1 and H_2 are the two Higgs superfields This mass, however, must be on the order of the electroweak scale (rather than more naturally the fundamental scale) if it is to generate the correct electroweak symmetry breaking. Related to the μ is also the μB parameter, a SUSY breaking term which is responsible for mass mixing between the two Higgs scalars. Thus all together, the mass terms in the Higgs sector are

$$\mu^2(|h_1|^2 + |h_2|^2) + \mu B(h_1 h_2 + h.c.). \tag{1.2}$$

For the consistency of electroweak breaking, all above parameters need to be on the order of the electroweak scale, and in particular, μB is in danger of being far greater if μ is not small.

The MSSM also introduces a whole new set of soft mass parameters which could potentially lead to new sources of CP violation and flavor changing neutral currents (FCNC). In particular, in flavor space, the squark mass matrices need not be diagonal in the same basis in which the quark mass matrices are diagonal. This potentially leads to flavor changing processes, unless the squark mass eigenstates are somehow aligned with the quark mass eigenstates. Thus, if the squarks are not extremely heavy, one has to generate new mechanisms to secure alignment.

Within the context of four dimensional models, there have been two proposals for the communication of SUSY breaking that have had partial success in addressing the

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above issues. In gravity mediated scenarios, the μ problem can be partially addressed using the Giudice-Masiero mechanism. Here, one assumes the this term is absent in the supersymmetric limit, and hence comes from Planck suppressed operators involving a SUSY breaking field coupling to the Higgses. This naturally generates parameters on the order of the electroweak scale. Gravity mediated scenarios, however, have no mechanism for generating flavor diagonal squark mass matrices, as gravitational interactions are generically not flavor symmetric. Indeed, one expects interactions between the SUSY breaking sector and the MSSM of the form:

$$\int d^4\theta \frac{1}{M_{pl}^2} \Sigma^\dagger \Sigma Q_i^\dagger Q_j, \tag{1.3}$$

where Σ is a SUSY breaking superfield with a non-zero F-term, and the indices on the quark fields are flavor indices. Such interactions therefore lead to dangerous FCNC processes.

The second proposal suggests that SUSY breaking is mediated instead through gauge interactions. Since gauge interactions are flavor blind, these scenarios do not suffer from FCNC problems. However, they face difficulties in generating μ and μB parameters of the right size. This is because both μ and μB receive similar suppression, and so μ turns out to be much smaller than B. It is thus necessary to find new SUSY mediation mechanisms that address both the μ problem and do not generate FCNC.

1.2 Branes and the sequestered sector models of SUSY breaking

In recent years, progress in string theory, and in particular the discovery of branes, has provided a whole new set of tools in constructions of higher dimensional models which address the above challenges. Initially, branes were seen as supersymmetric supergravity solitons of varying dimension. These solitons were mostly black hole type solutions (in the space transverse to the branes) which were also sources for

Ramond-Ramond gauge fields. It was later discovered that certain branes have a worldsheet description as open string end points. Namely, the endpoint of the open string must obey Neumann boundary conditions along the worldvolume of the brane, and Dirichlet boundary conditions in directions transverse to the brane. These so called Dirichlet-branes, where thus found to be dynamic, fluctuating objects - higher dimensional analogs of particles. Worldsheet calculation then showed that the open string D-brane vacuum provided a source for Ramond-Ramond fields, completing the identification of D-branes with the supergravity solitons. Through a web of string dualites, various other branes were consequently found to be dynamical objects. Unlike previously known field theory solitons, branes can posses rich worldvolume theories that contain spinors, scalars, and non-Abelian gauge fields. For the first time one could consider subspaces of spacetime which completely localize gauge theories resembling those of the Standard Model. Due to this fact, branes have had considerable impact on the study of field theory and have provided compelling and novel options for phenomenological models with extra dimensions.

The advantage of using branes in building such models is that they allow one to construct models with controlled communication between different sectors. These sectors may live on distinct branes, and thus be spatially separated. Choosing appropriate fields to live in the bulk between the branes provides new mechanisms for generating suppression of the interactions of these sectors.

One appealing example of this is the sequestered sector model for SUSY breaking of Randall and Sundrum [2]. These models were originally inspired by the Horava-Witten setup of $E_8 \times E_8$ heterotic M-theory, which is a low-energy description for the strongly coupled heterotic string. In this setup, one has eleven dimensional supergravity compactified on a S^1/Z_2 orbifold, where the radius of the eleventh dimension is linked to the coupling of the ten dimensional Heterotic string. While the bulk contains only supergravity fields, there are orbifold branes sitting at the orbifold fixed points which contain matter and gauge fields. In fact, fields transforming under the first E_8 sit at one of the two branes, while those transforming under the second E_8 live at the other orbifold brane. Upon compactification to five dimensions, one can

embed the GUT group into one of the E_8 factors, thereby taking the MSSM to be localized to an orbifold three brane within a five dimensional bulk. Similarly, the sequestered sector models include one extra dimension which is compacitified on a S^1/Z_2 orbifold. The branes are taken to sit at the two orbifold fixed points, with the bulk of the fifth dimension containing only the gravity multiplet of supergravity. The MSSM is localized to one of the orbifold branes, while the SUSY breaking sector is localized on the other brane. At energies below the compactification scale, which gives the separation of the two branes, one has the MSSM and sequestered sector matter coupled to $N = 1$ 4D supergravity.

The crucial difference between these models and the gravity mediated scenarios mentioned above, is that the extra dimension forbids Planck suppressed operators like (1.3). This is because the two sectors are separated by a distance on the order of the GUT scale and hence cannot have local, point like interactions. These Planck suppressed operators, upon SUSY breaking, were exactly the ones contributing to flavor non-diagonal squark masses. Thus by avoiding them, we ensure that there would be no FCNC processes. With the usual Planck suppressed interactions gone, the SUSY breaking is communicated instead through terms which result from quantum anomalies, now providing the dominant contribution. In particular, the terms that generate superparticle masses are related by supersymmetry to non-local terms involving the metric and matter fields that reproduce the scale anomaly. This method of communication of SUSY breaking is hence called Anomaly Mediation. Since the scale anomaly in quantum theories is connected to the running of the couplings, the superparticle masses are uniquely predicted in terms of the β functions and field anomalous dimensions (the γ functions) in a model independent way. The gaugino and scalar masses are given by [2, 3]

$$
\begin{aligned}
m_{\tilde{g}} &= -\frac{g^2}{16\pi^2} b_0 F \\
m_s^2 &= -\frac{1}{4} |F|^2 \left(\frac{d}{dln\mu} \gamma_s \right) .
\end{aligned}
\qquad (1.4)
$$

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Here, F is the gravitino mass, and b_0 is the one-loop beta function of the gauge group associated to the gaugino. This leads to largely flavor independent results for the squark masses, especially for the first two generations (where the Yukawa couplings are negligible), thus avoiding FCNC.

Unfortunately, anomaly mediation results in negative scalar partner mass squardes for theories which are not asymptotically free. As a consequence, the sleptons in the MSSM are in danger of Higgsing the photon. The goal of Chapter 2 of this dissertation, based on work done with Shadmi and Shirman [1] is to address this problem. We would like to provide positive contributions to the slepton masses entirely within a 4D framework, i.e. we will work below the compactification scale. We assume that the only source for SUSY breaking comes through anomaly mediation. The idea is then to introduce additional heavy fields at a intermediate scale, M, between the compactification scale and the scale F of SUSY breaking. Specifically, we will look for non-decoupling, heavy threshold, effects in anomaly mediation. These effects will turn out to be higher order in the SUSY breaking scale (i.e. $\sim F^4/M^2$), and will subsequently be used to correct the slepton masses. We will also use these higher order effects to produce natural μ and B parameters.

1.3 Branes and gravitational models with extra dimensions

Besides their impact on models of SUSY breaking, branes have also revolutionized gravitational models with extra dimensions. These more recent models were initially based on the observation that if the Standard Model fields were confined to a three dimensional brane, current experiments could not rule out the possibility of macroscopic size extra dimensions. The brane was crucial in these scenarios to insure that the SM interactions indeed remain four dimensional, with the more stringent experimental constraints coming from the new, higher dimensional, nature of gravity. The purpose of these models was to make the entirely novel suggestion that the hierar-

chy between the electroweak scale of a TeV, and the Planck energy scale could be of geometric origin. Namely, one assumes that the fundamental scale of the higher dimensional theory is a TeV (instead of Planck), thus making the electroweak scale a natural scale from the particle physics point of view. In other words, one does not need to worry about radiative corrections pushing the Higgs mass further than a TeV. The Planck scale is then a derived scale and is large due to the large volume of the extra dimensions. These type of models, however, suffered from a different type of hierarchy problem. Namely explaining the large size of newly introduced extra dimensions, as compared to the fundamental scale of physics.

Fortunately, it was found by Randall and Sundrum [25] that it is possible to avoid having very large length scales if one considers gravitational backgrounds with non-trivial curvature. In these so called warped backgrounds, the fascinating observation was made that the metric including the extra dimensions need not have a simple product structure. In other words, it is consistent with the Poincare symmetry of our particle physics world, to have the metric vary (or warp) along the extra dimensions. Randall and Sundrum constructed a model where the hierarchy came from an exponential warp factor in the metric without the need to introduce very large or small scales. This model, which we will call RS1, consists of two branes with finite tensions living in a world with a single, compact extra dimension. In a sense, this setup is reminiscent of the sequestered sector models, where matter is taken to live on orbifold branes, while the bulk between them contains only gravity. In addition, one allows for the possibility of a bulk cosmological constant. The important difference is that tensions of the branes are assumed to be non-zero, and hence one needs to take into account the back-reaction of the branes on the geometry. By tuning the bulk cosmological constant against the brane tensions, one can find the following Poincare invariant (flat brane) solution:

$$ds^2 = e^{-2r_c|\phi|/R}(\eta_{\mu\nu}dx^\mu dx^\nu) + r_c^2 d\phi^2. \tag{1.5}$$

Here, $R = \sqrt{-12M^3/\Lambda}$, where M is the 5D fundamental scale, and Λ is the bulk

cosmological constant (which is taken to be negative). The two branes are located at $\phi = 0$ and $\phi = \pi$, and have the tensions $+12M^3/R$, and $-12M^3/R$, respectively. Let us remark that having a negative tension brane is not an immediate catastrophe, as it taken to sit at an orbifold fixed point, and hence cannot fluctuate. The fine tuning of the tensions is associated with setting the 4D cosmological constant to zero. Thus, it is related to the usual cosmological constant hierarchy problem, and we will not attempt to address it here. We note that all of the above parameters are taken to be not much larger than the fundamental length scale, with the requirement that they are sufficiently large so that classical gravity may be trusted. We will now show that despite these fundamental scale parameters, we can naturally generate the electroweak scale without fine tuning.

Suppose that the SM Higgs is confined the negative tension brane at $\phi = \pi$. Its action is therefore given by

$$
\begin{aligned}
S = & \int d^4x \sqrt{-G(\phi = \pi)}\left(G^{\mu\nu}(\phi = \pi)\partial_\mu H \partial_\nu H + (H^2 - m_0)^2\right) \qquad (1.6)\\
= & \int d^4x\, e^{-4r_c\pi/R}\left(e^{2r_c\pi/R}\eta^{\mu\nu}\partial_\mu H \partial_\nu H + (H^2 - m_0)^2\right),
\end{aligned}
$$

where again we assume that the mass parameter is near the fundamental scale. We notice that in this action, the Higgs field does not have a canonical kinetic term. We therefore rescale the field, to obtain the following action for the properly normalized Higgs

$$
\int d^4x \left(\eta^{\mu\nu}\partial_\mu H \partial_\nu H + (H^2 - e^{-2r_c\pi/R}m_0)^2\right), \qquad (1.7)
$$

An interesting thing has occurred, the mass parameter in the action is rescaled by the warp factor. By choosing r_c to be slightly bigger than R (assumed to be near the fundamental scale), one can easily achieve a large hierarchy between the weak scale and the fundamental scale. This rescaling of the mass by the warp factor is characteristic of all mass parameters at the negative tension brane. Consequently, from the view point of an observer living on the negative tension brane, strong gravitational effects associated with the 5D fundamental scale, set in at around a TeV, making that the effective strong coupling scale. It remains to relate the fundamental scale of the 5D

theory, to the 4D Planck scale. At low energies we can safely assume that the metric will not vary significantly in the extra dimension. We therefore take the metric to have the zero mode profile

$$ds^2 = e^{-2r_c|\phi|/R}(g_{\mu\nu}(x)dx^\mu dx^\nu) + r_c^2 d\phi^2. \tag{1.8}$$

Inserting this into the five dimensional gravity action yields upon integration in the fifth dimension

$$S = \int d^4x \int_0^\pi d\phi M^3 \sqrt{-g} e^{-2r_c\phi/R} \mathcal{R}[g] = \int d^4x \sqrt{-g} \frac{M^3 R}{2}(1 - e^{-2r_c\pi/R})\mathcal{R}[g]. \tag{1.9}$$

Hence, we can identify $\frac{M^3 R}{2}(1 - e^{-2r_c\pi/R})$ with the 4D Planck mass, M_{pl}, which implies (given $r_c \sim 30R$ for generating the hierarchy) that the 5D fundamental scale is near the Planck scale. Thus the warp factor indeed generates the hierarchy between the Higgs mass and the 4D gravitational scale.

Perhaps equally exciting is the fact that the above warping allows for the extra dimension to be macroscopic, or even infinite, without observable consequences. We can already see an indication of this by taking $r_c \to \infty$ in (1.9), and noting that M_{pl} remains finite. In other words, even if the fifth dimension is infinite, 4D gravity does not decouple from the dynamics. Randall and Sundrum, hence, proposed a model [26] (dubbed RS2), which includes only a positive tension brane, with the negative tension brane removed to infinity. This model is appealing in that it requires no stabilization of moduli related to volume of the extra dimension, namely the radion field. In this non-compact model, the Standard Model fields are taken to be confined to an orbifold three brane and thus still have four dimensional physics. Gravity, while localized near the positive tension brane (or Planck brane), is still five dimensional in nature. Namely, there is a continuous Kaluza-Klein (KK) tower of graviton modes that start from zero mass. Their contribution is suppressed near the brane, but significant in the bulk as one moves away from the brane. Thus, unlike in standard compactifications, the low energy physics is not captured just by the zero mode.

In fact, in the early stages of examining the model, it was found that including only the zero mode leads to solutions which are singular near the horizon [91] (which appears once the negative tension brane is completely removed). The aim of Chapter 3, based on work done with Giddings and Randall [69], is to show that including the KK modes leads to consistent gravitational phenomenology. We do this by solving the five dimensional Einstein's equations in the RS2 background in the linearized approximation. As a result of this analysis we are able to estimate the shape of 5D black holes stuck to the brane, describe the metric profile due to general localized sources, and also find the deviations from 4D Einstein gravity resulting from the 5D physics. This analysis can also be applied to another non-compact model, the Lykken-Randall (LR) model, which combines the features of RS1 and RS2. In this model, as in RS1, there are two branes, the Planck brane and the TeV brane (where the SM lives). However, the TeV brane is assumed to have a negligible tension, and thus one can ignore its back-reaction on the geometry. Consequently, the metric background is that of RS2, but one still has the solution to the hierarchy problem of RS1. Chapter 3 will thus also include a discussion of the gravitational phenomena as seen by an observer living on the LR zero tension (probe) brane.

1.4 Holography and the low-energy theory of the non-compact models

An interesting problem related to the above non-compact models is to provide an effective 4D description of their low energy excitations. As discussed, the low energy physics of the non-compact models is quite rich, and includes a continuum of low lying KK modes. It was suggested by Maldacena and Witten, based on the similarity between RS2 and the near-horizon geometry of D-branes, that from AdS/CFT one learns that the effect of these KK modes can be accounted for by a CFT. Namely, instead of describing the 5D gravity of RS2, one can rather consider 4D gravity coupled to a fully quantum conformal field theory. This idea is motivated by the

observation that string theory propagating on an AdS_{d+1} background (RS2 is in effect truncated AdS_5) is holographically captured by a d-dimensional theory without gravity, living on the boundary of AdS_{d+1}. The isometries of the AdS_{d+1} space induce conformal transformations on its boundary, and therefore the holographic theory should be a conformally invariant one. The hope is that the string theory realization of RS2 will have the same features, and so for an observer sitting at the Planck brane (which acts as a boundary), the 5D physics will be holographically captured by a CFT. The purpose of Chapter 4, which is based on work done with Giddings [70], is to examine this possibility. In particular, we will provide a dictionary (within the limitations of a linearized gravity analysis) between bulk 5D energy sources, and their effective description in terms of the 4D CFT. We will show that for energies below the curvature scale of the RS2 background, our results are consistent with a CFT interpretation. For fun, we will then apply our results to the specific case of a particle free falling towards the RS2 horizon, while flying by a black hole stuck to the brane (the shape of which is discussed in Chapter 3). We will give the CFT description of this, as seen by the Planck brane observer, checking that the whole process can be understood using only 4D physics. Finally, we will attempt to extend the holographic interpretation to the TeV brane observer in the RS1 and LR models, concentrating on describing strong 5D gravity effects such as black hole production.

Chapter 2

Heavy Thresholds, Slepton Masses and the μ Term in Anomaly Mediated Supersymmetry Breaking

2.1 Introduction

Recently, it was pointed out that in the presence of any supersymmetry-breaking hidden sector, soft supersymmetry-breaking terms are generated for the observable fields through the super-Weyl anomaly [2, 3]. Because of their origin, these supersymmetry (SUSY) breaking terms are directly related to the scaling dimension of the relevant operator.

The soft terms are most readily obtained by working in a supergravity formulation that uses the superconformal, or super-Weyl compensator, Φ, a non-dynamical chiral superfield which allows one to write a manifestly super-Weyl invariant action at the classical level [4]. Essentially the role of Φ is to compensate for the non-trivial super-Weyl transformations of different action terms, while any Φ vev breaks the super-Weyl invariance. In particular, one may choose the lowest component Φ vev to be 1, leading

to a trivial modification of the action. However, for the discussion of supersymmetry breaking, we will focus on the auxiliary component of Φ. As was argued in [2], in the presence of supersymmetry breaking, this auxiliary component acquires a non-zero vev, which we denote by F, so that $\Phi \equiv 1 + F\theta^2$.

To recover the observable sector Lagrangian coupled to canonical supergravity, one then rotates Φ away through a super-Weyl rescaling. For a scale-invariant observable sector Lagrangian, Φ disappears altogether. However, if any explicit mass scale appears in the Lagrangian, it gets rescaled by Φ as $M \to M\Phi$, so that in the presence of supersymmetry breaking, various tree-level supersymmetry splitting masses appear.

Even if there are no explicit mass scales in the classical theory, some mass scale is generated quantum mechanically. In particular, if we consider the renormalized Lagrangian, the wave function renormalizations of the fields contain some cutoff dependence. The cutoff scale now appears multiplied by Φ. Expanding the wave function renormalization in F (which enters through Φ), one finds a soft supersymmetry breaking mass for the relevant field.[1]

Let us concentrate for now on the scalar masses squared. Since these masses squared enter through [2]

$$Z(\mu) = Z_0(\mu) \left(1 - \frac{1}{2}\gamma_i(\mu) \left(F\theta^2 + c.c. \right) + \frac{1}{4}\,\dot{\gamma}_i\,(\mu)\,|F|^2\,\theta^2\bar{\theta}^2 \right) , \qquad (2.1)$$

they have two immediately apparent features. First, the soft mass has no $\mathcal{O}(F^3)$ or higher contributions. Second, while the soft masses can be obtained through (2.1) at the UV cutoff of the theory and then evolved down to any lower scale, it is also possible to directly evaluate them at the low scale, again through (2.1). The reason is of course that the wave function renormalization already takes running effects into account. Thus, at any scale, the anomaly-mediated (AM) masses appear to be determined solely by the theory at that scale, specifically by the β-function and anomalous dimensions at that scale, with no memory of the theory at higher scales.

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[1] The extraction of these soft masses is completely analogous to the method of calculating soft masses in models of gauge-mediated supersymmetry breaking through wave function renormalizations of [6, 7].

This picture has been derived to leading order in the SUSY breaking parameter in a theory in which the only source of SUSY breaking is anomaly mediation.

If there are any additional contributions to the soft masses at high scales (such as the compactification scale), from, say, different gravitational sources, these contributions affect the running of the masses to lower scales.

Even if the only source of supersymmetry breaking in the theory is anomaly mediation, which generates soft masses near the UV cutoff of the theory which are given by (2.1), one may wonder about what happens when the theory also contains some heavy thresholds. The mass of the heavy fields, which we denote by M, is rescaled as $M \to M\Phi$. Therefore the heavy fields obtain tree-level supersymmetry splittings proportional to F. If the heavy fields interact with some of the light fields, they can then contribute to the light fields' soft masses at the loop level. Still, these contributions largely decouple [2, 5]. More specifically, the contribution of any heavy field decouples at order F^2. This should be apparent from our discussion above – since the AM soft masses can be read of the wave-function renormalizations, the effects of heavy thresholds should somehow be accounted for already. In section 2.2, we will show in detail how this decoupling occurs. However, we will argue that the heavy thresholds do not decouple completely. There are in fact two sources of non-decoupling.

First, the $\mathcal{O}(F^2)$ effects of the heavy threshold do not decouple if there is a light field associated with this threshold [5]. More specifically, this happens if the masses of the heavy fields are determined by some modulus, and this modulus is stabilized primarily by small supersymmetry breaking effects.

Second, if there are fields of mass M and supersymmetry splittings proportional to F that interact with the light fields, the loop-level soft masses that they induce contain contributions that are higher order in F. For example, scalar masses squared obtain contributions of order F^4/M^2. Furthermore, in many cases these contributions start at one-loop, whereas the F^2 contributions start at two-loops. Thus, if M is not much bigger than F, these contributions may be important.

Another interesting issue to address is the decoupling of D-terms. As we shall see in section 2.3, in anomaly mediation D-terms decouple to leading order in the SUSY

breaking, unlike in the usual case. Again, however, F^4/M^2 contributions to D-terms do not decouple.

The above observations may be used to address the main phenomenological problem of the minimal anomaly-mediation scenario [2]. In this scenario, the soft breaking terms of the supersymmetric standard model (SM) are generated purely by anomaly mediation, and are therefore given by (2.1). The slepton masses squared are proportional to the $SU(2)$, and $U(1)$ β functions, and are therefore negative. Ref. [2] invoked additional gravitational contributions to overcome this problem. Ref. [5] proposed a solution in which the soft spectrum is modified by the presence of a light (order F) modulus, which is massless in the supersymmetric limit. Another solution involves additional Higgs doublets that generate large Yukawa couplings for the sleptons [9]. Here we will instead introduce new heavy thresholds, within one or two orders of the supersymmetry breaking scale F, and utilize the resulting F^4 soft masses to generate positive slepton masses squared. Our model, which has as its basis a simple $U(1)$ theory, which we describe in Section 2.3, illustrates the decoupling of order F^2 contributions and the non-decoupling of F^4 terms. It also clarifies the issue of D-term decoupling in AM scenarios, as some of the contributions we will discuss can be understood as arising from the $U(1)$ D-term.

As we said above, the crucial contributions in our model are of the form F^4/M^2. In Section 2.4, we will show how, in a modification of our model, the relevant scale M is generated dynamically from F. In fact, as we will see, it is quite easy to generate scales that are close to the SUSY breaking scale using anomaly mediation. This fact could be useful for model building purposes. In Section 2.5, we put these pieces together and construct a modification of the SM in which sleptons obtain positive masses squared.

In Section 2.6, we again use the same ingredients, that is, non-decoupling $\mathcal{O}(F^4/M^2)$ contributions and a scale M that is generated dynamically from F, to naturally obtain a μ-term and a B-term of the correct size.

We close this Introduction with one comment. The mechanism of anomaly-mediated supersymmetry breaking exists in any theory with some supersymmetry

breaking sector. In particular, it exists in any 4-dimensional theory. However, in a 4-dimensional theory, higher order, M_p-suppressed operators that couple the observable and hidden sector, typically generate tree-level soft masses for the observable fields. These are the well known "hidden sector" contributions. These contributions are larger than the anomaly mediated contributions, which are loop-suppressed. In a 4-dimensional theory, it is hard to rule out the existence of the tree-level hidden sector contributions. However, in [2], it was shown that in $d > 4$ dimensions, such direct couplings of the observable and hidden sector can plausibly be absent. Thus, while our discussion of decoupling is independent of the dimensionality of the full theory, when we move on to model building we envision a situation in which the soft terms are purely anomaly mediated, as in sequestered sector models [2]. We implicitly assume then that the full theory has more than 4 dimensions, and the appropriate cutoff scale, at which the anomaly-mediated soft masses of the observable sector are generated, is the compactification scale.

2.2 The effect of mass thresholds on soft masses of light fields

We would like to study the impact of supersymmetric mass thresholds on anomaly mediated SUSY breaking terms. We assume that the heavy thresholds lie above the visible sector SUSY breaking scale, F, and below the UV cutoff scale, whether it is the Planck scale or the compactification scale. The decoupling of these thresholds can then be addressed in terms of an effective 4-dimensional field theory. We also assume that there are no sources of SUSY breaking other than anomaly mediation. A discussion of decoupling effects at leading order in the SUSY breaking recently also appeared in Ref. [5].

In the absence of heavy thresholds, slepton and squark soft masses follow directly

from the wave function renormalization [2]:

$$\int d^4\theta Z \left(\frac{\mu}{\Lambda(\phi^\dagger\phi)^{1/2}} \right) Q^\dagger Q \qquad (2.2)$$

$$= \int d^4\theta \, Z \left(\frac{\mu}{\Lambda} \right) \left(1 - \frac{1}{2}\gamma(F\theta^2 + c.c) + \frac{1}{4}(\partial_t\gamma) |F|^2 \theta^2\bar\theta^2 \right) Q^\dagger Q \ .$$

Here $\Phi = 1 + F\theta^2$ is the super-Weyl compensator, Λ is the UV cutoff, $t = \ln\mu$, and the anomalous dimension $\gamma = \partial \ln Z / \partial \ln \mu$. Rescaling the fields

$$Q \to Z^{-1/2} \exp\cdot(\frac{1}{2}\gamma F\theta^2)\, Q \ , \qquad (2.3)$$

to obtain canonical kinetic terms, and expanding the remaining exponent yields the following soft mass for the light field Q:

$$m_s^2 = -\frac{1}{4} |F|^2 (\partial_t\gamma) \ . \qquad (2.4)$$

We see that the soft masses are determined by the anomalous dimensions of the light fields. Thus, the soft masses (as well as other soft parameters) are largely insensitive to the details of the high energy theory.

However, in the presence of heavy mass thresholds, there may be additional contributions to the soft masses of the light fields. The theory contains explicit mass parameters which determine the heavy threshold. Due to the scaling anomaly, all such parameters will be accompanied by the compensator field Φ. The heavy fields then acquire tree-level supersymmetry-breaking mass splittings. As these fields are integrated out, their supersymmetry-breaking splittings may in principle generate soft terms for the light fields, through gauge or Yukawa interactions. We would like to understand to what extent such thresholds affect the masses of the light fields, and in particular, whether their effects completely decouple.

To study these questions, it will prove convenient to distinguish between two possibilities. One is that the heavy threshold, or in other words, the mass of the heavy fields associated with this threshold, appears as an explicit mass term in the

Lagrangian. The second is that the mass of the heavy field is given by the expectation value of some dynamical field. These two cases are different because vevs are determined dynamically and may depend on non-supersymmetric effects.

Let us first consider the case that the masses of all heavy fields arise from explicit mass terms in the superpotential. As explained above, any mass term M should be promoted to a superfield-valued $X = M\Phi$. We can then obtain the soft masses of the light fields at a scale $\mu < M$ from the relevant wave function renormalization $Z(\mu)$. Assume for simplicity that Z depends on a single coupling. Then, from the 1-loop relation

$$\partial_t Z = \frac{\alpha}{\pi} c \,, \tag{2.5}$$

where c is the appropriate quadratic Casimir, we first solve (2.5) for Z in the low energy theory, and then match boundary conditions at M. Using arguments of holomorphy and R-symmetry [6, 7], we then make the substitutions $M^2 \to X^\dagger X$ and $\Lambda^2 \to \Lambda^\dagger \Lambda$. Note that both X and Λ are superfield valued and contain the compensator Φ. This yields

$$Z\left(\alpha(\mu, X), \alpha(X)\right), \tag{2.6}$$

where,

$$\alpha(X)^{-1} = \alpha(\Lambda)^{-1} + \frac{b_H}{4\pi} \ln \frac{X^\dagger X}{\Lambda^\dagger \Lambda} = \alpha(\Lambda)^{-1} + \frac{b_H}{4\pi} \ln \frac{M^2}{\Lambda^2} \,, \tag{2.7}$$

$$\begin{aligned}
\alpha(\mu, X)^{-1} &= \alpha(X)^{-1} + \frac{b_L}{4\pi} \ln \frac{\mu^2}{X^\dagger X} \\
&= \alpha(\Lambda)^{-1} + \frac{b_L}{4\pi} \ln \frac{\mu^2}{\Lambda^2 \Phi^\dagger \Phi} + \frac{b_H - b_L}{4\pi} \ln \frac{M^2}{\Lambda^2} \,,
\end{aligned} \tag{2.8}$$

where b_H (b_L) is the one-loop beta function above (below) the scale M. Note that the Φ dependence cancels between X and Λ in the last term of (2.8). Therefore, the only contribution to the soft-mass squared, which is $\mathcal{O}(F^2)$, comes from the second term in the second line of eq. (2.8). Since this contribution is not related to the threshold M, the low-energy theory is insensitive to the mass splittings associated with the heavy threshold.

It is important to stress that while the previous discussion can be generalized to

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all orders in the coupling constants, it only holds to leading order in supersymmetry breaking, that is, $\mathcal{O}(F^2)$ corrections to the scalar-masses squared. We will return to the question of higher order corrections in the end of this section.

As we just saw, the crucial point for the decoupling of the heavy fields is that their mass scale M is rescaled by Φ in the same way that Λ is, so that the Φ dependence cancels in the ratio. This implies a specific relation between the SUSY masses and the soft masses of the heavy fields. We will refer to masses that are obtained by the rescaling of some mass-scale by Φ as being "aligned". For example, the mass of a chiral field is aligned if

$$\frac{\left| m^2_{scalar} - m^2_{fermion} \right|}{m_{fermion}} = F \, , \tag{2.9}$$

and $\text{Str} m^2 = 0$ in a supermultiplet.

The decoupling of the heavy threshold which we just saw may be understood as a cancellation between the anomaly-mediated contribution and the contribution of the heavy fields. To see that explicitly, consider an example in which the heavy fields are N_f pairs of fundamentals and antifundamentals of an $SU(N)$ gauge group, with supersymmetric mass M. We also assume that the light fields transform as fundamentals of the same $SU(N)$. Consider then the soft masses of the light fields, evaluated at a scale $\mu > M$,

$$m^2(\mu)_{\text{anomaly-mediated}} = 2c_0 b_H \frac{\alpha^2}{16\pi^2} F^2 \, , \tag{2.10}$$

with b_H and c_0 the one-loop beta function and one-loop anomalous dimension coefficients, respectively. To evaluate these masses at low-scales, we should evolve them down to the low-scale, but the only thing that evolves in (2.10) is the coupling α. However, as we go below the threshold M, we also need to integrate out the heavy fields, and they contribute to the soft masses of the light fields. Their contribution is the usual gauge-mediated contribution,

$$m^2(\mu)_{\text{gauge-mediated}} = 2c_0 N_F \frac{\alpha^2}{16\pi^2} F^2 \, , \tag{2.11}$$

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Adding the contributions (2.10) and (2.11), we find, at $\mu < M$,

$$m^2(\mu) = 2c_0\, b_L\, \frac{\alpha^2}{16\pi^2}\, F^2\ ,\tag{2.12}$$

since $b_L = b_H + N_f$. This is precisely the soft mass we would obtain when calculating directly in the low-energy theory below the threshold M, ignoring the heavy fields altogether. We see that the contribution of the heavy fields through anomaly mediation, which is proportional to N_f and arises through their contribution to the beta function, exactly cancels their contribution through gauge mediation. Our discussion can be easily generalized to models with Yukawa couplings between light and heavy fields.

This cancellation may also be seen diagrammatically in the Pauli-Villars regularization scheme. The regulator fields always have aligned masses $\Lambda\Phi$, because an explicit mass term must be put in for them. Now, when we consider any diagram contributing to the soft masses, with heavy fields running in the loop, there will be an analogous diagram with the regulator fields running in the loop. We can assume that the real fields have mass M, the regulators have mass Λ, and treat the splittings MF and ΛF as insertions. Then, to order F^2 (for which there is no dependence on the heavy mass), the heavy fields and the regulator fields have exactly opposite contributions to the soft masses. The contributions are guaranteed to be of exactly the same magnitude because the masses of both the heavy fields and the regulators are aligned.

Now let us consider the possibility that some SUSY masses originate from non-zero vevs. Take X to be a field which acquires a vev in the supersymmetric limit. Fields that couple to X obtain a SUSY preserving mass $C\langle X\rangle$, and soft mass splittings CF_X, with C some constant that depends on different couplings.[2] Such heavy fields decouple from the effective theory of the light degrees of freedom if their masses are aligned, that is, if $F_X = XF$. In the limit of unbroken SUSY, we write $X = v + \delta x$,

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[2]For simplicity we assume here that the coupling is renormalizable, but it is easy to repeat the following discussion for the general case.

where v is a background superfield whose lowest component is just the vev of X, while δx is the fluctuating part of X whose vev vanishes in the supersymmetric limit. Since v is determined by a combination of mass parameters in the superpotential, upon supersymmetry breaking, it acquires an F-term which is automatically aligned with the scalar component, $v \to v(1 + F\theta^2)$. On the other hand, when supersymmetry is broken, δx can obtain both a scalar vev and an auxiliary component vev, that are determined by non-holomorphic effects and thus are not necessarily aligned. Thus we need to understand under which conditions the vevs can become significantly non-aligned. The Lagrangian for δx has the form

$$\int d^4\theta \; \left[(1 - m_x^2\theta^4)(\delta x)^\dagger(\delta x) + \left(v(\Phi^\dagger - m_x^2\theta^4)(\delta x) + \text{c.c.}\right)\right] + \qquad (2.13)$$
$$\left(\int d^2\theta \tfrac{1}{2}\, m\,(\Phi - a\theta^2)(\delta x)^2 + \text{c.c.}\right) \; ,$$

where m is the mass of δx in the supersymmetric limit, while a and m_x^2 are soft mass parameters generated by anomaly mediation. Note that while we can treat m as a free parameter (in a given model, it is determined by the detailed form of $W(X)$), a and m_x are suppressed relative to F by a one loop factor, $a \sim m_x \ll F$. We see that the radiative soft parameters in the Lagrangian will modify X and F_X. Solving for $F_{\delta x}$ and extremizing the potential we find for the corrected values of X and F_X at the extremum

$$X = v\frac{m^2 + ma}{m^2 + mF + m_x^2 + ma} \; , \qquad F_X = v\frac{m^2 F + mm_x^2}{m^2 + mF + m_x^2 + ma} \; . \qquad (2.14)$$

It is clear that in the limit $m \to \infty$ we recover the results obtained for explicit mass parameters in the Lagrangian as should have been expected. On the other hand, in the limit $m \to 0$ the extremum is found at $X = F_X = 0$ independently of the value of m_x^2. This point may in fact be the maximum of the potential if $m_x^2 < 0$. In such a case there may exist a local minimum at non-zero X if m_x^2 changes sign as a function of v [5]. Moreover, a local minimum may exist at non-zero X for any sign of m_x^2 if cubic and higher order superpotential terms for δx are taken into account [5]. In any

case, as we show and quantify momentarily, for small m alignment is lost in a vacuum with non-vanishing X.

Assuming that the eigenvalues of the X mass matrix are positive, so that (2.14) gives a minimum of the potential, we can easily check whether the corrected values of X and F_X are aligned,

$$\frac{F_X}{X} = F\left(1 - \frac{a}{m} + \frac{a^2}{m^2} + \frac{m_x^2}{mF} + \ldots\right), \qquad (2.15)$$

where the dots stand for higher order terms in m_x^2 and a. We see that alignment is approximately preserved if

$$a \sim \frac{y^2}{16\pi^2}F \ll m, \qquad (2.16)$$

where y stands for the couplings of X. Note that for small m, m_x^2 gives a subleading contribution since it appears at two loops whereas a appears at one loop. Moreover, if X interacts sufficiently weakly (so that a, which is one loop suppressed, is sufficiently small), it is possible that the mass of δx is significantly below the SUSY breaking scale, $m \ll F$, yet alignment holds to a good approximation. We therefore conclude that heavy fields whose masses are generated by the vevs of dynamical fields decouple so long as all mass parameters (including the masses of the fields which obtain vevs) satisfy the hierarchy (2.16).

As we mentioned above, our discussion so far only involved the scalar masses-squared to leading order in the SUSY breaking, $\mathcal{O}(F^2)$. We do not expect anomaly-mediation to generate contributions that are higher order in F. To have the correct dimension, such corrections should be of the form $\left(\frac{F}{\Lambda}\right)^n F^2$, where n is even and Λ is a physical cutoff associated with the compactification scale. [3]

In contrast, when heavy fields of mass M are present in the theory, with supersymmetry breaking mass splittings, they generally generate contributions to the soft

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[3] As always in our discussions in this chapter our statement is limited to the case that there are no significant bulk gravity contributions to the soft masses. These necessarily contribute to the masses at the compactification scale and thus alter the UV boundary conditions on the masses. As a result the usual formulae for anomaly mediated soft masses cannot be applied. In other words, the UV physics does not decouple.

masses of the light fields that are of order F^4/M^2. In the presence of Yukawa or gauge interactions between heavy and light fields, such contributions, which we call "light-heavy mixing", may even appear at one loop. Thus, the decoupling of heavy thresholds is not complete even when the masses of the heavy fields are aligned. The decoupling holds to leading order in F^2, where there is complete cancellation between anomaly-mediated contributions and light-heavy mixing but at order F^4/M^2 and higher, there is no analogous cancellation simply because there is no anomaly-mediated contribution.

This non-decoupling can again be seen by considering the regulator fields as above. We can calculate the contributions of both the dynamical fields, whose mass is M, and the regulator fields, whose mass is some physical UV cutoff Λ by treating the SUSY breaking parameter F as a small insertion. However, subleading contributions in F are suppressed by some power of F/M in the case of the dynamical fields, and by the same power of F/Λ in the case of the regulators. For very large Λ, the regulator contributions are negligible, and can no longer cancel the contributions of the dynamical fields. Indeed, the F^4/M^2 contributions will play an important role in the models we will discuss in the following sections. We will use these contributions to generate positive masses-squared for the standard-model sleptons.

2.3 A $U(1)$ model

We will now construct a simple $U(1)$ theory which illustrates the discussion of the previous section. The $U(1)$ is higgsed, with some fields becoming massive. These massive fields then generate one-loop $\mathcal{O}(F^4/M^2)$ contributions to the soft masses of the remaining light fields through gauge and Yukawa interactions. These contributions do not decouple. Some of the relevant contributions can be viewed as arising from the D-term. Different $U(1)$-charged heavy fields receive different anomaly-mediated contributions to their soft masses, and as a result a non-zero $U(1)$ D-term is generated. Generally, such D-terms do not decouple from the low energy theory even when the heavy fields are integrated out [8]. As we will see in the case of anomaly mediation,

the D-term decouples to leading order in the SUSY breaking F^2, but not at higher orders.

Our $U(1)$ theory consists of the fields $h_\pm$, $\chi_\pm$, and $l_\pm$, with $U(1)$ charges ± 1, as well as the gauge singlets S, $n_{i=1,2}$, with the superpotential,

$$W = S\left(\lambda_1\, h_+ h_- - M^2\right) + y_1\, n_1\, h_+\, \chi_- + y_2\, n_2\, h_-\, \chi_+ \, , \qquad (2.17)$$

where λ_1, $y_{i=1,2}$ are couplings. The first term is needed to break the $U(1)$. As we will see the last two terms with different Yukawa couplings, y_1 and y_2, are required to generate positive soft masses squared for some light scalars. With this superpotential all fields except $l_\pm$ acquire mass, either through the Higgs mechanism or through Yukawa interactions. We will eventually identify $l_\pm$ with standard model fields.

We assume that the only source of supersymmetry breaking in the theory is gravitational, through anomaly-mediation. We can then can keep track of supersymmetry-breaking effects by rescaling $M \to M\Phi$, with $\Phi \equiv 1 + F\theta^2$ as before. We also assume that $F \ll M$. The potential is then given by:

$$V = \left|\lambda_1\, h_+ h_- - M^2\right|^2 + \lambda_1^2\, |S|^2\, \left(|h_+|^2 + |h_-|^2\right) - 2M^2\, F\, (S + S^*) + \dots \, , \quad (2.18)$$

where we left out terms that involve $n_{1,2}$, $\chi_\pm$, as we will be interested in a (potentially local) minimum where these fields do not obtain vevs. Note that because supersymmetry is broken, there is a tadpole for the scalar component of S, so that it develops a vev proportional to F,

$$S = -\frac{1}{\lambda_1} F + \mathcal{O}(\frac{F^3}{M^2}) \, , \qquad (2.19)$$

whereas $h_\pm$ obtain vevs $h = M/\sqrt{\lambda_1} + \mathcal{O}(F^2/M)$. The $U(1)$ is then Higgsed by the $h_\pm$ vevs, and at tree level we obtain a heavy vector multiplet with a vector of mass $M_{SUSY} = 2eh$, a scalar of mass $m^2 = M_{SUSY}^2 + 2S^2$, and two fermions of masses $\sqrt{M_{SUSY}^2 + S^2/4} \pm S/2$, where e is the $U(1)$ gauge coupling. In addition, χ_- mixes with n_1 to give a chiral multiplet with a fermion of mass $y_1 h$ and scalars of masses-squared $y^2 h^2 \pm y\lambda_1 Sh$, and similarly for the pair χ_+, n_2.

At low energies the theory contains only the fields $l_\pm$, with no renormalizable interactions. However, soft masses are generated for the $l_\pm$ scalars through gauge and Yukawa loops containing the heavy fields. As discussed in the previous section, to leading order in supersymmetry breaking, namely, order F^2, these soft masses vanish. We can see that by working directly in the low energy theory, which contains no renormalizable interactions, so that the anomalous dimensions of $l_\pm$ vanish. We will return to this point shortly and see how this can be understood from the point of view of the full theory.

However, even at one loop, both the heavy gauge multiplet and the heavy chiral fields generate contributions to the $l_\pm$ soft masses starting at order F^4/M^2.[4] Specifically, the gauge multiplet one-loop contribution to the $l_\pm$ scalar mass-squared is given by

$$
m_{\text{gauge}}^2 = q^2 \frac{1}{16\pi^2} 4e^4 h^2 \left[\ln\left(1 + \frac{x^2}{2e^2} \right) - \frac{8x}{\sqrt{16e^2 + x^2}} \ln\left(\sqrt{1 + \frac{x^2}{16e^2}} + \frac{x}{4e} \right) \right] ,
$$
(2.20)

where $x \equiv \frac{s}{h}$, and q is the relevant $U(1)$ charge, which for $l_\pm$ is $q = \pm 1$. This contribution arises purely from the $U(1)$ gauge interactions,

The contribution of the heavy matter fields $\chi_- - n_1$ to the $l_\pm$ scalar mass-squared is:

$$
\begin{aligned}
m_y^2 = \ & -\frac{q}{64\pi^2} y^4 h^2 \left(1 + \frac{x^2}{2e^2} \right)^{-1} \times \\
& \left[\left(2 + \frac{x}{y} + \frac{x^3}{y^3} \right) \ln\left(1 + \frac{x}{y} \right) + \left(2 - \frac{x}{y} - \frac{x^3}{y^3} \right) \ln\left(1 - \frac{x}{y} \right) \right] ,
\end{aligned}
$$
(2.21)

with $y = y_1$, and where again, for $l_\pm$, $q = \pm 1$. This contribution, although we denote it by the subscript y for the Yukawa superpotential coupling, arises from both superpotential interactions and D-term interactions. Similarly, the contribution of

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[4]Recall that unlike F^2 terms, such contributions to the soft masses cannot be read off the wave-function renormalizations.

the heavy matter fields $\chi_+ - n_2$ is

$$m_{y_2}^2 = - m_y^2|_{y=y_2} \ . \tag{2.22}$$

Note the relative minus sign between (2.21) and (2.22), which arises from the opposite signs of χ_+ and χ_-.

The contribution of (2.21) and (2.22) can also be understood as arising from the $U(1)$ D-term. The fields $h_\pm$ obtain soft-masses from one-loop diagrams with $\chi_\pm - n_{1,2}$ running in the loop. Since h_+ and h_- have different superpotential couplings ($y_1 \neq y_2$), their soft masses are also different. As a result, the h_+ and h_- vevs are shifted by different amounts, so that the $U(1)$ D-term is non-zero and proportional to the difference between the h_+ and h_- soft masses squared. This non-zero D-term then leads to soft masses for $l_\pm$, proportional to their $U(1)$ charges, which are precisely given by (2.21).

Let us now return to the F^2 contributions to the soft masses. As we already mentioned, if we work directly in the low-energy theory, which contains only $l_\pm$ with no interactions, the order-F^2 anomaly-mediated soft masses, can be read off the (trivial) wave function renormalizations of $l_\pm$ and therefore vanish. We could however try to derive this result starting from the full theory, above the scale of the heavy fields.

In the UV theory, the wave function renormalizations of $l_\pm$ depend on the gauge coupling, and lead to non-zero, gauge-coupling dependent soft masses for $l_\pm$, which we denote by m_H^2. Once we go below the heavy threshold, we have to add to m_H^2 the contribution of the gauge multiplet to the soft masses which comes from loops involving the heavy gauge multiplet, m_G^2. m_G^2 is the analogue of (2.20). However, it only comes in at the two-loop level, since at one loop there can be no F^2 contribution to the soft masses squared [6]. As was shown in [6], m_G^2 can also be read off the supersymmetric wave function renormalizations. Adding these two contributions, m_G^2 precisely cancels m_H^2.

The Yukawa-dependent contribution to the $l_\pm$ soft masses is a bit trickier, since the $l_\pm$ wave function renormalizations do not depend on the Yukawa couplings. As

discussed above, the Yukawa dependence enters through the D-term. As long as h_+ and h_- have different soft masses, the D-term is non-zero, and leads to soft masses for $l_\pm$. The relevant quantities to consider are then the $h_\pm$ soft masses. Again, we can read these off the $h_\pm$ wave function renormalizations in the full theory, but as we integrate out the heavy fields $\chi_+ - n_2$ and $\chi_- - n_1$, we obtain $\chi_\pm - n_{1,2}$ loops which precisely cancel the original contribution. Thus, the D-term decouples to order-F^2, since the radiative contributions to the $h_\pm$ soft masses vanish in the low-energy theory. In fact, it would be surprising if D-terms generated F^2 contributions to soft masses through anomaly-mediation. We expect to be able to obtain anomaly-mediated F^2 soft masses from wave function renormalizations, which certainly cannot capture D-term contributions. It is therefore reassuring to find that the D-term decouples at order F^2. This is in contrast to the usual case [8] where the leading order SUSY breaking effects do not decouple. As we have seen, however, there are non-decoupling effects at order $\mathcal{O}(F^4/M^2)$.

We would eventually like to identify l_+ with the standard model leptons, and to use the contributions we found to its soft mass, m^2_{gauge} and m^2_y, to compensate for the negative masses squared which the sleptons obtain in the minimal anomaly-mediated scenario [2]. However, m^2_{gauge} and m^2_y are proportional to F^4/M^2, so that would involve tuning the ratio of two unrelated scales, F and M. In the next section we will present a model in which a "large" scale $M \sim F/\lambda_0$, where λ_0 is some Yukawa coupling, is generated dynamically.

2.4   A two step model

Our goal is to find a mechanism which would naturally generate a mass scale which is somewhat larger than the SUSY breaking scale, yet parametrically is of the same order. In fact the model considered in the previous section suggests a way to achieve this goal. Our main observation is that the field S obtained a small vev of the order F/λ_0. For relatively small λ_0 this scale may be sufficiently large, so that the S threshold is supersymmetric, yet it is not parametrically large, and can therefore

play the role of the mass scale M of the previous section. Thus we are lead to the modification of our model which has the following superpotential[5]

$$W = X(\lambda_0 n^2 - \tilde{M}^2) + S\left(\lambda_1 h_+ h_- - \lambda_2 X^2\right) + y_1 n_1 h_+ \chi_- + y_2 n_2 h_- \chi_+ \,. \quad (2.23)$$

We now describe the role of the various terms in this superpotential. The last three terms are exactly the same as in the superpotential (2.17) with the substitution $M^2 \to \lambda_2 X^2$. This substitution is justified since X is heavy. Moreover, all other fields in the above superpotential become heavy at the minimum at least in the sense of eq. (2.16). If X obtains a non-vanishing vev, this sector of the theory will generate soft masses for $l_\pm$ exactly as in the previous section.

The first term in the superpotential (2.23) is designed to produce an X vev of the order F/λ_0. The large scale $\tilde{M}$ is necessary to generate this vev as well as other field vevs and eventually leads to the breaking of the $U(1)$ gauge symmetry. Such a large mass parameter may be naturally generated by various strong coupling effects in the microscopic theory. It is important, however, that our results are insensitive to the precise value of $\tilde{M}$.

Let us now analyze the model. Upon supersymmetry breaking the scalar potential has the form (neglecting the couplings to $\chi_\pm$)

$$\begin{aligned}
V = |\lambda_1 h_+ h_- - \lambda_2 X^2|^2 + |\lambda_1 S h_+|^2 + |\lambda_1 S h_-|^2 + \\
\left| -2\lambda_2 S X + \lambda_0 n^2 - \tilde{M}^2 \right|^2 + |2\lambda_0 X n|^2 - (2\tilde{M}^2 F X + c.\,c.) \,.
\end{aligned} \quad (2.24)$$

We wish to separate our analysis in two stages. First we set $\lambda_2 = 0$ and find vevs for n and X using our results from the previous section. Since these fields are heavy at the minimum we then assume that their vevs are not significantly shifted in the full theory. We will discuss under which conditions this assumption is true shortly. Then we turn on λ_2 and integrate out the heavy fields X, and n, and then we consider an effective theory for the lighter fields S, h_+ and h_-. Using our results from the previous section we find that $X = F/\lambda_0$. It is also easy to find that $F_X = F^2/\lambda_0$. The

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[5]This superpotential is the most general one preserving a $U(1)_R$ symmetry.

superpotential for the lighter fields now acquires the form (2.17) with the substitution $M = \sqrt{\lambda_2} X$. Note that the value of mass scale $\tilde{M}$ is indeed irrelevant for the dynamics of the lighter degrees of freedom, S and $h_\pm$. We now have to check that the mass parameter M can be promoted to a superspace valued background superfield with an F-term expectation value MF. Indeed,

$$\sqrt{\lambda_2} X = \sqrt{\lambda_2} \frac{F}{\lambda_0} + \sqrt{\lambda_2} \frac{F^2}{\lambda_0} \theta^2 = \frac{\sqrt{\lambda_2} F}{\lambda_0} (1 + F\theta^2) \ . \tag{2.25}$$

Thus we can use our results from the previous section for the soft masses of $l_\pm$ by substituting $M \to \sqrt{\lambda_2} F / \lambda_0$.

Finally, let us turn to the conditions for the existence of the desired minimum in the model (2.23). We note that the model possesses a flat direction parameterized by the S vev. In the limit of unbroken SUSY and in the region of the moduli space of interest S is heavy and can not acquire a vev until $h_+ \sim h_- \to 0$. However, when we turn SUSY breaking on, it is possible that one mass eigenvalue for the two real fields in the S supermultiplet will become negative, and the local minimum will not exist. We should, therefore, require that the F-type SUSY violating masses for S are much smaller that the supersymmetric contribution to the mass. For the supersymmetric mass we have $m_S^2 = 2\lambda_0^{-2}\lambda_1\lambda_2 F^2$, while the soft mass is $m_F^2 = \sqrt{2}\lambda_1 F_h = \sqrt{2\lambda_0^{-2}\lambda_1\lambda_2} F^2$. We easily see that a local minimum exists if

$$\frac{\lambda_0}{\sqrt{\lambda_1\lambda_2}} \ll 1 \ . \tag{2.26}$$

Note that the combination of couplings in (2.26) is exactly the quantity x which enters formulae (2.20) and (2.21) for the soft masses. We performed a numerical minimization of the scalar potential, and verified that the local minimum exists for a range of parameters when the ratio in (2.26) is of order or smaller than 0.1.

Table 2.1: Table 1: The field content of the model without the SM fields that are neutral under the $U(1)$.

	SM	$U(1)$
S	$(1,1,0)$	0
X	$(1,1,0)$	0
n	$(1,1,0)$	0
h_+	$(1,1,0)$	1
h_-	$(1,1,0)$	-1
$L \equiv l_+^{i=1..3}$	$(1,2,-1/2)$	1
$\chi_-^{i=1..3}$	$(1,2,1/2)$	-1
$n_1^{i=1..3}$	$(1,2,1/2)$	0
$\bar{l} \equiv l_+^{i=4..6}$	$(1,1,-1)$	1
$\chi_-^{i=4..6}$	$(1,1,,1)$	-1
$n_1^{i=4..6}$	$(1,1,-1)$	0

2.5 Correcting the slepton masses

As was pointed out in [2], the minimal AM scenario in which the sole origin of the superpartner masses is anomaly mediation gives negative slepton masses squared. Thus extra contributions to the slepton masses squared are required. We will now use the model we constructed in the last two sections to generate positive contributions to these masses. To do that we augment the SM gauge group by the $U(1)$ of the model we described earlier, and charge the lepton fields under this $U(1)$, with $U(1)$ charge +1. Our starting point is then the theory described in Section 2.3, with six copies of the field l_+ corresponding to the six SM lepton fields: $l_+^i = (1,2,-1/2,+1)$, $l_+^{i+3} = (1,1,-1,+1)$, where $i = 1\ldots3$ is a generation index, and the parenthesis indicate the SM$\times U(1)$ representation. Similarly, we also take six copies of the fields χ_-, with $\chi_-^i = (1,2,1/2,-1)$, and $\chi_-^{i+3} = (1,1,1,-1)$, and six copies of the field n_1 with $n_1^i = (1,2,-1/2,0)$, and $n_2^{i+3} = (1,1,1,0)$. It is easy to check that with this field content there are no SM$\times U(1)$ anomalies. The field content of the model is summarized in Table 1. The superpotential is then given as in (2.23),

$$W = X(\lambda_0 n^2 - \tilde{M}^2) + S\left(\lambda_1 h_+ h_- - \lambda_2 X^2\right) + y_1 \sum_{i=1}^{6} n_1^i \, h_+ \, \chi_-^i \, , \qquad (2.27)$$

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except that so far we have not added the fields l_-, χ_+ and n_2, so that the superpotential term containing them does not appear. The fields χ_-^i, n_1^i become heavy, with mass $y_1 h$. As discussed in Section 2.2, the sleptons then obtain the following soft masses

$$m_{slepton}^2 = m_{gauge}^2 + 9 m_y^2|_{y=y_1} \ , \tag{2.28}$$

where m_{gauge}^2 and m_y^2 are given in (2.20) and (2.21) with $q = 1$.

Let us now discuss the different mass contributions. Recall that the heavy matter contribution to the l_+ mass, m_y^2, is proportional to the charge of l_+, whereas m_{gauge}^2 is proportional to the square of the charge. In addition, m_{gauge}^2 is always negative. Thus, if we want to generate positive masses squared for all sleptons, they all have to have $U(1)$ charges of the same sign. Thus, we choose all leptons to have $U(1)$ charge $+1$, and identify them with the fields l_+. To cancel anomalies, we then add fields of the type χ_-, of $U(1)$ charge -1, and n_1. These fields then become heavy, and at one loop generate the contribution $m_y^2|_{y=y_1}$. Unfortunately, however, this contribution is also negative! Examining eq. (2.22), we see that in order to get a positive contribution, we need heavy fields of opposite $U(1)$ charge running in the loop, that is, fields of the type χ_+ (and their n_2 partners). Again anomaly considerations then require the presence of additional fields l_-. Unlike the χ's and the n's, these fields remain light, so that the simplest possibility is to identify them with some of the SM fields. We are therefore led to charging additional SM fields under the $U(1)$, with charges that are opposite in sign to the lepton charges. We will now discuss two possibilities of doing so. One in which the down antiquarks have charge -1, and the other in which the first and second generation quarks have charge -1. While we will be able to generate positive masses squared for the sleptons, we see from the required matter content that probably the most disappointing aspect of our model is that it can not be made consistent with grand unification. As explained above, the lepton fields all have the same $U(1)$ charge, and some other SM fields should have the opposite $U(1)$ charge. This automatically excludes both $SU(5)$ and $SO(10)$ unification. Moreover, the additional matter fields do not come in GUT representations, and even gauge

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Table 2.2: Table 2: Additional fields: down antiquarks without the SM fields that are neutral under the $U(1)$.

	SM	$U(1)$
$\bar{d} \equiv l_-^{i=1..3}$	$(\bar{3}, 1, -1/3)$	-1
$\chi_+^{i=1..3}$	$(3, 1, 1/3)$	-1
$n_2^{i=1..3}$	$(\bar{3}, 1, -1/3)$	0

coupling unification requires the introduction of extra matter at intermediate scales.

2.5.1 $U(1)$ charged leptons and down antiquarks

As discussed above, in order to obtain positive slepton masses-squared, we need heavy fields of positive $U(1)$ charge running in the loop. We can achieve that by identifying the SM down antiquarks with the fields l_-. Thus, there are three copies of the field l_-: $l_-^i = (\bar{3}, 1, -1/3, -1)$, which are accompanied by three copies of the field χ_+ with $\chi_+^i = (3, 1, 1/3, 1)$ and three copies of n_2 with $n_2^i = (\bar{3}, 1, -1/3, 0)$. This additional field content is summarized in Table 2.

The superpotential is then given exactly as in (2.23). For convenience we rewrite it here:

$$W = X(\lambda_0 n^2 - \tilde{M}^2) + S\left(\lambda_1 h_+ h_- - \lambda_2 X^2\right) + y_1 \sum_{i=1}^{6} n_1^i \, h_+ \, \chi_-^i + y_2 \sum_{i=1}^{3} n_2^i \, h_- \, \chi_+^i \; . \quad (2.29)$$

The fields χ_-^i, n_1^i (χ_+^i, n_2^i) all become heavy, with mass $y_1 h$ ($y_2 h$). The sleptons and down antiquarks obtain the following soft masses

$$
\begin{aligned}
m_{slepton}^2 &= m_{gauge}^2 + 9m_y^2|_{y=y_1} - 9m_y^2|_{y=y_2} \; , \\
m_d^2 &= m_{gauge}^2 - 9m_y^2|_{y=y_1} + 9m_y^2|_{y=y_2} \; .
\end{aligned}
\quad (2.30)
$$

Note that because the sleptons and down quarks have opposite $U(1)$ charges, their soft masses obtain opposite contributions from the heavy matter fields.

As explained in the beginning of this section, m_{gauge}^2 and m_y^2 are negative. Thus,

to get a positive contribution to the slepton masses squared, the third term in the first line in (2.30) should overcome the first two. We also point out that m^2_{gauge}, $m^2_{y_1}$, and $m^2_{y_2}$ all start at order $h^2 x^4 = s^4/h^2$. At this order, the dependence on the couplings e, y_i drops out, so that the $\mathcal{O}(x^4)$ terms cancel between $m^2_{y_1}$ and $m^2_{y_2}$.

It is clear from (2.30) that as long as the sleptons obtain a positive contribution to the mass squared, the down-type squarks get a negative contribution. We then have to ensure that the new contribution to the slepton mass is bigger than the one generated directly by anomaly-mediation, whereas the new contribution to the down squark mass is smaller than the one generated by anomaly-mediation. Thus, we need,

$$m^2_{slepton} > \left| m^2_{slepton,AM} \right| \; , \qquad \left| m^2_d \right| < m^2_{d,AM} \; , \tag{2.31}$$

where $m^2_{slepton,AM}$ and $m^2_{d,AM}$ are the slepton and down squark masses generated by anomaly-mediation in the absence of any heavy thresholds,

$$m^2_{slepton,AM} \sim 10^{-3} \frac{F^2}{16\pi^2} \; , \qquad m^2_{d,AM} \sim 10^{-1} \frac{F^2}{16\pi^2} \; . \tag{2.32}$$

Thus, the different couplings need to be tuned to satisfy this relation. It turns out that the tuning required is not drastic. First, no large hierarchy is required between the couplings e, y_1 and y_2 to obtain a positive $m^2_{slepton}$. Second, to satisfy (2.31), the coupling λ_0, which controls the size of h, can vary within an overall factor of around five.

We now turn to consider the SM Yukawa couplings. Since both the $SU(2)$-doublet and -singlet leptons have $U(1)$ charge $+1$, and the down quarks have $U(1)$ charge -1, the lepton and down Yukawas are not neutral under the $U(1)$. These Yukawas can however arise from non-renormalizable terms,

$$\frac{1}{M} h_+ H_d Q \bar{d} + \frac{1}{M^2} h^2_- H_d L \bar{l} \; , \tag{2.33}$$

where M is some higher scale. We are thus led to a model with nontrivial flavor structure, where the up Yukawas have no suppression, the down Yukawas are suppressed

Table 2.3: Table 3: Additional fields: 1st and 2nd generation quarks

	SM	$U(1)$
$Q \equiv l_-^{i=1,2}$	$(3, 2, 1/6)$	-1
$\chi_+^{i=1,2}$	$(\bar{3}, 2, -1/6)$	1
$n_2^{i=1,2}$	$(3, 2, 1/6)$	0

by one power of h/M, and the lepton Yukawas are suppressed by two powers of h/M.

2.5.2 $U(1)$-charged leptons and first generations quarks

An alternative to assigning $U(1)$ charge -1 to the SM down-type antiquarks, is to assign charge -1 to the first and second generation doublet quarks. We then have fields l_+^i, χ_-^i and n^i as in the previous subsection, as well as the the first and second generation quarks, which we identify with $l_-^{i=1,2} = (3, 2, 1/6, -1)$, $\chi_+^{i=1,2} = (3, 2, -1/6, 1)$ and $n_2^{i=1,2} = (3, 2, 1/6, 0)$. We summarize the additional field content of this model in Table 3.

The sleptons and first and second generation squarks now obtain the following contributions to their masses-squared:

$$
\begin{aligned}
m_{slepton}^2 &= m_{gauge}^2 + 9m_y^2|_{y=y_1} - 12m_{y_2}^2|_{y=y_2} \,, \\
m_d^2 &= m_{gauge}^2 - 9m_y^2|_{y=y_1} + 12m_y^2|_{y=y_2} \,.
\end{aligned}
\tag{2.34}
$$

Again, the third term on the first line gives a positive contribution, with the two other contributions negative. As in the last subsection, we can tune the various couplings so that the total slepton mass is positive, and the squark mass receives only a small (negative) correction.

The lepton Yukawa couplings again arise from non-renormalizable operators, and are suppressed by two powers of h/M. As for the quark Yukawa couplings, the third generation term is not suppressed, whereas the first two generations are suppressed by one power of h/M.

Note that, unlike in the previous subsection, the soft masses are no longer flavor-

blind: the soft masses of the first two generation squarks receive negative corrections and are smaller than the soft masses of the third generation squarks. However, these corrections can be chosen to be small, with no severe tuning of parameters. In addition, constraints on FCNC processes which involve the third generation are typically weaker.

2.6   The μ-term from anomaly mediated SUSY breaking

We now turn to the μ-term problem in AMSB. As has been noted in [2] this problem is much less severe than in gauge mediated models, and indeed several mechanisms generating μ and B terms of the correct order of magnitude have been proposed recently [2, 5, 9]. Here we propose a solution to this problem based on the use of the higher order (in SUSY breaking) correction as well as the observation that in AMSB models it is easy to generate a scale which is somewhat larger than gravitino mass, $m_{3/2} \sim F$.

We introduce the superpotential

$$W = \lambda_H S H_u H_d + \lambda_S S^3 + \lambda_N S N^2 + M N^2 \ , \tag{2.35}$$

where the mass parameter M is assumed to be generated dynamically as in Section 2.4, $M = F/y$ for some coupling constant y. In addition, the scalar components of N have soft tree level contributions to their masses, MF. Generally, the Higgs boson vevs lead to a vev (and therefore an effective μ term) for S through the superpotential (2.35). We will argue shortly that such a contribution does not affect the conclusions we will draw, and therefore, will neglect it throughout our discussion.

Anomaly mediation generates a positive contribution to the S singlet mass squared of the order

$$m_{AM}^2 \sim \frac{\lambda_S^4 + \lambda_H^4}{(16\pi^2)^2} F^2 \ . \tag{2.36}$$

Here and throughout this section we omit some order one numerical coefficients. On the other hand, a one loop negative mass squared for the singlet is generated due to the non-decoupling of the heavy states,

$$m_{F^4}^2 \sim -\frac{\lambda_N^2}{16\pi^2}\frac{F^4}{M^2} = -\frac{\lambda_N^2}{16\pi^2}\,y^2\,F^2 \ . \tag{2.37}$$

It is easy to see that the singlet mass will be negative as long as[6]

$$\lambda_N^2 y^2 > \frac{\lambda_H^4 + \lambda_S^4}{16\pi^2} \ , \tag{2.38}$$

where the right hand side indicates the order of magnitude only.

As a result both the scalar and the auxiliary components of S acquire vevs

$$S \ \sim \ \frac{1}{4\pi}\frac{\lambda_N y}{\lambda_S}F \ , \tag{2.39}$$

$$F_S \ \sim \ \frac{1}{16\pi^2}\frac{\lambda_N^2 y^2}{\lambda_S}F^2 \ .$$

Substituting these vevs into the superpotential (2.35) we find that both μ and B are generated

$$\mu \ \sim \ \frac{1}{4\pi}\frac{\lambda_H}{\lambda_S}y\lambda_N F \ , \tag{2.40}$$

$$B \ \sim \ \frac{1}{16\pi^2}\frac{\lambda_H}{\lambda_S}y^2\lambda_N^2 F^2 \ .$$

After S acquires a vev there is an additional contribution to B arising from the $S H_u H_d$ A-term, however, it is negligible when (2.38) is satisfied. It is easy to see that B and μ^2 are of the same order if

$$\lambda_H/\lambda_S \sim \mathcal{O}(1) \ . \tag{2.41}$$

---

[6]Remember that N is heavy, and as a result λ_N does not contribute to the S mass at order F^2.

We further need to require that the μ term is of the order of the weak scale,

$$\frac{1}{4\pi}\frac{\lambda_H}{\lambda_S}y\lambda_N F \sim \frac{\alpha_2}{4\pi}F \ . \tag{2.42}$$

This requirement together with (2.41) gives a condition on two Yukawa couplings $\lambda_N y \sim \alpha_2$. We note that this condition is quite compatible with the requirement that S has a negative mass squared.

Finally we observe that the Higgs vevs generate a mass term for S. This mass contribution is below the negative mass (2.37) by roughly $\lambda_H H_U/\mu$. Since λ_H can be arbitrary as long as it is comparable with λ_S, such a contribution is small compared to the negative mass generated by non-decoupling effects (which we can arrange to be between electroweak scale and $1\mathrm{T}eV$). Even in the case $\lambda_H \sim 1$, our qualitative conclusions remain valid, and both a μ and a B term of the correct order of magnitude are generated.

Having established that eq. (2.40) gives a leading contribution to μ and B it is possible to show that the physical phase $\phi = \arg(B\mu^*M_\lambda^*)$ vanishes. Here M_λ is the gaugino mass. Thus, this sector of the theory does not lead to a SUSY CP problem.

To conclude our discussion of the μ term, we observe that the superpotential (2.35) could be introduced in a gauge mediated model, with N being a messenger field. However, in calculable models of gauge mediation the scale of supersymmetry breaking is relatively large, while the messenger mass is at most suppressed by several loop factors relative to this scale. As a result the higher order contributions used here are too small to generate electroweak scale parameters. [7] In principle it is possible to generate a small mass for the messengers, however, this requires the introduction of a quite complicated structure and explicit mass scales, unlike with anomaly mediation where a mass scale somewhat larger than the SUSY breaking scale of the visible sector can naturally be generated.

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[7]This is not a problem in strongly coupled gauge mediated models with a low SUSY breaking scale. However, such models are non-calculable, and it is not possible to quantitatively analyze their spectrum.

2.7 Conclusions

In this chapter, we studied the decoupling of heavy thresholds in theories with anomaly-mediated supersymmetry breaking. To leading order in the supersymmetry breaking, such thresholds decouple. That is, the anomaly-mediated supersymmetry breaking terms at some low scale are independent of whether or not there are supersymmetric thresholds above that scale. These soft terms are thus quite robust.

It is possible to see this decoupling in several ways. For example, it can be understood as a cancellation between the following two quantities: The first is the contribution of the heavy fields to the anomaly mediated soft terms in the full theory. Recall that the AM soft terms depend on the beta function of the theory, which in the full theory reflects the presence of the heavy fields. The second is the direct radiative contribution, through gauge or Yukawa interactions, of the heavy fields to the soft terms of the light fields. This contribution is generated when the heavy fields are integrated out. These two contributions cancel exactly, so that below the scale of the heavy fields, they leave no trace on the soft terms at leading order in SUSY breaking. That these two contributions exactly cancel can be seen on a case by case basis, but it is most simply seen from the fact that the direct gauge- or Yukawa-mediated contributions of the heavy fields can be read off the wave function renormalizations [6, 7] in precisely the same way as the AM contributions.

Alternatively, the decoupling can be seen as a cancellation between "real" fields and their regulators.

When do heavy threshold not decouple? One obvious possibility is that they are not truly heavy [5]. That is, there is some light modulus associated with the heavy threshold whose mass comes mainly from SUSY breaking effects. It is worth pointing out that approximate decoupling persists even for a modulus much lighter than the SUSY breaking scale F so long as its mass is primarily determined by supersymmetric parameters.

However, there is additional non-decoupling even when all the heavy fields are truly heavy. That is because the purely anomaly-mediated soft terms only appear at

leading order in the SUSY breaking. For example, scalar masses squared are order F^2. In contrast, as we integrate out some heavy fields, they give direct contributions, again through loops, to the soft terms to all orders in the SUSY breaking. Scalar masses squared now have contributions of order F^4/M^2, where M is the heavy threshold. Moreover, these contributions typically appear at lower order in the loop expansion. For scalar masses squared, they can appear at one-loop, whereas the AM soft masses are two-loop contributions.

Having established that heavy supersymmetric thresholds do affect the soft terms at order F^4, we then use this fact to generate positive slepton masses. If the only source of SUSY breaking in the SM is anomaly mediation, and if there are no supersymmetric thresholds, the slepton masses squared are negative. However, we can charge the leptons (and another subset of SM fields) under a new $U(1)$ gauge symmetry, which is broken at a scale somewhat above the visible sector SUSY-breaking scale, and add some fields that obtain supersymmetric masses. In the presence of anomaly-mediation, these heavy fields also acquire SUSY-breaking masses, and contribute to slepton masses at order F^4. Interestingly, we are led to a model with some non-trivial flavor structure. Unfortunately, this model is not consistent with grand unification.

As another model building application, we used the F^4 contributions of heavy supersymmetric thresholds to generate acceptable μ- and B-terms. As we saw, this can be done quite simply in models of anomaly mediation, unlike in the case of gauge-mediation. In the latter case, an acceptable μ term typically leads to a B term that is too large.

In both these model building examples, we also use a simple mechanism that dynamically generates a scale that is naturally somewhat above the SUSY breaking scale through anomaly-mediation. We expect this fact to be useful for further model building applications.

Chapter 3

Linearized gravity in brane backgrounds

3.1 Introduction and summary

It is possible that the observed world is a brane embedded in a space with more non-compact dimensions. This proposal was made more concrete in the scenario advanced in [25, 26], where the problem of recovering four-dimensional gravity was addressed. (Earlier work appears in refs. [28], [27]–[29].) Further exploration of this scenario has included investigation of its cosmology [10, 65, 13, 66, 15, 16, 18], [57]–[67] and phenomenology [22, 23].[1]

In order to "localize" gravity to the brane, ref. [26] worked in an embedding space with a background cosmological constant, with total action of the form

$$S = \int d^5 X \sqrt{-G}(-\Lambda + M^3 \mathcal{R}) + \int d^4 x \sqrt{-g} \mathcal{L} \, . \tag{3.1}$$

G and X are the five-dimensional metric and coordinates, and g and x are the corresponding four-dimensional quantities with g given as the pullback of the five-dimensional metric to the brane. M is the five-dimensional Planck mass, and $\mathcal{R}$ denotes the five-dimensional Ricci scalar. The bulk space is a piece of anti-de Sitter

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[1]For a recent survey of some of these topics, see also [49].

space, with radius $R = \sqrt{-12M^3/\Lambda}$, which has metric

$$dS^2 = \frac{R^2}{z^2}(dz^2 + dx_4^2) \ . \tag{3.2}$$

The brane can be taken to reside at $z = R$, or in scenarios [76] with both a probe (or "TeV") brane and a Planck brane, this will be the location of the Planck brane. The horizon for observers on either brane is at $z = \infty$.

There are a number of outstanding questions with this proposal. One very interesting question is what black holes or more general gravitational fields, *e.g.* due to sources on the brane, look like, both on and off the brane. For example, consider a black hole formed from matter on the brane. From the low-energy perspective of an observer on the brane it should appear like a more-or-less standard four-dimensional black hole but one expects a five-dimensional observer to measure a non-zero transverse thickness. One can trivially find solutions that a four-dimensional observer sees as a black hole by replacing dx^2 with the Schwarzschild metric in (3.2). [2] However, these "tubular" solutions become singular at the horizon at $z = \infty$, suggesting that another solution be found.

Another related question concerns the dynamics of gravity. It was argued in [26] that four-dimensional gravitational dynamics arises from a graviton zero mode bound to the brane. Fluctuations in this zero mode correspond to perturbations of the form

$$dS^2 \rightarrow dS^2 + \frac{R^2}{z^2}h_{\mu\nu}dx^\mu dx^\nu \ . \tag{3.3}$$

where $h_{\mu\nu}$ is a function only of x,

$$h_{\mu\nu} = h_{\mu\nu}(x) \ . \tag{3.4}$$

Computing the lagrangian of such a fluctuation yields

$$\mathcal{R} \sim z^2 \partial h \partial h \ . \tag{3.5}$$

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[2]Such metrics were independently found in [91].

This and other measures of the curvature of the fluctuation generically grow without bound as $z \to \infty$. In particular, if we add a higher power of the curvature to the action, with small coefficient, as may be induced from some more fundamental theory of gravity, then generically divergences will be encountered. For example, one easily estimates

$$\mathcal{R}_{\mu\nu\lambda\sigma}\mathcal{R}^{\mu\nu\lambda\sigma} \sim z^4 \tag{3.6}$$

suggesting that Planck scale effects are important near the horizon. This would raise serious questions about the viability of the underlying scenario. These estimates are however incorrect as they neglect the non-zero modes.

Yet another question regards corrections to the 4d gravitational effective theory on the brane. We'd like to better understand the strength of corrections to Newton's Law and other gravitational formulae; some of the leading corrections have already been examined via the mode sum [26, 76]. Sufficiently large corrections could provide experimental tests of or constraints on these scenarios.

A final point addresses reinterpretation [75, 31, 73] of these scenarios within the context of AdS/CFT duality [35], [36]–[37]. In this picture, gravity in the bulk AdS off the brane can be replaced by $\mathcal{N} = 4$ super-Yang Mills theory on the brane. Witten [73] [3] has suggested that gravitational corrections from the bulk can be reinterpreted in terms of the loop diagrams in the SYM theory.

In order to address these questions, this chapter will give an analysis of linearized gravity in the background of [26]. We begin in section 3.2 with a derivation of the propagator for a scalar field in the brane background of [26], generalized to $d+1$ dimensions. This exhibits much of the physics with less complication than gravity. The propagator is the usual AdS propagator plus a correction term, and can be rewritten, for sources on the brane, in terms of a zero-mode contribution that produces d-dimensional gravity on the brane plus a correction from the "Kaluza-Klein" modes. For even $d \geq 4$ this term produces corrections of order $(R/r)^{d-2}$ at large distances r from the source.

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[3]See also [74].

In section 3.3 we perform a linearization of gravity about the $d+1$ dimensional brane background. For general matter source on the brane, the brane has non-zero extrinsic curvature, and a consistent linear analysis requires introduction of coordinates in which the bending of the brane is manifest, as exhibited in eq. (3.22). We outline the derivation of the graviton propagator, which can be written in terms of the scalar propagator of section 3.2. (For those readers interested primarily in the applications discussed in the subsequent sections, the results appear in eqs. (3.23), (3.24), and (3.26).) Special simplifying cases include treatment of sources restricted to the Planck brane, or living on a probe brane in the bulk.

In section 3.4 we discuss the asymptotics and physics of the resulting propagator. Linearized gravity on the Planck brane corresponds to d-dimensional linearized gravity, plus correction terms from Kaluza-Klein modes. As in the scalar case, these yield large-r corrections suppressed by $(R/r)^{d-2}$ for even $d \geq 4$ and by R/r for $d = 3$, in agreement with [77]. We also discuss corrections in the probe-brane scenario of [76]. We then find the falloff in the gravitational potential off the brane and thus deduce the shape in the extra dimensions of black holes bound to the brane or of more general gravitational fields. In particular, we find that black holes have a transverse size that grows with mass like $\log m$, compared to the usual result $m^{1/d-3}$ along the brane. Thus black holes have a pancake-like shape. We also check consistency of the linearized approximation, and check that higher-order curvature terms in the action in fact do not lead to large corrections, as the naïve analysis of the zero mode would indicate.

3.2 The massless scalar propagator

Much of the physics of linearized gravity in the scenario of [26] is actually found in the simpler case of a minimally coupled scalar field. Because of this, and because the scalar propagator is needed in order to compute the graviton propagator, this section will focus on computing the scalar Green function.

In much of this chapter we will work with the generalization to a $d+1$ dimensional

theory with a brane of codimension one. The scalar action, with source terms, takes the form

$$S = \int d^{d+1} X \sqrt{-G} \left[-\frac{1}{2}(\nabla \phi)^2 + J(X)\phi(X) \right] . \qquad (3.7)$$

Here we work in the brane background of [26]; for $z > R$ the metric is the $d+1$-dimensional AdS metric,

$$dS^2 = \frac{R^2}{z^2}(dz^2 + dx_d^2). \qquad (3.8)$$

Without loss of generality the brane can be located at $z = R$. This problem serves as a toy-model for gravity; for a given source $J(X)$, the resulting field $\phi(X)$ is analogous to the gravitational field of a fixed matter source. The field is given in terms of the scalar Green function, obeying

$$\Box \Delta_{d+1}(X, X') = \frac{\delta^{d+1}(X - X')}{\sqrt{-G}} . \qquad (3.9)$$

Analogously to the boundary conditions that we will find on the gravitational field, the scalar boundary conditions are taken to be Neumann,

$$\partial_z \Delta_{d+1}(x, x')_{|_{z=R}} = 0 ; \qquad (3.10)$$

these can be interpreted as resulting from the orbifold boundary conditions at the brane, or alternately as due to the energy density on the wall. The scalar field has a bound zero mode $\phi = \phi(x)$ analogous to that of gravity.

In order to solve (4.23), we first reduce the problem to solving an ordinary differential equation via a Fourier transform in the d dimensions along the wall,

$$\Delta_{d+1}(x, z; x', z') = \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \Delta_p(z, z'). \qquad (3.11)$$

The Fourier component must then satisfy the equation,

$$\frac{z^2}{R^2}(\partial_z{}^2 - \frac{d-1}{z}\partial_z - p^2)\Delta_p(z, z') = \left(\frac{z}{R}\right)^{d+1} \delta(z - z') . \qquad (3.12)$$

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Making the definitions $\Delta_p = (\frac{zz'}{R^2})^{\frac{d}{2}}\hat{\Delta}_p$ and

$$q^2 = -p^2 \,, \tag{3.13}$$

this becomes

$$\left(z^2\partial_z^2 + z\partial_z + q^2z^2 - \frac{d^2}{4}\right)\hat{\Delta}_p(z, z') = Rz\delta(z - z'). \tag{3.14}$$

For $z \neq z'$, the equation admits as its two independent solutions the Bessel functions $J_{\frac{d}{2}}(qz)$ and $Y_{\frac{d}{2}}(qz)$. Hence, the solution for $z < z'$ and for $z > z'$ must be linear combinations $\hat{\Delta}_<(z, z')$, $\hat{\Delta}_>(z, z')$ of these functions. Eq. (3.14) then implies matching conditions at $z = z'$:

$$\hat{\Delta}_<|_{z=z'} = \hat{\Delta}_>|_{z=z'} \tag{3.15}$$

$$\partial_z(\hat{\Delta}_> - \hat{\Delta}_<)|_{z=z'} = \frac{R}{z'} \,. \tag{3.16}$$

We begin with the Green function for $z < z'$. The boundary condition (3.10) translates to

$$\partial_z\left[z^{d/2}\hat{\Delta}_<\right]|_{z=R} = 0 \,. \tag{3.17}$$

This has solution

$$\begin{aligned}
\hat{\Delta}_< &= A(z')\left[Y_{\frac{d}{2}-1}(qR)J_{\frac{d}{2}}(qz) - J_{\frac{d}{2}-1}(qR)Y_{\frac{d}{2}}(qz)\right] \\
&= iA(z')\left[J_{\frac{d}{2}-1}(qR)H_{\frac{d}{2}}^{(1)}(qz) - H_{\frac{d}{2}-1}^{(1)}(qR)J_{\frac{d}{2}}(qz)\right] \,,
\end{aligned} \tag{3.18}$$

where $H^{(1)} = J + iY$ is the first Hankel function.

Next, consider the region $z > z'$. The boundary conditions at the horizon $z = \infty$ are analogous to the Hartle-Hawking boundary conditions and are inferred by demanding that positive frequency waves be ingoing there, implying[41]

$$\hat{\Delta}_> = B(z')H_{\frac{d}{2}}^{(1)}(qz) \,. \tag{3.19}$$

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The matching conditions (3.15) between the regions then become

$$iA(z')\left[J_{\frac{d}{2}-1}(qR)H_{\frac{d}{2}}^{(1)}(qz') - H_{\frac{d}{2}-1}^{(1)}(qR)J_{\frac{d}{2}}(qz')\right] = B(z')H_{\frac{d}{2}}^{(1)}(qz'), \qquad (3.20)$$

$$B(z')H_{\frac{d}{2}}^{(1)\prime}(qz') - iA(z')\left[J_{\frac{d}{2}-1}(qR)H_{\frac{d}{2}}^{(1)\prime}(qz') - H_{\frac{d}{2}-1}^{(1)}(qR)J_{\frac{d}{2}}'(qz')\right] = \frac{R}{qz'}.$$

The solution to these gives

$$\hat{\Delta}_p = i\frac{\pi}{2}R\left[J_{\frac{d}{2}-1}(qR)H_{\frac{d}{2}}^{(1)}(qz_<) - H_{\frac{d}{2}-1}^{(1)}(qR)J_{\frac{d}{2}}(qz_<)\right]\frac{H_{\frac{d}{2}}^{(1)}(qz_>)}{H_{\frac{d}{2}-1}^{(1)}(qR)}, \qquad (3.21)$$

where $z_>$ ($z_<$) denotes the greater (lesser) of z and z'. This leads to the final expression for the scalar propagator:

$$\Delta_{d+1}(x, z; x', z') = \qquad \frac{i\pi}{2R^{d-1}}(zz')^{\frac{d}{2}}\int\frac{d^dp}{(2\pi)^d}e^{ip(x-x')} \qquad (3.22)$$

$$\times\left[\frac{J_{\frac{d}{2}-1}(qR)}{H_{\frac{d}{2}-1}^{(1)}(qR)}H_{\frac{d}{2}}^{(1)}(qz)H_{\frac{d}{2}}^{(1)}(qz') - J_{\frac{d}{2}}(qz_<)H_{\frac{d}{2}}^{(1)}(qz_>)\right].$$

We note that the second term is nothing but the ordinary massless scalar propagator in AdS_{d+1}.

A case that will be of particular interest in subsequent sections is that where one of the arguments of Δ_{d+1} is on the Planck brane, at $z = R$. In this case, the propagator is easily shown to reduce to

$$\Delta_{d+1}(x, z; x', R) = \left(\frac{z}{R}\right)^{d/2}\int\frac{d^dp}{(2\pi)^d}e^{ip(x-x')}\frac{1}{q}\frac{H_{\frac{d}{2}}^{(1)}(qz)}{H_{\frac{d}{2}-1}^{(1)}(qR)}. \qquad (3.23)$$

For both points at $z = R$, a Bessel recursion relation gives a more suggestive result:

$$\Delta_{d+1}(x, R; x', R) = \int\frac{d^dp}{(2\pi)^d}e^{ip(x-x')}\left[\frac{d-2}{q^2R} - \frac{1}{q}\frac{H_{\frac{d}{2}-2}^{(1)}(qR)}{H_{\frac{d}{2}-1}^{(1)}(qR)}\right]. \qquad (3.24)$$

This can clearly be separated into the standard d-dimensional scalar propagator Δ_d,

with

$$\partial_\mu \partial^\mu \Delta_d(x, x') = \delta^d(x - x') \,, \tag{3.25}$$

which is produced by the zero-mode, plus a piece due to exchange of Kaluza-Klein states:

$$\Delta_{d+1}(x, R; x', R) = \frac{d-2}{R}\Delta_d(x, x') + \Delta_{KK}(x, x') \,. \tag{3.26}$$

Here

$$\Delta_{KK}(x, x') = -\int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \frac{1}{q} \frac{H^{(1)}_{\frac{d}{2}-2}(qR)}{H^{(1)}_{\frac{d}{2}-1}(qR)} \,. \tag{3.27}$$

Note that for $d = 3$, this gives the very simple result

$$\Delta_{KK} = i \int \frac{d^3 p}{(2\pi)^3} \frac{e^{ip(x-x')}}{q} = \frac{1}{2\pi^2 |x - x'|^2} \,. \tag{3.28}$$

(One must however be careful in treating the gravitational field as a perturbation in this case; recall that in $d = 3$ the potential is logarithmic, which is in a sense not a small perturbation on a Minkowski background.)

The effective action for exchange of ϕ fields between two sources as in (3.7) is given by the usual quadratic expression involving this propagator. While we'll postpone general discussion of the asymptotics of these expressions for large or small x or z until we discuss the graviton propagator, it is worth noting that at large distances on the brane, $|x - x'| \gg R$, the zero mode piece dominates and we reproduce the standard effective action for d-dimensional scalar exchange, plus subleading corrections from the Kaluza-Klein part.

3.3 Linearized gravity

3.3.1 General matter source

We next turn to a treatment of linearized gravity in the context of brane reduction. We again work with a $d+1$-dimensional theory, with action

$$S = \int d^{d+1}X\sqrt{-G}(M^{d-1}\mathcal{R} - \Lambda + \mathcal{L}_{\text{matter}}) - \int d^d x\sqrt{-g}\tau . \qquad (3.29)$$

Here $\mathcal{L}_{\text{matter}}$ may include both matter in the bulk and restricted to the brane. We have explicitly separated out the brane tension τ from the matter lagrangian. Away from the Planck brane, the vacuum solution is $d+1$-dimensional AdS space, with metric (3.8). The AdS radius is determined by solving Einstein's equations. Denote the Einstein tensor by $\mathcal{G}_{IJ}$; these then take the form

$$\mathcal{G}_{IJ} = \frac{1}{2M^{d-1}}\left[T_{IJ} - \Lambda G_{IJ} - \tau P_{IJ}\delta(X^{d+1} - X^{d+1}(x))\right] \qquad (3.30)$$

where P_{IJ} is the projection operator parallel to the brane, given in terms of unit normal n^I as

$$P_{IJ} = G_{IJ} - n_I n_J \qquad (3.31)$$

and $X^{d+1}(x)$ gives the position of the brane in terms of its intrinsic coordinates x^μ. Off the brane, (3.30) gives

$$R = \sqrt{\frac{-d(d-1)M^{d-1}}{\Lambda}}. \qquad (3.32)$$

As in [26], the brane tension is fine-tuned to give a Poincare-invariant solution with symmetric (orbifold) boundary conditions about the brane; this condition is

$$\tau = \frac{4(d-1)M^{d-1}}{R}. \qquad (3.33)$$

The location of the brane is arbitrary; we take it to be $z = R$.

The rest of this subsection will focus on deriving the linearized gravitational field
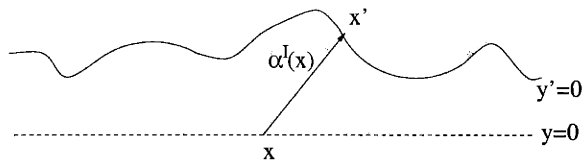
Figure 3-1: A generic deformation of the base surface leads to a redefinition of gaussian normal coordinates.

due to an arbitrary source; the results are presented in eqs. (3.23), (3.24), and (3.26) for readers not wishing to follow the details of the derivation. As we'll see, maintaining the linearized approximation requires choosing a gauge in which the brane is bent, with displacement given in eq. (3.22).

It is often easier to work with the coordinate y, defined by

$$z = Re^{y/R},\tag{3.34}$$

in which the AdS metric takes the form

$$ds^2 = dy^2 + e^{-2|y|/R}\eta_{\mu\nu}dx^\mu dx^\nu;\tag{3.35}$$

the brane is at $y = 0$, and we have written the solution in a form valid for all y.

It is convenient to describe fluctuations about (3.35) in Riemann normal (or hypersurface orthogonal) coordinates, which can be locally defined for an arbitrary spacetime metric which then takes the form

$$ds^2 = dy^2 + g_{\mu\nu}(x,y)dx^\mu dx^\nu.\tag{3.36}$$

The coordinate y picks out a preferred family of hypersurfaces, $y = const$. Such coordinates are not unique; the choice of a base hypersurface on which they are constructed is arbitrary. This base hypersurface may be taken to be the brane, but later another choice will be convenient.

In the case where the coordinates are based on the brane, small fluctuations in

the metric can be represented as

$$ds^2 = dy^2 + e^{-2|y|/R}[\eta_{\mu\nu} + h_{\mu\nu}(x,y)]dx^\mu dx^\nu \ . \tag{3.37}$$

Parameterize a deformation of the coordinates corresponding to changing the base hypersurface (see fig. 3-1) by

$$y' = y - \alpha^y(x,y) \ ; \ x^{\mu\prime} = x^{\mu\prime}(x,y) = x^\mu - \alpha^\mu(x,y) \tag{3.38}$$

and consider a small deformation in the sense that α^y is small. Working at $y > 0$, the condition that the metric takes Gaussian normal form (3.36) in the new coordinates is[4]

$$2\partial_{y'}\alpha^y + g_{\mu\nu}\frac{\partial x^\mu}{\partial y'}\frac{\partial x^\nu}{\partial y'} \ \ = 0 \tag{3.39}$$

$$\partial_{\mu'}\alpha^y + g_{\mu\nu}\frac{\partial x^\mu}{\partial y'}\frac{\partial x^\nu}{\partial x^{\mu\prime}} \ \ = 0 \ . \tag{3.40}$$

In the background metric (3.35), at $y < R$ this has general solution in terms of arbitrary small functions of x:

$$\alpha^y = \ \ \alpha^y(x) \tag{3.41}$$

$$\alpha^\mu(x,y) \ \ = -\tfrac{R}{2}\,e^{2y/R}\partial_\mu\alpha^y(x^\nu) + \beta^\mu(x) \ . \tag{3.42}$$

The corresponding small gauge transformation in the fluctuation $h_{\mu\nu}$ is

$$h_{\mu\nu} \to h'_{\mu\nu} = h_{\mu\nu} + \partial_\mu\alpha_\nu + \partial_\nu\alpha_\mu - \frac{2\alpha^y}{R}\,\eta_{\mu\nu} \ . \tag{3.43}$$

It is straightforward to expand Einstein's equations (3.30) in the fluctuation $h_{\mu\nu}$;

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[4]Here we have suppressed a subtlety that arises in trying to write a similar expression valid on both sides of the brane due to the discontinuity in the change of coordinates (3.38) across the brane

at linear order the result is

$$2\mathcal{G}_y^{(1)y} = -{}^{(d)}\mathcal{R} - \frac{(d-1)}{R}\,\partial_y h\epsilon(y) = \frac{T_y^y}{M^{d-1}} \tag{3.44}$$

$$2\mathcal{G}_\mu^{(1)y} = \partial_y \partial^\nu (h_{\mu\nu} - h\eta_{\mu\nu}) = \frac{T_\mu^y}{M^{d-1}} \tag{3.45}$$

$$\mathcal{G}_\nu^{(1)\mu} = {}^{(d)}\mathcal{G}_\nu^\mu + \frac{d\epsilon(y)}{2R}\,\partial_y(h_\nu^\mu - \delta_\nu^\mu h) - \frac{1}{2}\,\partial_y^2(h_\nu^\mu - \delta_\nu^\mu h) = T_\nu^\mu \frac{e^{2|y|/R}}{2M^{d-1}} \tag{3.46}$$

where $\mathcal{G}^{(1)}$ denotes the linear part of the Einstein tensor. In these and subsequent equations, indices on "small" quantities $h_{\mu\nu}$, $T_{\mu\nu}$, etc. are raised and lowered with $\eta_{\mu\nu}$. ${}^{(d)}\mathcal{R}$ and ${}^{(d)}\mathcal{G}_\nu^\mu$ represent the curvatures of the induced d-dimensional metric of (3.36) (which include the conformal factor and will subsequently be expanded in h), and we have used the definitions

$$\epsilon(y) = \begin{cases} 1 & \text{if } y > 0; \\ -1 & \text{if } y < 0 \end{cases} \tag{3.47}$$

and $h = h_\mu^\mu$.

Boundary conditions on $h_{\mu\nu}$ at the brane are readily deduced by integrating the equation (3.46) from just below to just above the brane – resulting in the Israel matching conditions [42] – and enforcing symmetry under $y \to -y$. If the energy momentum tensor includes a contribution from matter on the brane,

$$T_{\mu\nu}^{\text{brane}} = S_{\mu\nu}(x)\delta(y)\,, \quad T_{yy}^{\text{brane}} = T_{y\mu}^{\text{brane}} = 0 \tag{3.48}$$

then we find

$$\partial_y(h_{\mu\nu} - \eta_{\mu\nu}h)_{|y=0} = -\frac{S_{\mu\nu}(x)}{2M^{d-1}}\,. \tag{3.49}$$

The first step in solving Einstein's equations (3.44-3.46) is to eliminate ${}^{(d)}\mathcal{R}$ between the (yy) and $(\mu\mu)$ equations, resulting in an equation for h alone, (working on the $y > 0$ side of the brane)

$$\partial_y\left(e^{-2y/R}\partial_y h\right) = \frac{1}{(d-1)M^{d-1}}\left[T_\mu^\mu - (d-2)e^{-2y/R}\,T_y^y\right]\,. \tag{3.50}$$
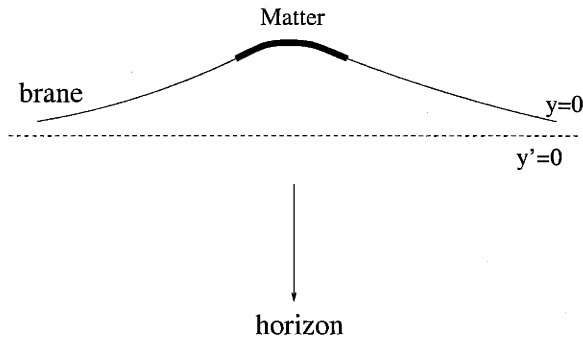
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Figure 3-2: In the presence of matter on the brane, the brane is bent

Conservation of T allows this to be rewritten

$$\partial_y \left[e^{-2y/R} \left(\partial_y h + \frac{R}{(d-1)M^{d-1}} T_{yy} \right) \right] = -\frac{R}{(d-1)M^{d-1}} \partial^\mu T_{\mu y} \ . \qquad (3.51)$$

This can be integrated with initial condition supplied by the trace of (4.26). There is however an apparent problem if $S^\mu_\mu \neq 0$ or $\partial^\mu T_{\mu y} \neq 0$: the resulting h grows exponentially, leading to failure of the linear approximation.

Fortunately this is a gauge artifact, resulting from basing coordinates on the brane. Indeed, non-vanishing $S_{\mu\nu}(x)$ produces extra extrinsic curvature on the brane; to avoid pathological growth in perturbations one should choose coordinates that are straight with respect to the horizon. In this coordinate system, the brane appears bent, as was pointed out in [79]. (See fig. 2). For simplicity consider the case where all matter is localized to $y < y_m$, for some y_m. First, the exponential growth in h due to the initial condition (4.26) can be eliminated by a coordinate transformation of the form (3.41). We may then integrate up in y until we encounter another source for this growth due to $T_{\mu y}$ on the RHS of (3.51, and kill that by again performing a small deformation of the Gaussian-normal slicing. We can proceed iteratively at increasing y in this fashion, with net result that the exponentially growing part of h can be eliminated by a general slice deformation satisfying

$$\partial_\mu \partial^\mu \alpha^y(x) = \frac{1}{2(d-1)M^{d-1}} \left[\frac{S^\mu_\mu(x)}{2} + R T_{yy}(0) - R \int_0^{y_m} dy \, \partial^\mu T_{\mu y} \right] \ ; \qquad (3.52)$$

in this equation and the remainder of the section, we work in the region on the $y > 0$

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side of the brane. In particular, consider the case $\partial^\mu T_{\mu y} = T_{yy}(0) = 0$; the solution to (3.52) then explicitly gives the bending of the brane due to massive matter on the brane.

In order to solve Einstein's equations we'll therefore work with the metric fluctuation $h'_{\mu\nu}$ in this gauge, which for small coordinate transformations is given by (??) (the spatial piece $\beta^\mu(x)$ is still arbitrary). Eq. (3.51) has first integral

$$\partial_y h' = -\frac{R}{(d-1)M^{d-1}}\left[T_{yy}(y) - e^{2y/R}\int_y^{y_m} dy\,\partial^\mu T_{\mu y}\right] \tag{3.53}$$

and can be solved by quadrature, up to the boundary conditions at $y = 0$. Eq. (3.45) is then

$$\partial_y \partial^\nu h'_{\mu\nu} = \partial_y \partial_\mu h' + \frac{T^y_\mu}{M^{d-1}} \tag{3.54}$$

and can be integrated, again given the boundary conditions, to give the longitudinal piece of $h'_{\mu\nu}$. Finally, linearizing $^{(d)}\mathcal{G}^\mu_\nu$ in (3.46) and defining

$$\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}\eta_{\mu\nu}h \tag{3.55}$$

gives

$$\begin{aligned}
\Box\bar{h}'_{\mu\nu} &= e^{2y/R}\left(-\eta_{\mu\nu}\partial^\lambda\partial^\sigma \bar{h}'_{\lambda\sigma} + \partial^\lambda\partial_\mu \bar{h}'_{\nu\lambda} + \partial^\lambda\partial_\nu \bar{h}'_{\mu\lambda}\right) \\
&+ \frac{\eta_{\mu\nu}}{2}e^{yd/R}\partial_y(e^{-yd/R}\partial_y h') \\
&- \frac{e^{2y/R}}{M^{d-1}}T_{\mu\nu}\,.
\end{aligned} \tag{3.56}$$

In this expression, $\Box$ is the *scalar* anti-de Sitter laplacian. All quantities on the right-hand side of (4.22) are known, so $h'_{\mu\nu}$ is determined in terms of the scalar Green function for the brane background, found in the preceding section. The metric deformation itself is given by trace-reversing,

$$h_{\mu\nu} = \bar{h}_{\mu\nu} - \frac{\bar{h}}{d-2}\eta_{\mu\nu}\,. \tag{3.57}$$

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Note, however, that (4.22) also suffers potential difficulty from exponentially growing sources. By (3.53) we see that for a bounded matter distribution, the trouble only lies in the terms involving $\partial^\nu \bar{h}'_{\mu\nu}$. Note also that outside matter $\partial_y \partial^\nu \bar{h}'_{\mu\nu} = 0$, from (4.21) and (3.53). Therefore, the remaining gauge invariance $\beta^\mu(x)$ in (3.41) can be used to set these contributions to zero just outside the matter distribution,

$$\partial^\nu \bar{h}'_{\mu\nu}|_{y=y_m} = 0 \ , \tag{3.58}$$

and the same holds for all $y > y_m$, eliminating the difficulty. Eq. (4.22) can then be solved for $h'_{\mu\nu}$ using the scalar Green function $\Delta_{d+1}(X, X')$, given in eq. (A.1).

This will give an explicit (but somewhat complicated) formula for the gravitational Green function, defined in general by

$$h_{IJ}(X) = \frac{1}{M^{d-1}} \int d^{d+1}X \sqrt{-G} \Delta_{IJ}^{KL}(X, X') T_{KL}(X') \ , \tag{3.59}$$

and which can be read off in this gauge from (3.53), (4.21), and (4.22). In order to better understand these results, the following two subsections will treat two special cases.

3.3.2   Matter source on the brane

Consider the case where the only energy-momentum is on the brane. In terms of the flat-space Green function Δ_d, (3.52) determines a brane-bending function of the form

$$\alpha^y(x) = \frac{1}{4(d-1)M^{d-1}} \int d^d x' \, \Delta_d(x, x') \, S^\mu_\mu(x') \ . \tag{3.60}$$

Eq. (3.53) and (4.21) then imply

$$\partial_y h' = 0 = \partial_y \partial^\nu h'_{\mu\nu} \ . \tag{3.61}$$

The gauge freedom $\beta^\mu(x)$ can then be used to set

$$\partial^\nu \bar{h}'_{\mu\nu} = 0 \tag{3.62}$$

and the remaining equation (4.22) becomes

$$\Box \bar{h}'_{\mu\nu} = 0 \ . \tag{3.63}$$

Boundary conditions for this are determined from the boundary condition (4.26) and the gauge shift induced by (3.60), and take the form

$$\partial_y(h'_{\mu\nu} - h'\eta_{\mu\nu})|_{y=0} = -\frac{S_{\mu\nu}}{2M^{d-1}} - 2(\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial_\lambda^2)\,\alpha^y \ . \tag{3.64}$$

In terms of the scalar Neumann Green function Δ_{d+1} of the preceding section, the solution is given by

$$h'_{\mu\nu}(X) = \bar{h}'_{\mu\nu}(X) = \frac{1}{2M^{d-1}} \int d^dx'\sqrt{-g}\,\Delta_{d+1}(X;x',0) \tag{3.65}$$
$$\left[S_{\mu\nu}(x') - \frac{1}{d-1}\,\eta_{\mu\nu}S(x') + \frac{1}{d-1}\,\frac{\partial_\mu\partial_\nu}{\partial_\lambda^2}\,S(x') \right]$$

where the first equality follows from tracelessness of $\bar{h}'_{\mu\nu}$; the source on the RHS is clearly transverse and traceless as well. Recall that in this gauge the brane is located at $y = -\alpha^y$.

The quantity $h'_{\mu\nu}$ is appropriate for discussing observations in the bulk, but a simpler gauge exists for observers on the brane. First note that integration by parts and translation invariance of $\Delta_{d+1}(x,0;x',0)$ implies

$$\int d^dx'\sqrt{-g}\,\Delta_{d+1}(x,0;x',0)\left[2\frac{\partial_\mu\partial_\nu}{\partial_\lambda^2} - \eta_{\mu\nu}\frac{\partial_\lambda^2}{\partial_\lambda^2}\right]S(x') \tag{3.66}$$
$$= (2\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial_\lambda^2)\int d^dx'\sqrt{-g}\,\Delta_{d+1}(x,0;x',0)\frac{1}{\partial_\lambda^2}S(x') \ ;$$

this, together with a gauge transformation using $\beta^\mu(x)$, can be used to eliminate the third term in (3.65). We then return to $\bar{h}_{\mu\nu}$ by inverting the gauge transformation

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(3.43) ; from the d-dimensional perspective the only gauge non-trivial piece is the α^y term, which we rewrite using (3.60). Thus, modulo d-dimensional gauge transformations,

$$\bar{h}_{\mu\nu}(x) = \frac{1}{2M^{d-1}} \int d^d x' \Big\{ \Delta_{d+1}(x, 0; x', 0) S_{\mu\nu}(x') - \tag{3.67}$$
$$\eta_{\mu\nu} \Big[\Delta_{d+1}(x, 0; x', 0) - \frac{(d-2)}{R} \Delta_d(x, x') \Big] \frac{S_\lambda^\lambda(x')}{2(d-1)} \Big\} \ .$$

Note from (3.24) that the zero-mode piece cancels in the term multiplying S_λ^λ. Writing the result in terms of the d-dimensional propagator and Kaluza-Klein kernel given in (3.27) then yields

$$\bar{h}_{\mu\nu}(x) = \frac{d-2}{2RM^{d-1}} \int d^d x' \Delta_d(x, x') S_{\mu\nu}(x') \tag{3.68}$$
$$+ \frac{1}{2M^{d-1}} \int d^d x' \Delta_{KK}(x, x') \Big[S_{\mu\nu}(x') - \frac{\eta_{\mu\nu}}{2(d-1)} S_\lambda^\lambda(x') \Big] \ .$$

The first term is exactly what would be expected from standard d-dimensional gravity, with Planck mass given by [5]

$$M_d^{d-2} = \frac{RM^{d-1}}{d-2} \ . \tag{3.69}$$

The second term contains the corrections due to the Kaluza-Klein modes.

3.3.3 Matter source in bulk

As a second example of the general solution provided by (4.22), suppose that the matter source is only in the bulk. This in particular includes scenarios with matter distributions on a probe brane embedded at a fixed y in the bulk [76].

By the Bianchi identities, Einstein's equations are only consistent in the presence of a conserved stress tensor. If we wish to consider matter restricted to a brane at constant y, a stabilization mechanism[6] must be present to support the matter at this

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[5] Our conventions are related to the standard ones for the gravitational coupling G (see *e.g.* [43]) by $M_4^2 = 1/16\pi G$ for four dimensions.
[6] See *e.g.* [80, 53, 67].

constant "elevation." Consider a stress tensor of the form

$$T_{\mu\nu} = S_{\mu\nu}(x)\delta(y - y_0) \tag{3.70}$$

which is conserved on the brane,

$$\partial^\mu S_{\mu\nu} = 0 . \tag{3.71}$$

For simplicity assume $T_{\mu y} = 0$. Energy conservation in bulk then states

$$\partial_y \left(e^{-dy/R}T_{yy}\right) = -\frac{e^{(2-d)y/R}}{R}S^\mu_\mu(x)\delta(y - y_0) . \tag{3.72}$$

A solution to this with $T_{yy} = 0$ for $y > y_0$ is

$$T_{yy} = \frac{e^{[2y_0+d(y-y_0)]/R}}{R}\theta(y_0 - y)S^\mu_\mu(x) . \tag{3.73}$$

We can think of this T_{yy} as arising from whatever physics is responsible for the stabilization.

Whether we consider matter confined to the brane in this way, or free to move about in the bulk, the results of this section give the linearized gravitational solution for a general conserved bulk stress tensor. Assuming for simplicity that $T_{\mu\nu} = T_{yy} = 0$ for $y > y_0$, and that $T_{\mu y} \equiv 0$, we can gauge fix such that (see (3.53),(4.21))

$$\partial^\mu \bar{h}_{\mu\nu} = 0 , \ \partial_y h = 0 \tag{3.74}$$

for $y > y_0$. Thus outside of matter, we see from eq. (4.22) that $h_{\mu\nu}$ again satisfies the scalar AdS wave equation. In particular, for matter concentrated on the probe brane at $y = y_0$, eq. (4.22) gives

$$\Box \bar{h}_{\mu\nu} = -\frac{e^{2y_0/R}}{M^{d-1}}\left[S_{\mu\nu} - \frac{1}{2(d-1)}\eta_{\mu\nu}S^\lambda_\lambda\right]\delta(y - y_0) - \frac{1}{M^{d-1}}\tilde{S}_{\mu\nu} , \tag{3.75}$$

where $\tilde{S}_{\mu\nu}$ arises from nonvanishing $\partial^\mu \bar{h}_{\mu\nu}$ for $y < y_0$ on the RHS of (4.22), resulting

from the stabilization mechanism. This has solution

$$\bar{h}_{\mu\nu}(X) \quad -\frac{1}{M^{d-1}} \int d^d x' \sqrt{-g} \Delta_{d+1}(X;x',y_0) e^{2y_0/R} \left[S_{\mu\nu}(x') - \frac{1}{2(d-1)} \eta_{\mu\nu} S_\lambda^\lambda(x') \right]$$
$$-\frac{1}{M^{d-1}} \int d^{d+1} X' \sqrt{-G} \Delta_{d+1}(X;X') \tilde{S}_{\mu\nu}(X') . \tag{3.76}$$

Again the graviton Green function $\Delta_{IJ}^{KL}(X,X')$ is given in terms of the scalar Green function Δ_{d+1} of (A.1).

3.4 Asymptotics and physics of the graviton propagator

We now turn to exploration of various aspects of the asymptotic behavior of the propagators given in the preceding two sections, both on and off the brane. This will allow us to address questions involving the strength of gravitational corrections, the shape of black holes, *etc.*

3.4.1 Source on "Planck" brane

We begin by examining the gravitational field seen on the "Planck" brane by an observer on the same brane. The relevant linearized field was given in (3.68). This clearly exhibits the expected result from linearized d-dimensional gravity, plus a correction term. The latter gives a subleading correction to gravity at long distances. This can be easily estimated: $x \gg R$ corresponds to $qR \ll 1$, where (3.27) and the small argument formula for the Hankel functions yields

$$\Delta_{KK}(x,x') \approx R \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \ln(qR) \propto R/r^4 \tag{3.77}$$

for $d = 4$, with $r = |x - x'|$. For $d > 4$, we need subleading terms in the expansion of the Hankel function. For even d, this takes the general form (neglecting numerical

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coefficients)

$$H_\nu^{(1)}(x) \sim x^{-\nu}(1 + x^2 + x^4 + \cdots) + x^\nu \ln x (1 + x^2 + x^4 + \cdots) . \tag{3.78}$$

Powers of q in the integrand of (3.27) yield terms smaller than powers of $1/r$ (contact terms, exponentially supressed terms). The leading contribution to the propagator comes from the logarithm, with coefficient the smallest power of q. This gives

$$\Delta_{KK}(x, x') \propto \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} q^{d-4} \ln(qR) \propto 1/|x - x'|^{2d-4} \tag{3.79}$$

for $d > 4$ and even. For odd d, the logarithm terms are not present in the expansion (3.78), and such corrections vanish. Thus for general even d, the dominant correction terms are supressed by a factor of $(R/r)^{d-2}$ relative to the leading term; these are swamped by post-newtonian corrections. Note that in the special case $d = 3$, Δ_{KK} was *exactly* given by eq. (3.28), yielding a correction term of order R/r for a static source, as noted in [77].

One can also examine the short-distance, $r \ll R$, behavior of the propagator, which is governed by the large-q behavior of the Fourier transform. In this case we find

$$\Delta_{KK}(x, x') \approx \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \frac{1}{p} \propto \frac{1}{|x - x'|^{d-1}} . \tag{3.80}$$

Here clearly the Kaluza-Klein term dominates, and gives the expected $d + 1$ dimensional behavior.

Next consider the asymptotics for $z \gg R$ and/or $|x - x'| \gg R$, with a source on the Planck brane. These are dominated by the region of the integral with $q < min(1/z, 1/|x - x'|)$. This means that a small argument expansion in qR can be made in the denominator of the propagator (A.3), and this gives

$$\Delta_{d+1}(x, z; x', R) \approx \frac{2\pi i}{R\Gamma\left(\frac{d}{2} - 1\right)} \left(\frac{z}{2}\right)^{d/2} \tag{3.81}$$

$$\int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} q^{d/2-2} H_{\frac{d}{2}}^{(1)}(qz) \left[1 + \mathcal{O}((qR)^{d-2} \log qR)\right] .$$

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In particular $z \gg |x - x'|$ gives $\exp\{ipx\} \approx 1$, and we find a falloff

$$\Delta_{d+1}(x, z; x', R) \sim \frac{1}{Rz^{d-2}} \tag{3.82}$$

in the propagator at large z.

Note that this means that in the physical case $d = 4$, for a static source, the potential falls like $1/z$ at large z. This calculation can be taken further to determine the asymptotic shape of the Green function and potential as a function of r and z; we do so only for the $d = 4$ potential though the calculation may be extended to other d. The static potential for a source at $x' = 0$ follows from (3.81) by integrating over time,

$$V(x, z) = \int dt \Delta_{4+1}(x, z; 0, R) \tag{3.83}$$

which then becomes

$$V(x, z) \approx \frac{\pi i z^2}{2R} \int \frac{d^3 p}{(2\pi)^3} e^{ipx} H_2^{(1)}(ipz) \ . \tag{3.84}$$

This integral is straightforward to perform, and yields

$$V(x, z) = -\frac{3}{4\pi} \frac{1}{Rz} \left(1 + \frac{2r^2}{3z^2}\right) \left(1 + \frac{r^2}{z^2}\right)^{-3/2} \left[1 + \mathcal{O}\left(\frac{R^2}{z^2}, \frac{R^2}{r^2}\right)\right] \ , \tag{3.85}$$

giving the large r and large z dependence. We will return to discuss implications of the $1/z$ behavior shortly.

3.4.2 Source in bulk or on probe brane

Similar results hold for gravitational sources in the bulk, for example due to matter on a probe brane as described in (3.76). As seen there, the detailed field depends on the form of the stabilization mechanism, but a general understanding follows from consideration of the scalar propagator entering into (3.76). Begin by considering the general scalar propagator (A.1) for $|x - x'| \gg R, z, z'$. This region is again governed

by the small-q expansion. At leading order in q, (A.1) again yields

$$\Delta_{d+1}(x, z; x, z') \approx \frac{d-2}{R} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \frac{1}{q^2} \ . \tag{3.86}$$

Note that the result is z independent, as in the analogous situation in Kaluza-Klein theory when we consider sources at large separation compared to the compact radius; the long-distance field is determined by the zero mode.

From this result we find that the gravitational potential energy between two objects at coordinate $z > R$ and with $d + 1$ dimensional masses m and m' behaves as

$$V(r) \propto \frac{1}{RM^{d-1}} \left(\frac{R}{z}\right)^2 \frac{mm'}{r^{d-3}} \ . \tag{3.87}$$

(Note that this potential only includes contributions from the two objects and not their stabilizing fields.) This can be rewritten in terms of the d-dimensional "physical" mass using

$$m_d = Rm/z \ . \tag{3.88}$$

For probe-brane scenarios [76], it is also important to understand the size of the corrections to this formula. This follows from (A.1) and the expansions (3.78) and (again neglecting numerical coefficients)

$$J_\nu(x) \sim x^\nu (1 + x^2 + x^4 + \cdots) \ . \tag{3.89}$$

Applying these to the first term in (A.1) yields (for even d)

$$\frac{J_{\frac{d}{2}-1}(qR)}{H_{\frac{d}{2}-1}(qR)} H_{\frac{d}{2}}^2(qz) \tag{3.90}$$

$$\sim \frac{R^{d-2}}{z^d} \frac{1}{q^2} \left[1 + q^2 R^2 + q^2 z^2 + (qR)^{d-2} \ln(qR) + q^d z^d \ln(qz)\right] \left\{1 + \mathcal{O}\left[(qz)^2, (qR)^2\right]\right\} \ .$$

Here we have dropped subdominant terms. The corrections involving simple powers of q again all integrate to yield contact terms at $x = x'$, so the dominant corrections at finite separation come from the logarithmic terms. These then give contributions

to the propagator of the form

$$\Delta(x, z; x', z) \sim \frac{1}{Rr^{d-2}} \left[1 + \left(\frac{R}{r} \right)^{d-2} + \frac{z^d}{r^d} + \cdots \right] . \qquad (3.91)$$

Combining this with a similar analysis of the second term in (A.1) using

$$J_{\frac{d}{2}}(qz) H_{\frac{d}{2}}(qz) \sim 1 + q^2 z^2 + \cdots + q^d z^d \ln(qz) + \cdots \qquad (3.92)$$

leads to an expansion of the form

$$\Delta(x, z; x', z') \sim \frac{1}{Rr^{d-2}} \left[1 + \frac{R^{d-2}}{r^{d-2}} + \frac{z^d}{r^d} + \frac{z^{2d}}{r^{2d}} \frac{r^{d-2}}{R^{d-2}} \right] \left[1 + \mathcal{O} \left(\frac{z^2}{r^2}, \frac{R^2}{r^2} \right) \right] \qquad (3.93)$$

for the propagator for even d. Which terms are dominant depends on the magnitude of r. At long distances, $r^2 > z^d / R^{d-2}$, the first term is dominant. In the physical case of $d = 4$, the corresponding scale is

$$r \sim z^2 / R \sim (10^{-4} eV)^{-1} \qquad (3.94)$$

in the scenario of [76], and at larger scales the corrections to the newtonian potential therefore go like $1/r^3$ with a Planck-size coefficient, and would be swamped by post-newtonian corrections, as with corrections on the Planck brane. On the other hand, at shorter scales than (3.94), the last term in (4.68) dominates. This gives a propagator correction of the form z^8 / r^8 in four dimensions.[7] This is the first correction that we could hope to measure in high-energy experiments.

It is interesting to investigate the asymptotics of the propagator in more detail. Concretely, consider a source at (x', z') with $z' \gg R$. In the limit r^2 and/or $z^2 \gg (z')^d / R^{d-2}$, or $z^d \ll (z')^2 R^{d-2}$, the first term in the propagator (A.1) dominates over the bulk AdS part given by the second term. Consequently, the behavior is given by expressions like (3.81) and (3.85) (where z is replaced by z' for the latter limit). On

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[7]This is in agreement with [76], and the subleading corrections can also be obtained from the mode sum.

the other hand, for $r^2, z^2 \ll (z')^d/R^{d-2}$, the bulk AdS portion dominates and hence determines the shape of the potential. This AdS propagator is explicitly given in terms of a hypergeometric function [20, 21]. This transition to bulk AdS behavior is that seen in (4.68). Indeed, the asymptotic behavior of the bulk AdS propagator for $r \gg z, z', R$,

$$G_{\text{AdS}} \sim \left[\frac{zz'}{r^2 + (z - z')^2} \right]^d , \qquad (3.95)$$

is what determines the z^{2d}/r^{2d} corrections discussed there.

Finally, the short-distance bulk asymptotics are also easily examined. Specifically, let $|x - x'| \ll R$ and $|z - z'| \ll R$; again, the result is governed by the large qR behavior of the Green function. From (A.1) we find

$$\Delta_{d+1}(x, z; x', z') \approx -\frac{i\pi}{4R^{d-1}}(zz')^{\frac{d}{2}} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} H^{(1)}_{\frac{d}{2}}(qz_>) H^{(2)}_{\frac{d}{2}}(qz_<) , \qquad (3.96)$$

where $H^{(2)} = J - iY$. Aside from non-standard boundary conditions, this is the usual propagator for anti-de Sitter space, and at short distances compared to R will reduce to the flat space propagator (see *e.g* [34]). In particular, suppose that $|x - x'| \ll z, z'$. Then we may also use the large qz expansion, which gives

$$\Delta_{d+1}(x, z; x', z') \approx -\frac{i}{2}\left(\frac{zz'}{R^2}\right)^{\frac{d-1}{2}} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')+iq(z_>-z_<)}\frac{1}{q} , \qquad (3.97)$$

resulting in

$$\Delta_{d+1}(x, z; x', z') \sim \left(\frac{zz'/R^2}{|x - x'|^2 + |z - z'|^2}\right)^{\frac{d-1}{2}} \qquad (3.98)$$

which is the expected behavior in $d + 1$ dimensions.

3.4.3 Off-brane profile of gravitational fields and black holes

These results can readily be applied to discuss some interesting properties of black holes and more general gravitational fields in the context of brane-localized gravity. In particular, one might ask what a black hole – or more general gravitational field – formed from collapsing matter on the brane looks like. In a naïve analysis, considering

only fluctuations in the zero mode (3.4), one finds that metrics of the form

$$ds^2 = \frac{R^2}{z^2}[dz^2 + g_{\mu\nu}(x)dx^\mu dx^\nu] \tag{3.99}$$

are solutions of Einstein's equations for general Ricci flat four-dimensional metric $g_{\mu\nu}(x)$. These solutions are, however, singular on the horizon at $z = \infty$, as was discussed in [91]. Nonsingular solutions require excitation of the other modes of the graviton. Although exact solutions are elusive except for $d = 3$ [77], the linear analysis of this chapter gives us the general picture.

Consider a massive object, $m \gg 1/R$, on the Planck brane. Without loss of generality its metric may be put in the form (3.37). The linear approximation is valid far from the object (for more discussion, see the next subsection). As we've seen in section 3.4.1, at long distances along the brane we recover standard linearized gravity, with potential of the form (3.87). Transverse to the brane, we use the expression (3.82), which, with (3.65), implies that

$$h_{00} \sim \frac{m}{M_d^{d-2} z^{d-3}} \tag{3.100}$$

(the extra power of z arises because we are considering the static potential). Note that the proper distance off the brane is given by y, defined in eq. (3.34). Thus in fact the metric due to an object on the brane falls exponentially with proper distance off the brane.

If the mass is compact enough to form a black hole, the corresponding horizon is the surface where (for static black holes) $h_{00}(x,y) = 1$. In the absence of an exact solution, the horizon is approximately characterized by the condition $h_{00} \sim 1$. Since we recover linearized d-dimensional gravity at long distances along the brane, the horizon location on the brane is given by the usual condition in terms of the d-dimensional Planck mass (3.69):

$$r_h^{d-3} \sim \frac{m}{M_d^{d-2}} \, . \tag{3.101}$$

Transverse to the brane (3.100) implies that the horizon is at $z_h \sim r_h$. Taking into account the exponential relation between y and z, a rough measure of the proper "size" of the black hole transverse to the brane is

$$y_h \sim R \log\left[\left(\frac{m}{M_d^{d-2}}\right)^{\frac{1}{d-3}} \frac{1}{R}\right] . \tag{3.102}$$

So while the size along the brane grows like $m^{1/d-3}$, the thickness transverse to the brane grows only like $\log m$. The black hole is shaped like a pancake.[8] This black pancake has a constant coordinate radius r_h in x coordinates for $z < z_h$, as seen in (3.86). From (3.8) we then see that the proper physical size scales with z as

$$r_h(z) = R r_h / z . \tag{3.103}$$

An amusing consequence of this picture of black holes is the possibility that matter on a collision course with a black hole from the four-dimensional perspective can in fact pass around it through the fifth dimension.[9] This suggests that from the perspective of the four-dimensional observer, matter can pass through a black hole! To investigate this more closely, consider the specific case of a black hole on the Planck brane. A bulk mode (graviton or other bulk matter) can bypass the black hole by traveling at $z > z_h$. Suppose first that the proper wavelength of the bulk mode is $\lambda > R$. From the four dimensional perspective of the Planck brane the wavelength is redshifted,

$$\lambda_d \sim \left(\frac{z}{R}\right)\lambda > z_h\left(\frac{\lambda}{R}\right) . \tag{3.104}$$

Thus for $\lambda > R$, such matter has a wavelength larger than the size of the black hole, which suggests that this process is difficult to distinguish from passage around the black hole in four dimensions. Next consider a mode with wavelength $\lambda < R$. Such a perturbation obeys the geometric optics approximation and will fall into the horizon

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[8]The value of z_h was guessed by [91] who nonetheless referred to the resulting object as a black cigar instead of a black pancake. Ref. [77] also independently found the black pancake picture in the special case of $d = 3$, where they were able to find an exact solution.

[9]We thank L. Susskind for this observation.

at $z = \infty$; if emitted from the Planck brane it will never return. However, although this particle is moving in the z direction it might be possible that a four-dimensional observer could observe it's projected gravitational field emerging from the far side of the black hole. We leave further investigation of this process for future work.

A semi-quantitative picture of the shape of gravitational fields due to sources in the bulk, e.g. on a probe brane, also follows from the asymptotics of the propagator as described in section 3.4.2. Again, let the source be at $x' = 0$ and $z' \gg R$. At short distances $r^2, z^2 \ll (z')^d / R^{d-2}$, the shape is given by the bulk AdS propagator (which, as expected, for $r \ll R$ and $z - z' \ll R$ reduces to the flat $d+1$ dimensional result as seen in (3.98).) At longer distances, r^2 and/or $z^2 \gg (z')^d / R^{d-2}$, or $z^d \ll (z')^2 R^{d-2}$, the zero mode begins to dominate with a shape given by (3.85) or its d-dimensional generalization. This determines the pancake shape discussed above.

3.4.4 Strength of perturbations

Lastly we turn to the question of consistency of both the linear approximation, as well as of the scenario of [26] within the context of a complete theory of quantum gravity, such as string theory (see the next section for comments on the latter). For simplicity we confine the discussion to the $d = 4$ case, although the results clearly extend.

If one were to base one's analysis solely on the properties of the zero-mode of (3.4), serious questions of consistency immediately arise. For one thing, the black tube metrics mentioned in the introduction become singular at the AdS horizon [91]. Alternately, trouble would be encountered when one considered consistency of linearized gravity, or of the overall scenario after higher-order corrections to gravity of the form

$$S \to S + \alpha \int d^5 X \sqrt{-G} \mathcal{R}^2 \tag{3.105}$$

are taken into account. The apparent trouble occurs due to the growth of the curvature of the zero-mode correction to the metric, (3.3).

To see the problems more directly, recall that the general structure of the curvature

scalar is

$$\mathcal{R} \sim G^{-2}\partial^2 G \sim z^2 \left(\partial^2 h + h\partial^2 h + h^2\partial^2 h + \cdots\right) \ . \tag{3.106}$$

Similarly, scalars comprised of p powers of the curvature will grow like z^{2p}. According to this naïve analysis, such terms would dominate the action of the zero mode, which would suggest the scenario is inconsistent. Likewise, consider the Einstein-Hilbert action,

$$S \sim \int d^5 X \frac{1}{z^3} \left(h\partial^2 h + h^2\partial^2 h + \cdots\right) \ . \tag{3.107}$$

Rescaling $h \to z^{3/2} h$ gives an expansion of the form

$$S \sim \int d^5 X \left(h\partial^2 h + z^{3/2} h^2\partial^2 h + \cdots\right) \tag{3.108}$$

with diverging expansion parameter, a similar problem. What these arguments do not account for is that far from the brane the zero-mode no longer dominates, and in fact for a static source the perturbation falls as $h \sim 1/z$ as seen in (3.100). This ensures that $\mathcal{R}^p$ corrections and non-linear corrections to linearized Einstein-Hilbert action are indeed suppressed, in accord with the intuition that a localized gravitational source produces a localized field, not a field that is strong at the horizon.

This addresses the question of potential strong gravitational effects near the horizon in the static case. We see that the source of the apparently singular results is the treatment of the zero mode in isolation; with the full propagator the field dies off as it should with z. This leaves open questions about potentially strong effects in dynamical processes. However, once again given the falloff of the propagator with z, we expect this situation to be very similar to that of scattering in the presence of a black hole, where for most local physics processes (e.g. near the brane), the existence of the black hole is irrelevant. However, while we have not attempted to fully describe particles that fall into the AdS horizon, and radiation that is potentially reemitted from the horizon, this may require confronting the usual puzzles of strongly coupled gravity.[10]

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[10]For related comments see [73].

3.5 Conclusion

The scenario posed in [26] has by now survived several consistency checks, including those of this chapter. In a linearized analysis, this chapter has outlined many interesting features of gravity and gravitational corrections in this scenario. These await a treatment in an exact non-linear analysis. Many other questions remain, including other aspects of phenomenology and cosmology, and that of a first-principles derivation of such scenarios in string theory.

Chapter 4

Effective theories and black hole production in warped compactifications

4.1 Introduction

It is an old idea that, as an alternative to compactification, the observed Universe instead lives on a brane in a higher-dimensional space. Such "branification" scenarios had however until recently been hard to realize, largely because of the difficulty of recovering four-dimensional gravitational dynamics. Two new approaches have changed this and at the same time suggested new views of the origin of the hierarchy of scales in physics. The first, pursued by [68], is a hybrid of branification and compactification, in which matter is confined to a brane and then large-radius compactification of the extra dimensions yields four-dimensional gravity at long distances.

A more recent approach utilizes warped compactifications to achieve effectively four-dimensional gravitational dynamics. A outline of such a picture has been provided by the RS2 model [72]. This utilizes a "Planck brane" that serves as the boundary of five-dimensional anti-de Sitter space, and the curvature of anti-de Sitter space effectively "localizes" low-energy gravity to the brane. Related models are

the RSI model [71] in which AdS is terminated above the horizon by a "negative tension brane," and the model of Lykken and Randall [76] in which visible sector matter lives on a probe brane. None of these are fundamental pictures as they do not provide a microscopic dynamics for the Planck, "negative-tension," and probe branes, but recent work in string theory has begun to provide descriptions of such objects. In particular [75] has given a geometrical realization of an object akin to a Planck brane, and [86, 84] have provided geometrical realizations of objects similar to "negative-tension" branes. At the same time, these models have been connected to renormalization group flows in four-dimensional gauge theories through the AdS/CFT correspondence.

In providing a new view of the hierarchy problem, either through large radius or other geometrical mechanisms, these scenarios suggest the exciting possibility that quantum gravity effects could be observed at scales far below the usual Planck scale, and perhaps even near the TeV scale. They also suggest the possibility of interesting new gravitational phenomena, particularly in scenarios with infinite extra dimensions (*e.g.* RS2) and with non-trivial curvature and horizon structure of the resulting spacetime.

Some aspects of this gravitational dynamics has been studied in [77, 78], [79]–[69]. In particular, [69] studied linearized gravity in the RS2 scenario, and gave both prescriptions for computing propagators and a general picture of the structure of black holes bound to the Planck brane. The latter were found to be pancake-like objects, whose transverse sizes are logarithmically smaller than their four-dimensional Schwarzschild radii. Cosmology of these scenarios has also been extensively studied (see *e.g.* [65, 66] [67]–[57]) with suggestions that they offer new approaches to the cosmological constant problem [59, 58, 64, 63, 62, 54, 12, 60], [51]–[61].

Many open questions remain, however, in the RS2 scenario and its variants. One set of questions centers on the four-dimensional representation of the five-dimensional dynamics. In particular, localization of gravity is not complete and in the RS2 scenario there is a gapless spectrum of analogs to Kaluza-Klein modes that are weakly coupled to excitations on the brane. Therefore a four-dimensional low-energy effec-

tive field theory does not follow from the usual Kaluza-Klein reasoning, and so one challenge has been to deduce what this effective theory is. It has previously been argued [75, 73, 74, 69, 85] that the bulk dynamics can be replaced via the AdS/CFT correspondence by a conformal field theory on the brane, and this suggests an answer, namely that the effective field theory is provided by conformal field theory coupled to the visible sector solely through gravity. This chapter amplifies on this statement, clarifies the role of the cutoff, which in RS2 is expected to be at the AdS radius scale, and provides one entry in the map between the five- and four-dimensional descriptions by computing a linearized approximation to the four-dimensional stress tensor corresponding to an arbitrary five-dimensional matter distribution. This stress tensor is both conserved and traceless. Corresponding statements should hold for other warped compactification scenarios, using realizations of the AdS/CFT correspondence in more general warped compactifications.

Given the novelties of the gravitational dynamics, for example the above picture of black holes, one is also prodded to investigate whether this field theory has unusual properties. For example, consider the following question [69]: [1] suppose that a particle is launched towards a black hole on the brane with zero four-dimensional impact parameter, but such that it follows a trajectory that misses the black hole through the fifth-dimension. Does this correspond in the four-dimensional perspective to matter that enters a black hole and exits the opposite side? This would surely be a radical departure from usual four-dimensional effective theory!

However, standard AdS/CFT reasoning suggests a more mundane answer. In the UV/IR correspondence outlined in [81], a state deep in AdS corresponds to a state in the far infrared of the corresponding field theory. This suggests that a falling particle corresponds to a state that spreads. Indeed, using our results for the stress tensor we find that in the four dimensional description, the falling particle corresponds to an expanding shell of CFT matter. The condition that the five-dimensional trajectory misses the black hole becomes the four-dimensional statement that the shell misses by expanding to a size larger than the black hole.

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[1] This question was asked by L.Susskind.

It is important to emphasize that the CFT description is an effective description, and another interesting set of questions therefore regards breakdown of the effective field theory and the question of whether strong gravitational dynamics – for example black hole formation – is observable at scales far below the four-dimensional Planck scale. We investigate the scales at which scattering experiments would be expected to encounter dynamics beyond the four-dimensional description in the three scenarios outlined, RS2, the LR probe brane scenario, and RS1. In particular, in the latter scenario with a certain set of assumptions it appears possible to create black holes that decay into observable matter in scattering experiments in the vicinity of the TeV scale. This exciting possibility deserves more theoretical investigation; in particular through construction of concrete models with the required properties.

In outline, section 4.2 of this chapter discusses conformal field theory as the 4d low-energy effective theory of RS2. Section 4.3 computes the linearized effective stress tensor of bulk matter, as well as solving a corresponding simpler problem of the 4d scalar profile of a five-dimensional scalar source. It also elaborates on the black hole flyby scenario mentioned above. Section 4.4 then discusses questions of the scale of breakdown for the 4d effective theories, and of the possibility of low-energy black hole production. Section 4.5 closes with conclusions.

4.2   The effective theory of RS2

We begin with a quick review of the RS2 scenario, and of its transcription into conformal field theory via the AdS/CFT correspondence [73, 74, 69] in which we will offer some refinements. The upshot of this discussion is that the low-energy effective field theory for the RS2 scenario consists of visible 4d matter gravitationally coupled to dark matter described by a cutoff CFT. Subsequent sections will explore consequences and extensions of this picture.

The RS2 scenario is of course just an example of a much broader class of warped compactifications, which have recently been widely studied both in the context of model building, and in the context of string theory and the correspondence between

renormalization group flows and supergravity geometries. While many of our comments will be made within the framework of this greatly simplified example (for which the only known microscopic construction is [75]), corresponding arguments should apply to other models including those with stringy realizations. In particular later sections will also comment on other variants of the RS2 scenario (those with a terminated AdS space (RS1) or with a probe or "TeV" brane) and their possible stringy realizations.

We therefore begin by considering the geometry with a single "Planck" brane. Although our central interest is dimension $d = 4$, most of the relevant formulas easily generalize and will be given in arbitrary dimension. We assume that matter fields, denoted by ψ, live only on this Planck brane. The action is

$$S = \int d^{d+1}X\sqrt{-G}(M^{d-1}\mathcal{R} - \Lambda) + \int d^d x\sqrt{-\gamma}\left[\mathcal{L}(\gamma, \psi) - \tau\right] \qquad (4.1)$$

where G, M, $\mathcal{R}$, and Λ are the $d + 1$ dimensional metric, Planck mass, curvature scalar, and cosmological constant respectively, γ is the induced metric on the Planck brane, $\mathcal{L}$ is the action of matter on the brane, and τ is the brane tension. The bulk AdS metric is

$$ds^2 = \frac{R^2}{z^2}\left(dz^2 + dx_d^2\right) \qquad (4.2)$$

in $d + 1$-dimensional coordinates $X = (x, z)$; here dx_d^2 is the d-dimensional Minkowski metric and the AdS radius R is given by

$$R = \sqrt{\frac{-d(d-1)M^{d-1}}{\Lambda}}. \qquad (4.3)$$

The brane tension is fine tuned to the value

$$\tau = \frac{4(d-1)M^{d-1}}{R} \qquad (4.4)$$

in order to maintain a Poincaré invariant Planck brane. We may take the Planck brane to reside at an arbitrary elevation $z = \rho$.

As argued in [71, 72, 69], at long distances compared to R, the gravitational dynamics appears d-dimensional. However, there is also a gapless spectrum of weakly-coupled bulk modes. An obvious question is what serves as a d-dimensional low-energy effective field theory describing the dynamics. Within string theory, an answer to this is provided by the conjectured AdS/CFT correspondence [73, 74, 69].

To see this, recall that the AdS/CFT correspondence equates the $d+1$-dimensional bulk gravity (or more precisely, string theory) functional integral to a generating function in the CFT. A regulator is provided by excluding the AdS volume outside $z = \rho$. Suppose that we put the fluctuating metric in a gauge such that near this boundary

$$ds^2 = \frac{R^2}{z^2} \left[dz^2 + g_{\mu\nu}(z, x) dx^\mu dx^\nu \right]. \tag{4.5}$$

The induced metric γ on the boundary $z = \rho$ is thus

$$ds_d^2 = \frac{R^2}{\rho^2} g_{\mu\nu}(\rho, x) dx^\mu dx^\nu \equiv \gamma_{\mu\nu} dx^\mu dx^\nu. \tag{4.6}$$

Define the functional integral over bulk metrics G for fixed boundary metric γ as

$$Z[\gamma, \rho] = \int_\gamma \mathcal{D}G e^{i \int d^{d+1} X \sqrt{-G}(M^{d-1}\mathcal{R} - \Lambda) + 2iM^{d-1} \int d^d x \sqrt{-\gamma} K} \tag{4.7}$$

where K is the extrinsic curvature of the boundary. The AdS/CFT correspondence then states that for small fluctuations about the flat boundary geometry, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$

$$\lim_{\rho \to 0} e^{-iS_{\text{grav}}[\gamma]} Z[\gamma, \rho] = \left\langle e^{i \int h_{\mu\nu} T^{\mu\nu}} \right\rangle_{\text{CFT}}. \tag{4.8}$$

Here S_{grav} is a counterterm action formed purely from the induced metric γ [82, 83]; in the case $d = 4$

$$S_{\text{grav}} = \int d^4 x \sqrt{-\gamma} \left[\frac{6M^3}{R} + \frac{RM^3}{2} \mathcal{R}(\gamma) - 2M^3 R^3 \log(\rho) \mathcal{R}_2(\gamma) \right] \tag{4.9}$$

where

$$\mathcal{R}_2 = -\frac{1}{8} \mathcal{R}_{\mu\nu} \mathcal{R}^{\mu\nu} + \frac{1}{24} \mathcal{R}^2. \tag{4.10}$$

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While the AdS/CFT correspondence was originally stated in terms of small fluctuations, a natural assumption is that it extends to more general boundary geometries. We therefore assume that the CFT generating functional can be written as a functional integral over the CFT degrees of freedom, which we collectively denote χ, in the background metric $g_{\mu\nu}$, and that the correspondence thus becomes

$$\lim_{\rho \to 0} e^{-iS_{\text{grav}}[\gamma]} Z[\gamma, \rho] = \int \mathcal{D}\chi e^{i \int d^d x \sqrt{-g} \mathcal{L}_{\text{CFT}}(g_{\mu\nu}, \chi)} . \qquad (4.11)$$

Following the ideas of the UV/IR correspondence [81], we connect this with the RS scenario by extending the conjecture to a statement with a finite cutoff, and assume that

$$e^{-iS_{\text{grav}}[\gamma]} Z[\gamma, \rho] = \int [\mathcal{D}\chi]_\rho e^{i \int d^d x \sqrt{-g} \mathcal{L}_{\text{CFT}}(g_{\mu\nu}, \chi)} \qquad (4.12)$$

where the on the RHS ρ provides the cutoff scale for the CFT. While a precise description of this cutoff in the language of the CFT is not known, for sake of intuition one may imagine that it is for example given by only considering fluctuations on scales Δx such that

$$g_{\mu\nu} \Delta x^\mu \Delta x^\nu > \rho^2 . \qquad (4.13)$$

In particular, notice that since the only dependence of the CFT on the scale of the metric is through the cutoff, this implies

$$\int [\mathcal{D}\chi]_\rho e^{i \int d^d x \sqrt{-g} \mathcal{L}_{\text{CFT}}(g_{\mu\nu}, \chi)} = \int [\mathcal{D}\chi]_R e^{i \int d^d x \sqrt{-\gamma} \mathcal{L}_{\text{CFT}}(\gamma_{\mu\nu}, \chi)} \qquad (4.14)$$

where on the RHS the cutoff is thought of as restricting to fluctuations with

$$\gamma_{\mu\nu} \Delta x^\mu \Delta x^\nu > R^2 . \qquad (4.15)$$

From (4.12) and (4.14) we therefore see that the integral over the bulk modes can be replaced by a correlator in the CFT, as originally proposed in [73, 74, 69], with a cutoff given by R. Specifically, d-dimensional dynamics is summarized by a

functional integral of the form

$$\int [\mathcal{D}\gamma \mathcal{D}\psi \mathcal{D}\chi]_R e^{i\int d^d x\sqrt{-\gamma}\left[\frac{1}{2}\mathcal{L}(\gamma,\psi)+\mathcal{L}_{CFT}(\gamma,\chi)+\mathcal{L}_{grav}(\gamma)-\tau\right]}(\dots) . \qquad (4.16)$$

For consistency with the cutoff (4.15) the other modes also presumably should have a corresponding cutoff, as indicated. One consistency check on this approach is cancellation of the brane tension τ by the corresponding term in S_{grav}, using (4.4). This indicates that the low-energy effective field theory for the system, up to the scale determined by R, is the theory of brane-matter gravitationally coupled to "dark" matter described by the CFT. The d-dimensional Planck mass follows from the d-dimensional version of (4.9), and is given by

$$M_d^{d-2} = \frac{RM^{d-1}}{d-2}. \qquad (4.17)$$

4.3   Effective stress tensor of bulk matter

We now investigate some of the consequences of the above identification of the CFT as the low-energy effective field theory for the RS2 scenario. In particular, we start by giving an entry in the bulk to boundary dictionary, by computing a linearized approximation to the CFT stress tensor corresponding to a perturbation in the bulk. We then investigate the particular case of a particle freely falling into the bulk.

Using this calculation, we discuss a test of the AdS/CFT correspondence and of our effective description of RS2: suppose that we shoot a particle towards a black hole with zero 4d impact parameter, but such that it will miss the black hole through the z direction. How does a 4d observer understand the failure of the black hole to absorb the particle?

4.3.1   General results

In this subsection we turn to the problem of deriving the d-dimensional brane stress tensor that corresponds to a general $d + 1$-dimensional bulk matter distribution. In

general this is a difficult problem, requiring solution of the bulk Einstein equations, so we will only give a linear treatment.

The basic strategy is as follows. Ref. [69] computes the linearized bulk gravitational field of a general matter perturbation. This in particular gives the linearized metric and therefore Einstein tensor induced on the brane. We can then read off the matter stress tensor from the right hand side of the d-dimensional Einstein equations along the brane.

Although the resulting stress tensor has a number of special properties, we have not yet found a particularly illuminating expression for it. However, in the next subsection we specialize to the case of a particle falling into the bulk; in the long distance approximation the corresponding stress tensor simplifies substantially.

In studying gravitational perturbations it proves convenient to introduce the proper "height" coordinate y, given by

$$z = Re^{y/R} \tag{4.18}$$

in terms of which the linearized metric takes the form

$$ds^2 = dy^2 + e^{-2y/R}(\eta_{\mu\nu} + h_{\mu\nu})dx^\mu dx^\nu . \tag{4.19}$$

Eqs. (3.20), (3.24), and (3.26) of [69] then give the linearized bulk Einstein equations in terms of the metric perturbation $\bar{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}h\eta_{\mu\nu}$ as:

$$\partial_y \left(e^{-2y/R}\partial_y h\right) = \frac{1}{(d-1)M^{d-1}}\left[T_\mu^\mu - (d-2)e^{-2y/R}T_y^y\right], \tag{4.20}$$

$$\partial_y \partial^\nu h_{\mu\nu} = \partial_y \partial_\mu h + \frac{T_\mu^y}{M^{d-1}} , \tag{4.21}$$

and

$$\begin{aligned}
\Box\bar{h}_{\mu\nu} &= \frac{\eta_{\mu\nu}}{2} e^{yd/R} \partial_y(e^{-yd/R}\partial_y h) \\
&+ e^{2y/R}(-\eta_{\mu\nu}\partial^\lambda\partial^\sigma\bar{h}_{\lambda\sigma} + \partial^\lambda\partial_\mu\bar{h}_{\nu\lambda} + \partial^\lambda\partial_\nu\bar{h}_{\mu\lambda})
\end{aligned}$$

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$$- \frac{e^{2y/R}}{M^{d-1}} T_{\mu\nu} \ . \tag{4.22}$$

The right hand side of (4.22) is determined by the stress tensor and the solutions of eqs.(4.20), (4.21). This equation can then be solved for $h_{\mu\nu}$ using the scalar Neumann Green function Δ_{d+1}, satisfying

$$\Box\Delta_{d+1}(X, X') = \frac{\delta^{d+1}(X - X')}{\sqrt{-G}} \ , \tag{4.23}$$

$$\partial_y\Delta_{d+1}(X, X')|_{y=0} = 0 \ , \tag{4.24}$$

and which was derived in [69]. In the present situation we need the retarded propagator rather than the Feynman propagator; the relation between these and approximate expressions for them are given in the appendix. The resulting expression for the metric has three terms arising from the three lines of (4.22). However, the second term is inessential as a short calculation shows it to be pure gauge on the brane. Therefore its contribution drops when we compute the Einstein tensor on the brane.

One must also specify boundary conditions at the brane; in the case of a surface stress tensor

$$T_{\mu\nu}^{\text{brane}} = S_{\mu\nu}(x)\delta(y) \ , \quad T_{yy}^{\text{brane}} = T_{y\mu}^{\text{brane}} = 0 \tag{4.25}$$

these become

$$\partial_y(h_{\mu\nu} - \eta_{\mu\nu}h)_{|y=0} = -\frac{S_{\mu\nu}(x)}{2M^{d-1}} \ . \tag{4.26}$$

In order to simplify the resulting expression for the metric, it is useful to rewrite the scalar Green function in terms of a new function F as

$$\Delta_{d+1}(y, x; y', x') = e^{(d-2)y'/R}\partial_{y'}\left[e^{-(d-2)y'/R}F_{y'}(y; x - x')\right] \ . \tag{4.27}$$

One nice property of this redefinition is immediate: one readily checks that

$$\int_0^\infty dy' e^{(2-d)y'/R}\Delta_{d+1}(X, X') = -F_0(y; x - x') \tag{4.28}$$

satisfies the equation for the d-dimensional Green function, and so

$$F_0(y; x - x') = -\Delta_d(x, x') .\qquad(4.29)$$

Using this and integrating by parts gives the contribution to $\bar{h}_{\mu\nu}$ from the first line in (4.22) as

$$\bar{h}_{\mu\nu}^{(1)}(x, y) = \frac{\eta_{\mu\nu}}{2(d-1)M^{d-1}} \int dV' \partial_{y'} F_{y'}(y; x - x') \left[e^{2y/R} T_\mu^\mu(X') - (d-2)T_y^y(X') \right] .$$

$$(4.30)$$

Eq.(4.30) combines with the terms induced by the stress tensor and the surface stress (4.25) to give a complete expression of the form

$$\bar{h}_{\mu\nu} = \quad -\frac{1}{M^{d-1}} \int dV' e^{dy'/R} \partial_{y'} \left[e^{-(d-2)y'/R} F_{y'}(y; x - x') \right] T_{\mu\nu}(X') \qquad(4.31)$$

$$+\frac{\eta_{\mu\nu}}{2(d-1)M^{d-1}} \int dV' \partial_{y'} F_{y'}(y; x - x') \left[e^{2y/R} T_\mu^\mu(X') - (d-2)T_y^y(X') \right]$$

$$+\bar{h}_{\mu\nu}^S + \bar{h}_{\mu\nu}^{\text{gauge}} .$$

Here $\bar{h}_{\mu\nu}^{\text{gauge}}$ is the piece that is pure gauge on the brane, mentioned above, and $\bar{h}_{\mu\nu}^S$ is the contribution due to the surface stress (we will see an example of this shortly).

For simplicity consider a purely bulk distribution ($S_{\mu\nu} = 0$). The four-dimensional effective stress tensor is readily computed from (4.31) via Einstein's equations,

$$T_{\mu\nu}^{\text{eff}} = 2M_d^{d-2(d)} \mathcal{G}_\nu^\mu = -\frac{RM^{d-1}}{d-2}(\partial^2 \bar{h}_{\mu\nu} + \eta_{\mu\nu}\partial^\alpha\partial^\beta \bar{h}_{\alpha\beta} - \partial_\mu\partial^\alpha \bar{h}_{\alpha\nu} - \partial_\nu\partial^\alpha \bar{h}_{\alpha\mu})|_{y=0} ,$$

$$(4.32)$$

with $\bar{h}_{\mu\nu}$ given by (4.31). The contribution of $\bar{h}_{\mu\nu}^{\text{gauge}}$ drops out.

One may expand out the expression (4.32) to write it explicitly in terms of the bulk stress tensor $T_{\mu\nu}$:

$$T_{\mu\nu}^{\text{eff}} = \frac{R}{d-2} \int dV' \left\{ \{ e^{dy'/R} \partial_{y'} \left[e^{-(d-2)y'/R} F_{y'}(0; x - x') \right] (\partial^2 T_{\mu\nu} + \eta_{\mu\nu}\partial^\alpha\partial^\beta T_{\alpha\beta} \right.$$

$$-\partial^\alpha\partial_\mu T_{\alpha\nu} - \partial^\alpha\partial_\nu T_{\alpha\mu})$$

$$\left. +\frac{1}{d-1}\partial_{y'} F_{y'}(0; x - x')(\partial_\mu\partial_\nu - \eta_{\mu\nu}\partial^2) \left[e^{2y'/R} T_\mu^\mu - (d-2)T_y^y \right] \right\}.(4.33)$$

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Note that $T_{\mu\nu}^{\text{eff}}$ satisfies two important properties. First, from its construction and the Bianchi identities, it is conserved:

$$\partial^\mu T_{\mu\nu}^{\text{eff}} = 0 \ . \tag{4.34}$$

Secondly, one may readily verify that it is traceless,

$$\eta^{\mu\nu} T_{\mu\nu}^{\text{eff}} = 0 \ , \tag{4.35}$$

which accords nicely with its interpretation as arising from a conformal field theory on the brane. Indeed, this easily follows from the (yy) Einstein equation, which states (*cf.* [69] eq. (3.14))

$$^{(d)}\mathcal{R} + \frac{(d-1)}{R} \partial_y h \epsilon(y) = -\frac{T_y^y}{M^{d-1}} \tag{4.36}$$

where $\epsilon(y)$ is a step function. On the brane $\partial_y h$ and T_y^y vanish (the former by (4.26)), implying $^{(d)}\mathcal{R}(y=0) = 0$, and thus $T^{\text{eff}} = 0$.

Note that in the above discussion we have said nothing about bending of the brane, which was described in [79, 69]. The reason for this is that we are interested in the metric on the brane, and for this it is best to work in a gauge where the brane is straight. In [69] the resulting metric was computed by first working in the bent gauge, and then transforming back, but an equivalent result is found by working directly in the straight gauge.[2]

4.3.2 Matter on the brane

In order to illustrate this equivalence – and because the result will be used in the next subsection – we'll compute the linearized metric and effective stress tensor due to matter on the brane in this approach. Specifically, suppose that there is a surface stress of the form (4.25), but that otherwise $T_{IJ} = 0$. The field eqs.(4.20-4.22) should then be solved subject to the boundary conditions (4.26). By tracing the latter can

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[2]For purposes of measurements on the brane, the apparent breakdown of the linearized approximation at $y \to \infty$ may be ignored; for another treatment of these matters see [92].

be rewritten in terms of $\bar{h}$, and take the form

$$\partial_y \bar{h}_{\mu\nu}|_{y=0} = -\frac{1}{2M^{d-1}}\left[S_{\mu\nu} - \frac{\eta_{\mu\nu}}{2(d-1)}S\right] . \tag{4.37}$$

By Green's theorem these give a contribution

$$\bar{h}^S_{\mu\nu}(X) = -\frac{1}{2M^{d-1}}\int d^d x' \Delta_{d+1}(X;0,x')\left[S_{\mu\nu}(x') - \frac{\eta_{\mu\nu}}{2(d-1)}S(x')\right] \tag{4.38}$$

to the metric. As above, the second term on the RHS of (4.22) is pure gauge, and the third term vanishes, so the remaining contribution comes from the first term. The trace equation (4.20) and the boundary condition (4.26) imply

$$\partial_y h = \frac{e^{2y/R}}{2(d-1)M^{d-1}}S , \tag{4.39}$$

which gives a contribution

$$\bar{h}^{(1)}_{\mu\nu} = \frac{\eta_{\mu\nu}(2-d)}{4(d-1)RM^{d-1}}\int d^d x' S(x')\int dy' e^{(2-d)y/R}\Delta_{d+1}(X,X') . \tag{4.40}$$

The integral over y' is eliminated by using the identities (4.28) and (4.29), and the combined expressions (4.38) and (4.40) yield

$$\bar{h}_{\mu\nu}(x) = \quad -\frac{1}{2M^{d-1}}\ \int d^d x'\Bigg\{ \Delta_{d+1}(x,0;x',0)S_{\mu\nu}(x') \tag{4.41}$$

$$- \quad \eta_{\mu\nu}\left[\Delta_{d+1}(x,0;x',0) - \frac{(d-2)}{R}\Delta_d(x,x')\right]\frac{S^\lambda_\lambda(x')}{2(d-1)}\Bigg\}$$

in agreement with [69]. In particular, this expression may be evaluated for a stress tensor corresponding to a point mass at rest on the brane at $\vec{x} = 0$,[3]

$$T_{tt} = 2m\delta^{d-1}(x)\delta(y) . \tag{4.42}$$

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[3]The extra factor of two is present because of the orbifold boundary conditions, and compensates the 1/2 in (4.16).

Using the long-distance expansion of the propagator [69],

$$\Delta_{d+1}(x, 0; x', 0) = \frac{d-2}{R} \Delta_d(x, x') \left[1 + \left(\frac{R^{d-2}}{r^{d-2}} \right) \right] \tag{4.43}$$

this gives the $d = 4$ expression

$$\bar{h}_{tt} = \frac{m}{2\pi R M^3 r} \left[1 + \mathcal{O} \left(\frac{R^2}{r^2} \right) \right] \ , \ \bar{h}_{ij} = \mathcal{O} \left(\frac{mR}{M^3 r^3} \right) \ . \tag{4.44}$$

4.3.3 The falling particle

The above expression (4.33) for the stress tensor appears rather complicated, but simplifies significantly in the long-distance limit. To illustrate this, we compute the effective stress tensor of a particle falling into the bulk. (The corresponding approximate metric has also been computed by Gregory, Rubakov, and Sibiryakov in [87].) This case will also apply to our later discussion of black hole flybys; by performing a boost along the brane we get a trajectory that can sail behind a black hole through the extra dimension.

Concretely, consider a particle of mass m that is stuck to the brane at $\vec{x} = 0$ until time $t = 0$ and then released and allowed to freely fall into the bulk. The trajectory for $t > 0$ is easily seen to be given by the equation

$$z^2 - t^2 = R^2 \ . \tag{4.45}$$

For $t < 0$ the only nonzero component of the stress tensor is given by (4.42). For $t > 0$ the stress tensor is given by the general formula

$$T_{IJ} = m \frac{dX_I}{d\tau} \frac{dX_J}{dt} \frac{\delta^{d-1}(x - x(t)) \delta(y - y(t))}{\sqrt{-G}} \ , \tag{4.46}$$

which in the present case gives nonvanishing components

$$T_{tt} = m e^{(d-2)y/R} \delta^{d-1}(\vec{x}) \delta(y - y(t)) \ , \tag{4.47}$$

$$T_{yy} = m\frac{t^2}{R^2}e^{(d-2)y/R}\delta^{d-1}(\vec{x})\delta(y-y(t)) \ , \tag{4.48}$$

and

$$T_{ty} = -m\frac{t}{R}e^{(d-2)y/R}\delta^{d-1}(\vec{x})\delta(y-y(t)) \ . \tag{4.49}$$

Therefore the contribution to the metric from the trajectory for $t < 0$ is a special case of the general surface-stress result of the preceding subsection, (4.41), with

$$S_{tt} = 2m\delta^{d-1}(x)\theta(-t) \ , \quad S_{\mu i} = 0. \tag{4.50}$$

The contribution to the metric from the second half of the trajectory is given by our formula (4.31). Specifically, rewriting the $t > 0$ stress tensor as

$$T_{IJ} = \mathcal{S}_{IJ}(t,y)\delta(y-y(t)) \ , \tag{4.51}$$

we find

$$\bar{h}_{\mu\nu}^{>}(x) = -\frac{1}{M^{d-1}} \int_{t'>0} d^d x' \Big\{ \partial_y \left[e^{-(d-2)y/R} F_y(0; x - x') \right] \mathcal{S}_{\mu\nu}(x', y) \tag{4.52}$$
$$-\frac{\eta_{\mu\nu}}{2(d-1)} e^{-(d-2)y/R} \partial_y F \left[\mathcal{S}_{\mu}^{\mu} - (d-2)e^{-2y/R}\mathcal{S}_{y}^{y} \right] \Big\}|_{y=y(t)}$$
$$+\bar{h}_{\mu\nu}^{\text{gauge}} \ .$$

The expression for the effective stress tensor follows directly from computing the Einstein tensor (4.32) from these expressions for the linearized metric. In order to gain some intuition for this expression, consider the approximation of distances and times much greater than the AdS scale R, which we've seen is the cutoff for the effective theory:

$$x^2 - t^2 \gg R^2 \ . \tag{4.53}$$

In this limit the Green function simplifies dramatically (see appendix),

$$F_y(0; x - x') \simeq \frac{1}{2\pi}\delta(z^2 + (x - x')^2)\theta(t - t') \ , \tag{4.54}$$

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and the trajectory (4.45) becomes

$$z = \tilde{R}e^{y/\tilde{R}} \simeq t \ . \tag{4.55}$$

Defining $r = |x|$, in $d = 4$ the approximate metric is then

$$\bar{h}_{\mu\nu} \simeq \frac{m}{2\pi M^3 Rr} \delta_\mu^t \delta_\nu^t \tag{4.56}$$

for $r > t$, and

$$\bar{h}_{\mu\nu} \simeq \frac{m}{2\pi M^3 R} \left[\left(\frac{3}{2t} - \frac{\vec{x}^2}{2t^3} \right) \delta_\mu^t \delta_\nu^t + \frac{t^2 - \vec{x}^2}{4t^3} \eta_{\mu\nu} \right] \tag{4.57}$$

for $r < t$, as in [87]. A straightforward computation shows that the Einstein tensor of both of these metrics vanishes! Thus the effective stress tensor is concentrated on the surface where they match, $r = t$. This stress tensor is

$$T_{\mu\nu}^{\text{eff}} \simeq \frac{m}{4\pi t^2} \delta(t - r) u^\mu u^\nu \tag{4.58}$$

where $u^\mu = x^\mu/t$.

The effective stress tensor of the conformal field theory configuration describing a falling particle is thus concentrated on a thin shell of radius r which expands outward with time, $r = t$. We can estimate the thickness of the shell by investigating the size of the leading corrections in the limit (4.53). One readily sees that the metric is corrected at order R^2/t^2, R^2/r^2, both due to corrections to the trajectory and to the Green function. This suggests that the thickness of the shell of CFT matter is the expected $\mathcal{O}(R)$, the cutoff length scale.

This spreading behavior appears to be quite generic, as one might expect from the IR/UV duality of the AdS/CFT correspondence. Another example of this behavior is provided by a falling charged particle coupled to a bulk gauge field, as investigated in [88]. Indeed, an even simpler example is provided by a falling particle coupled to

a bulk scalar field. Specifically, consider a Lagrangian

$$S = -\int dV \frac{1}{2}(\nabla\phi)^2 - q\int d\tau \phi(X(\tau)) \qquad (4.59)$$

with a coupling of a bulk scalar field ϕ to a particle of scalar charge q falling along a trajectory $X(\tau)$. This determines the scalar field,

$$\phi(X) = q\int d\tau \Delta_{d+1}(X, X(\tau)) \ . \qquad (4.60)$$

If we assume that the particle again follows the trajectory (4.45) and work at large distances as compared to R and with $d = 4$, then the field at $y = 0$ takes the approximate form

$$\phi(x, t) \simeq -\frac{q}{2\pi R}\left[\frac{1}{r}\theta(r-t) + \frac{4R^2 t}{(r^2-t^2)^2}\theta(t-r)\right]\left[1 + \mathcal{O}\left(\frac{R}{r}\right)^2\right] \ . \qquad (4.61)$$

If we compute the effective source, $J = \Box_4\phi$, we find it vanishes except at $r = t$. Again, subleading $\mathcal{O}(R)$ corrections appear to smooth this into a shell of thickness R.

Note that similar behavior was found by Horowitz and Itzhaki [89], who investigated the CFT stress tensor corresponding to a particle moving geodesically in the full, infinite AdS. This work also found a shell expanding outward at the speed of light. Indeed, the two calculations are directly related in the infrared limit, as discussed in appendix B.

This behavior can can also be understood directly in terms of the CFT using an argument due to Coleman and Smarr [90], which shows that a stress tensor that is conserved, traceless, and has positive energy density will be localized on the light cone. The basic idea for the proof is to show that the average squared energy radius,

$$\bar{r}(t)^2 = \frac{\int d^3 x \, r^2 T_{00}}{\int d^3 x \, T_{00}} \qquad (4.62)$$

satisfies

$$\frac{d^2\bar{r}^2}{dt^2} = 2 \qquad (4.63)$$

from which it immediately follows that a configuration initially localized at a point will expand on the light cone. Ref. [89] argues that the argument extends even to the quantum case, where the energy density may be negative, as long as the total energy is positive.

These results neglect the backreaction of gravity on the outgoing shell. It would be interesting to understand what dynamics results when strong self gravitation of the shell is included.

Vanishing of the Einstein tensor for the metric (4.57) at first sight leads to another puzzle. Specifically, suppose we consider a "bounce" trajectory, where the particle follows the trajectory (4.45) for *all* time. The calculation of the metric above is modified by extending to the trajectory for $t < 0$, but still yields a stress tensor that vanishes everywhere. This contradicts our expectation of a shell that collapses and then reexpands. However, note that this computation is not complete. The z coordinates only cover the region outside the AdS horizon, and thus this calculation would miss the contribution of the piece of the trajectory behind the past horizon. If this is not included, energy-momentum conservation is violated at the horizon, and consequently gravity cannot be consistently coupled. A correct calculation includes this piece, but also requires more information about the structure of the Green function. Specifically, one needs to know what boundary conditions it obeys in the far past. In order to make predictions in the RS2 scenario, one needs to understand the physics determining the boundary conditions at the past horizon. Correspondingly, in CFT language one needs to know in what state the CFT sector began.

4.3.4 Black hole flybys

We now have the necessary tools to discuss particles flying past black holes through the bulk; for simplicity we discuss the four-dimensional case. Specifically, suppose that there is a black hole of mass m located at $\vec{x} = 0$ and that a particle is shot at it
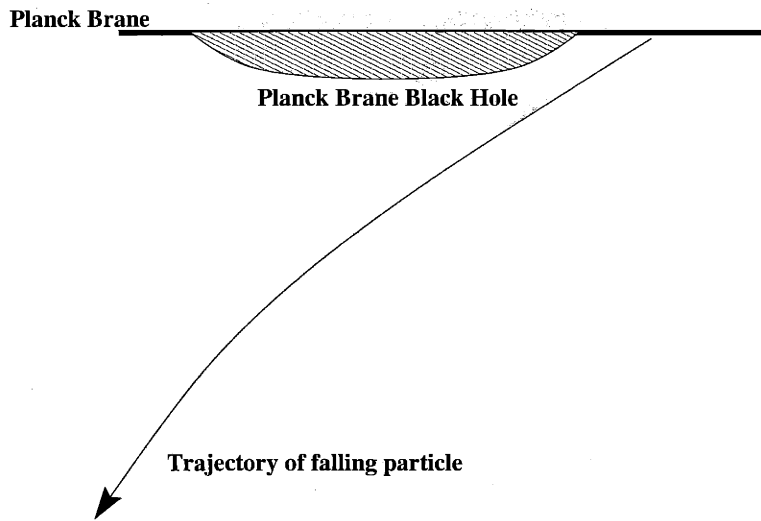
Figure 4-1: A particle trajectory that misses a black hole on the brane because of its motion in the extra dimension.

with zero four-dimensional impact parameter, but is allowed to fall in z long enough that it misses the black hole by passing it in the z direction (see fig. 1). How does a four dimensional observer describe such an experiment, and in particular does one see radical departures from usual gravitational dynamics, such as matter entering and then escaping a 4d event horizon?

The answer to the latter question is, of course, no. Indeed, a black hole with mass and 4d Schwarzschild radius m has a horizon extending to $z_h \sim m$ in the bulk picture. In order for the particle to miss the black hole, the particle must have $z \gg m$ when $\vec{x} = 0$. As we've seen above, in the CFT description the particle corresponds to a shell of CFT matter. If it has reached $z \gg m$ by the time it reaches the black hole at $\vec{x} = 0$, then the shell has expanded to size $r \gg m$ by the time it has reached the black hole, and is continuing to expand outward. No novel physics need be invoked to explain why the shell is not absorbed by the black hole.[4] The process has a perfectly adequate four-dimensional effective description in terms of the matter conformal field theory coupled to four-dimensional gravity.

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[4]Note, however, that a small piece of the shell may be absorbed by the black hole; a quantum treatment of the bulk should yield a corresponding result.

4.4 Breakdown of EFT; cutoffs, strong gravity, and black hole production

Section 4.2 argued that at low energies the RS scenario is equivalent to coupling ordinary matter to a hidden CFT. Section 4.3 provided illustrations of this statement. An obvious question regards the limitations of this description. At what scale does it fail? Is there any practical advantage or consequence of the five-dimensional description? And what conclusions can one draw about strong gravitational phenomena, such as production of black holes in high-energy scattering?

In this section we will first consider the scenario with a single Planck brane, and then comment on extensions of the discussion to scenarios with an added probe or "TeV" brane or with AdS terminated by a brane-like object at large z (like the "negative tension" brane proposal of [71]).

4.4.1 Scattering on the Planck brane

The four-dimensional effective action for the scenario with a single Planck brane is given in (4.16). Recall that the fundamental parameters determine the 4d Planck mass by the relation (4.17). We would like to understand what this scenario predicts for high-energy scattering.

The simplest assumption (if one is not trying to solve the hierarchy problem) is that the five-dimensional Planck scale and inverse AdS radius, and hence the four-dimensional Planck scale, are all comparable:

$$M \sim 1/R \sim M_4 \ . \tag{4.64}$$

In that case all new physics is clearly at the Planck scale. How much can this statement be relaxed? One would expect observable deviations in microgravity experiments – as in the scenario of [68] – for $R \sim 1mm$. This puts a lower bound of $M > 10^8 GeV$ on the five-dimensional Planck scale.

Consider now high energy scattering of particles confined to the brane. From

the bulk perspective, we see that at distances $\ll R$ the dynamics is effectively five-dimensional. This is mirrored in the four-dimensional description of (4.16; energies above $1/R$ are past the cutoff and the cutoff CFT description is incomplete.

Does this mean that we can see what a 4d observer would interpret as strong gravitational phenomena at energies just above $1/R$? Clearly not, except when the parameters satisfy $1/R \sim M$, in which case we are at Planckian 4d energies anyway. Consider for example black hole production. There are two types of black holes that one might produce. The first type is the AdS/Schwarzschild black hole, which moves freely in the bulk, and in general will fall towards the AdS horizon once produced. The threshold for producing such black holes is the 5d Planck energy $M > 10^8 GeV$. The second type of black hole is bound to the brane, as described in [91, 69]. The horizon radius of such a black hole is $r_h \sim m/M_4^2$; this should be larger than the 5d Planck length which implies $m > RM^2$. Since $RM > 1$, the threshold is again at M or above.

From this discussion we see that scattering pushes beyond the cutoff scale at the threshold $1/R$ and in the bulk perspective begins to explore the extra dimension. While this may have visible consequences through production of the Kaluza-Klein modes, strong gravitational dynamics such as black hole production has a much higher threshold of M, which in the most "optimistic" scenario of $M \sim 10^8 GeV$ is still a long ways off.

4.4.2   Scattering on a probe brane

The preceding Planck-brane scenario is not favored from the viewpoint of low-energy phenomenology in any case, given the expected relation (4.64) between scales. Scenarios which try to generate the hierarchy via the exponential warp factor show more promise. Consider first the probe brane scenario of [76]. Here 4d matter is taken to reside on a "TeV" brane at an elevation $z = \rho_T$; the Planck brane is again at $z = \rho$. This brane must be stabilized by a mechanism such as in [80, 53]. The 4d Planck mass is again given by (4.17), but matter on the TeV brane has its energy redshifted by ρ/ρ_T relative to the Planck brane. If $\rho/\rho_T \sim TeV/M_4$, this gives a mechanism

to generate TeV scale effective masses from particles with fundamentally Planckian masses.

To elaborate on these comments, note that in giving a four-dimensional description of the physics it is necessary to specify a reference frame at a definite value of z in terms of which four-dimensional energies are measured. The natural frame to use is that of the Planck brane, as this is where the 4d graviton bound state is supported. Then if we consider an energy E_{prop} as measured by an observer at another value of z, it will be redshifted so that the energy in the frame of the Planck brane is $E = \rho E_{\text{prop}}/z$. In particular, a particle of mass m at rest in the frame at z will have an energy $m\rho/z$ relative to the Planck brane, and that will be interpreted as its four-dimensional mass.

The Lagrangian in this scenario is expected to take the form

$$
\begin{aligned}
S = \int d^{d+1}X \quad & \sqrt{-G}(M^{d-1}\mathcal{R} - \Lambda + \mathcal{L}_{\text{stab}}) \\
+ \quad & \int d^d x \left[\sqrt{-\gamma(x,\rho_T)}\mathcal{L}(\gamma(x,\rho_T),\psi) - \sqrt{-\gamma(x,\rho)}\tau \right] .
\end{aligned}
\tag{4.65}
$$

Here we denote by $\mathcal{L}_{\text{stab}}$ the Lagrangian of the stabilizing fields; we assume that beyond stabilizing the brane the don't qualitatively affect our conclusions.

What is the CFT description of this scenario? Here we encounter subtleties beyond the derivation of (4.16). Specifically, the action depends on the metric at $z = \rho_T$. In attempting to relate the bulk functional integral to the boundary CFT we have to confront the non-trivial z-dependence of γ, and in particular give a CFT prescription for computing the metric in the bulk. We have not yet found a convincing prescription to derive such off-shell information from the AdS/CFT correspondence.

In the absence of such a prescription we will consider two approaches to this problem. The first is to work with long-wavelength excitations of the theory such that the simple scaling approximation

$$
\gamma(x,\rho) \simeq \frac{\rho_T^2}{\rho^2}\gamma(x,\rho_T)
\tag{4.66}
$$

holds. We use this equation to replace $\gamma(x, \rho_T)$ by $\overset{\circ}{\gamma}(x, \rho)$ in the Lagrangian for matter on the TeV brane. This effectively rescales parameters of dimension δ in that Lagrangian by a factor $(\rho/\rho_T)^\delta$ (c.f. [76]). Rewriting the functional integral as in section 4.2 produces a 4d effective action analogous to (4.16) in the Planck brane scenario,

$$S_{TeV} = \int d^4x \sqrt{-\gamma} \left[\mathcal{L}(\gamma, \psi, m\rho/\rho_T) + \mathcal{L}_{\text{CFT}}(\gamma, \chi) + \mathcal{L}_{\text{grav}}(\gamma) - \tau \right] . \qquad (4.67)$$

Here we have explicitly indicated the rescaling of a typical mass parameter in the matter Lagrangian. Again $\mathcal{L}_{\text{CFT}}(\chi)$ represents the Lagrangian of "dark" CFT matter, and $\mathcal{L}_{\text{grav}}$, given by (4.9), is the gravitational action.

The simple approximation (4.66) fails at short wavelengths, where the z dependence becomes non-trivial. This effect can be estimated from the long-distance expansion of the propagator [69],

$$\Delta(x, z; x', z') \sim \frac{1}{Rr^{d-2}} \left[1 + \frac{R^{d-2}}{r^{d-2}} + \frac{z^d}{r^d} + \frac{z^{2d}}{r^{2d}} \frac{r^{d-2}}{R^{d-2}} \right] \left[1 + \mathcal{O}\left(\frac{z^2}{r^2}, \frac{R^2}{r^2} \right) \right] . \quad (4.68)$$

In $d = 4$, the correction due to the last term becomes large at distances

$$r \sim \left(\frac{\rho_T}{\rho} \right)^{4/3} R , \qquad (4.69)$$

or at about 10 Fermi for $\rho/\rho_T \sim TeV/M$.

In order to understand the origin of corrections at this scale, first let's recall a similar phenomenon in the large scale compactification scenario of [68]. If one for example considers such a compactification with two extra dimensions of size $\mathcal{O}(mm)$, gravitational experiments performed at shorter scales reveal the six-dimensional nature of spacetime: the part of the four-dimensional effective Lagrangian describing the gravitational sector breaks down. One way of understanding this is to note that sources with shorter wavelengths than a millimeter will generically have coupling to the Kaluza-Klein modes that is comparable to the coupling to the gravitational zero mode; summing over these modes produces the six-dimensional gravitational field.

While the gravitational part of the 4d effective action breaks down, nonetheless the gauge part of the effective action remains four-dimensional up to scales of order a TeV where gravity itself becomes strongly coupled.

A similar phenomena occurs here. At scales given by (4.69), the couplings of the TeV brane matter to the continuum analogs of the Kaluza-Klein states become comparable to the coupling to the four-dimensional graviton. This means that in the gravitational sector the 4d effective theory fails, but of course the gauge part of the theory remains four-dimensional up to the TeV scale. The stress tensor of the TeV brane matter acts as the source for these couplings to the Kaluza-Klein modes.

Corresponding statements can be made in the CFT, and will tell us the form of the corrections to the action (4.67) that are responsible for its failure as a 4d effective description. In particular, we expect that corresponding to the couplings to the KK modes, a term is induced in (4.67) in which there is a direct coupling of the stress tensor of the TeV brane matter to the stress tensor of the CFT, and by scaling the coefficient of this should include a factor of $(\rho_T/\rho)^4$. Such terms are responsible for the breakdown of the gravitational part of the 4d effective theory.

A second approach would be to use the holographic renormalization group [55] to evolve the Lagrangian from the Planck brane to the TeV brane. We would expect this to produce similar results, namely a gravitationally coupled CFT with a cutoff scale $\sim$10 MeV. We expect the $T_\chi T_\psi$ terms described above to be present in the Lagrangian at the Planck scale, and then to be rescaled by the renormalization group flow. A better understanding along these lines of the relationship between operators at different z would also clearly be illuminating for our fundamental understanding of holography in the AdS/CFT correspondence.

As in the scenario of [68], there is a distinction between the scale at which the 5d nature of gravity becomes important and the scale at which gravity becomes strongly coupled. A particularly interesting question is when do we expect to be able to manufacture configurations which would manifest signatures for black holes that we as four-dimensional observers could see?

Within the context of the TeV-brane scenario, there are again two kinds of black

hole solutions known. The first is the AdS-Schwarzschild black hole. The minimum energy to create these should be $\mathcal{O}(M)$. A collision of TeV brane matter with a proper energy of this magnitude is a collision at the TeV energy scale as measured with respect to the four-dimensional observer. However, it appears that such black holes are not bound to the brane. In the probe-brane limit, where the gravitational backreaction is neglected, this is manifest, but even taking into account the small energy density of the probe brane it seems likely that the binding of the black hole to the brane will be overcome by the gravitational pull of the black hole towards the AdS horizon.[5] While a complete analysis of this requires detailed investigation of stabilized probe brane scenarios, it appears that such a black hole will therefore generically fall towards the AdS horizon, and that the 4d observer will therefore not perceive it as a black hole. In the CFT picture, such black holes will be perceived as complex excitations of the CFT which spread out over time, and it is very unlikely that their signature can be experimentally disentangled from other excitations of the CFT.

The second type of solution is the black hole on the Planck brane. These are truly perceived as 4d black holes. However, given the relation (4.64) between scales, the minimum energy to create such a black hole is again of order the Planck energy. The TeV brane scenario doesn't seem to allow access to what a 4d observer would perceive as strongly coupled gravitational dynamics at lower scales.

In fact, notice the following novelty. A small black hole bound to the Planck brane will not intersect the TeV brane, until its horizon reaches the TeV^{-1} size. Therefore matter moving on the TeV brane may bypass such a black hole, in a close analogy to the black hole flybys discussed in sec. 3.2. (See fig. 2.) In other words, 4d observers made of TeV brane matter have difficulty resolving sub-TeV size black holes. How is this interpreted in four-dimensional language?

To really study this question requires a detailed model of the stabilization and the TeV brane matter. However, a plausible answer to this also comes directly from our

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[5]There may be interesting transitory effects – such as stretching and then recoil of the probe brane – that we leave for future investigation.
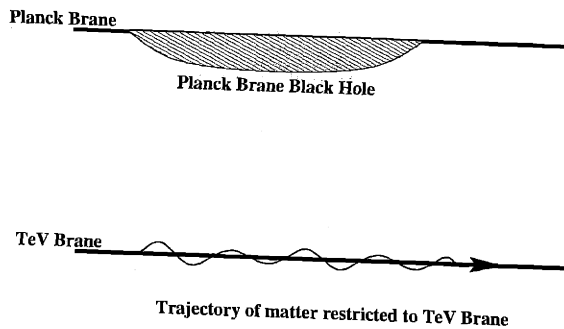
Figure 4-2: A particle moving on a probe brane can bypass a small black hole localized on the Planck brane.

earlier discussion. Matter passing a black hole by moving on the TeV brane should be interpreted in the 4d perspective as matter smeared out on the TeV scale. Such matter has a small probability of probing a black hole with a radius much less than $1/TeV$.

4.4.3 The RS1 scenario

Another interesting possibility is that AdS is terminated at both ends in z. The outlines of such a picture was suggested in Ref. [71] with a idealized lower brane taken to have a finely-tuned negative tension.

Recent developments in string theory have suggested a concrete means to construct geometries with similar properties. Specifically [86, 84] describe geometries that terminate at a definite value of z. These geometries do not arise from negative tension objects, or even singular branes, but rather are rounded off at the maximal z in a smooth geometry that uses the extra dimensions of string theory in a non-trivial way.

Refs. [86, 84] do not have a simultaneous microscopic construction of the analog of a Planck brane. However, one can envision building such a model by using Verlinde's geometric realization [75] of the Planck brane as a piece of a compactification manifold on the ultraviolet end, and then realize the IR brane as in [86, 84] or in a variant of these scenarios producing other low-energy dynamics. There may of course

be other inequivalent stringy constructions of such doubly-terminated AdS spaces. Constructing detailed models of this kind is an interesting problem for the future.

The models of [86, 84] have explicit gauge theory duals. If one constructs a model with a geometric "Planck brane," one would expect these to be modified at the UV end and depend on the internal structure of the compactification manifold. Nonetheless, these gauge theories should serve as good effective theories at lower scales, in parallel with our earlier discussion.

Such a scenario – or others with a microscopic realization of an IR brane – may have very interesting consequences for the observability of strong gravitational phenomena. Assume that in such a construction there is a gauge theory sector that we may think of as being truly localized in the vicinity of the maximal z which we take to be $z \simeq \rho_T$. This would then realize what was referred to as matter living on the "negative tension brane" of ref. [71]. As explained above, energies at $z \simeq \rho_T$ are redshifted relative to those at the Planck brane, and so if a suitable way is found of stabilizing the separation between the branes, TeV scale scattering corresponds to a proper energy comparable to M, the five-dimensional Planck scale, if the scattering takes place at $z \simeq \rho_T$. Thus scattering at this scale should begin to make black holes. These should be similar to AdS-Schwarzschild solutions, or to the analogous solutions in the new geometry of [86, 84]. (For an explicit formula for the smooth metric in question, see sec. 5.1 of [86].)

In the probe-brane picture, these black holes were expected to fall off the brane and into the horizon at $z = \infty$. Now this is not possible since the geometry terminates at $z \simeq \rho_T$. One expects such a black hole to undergo approximately geodesic motion in this vicinity, and ultimately to evaporate.

Note that one may achieve a clean separation of scales in situations where the AdS radius R is larger than the 5d Planck length M^{-1}. In this situation (which can be achieved by taking large 't Hooft parameter $g^2 M$ – here only M is the dimension of $SU(M)$ – in [86]), there exist black holes larger than the Planck size but smaller than the AdS radius. These would have an approximately (five-dimensional) Schwarzschild description.

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In the probe brane picture, the evaporation of 5d black holes was expected to be nearly exclusively into bulk modes, since the black hole becomes well separated from the TeV brane and so will not couple to its excitations. However, in the present picture, the black hole remains in the vicinity of the analog of the IR brane and this suggests that there is no reason for it to decouple from the matter modes in this vicinity. Indeed, in the idealized "negative tension brane" picture, gauge modes on the brane will directly see the black hole metric. Therefore, with this assumption, on general grounds one expects the black hole to Hawking radiate into all available modes, including the visible matter sector modes. As explained in [56], the radiation in the visible sector is generically expected to be important.

This suggests an interesting scenario in which a black hole could be created at an accelerator operating in the vicinity of the TeV scale. Assuming the black hole is sufficiently coupled to the visible modes, these would provide a channel for the Hawking decay and provide an observational window on this process. One would observe such an object by looking for the characteristic approximately thermal spectrum – with increasing temperature – of the Hawking process.

The basic assumptions that could lead to this possibility being realized are 1) that one has a geometry effectively terminated at a maximal z corresponding to the TeV scale, 2) that one has a description of visible-sector matter localized to the vicinity of this maximal z, and 3) that black holes near the maximal z couple to the visible sector. Whether these assumptions will hold in models based on the ideas of [86, 84] remains to be seen, but they plausibly do, and there may also be other models with these properties, for which the creation and visible-sector decay of TeV-scale black holes seems a generic prediction.

4.5 Conclusions

This chapter has investigated the interplay between the four- and five-dimensional descriptions of the physics of warped compactifications. In the simplified example of the RS2 scenario, at distances long as compared to the AdS radius R there is a four-

dimensional effective description of the dynamics given by observable brane matter coupled gravitationally to a sector described by a conformal field theory. At shorter distances the derivation of this description fails. One expects similar 4d effective descriptions for other warped compactification scenarios.

One element of the correspondence between the 4d and 5d descriptions is supplied by the computation of the 4d stress tensor corresponding to a 5d matter distribution. At the linear level we have given a formula for this stress tensor. We have also investigated an amusing scenario that illustrates the interplay between the 4d and 5d descriptions, that of a particle passing a black hole through the fifth dimension, with a corresponding 4d description in terms of a matter distribution expanding into a shell larger than the black hole.

Finally, we have explored situations in which strong gravitational dynamics may give important modifications to the 4d description. In particular, in scenarios where the hierarchy is addressed by visible matter being effectively localized to a large z in AdS space, one potentially has access to strong gravitational dynamics such as black hole formation at TeV energy scales. In probe brane scenarios this may not lead to observable effects since the resulting black hole seems to rapidly decouple from the visible sector by falling off the brane, but scenarios with AdS terminated at this maximal z show much more promise as such a black hole should stay localized in the vicinity of the maximal z. This leads to the possibility of creation and observable Hawking decay of a black hole in the vicinity of the TeV scale. It would be particularly interesting to find extensions of the work of [75] and [86, 84] which give explicit string theory realizations of such RS1 type scenarios.

Chapter 5

Summary and Outlook

We have found through the course of this work that branes provide new tools for addressing the hierarchy problem. In particular, within a supersymmetric solution to the hierarchy, they allow one to consider models where the dominant contribution to SUSY breaking comes from anomaly mediation. The resulting spectrum for the superpartners is quite predictive and model independent. Minimal anomaly mediation suffers from having negative slepton mass squareds, however, we have given an existence proof that one can provide sufficient positive contributions entirely within anomaly mediation. We have also shown how to generate acceptable μ and B parameters, insuring proper electroweak breaking. As a result, we have shown that one can construct a model that protects the hierarchy against radiative corrections, and has a spectrum which is consistent with current observations, including the bounds on FCNC.

Unfortunately, our solution is quite complicated and involves a host of new heavy fields, which do not allow for unification. There has been another proposal for generating positive contributions within a 4D framework which is perhaps simpler and does allow GUTs [5]. It, however, has a very light field of mass comparable to that of the MSSM fields, and also suffers from possible cosmological problems of why we live in the vacuum with positive slepton masses. This is because in this proposal the sleptons, in fact, have negative mass squareds above the heavy threshold scale. Thus, there is still an important challenge to find a solution to the negative slepton mass

problem, which is simple, elegant, and is completely consistent.

Perhaps related to this, it would be interesting to better understand the 5D dynamics that leads to anomaly mediation. Thus far, we have been working entirely within the low-energy, 4D effective theory below the compactification scale. It would be interesting to construct explicit 5D theories where the interactions of anomaly mediation can be seen directly. This might also help in providing solutions to the slepton mass problem by including contributions from 5D bulk fields which are naturally of the right size and sign. Also, a realization of this scenario from a string compactification with a limited number of moduli fields lighter than 5D compactification scale is important, and might shed some light on the physics.

We have also seen that the gravitational phenomena associated with curved brane backgrounds are quite novel. We have shown that even with an infinite extra dimension, one can have models which are perfectly consistent, and conform to current observations. This possibility is thus especially important to keep in mind when considering possible string compactifications. Indeed, it would be interesting to find a stringy realization of warped solutions similar to the RS models. Perhaps, it would be possible to construct a string background with some type of brane defect, localizing gravity only near its vicinity, without having any compact directions at all. This would then be a very exciting solution to the usual moduli problem associated with a complicated compact manifold.

In addition, we have seen that 5D warped backgrounds might have equivalent, holographic, interpretations as 4D field theories. As shown by our toy example of a falling particle, this might be an important insight for simplifying the analysis of certain problems. In other words, one can use features generic to any CFT to make statements about gravitational solutions, that would be non-trivial to derive from the full 5D gravity theory. This knowledge was also crucial to describing the otherwise intractable strong coupling physics for a TeV brane observer above a TeV. Much remains to be done to fully understand the implications of holography. Holography could also be a useful tool in addressing the important issue of GUTs, in the warped backgrounds which solve the hierarchy problem.

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Appendix A

The retarded Green function

In this appendix we describe some properties of the scalar Green function for the RS2 geometry. This was given in [69] and takes the form

$$\Delta_{d+1}(x, z; x', z') = \frac{i\pi}{2R^{d-1}}(zz')^{\frac{d}{2}} \int \frac{d^d p}{(2\pi)^d} e^{ip(x-x')} \tag{A.1}$$

$$\times \left[\frac{J_{\frac{d}{2}-1}(qR)}{H^{(1)}_{\frac{d}{2}-1}(qR)} H^{(1)}_{\frac{d}{2}}(qz) H^{(1)}_{\frac{d}{2}}(qz') - J_{\frac{d}{2}}(qz_<) H^{(1)}_{\frac{d}{2}}(qz_>) \right] \tag{A.2}$$

For the following discussion it is most convenient to use the z coordinate, related to y by (4.18).

The scalar propagator with $d = 4$ and one point on the boundary is given by [69]

$$\Delta_{4+1}(x, z; x', R) = \left(\frac{z}{R}\right)^2 \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{q} \frac{H^{(1)}_2(qz)}{H^{(1)}_1(qR)} , \tag{A.3}$$

where $q^2 = -p^2$. As in eq. (4.27), let us define a function F,

$$\Delta_{4+1}(R, x; z', x') = \frac{z'^3}{R} \partial_{z'} \left[\frac{F_{z'}(R; x - x')}{z'^2} \right] . \tag{A.4}$$

Hence, F is given as Fourier transform of Hankel functions,

$$F_{z'}(R; x - x') = -\frac{z'}{R} \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{q^2} \frac{H^{(1)}_1(qz')}{H^{(1)}_1(qR)} . \tag{A.5}$$

In our conventions (given by (4.23) the Feynman propagator is

$$\Delta_F(X, X') = -i \left[\theta(t - t') \Delta^+(X, X') + \theta(t' - t) \Delta^-(X, X') \right] , \qquad (A.6)$$

where Δ^+ is the Wightman function $\langle \phi(X)\phi(X') \rangle$ and Δ^- is its hermitian conjugate. The retarded Green function is defined as

$$\Delta_R(X, X') = -i\theta(t - t') \left[\Delta^+(X, X') - \Delta^-(X, X') \right] ; \qquad (A.7)$$

this manifestly vanishes for $t < t'$ and can easily be shown to obey (4.23). We therefore find that

$$\Delta_R(X, X') = 2\mathrm{Re}\Delta_F(X, X')\theta(t - t') . \qquad (A.8)$$

In order to compute the asymptotic retarded Green function, note that in the long distance approximation $(qR << 1)$, F reduces to the following form,

$$F_{z'}(R; x - x') \approx -\frac{\pi i z'}{2} \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{q} H_1^{(1)}(qz') . \qquad (A.9)$$

We then perform a Euclidean rotation on the above integral, giving

$$F_{z'}(R; x - x') \quad \approx \frac{\pi i z'}{2} \int \frac{d^4 p}{(2\pi)^4} e^{ip(x-x')} \frac{1}{p} H_1^{(1)}(ipz') \qquad (A.10)$$

$$\approx \frac{i}{4\pi^2} \frac{1}{(x - x')^2 + z^2} . \qquad (A.11)$$

The Feynman prescription is then to replace $(x - x')^2$ with $(x - x')^2 + i\epsilon$. Making this replacement, and taking the real part gives

$$F_{z'}^{Ret}(R; x - x') \approx \frac{1}{2\pi} \delta((x - x')^2 + z'^2)\theta(t - t') , \qquad (A.12)$$

which finally yields the retarded scalar propagator,

$$\Delta_{4+1}(R, x; z', x') \approx \frac{1}{\pi R} \left[z'^2 \delta'((x - x')^2 + z'^2) - \delta((x - x')^2 + z'^2) \right])\theta(t - t') . \quad (A.13)$$

Appendix B

The infrared limit

As we saw in the text, our calculation of the effective source on the boundary produces an expanding shell in the infrared limit. This is true for scalar, vector, and graviton fields. This is also the result that Horowitz and Itzhaki found in [89], using the boundary conditions appropriate for infinite AdS rather than the brane boundary conditions (4.26). In this appendix we sketch the relation between the calculations. For simplicity we only treat the scalar case although the derivation extends to the other cases.

In our calculation with brane boundary conditions, we solve the bulk equation

$$\Box_{d+1}\phi = T \; , \tag{B.1}$$

with the Neumann condition

$$\partial_n\phi|_{z=\rho} = 0 \; . \tag{B.2}$$

Here T is the scalar source, in the text given by the falling particle, and ∂_n denotes the normal derivative. The effective boundary source is found by restricting this solution to the boundary and computing its Laplacian:

$$J = \Box_d\phi|_\partial \; . \tag{B.3}$$

Another way to get the same solution is to solve (B.1) subject to the Dirichlet

boundary condition

$$\phi|_\partial = \varphi \ . \tag{B.4}$$

The solution is given in terms of the Dirichlet Green function as

$$\phi(X) = \int dV' \Delta^D_{d+1}(X, x') T(X') + \oint_\partial dn' \partial_{n'} \Delta^D_{d+1}(X, X') \varphi(x') \ . \tag{B.5}$$

The effective boundary action for φ is computed by evaluating the $d+1$-dimensional action of this solution, which using the bulk equation of motion becomes

$$S[\varphi] = -\frac{1}{2} \oint_\partial dn' \varphi \partial_{n'} \phi \ . \tag{B.6}$$

The boundary equation of motion for φ then states

$$\partial_n \phi|_\partial = 0 \ . \tag{B.7}$$

Thus a solution of the Dirichlet boundary problem such that the boundary field satisfies the boundary equations of motion corresponds to a solution of the Neumann boundary problem.

In the latter approach the effective boundary source can be read off from the boundary equation of motion. Inserting the second term of (B.5) into (B.7) gives the kinetic operator acting on φ, which becomes $\Box_d$ in the long distance limit. Thus in this limit (B.7) states

$$J = \Box_d \varphi \propto \partial_z \phi_D|_\partial \ , \tag{B.8}$$

where ϕ_D, the first term in (B.5), is the solution to (B.1) with Dirichlet boundary conditions, $\phi_D|_\partial = 0$. In the limit as the cutoff is removed, $\rho \to 0$, this corresponds to the desired solution in infinite AdS. And aside from a rescaling, $\partial_z \phi_D$ corresponds to the source on the boundary, which if we had been discussing the metric would be the boundary stress tensor of ([89]).

It is also possible to check the relationship to ([89]) directly, by acting with $\Box_d$ on (4.60), using the eq. (4.23) to eliminate the d-dimensional laplacian in favor of

y derivatives, and then using the fact that in the infrared limit (A.1) is the standard bulk propagator plus a y-independent piece which therefore doesn't contribute.

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