Control of a Simulated, Three-Dimensional Bipedal Robot to Initiate Walking, Continue Walking, Rock Side-to-Side, and Balance

By

Allen S. Parseghian

B.S., Electrical Engineering and Computer Sciences (1998)
University of California, Berkeley

Submitted to the Department of Electrical Engineering and Computer Science
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Author ....
Department of Electrical Engineering and Computer Science
August 4, 2000

Certified by ....
Gill A. Pratt
Assistant Professor of Electrical Engineering and Computer Science
Thesis Supervisor

Accepted by ....
Arthur C. Smith
Chairman, Department Committee on Graduate Students

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Abstract

Physical-based control using center of mass, center of pressure, and foot placement is used to enable a simulated twelve-degree of freedom, seven-link, three-dimensional bipedal robot to lean sideways, pick up its foot and start walking on a flat surface.

Energy analysis is used to compel the same simulated robot to do a side-to-side rocking motion and eventually come to a stop. If the robot is pushed hard enough, it will raise its leg that is in the air in the frontal plane to prevent itself from falling.

Center of mass and center of pressure analysis is used to enable the same robot to balance on one foot and stand.

Thesis Supervisor: Gill A. Pratt
Title: Assistant Professor of Electrical Engineering and Computer Science

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I thank my adviser, Gill A. Pratt for allowing me to explore the idea of 3D walking.

Thanks to all the members of the Leglab for all their tremendous support and advice.

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I'd like to dedicate this thesis to my parents and my sister for their unbelievable support throughout my school years and believing in me.

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Chapter 1

Introduction

Proving the stability of certain systems using the present control techniques could be a very painstaking process, if not impossible. Dynamically walking two-legged robots are systems that belong to this category which are highly nonlinear and naturally unstable so that legged locomotion researchers are yet to come up with a convincing, mathematically based control system that can fully explain why a biped is able to walk or fail to walk continuously. Bipeds are multi-input, multi-output systems that are both continuous and discrete. While in single support, the system operates in a continuous fashion, as soon as the support leg switches, there is discreteness, as well.

In order to reduce the complexity of the bipedal robotics systems, most researchers have only settled for building/simulating planar bipeds i.e. bipeds that can only walk forward or backward in the sagittal plane. These kinds of bipeds lack the roll and the yaw degrees of freedoms that would allow them to operate in the three-dimensional space, therefore these types of robots have to be connected to an external stationary device such as a boom in order to be contained in a plane. Simulation is an extremely useful tool to explore a control system for a bipedal robot, especially if it has all the necessary degrees of freedom in order for it to be able to walk in the three-dimensional world. Great design of a biped can contribute significantly to a successful bipedal locomotion. In this thesis, mostly physical intuition will be used to control a simulated three-dimensional bipedal robot to walk, rock side-to-side, balance on one leg, and stand on both legs.

1.1 Background

Many researchers have studied legged locomotion by simulating, building, and controlling walking, hopping, and running robots. Simple controllers can be used and natural dynamics can be exploited to enable bipedal robots to perform complicated tasks such as walking ([14], [15], [16], [17], [18]). There have been quite many passive walking robots/toys built such that they completely rely on their natural dynamics and the gravitational force in order to be able to operate. McGeer explored passive walking and showed that a system that has no sensors, actuators, or any sort of a brain can walk downhill, if appropriate hardware geometry is used ([12]). Jessica Hodgins [8] has too applied passive strategies in her running biped simulations. One of the advantages to these passive walkers is that it is easy to build them and also they do not require actuators, sensors, or computers in order to make them move, but these robots have limited capabilities such as they cannot walk up a slope.

Pratt et al. [17] used a technique called "Virtual Model Control" to enable their planar biped, Spring Flamingo to walk. This technique employs virtual springs and dampers to describe interactive force behavior. The robot behaves as if those components were in fact attached to it. The virtual components exert virtual forces, which are transformed into real torques at the joints of the robot via Jacobian transformation matrix. The advantage to this technique is that the controller is mostly intuitive and easy to understand.

There have also been quite many robots built that are fully power-operated without use of natural dynamics ([2], [7]). One of the advantages to these types of robots is that they have wide range of capabilities such as walking on a rough terrain, but these robots can have unnatural looking motions due to limitations in their actuators. Also the control of these types of robots can become quite complicated especially if the controller requires an exact dynamic model of the system. Controlling a fully powered three-dimensional biped that is fully dependent upon its dynamic model is quite a complicated task because it requires extremely complicated dynamics equations of motion in order to describe its motion.

There have been robots such that their control is based on their certain joints and/or certain points on their structure track pre-specified trajectories ([4], [6], [7], [9]). One of the advantages to this approach is that the controller is relatively simple since all the trajectories are known, but if there is a slight change in the shape of the robot or the terrain on which the robot walks, the controller may not work any longer and it will usually require supplemental control in addition to trajectory tracking.

Kun et al. [11] used CMAC neural networks to control the lateral (sideways) lean angle, hip motion in the sagittal plane, and lateral roll of the ankles while the robot is in double support. One of the advantages of employing neural networks in biped control is that it will most likely result in a motion, which is fairly close to the desired one. One disadvantage is that it might take the robot several iterations until the goal is achieved.

Yamaguchi et al. [9] employed a heavy trunk with 2 degrees of freedom to ensure dynamically that the Zero Moment Point (ZMP) of the robot stayed within the polygon of the support foot. One advantage to this approach is that it will give us an extra link, which can contribute to controlling the robot successfully, but at the same time it might cause a very unnatural looking motion. The other disadvantage is that it will increase the weight of the robot.

Many dynamically and statically stable bipeds have been built and controlled, but the only robots that have been built which resemble the structure of an adult human closer than any other biped is the Honda company biped robots, P2 and P3 ([7]). P3 can perform several complicated tasks such as walking on a flat ground, turning, walking up/down stairs, balancing, and pushing objects around all in three-dimensional space without being held by any external devices such as a boom.

Figure 1-1 shows pictures of some previously powered bipedal robots.

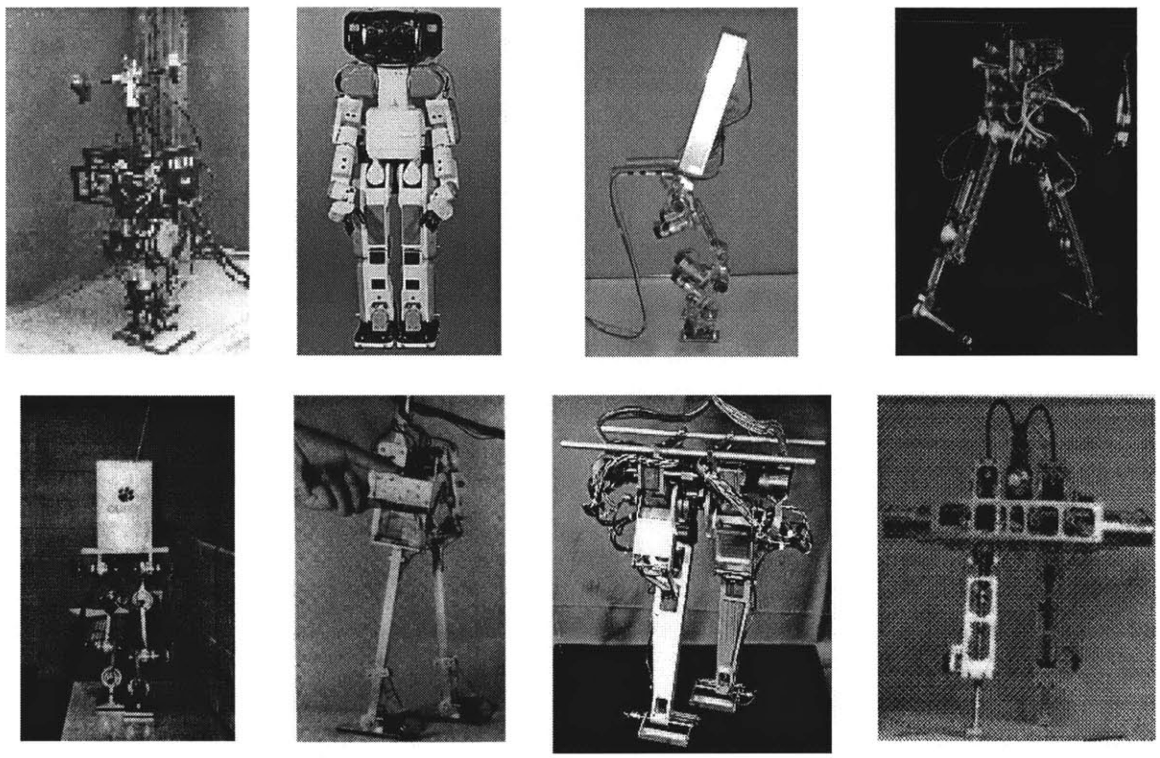

Figure 1-1: Some bipedal robots. From left to right: WL-10RV1 from Waseda, P2 from Honda, Toddler from UNH, the Moscow State University Biped, SD-2 from Clemson and Ohio State, Biper from University of Tokyo, Meltran II from Mechanical Engineering Lab in Tsukuba, and Timmy from Harvard.

The control method described in this thesis differs from the others in that it is very simple to understand, it does not require dynamic calculations of the robot, it calculates the position of the center of mass and center of pressure of the robot at every instance in such a way that couplings between joints are taken into account. Each joint, triggered by finite state machine conditions, is servoed independent from the others therefore making the control more intuitive. Position of the center of mass and center of pressure of the robot are controlled using ankles, therefore every time the ankles are servoed, the couplings between all the joint of the robot are taken into account. A simple sideways foot placement control is used which is a function of sideways velocity of the center of mass of the robot and the sideways displacement of the position of the center of mass of the robot's body with respect to the support foot. Natural dynamics is exploited to simplify the sagittal (forward / backward) plane control. A block diagram describing the control strategy in this thesis is shown in Figure 1-2.

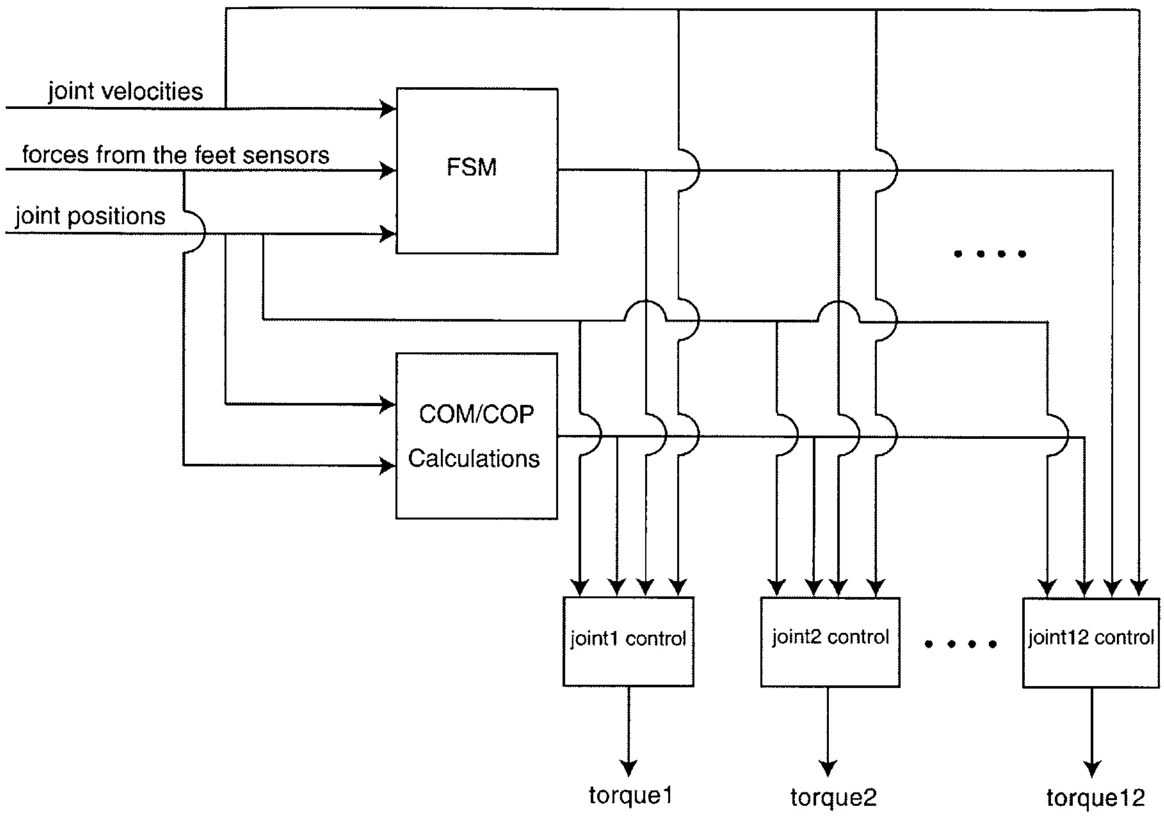

Figure 1-2: Diagram illustrating the general control technique used in this thesis.

1.2 Thesis Contents

This thesis is organized as follows:

Chapter 2 describes model of the robot

Chapter 3 describes natural dynamics of the robot

Chapter 4 describes the simulation algorithms for walking initiation, walking continuation, balancing on one foot, standing

Chapter 5 conclusions and future work

Chapter 2

Robot Model

2.1 Model background

The simulation model of this robot is based on the actual hardware design of the MIT Leglab biped, M2. The previous version of the Leglab biped, Spring Flaming, was a planar robot with a total of six degrees of freedom, one joint at each hip, one at each knee, and one at each ankle. The robot was connected to a boom in order to prevent the biped from falling side-to-side. The newer generation of the Leglab biped, M2, is supposed to be able to walk freely without being held by any external devices.

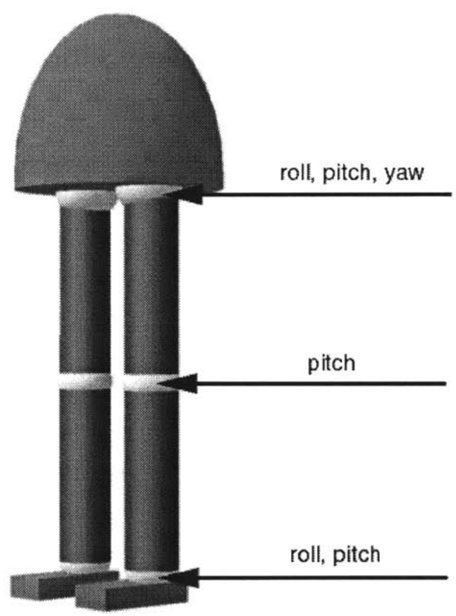

Figure 2-1: M2, the three-dimensional bipedal robot has three degrees of freedom at each hip, one degree of freedom at each knee, and two degrees of freedom at each ankle.

For many years, the tradition in the Leglab has been so that every robot is simulated and controlled first, where the simulation model is created based on the specifications of the actual hardware of the robot before the control code is tested on the actual hardware. Figure 2-1 shows the simulation cartoon model of this biped. As it can be seen it this figure, there are three degrees of freedom at each hip (roll, pitch, and yaw), one degree of freedom at each knee (pitch), and two degrees of freedom at each ankle (roll, pitch) for a total of twelve degrees of freedom. This three-dimensional seven-link biped possesses all the degrees of freedoms required in order to freely traverse in the three-dimensional world, including turning.

2.2 Links Specifications

The specifications of each link (mass, length, height, and width) are chosen to match an average male adult human, especially those of the designer's (Daniel Paluska).

2.2.1 Feet

The biped model has 2 rectangular feet as shown in Figure 2-2 with the specifications shown in Table 2-1.

Mass (kg)	Length (m)	Width (m)	Height (m)	Ankle to Toe (m)	Ankle to Heel (m)
0.562	0.203	0.0889	0.0641	0.152	0.051

Table 2-1: Feet specification of M2.

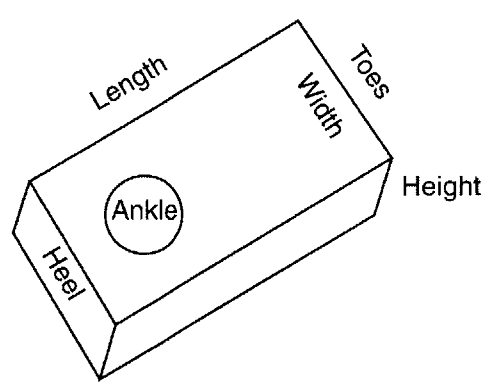

Figure 2-2: A rectangular foot has been used for M2.

Where the mass is in kilograms and all the lengths are in meters.

The contribution of the feet in natural-looking walking is extremely critical, for instance, if the feet are designed to be too wide, it might result in a very unnatural-looking landing of the foot. Obviously, feet cannot be too narrow either since the side-to-side control would become very challenging. Feet cannot be too long either since foot clearance in the swing phase would be a difficult task to achieve. Feet also play a major role in the toe-off state, where the robot's back-foot pushes against the ground in order for the robot to move forward and go into its opposite single support state, therefore if the feet are too narrowly designed, this task may not be completed successfully as the robot's feet can easily be twisted. The original design of the hardware of the foot included toes as well which was a triangular piece attached to the front of the foot. Since we were uncertain about the stability issues of the robot with toes, we decided to stick with simple rectangular feet.

2.2.2 Shins

The biped has two cylindrical shins as shown in Figure 2-3 that at the lower end are connected to the feet to form the ankle joints. The shin specifications are listed in Table 2-2.

Mass (kg)	Length (m)	Radius (m)
2.72	0.432	0.051

Table 2-2: Shins specifications of M2.

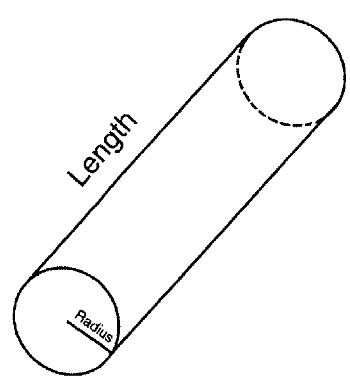

Figure 2-3: Cylindrical shins have been used on M2.

On the hardware of the robot, carbon fiber tubes are used to keep the weight as low as possible. Two actuators are attached on each shin to servo each ankle. With the actuators mounted, each shin weighs about 2.7 kg.

2.2.3 Thighs

Thighs are exactly similar to the shins. At the lower end, each thigh link is connected to the upper end of the appropriate shin to form the knee joints.

2.2.4 Body

The body has too parts. The lower part of the body consists of a short cylinder as shown in Figure 2-4 and the upper part of the body which is connected right on top of its lower part is a semi-ellipsoid. The body specifications are shown in Table 2-3.

Mass (kg)	Cylinder Height (m)	Cylinder Radius (m)	Semi-Ellipsoid Height (m)
12.7	0.0508	0.228	0.4572

Table 2-3: Body specifications of M2.

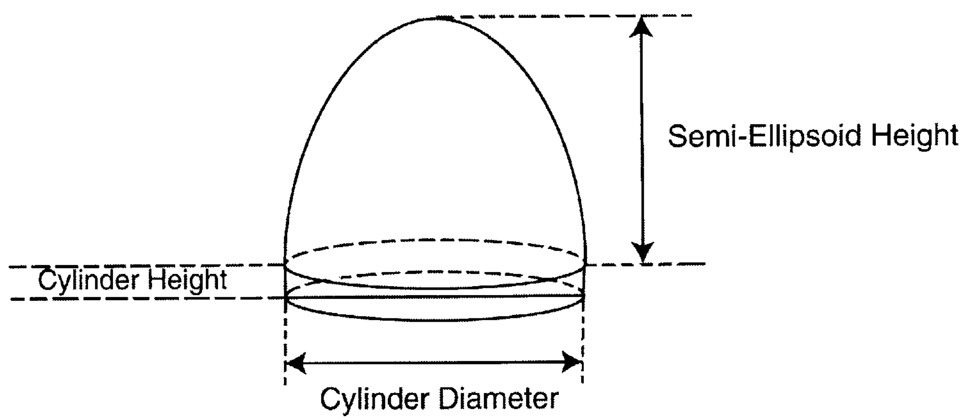

Figure 2-4: The body of M2 has a semi-ellipsoidal shape.

The upper ends of the thighs are connected at the two hip joints shown in Figure 2-5. The length of the body is critical in stable control i.e. a taller body is challenging to control since gravity can easily tip it over.

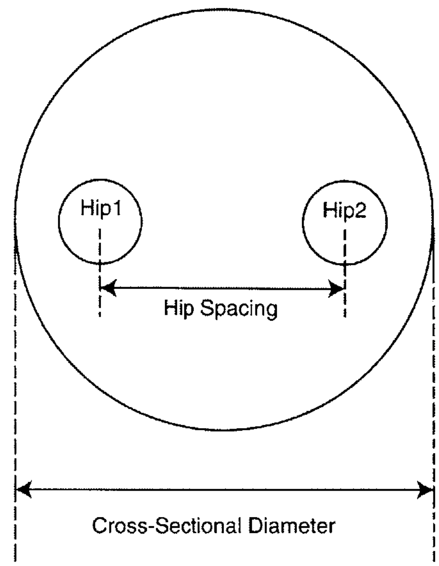

Figure 2-5: Cross-section of the body of M2.

2.3 Joint Characteristics

2.3.1 Ankles

Each ankle has two degrees of freedom (roll and pitch). A universal joint has been used to form this joint. The pitch degree of freedom allows the robot to move its feet up and down, while the roll degree of freedom allows the robot to move its feet side-to-side. Although ankle roll is not necessary in 3D walking if hip roll joint is present, but its availability allows the robot's feet to stay flat on the ground during almost throughout the entire single support phase. Ankle roll can too contribute to the control of the biped such that it will not fall sideways while walking. At each ankle pitch, it is assumed that a virtual spring is attached between the foot and the shin which enables the robot's heel to come off the ground naturally as its weight is transferred forward. A more detailed explanation on virtual spring of the ankle pitch will be given in chapter 3 of the thesis.

Figure 2-6 shows how each ankle pitch of the actual hardware is constructed. The two actuators attached on each shin servo the ankle pitch and roll. If only ankle pitch torque is desired, they both output equal forces in the same directions, and if only ankle roll torque is desired, they both output equal forces but in opposite directions. If both ankle pitch and roll torques are desired, the forces are related in a more complicated way which is outside the context of this thesis since the simulation uses a model such that for every joint there is a motor directly servoing it.

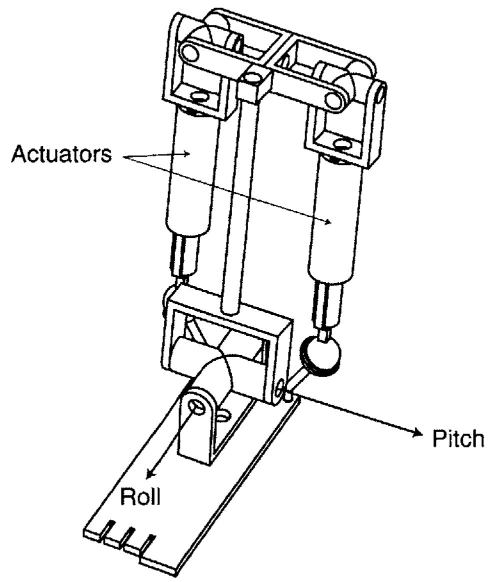

Figure 2-6: Ankle joint of M2 has two degrees of freedoms, roll and pitch.

2.3.2 Knees

Each knee has one degree of freedom (pitch), which is made of a pin joint (Figure 2-7). Just like the case in the humans, the knee is limited by a stop that does not allow the shin to bend out where out is defined the direction in which the swing shin is rotating up. Therefore a knee stop is used in the simulation model in order for us to be able to lock the knees as soon as the leg is straightened during landing and support phases.

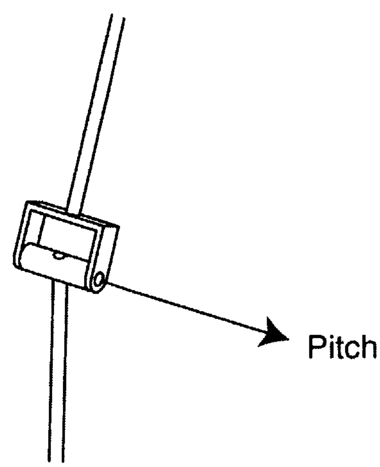

Figure 2-7: Knee joint of M2.

2.3.3 Hips

Each hip has three degrees of freedom (roll, pitch, and yaw). There is a universal joint used for the roll and the yaw degrees of freedom, and a pin joint for the pitch degree of freedom, which is based on the design of the actual hardware as shown in Figure 2-8.

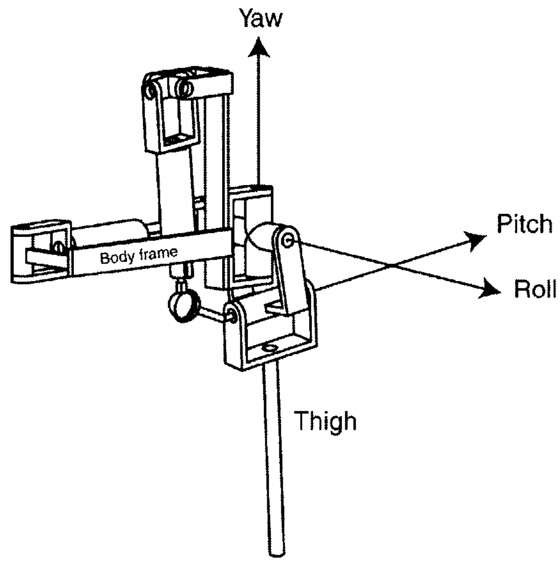

Figure 2-8: Each hip joint of M2 has three degrees of freedoms, roll, pitch and yaw.

The pitch degree of freedom allows the robot to swing its leg forward and backward, the roll degree of freedom provides the side-to-side motion of the leg which the robot needs in order to place its foot where it can prevent itself from falling sideways, and the yaw degree of freedom is the twist which is required for the robot to be able to turn.

2.4 Summery

The robot model has seven links and twelve degrees of freedom, which allows the biped to traverse in the 3D world. There are three degrees of freedom on each hip, one degree of freedom on each knee, and two degrees of freedom on each ankle. This biped is meant to have all necessary degrees of freedom in order to walk as naturally as possible without being held by an external object. Figure 2-9 shows a drawing model of the robot's leg with all the degrees of freedom.

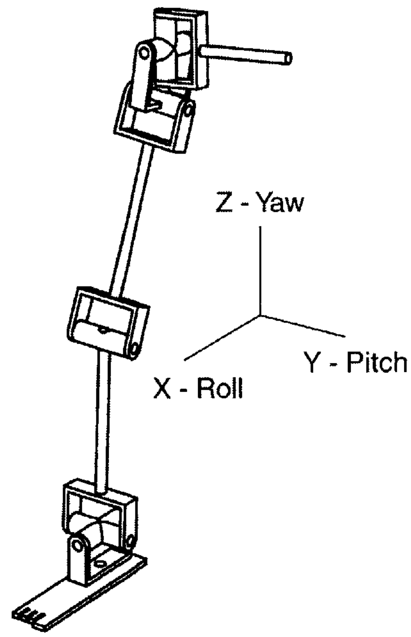

Figure 2-9: Model of a leg of M2 showing all the degrees of freedoms.

Chapter 3

Robot's Natural Dynamics

Many researchers such as McGeer [12], Goswami et al [5], and Garcia et al [3] have exploited natural dynamics to make their walking machines walk passively meaning that their machines rely completely on their natural dynamics and gravitational force in order to traverse along. Powerful design of a robot can simplify the control significantly by making use of natural dynamics. For instance, spinning an object about its small and large axis is naturally stable and requires no complicated control system. Pratt et al have employed natural dynamics in order to make a powered planar bipedal robot walk. They have also shown that natural dynamics can simplify control of a powered planar biped significantly.

In this chapter, the idea of natural dynamics is extended to the three-dimensional simulated bipedal robot, M2. First, the natural dynamics mechanisms will be explained and later in chapter 4, they will be exploited in order to control M2.

3.1 Springy Ankle

The ankle of the hardware of M2 contains a rubber stop that serves as an ankle limit, which enables the robot's heel to come off the ground passively as the robot's center of mass is moving forward. In the simulation model of M2, a virtual quadratic spring is used in order to serve this purpose. Figure 3-1 illustrates how a compliant ankle helps the heel to come off the ground. As the robot's center of mass is moving forward, the spring gets compressed, the center of pressure moves to the toes, as a result of that the heel lifts off the ground. Combination of springy ankle and active control will allow the robot's toe to come off the ground. As soon as the heel of the robot lifts off, the ankle pitch is servoed to open up, as a result of that the toes push off against the ground, which helps the robot to go into toe-off state. There are of course differences between a rubber stop and model of the spring used in the simulation, therefore appropriate adjustments need to be made for the robot's heel to lift off at the right time. A late lift off can cause the robot to not get over its apex, and an early lift or a hard push can cause the body roll to go unstable.

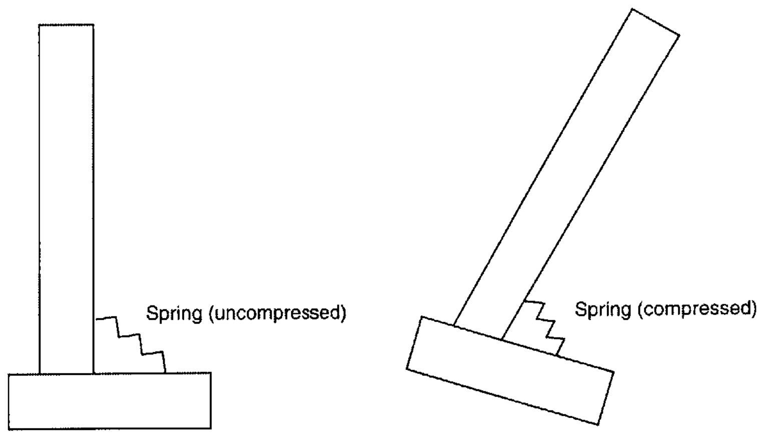

Figure 3-1: Virtual spring is used at M2's ankle pitch joint.

3.2 Knee Stop

Walking with a straightened support knee is simpler to control since the robot can be modeled as an inverted pendulum. A knee stop is used so that during walking, every time the knee is straightened right before touchdown or during support, the knee is servoed to a locked position, which creates a reliable and strong support leg. Figure 3-2 illustrates the knee stop. On the hardware of the robot, rubber-stop are used so that when the knee is straightened, there will be soft contact between the shin and the thigh, where in the simulation, damping is used right before the swing leg is straightened so that the shin will not bang into the knee stop too violently. As soon as the knee is straightened, stiff proportional gain is used to ensure the knee is locked.

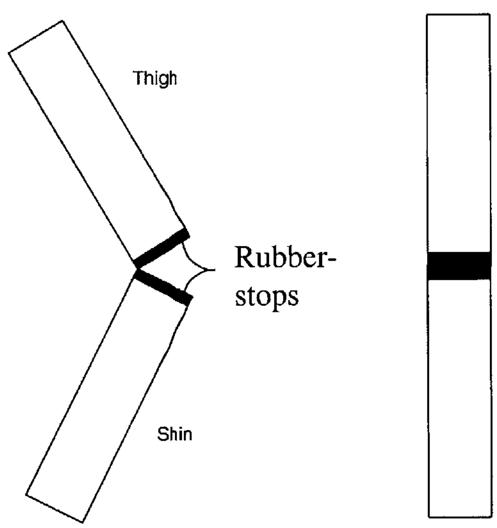

Figure 3-2: Knee strop is used to prevent the knee from inverting.

3.3 Passive Swing

In the swing phase, Pratt et al. [15] showed that the shin can swing passively while the swing hip pitch is servoed forward. This makes the control easier in a sense that the active torque on the swing knee can be turned off and let the natural dynamics of the swing shin take over. Figure 3- 3 illustrates how the shin is swung forward. As soon as the knee is straightened, it is locked against its stop to maintain its straightened shape.

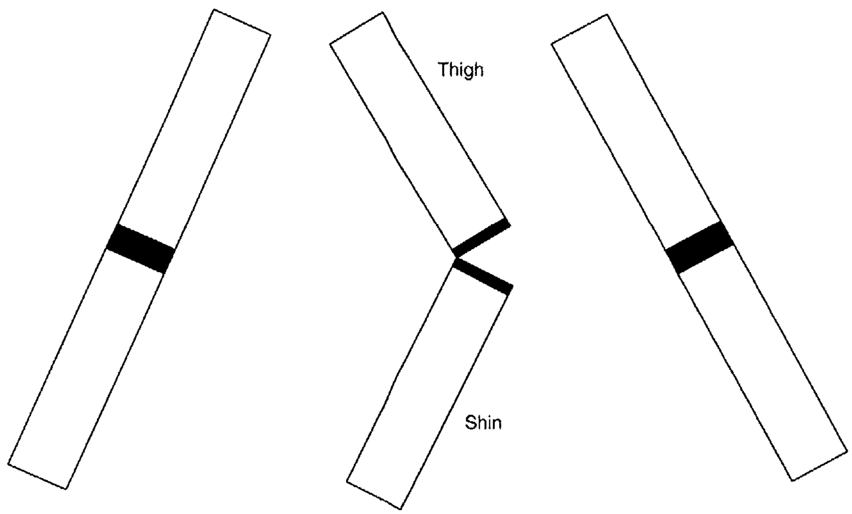

Figure 3- 3: As the thigh of the robot is swung forward, due to natural dynamics of the biped, the shin too swings which causes the knee to straighten.

Chapter 4

Robot Control

4.1 Simulation Algorithm For Walking Initiation

A Finite State Machine (FSM), comprising two states, is used for walking initiation control algorithm as shown in Figure 4-1. In the first state (Leaning Sideways), the robot uses one of its ankle rolls joints (in this case, the right one) to push against the ground (by twisting the right foot) and as a result of that, the robot leans to the opposite side. During the whole time that the robot is in state 1, all its joints are controlled using proportional-derivative controller. The body is controlled to have an upright position by servoing the hips while both of its legs are leaning sideways as show in Figure 4-2. The knees are in the locked position the whole time. The robot keeps pushing against the ground in the frontal plane until the position of its center of mass, which is measured from the left ankle falls on top of its left foot. This is when the biped goes into state 2 (Pick up Foot).

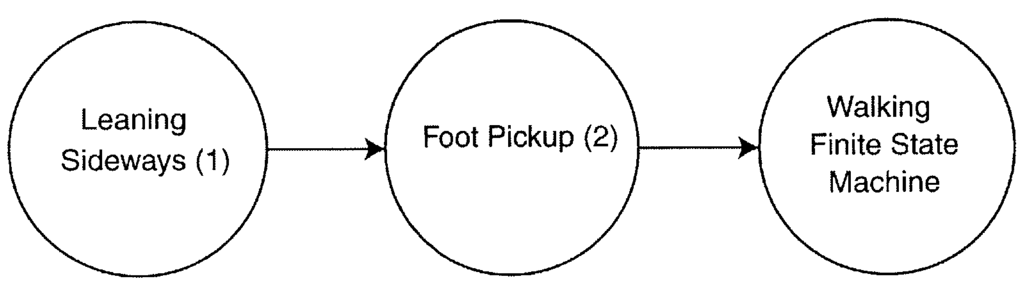

Figure 4-1: Finite state machine is used for walking initiation.

In state 2 (Figure 4-3), most of the robot's weight has been taken off of its right foot, which makes it plausible for the robot to pick it up by driving its right hip pitch joint to a desired position. The knee joint of the right leg is bent at the same time while the left knee maintains its locked position. The right foot is controlled to stay parallel with the ground to ensure foot clearance. Left ankle pitch is servoed to maintain the center of mass of the robot at a desired position in the sagittal plane so that the robot will not fall forward or backward. Left ankle roll is used to control the position of the center of mass of the robot in the frontal plane so that it won't fall to the side. As soon as the position of the right hip pitch joint reaches a certain threshold, the robot goes into a different state, which is when it starts walking.

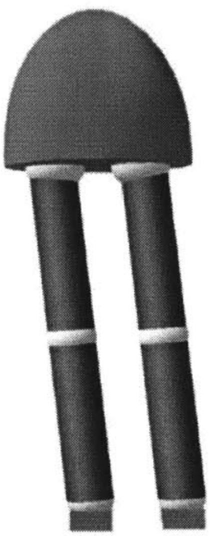

Figure 4-2: Robot is leaning to the side in state 1.

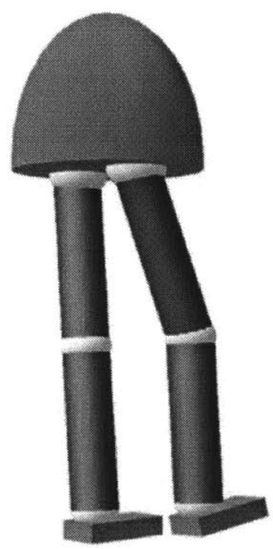

Figure 4-3: Robot is picking up its foot in state 2.

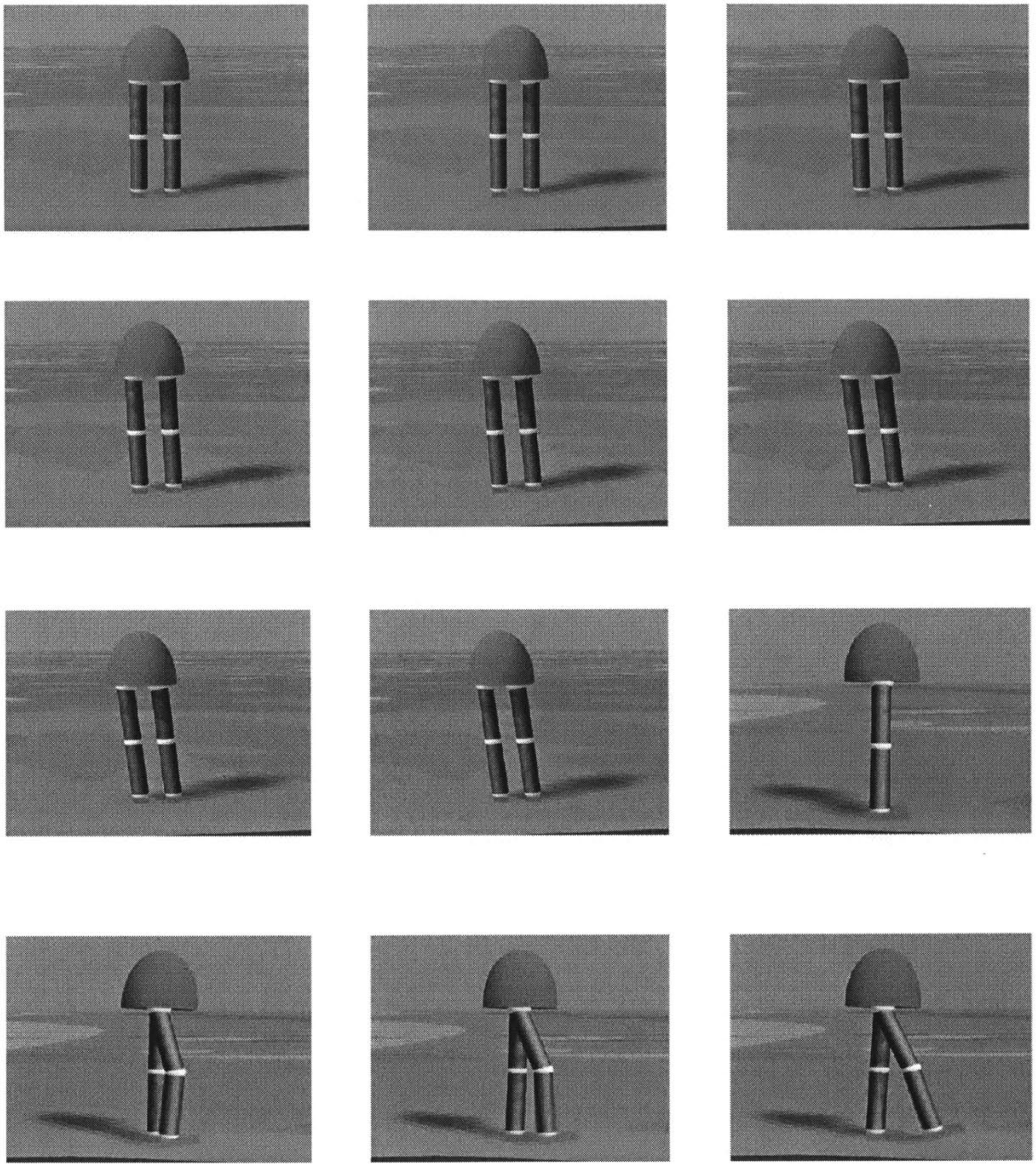

Figure 4-4: The whole walking initiation process is displayed in two different angles.

4.2 Walking Continuation

4.2.1 Analysis

The location of the center of mass of the robot is calculated in both frontal and sagittal planes and later used in the walking control algorithm.

For the calculations in the sagittal plane (x-z), consider Figure 4-5.

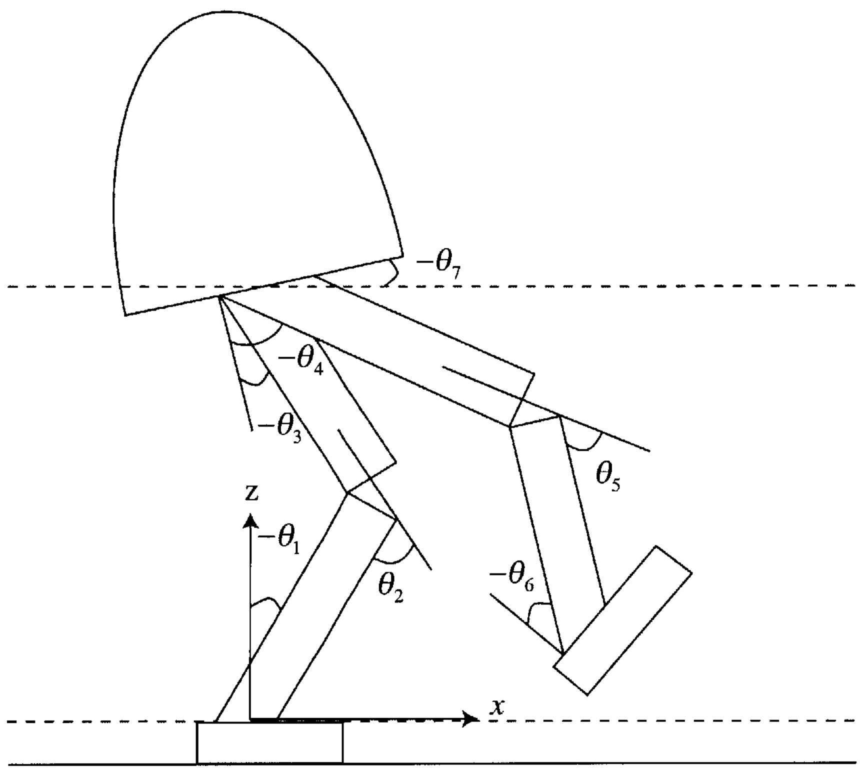

Figure 4-5: Geometric drawing of M2 in x-z plane.

where

$$\theta_1 = q\_la\_pitch$$
$$\theta_2 = q\_lk$$
$$\theta_3 = q\_lh\_pitch$$
$$\theta_4 = q\_rh\_pitch$$
$$\theta_5 = q\_rk$$
$$\theta_6 = q\_ra\_pitch$$
$$\theta_7 = q\_pitch$$

x_i is the position of the center of mass of link i in the x-z plane measured with respect to the x-z reference frame located at ankle joint as shown in Figure 4-5. When the left foot is the support foot, x_i_L is used and when the right foot is the support foot, x_i_R is used.

The links are defined as follows:
i = 1 represents the left foot
i = 2 represents the left shin
i = 3 represents the left thigh
i = 4 represents the body
i = 5 represents the right thigh
i = 6 represents the right shin
i = 7 represents the right foot

the expressions for x_i_L for i=1,2,3,4,5,6, and 7 are:

$$x_1\_L = FOOT\_FORWARD - 0.5FOOT\_LENGTH$$

$$x_2\_L = -(0.5)(0.5)L\_SL\_L\_x\sin(q\_la\_pitch)$$

$$x_3\_L = 2x_2\_L - (0.5)(0.5)L\_SL\_L\_x\sin(q\_la\_pitch + q\_lk)$$

$$x_4\_L = 2x_2\_L - 0.5L\_SL\_L\_x\sin(q\_la\_pitch + q\_lk) - CG\_Z\_OFFSET\cos(q\_roll)\sin(q\_pitch)$$

$$x_5\_L = 2x_2\_L - 0.5L\_SL\_L\_x\sin(q\_la\_pitch + q\_lk) - (0.5)(0.5)L\_SL\_R\_x\sin(q\_pitch + q\_rh\_pitch)$$

$$x_6\_L = 2x_2\_L - 0.5L\_SL\_L\_x\sin(q\_la\_pitch + q\_lk) - (0.5)(0.5)L\_SL\_R\_x\sin(q\_pitch + q\_rh\_pitch) - (0.5)(0.5)L\_SL\_R\_x\sin(q\_pitch + q\_rh\_pitch + q\_rk)$$

$$x_7\_L = 2x_2\_L - 0.5L\_SL\_L\_x\sin(q\_la\_pitch + q\_lk) - (0.5)(0.5)L\_SL\_R\_x\sin(q\_pitch + q\_rh\_pitch) - (0.5)L\_SL\_R\_x\sin(q\_pitch + q\_rh\_pitch + q\_rk) + x_1\_L\cos(q\_pitch + q\_rh\_pitch + q\_rk + q\_ra\_pitch)$$

where

$$L\_SL\_L\_x = 2SHIN\_LENGTH\cos(q\_roll + q\_lh\_roll)$$

which is the length of the projection of the support leg onto the x-axis in the x-z plane when the left leg is the support leg.

and

$$L\_SL\_R\_x = 2SHN\_LENGTH\cos(q\_roll + q\_rh\_roll)$$

which is the length of the projection of the support leg on the x-axis in the x-z plane when the right leg is the support leg.

similarly

$$x_1\_R = FOOT\_FORWARD - 0.5FOOT\_LENGTH$$
$$x_2\_R = -(0.5)(0.5)L\_SL\_R\_x\sin(q\_ra\_pitch)$$
$$x_3\_R = 2x_2\_R - (0.5)(0.5)L\_SL\_R\_x\sin(q\_ra\_pitch + q\_rk)$$
$$x_4\_R = 2x_2\_R - 0.5L\_SL\_R\_x\sin(q\_ra\_pitch + q\_rk) - CG\_Z\_OFFSET\cos(q\_roll)\sin(q\_pitch)$$
$$x_5\_R = 2x_2\_R - 0.5L\_SL\_R\_x\sin(q\_ra\_pitch + q\_rk) - (0.5)(0.5)L\_SL\_L\_x\sin(q\_pitch + q\_lh\_pitch)$$
$$x_6\_R = 2x_2\_R - 0.5L\_SL\_R\_x\sin(q\_ra\_pitch + q\_rk) - (0.5)(0.5)L\_SL\_L\_x\sin(q\_pitch + q\_lh\_pitch) - (0.5)(0.5)L\_SL\_L\_x\sin(q\_pitch + q\_lh\_pitch + q\_lk)$$
$$x_7\_R = 2x_2\_R - 0.5L\_SL\_R\_x\sin(q\_ra\_pitch + q\_rk) - (0.5)(0.5)L\_SL\_L\_x\sin(q\_pitch + q\_lh\_pitch) - (0.5)L\_SL\_L\_x\sin(q\_pitch + q\_lh\_pitch + q\_lk) + x_1\_R\cos(q\_pitch + q\_lh\_pitch + q\_lk + q\_la\_pitch)$$

For the calculations in the frontal plane (y-z), consider Figure 4-6 shown below:

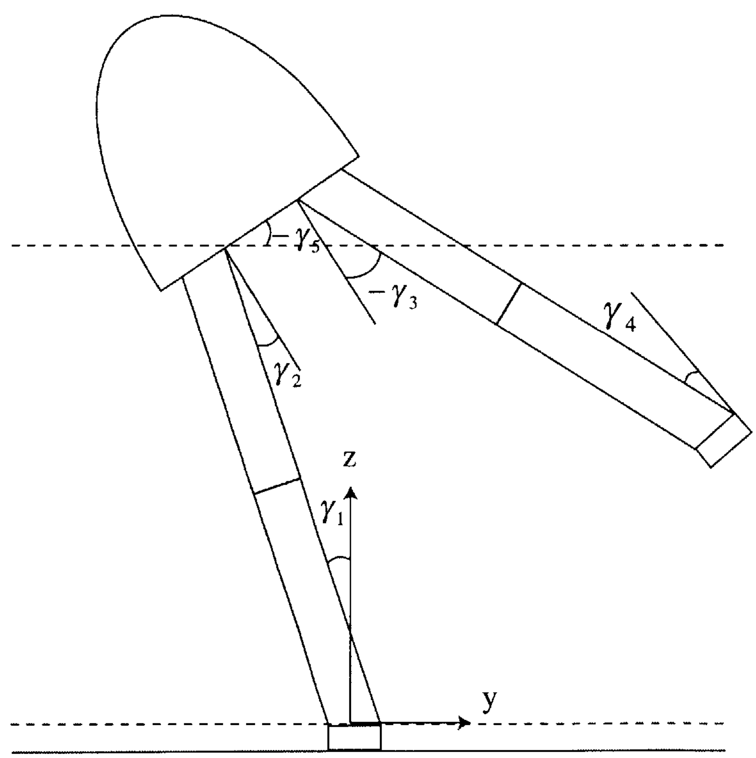

Figure 4-6: Geometric drawing of M2 in the y-z plane.

where

$\gamma_1 = q\_la\_roll$
$\gamma_2 = q\_lh\_roll$
$\gamma_3 = q\_rh\_roll$
$\gamma_4 = q\_ra\_roll$
$\gamma_5 = q\_roll$

y_j is the position of the center of mass of link j in the y-z plane measured with respect to the z-y reference frame located at the ankle joint as shown in Figure 4-6. When the left foot is the support foot, y_i_L is used and when the right foot is the support foot, y_i_R is used.

The links are defined as follows:

j = 1 represents the left shin
j = 2 represents the left thigh
j = 3 represents the body
j = 4 represents the right thigh
j = 5 represents the right shin
j = 6 represents the right foot

the expressions for y_i's for i=1,2,3,4,5, and 6 are

$$y_1\_L = -0.5L\_SHIN\_L\sin(q\_roll + q\_lh\_roll)$$
$$y_2\_L = 2y_1\_L - 0.5L\_THIGH\_L\sin(q\_roll + q\_lh\_roll)$$
$$y_3\_L = 2y_1\_L - L\_THIGH\_L\sin(q\_roll + q\_lh\_roll) - 0.5HIP\_SPACING\cos(q\_roll) + CG\_Z\_OFFSET\cos(q\_pitch)\sin(q\_roll)$$
$$y_4\_L = 2y_1\_L - L\_THIGH\_L\sin(q\_roll + q\_lh\_roll) - HIP\_SPACING\cos(q\_roll) - 0.5L\_T\_S\_R\sin(q\_roll + q\_rh\_roll)$$
$$y_5\_L = 2y_1\_L - L\_THIGH\_L\sin(q\_roll + q\_lh\_roll) - HIP\_SPACING\cos(q\_roll) - (L\_T\_S\_R + 0.5L\_S\_S\_R)\sin(q\_roll + q\_rh\_roll)$$
$$y_6\_L = 2y_1\_L - L\_THIGH\_L\sin(q\_roll + q\_lh\_roll) - HIP\_SPACING\cos(q\_roll) - (L\_T\_S\_R + L\_S\_S\_R)\sin(q\_roll + q\_rh\_roll) - 0.5FOOT\_HEIGHT\sin(q\_roll + q\_rh\_roll + q\_ra\_roll)$$

where

$$L\_SHIN\_L = SHIN\_LENGTH\cos(q\_la\_pitch)$$
$$L\_THIGH\_L = SHIN\_LENGTH\cos(q\_pitch + q\_lh\_pitch)$$

which are the lengths of the projections of the shin and thigh of the support leg on the y-axis in the y-z plane, if the left leg is the support leg, respectively.

and

$$L\_T\_S\_R = SHIN\_LENGTH\cos(q\_pitch + q\_rh\_pitch)$$
$$L\_S\_S\_R = SHIN\_LENGTH\cos(q\_pitch + q\_rh\_pitch + q\_rk)$$

which are the lengths of the projections of the thigh and shin of the swing leg on the y-axis in the y-z plane, if the right leg is the sing leg, respectively.

Similarly

$$\begin{aligned}
y_1_R &= -0.5L_SHIN_R\sin(q_roll + q_rh_roll) \\
y_2_R &= 2y_1_R - 0.5L_THIGH_R\sin(q_roll + q_rh_roll) \\
y_3_R &= 2y_1_R - L_THIGH_R\sin(q_roll + q_rh_roll) + \\
&\quad 0.5HIP_SPACING\cos(q_roll) - CG_Z_OFFSET\cos(q_pitch)\sin(q_roll) \\
y_4_R &= 2y_1_R - L_THIGH_R\sin(q_roll + q_rh_roll) + \\
&\quad HIP_SPACING\cos(q_roll) + 0.5L_T_S_L\sin(q_roll + q_lh_roll) \\
y_5_R &= 2y_1_R - L_THIGH_R\sin(q_roll + q_rh_roll) + HIP_SPACING\cos(q_roll) + \\
&\quad (L_T_S_RL + 0.5L_S_S_L)\sin(q_roll + q_lh_roll) \\
y_6_R &= 2y_1_R - L_THIGH_R\sin(q_roll + q_rh_roll) + HIP_SPACING\cos(q_roll) + \\
&\quad (L_T_S_L + L_S_S_L)\sin(q_roll + q_lh_roll) + \\
&\quad 0.5FOOT_HEIGHT\sin(q_roll + q_lh_roll + q_la_roll)
\end{aligned}$$

where

$$L\_SHIN\_R = SHIN\_LENGTH\cos(q\_ra\_pitch)$$
$$L\_THIGH\_R = SHIN\_LENGTH\cos(q\_pitch + q\_rh\_pitch)$$

which are the lengths of the projections of the shin and thigh of the support leg on the y-axis in the y-z plane, if the right leg is the support leg, respectively.

and

$$L\_T\_S\_L = SHIN\_LENGTH\cos(q\_pitch + q\_lh\_pitch)$$
$$L\_S\_S\_L = SHIN\_LENGTH\cos(q\_pitch + q\_lh\_pitch + q\_lk)$$

which are the lengths of the projections of the thigh and shin of the swing leg on the y-axis in the y-z plane, if the left leg is the sing leg, respectively.

4.2.2 Simulation Algorithm

A finite state machine is used to perform the walking algorithm. The states are related as shown in Figure 4-7. A detailed control description of each joint in every state is explained below. All the positions of the joints have the form "q_*joint_name*". All the angular velocities of the joints have the form "qd_*joint_name*". All the desired positions of the joints have the form "q_d_*joint_name*". All the desired angular velocities of the joints have the form "qd_d_*joint_name*"

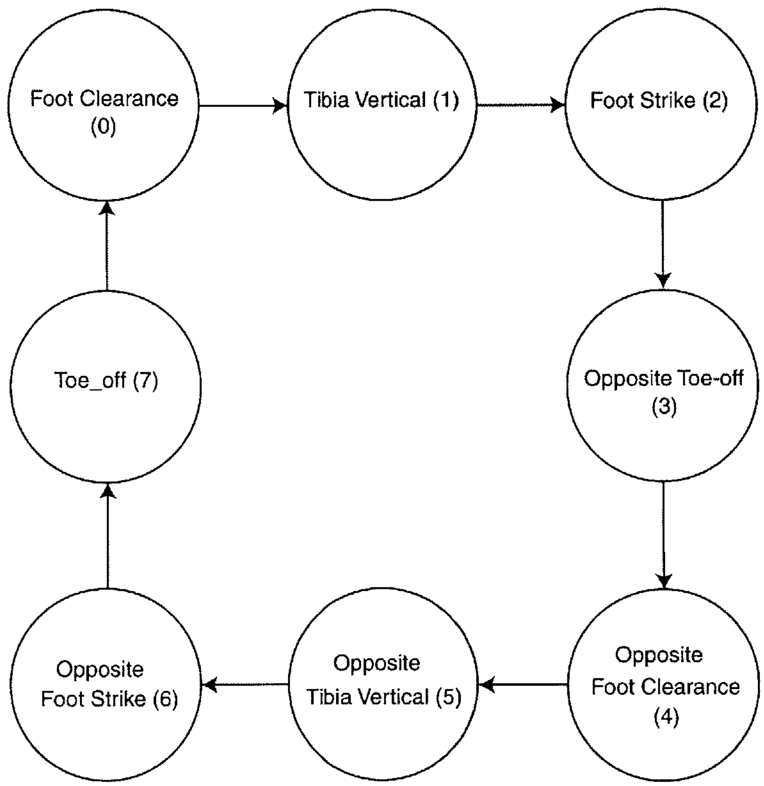

Figure 4-7: Finite state machine with eight states is used to perform the walking algorithm.

Foot Clearance

This state is initiated when the right ankle pitch exceeds a certain angle while pushing against the ground (Figure 4-8). The left leg is the support leg while the right leg is in the beginning of its swing phase. The control is as follows:

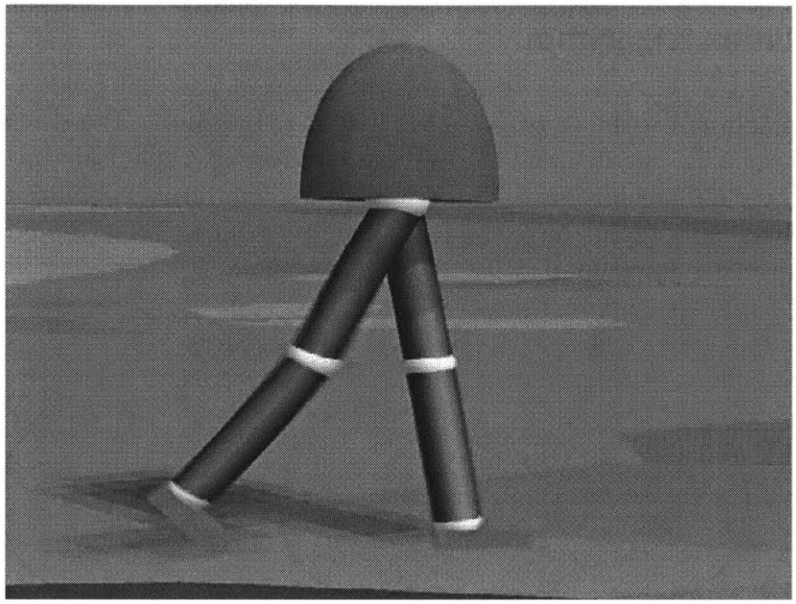

Figure 4-8: Initiation of state 0.

Body Roll: Body roll is controlled at the left hip roll by using both Proportional-Derivative (PD) and a feed forward term, which counters the effect of the gravitational force.

$$\begin{aligned} tau\_lh\_roll = {} & k\_roll(q\_roll - q\_d\_roll) + b\_roll(qd\_roll - qd\_d\_roll) + \\ & (0.5(HIP\_SPACING)\cos(q\_roll) + \\ & (CG\_Z\_OFFSET)\sin(q\_roll))(grav\_force\_body\_roll\_l) \end{aligned}$$

where

HIP_SPACING is the distance between the two hip joints and *CG_Z_OFFSET* is the distance from the center of mass of the body to the base of the ellipsoid.

Parameter	Value
k_roll	60
b_roll	20
grav_force_body_roll_l	225
q_d_roll	0
qd_d_roll	0

Table 4-1: Control parameters of body roll in state 0.

Body Pitch: Body pitch is controlled by servoing the left hip pitch joint using a PD controller such that it stays parallel to the ground. An offset is used to ensure upright position of the body.

$$tau\_lh\_pitch = k\_pitch(q\_pitch - q\_d\_pitch) + b\_pitch(qd\_pitch - qd\_d\_pitch)$$

where

Parameter	Value
k_pitch	60
b_pitch	20
q_d_pitch	0.1
qd_d_pitch	0

Table 4-2: Control parameters of body pitch in state 0.

Body Yaw: Body yaw is controlled by servoing the left hip yaw joint using a PD controller such that it maintains the least amount of twist.

$$tau\_lh\_yaw = k\_yaw(q\_yaw - q\_d\_yaw) + b\_yaw(qd\_yaw - qd\_d\_yaw)$$

where

Parameter	Value
k_yaw	60
b_yaw	20
q_d_yaw	0
qd_d_yaw	0

Table 4-3: Control parameters of body yaw in state 0.

Left Knee: It is made sure that the left knee is servoed against its stop so that it is tightly locked. This allows the robot to be like an inverted pendulum, which will provide an easy transition from single support to double support.

$$tau\_lk = k\_lk(q\_d\_lk - q\_lk) + b\_lk(qd\_d\_lk - qd\_lk)$$

where

Parameter	Value
k_lk	30
b_lk	10
q_d_lk	0
qd_d_lk	0

Table 4-4: Control parameters of left knee in state 0.

Left Ankle Roll: Position of the center of pressure is calculated from the left ankle. The desired position of the center of pressure is a function of the position of the center of mass error and sideways translational velocity of a point on the left hip joint.

$$L\_com\_err\_y = y\_com\_L + (frac\_foot\_width)(FOOT\_WIDTH)$$
$$L\_cent\_press\_y\_des = (b\_ank\_yd)(lhip\_proj\_yd) + (k\_com\_err\_y)(L\_com\_err\_y)$$

if $(L\_cent\_press\_y\_des > 0.5(FOOT\_WIDTH))$
$\quad L\_cent\_press\_y\_des = 0.5(FOOT\_WIDTH)$
if $(L\_cent\_press\_y\_des < -0.5(FOOT\_WIDTH))$
$\quad L\_cent\_press\_y\_des = -0.5(FOOT\_WIDTH)$

$$tau\_la\_roll = k\_la\_roll\_cop(L\_cent\_press\_y\_des - y\_cop\_L)$$

Parameter	Value
k_la_roll_cop	100
b_ank_yd	-1
k_com_y	1
frac_foot_width	0.22

Table 4-5: Control parameters of left ankle roll in state 0.

Left Ankle Pitch: Velocity control is used to slow down the robot if it is moving with a velocity higher than a certain threshold.

```
body_speed_control_torque(vel)
     double vel;
{
  if ( vel > vel_threshold )
    return ((vel_gain)(vel^2));
  else return (0);
}
```

A quadratic virtual spring is used to enable the robot's heel to come off the ground as the biped's weight is shifted forward.

```
double pass_ank_pitch_torque(pos,vel)
     double pos,vel;
{
  if ( pos < ank_pitch_lim_set )
    return ((ank_pitch_lim_gain) (ank_pitch_lim_set - pos)^2);
  else return (0);
}
```

$$tau\_la\_pitch = pass\_ank\_pitch\_torque(q\_la\_pitch, qd\_la\_pitch) + body\_speed\_control\_torque(x\_vel)$$

where

x_vel is the transnational velocity of the robot in the sagittal plane.

Parameter	Value
vel_threshold	0.91
vel_gain	40
ank_pitch_lim_set	0
ank_pitch_lim_gain	1000

Table 4-6: Control parameters of left ankle pitch in state 0.

Right Hip Pitch: Right hip pitch joint is servoed to a desired position such that neither overshoot nor undershoot is achieved. An overshoot can cause the robot's foot to land too hard. An undershoot will result in a short swing which would mean no foot clearance.

$$tau\_rh\_pitch = k\_rh\_pitch(q\_d\_rh\_pitch - q\_rh\_pitch) + b\_rh\_pitch(qd\_d\_rh\_pitch - qd\_rh\_pitch)$$

where

$$q\_d\_rh\_pitch = -q\_pitch + q\_d\_rh\_pitch\_final$$

Parameter	Value
k_rh_pitch	55
b_rh_pitch	20
q_d_rh_pitch_final	-0.5
qd_d_rh_pitch	0

Table 4-7: Control parameters of right hip pitch in state 0.

Right Knee: The right knee torque is completely shut down, therefore the shin of the swing leg is swung forward passively due to the servoing of the right hip pitch.

$$tau\_rk = 0$$

Right Ankle Roll: Right ankle roll is simply servoed to be held aligned with the shin of the swing leg.

$$tau\_ra\_roll = k\_ra\_roll(q\_d\_ra\_roll - q\_ra\_roll) + b\_ra\_roll(qd\_d\_ra\_roll - qd\_ra\_roll)$$

Parameter	Value
k_ra_roll	4
b_ra_roll	1
q_d_ra_roll	0
qd_d_ra_roll	0

Table 4-8: Control parameters of right ankle roll in state 0.

Right Ankle Pitch: Right ankle pitch is simply servoed to be pointing up with respect to the shin of the swing leg to ensure foot clearance.

$$tau\_ra\_pitch = k\_ra\_pitch(q\_d\_ra\_pitch - q\_ra\_pitch) + b\_ra\_pitch(qd\_d\_ra\_pitch - qd\_ra\_pitch)$$

Parameter	Value
k_ra_pitch	7
b_ra_pitch	1
q_d_ra_pitch	-0.3
qd_d_ra_pitch	0

Table 4-9: Control parameters of right ankle pitch in state 0.

Tibia Vertical

This state is initiated when the left ankle pitch angle and right knee fall below a certain threshold (Figure 4-9). Most of the control in this state is similar to the control in the "foot clearance" state. If there are any differences in their parameters, they are shown in new tables.

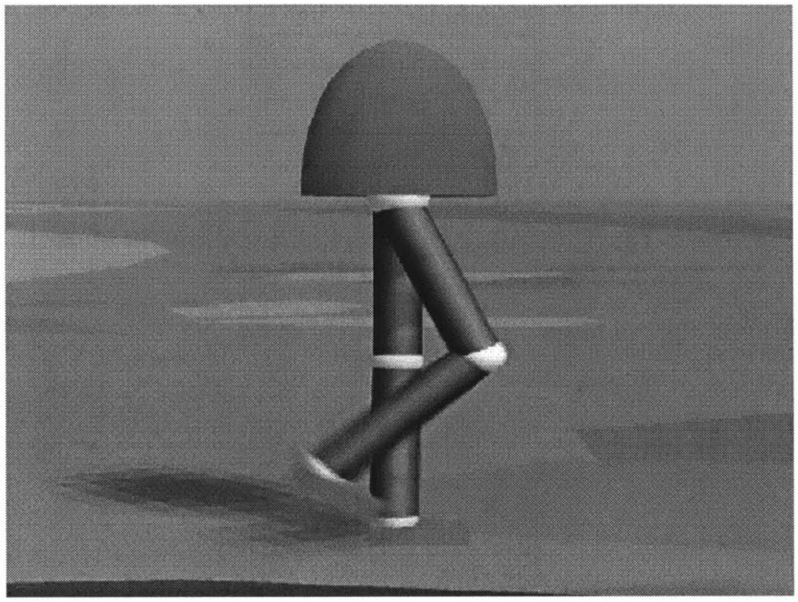

Figure 4-9: Initiation of state 1.

Body Roll: Similar control to the "Foot Clearance" state.

Parameter	Value
k_roll	60
b_roll	10
grav_force_body_roll_l	230
q_d_roll	0
qd_d_roll	0

Table 4-10: Control parameters of body roll in state 1.

Body Pitch: Similar control to the "Foot Clearance" state.

Parameter	Value
k_pitch	60
b_pitch	20
q_d_pitch	0
qd_d_pitch	0

Table 4-11: Control parameters of body pitch in state 1.

Body Yaw: Similar control to the "Foot Clearance" state.

Parameter	Value
k_yaw	60
b_yaw	20
q_d_yaw	0
qd_d_yaw	0

Table 4-12: Control parameters of body yaw in state 1.

Left Knee: Similar control to the "Foot Clearance" state.

Parameter	Value
k_lk	30
b_lk	10
q_d_lk	0
qd_d_lk	0

Table 4-13: Control parameters of left knee in state 1.

Left Ankle Roll: Similar control to the "Foot Clearance" state.

Parameter	Value
k_la_roll_cop	100
b_ank_yd	-1
k_com_y	1
frac_foot_width	0.2

Table 4-14: Control parameters of left ankle roll in state 1.

Left Ankle Pitch: Only a quadratic virtual spring is used to enable the heel of the support foot to lift off.

$$tau\_la\_pitch = pass\_ank\_pitch\_torque(q\_la\_pitch, qd\_la\_pitch)$$

Right Hip Pitch: Similar control to the "Foot Clearance" state.

Parameter	Value
k_rh_pitch	50
b_rh_pitch	20
q_d_rh_pitch_final	-0.5
qd_d_rh_pitch	0

Table 4-15: Control parameters of right hip pitch in state 1.

Right Knee: Similar control to the "Foot Clearance" state.

Right Ankle Roll: Similar control to the "Foot Clearance" state.

Right Ankle Pitch: Similar control to the "Foot Clearance" state.

Foot Strike

This state is initiated when the left ankle pitch angle and right knee fall below a certain threshold (Figure 4-10). This state is divided into two parts. First when the right foot is still in the air, and next, when the right heel lands on the ground.

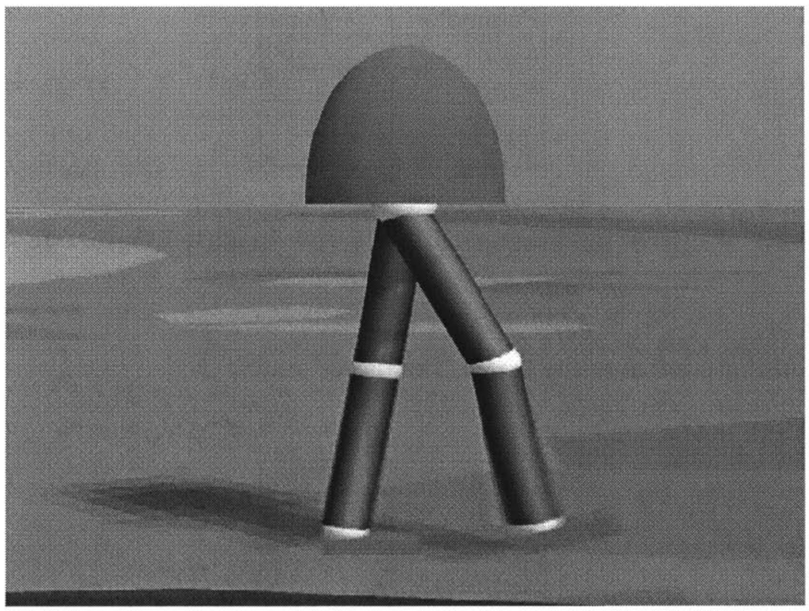

Figure 4-10: Initiation of state 2.

When the Right Foot is in the Air

Body Roll: Similar control to the "Foot Clearance" state.

Parameter	Value
k_roll	60
b_roll	20
grav_force_body_roll_l	270
q_d_roll	0
qd_d_roll	0

Table 4-16: Control parameters of body roll in state 2 before foot strike.

Body Pitch: Similar control to the "Foot Clearance" state.

Parameter	Value
k_pitch	60
b_pitch	20
q_d_pitch	0.1
qd_d_pitch	0

Table 4-17: Control parameters of body pitch in state 2 before foot strike.

Body Yaw: Similar control to the "Foot Clearance" state.

Parameter	Value
k_yaw	60
b_yaw	20
q_d_yaw	0
qd_d_yaw	0

Table 4-18: Control parameters of body yaw in state 2 before foot strike.

Left Knee: Similar control to the "Foot Clearance" state.

Parameter	Value
k_lk	30
b_lk	10
q_d_lk	0
qd_d_lk	0

Table 4-19: Control parameters of left knee in state 2 before foot strike.

Left Ankle Roll: Similar control to the "Foot Clearance" state.

Parameter	Value
k_la_roll_cop	100
b_ank_yd	-1
k_com_y	1
frac_foot_width	0.7

Table 4-20: Control parameters of left ankle roll in state 2 before foot strike.

Left Ankle Pitch: Similar control to the "Tibia Vertical" state.

Right Hip Pitch: Similar control to the "Foot Clearance" state.

Parameter	Value
k_rh_pitch	55
b_rh_pitch	20
q_d_rh_pitch_final	-0.1
qd_d_rh_pitch	0

Table 4-21: Control parameters of right hip pitch in state 2 before foot strike.

Right Knee: Damping is used in order to slow down the velocity of the swing of the shin so that it will not bang into the knee stop too harshly. A proportional term is used to ensure the knee is straightened.

$$tau\_rk = k\_rk(q\_d\_rk - q\_rk) + b\_rk(qd\_d\_rk - qd\_rk)$$

Parameter	Value
k_rk	4.3
b_rk	1.4
q_d_rk	0
qd_d_rk	0

Table 4-22: Control parameters of right knee in state 2 before foot strike.

Right Ankle Roll: Similar control to the "Foot Clearance" state.

Right Ankle Pitch: Similar control to the "Foot Clearance" state.

Parameter	Value
k_ra_pitch	4
b_ra_pitch	1
q_d_ra_pitch	0
qd_d_ra_pitch	0

Table 4-23: Control parameters of right ankle pitch in state 2 before foot strike.

When the Right Heal is on the Ground

Body Roll: Similar control to the "Foot Clearance" state.

Parameter	Value
k_roll	60
b_roll	20
grav_force_body_roll_l	170
q_d_roll	0
qd_d_roll	0

Table 4-24: Control parameters of body roll in state 2 after foot strike.

Body Pitch: Similar control to the case when the right foot is in the air.

Body Yaw: Similar control to the case when the right foot is in the air.

Left Knee: Similar control to the case when the right foot is in the air.

Left Ankle Roll: Similar control to the case when the right foot is in the air.

Left Ankle Pitch: In addition to employing a virtual spring, the robot pushes against the ground in order to shift its weight forward. This process is accomplished by servoing the robot's left ankle pitch to a desired position.

$$tau\_la\_pitch = pass\_ank\_pitch\_torque(q\_la\_pitch, qd\_la\_pitch) + \\ k\_la\_pitch(q\_d\_la\_pitch - q\_la\_pitch)$$

Parameter	Value
k_la_pitch	30
q_d_la_pitch	0.3

Table 4-25: Control parameters of left ankle pitch in state 2 after foot strike.

Right Hip Pitch: Since the right foot is on the ground now, the right hip pitch is used to control position of the body. Similar control to the left hip pitch joint.

Right Knee: In order to make sure that knee stays locked, higher gains are used on the knee joint control. Similar control to the left knee.

Parameter	Value
k_rk	30
b_rk	10
q_d_rk	0
qd_d_rk	0

Table 4-26: Control parameters of right knee in state 2 after foot strike.

Right Ankle Roll: Low gains are used in the PD controller to neither allow the foot to twist nor influence the landing posture of the robot.

Parameter	Value
k_ra_roll	4
b_ra_roll	1
q_d_ra_roll	0
qd_d_ra_roll	0

Table 4-27: Control parameters of right ankle roll in state 2 after foot strike.

Right Ankle Pitch: Low gains are used in the PD controller to allow the right foot of the robot to flatten on the ground without much resistance.

Parameter	Value
k_ra_pitch	1
b_ra_pitch	0.5
q_d_ra_pitch	0
qd_d_ra_pitch	0

Table 4-28: Control parameters of right ankle pitch in state 2 after foot strike.

Opposite Toe-Off

This state is initiated when the left heel has come off the ground and the total forces on the left toe fall below a certain threshold (Figure 4-11).

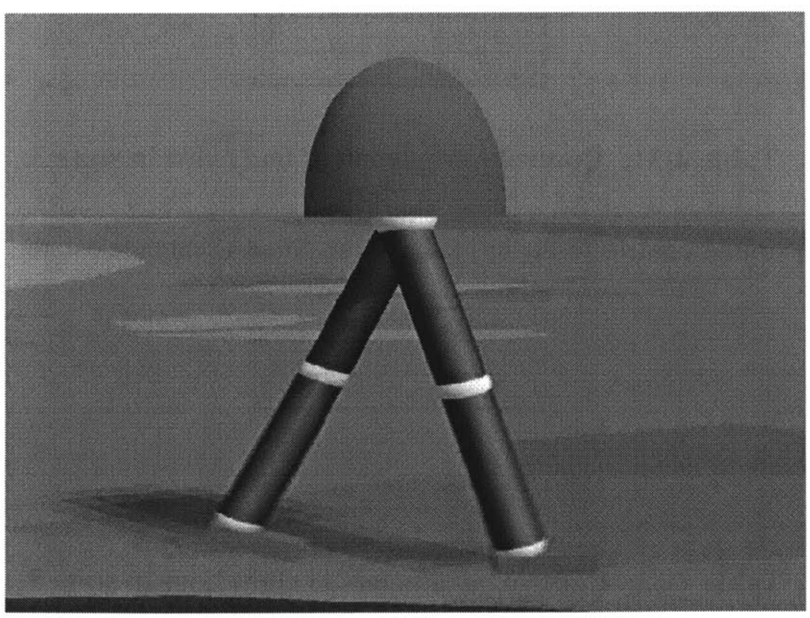

Figure 4-11: Initiation of state 3.

Body Roll: Similar control to the left hip roll in the "Foot Clearance" state.

Parameter	Value
k_roll	60
b_roll	20
grav_force_body_roll_r	75
q_d_roll	0
qd_d_roll	0

Table 4-29: Control parameters of body roll in state 3.

Body Pitch: Similar control to the left hip pitch in the "Foot Clearance" state.

Parameter	Value
k_pitch	60
b_pitch	20
q_d_pitch	-0.2
qd_d_pitch	0

Table 4-30: Control parameters of body pitch in state 3.

Body Yaw: Similar control to the left hip yaw in the "Foot Clearance" state.

Parameter	Value
k_yaw	80
b_yaw	20
q_d_yaw	0
qd_d_yaw	0

Table 4-31: Control parameters of body yaw in state 3.

Right Knee: Similar control to the left knee in the "Foot Clearance" state.

Parameter	Value
k_rk	30
b_rk	10
q_d_rk	0
qd_d_rk	0

Table 4-32: Control parameters of right knee in state 3.

Right Ankle Roll: Similar control to the left ankle roll in the "Foot Clearance" state.

Parameter	Value
k_ra_roll_cop	15
b_ank_yd	-1
k_com_y	1
frac_foot_width	-1.1

Table 4-33: Control parameters of right ankle roll in state 3.

Right Ankle Pitch: No control is used.

$tau\_ra\_pitch = 0$

Left Hip Pitch: Similar control to the right hip pitch.

Parameter	Value
k_pitch	40
b_pitch	20
q_d_pitch	-0.2
qd_d_pitch	0

Table 4-34: Control parameters of left hip pitch in state 3.

Left Knee: Similar control to the right knee.

Parameter	Value
k_lk	30
b_lk	10
q_d_lk	0
qd_d_lk	0

Table 4-35: Control parameters of left knee in state 3.

Left Ankle Roll: Similar control to the right ankle roll in the "Foot Clearance" state.

Parameter	Value
k_la_roll	10
b_la_roll	2
q_d_la_roll	0
qd_d_la_roll	0

Table 4-36: Control parameters of left ankle roll in state 3.

Left Ankle Pitch: Similar to the "Foot Strike" case when the right foot was on the ground.

Parameter	Value
k_la_pitch	30
q_d_la_pitch	0.5

Table 4-37: Control parameters of left ankle pitch in state 3.

The following four states are the exact replica of the four states mentioned above, except that the role of the left and right joints/links are reversed.

Opposite Foot Clearance

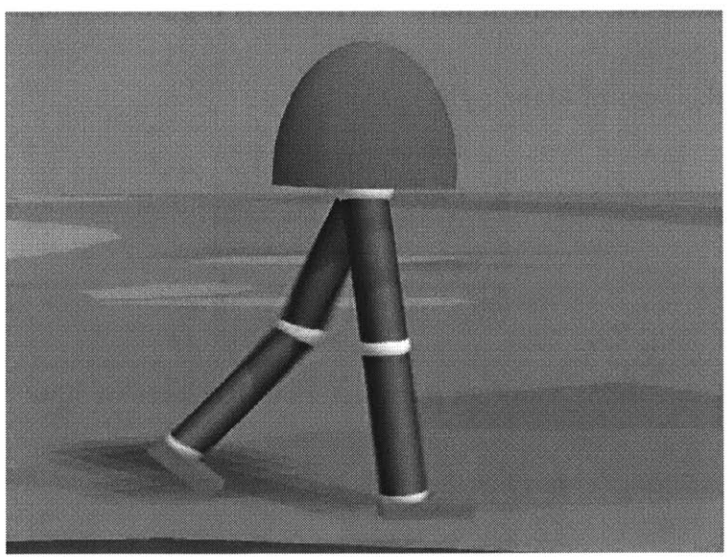

Figure 4-12: Initiation of state 4.

Opposite Tibia Vertical

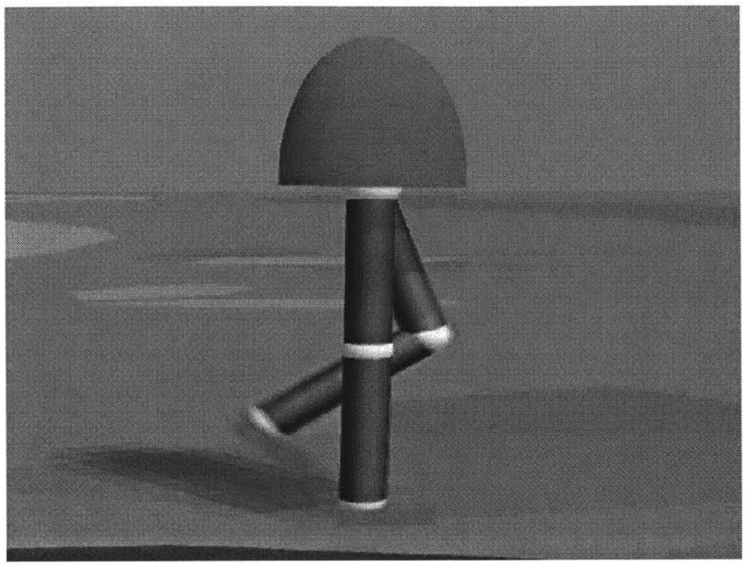

Figure 4-13: Initiation of state 5.

Opposite Foot Strike

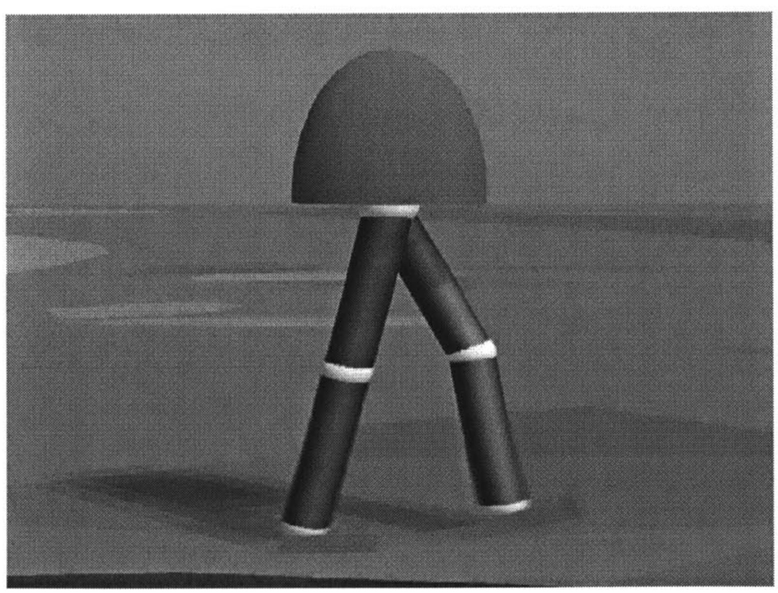

Figure 4-14: Initiation of state 6.

Toe-Off

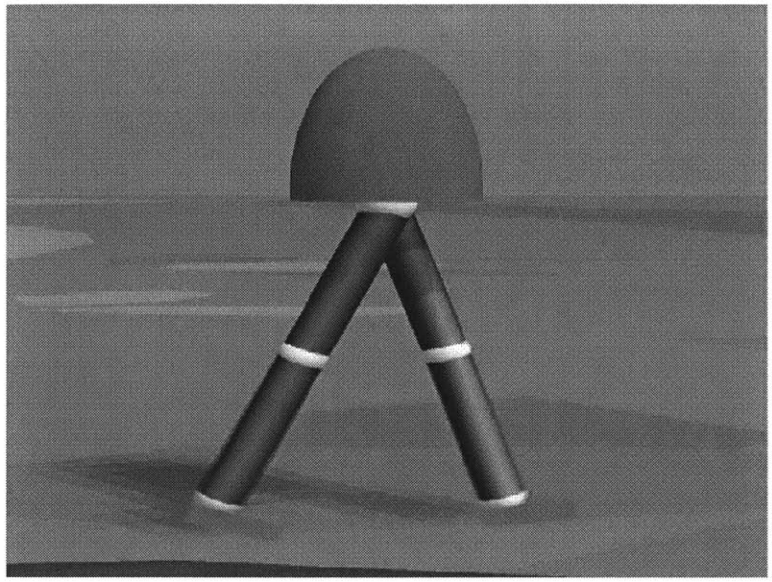

Figure 4-15: Initiation of state 7.

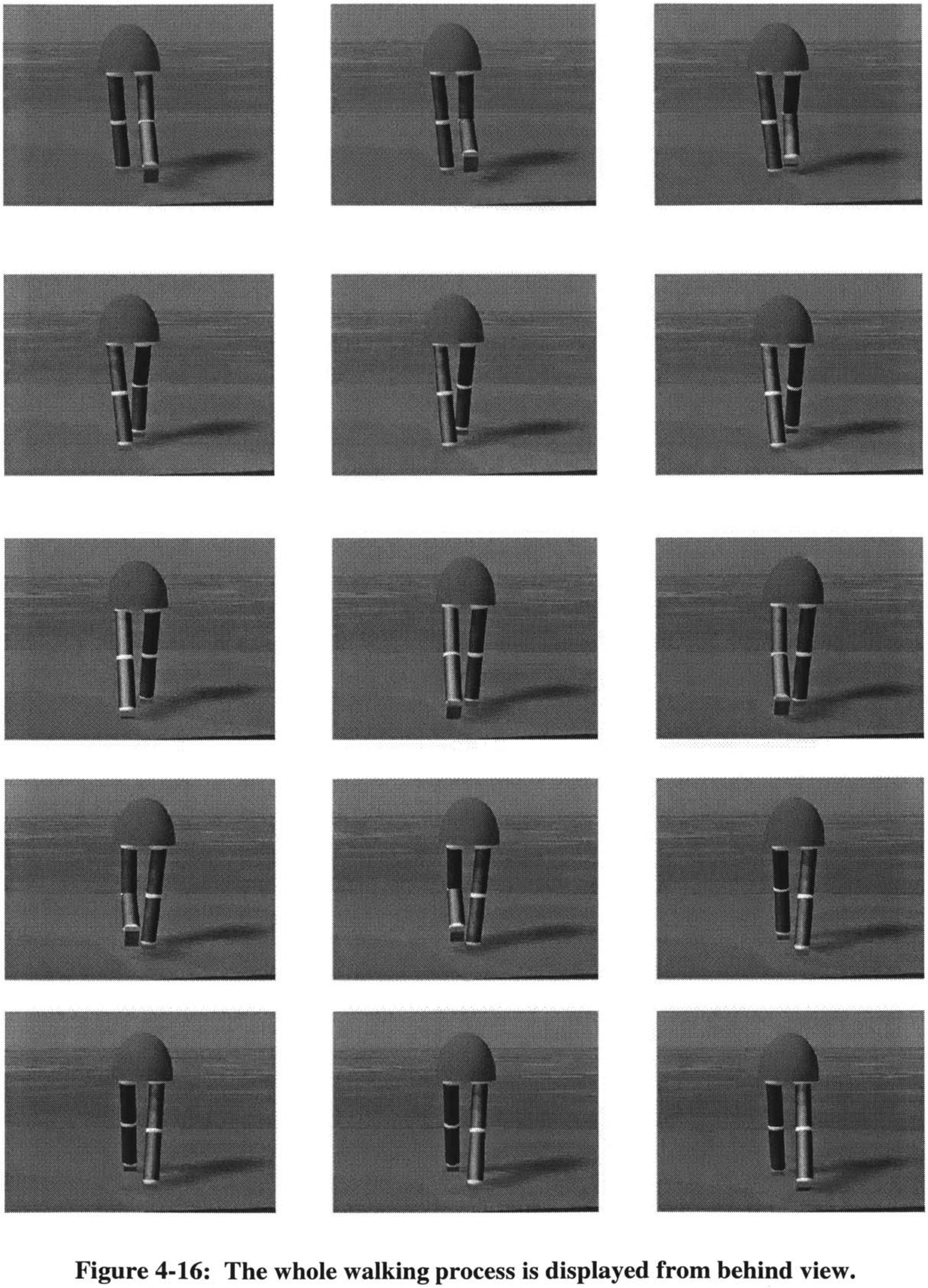

Figure 4-16: The whole walking process is displayed from behind view.

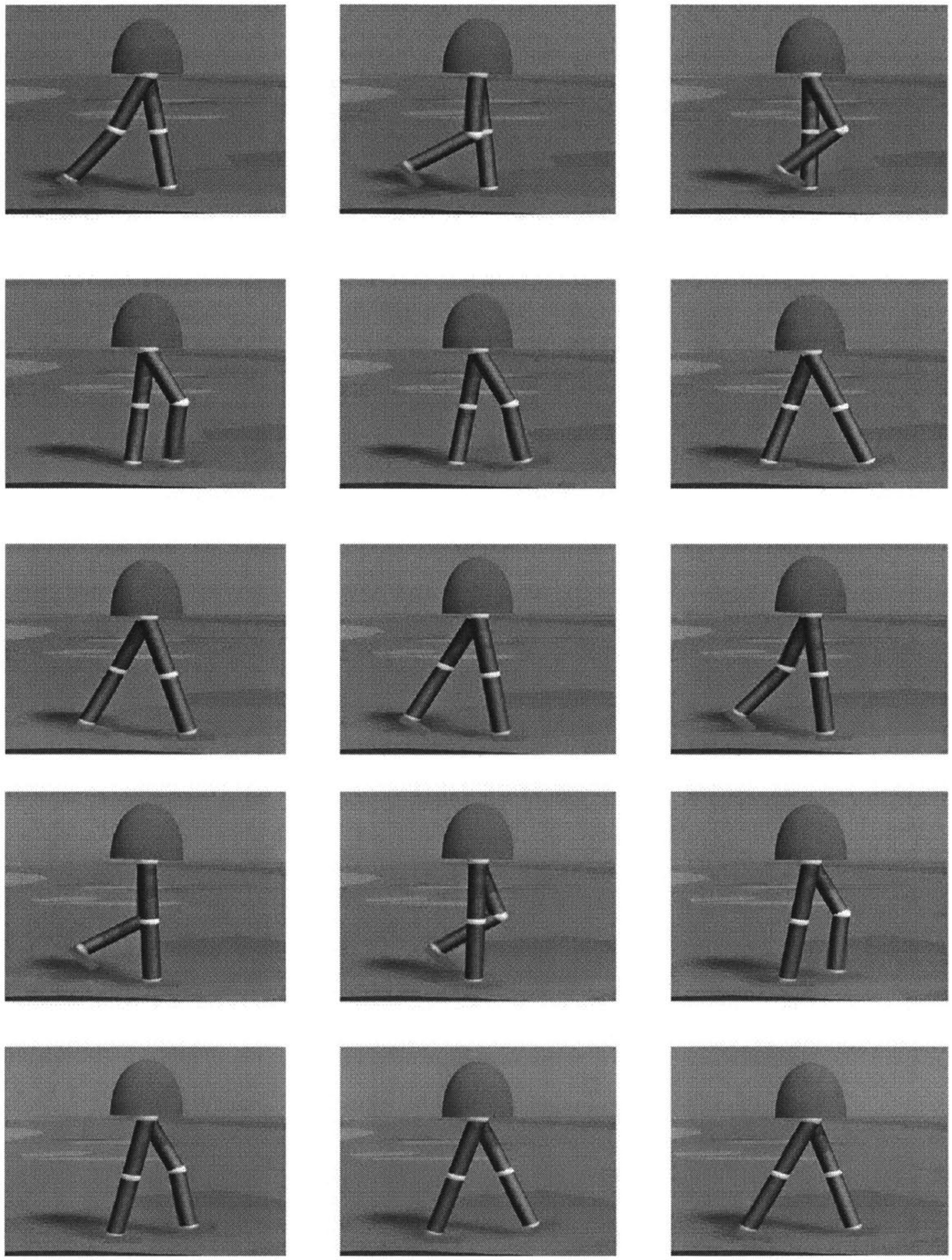

Figure 4-17: The whole walking process is displayed from side view.

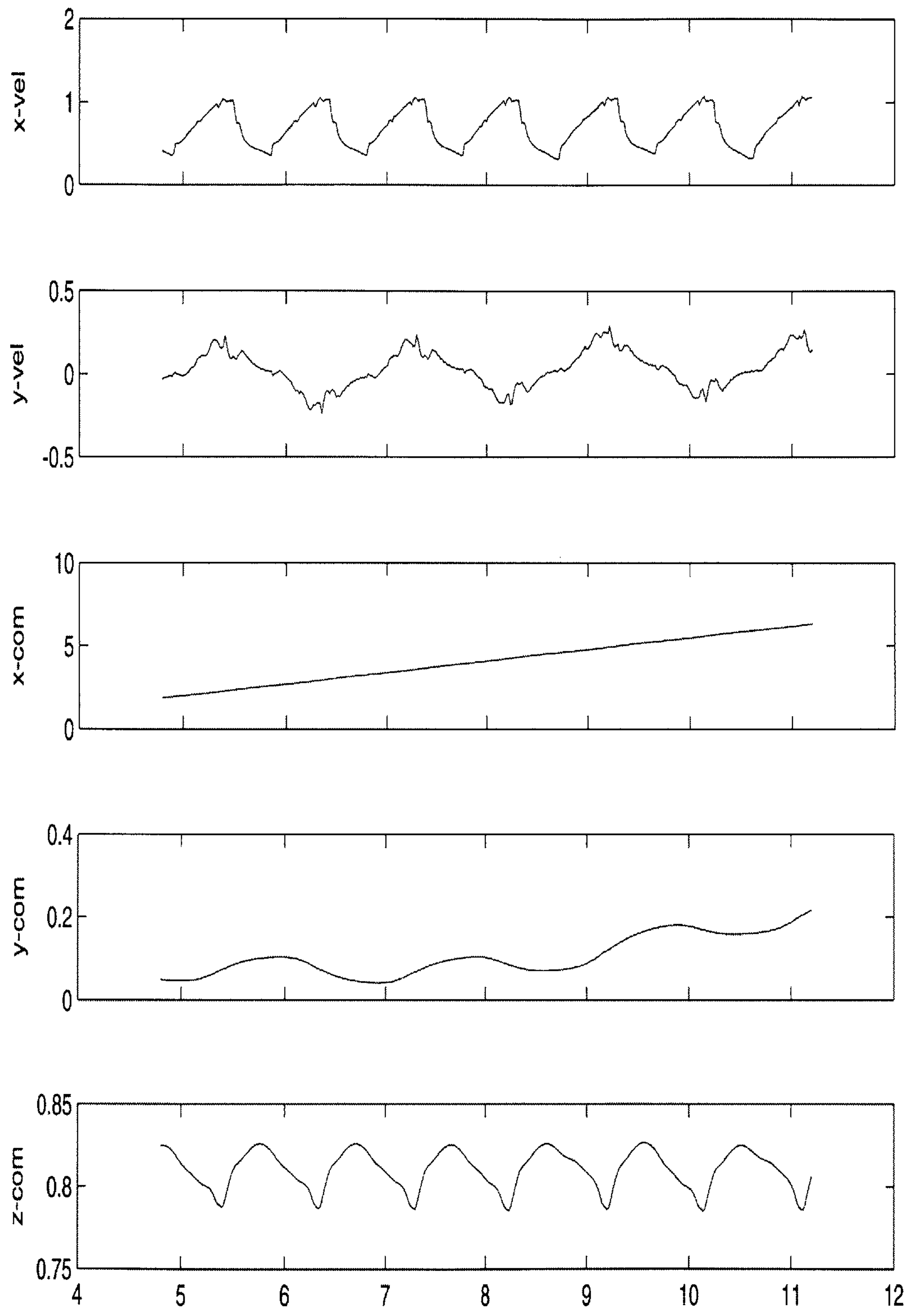

Figure 4-18: Forward/sideways velocities and position of center of mass of M2 while it is walking.

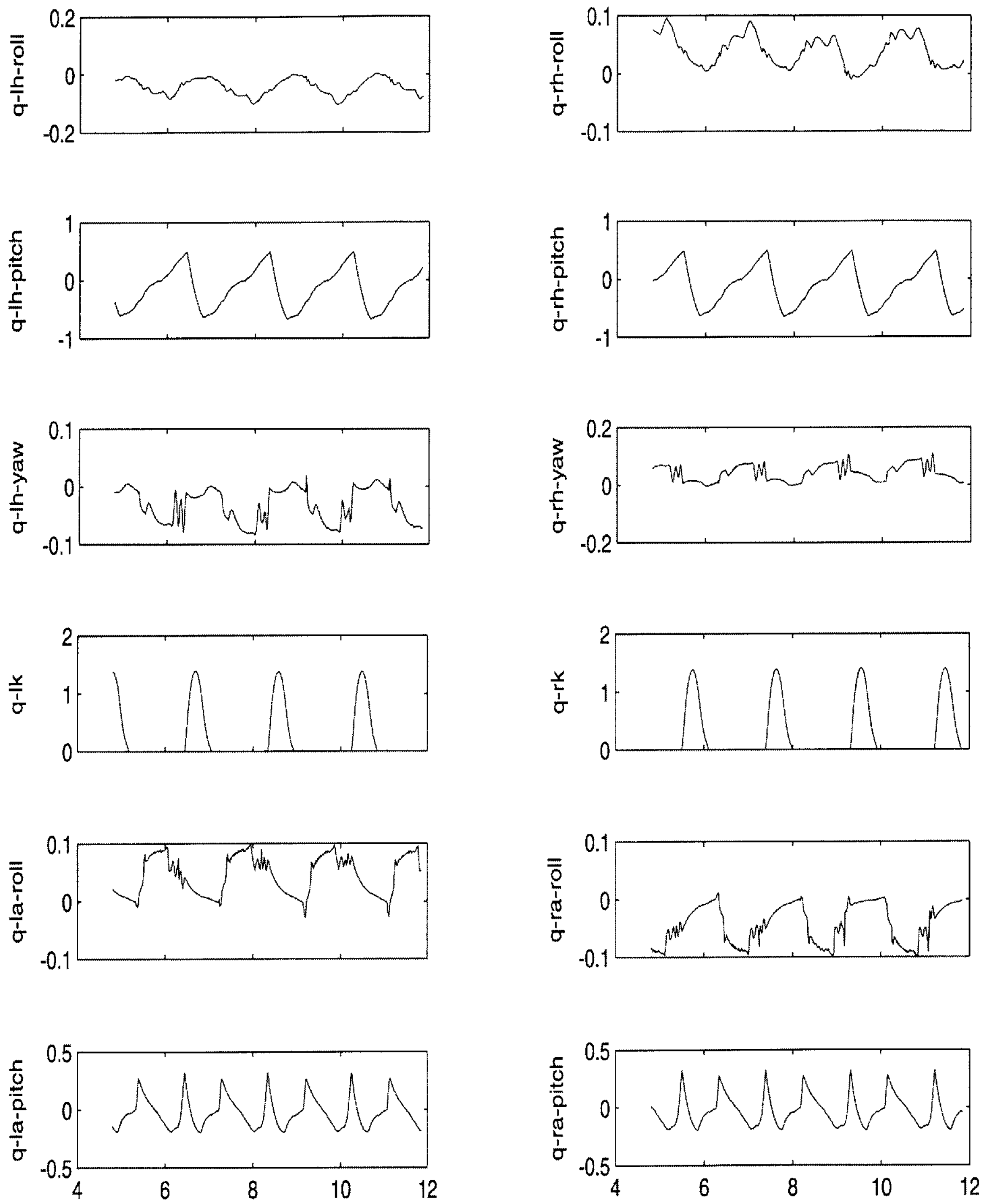

Figure 4-19: Position of M2 joints while it is walking.

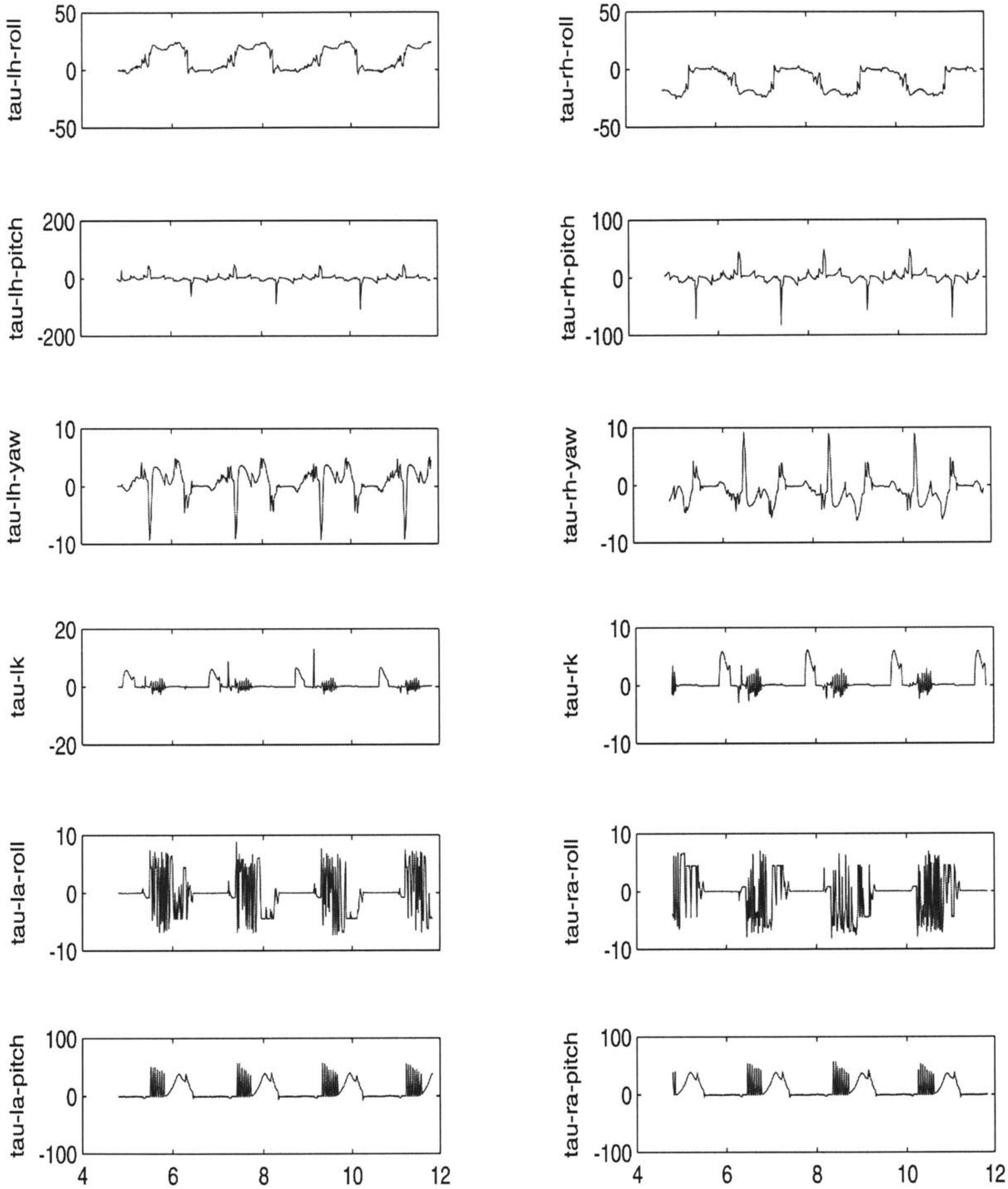

Figure 4-20: Torques at the M2 joints while it is walking.

4.2.3 Robustness

One way to measure the robustness of a biped control algorithm is to exert an external force on the robot in different directions. The following robustness tests have been applied while the robot was in the state shown in Figure 4-21:

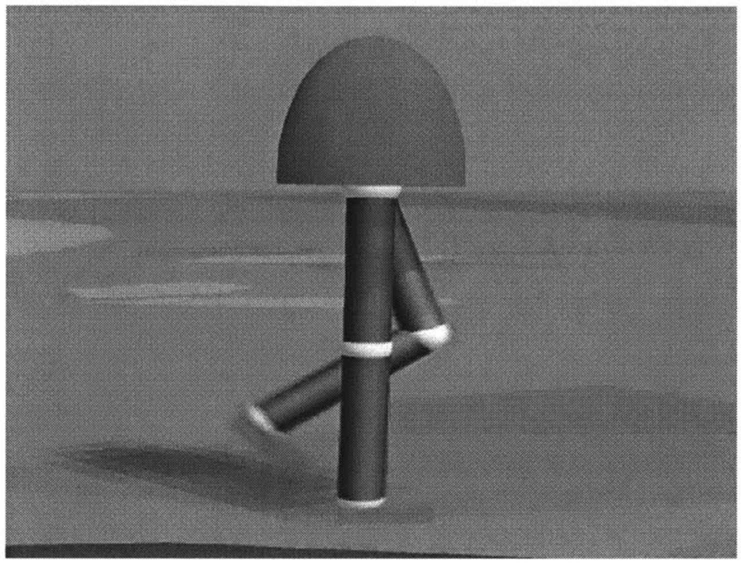

Figure 4-21: External forces were applied on the robot when it was in this configuration.

1) A Force of 25 N (5.6 lbs) in the Positive X Direction (Forward)

The robot was given a 25 N (5.6 lb) bump (for 10 ms) at the center of mass of its body from behind. This causes the forward velocity of the robot to increase (Figure 4-22), but we believe that due to the natural dynamics of the biped, it takes a faster swing, which allows the robot to recover from the bump. A force greater than 25 N would make the robot fall.

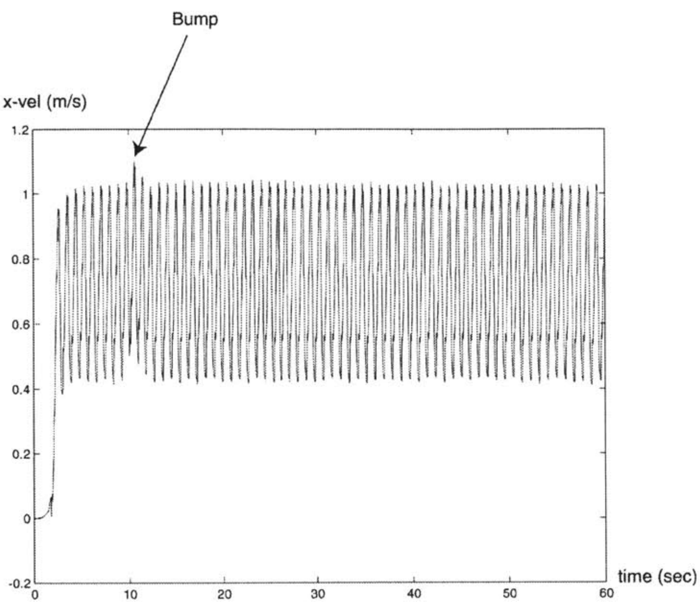

Figure 4-22: Forward velocity of the robot when an external force in the positive X direction was applied.

2) A Force of 9 N (2.03 lbs) in the Positive Y Direction (towards left)

The robot was given a 9 N (2.03 lb) bump (for 10 ms) at the center of mass of its body from right. The sideways foot placement control helps the robot to recover from this push. Figure 4-23 shows the position of the center of mass of the robot in the frontal plane (y-z) measured from a fixed point. A force greater than 9 N would make the robot fall.

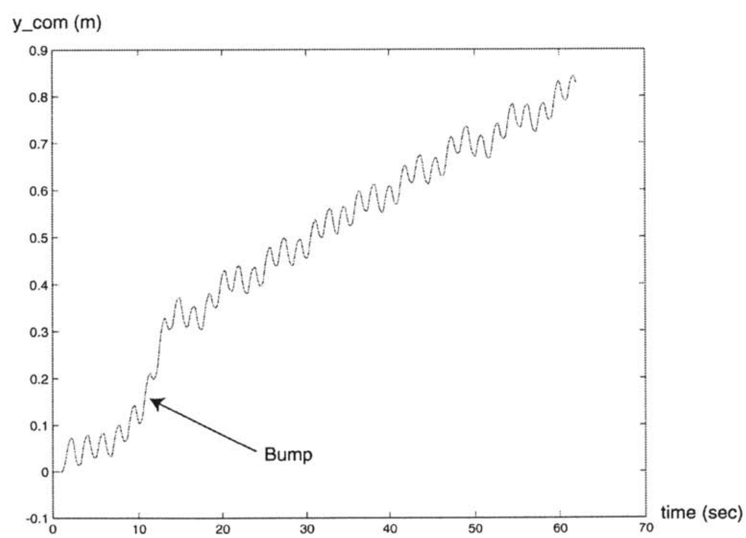

Figure 4-23: Position of the center of mass of the robot measured from a fixed point when an external force in the positive Y direction was applied.

3) A Force of 100 N (22.5 lbs) in the Negative Z Direction (downward)

The robot was given a 100 N (22.5 lb) bump (for 10 ms) at the center of mass of its body from top. Figure 4-24 shows the position of the center of mass of the robot in the frontal plane (y-z) measured from a fixed point. A force greater than 100 N would make the robot fall.

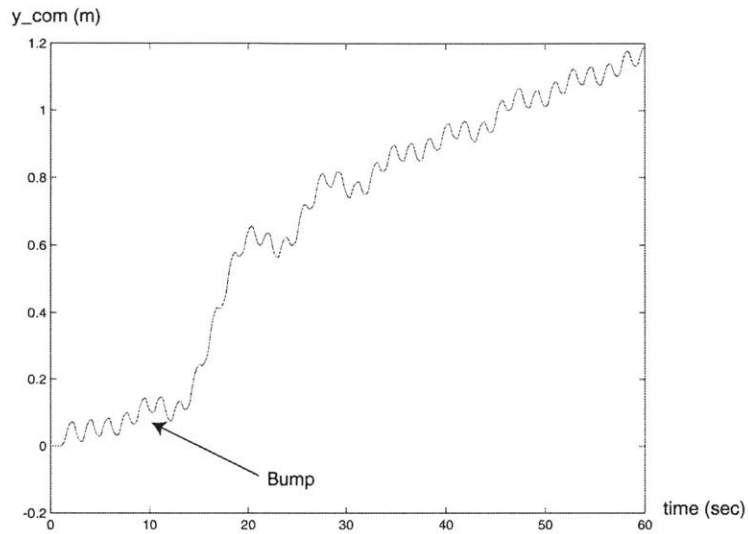

Figure 4-24: Position of the center of mass of the robot measured from a fixed point when an external force in the positive Y direction was applied.

4) %90 of All three Forces

This time %90 of each force in cases 1,2, and 3 are exerted on the robot simultaneously (for 10 ms). Figure 4-25 shows the forward velocity of the robot in the sagittal plane (x-z) and Figure 4-26 shows the position of the center of mass of the robot in the frontal plane (y-z) measured from a fixed point. If the exact same forces were applied from cases 1,2, and 3 the robot would fall due to the coupling between the joints.

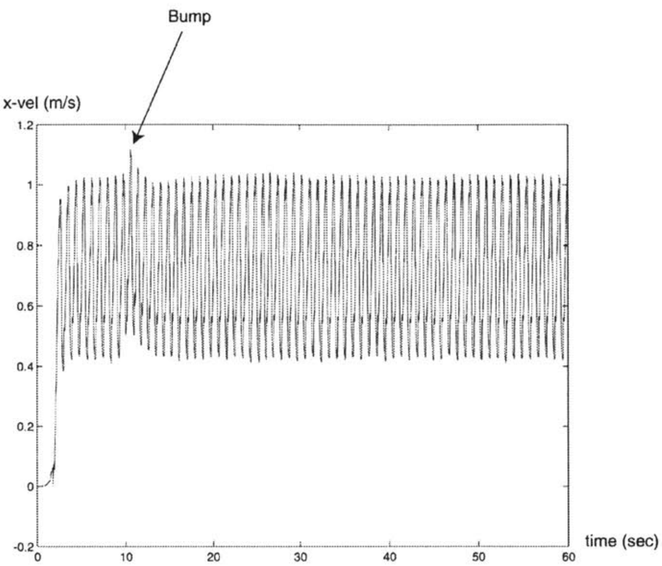

Figure 4-25: Forward velocity of the robot when an external force in the positive X, Y, and Z directions were applied.

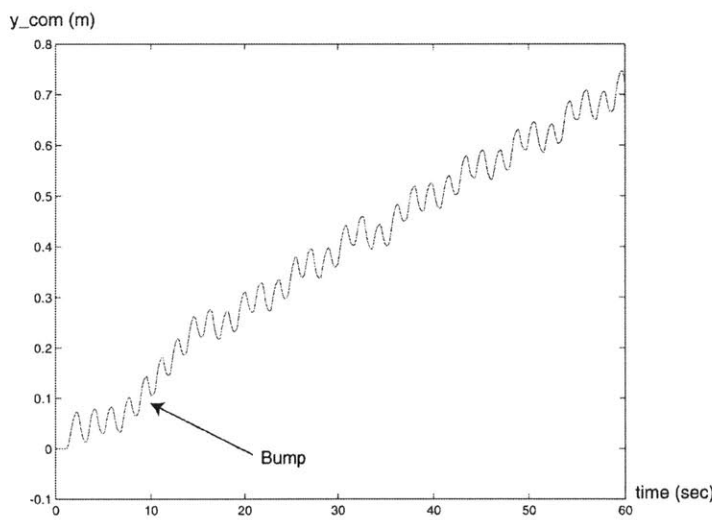

Figure 4-26: Position of the center of mass of the robot measured from a fixed point when an external force in the positive X, Y, and Z directions were applied.

4.3 Side-to-side Rocking

Side-to-side balancing of a three-dimensional bipedal robot while standing on one foot is a very challenging task, because the stability range is quite narrow. Ankle roll can only contribute to the balance as long as the center of mass of the robot is on top of its support foot. One way to help stabilize the robot side-to-side is to use the hip roll of the leg that is in the air, so that if needed the robot can kick its leg out the opposite direction of fall. Energy analysis will be done and used in control to achieve this task.

4.3.1 Energy Analysis

In the single support state, the robot is modeled as a simple inverted pendulum rotating about the ankle of the support foot. The robot's total mass is modeled as a lumped mass at its center of mass, which is connected to the ankle of the support foot as shown in Figure 4-27. When the robot flares its hips, its center of mass moves away from the support foot, and the moment of inertia of the robot increases. Assuming the robot can change its configuration instantaneously then the angular momentum will be conserved while the length and the angular momentum of the inverted pendulum increase. Suppose that initially, as shown in Figure 4-27, the inverted pendulum has moment of inertia I_i, mass m at a radius r_i, which is located at an angle θ_i with an angular velocity of ω_i subject to gravity g. Then the robot's initial total energy (kinetic and potential energies) is

(4-3-1) $$TE_i = KE_i + PE_i$$

where

(4-3-2) $$KE_i = \frac{(I_i + mr_i^2)\omega_i^2}{2}$$

(4-3-3) $$PE_i = mgr_i \sin\theta_i$$

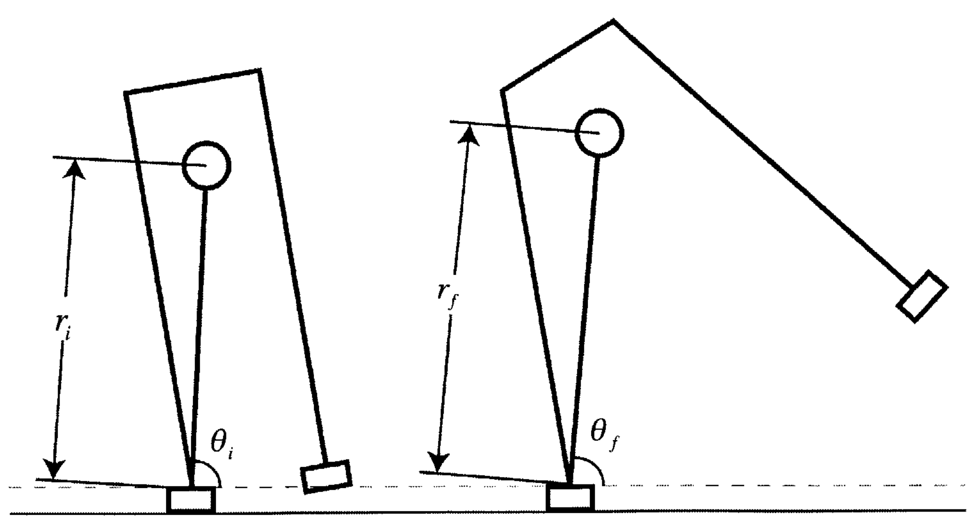

Figure 4-27: M2 kicking its leg out in order to balance.

where $I_i + mr_i^2$ is the moment of inertia of the center of mass of the robot about the ankle of the support foot.

For the final configuration shown in Figure 4-27, the total energy is

(4-3-4) $$TE_f = KE_f + PE_f$$

where

(4-3-5) $$KE_f = \frac{(I_f + mr_f^2)\omega_f^2}{2}$$

(4-3-6) $$PE_f = mgr_2 \sin\theta_f$$

Conservation of momentum through configuration change implies

(4-3-7) $$H_i = H_f$$

(4-3-8) $$(I_i + mr_i^2)\omega_i = (I_f + mr_f^2)\omega_f$$

solving for ω_f will give us

(4-3-9) $$\omega_f = \frac{(I_i + mr_i^2)\omega_i}{(I_f + mr_f^2)}$$

substituting (4-3-9) into (4-3-5) will result in

(4-3-10) $$KE_f = \frac{(I_i + mr_i^2)^2\omega_i^2}{2(I_f + mr_f^2)}$$

When the robot raises its leg, r_f becomes greater than r_i which implies that I_f becomes greater than I_i and as a result of this, as it can be seen in Equation (4-3-10), KE_f decreases. Therefore, raising a leg can decrease the robot's sideways velocity, which implies that the robot can prevent itself from falling.

Now consider the inverted pendulum shown in Figure 4-28. The kinetic and potential energies of this configuration respectively are

(4-3-11) $$KE_i = \frac{(I_i + ml^2)\omega^2}{2}$$

(4-3-12) $$PE_i = mgl\sin\theta$$

Now suppose that the inverted pendulum is rotating about a pin joint O in the counter clockwise direction and reaches its apex as shown in Figure 4-28. The kinetic and potential energies of the inverted pendulum when it is at its highest point is

(4-3-13) $$KE_f = \frac{(I_f + ml^2)\omega_f^2}{2}$$

(4-3-14) $$PE_f = mgl$$

Assuming there are no energy losses in the system, we can use the equation for conservation of energy to obtain

(4-3-15) $$TE_i = TE_f$$

(4-3-16) $$KE_i + PE_i = KE_f + PE_f$$

(4-3-17) $$KE_f = KE_i + PE_i - PE_f$$

We want $$KE_f \le 0 \Rightarrow KE_i + PE_i - PE_f \le 0 \Rightarrow$$

(4-3-18) $$KE_i \le PE_f - PE_i$$

Where KE_i, PE_i, and PE_f are as shown in Equations (4-3-11), (4-3-12), and (4-3-14), respectively.

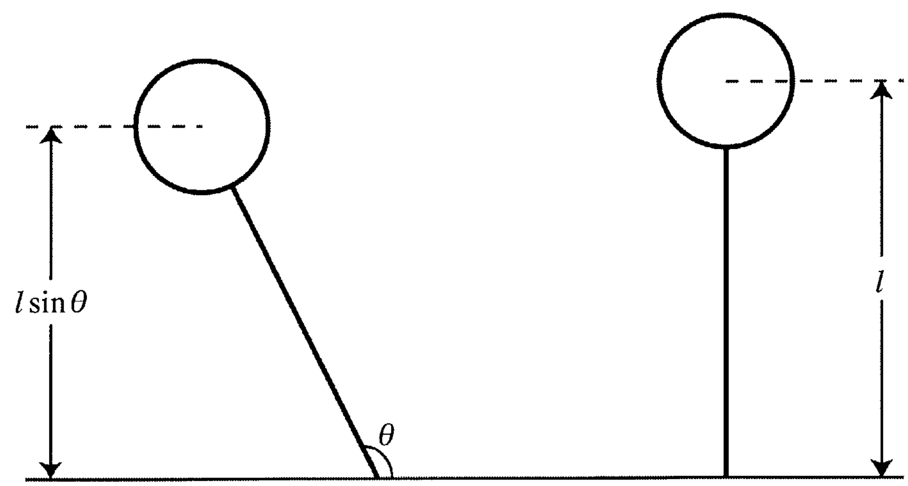

Figure 4-28: M2 is modeled as an inverted pendulum, which can rotate about its support ankle.

4.3.2 Simulation Algorithm

A finite state machine, comprising six states, is used for side-to-side rocking algorithm as shown in Figure 4-29.

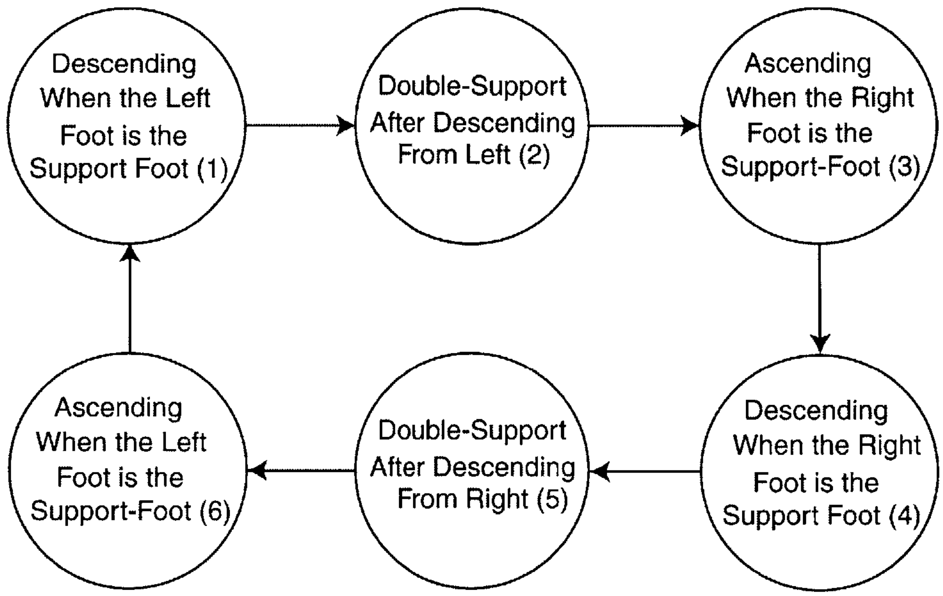

Figure 4-29: Finite State Machine with six states is used to achieve the side-to-side rocking motion.

When the robot is in state 1 (single support and descending as shown in Figure 4-30), there is no torque at the left ankle roll joint. The right leg, which is in the air, is controlled such that it remains aligned with the body. The position of the body is controlled using the left hip roll joint such that it is aligned with the left leg, which is the support leg. The torque that compensates for the torque generated by the gravitational force on the body is applied at the left hip roll too. As soon as robot's right foot hits the round, the robot goes into double-support state (state 2 shown in Figure 4- 31). In this state, the torque at the right ankle roll joint shuts down allowing the robot to rotate about that particular joint freely. As soon as most of the weight of the robot is transferred onto the right foot (the support foot), the left ankle pitch and the right hip roll joints are servoed so that the robot pushes against the ground with its left foot and the body of the robot slightly rotates in the clockwise direction. These two actions will help the robot go into state3 (ascending while the right foot is the support foot).

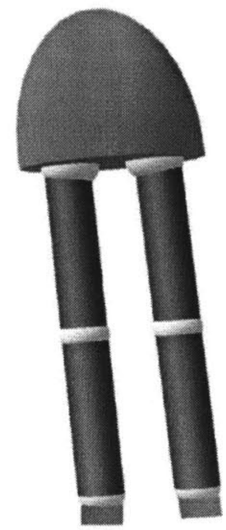

Figure 4-30: M2 is descending when the left foot is the support foot.

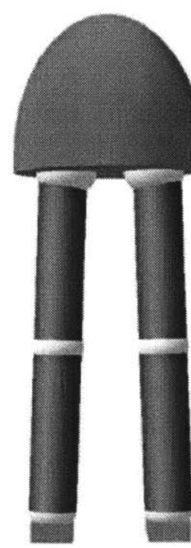

Figure 4- 31: M2 is in double-support state after descending from left.

In state 3, first, Inequality (4-3-18) is checked. If this inequality holds, this state will be similar to state 1 except that the role of the right leg and the left leg will be reversed, otherwise the robot will raise its left leg (Figure 4-32) by servoing the left hip roll joint to a desired angle which is proportional to the kinetic energy of the robot. States 4, 5, and 6 are same as 1, 2, and 3, respectively except that the role of right leg and left leg is reversed. Throughout this simulation, the robot uses its ankle pitch joints to control the position of its center of mass in the sagittal plane to maintain its sagittal balance.

Figure 4-32: M2 is ascending when the right foot is the support foot.

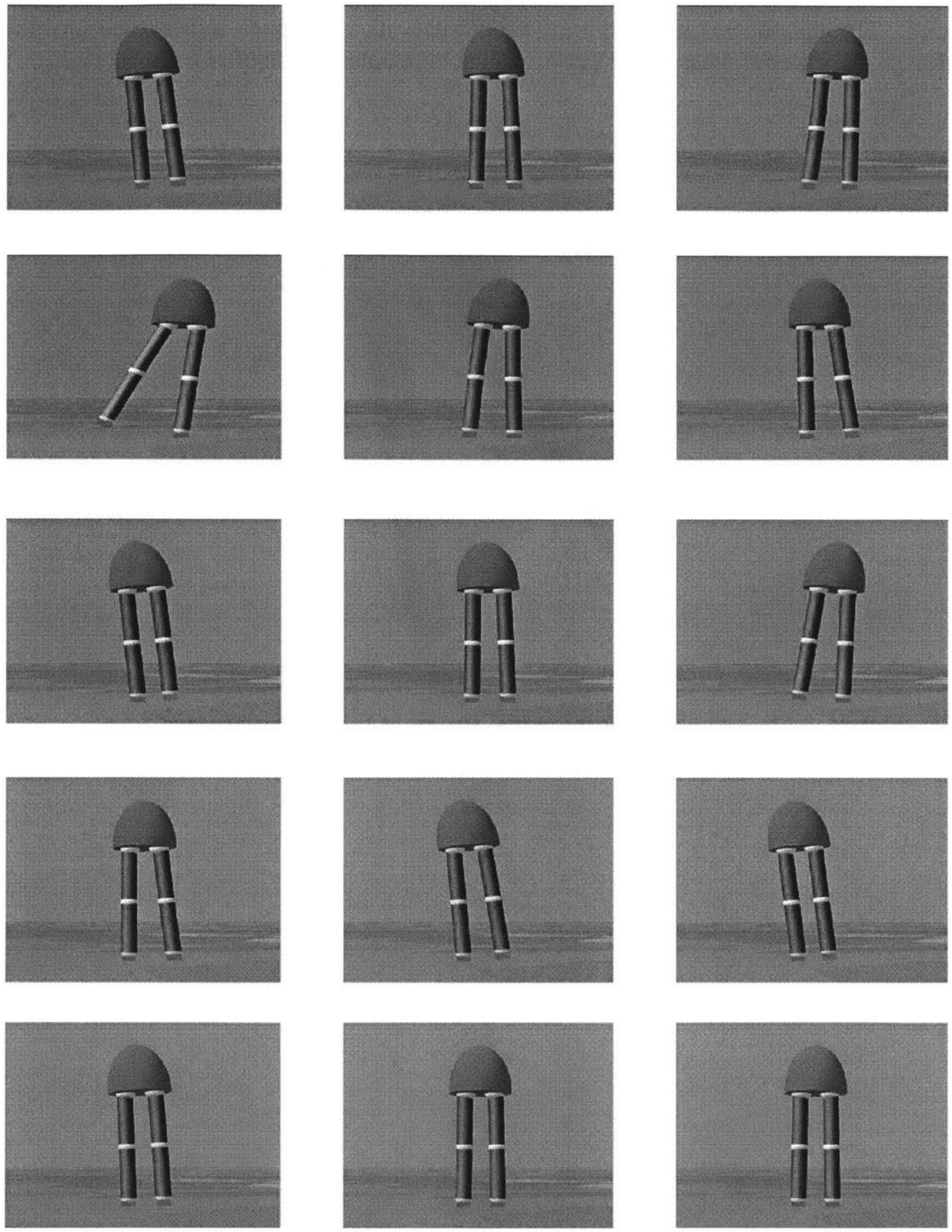

Figure 4-33: The whole side-to-side rocking process is displayed.

4.4 Balancing

4.4.1 On One Leg

Balancing on one leg is performed using control of the support ankle only. The goal of this approach is to show that stability can be achieved by using only ankle torque. The body of the robot is controlled such that it maintains a desired angle with respect to the ground using a simple PD controller and a counter gravitational torque as feed forward (as shown in the walking algorithm). The right leg is controlled to maintain a desired angle such that it sticks out from the body.

Ankle pitch is servoed to control the position of the center of mass of the whole robot in the sagittal plane such that it falls a few centimeters in front with respect to the support ankle. Similarly, ankle roll is servoed to control the position of the center of mass of the robot in the frontal plane such that it falls right on top of the support ankle.

The yaw and the knee joints are servoed such that it is ensured they are locked hard enough.

Figure 4-34: M2 balancing on one foot.

Initially the robot is in a position as shown in Figure 4-34. Once it is let go, the control described above achieves balance on one leg.

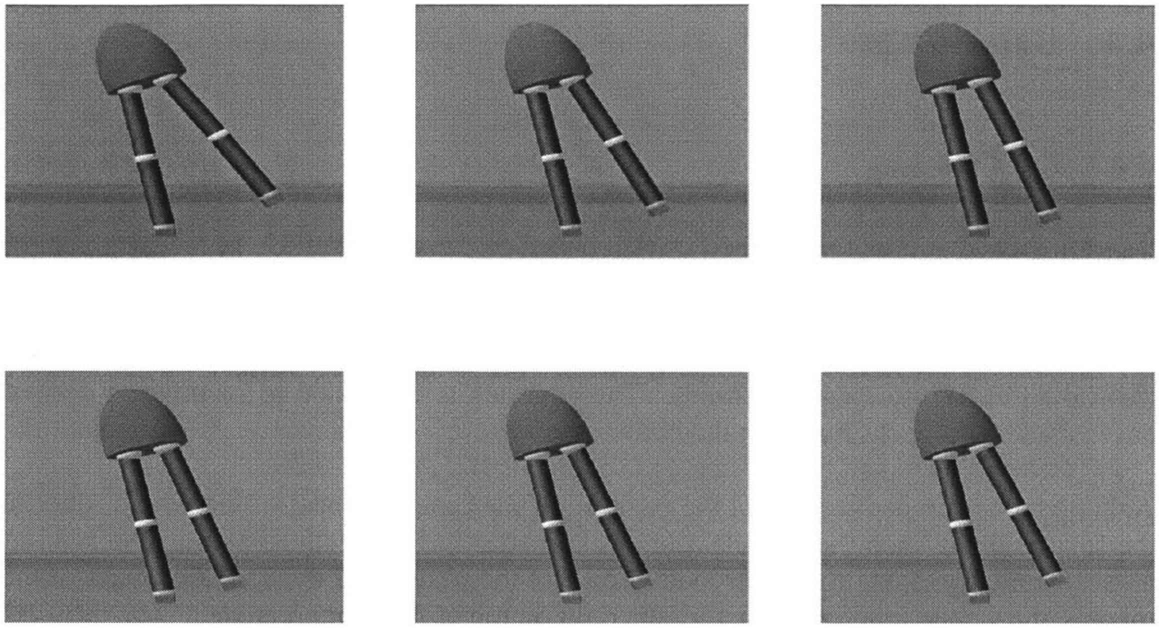

Figure 4-35: The whole balancing process is displayed.

4.4.2 Standing

Standing is similar to balancing on one leg with the exception of ankle roll and hip roll. No ankle roll torque is used. Instead, hip roll torque is used as a switch. Every time the center of mass of the robot passes the point in the middle of the feet towards a certain direction, the appropriate hip is activated to resist the motion. For example, if the robot is pushed to the left, the left hip roll control is activated to push the position of the center of mass of the body in the sagittal plane back to the middle.

Chapter 5

Conclusions and Future Work

The following conclusions can be drawn from this project:

1) Certain Complicated systems such as three-dimensional bipeds do not necessarily need to have complicated controls systems in order to accomplish the desired task.

2) Passive dynamics can be used in order to reduce the complexity of the control system.

3) Roll stability can be achieved by controlling the position of the center of mass and center of pressure using ankle roll.

4) A strong foot placement controller can make the walking robust by a considerable amount.

5) Derivations of dynamics equations are not necessary to compel a robot to walk; most joints can be treated as decoupled.

The control algorithms described in this thesis are currently being tested on the actual hardware of the robot and the preliminary results have been promising. The major problem we are currently dealing with is the noisiness of the velocity signals, which limits us to use low damping parameters, which makes the roll control more challenging. A powerful filter can be quite helpful. Robustness measurement of a biped control algorithm can be carried out in more extensive ways such as applying different forces in different directions at different times while the robot is in different states.

The final goal is to compel the robot to not only walk in a straight line but also in a circle or get it to turn, which would require a more robust controller for foot placement along with hip yaw. Currently the hardware of M2 is standing successfully and has completed its walking initiation.

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