Wavelength Switching and Routing through Evanescently Induced Absorption

by

Michael Robert Watts

Submitted to the Department of Electrical Engineering and Computer Science in partial fulfillment of the requirements for the degree of

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Abstract

High index contrast optical micro-ring resonators have previously been proposed for large scale integrated optical circuits. Additionally, absorption has been shown to be a useful mechanism for switching ring resonators. In this thesis, we propose Add / Drop and Cross-Connect architectures utilizing ring resonator switching elements. The performance requirements of the elements necessary to build Add / Drop and Cross-Connect switches are considered in detail.

The on-state requirements of the switching elements are met by a high index contrast silicon-core silica-clad series coupled ring resonator pair. To avoid multimode excitation and the resultant multi-peaked filter responses commonly associated with ring resonators, the resonator guides are designed with effectively single-mode single-polarization waveguides. A flat filter passband, a steep roll-off, and a 10 Terahertz free spectral range are obtained, therein lending the design to large channel count Wavelength Division Multiplexed communication (WDM) systems.

The electro-absorptive techniques considered in prior works, are shown to provide insufficient absorption for large scale switching applications. With the aim of providing greater absorption, and therein truly shutting down the resonator, we consider the alternate approach of using evanescent interaction with the resonator to selectively induce absorption into the otherwise transparent resonator guide. The complex refractive index, thickness, and separation of an absorbing membrane are all considered as open variables in the design. Through both perturbational analysis and the Finite Difference Time Domain (FDTD) technique, we show that at a separation of less than 0.1μm from the resonator waveguide, a thin (0.2μm) absorbing membrane with a refractive index similar to that of indium arsenide provides an order of magnitude more absorption than electro-absorptive based techniques. As a result of the increased absorption in the resonator guide, the interaction between the resonator and the bus waveguides is fully destroyed. The signal passes by the resonator losing only 0.15dB of its power, only 0.05dB greater than the theoretical minimum. And, at an absorber-resonator separation of only 0.8μm, the resonator recovers to its original unperturbed state. Importantly, the small membrane size, thickness and required range of motion, enable the use of micromechanical actuation. A preliminary micromechanical design is presented.

On a more general note, a novel design for a polarization rotator is developed and the operation verified through both coupling of modes analysis and the finite difference time domain technique. The polarization rotator is shown to be both low loss and broadband,
and forms an integral component of our approach for handling the inherently polarization sensitive high index contrast waveguides used in our chip architecture.

Thesis Supervisor: William P. Kelleher
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Chapter 1

Introduction

Low loss optical fiber was first demonstrated in the late 1970’s [1]. Since then researchers have worked to improve the transmission characteristics of the fiber and to develop the necessary components for making very high bandwidth transmission possible. Initial systems carried only a single signal per fiber and required the use of electrical regenerators to restore the signal fidelity. However, with only a single optical signal, the bandwidth of such systems was limited to the electrical bandwidths of the signal generation and detection systems. Multiple optical signals of differing carrier frequencies could be transmitted along a single fiber (i.e. Wavelength Division Multiplexing (WDM)) to improve fiber bandwidth utilization, however, the signals would have to be separated, detected, and regenerated independently at each regeneration site. A regeneration site required as many lasers, modulators and detectors as there were signals on the fiber. With regeneration sites spaced every 25km the cost of implementing WDM systems was prohibative. A pivotal change in the development of optical communication systems came about in the late 1980s with the introduction of the erbium amplifier [2]. The electrical regenerators of the previous era could now be replaced by a broadband optical amplifier and signals of differing carrier frequencies could now be amplified in parallel, without the need for individual detection and regeneration components. Spurred by the advent of the optical amplifier, WDM or dense wavelength division multiplexing (DWDM) for closely spaced signal channels, became the norm in the industry. WDM has been so successful that it has resulted in the demonstration of Terabit long distance transmission systems [3], roughly enough bandwidth to carry all the voice and data traffic in the US on a single fiber at the time of this publication. The
general implementation of a WDM system is depicted in Figure 1-1.

![Diagram of WDM system](image)

**Figure 1-1: Wavelength Division Multiplexed (WDM) optical transmission line.**

Of course, one fiber can hardly direct all the signals to their appropriate destinations. So, signals from multiple fibers are connected at various intersection points around the world referred to as nodes. Currently, at the nodes, the signals on the fibers are first demultiplexed, detected, switched electronically, regenerated, and then multiplexed back onto an outgoing fiber. Here again, the required number of lasers, modulators, and detectors scales with the number of signals incident on the node. It appears that a striking similarity exists between the electronic regeneration sites of the past and the electronic switching and routing nodes of the present. In a similar fashion, an all optical solution, capable of routing the incident signals without first converting them to electrical form, holds great promise for reducing the cost of many of these nodes (Figure 1-2). Such a device, termed a Cross-Connect switch has the ability to direct any input signal to any output fiber. It is important to note that such a Cross-Connect has a slightly different functionality than its electronic counterpart, the electronic router. Electronic routers have the capability of using Time Division Multiplexing (TDM), and thus switch data on a packet by packet basis to make maximal use the available bandwidth. In contrast, on account of the lack of a scalable optical memory and often slow switching times, optical switches cannot route individual data packets, but must provision whole wavelength channels at a time. Although at present this is an advantage that electronic routers have over optical cross-connects, the general trend toward greater bandwidth and resultant larger packet size minimizes the benefits of the increased switching granularity offered by the electronic router.

The application of all optical switches is not limited to the large scale cross-connect application. Considerable evidence suggests that whole wavelength provisioning may be
Figure 1-2: Cross-connect switching node with $M$ input and output fibers each carrying $N$ wavelength channels.

An ability to pull an incoming signal off of a fiber while placing an outgoing signal onto the same fiber is an essential element of such systems. The added functionality of being able to actively provision the available bandwidth would be extremely beneficial. However, here again, to do so with electronic switches would require the detection and regeneration of all the signals on the fiber, a greatly prohibative task. An optical switch on the other hand could selectively pull off a single wavelength without affecting the other channels on the fiber. This type of switch is known as an optical Add/Drop switch (Figure 1-3).

Figure 1-3: Add/Drop Switch

As optical networks expand into the metro and local area networks, the density of nodes will inevitably increase and optical Add/Drop and Cross-Connect switches will become increasingly important. Anticipating the demand, commercial vendors have begun selling optical switches. Examples include the Lucent’s Wavestar™ LambdaRouter [5] and Agilent’s Photonic Switching Platform [6]. However, these along with most other switches being developed, are pure space switches with no wavelength selectivity. That is, the wavelength channels on the fiber must first be separated before being redirected by the switching fabric.
After exiting the switching fabric, the signals must then be multiplexed at the output before entering the outgoing fiber.

A more useful switch is what is commonly referred to as a wavelength switch, a switch capable of redirecting a single wavelength channel without affecting the other channels on the guide. B. E. Little et al. [7] proposed using absorption to selectively kill the resonance of an integrated optic ring resonator based Add/Drop filter and therein form a wavelength selective switch. However, as we will demonstrate, the absorption offered by electro-absorptive techniques, cited as possible mechanisms for introducing the absorption, provide insufficient absorption for all but small scale applications. To induce greater absorption we consider the use of evanescent interaction with the resonator field using a micromechanically actuated absorbing membrane. However, in order to develop system requirements, we first present architectures for both Add/Drop and Cross-Connect switches utilizing ring resonator switching elements, and consider the necessary performance specifications for both the stand alone resonator and the resonator based switch. Further, we develop a detailed resonator design and use it as a basis for designing the micromechanical based switch. Additionally, we present a method for handling the large inherent polarization sensitivity of the high index contrast waveguides required for ring resonators WDM filters. Included within this discussion, is, to our knowledge, a unique design for an integrated optic polarization rotator required to implement our approach for handling the polarization sensitivity. Finally, in order to verify the design of the polarization rotator, we develop a Finite Difference Time Domain (FDTD) code utilizing a sliding computational domain.
Chapter 2

Wavelength Switching and Routing with Ring Resonators

In this chapter we review the operational characteristics of ring resonators and develop the requirements on the resonator performance for Add/Drop and Cross-Connect switches used in wavelength division multiplexed communication systems.

2.1 Ring Resonators

In order to develop the intuition necessary to properly design resonators, tune their characteristics, and develop Add/Drop and Cross-Connect chip architectures, we must first consider some resonator fundamentals. Resonators are useful in wavelength division multiplexed (WDM) systems because they allow for the ability to interchange a single wavelength channel from two otherwise mutually orthogonal spatial modes. Moreover, resonators offer the added functionality of switching through a simple introduction of absorption in the resonant cavity [7]. Ring resonators are an attractive resonator design because they may offer the finest linewidths and lowest loss for a given device size. These properties have led to the suggestion that ring resonators may form the fundamental building blocks for very large scale integrated (VLSI) optics [8]. Ring resonators also possess the interesting property of providing complete power transfer on resonance between a pair of side coupled bus waveguides, a feat not possible with standing wave resonators.

Conceptually, a ring resonator is formed by bending a dielectric waveguide to close upon itself, and subsequently bringing a pair of bus waveguides into proximity with the guide that
now forms a ring (Figure 2-1). In general, the ports of the resonator are classified as input, throughput and drop. However, since the resonator may be excited by waves emanating from any of the available ports, such designations are relative and can be explained in the following manner. For a given input port, the corresponding throughput port is located at the far end of the input port bus waveguide while the drop port is located at the near end of the adjacent bus waveguide. In general, the drop port field represents the field to be pulled off the bus waveguide whereas the field at the throughput port is intended to be left largely unaffected by the presence of the resonator. The excitation of the resonator occurs through

\[
\begin{align*}
\text{Drop} & \quad \lambda_1 \\
\lambda_1, \lambda_2, \ldots \lambda_M & \quad \lambda_2, \lambda_3, \ldots \lambda_M
\end{align*}
\]

Figure 2-1: Ring resonator of resonant wavelength \( \lambda_1 \)

the generation of a polarization in the ring by the evanescent field in the bus waveguide. As the field generated in the ring propagates it deteriorates as a result of absorption and scattering in the ring, and coupling losses to the output waveguide. Conversely, with each pass by the input waveguide, the resonant field interferes constructively in the resonator waveguide with the induced polarization, thereby growing in amplitude. The resonant field in the resonator continues to grow until an equilibrium is reached whereby the power loss through absorption, scattering, and external coupling to the output waveguide is equal to the power gain from coupling to the input waveguide. If there exists equal coupling to the input and output waveguides and there are no internal losses in the resonator, the power transfer on resonance will be complete. However, in high \( Q \) resonators where there is necessarily significant travel in the resonator, even small losses in the resonator guide lead to significant losses in the drop field.

2.1.1 Coupling of Modes in Time

Although exact solutions for resonators can in general be obtained by taking an infinite summation over the reflections incurred at the resonator boundaries, we opt for the lumped oscillator approach of Haus [9] and Little et al. [10]. This approach neglects field envelope
variations within the resonator and is therefore only valid in the limit of large $Q$ (i.e. $Q >> 1$), however, it enables greater physical insight than the exact approach while providing highly accurate results for the vast majority of the computations we will encounter.

The resonator can be considered from both power and energy pictures. However, because the energy picture is foreign to most, it is instructive to relate the two. To begin, consider a field amplitude in the resonator $A(t)$ normalized such that $|A(t)|^2$ represents the time averaged power passing through any cross section of the resonator waveguide at time $t$. The total energy in the resonator $|a(t)|^2$ is then related to the power flow through the group velocity $v_g$ and the ring radius $R$.

\[ |a(t)|^2 = |A(t)|^2 \frac{2\pi R}{v_g} \tag{2.1} \]

Over time the energy in the resonator will decay due to both internal losses and external coupling. The time constant for the combined losses $\tau$ is then related to the individual losses through

\[ \frac{1}{\tau} = \frac{1}{\tau_e} + \frac{1}{\tau_d} + \frac{1}{\tau_l} \tag{2.2} \]

where $\tau_e$ is the decay due to the coupling to the input waveguide, $\tau_d$ is the decay due to the coupling to the output waveguide, and $\tau_l$ is the decay due to scattering, absorption, and radiation losses in the resonator. In addition to the losses imposed by external coupling, the energy amplitude grows by an amount $-j\mu s_i$ where $\mu$ is the energy coupling coefficient to the input guide and $s_i$ is the amplitude of the incident field. From coupling of modes in time, if the losses and gain are small, the energy amplitude evolves according to

\[ \frac{d}{dt} a = \left(j\omega_0 - \frac{1}{\tau} \right) a - j\mu s_i \tag{2.3} \]

A similar equation can be arrived at in the power picture

\[ \frac{d}{dt} A = \left(j\omega_0 - \frac{1}{\tau} \right) A - j\frac{v_g}{2\pi R} \kappa s_i \tag{2.4} \]

where we have replaced $\sqrt{\frac{v_g}{2\pi R}\mu}$ by $\frac{v_g}{2\pi R}\kappa$ where $\kappa$ is the field coupling coefficient. For the case where there is no loss, no detector guide, and no incident signal, we find from (2.3)
that the resonator energy decays as

\[ |a(t)|^2 = |a(0)|^2 e^{-\frac{2t}{\tau_e}} \quad (2.5) \]

and from (2.4) that the throughput field \( s_t \) is related to the resonator amplitude by

\[ s_t = \kappa A(t) \quad (2.6) \]

Using power conservation, we equate the derivative of (2.5) with the squared modulus of (2.6) to arrive at the following relation between \( \kappa, \mu \) and \( \tau_e \)

\[ \frac{2}{\tau_e} = |\kappa|^2 \frac{v_g}{2\pi R} = |\mu|^2 \quad (2.7) \]

We can now determine the frequency response of the energy amplitude by inserting (2.3) into (2.7) and solving for \( a \).

\[ a = \frac{-j \sqrt{\frac{2}{\tau_e}}}{j(\omega - \omega_0) + \frac{1}{\tau} s_i} \quad (2.8) \]

The throughput field is given by the incident field \( s_i \) minus the loss due to coupling from the input guide to the resonator \( (\kappa s_i) \) plus the gain due to coupling from the resonator back into the input guide \( (-j\mu a) \). Neglecting the loss due to coupling to the resonator, the throughput field is simply

\[ s_t = s_i - j\mu a \quad (2.9) \]

The drop field is found similarly, however, here the resonator energy amplitude \( a \) is assumed to be the only source term.

\[ s_d = -j\mu a \quad (2.10) \]

From (2.8) and (2.9) we find the throughput field to be

\[ s_t = \frac{j(\omega - \omega_0) + \frac{1}{\tau} - \frac{2}{\tau_e} s_i}{j(\omega - \omega_0) + \frac{1}{\tau} s_i} \quad (2.11) \]

and with (2.10) we find the drop field to be

\[ s_d = \frac{-\frac{2}{\tau_e} s_i}{j(\omega - \omega_0) + \frac{1}{\tau} s_i} \quad (2.12) \]
The quality factor or $Q$ of a resonator which is traditionally defined as

$$Q = \frac{\omega}{\Delta \omega}$$  \hspace{1cm} (2.13)

can be related to the decay time constant $\tau$ by taking the squared modulus of the energy amplitude $a$ (2.8), setting $\omega - \omega_0$ to $1/\tau$, and then noting that the Full Width Half Maximum (FWHM) bandwidth $\Delta \omega$ is equal to $2/\tau$.

$$Q = \frac{\omega \tau}{2}$$  \hspace{1cm} (2.14)

Internal $Q_{int}$ and external $Q_{ext}$ quality factors can then be defined according to,

$$Q_{int} = \frac{\omega F \tau d}{2(\tau e + \tau d)}$$  \hspace{1cm} (2.15a)

$$Q_{ext} = \frac{\omega F}{2}$$  \hspace{1cm} (2.15b)

respectively, where the internal $Q$ characterizes the intrinsic quality of the resonator and the external $Q$ characterizes the coupling.

To illustrate the response of a simple ring resonator filter, we consider as an example, the throughput (2.11) and drop (2.12) port responses for a resonator with a resonant frequency of $\nu_0 \approx 2 \cdot 10^{14}$ Hz (i.e. $\lambda \approx 1.5 \mu$m) and an external $Q$ of $10^4$ are plotted in (Figure 2-2). Internal $Q'$s of $10^4$, $10^5$, and $10^6$ were used in the calculations. The responses are Lorentzian, and therefore highly peaked and not very useful for WDM filtering applications.
Figure 2-2: Throughput and drop port responses for a single ring resonator as a function of the resonator internal $Q$.
2.1.2 Higher Order Filter Responses

Higher order filters can provide broader, more sharply defined bandpasses. A basic review of higher order filters is provided herein. More detailed approaches are presented in [10, 11, 12, 13]. Higher order filters can be constructed by simply coupling resonators in series or in parallel (Figure 2-3). The set of differential equations describing the evolution of the

\begin{align*}
\frac{d}{dt}a_1 &= \left(j\omega_1 - \frac{1}{\tau_1}\right)a_1 - j\mu_1 s_1 - j\mu_2 a_2 \\
\frac{d}{dt}a_2 &= \left(j\omega_2 - \frac{1}{\tau_2}\right)a_2 - j\mu_1 a_1
\end{align*}

Figure 2-3: Higher order ring resonator Add/Drop filters formed from (a) series and (b) parallel coupled resonators

energies in the resonators can be arrived at by simply adding the additional field components resulting from the presence of the additional resonator to the original differential equation (2.3) for a single resonator. For a pair of series coupled resonators the resultant differential equations are,
where \(a_1\) and \(a_2\) denote the energies of the resonators and the coupling between the resonators is expressed by the mutual coupling coefficient \(\mu_1 = \kappa_1 \sqrt{\frac{v_{21} v_{22}}{4 \pi^2 R_1 R_2}}\). The set of coupled differential equations can be readily solved to give the energy amplitudes in the resonators.

\[
a_1 = \frac{-j \mu s_i}{j \Delta \omega_1 + \frac{1}{\tau_1} + \frac{\mu_1^2}{j \Delta \omega_2 + \frac{1}{\tau_2}}} \tag{2.17a}
\]
\[
a_2 = \frac{-j \mu a_1}{j (\Delta \omega_2) + \frac{1}{\tau_2}} \tag{2.17b}
\]

The throughput and drop port responses are then calculated in the same manner as before, however, now the throughput response is determined entirely by the energy amplitude in the first resonator \(a_1\)

\[
s_t = s_i - j \mu a_1 \tag{2.18}
\]

and the drop port response is determined entirely by the energy amplitude in second resonator \(a_2\).

\[
s_d = -j \mu a_2 \tag{2.19}
\]

For a pair of parallel coupled resonators, the differential equations take the form

\[
\frac{d}{dt} a_1 = \left( j \omega_1 - \frac{1}{\tau_1} \right) a_1 - j \mu s_i - j \mu \left( -j \mu a_2 e^{-j \beta L} \right) \tag{2.20a}
\]
\[
\frac{d}{dt} a_2 = \left( j \omega_2 - \frac{1}{\tau_2} \right) a_2 - j \mu (s_i - j \mu a_1) e^{-j \beta L} \tag{2.20b}
\]

where \(L\) represents the separation between the resonators and \(\beta\) is the propagation constant of the guide. Equations (2.20a) and (2.20b) can be solved for the energy amplitudes \(a_1\) and \(a_2\).

\[
a_1 = -j \mu \frac{1 - \frac{\mu_1^2 e^{-j \beta L}}{j \Delta \omega_2 + \frac{1}{\tau_2}}} {j \Delta \omega_1 + \frac{1}{\tau_1} + \frac{\mu_1^2}{j \Delta \omega_2 + \frac{1}{\tau_2}}} s_i \tag{2.21a}
\]
\[
a_2 = -j \mu \frac{(s_i - j \mu a_1) e^{-j \beta L}}{j (\Delta \omega_2) + \frac{1}{\tau_2}} \tag{2.21b}
\]

The throughput response is now determined by the sum of the phase delayed power coupled out of the first resonator \((s_i - j \mu a_1) e^{-j \beta L}\) and the power coupled out of the second resonator.
\[ s_t = (s_i - j\mu a_1)e^{-j\beta L} - j\mu a_2 \]  
(2.22)

And, in a similar manner, the drop port response is simply the sum of the phase delayed power coupled out of the second resonator \(-j\mu a_2e^{-j\beta L}\) and the power coupled out of the first resonator \(-j\mu a_1\).

\[ s_d = -j\mu a_2e^{-j\beta L} - j\mu a_1 \]  
(2.23)

Although the resonator energy amplitudes for the series and parallel coupled resonators are somewhat different, the throughput and drop port responses can in fact be made to be quite similar. Maximally flat responses can be obtained for each by setting the mutual coupling coefficient for the series coupled resonators \(\mu_1\) to \(\mu^2/2\) and by setting the separation between the resonators to \(L = (\pi N)/(2\beta)\) where \(N\) is a positive valued integer for the parallel coupled resonators. Responses for the series and parallel cases with resonator external \(Q's\) of \(10^4\) and resonant frequencies of \(\nu_0 \approx 2 \cdot 10^{14}\)Hz (i.e. \(\lambda \approx 1.5\mu m\)) are plotted in Figures 2-4 and 2-5, respectively, for three values of the resonator internal \(Q\). Both series and parallel coupled resonators exhibit wider bandwidths with sharper fall offs than single resonators, but both are significantly more sensitive to the internal \(Q's\) of the resonators.

Although the responses for both the series and parallel coupled cases appear to be quite similar, there is in fact, a very important difference. At optical frequencies it is impossible to operate on the lowest order mode of the resonator. In order to obtain a sufficiently high resonant frequency a higher order mode must be excited. The excited resonant frequency is therefore,

\[ \omega_m = m\omega_0 \]  
(2.24)

where \(m\) is the mode number and the separation between resonant peaks, the Free Spectral Range (FSR), is simply equal to the fundamental resonance \(\nu_0\) where \(\nu_0 = 2\pi/\omega_0\).

\[ \text{FSR} = \nu_0 \]  
(2.25)

A large FSR can therefore be obtained by designing a resonator with a high fundamental frequency. However, as will be discussed in the subsequent chapter, this generally proves to be a difficult task. An alternate approach can be used for series coupled, but not parallel coupled or stand alone resonators, to extend the FSR. Essentially, for resonators coupled
in series, the FSR can be extended over the entire communications bandwidth by simply using resonators with different resonant frequencies (i.e., the Vernier effect). To observe this, we calculated the response of a pair of series coupled rings with fundamental resonances of $4.96 \cdot 10^{12}\text{Hz}$ and $3.17 \cdot 10^{12}\text{Hz}$. The resonances of the resonators coincide for a center frequency of $\nu_0 \approx 2 \cdot 10^{14}\text{Hz}$ (i.e., $\lambda \approx 1.5\mu\text{m}$) and mode numbers 39 and 61, respectively. The responses of non-coincident resonances were included in the calculations and the results are plotted in Figure 2-6. It is evident from the drop port response that the unwanted resonances are substantially suppressed over the entire band. Optical communication systems typically utilize bandwidths that extend well beyond even the 20THz range plotted in the figure. It is essential that the filters used for WDM applications possess FSR’s that are equally large or else more than one signal would be pulled off by a single filter. Because of both their box-like filter response and potentially large FSR, from here on out, we only consider the use of series coupled resonators.
Figure 2-4: Throughput and drop port responses for a pair of series coupled ring resonators as a function of the resonator internal $Q$
Figure 2-5: Throughput and drop port responses for a pair of parallel coupled ring resonators as a function of the internal resonator $Q$.
Figure 2-6: The broadband throughput and drop port responses for a pair of series coupled ring resonators utilizing the Vernier effect.
2.1.3 Switching Through Absorption

B. E. Little et al. [7] showed that with sufficient absorption, the field inside the resonator can be reduced to a point where the necessary interaction between and the resonator field and that of the bus waveguide is negligible. The resonator is then effectively switched off. This effect can be demonstrated quite easily by simply lowering the internal $Q$ of the resonators. We do so for a pair of series coupled resonators with an external $Q$ of $10^4$ and resonant frequency of $\nu_0 \approx 2 \cdot 10^{14}$Hz (i.e. $\lambda \approx 1.5 \mu$m) and plot the responses in Figure (2-7). It is evident from the plots that the response of the resonator can in fact be greatly suppressed if sufficient absorption is available.
Figure 2-7: Throughput and drop port responses for a pair of series coupled ring resonators with low internal $Q'\text{s}$
2.2 Wavelength Switching and Routing

The general trend in fiber optic communication systems has been directed toward greater and greater utilization of the fiber bandwidth. Channel counts of over 100 are being considered [14] and data rates within a channel have increased from under 1 Gib/s to proposed systems operating at 40 Gib/s [15]. However, at the time of this publication, WDM systems are implementing 10 Gbit/s channels spaced roughly 100GHz apart, and channel counts are generally below 100. Thus, in considering the performance specifications for Add / Drop and Cross-Connect switches, we use these specifications while keeping in mind that future systems will likely possess both greater channel counts and higher data rates. In resonators, it is the $Q$ that determines the bandwidth ($Q = \omega/\Delta\omega$). Since the center frequency for fiber optic communications is roughly $2 \cdot 10^{14}$Hz, an external $Q$ of $10^4$ provides a bandpass of roughly 20GHz. Thus, the choice of $Q_{\text{ext}} = 10^4$ used in the calculations in the previous section was not arbitrary. In the current section, we consider the prior results in the context of Add/Drop and Cross-Connect applications.

2.2.1 The Add/Drop Application

The basic layout for an Add/Drop switch with complete functionality to interchange wavelength channels between a pair of fibers is presented in Figure 2-8. Although the waveguide crossing is unnecessary for the Add/Drop application, it will become necessary when considering the Cross-Connect application. Also, it is worth noting that the single resonators depicted in the figure are meant to be representative of any resonator type used, and not meant to imply that single resonators would be used in the application. The output state of

![Figure 2-8: Non-blocking Add/Drop switch](image-url)
the device may be altered by simply turning resonators on and off (Figure 2-9). In the figure, black is used to designate turned off resonators and $\lambda^N_M$ refers to the $Mth$ wavelength channel emanating from the $Nth$ input fiber. The total on-chip loss and cross-talk level amongst

![Diagram](image)

Figure 2-9: Operation of a Non-blocking Add / Drop switch with all resonators except the one turned off

channels represent the two most important performance specifications for Add/Drop and Cross-Connect switches. Both impact the bit error rate at the fiber termination substantially. However, it is difficult to specify maximum allowable loss and cross-talk levels because such specifications are heavily system dependent. Certainly, a tradeoff between performance and functionality exists. Still, it is reasonable to assume that the loss should not exceed 10dB and the cross-talk levels -20dB or regenerators may be required at the output.

We begin by considering the loss specifications for an Add/Drop switch. Neglecting coupling and propagation losses in the bus waveguides, loss can be incurred in any of the three possible interactions with the resonators (i.e. non-resonant, resonant, and switched-off). Since only a single on-resonant interaction can take place, we assume that the loss incurred in this interaction is sufficiently small to be neglected. This assumption is accurate for a pair of series coupled resonators with internal $Q's$ of $10^5$ or more (Figure 2-4). In order to isolate effects of non-resonant and switched-off interactions, we consider the extreme cases in which either all the resonators are switched on or all the resonators are switched off. In the former case, a maximum of $2(M - 1)$ non-resonant interactions can take place. For a maximum allowable loss of 10dB, the loss per non-resonant interaction must be less than $\frac{10}{2(M - 1)}$dB or for a 100 wavelength system, a maximum permissible loss of 0.05dB per interaction. However, since only adjacent channel resonators contribute significantly to the
throughput loss, this specification is perhaps overly strict. Still at 100GHz away from the center frequency, the throughput loss resulting from a pair of series coupled resonators can be as low as 0.02dB (Figure 2-4, $Q = 10^5$), and thus the throughput loss resulting from non-resonant interactions is not a significant concern.

With all of the resonators turned off, $M$ switched-off interactions occur. Therefore, a 10dB loss specification dictates a loss per switched-off interaction of less than $\frac{10}{M}$ dB, or for a 100 channel system a maximum loss of 0.1dB per interaction. The throughput responses for a pair of resonators coupled in series with internal $Q's$ of 10, 100, and 1000 are plotted in Figure 2-7. Although difficult to determine from the figure, the loss resulting from resonators with $Q's$ of 10, 100, and 1000, is 0.017dB, 0.17dB, and 1.7dB, respectively. However, the power lost on the first pass by the resonator $|\kappa|^2$ was left out in order for the analysis to be generally applicable. This loss must be added onto that shown in Figure 2-7 and can be expressed in terms of the resonator parameters $\Delta \omega$, $R$, and $v_g$.

\[
Loss = \Delta \omega \frac{2\pi R}{v_g}
\] (2.26)

For a ring with an extremely small radius of 3\(\mu\)m, a moderate bandwidth of 20GHz, and a group velocity of 10^8 m/s, the minimum loss for an absorptive based switch is 0.1dB. It is therefore of little value to attempt to achieve $Q's$ below 10. In order to consider possible techniques for achieving such low $Q's$, it is necessary to relate these $Q$ values to waveguide loss specifications in dB/cm. To do so, we express the $Q$ in terms of the imaginary component of the resonant frequency $\omega_i$

\[
Q = \frac{\omega}{2\omega_i}
\] (2.27)

where $\omega_i = 1/\tau$ and $\omega_0 = \omega + j\omega_i$. By then expanding the resonator frequency with a Taylor series about the real component of the propagation constant $\beta_0$,

\[
\omega(\beta) = \omega(\beta_0) + (\beta - \beta_0) \frac{\partial \omega}{\partial \beta} \bigg|_{\beta_0} + \cdots
\approx \omega(\beta_0) + (\beta - \beta_0) v_g
\] (2.28)

the imaginary component of the resonator frequency can be related to the imaginary com-
ponent of the propagation constant through the group velocity.

\[ \omega_i = \alpha v_g \]  

(2.29)

The \( Q \) can then be expressed in terms of \( \alpha \),

\[ Q = \frac{\omega}{2\alpha v_g} \]  

(2.30)

and by taking the logarithm of the power decay \( \exp(-2\alpha z) \), we determine that absorptions of 5200dB/cm and 52000dB/cm are required to achieve \( Q's \) of 100 and 10, respectively (assuming \( v_g \approx 10^8 \)). To put these requirements into perspective, the Franz-Keldysh effect, cited as possible mechanism for the introduction of absorption into a resonator [7], produces a maximum of \((<2500dB/cm [16])\). A 100 channel system requires a \( Q \) substantially less than 100 or a waveguide loss substantially greater than 5200dB/cm in order to approach the 10dB loss specification. Current electro-absorptive techniques are therefore unable to produce sufficient loss.

The cross-talk level is more difficult to determine than the loss in the device because individual cross-talk components can add constructively or destructively depending on their phase relationship. To remain conservative, we assume the worst case scenario in which all cross-talk components add constructively. We further assume that a cross-talk level of -20dB is acceptable. The primary cross-talk components arise from off-resonant coupling to adjacent channels. From Figure 2-4, we see that for a pair of series coupled resonators the drop port filter function rolls off to a level of -35dB at 100GHz away from the center of the filter band. This is considerably lower than the -20dB specification. Imperfect extinction in a switched-off resonator also results in cross-talk. However, if we again consider a pair of series coupled resonators, we see from the drop port response in Figure 2-7 that even with \( Q's \) as high as 100, the residual power has been reduced by over -70dB, and is therefore negligible even in large scale systems. Still, other components arise due to scattering effects, however, these are difficult to model and specific to the resonator / switch design.

To first order, it appears that a large scale Add/Drop switch formed from series coupled ring resonator switching elements is feasible so long as the internal resonator \( Q's \) approach \( 10^5 \) and the switched-off \( Q's \) are substantially below 100. However, in order to achieve a sufficiently low switched-off \( Q \), a new approach for the introduction of absorption must be
considered. We do so in Chapter 4.

2.2.2 The Cross-Connect Application

Conceptually a Cross-Connect switching matrix can be built from the Add/Drop switches presented in the previous section. In such a matrix the vast majority, \( \geq (N - 1) \times M \), of the resonators must necessarily be switched off since there are \( N \) times as many resonators as there are signals. As a result of the inherent loss imposed by absorptive based resonator switches (2-7) it is difficult to imagine large scale Cross-Connects based purely on this principle. If we assume a fundamental loss of 0.1dB, at most a 10 wavelength channel system with 10 input and output fibers could be implemented. In order to develop Cross-Connects on a larger scale, the coupling to the resonator itself would have to be destroyed. Although not discussed in this thesis, this may be a topic of future work.

2.3 Summary

We have presented possible architectures for Add/Drop and Cross-Connect switches and considered the requirements on both the resonator spectral response and the degree of loss required to switch the resonators. It appears that a pair of series coupled ring resonators with internal \( Q' \)s of \( 10^8 \) are sufficient for the application. And, switched-off \( Q' \)s substan-
ially below 100 are necessary for the Add / Drop application. Current electro-absorptive techniques are unable to introduce sufficient loss to achieve these $Q$ values. However, this is not a fundamental limit. In Chapter 4, we show that the fundamental limit can be approached through evanescent interaction with the resonator. The Cross-Connect application is, however, limited by the fundamental loss imposed by the absorptive based switching technique. In order to implement large scale Cross-Connects with resonators, the coupling to the resonator itself must be destroyed.
Chapter 3

Resonator Design

In the previous chapter we considered some basic resonator theory, and in the process demonstrated the need to use a pair of resonators cascaded in series in order to obtain sufficiently flat filter responses, free spectral range, and roll off for WDM applications. Moreover, we showed that in order to maintain sufficiently low losses in the drop port, $Q's$ on the order of $10^5$ are necessary. In the current chapter, we consider the details of the resonator design required to achieve the necessary $Q's$. In particular, we consider the relevant waveguide parameters and resonator geometry.

3.1 The Waveguides

Since the degree of integration and the Free Spectral Range (FSR) of the resonator increase with decreased ring radius, it is generally desirable to minimize the ring radius. However, in order to maintain phase fronts that are perpendicular to the direction of propagation, the field beyond a critical distance from the center of the ring would have to travel at a velocity greater than the speed of light in the medium. Since phase fronts perpendicular to the ring radius can therefore not be maintained, all rings must radiate. And, because weakly confined fields extend further into the cladding region, such fields produce greater degrees of radiation when compared to more tightly confined fields traversing the same bend. To produce these tightly confined fields, high index contrast waveguides are required.
3.1.1 Slab Waveguides

Before considering bent and three dimensional waveguides, it is instructive to first review the simple case of a dielectric slab waveguide, the basic structure of which is presented in Figure 3-1. To analyze the structure, we begin by considering Maxwell’s equations.

\[ \mathbf{V} \times \mathbf{E} = -\mu \frac{\partial}{\partial t} \mathbf{H} \]  \hspace{1cm} (3.1a)

\[ \mathbf{V} \times \mathbf{H} = \varepsilon \frac{\partial}{\partial t} \mathbf{E} + \mathbf{J} \]  \hspace{1cm} (3.1b)

\[ \mathbf{V} \cdot \varepsilon \mathbf{E} = \rho \]  \hspace{1cm} (3.1c)

\[ \mathbf{V} \cdot \mu \mathbf{H} = 0 \]  \hspace{1cm} (3.1d)

Since the structure is formed from dielectrics, the current density is zero everywhere \( \mathbf{J} = \mathbf{0} \). The wave equations for the electric and magnetic fields are developed by inserting (3.1b) into (3.1a), and (3.1a) into (3.1b), respectively. We develop the wave equation for the electric field, but note that the wave equation for the magnetic field can be developed in identical fashion resulting in an equation of identical form. Inserting (3.1b) into (3.1a), and making use of the vector identity,

\[ \mathbf{V} \times \mathbf{v} \times \mathbf{A} = \mathbf{V}(\mathbf{v} \cdot \mathbf{A}) - \mathbf{v}^2 \mathbf{A} \]  \hspace{1cm} (3.2)

the wave equation for the electric field becomes

\[ \nabla^2 \mathbf{E} - \nabla (\nabla \cdot \mathbf{E}) = \mu \varepsilon \frac{\partial^2}{\partial t^2} \mathbf{E} \]  \hspace{1cm} (3.3)
From the divergence relation \( \nabla \cdot \varepsilon \mathbf{E} = 0 \), we note that \( \nabla \cdot \mathbf{E} = 0 \) in any given region of the structure. Thus (3.3) simplifies to the Helmholtz equation.

\[
\nabla^2 \mathbf{E} + \mu \varepsilon \omega^2 \mathbf{E} = 0 \\
\tag{3.4}
\]

For the \( \mathbf{H} \) field, the wave equation is quite simply

\[
\nabla^2 \mathbf{H} + \mu \varepsilon \omega^2 \mathbf{H} = 0 \\
\tag{3.5}
\]

The bounded solutions (modes) of (3.4) and (3.5) have the following general form

\[
e(y), h(y) = \begin{cases} 
A e^{-\alpha_2 y} & y > 0 \\
B \cos(k_y y + \phi) & -T \leq y \leq 0 \\
C e^{\alpha_3 (y+T)} & y < -T
\end{cases} \\
\tag{3.6}
\]

for \( \mathbf{E} \) and \( \mathbf{H} \) described by

\[
\mathbf{E} = a e(y) e^{j(\omega t - \beta z)} \\
\tag{3.7a}
\]
\[
\mathbf{H} = b h(y) e^{j(\omega t - \beta z)} \\
\tag{3.7b}
\]

and can be categorized by their polarization state as either transverse electric (TE) for an \( \hat{x} \) directed electric field or transverse magnetic (TM) for an \( \hat{x} \) directed magnetic field. In order to solve for the terms \( A, B, C, k_y, \alpha_2, \) and \( \alpha_3 \), the boundary conditions must be considered. The boundary conditions for the tangential electric and magnetic fields are determined from the integral forms of Faraday’s (3.8a) and Ampere’s (3.8b) Laws.

\[
\int_C \mathbf{E} \cdot da = -\int_A \frac{\partial}{\partial t} \mathbf{H} \cdot da \\
\tag{3.8a}
\]
\[
\int_C \mathbf{H} \cdot da = \int_A \frac{\partial}{\partial t} \mathbf{E} \cdot da + \int_A \sigma \mathbf{E} \cdot da \\
\tag{3.8b}
\]

By taking the area of integration \( A \) separately to be rectangles oriented along the \( \hat{x} \) and \( \hat{z} \) directions across the boundary and forcing the height of the rectangles to zero, the right hand sides of (3.8a) and (3.8b) go to zero in each case and we are left with the condition
that the tangential electric and magnetic fields are continuous across the boundaries.

\[ E_x^{(i)} = E_x^{(i+1)} \]  \hspace{1cm} (3.9a)
\[ E_z^{(i)} = E_z^{(i+1)} \]  \hspace{1cm} (3.9b)
\[ H_x^{(i)} = H_x^{(i+1)} \]  \hspace{1cm} (3.9c)
\[ H_z^{(i)} = H_z^{(i+1)} \]  \hspace{1cm} (3.9d)

By then using these conditions in conjunction with the differential form of Faraday’s (3.1a) and Ampere’s Laws (3.1b), we find that the derivative of the \( \hat{x} \) polarized electric field is continuous across the boundaries, but that the derivative of the \( \hat{x} \) polarized magnetic field is not.

\[ \frac{\partial}{\partial y} E_x^{(i)} = \frac{\partial}{\partial y} E_x^{(i+1)} \]  \hspace{1cm} (3.10a)
\[ \frac{1}{\varepsilon_i} \frac{\partial}{\partial y} H_x^{(i)} = \frac{1}{\varepsilon_{i+1}} \frac{\partial}{\partial y} H_x^{(i+1)} \]  \hspace{1cm} (3.10b)

Using the relations (3.9a) and (3.9c) in conjunction with (3.6), we find that for the both the TE and TM polarized modes the field coefficients \( A, B, \) and \( C \) must satisfy

\[ A = B \cos(\phi) \]  \hspace{1cm} (3.11a)
\[ C = B \cos(\phi - k_y T) \]  \hspace{1cm} (3.11b)

which leads to the following field distribution for either case.

\[ e(y), h(y) = \begin{cases} 
\cos(\phi) e^{-\alpha_2 y} & \text{if } y > 0 \\
\cos(k_y y + \phi) & \text{if } -T \leq y \leq 0 \\
\cos(\phi - k_y T) e^{\alpha_3 (y+T)} & \text{if } y < -T 
\end{cases} \]  \hspace{1cm} (3.12)

Using (3.10a) and 3.6, we find that the TE mode must also satisfy

\[ \alpha_2 A = k_y B \sin(\phi) \]  \hspace{1cm} (3.13a)
\[ \alpha_3 C = -k_y B \sin(\phi - k_y T) \]  \hspace{1cm} (3.13b)
and similarly using (3.10b) and 3.6, we find that the TM mode must satisfy

\[ \varepsilon_1 \alpha_2 A = \varepsilon_2 k_y B \sin(\phi) \]  
\[ \varepsilon_1 \alpha_3 C = -\varepsilon_3 k_y B \sin(\phi - k_y T) \]

For the TE case, we take the ratios of the equations (3.13) and (3.11) and for the TM case, the ratios of (3.14) and (3.11) to arrive at the following set of equations describing the boundary conditions for the TE modes,

\[ \tan(\phi) = \frac{\alpha_2}{k_y} \]  
\[ \tan(\phi - k_y T) = \frac{\alpha_3}{k_y} \]

and for the TM modes,

\[ \tan(\phi) = \frac{\varepsilon_1 \alpha_2}{\varepsilon_2 k_y} \]
\[ \tan(\phi - k_y T) = \frac{\varepsilon_1 \alpha_3}{\varepsilon_3 k_y} \]

respectively. By inverting these relations, the eigenvalue equations for the TE (3.17a) and TM (3.17b) cases are determined.

\[ k_T = m\pi + \tan^{-1} \frac{\alpha_2}{k_y} + \tan^{-1} \frac{\alpha_3}{k_y} \]  
\[ k_T = m\pi + \tan^{-1} \left( \frac{n_1}{n_2} \right)^2 \frac{\alpha_2}{k_y} + \tan^{-1} \left( \frac{n_1}{n_3} \right)^2 \frac{\alpha_3}{k_y} \]

In order to establish these relations in terms of a single eigenvalue, we insert (3.7) into the wave equation (3.4) and derive the dispersion relations.

\[ \beta^2 + k_y^2 = \left( \frac{\omega}{c n_1} \right)^2 \]  
\[ \beta^2 - \alpha_2^2 = \left( \frac{\omega}{c n_2} \right)^2 \]  
\[ \beta^2 - \alpha_3^2 = \left( \frac{\omega}{c n_3} \right)^2 \]
The dispersion relations can then be used to write (3.17) in terms of the propagation constant $\beta$.

For a symmetric case (i.e. $n_2 = n_3$), a cutoff condition for the higher order modes can readily be determined. Setting $m = 1$ and noting that at cutoff the evanescent decay parameter $\alpha$ tends toward zero, we find that the first order TE and TM modes are cutoff provided the thickness of the guide $T$ is smaller than a specified value (3.19).

$$T < \frac{\lambda}{2\sqrt{n_{core}^2 - n_{clad}^2}}$$

(3.19)

### 3.1.2 Rectangular Waveguides - The Effective Index Method

Rectangular dielectric waveguides do not possess closed form analytic solutions. Approximate techniques are therefore necessary to arrive at analytic solutions. The easiest, and thus perhaps most valuable of these techniques is the Effective Index Method (EIM) [17]. The approach begins by first separating the field (electric or magnetic) distribution $\psi$ into two components

$$\psi(x,y) = \phi(x)\chi(x,y)$$

(3.20)

where $\phi(x)$ represents the separable component of the $x$ variation in the field, and $\chi(x,y)$...
represents the inseparable component. Alternatively, the separable component of \( y \) could be explicited.

\[
\psi(x, y) = \phi(x, y)\chi(y)
\]  

(3.21)

However, if we compare the approaches, we see that the asymmetry of the structure (Figure 3-2) forces the variation in \( \phi(x, y) \) with respect to \( y \) to be much greater than the variation of \( \chi(x, y) \) with respect to \( x \). The later approach is therefore less effective at separation of the \( x \) and \( y \) variations of \( \psi \). Inserting (3.20) into the wave equation,

\[
\frac{\partial^2}{\partial x^2} \psi + \frac{\partial^2}{\partial y^2} \psi + \left[ k_0^2 n^2(x, y) - \beta^2 \right] \psi = 0
\]  

(3.22)

we arrive at the following set of coupled equations

\[
\frac{\partial^2}{\partial x^2} \phi + \left( \frac{2}{\chi} \frac{\partial}{\partial x} \chi \right) \frac{\partial}{\partial x} \phi + \left[ k_0^2 N^2(x) + \frac{1}{\chi} \frac{\partial^2}{\partial x^2} \chi \right] \phi = \beta^2 \phi
\]  

(3.23a)

\[
\frac{\partial^2}{\partial y^2} \chi + k_0^2 n^2(x, y) = N^2(x)k_0^2
\]  

(3.23b)

Assuming the partial derivatives of \( \chi \) with respect to \( x \) are sufficiently small that they may be neglected, an assumption which is valid for \( b >> a \) and low index contrast structures, the set of coupled differential equations is reduced to a set of uncoupled differential equations.

\[
\frac{\partial^2}{\partial y^2} \chi + k_0^2 n^2(x, y) = N^2(x)k_0^2 \chi
\]  

(3.24a)

\[
\frac{\partial^2}{\partial x^2} \phi + k_0^2 N^2(x) \phi = \beta^2 \phi
\]  

(3.24b)

These equations are equivalent to the wave equations for slab waveguides with index profiles \( n(x, y) \) and \( N(x) \), where \( n(x, y) \) is the original and \( N(x) \) is the derived index distribution. Thus, conceptually, the approach begins by solving for the propagation constant \( \beta_y \) of a slab waveguide of width \( a \), core index \( n_1 \), and cladding index \( n_2 \). And, then solving for a slab waveguide of width \( b \), a core index \( N_{\text{eff}} \) equal to \( \lambda \beta_y / 2\pi \), and a cladding index of \( n_2 \) (Figure 3-2).
3.1.3 Bend Effects

The effect of index contrast on bend performance can be determined by considering the simple two dimensional case of a bent waveguide. The approach is presented in detail in Appendix A. The analysis begins by using a conformal mapping to effectively straighten the curved waveguide. In doing so, the index takes on an exponential dependence with radius, which to first order is linearized so as to allow for simple Airy function eigenmode solutions. The loss is then determined using the WKB approximation for the tunneling of the photons from the core to the exponentially increasing index in the cladding.

The analysis presented in the appendix was formulated in terms of the spatial dependencies of the field \( \beta = \beta_r + j\alpha \) in order to agree with the established literature. Since we are ultimately interested in the effect of the bend radius on the resonator \( Q \), we use the relationship between the \( Q \) and the loss parameter \( \alpha \) developed in the previous chapter to relate the two.

\[
Q = \frac{\omega}{2\alpha v_g}
\]  

(3.25)

To prevent the appearance of multi-peaked filter responses, the resonator should be formed from single mode waveguides. The modes and loss parameters for three values of \( \Delta n = n_{core} - n_{clad} \) with a cladding index set to that of silica (1.44) and a width adjusted to just maintain single mode operation (3.19), were solved for as a function of the radius of curvature of the guides for a TM slab mode (TE and TM modes exhibit similar dependencies). The \( Q's \) were calculated using (3.25), and the results are plotted in Figure 3-3. The results show that the impact of the index contrast is in fact quite significant. With an index contrast of \( \Delta n = 0.1 \), a \( Q \) only slightly greater than \( 10^3 \) is achievable even with a bend radii of 100\(\mu m\). However, with an index contrast of \( \Delta n = 0.5 \), a \( Q \) of \( 10^6 \) is possible with a bend radius of only 20\(\mu m\). And, with an index contrast of \( \Delta n = 1.5 \), a \( Q \) of \( 10^6 \) can be obtained with a bend radius of only 5\(\mu m\). The minimum device size is therefore greatly affected by the index contrast. And, since small resonator sizes are necessary for large scale integration and for maintaining a large free spectral range, high index contrast waveguides are an essential element of Add/Drop and Cross-Connect switching architectures formed from ring resonators.
Figure 3-3: $Q$ as a function of bend radius for three values of index contrast

3.1.4 Bend Effects in Three Dimensions

Rectangular waveguides in general possess modes with predominant polarizations in both the TE and TM directions. Since both modes possess components of all three polarizations, neither mode is purely transverse electric or transverse magnetic. Therefore, we denote the mode that is predominantly TE polarized, the TE-like mode, and the mode that is predominantly TM polarized, the TM-like mode, where transverse is in reference to the plane of the substrate (Figure 3-4). For a square waveguide, the symmetry of the structure dictates that the TE-like and TM-like modes be degenerate. Unfortunately, in a bend, the symmetry of the structure is broken forcing the TE-like and TM-like modes to propagate at different speeds therein causing ring resonators to be inherently polarization sensitive. To demonstrate the importance of this effect, we consider a $3\mu$m × $3\mu$m silicon core silica clad waveguide. Despite the square guide geometry, the effective index method (EIM) (Appendix B) can be used to collapse the guide onto a two-dimensional space. We do so, and then use conformal transformations (Appendix A) to determine the bend effects. Again using a
Figure 3-4: Principal polarizations of the (a) TE-like and (b) TM-like modes in a buried rectangular waveguide

Taylor series to expand about the unperturbed propagation constant $\beta_0$,

$$\omega(\beta) = \omega(\beta_0) + (\beta - \beta_0) \frac{\partial \omega}{\partial \beta}|_{\beta_0} + \cdots$$

$$\approx \omega(\beta_0) + (\beta - \beta_0)v_g$$  \hspace{1cm} (3.26)

the effect on the resonant frequency is calculated and the results are presented in Figure 3-5.

The calculations indicate that even at relatively modest bends (given the index contrast), the frequency difference between the modes is in the range of 100's of gigahertz. Since it is essentially impossible to excite only a single mode in a multimode waveguide, and since TE-like and TM-like modes have been shown to couple through bends and slanted sidewalls [27], resonators which propagates both modes generally possess multi-peaked filter responses. Although it may be possible to compensate for the bend effects with guide geometry, the coupling coefficients to the resonators would have to be adjusted through the same mechanism. Adjusting for both polarization effects with a single guide geometry could prove to be a difficult task.

3.1.5 Designing a Single Mode Waveguide

To avoid the pitfalls of polarization effects, we consider the possibility of essentially eliminating the propagation of one of the modes and/or preventing any coupling between the
Figure 3-5: Calculated frequency difference between the two polarization modes in a ring resonator formed from a symmetric silicon-core silica-clad waveguide as a function of the radius of the ring. The jaggedness of the curve is a result of the inaccuracy of the root solver.

modes. We begin by considering the coupling between a pair of modes with electric fields described by

\begin{align*}
\mathbf{E}_1 &= a_1(z)e_1(x,y)e^{-j\beta_1 z} \\
\mathbf{E}_2 &= a_2(z)e_2(x,y)e^{-j\beta_2 z}
\end{align*}

(3.27a) (3.27b)

where \(a_1(z)\) and \(a_2(z)\) are slowly varying functions of \(z\) and \(e_1(x,y)\) and \(e_2(x,y)\) contain the normalized distributions of the \(\hat{x}\), \(\hat{y}\) and \(\hat{z}\) components of the electric field for modes 1 and 2, respectively. With an initial excitation of mode 1 of amplitude \(a_1(0)\) and no initial excitation of mode 2, the solutions as determined by coupling of modes theory [9], exhibit the following sinusoidal dependencies as a function of \(z\)

\[ a_1(z) = a_1(0) \left[ \cos(\gamma z) - \frac{\Delta \beta}{2\gamma} \sin(\gamma z) \right] \]  

(3.28a)
\[ a_2(z) = -j a_1(0) \frac{\kappa_{12}}{\gamma} \sin(\gamma z) \]  

(3.28b)

where the coupling coefficient \( \kappa_{12} \) is defined by,

\[ \kappa_{12} = -j \frac{\omega}{4} \int \int (\varepsilon - \varepsilon_{\text{clad}}) \mathbf{e}_1 \cdot \mathbf{e}_2^* dxdy \]  

(3.29)

the term \( \gamma \) by,

\[ \gamma = \sqrt{\kappa_{12}^2 + \left( \frac{\Delta\beta}{2} \right)^2} \]  

(3.30)

and \( \Delta\beta = \beta_1 - \beta_2 \). By taking the modulus squared of (3.28a) and (3.28b) and setting \( z = \pi/2\gamma \), we find that the maximum fractional power coupled between a pair of modes is simply

\[ \frac{P_2}{P_1 + P_2} = \frac{\kappa_{12}^2}{\kappa_{12}^2 + \left( \frac{\Delta\beta}{2} \right)^2} \]  

(3.31)

From (3.31) it is evident that the coupling between any two modes, whether they be guided or unguided, is influenced by two factors, the coupling coefficient \( \kappa_{12} \) and the difference between the propagation constants \( \Delta\beta \). The coupling coefficient determines the rate of transfer and \( \Delta\beta \) determines the degree of phase matching between the modes. It is the ratio of \( \Delta\beta \) to \( \kappa_{12} \) that determines the maximum power transfer between the modes. By forcing \( \Delta\beta >> \kappa_{12} \), the coupling between the modes is effectively eliminated. Thus, what matters is not whether the guide is truly single mode, but whether the secondary modes, guided or unguided, are somehow strongly coupled to or closely phasematched to the primary mode.

To enhance the difference in the rates of propagation of the TE-like and TM-like modes and therein minimize the potential power transfer between the modes, we propose using a high-index-contrast large-aspect-ratio structure (Figure 3-6). If we consider the effective

![Figure 3-6: Buried large aspect ratio rectangular waveguide of height a and width b](image-url)
index method discussed in Section 3.1.2, we see that on account of the large aspect ratio of the structure, the boundary conditions at \( y = a/2 \) and \( y = -a/2 \) dominate. Thus, the TE-like and TM-like modes maintain much of the differences of the TE and TM modes of a slab of width \( a \). The transcendental eigenvalue equations for the fundamental TE and TM modes of a symmetric dielectric slab of width \( a \) are from (3.17),

\[
\tan \left( k_{y}^{TE} \frac{a}{2} \right) = \frac{\alpha_{y}^{TE}}{k_{y}^{TE}} \tag{3.32a}
\]

\[
\tan \left( k_{y}^{TM} \frac{a}{2} \right) = \frac{n_{1}^{2} \alpha_{y}^{TM}}{n_{2}^{2} k_{y}^{TM}} \tag{3.32b}
\]

respectively. The limiting behaviour (i.e. as \( k_{y} a / 2 \to 0 \)) of (3.32a) and (3.32b) can be examined in detail if we approximate the tangent by the first term of the MacLauren series expansion.

\[
k_{y}^{TE} a \approx \frac{\alpha_{y}^{TE}}{k_{y}^{TE}} \tag{3.33a}
\]

\[
k_{y}^{TM} a \approx \frac{n_{1}^{2} \alpha_{y}^{TM}}{n_{2}^{2} k_{y}^{TM}} \tag{3.33b}
\]

By making use of the dispersion relations,

\[
\beta^{2} - \alpha_{y}^{2} = k_{1}^{2} \tag{3.34a}
\]

\[
\beta^{2} + k_{y}^{2} = k_{2}^{2} \tag{3.34b}
\]

we arrive at the following quadratic equations in \( \alpha_{y} \)

\[
\left( \frac{\alpha_{y}^{TE}}{a} \right)^{2} + 2 \frac{\alpha_{y}^{TE}}{a} - (k_{1}^{2} - k_{2}^{2}) = 0 \tag{3.35a}
\]

\[
\left( \frac{\alpha_{y}^{TM}}{a} \right)^{2} + 2 \left( \frac{n_{1}^{2}}{n_{2}} \right) \frac{\alpha_{y}^{TM}}{a} - \left( \frac{n_{1}^{2}}{n_{2}} \right)^{2} (k_{1}^{2} - k_{2}^{2}) = 0 \tag{3.35b}
\]

which possess the following positive valued roots

\[
\alpha_{y}^{TE} = \sqrt{\frac{1}{a^{2}} + (k_{1}^{2} - k_{2}^{2}) - \frac{1}{a}} \tag{3.36a}
\]

\[
\alpha_{y}^{TM} = \sqrt{\left( \frac{n_{1}}{n_{2}} \right)^{6} \frac{1}{a^{6}} + (k_{1}^{2} - k_{2}^{2}) - \left( \frac{n_{1}}{n_{2}} \right)^{6} \left( \frac{1}{a} \right)} \tag{3.36b}
\]
From (3.36a) and (3.36b), we see that neither mode cuts off, but rather \(\alpha_y\) goes to zero and the modes spread out as \(a\) approaches zero. However, clearly \(\alpha_y\) for the TM mode approaches zero faster than it does for the TE mode, and, from the dispersion relation (3.34a), we see that as \(\alpha_y\) approaches zero, \(\beta\) approaches its minimum value of \(k_2\). Applying this logic to the TE-like and TM-like modes of a rectangular waveguide, we can argue that for a small guide height \(a\) the propagation constant of the TM-like mode can be driven towards \(k_2\) while the TE-like mode is left relatively unaffected. The effect is maximized for large index differences between the core and the cladding \(n_1 > n_2\). Moreover, as \(\alpha_y\) tends toward zero and the TM-like mode spreads out, the overlap integral between the TM-like mode and any nearby tightly confined mode is ultimately reduced to zero. Thus, by taking advantage of the disparity in the limiting behavior of the TE-like and TM-like modes, the difference in the propagation constants \(\Delta\beta\) can be maximized while the coupling coefficient \(\kappa\) to the undesirable TM-like mode is minimized. In total, high-index-contrast large-aspect-ratio double-moded waveguides are relatively immune to potential coupling between the TE-like and TM-like modes, and, due to the weak confinement of the TM-like mode, effectively single-moded.

### 3.1.6 Mode calculations

Our primary interest is in developing waveguides with excellent bend radii performance. By choosing a high-index-contrast silicon-core \((n_1 = 3.48)\) silica-clad \((n_2 = 1.44)\) waveguide with a large aspect ratio, we anticipate both low loss and low cross polarization coupling through bends. In designing a large aspect ratio waveguide, a choice must be made between making the width greater than the height or vice versa. Since maintaining straight sidewalls presents a major difficulty in fabrication, it is in fact much easier to form guides with larger widths than heights. For this reason, we choose to make the guide height as small as possible. For the purposes of developing some basis for choosing the height of the guide, we calculate and plot (Figure 3-7) the effective index for the lowest order mode of a silicon-core silica-clad slab waveguide as a function of the guide height. It is important to note that the range of values plotted is actually greater than the useful range since the cutoff for higher order modes occurs at a height of 0.23\(\mu\)m. From the figure, we see that for a slab guide of height 0.2\(\mu\)m, the TE and TM propagation constants differ remarkably, and that only minor gains are made in going to smaller heights. In order to maintain a good balance between
Figure 3-7: Propagation constants of the TE and TM modes of a slab waveguide with core index 3.48 and cladding index 1.44 as a function of the guide height

a high effective index and a large $\Delta \beta$, we chose a guide height of 0.2$\mu$m. The guide width is then appropriately chosen by maintaining as large a width, and thus effective index, as possible while maintaining cutoff of higher order modes. To allow for the guide to later be perturbed evanescently, we considered a guide with a slightly different geometry than that depicted in Figure 3-6. Instead of being a purely buried waveguide, the top cladding is set to the index of air (Figure 3-8). Since vector interactions are generally significant in high index contrast waveguides, we used a modesolver to accurately assess the appearance of higher order modes [34, 20]. A width of 0.6$\mu$m (aspect ratio of 3:1) was obtained with only the lowest order TE-like and TM-like modes propagating. The distributions of the modes are presented in Figure 3-9. The results of the modesolver confirm our predictions of a well confined TE-like mode ($N_{\text{eff}} = 2.47$) and a poorly confined TM-like mode ($N_{\text{eff}} = 1.58$) with most of its energy spilling into the cladding (Figure 3-9). With such large differences in propagation, we expect very little power transfer between the modes, even under the most extreme bends. Moreover, due to the weak confinement of the TM-like mode, in
Figure 3-8: High index contrast large aspect ratio waveguide used for resonator

any fabricated device it will suffer extreme losses due to surface roughness, coupling to the substrate, and bend losses. It is therefore reasonable to consider our waveguide to be effectively single moded. Fabrication and measurement of filter responses will ultimately prove or disprove these predictions.
Figure 3-9: Modal distributions for the TE-like (a) and TM-like (b) modes of the high-index-contrast large-aspect-ratio waveguides to be used in the resonator
In contrast, the tight confinement of the TE-like mode should enable low loss guidance even around very tight bends. To prove this assertion, we again make use of the effective index method and conformal transformations, and calculate the effect of the bend radius on the resonator $Q$ (Figure 3-10). It appears that our predictions are accurate. According to the theory $Q$'s of over $10^6$ can be achieved with bend radii of only 1$\mu$m.

Figure 3-10: Calculated $Q$ as a function of the radius of a resonator formed from the guides in Figure 3-8
3.2 The Resonator

With the waveguides designed, it is time to consider the resonator geometry. Paramount to the design is the bend radius of the resonator. Although in the previous section we showed that bend radii as small as 1μm may be possible, we know that in practice the effects of the bends can be greatly enhanced due to surface roughness and fabrication errors. Additionally, since vector effects were not taken into account in the analysis, we set the minimum resonator radius to ≈ 2.5μm in order to be safe. In a fabricated device, this may in fact still be too tight a bend. However, such a small device allows for a large free spectral range, and as a result, good use of the Vernier effect since the fundamental resonances of the two resonators can then be made to be quite far apart and the overlap of the adjacent modes quite minimal.

A demonstration of a spectrum that is possible with such small rings is presented in 3-11. The resonator was formed from a pair of resonators with radii of 2.54μm and 3.97μm cascaded in series. The spectrum was obtained using coupling of modes in time (Section 2.1.2), but included the effects of all the excitations of modes in the resonators, not just the mode of interest. The drop port response possess multiple peaks, however, all peaks except the one of interest are close to 30dB down over a 10THz range in either direction (100 channel spacings). As a result of the clean drop port response, only the primary dip in transmission is noticeable in the throughput response. Although on an actual Add/Drop or Cross-Connect switch, the resonator sizes would have to vary with the wavelength to be dropped, these variations would be minor. Therefore, we take as a baseline design the resonator radii used in these calculations.
Figure 3-11: The throughput and drop port responses of a pair of series coupled ring resonators of radii 2.54μm and 3.97μm. The Vernier effect is used to suppress unwanted resonances.
3.2.1 Resonator / Bus Waveguide Separation

![Diagram of resonator and bus waveguide separation](image)

Figure 3-12: Geometry considerations for determining the separation between the resonator and bus waveguides

In the previous chapter, we showed that a bandwidth of roughly 20GHz could be obtained by setting the external $Q$ of the resonator to $10^4$. In order to determine the necessary separation between the bus waveguides and the resonator to achieve an external $Q$ of $10^4$, we use coupling of modes theory. Since the distance between the bus waveguide and the resonator waveguide varies over the interaction (Figure 3-12), the coupling coefficient must vary as well. This variation can be formulated in terms of the angle between the normal made by the ring radius and the bus waveguide,

$$\Delta(\theta) = 2R\sin^2\frac{\theta}{2}$$  \hspace{1cm} (3.37)

and the total coupling to the ring can then be determined by integrating over $180^\circ$.

$$\kappa = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \kappa_{12}(s + \Delta(\theta)) Rd\theta$$  \hspace{1cm} (3.38)

where $\kappa_{12}$ is the usual coupling coefficient (3.29) from coupling of modes in space. In Chapter 2 we saw that the coupling coefficient is related to the decay rate $\tau_e$.

$$\tau_e = \frac{4\pi R}{|\kappa|^2 v_g}$$  \hspace{1cm} (3.39)

Assuming the resonator is lossless and coupled equally to the bus waveguides $\tau_e = \tau_d$, the total decay rate in the resonator is $\tau = \tau_e/2$ and the $Q$ of the resonator is related to the
coupling coefficient through

\[ Q = \pi \frac{\omega_0 R}{|\kappa|^2 v_g} \]  

(3.40)

Using the guides of Figure 3-8 and their calculated modes (Figure 3-9), the external \( Q \) was calculated as a function of the separation between the bus waveguides and the resonator. The results are plotted in Figure (3-13). We find that a separation of roughly 0.12\( \mu \)m leads to an external \( Q \) of \( 10^4 \) for either ring.
Figure 3-13: The external $Q$ is plotted as a function of the separation between the resonators and the bus waveguides for rings of radii $2.5\mu m$ and $4\mu m$. 

$R = 2.5\mu m$

$R = 4\mu m$
3.2.2 Resonator / Resonator Separation

Figure 3-14: Diagram associated with the calculation of the mutual energy coupling coefficient $\mu_1$

In addition to the separation between the rings and the bus waveguides, the separation between the rings themselves must be determined. While the separation between the rings and the bus waveguides determines the bandwidth of the response, the separation between the rings determines the shape of the response. To obtain a maximally flat response, it is necessary to set the mutual energy coupling coefficient $\mu_1$ to [10]

$$\mu_1 = 0.5\mu^2 = 1/\tau_c$$

(3.41)

The mutual energy coupling coefficient is related to the power coupling coefficient $\kappa_1$ through

$$|\mu_1| = |\kappa_1|^2 \sqrt{\frac{v_{g1}v_{g2}}{4\pi^2 R_1 R_2}}$$

(3.42)

which can again be calculated from the usual coupling of modes in space coupling coefficient $\kappa_{12}$ which is a function of the total separation between the rings $s + \Delta_1(\theta) + \Delta_2(\theta)$

$$\kappa_1 = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \kappa_{12} (s + \Delta_1(\theta) + \Delta_2(\theta)) R d\theta$$

(3.43)
where we have integrated over the angle $\theta_1$ and related the angles $\theta_1$ and $\theta_2$ using the equation,

$$R_1 \sin(\theta_1) = R_2 \sin(\theta_2)$$  \hspace{1cm} (3.44)

developed from geometry considerations. We calculate and plot (Figure 3-15) the mutual coupling coefficient for resonators with radii of 2.54$\mu$m and 3.97$\mu$m. For a $Q$ of $10^4$ and a resonant frequency of $2 \times 10^{14}$, $\tau_e$ is equal to $2\pi \cdot 10^{-10}$. From the plot we find that $\mu_1 = 1/\tau_e$ and maximally flat response is achieved for a separation between the rings of 0.27$\mu$m.

Figure 3-15: The mutual coupling coefficient as a function of the separation between the rings.
3.2.3 Resonator Geometry and Fabrication

The basic resonator and waveguide geometries are presented in Figure 3-16. A distance of 1μm was left between the core of the waveguide and the substrate. At this separation, we find from the calculated modes that the intensity of the TE-like mode has decayed by more than 90dB. Therefore, any coupling to the substrate in the primary TE-like mode should be negligible.

Although not discussed in detail here, the waveguides could be formed in a three step process (Figure 3-17). An SiO$_2$ layer must first be grown on a silicon wafer. The waveguide patterns could then be etched into the SiO$_2$. A silicon layer could then be deposited to fill in the etched regions and subsequently planarized. Alternatively, a silicon on insulator (SOI) process could be used.
Top View
(Resonator)

\[ \begin{align*}
  s_1 &= s_2 = 0.12\mu m \\
  s_m &= 0.27\mu m \\
  R_1 &= 2.54\mu m \\
  R_2 &= 3.97\mu m
\end{align*} \]

Side View
(Waveguide)

Figure 3-16: Diagram of proposed resonator
Grow SiO₂ Layer on Silicon Wafer

Pattern and Etch Out Waveguide Regions

Deposit Silicon Waveguides and Planarize

Figure 3-17: Proposed method of fabricating resonator
3.3 Summary

We have shown that for achieving both dense integration and a large free spectral range, high index contrast waveguides are an essential element of ring resonator based integrated optics. Using high index contrast waveguides, a resonator theoretically capable of achieving all of the performance goals stated in Chapter 2, was designed from the ground up. Large-aspect-ratio high-index-contrast waveguides were used to inhibit coupling to, and propagation of, the TM-like mode and therein prevent the mutlipeaked filter responses commonly associated with ring resonators. However, since fiber optic communication is generally carried out with signals that possess random polarization states, we must consider how to couple the random polarization state entering the chip to the single polarization state available on the chip. A method for doing so will be presented in Chapter 5.
Chapter 4

Switch Design

B. E. Little et al. [7] proposed using absorption to selectively kill the resonance of a ring resonator based Add/Drop filter and therein form a wavelength selective switch. In the paper, electro-absorption arising from the Franz-Keldysh effect [21] or other multiple quantum well effects [16] in a semiconductor ring resonator guide were cited as possible mechanisms for the introduction of absorption into an otherwise transparent guide. However, such material systems are complex, requiring epitaxial growth and careful control over layer thickness and composition, and provide only limited absorption (<2500dB/cm [16]). As was demonstrated in Chapter 2, an absorption substantially greater than 5200dB/cm is necessary to turn off a resonator sufficiently for use in large scale switching applications. Recognizing the limited absorption offered by electro-absorptive effects, B. E. Little suggested the possibility of using a MEMS actuated evanescent absorber to selectively introduce loss into the ring. Since the complex refractive index of a material outside the guide can be designed independently, it was anticipated that much greater absorption coefficients might be possible through evanescent interaction. Moreover, since most integrated optic materials are amorphous in structure and can be deposited with much simpler processes, but lack the requisite semiconductor properties for electro-absorptive effects, evanescent interaction would enable a wider range of materials along with simpler fabrication techniques. In this chapter, we consider in detail the potential of a MEMS actuated absorbing membrane for switching a ring resonator. Specifically, we develop the underlying theory governing the material selection for the evanescent absorber, present projected changes in resonator $Q$, discuss the basic mechanics governing the implementation of a MEMS actuated absorbing
structure, and present a preliminary design of such a device.

4.1 Optical Considerations of the Absorbing Membrane

The basic concept of using an absorber to interact with the evanescent field of a resonator is presented in Figure 4-1. With the absorbing membrane placed far from the resonator, the interaction with the evanescent field is negligible and the resonant wavelength is dropped in the usual fashion. In contrast, with the absorber positioned very close to the resonator guide, the interaction becomes significant and the resonator is effectively turned off allowing the resonant wavelength to pass by undisturbed.

![MEMS based resonator switch](image)

Figure 4-1: MEMS based resonator switch (a) in the on state and (b) in the off state

The degree to which this approach works is heavily dependent on the choice of refractive index of the absorber material. Conceptually, it is straightforward to envision that index matching the membrane to the core of the resonator guide and positioning it sufficiently close to the ring will provide an almost immediate dissipation of the energy stored in the cavity. However, such an approach could lead to considerable crosstalk in a densely arrayed system of resonators since the power coupled out of one resonator would be available to couple into an adjacent resonator. It is for this reason that we have restricted ourselves to absorbing membranes. Moreover, since the structure must be mobile, we further restrict ourselves to thin membranes ($T \sim \lambda$). In order to absorb the field within a thin membrane, the imaginary component of the membrane refractive index must be significant, and perfect index matching can no longer be maintained.
4.1.1 The Slab Resonator

Since the choice of the membrane index and thickness will greatly impact the performance of the device, it is essential to model the effects of each. However, the complex geometry of the proposed system (Figure 4-1) lacks an analytic solution. To first order, the waveguide/absorber system can be approximated by its two dimensional vertical cross-section. Conceptually, a resonator is then formed by wrapping the vertical cross-section around to form a ring (Figure 4-2) while being careful to ensure a total resonator length equal to an integer number of half wavelengths.

![Image of Slab Resonator Diagram]

Figure 4-2: Subsection of a 2D ring resonator model

The evanescent field of a slab mode decays exponentially with distance from the guide core-cladding boundary. As a result, maximal interaction with the field is obtained by placing the absorber in direct contact with the waveguide core. Additionally, reflections off of either interface of the absorber serve to minimize the interaction. To isolate the effect of reflections off the lower boundary of the absorber while illustrating the ultimate potential of this approach, we place an infinitely thick absorber in direct contact with waveguide core. In the presence of the absorber, the guide radiates and therefore does not possess guided modes in the traditional sense. Still, the self-consistent solutions of the guide must obey the boundary conditions and be solutions to the wave equation. As such, the original eigenvalue equations developed in Chapter 3,

\[ k_y a = m\pi + \tan^{-1} \frac{\alpha_2}{k_y} + \tan^{-1} \frac{\alpha_3}{k_y} \]  

\[ (4.1a) \]
\[ k_y a = m\pi + \tan^{-1} \left( \frac{n_1}{n_2} \right)^2 \frac{\alpha_2}{k_y} + \tan^{-1} \left( \frac{n_1}{n_3} \right)^2 \frac{\alpha_3}{k_y} \] (4.1b)

still hold. However, to account for the radiation, the solutions, often called quasi-modes, must take on complex eigenvalues. We consider only the transverse electric field since this is the predominant field in our waveguides. Using the dispersion relations (4.2)

\[
\beta^2 + k_y^2 = \left( \frac{\omega}{c n_1} \right)^2 \] (4.2a)
\[
\beta^2 - \alpha_2^2 = \left( \frac{\omega}{c n_2} \right)^2 \] (4.2b)
\[
\beta^2 - \alpha_3^2 = \left( \frac{\omega}{c n_3} \right)^2 \] (4.2c)

(4.1a) can be expressed with \( \omega \) and \( \beta \) as the only unknowns.

\[
\sqrt{\left( \frac{\omega}{c n_1} \right)^2 - \beta^2} a = m\pi + \tan^{-1} \left( \frac{\beta^2 - \left( \frac{\omega}{c n_2} \right)^2}{\left( \frac{\beta^2 - \left( \frac{\omega}{c n_1} \right)^2}{\sqrt{\frac{\beta^2 - \left( \frac{\omega}{c n_3} \right)^2}{\left( \frac{\omega}{c n_1} \right)^2 - \beta^2}}} \right)^2 + \tan^{-1} \left( \frac{\beta^2 - \left( \frac{\omega}{c n_3} \right)^2}{\left( \frac{\omega}{c n_1} \right)^2 - \beta^2} \right) \] (4.3)

A choice exists between fixing \( \omega \) and solving for a complex propagation constant \( \beta \), or, alternatively, fixing \( \beta \) and solving for a complex \( \omega \). Since the properties of a resonator are most readily expressed in terms of the resonator \( Q \), which is related to the real and imaginary components of the resonant frequency through,

\[ Q = \frac{\omega_r}{2\omega_i} \] (4.4)

it is most efficient to solve directly for the complex resonant frequency \( (\omega_0 = \omega_r + j\omega_i, \text{ where } \omega_i = 1/\tau) \) of the structure. However, in order to ensure that the resultant real component of the resonant frequency is close to the frequency of interest, knowledge of the spatial dependence of the field is necessary. Therefore, we fix \( \beta \) to that of the unperturbed system, and use a root searching algorithm to solve (4.3) for \( \omega \). We model our guide by a slab with the same core index (3.48), lower cladding index (1.44), and height \( (a = 0.2\mu m) \) as the full three dimensional structure (Figure 3-8). And, using the aforementioned approach, solve for the \( Q \) of the resonator as a function imaginary component of the absorber refractive index for three values of the real component of the absorber refractive index (Figure 4-3).
Figure 4-3: (a) Diagram of model and (b) plot of $Q$ as a function of the imaginary component of the absorber refractive index for three values of the real part of the absorber refractive index.
It should be noted that although the imaginary component of the index is shown in the figure to possess a positive sign, this sign is relative and dependent on the choice of the sign in the exponent of the exponential term $e^{-j(\omega t - k_r)}$. The important point is to choose a sign that ensures decay. The plots demonstrate that the $Q$ is greatly reduced for real values of the refractive index equal to or greater than the index of the core, and for low values of the imaginary component. The effect is lessened when the real component of the absorber index is set significantly below the waveguide core index due to substantial internal reflection at the interface. Similarly, the effect is also lessened for large values of the imaginary component of the refractive index due to external reflection at the interface. The calculations demonstrate the ability of the technique to achieve $Q$'s below 10, and thus compatibility with the Cross-Connect switching requirements discussed in Chapter 2. However, the situation depicted here is somewhat unrealistic. In a real device, the absorber would have to be thin and the separation between the absorber and the resonator nonzero (i.e. to prevent sticktion at the interface).

To present a more realistic situation, we consider an absorber/guide separation of 50nm and leave the absorber height as an open variable. The problem is now considerably more complicated, and, although a full analytic solution can developed in the traditional manner of matching boundary conditions and using the wave equation, it would provide little additional insight. An eigenvalue equation can be arrived at more readily by translating the impedance from upper boundary of the absorber using (4.5)

\[ Z(y) = \frac{Z(0) - jZ_0 \tan(k_y \Delta y)}{Z_0 - jZ(0) \tan(k_y \Delta y)} \]  

and matching it to the impedance at the lower boundary of the guide [9]. The structure is in general multi-moded. However, the quasi-mode with most of its energy concentrated in the guide as opposed to the absorber, propagates with the lowest loss, and possesses the greatest overlap with the mode of the unperturbed guide. The impact of the other modes on the resonator $Q$ is minimal, and is therefore neglected. Using the lowest loss quasi-mode, we calculate the resonance in the same manner as before plot the dependence on the imaginary component of the index for three absorber widths (Figure 4-4)
Figure 4-4: (a) Diagram of model and (b) plot of $Q$ as a function of the imaginary component of the absorber refractive index for three values of the absorber thickness 0.2$\mu$m, 0.5$\mu$m and 0.8$\mu$m.
The $Q'$s are slightly higher than they were for the slab case, but still remain remarkably close to the $Q$ of 10 required for the Cross-Connect application. In particular, a minimum is reached for an imaginary component of the index equal to $\sim 0.5$. As should be expected, the $Q'$s for the three thicknesses converge as the imaginary index increases, a consequence of a field totally absorbed after one round trip inside the absorber. The fact that the $Q'$s are similar for all three thicknesses even at relatively low values of the imaginary index, suggests the possibility of using a very thin absorber. Of the standard materials that are available, Indium Arsenide (InAs) represents a reasonable match with an index of $3.51 + 0.2j$ in the $1.55\mu m$ regime [22]. A $Q$ of 28 is possible with a $0.2\mu m$ thick InAs membrane. Lower $Q'$s can be achieved by increasing the absorber thickness or modifying the material, however, scattering and increased bend losses resulting from the presence of the absorber are likely to lower the $Q$ to an even greater extent.

Since we aim to actuate the structure micromechanically, it is important to consider the range of motion required to turn the resonator from an on to an off state. Assuming a membrane thickness of $0.2\mu m$, and an absorber index of $3.51 + 0.2j$, the resonator $Q$ is solved for and plotted as a function of absorber separation in Figure 4-5. At a separation of only $0.8\mu m$, the resonator returns to a potential $Q$ of greater than $10^6$, thereby lending itself nicely to a MEMS implementation.
Figure 4-5: (a) Diagram of model and (b) plot of resonator Q as a function of the separation of the absorber.
4.1.2 FDTD Simulations

Before setting out on the time consuming project of fabricating the device, it is desirable to generate a greater degree of certainty that the device will work as projected. The Finite Difference Time Domain (FDTD) technique (Appendix B) is useful for this purpose. However, to model the whole structure would require computational resources far greater than what were available to us. In a manner similar to the approach taken in Section 4.1.1, we simplify the problem, modeling the ring resonator by the simple standing wave resonator depicted in Figure 4-6. Perfectly conducting mirrors are placed at a separation of $\lambda_{\text{eff}}/2$

![Figure 4-6: Model resonator used in FDTD simulations](image)

where $\lambda_{\text{eff}} = \lambda/N_{\text{eff}}$ is the effective wavelength in the guide, thereby supporting the lowest order mode while minimizing the device length and the required computational resources. Despite the reduction in complexity, with the exception of bend induced effects, essentially all of the characteristics of the three dimensional ring resonator are maintained.

The basic guide developed for our resonator was used in the simulation with the minor exception that the cladding index was taken to be 1.44 everywhere in order to simplify the geometry. Absorber separations of 0 and 0.1\,\mu m were used. A grid spacing of 0.02\,\mu m and a time step of $1.3dx/c\sqrt{3}$ were used in the calculations. The electric energy in the guide was monitored and recorded at each time step and the results are plotted in Figure 4-7. From the
plots, we see that the electric energy oscillates with both fast and slow time constants as it decays. The fast oscillations in the electric energy are simply due to the oscillation between the energy storage in the electric and magnetic fields. The slower oscillations are, however, due to the exchange of energy between the resonator and the absorber. Exponential decays were fitted to the peaks of the slower oscillations since these peaks represent the maximum amount of energy that could possibly be left in the resonator at a given time. From the curve fits, the imaginary components of the frequency were found to be $2 \cdot 10^{13}$ Hz and $1.5 \cdot 10^{13}$ Hz for the case of 0 and 0.1μm separations, respectively. The corresponding $Q$'s are roughly 30 and 40 for the two separations, thereby agreeing quite closely with the analysis of Section 4.1.1. It is worth noting that in both case some residual energy exhibiting a much slower time constant remained in the cavity. This is believed to be a result of light scattered at the beginning of the simulation not being able to exit the computational domain due to the use of conducting boundary conditions. A slower decay rate would then be expected due to reduced interaction with the absorber. This hypothesis is bolstered by cross-sectional images of the square of the fields presented in Figure 4-8. The absorber separation in the simulation was 0.1μm and the distributions were obtained by taking the square of each of the electric field components and then summing each along the length of the cavity. The images demonstrate that scattering plays a significant role, and is therefore a likely cause of the residual energy remaining in the cavity (Figure 4-7).
Figure 4-7: Results of FDTD simulations for the structure presented in Figure 4-6 for absorber separations of 0 and 0.1\(\mu\)m.
Figure 4-8: Images obtained by squaring FDTD field outputs of a standing wave resonator and then summing along the length of the cavity
4.2 Implementation of the Absorbing Membrane

4.2.1 Mechanical Considerations

The mechanical considerations are in fact quite complex, and largely beyond the scope of this work. Thus, we aim to simply demonstrate the feasibility of using electrostatic actuation for inducing the required deflection and present a preliminary design. As it turns out the temporal response of the switch is intimately tied to its mechanical stiffness. Thus, we touch upon both the static and dynamic characteristics of this MEMS device with the aim of providing an estimate of both the voltages required and the switch response time. Since the required movements are so slight, it is reasonable to consider simply bending the membrane without fear of plastic deformation. We present a simple clamped beam analysis of the mechanics of the switching motion. For a uniformly loaded doubly clamped beam (Figure 4-9) of width $w$, thickness $T$, length $l$, and Young's modulus $E$, the force required

\[ F_y \]

\[ \Delta s \]

\[ s \]

Figure 4-9: Model of doubly clamped beam (a) under no load and (b) under a uniformly applied load.
to deflect the beam a distance $\Delta s$ is given by [25]

$$F = k_s \Delta s$$  \hspace{1cm} (4.6)

where $k_s$ is defined as

$$k_s = 32 \frac{EwT^3}{l^3}$$  \hspace{1cm} (4.7)

To generate this force, we consider the force induced by applying a potential $V$ between the membrane and the substrate. The force required to change the separate of the plates of a capacitor by a distance $dy$ is given by the change in stored electric energy with the capacitor for a fixed voltage.

$$F_y = \frac{dW_e}{dy} \bigg|_{V \ fixed}$$  \hspace{1cm} (4.8)

Recalling that the electric energy stored within a capacitor is given by

$$W_e = \frac{CV^2}{2} = \frac{\varepsilon AV^2}{2y}$$  \hspace{1cm} (4.9)

where $\varepsilon$ is the dielectric constant of the medium, $A$ is the area of the plates and $s$ is the separation, the force can be expressed in terms of the applied voltage $V$ by inserting (4.9) into (4.8).

$$F_y = -\frac{k_c}{s^2}$$  \hspace{1cm} (4.10)

where

$$k_c = \frac{\varepsilon AV^2}{2}$$  \hspace{1cm} (4.11)

Equating (4.6) and (4.10), the voltage required to hold the beam at a deflection of $\Delta s$ given an initial separation of $s_0$ (current separation $s$ where $s = s_0 - \Delta s$) can be determined.

$$V = \sqrt{\frac{64 \Delta s(s - \Delta s)^2}{\varepsilon l^4}} \frac{ET^3}{s_0}$$  \hspace{1cm} (4.12)

By setting the derivative of (4.12) with respect to $s$ to zero, the separation requiring the maximum voltage to maintain the position of the beam is found to be $s = 2s_0/3$. The maximum voltage is then readily determined.

$$V_{max} = \frac{16}{3} \sqrt{\frac{s_0^3 ET^3}{3l^4 \varepsilon}}$$  \hspace{1cm} (4.13)
In addition to the maximum voltage required to bend the membrane, the switching time required to do so is of key interest. In the present configuration, the time required for the membrane to relax back to its initial state is likely to determine the switching time. The beam cannot be allowed to relax back freely, but rather must be damped in order to prevent excitation of the natural frequencies of the beam and resulting oscillations in the position of the membrane. In order to ensure such oscillations are of little concern, the damping must be sufficient to prevent excitation of the lowest order mode of the membrane, the frequency of which is given below in (4.14) [26].

\[ f_0 = 3.6 \sqrt{\frac{EI}{ml^2}} \]  

In Figure 4-10, the maximum applied voltage and the fundamental resonance are plotted as a function of the length of the membrane for the three thicknesses considered earlier (i.e. 0.2\( \mu \)m, 0.5\( \mu \)m, and 0.8\( \mu \)m) assuming material properties of bulk InAs \((E = 5.8 \cdot 10^{11} \text{dyn/cm}^2, \rho = 5.68 \text{g/cm}^2 [23, 24])\). Since voltages above 10V are unrealistic and since the length of the membrane should be kept close to the size of the resonator (10\( \mu \)m diameter), the thickness of the membrane should not exceed 0.2\( \mu \)m. The length of the membrane could then be kept to around 40\( \mu \)m, corresponding to a fundamental resonance of greater than 400 kHz. A switching time on the order of a few micro-seconds might then be possible. To demonstrate this potentially fast switching time, we make a summation of the forces on the beam

\[ m \frac{d^2s}{dt^2} + \gamma \frac{ds}{dt} + k_s(s - s_0) + \frac{k_c}{s^2} = 0 \]  

and use the Runge-Kutta technique to solve the differential equation numerically. The results are plotted in Figure 4-11. A voltage of 12V was used to generate the turn-on time and the damping \( \gamma \) was adjusted to prevent oscillations. From the plot, we see that both turn-on and turn-off times are in the vicinity of a few microseconds.
Figure 4-10: Plots of the required voltage and fundamental resonance of the membrane as a function of the membrane length for three membrane thicknesses
Figure 4-11: Turn-on and turn-off responses for the switch with an applied voltage of 12V and a damping term adjusted to prevent oscillations

A perspective view and a diagram of the proposed design are presented in Figures 4-12 and 4-13, respectively. A 1\(\mu\)m lateral gap between the bus waveguides and the absorbing membrane was left to prevent interaction between them.
Figure 4-12: Perspective view of the resonator switch in the (a) on and (b) off states.
Figure 4-13: Top and side views of the switch
4.2.2 Fabrication

An approach for fabricating the absorbing membrane over an existing ring resonator device is depicted in Figures 4-14 through 4-17. The first components to be fabricated are the lower electrodes. As depicted (Figure 4-14), the electrodes are first deposited and then patterned and etched. However, depending on the material selection, a liftoff process could be used as well. Poly-silicon or a metal such as aluminum are potential electrode materials. The second step in fabrication is deposition, patterning and etching of the membrane supports (Figure 4-15). The membrane supports must be formed from a dielectric to prevent shorting of the device. Silica is a likely membrane support material. A sacrificial layer is then deposited and planarized so that the membrane supports are showing (Figure 4-16). The InAs membrane is then deposited on top of the sacrificial layer (Figure 4-17). The membrane is then patterned and the sacrificial layer is under-etched away. The absorbing membrane is left supported by the membrane support structures. If a low temperature process such as sputtering is found to yield sufficient material properties for the InAs membrane, then a simple polymer sacrificial layer could be used.

4.3 Summary

We have shown both through approximate analytic techniques and the highly accurate finite difference time domain technique, that $Q'$s of close to 30 should be achievable through evanescent interaction with a thin (0.2μm) InAs membrane. If we consider the analysis of Chapter 2, the associated loss in the throughput port of the switch should be about 0.15dB, only 0.05dB above the theoretical minimum. For comparison, a switch utilizing electro-absorption would incur a loss of more than 0.45dB, a factor of 3 greater. Additionally, through basic mechanics we showed that with a voltage of only 12 volts and a damping coefficient adjusted to prevent oscillations, the membrane turn-on and turn-off times could be as short as a few microseconds.
Figure 4-14: Fabrication steps for forming the lower electrodes
Deposit Support Material

Pattern and Etch Supports

Figure 4-15: Fabrication steps for forming the membrane supports
Figure 4-16: Fabrication steps for preparing the sacrificial layer
Figure 4-17: Fabrication steps for forming the absorbing membrane
Chapter 5

Polarization

In the previous chapters we considered many issues specific to the design of ring resonators and ring resonator switching elements. In the process, we designed waveguides that effectively propagate only a single polarization state. However, the signals emanating from an optical fiber possess random polarization states. In order to prevent signal fading, a method for coupling the random polarization state leaving the fiber into the well defined polarization on the chip is required. Importantly, such an approach has wide range applicability since essentially all high index contrast integrated optic components share this polarization sensitivity. In this chapter, we develop such an approach. In order to implement our approach a polarization rotator is necessary. The design and verification of a novel integrated optic polarization rotator is the focus of this chapter.

5.1 Polarization Approach

In Chapter 3 we demonstrated that high index contrast dielectric waveguides exhibit severe polarization sensitivities which generally preclude the possibility of performing identical operations on the TE-like and TM-like modes of the guide with single on-chip structures. Thus, in order for high index contrast devices to be incorporated into present systems, the random polarization states exiting the fiber must either be controlled at the input, or separated, operated upon in parallel, and recombined at the output of the chip. However, the multiplicity of wavelength channels in WDM systems and their associated random polarization states makes the task of individually measuring and controlling the polarization states, daunting, at best. We have therefore found it necessary to split the polarization states
and operate on them in parallel. The challenge of maintaining identical operations in parallel is simplified considerably if the two input polarization states possess the same on-chip polarization. This can be accomplished by rotating one of the input polarizations by 90°, and then rotating it back to its original polarization at the output to prevent interference between the fields (see Figure 5-1). Necessary to this operation is a set of integrated optic polarization splitters/combiners, polarization rotators, and single mode waveguides. Designs for polarization splitters/combiners utilizing standard materials and geometries are plentiful [31, 32, 33]. However, to date, most designs for integrated optic polarization rotators have required non-traditional materials and/or fabrication techniques [27, 28, 29]. In particular, many designs require the precise fabrication of angled waveguide sidewalls, a task not easily accomplished in most material systems. Therefore, in the subsequent section we consider the design of a polarization rotator that lends itself to existing planar fabrication techniques. At the end of the chapter, we detail the entire process of mode transformations, and present an approach for fabrication.

5.2 Polarization rotator

Much of the analysis required for the design of the polarization rotator was already performed in Chapter 3 when designing the resonator waveguide. For convenience, we repeat some of the basic results here. Essentially, we wish to couple a TE-like mode of one guide to a TM-like of another and therein induce polarization rotation. However, the coupling between the orthogonally polarized modes could be reduced considerably by coupling be-
tween like polarized modes. To prevent this, it is necessary to design the waveguides in a manner such that coupling only takes place between orthogonally polarized modes. From the general coupling of modes considerations of Chapter 3, we found that the maximum fractional power coupled between a pair of modes is simply

\[ \frac{P_2}{P_1 + P_2} = \frac{\kappa_{12}^2}{\kappa_{12}^2 + \left( \frac{\Delta \beta}{2} \right)^2} \]  

(5.1)

and thus, the coupling between any two modes, is influenced by two factors, the coupling coefficient \( \kappa_{12} \) and the difference between the propagation constants \( \Delta \beta \). The coupling coefficient determines the rate of transfer and \( \Delta \beta \) determines the degree of phase matching between the modes. It is the ratio of \( \Delta \beta \) to \( \kappa_{12} \) that determines the maximum power transfer between the modes. By forcing \( \Delta \beta \gg \kappa_{12} \), the coupling between the modes is effectively eliminated. To ensure this result, we again design waveguides with a high index contrast and a large aspect ratio (Figure 5-2). The guide we consider is remarkably similar to the guide we used in the design of the resonator, the only difference being the cladding is entirely formed out of silica. The modes of the structure are also quite similar with a TE-like mode is again well confined with an effective index of 2.49 and a TM-like that is poorly confined with an effective index of 1.69 (Figure 5-3).

The fact that a TM-like mode can be coupled to a TE-like mode may appear to be counter-intuitive. However, if we again consider the coupling of modes formalism presented earlier, we see that the coupling coefficient \( \kappa_{12} \) is formed from an overlap integral of the dot product of the modal distributions.

\[ \kappa_{12} = -j \frac{\omega}{4} \int \int (\varepsilon - \varepsilon_{clad}) \mathbf{e}_1 \cdot \mathbf{e}_2^* dxdy \]

Figure 5-2: Buried large aspect ratio rectangular waveguide of height \( a \) and width \( b \)
This is an important result since it tells us that even modes with orthogonal principal states of polarization can be coupled through their minor field components. By examining the field pattern of the TE-like mode of our waveguide (Figure 5-3), we see that the \( \hat{z} \)-component of the electric field represents the only significant minor field contribution. Thus, if we wish to couple a TM-like mode to our TE-like mode we must do so through the interaction of the \( \hat{z} \) electric fields. If we imagine rotating our guide by 90° we can form a mode with an orthogonal principal polarization state to our original mode. By then bringing the two guides into proximity and offsetting their center positions relative to one another (to prevent a non-zero overlap integral) these modes can be coupled through the \( \hat{z} \)-components of the electric field, therein forming a polarization rotator. One scheme for doing so is presented in Figure 5-4.

The degree to which such a scheme will convert the input TM-like polarization state into a TE-like polarization state at the output is determined by the interaction of the propagating modes in the guides. Since we previously designed our guides so that the TE-like and TM-like modes exhibited large differences in their rates of propagation, we expect little coupling between the TE-like mode of guide 1 and the TE-like mode of guide 2 (i.e. the
Figure 5-4: Concept for coupling the TM-like mode of Guide 2 to the TE-like mode of guide 1 and therein form a polarization rotator.

TM-like mode of guide 1 rotated by $90^\circ$). Conversely, the TE-like mode of guide 1 couples strongly to the TM-like mode of guide 2. Therefore, we expect almost complete transfer of power between the TM-like mode of guide 2 and the TE-like mode of guide 1. Using the previously calculated modal distributions, we performed a coupling of modes analysis on the structure as a function of the separation of the guide. The length for complete required for $90^\circ$ of rotation is plotted in Figure 5-5 as a function of the separation between the guides.
Figure 5-5: Rotation length as a function of the guide separation using coupling of modes analysis
5.2.1 Numerical Simulation

Coupling of modes provides, at best, only approximate solutions. More accurate results can be obtained through numerical simulations. For forward propagating fields, the beam propagation method (BPM) is generally the technique of choice for designing integrated optic components since only field cross-sections need to be computed. Many variations of BPM exist [35, 36, 37], however, all solve the paraxial wave equation in one form or another. For rapidly varying wide angle fields, the second derivative term normally neglected in the paraxial wave equation must be reinstated to some degree to arrive at accurate results. Typically, this is done by approximating the second derivative with Padé approximates. Arguably the leading experts in the field, Apollo Photonics, generously provided us with a wide angle BPM program with high order Padé approximates implemented. However, after numerous attempts at propagating the field using the Apollo BPM program we found that although the program converged for several iterations in a row, ultimately, we were unable to get the program to converge in a consistent manner. On account of the tight confinement of our mode, it appears that even with the use of high order Padé terms, BPM could not provide accurate results.

Therefore, we again consider the use of the Finite Difference Time Domain (FDTD) technique which represents a more rigorous albeit more computationally intensive approach for propagating an electromagnetic field (Appendix B). To minimize the calculation time, as small a structure should be simulated as possible. From our coupling of modes analysis, we know that the minimum transfer length is obtained when the guides are in contact. However, such large perturbations to the original modes negate the validity of the coupling of modes analysis. Moreover, coupling between the TE-like modes of the guides may become significant and thereby destroy the polarization rotation as the coupling coefficient $\kappa_{12}$ approaches the difference in the propagation constants $\Delta \beta$. For purposes of minimizing the device length and resultant computation time while still demonstrating the performance of our device, we chose to simulate a small but significant separation of 0.1$\mu$m. Coupling of modes predicts an associated transfer length of roughly 80$\mu$m. The grid cross-section was chosen to be $2 \times 2.2\mu$m leaving a gap of greater than 0.6$\mu$m on all sides of the structure, a distance for which the intensity had decayed to roughly $10^{-5}$ of its maximum. The center wavelength to be simulated was 1.55$\mu$m or $\lambda/N_{eff} \approx 0.6\mu$m in the guide. Erring on the side
of caution, we chose a grid of 0.02μm for our simulations corresponding to a grid spacing of λ/30. A time step of 1.3dx/c√3 was used. With the stated values for the grid, simulating the whole structure would require 100 × 120 × 4000 grid points or over 2 Gigabites of memory. This is considerably more memory than we had available, and would have taken far too long to calculate even on an available Cray T90 supercomputer. To reduce the computational burden, we decided to input a short pulse, fit the computational domain around the pulse such that the computational walls are placed where the field is negligible, and slide the computational domain with the propagating pulse (Figure 5-6). Note that such an approach is only valid for forward propagating fields. To further minimize the computational burden, simple conducting boundary conditions were chosen over the more elegant Perfectly Matched Layer (PML) boundaries. In order to minimize radiation out

![Figure 5-6: Method for reducing the computational burden by sliding the computational domain with the propagating field.](image)

of our structure, we used the previously calculated modal field distributions to define the input field in cross-section. The envelope of the field was then chosen to be a gaussian pulse of the form

$$\Psi(z) = \exp \left( -\frac{z^2}{2\Delta^2} \right)$$

(5.2)

with ∆ = 0.5μm. Such a short pulse enabled the use of a small computational box (6μm) and provided a high degree of spectral information. By choosing a time-space relationship of cos(ωt − βz) for the $E_x$ and $E_y$ components of the field, using these fields to derive the $E_z$ and the $H$ field components, and then setting $t = 0$, we forced the field to propagate in
the $+\hat{z}$ direction.

The simulations were performed with the input field in guide 1, and thus the transfer is from TE-like to TM-like. However, through reciprocity and time reversal, it is clear that the TM-like to TE-like conversion would be identical. Images of the cross-sectional intensities are presented in Figure 5-8. At a distance of 41.6$\mu$m, roughly half of the power has transferred over to guide 2, and at a distance of 83.2$\mu$m essentially all of the power has transferred over with the polarization having been effectively rotated by $90^\circ$. For our concept to work, the polarization rotator must be sufficiently broadband to cover the entire telecommunications band. In order to determine the spectral behavior of the device, it is first necessary to determine the time and position for which full power transfer occurs. The power of an electromagnetic field propagating in the $\hat{z}$ direction, is found from the Pointing vector to be

$$P = \frac{1}{2} \int \int \mathbf{E} \times \mathbf{H}^* \cdot \hat{z} \, dx \, dy$$

(5.3)

Using Maxwell’s equations we can write (5.3) entirely in terms of the electric field. By further taking advantage of the symmetry properties of the mode, we can write

$$P = \frac{\beta}{2\omega\mu} \int \int \left( |E_x|^2 + |E_y|^2 \right) \, dx \, dy$$

(5.4)

To calculate the power within the mode, we express the field as an infinite summation of modes with amplitude coefficients $a_n$

$$\mathbf{E} = \sum_n a_n \mathbf{e}_n$$

(5.5)

The amplitude coefficient $a_m$ of the field contained within a mode $m$, is found by dotting both sides of (5.5) by $e_m^*$ and taking the double integral of both sides.

$$\int \int e_m^* \cdot \mathbf{E} \, dx \, dy = \int \int \sum_n a_n e_m^* \cdot e_n \, dx \, dy$$

(5.6)

Using the principle of orthogonality and rearranging, we have

$$a_m = \frac{\int \int e_m^* \cdot \mathbf{E} \, dx \, dy}{\int \int e_m \cdot e_m^* \, dx \, dy}$$

(5.7)
By combining (5.4) with (5.7), the power propagating within a mode $m$ can be determined

$$P_m = \frac{\beta}{2\omega \mu} \left| \int \int e_{xm}^* E_x dx dy \right|^2 + \left| \int \int e_{ym}^* E_y dx dy \right|^2 $$

(5.8)

This calculation was performed at every $z$ position in the grid and for every time step $n$. To determine an aggregate value for each time step, a summation over $z$ was performed. The results are plotted in Figure 5-7. Approximate position values were obtained by keeping track of the number computation domain slides. Maximal transfer was obtained at distance of roughly 83$\mu$m with almost 75% of the energy propagating in guide 2. The noise in the modal energy in guide 1 at the beginning of the simulation is due to the non-adiabatic transition used in the simulation. That is, the modal field of guide 1 is used at the input, but the presence of guide 2 is in fact a large perturbation which caused reflections and scattering. The scattering is a likely cause of the minor loss made in the rotation of the field. Although it appears from the plot that the power transfer was incomplete, this is in fact difficult to determine because the modes are not distinct. However, it is likely that some power was not transferred over since the structure is not diagonally symmetric. Said otherwise, the perturbation on guide 1 from the presence of guide 2 is not equivalent to the perturbation on guide 2 from the presence of guide 1. This could easily be corrected by making the perturbation smaller or, alternatively, shifting guide 2 down by the height of guide 1.

Spectral information was obtained by invoking a Discrete Fourier Transform (DFT) of both the input and output modal fields and dividing the output response by the input response.

$$T(\omega) = \frac{(\int a_{1n}^{\text{out}} \cos(\omega t) dt)^2 + (\int a_{1n}^{\text{out}} \sin(\omega t) dt)^2}{(\int a_{1n}^{\text{in}} \cos(\omega t) dt)^2 + (\int a_{1n}^{\text{in}} \sin(\omega t) dt)^2}$$

(5.9)

The results are presented in Figure 5-9. We see that the response of the device is in fact extremely broadband.
Figure 5-7: Energy transfer between the guides of the Polarization Rotator as a function of time (upper) and approximate distance (lower)
Figure 5-8: FDTD obtained cross-sectional intensity patterns
Figure 5-9: Spectrum of Polarization Rotator obtained through a discrete Fourier transform in the FDTD simulation
5.3 Concept overview

At the beginning of this chapter we promised to provide an explanation for how to process the random polarization from the fiber into a pair of separate but identical TE-like polarization states. The first operation to be performed on the field is to separate the polarizations. A basic polarization splitter can be formed by simply coupling a pair of waveguides and taking advantage of the naturally occurring difference in the TE and TM coupling coefficients (Figure 5-10). It is essential that such a device be broadband. The degree to which this is possible is determined by the frequency dependence of the coupling coefficient and the number of transfers required for the polarizations to be aligned in opposite guides. The less frequency dependent the coupling coefficient is, and the fewer the number of transfers, the more broadband the polarization splitter. Certainly, this is only one potential approach for splitting the polarization states and that many other designs have been developed[].

Once the polarizations have been separated, the fields must be coupled to the primary waveguides on the chip. For the TE-like polarization, this can be accomplished by adiabatically eroding away the lower portion of the guide (Figure 5-11). By eroding the lower portion of the guide in width rather than height, standard planar fabrication techniques can be utilized. In a similar manner (Figure 5-12), the TM-like polarized mode is conditioned into a shape consistent with guide 2 in Figure 5-4. The TM-like mode is then converted to a TE-like mode through our polarization rotator.

![Coupler based polarization splitter](image)

Figure 5-10: Coupler based polarization splitter
Figure 5-11: Mode transformer for the TE-like mode

Figure 5-12: Mode transformer for the TM-like mode

Figure 5-13: Coupler based polarization rotator
At the output of the device, the process is reversed.

5.4 Summary

We have presented an approach for handling the large polarization sensitivity inherent to high index contrast waveguides. Additionally, we have designed an integrated optic polarization rotator and through finite difference time domain simulations have shown the design to be both low loss and broadband. In the process, we developed an approach for minimizing the computational burden of the FDTD technique for forward propagating fields.
Chapter 6

Conclusion

We began by considering architectures for Add/Drop and Cross-Connect switches utilizing ring resonator switching elements. Absorption was considered in detail as a possible switching mechanism, and we showed that the fundamental loss imposed by absorption based switching (~0.1dB) limits the number of switching elements that a signal may pass by to under 100. Despite this result, useful Add/Drop and small scale Cross-Connect switches can be constructed with absorption based switching elements. However, electro-absorptive techniques considered in prior works were shown to provide insufficient absorption and thus induced much higher losses (≥ 0.45dB) than the theoretical limit. Additionally, we discussed the importance of having ring resonator elements that possessed flat top responses, sharp roll-off and a large free spectral range. A pair of series coupled ring resonators were shown to hold potential for meeting these requirements.

For the purpose of demonstrating an ability to meet the required filter response characteristics, and fixing a design with which to design a switch, we considered the design of the resonator in detail. The resonator waveguides were designed with the aim of providing exceptional bend performance. High index-contrast large-aspect ratio waveguides were used for this purpose. The design suppresses the propagation of the TM-like mode, and therein inhibits coupling between the TE-like and TM-like modes and the multi-peaked filter responses commonly associated with ring resonators. The bend losses as predicted through the effective index method and conformal transformations were shown to be very low with $Q'$s of $10^6$ even at a ring radius of only 1μm. To remain conservative, the resonator was designed with ring radii of 2.54μm and 3.97μm and through the Vernier effect, a free spec-
tral range of greater than 10THz was predicted. The separations between the resonators themselves and between the resonators and the bus waveguides were determined through two dimensional coupling of modes analysis, thereby completing the resonator design.

In order to obtain higher levels of absorption and thus lower device loss, we considered in detail the use of evanescent interaction for the introduction of loss into the resonator. The complex refractive index, thickness, and separation of an absorbing membrane were considered as open variables in the design of the absorber. Through both perturbational analysis and the Finite Difference Time Domain (FDTD) technique, we showed that at an absorber/resonator separation of less than 0.1μm a membrane only 0.2μm thick made from a semiconducting material with complex refractive index similar to indium arsenide can provide an order of magnitude more absorption than electro-absorptive based techniques, leading to losses of only 0.15dB per interaction. Moreover, at an absorber separation of only 0.8μm, the resonator $Q$ is predicted to reach its unperturbed level of greater than $10^6$. As a result of the small membrane thickness and short range of travel, a voltage of only ~10V was determined to be sufficient to actuate the membrane through electrostatic micromechanical actuation. And, with an applied voltage of only 12V and a damping coefficient adjusted to prevent oscillations, turn-on and turn-off times on the order a few microseconds were shown to be possible.

The final component of this work was the discussion of an approach for handling the inherent polarization sensitivity of the high index contrast waveguides used in the resonator design and the design of an integrated optic polarization rotator. Essentially, our approach is to split the polarization states, rotate one, perform separate but identical operations on each, and then rotate the polarization state back to its original state. An integral component of this approach is the design of an integrated optic polarization rotator. The polarization rotator we developed which is based on the coupling of dissimilar high index contrast large aspect ratio waveguides, is, to our knowledge, unique. The device was analyzed through coupling of modes analysis and the finite difference time domain technique. In order to minimize the computational burden, a novel approach to the finite difference time domain was implemented. Since the field was known to be forward propagating, the computational domain was moved with the propagating pulse, thereby reducing the memory and calculation requirements by over an order of magnitude. The simulations showed that the rotator is both low loss and broadband.
In total, we have developed both high level switching architectures and a detailed design for implementing a ring resonator based switching matrix. The next step in this work is clearly the fabrication of both the resonator based switch and the polarization rotator.
Appendix A

Review of Conformal Transformations of Curved Waveguides

Analytic techniques for determining the eigenvalues and eigenfunctions (modes) of curved waveguides are well developed [39, 40, 41]. Provided herein is a review of the approach taken in [39] and [41] for the two dimensional case. We begin with the vector wave equations for the TE case

$$\nabla \times \nabla \times \mathbf{E} - k^2 \mathbf{E} = 0 \quad (A.1)$$

and for the TM case.

$$\nabla \times \nabla \times \mathbf{H} - k^2 \mathbf{H} = 0 \quad (A.2)$$

Using the vector identity

$$\nabla \times \nabla \times \mathbf{A} = \nabla (\nabla \cdot \mathbf{A}) - \nabla^2 \mathbf{A} \quad (A.3)$$

and taking the term $\nabla \cdot \mathbf{E} = 0$ for the 2D TE case and $\nabla \cdot \mathbf{H} = 0$ for the 2D TM case we arrive at the scalar wave equation.

$$\nabla^2 \psi + k^2 \psi = 0 \quad (A.4)$$
Since we are seeking the solution to the circular guide depicted in Figure A-1a, it is beneficial to use the polar form of the scalar wave equation.

\[
\frac{\partial^2 \psi}{\partial r^2} + \frac{1}{r} \frac{\partial \psi}{\partial r} + \frac{1}{r^2} \frac{\partial^2 \psi}{\partial \phi^2} + k^2 \psi = 0
\] (A.5)

As written, a solution to the wave equation is not obvious. However, if we introduce the following change of variables

\[
u = R_2 \ln(r/R_2), \quad v = R_2 \phi
\] (A.6)

(Note: \(R_1\) is equally applicable) and allow for the refractive index profile to take on an effective refractive index profile of

\[n(u) = ne^{\frac{u}{R_2}}\] (A.7)

the wave equation for the new coordinate system \((u, v)\) becomes analogous to the wave equation in cartesian coordinates

\[
\frac{\partial^2 \psi}{\partial u^2} + \frac{\partial^2 \psi}{\partial v^2} + k^2(u)\psi = 0
\] (A.8)

We have made what is commonly referred to as a conformal transformation. In the new coordinate system, the curved guides have been effectively straightened (Figure A-1b).

The wave equation can be simplified further by isolating the \(u\) and \(v\) components of \(\psi\). However, as a consequence of the circular symmetry of the original problem, in order for \(\psi(u)\) to remain invariant along \(v\), \(k_v\) must vary along \(u\). Thus, the \(u\) and \(v\) components of \(\psi\) are inherently coupled. However, we assume that the components are separable and that the field propagates with a propagation constant \(k_v\), where \(k_v\) is now taken to be the propagation constant at the outer edge \((R_2)\) of the guide.

\[
\psi(u, v) = \psi(u)e^{jk_v v}
\] (A.9)

The wave equation simplifies to

\[
\frac{\partial^2 \psi(u)}{\partial u^2} = \left(k_v^2 - k_i^2 e^{2u/R_2}\right) \psi(u)
\] (A.10)
The solution can be obtained directly using the WKB approximation [40] or, alternatively, by linearizing the exponential term [39] [41]. We have chosen the later approach for its simplicity. By expanding the exponential in a Taylor series about $u = R_2$, keeping only the first two terms, and defining the variables $\varepsilon = k_v^2$, $c_i = k_i^2$, and $d_i = \frac{2k_i^2}{R_2}$, the wave equation becomes

$$\frac{\partial^2 \psi(u)}{\partial u^2} = (\varepsilon - c_i - d_i u) \psi(u)$$

(A.11)

By further making the change of variables $Z_i = (\varepsilon - c_i - d_i u)/d_i^{2/3}$ the wave equation reduces to the Airy equation

$$\frac{\partial^2 \psi(Z_i)}{\partial Z_i^2} = -Z_i \psi(Z_i)$$

(A.12)

We have reduced the complex polar wave equation to an equation with a known solution (i.e. a linear combination of Airy functions)

$$\psi(Z_i) = a_i Ai(Z_i) + b_i Bi(Z_i)$$

(A.13)

To arrive at the total solution, boundary conditions must be considered. For the TE case, the electric and magnetic fields must be continuous at the boundaries. Thus, using the
such that \( Z_i^- \) and \( Z_i^+ \) denote \( Z \) immediately to the left and right of the \( i \)th boundary, respectively, and \( \Delta u_i \) corresponds to the thickness of the \( i \)th layer, we arrive at the following set of relations

\[
\begin{align*}
    a_i Ai(Z_i^-) + b_i Bi(Z_i^-) &= a_{i+1} Ai(Z_i^+) + b_{i+1} Bi(Z_i^+) \quad \text{(A.14a)} \\
    \left\{ a_i Ai'(Z_i^-) + b_i Bi'(Z_i^-) \right\} d_i^{1/3} &= \left\{ a_{i+1} Ai'(Z_i^+) + b_{i+1} Bi'(Z_i^+) \right\} d_{i+1}^{1/3} \quad \text{(A.14b)}
\end{align*}
\]

and applying the boundary conditions for the TM case, we find similarly that

\[
\begin{align*}
    a_i Ai(Z_i^-) + b_i Bi(Z_i^-) &= a_{i+1} Ai(Z_i^+) + b_{i+1} Bi(Z_i^+) \quad \text{(A.15)} \\
    \left\{ a_i Ai'(Z_i^-) + b_i Bi'(Z_i^-) \right\} n_i^2 d_i^{1/3} &= \left\{ a_{i+1} Ai'(Z_i^+) + b_{i+1} Bi'(Z_i^+) \right\} n_{i+1}^2 d_{i+1}^{1/3} \quad \text{(A.16)}
\end{align*}
\]

Solving (A.14a) and (A.14b) gives

\[
\begin{pmatrix}
    a_i \\
    b_i
\end{pmatrix} =
\begin{pmatrix}
    p_i & q_i \\
    r_i & s_i
\end{pmatrix}
\begin{pmatrix}
    a_{i+1} \\
    b_{i+1}
\end{pmatrix}
\quad \text{(A.17)}
\]

where

\[
\begin{align*}
    p_i &= \pi \left\{ Ai(Z_i^+) Bi'(Z_i^-) - \left( \frac{d_{i+1}}{d_i} \right)^{1/3} Ai'(Z_i^+) Bi(Z_i^-) \right\} \\
    q_i &= \pi \left\{ Bi(Z_i^+) Bi'(Z_i^-) - \left( \frac{d_{i+1}}{d_i} \right)^{1/3} Bi'(Z_i^+) Bi(Z_i^-) \right\} \\
    r_i &= -\pi \left\{ Ai(Z_i^+) Ai'(Z_i^-) - \left( \frac{d_{i+1}}{d_i} \right)^{1/3} Ai'(Z_i^+) Ai(Z_i^-) \right\} \\
    s_i &= -\pi \left\{ Bi(Z_i^+) Ai'(Z_i^-) - \left( \frac{d_{i+1}}{d_i} \right)^{1/3} Bi'(Z_i^+) Ai(Z_i^-) \right\}
\end{align*}
\]
for the TE case. Similarly, solving (A.15a) and (A.16b) we have for the TM case.

\[
p_i = \pi \left\{ A_i(Z_i^+)B_i'(Z_i^-) - \left( \frac{n_i}{n_{i+1}} \right)^2 \left( \frac{d_{i+1}}{d_i} \right)^{1/3} A_i'(Z_i^+)B_i(Z_i^-) \right\}
\]

\[
q_i = \pi \left\{ B_i(Z_i^+)B_i'(Z_i^-) - \left( \frac{n_i}{n_{i+1}} \right)^2 \left( \frac{d_{i+1}}{d_i} \right)^{1/3} B_i'(Z_i^+)B_i(Z_i^-) \right\}
\]

\[
r_i = -\pi \left\{ A_i(Z_i^+)A_i'(Z_i^-) - \left( \frac{n_i}{n_{i+1}} \right)^2 \left( \frac{d_{i+1}}{d_i} \right)^{1/3} A_i'(Z_i^+)A_i(Z_i^-) \right\}
\]

\[
s_i = -\pi \left\{ B_i(Z_i^+)A_i'(Z_i^-) - \left( \frac{n_i}{n_{i+1}} \right)^2 \left( \frac{d_{i+1}}{d_i} \right)^{1/3} B_i'(Z_i^+)A_i(Z_i^-) \right\}
\]

We have developed the general solution for a multi-layer curved structure. From (A.17), the distribution at any layer can be determined if the distribution of a single layer is known.

We are interested in the three layer system depicted in Figure A-1 for which the field is guided in layer II. The system matrix is found by multiplying the matrices of the first and second layers together to obtain

\[
\begin{pmatrix}
M_{11} & M_{12} \\
M_{21} & M_{22}
\end{pmatrix} =
\begin{pmatrix}
p_{1}p_{2} + q_{1}r_{2} & p_{1}q_{2} + q_{1}s_{2} \\
r_{1}p_{2} + s_{1}r_{2} & r_{1}q_{2} + s_{1}s_{2}
\end{pmatrix}
\]

where the term \( k^2 = \frac{\varepsilon}{\varepsilon_{o}} \) is the eigenvalue to be determined. As formulated, \( k_{v} \) represents \( \beta \), the real part of the complex propagation constant.

The field must decay to zero as \( u \to \pm \infty \). Therefore, from the properties of the Airy functions \( A_i \) and \( B_i \), we find that \( a_3 = b_1 = 0 \) which leads to the eigenvalue equation \( M_{22} = 0 \). The imaginary part, or loss coefficient, \( \alpha \), can be obtained readily by forming an analogy with the quantum mechanics problem of tunneling through a barrier [42]. The barrier in this case is the effective index profile forming the potential between regions II and III as depicted in Figure 1(b). In the WKB approximation, the power transmission coefficient, \( T \),
is given by
\[ T = e^{-\int_{u_0}^{u_2} \kappa(u) du} \]  
(A.20)

where
\[ \kappa(u) = \sqrt{\beta^2 - k_0^2 e^{\frac{2\pi u}{\lambda}}} \]  
(A.21)

Since only a single reflection off of the outer wall of the waveguide occurs in a travel length of one effective wavelength \( \frac{2\pi}{\beta} \), the power lost is equal to \( 1 - T \). Thus, a correspondance can be made between \( \alpha \) and \( 1 - T \).

\[ e^{-2a \frac{\pi}{\beta}} = 1 - e^{-\int_{u_0}^{u_2} \kappa(u) du} \]  
(A.22)

Solving for \( \alpha \), we find
\[ \alpha = -\frac{\beta}{\pi} \ln \left( 1 - e^{-\int_{u_0}^{u_2} \kappa(u) du} \right) \]  
(A.23)

We have thus determined approximate relationships for the real and imaginary components, \( \beta \) and \( \alpha \), respectively, of the complex propagation constant.
Appendix B

The Finite Difference Time Domain Technique

A complete description of the FDTD technique is presented in the book by A. Taflove [38]. The bare essentials of the approach are reviewed here. The FDTD technique represents a discretized implementation of Maxwell’s equations with no inherent accuracy limitations or need for root searching algorithms. The fields are updated in time rather than space and N-dimensional grids are required for N-dimensional problems. The computational grid is arranged such that divergence operations $\nabla \cdot \mathbf{D} = \rho$ and $\nabla \cdot \mathbf{B} = 0$ are naturally maintained. Additionally, the electric and magnetic field components are offset from one another by a grid point so as to enable second order accuracy in the derivatives of Maxwell’s equations. As an example of how the Maxwell’s equations are discretized, consider Maxwell’s equation for the $E_z$ field component in a dielectric.

$$\frac{\partial}{\partial t} E_z = \frac{1}{\varepsilon} \left[ \frac{\partial}{\partial x} H_y - \frac{\partial}{\partial y} H_x \right] \quad (B.1)$$

For use in FDTD, equation (B.1) is discretized by central differences in both time and space.

$$\frac{E_z^{n+1}_{i,j,k} - E_z^n_{i,j,k}}{\Delta t} = \frac{1}{\varepsilon_{i,j,k}} \left( \frac{H_z^{n+1/2}_{i,j+1/2,k} - H_z^{n+1/2}_{i,j-1/2,k}}{\Delta y} + \frac{H_y^{n+1/2}_{i+1/2,j,k} - H_y^{n+1/2}_{i-1/2,j,k}}{\Delta x} \right) \quad (B.2)$$
The rest of the field components are found similarly, such that the updated fields become

\[
\begin{align*}
H_x^{n+1/2}_i,j,k & = H_x^{n-1/2}_i,j,k + \frac{\Delta t}{\mu_{i,j,k}} \left( \frac{E_y^{n+1/2}_i,j,k+1/2 - E_y^{n+1/2}_i,j,k-1/2}{\Delta z} + \frac{E_z^{n+1/2}_i,j,k+1/2 - E_z^{n+1/2}_i,j,k-1/2}{\Delta y} \right) \\
H_y^{n+1/2}_i,j,k & = H_y^{n-1/2}_i,j,k + \frac{\Delta t}{\mu_{i,j,k}} \left( \frac{E_x^{n+1/2}_i,j,k+1/2 - E_x^{n+1/2}_i,j,k-1/2}{\Delta z} + \frac{E_z^{n+1/2}_i,j,k+1/2 - E_z^{n+1/2}_i,j,k-1/2}{\Delta x} \right) \\
H_z^{n+1/2}_i,j,k & = H_z^{n-1/2}_i,j,k + \frac{\Delta t}{\mu_{i,j,k}} \left( \frac{E_x^{n+1/2}_i,j,k+1/2 - E_x^{n+1/2}_i,j,k-1/2}{\Delta y} + \frac{E_y^{n+1/2}_i,j,k+1/2 - E_y^{n+1/2}_i,j,k-1/2}{\Delta x} \right)
\end{align*}
\]

\[
\begin{align*}
E_x^{n+1}_i,j,k & = E_x^{n}_i,j,k + \frac{\Delta t}{\varepsilon_{i,j,k}} \left( \frac{H_y^{n+1/2}_i,j,k+1/2 - H_y^{n+1/2}_i,j,k-1/2}{\Delta z} + \frac{H_z^{n+1/2}_i,j,k+1/2 - H_z^{n+1/2}_i,j,k-1/2}{\Delta x} \right) \\
E_y^{n+1}_i,j,k & = E_y^{n}_i,j,k + \frac{\Delta t}{\varepsilon_{i,j,k}} \left( \frac{H_x^{n+1/2}_i,j,k+1/2 - H_x^{n+1/2}_i,j,k-1/2}{\Delta z} + \frac{H_z^{n+1/2}_i,j,k+1/2 - H_z^{n+1/2}_i,j,k-1/2}{\Delta y} \right) \\
E_z^{n+1}_i,j,k & = E_z^{n}_i,j,k + \frac{\Delta t}{\varepsilon_{i,j,k}} \left( \frac{H_x^{n+1/2}_i,j,k+1/2 - H_x^{n+1/2}_i,j,k-1/2}{\Delta y} + \frac{H_y^{n+1/2}_i,j,k+1/2 - H_y^{n+1/2}_i,j,k-1/2}{\Delta x} \right)
\end{align*}
\]

where \( i, j, \) and \( k \) refer to the \( x, y \) and \( z \) grid points and \( n \) refers to the current time step.

As might be expected, for discretizations in space and time approach zero, the algorithm becomes exact. However, a less obvious result is that algorithm also becomes exact for a time step of \( \Delta t = \Delta x / (c\sqrt{3}) \), which Taflove refers to as the *magic time step*. Importantly, this value of \( \Delta t \) also represents the upper bound for stable operation of the algorithm. For time steps greater than the magic time step, the field grows without bound. And, for time steps less than the magic time step, numerical dispersion creeps into the propagating field. It is therefore desirable to choose a time step that is close to, but slightly less than the magic time step. Residual numerical dispersion can then be minimized by choosing a sufficiently fine spatial grid. However, clearly the use of finely spaced grid points increases the number of calculations required to update the field. Therefore, it is important to choose a fine, but not overly fine grid spacing. Taflove demonstrates that accurate results can be obtained with a spatial discretization of \( \lambda/20 \) and a time step of \( \Delta t = \Delta x / (2c) \).

The program used to calculate the index distribution and the actual FDTD programs used to calculate the absorber and polarization rotator structures are presented in the subsequent pages.
!This is the FDTD program used to calculate the impact of the absorber on the resonator.

dimension Cax(102,102,17), Cay(102,102,17), Caz(102,102,17)
dimension Cbx(102,102,17), Cby(102,102,17), Cbz(102,102,17)
dimension Hx(17,102,102), Hy(17,102,102), Hz(17,102,102), Cb(102,102), Ca(102,102)
dimension Ex(17,102,102), Ey(17,102,102), Ez(17,102,102)
dimension IEx(17,102,102), IEy(17,102,102), IEz(17,102,102)
dimension SIE(102,102), SIEy(102,102), SIEz(102,102)
dimension SExz(17,102), SEy_xz(17,102), SEz_xz(17,102)
dimension Ax(102,102), Ay(102,102), Az(102,102), Energy(10000), Energy1(10000)
dimension savewhen(600), pullwhen(5000), pushwhen(200)
dimension Exout(60), Eyout(60), Ezout(60), Ixout(60), Iyout(60), Izout(60)
real(8) Hx, Hy, Hz, Ex, Ey, Ez, Ax, Ay, Az, Ca
real(8) Cb, IEx, IEy, IEz, SIE, SIEy, SIEz, SExz, SEy_xz, SEz_xz
real(8) eps0, ncore, nclad, beta, lambda, c, pi, mu0, omega, dx, dy, dz, dt, delta, Energy1
real(8) time_dif, time_begin, time_end, savewhen, pullwhen, pushnumber
integer(4) mmax, n, nmax, xmax, ymax, zmax, xmaxp, ymaxp, zmaxp, ix, iy, iz
integer(4) nowsave, pullwhen, nowpull, pushwhen, iter, savewhen
real(8) epscore, epsclad, try, Energy
integer(4) xmaxAx, ymaxAx, xmaxAy, ymaxAy, xmaxAz, ymaxAz
character(27) Exout, Eyout, Ezout, Ixout, Iyout, Izout
INTEGER(4) status, MaxEz

nmax = 5000
savenumber = 500.0

!Set the core and cladding indices
ncore = 3.48
nclad = 1.44

!Set the grid size
xmax = 100
ymax = 100
zmax = 15

!Determine when to save
do i = 1, ceiling(nmax/savenumber)
savewhen(i) = i*int(savenumber)
print*, savewhen(i)
enddo

!Set the output file names
Exout(1) = 'c:\matlabrl2\work\Exout001r'
Exout(2) = 'c:\matlabrl2\work\Exout011r'
Exout(3) = 'c:\matlabrl2\work\Exout021r'
Exout(4) = 'c:\matlabrl2\work\Exout031r'
Exout(5) = 'c:\matlabrl2\work\Exout041r'
Exout(6) = 'c:\matlabrl2\work\Exout051r'
Exout(7) = 'c:\matlabrl2\work\Exout061r'
Exout(8) = 'c:\matlabrl2\work\Exout071r'
Exout(9) = 'c:\matlabrl2\work\Exout081r'
!%Define grid inputs
dx = .02
dy = dx
dz = dx

!Define basic constants
pi = 3.1415927
eps0 = 8.854187*0.000000000001*0.000001
mu0 = 4*pi*0.0000001*0.000001
c = sqrt(1/(eps0*mu0))
lambda = 1.55
omega = 2*pi*c/lambda
dt = 1.3*dx/(c*sqrt(3.0)) !Time step

!Define region size and material properties
blay = 1
xmaxp = xmax+2*blay
ymaxp = ymax+2*blay
zmaxp = zmax+2*blay

xmaxAx = 101
ymaxAx = 100
xmaxAy = 100
ymaxAy = 101
xmaxAz = 101
ymaxAz = 101

!! Read in the Modal Distribution
Ax = 0
Ay = 0
Az = 0

OPEN (1, FILE = 'c:\matlabr12\work\Ax')
READ(1,*) ((Ax(i,j),j=1,xmaxAx),i=1,ymaxAx)
close(1)

OPEN (2, FILE = 'c:\matlabr12\work\Ay')
READ(2,*) ((Ay(i,j),j=1,xmaxAy),i=1,ymaxAy)
close(2)

OPEN (3, FILE = 'c:\matlabr12\work\Az')
READ(3,*) ((Az(i,j),j=1,xmaxAz),i=1,ymaxAz)
close(3)

!Read in the index information
!allocate(Ca(xmaxp,ymaxp), Cb(xmaxp,ymaxp))
open(4,FILE = 'c:\matlabr12\work\Cax.txt')
do ix=1,xmaxp
do iy=1,ymaxp
do iz =1,zmaxp
   read(4,*) (Cax(ix,iy,iz))
   enddo
endo
doendo
close(4)

open(5,FILE = 'c:\matlabr12\work\Cbx.txt')
do ix=1,xmaxp
do iy=1,ymaxp
   do iz =1,zmaxp
      read(5,*) (Cbx(ix,iy,iz))
      enddo
   enddo
endo
close(5)

open(6,FILE = 'c:\matlabr12\work\Cay.txt')
do ix=1,xmaxp
do iy=1,ymaxp
   do iz =1,zmaxp
      read(6,*) (Cay(ix,iy,iz))
      enddo
   enddo
endo
close(6)

open(7,FILE = 'c:\matlabr12\work\Cby.txt')
do ix=1,xmaxp
do iy=1,ymaxp
   do iz =1,zmaxp
      read(7,*) (Cby(ix,iy,iz))
      enddo
   enddo
endo
close(7)

open(8,FILE = 'c:\matlabr12\work\Caz.txt')
do ix=1,xmaxp
do iy=1,ymaxp
  do iz =1,zmaxp
    read(8,*) (Caz(ix,iy,iz))
  enddo
enddo
close(8)

open(9,FILE =c:\matlabrl2\work\Cbz.txt')
  do ix=1,xmaxp
    do iy=1,ymaxp
      do iz =1,zmaxp
        read(9,*) (Cbz(ix,iy,iz))
      enddo
    enddo
  enddo
close(9)

beta = 2.49*2*pi/lambda
print*,beta,dz

!! Set the longitudinal field distribution
Ex = 0
Ey = 0
Ez = 0
Hx = 0
Hy = 0
Hz = 0
do ix = 1,xmaxp-1
  do iy = 1,ymaxp-1
    do iz = 1,zmaxp-1
      Ex(iz,iy,ix) = (Ax(ix,iy)*dt/(mu0*dx))*cos(beta*(iz-zmaxp/2-1/2)*dz)
      Ey(iz,iy,ix) = ((Ay(ix,iy))*dt/(mu0*dx))*cos(beta*(iz-zmaxp/2-1/2)*dz)
      Ez(iz,iy,ix) = ((Az(ix,iy))*dt/(mu0*dx))*sin(-beta*(iz-zmaxp/2)*dz)
      Hx(iz,iy,ix) = (1/(omega*mu0))*cos(beta*(iz-zmaxp/2)*dz)*(-beta*Ay(ix,iy)+Az(ix,iy-1))/dy
      Hy(iz,iy,ix) = (1/(omega*mu0))*cos(beta*(iz-zmaxp/2)*dz)*(-(Az(ix,iy)-Az(ix-1,iy))/dx+beta*Ax(ix,iy))
      Hz(iz,iy,ix) = -(1/(omega*mu0))*sin(-beta*(iz-zmaxp/2-1/2)*dz)*((Ay(ix,iy)-Ay(ix-1,iy))/dx - (Ax(ix,iy) - Ax(ix,iy-1))/dy)
    enddo
  enddo
enddo

!General Finite Differencing Begins
Energy = 0
Energy1 = 0
print*,c

iter=0
!loop
call cpu_time(time_begin)
!BEGIN
!FINITE
!DIFFERENCING
do n = 1,nmax
  do ix = 2,xmaxp-1
    do iy = 2,ymaxp-1
      do iz = 2,zmaxp-1
        Hx(iz,iy,ix) = Hx(iz,iy,ix) + Ey(iz+1,iy,ix) - Ey(iz,iy,ix) - Ez(iz,iy,ix) + Ez(iz,iy-1,ix)
        Hy(iz,iy,ix) = Hy(iz,iy,ix) + Ez(iz,iy,ix) + Ez(iz,iy,ix-1) - Ex(iz+1,iy,ix) - Ex(iz,iy,ix)
        Hz(iz,iy,ix) = Hz(iz,iy,ix) + Ex(iz,iy,ix) + Ex(iz,iy-1,ix) - Ez(iz,iy,ix) - Ez(iz,iy,ix-1)
      ! Energy1(n) = Energy1(n) + Hx(iz,iy,ix)**2 + Hy(iz,iy,ix)**2 + Hz(iz,iy,ix)**2
      enddo
    enddo
  enddo
enddo

do ix = 2,xmaxp-1
  do iy = 2,ymaxp-1
    do iz = 2,zmaxp-1
      Ex(iz,iy,ix) = Cax(ix,iy,iz)*Ex(iz,iy,ix) + Cbx(ix,iy,iz)*(Hz(iz,iy+1,ix)-Hz(iz,iy,ix) + Hy(iz-1,iy,ix))
      Ey(iz,iy,ix) = Cay(ix,iy,iz)*Ey(iz,iy,ix) + Cby(ix,iy,iz)*(Hx(iz,iy,ix)-Hx(iz-1,iy,ix) + Hz(iz,iy,ix+1))
      Ez(iz,iy,ix) = Caz(ix,iy,iz)*Ez(iz,iy,ix) + Cbz(ix,iy,iz)*(Hy(iz,iy,ix+1)-Hy(iz,iy,ix) - Hx(iz,iy+1,ix))
    enddo
  enddo
enddo

do ix = (xmaxp-30)/2+1, (xmaxp+30)/2-1
  do iy = (ymaxp-10)/2+1, (ymaxp+10)/2-1
    do iz = 4,zmaxp-3
      Energy(n) = Energy(n) + Ex(iz,iy,ix)**2 + Ey(iz,iy,ix)**2 + Ez(iz,iy,ix)**2
      Energy1(n) = Energy1(n) + Hy(iz,iy,ix)**2 + Hz(iz,iy,ix)**2
    enddo
  enddo
enddo

!General Finite Differencing Ends

!END
!FINITE
!DIFFERENCING

!Begin Saving
try = real(n)/savenumber-.001
nowsave=ceiling(try)
print*,savewhen(nowsave),nowsave
if(savewhen(nowsave).eq.n) then
  iter = iter+1
endif
do ix = 1,xmaxp
    do iy = 1,ymaxp
        do iz = 1,zmaxp
            IE(iy,ix,iz) = (mu0*dx)/(dt*dt)*Ex(iy,ix,iz)*Ex(iy,ix,iz)
            IEy(iy,ix,iz) = (mu0*dx)/(dt*dt)*Ey(iy,ix,iz)*Ey(iy,ix,iz)
            IEz(iy,ix,iz) = (mu0*dx)/(dt*dt)*Ez(iy,ix,iz)*Ez(iy,ix,iz)
        enddo
    enddo
enddo

SIE = Sum(IEx, dim=1)
SIEy = Sum(IEy, dim=1)
SIEz = Sum(IEz, dim=1)

open(I0, FILE = Ixout(iter))
    do ix = 1,xmaxp
        do iy = 1,ymaxp
            write(I0,*) SIE(iy,ix)
        enddo
    enddo
close(I0)

open(I10, FILE = Iyout(iter))
    do ix = 1,xmaxp
        do iy = 1,ymaxp
            write(I10,*) SIEy(iy,ix)
        enddo
    enddo
close(I10)

open(I20, FILE = Izout(iter))
    do ix = 1,xmaxp
        do iy = 1,ymaxp
            write(I20,*) SIEz(iy,ix)
        enddo
    enddo
close(I20)

SE_xz = Sum(((mu0*dx)/dt)*Ex, dim=2)
SEy_xz = Sum(((mu0*dx)/dt)*Ey, dim=2)
SEz_xz = Sum(((mu0*dx)/dt)*Ez, dim=2)

open(I30, FILE = Exout(iter))
    do ix = 1,xmaxp
        do iz = 1,zmaxp
            write(I30,*) SE_xz(iz,ix)
        enddo
    enddo
close(I30)
open(50,FILE = Eyou(iter))
  do ix = 1,xmaxp
    do iz = 1,zmaxp
      write(50,*) SEy_xz(iz,ix)
    enddo
  enddo
close(50)

open(60,FILE = Ezout(iter))
  do ix = 1,xmaxp
    do iz = 1,zmaxp
      write(60,*) SEz_xz(iz,ix)
    enddo
  enddo
close(60)
endif
endo
doo
call cpu_time(time_end)
time_dif = time_end-time_begin

open(70,FILE = 'c:\matlabr12\work\Energy03')
  do n = 1,nmax
    write(70,*) Energy(n)
  enddo
close(70)
print*,time_dif

open(80,FILE = 'c:\matlabr12\work\Energy1a')
  do n = 1,nmax
    write(80,*) Energy1(n)
  enddo
close(80)
print*,time_dif

END
This program was used to generate the index profile used for determining the effect of the absorber on a standing wave resonator.

INTEGER(2) i, j, status, MaxEz
real(4), allocatable :: Ca(:, :), Cb(:, :)
real(4), allocatable :: Cax(:, :), Cbx(:, :), Cay(:, :), Cby(:, :), Caz(:, :), Cbz(:, :)
real(4), allocatable :: epsx(:, :), epsy(:, :), epsz(:, :), eps(:, :, :)
real(4), allocatable :: sigmax(:, :), sigmay(:, :), sigmaz(:, :), sigma(:, :, :)
real(4) eps0, lambda, c, mu0, omega, pi, dx, dy, dt, delta, epscore, epsclad
real(4) ncore, nelad, sigma0, sigmaabs, width, separation, epsclada
integer ib, blay, ix, iy, xmax, ymax, zmax, xmaxp, ymaxp, zmaxp
integer nwidth, nseparation, nwidth_u
real(4) Caxc, Cayc, Cazc, Cbxc, Cbyc, Cbzc
integer nheight, nheight_1, nheight_u
real(4) height

! Define basic constants
pi = 3.1415927
eps0 = 8.854187*0.000000000001*0.000001
mu0 = 4*pi*0.0000001*0.000001

! Define grid inputs
dx = .02
dy = dx
dz = dx
dt = 1.3*dx/(c*sqrt(3.0))
delta = .4

! Define region size and material properties
xmaxp = xmax+2*blay
ymaxp = ymax+2*blay
zmaxp = zmax + 2*blay
alphab = 10
separation = 0.1

sigmab = 10000000*eps0*nclad*nclad*omega
sigma0 = 0
sigma_abs = 2*eps0*omega*0.2
epscore = eps0*ncore*ncore
epsclad = eps0*nclad*nclad
epsclada = eps0*3.51**2

allocate(eps(xmaxp,ymaxp,zmaxp), epsx(xmaxp,ymaxp,zmaxp), epsy(xmaxp,ymaxp,zmaxp), epsz(xmaxp,ymaxp,zmaxp))
allocate(sigma(xmaxp,ymaxp,zmaxp), sigmax(xmaxp,ymaxp,zmaxp), sigmay(xmaxp,ymaxp,zmaxp), sigmaz(xmaxp,ymaxp,zmaxp))
allocate(Cax(xmaxp,ymaxp,zmaxp), Cay(xmaxp,ymaxp,zmaxp), Caz(xmaxp,ymaxp,zmaxp))
allocate(Cbx(xmaxp,ymaxp,zmaxp), Cby(xmaxp,ymaxp,zmaxp), Cbz(xmaxp,ymaxp,zmaxp))

! Set the guide dimensions
nwidth = width/dx
nheight = height/dy
nwidth_l = (xmaxp-nwidth)/2
nwidth_u = (xmaxp+nwidth)/2
nheight_l = (ymaxp-nheight)/2
nheight_u = (ymaxp+nheight)/2
nseparation = floor(separation/dx)

! The guide
eps = epsclad
sigma = 0
do iz = 1,zmaxp
  do iy = (ymaxp-nheight)/2+1, (ymaxp+nheight)/2-1
    do ix = (xmaxp-nwidth)/2+1, (xmaxp+nwidth)/2-1
      eps(ix,iy,iz) = epscore
    enddo
  enddo
enddo

! The absorber
do iz = 1,zmaxp
  do iy = (ymaxp-nheight)/2+1-nseparation-nheight, (ymaxp+nheight)/2-1-nseparation-nheight
    do ix = 1,xmaxp
      sigma(ix,iy,iz) = sigma_abs
      eps(ix,iy,iz) = epsclada
    enddo
  enddo
enddo

! Boundaries - perfectly conducting
do ix = 1,xmaxp
do iy = 1,ymaxp
  sigma(ix,iy,1) = sigmab
  sigma(ix,iy,zmaxp) = sigmab
  sigma(ix,iy,2) = sigmab
  sigma(ix,iy,zmaxp-1) = sigmab
enddo
enddo

do iy = 1,xmaxp
  do iz = 1,zmaxp
    sigma(1,iy,iz) = sigmab
    sigma(xmaxp,iy,iz) = sigmab
    sigma(2,iy,iz) = sigmab
    sigma(xmaxp-1,iy,iz) = sigmab
  enddo
enddo

do ix = 1,xmaxp
  do iz = 1,zmaxp
    sigma(ix,1,iz) = sigmab
    sigma(ix,ymaxp,iz) = sigmab
    sigma(ix,2,iz) = sigmab
    sigma(ix,ymaxp-1,iz) = sigmab
  enddo
enddo

do ix = 2,xmaxp-1
  do iy = 2,ymaxp-1
    do iz = 2,zmaxp-1
      epsx(ix,iy,iz) = (eps(ix,iy+1,iz)+eps(ix,iy,iz))/2
      sigmax(ix,iy,iz) = (sigma(ix,iy+1,iz)+sigma(ix,iy,iz))/2
      epsy(ix,iy,iz) = (eps(ix+1,iy,iz)+eps(ix,iy,iz))/2
      sigmay(ix,iy,iz) = (sigma(ix+1,iy,iz)+sigma(ix,iy,iz))/2
      epsz(ix,iy,iz) = ((eps(ix+1,iy+1,iz+1)+eps(ix+1,iy,iz+1)+eps(ix,iy,iz+1)+eps(ix,iy+1,iz+1))/8) &
                      + ((eps(ix+1,iy+1,iz)+eps(ix+1,iy,iz)+eps(ix,iy,iz)+eps(ix,iy+1,iz))/8)
      sigmaz(ix,iy,iz) = ((sigma(ix+1,iy+1,iz+1)+sigma(ix+1,iy,iz-1)+sigma(ix,iy,iz+1)+sigma(ix,iy+1,iz))/8) &
                      + ((sigma(ix+1,iy+1,iz)+sigma(ix+1,iy,iz)+sigma(ix,iy,iz)+sigma(ix,iy+1,iz))/8)
    enddo
  enddo
enddo

!Boundary
do ix = 1,xmaxp
  do iy = 1,ymaxp
    do iz = 1,zmaxp
      Cax(ix,iy,iz) = (1-(sigmax(ix,iy,iz)*dt)/(2*epsx(ix,iy,iz)))/(1+(sigmax(ix,iy,iz)*dt)/(2*epsx(ix,iy,iz)))
    enddo
  enddo
enddo
Cay(ix,iy,iz) = (1-(sigmay(ix,iy,iz)*dt)/(2*epsy(ix,iy,iz)))/(1+(sigmay(ix,iy,iz)*dt)/(2*epsy(ix,iy,iz)))
Caz(ix,iy,iz) = (1-(sigmaz(ix,iy,iz)*dt)/(2*epsz(ix,iy,iz)))/(1+(sigmaz(ix,iy,iz)*dt)/(2*epsz(ix,iy,iz)))

Cbx(ix,iy,iz) = ((dt*dt)/(mu0*epsx(ix,iy,iz)*dx*dx))/(1+sigmax(ix,iy,iz)*dt/(2*epsx(ix,iy,iz)))
Cby(ix,iy,iz) = ((dt*dt)/(mu0*epsy(ix,iy,iz)*dx*dx))/(1+sigmay(ix,iy,iz)*dt/(2*epsy(ix,iy,iz)))
Cbz(ix,iy,iz) = ((dt*dt)/(mu0*epsz(ix,iy,iz)*dx*dx))/(1+sigmaz(ix,iy,iz)*dt/(2*epsz(ix,iy,iz)))

allocate(Ca(xmaxp,ymaxp),Cb(xmaxp,ymaxp))
do ix = 1,xmaxp
do iy = 1,ymaxp
Ca(ix,iy) = Cax(ix,iy,10)
Cb(ix,iy) = Cbx(ix,iy,10)
enddo
enddo

allocate(Ca(xmaxp,ymaxp),Cb(xmaxp,ymaxp))
do ix = 1,xmaxp
do iy = 1,ymaxp
Ca(ix,iy) = Cax(ix,iy,10)
Cb(ix,iy) = Cbx(ix,iy,10)
enddo
enddo

!print*,Cb(50,50), Cb(90,50), Cb(50,90), Cb(50,10), Cb(10,50), epsclad, epscore
open(20,FILE = 'c:\matlabrl2\work\Cax.txt')
  do ix=,xmaxp
do iy=1,ymaxp
  do iz =1,zmaxp
    write(20,*) (Cax(ix,iy,iz))
  enddo
  enddo
enddo
close(20)

open(30,FILE = 'c:\matlabrl2\work\Cbx.txt')
  do ix=1,xmaxp
do iy=1,ymaxp
  do iz =1,zmaxp
    write(30,*) (Cbx(ix,iy,iz))
  enddo
enddo
close(30)

open(40,FILE = 'c:\matlabrl2\work\Cay.txt')
  do ix=1,xmaxp
do iy=1,ymaxp
  do iz =1,zmaxp
    write(40,*) (Cay(ix,iy,iz))
  enddo
enddo
close(40)

open(50,FILE = 'c:\matlabrl2\work\Cby.txt')
  do ix=1,xmaxp
do iy=1,ymaxp
    do iz = 1,zmaxp
        write(50,*) (Cby(ix,iy,iz))
    enddo
enddo
enddo

end

end
! This is a Finite Difference Time Domain program for propagating electromagnetic fields
! The program allows for the computational domain to be moved with the propagating field
! and was used to simulate the polarization rotator.

! Begin by defining the variables to be used

**dimension** Hx(302,102,112), Hy(302,102,112), Hz(302,102,112), Cb(112,102), Ca(112,102)
**dimension** Ex(302,102,112), Ey(302,102,112), Ez(302,102,112)
**dimension** IEx(302,102,112), IEy(302,102,112), IEz(302,102,112)
**dimension** SIE(102,112), SIEy(102,112), SIEz(102,112)
**dimension** SEx_xz(302,112), SEy_xz(302,112), SEz_xz(302,112)
**dimension** Ax(102,102), Ay(102,102), Az(102,102)
**dimension** savewhen(600), pullwhen(5000), pushwhen(200),
Exout(60), Eyout(60), Ezout(60), Ixout(60), Iyout(60), Izout(60)
**dimension** glx(102,112), gly(102,112), glz(102,112), g2x(102,112), g2y(102,112), g2z(102,112)
**dimension** Overlap1(302,24000), Overlap2(302,24000), Field1(302,24000), Field2(302,24000)
**real(4)** glx, gly, glz, g2x, g2y, g2z, Overlap1, Overlap2
**real(4)** Hx, Hy, Hz, Ex, Ey, Ez, Ax, Ay, Az, Ca, Cb, IEx, IEy, IEz, SIE, SIEy, SIEz, SEx_xz, SEy_xz, SEz_xz
**real(4)** eps0, ncore, nclad, beta, lambda, c, pi, nu0, omega, dx, dy, dz, dt, delta
**real(4)** time_dif, time_begin, time_end, savewhen, pullwhen, pushnumber
**integer(4)** nmax, n, xmax, ymax, zmax, xmaxp, ymaxp, zmaxp, ix, iy, iz
**integer(4)** nowsave, pullwhen, pushwhen, pushnumber, savewhen
**real(4)** epscore, epsclad, try, Field1, Field2
**integer(4)** xmaxAx, ymaxAx, xmaxAy, ymaxAy, xmaxAz, ymaxAz
**character(26)** Exout, Eyout, Ezout, Ixout, Iyout, Izout, Ovout1, Ovout2
**INTEGER(4)** status, MaxEz, nz

! Set the number of time steps
nmax = 2400
! Tell the computer on what time step to save the field/intensity distributions
savewhen = 400.0
! Tell the computer how often to shift the field/computational domain
pullnumber = 10.0

! Take inputs
! Inputs
ncore = 3.48
nclad = 1.44
xmax = 110
ymax = 100
zmax = 300

do i = 1, ceiling(nmax/savewhen)
savewhen(i) = i*int(savewhen)
print*, Savewhen(i)
endo

do i = 1, floor(nmax/pullnumber)+1
pullwhen(i) = i*int(pullnumber)
endo
! do n = 1, floor(nmax/pushnumber)+1

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!pushwhen(n) = n*int(pushnumber)
!enddo

n=0

!Name the output files
Ovout1 = 'c:\matlabr12\work\Ovout001'
Ovout2 = 'c:\matlabr12\work\Ovout002'

Exout(1)='c:\matlabr12\work\Exout001'
Exout(2)='c:\matlabr12\work\Exout001'
Exout(3)='c:\matlabr12\work\Exout001'
Exout(4)='c:\matlabr12\work\Exout001'
Exout(5)='c:\matlabr12\work\Exout001'
Exout(6)='c:\matlabr12\work\Exout001'
Exout(7)='c:\matlabr12\work\Exout001'
Exout(8)='c:\matlabr12\work\Exout001'
Exout(9)='c:\matlabr12\work\Exout001'
Exout(10)='c:\matlabr12\work\Exout001'

Eyout(1)='c:\matlabr12\work\Eyout001'
Eyout(2)='c:\matlabr12\work\Eyout001'
Eyout(3)='c:\matlabr12\work\Eyout001'
Eyout(4)='c:\matlabr12\work\Eyout001'
Eyout(5)='c:\matlabr12\work\Eyout001'
Eyout(6)='c:\matlabr12\work\Eyout001'
Eyout(7)='c:\matlabr12\work\Eyout001'
Eyout(8)='c:\matlabr12\work\Eyout001'
Eyout(9)='c:\matlabr12\work\Eyout001'
Eyout(10)='c:\matlabr12\work\Eyout001'

Ezout(1)='c:\matlabr12\work\Ezout001'
Ezout(2)='c:\matlabr12\work\Ezout001'
Ezout(3)='c:\matlabr12\work\Ezout001'
Ezout(4)='c:\matlabr12\work\Ezout001'
Ezout(5)='c:\matlabr12\work\Ezout001'
Ezout(6)='c:\matlabr12\work\Ezout001'
Ezout(7)='c:\matlabr12\work\Ezout001'
Ezout(8)='c:\matlabr12\work\Ezout001'
Ezout(9)='c:\matlabr12\work\Ezout001'
Ezout(10)='c:\matlabr12\work\Ezout001'

Ixout(1)='c:\matlabr12\work\Ixout001'
Ixout(2)='c:\matlabr12\work\Ixout001'
Ixout(3)='c:\matlabr12\work\Ixout001'
Ixout(4)='c:\matlabr12\work\Ixout001'
Ixout(5)='c:\matlabr12\work\Ixout001'
Ixout(6)='c:\matlabr12\work\Ixout001'
Ixout(7)='c:\matlabr12\work\Ixout001'
Ixout(8)='c:\matlabr12\work\Ixout001'
Ixout(9)='c:\matlabr12\work\Ixout001'
blay = 1

!%Define grid inputs
dx = .02
dy = dx
dz = dx

!Define basic constants
pi = 3.1415927
eps0 = 8.854187*0.000000000001*0.000001
mu0 = 4*pi*0.0000001*0.000001
c = sqrt(1/(eps0*mu0))
lambda = 1.55
omega = 2*pi*c/lambda
!sigma0 = 0

dt = (1.3017 + 0.002*1.3017)*dx/(c*sqrt(3.0))
delta = 0.5

!Define region size and material properties
xmaxp = xmax+2*blay
ymaxp = ymax+2*blay
zmaxp = zmax+2*blay

xmaxAx = 101
ymAx = 100
xmaAx = 100
ymAy = 101
xmaxAy = 101
ymAz = 101
xmaxAz = 101

!! Read in the Modal Distribution
allocate(Ca(xmaxp,ymaxp), Cb(xmaxp,ymaxp))

Ax = 0
Ay = 0
Az = 0

! Import the field distributions
OPEN (1, FILE = 'c:\matlabr12\work\Ax')
READ(1,*) ((Ax(i,j),j=1,xmaxAx),i=1,ymaxAx)
close(1)

OPEN (2, FILE = 'c:\matlabr12\work\Ay')
READ(2,*) ((Ay(i,j),j=1,xmaxAy),i=1,ymaxAy)
close(2)

OPEN (3, FILE = 'c:\matlabr12\work\Az')
READ(3,*) ((Az(i,j),j=1,xmaxAz),i=1,ymaxAz)
close(3)

! Import the refractive index information
OPEN (4, FILE = 'c:\matlabr12\work\Cb.txt')
READ(4,*) ((Cb(i,j),j=1,ymaxp),i=1,xmaxp)
close(4)

OPEN (5, FILE = 'c:\matlabr12\work\Ca.txt')
READ(5,*) ((Ca(i,j),j=1,ymaxp),i=1,xmaxp)
close(5)

! print*,Cb(50,50),Cb(10,50)

! Set the initial propagation constant
beta = 2.5*2*pi/lambda

! Center the E-field over the guide
Ax = eoshift(eoshift(Ax,shift=6,dim=7),shift=5,dim=3)
Ay = eoshift(eoshift(Ay,shift=6,dim=1),shift=5,dim=4)
Az = eoshift(eoshift(Az,shift=6,dim=1),shift=6,dim=4)

! Establish the H field
do ix = 5,xmaxp-4
  do iy = 5,ymaxp-4
    glx(iy,ix) = (-beta*Ay(iy,ix) + (Az(iy,ix) - Az(iy-1,ix))/dy)
    gly(iy,ix) = (-Az(iy,ix) - Az(iy-1,ix))/dx + beta*Ax(iy,ix))
\( g_{1z}(iy,ix) = \frac{-((Ay(ix,iy)-Ay(ix-1,iy))/dx - (Ax(ix,iy) - Ax(ix,iy-1))/dy)}{\Delta y} \)

```
enddo
enddo

!! Set the longitudinal field distribution
Ex = 0
Ey = 0
Ez = 0
Hx = 0
Hy = 0
Hz = 0

do ix = 5,xmaxp-4
  do iy = 5,ymaxp-4
    do iz = 5,zmaxp-4
      Ex(iz,iy,ix) = (Ax(ix,iy)*dt/(mu0*dx))*cos(beta*(iz-zmaxp/2-1/2)*dz)
                      & *exp(-((iz-zmaxp/2-1/2)*dz)*((iz-zmaxp/2-1/2)*dz)/(2*delta*delta))
      Ey(iz,iy,ix) = ((Ay(ix,iy))*dt/(mu0*dx))*cos(beta*(iz-zmaxp/2-1/2)*dz)
                      & *exp(-((iz-zmaxp/2-1/2)*dz)*((iz-zmaxp/2-1/2)*dz)/(2*delta*delta))
      Ez(iz,iy,ix) = ((Az(ix,iy))*dt/(mu0*dx))*sin(-beta*(iz-zmaxp/2)*dz)
                      & *exp(-((iz-zmaxp/2)*dz)*((iz-zmaxp/2)*dz)/(2*delta*delta))
      Hx(iz,iy,ix) = (1/(omega*mu0))*cos(beta*(iz-zmaxp/2)*dz)*gLx(iy,ix) & *exp(-((iz-zmaxp/2)*dz)*((iz-zmaxp/2)*dz)/(2*delta*delta))
      Hy(iz,iy,ix) = (1/(omega*mu0))*cos(beta*(iz-zmaxp/2)*dz)*gLy(iy,ix) & *exp(-((iz-zmaxp/2)*dz)*((iz-zmaxp/2)*dz)/(2*delta*delta))
      Hz(iz,iy,ix) = (1/(omega*mu0))*sin(-beta*(iz-zmaxp/2-1/2)*dz)*gLz(iy,ix) & *exp(-((iz-zmaxp/2-1/2)*dz)*((iz-zmaxp/2-1/2)*dz)/(2*delta*delta))
    enddo
  enddo
enddo

!! H mode for guide 2

do ix = 1,xmaxp-10
  do iy = 1,ymaxp-14
    g2x(ix+9,ymaxp-iy+10) = gLy(iy+4,ix)
    g2y(ix+9,ymaxp-iy+10) = gLx(iy+4,ix)
    g2z(ix+9,ymaxp-iy+10) =gLz(iy+4,ix)
  enddo
enddo

! General Finite Differencing Begins

print*,c

iter=0
! loop
  call cpu_time(time_begin)
  do n = 1,nmax
    print*,n
```
!Determine whether or not to shift the field/computational domain
nowpull = floor(n/pullnumber)
if (pullwhen(nowpull).eq.n) then
  do ix = 2,xmaxp-2
    do iy = 2,ymaxp-2
      do iz = 2,zmaxp-2
        Hx(iz,iy,ix) = Hx(iz+2,iy,ix)
        Hy(iz,iy,ix) = Hy(iz+2,iy,ix)
        Hz(iz,iy,ix) = Hz(iz+2,iy,ix)
        Ex(iz,iy,ix) = Ex(iz+2,iy,ix)
        Ey(iz,iy,ix) = Ey(iz+2,iy,ix)
        Ez(iz,iy,ix) = Ez(iz+2,iy,ix)
      enddo
    enddo
  endif
!BEGIN
!FINITE
!DIFFERENCING
  do ix = 2,xmaxp-1
    do iy = 2,ymaxp-1
      do iz = 2,zmaxp-1
        Hx(iz,iy,ix) = Hx(iz,iy,ix) + Ey(iz+1,iy,ix) - Ey(iz,iy,ix) - Ez(iz,iy,ix) + Ez(iz,iy-1,ix)
        Hy(iz,iy,ix) = Hy(iz,iy,ix) + Ez(iz,iy,ix) - Ez(iz,iy,ix-1) - Ex(iz+1,iy,ix) + Ex(iz,iy,ix)
        Hz(iz,iy,ix) = Hz(iz,iy,ix) + Ex(iz,iy,ix) - Ex(iz,iy,ix-1) - Ey(iz+1,iy,ix) + Ey(iz,iy,ix-1)
      enddo
    enddo
  enddo
!Calculate the overlap integrals for the two guides
  Overlap1(iz,n) = Overlap1(iz,n) + (Hx(iz,iy,ix)*glx(iy,ix)) + & (Hy(iz,iy,ix)*gly(iy,ix)) + (Hz(iz,iy,ix)*glz(iy,ix))
  Overlap2(iz,n) = Overlap2(iz,n) + (Hx(iz,iy,ix)*g2x(iy,ix)) + & (Hy(iz,iy,ix)*g2y(iy,ix)) + (Hz(iz,iy,ix)*g2z(iy,ix))
  enddo
enddo
!Set up perfectly conducting boundary conditions at the z = 0 and z = zmaxp boundaries
  Ex(2,iy,ix) = 0
Ex(2,iy,ix) = ((Ca(ix,iy+1)+Ca(ix,iy))/2)*Ex(iz,iy,ix) + & ((Cb(ix,iy+1)+Cb(ix,iy))/2)*(Hz(iz,iy,ix)-Hz(iz,iy+1,ix)-Hy(iz,iy,ix)+Hy(iz-1,iy,ix))
  Ey(iz,iy,ix) = ((Ca(ix,iy+1)+Ca(ix,iy))/2)*Ey(iz,iy,ix) + & ((Cb(ix,iy+1)+Cb(ix,iy))/2)*Hx(iz,iy,ix)-Hx(iz+1,iy,ix)-Hz(iz,iy,ix)+Hz(iz-1,iy,ix)
  Ez(iz,iy,ix) = ((Ca(ix+1,iy)+Ca(ix,iy))/2)*Ez(iz,iy,ix) + & ((Cb(ix+1,iy)+Cb(ix,iy))/2)*(Hy(iz,iy,ix)-Hy(iz-1,iy,ix)+Hx(iz,iy,ix)+Hx(iz-1,iy,ix))
    enddo
!Set up perfectly conducting boundary conditions at the z = 0 and z = zmaxp boundaries
  Ex(2,iy,ix) = 0

Ey(2,iy,ix) = 0
Ez(2,iy,ix) = 0

Ex(3,iy,ix) = 0
Ey(3,iy,ix) = 0
Ez(3,iy,ix) = ((Cb(ix+1,iy+1)+Cb(ix+1,iy)+Cb(ix,iy)+Cb(ix,iy+1))/8) & *(Hy(3,iy,ix+1)-Hy(3,iy,ix) - Hx(3,iy+1,ix) + Hx(3,iy,ix))

Ex(zmaxp-2,iy,ix) = ((Ca(ix,iy+1)+Ca(ix,iy))/2)*Ex(zmaxp-2,iy,ix) + & ((Cb(ix,iy+1)+Cb(ix,iy))/2)*(Hz(zmaxp-2,iy+1,ix)-Hz(zmaxp-2,iy,ix) - & Hy(zmaxp-2,iy,ix) + Hy(zmaxp-3,iy,ix))
Ey(zmaxp-2,iy,ix) = ((Ca(ix+1,iy)+Ca(ix,iy))/2)*Ey(zmaxp-2,iy,ix) + & ((Cb(ix+1,iy)+Cb(ix,iy))/2)*(Hx(zmaxp-2,iy,ix)-Hx(zmaxp-3,iy,ix) - & Hz(zmaxp-2,iy,ix+1) + Hz(zmaxp-2,iy,ix))
Ez(zmaxp-2,iy,ix) = 0

Ex(zmaxp-1,iy,ix) = 0
Ey(zmaxp-1,iy,ix) = 0
Ez(zmaxp-1,iy,ix) = 0
endo
write(10,*) SIEy(iy,ix)
    enddo
  enddo
close(10)

open(20,FILE = Iyout(iter))
  do ix = 1,xmaxp
    do iy = 1,ymaxp
      write(20,*) SIEy(iy,ix)
    enddo
  enddo
close(20)

open(30,FILE = Izout(iter))
  do ix = 1,xmaxp
    do iy = 1,ymaxp
      write(30,*) SIEz(iy,ix)
    enddo
  enddo
close(30)

!Calculate and save the x-z field distributions
SEx_xz = Sum((mu0*dx)/dt)*Ex,dim=2)
SEy_xz = Sum((mu0*dx)/dt)*Ey,dim=2)
SEz_xz = Sum((mu0*dx)/dt)*Ez,dim=2)

open(40,FILE = Exout(iter))
  do ix = 1,xmaxp
    do iz = 1,zmaxp
      write(40,*) SEx_xz(iz,ix)
    enddo
  enddo
close(40)

open(50,FILE = Eyout(iter))
  do ix = 1,xmaxp
    do iz = 1,zmaxp
      write(50,*) SEy_xz(iz,ix)
    enddo
  enddo
close(50)

open(60,FILE = Ezout(iter))
  do ix = 1,xmaxp
    do iz = 1,zmaxp
      write(60,*) SEz_xz(iz,ix)
    enddo
  enddo
close(60)
endif
enddo

call cpu_time(time_end)
time_dif = time_end-time_begin

print*,time_dif

!Save the overlap integrals
open(110,FILE = 'c:\matlabr12\work\Ovout1')
do n = 1,nmax
   do iz = 1,zmaxp
      write(110,*) Overlap1(iz,n)
   enddo
enddo
close(110)

open(120,FILE = 'c:\matlabr12\work\Ovout2')
do n = 1,nmax
   do iz = 1,zmaxp
      write(120,*) Overlap2(iz,n)
   enddo
enddo
close(120)

END
This program was used to set up the index profile for the polarization rotator.

INTEGER(2) i, j, status, MaxEz
real(4), allocatable :: Ca(:, :), Cb(:, :)
real(4), allocatable :: epsx(:, :), epsy(:, :), epsz(:, :)
real(4) eps0, lambda, c, mu0, omega, pi, dx, dy, dt, delta
real(4) ncore, nclad, sigma0, width, separation
integer ib, blay, ix, iy, xmax, ymax, xmaxp, ymaxp
integer nwidth, nseparation, nwidth_l, nwidth_u
real(4) Caxc, Cayc, Cazc, Cbxc, Cbyc, Cbz
integer nheight, nheight_l, nheight_u
real(4) height

! Define basic constants
pi = 3.1415927
eps0 = 8.854187*0.000000000001*0.000001
mu0 = 4*pi*0.0000001*0.000001

! Define grid inputs
dx = .02
dy = dx
dz = dx
dt = (1.3017 + .002*1.3017)*dx/(c*sqrt(3.0))
delta = .4

! Define region size and material properties
xmaxp = xmax + 2*blay
ymaxp = ymax + 2*blay
zmaxp = ymax + 2*blay
alpha = 10
\[ \text{sigmab} = 1000000 \times \varepsilon_0 \times n_{\text{clad}} \times n_{\text{clad}} \times \omega \]
\[ \sigma_0 = 0 \]
\[ \varepsilon_{\text{score}} = \varepsilon_0 \times n_{\text{core}} \times n_{\text{core}} \]
\[ \varepsilon_{\text{clad}} = \varepsilon_0 \times n_{\text{clad}} \times n_{\text{clad}} \]

! Set the guide dimensions
\[ n_{\text{width}} = \text{width}/dx \]
\[ n_{\text{height}} = \text{height}/dy \]
\[ n_{\text{width}}_l = (x_{\text{maxp}} + n_{\text{width}})/2 \]
\[ n_{\text{width}}_u = (x_{\text{maxp}} - n_{\text{width}})/2 \]
\[ n_{\text{height}}_l = (y_{\text{maxp}} + n_{\text{height}})/2 \]
\[ n_{\text{height}}_u = (y_{\text{maxp}} - n_{\text{height}})/2 \]
\[ n_{\text{separation}} = \text{floor}(\text{separation}/dx) - 1 \]

allocate(Ca(x_{\text{maxp}}, y_{\text{maxp}}), Cb(x_{\text{maxp}}, y_{\text{maxp}}))

\[ \text{Ca} = (1 - (\sigma_0 \times \text{dt})/(2 \times \varepsilon_{\text{clad}}))/(1 + (\sigma_0 \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
\[ \text{Cb} = ((\text{dt} \times \text{dt})/(\mu_0 \times \varepsilon_{\text{core}} \times dx \times dx)) \]

!guide 1
\[ \text{do} \quad \text{ix} = (x_{\text{maxp}} - 10 + n_{\text{width}})/2 + 1 - 5, (x_{\text{maxp}} + n_{\text{width}})/2 - 1 - 5 \]
\[ \text{do} \quad \text{iy} = (y_{\text{maxp}} + n_{\text{height}})/2 + 1 - 10, (y_{\text{maxp}} + 10 + n_{\text{height}})/2 - 1 - 10 \]
\[ \text{Ca}(\text{ix}, \text{iy}) = (1 - (\sigma_0 \times \text{dt})/(2 \times \varepsilon_{\text{core}}))/(1 + (\sigma_0 \times \text{dt})/(2 \times \varepsilon_{\text{core}})) \]
\[ \text{Cb}(\text{ix}, \text{iy}) = ((\text{dt} \times \text{dt})/(\mu_0 \times \varepsilon_{\text{core}} \times dx \times dx)) \]
\[ \text{enddo} \]
\[ \text{enddo} \]

!guide 2
\[ \text{do} \quad \text{ix} = (x_{\text{maxp}} - 10 + n_{\text{width}} + 2 \times n_{\text{separation}})/2 + 1 - 5, (x_{\text{maxp}} + n_{\text{width}} + 2 \times n_{\text{separation}})/2 - 1 - 5 \]
\[ \text{do} \quad \text{iy} = (y_{\text{maxp}} + n_{\text{height}} + 2 \times n_{\text{separation}})/2 + 1 - 10, (y_{\text{maxp}} + 10 + n_{\text{height}} + 2 \times n_{\text{width}})/2 - 1 - 10 \]
\[ \text{Ca}(\text{ix}, \text{iy}) = (1 - (\sigma_0 \times \text{dt})/(2 \times \varepsilon_{\text{core}}))/(1 + (\sigma_0 \times \text{dt})/(2 \times \varepsilon_{\text{core}})) \]
\[ \text{Cb}(\text{ix}, \text{iy}) = ((\text{dt} \times \text{dt})/(\mu_0 \times \varepsilon_{\text{core}} \times dx \times dx)) \]
\[ \text{enddo} \]
\[ \text{enddo} \]

\[ \text{do} \quad \text{ix} = 1, x_{\text{maxp}} \]
\[ \text{Ca}(\text{ix}, 1) = (1 - (\sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}}))/(1 + (\sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
\[ \text{Ca}(\text{ix}, y_{\text{maxp}}) = (1 - (\sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}}))/(1 + (\sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
\[ \text{Cb}(\text{ix}, 1) = ((\text{dt} \times \text{dt})/(\mu_0 \times \varepsilon_{\text{core}} \times dx \times dx))/(1 + \sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
\[ \text{Cb}(\text{ix}, y_{\text{maxp}}) = ((\text{dt} \times \text{dt})/(\mu_0 \times \varepsilon_{\text{core}} \times dx \times dx))/(1 + \sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
\[ \text{enddo} \]

\[ \text{do} \quad \text{iy} = 1, y_{\text{maxp}} \]
\[ \text{Ca}(1, \text{iy}) = (1 - (\sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}}))/(1 + (\sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
\[ \text{Ca}(x_{\text{maxp}}, \text{iy}) = (1 - (\sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}}))/(1 + (\sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
\[ \text{Cb}(1, \text{iy}) = ((\text{dt} \times \text{dt})/(\mu_0 \times \varepsilon_{\text{core}} \times dx \times dx))/(1 + \sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
\[ \text{Cb}(x_{\text{maxp}}, \text{iy}) = ((\text{dt} \times \text{dt})/(\mu_0 \times \varepsilon_{\text{core}} \times dx \times dx))/(1 + \sigma_{\text{mab}} \times \text{dt})/(2 \times \varepsilon_{\text{clad}})) \]
$$Cb(x_{\text{maxp}}, iy) = \frac{(dt^2)}{(\mu_0 \varepsilon_{\text{core}} dx^2)} / (1 + \frac{\sigma_b dt}{2 \varepsilon_{\text{clad}}})$$

$$Ca(2, iy) = \frac{1 - (\sigma_b dt)}{(2 \varepsilon_{\text{clad}})} / (1 + (\sigma_b dt)(2 \varepsilon_{\text{clad}}))$$
$$Ca(x_{\text{maxp}-1}, iy) = \frac{1 - (\sigma_b dt)}{(2 \varepsilon_{\text{clad}})} / (1 + (\sigma_b dt)(2 \varepsilon_{\text{clad}}))$$

$$Cb(2, iy) = \frac{(dt^2)}{(\mu_0 \varepsilon_{\text{core}} dx^2)} / (1 + \sigma_b dt / (2 \varepsilon_{\text{clad}}))$$
$$Cb(x_{\text{maxp}-1}, iy) = \frac{(dt^2)}{(\mu_0 \varepsilon_{\text{core}} dx^2)} / (1 + \sigma_b dt / (2 \varepsilon_{\text{clad}}))$$

enddo

open(200, FILE = 'c:\matlabrl2\work\Ca.txt')
    do ix= 1, xmaxp
        do iy=1, ymaxp
            write(200, *) (Ca(ix,iy))
        enddo
    enddo
enddo
close(200)

open(300, FILE = 'c:\matlabrl2\work\Cb.txt')
    do ix= 1, xmaxp
        do iy=1, ymaxp
            write(300, *) (Cb(ix,iy))
        enddo
    enddo
enddo
close(300)
end
Bibliography


[20] The modesolver was written by Milos Popovic


[42] H. Kroemer, Quantum Mechanics, New Jersey: Prentice-Hall, Ch. 6, (1994)