A System study on the use of Aspirated Technology in Gas Turbine Engines

by

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ABSTRACT

Increasing aircraft engine efficiency and reducing the engines weight have driven innovation in the aircraft engine business since its inception. By simply looking at the Brayton cycle increasing the compressor pressure ratio can bring about an increase in efficiency. To achieve this high-pressure ratio, multi-stage axial compressors are used, which tend to be both heavy and expansive. Increasing the number of stages in an axial compressor can increase the pressure ratio and therefore the thermal efficiency; however as the number of stages increases, the engine weight, cost and length also increase, all of which are detrimental to the overall aircraft performance. Recent work by Kerrebrock, Merchant, and Schuler, has led to the possibility of achieving high pressure ratios with a reduction in the number of stages. These compressors use aspiration, or suction on the surface of the blades and endwalls, to keep the boundary layer attached over a greater percentage of the blade chord. Keeping the boundary layer attached longer allows the each blade row to be more highly loaded than the equivalent non-aspirated blade. This higher loading means fewer stages are needed to achieve a given pressure rise.

The extracted air is brought inside the blade where it is removed at a convenient location. This bleed air can contain a substantial amount of energy that can be used for numerous purposes on the aircraft or engine. Recovery of the bleed flow and its disposition are important factors in the success of aspirated compressor technology. In this study it is assumed the bleed air can be used for threes purposes: its is returned to the turbine as cooling air, expanded overboard to augment the engine thrust or used to perform "auxiliary work" in a different part of the aircraft. The thermodynamic efficiency (as measured by the specific impulse) and the installed efficiency of the compression system were calculated for different engine/fan configurations and compared with equivalent non-aspirated engines. This allows the effects of aspiration to be quantified and can be used to assess if aspiration is viable for a specific setting.

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In the lab I would first like to thank Professor Kerrebrock, my advisor, whose advice and help were a substantial played as substantial part in the completion of this thesis. I would like to thank Dr. Ali Merchant, for his help with the one-dimensional flow model and my numerous questions on aerodynamics. I would finally like to thank, Ms. Jodie Kirchner whose Excel spread sheet of her counter-rotating compressor mean line design saved me a lot of time and helped me improve my own Excel programming skills.

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Nomenclature

Roman

A	Area
C_{pc}	Specific heat capacity at constant pressure in the Compressor
C_{pt}	Specific heat capacity at constant pressure in the turbine
D	Diffusion factor
D	Hydraulic diameter
F	Thrust
I	Rothalpy
I	Specific Impulse
P	Static pressure
P_t	Total pressure
P_{tre1}	Total pressure of 1st stage rotor core compressor bleed
P_{tre2}	Total pressure of 2nd stage rotor core compressor bleed
P_{tre3}	Total pressure of 3rd stage rotor core compressor bleed
P_{tse1}	Total pressure of 1st stage stator core compressor bleed
P_{tse2}	Total pressure of 2nd stage stator core compressor bleed
P_{tse3}	Total pressure of 3rd stage stator core compressor bleed
M	Mach number
R_c	gas constant for air in the compressor
R_t	gas constant for air in the turbine
S	Specific fuel consumption

T	Static temperature
T_t	Total temperature
T_{tre1}	Total temperature of 1st stage rotor core compressor bleed
T_{tre2}	Total temperature of 2nd stage rotor core compressor bleed
T_{tre3}	Total temperature of 3rd stage rotor core compressor bleed
T_{tse1}	Total temperature of 1st stage stator core compressor bleed
T_{tse2}	Total temperature of 2nd stage stator core compressor bleed
T_{tse3}	Total temperature of 3rd stage stator core compressor bleed
V	Velocity in an absolute coordinates
W	Velocity relative to the rotating coordinate system
a	Speed of sound
e	Polytropic efficiency
f	fuel/air ratio
f	fanning friction factor
h	static enthalpy
h_{pr}	low heating value of fuel
$\dot{m}$	mass flow rate
r	radius
v	Swirl velocity

Greek

α	Bypass Ratio
Ω	rotational speed
π	Pressure Ratio

τ	Temperature ratio
ρ	density
ε	Fractional bleed extraction
γ	Ratio of specific heats
η	Efficiency

Subscripts

b	burner
c	core compressor
d	inlet
f	fan
m	mixed
n	nozzle
p	propulsive
r1	1st stage rotor bleed
r2	2nd stage rotor bleed
r3	3rd stage rotor bleed
rf	Fan rotor bleed
s1	1st stage stator bleed
s2	2nd stage stator bleed
s3	3rd stage stator bleed
sf	Fan stator bleed
t	turbine
t	thermal

Chapter 1: Introduction

Since the invention of the gas turbine aircraft engine, airlines, the military and the engine manufacturers have all aggressively pushed to improve the engines overall efficiency and reduce the engines weight. From a simple analysis of the Brayton cycle, increasing the compressor pressure ratio can bring about an increase in efficiency. Most commercial engines that operate on the Brayton cycle currently use multi-stage axial compressors to achieve high pressure ratios. However, as the number of stages used increases, the engine weight, cost and length also increases, all of which are detrimental to the overall aircraft performance. One solution to this problem is to use an aspirated compressor. Blades employing aspiration will decrease both the length of the compressor and the number of blades/disk.

The aspirated compressor uses boundary layer control on the blades and the endwalls, allowing an increase in the amount of work done by each blade row at a given loss level. Keeping the flow attached longer and minimizing the wakes results in lower losses. Boundary layer control is implemented by "sucking" a portion of the blade boundary layer at a critical location just downstream of the shock impingement location. Suction causes the boundary layer to remain attached further downstream of the shock. Aspiration allows for a blade row to function at a higher diffusion factor (0.7) with the viscous losses of a normal blade operating closer to a diffusion factor of 0.5. Diffusion factor is a measure of the turning done by a blade and is defined by

$$D = 1 - \frac{V_c}{V_b} + \frac{|v_c - v_b|}{2\sigma V_b}$$

Where σ is the solidity.

$$\sigma = \frac{c}{s}$$

Suction is accomplished by means of a single slot on the blade suction surface, close to the point of shock impingement. The extracted flow is then brought inside the blade where it can flow radially outwards and be removed from the tip or radially inwards where it can be removed at a convenient location. This suction air can contain a substantial amount of energy (especially if bled from the latter stages of a compressor) that can be used for numerous purposes on the airplane or returned to the engine as turbine cooling air. Recovery of the bleed flow and its disposition in the engine system are important factors in the success of aspirated compressor technology.

If the bleed flow is discarded and it's potential contributions to the overall system ignored, the penalty for using aspirated technology can be high. Although the weight of an engine can be reduced by the use of fewer stages in the compressor, the penalty paid in Specific Impulse may over ride these weight benefits for many applications. This study looks at the effect the bleeds have on the engines thermodynamic efficiency and compares the specific impulse of the aspirated compressor configuration to that achieved by a similar (i.e. same π_c, T_{t4}, flight Mach number etc.) non-aspirated engine. In the comparison, it is assumed that the aspirated bleeds are returned to the engine cycle as turbine cooling air (Core bleed air) or expanded overboard to recover the momentum drag (Fan bleed air). In the analysis of the Turbojet Cycle the bleed air is used for some type of non-specified Auxiliary work.

Another, important issue which bears on the systems analysis, is the conditions of extracted air when it leaves the blade. This study outlines a technique used to calculate the exit conditions (P_t, T_t, Mach number etc.) of the bleed air. Full recovery of the total exit pressure will not be possible and the recovery factor will depend on diffusion system. Knowing these temperatures and pressures, an equivalent isentropic compressor efficiency, defined as the ratio of the total available work in the compressed air (both bleed air and through flow air) to the total work carried out compressing the air, can be calculated. This equivalent compressor efficiency will depend on the recovery factor of the diffusion system, as this will dictate the work available in the bleed air.

In the Turbofan cycle analysis the core aspirated air is returned to the cycle as turbine cooling air. To carry out this mixing analysis the temperature and the pressure of the aspirated air is needed, to calculate the mixed out temperature and pressure. Another use for the exit conditions of the aspirated air is to calculate the amount of auxiliary work, by assuming the bleed air is expanded to the compressor inlet pressure P_{t2}. Calculation of the bleed air exit conditions is performed using a one dimensional compressible flow approximation to the flow in the bleed slot.

Chapter 2: Cycle Analysis

2.0: Introduction

In this section, an outline of the steps involved in a parametric cycle analysis of both aspirated engine configurations and non-aspirated configurations are provided. The non-aspirated engines will act as a baseline configuration and can be used to compare the effects of aspiration (i.e. changing the number of stages with bleed, increasing or decreasing the amount of bleed, pressure recovery achievable etc.). Four different engine configurations will be discussed:

1. Mixed exhaust Turbofan.
2. Mixed exhaust Turbofan with aspirated core compressor and fan.
3. Turbojet with compressor discharge extraction.
4. Turbojet with compressor discharge extraction and aspirated core compressor.

For the aspirated engines (cases 2 & 4), it will be assumed, in both cases that the core is an aspirated 3-stage counter-rotating compressor. All threes stages of the counter rotating compressor are aspirated; aspiration on each stage implies bleed air is removed from both the rotor and the stator. Approximately 1% of the core mass flow is removed by each aspirated blade row.

In both the aspirated engines the bleed air is not simply discarded but returned to the cycle in some form. For the mixed exhaust turbofan the core aspirated air is retuned to the turbine where it is mixed out with the through flow air, while the bleed air form the fan is expanded overboard to recover it's momentum drag.

To make the comparison between the aspirated turbojet and the non-aspirated case more meaningful it will also be assumed that both units do an equal amount of "Auxiliary Work". For the non-aspirated engine, extracting a percentage of the Compressor discharge Air (CDA) will perform this "Auxiliary Work". For the aspirated engine, using a combination of the bleed air and a percentage of the CD air will perform the "Auxiliary Work". The cycle analysis is based on the methods discussed in

1. "***Elements of Gas Turbine Propulsion***" by Jack D. Mattingly
2. "***Aircraft Engines and Gas Turbines***" by Jack L. Kerrebrock

2.1: Mixed Exhaust Turbofan

A schematic of the mixed exhaust turbofan is shown in Figure 2.1. In the turbofan engine, a part of the airflow through the fan bypasses the core; the remainder passes through the compressor, the combustor, and the turbine of the gas generator. The separate airstreams mix before exiting through the nozzle. For convenience in the cycle analysis, the overall compression ratio through the fan and the compressor is denoted by π_c. This is in fact the product of the fan and the compressor pressure ratios. The pressure ratio of the fan alone is denoted by π_f. The schematic shows a normal turbofan with no aspiration or bleed extraction. What follows below is an outline, providing a description of the main steps in the analysis; in fact these steps will form the basis upon which the other 3 cycles are analyzed.

1. Starting with an equation for uninstalled engine thrust, we rewrite this equation in terms of the total pressure and total temperature ratios: the ambient pressure P_0, the temperature T_0, and the speed of sound a_0, and the flight Mach number M_0 as follows:

$$F = (\dot{m}_9 V_9 - \dot{m}_0 V_0) + A_9 (P_9 - P_0)$$

$$\frac{F}{\dot{m}_0} = a_0 \left(\frac{\dot{m}_9}{\dot{m}_0} \frac{V_9}{a_0} - M_0 \right) + \frac{A_9 P_9}{\dot{m}_0} \left(1 - \frac{P_0}{P_9} \right)$$

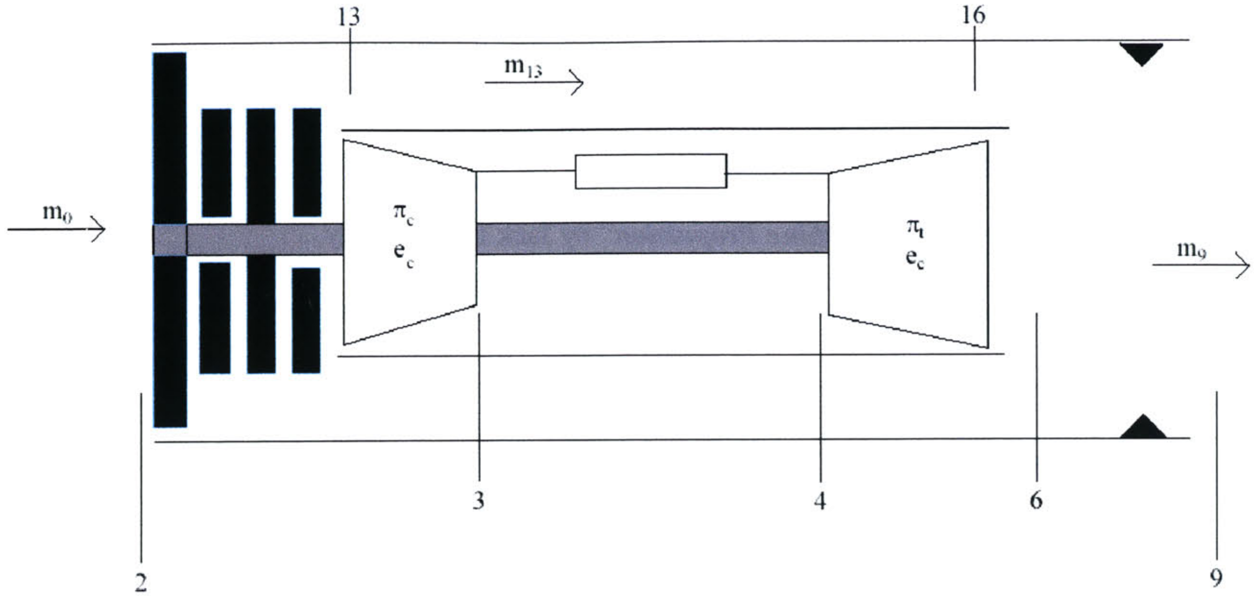


Figure 2.1: Mixed Exhaust Turbofan

2. Express the velocity ratio V_9/a_0 in terms of Mach numbers temperatures, and gas properties of states *0* and *9*:

$$\left(\frac{V_9}{a_0}\right)^2 = \frac{a_9^2 M_9^2}{a_0^2} = \frac{\gamma_9 R_9 T_9}{\gamma_0 R_0 T_0} M_9^2$$

3. Find the exit Mach number M_9. Since

$$P_{t9} = P_9 \left(1 + \frac{\gamma - 1}{2} M_9^2\right)^{\gamma_t/\gamma_t - 1}$$

Then

$$M_9^2 = \frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t - 1)/\gamma_t} - 1\right]$$

Where

$$\frac{P_{t9}}{P_9} = \frac{P_0}{P_9} \pi_r \pi_d \pi_c \pi_b \pi_t \pi_m$$

4. Find the temperature ratio T_9/T_0:

$$\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{T_{t9}/T_9} = \frac{T_{t9}/T_0}{\left(P_{t9}/P_9\right)^{(\gamma_t-1)/\gamma_t}}$$

Where

$$\frac{T_{t9}}{T_0} = \tau_r \tau_d \tau_c \tau_b \tau_t \tau_m$$

5. Apply the first law of thermodynamics to the combustor, and find an expression for the fuel/air ratio f in terms of τ's etc:

$$\dot{m}_3 C_{pc} T_{t3} + \eta_b \dot{m}_{fuel} h_{pr} = \dot{m}_4 C_{pt} T_{t4}$$

6. When applicable, find an expression for the total temperature ratio across the turbine τ_t by relating the turbine power output to the compressor, fan and/or aspirated air requirements. This allows τ_t to be evaluated in terms of other variables.
7. Evaluate the specific thrust using the above results.
8. Evaluate the specific impulse I, using the results for the specific thrust and the fuel/air ratio:

$$I = \frac{F_{total}/\dot{m}_0}{f\,g}$$

9. Develop expressions for the thermal and propulsive efficiencies.

2.1.1: Cycle Analysis

This section outlines the cycle analysis of the mixed exhaust turbofan. Component losses, the mass flow rate of the fuel through the components, the variation of specific heats and losses due to the mixing of the fan and core flows in the exhaust are all included. The analysis will assume one-dimensional flow at the entrance and exit of each component. The variation of the specific heat will be approximated by assuming a perfect gas with constant specific heat c_{pc} upstream of the main burner and a perfect gas with different constant specific heat c_{pt} downstream of the main burner. As alluded to earlier the analysis will be brief and only an outline of the most important steps is included. Detailed analysis of this cycle is provided in both of the references given above. The turbofan with station numbering is shown in Fig 2-1.

The gas flow through the core engine is $\dot{m}_c$, and the gas flow through the fan is $\dot{m}_f$. The ratio of the fan flow to the core flow is defined as the ***bypass ratio*** and is given the symbol alpha α. Thus

$$\alpha = \frac{\dot{m}_f}{\dot{m}_c}$$

The total gas flow is given the symbol $\dot{m}_0$. Thus

$$\dot{m}_0 = \dot{m}_c + \dot{m}_f = (1+\alpha)\dot{m}_c$$

Uninstalled Thrust:

$$F = (\dot{m}_9 V_9 - \dot{m}_0 V_0) + A_9 (P_9 - P_0)$$

Assume the engine exhaust nozzle expands the gas to the ambient pressure $P_0 = P_9$. Therefore

$$\boxed{\frac{F}{\dot{m}_0 a_0} = \left(\frac{\dot{m}_9}{\dot{m}_0}\frac{V_9}{a_0} - M_0\right)} \qquad \textbf{Eqn. (2.1)}$$

Where

$$\boxed{\frac{\dot{m}_9}{\dot{m}_0} = 1 + \frac{f}{(1+\alpha)}}$$ **Eqn. (2.2)**

Placing **Eqn. (2.2)** into **Eqn. (2.1)** gives

$$\boxed{\frac{F}{\dot{m}_0 a_0} = \left[\left(1 + \left(\frac{f}{1+\alpha}\right)\right)\frac{V_9}{a_0} - M_0\right]}$$ **Eqn. (2.3)**

Calculation of $(V_9 / a_0)^2$

For the turbojet cycle, this equation becomes

$$\left(\frac{V_9}{a_0}\right)^2 = \frac{\gamma_9 R_9 T_9}{\gamma_0 R_0 T_0} M_9^2$$ **Eqn. (2.4)**

Calculation of f:

$$f = \frac{\dot{m}_{fuel}}{\dot{m}_c}$$

Application of the 1st law of thermodynamics to the control volume about the combustor shown below in Figure 2-2 gives:

$$\boxed{\dot{m}_c C_{pc} T_{t3} + \eta_b \dot{m}_{fuel} h_{pr} = \dot{m}_4 C_{pt} T_{t4}}$$ **Eqn. (2.5)**

Where $\mathbf{h_{pr}}$ is the thermal energy released by the fuel during combustion.

$$\dot{m}_4 = \dot{m}_c (1+f)$$

Using the above relations in

$$\boxed{f = \frac{[\tau_r \tau_c - \tau_\lambda]}{\left[\tau_\lambda - \left(\eta_b h_{pr} \big/ C_{pc} T_0\right)\right]}}$$ **Eqn. (2.6)**

Where

$$\tau_\lambda = \frac{C_{pt} T_{t4}}{C_{pc} T_0}$$

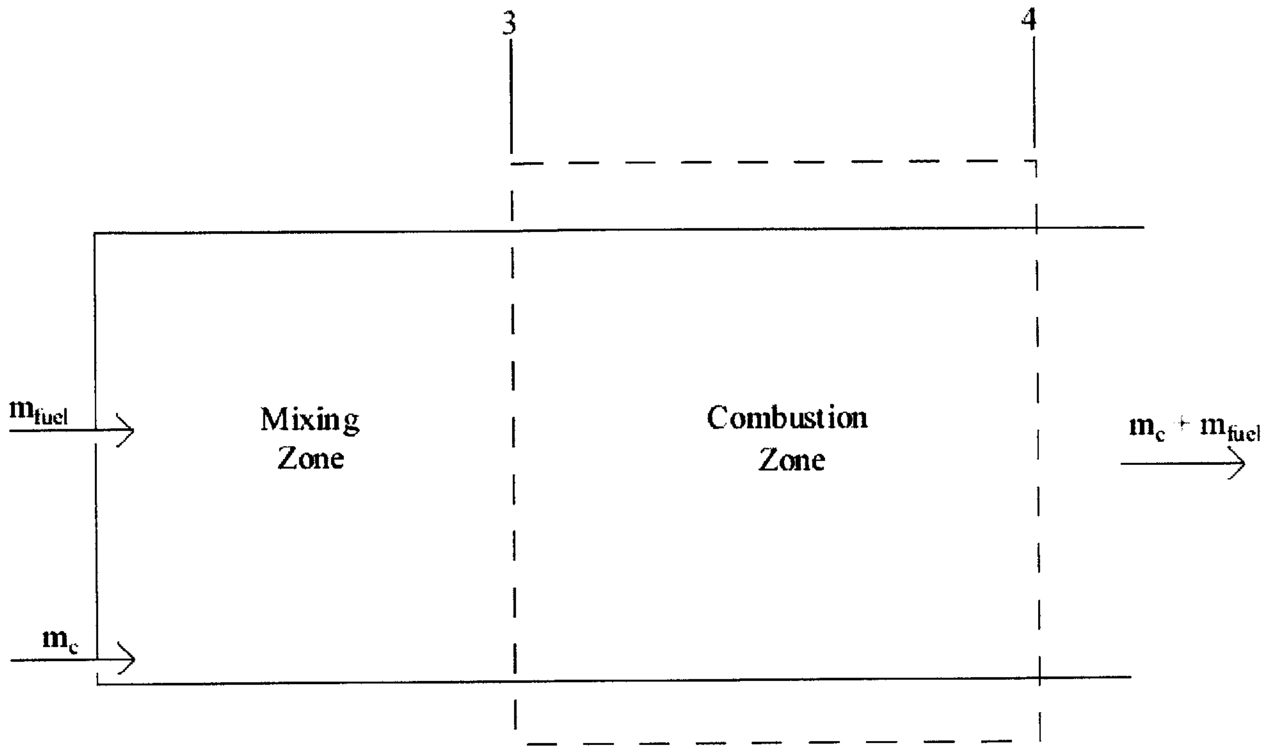


Figure 2.2: Combustor Model

Calculation of τ_t:

The power balance between the turbine, compressor and fan gives

Power into compressor + Power into Fan = Net Power from turbine

$$\dot{m}_c C_{pc}\left(T_{t3}-T_{t2}\right)+\dot{m}_f C_{pc}\left(T_{t13}-T_{t2}\right)=\dot{m}_4 C_{pt}\left(T_{t4}-T_{t5}\right)$$

Solving for the turbine temperature ratio gives

$$\boxed{\tau_t = 1-\left[\frac{1}{(1+f)}\,\frac{\tau_r}{\tau_\lambda}\,\left[\left(\tau_c-1\right)+\alpha\left(\tau_f-1\right)\right]\right]} \qquad \textbf{Eqn. (2.7)}$$

This expression allows solution for τ_t, from which we then obtain π_t by using the polytropic efficiency.

Calculation of α:

Fluid dynamics requires equal static pressures at stations 6 and 16. Normal design of the mixer has the Mach numbers of the two entering streams nearly equal. For this analysis, we assume the total pressures of the two entering streams are equal, or

$$P_{t6} = P_{t16}$$

Assuming isentropic flow in the bypass duct from 13 to 16, we can write

$$\pi_c \pi_b \pi_t = \pi_f$$

Or

$$\tau_t = \pi_t^{(\gamma_t - 1)e_t/\gamma_t} = \left(\frac{\pi_f}{\pi_c \pi_b} \right)^{(\gamma_t - 1)e_t/\gamma_t} \qquad \textbf{Eqn. (2.8)}$$

Eqn. (2.7) and **(2.8)** can be solved to obtain the bypass ratio α or the fan temperature ratio τ_f in terms of the known quantities. In this analysis it is assumed that τ_f is given soothe solution for the bypass ratio gives

$$\boxed{\alpha = \frac{(1+f)(\tau_\lambda/\tau_r)\left\{1 - \left[\pi_f/(\pi_c \pi_b)\right]^{(\gamma_t - 1)e_t/\gamma_t}\right\} - (\tau_c - 1)}{\tau_f - 1}} \qquad \textbf{Eqn. (2.9)}$$

Mixed Exhaust Stream

Before exiting the engine through the nozzle, the bypass flow and the core flow are mixed in the exhaust. The mixing of these two streams will play a role in the overall engine efficiency. The temperature and pressure ratios of the mixer are defined as

$$\tau_m = \frac{T_{t6A}}{T_{t6}} \quad \textit{and} \quad \pi_m = \frac{P_{t6A}}{P_{t6}} \qquad \textbf{Eqn. (2.10)}$$

The mixer temperature ratio τ_m will be obtained from an energy balance, while the total pressure ratio π_m will be obtained from an analysis of a constant area ideal mixer.

Ideal Mixer Analytic Model

Figure 2.3 shows an ideal (no wall friction) subsonic constant area mixer, with primary and secondary streams of calorically perfect gases having different c_p and γ values. The flow is assumed to be one-dimensional, and the subscripts 6, 16, and 6A are used for the core, bypass, and mixed streams, respectively. We assume that the c_p and γ values of the core and the bypass streams are known as well as M_6 and the following ratios:

$$\frac{T_{t16}}{T_{t6}} \quad \frac{P_{t16}}{P_{t6}} \quad and \quad \alpha' \equiv \frac{\dot{m}_{16}}{\dot{m}_6}$$

To obtain the properties of the mixed stream, the conservation laws for the ideal mixer were used:

Conservation of Mass

$$\dot{m}_6 + \dot{m}_{16} = \dot{m}_{6A} \qquad \textbf{Eqn. (2.11)}$$

Conservation of Energy

$$\dot{m}_6 C_{p6} T_{t6} + \dot{m}_{16} C_{p16} T_{t16} = \dot{m}_{6A} C_{p6A} T_{t6A} \qquad \textbf{Eqn. (2.12)}$$

Momentum

$$P_6 A_6 + \dot{m}_6 V_6 + P_{16} A_{16} + \dot{m}_{16} V_{16} = P_{6A} A_{6A} + \dot{m}_{6A} V_{6A} \qquad \textbf{Eqn. (2.13)}$$

Constant Area

$$A_6 + A_{16} = A_{6A} \qquad \textbf{Eqn. (2.14)}$$

Using these **Eqn's 2.11** to **2.14,** the mixer total temperature ratio τ_m is

$$\boxed{\tau_m \equiv \frac{T_{t6A}}{T_{t6}} = \frac{C_{p6}}{C_{p6A}} \frac{1 + \alpha'\left(C_{p16}/C_{p6}\right)\left(T_{t16}/T_{t6}\right)}{1 + \alpha'}} \qquad \textbf{Eqn. (2.15)}$$

Using the momentum equation and τ_m a value for M_{6A} can be calculated (see Ref. 2, Mattingly, Chapter 7). Using this value an expression for the mixer total pressure ratio in

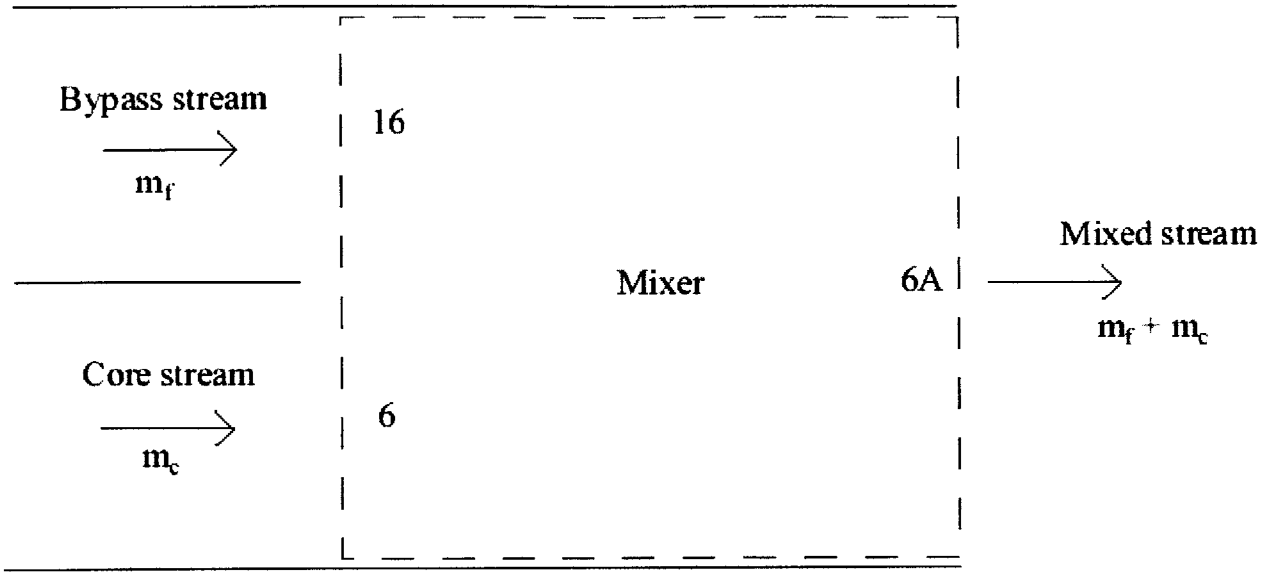


Figure 2.3: Ideal constant-area mixer

terms of τ_m and the other flow properties at station 6, 16 and 6A can be obtained

$$\boxed{\pi_m \equiv \frac{P_{t6A}}{P_{t6}} = \frac{(1+\alpha')\sqrt{\tau_m}}{1+A_{16}/A_6}\frac{MFP(M_6,\gamma_6,R_6)}{MFP(M_{6A},\gamma_{6A},R_{6A})}}$$ **Eqn. (2.16)**

Where the mass flow parameter (*MFP*) is defined as

$$\frac{\dot{m}\sqrt{T_t}}{P_t A} \equiv MFP(M,\gamma,R) = \frac{M\sqrt{\gamma/R}}{\left\{1+\left[(\gamma-1)/2\right]M^2\right\}^{(\gamma+1)/[2(\gamma-1)]}}$$

Calculation of Propulsive Efficiency

The propulsive efficiency is defined as

$$\eta_p = \frac{TV_0}{\dot{W}_{out}}$$

Where T = thrust of propulsion system

V_0 = Velocity of aircraft

$\dot{W}_{out}$ = Net power out of engine

It can be shown that

$$\boxed{\eta_P = \frac{2V_0\left(F/\dot{m}_0\right)}{a_0^2\left[\left(1+\left(f/1+\alpha\right)\right)\left(V_9/a_0\right)^2 - M_0^2\right]}}$$ **Eqn. (2.17)**

Calculation of Thermal Efficiency

The thermal efficiency is defined as

$$\eta_T = \frac{\dot{W}_{out}}{Q_{in}}$$

Where $\dot{W}_{out}$ = Net power out of engine

$\dot{Q}_{in}$ = Rate of thermal energy release $\left(\dot{m}_f h_{pr}\right)$

This leads to the following expression

$$\boxed{\eta_T = \frac{a_0^2\left(1+\alpha\right)\left[\left(1+\left(f/\left(1+\alpha\right)\right)\right)\left(V_9/a_0\right)^2 - M_0^2\right]}{2fh_{pr}}}$$ **Eqn. (2.18)**

Calculation of Specific Impulse

The specific Impulse is defined as

$$I = \frac{F_{total}}{\dot{m}_{fuel}\, g}$$

Where F = Total thrust of the engine

$\dot{m}_{fuel}$ = Mass flow rate of fuel

g = acceleration due to gravity

$$\boxed{I = \frac{F/\dot{m}_0}{fg}}$$ **Eqn. (2.19)**

2.2: Aspirated Turbofan

A schematic of the aspirated turbofan is shown in Figure 2.4a and 2.4b. As mentioned in the introduction, it will be assumed that a 3-stage counter-rotating core compressor is used in the engine. Aspiration is applied to each stage of the core compressor. This means that a portion of the air is bled from both the rotor and the stator on all three stages. On the first stage the air is taken radially outward ($\varepsilon_{r1}\dot{m}_c$ & $\varepsilon_{s1}\dot{m}_c$) as indicated in Figure 2.4a, while the air is taken radially inwards on the final two stages ($\varepsilon_{r2}\dot{m}_c, \varepsilon_{s2}\dot{m}_c, \varepsilon_{r3}\dot{m}_c, \varepsilon_{s3}\dot{m}_c$). Like the core compressor the fan stage is also aspirated, with bleeds being taken from the rotor and the stator. In both cases the bleed air is taken radially outwards.

As can be seen in Figure 2.4b the aspirated air from the core is returned to the cycle as turbine cooling air. It is assumed that each bleed can be admitted to the turbine at a pressure equal to the bleeds total pressure. Therefore, for cycle purposes the bleed airflow $\varepsilon_{s3}\dot{m}_c$ is modeled as being introduced at a pressure equal to its total pressure (P_{tse3}) and fully mixed in coolant mixer 1 (see Figure 2.4b). No total pressure loss is assumed for coolant mixer 1. The mixing of the cooling air causes a reduction in the total temperature of the through flow air, while increasing the mass flow of air through the turbine. The other compressor bleeds are treated similarly; each is introduced at a pressure equal to its total pressure and fully mixed out in a coolant mixer (see Figure 2.4b).

The fan extraction air is also used to augment the cycle. It is assumed that this air can be expanded through a nozzle to the ambient pressure P_0, thereby recovering the momentum drag of the air and increasing the thrust of the engine. Table 2-1 details each of the bleeds, the direction of extraction and how they augment the engine cycle.

In the analysis it is assumed that the pressure ratio and isentropic efficiency for each

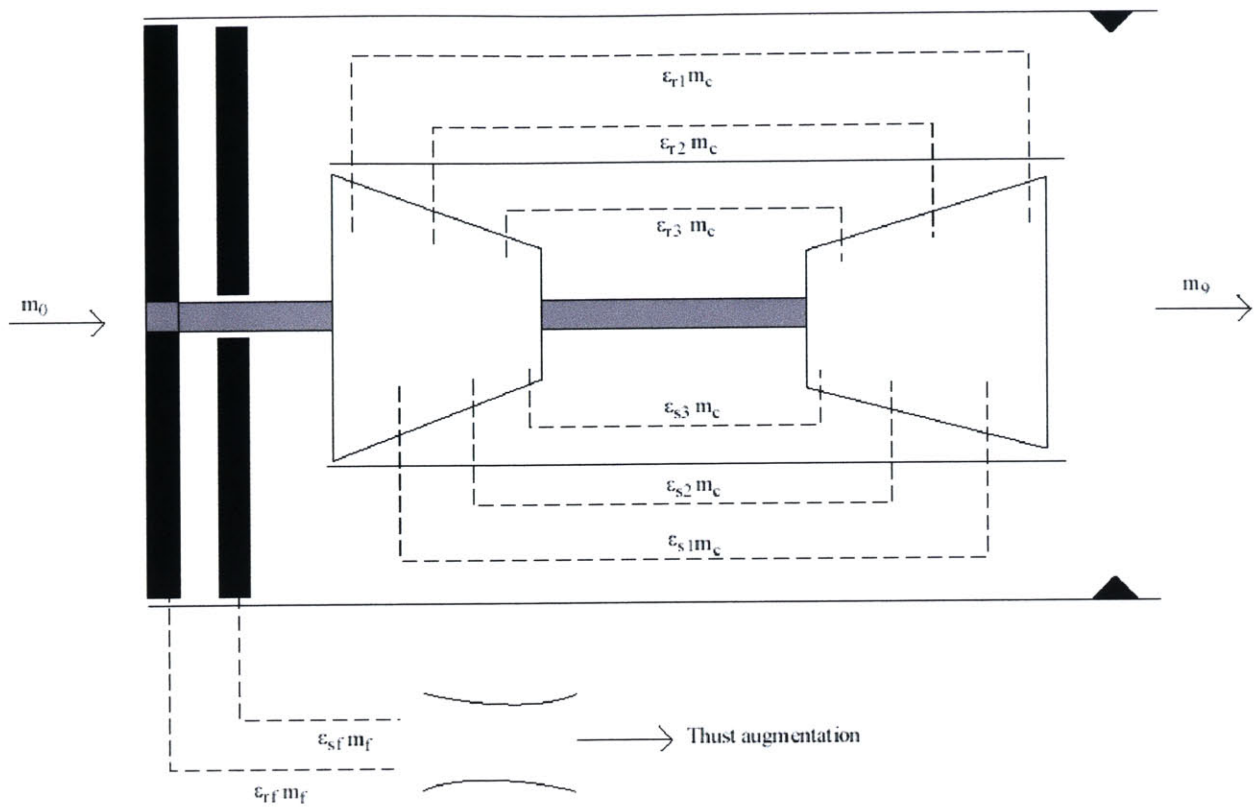


Figure 2.4a: Aspirated Turbofan

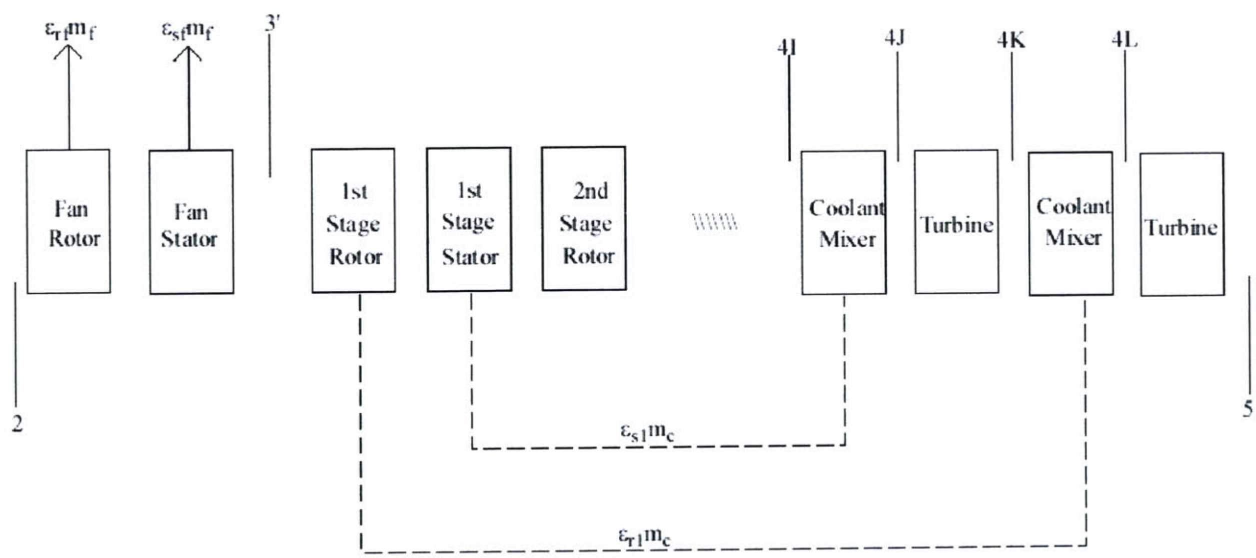


Figure 2.4b: Bleed and Turbine cooling Air Flows

stage is known. These values can be used to calculate to calculate an overall polytropic efficiency for the compressor. When comparing the aspirated turbofan with the non-aspirated turbofan, the compressor pressure ratio across both units is assumed equal

$$\pi_f \pi_1 \pi_2 \pi_3 = (\pi_c)_{aspirated} = (\pi_c)_{non-aspirated}$$

as are the overall polytropic efficiencies of both core compressors, the fan pressure ratios (π_f) and the fan polytropic efficiencies.

Nomenclature	Extraction Point	Direction of Extraction	Cycle Augmentation
$\varepsilon_{r1}\dot{m}_c$	1st Stage Rotor	Radially Outwards	Turbine Cooling Air
$\varepsilon_{s1}\dot{m}_c$	1st Stage Stator	Radially Outward	Turbine Cooling Air
$\varepsilon_{r2}\dot{m}_c$	2nd Stage Rotor	Radially Inward	Turbine Cooling Air
$\varepsilon_{s2}\dot{m}_c$	2nd Stage Stator	Radially Inward	Turbine Cooling Air
$\varepsilon_{r3}\dot{m}_c$	3rd Stage Rotor	Radially Inward	Turbine Cooling Air
$\varepsilon_{s3}\dot{m}_c$	3rd Stage Stator	Radially Inward	Turbine Cooling Air
$\varepsilon_{rf}\dot{m}_f$	Fan Rotor	Radially Outward	Expanded to P_0
$\varepsilon_{sf}\dot{m}_f$	Fan Stator	Radially Outward	Expanded to P_0

Table 2-1: Aspirated Turbofan Bleeds

2.2.1: Cycle Analysis

This section will compute the behavior of the aspirated turbofan including component losses, the mass flow rate of the fuel through the components, and the variation of specific heats. It also takes the work performed on the bleed air into account. This is an important difference between the cycle analysis performed on the non-aspirated turbofan.

For example the bleed air removed from the 1st stage rotor is taken radially outward and in so doing, work is performed on the air; the air is "pumped" similar to the work done by a centrifugal compressor. This "centrifugal work" increases the total temperature of the bleed air to a temperature greater than T_{t2}. The temperature rise can be greater than or less than the stage temperature rise. This means that the work performed on the bleed air must be treated separately in the cycle analysis.

For the second and third stage rotors where the bleed air is taken radially inward, the bleed air performs work analogous to a centrifugal turbine. Again like the bleed air taken radially outward this air must be treated separately in the cycle analysis. The calculation of the temperatures and pressures of the bleed air after they have been taken inward or outward is performed using a 1-D compressible flow code, detail in Section 3.

Another important difference between the aspirated case and the non-aspirated case is the inclusion of the momentum drag derived from the expansion of the fan bleed air to P_0. Assuming that this air is expanded through a nozzle to produce thrust allows the fan bleed air to be incorporated into the cycle. This can seen in the uninstalled thrust term below.

Like the non-aspirated turbofan the analysis will assume one-dimensional flow at the entrance and exit of each component. The variation of the specific heat will be approximated by assuming a perfect gas with constant specific heat c_{pc} upstream of the main burner and a perfect gas with different constant specific heat c_{pt} downstream of the main burner. The symbols used during the analysis are explained in the table of nomenclature. Again as in the case of the non-aspirated engine the analysis will be brief and only an outline of the most important steps is included. Detailed analysis of this cycle is provided in **Appendix A**. The aspirated turbofan with station numbering is shown in Figure 2-4a. The bleed and turbine cooling air flow is shown in Figure 2-4b.

Uninstalled Thrust:

$$F = \left(\dot{m}_9 V_9 - \dot{m}_0 V_0\right) + A_9\left(P_9 - P_0\right) + \left(Momentum\ Drag\ of\ \ Aux.\ Work\right)$$

Note: The thrust developed by expanding the Fan bleed air to P_0 will be calculated latter and will be removed from the thrust calculation for now.

Assume the engine exhaust nozzle expands the gas to the ambient pressure $P_0 = P_9$. Therefore we return to **Eqn. 2.1**

$$\boxed{\frac{F}{\dot{m}_0 a_0} = \left(\frac{\dot{m}_9}{\dot{m}_0}\frac{V_9}{a_0} - M_0\right)} \qquad \textbf{Eqn. (2.1)}$$

Calculation of $\frac{\dot{m}_9}{\dot{m}_0}$***:***

The fan bleed air is not returned to cycle directly and as mentioned above will be treated separately.

$$\dot{m}_9 = \dot{m}_c + \dot{m}_f + \dot{m}_{fuel} - \varepsilon_{rf}\dot{m}_f - \varepsilon_{sf}\dot{m}_f$$

After some manipulation one gets

$$\frac{\dot{m}_9}{\dot{m}_0} = \frac{1}{(1+\alpha)}\left[1 + f + \alpha\left(1 - \left(\varepsilon_{rf} + \varepsilon_{sf}\right)\right)\right] \qquad \textbf{Eqn. (2.20)}$$

Placing **Eqn. 2.20** into **Eqn. 2.1** gives

$$\boxed{\frac{F}{\dot{m}_0 a_0} = \left\{ \left(\frac{1 + f + \alpha\left(1 - \left(\varepsilon_{rf} + \varepsilon_{sf}\right)\right)}{1+\alpha} \right) \frac{V_9}{a_0} - M_0 \right\}} \qquad \textbf{Eqn. (2.21)}$$

Calculation of f:

$$f = \frac{\dot{m}_{fuel}}{\dot{m}_c}$$

Application of the 1st law of thermodynamics to the control volume about the combustor shown in Figure 2-2 gives:

$$\boxed{\dot{m}_3 C_{pc} T_{t3} + \eta_b \dot{m}_{fuel} h_{pr} = \dot{m}_4 C_{pt} T_{t4}} \qquad \textbf{Eqn. (2.5)}$$

The mass flow of the air entering the combustor is reduced by aspiration. This air is not returned to the cycle until the turbine. Therefore

$$\dot{m}_3 = \dot{m}_c \left(1 - \sum \varepsilon_i\right)$$

and

$$\dot{m}_4 = \dot{m}_c \left(\left(1 - \sum \varepsilon_i\right) + f\right)$$

Where $\sum \varepsilon_i = \varepsilon_{r1} + \varepsilon_{s1} + \varepsilon_{r2} + \varepsilon_{s2} + \varepsilon_{r3} + \varepsilon_{s3}$ and the mass flow out of the combustor is increased only by the addition of fuel.

$$\boxed{f = \frac{\left(1 - \sum \varepsilon_i\right) \left[\tau_r \tau_c - \tau_\lambda\right]}{\left[\tau_\lambda - \left(\eta_b h_{pr} \middle/ C_{pc} T_0 \right)\right]}} \qquad \textbf{Eqn. (2.22)}$$

Calculation of τ_t:

Like the non-aspirated case the calculation of τ_t involves a power balance between the compressor, fan and the turbine.

Power into compressor + Power into Fan = Net Power from turbine

As mentioned above calculating the power consumed by both the fan and the core compressor, the effects of aspiration must be taken into account. For example the bleed air removed from the 1st stage rotor is taken radially outward and in so doing, work is performed on the air; the air is "pumped" similar to the work done by a centrifugal compressor. This "centrifugal work" increases the total temperature of the bleed air to a temperature greater than T_{t2}. The temperature rise can be greater than or less than the stage temperature rise.

This means that the work performed on each bleed air must be treated separately in the cycle analysis. Figure 2.5 shows the fan and compressor bleeds, plus the nomenclature used in the analysis

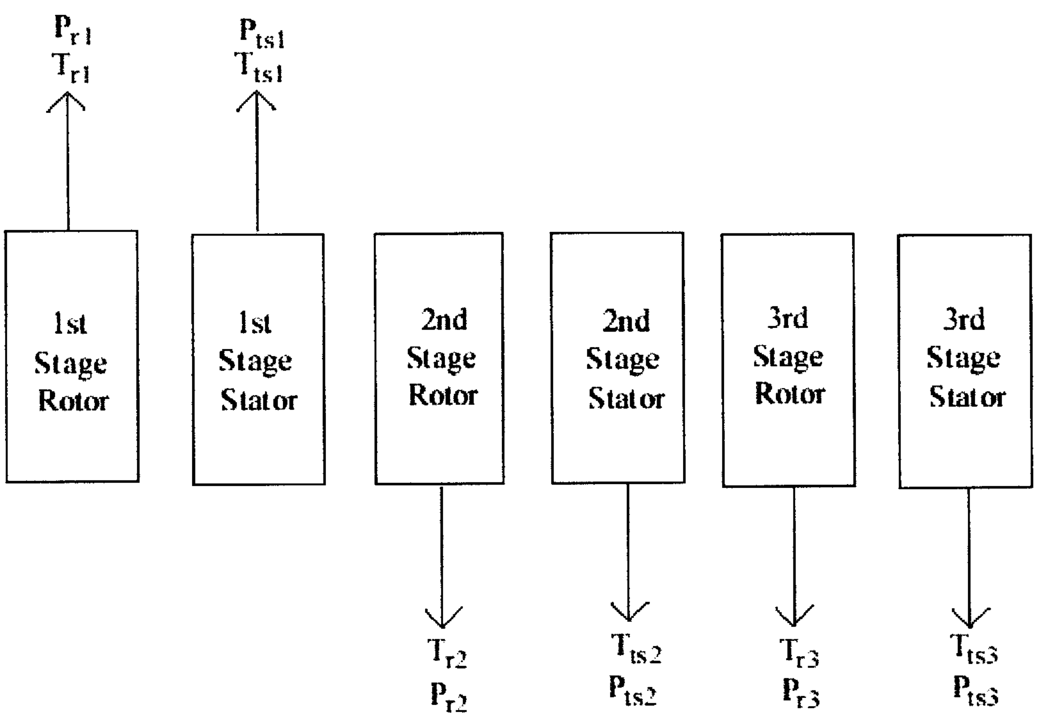


Figure 2.5: Counter-rotating Compressor

<u>Compressor & Fan Work</u>

To calculate the work performed on all the air that passes through the compression system it is necessary to account for the work performed on each of the bleeds. The fan has two bleeds and the core compressor has 6 bleeds, as shown in Table 2-1. Work performed on each bleed depends on the total temperature rise achieved. For the rotor bleeds temperature rise is calculated using a 1-D compressible flow calculation, detailed in Section 3. The temperature of the stator bleeds and the through flow air can be found using the isentropic or polytropic efficiency and the pressure rise.

Therefore

1. Through Flow of Core Compressor

$$\dot{m}_c\left(1-\sum\varepsilon_i\right)C_{pc}\left(T_{t3}-T_{t2}\right) \qquad \textbf{Eqn. (2.23)}$$

2. 1st Stage Rotor Bleed

$$\varepsilon_{r1}\dot{m}_c C_{pc}\left(T_{r1}-T_{t2}\right) \qquad \textbf{Eqn. (2.24)}$$

3. 1st Stage Stator Bleed

$$\varepsilon_{s1}\dot{m}_c C_{pc}\left(T_{ts1}-T_{t2}\right) \qquad \textbf{Eqn. (2.25)}$$

4. 2nd Stage Rotor Bleed

$$\varepsilon_{r2}\dot{m}_c C_{pc}\left(T_{r2}-T_{t2}\right) \qquad \textbf{Eqn. (2.26)}$$

5. 2nd Stage Stator Bleed

$$\varepsilon_{s2}\dot{m}_c C_{pc}\left(T_{ts2}-T_{t2}\right) \qquad \textbf{Eqn. (2.27)}$$

6. 3rd Stage Rotor Bleed

$$\varepsilon_{r3}\dot{m}_c C_{pc}\left(T_{r3}-T_{t2}\right) \qquad \textbf{Eqn. (2.28)}$$

7. 3rd Stage Stator Bleed

$$\varepsilon_{s3}\dot{m}_c C_{pc}\left(T_{ts3}-T_{t2}\right) \qquad \textbf{Eqn. (2.29)}$$

8. Fan through flow

$$\dot{m}_f\left(1-\varepsilon_{rf}\right)C_{pc}\left(T_{t13}-T_{t2}\right) \qquad \textbf{Eqn. (2.30)}$$

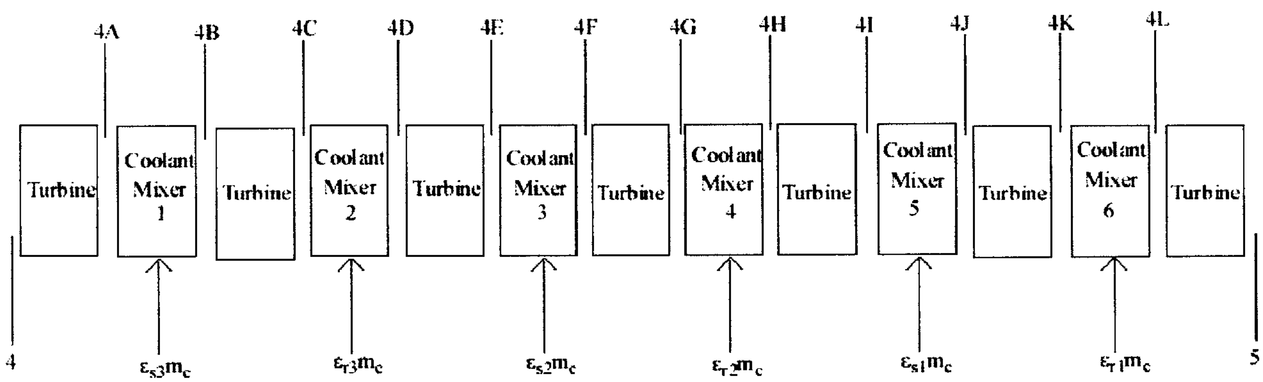
9. Fan Rotor Bleed

$$\varepsilon_{rf}\dot{m}_f C_{pc}\left(T_{tf} - T_{t2}\right) \qquad \textbf{Eqn. (2.31)}$$

Power input to the fan stator bleed is accounted for **Eqn. 2.30**; no work is performed on the air during the extraction process.

Turbine Power

The aspirated air from the compressor is returned to the turbine as cooling air. The air enters the turbine at the total pressure of the bleed air and is mixed with the through flow air. This mixing reduces the temperature of the through flow, while increasing the mass flow of air through the turbine. Because the mass flow of air through the turbine changes due to the addition of the aspirated air, the power extracted from the turbine must be calculated in steps. Figure 2.6 shows how the mass flow through the turbine changes with the addition of the bleed flow air from the compressor.



$m_4 = m_{4A} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1}+\varepsilon_{T2}+\varepsilon_{s2}+\varepsilon_{T3}+\varepsilon_{s3})) + f)$

$m_{4B} = m_{4C} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1}+\varepsilon_{T2}+\varepsilon_{s2}+\varepsilon_{T3})) + f)$

$m_{4D} = m_{4E} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1}+\varepsilon_{T2}+\varepsilon_{s2})) + f)$

$m_{4F} = m_{4G} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1}+\varepsilon_{T2})) + f)$

$m_{4H} = m_{4I} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1})) + f)$

$m_{4J} = m_{4K} = m_c\,(1-(\varepsilon_{T1}) + f)$

$m_{4L} = m_5 = m_c\,(1 + f)$

Figure 2.6: Changing mass flow through the turbine

Expansion of the air through the turbine and the work produced by this expansion is divided into 7 sections, one expansion to each of the six bleed air total pressures and a seventh to P_{t5} (i.e.

Expansion from P_{t4} to P_{tse3} (the total pressure of the 3rd stage stator bleed) etc.). After expansion the temperature of the through flow air can be calculated by assuming a polytropic efficiency for each step of the expansion. Mixing between the bleed air and the through flow air then takes place and a mixed out temperature based on mass is calculated. No pressure loss takes place during the mixing. Figure 2.7 shows a schematic of the expansion and the mixing on a T-S diagram.

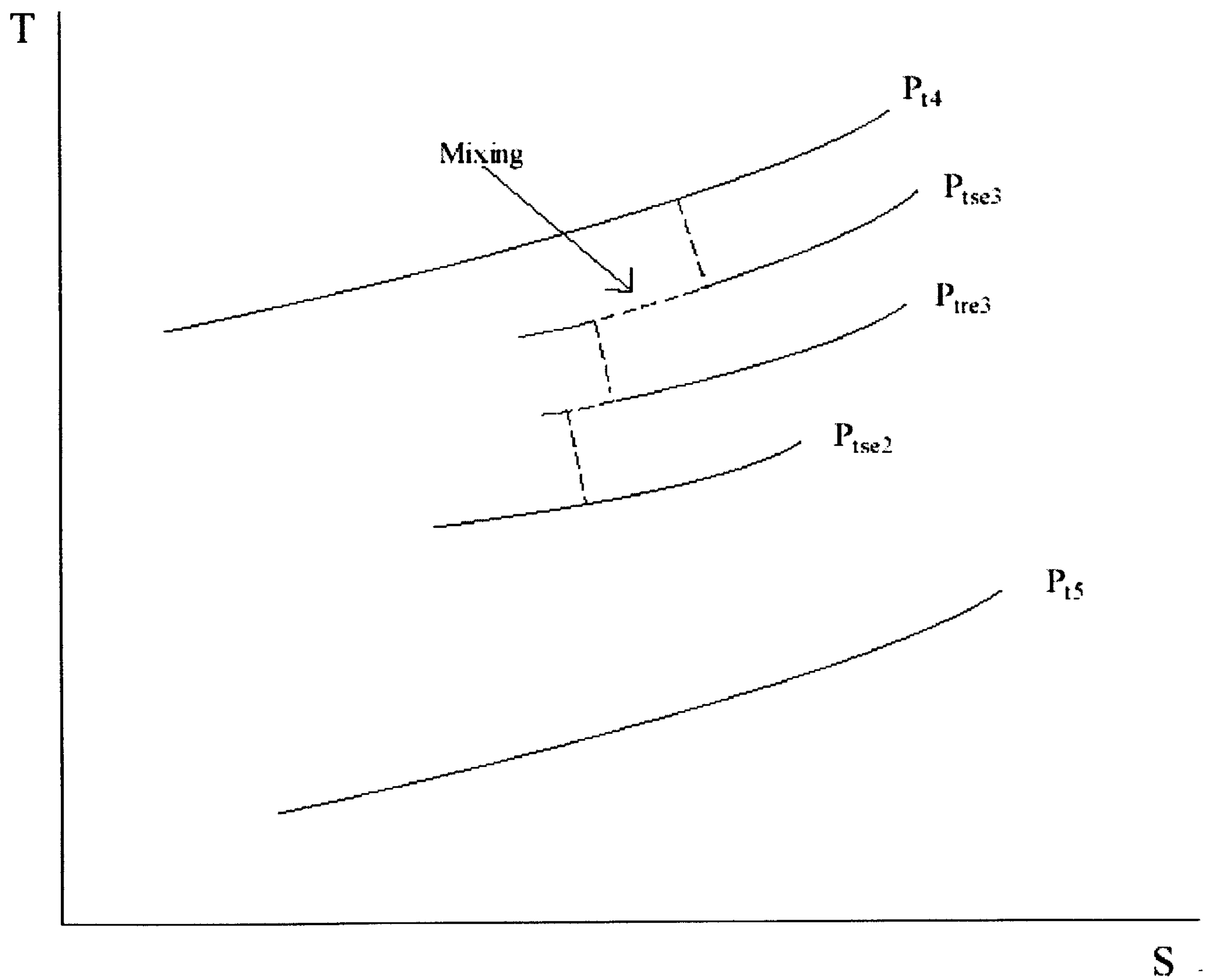


Figure 2.7: Stepwise expansion through the turbine

The steps involved in calculating the final mixed out temperature are shown here for the case of expansion from P_{t4} (Total pressure out of combustor) to P_{tse3} (Total pressure of the 3rd stage stator bleed air). Both P_{t4} $\left(P_0\pi_r\pi_d\pi_c\pi_b\right)$ and P_{tse3} (Calculated using the 1-D compressible flow analysis talked about in Section 3) are known, as is the polytropic efficiency of the expansion η_p, which will allow us to calculate the temperature of the through flow $\left(\text{to be known as } T_{tt1}\right)$ air before mixing.

$$\frac{T_{tt1}}{T_{t4}} = \left(\frac{P_{tse3}}{P_{t4}}\right)^{(\gamma_t-1)/(\eta_p\gamma_t)} \qquad \textbf{Eqn. (2.32)}$$

Once T_{tt1} is known, the mixed out temperature $\left(\text{to be known as } T_{tm1}\right)$ can be calculated based on the mass of the through flow air and the bleed air.

$$\dot{m}_4 C_{pt} T_{tt1} + \varepsilon_{s3}\dot{m}_c C_{pc} T_{tse3} = \left(\dot{m}_4 + \varepsilon_{s3}\dot{m}_c\right) T_{tm1}$$

Where $\dot{m}_4 = \left(\left(1-\sum\varepsilon_i\right)+f\right)\dot{m}_c$ gives

$$T_{tm1} = \frac{\left[\left(\left(1-\sum\varepsilon_i\right)+f\right)C_{pt}T_{tt1} + \varepsilon_{s3}C_{pc}T_{tse3}\right]}{\left[\left(\left(1-\sum\varepsilon_i\right)+f\right)+\varepsilon_{s3}\right]C_{pt}} \qquad \textbf{Eqn. (2.33)}$$

Where $\sum\varepsilon_i = \varepsilon_{r1}+\varepsilon_{s1}+\varepsilon_{r2}+\varepsilon_{s2}+\varepsilon_{r3}+\varepsilon_{s3}$

A similar analysis can be performed for each of the six expansions to match the total pressures of the bleeds. Each expansion performs a certain percentage of the overall turbine work. The exact amount of work accomplished depends on the mass flow and the temperature change. So for example, the work accomplished by the expansion from P_{t4} (Total pressure out of combustor) to P_{tse3} (Total pressure of the 3rd stage stator bleed air) is

$$\dot{m}_4 C_{pt}\left(T_{t4}-T_{tt1}\right) = \dot{m}_4 C_{pt} T_{t4}\left(1-\left(\frac{T_{tt1}}{T_{t4}}\right)\right)$$

But from **Eqn. (2.32)** we get the following

$$\dot{m}_4 C_{pt} T_{t4}\left(1-\left(\frac{P_{tse3}}{P_{t4}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right) \qquad \textbf{Eqn. (2.34)}$$

Notice the work is calculated using T_{tt1} not T_{tm1}, the mixed out temperature. As the Figure 2.7 illustrates the expansion through the turbine is modeled as an expansion followed by a constant pressure mixing process, followed by another expansion etc.

Changes in the mass flow and the effects of mixing are illustrated in the calculation of the work performed in the second expansion through the turbine; the expansion from P_{tse3} (Total pressure of the 3rd stage stator bleed air) to P_{tr3} (Total pressure of the 3rd stage rotor bleed air). The work done is

$$\left(\dot{m}_4+\varepsilon_{s3}\dot{m}_c\right)C_{pt}\left(T_{tm1}-T_{tt2}\right)$$

Similar to **Eqn. (2.32)** this can be rewritten as

$$\left(\dot{m}_4+\varepsilon_{s3}\dot{m}_c\right)C_{pt}T_{tm1}\left(1-\left(\frac{P_{tr3}}{P_{tse3}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right) \qquad \textbf{Eqn. (2.35)}$$

From the above equation it can be seen how the effects of mass addition $\left(\dot{m}_4+\varepsilon_{s3}\dot{m}_c\right)$ and mixing $\left(\text{the } T_{tm1} \text{ term}\right)$ contribute. Values of P_{tr3} and P_{tse3} can be found using the 1-D compressible flow calculation outline in Section 3. All of the stepwise expansions are modeled in a similar manner and each contributes to the power balance.

<u>Power Balance</u>

A power balance between the compressor, Fan and the turbine must be performed in order to obtain a value for T_{t5} or P_{t5} the turbine exhaust temperature or pressure. **Eqn.'s (2.23) - (2.31)** give the power required by the fan and the compressor; **Eqn.'s (2.34), (2.35)** and similar equations give the power out of the turbine. A balance results in an equation that can be solved for T_{t5} or P_{t5}.

$$\dot{m}_c\left(1-\sum\varepsilon_i\right)C_{pc}\left(T_{t3}-T_{t2}\right)+.........+\varepsilon_{rf}\dot{m}_f C_{pc}\left(T_{tf}-T_{t2}\right)$$

$$=$$

$$\dot{m}_4 C_{pt}T_{t4}\left(1-\left(\frac{P_{tse3}}{P_{t4}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right)+.........+\left(\dot{m}_4+\left(\varepsilon_{s3}+\varepsilon_{r3}+\varepsilon_{s2}+\varepsilon_{r2}+\varepsilon_{s1}+\varepsilon_{r1}\right)\dot{m}_c\right)C_{pt}T_{tm6}\left(1-\left(\frac{P_{t5}}{P_{tr1}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right)$$

Calculation of α:

Like the non-aspirated turbofan fluid dynamics requires equal static pressures at stations 6 and 16, so for this analysis, we assume the total pressures of the two entering streams are equal, or

$$P_{t6}=P_{t16}$$

Assuming isentropic flow in the bypass duct from 13 to 16, we can write

$$\pi_c\pi_b\pi_t=\pi_f$$

However unlike the non-aspirated turbofan, no simple closed form solution for α exists due to the changing mass flow through the turbine. This means the bypass ratio α is calculated by iteration. In the above relationship the only term effected by α is π_t, so one approach to calculating the bypass ratio is to vary α until the relationship $\pi_c\pi_b\pi_t=\pi_f$ is satisfied to two decimal places.

Mixed Exhaust Stream

The mixing analysis for the aspirated turbofan is identical to the analysis performed on the non-aspirated turbofan. The mixing of these two streams will play a role in the overall engine efficiency. The temperature and pressure ratios of the mixer are defined as

$$\tau_m=\frac{T_{t6A}}{T_{t6}} \qquad \text{and} \qquad \pi_m=\frac{P_{t6A}}{P_{t6}} \qquad \textbf{Eqn. (2.10)}$$

The mixer temperature ratio τ_m will be obtained from an energy balance, while the total pressure ratio π_m will be obtained from an analysis of a constant area ideal mixer.

The only change is to the $\alpha' \equiv \frac{\dot{m}_{16}}{\dot{m}_6}$ term. $\dot{m}_6$ is the same, but the $\dot{m}_{16}$ term is different as both the fan rotor and stator are aspirated. Therefore

$$\alpha' = \frac{\dot{m}_f\left(1-\left(\varepsilon_{rf}+\varepsilon_{sf}\right)\right)}{\dot{m}_c+\dot{m}_{fuel}} = \frac{\alpha\left(1-\left(\varepsilon_{rf}+\varepsilon_{sf}\right)\right)}{1+f}$$

This value for α' can be used throughout the rest of the mixer calculation.

Calculation of the thrust from Fan Bleeds

As discussed at the start of the analysis it is assumed that the fan bleed air is expanded overboard to recover it's momentum drag. This produces thrust, which adds to the core thrust calculated in **Eqn. (2.1)**. For the rotor bleed air

$$F_{fan\ rotor} = \varepsilon_{rf}\dot{m}_f\left(u_{fan\,rotor}-V_0\right)$$ **Eqn. (2.36)**

Assume the air is expanded isentropically

$$P_{tf} = P_0\left(1+\frac{\gamma_c-1}{2}M_{fr}^2\right)^{\frac{\gamma_c}{\gamma_c-1}}$$

After some manipulation it can be shown that

$$M_{rf}^2 = \frac{2}{\gamma_c-1}\left(\tau_{rf}-1\right)$$ **Eqn. (2.37)**

From **Eqn. (2.36)**

$$\frac{F_{fan\,rotor}}{\dot{m}_o a_o} = \varepsilon_{rf}\left(\frac{\alpha}{\alpha+1}\right)\left(\frac{u_{fan\,rotor}}{u_0}M_0-M_0\right)$$

Giving

$$\frac{F_{fan\,rotor}}{\dot{m}_0 a_0} = \varepsilon_{rf}\left(\frac{\alpha}{1+\alpha}\right)\left(\sqrt{\frac{T_{rf}}{T_0}}\ \sqrt{\frac{2}{\gamma_c-1}\left(\tau_{cen}-1\right)}-M_0\right)$$ **Eqn. (2.38)**

Where T_{rf} is the static temperature of the fan bleed air, which can be found using the 1-D compressible flow code of Section 3. While $\tau_{cen} = \frac{T_{tf}}{T_{t2}}\frac{T_{t2}}{T_0} = \tau_{rf}\tau_r$. The thrust of the stator bleed is calculated in a similar fashion. The sum of the two thrusts is called $\frac{F_{fan}}{\dot{m}_0 a_0}$.

Calculation of Total Thrust F_{total}

$$\frac{F_{total}}{\dot{m}_0 a_0} = \frac{F}{\dot{m}_0 a_0} + \frac{F_{fan}}{\dot{m}_0 a_0} \qquad \textbf{Eqn. (2.39)}$$

Calculation of Specific Impulse

$$I = \frac{F_{total}/\dot{m}_o}{f\,g} \qquad \textbf{Eqn. (2.40)}$$

2.3: <u>Aspirated Turbojet</u>

A schematic of the aspirated turbojet is shown in Figure 2.8. Like the aspirated turbofan, it will be assumed that a 3-stage counter-rotating compressor is used in the engine. Aspiration is applied to each stage of the compressor. This means that a portion of the air is bled from both the rotor and the stator on all three stages. On the first stage the air is taken radially outward ($\varepsilon_{r1}\,\dot{m}_0$ & $\varepsilon_{s1}\,\dot{m}_0$) as indicated in Figure 2.8, while the air is taken radially inwards on the final two stages ($\varepsilon_{r2}\,\dot{m}_0, \varepsilon_{s2}\dot{m}_0, \varepsilon_{r3}\,\dot{m}_0, \varepsilon_{s3}\dot{m}_0$). Also, like the case of the non-aspirated turbojet a portion of the compressor discharge air is also removed ($x'\dot{m}_0$).

Both the bleed air and the compressor discharge air are used to perform "Auxiliary Work". It is assumed that all of the bleed air can be used for this purpose. To perform this "Auxiliary Work" the bleed air and the compressor discharge air are expanded to the compressor inlet pressure P_{t2}. To allow a comparison between the Non-Aspirated turbojet and the Aspirated

Turbojet, the amount of auxiliary work performed by both engines is equal (i.e. the work performed by $x'\dot{m}_0, \varepsilon_{r1}$ *etc.* is equal to the work performed by $x\dot{m}_0$ in the non-aspirated engine). Therefore

$$(\text{Auxiliary work})_{\textbf{aspirated}} = (\text{Auxiliary Work})_{\textbf{non-aspirated}}$$

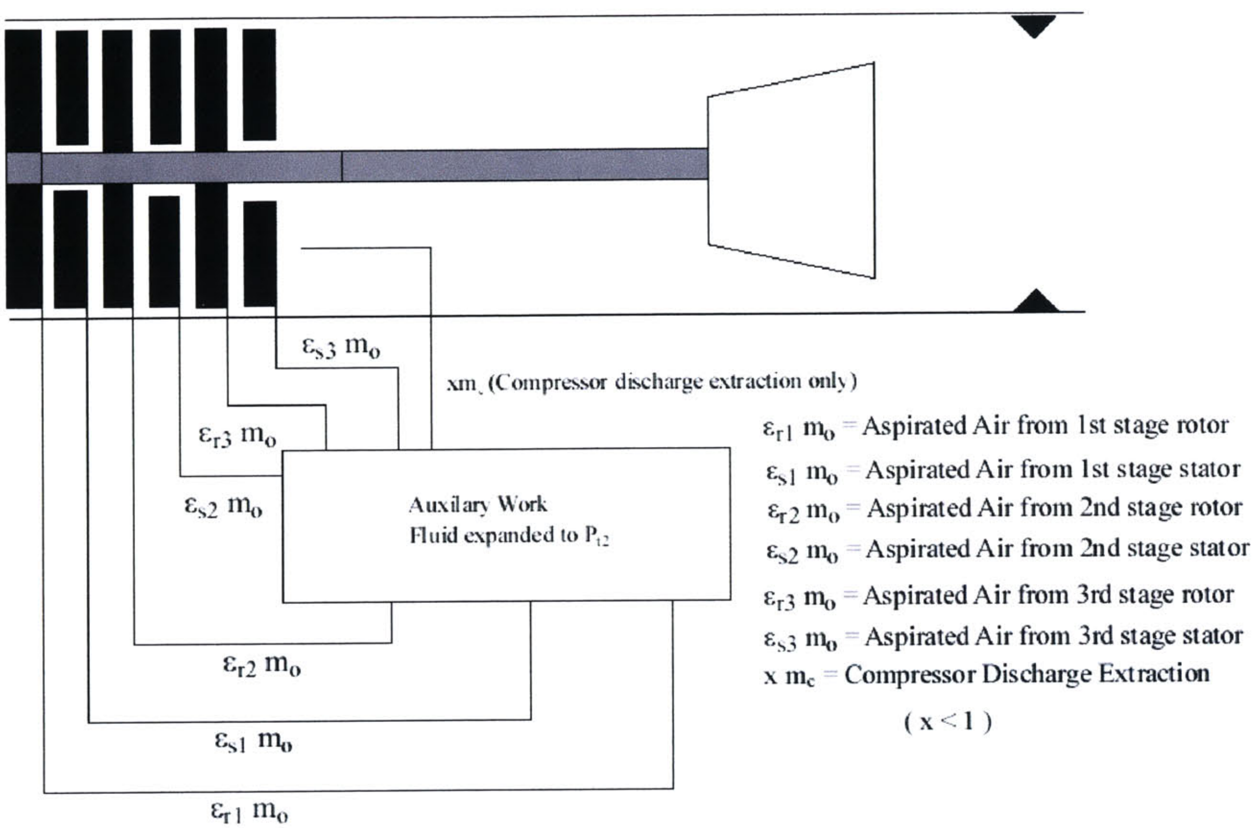


Figure 2.8: Aspirated Turbojet

This means that the compressor discharge extraction ($x'\dot{m}_0$) for the aspirated case is smaller that that for the non-aspirated case. The calculation of $x'\dot{m}_0$ is detailed in the cycle analysis below. As in the Non-Aspirated case, after being expanded to P_{t2}, the air is then expanded further to P_0, to recover the momentum drag, adding to the thrust of the engine. Table 2-2 details each of the bleeds and extraction for the aspirated turbojet.

In the analysis it is assumed that pressure ratio and isentropic efficiency for each stage is known and the overall compressor polytropic efficiency is also known. When comparing the aspirated turbojet with the non-aspirated turbojet, the compressor pressure ratio across both units is assumed equal

$$\pi_1 \pi_2 \pi_3 = (\pi_c)_{aspirated} = (\pi_c)_{non-aspirated}$$

Nomenclature	Place of Extraction	Direction of Extraction
$\varepsilon_{r1}\, \dot{m}_0$	1st Stage Rotor Bleed	Radially Outwards
$\varepsilon_{s1}\, \dot{m}_0$	1st Stage Stator Bleed	Radially Outward
$\varepsilon_{r2}\, \dot{m}_0$	2nd Stage Rotor Bleed	Radially Inward
$\varepsilon_{s2}\, \dot{m}_0$	2nd Stage Stator Bleed	Radially Inward
$\varepsilon_{r3}\, \dot{m}_0$	3rd Stage Rotor Bleed	Radially Inward
$\varepsilon_{s3}\, \dot{m}_0$	3rd Stage Stator Bleed	Radially Inward
$x' \dot{m}_0$	Compressor Discharge Extraction	N/A

Table 2-2: Aspirated Turbojet Bleeds

The cycle analysis for this cycle is similar to that performed on the turbojet and will not be repeated here. The details can be found in Appendix B.

Chapter 3: Calculation of flow properties

3.0: Introduction

If the bleed flow is discarded and its contribution to the overall system ignored, the penalty for using aspirated technology can be high. As mentioned in Section 2, an important component of the cycle analysis is using the bleed air in another part of the cycle, whether to cool the turbine or augment the engine thrust. Therefore it is essential to know the temperature and pressure of the bleed air as it leaves the rotor or stator blade. These conditions dictate at what pressure and temperature the bleed air mixes with the turbine through flow air or how much thrust can be produced by an expansion to P_0. For the aspirated turbojet the bleed air is used to perform "Auxiliary Work" and to contribute to the core thrust of the engine. The effective use of the bleed air is an important element in improving the Specific Impulse and efficiency of an engine using aspiration.

To model the bleed airflow in the blade internal passage, a quasi one-dimensional compressible flow model is utilized. In what follows below the model will be developed for the rotor in a non-inertial rotating coordinate system, but can easily be generalized to the stator by setting the $\boldsymbol{\Omega r}$ term to zero. Flow in the bleed passage is assumed to be *steady* and *one-dimensional* or, more precisely, steady and quasi-one-dimensional. All the flow variables of interest, such as pressure and density, will be treated according to the one-dimensional model; that is, in at any given cross section all flow variables have constant values. Thus all properties are assumed to vary only along the axis of the bleed slot. Using this one-dimensional approximation the model will calculate the pressure, temperature, density and exit Mach number of the bleed air as it leaves the suction slot. The one-dimensional flow concept as employed below, is an approximation only as far as the flow model is concerned, but not insofar as the governing flow equations are concerned. Once the approximation of uniform flow properties at each cross section is made, the integral forms of the governing equations can be applied to the simplified model.

The one-dimensional flow approximation is exact for the flow through an infinitesimal stream tube. Thus, many of the general features that characterize large-scale one-dimensional flows are also present along the streamlines of a multidimensional flow. In general, the one-dimensional approximation is reasonable if the rate of change of the flow driving potential is small in the direction of flow. Examples of the driving potentials are: *area change*, *wall friction*, and *mass addition*. Furthermore, the radius of curvature of the flow passage should be large (this does not apply to this analysis as the internal bleed passages are all straight), and the profiles of the flow properties should remain similar at each flow cross section. Finally, it should be noted that the one-dimensional model considers changes only in the average or bulk values of the flow properties in the direction of flow; it disregards completely the variations in the flow properties in the direction normal to the streamlines.

In a rotating coordinate system, the Coriolis force $(2 \times \Omega \times W)$ acts normal to the direction of rotation and flow. For the bleed passages, the Coriolis force acts normal to the flow direction and hence is ignored in the analysis. To account for the secondary flow effects associated with the Coriolis force, a correction factor is used to increase the value of the **Fanning friction factor**. The basic principle and ideas used to calculate the correction factors are presented below, with appropriate references.

The analysis presented below is a modification (changed to a rotating coordinate system) of the work presented in

1. **"Gas Dynamics, Vol. 1"** by Maurice J. Zucrow & Joe D. Hoffman

2. **"The Dynamics and Thermodynamics of Compressible Fluid Flow, Vol. 1"** by A.H. Shapiro

3.1: One-dimensional model

Suction on the blade surface is accomplished by means of a single slot on the suction surface close to the point of shock impingement. The extracted flow is then brought inside the blade where a hollow internal passage allows it to flow radially outwards or radially inwards. As the air moves radially outward or inward the internal passages area can change or remain constant. Figure 3.1 illustrates schematically the physical model for the bleed passage. The bleed air is "sucked" into the passage causing a change in mass flow, and rothalpy. These changes can in turn lead to changes in the static temperature, the velocity, static pressure and density of the flow. The exact value of the changes and whether they increase or decrease is a function of the driving potentials. The independent driving potentials for the flow are:

1. Area change dA.
2. Wall friction δF_f.
3. Heat Transfer δQ.
4. Work δW.
5. Mass addition $d\dot{m}$.
6. Body forces caused by the rotational effects $\rho A \Omega^2 r\, dr$.

As mentioned in the introduction, the Coriolis force plays no direct role in the analysis and will be accounted for in the friction factor. For the assumed one-dimensional flow, each of the variables P, ρ, and W is uniform over any arbitrary cross-sectional area. For the element of differential volume of length dr, in Figure 3.1, the surfaces over which the control volume equations must be integrated are the inlet area A, the exit area $A+dA$, and the stream tube boundary area $(dA/\sin\alpha)$, where α is the angle made by the passage boundary with respect to the r axis.

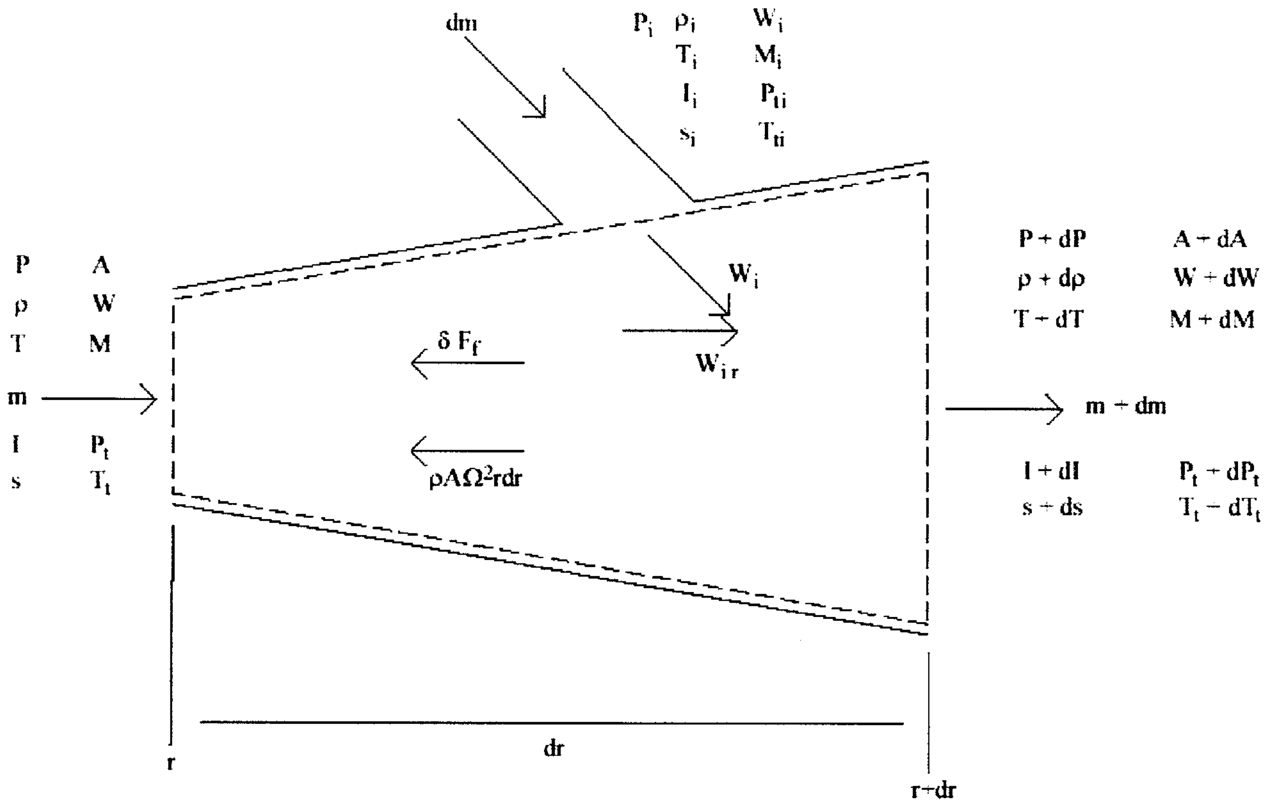


Figure 3.1: Model for Bleed Passage Flow

By convention, all of the flow properties on the surfaces across which mass enters will be assigned the nominal values P, V, $V^2/2$, ρ, etc., and it will be assumed that positive changes in these properties occur in the direction of flow. Thus, at the exit area, the properties are P+dP, V+dV, ρ + dρ, etc. In addition, positive changes in the driving potentials are assumed. Hence, the inlet area is A and the exit area is A+dA. In a specific flow situation, any or all of the property changes may turn out to be negative.

To summarize, the following assumptions are made about the flow:

1. One-dimensional flow. Each of the variables P, ρ, and W is uniform over any arbitrary cross-sectional area.
2. Steady flow. Flow is steady in the reference frame of the rotor or the stator.
3. The effects of gravity are negligible. In this analysis the contribution of the body force caused by gravity is small compared to the other forces.

4. Air is modeled as a perfect gas (obeys the equation of state $P = \rho RT$ and constant specific heats C_p and C_v).

5. The angular velocity Ω is constant. It is assumed no angular acceleration takes place.

Note: All static properties will be denoted by a upper case letter (i.e. Static temperature = T, Static pressure = P), while stagnation or Total quantities will be denoted by an upper case letter and a subscript t(i.e. Total temperature = T_t, Total pressure = P_t). The analysis below is done for the case of the rotor. This means that the analysis takes place in a non-inertial coordinate system and most of the quantities are values with respect to the rotating coordinate system (i.e. W is the velocity of the air relative to the rotating coordinate system for the rotor and the velocity of the air relative to the stationary coordinate system for the stator).

Applying the continuity, momentum and energy equation in rotating coordinates to the passage in Figure 3.1 gives the following:

Continuity Equation

$$\dot{m} = \rho A W \qquad \textbf{Eqn. (3.1)}$$

Differentiating **Eqn. (3.1)** yields

$$\boxed{\frac{d\dot{m}}{\dot{m}} = \frac{d\rho}{\rho} + \frac{dA}{A} + \frac{dW}{W}} \qquad \textbf{Eqn. (3.2)}$$

Where $d\dot{m}$ is the rate of mass addition to the flow. $\dot{m}$ is the mass flow rate of the through flow air. W is the velocity of the air relative to the rotating coordinate system. For a stator W is the velocity of the air relative to the stationary coordinate system.

Momentum Equation

The integral form of the momentum equation for flow in a rotating coordinate system is

$$\int_{\mathcal{V}}\left(2\Omega\times\widetilde{W}\right)\rho\, d\mathcal{V}+\int_{\mathcal{V}}\left(\Omega\times\Omega\times r\right)\rho\, d\mathcal{V}-\int_{\mathcal{S}}P d\mathcal{S}+F_{shear}=\frac{d}{dt}\int_{\mathcal{V}}\rho\widetilde{W}d\mathcal{V}+\int_{\mathcal{S}}\widetilde{W}\left(\rho\widetilde{W}\bullet d\mathcal{S}\right)\quad \textbf{Eqn.(3.3)}$$

From **Eqn. (3.3)** it is evident that the equation of motion of a fluid in a rotating system is identical in form to the equation in the absolute frame of reference, provided a fictitious body force (per unit mass) equal to $\left(2\Omega\times\widetilde{W}+\Omega\times\Omega\times r\right)$ acts on the fluid in addition to the body and surface forces. The term $2\Omega\times\widetilde{W}$ is the Coriolis force, and $\Omega\times\Omega\times r$ is the centrifugal force due to the systems rotation. The Coriolis force $2\Omega\times\widetilde{W}$ acts in a plane normal to Ω and $\widetilde{W}$; thus for the bleed passage in Figure 3.1, with the Ω component normal to the plane of the passage, the Coriolis force acts in the plane of the passage, normal to the *r*-component of velocity. Hence the Coriolis force plays no direct role in the 1-D momentum equation.

Applying **Eqn. (3.3)** in the *r*-direction, assuming steady flow gives:

$$PA-\left(P+dP\right)\left(A+dA\right)+\left(P+\frac{dP}{2}\right)dA-\rho A\Omega^{2}rdr=\left(\dot{m}+d\dot{m}\right)\left(W+dW\right)-\dot{m}W-d\dot{m}V_{ir}+\delta F_{f}$$

On the stream tube boundary the average static pressure intensity is $P+dP/2$, which acts on the area $dA/\sin\alpha$. However, only the component of the force acting in the direction of flow is desired. Consequently, the total force acting on the boundary surface of the stream tube is given by $\left(P+dP/2\right)dA$. Neglecting higher order terms $\left(d\dot{m}\,dW\ \&\ dPdA\right)$ the above equation reduces to

$$AdP+\dot{m}dW+d\dot{m}\left(W-V_{ir}\right)+\delta F_{f}-\rho A\Omega^{2}rdr=0 \qquad \textbf{Eqn. (3.4)}$$

Using the continuity equation $\left(\dot{m}=\rho AW\right)$ and dividing by A gives

$$dP+\rho WdW+\frac{\delta F_{f}}{A}+\rho W^{2}\left(1-y\right)\frac{d\dot{m}}{\dot{m}}-\rho\Omega^{2}rdr=0 \qquad \textbf{Eqn. (3.5)}$$

Where $y = \frac{V_{ir}}{W}$. If $V_{ir} = 0$, then $y = 0$, and the mass addition is added so that it enters the main gas stream in a direction normal to the mainstream velocity W. This means all of the momentum of the injected fluid is lost. If $V_{ir} = W$, then $y = 1$, and the mass addition has the same velocity as the main gas stream.

The **wall friction force** δF_f in **Eqn. (3.5)** may be expressed in terms of the hydraulic characteristics of the flow passage and an experimental **friction coefficient** f. Let *(WP)* denote the average value of the **wetted perimeter** for the flow passage. The length of the fluid element is *dr*, so that the area of the surface wetted by the fluid element is *(WP) dr*.

Let $\mathscr{D}$ denote the **hydraulic diameter** then, by definition

$$\frac{\mathscr{D}}{4} = \frac{\textit{Wetted Area}}{\textit{Wetted Perimeter}} = \frac{A}{(WP)} \qquad \textbf{Eqn. (3.6)}$$

The friction coefficient **f** is defined by the ***Fanning equation***. Thus

$$f \equiv \frac{\text{tangential force}}{\frac{1}{2}\rho W^2 (\textit{wetted area})} = \frac{\delta F_f}{\frac{1}{2}\rho W^2 (WP)\,dr} = \frac{\tau_w}{\frac{1}{2}\rho W^2} \qquad \textbf{Eqn. (3.7)}$$

Where τ_w is the shear stress at the wall. Hence,

$$\delta F_f = f\frac{\rho W^2}{2}(WP)\,dr = \frac{\rho W^2}{2}\left(\frac{4f\,dr}{\mathscr{D}}\right)A \qquad \textbf{Eqn. (3.8)}$$

Using **Eqn. (3.8)** the momentum equation now becomes

$$\boxed{dP + \rho W dW + \frac{\rho W^2}{2}\left(\frac{4f\,dr}{\mathscr{D}}\right) + \rho W^2 (1-y)\frac{d\dot{m}}{\dot{m}} - \rho\Omega^2 r dr = 0} \qquad \textbf{Eqn. (3.9)}$$

As mentioned earlier, the Fanning friction factor will be modified to account for the effects of the Coriolis force. Basically, ***f*** is calculated using a standard correlation and modified by a correction factor, to account for the effects of rotation. The procedure is detailed latter.

Assuming the air is a perfect gas, allows us to express **Eqn. (3.9)** in terms of the flow Mach number M relative to the rotating coordinate system. Multiplying through by $\frac{1}{P}$ and noting that $\frac{\gamma P}{\rho} = a^2$, we obtain

$$\frac{dP}{P} + \frac{\rho\gamma}{P\gamma}\frac{W^2}{W^2} d\left(\frac{W^2}{2}\right) + \frac{\rho\gamma}{P\gamma}\frac{W^2}{2}\left(\frac{4fdr}{\mathscr{D}}\right) + \frac{\rho W^2}{P}(1-y)\frac{d\dot{m}}{\dot{m}} - \frac{\Omega^2 r dr}{W^2}\frac{\rho W^2}{P} = 0$$

Using the fact that $\frac{\rho W^2}{p} = \gamma M^2$ and $\frac{\gamma p}{\rho} = a^2$ gives

$$\frac{dP}{P} + \frac{\gamma M^2}{2}\frac{dW^2}{W^2} + \frac{\gamma M^2}{2}\left(\frac{4fdr}{D}\right) + \gamma M^2 (1-y)\frac{d\dot{m}}{\dot{m}} - \gamma M^2 \frac{\Omega^2 r dr}{W^2} = 0 \qquad \textbf{Eqn. (3.10)}$$

Since $W^2 = M^2 a^2$ and $da^2/a^2 = dT/T$, the second term in **Eqn. (3.10)** becomes

$$\frac{\gamma M^2}{2}\frac{dW^2}{W^2} = \frac{\gamma}{2} dM^2 + \frac{\gamma M^2}{2}\frac{dT}{T} \qquad \textbf{Eqn. (3.11)}$$

Substituting **Eqn. (3.11)** into **Eqn. (3.10)** yields

$$\boxed{\frac{dP}{P} + \gamma M^2 \frac{dM}{M} + \frac{\gamma M^2}{2}\frac{dT}{T} + \frac{\gamma M^2}{2}\left(\frac{4fdr}{\mathscr{D}}\right) + \gamma M^2 (1-y)\frac{d\dot{m}}{\dot{m}} - \gamma M^2 \frac{\Omega^2 r^2}{W^2}\frac{dr}{r} = 0} \qquad \textbf{Eqn. (3.12)}$$

Energy equation

For steady flow in a non-inertial, rotating coordinate system the energy equation has the form

$$\dot{W}_{shaft} - \dot{Q} + \int_{\mathscr{A}} \left(h + \frac{\widetilde{W}^2}{2} - \frac{(\Omega r)^2}{2}\right)\left(\rho \widetilde{W} \bullet d\mathscr{A}\right) = 0 \qquad \textbf{Eqn. (3.13)}$$

The quantity $h + \frac{\widetilde{W}^2}{2} - \frac{(\Omega r)^2}{2}$ is known as the *rothalpy*. In a rotating coordinate system, rothalpy has properties analogous to stagnation enthalpy in stationary coordinates. In a moving passage the rothalpy is constant provided:

1. The flow is steady in the rotating frame;
2. No work is done on the flow in the rotating frame;
3. There is no heat flow to or from the flow.

However, looking at Figure 3.1 it can be seen that the rothalpy of the fluid in the bleed passage can be changed by the addition of mass $(d\dot{m})$ from outside. The rothalpy of the air outside the slot could be higher or lower and hence once mixed with the main stream will change the rothalpy of the flow.

Applying **Eqn. (3.13)** to the deferential control volume in Figure 3.1 gives:

$$\delta W - \delta Q + (\dot{m} + d\dot{m})\left[h + dh + \frac{W^2}{2} + d\left(\frac{W^2}{2}\right) - \left(\frac{(\Omega r)^2}{2} + \frac{d(\Omega r)^2}{2}\right)\right]$$

$$-\dot{m}\left(h + \frac{W^2}{2} - \frac{(\Omega r)^2}{2}\right) - d\dot{m}\left(h_i + \frac{V_i^2}{2} - \frac{(\Omega r)^2}{2}\right) = 0 \quad \textbf{Eqn. (3.14)}$$

Combining terms, and neglecting products of differentials, and dividing by $\dot{m}$ gives,

$$\delta W - \delta Q + dh + d\left(\frac{W^2}{2}\right) - d\left(\frac{(\Omega r)^2}{2}\right)$$

$$+\left[\left(h + \frac{W^2}{2} - \left(\frac{(\Omega r)^2}{2}\right)\right) - \left(h_i + \frac{V_i^2}{2} - \left(\frac{(\Omega r_i)^2}{2}\right)\right)\right]\frac{d\dot{m}}{\dot{m}} = 0 \quad \textbf{Eqn. (3.15)}$$

The term in the square brackets arises because of the difference in the rothalpy I of the main stream $\left(h + \frac{W^2}{2} - \frac{(\Omega r)^2}{2}\right)$ and the rothalpy I_i $\left(h_i + \frac{V_i^2}{2} - \frac{(\Omega r_i)^2}{2}\right)$ of the mass addition stream.

Define the parameter

$$dI_i = (I - I_i)\frac{d\dot{m}}{\dot{m}}$$

Substituting into **Eqn. (3.15)** and noting that for the flow in the bleed passage of the blade $\delta Q = 0$ and $\delta \mathrm{W} = 0$ gives

$$\boxed{dI + dI_i = 0} \qquad \textbf{Eqn. (3.16)}$$

Like the momentum equation, **Eqn. (3.16)** may be simplified by assuming the air is a perfect gas for which $h = C_p T$ gives

$$dI_i = C_p \left[\left(T + \frac{W^2}{2C_p} - \left(\frac{(\Omega r)^2}{2C_p} \right) \right) - \left(T_i + \frac{V_i^2}{2C_p} - \left(\frac{(\Omega r_i)^2}{2C_p} \right) \right) \right] \frac{d\dot{m}}{\dot{m}}$$

Therefore from **Eqn. (3.16)**

$$dI = -C_p \left[\left(T + \frac{W^2}{2C_p} - \left(\frac{(\Omega r)^2}{2C_p} \right) \right) - \left(T_i + \frac{V_i^2}{2C_p} - \left(\frac{(\Omega r_i)^2}{2C_p} \right) \right) \right] \frac{d\dot{m}}{\dot{m}} \qquad \textbf{Eqn. (3.17)}$$

From **Eqn. (3.17)** it can be seen that if the rothalpy of the air "sucked" into the bleed passage $(d\dot{m})$ is greater than the rothalpy of the main flow; an increase in the rothalpy takes place.

$$dI > 0$$

The opposite case is also true, if the rothalpy of the air "sucked" into the bleed passage $(d\dot{m})$ is lower than the rothalpy of the main flow; a decrease in the rothalpy takes place.

$$dI < 0$$

Equation of State

For a perfect gas, $P = \rho RT$, logarithmic differentiation yields

$$\boxed{\frac{dP}{P} = \frac{d\rho}{\rho} + \frac{dT}{T}} \qquad \textbf{Eqn. (3.18)}$$

Mach Number definition

For a perfect gas, $M = \dfrac{W}{\sqrt{\gamma RT}}$. This is the Mach number relative to the rotating coordinate system.

$$\boxed{\frac{dM}{M}=\frac{dW}{W}-\frac{1}{2}\frac{dT}{T}}$$ **Eqn. (3.19)**

Rothalpy

As shown before the rothalpy is defined as

$$I=h+\frac{\widetilde{W}^2}{2}-\frac{(\Omega r)^2}{2}$$

For a perfect gas, with enthalpy defined as $h=C_pT$ this can be written as

$$I=C_p\left(T+\frac{W^2}{2C_p}\right)-\frac{(\Omega r)^2}{2}$$ **Eqn. (3.20)**

The term in brackets is the stagnation temperature of the air in the rotating coordinate system.

This term can be written as follows:

$$T_t=T+\frac{W^2}{2C_p}=T\left(1+\frac{\gamma-1}{2}M^2\right)$$

Rewriting **Eqn. (3.20)** gives

$$I=C_pT\left(1+\frac{\gamma-1}{2}M^2\right)-\frac{(\Omega r)^2}{2}$$ **Eqn. (3.21)**

Differentiating **Eqn. (3.21)** and then dividing both sides by the rothalpy I leads to

$$\frac{dI}{I}=C_pdT\frac{\left(1+\frac{\gamma-1}{2}M^2\right)}{I}+\frac{C_pT}{I}\left((\gamma-1)MdM\right)-\frac{\Omega^2r}{I}dr$$ **Eqn. (3.22)**

Dividing both sides of **Eqn. (3.21)** by I gives

$$1=\frac{C_pT}{I}\left(1+\frac{\gamma-1}{2}M^2\right)-\frac{(\Omega r)^2}{2I}$$

After some rearrangement this becomes

$$\frac{\left(1+\frac{\gamma-1}{2}M^2\right)}{I}=\frac{1}{C_pT}\left(1+\frac{(\Omega r)^2}{2I}\right)$$ **Eqn. (3.23)**

Putting **Eqn. (3.23)** into **(3.22)** and using a relationship for $\frac{C_p T}{I}$ derived from **Eqn. (3.23)**, gives

$$\frac{dI}{I} = \frac{dT}{T}\left(1+\frac{(\Omega r)^2}{2I}\right) + \frac{((\gamma-1)\,MdM)}{\left(1+\frac{\gamma-1}{2}M^2\right)}\left(1+\frac{(\Omega r)^2}{2I}\right) - \frac{(\Omega r)^2}{I}\frac{dr}{r}$$

Manipulating this expression to fit into the framework outlined in **Zucrow & Shapiro** means getting an expression for the change in the total temperature relative to the rotating coordinates in terms of the rothalpy. So doing this manipulation gives

$$\boxed{\frac{\frac{dI}{I} + \frac{(\Omega r)^2}{I}\frac{dr}{r}}{\left(1+\frac{(\Omega r)^2}{2I}\right)} = \frac{dT}{T} + \frac{(\gamma-1)M^2}{\left(1+\frac{\gamma-1}{2}M^2\right)}\frac{dM}{M}}$$

Eqn. (3.24)

Relative Stagnation Pressure

$$\frac{P_t}{P} = \left(1+\frac{\gamma-1}{2}M^2\right)^{\frac{\gamma}{\gamma-1}}$$

An expression for the relative change in the stagnation pressure dP_t/P_t may be obtained by logarithmic differentiation of the above expression. Thus,

$$\boxed{\frac{dP_t}{P_t} = \frac{dP}{P} + \frac{\gamma M^2}{1+\frac{\gamma-1}{2}M^2}\frac{dM}{M}}$$

Eqn. (3.25)

Impulse Function

The impulse function for a perfect gas is defined in "**Zucrow**" and "**Shapiro**".

$$F = PA\left(1+\gamma M^2\right)$$

Like the above equations an expression for the relative change in the impulse function can be obtained by logarithmic differentiation. Thus,

$$\boxed{\frac{dF}{F}=\frac{dP}{P}+\frac{dA}{A}+\frac{2\gamma M^2}{1+\gamma M^2}\frac{dM}{M}}$$ **Eqn. (3.26)**

Entropy Change

$$ds = C_p\, d\left\{\ln\left[\frac{T}{P^{(\gamma-1)/\gamma}}\right]\right\}$$

By logarithmic differentiation this expression becomes

$$\boxed{\frac{ds}{C_p}=\frac{dT}{T}-\frac{\gamma-1}{\gamma}\frac{dP}{P}}$$ **Eqn. (3.27)**

3.1.1: Influence Coefficients

Equations **(3.2)**, **(3.12)** and **(3.18)-(3.27)** comprise a set of eight equations relating the changes in the eight flow properties, $dP/P, d\rho/\rho, dT/T$ etc.to the four independent variables or *driving potentials* dA/A, $\dfrac{\dfrac{dI}{I}+\dfrac{(\Omega r)^2}{I}\dfrac{dr}{r}}{\left(1+\dfrac{(\Omega r)^2}{2I}\right)}$, $\left(\dfrac{4fdr}{\mathscr{D}}-\dfrac{2\Omega^2 r^2}{W^2}\dfrac{dr}{r}\right)$ and $d\dot{m}/\dot{m}$. The eight equations are linear in the derivatives of the flow properties and the driving potentials, allowing the equations to be solved simultaneously. For convenience sake, the term involving the rothalpy change will be written as *J*:

$$J=\frac{\dfrac{dI}{I}+\dfrac{(\Omega r)^2}{I}\dfrac{dr}{r}}{\left(1+\dfrac{(\Omega r)^2}{2I}\right)}$$

Equations **(3.2)**, **(3.12)** and **(3.18)-(3.27)** can be written in matrix form with the flow properties as the dependent variables and the driving potentials on the right hand side.

$$\begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & \frac{\gamma M^2}{2} & 0 & \gamma M^2 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{2} & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & \frac{(\gamma-1)M^2}{\psi} & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & \frac{\gamma M^2}{\psi} & -1 & 0 & 0 \\ -1 & 0 & 0 & 0 & -\frac{2\gamma M^2}{1+\gamma M^2} & 0 & 1 & 0 \\ \frac{\gamma-1}{\gamma} & 0 & -1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \frac{dP}{P} \\ \frac{d\rho}{\rho} \\ \frac{dT}{T} \\ \frac{dW}{W} \\ \frac{dM}{M} \\ \frac{dP_t}{P_t} \\ \frac{dF}{F} \\ \frac{ds}{C_p} \end{bmatrix} = \begin{bmatrix} \frac{d\dot{m}}{\dot{m}} - \frac{dA}{A} \\ K+L \\ 0 \\ 0 \\ J \\ 0 \\ \frac{dA}{A} \\ 0 \end{bmatrix}$$

Eqn. (3.28)

Where

$$\psi \equiv 1 + \frac{\gamma-1}{2} M^2$$ **Eqn. (3.29)**

$$K = -\frac{\gamma M^2}{2}\left[\left(\frac{4fdr}{D}\right) - \frac{2\Omega^2 r^2}{W^2}\frac{dr}{r}\right]$$ **Eqn. (3.30)**

$$L = -\gamma M^2 (1-y)\frac{d\dot{m}}{\dot{m}}$$ **Eqn. (3.31)**

$$J = \frac{\frac{dI}{I} + \frac{(\Omega r)^2}{I}\frac{dr}{r}}{\left(1 + \frac{(\Omega r)^2}{2I}\right)}$$ **Eqn. (3.32)**

Eqn. (3.28) may be written as

$$Ax = b$$

Where x is the vector of the eight flow property changes and b is the vector of changes in the driving potentials.

A system of equations similar to those shown in **Eqn. (3.28)** is solved in both **Zucrow** and **Shapiro**. The A matrix and the x vector are exactly the same and the only differences being in b the vector of driving potentials. Using the technique outlined the system of equations can be solved and a table of influence coefficients is present in Table 3.1. Anyone of the changes in the flow properties listed in the first column of Table 3.1 may be related to the independent driving potentials listed across the top of the table by multiplying the driving potential by the tabulated expression located at the intersection of the row containing the desired flow property change and the column containing the desired driving potential. For the flow in the bleed passage, all the driving potentials will contribute to the change in each flow property. Each influence coefficient represents the partial derivative of the variable in the left-hand column in Table 3.1 with respect to the variable in the top row. They determine the influence of each driving potential on the change in each flow property. An example of the effect of each influence coefficient on the change in Mach number M is given under Table 3.1.

Flow Properties	$\frac{dA}{A}$	$\left[\left(\frac{4fdr}{D}\right)-\frac{2\Omega^2r^2}{W^2}\frac{dr}{r}\right]$	$\frac{\frac{dI}{I}+\frac{(\Omega r)^2}{I}\frac{dr}{r}}{\left(1+(\Omega r)^2/2I\right)}$	$\frac{d\dot{m}}{\dot{m}}$
$\frac{dM}{M}$	$-\frac{\psi}{1-M^2}$	$\frac{\gamma M^2\psi}{2\left(1-M^2\right)}$	$\frac{\left(1+\gamma M^2\right)\psi}{2\left(1-M^2\right)}$	$\frac{\psi\left[\left(1+\gamma M^2\right)-y\gamma M^2\right]}{1-M^2}$
$\frac{dP}{P}$	$\frac{\gamma M^2}{1-M^2}$	$-\frac{\gamma M^2\left[1+(\gamma-1)M^2\right]}{2\left(1-M^2\right)}$	$-\frac{\gamma M^2\psi}{1-M^2}$	$-\frac{\gamma M^2\left[2\psi(1-y)+y\right]}{1-M^2}$
$\frac{d\rho}{\rho}$	$\frac{M^2}{1-M^2}$	$-\frac{\gamma M^2}{2\left(1-M^2\right)}$	$-\frac{\psi}{1-M^2}$	$-\frac{\left[(\gamma+1)M^2-y\gamma M^2\right]}{1-M^2}$
$\frac{dT}{T}$	$\frac{(\gamma-1)M^2}{1-M^2}$	$-\frac{\gamma(\gamma-1)M^4}{2\left(1-M^2\right)}$	$\frac{\left(1-\gamma M^2\right)\psi}{1-M^2}$	$-\frac{(\gamma-1)M^2\left[\left(1+\gamma M^2\right)-y\gamma M^2\right]}{1-M^2}$
$\frac{dW}{W}$	$-\frac{1}{1-M^2}$	$\frac{\gamma M^2}{2\left(1-M^2\right)}$	$\frac{\psi}{1-M^2}$	$\frac{\left[\left(1+\gamma M^2\right)-y\gamma M^2\right]}{1-M^2}$
$\frac{dP_t}{P_t}$	0	$-\frac{\gamma M^2}{2}$	$-\frac{\gamma M^2}{2}$	$-\gamma M^2(1-y)$

Table 3.1: Influence coefficients for 1-D steady flow in rotating coordinates

$$\frac{dM}{M}=\frac{\left(1+\frac{\gamma-1}{2}M^2\right)}{1-M^2}\left\{-\frac{dA}{A}+\frac{\gamma M^2}{2}\left[\left(\frac{4fdr}{\mathscr{D}}\right)-\frac{2(\Omega r)^2}{W^2}\frac{dr}{r}\right]+\frac{\left(1+\gamma M^2\right)}{2}\left[\frac{\frac{dI}{I}+\frac{(\Omega r)^2}{I}\frac{dr}{r}}{\left(1+\left((\Omega r)^2/2I\right)\right)}\right]+\left[\left(1+\gamma M^2\right)-y\gamma M^2\right]\frac{d\dot{m}}{\dot{m}}\right\}$$

3.1.2: Calculation of the Friction Coefficient

When the aspirated air, flows through the bleed passage of the rotor, a Coriolis force acts on the air as it travels radially outward or inward. The magnitude of the Coriolis force per unit volume is $\left(2\Omega\times W\right)$ and it acts in a plane normal to Ω and $\widetilde{W}$; thus for the bleed passage in Figure 3.1, with the Ω component normal to the plane of the passage, the Coriolis force acts in the plane of the passage, normal to the *r*-component of velocity.

The rotation induced Coriolis force leads to a secondary from the leading to the trailing edge of the passage across the central core; this mass flux is balanced by flow in the opposite direction in the boundary layers adjacent to the passage walls. This produces the secondary flow vortices normally associated with flow in rotating passages. Figure 3.2 gives a schematic of the secondary flow induced in the bleed passage. This secondary flow increases the frictional resistance experienced by the through flow air. To account for this the fanning friction coefficient will be modified using a correction factor, whose size depends on the Reynolds number based on the mean radial velocity $\mathrm{Re}=\frac{W\mathscr{D}}{\nu}$ and a rotational Reynolds number based on the angular velocity of the blade, $J_d=\frac{\Omega\mathscr{D}^2}{\nu}$.

Details of the correlations discussed here can be found in **Morris** (Ref 5), **Ito et al.** (Ref 6), **Mori et al.** (Ref. 7) and **Chew** (Ref. 8). The analysis was carried out for both fully developed laminar and turbulent flows, although for the bleed passage the flow was assumed to be turbulent.

In calculating the friction factors for the rotating channel the flow was divided into two regions:

1. A thin boundary layer near the wall of the passage.
2. A frictionless core where the flow is influenced by inertial and pressure forces alone.

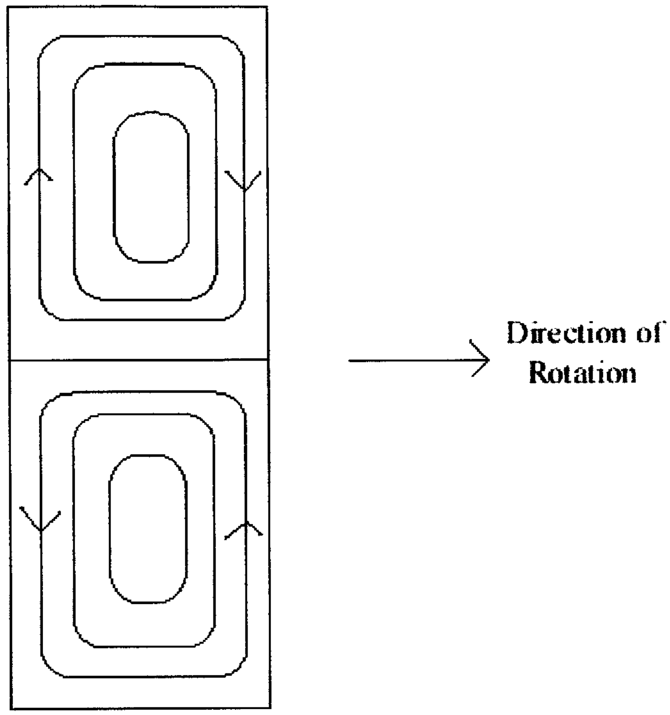


Figure 3.2: Secondary flow in the bleed passage

From an analysis of the frictionless core the velocity of the secondary flow in the core can be calculated, and hence the mass flux of the secondary flow. From continuity, this mass flux is balanced by flow in the opposite direction in the boundary layers adjacent to the passage walls. The authors then assume velocity profiles for the flow in the core and the boundary layer. The assumed core flow profile is skewed, to account for the Coriolis effects, while the boundary layer profile depends on whether the flow is turbulent or laminar. The secondary mass flux and the assumed velocity profile allow the velocity distribution within the boundary layer to be calculated. Using an integral analysis, the shear stress at the wall can be calculated, which in turn allows the friction factor to be calculated.

In the references, the authors have produced correlations that give the ratio of the friction coefficient for a rotating page to the fanning friction coefficient for a stationary pipe. The value of this ratio depends on numerous factors but numerous factors but for flows considered in this analysis the ratio is greater than 1, which implies the losses due to friction in the rotating passage are greater than those in a stationary passage.

Fanning Friction Coefficient

As noted above references 5-7 give the ratio of the rotating friction factor to the fanning friction factor. So in order to calculate ***f***, the fanning friction factor must be known. The fanning friction coefficient is calculated using the formula proposed by Haaland and taken from **"Viscous Fluid Flow"** by White (Ref 9).

$$f = \frac{\Lambda}{4}$$

$$\frac{1}{\Lambda^{\frac{1}{2}}} \approx -1.8\log_{10}\left[\frac{6.9}{\mathrm{Re}_{\mathscr{D}}} + \left(\frac{k/\mathscr{D}}{3.7}\right)^{1.11}\right]$$

Where $\mathscr{D}$ is the hydraulic diameter and k is the roughness of the slot.

For the case of fully developed turbulent flow, an empirical correlation proposed by Ito and Nanbu (Ref. 6) was used in calculating the rotating friction factor (f_r). The correlation is

$$\frac{f_r}{f} = 0.942 + 0.058\left[\frac{J_d^2}{\mathrm{Re}}\right]^{0.282} \qquad \textbf{Eqn. (3.33)}$$

Where $J_d = \frac{\Omega \mathscr{D}^2}{\nu}$ and $\mathrm{Re} = \frac{W\mathscr{D}}{\nu}$. This correlation is valid in the range

$$1 \le \frac{J_d^2}{\mathrm{Re}} \le 5 \mathrm{x} 10^5$$

Which is within the range of the flow of the flow through the bleed passage. The increase in the value of the rotational friction factor f_r are document in Chapter 6 of Ref. 8.

3.2: Numerical Integration

The differential equations in Table 3.1 must be integrated numerically for specific initial and boundary conditions. The initial conditions for this problem were the Mach number, static temperature and static pressure at the bottom of the slot. A discussion of how these values were calculated is given below. The boundary conditions are the conditions of the air outside the suction slot. These conditions can be got from the CFD analysis of the blade shape or can be calculated as discussed in Chapter 4. A Forward Euler technique was used to integrate each of the equations.

Conditions outside the slot

The following conditions need to be specified outside the suction slot

1. Rothalpy of the air outside the slot (or total enthalpy for stationary blades).
2. Static pressure P_i.
3. Air density ρ_i.

The slot is discretized as shown in Figure 3.3 below, with the above conditions specified at each location. Knowledge of the conditions outside the slot allow the changes in dI and $d\dot{m}$ to be calculated.

From **Eqn. (3.16)** the change in the rothalpy due to mass addition can be calculated

$$dI + dI_i = 0$$

Where

$$dI_i = C_p\left[\left(T_i + \frac{W^2}{2C_p} - \left(\frac{(\Omega r)^2}{2C_p}\right)\right) - \left(T_{ei} + \frac{V_{ei}^2}{2C_p} - \left(\frac{(\Omega r_i)^2}{2C_p}\right)\right)\right]\frac{d\dot{m}}{\dot{m}}$$

Where T_{ei} is the static temperature outside the suction slot.

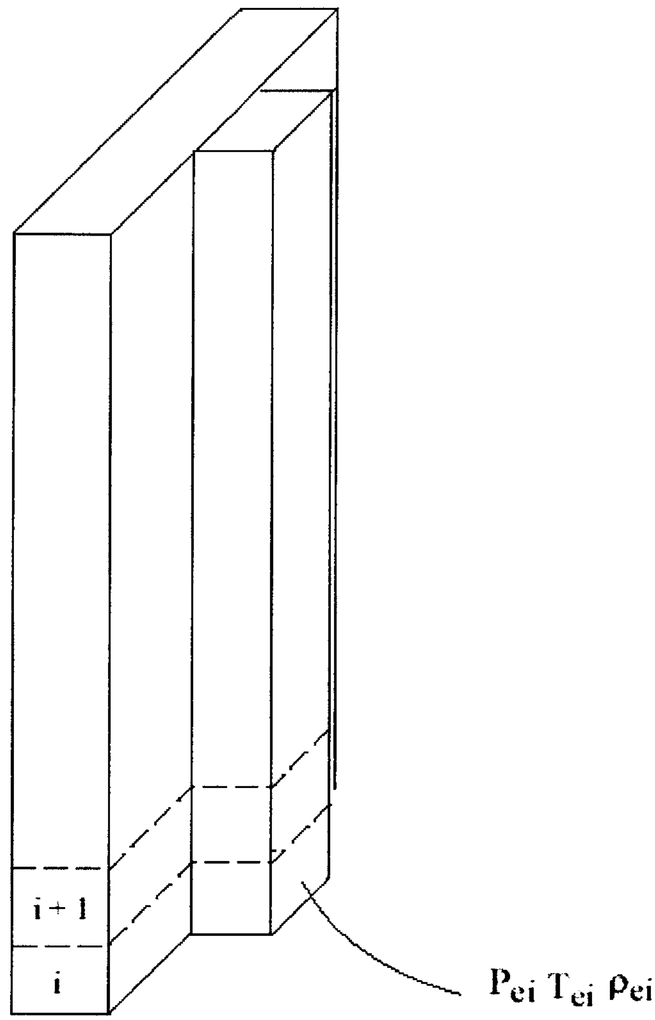


Figure 3.3: Discretized slot with conditions outside the slot specified

The amount of mass "sucked" in to the bleed passage of the rotor or stator blade is determined by the pressure difference between the static pressure outside the slot and the static pressure inside the slot. This difference allows $d\dot{m}$, (the amount of mass added at each section) to be calculated.

Aspiration is implemented by "sucking" a portion of the blade boundary layer, causing the boundary layer to remain attached further downstream. In the boundary layer, just above the blade surface the fluid velocity is approximately zero (in the rotating coordinate system for the rotor). This means that the static pressure is equal to the total pressure just above the blade surface.

For the air to be sucked into the slot, the static pressure inside the slot must be less than the static pressure outside the slot. Luckily this is the case for the aspirated compressor. The factors that dictate the amount of mass $d\dot{m}$ "sucked" into the slot are

- Static Pressure difference between the inside and outside of the slot.
- Area of the slot.
- Density of the air outside the slot.

Figure 3.4, below shows a schematic of the flow entering the slot. The first step is to calculate the Mach number M_i of the air sucked into the bleed passage. This is done by assuming an isentropic expansion through the slot from the "Total pressure" P_{te} outside the slot to the static pressure P_i inside the slot.

$$M_i = \sqrt{\frac{2}{\gamma-1}\left(\left(\frac{P_e}{P_i}\right)^{\frac{\gamma-1}{\gamma}} - 1\right)} \qquad \textbf{Eqn. (3.34)}$$

Using this Mach number, the Area of the entrance slot, the external total pressure and temperature an estimate for the mass into the slot can be calculated as follows:

$$\boxed{d\dot{m}_i = dA_i\, M_i\, P_e\left(\frac{\gamma}{RT_e}\right)^{\frac{1}{2}}\left(1+\frac{\gamma-1}{2}M^2\right)^{-(\gamma+1)/2(\gamma-1)}} \qquad \textbf{Eqn. (3.35)}$$

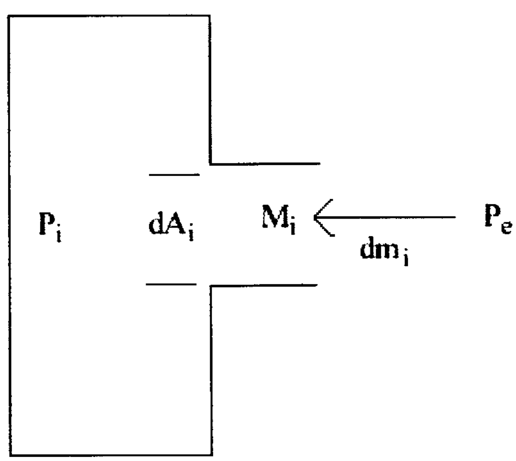


Figure 3.4: Calculation of Mass flow into slot

Initial Conditions

An initial value for the Mach number M, the static pressure and temperature must be specified. To calculate the initial conditions some assumptions about the conditions at the bottom of the bleed passage were necessary. The static pressure is assumed to be 12% less than the pressure outside the slot. This then allows a value of $d\dot{m}$ to be calculated. The static temperature inside the passage is assumed equal to the static temperature outside the slot. The values of static temperature and pressure allow the density of the air in the bleed passage to be calculated. Using continuity $\rho AW = d\dot{m}$ a value for W, can be found and using the static temperature to calculate a speed of sound, a value for $M_{initial}$ to be calculated. A schematic illustrating the ideas behind this calculation is shown below in Figure 3.5.

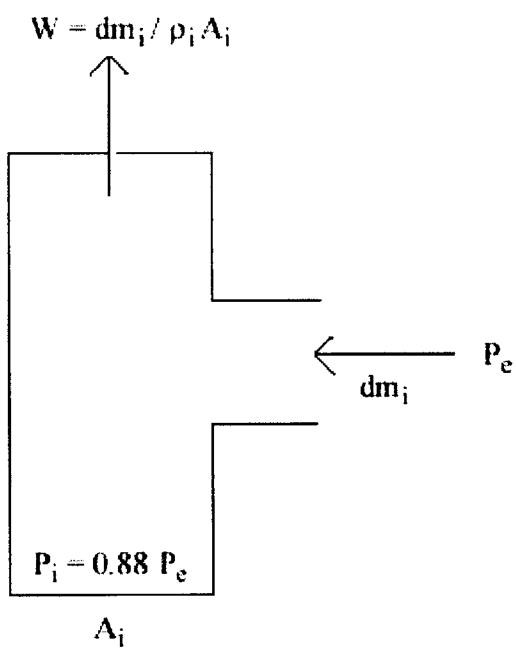


Figure 3.5: Calculation of Initial Conditions

Chapter 4: Results

4.1: Introduction

In this section the effects of aspiration on the performance of the compression system and the overall engine are presented. To illustrate these effects, the systems analysis discussed in section 2 is used to compare the performances of both the aspirated and the non-aspirated engine. Five different engine/fan configurations are presented, to illustrate and understand the effects of aspiration. The five configurations are

1. Potential QSP configured Turbofan.
   - (i) Aspirated Fan and Aspirated core compressor (see **Figure 4.1**);
   - (ii) Aspirated Core compressor only (see **Figure 4.6**);
   - (iii) Aspirated Fan only (see **Figure 4.8**).
2. MIT low speed Fan (see **Figure 4.11**).
3. Aspirated Turbojet performing auxiliary work (see **Figure 4.13**).

At the heart of the analysis is the potential QSP (Quiet Supersonic Platform) configured turbofan. This engine is analyzed with aspiration applied to either the fan, core compressor or both. Analyzing the three separate configurations allows the effects of aspiration on each component to be recognized and understood more clearly. Two other configurations are analyzed, the MIT low speed fan and the aspirated turbojet performing "auxiliary work". The MIT low speed fan is an aspirated fan stage; with a rotor tip speed of 228 m/s. This stage has been built and tested in the MIT blow down compressor, where accurate values for the through flow efficiency have been calculated. Using these efficiency numbers and values for the conditions of the bleed air, the fans installed efficiency was calculated (the concept of installed efficiency is discussed latter). In chapter 2, the concept of the aspirated turbojet (see **Section 2.2, Figure 2.8**) was introduced. The bleed air in this engine is used to perform "auxiliary work" in another part of the engine or aircraft. This idea is presented as an alternative approach to the analysis of the turbofan.

The performance of the aspirated engine depends on whether the bleed air can be used in the cycle for some other function. In section 2 we assumed for the turbofan that the air returned to the turbine as cooling air (core compressor bleed air) or expanded overboard to recover the momentum drag (Fan bleed air). In the case of the turbojet, performance depends on the amount of "Auxiliary Work" the bleed air could perform.

What dictates whether the bleed air can be returned to the turbine or the amount of thrust the air can produce is its pressure and temperature. The total pressure and temperature of the bleed air leaving the blade is calculated using the techniques outlined in Section 3. This analysis accounts for the losses in the blade passage (i.e. losses inherent in the aspiration process) and gives the total temperature and pressure leaving the blade. However, to use this air for a useful purpose the air must be diffused and piped to the area where it can be returned to the cycle or expanded overboard. This diffusion process involves losses that depend on the diffuser design and the specific routing of the air. To account for these losses a pressure recovery factor is introduced. The value of the recovery factor varies between 0 – 1 (0 for complete loss of the bleed air, 1 for complete recovery).

Two measures of performance were used to compare the aspirated to the non-aspirated technology. Specific Impulse was used to compare the overall engine efficiencies, while compressor efficiency was used to compare the compression systems. For the aspirated configuration, an "***Installed compressor efficiency***" was defined. This concept will be explained below.

4.2: Bleed Air Calculations

During this study detailed designs of the rotor and stator blades to be used in the counter-rotating compressor had not been completed. Without this information values for the ***Relative Total temperature and Static pressure*** for conditions outside the bleed passage on the rotor blades were not available. Other information such as the placement of the bleed slot on the suction surface of the blade, the dimensions and area distribution of the slot were also not available.

In this analysis, the bleed slot was assumed to be located on the suction surface at 50% of the chord length. The opening on the blade surface through which the bleed is extracted was assumed to vary between 1mm – 2mm in width. This parameter was varied to control the mass flow of air into the slot (see **Section 3.2**, where the factors that influence the mass flow into the slot are discussed). The internal passage and it's opening were assumed to occupy the top 30% of the blade span. For example if the hub/tip ratio was 0.5, the slot extended from, $0.85r_{tip}$ to r_{tip}.

To overcome the problem of no static pressure and total temperature outside the bleed passage, estimates were calculated. An estimate of the total temperature outside the slot was calculated using Euler's equation. With the value for total temperature, the total pressure was calculated by assuming an isentropic efficiency for the temperature rise and using this to get an estimate of the total pressure outside the bleed passage.

The swirl component of the air close to the blades surface is approximately equal to the velocity of the blade Ωr. If the swirl of the air entering the blade is assumed to be zero, Euler's equation can be used to estimate the rise in total temperature based on the change in the swirl component of the air.

$$C_p \Delta T_t = \Omega r \left(v - v_{in}\right) = \left(\Omega r\right)^2$$

The total temperature into each stage can be calculated using the stage pressure ratio (π_1, π_2, π_3) and the stage isentropic efficiencies (η_1, η_2, η_3). The total pressure outside the slot, can be calculated using

$$P_t = P_{tin} + \overline{\rho}(\Omega r)^2$$

Where $\overline{\rho}$ is the average of the fluid density; the average of the air density before the blade row and the density after the blade row. What drives the flow into the bleed passage is not the total pressure but the static pressure difference between the outside of the slot and the inside. The static pressure outside the slot is found using an estimate for the fluid Mach number, M_{fluid}

$$P = \frac{P_t}{\left(1 + \left(\frac{\gamma - 1}{2}\right) M_{fluid}^2\right)^{\gamma/(\gamma-1)}}$$

The fluid Mach number M_{fluid} contains two components, the swirl component and the axial component. No radial component is assumed. The swirl component is equal to Ωr and the axial velocity is assumed constant equal to the inlet Mach number M_{in}. Therefore

$$M_{fluid} = \sqrt{(\text{Swirl component})^2 + (\text{Axial Component})^2}$$

$$M_{fluid} = \sqrt{\left(\frac{\Omega r}{\sqrt{\gamma R T}}\right)^2 + M_{in}^2}$$

For this study the axial Mach number M_{in} is set equal to 0.6. This is the value being used for the preliminary design of the counter-rotating compressor.

4.3: Installed Efficiency

As stated earlier, to understand and see the effects of aspiration on the overall turbofan cycle two measures of performance will be used.

1. Specific Impulse was used to compare the overall engine efficiencies
2. Compressor efficiency was used to compare the compression systems.

Specific impulse was defined in Section 2 and is repeated here for clarity. *Specific Impulse* is defined as the number of units of thrust produced per unit of fuel weight flow rate (Kerrebrock). For the turbofan analyzed in Section 2 this gives

$$I = \frac{F_{total}}{\dot{m}_{fuel} g}$$

Where F_{total} = Total thrust of the engine

$\dot{m}_{fuel}$ = Mass flow rate of fuel

g = acceleration due to gravity

For the aspirated compressor an equivalent isentropic efficiency is defined. This new efficiency takes the bleeds into account and will be called the "***Installed Efficiency***".

Effects of Bleeds on Efficiency

For a normal non-aspirated compressor, the isentropic efficiency is defined as

$$\eta_{isen} = \frac{\text{Ideal work of compression for a given } \pi_c}{\text{Actual work of compression for a given } \pi_c}$$

This leads to the familiar expression found in references 1 and 2.

$$\eta_{isen} = \frac{\pi_c^{(\gamma-1)/\gamma} - 1}{\tau_c - 1}$$

For the aspirated compressor the bleeds must be accounted for to give a more revealing measure of the efficiency. If the bleeds are treated as a set of compressors operating in parallel, each producing a stagnation pressure ratio and a stagnation temperature ratio, the installed efficiency can be defined as

$$\eta_{installed} = \frac{\text{Total available work}}{\text{Total work performed}}$$

The total available work will be estimated by assuming the air can be expanded isentropically to the local ambient pressure, while the work performed will be the total temperature rise during the compression. If the $\mathbf{C_{pc}}$ for both the through flow air and the bleeds are equal, the following relationship, the efficiency can be defined as

$$\eta_{installed} = \frac{\dot{m}_0\left(\pi_0^{(\gamma-1)/\gamma} - 1\right) + \sum \dot{m}_i\left(\pi_i^{(\gamma-1)/\gamma}\right)}{\dot{m}_0\left(\tau_0 - 1\right) + \sum \dot{m}_i\left(\tau_i - 1\right)}$$

$\dot{m}_0$ is mass flow through the core compressor, and the subscript $i = 0$ represents the core flow.

Note: For the core compressor of the turbofan $\dot{m}_0 = \dot{m}_c$

Define the ratio of the core mass flow to the bleed mass flow as ε_i

$$\varepsilon_i = \frac{\dot{m}_i}{\dot{m}_0}$$

After some manipulation the Installed efficiency of the aspirated compressor is

$$\eta_{installed} = \frac{\eta_0\left(\tau_0 - 1\right) + \sum \varepsilon_i\left(\pi_i^{(\gamma-1)/\gamma} - 1\right)}{\left(\tau_0 - 1\right) + \sum \varepsilon_i\left(\tau_i - 1\right)} \qquad \textbf{Eqn. (4.1)}$$

Where η_0 = the isentropic efficiency of the core flow compression. From **Eqn (4.1)** it can be seen that the equivalent compressor efficiency equals the isentropic efficiency of the core flow compression system if the bleed compressions are carried out isentropically.

$$\sum \varepsilon_i \left(\pi_i^{(\gamma-1)/\gamma} - 1 \right) = \sum \varepsilon_i \left(\tau_i - 1 \right)$$

However, the compression of the bleed air is inefficient and the equivalent compressor efficiency will be less than the isentropic efficiency of the through flow air.

Before going onto the actual engine comparisons, another point about efficiencies will be addressed. In the results to follow, it was assumed throughout that the through flow air through the aspirated compressor was the same as that of the non-aspirated compressor. This may not be true as the aspirated compressor may have higher polytropic efficiency. The reason for this higher polytropic efficiency is the fact air bleed off due to aspiration is the high entropy air in the boundary layer of the fluid in contact with the blade surface. Removing this air has the effect of reducing the temperature into the next stage of the compressor and thereby can lead to an increase in polytropic efficiency. This concept is discussed in much greater detail in Ref 10 (Kerrebrock et. al, 97-GT-525)

4.4: QSP Configured Turbofan

To clearly see the effects of aspiration on the cycle, both the non-aspirated and aspirated engines were identical in almost all respects except for the presence of aspiration. This means that both engines were assumed to be flying at the same altitude and speed, T_{t4} for both units was equal; the polytropic efficiency of the fan, compressor and turbine on both units was assumed to be the same etc. Table 4.1 below, shows the configuration of each turbofan engine.

As mentioned in Section 2, the aspirated turbofan uses a counter-rotating compressor. This compressor has 3 stages and each stage is aspirated. The bleed air is taken radially outward for Stage 1 and radially inwards for Stages 2 & 3. Table 4.1, gives the overall pressure ratio and isentropic efficiency for each stage, while Table 4.2 below gives a more detailed breakdown of the pressure rise and temperature rise across the aspirated compressor stages.

For convenience in the cycle analysis, the overall compression ratio through the fan and the compressor is denoted by π_c. This is in fact the product of the fan and the compressor pressure ratios. For the aspirated compressor this works out to be

$$\pi_c = \pi_f \, \pi_1 \, \pi_2 \, \pi_3 = 32.827$$

The pressure ratio of the fan alone is denoted by π_f.

For stages 2 & 3 of the counter-rotating compressor, the bleed air is taken radially inward. It is assumed that the inward flow is extracted for turbine cooling at 15% of the blade tip radius. As table 4.1 shows, for the aspirated compressor, 1% of the main bleed flow was extracted from each blade row (i.e. $\varepsilon_{r1} = 0.01$ etc.). This air is returned to the cycle as turbine cooling air or in the case of the fan extraction air is expanded through a nozzle to recover the momentum drag.

	Non-Aspirated Turbofan	Aspirated Turbofan (Counter-rotating compressor)
Flight Mach No.	2	2
Altitude	50,000 ft	50,000 ft
T_{t4}	1922 K	1922 K
π_d	0.925	0.925
π_c	32.83	32.83
π_f	1.776	1.776
$(\Omega r)_{tip}$	457 m/s	457 m/s
Stage 1, π_1, η_1	N/A	3.325, 0.94
Stage 2, π_2, η_2	N/A	2.66, 0.9
Stage 3, π_3, η_3	N/A	2.09, 0.9
e_c	0.916	0.916
e_f	0.9	0.9
e_t	0.9	0.9
ε_{rf}	N/A	$0.01\,\dot{m}_f$
ε_{sf}	N/A	$0.01\,\dot{m}_f$
ε_{r1}	N/A	$0.01\,\dot{m}_c$
ε_{s1}	N/A	$0.01\,\dot{m}_c$
ε_{r2}	N/A	$0.01\,\dot{m}_c$
ε_{s2}	N/A	$0.01\,\dot{m}_c$
ε_{r3}	N/A	$0.01\,\dot{m}_c$
ε_{s3}	N/A	$0.01\,\dot{m}_c$
h_{pr}	42800 KJ/kg	42800 KJ/kg

Table 4.1: QSP Configured Turbofan

Stage	Rotor	Stator	Overall
π_1	3.5	0.95	3.325
τ_1	1.44	1	1.44
η_{isen}	-	-	0.94
π_2	2.8	0.95	2.66
τ_2	1.36	1	1.36
η_{isen}	-	-	0.9
π_3	2.2	0.95	2.09
τ_3	1.26	1	1.26
η_{isen}	-	-	0.9

Table 4.2: Aspirated Compressor

Using the code discussed in Section 3, the values of the total pressure and temperature for each of the aspirated stages were calculated and are listed below in Table 4.3. These values represent the temperature and pressure of the air as it leaves the bleed passage of the blade. The losses that the bleed air experiences in the bleed passage are intrinsic to the aspiration process. Once the air leaves the blade, it can be used for other purposes such as turbine cooling etc. The amount of pressure recovery that can be achieved once the air has left the blade passage will depend on the diffusion system and the final destination of the air.

As can be seen in **Eqn. (4.1)** the value of pressure recovery can plays an important role in the efficiency of the compression system. If none of the bleed air can be recovered for useful work, **Eqn. (4.1)** becomes

$$\eta_{eq} = \frac{\eta_0(\tau_0 - 1)}{(\tau_0 - 1) + \sum \varepsilon_i(\tau_i - 1)}$$

So to understand the full effects of aspiration the Specific impulse and equivalent compressor efficiency for the aspirated turbofan are calculated for various levels of Pressure recovery.

Stage Component	Total Pressure	Total temperature
1st Stage Rotor	$P_{tre1} = 2.998 \text{ x } 10^5 \ N/m^2$	$T_{tre1} = 664$ K
1st Stage Stator	$P_{tse1} = 3.06 \text{ x } 10^5 \ N/m^2$	$T_{tse1} = 674$ K
2nd Stage Rotor	$P_{tre2} = 3.45 \text{ x } 10^5 \ N/m^2$	$T_{tre2} = 688$ K
2nd Stage Stator	$P_{tse2} = 9.96 \text{ x } 10^5 \ N/m^2$	$T_{tse2} = 922$ K
3rd Stage Rotor	$P_{tre3} = 1.12 \text{ x } 10^6 \ N/m^2$	$T_{tre3} = 931$ K
3rd Stage Stator	$P_{tse3} = 2.14 \text{ x } 10^6 \ N/m^2$	$T_{tse3} = 1155$ K
Fan Rotor	$P_{rf} = 1.7 \text{x } 10^5 \ N/m^2$	$T_{rf} = 654$K
Fan Stator	$P_{sf} = 8.57 \text{x } 10^4 \ N/m^2$	$T_{sf} = 468$K

Table 4.3: Bleed air temperature and pressure for QSP configured turbofan

As a reference for the pressures and temperatures in Table 4.3,

$$P_{t2} = 8.77 \text{x } 10^4 \ N/m^2 \ \ \& \ \ T_{t2} = 388.8\text{K}$$

The air is used for turbine cooling or is expanded overboard to recover its momentum drag. The full cycle calculations for the 3 differently configured QSP turbofans and the Non-aspirated turbofan in Appendix D, for various levels of pressure recovery.

4.4.1: QSP Turbofan-Aspirated Fan & Core Compressor

In the introduction to this chapter, it was mentioned that three different QSP turbofan configurations would be analyzed and their performance compared to a non-aspirated mixed exhaust turbofan. The first of these configurations is shown in Figure 4.1 below.

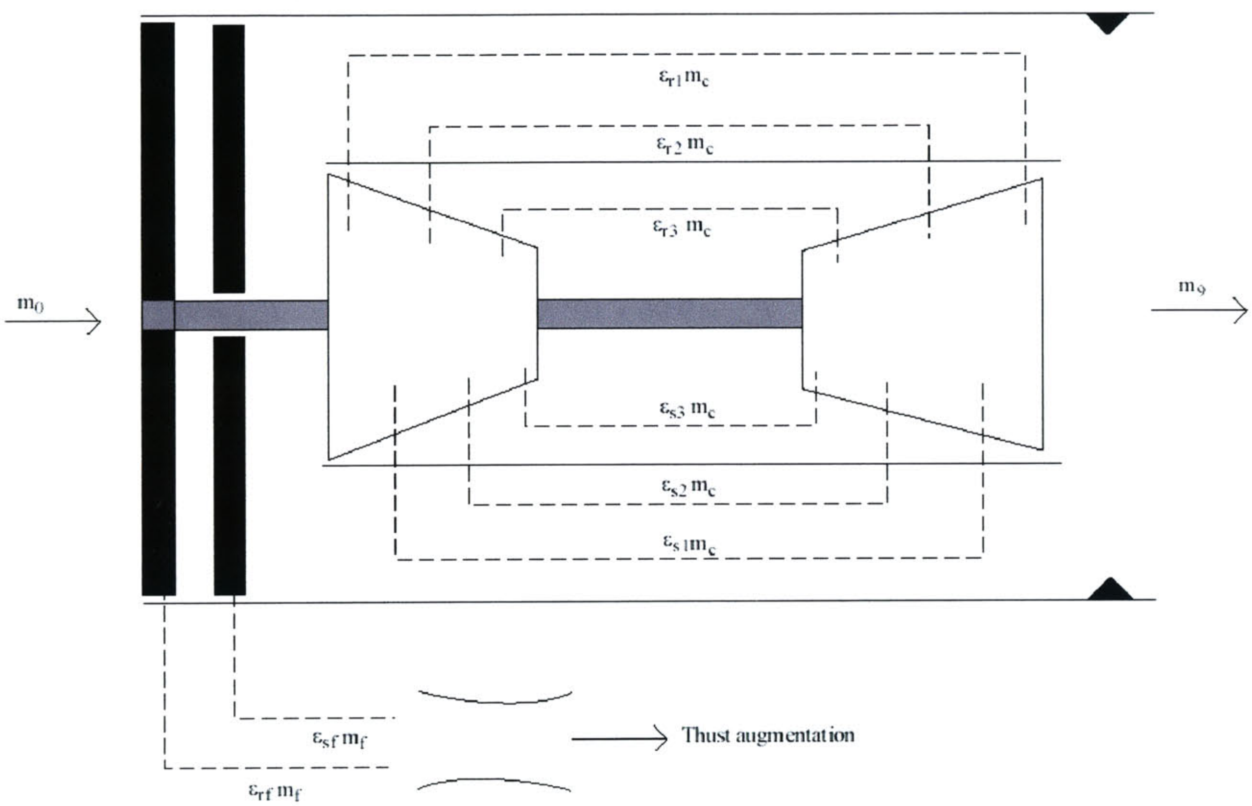


Figure 4.1: QSP turbofan with Fan and core aspiration

As can be seen from Figure 4.1, the engine contains and aspirated fan and a 3 stage aspirated counter-rotating compressor. The flight conditions, the percentage air removed by aspiration, the compressor pressure ratio, T_{t4} etc. are all detailed in **Table 4.1**.

Figure 4.2, compares the Specific Impulse for the QSP configured turbofan and non-aspirated turbofan versus pressure recovery. Table 4.4 gives the actual values of Specific Impulse for both the aspirated and non-aspirated case. For figure 4.2 the pressure recovery of the fan was assumed to be 100% and the effect of aspiration on the core was studied. At 100% pressure recovery, it can be seen that the specific impulse of the engine with the aspirated fan and core compressor is less than that of the non-aspirated case. This is because of the frictional losses present in the internal bleed passage.

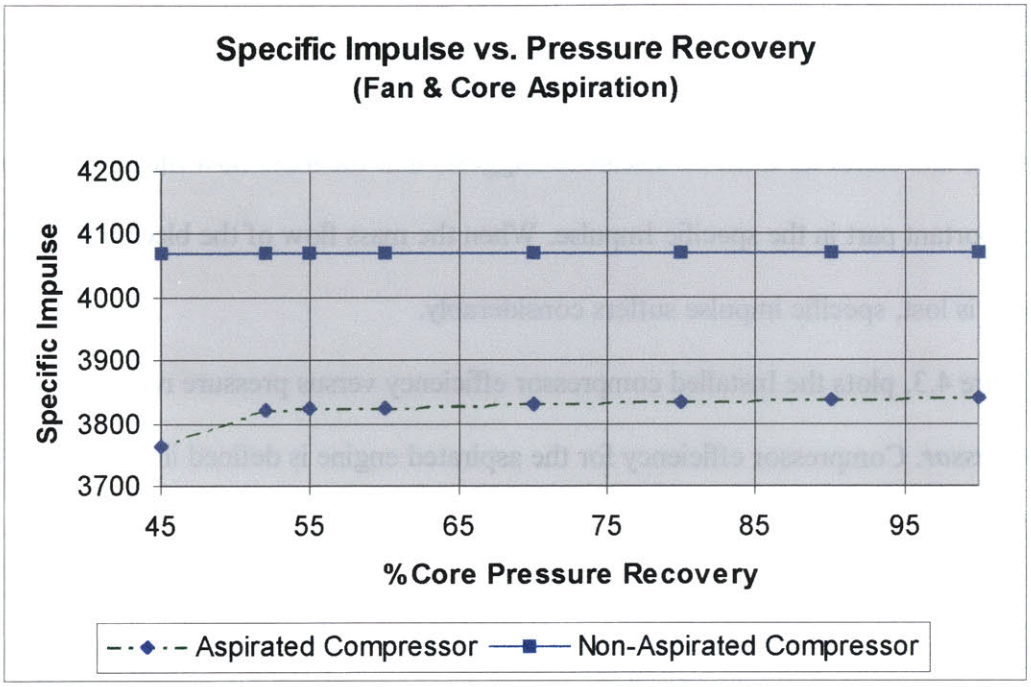


Figure 4.2: Specific Impulse vs. Pressure Recovery for QSP turbofan

(See Table 4.2 for engine configuration)

% Pressure Recovery	Specific Impulse (Aspirated Compressor)	Specific Impulse (Non-Aspirated Compressor)
100	3840 sec	4070
90	3837 sec	4070
80	3834 sec	4070
70	3830 sec	4070
60	3825 sec	4070
55	3823 sec	4070
52	3821 sec	4070
45	3763 sec	4070

Table 4.4: Specific Impulse vs. Pressure Recovery for QSP turbofan

Pressure recovery does not play a significant factor in the value of specific impulse until a pressure recovery of 52% is reached. At this point reducing the pressure of the 1st stage rotor air 48% means that the air cannot be returned to the turbine as cooling air as its pressure is too low. Below this pressure recovery the air from the 1st stage rotor and stator is discarded and plays is not returned to the cycle, hence the fall off in Specific Impulse.

The fact that pressure recovery does not significantly effect the specific impulse until the 1st stage rotor and stator air must be discarded suggests that the mass flow through the engine plays an important part in the specific Impulse. When the mass flow of the bleed air from the 1st stage bleeds is lost, specific impulse suffers considerably.

Figure 4.3, plots the Installed compressor efficiency versus pressure recovery, for the ***core compressor***. Compressor efficiency for the aspirated engine is defined in **Eqn. (4.1)**.

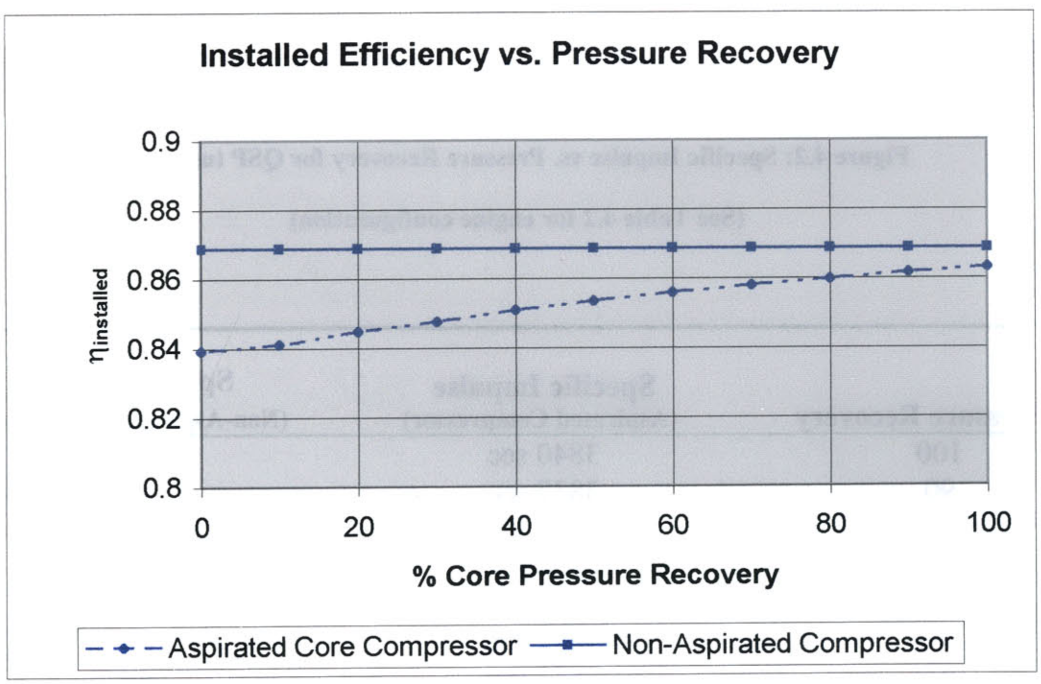


Figure 4.3: Core Installed Efficiency vs. Pressure Recovery for aspirated counter rotating compressor (see Table 4.2 & Table 4.3)

Like Figure 4.2, even with 100% pressure recovery the compressor efficiency of the aspirated compressor is less than that of the non-aspirated core, due to the losses from the bleed passages. At a pressure recovery of approximately 60% the compressor isentropic efficiency is reduced by approximately 1%. Table 4.5, below dives the exact values of compressor efficiency as pressure recovery changes. When calculating the available work, it is assumed the bleed air is expanded isentropically to P_{t2}.

% Pressure Recovery	Installed Efficiency (Aspirated)	Compressor Efficiency (Non-Aspirated)
100	0.863	0.869
90	0.862	0.869
80	0.86	0.869
70	0.858	0.869
60	0.856	0.869
50	0.854	0.869
40	0.853	0.869
30	0.848	0.869
20	0.845	0.869
10	0.841	0.869
0	0.839	0.869

Table 4.5: Core Compressor Installed Efficiency vs. Pressure Recovery

To further illustrate the effects of aspiration on the compression system of the counter-rotating compressor the equivalent efficiency for the 1st and 3rd stages was calculated for various levels of pressure recovery. The results are shown in Figure 4.4 & 4.5. In both cases the baseline efficiency against which both stages are based is the isentropic efficiency of the through flow air. So from Table 4.1, the isentropic efficiency of the 1st stage is $\eta_1 = 0.94$, while for the 3rd stage the isentropic efficiency is $\eta_3 = 0.9$.

Each graph "breaks out" the effect of aspiration on the compressor on a stage-by-stage basis. Again like Figure 4.3, even with 100% pressure recovery the stage efficiency of the aspirated stages is less than that of the non-aspirated stages, due to the losses from the bleed passages. When calculating the available work in the bleed and through flow air, it is assumed the air is expanded to P_{t2}.

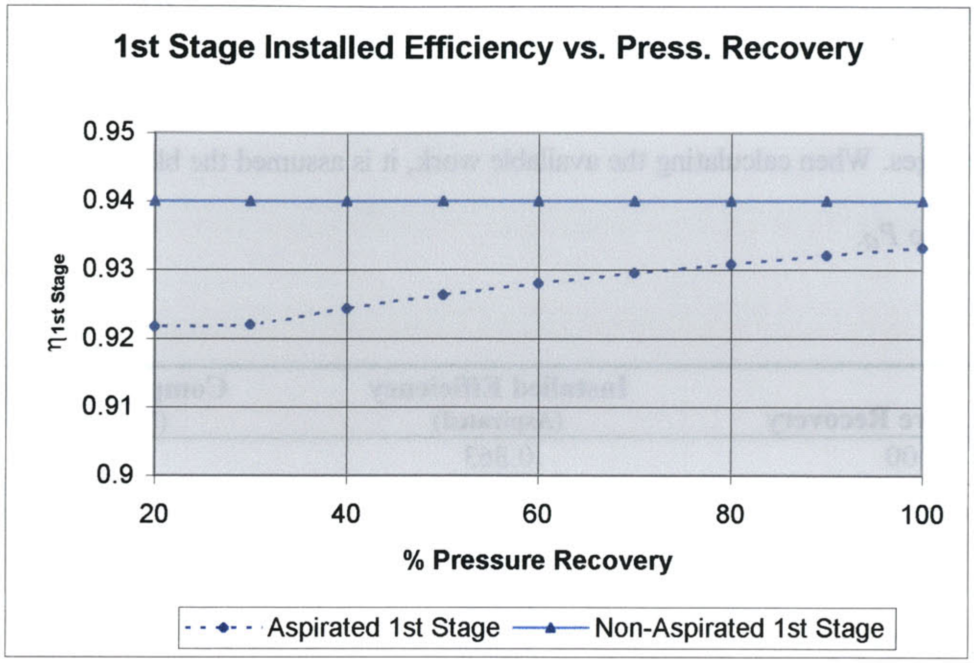


Figure 4.4: 1st Stage Installed Efficiency vs. Pressure Recovery

(see Table 4.2 & Table 4.3)

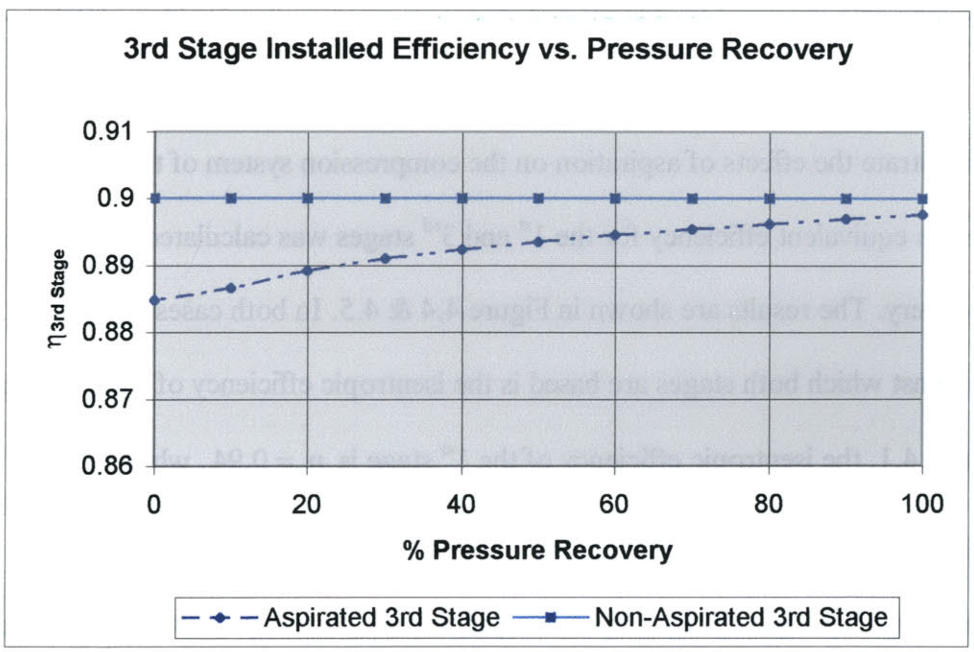


Figure 4.5: 3rd Stage Installed Efficiency vs. Pressure Recovery

(see Table 4.2 & Table 4.3)

4.4.2: QSP Turbofan-Aspirated Core Compressor Only

The second configuration for the QSP configured turbofan is the case where no aspiration is applied to the fan, but the core compressor remains a 3-stage aspirated counter-rotating compressor. This configuration is shown in Figure 4.6 below.

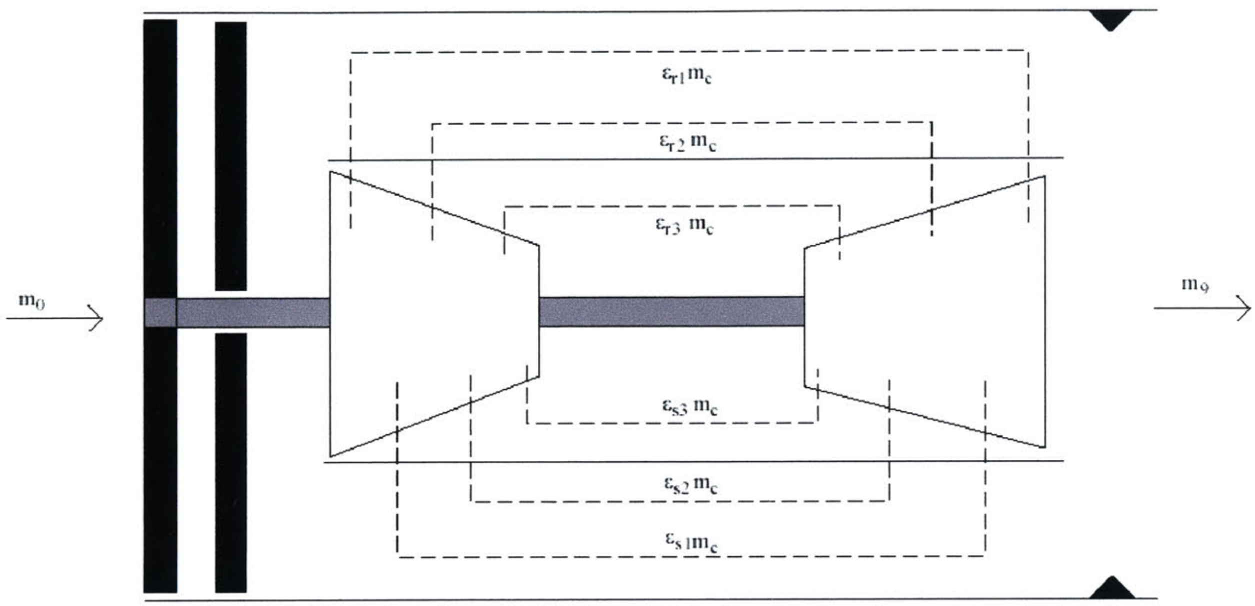


Figure 4.6: QSP turbofan with Core Aspiration only

A QSP configured turbofan with core aspiration was analyzed to isolate the impact core aspiration has on the Specific Impulse of the overall engine. The pressure ratio and polytropic efficiency of the non-aspirated fan was equal to that of the aspirated fan. Except for the replacement of the aspirated fan, all the other engine parameters were equal to the QSP turbofan with fan and core compressor aspiration. The engine parameters for the engine in Figure 4.6 are shown in Table 4.6 below.

	Non-Aspirated Turbofan	Aspirated Turbofan (Counter-rotating compressor)
Flight Mach No.	2	2
Altitude	50,000 ft	50,000 ft
T_{t4}	1922 K	1922 K
π_d	0.925	0.925
π_c	32.83	32.83
e_c	0.916	0.916
π_f	1.776	1.776
e_f	0.9	0.9
$(\Omega r)_{tip}$	457 m/s	457 m/s
Stage 1, π_1, η_1	N/A	3.325, 0.94
Stage 2, π_2, η_2	N/A	2.66, 0.9
Stage 3, π_3, η_3	N/A	2.09, 0.9
e_t	0.9	0.9
ε_{r1}	N/A	0.01 $\dot{m}_c$
ε_{s1}	N/A	0.01 $\dot{m}_c$
ε_{r2}	N/A	0.01 $\dot{m}_c$
ε_{s2}	N/A	0.01 $\dot{m}_c$
ε_{r3}	N/A	0.01 $\dot{m}_c$
ε_{s3}	N/A	0.01 $\dot{m}_c$
h_{pr}	42800 KJ/kg	42800 KJ/kg

Table 4.6: QSP turbofan with Aspirated Core compressor only

The bleed air from the compressor is returned to the cycle as cooling air for the turbine. The conditions of the bleed air are the same as those calculated for the QSP turbofan with fan and core aspiration and are listed in Table 4.3. The basis for comparison with the non-aspirated turbofan will be the specific impulse as the installed compressor efficiency is the same as that for QSP turbofan with fan and core aspiration (see Figure 4.3). Figure 4.7 shows the Specific impulse versus pressure recovery for the engine in Figure 4.6, while Table 4.7 gives the actual values.

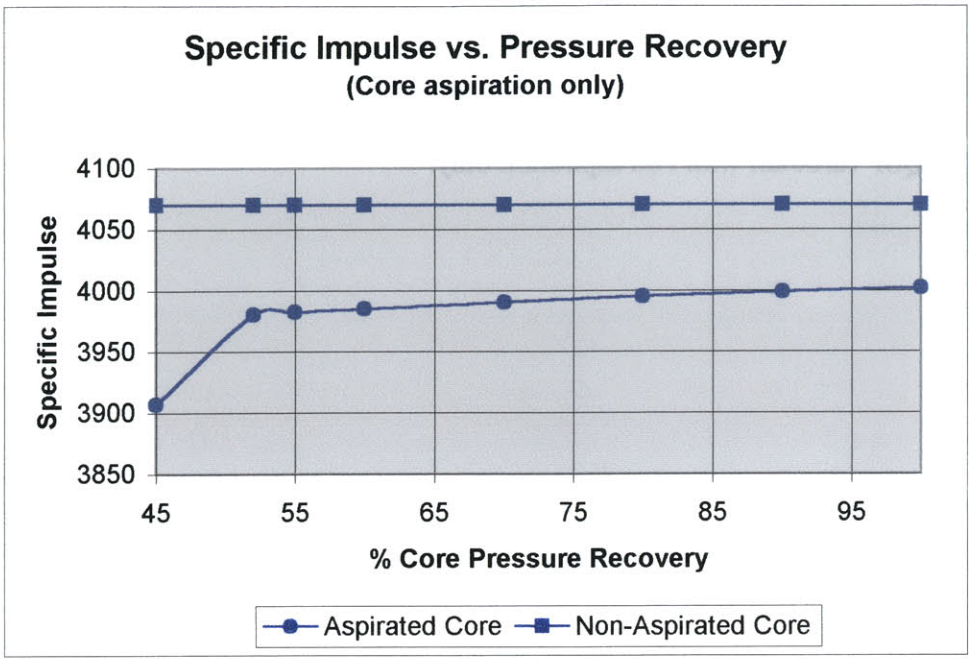


Figure 4.7: Specific Impulse vs. Pressure Recovery for QSP turbofan with Core aspiration only

(See table 4.6 & Table 4.3)

% Pressure Recovery	**Specific Impulse** (Aspirated Core Compressor)	**Specific Impulse** (Non-Aspirated Compressor)
100	4002 sec	4070
90	3999 sec	4070
80	3995 sec	4070
70	3990 sec	4070
60	3985 sec	4070
55	3982 sec	4070
52	3980 sec	4070
45	3907 sec	4070

Table 4.7: Specific Impulse vs. Pressure Recovery for QSP turbofan with core aspiration only

From Figure 4.7, it can be seen that the specific impulse of the QSP turbofan with the aspirated core only is substantially better than the specific impulse for the QSP turbofan with fan and core aspiration (Figure 4.2). This suggests that the biggest penalty in specific impulse arises in the fan aspiration. With this fact in mind we move onto look at the effect on the overall engine efficiency of a QSP turbofan with Fan aspiration only.

4.4.3: QSP Turbofan-Aspirated Fan Only

A turbofan with aspiration applied to the fan rotor and stator only, was analyzed to understand the effect this would have on the overall engine efficiency. From Figure 4.2 and Figure 4.7, the fan aspiration appears to be the major cause of the decrease in Specific Impulse. The engine configuration is shown schematically in Figure 4.8.

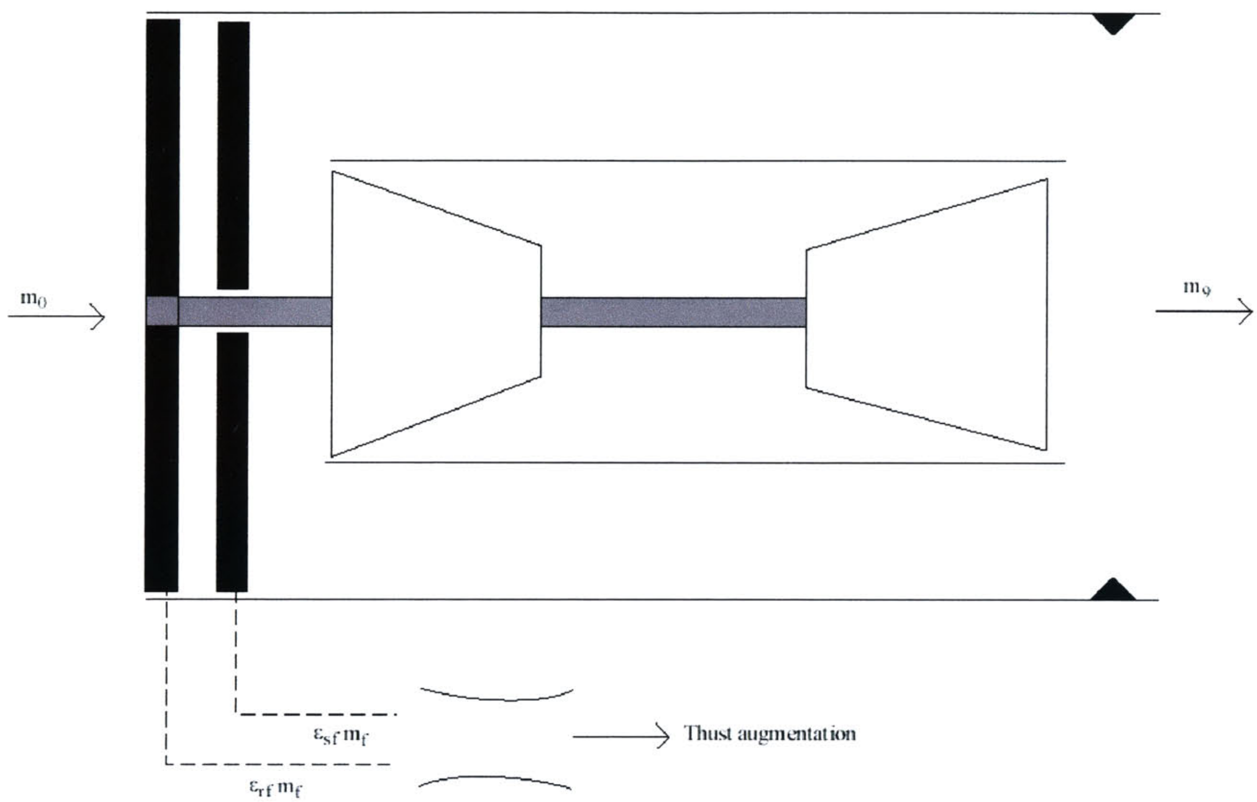


Figure 4.8: QSP turbofan with Fan Aspiration only

The rest of the engine parameters were the same as those in Table 4.1. Basically the counter-rotating compressor discussed in above is removed from the engine and replaced with the non-aspirated core. The condition of the aspirated air is the same as that for the QSP turbofan with Fan and core aspiration and can be found in Table 4.3. The engine parameters are listed below in Table 4.8.

	Non-Aspirated Turbofan	Aspirated Turbofan (Fan aspiration only)
Flight Mach No.	2	2
Altitude	50,000 ft	50,000 ft
T_{t4}	1922 K	1922 K
π_d	0.925	0.925
π_c	32.83	32.83
e_c	0.916	0.916
π_f	1.776	1.776
e_f	0.9	0.9
e_t	0.9	0.9
ε_{rf}	N/A	$0.01\,\dot{m}_f$
ε_{sf}	N/A	$0.01\,\dot{m}_f$
h_{pr}	42800 KJ/kg	42800 KJ/kg

Table 4.8: QSP Turbofan with Aspirated Fan only

The bleed air from the rotor and the stator are expanded to P_0 to recover the momentum drag. This expansion produces thrust that is added to the thrust produced by the through flow air (see section 2). The basis for comparison between the non-aspirated turbofan and the aspirated turbofan will be the specific impulse and the "***Installed Compressor Efficiency***". Figure 4.9 shows the Specific Impulse versus pressure recovery, while Table 4.9 gives the numerical values for specific impulse.

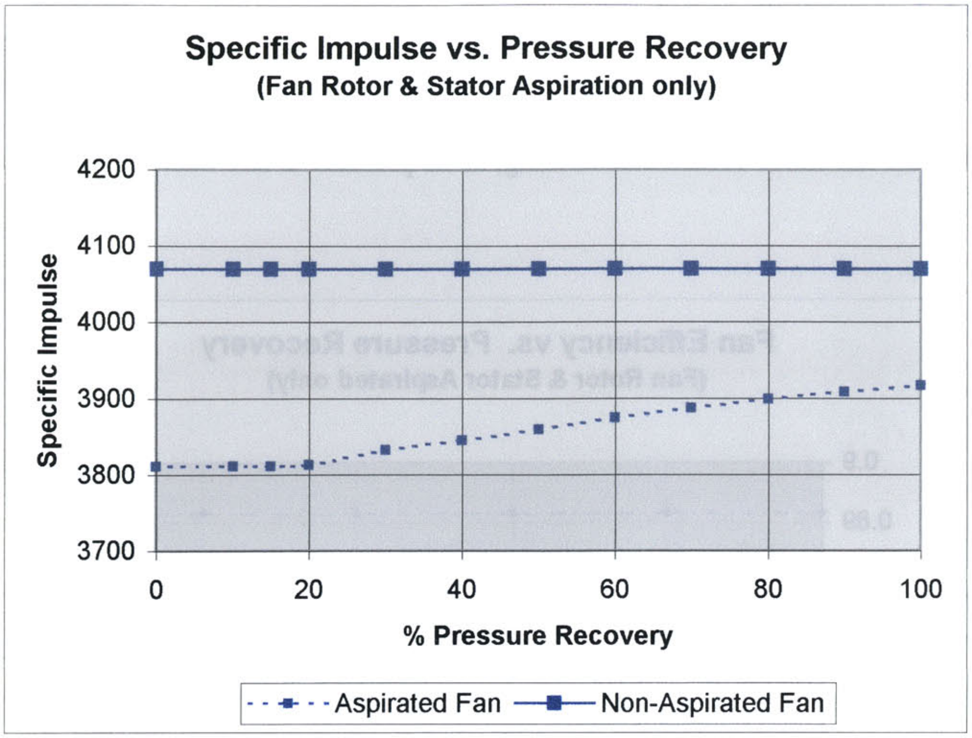


Figure 4.9: Specific Impulse vs. Pressure recovery for QSP turbofan with Aspirated Fan only

(See Table 4.8 & Table 4.3)

% Pressure Recovery	Specific Impulse (Aspirated Core Compressor)	Specific Impulse (Non-Aspirated Compressor)
100	3917 sec	4070
90	3908 sec	4070
80	3898 sec	4070
70	3887 sec	4070
60	3874 sec	4070
50	3859 sec	4070
40	3844 sec	4070
30	3832 sec	4070
20	3813 sec	4070
10	3811 sec	4070
0	3811 sec	4070

Table 4.9: Specific Impulse vs. Pressure Recovery for QSP turbofan with Aspirated Fan only

(See Table 4.8 & Table 4.3)

Figure 4.10 below shows the Fan isentropic efficiency as a function of pressure recovery. When calculating the available work it was assumed the air was expanded to P_{t2}. This means that the bleed air from the fan stator contributes nothing, as its pressure is less than P_{t2} (see Table 4.3).

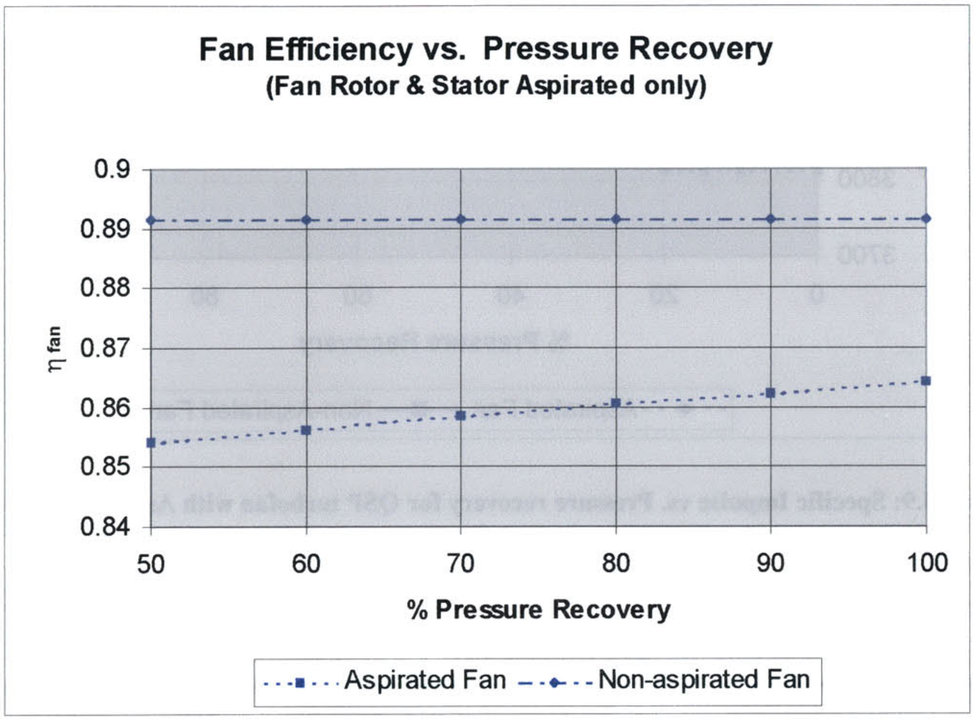


Figure 4.10: Fan Efficiency vs. Pressure Recovery (Aspirated Fan only)

(See Table 4.8 & Table 4.3)

From Figure 4.11 it can be seen that the specific impulse of the QSP turbofan with the fan aspiration, is less than that for the non-aspirated turbofan. Looking at Figures 4.2, 4.7 and 4.9, it can be surmised that the major contributor to the reduction in specific impulse noticed in Figure 4.2 is the aspirated fan. This indicates that returning the air directly to the cycle, as turbine-cooling air is less detrimental to the cycle than expanding the air overboard to augment the thrust.

4.5: MIT Low Speed Fan

As mentioned in the introduction a low speed fan stage has been designed, built and tested at the MIT blow down compressor. Experimental values of the stage pressure and temperature ratio's have been found and hence a value for the isentropic efficiency. The bleed air in this low speed fan is taken radially outwards for both the rotor and the stator. The rotor has a tip shroud to facilitate the removal of the bleed air. Figure 4.11 shows a schematic of the low speed fan.

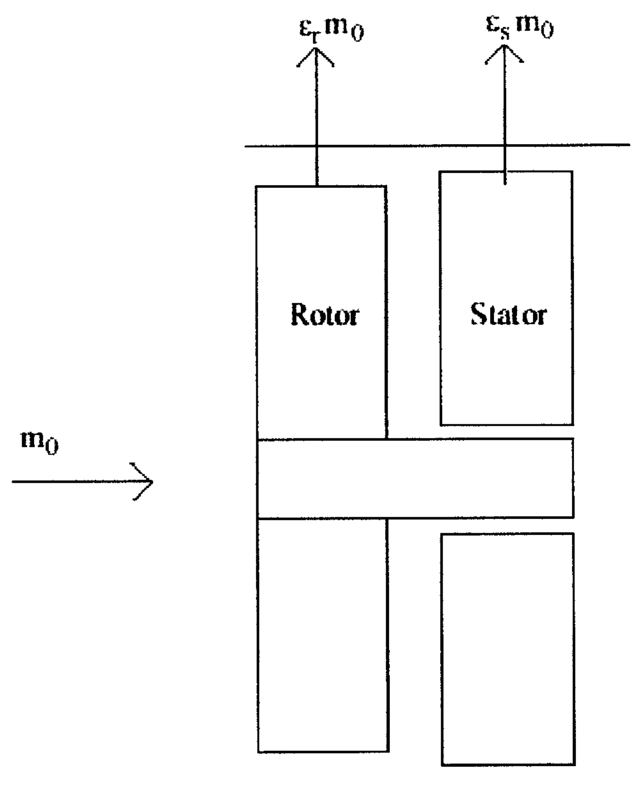


Figure 4.11: MIT Low speed aspirated Fan Stage

For this low speed stage it was assumed that 2% of the total mass flow was removed by aspiration (i.e. 1% by the Rotor and 1% by the Stator). Using the code described in section 3, the total temperature and pressure of the bleed air for both the rotor and stator were calculated and used to calculate the "***Installed Efficiency***" for the fan stage.

The testing of the fan stage was done at atmospheric pressure and for this analysis P_{t2} was assumed equal to atmospheric pressure ($P_{t2} = 1.013 \times 10^5\ N/m^2$). Table 4.10 presents low speed fan data.

Component	Data
Tip Speed	228.6 m/s
π_{stage}	1.57
π_{rotor}	1.62
π_{stator}	0.97
η_{stage}	0.9
P_{t2}	1.013×10^5 N/m^2
Rotor Bleed Air	
ε_r	0.01
P_{tr}	1.0538×10^5 N/m^2
T_{tr}	337.5 K
Stator Bleed Air	
ε_s	0.01
P_{ts}	1.0862×10^5 N/m^2
T_{ts}	332 K

Table 4.10: MIT Low speed Aspirated Fan conditions

Using the information in Table 4.10, the "***Installed Efficiency***" of the fan was calculated for various levels of bleed air pressure recovery. From the table it can be seen that the pressure rise in the bleed passage for both bleeds is low and very inefficient. So the available work that the bleed air can perform is low. The installed Fan efficiency versus pressure recovery is shown in Figure 4.12.

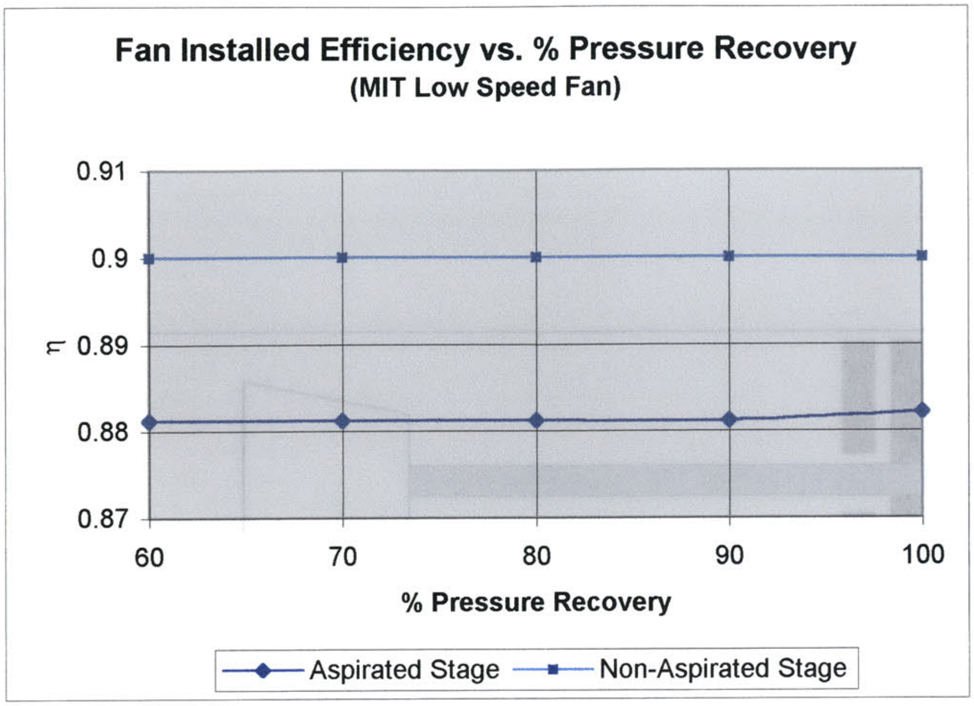


Figure 4.12: MIT Low speed fan installed efficiency vs. Pressure Recovery (see Table 4.10)

4.6 Aspirated Turbojet performing Auxiliary Work

To see the effects of aspiration on the Specific impulse and efficiency for a turbojet configuration, a comparison between the Aspirated turbojet and the Non-aspirated turbojet discussed in Section 2 was performed. A schematic of the aspirated turbojet is shown in Figure 4.13.

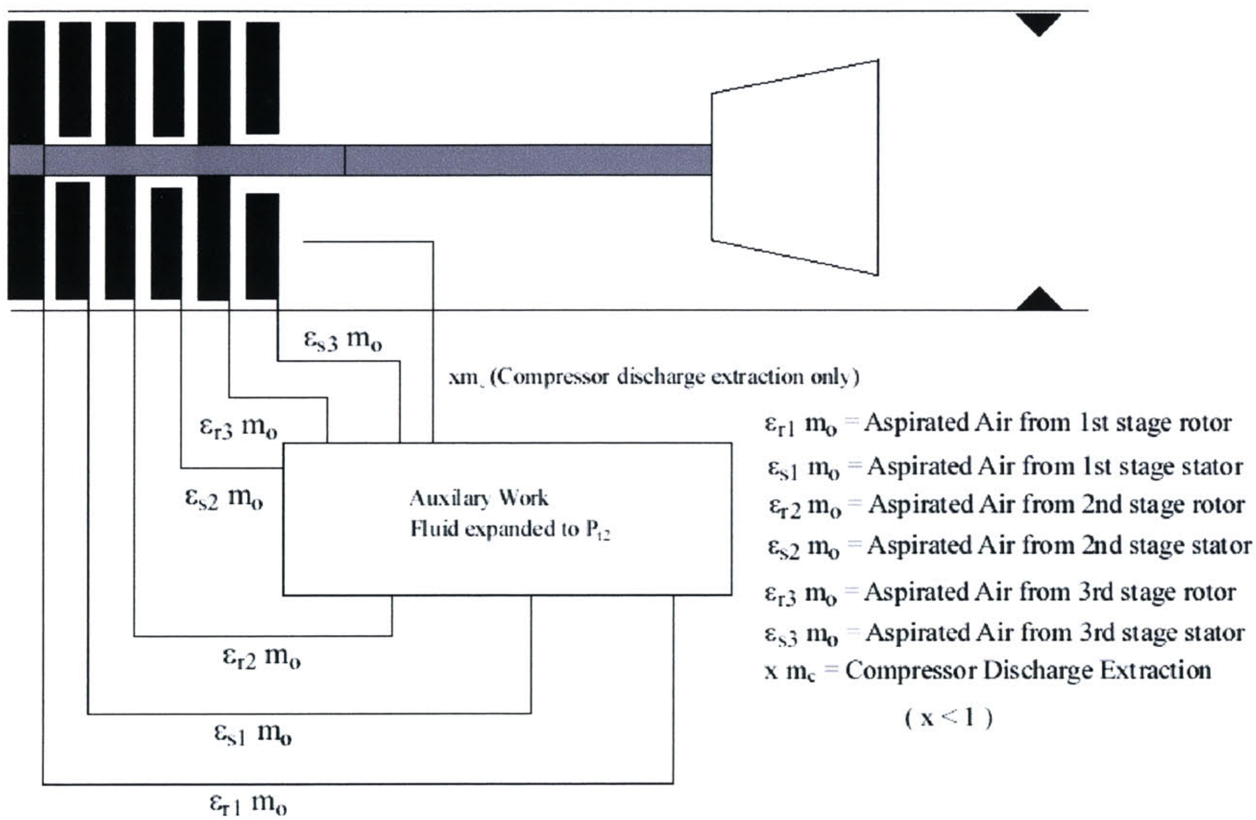


Figure 4.13: Aspirated turbojet performing Auxiliary work

Both engines had to perform equal amounts of "Auxiliary Work". Comparisons were made over a range of "Auxiliary work outputs". Both engines were assumed to be flying at the same altitude and speed. It is also assumed that 100% of the bleed air is capable of performing auxiliary work. Table 4.11 below compares the engine configurations of both engines.

	Non-Aspirated Turbojet	Aspirated Turbojet (Counter-rotating compressor)
Flight Mach No.	2	2
Altitude	40,000 ft	40,000 ft
T_{t4}	1922 K	1922 K
π_d	0.925	0.925
π_c	19.2	19.2
$(\Omega r)_{tip}$	457 m/s	457 m/s
Stage 1, π_1, η_1	N/A	3.2, 0.9
Stage 2, π_2, η_2	N/A	3.0, 0.89
Stage 3, π_3, η_3	N/A	2.0, 0.89
e_c	0.9	0.9
e_t	0.9	0.9
ε_{r1}	N/A	0.01
ε_{s1}	N/A	0.01
ε_{r2}	N/A	0.01
ε_{s2}	N/A	0.01
ε_{r3}	N/A	0.01
ε_{s3}	N/A	0.01
h_{pr}	42800 KJ/kg	42800 KJ/kg

Table 4.11: Aspirated Turbojet performing auxiliary work

4.6.1 Aspirated Turbojet

As mentioned in Section 2, the aspirated turbojet uses a counter-rotating compressor. This compressor has 3 stages and each stage is aspirated. The bleed air is taken radially outward for Stage 1 and radially inwards for Stages 2 & 3. The radial inflow is extracted for "Auxiliary Work" at 0.15 tip radius. As table 4.11 shows, 1% of the main bleed flow was extracted from each blade row (i.e. $\varepsilon_{r1} = 0.01$ etc.).

Using the code discussed in Section 3, the values of the total pressure and temperature for each of the aspirated stage was calculated. To allow for losses in diffusing the air from it was assumed that $1/3$ of the dynamic head was lost. With this loss in total pressure the final values of the total usable pressure and temperature are listed below in Table 4.12.

Stage Component	Total Pressure	Total temperature
1st Stage Rotor	$P_{r1} = 3.03 \text{ x } 10^5\ N/m^2$	$T_{r1} = 608$ K
1st Stage Stator	$P_{ts1} = 3.27 \text{ x } 10^5\ N/m^2$	$T_{ts1} = 558$ K
2nd Stage Rotor	$P_{r2} = 3.51 \text{ x } 10^5\ N/m^2$	$T_{r2} = 568$ K
2nd Stage Stator	$P_{ts2} = 1.41 \text{ x } 10^6\ N/m^2$	$T_{ts2} = 817$ K
3rd Stage Rotor	$P_{r3} = 1.3 \text{ x } 10^6\ N/m^2$	$T_{r3} = 827$ K
3rd Stage Stator	$P_{ts3} = 2.9 \text{ x } 10^6\ N/m^2$	$T_{ts3} = 1019$ K

Table 4.12: Conditions of the air used for "Auxiliary Work"

A comparison between the non-aspirated turbojet and the aspirated turbojet was performed using the data in Table 4.12. As has been mentioned numerous times before this air was expanded to P_{t2} to perform "Auxiliary Work". The amount of "Auxiliary work" performed by both engines is equal.

$$(\text{Auxiliary work})_{\textbf{aspirated}} = (\text{Auxiliary Work})_{\textbf{non-aspirated}}$$

this means the compressor discharge extraction $(x'\dot{m}_0)$ for the aspirated case is smaller than that for the non-aspirated case $(x\dot{m}_0)$. The quantity $(x\dot{m}_0)$ gives an indication of how much "Auxiliary Work" both engines perform. Figure 4.14 below gives a comparison of the specific impulse for both engines as the "Auxiliary Work" is increased.

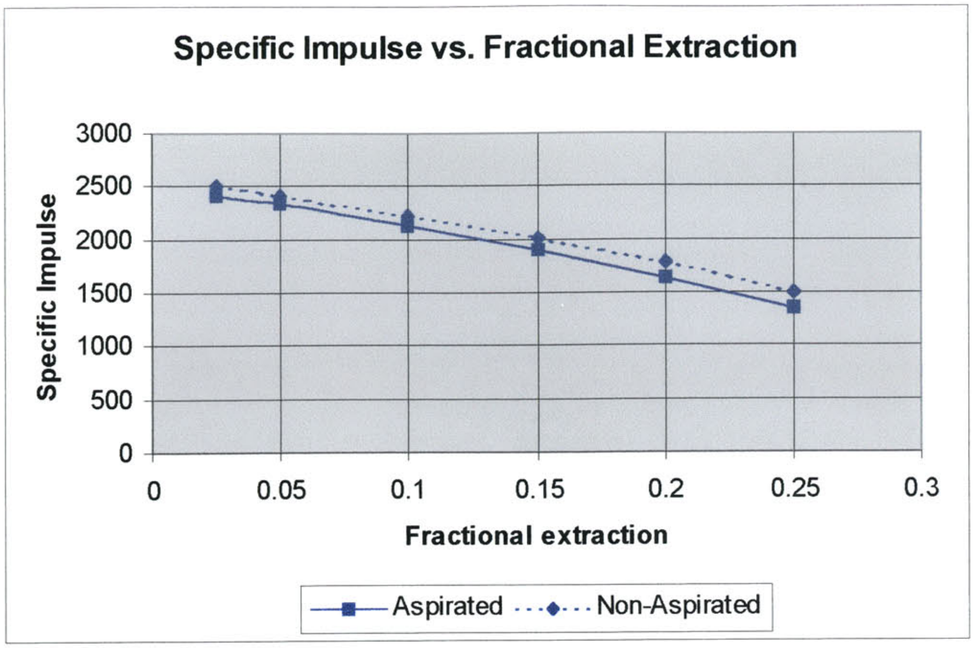


Figure 4.14: Specific Impulse comparison for aspirated turbojet performing Auxiliary work

(See Table 4.11 & Table 4.12)

It can be seen that the effects of aspiration have a minimal influence on the Specific impulse as the amount of "Auxiliary work" increases. The same effect can be seen in Figure 4.15 for the overall efficiency.

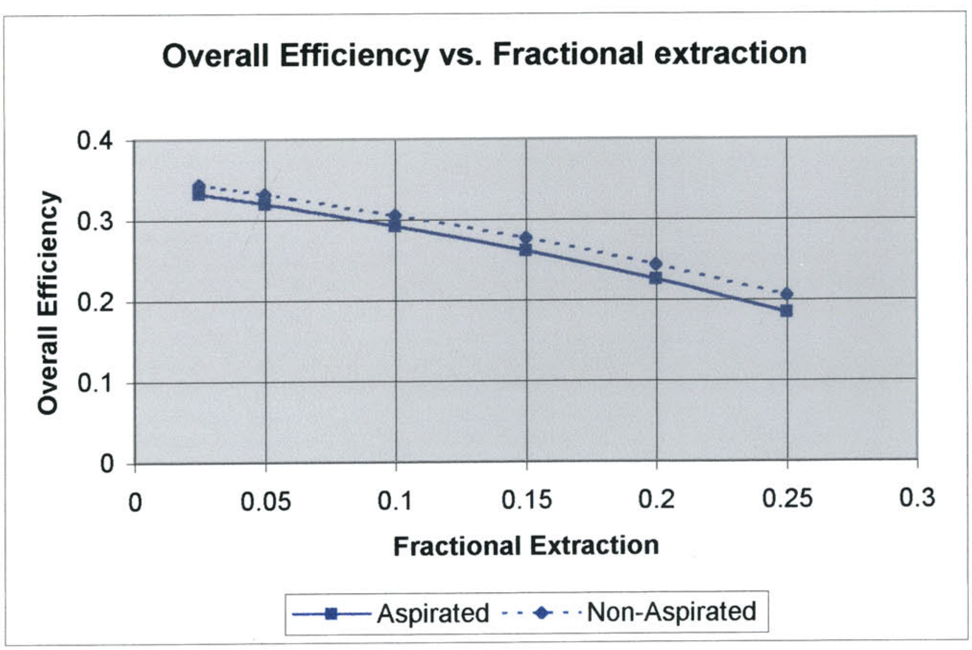


Figure 4.15: Overall Efficiency for aspirated turbojet performing auxiliary work

Chapter 5: Conclusions

Five different engine/fan configurations were studied, to illustrate and understand the effects of aspiration. The five configurations are repeated here

1. Potential QSP configured Turbofan.
   - (i) Aspirated Fan and Aspirated core compressor (see **Figure 4.1**);
   - (ii) Aspirated Core compressor only (see **Figure 4.6**);
   - (iii) Aspirated Fan only (see **Figure 4.8**).
2. MIT low speed Fan (see **Figure 4.11**).
3. Aspirated Turbojet performing auxiliary work (see **Figure 4.13**).

For the QSP configured turbofan with fan and core aspiration, ***1% bleed*** on each blade row and ***100% pressure recovery*** for the aspirated air, the **specific fuel consumption is 5.5% higher** than an equivalent non-aspirated turbofan. The bleed air from the fan is expanded overboard to augment the thrust while the bleed air from the core is returned to the turbine as cooling air.

The second case is the QSP configured turbofan with only an aspirated core (i.e. no fan aspiration), ***1% bleed*** on each blade row and ***100% pressure recovery*** for the aspirated air, **the specific fuel consumption is 1.7% higher** than an equivalent non-aspirated turbofan. The bleed air is returned to the turbine as cooling air.

For the QSP configured turbofan with only fan aspiration (i.e. no core compressor aspiration), ***1% bleed*** on each blade row and ***100% pressure recovery*** for the aspirated air, **the specific fuel consumption is 3.75% higher** than an equivalent non-aspirated turbofan. The bleed air is expanded overboard to augment thrust.

From the previous three cases, it can be surmised that aspirating the fan has the largest impact on the specific fuel consumption. This arises due to the high losses experienced by the aspirated air as it is "pumped up" in the bleed passage.

For the experiments performed on the MIT low speed fan, if ***1% bleed*** was removed from each of the blade rows and ***100% pressure recovery*** was achieved for the aspirated air, **the installed efficiency is 1.8% lower** than the non-aspirated fan. Again like the fan in the QSP configuration, this decrease in efficiency arises due to the high losses experienced by the aspirated air as it travels through the bleed passage.

Finally for the aspirated turbojet performing auxiliary work, **the specific fuel consumption is 4.5% higher** at a **fractional extraction of 0.1** (i.e. 10% of the compressor discharge is removed for auxiliary work).

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Appendix A

A.1: Cycle Analysis for Aspirated mixed exhaust turbofan

This section will compute the behavior of the aspirated turbofan including component losses, the mass flow rate of the fuel through the components, and the variation of specific heats. It also takes the work performed on the bleed air into account. This is an important difference between the cycle analysis performed on the non-aspirated turbofan.

Again as in the case of the non-aspirated engine the analysis will be brief and only an outline of the most important steps is included. The aspirated turbofan with station numbering is shown in Figure 2-4a. The bleed and turbine cooling airflow is shown in Figure 2-4b.

Uninstalled Thrust:

$$F = \left(\dot{m}_9 V_9 - \dot{m}_0 V_0\right) + A_9\left(P_9 - P_0\right) + \left(Momentum \quad Drag \quad of \;\; Fan \;\; bleeds\right)$$

Note: The thrust developed by expanding the Fan bleed air to P_0 will be calculated latter and will be removed from the thrust calculation for now.

Assume the engine exhaust nozzle expands the gas to the ambient pressure $P_0 = P_9$. Therefore we return to **Eqn. 2.1**

$$\boxed{\frac{F}{\dot{m}_0 a_0} = \left(\frac{\dot{m}_9}{\dot{m}_0}\frac{V_9}{a_0} - M_0\right)} \qquad \textbf{Eqn. (A.1)}$$

Calculation of $\frac{\dot{m}_9}{\dot{m}_0}$***:***

The fan bleed air is not returned to cycle directly and as mentioned above will be treated separately.

$$\dot{m}_9 = \dot{m}_c + \dot{m}_f + \dot{m}_{fuel} - \varepsilon_{rf}\dot{m}_f - \varepsilon_{sf}\dot{m}_f$$

After some manipulation one gets

$$\frac{\dot{m}_9}{\dot{m}_0} = \frac{1}{(1+\alpha)}\left[1 + f + \alpha\left(1 - \left(\varepsilon_{rf} + \varepsilon_{sf}\right)\right)\right] \qquad \textbf{Eqn. (A.2)}$$

Placing **Eqn. (A.2)** into **Eqn. (A.1)** gives

$$\boxed{\frac{F}{\dot{m}_0 a_0} = \left\{\left(\frac{1 + f + \alpha\left(1 - \left(\varepsilon_{rf} + \varepsilon_{sf}\right)\right)}{1+\alpha}\right)\frac{V_9}{a_0} - M_0\right\}} \qquad \textbf{Eqn. (A.3)}$$

Calculation of $(V_9 / a_0)^2$

$$\left(\frac{V_9}{a_0}\right)^2 = \frac{a_9^2 M_9^2}{a_0^2} = \frac{\gamma_9 R_9 T_9}{\gamma_0 R_0 T_0} M_9^2$$

For the turbojet cycle, this equation becomes

$$\boxed{\left(\frac{V_9}{a_0}\right)^2 = \frac{\gamma_t R_t T_t}{\gamma_c R_c T_c} M_9^2} \qquad \textbf{Eqn. (A.4)}$$

Calculation of M_9^2

$$\boxed{M_9^2 = \frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t - 1)/\gamma_t} - 1\right]} \qquad \textbf{Eqn. (A.5)}$$

Where

$$\boxed{\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r \pi_d \pi_c \pi_b \pi_t \pi_n} \qquad \textbf{Eqn. (A.6)}$$

Calculation of T_9 / T_0

$$\boxed{\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{\left(P_{t9}/P_9\right)^{(\gamma_t-1)/\gamma_t}}} \qquad \textbf{Eqn. (A.7)}$$

Where

$$\frac{T_{t9}}{T_0} = \tau_r \tau_d \tau_c \tau_b \tau_t \tau_n$$

Calculation of f:

$$f = \frac{\dot{m}_{fuel}}{\dot{m}_c}$$

Application of the 1st law of thermodynamics to the control volume about the combustor shown in Figure 2-2 gives:

$$\boxed{\dot{m}_3 C_{pc} T_{t3} + \eta_b \dot{m}_{fuel} h_{pr} = \dot{m}_4 C_{pt} T_{t4}} \qquad \textbf{Eqn. (A.8)}$$

The mass flow of the air entering the combustor is reduced by aspiration. This air is not returned to the cycle until the turbine. Therefore

$$\dot{m}_3 = \dot{m}_c \left(1 - \sum \varepsilon_i\right)$$

and

$$\dot{m}_4 = \dot{m}_c \left(\left(1 - \sum \varepsilon_i\right) + f\right)$$

Where $\sum \varepsilon_i = \varepsilon_{r1} + \varepsilon_{s1} + \varepsilon_{r2} + \varepsilon_{s2} + \varepsilon_{r3} + \varepsilon_{s3}$ and the mass flow out of the combustor is increased only by the addition of fuel.

$$\boxed{f = \frac{\left(1 - \sum \varepsilon_i\right) \left[\tau_r \tau_c - \tau_\lambda\right]}{\left[\tau_\lambda - \left(\eta_b h_{pr} \big/ C_{pc} T_0\right)\right]}} \qquad \textbf{Eqn. (A.9)}$$

Calculation of τ_t:

Like the non-aspirated case the calculation of τ_t involves a power balance between the compressor, fan and the turbine.

Power into compressor + Power into Fan = Net Power from turbine

As mentioned in Chapter 2 calculating the power consumed by both the fan and the core compressor, the effects of aspiration must be taken into account. For example the bleed air removed from the 1st stage rotor is taken radially outward and in so doing, work is performed on the air; the air is "pumped" similar to the work done by a centrifugal compressor. This "centrifugal work" increases the total temperature of the bleed air to a temperature greater than T_{t2}. The temperature rise can be greater than or less than the stage temperature rise.

This means that the work performed on each bleed air must be treated separately in the cycle analysis. Figure A1 shows the fan and compressor bleeds, plus the nomenclature used in the analysis

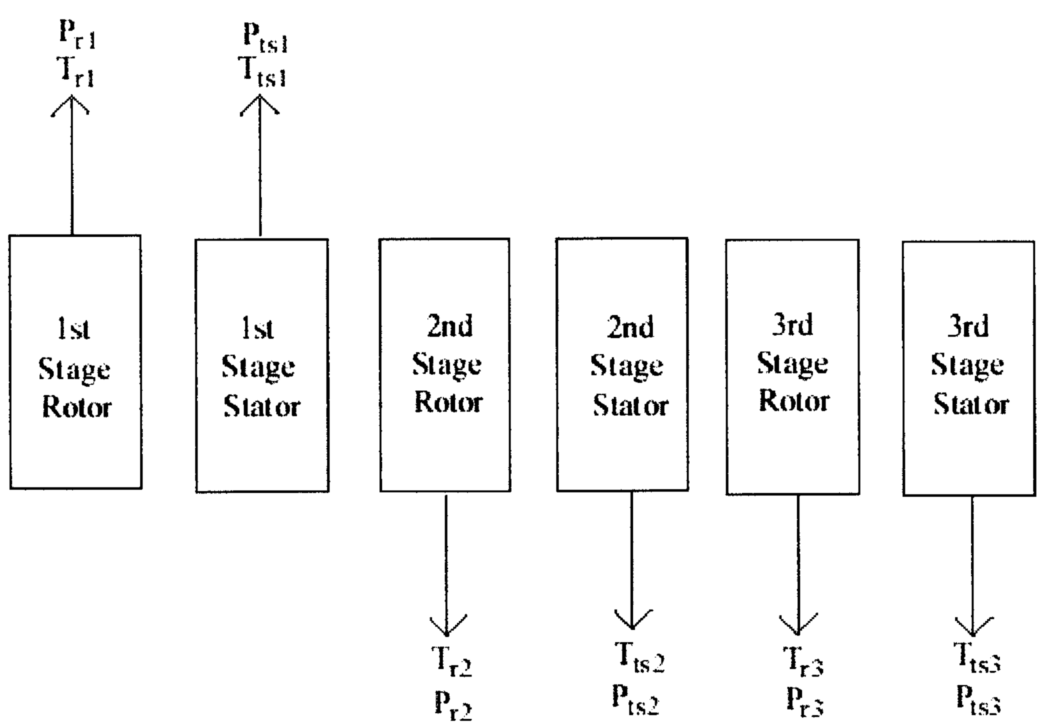


Figure A1: Counter-rotating Compressor

Compressor & Fan Work

To calculate the work performed on all the air that passes through the compression system it is necessary to account for the work performed on each of the bleeds. The fan has two bleeds and the core compressor has 6 bleeds, as shown in **Table 2-1**. Work performed on each bleed depends on the total temperature rise achieved. For the rotor bleeds temperature rise is calculated using a 1-D compressible flow calculation, detailed in Chapter 3. The temperature of the stator bleeds and the through flow air can be found using the isentropic or polytropic efficiency and the pressure rise.

Therefore

1. Through Flow of Core Compressor

$$\dot{m}_c\left(1-\sum\varepsilon_i\right)C_{pc}\left(T_{t3}-T_{t2}\right) \qquad \textbf{Eqn. (A.10)}$$

2. 1st Stage Rotor Bleed

$$\varepsilon_{r1}\dot{m}_c C_{pc}\left(T_{r1}-T_{t2}\right) \qquad \textbf{Eqn. (A.11)}$$

3. 1st Stage Stator Bleed

$$\varepsilon_{s1}\dot{m}_c C_{pc}\left(T_{ts1}-T_{t2}\right) \qquad \textbf{Eqn. (A.12)}$$

4. 2nd Stage Rotor Bleed

$$\varepsilon_{r2}\dot{m}_c C_{pc}\left(T_{r2}-T_{t2}\right) \qquad \textbf{Eqn. (A.13)}$$

5. 2nd Stage Stator Bleed

$$\varepsilon_{s2}\dot{m}_c C_{pc}\left(T_{ts2}-T_{t2}\right) \qquad \textbf{Eqn. (A.14)}$$

6. 3rd Stage Rotor Bleed

$$\varepsilon_{r3}\dot{m}_c C_{pc}\left(T_{r3}-T_{t2}\right) \qquad \textbf{Eqn. (A.15)}$$

7. 3rd Stage Stator Bleed

$$\varepsilon_{s3}\dot{m}_c C_{pc}\left(T_{ts3}-T_{t2}\right) \qquad \textbf{Eqn. (A.16)}$$

8. Fan through flow

$$\dot{m}_f\left(1-\varepsilon_{rf}\right)C_{pc}\left(T_{t13}-T_{t2}\right) \qquad \textbf{Eqn.(A.17)}$$

9. Fan Rotor Bleed

$$\varepsilon_{rf}\dot{m}_f C_{pc}\left(T_{tf}-T_{t2}\right) \qquad \textbf{Eqn. (A.18)}$$

Where

$$T_{ts1}=T_{t2}\left(1+\left(\frac{1}{\eta_1}\left(\left(\pi_f\pi_1\right)^{(\gamma_c-1)/\gamma_c}-1\right)\right)\right)$$

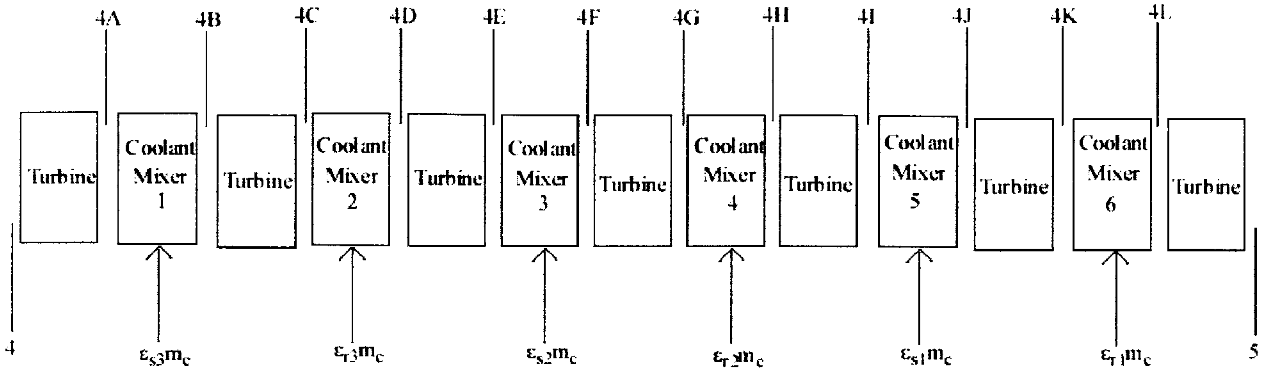
$$T_{ts2}=T_{ts1}\left(1+\left(\frac{1}{\eta_2}\left(\pi_2^{(\gamma_c-1)/\gamma_c}-1\right)\right)\right)$$

$$T_{ts3}=T_{ts2}\left(1+\left(\frac{1}{\eta_3}\left(\pi_3^{\frac{\gamma_c-1}{\gamma_c}}-1\right)\right)\right)$$

Power input to the fan stator bleed is accounted for **Eqn. (A.18)**; no work is performed on the air during the extraction process.

Turbine Power

The aspirated air from the compressor is returned to the turbine as cooling air. The air enters the turbine at the total pressure of the bleed air and is mixed with the through flow air. This mixing reduces the temperature of the through flow, while increasing the mass flow of air through the turbine. Because the mass flow of air through the turbine changes due to the addition of the aspirated air, the power extracted from the turbine must be calculated in steps. Figure A.2 shows how the mass flow through the turbine changes with the addition of the bleed flow air from the compressor.



$m_4 = m_{4A} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1}+\varepsilon_{T2}+\varepsilon_{s2}+\varepsilon_{T3}+\varepsilon_{s3})) + f)$

$m_{4B} = m_{4C} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1}+\varepsilon_{T2}+\varepsilon_{s2}+\varepsilon_{T3})) + f)$

$m_{4D} = m_{4E} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1}+\varepsilon_{T2}+\varepsilon_{s2})) + f)$

$m_{4F} = m_{4G} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1}+\varepsilon_{T2})) + f)$

$m_{4H} = m_{4I} = m_c\,((1-(\varepsilon_{T1}+\varepsilon_{s1})) + f)$

$m_{4J} = m_{4K} = m_c\,(1-(\varepsilon_{T1}) + f)$

$m_{4L} = m_5 = m_c\,(1 + f)$

Figure A.2: Changing mass flow through the turbine

Expansion of the air through the turbine and the work produced by this expansion is divided into 7 sections, one expansion to each of the six bleed air total pressures and a seventh to P_{t5} (i.e. Expansion from P_{t4} to P_{tse3} (the total pressure of the 3rd stage stator bleed) etc.). After expansion the temperature of the through flow air can be calculated by assuming a polytropic efficiency for each step of the expansion. Mixing between the bleed air and the through flow air then takes place and a mixed out temperature based on mass is calculated. No pressure loss takes place during the mixing. Figure 2.7 shows a schematic of the expansion and the mixing on a T-S diagram, while figure A.3 shows the pressure and temperature designations as the air goes through the turbine.

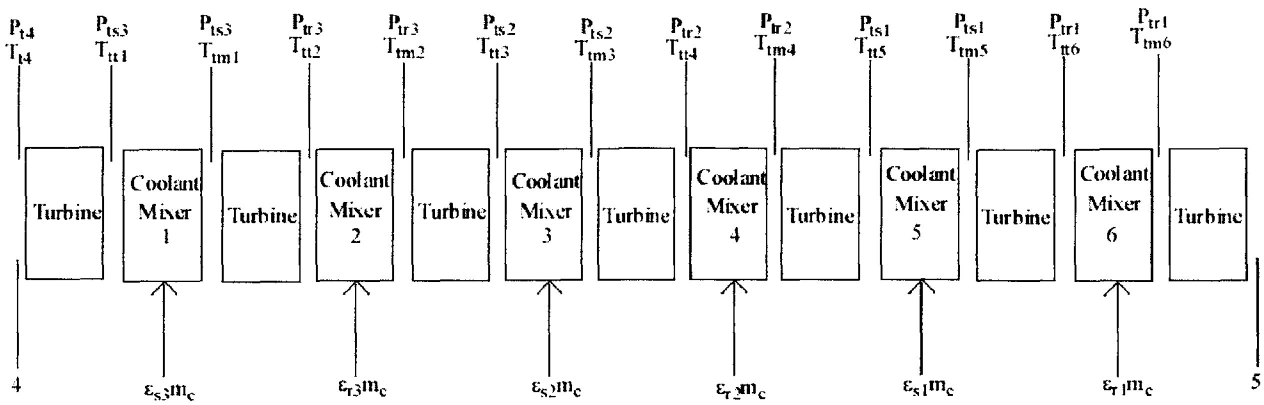


Figure A.3: Nomenclature of the air flow through the turbine

The steps involved in calculating the final mixed out temperature are shown here for the case of expansion from P_{t4} (Total pressure out of combustor) to P_{tse3} (Total pressure of the 3rd stage stator bleed air). Both P_{t4} $(P_0\pi_r\pi_d\pi_c\pi_b)$ and P_{tse3} (Calculated using the 1-D compressible flow analysis talked about in Section 3) are known, as is the polytropic efficiency of the expansion η_p, which will allow us to calculate the temperature of the through flow (to be known as T_{tt1}) air before mixing.

$$\frac{T_{tt1}}{T_{t4}} = \left(\frac{P_{tse3}}{P_{t4}}\right)^{(\gamma_t - 1)/(\eta_p \gamma_t)} \qquad \textbf{Eqn. (A.19)}$$

Once T_{tt1} is known, the mixed out temperature (to be known as T_{tm1}) can be calculated based on the mass of the through flow air and the bleed air.

$$\dot{m}_4 C_{pt} T_{tt1} + \varepsilon_{s3}\dot{m}_c C_{pc} T_{tse3} = (\dot{m}_4 + \varepsilon_{s3}\dot{m}_c) T_{tm1}$$

Where $\dot{m}_4 = \left(\left(1 - \sum \varepsilon_i\right) + f\right)\dot{m}_c$ gives

$$T_{tm1} = \frac{\left[\left(\left(1 - \sum \varepsilon_i\right) + f\right) C_{pt} T_{tt1} + \varepsilon_{s3} C_{pc} T_{tse3}\right]}{\left[\left(\left(1 - \sum \varepsilon_i\right) + f\right) + \varepsilon_{s3}\right] C_{pt}} \qquad \textbf{Eqn. (A.20)}$$

Where $\sum \varepsilon_i = \varepsilon_{r1} + \varepsilon_{s1} + \varepsilon_{r2} + \varepsilon_{s2} + \varepsilon_{r3} + \varepsilon_{s3}$

A similar analysis can be performed for each of the six expansions to match the total pressures of the bleeds. The temperatures through the turbine are

$$\frac{T_{tt2}}{T_{tm1}} = \left(\frac{P_{tre3}}{P_{tse3}}\right)^{(\gamma_t-1)/(\eta_p\gamma_t)} \qquad \textbf{Eqn. (A.21)}$$

$$T_{tm2} = \frac{\left[\left(\left(1-\sum\varepsilon_i\right)+f+\varepsilon_{s3}\right)C_{pt}T_{tt2}+\varepsilon_{r3}C_{pc}T_{tre3}\right]}{\left[\left(\left(1-\sum\varepsilon_i\right)+f\right)+\varepsilon_{s3}+\varepsilon_{r3}\right]C_{pt}} \qquad \textbf{Eqn. (A.22)}$$

$$\frac{T_{tt3}}{T_{tm2}} = \left(\frac{P_{tse2}}{P_{tre3}}\right)^{(\gamma_t-1)/(\eta_p\gamma_t)} \qquad \textbf{Eqn. (A.23)}$$

$$T_{tm3} = \frac{\left[\left(\left(1-\sum\varepsilon_i\right)+f+\varepsilon_{s3}+\varepsilon_{r3}\right)C_{pt}T_{tt3}+\varepsilon_{s2}C_{pc}T_{tse2}\right]}{\left[\left(\left(1-\sum\varepsilon_i\right)+f\right)+\varepsilon_{s3}+\varepsilon_{r3}+\varepsilon_{s2}\right]C_{pt}} \qquad \textbf{Eqn. (A.24)}$$

$$\frac{T_{tt4}}{T_{tm3}} = \left(\frac{P_{tre2}}{P_{tse2}}\right)^{(\gamma_t-1)/(\eta_p\gamma_t)} \qquad \textbf{Eqn. (A.25)}$$

$$T_{tm4} = \frac{\left[\left(\left(1-\sum\varepsilon_i\right)+f+\varepsilon_{s3}+\varepsilon_{r3}+\varepsilon_{s2}\right)C_{pt}T_{tt4}+\varepsilon_{r2}C_{pc}T_{tre2}\right]}{\left[\left(\left(1-\sum\varepsilon_i\right)+f\right)+\varepsilon_{s3}+\varepsilon_{r3}+\varepsilon_{s2}+\varepsilon_{r2}\right]C_{pt}} \qquad \textbf{Eqn. (A.26)}$$

$$\frac{T_{tt5}}{T_{tm4}} = \left(\frac{P_{tse1}}{P_{tre2}}\right)^{(\gamma_t-1)/(\eta_p\gamma_t)} \qquad \textbf{Eqn. (A.27)}$$

$$T_{tm5} = \frac{\left[\left(\left(1-\left(\varepsilon_{s1}+\varepsilon_{r1}\right)\right)+f\right)C_{pt}T_{tt5}+\varepsilon_{s1}C_{pc}T_{tse1}\right]}{\left[\left(\left(1-\varepsilon_{r1}\right)+f\right)\right]C_{pt}} \qquad \textbf{Eqn. (A.28)}$$

$$\frac{T_{tt6}}{T_{tm5}} = \left(\frac{P_{tre1}}{P_{tse2}}\right)^{(\gamma_t-1)/(\eta_p\gamma_t)} \qquad \textbf{Eqn. (A.29)}$$

$$T_{tm6} = \frac{\left[\left(\left(1-\varepsilon_{r1}\right)+f\right)C_{pt}T_{tt6}+\varepsilon_{r1}C_{pc}T_{tre1}\right]}{\left[\left(1+f\right)\right]C_{pt}} \qquad \textbf{Eqn. (A.30)}$$

Each expansion performs a certain percentage of the overall turbine work. The exact amount of work accomplished depends on the mass flow and the temperature change. So for example, the work accomplished by the expansion from P_{t4} (Total pressure out of combustor) to P_{tse3} (Total pressure of the 3rd stage stator bleed air) is

$$\dot{m}_4 C_{pt}\left(T_{t4}-T_{tt1}\right)=\dot{m}_4 C_{pt} T_{t4}\left(1-\left(\frac{T_{tt1}}{T_{t4}}\right)\right)$$

But from **Eqn. (A.19)** we get the following

$$\dot{m}_4 C_{pt} T_{t4}\left(1-\left(\frac{P_{tse3}}{P_{t4}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right) \qquad \textbf{Eqn. (A.31)}$$

Notice the work is calculated using T_{tt1} not T_{tm1}, the mixed out temperature. As Figure A.3 illustrates the expansion through the turbine is modeled as an expansion followed by a constant pressure mixing process, followed by another expansion etc.

Changes in the mass flow and the effects of mixing are illustrated in the calculation of the work performed in the second expansion through the turbine; the expansion from P_{tse3} (Total pressure of the 3rd stage stator bleed air) to P_{tr3} (Total pressure of the 3rd stage rotor bleed air). The work done is

$$\left(\dot{m}_4+\varepsilon_{s3}\dot{m}_c\right)C_{pt}\left(T_{tm1}-T_{tt2}\right)$$

Similar to **Eqn. (A.31)** this can be rewritten as

$$\left(\dot{m}_4+\varepsilon_{s3}\dot{m}_c\right)C_{pt}T_{tm1}\left(1-\left(\frac{P_{tr3}}{P_{tse3}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right) \qquad \textbf{Eqn. (A.32)}$$

From the above equation it can be seen how the effects of mass addition $\left(\dot{m}_4+\varepsilon_{s3}\dot{m}_c\right)$ and mixing $\left(\text{the } T_{tm1} \text{ term}\right)$ contribute. Values of P_{tr3} and P_{tse3} can be found using the 1-D compressible flow calculation outline in Section 3. All of the stepwise expansions are modeled in a similar manner and each contributes to the power balance.

Power Balance

A power balance between the compressor, Fan and the turbine must be performed in order to obtain a value for T_{t5} or P_{t5} the turbine exhaust temperature or pressure. **Eqn.'s (A.10) - (A.18)** give the power required by the fan and the compressor; **Eqn.'s (A.31), (A.32)** and similar equations give the power out of the turbine. A balance results in an equation that can be solved for T_{t5} or P_{t5}.

$$\dot{m}_c\left(1-\sum\varepsilon_i\right)C_{pc}\left(T_{t3}-T_{t2}\right)+\varepsilon_{r1}\dot{m}_cC_{pc}\left(T_{tr1}-T_{t2}\right)+\varepsilon_{r2}\dot{m}_cC_{pc}\left(T_{tr2}-T_{t2}\right)+\varepsilon_{r3}\dot{m}_cC_{pc}\left(T_{tr3}-T_{t2}\right)$$

$$\varepsilon_{s1}\dot{m}_cC_{pc}\left(T_{ts1}-T_{t2}\right)+\varepsilon_{s2}\dot{m}_cC_{pc}\left(T_{ts2}-T_{t2}\right)+\varepsilon_{s3}\dot{m}_cC_{pc}\left(T_{ts3}-T_{t2}\right)+\dot{m}_f\left(1-\varepsilon_{rf}\right)C_{pc}\left(T_{t13}-T_{t2}\right)$$

$$\varepsilon_{rf}\dot{m}_fC_{pc}\left(T_{trf}-T_{t2}\right)$$

$$=$$

$$\dot{m}_4C_{pt}T_{t4}\left(1-\left(\frac{P_{tse3}}{P_{t4}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right)+\left(\dot{m}_4+\left(\varepsilon_{s3}\right)\dot{m}_c\right)C_{pt}T_{tm1}\left(1-\left(\frac{P_{tr3}}{P_{tse3}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right)+$$

$$\left(\dot{m}_4+\left(\varepsilon_{s3}+\varepsilon_{r3}\right)\dot{m}_c\right)C_{pt}T_{tm2}\left(1-\left(\frac{P_{tse2}}{P_{tr2}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right)+\left(\dot{m}_4+\left(\varepsilon_{s3}+\varepsilon_{r3}+\varepsilon_{s2}\right)\dot{m}_c\right)C_{pt}T_{tm3}\left(1-\left(\frac{P_{tr2}}{P_{tse2}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right)$$

$$+$$

$$\left(\dot{m}_4+\left(\varepsilon_{s3}+\varepsilon_{r3}+\varepsilon_{s2}+\varepsilon_{r2}\right)\dot{m}_c\right)C_{pt}T_{tm4}\left(1-\left(\frac{P_{tse1}}{P_{tr1}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right)+\left(\dot{m}_4+\left(\varepsilon_{s3}+\varepsilon_{r3}+\varepsilon_{s2}+\varepsilon_{r2}+\varepsilon_{r1}\right)\dot{m}_c\right)C_{pt}T_{tm5}\left(1-\left(\frac{P_{tr1}}{P_{tse1}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right)$$

$$+$$

$$\left(\dot{m}_4+\left(\varepsilon_{s3}+\varepsilon_{r3}+\varepsilon_{s2}+\varepsilon_{r2}+\varepsilon_{s1}+\varepsilon_{r1}\right)\dot{m}_c\right)C_{pt}T_{tm6}\left(1-\left(\frac{P_{t5}}{P_{tr1}}\right)^{(\gamma_t-1)/\eta_p\gamma_t}\right) \qquad \textbf{Eqn. (A.33)}$$

Dividing both sides of **Eqn. (A.33)** by $\dot{m}_cC_{pc}$ and remembering that

$$\dot{m}_4=\dot{m}_c\left(\left(1-\sum\varepsilon_i\right)+f\right)$$

an expression for $\left(\frac{P_{t5}}{P_{tr1}}\right)^{(\gamma-1)/\eta_p\gamma} = \left(\frac{T_{t5}}{T_{tm6}}\right) = [/////////]$ can be obtained. This can be used to calculate

T_{t5} and hence τ_t.

Calculation of α:

Like the non-aspirated turbofan fluid dynamics requires equal static pressures at stations 6 and 16, so for this analysis, we assume the total pressures of the two entering streams are equal, or

$$P_{t6} = P_{t16}$$

Assuming isentropic flow in the bypass duct from 13 to 16, we can write

$$\pi_c \pi_b \pi_t = \pi_f$$

However unlike the non-aspirated turbofan, no simple closed form solution for α exists due to the changing mass flow through the turbine. This means the bypass ratio α is calculated by iteration. In the above relationship the only term effected by α is π_t, so one approach to calculating the bypass ratio is to vary α until the relationship $\pi_c \pi_b \pi_t = \pi_f$ is satisfied to two decimal places.

Mixed Exhaust Stream

The mixing analysis for the aspirated turbofan is identical to the analysis performed on the non-aspirated turbofan. The mixing of these two streams will play a role in the overall engine efficiency. The temperature and pressure ratios of the mixer are defined as

$$\tau_m = \frac{T_{t6A}}{T_{t6}} \qquad and \qquad \pi_m = \frac{P_{t6A}}{P_{t6}} \qquad \textbf{Eqn. (A.34)}$$

The mixer temperature ratio τ_m will be obtained from an energy balance, while the total pressure ratio π_m will be obtained from an analysis of a constant area ideal mixer.

The only change is to the $\alpha' \equiv \frac{\dot{m}_{16}}{\dot{m}_6}$ term. $\dot{m}_6$ is the same, but the $\dot{m}_{16}$ term is different as both the fan rotor and stator are aspirated. Therefore

$$\alpha' = \frac{\dot{m}_f\left(1-\left(\varepsilon_{rf}+\varepsilon_{sf}\right)\right)}{\dot{m}_c+\dot{m}_{fuel}} = \frac{\alpha\left(1-\left(\varepsilon_{rf}+\varepsilon_{sf}\right)\right)}{1+f}$$

This value for α' can be used throughout the rest of the mixer calculation.

Calculation of Total Thrust F_{total}

$$\frac{F_{total}}{\dot{m}_0 a_0} = \frac{F}{\dot{m}_0 a_0} + \frac{F_{fan}}{\dot{m}_0 a_0} \qquad \textbf{Eqn. (A.35)}$$

Calculation of Specific Impulse

$$I = \frac{F_{total}/\dot{m}_o}{f\,g} \qquad \textbf{Eqn. (A.36)}$$

Appendix B

B.1: Cycle Analysis for Aspirated Turbojet

This section will compute the behavior of the aspirated turbojet engine including component losses, the mass flow rate of the fuel through the components, and the variation of specific heats. It also takes the work performed on the bleed air into account. This is an important difference between the cycle analysis performed on the non-aspirated turbojet.

For example the bleed air removed from the 1st stage rotor is taken radially outward and in so doing, work is performed on the air; the air is "pumped" similar to the work done by a centrifugal compressor. This "centrifugal work" increases the total temperature of the bleed air to a temperature greater than T_{t2}. The temperature rise can be greater than or less than the stage temperature rise. This means that the work performed on the bleed air must be treated separately in the cycle analysis.

For the second and third stage rotors where the bleed air is taken radially inward, the bleed air performs work analogous to a centrifugal turbine. Again like the bleed air taken radially outward this air must be treated separately in the cycle analysis. The calculation of the temperatures and pressures of the bleed air after they have been taken inward or outward is performed using a 1-D compressible flow code, detail in Section 3.

Like the non-aspirated turbojet the analysis will assume one-dimensional flow at the entrance and exit of each component. The variation of the specific heat will be approximated by assuming a perfect gas with constant specific heat c_{pc} upstream of the main burner and a perfect gas with different constant specific heat c_{pt} downstream of the main burner. The symbols used during the analysis are explained in the table of nomenclature. The aspirated turbojet with station numbering is shown in Figure 2-3.

Uninstalled Thrust:

$$F = (\dot{m}_9 V_9 - \dot{m}_0 V_0) + A_9 (P_9 - P_0) + (Momentum\ Drag\ of\ Aux.\ Work)$$

Note: The thrust developed by expanding the auxiliary work air from P_{t2} to P_0 will be calculated latter and will be removed from the thrust calculation for now.

Assume the engine exhaust nozzle expands the gas to the ambient pressure $P_0 = P_9$. Therefore

$$\boxed{\frac{F}{\dot{m}_0 a_0} = \left(\frac{\dot{m}_9}{\dot{m}_0}\frac{V_9}{a_0} - M_0\right)} \qquad \textbf{Eqn. (B.1)}$$

Calculation of $\frac{\dot{m}_9}{\dot{m}_0}$***:***

$$\dot{m}_9 = \dot{m}_0 + \dot{m}_{fuel} - \varepsilon_{r1}\dot{m}_0 - \varepsilon_{s1}\dot{m}_0 - \varepsilon_{r2}\dot{m}_0 - \varepsilon_{s2}\dot{m}_0 - \varepsilon_{r3}\dot{m}_0 - \varepsilon_{s3}\dot{m}_0 - x'\dot{m}_0$$

$$\boxed{\therefore \frac{\dot{m}_9}{\dot{m}_0} = \left(1 - \left(\sum \varepsilon_i + x'\right)\right) + f} \qquad \textbf{Eqn. (B.2)}$$

Where $\sum \varepsilon_i = \varepsilon_{r1} + \varepsilon_{s1} + \varepsilon_{r2} + \varepsilon_{s2} + \varepsilon_{r3} + \varepsilon_{s3}$

Placing **Eqn. (B.2)** into **Eqn. (B.1)** gives

$$\boxed{\frac{F}{\dot{m}_0 a_0} = \left[\left(\left(1 - \left(\sum \varepsilon_i + x'\right)\right) + f\right)\frac{V_9}{a_0} - M_0\right]} \qquad \textbf{Eqn. (B.3)}$$

Calculation of $(V_9 / a_0)^2$

For the aspirated turbojet cycle,

$$\boxed{\left(\frac{V_9}{a_0}\right)^2 = \frac{\gamma_t R_t T_t}{\gamma_c R_c T_c} M_9^2} \qquad \textbf{Eqn. (B.4)}$$

Calculation of M_9^2

$$\boxed{M_9^2 = \frac{2}{\gamma_t - 1}\left[\left(\frac{P_{t9}}{P_9}\right)^{(\gamma_t-1)/\gamma_t} - 1\right]} \qquad \textbf{Eqn. (B.5)}$$

Where
$$\boxed{\frac{P_{t9}}{P_9} = \frac{P_0}{P_9}\pi_r\pi_d\pi_c\pi_b\pi_t\pi_n} \qquad \textbf{Eqn. (B.6)}$$

Calculation of T_9 / T_0

$$\boxed{\frac{T_9}{T_0} = \frac{T_{t9}/T_0}{\left(P_{t9}/P_9\right)^{(\gamma_t-1)/\gamma_t}}} \qquad \textbf{Eqn. (B.7)}$$

Where
$$\frac{T_{t9}}{T_0} = \tau_r\tau_d\tau_c\tau_b\tau_t\tau_n$$

Calculation of f:

$$f = \frac{\dot{m}_{fuel}}{\dot{m}_0}$$

Application of the 1st law of thermodynamics to the control volume about the combustor shown in Figure 2-2 gives:

$$\boxed{\dot{m}_3 C_{pc} T_{t3} + \eta_b \dot{m}_{fuel} h_{pr} = \dot{m}_4 C_{pt} T_{t4}} \qquad \textbf{Eqn. (B.8)}$$

Where $\mathbf{h_{pr}}$ is the thermal energy released by the fuel during combustion.

$$\dot{m}_3 = \dot{m}_0\left(1 - \left(\sum \varepsilon_i + x'\right)\right)$$

$$\dot{m}_4 = \dot{m}_3 + \dot{m}_{fuel}$$

$$\Rightarrow \dot{m}_4 = \dot{m}_0\left(\left(1 - \left(\sum \varepsilon_i + x'\right)\right) + f\right)$$

Using the above relations in **Eqn. (B.8)**

$$\dot{m}_0\left(1 - \left(\sum \varepsilon_i + x'\right)\right) C_{pc} T_{t3} + \eta_b \dot{m}_{fuel} h_{pr} = \dot{m}_0\left(\left(1 - \left(\sum \varepsilon_i + x'\right)\right) + f\right) C_{pt} T_{t4}$$

Dividing the above expression by $\dot{m}_0 C_{pc} T_0$ and solving for ***f*** gives

$$\boxed{f = \frac{\left(1-\left(\sum \varepsilon_i + x'\right)\right)\left[\tau_r \tau_c - \tau_\lambda\right]}{\left[\tau_\lambda - \left(\eta_b h_{pr} \big/ C_{pc} T_0\right)\right]}} \qquad \textbf{Eqn. (B.9)}$$

Calculation of τ_t:

Like the non-aspirated case the calculation of τ_t involves a power balance between the compressor and the turbine.

Compressor Work = Power from turbine

To make the analysis clearer the steps involve in calculating the work for each portion of the compressor flow is shown below. Figure A.1 shows the nomenclature and location of the temperatures referenced in the analysis.

Compressor Work

1. Through Flow

$$\dot{m}_0\left(1-\sum \varepsilon_i\right)C_{pc}\left(T_{t3}-T_{t2}\right) \qquad \textbf{Eqn. (B.10)}$$

2. 1st Stage Rotor Bleed

$$\varepsilon_{r1}\dot{m}_0 C_{pc}\left(T_{r1}-T_{t2}\right) \qquad \textbf{Eqn. (B.11)}$$

3. 1st Stage Stator Bleed

$$\varepsilon_{s1}\dot{m}_0 C_{pc}\left(T_{ts1}-T_{t2}\right) \qquad \textbf{Eqn. (B.12)}$$

4. 2nd Stage Rotor Bleed

$$\varepsilon_{r2}\dot{m}_0 C_{pc}\left(T_{r2}-T_{t2}\right) \qquad \textbf{Eqn. (B.13)}$$

5. 2nd Stage Stator Bleed

$$\varepsilon_{s2}\dot{m}_0 C_{pc}\left(T_{ts2}-T_{t2}\right) \qquad \textbf{Eqn. (B.14)}$$

6. 3rd Stage Rotor Bleed

$$\varepsilon_{r3}C_{pc}(T_{r3}-T_{t2}) \qquad \textbf{Eqn. (B.15)}$$

7. 3rd Stage Stator Bleed

$$\varepsilon_{s3}C_{pc}(T_{ts3}-T_{t2}) \qquad \textbf{Eqn. (B.16)}$$

Where
$$T_{ts1}=T_{t2}\left(1+\left(\frac{1}{\eta_1}\left(\pi_1^{\frac{\gamma_c-1}{\gamma_c}}-1\right)\right)\right)$$

$$T_{ts2}=T_{ts1}\left(1+\left(\frac{1}{\eta_2}\left(\pi_2^{\frac{\gamma_c-1}{\gamma_c}}-1\right)\right)\right)$$

$$T_{ts3}=T_{ts2}\left(1+\left(\frac{1}{\eta_3}\left(\pi_3^{\frac{\gamma_c-1}{\gamma_c}}-1\right)\right)\right)$$

T_{r1}, T_{r2}, T_{r3} are calculated using a 1-D compressible flow calculation, detail in Section 3.

Turbine Power

$$\dot{m}_4C_{pt}(T_{t4}-T_{t5})$$

Where $\dot{m}_4=\left(\left(1-\left(\sum\varepsilon_i+x'\right)\right)+f\right)\dot{m}_0$ gives

$$\dot{m}_0\left(\left(1-\left(\sum\varepsilon_i+x'\right)\right)+f\right)C_{pt}(T_{t4}-T_{t5}) \qquad \textbf{Eqn. (B.17)}$$

Power Balance

Using **Eqn.'s (B.10)-(B.17)** to perform a power balance between the compressor and the turbine plus some algebraic manipulation (taking T_{t2} out of the brackets and both sides by $\dot{m}_0C_{pc}T_{t2}$)

gives

$$\left(1-\sum\varepsilon_i\right)(\tau_c-1)+\varepsilon_{r1}(\tau_{r1}-1)+\varepsilon_{s1}(\tau_{s1}-1)+\varepsilon_{r2}(\tau_{r2}-1)+\varepsilon_{s2}(\tau_{s2}-1)+\varepsilon_{r3}(\tau_{r3}-1)+\varepsilon_{s3}(\tau_{s3}-1)$$

$$=\frac{\dot{m}_4C_{pt}}{\dot{m}_0C_{pc}}\frac{T_{t4}}{T_{t2}}(1-\tau_t) \qquad \textbf{Eqn. (B.18)}$$

Using $\dot{m}_4 = \left(\left(1-\left(\sum \varepsilon_i + x'\right)\right) + f\right)\dot{m}_0$ gives

$$\tau_t = 1 - \left\{ \left(\left(\frac{1}{\left[1-\left(\left(\sum \varepsilon_i + x'\right)+f\right)\right]} \right) \left(\frac{\tau_r}{\tau_\lambda} \right) \right) \left(\begin{array}{l} \left(1-\sum \varepsilon_i\right)\left(\tau_c - 1\right) + \varepsilon_{r1}\left(\tau_{r1}-1\right) + \varepsilon_{s1}\left(\tau_{s1}-1\right) + \varepsilon_{r2}\left(\tau_{r2}-1\right) \\ + \varepsilon_{s2}\left(\tau_{s2}-1\right) + \varepsilon_{r3}\left(\tau_{r3}-1\right) + \varepsilon_{s3}\left(\tau_{s3}-1\right) \end{array} \right) \right\}$$

Eqn. (B.19)

Calculation of $x'\dot{m}_0$

As mention above both the bleed air and the compressor discharge air are used to perform "Auxiliary Work". It is assumed that all of the bleed air can be used for this purpose. To perform this "Auxiliary Work" the bleed air and the compressor discharge air are expanded ***isentropically*** to the compressor inlet pressure P_{t2}. To allow a comparison between the Non-Aspirated turbojet and the Aspirated Turbojet, the amount of auxiliary work performed by both engines is equal (i.e. the work performed by $x'\dot{m}_0, \varepsilon_{r1}$ *etc.* is equal to the work performed by $x\dot{m}_0$ in the non-aspirated engine).

(Auxiliary work)$_{\textbf{aspirated}}$ =(Auxiliary Work)$_{\textbf{non-aspirated}}$

This means that the compressor discharge extraction ($x'\dot{m}_0$) for the aspirated case is smaller that that for the non-aspirated case. The calculation of $x'\dot{m}_0$ is detailed below.

The bleed and the compressor discharge air reach their final conditions by undergoing non-ideal compressions. This means that when they are expanded isentropically to P_{t2}, the temperature of the air after the expansion is greater than T_{t2} (see Figure 2.5). In order to calculate the work performed the pressure and temperature of the air at the extraction point must be known (see Figure 2.4). As has been mentioned earlier this pressure is calculated using a 1-D compressible flow model detailed in Section 3.

Like the τ_t calculation the work performed by each portion of the compressor air will be detailed separately for clarity.

Figure 2.5: Auxiliary Work expansion

<u>Aspirated Compressor</u>

1. Through Flow

$$x'\dot{m}_0 C_{pc} T_{t3}\left(1-\left(\frac{P_{t2}}{P_{t3}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right) \qquad \textbf{Eqn. (B.20)}$$

2. 1st Stage Rotor Bleed

$$\varepsilon_{r1}\dot{m}_0 C_{pc} T_{r1}\left(1-\left(\frac{P_{t2}}{P_{r1}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right) \qquad \textbf{Eqn. (B.21)}$$

3. 1st Stage Stator Bleed

$$\varepsilon_{s1}\dot{m}_0 C_{pc} T_{ts1}\left(1-\left(\frac{P_{t2}}{P_{s1}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$ **Eqn. (B.22)**

4. 2nd Stage Rotor Bleed

$$\varepsilon_{r2}\dot{m}_0 C_{pc} T_{r2}\left(1-\left(\frac{P_{t2}}{P_{r2}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$ **Eqn. (B.23)**

5. 2nd Stage Stator Bleed

$$\varepsilon_{s2}\dot{m}_0 C_{pc} T_{ts2}\left(1-\left(\frac{P_{t2}}{P_{s2}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$ **Eqn. (B.24)**

6. 3rd Stage Rotor Bleed

$$\varepsilon_{r3}\dot{m}_0 C_{pc} T_{r3}\left(1-\left(\frac{P_{t2}}{P_{r3}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$ **Eqn. (B.25)**

7. 3rd Stage Stator Bleed

$$\varepsilon_{s3}\dot{m}_0 C_{pc} T_{ts3}\left(1-\left(\frac{P_{t2}}{P_{s3}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$ **Eqn. (B.26)**

<u>Non-Aspirated Compressor</u>

$$x\dot{m}_0 C_{pc} T_{t3}\left(1-\left(\frac{P_{t2}}{P_{t3}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$ **Eqn. (B.27)**

From **Eqn. (B.20)-(B.27)** an energy balance can be established to calculate x'

(Auxiliary work) $_{\text{aspirated}}$ =(Auxiliary Work) $_{\text{non-aspirated}}$

$$x\dot{m}_0 C_{pc} T_{t3}\left(1-\left(\frac{P_{t2}}{P_{t3}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right) = x'\dot{m}_0 C_{pc} T_{t3}\left(1-\left(\frac{P_{t2}}{P_{t3}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right) + \varepsilon_{r1}\dot{m}_0 C_{pc} T_{r1}\left(1-\left(\frac{P_{t2}}{P_{r1}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$

$$+ \varepsilon_{s1}\dot{m}_0 C_{pc} T_{ts1}\left(1-\left(\frac{P_{t2}}{P_{s1}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right) + \varepsilon_{r2}\dot{m}_0 C_{pc} T_{r2}\left(1-\left(\frac{P_{t2}}{P_{r2}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$

$$+ \varepsilon_{s2}\dot{m}_0 C_{pc} T_{ts2}\left(1-\left(\frac{P_{t2}}{P_{s2}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right) + \varepsilon_{r3}\dot{m}_0 C_{pc} T_{r3}\left(1-\left(\frac{P_{t2}}{P_{r3}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right)$$

$$+ \varepsilon_{s3}\dot{m}_0 C_{pc} T_{ts3}\left(1-\left(\frac{P_{t2}}{P_{s3}}\right)^{\frac{\gamma_c-1}{\gamma_c}}\right) \qquad \textbf{Eqn. (B.28)}$$

Eqn. (B.28) allows x' to be calculated.

Calculation of the Auxiliary Work Thrust

After performing the Auxiliary Work all seven bleeds are expanded from P_{t2} to P_0 to recover the momentum drag. Like the fan bleeds for the turbofan the thrust is calculated as shown below:

$$\frac{F_{CD}}{\dot{m}_0 a_0} = x'\left[\left(\sqrt{\frac{T_{CD}}{T_0}}\sqrt{\frac{2}{\gamma_c-1}\left(\tau_r-1\right)}\right)-M_0\right] \qquad \textbf{Eqn. (B.29)}$$

The thrust for the other bleeds can be calculated in a similar fashion with x' being replaced (ε_{r1} etc.) and the T_{CD} being corrected.

The sum of the individual thrust is called $\frac{F_{ax}}{\dot{m}_0 a_0}$

Calculation of Total Thrust F_{total}

$$\boxed{\frac{F_{total}}{\dot{m}_0 a_0} = \frac{F}{\dot{m}_0 a_0} + \frac{F_{ax}}{\dot{m}_0 a_0}}$$

Eqn. (B.30)

Calculation of Propulsive Efficiency

It can be shown that

$$\boxed{\eta_p = \frac{2V_0\left(F_{total}/\dot{m}_0\right)}{a_0^2\left[\left(\left(1-\left(\sum\varepsilon_i + x'\right)\right)+f\right)\left(V_9/a_0\right)^2 - M_0^2\right]}}$$

Eqn. (B.31)

Calculation of Thermal Efficiency

It can be shown that

$$\boxed{\eta_T = \frac{a_0^2\left[\left(\left(1-\left(\sum\varepsilon_i + x'\right)\right)+f\right)\left(V_9/a_0\right)^2 - M_0^2\right]}{2fh_{pr}}}$$

Eqn. (B.32)

Calculation of Specific Impulse

$$\boxed{I = \frac{F_{total}/\dot{m}_0}{f\,g}}$$

Eqn. (B.33)

Appendix C

This appendix contains data sheets for the QSP configured turbofan with an aspirated fan and core compressor. The nomenclature is fairly self-explanatory, but some comments are provided to aid understanding.

Many of the mixing quantities such as $\mathbf{M_{6A}}$, $\boldsymbol{\phi(M_6,\gamma_6)}$ etc. are defined in Chapter 7, of Mattingly's "Elements of Gas Turbine Propulsion". The nomenclature used in the data sheets to describe the mixing of the two gas streams in the engine exhaust, is identical to the nomenclature used by Mattingly.

The quantities defined as π_{tre1}, π_{tse1}, π_{tre2}, π_{tse2} etc. are the ratios of the total pressure changes across the relevant blade row in the counter-rotating compressor. Figure C.1 below gives an example of the how these 6 Pi terms are defined for the counter-rotating compressor. Figure C.1 shows the 1st stage of the of the core counter-rotating compressor. The quantity P_{tin} can be calculated as follows:

$$P_{tin} = P_0 \pi_r \pi_d \pi_f$$

and the quantity P_{trout} can be calculated as

$$P_{trout} = P_0 \pi_r \pi_d \pi_f \pi_{r1}$$

where π_{r1} is the pressure rise across the 1st stage rotor. These quantities are known before the cycle analysis is started. The quantities, P_{tr1} and P_{tse1} can be calculated using the code in Chapter 3, with P_{tin} and P_{trout} as inputs to the code. With this information we can mow define π_{tre1} and π_{tse1} as

$$\pi_{tr1} = \frac{P_{tr1}}{P_{tin}} \quad \& \quad \pi_{tse1} = \frac{P_{tse1}}{P_{trout}}$$

The other pi quantities are defined in a similar manner.

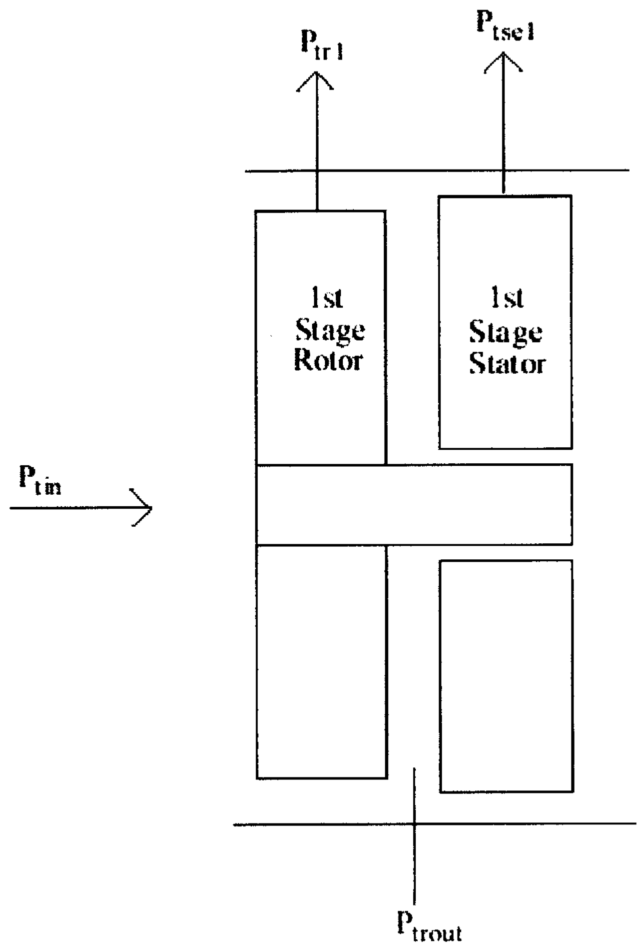


Figure C.1: Total pressure into and out of the 1st stage of the aspirated counter-rotating compressor

Specific Impulse is referred to as **I**, specific fuel consumption is referred to as **S** and the thermal and propulsive efficiencies are referred to as η_t and η_p respectively.

M_0	T_0	γ_c	C_{pc}	γ_t	C_{pt}	h_{pr}	π_d	π_b	π_n	η_m
2	216	1.4	1004	1.3	1239	42800000	0.925	1	0.98	1
T_{t4}	π_c	τ_f	η_b	e_f	e_c	η_{tstage}	R_c	Rt	P_0	**Press. Recov.**
1922	32.8277	1.2	1	0.9	0.91635	0.89	286.8571	285.9231	12112	**100%**
a_0	V_0	M_6	ε_{rf}	ε_{sf}	ε_{r1}	ε_{s1}	ε_{r2}	ε_{s2}	ε_{r3}	ε_{s3}
294.52606	589.0521	0.6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
π_{r1}	π_{r2}	π_{r3}	P_{t4}	π_{s3e}	π_{r3e}	π_{s2e}	π_{r2e}	T_{tse3}	T_{tre3}	T_{tse2}
3.5	2.8	2.2	2877742	0.7084	0.8101	0.6874	0.6668	1155	931.3173	921.89
T_{tre2}	T_{tse1}	T_{tre1}	π_{s1}	π_{s2}	π_{s3}	τ_{cen}	η_{cen}	π_{tr1e}	π_{ts1e}	
688.45	674.23	664.81	0.95	0.95	0.95	1.678	0.3087	1.9257	0.5709	

Turbofan with Aspirated Fan & 3 stage Core Compressor - Aspirated air from All Stages returned to the turbine

τ_r	1.8	π_r	7.824449	P_{t16}/P_{t6}	0.999945	τ_λ	10.98088203
π_f	1.77591	α'	3.612133	M_{16}	0.579933	τ_c	2.970008943
η_f	0.891598	C_{p6A}	1054.953	$\phi(M_6,\gamma_6)$	0.176072	η_c	0.868784
α	3.7906	R_{6A}	286.6546	$\phi(M_{16},\gamma_{16})$	0.165917	f	0.028419549
P_{t9}/P_9	12.38269	γ_{6A}	1.373103	Φ	0.180145	τ_t	0.532333303
P_0/P_9	1	T_{t16}/T_6	0.456006	M_{6A}	0.637666	π_t	0.054100874
M_9	2.290941	τ_m	0.594532	A_{16}/A_6	2.420598	η_t	0.932840555
V_9/a_0	2.766613	T_{t9}	608.2919	$MFP(M_6)$	0.033071	$MFP(M_{6A})$	0.034976895
T_{t2}	388.8	T_9/T_0	1.575688	π_M	0.982989	π_{cen}	1.944766323
T_{tt1}	1809.581	T_{tt2}	1574.226	T_{tt3}	1530.027	T_{tt4}	1224.373685
T_{tm1}	1800.652	T_{tm2}	1565.935	T_{tm3}	1522.185	T_{tm4}	1217.764319
T_{tse3}/T_{t4}	0.94151	T_{tre3}/T_{tse3}	0.874253	T_{tse2}/T_{tre3}	0.977069	T_{tre2}/T_{tse2}	0.80435299
P_{tse3}/P_{t4}	0.745684	P_{tre3}/P_{tse3}	0.519801	P_{tse2}/P_{tre3}	0.893197	P_{tre2}/P_{tse2}	0.346440002
T_{tt5}	1192.033	P_{tse1}/P_{tre2}	0.901241	T_{tt6}	1176.733	P_{tre1}/P_{tse1}	0.963741461
T_{tm5}	1185.693	T_{tse1}/T_{tre2}	0.97887	T_{tm6}	1170.529	T_{tre1}/T_{tse1}	0.992443387
P_{t5}/P_{tre1}	0.51932	T_{t5}	1023.145	τ_t	0.532333	π_t	0.054100874
$\pi_c\,\pi_b\,\pi_t$	1.776007	T_{fan}	521.97	$T_{fanstator}$	443.628	π_{stator}	0.5505
		T_{rotor1}	545.38	$T_{stator1}$	638.3		

$\mathbf{F/(m_0a_0)}$	*0.758759*	$\mathbf{F/m_0}$	*223.4744 N*	**S**	*26.54602 mg/(N.s)*
η_p	*0.845055*	η_T	*0.629296*	**I**	*3840.003sec*

M_0	T_0	γ_c	C_{pc}	γ_t	C_{pt}	h_{pr}	π_d	π_b	π_n	η_m
2	216	1.4	1004	1.3	1239	42800000	0.925	1	0.98	1
T_{t4}	π_c	τ_f	η_b	e_f	e_c	η_{tstage}	R_c	Rt	P_0	**Press. Recov.**
1922	32.8277	1.2	1	0.9	0.91635	0.89	286.8571	285.9231	12112	**90%**
a_0	V_0	M_6	ε_{rf}	ε_{sf}	ε_{r1}	ε_{s1}	ε_{r2}	ε_{s2}	ε_{r3}	ε_{s3}
294.52606	589.0521	0.6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
π_{r1}	π_{r2}	π_{r3}	P_{t4}	π_{s3e}	π_{r3e}	π_{s2e}	π_{r2e}	T_{tse3}	T_{tre3}	T_{tse2}
3.5	2.8	2.2	2877742	0.7084	0.8101	0.6874	0.6668	1155	931.3173	921.89
T_{tre2}	T_{tse1}	T_{tre1}	π_{s1}	π_{s2}	π_{s3}	τ_{cen}	η_{cen}	π_{tr1e}	π_{ts1e}	
688.45	674.23	664.81	0.95	0.95	0.95	1.678	0.3087	1.9257	0.5709	

Turbofan with Aspirated Fan & 3 stage Core Compressor - Aspirated air from All Stages returned to the turbine

τ_r	1.8	π_r	7.824449	P_{t16}/P_{t6}	0.999945	τ_λ	10.98088203
π_f	1.77591	α'	3.602508	M_{16}	0.579932	τ_c	2.970008943
η_f	0.891598	C_{p6A}	1055.059	$\phi(M_6,\gamma_6)$	0.176072	η_c	0.868784
α	3.7805	R_{6A}	286.6542	$\phi(M_{16},\gamma_{16})$	0.165917	f	0.028419549
P_{t9}/P_9	12.38216	γ_{6A}	1.373051	Φ	0.180178	τ_t	0.532662969
P_0/P_9	1	T_{t16}/T_6	0.455724	M_{6A}	0.637816	π_t	0.054100893
M_9	2.290915	τ_m	0.594597	A_{16}/A_6	2.413403	η_t	0.93213524
V_9/a_0	2.767604	T_{t9}	608.7357	$MFP(M_6)$	0.033071	$MFP(M_{6A})$	0.034980909
T_{t2}	388.8	T_9/T_0	1.576853	π_M	0.982947	π_{cen}	1.944766323
T_{tt1}	1770.844	T_{tt2}	1540.706	T_{tt3}	1497.606	T_{tt4}	1198.557426
T_{tm1}	1762.31	T_{tm2}	1532.754	T_{tm3}	1490.089	T_{tm4}	1192.204067
T_{tse3}/T_{t4}	0.921355	T_{tre3}/T_{tse3}	0.874253	T_{tse2}/T_{tre3}	0.977069	T_{tre2}/T_{tse2}	0.80435299
P_{tse3}/P_{t4}	0.671116	P_{tre3}/P_{tse3}	0.519801	P_{tse2}/P_{tre3}	0.893197	P_{tre2}/P_{tse2}	0.346440002
T_{tt5}	1167.013	P_{tse1}/P_{tre2}	0.901241	T_{tt6}	1152.146	P_{tre1}/P_{tse1}	0.963741461
T_{tm5}	1160.918	T_{tse1}/T_{tre2}	0.97887	T_{tm6}	1146.181	T_{tre1}/T_{tse1}	0.992443387
P_{t5}/P_{tre1}	0.577023	T_{t5}	1023.778	τ_t	0.532663	π_t	0.054100893
$\pi_c\,\pi_b\,\pi_t$	1.776008	T_{fan}	521.97	$T_{fanstator}$	443.628	π_{stator}	0.5505
		T_{rotor1}	545.38	$T_{stator1}$	638.3		

$F/(m_0a_0)$	*0.759789*	F/m_0	*223.7778 N*	**S**	*26.56605 mg/(N.s)*
η_p	*0.844891*	η_T	*0.628913*	**I**	*3837.109sec*

M_0	T_0	γ_c	C_{pc}	γ_t	C_{pt}	h_{pr}	π_d	π_b	π_n	η_m
2	216	1.4	1004	1.3	1239	42800000	0.925	1	0.98	1
T_{t4}	π_c	τ_f	η_b	e_f	e_c	η_{tstage}	R_c	Rt	P_0	**Press. Recov.**
1922	32.8277	1.2	1	0.9	0.91635	0.89	286.8571	285.9231	12112	**80%**
a_0	V_0	M_6	ε_{rf}	ε_{sf}	ε_{r1}	ε_{s1}	ε_{r2}	ε_{s2}	ε_{r3}	ε_{s3}
294.52606	589.0521	0.6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
π_{r1}	π_{r2}	π_{r3}	P_{t4}	π_{s3e}	π_{r3e}	π_{s2e}	π_{r2e}	T_{tse3}	T_{tre3}	T_{tse2}
3.5	2.8	2.2	2877742	0.7084	0.8101	0.6874	0.6668	1155	931.3173	921.89
T_{tre2}	T_{tse1}	T_{tre1}	π_{s1}	π_{s2}	π_{s3}	τ_{cen}	η_{cen}	π_{tr1e}	π_{ts1e}	
688.45	674.23	664.81	0.95	0.95	0.95	1.678	0.3087	1.9257	0.5709	

Turbofan with Aspirated Fan & 3 stage Core Compressor - Aspirated air from All Stages returned to the turbine

τ_r	1.8	π_r	7.824449	P_{t16}/P_{t6}	0.99996	τ_λ	10.98088203
π_f	1.77591	α'	3.59155	M_{16}	0.579953	τ_c	2.970008943
η_f	0.891598	C_{p6A}	1055.181	$\phi(M_6,\gamma_6)$	0.176072	η_c	0.868784
α	3.769	R_{6A}	286.6537	$\phi(M_{16},\gamma_{16})$	0.165921	f	0.028419549
P_{t9}/P_9	12.38149	γ_{6A}	1.372991	Φ	0.18022	τ_t	0.533038331
P_0/P_9	1	T_{t16}/T_6	0.455403	M_{6A}	0.638005	π_t	0.054100065
M_9	2.290883	τ_m	0.594674	A_{16}/A_6	2.405126	η_t	0.931328718
V_9/a_0	2.768736	T_{t9}	609.243	MFP(M_6)	0.033071	MFP(M_{6A})	0.034986065
T_{t2}	388.8	T_9/T_0	1.578187	π_M	0.982908	π_{cen}	1.944766323
T_{tt1}	1728.52	T_{tt2}	1504.082	T_{tt3}	1462.184	T_{tt4}	1170.35098
T_{tm1}	1720.419	T_{tm2}	1496.5	T_{tm3}	1455.022	T_{tm4}	1164.277331
T_{tse3}/T_{t4}	0.899334	T_{tre3}/T_{tse3}	0.874253	T_{tse2}/T_{tre3}	0.977069	T_{tre2}/T_{tse2}	0.80435299
P_{tse3}/P_{t4}	0.596547	P_{tre3}/P_{tse3}	0.519801	P_{tse2}/P_{tre3}	0.893197	P_{tre2}/P_{tse2}	0.346440002
T_{tt5}	1139.676	P_{tse1}/P_{tre2}	0.901241	T_{tt6}	1125.282	P_{tre1}/P_{tse1}	0.963741461
T_{tm5}	1133.85	T_{tse1}/T_{tre2}	0.97887	T_{tm6}	1119.579	T_{tre1}/T_{tse1}	0.992443387
P_{t5}/P_{tre1}	0.649141	T_{t5}	1024.5	τ_t	0.533038	π_t	0.054100065
$\pi_c\,\pi_b\,\pi_t$	1.775981	T_{fan}	521.97	$T_{fanstator}$	443.628	π_{stator}	0.5505
		T_{rotor1}	545.38	$T_{stator1}$	638.3		

$F/(m_0a_0)$	*0.760965*	**F/m_0**	*224.1239 N*	**S**	*26.58898 mg/(N.s)*
η_p	*0.844705*	η_T	*0.628475*	**I**	*3833.798sec*

M_0	T_0	γ_c	C_{pc}	γ_t	C_{pt}	h_{pr}	π_d	π_b	π_n	η_m
2	216	1.4	1004	1.3	1239	42800000	0.925	1	0.98	1
T_{t4}	π_c	τ_f	η_b	e_f	e_c	η_{tstage}	R_c	Rt	P_0	**Press. Recov.**
1922	32.8277	1.2	1	0.9	0.91635	0.89	286.8571	285.9231	12112	**70%**
a_0	V_0	M_6	ε_{rf}	ε_{sf}	ε_{r1}	ε_{s1}	ε_{r2}	ε_{s2}	ε_{r3}	ε_{s3}
294.52606	589.0521	0.6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
π_{r1}	π_{r2}	π_{r3}	P_{t4}	π_{s3e}	π_{r3e}	π_{s2e}	π_{r2e}	T_{tse3}	T_{tre3}	T_{tse2}
3.5	2.8	2.2	2877742	0.7084	0.8101	0.6874	0.6668	1155	931.3173	921.89
T_{tre2}	T_{tse1}	T_{tre1}	π_{s1}	π_{s2}	π_{s3}	τ_{cen}	η_{cen}	π_{tr1e}	π_{ts1e}	
688.45	674.23	664.81	0.95	0.95	0.95	1.678	0.3087	1.9257	0.5709	

Turbofan with Aspirated Fan & 3 stage Core Compressor - Aspirated air from All Stages returned to the turbine

τ_r	1.8	π_r	7.824449	P_{t16}/P_{t6}	0.999971	τ_λ	10.98088203
π_f	1.77591	α'	3.578781	M_{16}	0.579968	τ_c	2.970008943
η_f	0.891598	C_{p6A}	1055.324	$\phi(M_6,\gamma_6)$	0.176072	η_c	0.868784
α	3.7556	R_{6A}	286.6531	$\phi(M_{16},\gamma_{16})$	0.165925	f	0.028419549
P_{t9}/P_9	12.38073	γ_{6A}	1.372921	Φ	0.180266	τ_t	0.533475709
P_0/P_9	1	T_{t16}/T_6	0.455029	M_{6A}	0.638219	π_t	0.054099449
M_9	2.290846	τ_m	0.594766	A_{16}/A_6	2.395526	η_t	0.930390406
V_9/a_0	2.77006	T_{t9}	609.8371	$MFP(M_6)$	0.033071	$MFP(M_{6A})$	0.034991844
T_{t2}	388.8	T_9/T_0	1.579748	π_M	0.98286	π_{cen}	1.944766323
T_{tt1}	1681.759	T_{tt2}	1463.619	T_{tt3}	1423.049	T_{tt4}	1139.187659
T_{tm1}	1674.136	T_{tm2}	1456.447	T_{tm3}	1416.278	T_{tm4}	1133.423041
T_{tse3}/T_{t4}	0.875004	T_{tre3}/T_{tse3}	0.874253	T_{tse2}/T_{tre3}	0.977069	T_{tre2}/T_{tse2}	0.80435299
P_{tse3}/P_{t4}	0.521979	P_{tre3}/P_{tse3}	0.519801	P_{tse2}/P_{tre3}	0.893197	P_{tre2}/P_{tse2}	0.346440002
T_{tt5}	1109.474	P_{tse1}/P_{tre2}	0.901241	T_{tt6}	1095.602	P_{tre1}/P_{tse1}	0.963741461
T_{tm5}	1103.944	T_{tse1}/T_{tre2}	0.97887	T_{tm6}	1090.187	T_{tre1}/T_{tse1}	0.992443387
P_{t5}/P_{tre1}	0.741866	T_{t5}	1025.34	τ_t	0.533476	π_t	0.054099449
$\pi_c\ \pi_b\ \pi_t$	1.775961	T_{fan}	521.97	$T_{fanstator}$	443.628	π_{stator}	0.5505
		T_{rotor1}	545.38	$T_{stator1}$	638.3		

$F/(m_0a_0)$	*0.762341*	**F/m_0**	*224.5293 N*	**S**	*26.61576 mg/(N.s)*
η_p	*0.844487*	η_T	*0.627965*	**I**	*3829.941sec*

M_0	T_0	γ_c	C_{pc}	γ_t	C_{pt}	h_{pr}	π_d	π_b	π_n	η_m
2	216	1.4	1004	1.3	1239	42800000	0.925	1	0.98	1
T_{t4}	π_c	τ_f	η_b	e_f	e_c	η_{tstage}	R_c	Rt	P_0	**Press. Recov.**
1922	32.8277	1.2	1	0.9	0.91635	0.89	286.8571	285.9231	12112	**60%**
a_0	V_0	M_6	ε_{rf}	ε_{sf}	ε_{r1}	ε_{s1}	ε_{r2}	ε_{s2}	ε_{r3}	ε_{s3}
294.52606	589.0521	0.6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
π_{r1}	π_{r2}	π_{r3}	P_{t4}	π_{s3e}	π_{r3e}	π_{s2e}	π_{r2e}	T_{tse3}	T_{tre3}	T_{tse2}
3.5	2.8	2.2	2877742	0.7084	0.8101	0.6874	0.6668	1155	931.3173	921.89
T_{tre2}	T_{tse1}	T_{tre1}	π_{s1}	π_{s2}	π_{s3}	τ_{cen}	η_{cen}	π_{tr1e}	π_{ts1e}	
688.45	674.23	664.81	0.95	0.95	0.95	1.678	0.3087	1.9257	0.5709	

Turbofan with Aspirated Fan & 3 stage Core Compressor - Aspirated air from All Stages returned to the turbine

τ_r	1.8	π_r	7.824449	P_{t16}/P_{t6}	0.999965	τ_λ	10.98088203
π_f	1.77591	α'	3.563534	M_{16}	0.579959	τ_c	2.970008943
η_f	0.891598	C_{p6A}	1055.495	$\phi(M_6,\gamma_6)$	0.176072	η_c	0.868784
α	3.7396	R_{6A}	286.6525	$\phi(M_{16},\gamma_{16})$	0.165923	f	0.028419549
P_{t9}/P_9	12.37992	γ_{6A}	1.372836	Φ	0.180318	τ_t	0.533997952
P_0/P_9	1	T_{t16}/T_6	0.454584	M_{6A}	0.63845	π_t	0.054099794
M_9	2.290806	τ_m	0.594879	A_{16}/A_6	2.384191	η_t	0.929274492
V_9/a_0	2.771654	T_{t9}	610.5507	$MFP(M_6)$	0.033071	$MFP(M_{6A})$	0.034998019
T_{t2}	388.8	T_9/T_0	1.581621	π_M	0.982788	π_{cen}	1.944766323
T_{tt1}	1629.348	T_{tt2}	1418.267	T_{tt3}	1379.185	T_{tt4}	1104.259148
T_{tm1}	1622.261	T_{tm2}	1411.553	T_{tm3}	1372.854	T_{tm4}	1098.840899
T_{tse3}/T_{t4}	0.847736	T_{tre3}/T_{tse3}	0.874253	T_{tse2}/T_{tre3}	0.977069	T_{tre2}/T_{tse2}	0.80435299
P_{tse3}/P_{t4}	0.447411	P_{tre3}/P_{tse3}	0.519801	P_{tse2}/P_{tre3}	0.893197	P_{tre2}/P_{tse2}	0.346440002
T_{tt5}	1075.622	P_{tse1}/P_{tre2}	0.901241	T_{tt6}	1062.337	P_{tre1}/P_{tse1}	0.963741461
T_{tm5}	1070.425	T_{tse1}/T_{tre2}	0.97887	T_{tm6}	1057.245	T_{tre1}/T_{tse1}	0.992443387
P_{t5}/P_{tre1}	0.865516	T_{t5}	1026.344	τ_t	0.533998	π_t	0.054099794
$\pi_c\,\pi_b\,\pi_t$	1.775972	T_{fan}	521.97	$T_{fanstator}$	443.628	π_{stator}	0.5505
		T_{rotor1}	545.38	$T_{stator1}$	638.3		

$F/(m_0a_0)$ *0.763996* **F/m_0** *225.0168 N* **S** *26.64775 mg/(N.s)*

η_p *0.844224* **η_T** *0.627358* **I** *3825.344sec*

M_0	T_0	γ_c	C_{pc}	γ_t	C_{pt}	h_{pr}	π_d	π_b	π_n	η_m
2	216	1.4	1004	1.3	1239	42800000	0.925	1	0.98	1
T_{t4}	π_c	τ_f	η_b	e_f	e_c	η_{tstage}	R_c	Rt	P_0	**Press. Recov.**
1922	32.8277	1.2	1	0.9	0.91635	0.89	286.8571	285.9231	12112	**55%**
a_0	V_0	M_6	ε_{rf}	ε_{sf}	ε_{r1}	ε_{s1}	ε_{r2}	ε_{s2}	ε_{r3}	ε_{s3}
294.52606	589.0521	0.6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
π_{r1}	π_{r2}	π_{r3}	P_{t4}	π_{s3e}	π_{r3e}	π_{s2e}	π_{r2e}	T_{tse3}	T_{tre3}	T_{tse2}
3.5	2.8	2.2	2877742	0.7084	0.8101	0.6874	0.6668	1155	931.3173	921.89
T_{tre2}	T_{tse1}	T_{tre1}	π_{s1}	π_{s2}	π_{s3}	τ_{cen}	η_{cen}	π_{tr1e}	π_{ts1e}	
688.45	674.23	664.81	0.95	0.95	0.95	1.678	0.3087	1.9257	0.5709	

Turbofan with Aspirated Fan & 3 stage Core Compressor - Aspirated air from All Stages returned to the turbine

τ_r	1.8	π_r	7.824449	P_{t16}/P_{t6}	1.000008	τ_λ	10.98088203
π_f	1.77591	α'	3.554862	M_{16}	0.580016	τ_c	2.970008943
η_f	0.891598	C_{p6A}	1055.593	$\phi(M_6,\gamma_6)$	0.176072	η_c	0.868784
α	3.7305	R_{6A}	286.6521	$\phi(M_{16},\gamma_{16})$	0.165937	f	0.028419549
P_{t9}/P_9	12.37924	γ_{6A}	1.372788	Φ	0.180358	τ_t	0.534294978
P_0/P_9	1	T_{t16}/T_6	0.454332	M_{6A}	0.638639	π_t	0.054097459
M_9	2.290774	τ_m	0.594946	A_{16}/A_6	2.37748	η_t	0.928629529
V_9/a_0	2.772557	T_{t9}	610.9585	$MFP(M_6)$	0.033071	$MFP(M_{6A})$	0.035003261
T_{t2}	388.8	T_9/T_0	1.582697	π_M	0.982777	π_{cen}	1.944766323
T_{tt1}	1600.489	T_{tt2}	1393.295	T_{tt3}	1355.033	T_{tt4}	1085.026335
T_{tm1}	1593.697	T_{tm2}	1386.834	T_{tm3}	1348.943	T_{tm4}	1079.798809
T_{tse3}/T_{t4}	0.83272	T_{tre3}/T_{tse3}	0.874253	T_{tse2}/T_{tre3}	0.977069	T_{tre2}/T_{tse2}	0.80435299
P_{tse3}/P_{t4}	0.410126	P_{tre3}/P_{tse3}	0.519801	P_{tse2}/P_{tre3}	0.893197	P_{tre2}/P_{tse2}	0.346440002
T_{tt5}	1056.983	P_{tse1}/P_{tre2}	0.901241	T_{tt6}	1044.019	P_{tre1}/P_{tse1}	0.963741461
T_{tm5}	1051.969	T_{tse1}/T_{tre2}	0.97887	T_{tm6}	1039.106	T_{tre1}/T_{tse1}	0.992443387
P_{t5}/P_{tre1}	0.944159	T_{t5}	1026.915	τ_t	0.534295	π_t	0.054097459
$\pi_c\,\pi_b\,\pi_t$	1.775895	T_{fan}	521.97	$T_{fanstator}$	443.628	π_{stator}	0.5505
		T_{rotor1}	545.38	$T_{stator1}$	638.3		

$F/(m_0a_0)$	*0.764935*	**F/m_0**	*225.2934 N*	**S**	*26.66623 mg/(N.s)*
η_P	*0.844075*	η_T	*0.627007*	**I**	*3822.692sec*

M_0	T_0	γ_c	C_{pc}	γ_t	C_{pt}	h_{pr}	π_d	π_b	π_n	η_m
2	216	1.4	1004	1.3	1239	42800000	0.925	1	0.98	1
T_{t4}	π_c	τ_f	η_b	e_f	e_c	η_{tstage}	R_c	Rt	P_0	**Press. Recov.**
1922	32.8277	1.2	1	0.9	0.91635	0.89	286.8571	285.9231	12112	**52%**
a_0	V_0	M_6	ε_{rf}	ε_{sf}	ε_{r1}	ε_{s1}	ε_{r2}	ε_{s2}	ε_{r3}	ε_{s3}
294.52606	589.0521	0.6	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
π_{r1}	π_{r2}	π_{r3}	P_{t4}	π_{s3e}	π_{r3e}	π_{s2e}	π_{r2e}	T_{tse3}	T_{tre3}	T_{tse2}
3.5	2.8	2.2	2877742	0.7084	0.8101	0.6874	0.6668	1155	931.3173	921.89
T_{tre2}	T_{tse1}	T_{tre1}	π_{s1}	π_{s2}	π_{s3}	τ_{cen}	η_{cen}	π_{tr1e}	π_{ts1e}	
688.45	674.23	664.81	0.95	0.95	0.95	1.678	0.3087	1.9257	0.5709	

Turbofan with Aspirated Fan & 3 stage Core Compressor - Aspirated air from All Stages returned to the turbine

τ_r	1.8	π_r	7.824449	P_{t16}/P_{t6}	1.000022	τ_λ	10.98088203
π_f	1.77591	α'	3.549145	M_{16}	0.580035	τ_c	2.970008943
η_f	0.891598	C_{p6A}	1055.658	$\phi(M_6,\gamma_6)$	0.176072	η_c	0.868784
α	3.7245	R_{6A}	286.6518	$\phi(M_{16},\gamma_{16})$	0.165941	f	0.028419549
P_{t9}/P_9	12.37886	γ_{6A}	1.372756	Φ	0.180382	τ_t	0.534490819
P_0/P_9	1	T_{t16}/T_6	0.454165	M_{6A}	0.638747	π_t	0.054096691
M_9	2.290755	τ_m	0.59499	A_{16}/A_6	2.37314	η_t	0.928207436
V_9/a_0	2.773157	T_{t9}	611.2282	$MFP(M_6)$	0.033071	$MFP(M_{6A})$	0.035006193
T_{t2}	388.8	T_9/T_0	1.583407	π_M	0.982761	π_{cen}	1.944766323
T_{tt1}	1582.157	T_{tt2}	1377.432	T_{tt3}	1339.69	T_{tt4}	1072.809391
T_{tm1}	1575.552	T_{tm2}	1371.132	T_{tm3}	1333.754	T_{tm4}	1067.703014
T_{tse3}/T_{t4}	0.823183	T_{tre3}/T_{tse3}	0.874253	T_{tse2}/T_{tre3}	0.977069	T_{tre2}/T_{tse2}	0.80435299
P_{tse3}/P_{t4}	0.387756	P_{tre3}/P_{tse3}	0.519801	P_{tse2}/P_{tre3}	0.893197	P_{tre2}/P_{tse2}	0.346440002
T_{tt5}	1045.142	P_{tse1}/P_{tre2}	0.901241	T_{tt6}	1032.384	P_{tre1}/P_{tse1}	0.963741461
T_{tm5}	1040.245	T_{tse1}/T_{tre2}	0.97887	T_{tm6}	1027.584	T_{tre1}/T_{tse1}	0.992443387
P_{t5}/P_{tre1}	0.998615	T_{t5}	1027.291	τ_t	0.534491	π_t	0.054096691
$\pi_c\ \pi_b\ \pi_t$	1.77587	T_{fan}	521.97	$T_{fanstator}$	443.628	π_{stator}	0.5505
		T_{rotor1}	545.38	$T_{stator1}$	638.3		

$\mathbf{F/(m_0a_0)}$	*0.765558*	$\mathbf{F/m_0}$	*225.4769 N*	**S**	*26.67837 mg/(N.s)*
η_p	*0.843977*	η_T	*0.626777*	**I**	*3820.953sec*