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Abstract— **We apply relaxation procedures for polynomial optimization problems to shipboard distribution system design, and obtain new convex formulations for the AC case.**

I. INTRODUCTION

Transmission system planning is a network design problem in which lines are selected from a candidate set to meet certain physical requirements while minimizing investment and operational costs [5], [7]. Linearized or 'DC' power flow is a standard simplification of [9], since the AC flow is too complicated a representation for optimization. In network design problems where the existence of a line may be a variable, however, even linearized power flow creates a nonlinear, and further, non-convex problem. Additionally, in some applications like a ship electrical system, resistive characteristics are no longer negligible compared to reactances because of short lines, and the DC simplification is invalid for any purpose.

These difficulties are traditionally handled via linear approximations, namely the so called transportation and disjunctive models [7]; only recently has the AC problem been approached in full [6]. Here we present a new linear model for AC transmission planning, and apply it to several benchmark design cases, and to a smaller, representative shipboard distribution system. We also briefly describe the design problem for multiple operating scenarios.

Our objective is to develop and apply new algorithms for power distribution design problems, that will improve accuracy and efficiency across several application areas. We utilize 'lift-and-project' relaxation procedures in our development, specifically that of [8]. A relaxation is an approximation to an optimization problem which always bounds the minimum below (or maximum above), and is typically easier to solve than the original problem. The phrase lift-and-project refers to lifting an optimization problem to a higher dimensional space via the introduction of new variables, and then projecting the lifted problems solution back onto the original variables. Relaxed solutions are often suboptimal or infeasible for the original problem, but can contain a significant portion of the true optimal solution, thus reducing the size of the original problem, which may be intractable when approached directly. Through examples, we show that our formulation achieves very good accuracy with low computational cost.

Numerical results are given on a standard terrestrial test system and a shipboard distribution system.

II. TRANSMISSION SYSTEM PLANNING MODELS

There has been little work to date on transmission system planning using AC power flow. A notable recent approach is [6], in which a full AC model is solved by an interior point method in tandem with a constructive heuristic algorithm. We now derive linear models for AC transmission system planning which are similar in structure and size to existing linear models models [7].

We are given the following problem parameters: line investment vector c, a vector of real and reactive generation and demand limits $\underline{p}, \overline{p}, \underline{q},$ and $\overline{q}$, normalized flow limits $\overline{s}$, existing network ξ^0, and line construction limits $\overline{\xi}$. Let Γ denote the set of buses, Ω_0 the set of existing lines, and Ω the set of candidate lines. We follow the notational conventions that unless otherwise specified, single subscripts denote members of Γ, double subscripts members of Ω, and $i \sim j$ summation over $\Omega_0 \cup \Omega$. Let s, v, and y respectively denote complex powers, voltages and admittances. The basic AC power flow model is given by

$$
\begin{aligned}
\text{NLAC} \quad & \min_{\xi,s,v} \quad \sum_{i \sim j} c_{ij}\xi_{ij} \\
\text{s.t.} \quad & s_{ij} = \left(\xi_{ij}^0 + \xi_{ij}\right)\left(v_i v_i^* y_{ij}^* - v_i v_j^* y_{ij}^*\right) \\
& \underline{p}_i \leq \text{Re}\sum_j s_{ij} \leq \overline{p}_i \\
& \underline{q}_i \leq \text{Im}\sum_j s_{ij} \leq \overline{q}_i \\
& \underline{v}_i \leq |v_i| \leq \overline{v}_i \\
& |s_{ij}| \leq \left(\xi_{ij}^0 + \xi_{ij}\right)\overline{s}_{ij} \quad (i,j) \in \Omega_0 \cup \Omega \\
& 0 \leq \xi_{ij} \leq \overline{\xi}_{ij}, \quad \xi_{ij} \in \mathbb{N}
\end{aligned}
$$

Note that although line variables and parameters are non-directional, i.e. $\xi_{ij} = \xi_{ji}$, $\overline{s}_{ij} = \overline{s}_{ji}$ and so on, sending and receiving power flows s_{ij} and s_{ji} are not.

A. Linear AC models

We first must rewrite NLAC in terms of real, polynomial constraints so we may begin to build a relaxation. Let $y = g+jb$, $v = w+jx$, $s = p+jq$, and let $b^s = b+b^{sh}$, where b^{sh} is

the line shunt susceptance. Applying the relaxation procedure of [8], NLACS is then is given by

$$\text{NLACS} \quad \min_{\xi,p,q,w,x} \quad \sum_{i\sim j} c_{ij}\xi_{ij}$$

$$\text{s.t.} \quad p_{ij} = \left(\xi_{ij}^0 + \xi_{ij}\right)\left(b_{ij}(w_j x_i - w_i x_j)\right.$$
$$\left. -g_{ij}(x_i x_j + w_i w_j) + g_{ij}(w_i^2 + x_i^2)\right)$$
$$q_{ij} = \left(\xi_{ij}^0 + \xi_{ij}\right)\left(g_{ij}(w_j x_i - w_i x_j)\right.$$
$$\left. +b_{ij}(x_i x_j + w_i w_j) - b_{ij}^s(w_i^2 + x_i^2)\right)$$
$$\underline{p}_i \le \sum_j p_{ij} \le \overline{p}_i$$
$$\underline{q}_i \le \sum_j q_{ij} \le \overline{q}_i$$
$$\underline{v}_i^2 \le w_i^2 + x_i^2 \le \overline{v}_i^2$$
$$\sqrt{p_{ij}^2 + q_{ij}^2} \le \left(\xi_{ij}^0 + \xi_{ij}\right)\overline{s}_{ij}$$
$$(i,j) \in \Omega_0 \cup \Omega$$
$$0 \le \xi_{ij} \le \overline{\xi}_{ij}, \quad \xi_{ij} \in \mathbb{N}$$

The line capacity constraint represents a slight obstacle: although it can be expressed polynomially, fourth order products of voltage variables must be included, rendering the size of the resulting relaxation impractically large. We instead approximate them so that p and q are involved linearly; a few options are apparent. To keep the formulation general, we introduce the constants τ^1 and τ^2 and replace (1) with

$$\tau_{ij}^1 |p_{ij}| + \tau_{ij}^2 |q_{ij}| \le \left(\xi_{ij}^0 + \xi_{ij}\right)\overline{s}_{ij}$$

Although we have only used a single constraint in our approximation, any piecewise linear approximation may be used.

Define the new variables:

$$\alpha_i = w_i^2 + x_i^2$$
$$\delta_{ij} = \xi_{ij}\left(w_i^2 + x_i^2\right)$$
$$\mu_{ij} = b_{ij}(w_j x_i - w_i x_j) - g_{ij}(x_i x_j + w_i w_j)$$
$$+g_{ij}(w_i^2 + x_i^2)$$
$$\nu_{ij} = g_{ij}(w_j x_i - w_i x_j) + b_{ij}(x_i x_j + w_i w_j)$$
$$-b_{ij}^s(w_i^2 + x_i^2)$$
$$\phi_{ij} = \xi_{ij}\left(b_{ij}(w_j x_i - w_i x_j) - g_{ij}(x_i x_j + w_i w_j)\right.$$
$$\left. +g_{ij}(w_i^2 + x_i^2)\right)$$
$$\psi_{ij} = \xi_{ij}\left(g_{ij}(w_j x_i - w_i x_j) + b_{ij}(x_i x_j + w_i w_j)\right.$$
$$\left. -b_{ij}^s(w_i^2 + x_i^2)\right)$$

These new variables have implicit constraints given by

$$g_{ij}(\mu_{ij} - \mu_{ji}) - b_{ij}(\nu_{ij} - \nu_{ji})$$
$$= \left(g_{ij}^2 + b_{ij}b_{ij}^s\right)(\alpha_i - \alpha_j)$$
$$b_{ij}(\mu_{ij} + \mu_{ji}) + g_{ij}(\nu_{ij} + \nu_{ji})$$
$$= \left(g_{ij}b_{ij} - g_{ij}b_{ij}^s\right)(\alpha_i + \alpha_j)$$
$$g_{ij}(\phi_{ij} - \phi_{ji}) - b_{ij}(\psi_{ij} - \psi_{ji})$$
$$= \left(g_{ij}^2 + b_{ij}b_{ij}^s\right)(\delta_{ij} - \delta_{ji})$$
$$b_{ij}(\phi_{ij} + \phi_{ji}) + g_{ij}(\psi_{ij} + \psi_{ji})$$
$$= \left(g_{ij}b_{ij} - g_{ij}b_{ij}^s\right)(\delta_{ij} - \delta_{ji})$$

Let Φ denote the set on which the variables μ, ν, ϕ, ψ, α, and δ satisfy these equalities. Forming constraints containing up to second-order terms and substituting the new variables, we have

$$\text{LAC} \quad \min_{\xi,\mu,\nu,\phi,\psi,\alpha,\delta} \quad \sum_{i\sim j} c_{ij}\xi_{ij}$$

$$\text{s.t.} \quad \{\mu, \nu, \phi, \psi, \alpha, \delta\} \in \Phi$$
$$\underline{p}_i \le \sum_j \xi_{ij}^0 \mu_{ij} + \phi_{ij} \le \overline{p}_i$$
$$\underline{q}_i \le \sum_j \xi_{ij}^0 \nu_{ij} + \psi_{ij} \le \overline{q}_i$$
$$\underline{v}_i^2 \le \alpha_i \le \overline{v}_i^2$$
$$\underline{v}_i^2 \xi_{ij} \le \delta_{ij} \le \overline{v}_i^2 \xi_{ij}$$
$$\underline{v}_i^2\left(\overline{\xi}_{ij} - \xi_{ij}\right) \le \overline{\xi}_{ij}\alpha_i - \delta_{ij} \le \overline{v}_i^2\left(\overline{\xi}_{ij} - \xi_{ij}\right)$$
$$\tau_{ij}^1 |\mu_{ij}| + \tau_{ij}^2 |\nu_{ij}| \le \overline{s}_{ij} \quad (i,j) \in \Omega_0$$
$$\tau_{ij}^1 |\phi_{ij}| + \tau_{ij}^2 |\psi_{ij}| \le \overline{s}_{ij}\xi_{ij}$$
$$\tau_{ij}^1 \left|\overline{\xi}_{ij}\mu_{ij} - \phi_{ij}\right| + \tau_{ij}^2 \left|\overline{\xi}_{ij}\nu_{ij} - \psi_{ij}\right|$$
$$\le \overline{s}_{ij}\left(\overline{\xi}_{ij} - \xi_{ij}\right) \quad (i,j) \in \Omega_0$$
$$0 \le \xi_{ij} \le \overline{\xi}_{ij}, \quad \xi_{ij} \in \mathbb{N}$$

B. DC systems

Thus far we have exclusively focused on AC systems. DC distribution systems are also under consideration for the electric ship [2], and give rise to an analogous transmission planning problem; however, in a DC system, assuming prescribed bus voltages v, the basic model is entirely linear without modification. Although it is straightforward, we include it here for completeness. Let f be a line current, and let r be line resistance. For conciseness we assume current generation and loading, but note that buses with prescribed power generation or consumption can be accommodated without sacrificing linearity. The DC distribution design model is given by

$$\text{DC} \quad \min_{\xi,f} \quad \sum_{i\sim j} c_{ij}\xi_{ij} \tag{1}$$

$$\text{s.t.} \quad \sum_{j:i\sim j} f_{ij} = p_i \tag{2}$$
$$r_{ij}f_{ij} - (\xi_{ij}^0 + \xi_{ij})(v_i - v_j) = 0$$
$$|f_{ij}| \le (\xi_{ij}^0 + \xi_{ij})\overline{f}_{ij} \tag{3}$$
$$0 \le \xi_{ij} \le \overline{\xi}_{ij}, \quad \xi_{ij} \in \mathbb{N} \tag{4}$$

III. Applications

A. Multiple scenarios

Unlike a terrestrial system, a ship may encounter multiple highly different sets of loads, each occurring independently, for example traveling at high speed and combat. A conservative approach would be to simply optimize with each load bus consuming the maximum power over all scenarios; this however, may lead to highly conservative designs.

We can instead produce designs that are not overly conservative by creating constraints and variables for each scenario, and optimizing the same objective. Suppose we are given n_S

scenarios, and for each scenario $k = 1, ..., n_S$ we have a set of minimum and maximum bus power levels at each bus i, $\underline{p}_i^k$, $\overline{p}_i^k$, $\underline{q}_i^k$, and $\overline{q}_i^k$.

Using a separate set of variables for each set of power levels, we have the following multiple scenario transmission planning problem:

$$\text{MSAC} \quad \min_{\xi, s, v} \quad \sum_{i \sim j} c_{ij} \xi_{ij}$$

$$\text{s.t.} \quad s_{ij}^k = \left(\xi_{ij}^0 + \xi_{ij}\right)\left(v_i^k v_i^{k*} y_{ij}^* - v_i^k v_j^{k*} y_{ij}^*\right) \quad \forall k$$

$$\underline{p}_i^k \leq \text{Re} \sum_j s_{ij}^k \leq \overline{p}_i^k \quad \forall k$$

$$\underline{q}_i^k \leq \text{Im} \sum_j s_{ij}^k \leq \overline{q}_i^k \quad \forall k$$

$$\underline{v}_i \leq \left|v_i^k\right| \leq \overline{v}_i \quad \forall k$$

$$\left|s_{ij}^k\right| \leq \left(\xi_{ij}^0 + \xi_{ij}\right)\overline{s}_{ij} \quad (i,j) \in \Omega_0 \cup \Omega, \forall k$$

$$0 \leq \xi_{ij} \leq \overline{\xi}_{ij}, \quad \xi_{ij} \in \mathbb{N}$$

From here, relaxations may be developed by identically applying the procedures of the previous section.

B. Computational examples

In this section we compare the performance of our models to existing approaches. Mixed integer linear programs were solved using the modeling language AMPL [3] and solver CPLEX [1].

1) Terrestrial system: We demonstrate LAC on two example systems from [6], which are AC versions of the Garver's six bus system [4] and a Brazilian system. Tables I, and II show the objective value and solution reported for the nonlinear approach in [6] (NL) and obtained by the linear model LAC with $\tau^2 = 1$ and $\tau^2 = 0$ for all $(i, j) \in \Omega$. Running times in seconds are reported for each linear model on the two larger systems. In the 'line additions' section of each table, the left column indicates which line a given row corresponds to, and the other columns how many additions to that line were made by each algorithm; lines not listed where changed by none of the algorithms. Note that we do not consider reactive power source allocation, and so our solutions for the latter two examples correspond to slightly different scenarios than those in [6].

TABLE I

GARVER'S SIX BUS SYSTEM

Model	NL [6]	LAC, $\tau^2 = 1$
Obj.	260, 160	190, 80
Line additions without existing network		
1 - 5	1	1
2 - 3	1	2
2 - 6	3	1
3 - 5	2	2
4 - 6	3	2
Line additions to existing network		
2 - 6	2	2
3 - 5	2	1
4 - 6	2	0

TABLE II

BRAZILIAN SYSTEM

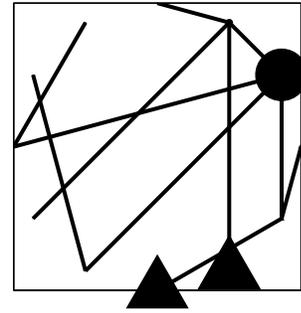| Model	NL [6]	LAC, $\tau^2 = 1$	LAC, $\tau^2 = 0$
Obj.$\times 10^{-9}$	10.258	10.800	8.254
Time	-	7.4	3.3
Line additions			
5 - 6	2	2	0
5 - 11	0	0	2
6 - 46	1	1	0
11 - 46	0	0	1
12 - 14	0	1	0
14 - 26	0	1	0
19 - 25	1	0	0
20 - 21	1	2	2
20 - 23	0	2	1
24 - 25	1	0	0
28 - 31	0	1	1
31 - 32	0	1	1
42 - 43	1	2	2
42 - 44	1	0	0

Fig. 1.   Shipboard layout

For each system, the linear model solutions are reasonably similar to the nonlinear ones (which are not necessarily optimal). By setting $\tau^2 = 1$, a more conservative solution is obtained, which can in fact have a higher objective than the nonlinear solutions, whereas setting $\tau^2 = 0$ yields relaxed solutions in considerably less time.

2) Shipboard power system: Lastly we consider an 12-bus idealized shipboard distribution system with a main generator (circle), two propulsive loads (triangles), and smaller loads distributed concentrically around the diagram in Fig. 1.

We can see that without a reliability objective, the optimal network is a tree. To increase robustness, one might, for example, include constraints of the form $\sum_{j:j \sim i} \xi_{ij}^0 + \xi_{ij} \geq c$ in the formulations given, to ensure that a vital bus i is connected by at least c lines.

IV. CONCLUSION

We have applied a general framework by which to generate relaxations of transmission planning problems without making any physical assumptions. Using this framework, the transmission system planning problem has been adapted to an AC shipboard distribution system context, within which many traditional simplifying assumptions are not valid. The resulting nonconvex constraints are relaxed via lift-and-project proce-

dures, resulting in various mixed-integer conic optimization models. The models are further tuned to shipboard applications by the inclusion of multiple load scenarios in the formulation. Examples were given demonstrating the behavior of each of the models on a terrestrial and shipboard example. On the terrestrial systems, similar results to far more computationally expensive approaches are achieved in drastically less time.

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