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Firm Characteristics and Stock Returns: The Role of Investment-Specific Shocks

Leonid Kogan* Dimitris Papanikolaou†

Abstract

We provide a unified explanation for several apparent anomalies in the cross-section of asset returns, namely the failure of the CAPM to account for the cross-sectional relation between average stock returns and firm valuation ratios, past investment, profitability, market beta, or idiosyncratic volatility. Using a calibrated structural model, we argue that these characteristics are imperfect proxies for the share of growth opportunities to firm value, which determines firms' exposures to capital-embodied shocks, and risk premia. Return differences among firms sorted on the above characteristics are largely driven by the same systematic factor related to embodied technology shocks.

*MIT Sloan School of Management and NBER, lkogan@mit.edu

†Kellogg School of Management and NBER, d-papanikolaou@kellogg.northwestern.edu

1 Introduction

Recent empirical work has identified a number of firm characteristics that forecast future stock returns. For instance, firms with higher investment rates (IK), Tobin's Q , price to earnings (PE), idiosyncratic volatility (IVOL) and market beta (MBETA) earn abnormally low returns. There is strong evidence of comovement in stock returns of firms with similar characteristics – even across industries – that is unrelated to their exposures to the market portfolio. These empirical patterns are often termed ‘anomalies’, based on the failure of existing models to rationalize both the dispersion in risk premia and the comovement in returns resulting from sorting firms on the above characteristics. We argue that these five firm characteristics are informative about the cross-section of stock returns because they are related to firms’ growth opportunities.¹ We then provide a unified explanation for empirical return patterns associated with these firm characteristics by extending the theoretical model of Kogan and Papanikolaou (2012b).²

We start by documenting that the empirical return patterns generated by sorting firms on each of the five characteristics are closely related to each other. Specifically, firm portfolios formed on each of these characteristics share a common factor structure. After removing their exposure to the market portfolio, not only do high-IK firms comove with other high-IK firms, but more importantly, they also comove with firms that have high Q , PE, IVOL, and MBETA. Thus, these five firm characteristics are likely correlated with firms’ exposures to the same common risk factor, that is *not* captured by the market. We construct an empirical factor model by extracting the first principal component from the pooled cross-section of portfolio returns, after removing the market

¹The potential for the firm characteristics we consider to be correlated with firms’ growth opportunities is apparent from the existing literature. For instance, firms with more growth opportunities are likely to invest more. Furthermore, such firms are likely to have higher valuation ratios (Tobin's Q , price-earnings ratios) since their market value includes the NPV of future investment projects which is not reflected in book values or current earnings. In addition, most real options models imply that growth opportunities have higher market betas than assets in place. Finally, the literature has informally connected growth opportunities to the firms’ idiosyncratic risk, appealing to the intuition that there is more uncertainty about firms’ growth opportunities than their assets in place (see, e.g., Myers and Majluf, 1984; Bartram, Brown, and Stulz, 2011).

²Kogan and Papanikolaou show theoretically that firms deriving most of their value from growth opportunities – rather than existing assets – face a higher exposure to technological shocks embodied in new capital goods. Papanikolaou (2011) argues that technological shocks are priced and carry a negative premium; the marginal value of wealth is higher in states with good real investment opportunities.

component. This constructed return factor, along with the market portfolio, captures a significant amount of variation in realized portfolio returns and cross-sectional differences in risk premia among the characteristic-sorted portfolios.

We show that the return risk factor we uncover is related to investment-specific technology (IST) shocks. IST shocks capture the idea that technological change is embodied in new productive capital. Using proxies for the IST shock based on real variables and stock returns, we document that, i) the five characteristics we study are related to future IST risk exposures, even controlling for lagged empirical estimates of risk; ii) heterogeneity in IST risk exposures accounts for a significant fraction of the differences in average returns associated with the five characteristics; iii) firms with characteristics associated with high growth opportunities exhibit higher sensitivity of firm investment to IST shocks; and iv) following a positive IST shock, such firms experience higher output growth. These findings suggest that heterogeneity in firms' exposures to IST shocks provides a unified explanation for the empirical patterns associated with the five firm characteristics we analyze.

To assess whether our proposed explanation is quantitatively plausible, we calibrate a structural model based on Kogan and Papanikolaou (2012b). The model replicates the above empirical patterns with a common set of structural parameters. It generates empirically realistic average return spreads between firms with high and low Tobin's Q, investment rates, earnings-to-price ratios, market betas, and idiosyncratic volatility. Further, it replicates the return comovement among these portfolios and the resulting failure of the CAPM to price the portfolio returns. In addition, our model reproduces the common empirical finding that firm characteristics contain additional information about risk premia relative to risk exposures estimated using stock return data.

Our model differs from Kogan and Papanikolaou (2012b) along two significant dimensions. First, we allow firms' growth opportunities to be imperfectly observable, formalizing the intuition that investors have higher uncertainty about the future growth prospects of the firm than its existing operations. The revelation of information about firms' future growth opportunities contributes to their idiosyncratic return variation. This mechanism is necessary to replicate the negative relation between idiosyncratic return uncertainty and future stock returns, and the pattern of return

comovement of firms with similar levels of idiosyncratic volatility. All else equal, firms with more growth opportunities are likely to have higher idiosyncratic volatility of returns, and therefore lower risk premia.

Second, we disentangle the profitability of existing assets from the firm’s growth opportunities by eliminating the correlation between the idiosyncratic profitability of current and future projects in the model of Kogan and Papanikolaou (2012b). This modification emphasizes an important conceptual difference between our theoretical framework and the neoclassical model of the firm. In the neoclassical model, investment simply scales the size of the existing firm. Hence, the profitability of existing assets is highly correlated with firms’ growth opportunities. By contrast, in our setting firms grow by acquiring heterogeneous projects; thus, current profitability and growth opportunities are imperfectly related. As a result, marginal and average Tobin’s Q are different; the return to new investment is distinct from the return of the firm; and the discount rate used to value new projects is distinct from the risk premium of the firm. Though subtle, these conceptual differences lead to different empirical predictions. This modification to the model in Kogan and Papanikolaou (2012b) allows us to simultaneously match the negative relation between future stock returns and investment or price-earnings ratios.³

To further illustrate the differences between our theoretical framework and the neoclassical model, we consider the ‘profitability premium’ of Novy-Marx (2012), who documents that profitable firms experience higher future stock returns than unprofitable firms. Moreover, he finds that controlling for profitability increases the performance of ‘value’ strategies. Novy-Marx (2012) argues that both of these patterns present a challenge to existing explanations of the value premium that combine neoclassical production functions with operating leverage and adjustment costs (e.g. Zhang, 2005). In these models, highly profitable firms have lower systematic risk than unprofitable firms, leading to lower risk premia. In contrast with such models, our model is consistent with both of the patterns documented in Novy-Marx (2012). In our model, firms with highly profitable projects derive most of their market value from existing assets rather than growth opportunities, hence, those firms have

³Absent this modification to the model of Kogan and Papanikolaou (2012b), the relation between risk premia and investment or price-earnings ratios is flat. Please see Table A.20 in the internet appendix for more details.

lower exposure to IST shocks and higher average stock returns. Further, because valuation ratios such as Tobin's Q are affected by profitability of assets in place, controlling for firm profitability strengthens the relation between Q and growth opportunities. Using our baseline calibration, we show that our model quantitatively replicates the findings of Novy-Marx (2012). Further, differences in IST risk exposures largely account for the differences in average returns among profitability portfolios, or of value strategies controlling for profitability.

Our work provides a unified explanation for several empirical patterns extensively documented in the literature, including the relation between average returns and: price-earnings ratios (Rosenberg, Reid, and Lanstein, 1985; Basu, 1977; Haugen and Baker, 1996); market-to-book ratios and Tobin's Q (Fama and French, 1992; Lakonishok, Shleifer, and Vishny, 1994); investment rates (Titman, Wei, and Xie, 2004; Anderson and Garcia-Feijo, 2006); profitability (Fama and French, 2006; Novy-Marx, 2012); idiosyncratic return volatility (Ang, Hodrick, Xing, and Zhang, 2006, 2009); as well as the fact that the security market line is weakly downward sloping (Black, Jensen, and Scholes, 1972; Frazzini and Pedersen, 2010; Baker, Bradley, and Wurgler, 2011; Hong and Sraer, 2012).

Previous work has argued that the profitability of trading strategies based on valuation ratios is largely mechanical (e.g. Ball, 1978; Berk, 1995). The argument is that, controlling for expected growth in cash flows, firms with lower risk premia have higher valuations. Hence, valuation ratios help identify variation in risk premia in the cross-section of firms. This argument is agnostic about the exact economic source of cross-sectional differences in risk premia.

The strong patterns of return comovement among firms with similar characteristics have been interpreted as evidence that observed cross-sectional differences in average stock returns are due to differences in systematic risk exposures (e.g. Fama and French, 1993).⁴ However, the economic origins of these empirical return factors are yet to be fully understood. The ICAPM (Merton, 1973) or APT (Ross, 1976) are typically invoked as theoretical justifications for empirical multifactor models. However, these studies do not address why firm characteristics are correlated with return exposures to the empirical risk factors. We contribute to this literature by showing that IST shocks

⁴Specifically, zero-net-investment strategies using the extreme characteristic-sorted portfolios typically account for a significant share of the realized return variation and cross-sectional differences in average returns across the sorted portfolios (see e.g. Fama and French, 1993; Chen, Novy-Marx, and Zhang, 2010; Novy-Marx, 2012).

give rise to a systematic return factor related to several prominent firm characteristics related to growth opportunities.

Investment-specific technical change is a key ingredient in our framework. Papanikolaou (2011) demonstrates that in a general equilibrium model with a representative agent, IST shocks are positively correlated with the stochastic discount factor – implying a negative price of risk for IST shocks – if the elasticity of intertemporal substitution is lower than the reciprocal of risk aversion. Kogan, Papanikolaou, and Stoffman (2012) study capital embodied shocks in an economy with heterogeneous agents and incomplete markets and find similar results under much weaker preference parameter restrictions. Kogan and Papanikolaou (2012b) study the relation between growth opportunities and IST shocks in partial equilibrium. Li (2011) and Yang (2011) study the link between IST shocks and momentum and the commodity basis spread, respectively.

Our work is related more broadly to models with production that aim to link average returns to firm characteristics.⁵ The main insight from this literature is that firm characteristics and risk exposures are both endogenously related to the state of the firm. Our framework differs from existing models along two important dimensions. First, many of the existing models feature a single aggregate shock, implying that firms’ risk premia are highly correlated with their conditional market betas. Second, in the most of the existing models, growth opportunities and profitability are tightly linked. As a result, these models cannot simultaneously replicate the positive relation between profitability and returns and the negative relation between returns and past investment or valuation ratios.

The difficulty of existing models with production in reproducing the negative relation between market betas or idiosyncratic volatility and future returns has led to several recent candidate explanations based on market frictions (e.g., Frazzini and Pedersen, 2010; Baker et al., 2011; Hong and Sraer, 2012) or incomplete information (e.g. Armstrong, Banerjee, and Corona, 2012). However, these explanations need additional assumptions to generate comovement of firms with similar levels of idiosyncratic return volatility or market beta that is unrelated to market movements. In contrast,

⁵Examples include Berk, Green, and Naik (1999); Gomes, Kogan, and Zhang (2003); Carlson, Fisher, and Giammarino (2004, 2006); Zhang (2005); Bazdrech, Belo, and Lin (2012). See Kogan and Papanikolaou (2012a) for a recent survey of the related literature.

our calibrated model replicates the patterns in both average returns and systematic risk across portfolios of firms sorted on market betas and idiosyncratic return volatility jointly with similar empirical patterns based on firm investment, profitability and valuation ratios in an environment with frictionless financial markets.

2 Empirical findings

We study five empirical patterns in stock returns that have received considerable attention in the literature. Specifically, we focus on the negative relation between future stock returns and market beta, past investment, idiosyncratic volatility and valuation ratios such as Tobin's Q and price to earnings. We select these particular empirical patterns based on our hypothesis that the above characteristics are correlated with firms' growth opportunities in the cross-section, which should help explain the observed differences in average stock returns and return comovement among the portfolios of firms with similar characteristics.

We begin by summarizing the relevant empirical evidence on the relations between the above firm characteristics and stock returns. Then, we show that the seemingly puzzling return patterns arising from sorting firms on each of these characteristics share a common risk-based explanation. First, we show that a single common return factor extracted from the pooled cross-section of characteristics-sorted portfolios – along with the market portfolio – prices this cross-section, and that this common factor is related to IST shocks. Second, we show that differences in IST risk exposure among firms account for a substantial fraction of cross-sectional variation of risk premia and CAPM pricing errors across the characteristic-sorted portfolios. Third, we show that these five characteristics predict future IST risk exposures, even controlling for lagged empirical estimates of such exposures.

2.1 Firm characteristics and risk premia

We begin by summarizing the empirical patterns documented in the literature regarding the link between asset returns and the characteristics we consider. We describe the details on the construction

of these characteristics in Appendix A. To be consistent with the model of Kogan and Papanikolaou (2012b), we omit firms in the investment sector. Doing so does not materially affect our analysis, since the investment sector is small; investment firms account for between 15% to 20% of the total market capitalization in the 1964 to 2008 period.

Table 1 reports the return spread between the top and bottom decile portfolio of firms sorted on these characteristics. There is a declining pattern of average returns across the characteristics-sorted portfolios; the difference in average returns ranges from -2% for the high-MBETA minus low-MBETA portfolios to -8.9% for the high- minus low-PE portfolios. Further, for each one of these five characteristics, the top decile portfolios have higher market betas than the bottom decile portfolios. As a result, the CAPM severely misprices these portfolios; the CAPM alphas of the top minus bottom decile portfolios range from -5.7% for the portfolios sorted on market beta to -10.8% for the portfolios sorted on idiosyncratic volatility.

2.2 Firm characteristics and return comovement

A key piece of the puzzle is that firms with similar characteristics comove with each other. As we see in Table 1, the portfolios formed by going long the top decile and short the bottom decile are substantially volatile; the annual standard deviations range from 19.7% to 37.1%. Yet, their market exposure does not fully account for their systematic risk; the CAPM R^2 ranges from 6.6% for the Q -sort to 25.9% for the MBETA-sort. Hence, these diversified long-short portfolios have exposure to systematic sources of risk that is not fully captured by the market portfolio. This pattern is particularly striking for the MBETA-sort; grouping firms based on their market exposures results in portfolios that have systematic risk that is *not* spanned by the market portfolio. Thus, a firm's market exposure is cross-sectionally related to its exposure to a systematic risk factor *unrelated* to the market. This pattern is not driven by variation in leverage across these portfolios.⁶

Our first result is that exposure to a common risk factor accounts for a substantial fraction of this comovement across the five characteristic sorts. To isolate this second source of systematic risk,

⁶We compute the median book leverage within each decile portfolio. We find that leverage is, if anything, negatively related to MBETA, IVOL, P/E, I/K and Q; see Table A.1 in the internet appendix for more details.

we first remove the effect of the market factor by constructing residuals from a market model. We normalize the residuals to unit standard deviation, and extract the first principal component from these residuals in each of the five cross-sections. To study comovement *across* the characteristic sorts, we extract the first principal component from a pooled cross-section of twenty portfolios that includes portfolios 1, 2, 9, and 10 from each sort.⁷

The top panel of Table 2 summarizes the degree of return comovement. As we see, there is substantial comovement of firms with similar characteristics, consistent with our findings above. For each characteristic, the normalized eigenvalue associated with the first principal component ranges from 31% to 52%. These results illustrate the common finding in the literature that there are return factors associated with each firm characteristic. The existence of these return factors is often interpreted as an indication that the CAPM alphas associated with various firm characteristics could be generated by the exposure of firms to some source of systematic risk missing from the single-factor market model (see e.g. Fama and French, 1993).

The more striking evidence in the top panel of Table 2 is that there is substantial comovement *among* these IK-, PE-, Q-, MBETA-, and IVOL-factors. The first principal component *PC1* extracted from the pooled cross-section of extreme decile portfolios is essentially the average of long-short portfolios across the IK, PE, Q, MBETA, and IVOL sorts, as we see in Table 3. The correlation between each individual factor and the first principal component of the pooled cross-section ranges from 47% for idiosyncratic volatility to 92% for investment. Hence, not only do high-IK firms comove more with other high-IK firms, but these firms also comove with high PE, Q, MBETA and to some extent high IVOL firms.⁸ The magnitude of this common source of return variation is substantial: the normalized eigenvalue associated with the first principal component from the pooled cross-section of twenty portfolios is 33%. This pattern suggests that the missing risk factor is largely common across the five characteristic sorts.

⁷As a robustness check, we have repeated the same analysis using monthly returns and the entire cross-section of fifty portfolios; results are similar.

⁸This result is not driven by the same stocks being ranked similarly using each of the above characteristics – correlations among portfolio assignments using various characteristics are low; pairwise correlations range from 11.3% to 38.1%, with the exception of the Tobin’s *Q* and price to earnings ratio pair which is 63%. Please see Table A.4 in the internet appendix for more details.

Importantly, we find that the common factor in these characteristic-sorted portfolio returns is closely related to IST shocks. We focus on two measures of capital-embodied technical change. The first measure of IST shocks is constructed from the change Δz_t^I in the detrended quality-adjusted relative price of new capital goods; see Kogan and Papanikolaou (2012b) for more details. Our second measure relies on the relative stock returns of investment and consumption producers (IMC); see Papanikolaou (2011) for the details of this classification procedure. We compute correlations between *PC1* and the measures of the IST shock in the bottom panel of Table 2. The common factor extracted from the pooled cross-section has correlation 69% with the IMC portfolio and 38% with the price of equipment shock Δz^I . Finally, this common factor is highly correlated (-82%) with the HML portfolio of Fama and French (1993).

Our findings indicate the presence of a common source of return variation across the portfolios sorted on these five characteristics related to growth opportunities. Since only two of the five characteristics involve market prices, this comovement cannot be mechanically attributed to movements in market prices.⁹ In addition, this pattern is mainly driven by within-industry variation in characteristics, hence it cannot be attributed to industry-specific factors.¹⁰ Further, this finding is not driven by small firms.¹¹ Last, our results are robust to focusing on industries that are more likely to be capital intensive.¹²

⁹Daniel and Titman (1997) argue that the comovement of firms with similar book-to-market ratios could be spurious; firms that end up in the growth portfolio are likely to have exposure to the same risk factors because the sorting characteristic is based on market prices. However, this argument does not explain why firms with similar investment rates, market betas and idiosyncratic volatility comove with each other, nor why firms in the top quintile of characteristic G_i comove with firms in the top quintile of characteristic $G_{j \neq i}$.

¹⁰We repeat our portfolio sorts within the Fama and French (1997) 17-industry classifications. We find that our results on comovement and dispersion in risk premia are similar or stronger for all the considered characteristics, with the exception of IVOL. Sorting firms on IVOL within industries produces a significantly smaller spread in average returns (2.2%) and CAPM alphas (4.7%) relative to the unconditional sort. In addition, the first principal component extracted from the cross-section of within-industry sorted firms is very weakly correlated with the other cross-sections. These results suggest that there is substantial intra-industry variation in idiosyncratic volatility that is not related to firms' growth opportunities. See Tables A.22 to A.24 in the internet appendix.

¹¹We repeat our analysis after eliminating the bottom 20% of firms in terms of market capitalization every year. We find that our results are similar – and in some cases stronger – on this sub-sample, and thus unlikely to be driven by the smallest firms. See Tables A.25 to A.27 in the internet appendix.

¹²We repeat our empirical analysis excluding the firms that produce services (industries 14-17 according to the Fama and French (1997) 17-industry classification scheme). We find that our results are somewhat stronger in this subsample, see Tables A.28 to A.30 in the internet appendix.

2.3 Risk premia and IST-risk exposures

The common return factor $PC1$ extracted from the market residuals of the characteristic sorted portfolios earns a negative risk premium. The annualized Sharpe ratio is equal to -0.51. Absence of arbitrage implies that portfolios loading positively on this factor must earn relatively low average returns.

First, we verify that this return factor prices the returns on each set of portfolios. We use a two-factor model including the market portfolio and $PC1$:

$$R_{pt} - r_{ft} = \alpha_p + \beta_{mkt,p}(R_{mkt,t} - r_{ft}) + \beta_{z,p}R_{pc1,t} + \varepsilon_{pt} \quad (1)$$

As we show in Table 4 and Figure 1, the two-factor model (1) captures the spreads in average returns in the cross-sections sorted by Q, P/E, I/K, IVOL, and MBETA. The estimates of α are small across the decile portfolios and the Gibbons, Ross, and Shanken (1989) (GRS) test p-values are greater than 10% in each cross-section. However, one limitation of this test is that it focuses on the pricing properties of the constructed return factor rather than on its economic source. Thus, we next explore whether dispersion in measures of IST risk is equally successful in accounting for the dispersion in risk premia across the characteristic-sorted portfolios.

We estimate the stochastic discount factor (SDF) in the model of Kogan and Papanikolaou (2012b)

$$m = a - \gamma_x \Delta x - \gamma_z \Delta z. \quad (2)$$

We normalize Δx and Δz to unit standard deviation; hence γ_x and γ_z can be interpreted as the Sharpe ratio of a test asset perfectly correlated with Δx and Δz respectively. We estimate (2) using the generalized method of moments (GMM). We use the model pricing errors as moment restrictions

$$E[R_i^e] = -cov(m, R_i^e), \quad (3)$$

where R_i^e denotes the excess return of portfolio i over the risk-free rate.¹³ As test assets, we use

¹³See Cochrane (2005) for details.

decile portfolios 1, 2, 9 and 10 from each of the five cross-sections. We report first-stage GMM estimates using the identity matrix to weigh moment restrictions, and adjust the standard errors using the Newey-West procedure with a maximum of three lags. As a measure of fit, we report the sum of squared errors (SSQE) and the mean absolute pricing error (MAPE) from the Euler equations (3).

We proxy for IST shocks with the changes in the detrended relative price of new equipment, Δz^I .¹⁴ For the neutral technology shock x , we use the change in the log total factor productivity in the consumption sector from Basu, Fernald, and Kimball (2006). We also consider specifications of the SDF based on portfolio returns. In particular, we use two-factor specifications with the market portfolio and either the IMC portfolio, or the HML portfolio; both of these two-factor models span the same linear subspace as the two technology shocks x and z in the model of Kogan and Papanikolaou (2012b).

As we see in Table 5, cross-sectional differences in IST risk exposures among the test portfolios account for a sizable portion of the differences in their average returns. Column (1) shows that the specification with only the disembodied shock x produces large pricing errors (4.23%), similar to the CAPM (3.61%). In contrast, adding the equipment-price shock as a proxy for the IST shock in column (2) reduces the pricing errors to 0.92%. Furthermore, adding IMC or HML portfolio returns to the market return in columns (4) and (5) reduces the pricing errors to 0.75% and 1.40% respectively.

The estimated market price of the IST shock in column (2) is negative, $\hat{\gamma}_z = -1.35$, and statistically significant, implying a negative relation between average returns on the characteristic-sorted portfolios and their IST shock exposures. Using the IMC or HML portfolio to proxy for the IST risk – columns (4) and (5) – leads to lower estimates of $\hat{\gamma}_z$, equal to -0.71 and -0.65 respectively. These point estimates are higher in magnitude than the estimates in Kogan and Papanikolaou (2012b), but the difference is not statistically significant.

In Figure 1 we compare the performance of the CAPM, the empirical factor model (MKT and

¹⁴We also consider an additional empirical proxy for IST shocks – changes in the aggregate investment-to-consumption ratio (as in Kogan and Papanikolaou (2012b)). We find that using this proxy for IST shocks leads to similar empirical findings.

PC1) and two specifications of the SDF; first, the one including real variables (Δx and Δz^I), and second, one including portfolio returns (MKT and IMC). As we see, the last three models do a comparable job pricing the cross-section of 50 characteristic sorted portfolios.

2.4 Characteristics and IST-risk exposures

Here, we provide further evidence that the five characteristics that we study are related to risk premia because they proxy for the firm’s exposure to IST shocks. Grouping firms into portfolios entails an information loss (Ang, Liu, and Schwarz, 2010); hence, here, we perform tests using individual stocks. In general, the major difficulty in disentangling a direct effect of characteristics on average stock returns versus an indirect effect through risk exposures is that risk exposures are often a function of characteristics. To address this issue, researchers have proposed several methodologies that incorporate information from firm characteristics in the measurement of conditional risk exposures (see e.g. Ferson and Harvey, 1991, 1999; Shanken, 1990; Daniel and Titman, 1997; Lewellen, 1999; Avramov and Chordia, 2006).

In our case, IST risk exposures depend on the ratio of growth opportunities to firm value, $PVGO/V$, which varies over time depending on the current state of the firm. Since the real proxies for the IST shock are only available at low frequencies, we cannot estimate time-varying firm-level exposures to IST shocks directly, using regressions of stock returns on Δz^I . Hence, we use IMC portfolio as a factor-mimicking portfolio for IST shocks and use IMC-betas estimated at higher frequencies in combination with firm characteristics. Our approach therefore combines the methodology of Ferson and Harvey (1999) with the mixed data sampling methodology of Ghysels, Santa-Clara, and Valkanov (2005).

We first form conditional IST risk exposures by combining the information in the firm characteristics related to growth opportunities G_{ft} with direct measures of IMC betas using only stock returns

$$BIMC_{ft} = \alpha_t + bG_{ft-1} + \rho BIMC_{ft-1} + u_{ft}, \quad (4)$$

where $G \in \{Q, IK, EP, MBETA, IVOL\}$ denotes the set of characteristics we use to instrument for

the conditional IMC beta, and *BIMC* refers to estimates of IMC-beta using weekly stock returns (see Appendix A for details). Here, we use the earnings-price rather than the price-earnings ratio to minimize the effect of outliers arising from firms with earnings close to zero. We include a year-fixed effect a_t and cluster the standard errors by firm and year.

We find that firm characteristics are informative about future IST risk exposures, even controlling for lagged estimates of IMC beta estimated using stock return data alone. As we see in Table 6, each one of the five characteristics has predictive power for the firms' future exposures to IST shocks. The last column of Table 6 shows that in a joint regression almost all firm characteristics are still significant predictors of future IMC betas, with the exception of the price-earnings ratio whose predictive power is subsumed by the other variables.

The additional information content of characteristics arises for two reasons, both quite general. First, IMC betas are measured with error; if the measurement error is i.i.d. over time, firm characteristics that are correlated with the true betas are informative, even controlling for the statistical beta estimates. Second, IMC betas change over time. Hence past IMC betas are not sufficient statistics for the future IST exposures even if they were measured without error. Certain firm characteristics can help estimate changes in IMC betas because they are measured as the end-of-period value in year t , whereas covariances are measured using data over the entire year t . Hence, characteristics may contain more up-to-date information about growth opportunities (and hence IST exposure) than realized return covariances.¹⁵

Next, we evaluate the extent to which the predictive relation between future stock returns and these five firm characteristics is driven by the fact that these characteristics proxy for IST shock exposures. To minimize look-ahead bias we conduct our analysis using a split sample. Specifically, we first estimate (4) using the first half of the sample (1963-1985). We use the specification including all five characteristics and the lagged IMC-betas. Using the point estimates from the first half of the sample, we construct predicted IMC-betas in the second half of the sample (1986-2008). We then estimate Fama-MacBeth regressions in the second half of the sample, using the cross-section of individual firms, with the right-hand-side variables being firm characteristics and the forecasts of

¹⁵Berk et al. (1999) make similar points in the context of a structural model.

future IMC betas

$$R_{ft} = \alpha_t + bG_{ft-1} + \gamma E_{t-1}[BIMC_{ft}] + \varepsilon_{ft}. \quad (5)$$

Table 7 shows that controlling for IST risk exposure significantly weakens the ability of these five characteristics to predict returns. In each panel *a* to *e*, in the first column we replicate the predictive relation between the five firm characteristics and average returns. With the exception of market beta, each of the remaining four characteristics are statistically significantly related to future stock returns. Including the market beta in the specification – as we see from the second column in panels *b* to *e* – does not significantly affect the predictive ability of characteristics. Consistent with the findings in the literature (see e.g. Daniel and Titman, 1997), not only does the CAPM fail, but variation in market betas unrelated to the included characteristics is uncorrelated with future returns.¹⁶ In the last column of panel *a*, we see that after controlling for IST risk exposures, variation in market beta is associated with a positive risk premium. This suggests that IST shocks represent an important missing risk factor in the univariate relation between market betas and average returns. Last, in panels *b* to *e*, controlling for IST risk exposures substantially reduces the predictive power of firm characteristics; the only characteristic that retains a statistically significant predictive ability is the earnings-to-price ratio, but the point estimates are reduced by a factor of two.

2.5 Characteristics and growth opportunities

Our empirical analysis above links five firm characteristics to IST risk exposures. Further, it shows that their relation to IST risk exposures accounts to a large extent for the ability of these characteristics to predict future stock returns. Next, we directly test our starting hypothesis that the five firm characteristics we consider are correlated with firms' richness in growth opportunities.

Firms' growth opportunities are not observable directly, hence we rely on indirect tests. Firms with more growth opportunities are better positioned to take advantage of positive IST shocks. Hence, we expect that firms with more growth opportunities should, first, increase investment by a

¹⁶Including a predicted market beta estimated in a similar fashion as equation (4) leads to negative and statistically significant coefficients on the CAPM beta. See Table A.6 in the internet appendix.

larger amount following a positive IST shock; and, second, experience an increase in output growth as they acquire more capital relative to firms with few growth opportunities.¹⁷

Characteristics and the response of investment to IST shocks

First, we compare the response of firm investment to IST shocks as a function of the characteristics proxying for growth opportunities. Following Kogan and Papanikolaou (2012b), we estimate the following specification

$$i_{ft} = b_1 \Delta z_{t-1} + \sum_{d=2}^5 b_d D(G_{f,t-1})_d \Delta z_{t-1} + \rho i_{t-1} + \gamma X_{ft-1} + u_t, \quad (6)$$

where i_t is the firm's investment rate; $\Delta z \in \{\Delta z^I, R^{imc}\}$ is the measure of the IST shock.¹⁸ The dummy variable $D(G_f)_d$ takes the value one if the firm's characteristic $G_f \in \{Q_f, IK_f, PE_f, MBETA_f, IVOL_f\}$ belongs to the quintile d in year $t-1$. We standardize all right-hand side variables to zero mean and unit standard deviation. We account for unobservable time and firm effects by clustering standard errors by firm and year (see Petersen (2009)). The vector X includes the dummy variables $D(G_f)$ and industry fixed effects. We show the results in Table 8; the coefficient of interest is b_5 which captures the differential response of investment to the IST shock across firms in the top and bottom quintile in terms of the five characteristics.

We find that firms in the top quintile in terms of Q; investment; price-earnings; market beta; and idiosyncratic volatility generally exhibit investment behavior that is more sensitive to our two proxies for the IST shock, compared to firms in the bottom quintile. In nine out of the ten specifications, the coefficient b_5 is statistically significant. Further, the economic magnitude is substantial. The point estimates imply that a single-standard-deviation IST shock leads to an increase in the investment to capital ratio of firms in the top quintile by 0.3% to 1.2%, relative to

¹⁷The analysis in this section is related to Kogan and Papanikolaou (2012b), who document that investment of firms with similar market-to-book ratios or IMC-betas exhibit correlated investment responses to IST shocks. Here, we extend the same analysis to all five characteristics. Further, we extend their analysis to study the response of future output growth to IST shocks.

¹⁸Using the common risk factor in the cross-section of characteristic portfolios ($PC1$) as proxy for the IST shock leads to quantitatively similar results. See Table A.9 in the internet appendix.

firms in the bottom quintile; for comparison, the median investment rate in our sample is 10.6%. This dispersion in investment responses conditional on G_f suggests that these firm characteristics are indeed related to firms' investment opportunities, consistent with our conjecture.

Characteristics and the response of firm output to IST shocks

Second, we examine whether firms in the top quintiles in terms of these five characteristics experience an acceleration in output growth in response to a positive IST shock. Motivated by the theoretical model of Kogan and Papanikolaou (2012b), we estimate the response of output growth to the IST shock using the following specification:

$$y_{f,t+k} - y_{ft} = b_1 \Delta z_t + \sum_{d=2}^5 b_d D(G_{ft})_d \Delta z_t + \rho y_{ft} + \gamma X_{ft-1} + u_{f,t+k}, \quad (7)$$

where $y_{f,t}$ is log firm output, defined as firm sales (sale) plus change in inventories (invt), scaled by average output across firms to ensure stationarity; the vector X of controls includes growth-quintile and industry dummies.¹⁹ We cluster standard errors by firm and by year. We estimate equation (7) for horizons of 1 to 6 years.

As before, we focus on the coefficient $b_5(k)$, which captures the differential impact of an IST shock on k -period output growth between firms in the top (G_5) and the bottom (G_1) quintile. In Figure 3, we plot the estimated coefficients along with the 90% confidence intervals. In nine out of the ten IST-shock / characteristic combinations, a positive IST shock is associated with an increase in output growth of firms in the top quintile relative to firms in the bottom quintile. This differential response of future revenue suggests that these new investments firms make in response to IST shock indeed lead to higher future revenue, and lends further support to our view that these five firm characteristics are related to growth opportunities.

¹⁹Our choice of specification is guided by the theoretical model of Kogan and Papanikolaou (2012b): we scale firm output by average output across firms to absorb the effect of disembodied productivity shocks and ensure that the variable is stationary; we include one lag of output level because relative firm profitability is mean-reverting.

2.6 Relation to existing theories

Here, we relate our findings to existing structural models of firm returns. The first generation builds on real option models. The common theme in these models is that the firm’s exposure to systematic shocks depends on its asset composition. Growth opportunities are a levered claim on the firm’s assets in place and therefore earn higher risk premia (e.g. Gomes et al., 2003). The introduction of operating leverage alters this relation, but the results are highly sensitive to the nature of capital adjustment costs (e.g. Carlson et al., 2004; Zhang, 2005; Hackbarth and Johnson, 2012). More problematic for our purposes, these models imply that returns have a conditional one-factor structure, hence the market portfolio should absorb the comovement associated with characteristics. In the data, it does not.

Models with multiple sources of risk are a promising alternative. Berk et al. (1999) features two aggregate risks, shocks to productivity and interest rates. Relatedly, Lettau and Wachter (2007) and Santos and Veronesi (2010) feature exogenous dividend dynamics that are exposed to cash flow and discount rate risk. These models generate comovement of firms with similar cash-flow duration and risk exposures. Since growth opportunities likely have higher duration than assets in place, these models could be consistent with our results on portfolio returns comovement. We study a different economic mechanism; hence we connect this comovement in returns, investment and output to empirical measures of IST shocks constructed using the price of equipment and IMC portfolio returns.²⁰

Our findings in this section suggest that these five ‘anomalies’ documented in the empirical literature are *qualitatively* consistent with the model of Kogan and Papanikolaou (2012b). However, as a framework to assess whether their mechanism is a quantitatively plausible explanation for these patterns, their model suffers from two shortcomings. First, the profitability of existing assets is strongly related to the marginal return on investment. Hence, price-earnings ratios and investment

²⁰In general equilibrium, the price of equipment could endogenously respond to preference shocks or changes in risk. However, this model would likely generate several counterfactual implications. For instance, a reduction in discount rates represents a positive demand shocks to capital and thus leads to a higher price and quantity of investment. In the data, the two are negatively related (see Greenwood, Hercowitz, and Krusell, 1997, and also our findings in Section 2.5). Further, an increase in the price of equipment would be positively correlated with a portfolio long high-growth and short low-growth firms; in the data, the opposite is true (see Section 2.2).

is uninformative about the share of growth opportunities in firm value or risk premia (see Table A.20 in the internet appendix). Second, there is no link between idiosyncratic volatility and growth opportunities. Next, we extend their model to address both of these issues.

3 The Model

In this section we explore whether an extended version of the model of Kogan and Papanikolaou (2012b) can *quantitatively* replicate the evidence in Section 2 using a common set of structural parameters. We introduce two modifications to their model. First, to weaken the relation between profitability and investment opportunities, we eliminate the firm-specific shock. This modification amplifies the difference between our framework and the neoclassical setup. Second, to link firm's idiosyncratic volatility to growth opportunities, we introduce uncertainty about the firm's growth opportunities. To conserve space, we briefly describe the main elements of the model, and refer the reader to Kogan and Papanikolaou (2012b) for more details.

3.1 Production and investment

There are two sectors of production, a sector producing consumption goods and a sector producing investment goods. Both sectors feature a continuum of measure one of infinitely lived competitive firms financed only by equity. We focus on the sector producing consumption goods; we use the investment-goods sector to illustrate the connection between the IMC portfolio and the IST shock in the model.

Assets in Place

Each consumption firm owns a finite number of individual projects. We index individual firms by $f \in [0, 1]$. We denote the set of projects owned by firm f at time t by \mathcal{J}_{ft} . Project j produces a flow of output equal to

$$y_{fjt} = u_{jt} x_t K_j^\alpha, \tag{8}$$

where K_j is physical capital chosen irreversibly at the project j 's inception date, u_{jt} is the project-specific component of productivity, and x_t is the disembodied productivity shock affecting output of all existing projects. There are decreasing returns to scale at the project level, $\alpha \in (0, 1)$. Projects expire independently at rate δ .

The project-specific component of productivity u and the the disembodied shock x evolve according to

$$du_{jt} = \theta_u(1 - u_{jt}) dt + \sigma_u \sqrt{u_{jt}} dB_{jt}, \quad (9)$$

$$dx_t = \mu_x x_t dt + \sigma_x x_t dB_{xt}, \quad (10)$$

where dB_{jt} and dB_{xt} are independent standard Brownian motions.

New projects

Consumption firms acquire new projects according to a Poisson count process N_{ft} with a firm-specific arrival rate λ_{ft} . The firm-specific arrival rate of new projects has two components, $\lambda_{ft} = \lambda_f \cdot \tilde{\lambda}_{f,t}$. The first component of firm arrival rate λ_f is constant over time and determines the size of the firm in the long-run. The second component $\tilde{\lambda}_{f,t}$ captures the current state of the firm in terms of investment opportunities. We assume that $\tilde{\lambda}_{f,t}$ follows a two-state, continuous-time Markov process $\tilde{\lambda}_{f,t} \in [\lambda_L, \lambda_H]$ with instantaneous transition probabilities μ_L and μ_H . Without loss of generality, we impose the normalization $E[\tilde{\lambda}_{f,t}] = 1$.

Faced with a new project at time t , firms make a take-it-or-leave-it decision; firms choose the scale of investment K_j and buy capital at a price p^I . Given (8), it is optimal for the firm to always choose a positive amount of investment K_j when it acquires a new project. At the time of investment, the project-specific component of productivity is at its long-run average value, $u_{jt} = 1$. The price of investment goods is equal to $p_t^I = z_t^{-1} x_t$, where

$$dz_t = \mu_z z_t dt + \sigma_z z_t dB_{zt}, \quad (11)$$

and $dB_{zt} \cdot dB_{xt} = 0$.

The z shock is the embodied, investment-specific shock in our model. A positive change in z reduces the cost of new capital goods and thus leads to an improvement in investment opportunities.

Investment Sector

The investment firms produce the demanded quantity of capital goods at the current unit price p_t^I , and have a constant profit margin ϕ .

3.2 Learning

An important feature that distinguishes our model from Kogan and Papanikolaou (2012b) is that the firms' ability to acquire new investment opportunities λ_{ft} is not observable. Market participants observe a long history of the economy, hence they know the firm-specific long-run mean λ_f . However, investors do not observe whether the firm is currently in the high-growth ($\tilde{\lambda}_{ft} = \lambda_H$) or low-growth ($\tilde{\lambda}_{ft} = \lambda_L$) phase. Thus, we model $\tilde{\lambda}_{ft}$ as an unobservable, latent process.

The market learns about the firm's growth opportunities through two channels. First, market participants observe a noisy public signal e_{ft} of λ_{ft} ,

$$de_{ft} = \lambda_{ft} dt + \sigma_e dZ_{ft}^e. \quad (12)$$

Second, the market updates its beliefs about λ_{ft} by observing the firm's investment decisions. Since the firm always finds it optimal to invest when acquiring a project, it is sufficient to observe the cumulative number of projects undertaken by the firm.

We derive the evolution of the probability p_{ft} that the firm is in the high growth state $\lambda_{ft} = \lambda_f \lambda_H$ using standard results on filtering for point processes (see, e.g. Liptser and Shiryaev, 2001),

$$dp_{ft} = \left((1 - p_{ft})\mu_H - p_{ft}\mu_L \right) dt + p_{ft} \left(\lambda_f \lambda_H - \hat{\lambda}_{ft} \right) \left(dM_{ft} + h_e d\tilde{Z}_{ft}^e \right), \quad (13)$$

where $h_e \equiv \sigma_e^{-1}$ is the precision of the public signal and

$$\hat{\lambda}_{ft} \equiv p_{ft}\lambda_f \lambda_H + (1 - p_{ft})\lambda_f \lambda_L \quad (14)$$

is the market's unbiased estimate of the arrival rate of the firm's investment opportunities. The investor updates his belief about $\hat{\lambda}_{ft}$ based on the two martingale processes \tilde{Z}^e and M ,

$$d\tilde{Z}_{ft}^e = h_e \left(de_{ft} - \hat{\lambda}_{ft} dt \right), \quad (15)$$

$$dM_{ft} = \bar{\lambda}_{ft}^{-1} \left(dN_{ft} - \hat{\lambda}_{ft} dt \right), \quad (16)$$

The market learns about λ_{ft} using the demeaned public signal \tilde{Z}_{ft}^e , adjusting upwards its beliefs about the firm's growth opportunities λ_{ft} whenever the firm invests, $dN_{ft} = 1$. Since stock returns contain news about firm investment opportunities, de_{ft} , this mechanism implies that stock returns will predict investment in the model, even though the productivity of new projects u is unrelated to the profitability of existing projects. Further, the information revealed in the public signal (12) increases the idiosyncratic volatility of the firm. Hence, firms with more growth opportunities are likely to have higher idiosyncratic volatility.

3.3 Valuation

We denote the stochastic discount factor as π_t . For simplicity, we assume that the two aggregate shocks x_t and z_t have constant prices of risk, γ_x and γ_z respectively. The risk-free interest rate r_f is also constant. Then,

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \gamma_x dB_{xt} - \gamma_z dB_{zt}. \quad (17)$$

The stochastic discount factor 17 is motivated by the general equilibrium model with IST shocks in Papanikolaou (2011). IST shocks endogenously affect the representative household's consumption stream, and hence they are priced in equilibrium.

The market value of a firm is the sum of the value of its existing projects and the value of its future growth opportunities. Following the standard convention, we call the first component of firm value *the value of assets in place*, VAP_{ft} , and the second component *the present value of growth*

opportunities, $PVGO_{ft}$. The value of a firm's assets in place is the value of its existing projects

$$VAP_{ft} = \sum_{j \in \mathcal{J}_{ft}} p(u_{jt}, x_t, K_j) = x_t \sum_{j \in \mathcal{J}_{ft}} A(u_{jt}) K_j^\alpha. \quad (18)$$

where $p(u_{jt}, x_t, K_j) = A(u) x_t K_j^\alpha$ is the market value of an existing project and $A(u)$ is a function. The value of assets in place is independent of the IST shock z and loads only on the disembodied shock x .

The present value of growth opportunities equals the expected discounted NPV of future investments

$$PVGO_{ft} = E_t \left[\int_t^\infty \frac{\pi_s}{\pi_t} (\lambda_{fs} NPV_t) ds \right] = z_t^{\frac{\alpha}{1-\alpha}} x_t (G_L + p_{ft} (G_H - G_L)), \quad (19)$$

where G_H , G_L and ρ are constants, and NPV denotes the net present value of new investments

$$NPV_t \equiv \max_{K_j} p(1, x_t, K_j) - p_t^I K_j. \quad (20)$$

The present value of growth opportunities depends positively on aggregate productivity x and the IST shock z , because the latter affects the profitability of new projects.

Adding the two pieces (18) and (19), the total value of the firm is equal to

$$V_{ft} = x_t \sum_{j \in \mathcal{J}_{ft}} A(u_{jt}) K_j^\alpha + z_t^{\frac{\alpha}{1-\alpha}} x_t (G_L + p_{ft} (G_H - G_L)). \quad (21)$$

Next, we illustrate the mechanism generating return comovement and cross-sectional dispersion in risk premia in our model. Examining equation (21), we see that all firms have the same sensitivity to the disembodied productivity shock, $\beta_{ft}^x = 1$. In contrast, the firm's stock return sensitivity to the IST shock z is a function of the ratio of firm growth opportunities to firm value, $PVGO/V$

$$\beta_{ft}^z = \frac{\alpha}{1-\alpha} \frac{PVGO_{ft}}{V_{ft}}. \quad (22)$$

The fact that the firm sensitivities to x and z are not perfectly correlated in the cross-section implies that realized stock returns have a conditional two-factor structure, giving rise to comovement among firms with similar levels of $PVGO/V$.²¹

The firm's risk premium is determined by the share of growth opportunities in firm value

$$\frac{1}{dt}\mathbf{E}_t[R_{ft}] - r_f = \gamma_x\sigma_x + \frac{\alpha}{1-\alpha}\gamma_z\sigma_z \frac{PVGO_{ft}}{V_{ft}}. \quad (23)$$

Whether a firm's expected return is increasing or decreasing in the share of growth opportunities in firm value depends on the sign of the risk premium γ_z of the IST shock. A negative sign for γ_z is consistent with the empirical evidence in Section 2 and the general equilibrium models of Papanikolaou (2011) and Kogan et al. (2012).

The fundamental variable summarizing firm heterogeneity in IST risk exposures – and thus risk premia – is the ratio of growth opportunities to firm value $PVGO/V$. This ratio evolves endogenously as a function of the market's belief's about the likelihood of acquiring new projects $\hat{\lambda}_{ft}$; the history of project arrival and expiration; and the level of idiosyncratic productivity u . Even though this ratio is unobservable, it is related to several observable firm characteristics. Next, we explore this link in more detail.

3.4 Growth Opportunities and Firm Characteristics

A key advantage of a structural model is that it allows for an explicit connection between firm characteristic and the fundamental variables summarizing the state of the firm, here the ratio of growth opportunities to firm value. This connection is particularly valuable in our case, since the theoretical link between certain characteristics and growth opportunities can be subtle. For instance, in the model of Carlson et al. (2004), firm investment is negatively related to the share of growth opportunities in firm value.

Here, we derive the explicit relation between several firm characteristics and the ratio of growth

²¹The presence of two aggregate shocks is a necessary, but not sufficient, condition for a conditional two-factor structure in returns. An additional requirement is that firm exposures to these two shocks are not perfectly cross-sectionally correlated. For instance, if it were the case that $\forall f, \beta_{ft}^z = a_t \beta_{ft}^x$, then the single factor $F = a_t dB_{zt} + dB_{xt}$ would be a sufficient statistic for all systematic risk.

opportunities to firm value $PVGO/V$ in our structural model.

IMC beta

We begin by reiterating the result in Kogan and Papanikolaou (2012b), namely that a firm's stock return beta with the portfolio long firms producing investment goods and short firms producing consumption goods (IMC portfolio) is related to the share of growth opportunities in firm value

$$\beta_{ft}^{imc} = \beta_{0t}^{-1} \left(\frac{PVGO_{ft}}{V_{ft}} \right), \quad (24)$$

where $\beta_{0t} \equiv \left(\int_0^1 VAP_{ft} df \right) / \left(\int_0^1 V_{ft} df \right)$ is a term that depends on the fraction of aggregate value that is due to growth opportunities in the consumption sector, which affects the IMC portfolio's beta with respect to the z -shock. As we saw in equation (22), a firm's IST risk exposure is a function of the share of growth opportunities in firm value. Since the IST shock is spanned by the IMC portfolio, firm IMC betas are informative about the share of firm value due to growth opportunities, $PVGO/V$.

Market Beta

The firm's exposure to the market portfolio is an increasing function of the share of growth opportunities in firm value,

$$\beta_{ft}^M = B_{0t} + B_{1t} \frac{PVGO_{ft}}{V_{ft}}, \quad (25)$$

where $B_{0t} > 0$, $B_{1t} > 0$ are functions that only depend on the aggregate state of the economy.

In our model, differences in firm market exposures arise only due to differences in IST risk exposures. Absent any other form of risk heterogeneity in our model, not only the CAPM fails, but the relation between market beta and risk premia has the same sign as the price of IST shocks, γ_z , which we estimate to be negative.²²

²²In more general settings, for instance in models with operating leverage, firms may have heterogeneous exposure to the disembodied shock x . In this case, market betas will vary for reasons not directly related to the ratio of growth

Investment rate

The firm's investment rate, measured as the ratio of cumulative capital expenditures over an interval Δ

$$INV_{f,t+\Delta} = \int_t^{t+\Delta} p_s^I K^*(z_s) dN_{fs}, \quad (26)$$

to the lagged replacement cost of its capital stock, B_{ft} , is related to the ratio of growth opportunities to firm value through two channels. The first channel leads to a cross-sectional link: firms with more growth opportunities will on average tend to have higher investment rates. The second channel results in the time series relation: once a given firm acquires a project, the market revises upward its estimate of the firm's growth opportunities – see in equation (13) – leading to higher $PVGO/V$. Here, we should emphasize that the discount rate the firm applies to the valuation of new projects is constant and distinct from the discount rate of the firm; hence, this mechanism linking investment to discount rates is conceptually distinct from mechanisms proposed in other studies.²³

Price-to-Earnings

The firm's price to earnings ratio is negatively related to share of growth opportunities to firm value. In the limiting case where the profitability of projects is highly persistent, $\theta_u \ll 1$, current profitability is a good measure of the value of assets in place

$$VAP_{ft} = x_t \sum_{j \in \mathcal{J}_{ft}} a_0 K_j^\alpha + a_1 E_{ft} \approx a_1 E_{ft} \quad \text{if } \theta_u \ll 1, \quad (27)$$

opportunities to firm value and the security market could be flat or even upward sloping. However, even in these settings, CAPM will not hold and the relation between CAPM alphas and CAPM betas will be negative as long as $\gamma_z < 0$.

²³In particular, in Carlson et al. (2004) and Carlson et al. (2006) growth opportunities have higher risk premia than assets in place. Investment converts growth opportunities to assets in place, so following an increase in investment, the same firm has a higher mix of VAP/V and therefore lower risk premia. In our model, the opposite is true, that is, growth opportunities have *lower* risk premia than assets in place, consistent with the empirical evidence on the value premium. In Zhang (2005), operating leverage leads to a negative relation between profitability and systematic risk, as captured by market beta. Consequently, investment – which is increasing in firm profitability – is negatively related to market beta and therefore risk premia.

where $E_{ft} \equiv x_t \sum_{j \in \mathcal{J}_{ft}} u_{jt} K_j^\alpha$. In this case, the price to earnings ratio equals

$$\frac{V_{ft}}{E_{ft}} \approx a_1 \left(1 - \frac{PVGO_{ft}}{V_{ft}} \right)^{-1} \quad \text{if } \theta_u \ll 1, \quad (28)$$

Firms with high earnings to price ratios derive a higher fraction of their value from existing assets. The value of these firms is less sensitive to IST shocks than firms that derive most of their value from growth opportunities.

Idiosyncratic volatility

In our model, the idiosyncratic variance of the firm return is a weighted average of the idiosyncratic variance of assets in place and growth opportunities

$$IVOL_{ft}^2 = C(x_t, \{K_j, u_{jt}\}_{j \in \mathcal{J}_{ft}}) \left(\frac{VAP_{ft}}{V_{ft}} \right)^2 + H(p_{ft}) \left(\frac{PVGO_{ft}}{V_{ft}} \right)^2. \quad (29)$$

The term C captures the uncertainty in firm value due to fluctuations in the project-specific level of productivity and the potential decline in firm value due to expiration of existing projects. The term H captures the uncertainty due to the arrival of information about the firm's growth opportunities, and changes in firm value due to acquisition of new projects.

The sign of the relation between $PVGO/V$ and firm's idiosyncratic return volatility depends on the relative strength of these two terms, C and H . If a firm holds a large portfolio of projects, then the C term is likely to be small since risk at the project level is diversified away. In this case, news about future investment opportunities become the dominant source of idiosyncratic risk, implying that firms with more growth opportunities will have higher idiosyncratic volatility.

Tobin's Q

The firm's average Tobin's Q , defined as the market value of the firm V_f over the replacement cost of its capital stock, $B_{ft} = p_t^I \sum_{j \in \mathcal{J}_{ft}} K_j$, is

$$Q_{ft} \approx a_1 \left(1 - \frac{PVGO_{ft}}{V_{ft}} \right)^{-1} \times \frac{E_{ft}}{B_{ft}} \quad (30)$$

where we used the approximation (27).

A firm's average Tobin's Q is positively related to the ratio of growth opportunities to firm value. However, average Q is a noisy measure of growth opportunities, since it also depends on the profitability of existing projects. Controlling for profitability E/B strengthens the link between Tobin's Q and $PVGO/V$. This relation illustrates the fact that average and marginal Q are different in our model, and this difference plays an important role in replicating the empirical patterns between average returns, profitability and Q as we see in Section 5.

4 Calibration

Here, we explore the ability of the model to quantitatively replicate the empirical relation between firm characteristics, risk premia and risk.

4.1 Parameter choice and calibration

Our model features a total of 18 parameters. Some of these parameters are determined by a priori evidence. In particular, we set the project expiration rate $\delta = 10\%$, to be consistent with commonly used values for the depreciation rate. We set the interest rate to $r_f = 3\%$, which is close to the historical average risk-free rate. We pick the price of risk of the IST shock $\gamma_z = -0.57$ to match the estimate of the price of risk of IST shocks estimated using the cross-section of industry portfolios in Kogan and Papanikolaou (2012b).²⁴ We verify that under this choice, the average return on

²⁴Our choice of $\gamma_z = -0.57$ is lower than the point estimate in Section 2; however, the calibrated parameter value for the price of IST shocks is still within the empirical confidence interval. Further, our choice of γ_z lies between the two estimates of the implied average price of risk of capital-embodied shocks in the general equilibrium models of Papanikolaou (2011) and Kogan et al. (2012), which are equal to -0.1 and -0.9 respectively.

the value factor HML in the calibrated model matches the historical returns on the value factor constructed using consumption-sector firms. We choose the price of the disembodied shock $\gamma_x = 1.77$ to match the historical equity premium.

We select the next set of parameters to approximately match 18 aggregate and firm-specific moments. We choose the parameters governing the dynamics of the two aggregate shocks ($\mu_x = 0.5\%$, $\sigma_x = 7.0\%$) and ($\mu_z = 0.3\%$, $\sigma_z = 4.20\%$) to match the first two moments of the aggregate dividend growth and investment growth. We select the parameters governing project cash flows ($\theta_u = 0.03$, $\sigma_u = 1.25$) to jointly match the serial autocorrelation and the cross-sectional distribution of firm-specific profitability and Tobin's Q. We calibrate the returns-to-scale parameter $\alpha = 0.85$ to match the sensitivity of investment to log Tobin's Q. We choose the profit margin of investment firms $\phi = 0.075$ to match the relative size of the consumption and investment sectors in the data.

We calibrate the dynamics of the stochastic component of the firm-specific arrival rate ($\mu_H = 0.05$, $\mu_L = 0.25$, and $\lambda_H = 5.1$) to match the time-series autocorrelation and cross-sectional dispersion of the firm-specific investment rates. We model the distribution of mean project arrival rates $\lambda_f = E[\lambda_{ft}]$ across firms as a uniform distribution $\lambda_f \sim U[\underline{\lambda}, \bar{\lambda}]$. We calibrate the parameters of the distribution of λ_f ($\underline{\lambda} = 5$, $\bar{\lambda} = 25$) to match the average investment rate and the cross-sectional distribution of the investment rate, Tobin's Q, and firm profitability. We select the parameters governing the precision of the public signal $\sigma_e = 0.15$ to match the correlation between firms' investment and their past stock returns.

We simulate the model at a weekly frequency ($dt = 1/52$) and time-aggregate the data to form annual observations. We estimate the firms' idiosyncratic volatility IVOL and MBETA in simulated data using weekly returns, replicating exactly our empirical procedure. We simulate 1,000 samples of 2,000 firms over a period of 100 years. We drop the first half of each simulated sample to eliminate the dependence on initial values.

In Table 9, we compare the estimated moments in the data to the median moment estimates and the 5th and 95th percentiles in simulated data. In most cases, the median moment estimate of the model is close to the empirical estimate. Further, even though the model features no firm-specific productivity shock as in Kogan and Papanikolaou (2012b), it has no difficulty replicating the

persistence of investment and profitability.

4.2 Results in simulated data

Here, we explore the ability of the model to quantitatively replicate the empirical patterns we document in Section 2. Specifically, we focus on the empirical relation between firm characteristics and risk premia; the pattern of comovement in stock returns, investment and output; and the failure of the CAPM. Focusing on the ability of the model to simultaneously replicate these empirical patterns using a common set of structural parameters allows us to test whether dispersion in IST risk exposures due to dispersion in the share of growth opportunities to firm value provides a quantitatively plausible explanation for the link between average returns and Q, P/E, I/K, MBETA and IVOL.

Firm characteristics, return comovement and risk premia

First, we explore the ability of the model to replicate the empirical relations between firm characteristics and stock returns. We repeat the analysis in Section 2.1 in simulated data from the model. We summarize the results in Table 10. As we see, the model generates differences in risk premia across the decile portfolios that are comparable to the data; the differences in average return between the top and bottom decile portfolio range from -4.8% for the I/K sort to -7.2% for P/E sort. Further, the model reproduces the empirical failure of the CAPM, generating substantial alphas (-6.2% to -11.9%).

In addition, the model replicates the patterns of comovement among the characteristic-sorted portfolio. However, the market portfolio absorbs a larger share of this comovement than in the data; the R^2 in the market model regressions range from 51% to 55% in simulated data, versus 7% to 26% in the data. This pattern occurs because IST risk accounts for a fairly large component of the variance of the market portfolio in the model. Introducing additional sources of risk in the market portfolio unrelated to growth opportunities would further lower the R^2 in market model regressions.

In Figure 2, we summarize the performance of the model in replicating the empirical relation between firm characteristics and average excess returns, CAPM alpha and beta. In both the model

and the data, average excess returns are declining across the Q, I/K, P/E, MBETA, and IVOL sorts; in contrast, CAPM betas are increasing across these five sorts – in both the data and the model – implying that CAPM alphas and betas are cross-sectionally negatively correlated. Comparing the empirical point estimates to the 5-th, 50-th and 95-th percentiles of point estimates across model simulations, we see that the model performs quite well. In most cases, the empirical estimates are within the model confidence intervals.

Additional results

To further investigate the quantitative performance of the model, we replicate additional empirical relations – sections 2.4 and 2.5 – in simulated data. To conserve space, we briefly summarize our findings here and describe the details in the internet appendix.

First, we explore whether it is also the case in the model that firm characteristics contain information about IST risk exposures in addition to estimated IMC-betas. We estimate equation (4) in simulated data and find that characteristics are indeed informative for IST risk exposures, even controlling for lagged empirical estimates (Table A.2 in the internet appendix). Second, we replicate our Fama-McBeth analysis in data generated by the model; the results are comparable to the empirical estimates (Table A.3 in the internet appendix). Third, we compare the magnitude of the response of investment to IST shock between the model and the data; we find quantitatively similar results (Table A.10 in the internet appendix). Last, we verify that the model produces realistic responses in output growth to IST shocks, conditional on firm characteristics; we estimate equation (7) in simulated data and find similar results (Figure A.1 in internet appendix). We conclude that our calibrated model succeeds in replicating the key features of stock returns, investment and output growth.

4.3 Inspecting the mechanism: the role of learning

Our model extends the model of Kogan and Papanikolaou (2012b) to introduce learning about firm growth opportunities. This feature captures the idea that investors have more uncertainty about the firm’s future growth prospects than the profitability of its existing assets. As a result, this learning

channel generates a strong link between idiosyncratic volatility and $PVGO/V$. To illustrate the effect of this mechanism on the relation between characteristics and returns, we simulate a version of the model where growth opportunities are perfectly observable to market participants, leaving all other parameters to their baseline values.

We verify that allowing for learning about growth prospects is crucial in generating a negative relation between the firm's idiosyncratic volatility and risk premia. If firm's growth opportunities are perfectly observable to market participants, then the relation between idiosyncratic volatility and average returns is weakly positive and statistically insignificant. In this case, the project-specific shock u dominates the firms' idiosyncratic return variation, and thus the relation between idiosyncratic volatility and $PVGO/V$ becomes weak. We find that the learning channel does not significantly impact the link between average returns and price-to-earnings, market beta, or Q . The link between investment and average returns is somewhat weaker, since, without learning, investment is not informative about firm's future growth opportunities. See Table A.19 in the internet appendix for more details.

5 Profitability and risk premia

Here, we focus on one aspect of our model that differentiates it from most existing asset pricing models based on the neoclassical model of the firm – the relation between profitability, growth opportunities, and risk premia.

5.1 Profitability and growth opportunities

The relation between profitability, growth opportunities and risk premia is one key aspect that differentiates our setup from the neoclassical model of the firm with adjustment costs. In the neoclassical setting – often referred to as q -theory – investment simply scales the size of the firm. Thus, high profitability of current assets implies high marginal returns to investment, and therefore more growth opportunities and higher Tobin's Q . In that model, profitability, investment, and Tobin's Q are very strongly positively related to each other; further, each one of these characteristics

is a sufficient statistic for the firm’s conditional market beta and thus its risk premium.

The empirical evidence presents a challenge to models with neoclassical production functions. Specifically, the empirical relation between profitability and future stock returns is positive (Novy-Marx, 2012); in contrast, the relation between future stock returns and past investment is negative (Titman et al., 2004; Anderson and Garcia-Feijo, 2006). Structural models with neoclassical production functions predict that these relations should have the same sign. Further, Novy-Marx (2012) documents that, controlling for profitability, the spread in average returns between value and growth firms increases. Hence, a firm’s profitability cannot be a summary statistic for its risk premium.

In our setting, firms invest by acquiring projects that are heterogenous in productivity. Consequently, the profitability of existing projects is not a sufficient statistic for the firm’s investment opportunities; marginal and average Tobin’s Q are quite different. Because of this distinction in the specification of growth opportunities, our model’s predictions for stock return dynamics differ substantially from those of the neoclassical models. In particular, firms with profitable projects (high u) are tend to have low ratios $PVGO/V$, since the value of existing assets accounts for a larger share of firm value. Hence, these firms have higher average returns. Further, as we see in equation (30), controlling for current profitability strengthens the link between Tobin’s Q and $PVGO/V$ in our model, which is qualitatively consistent with the findings of Novy-Marx (2012).

Our calibrated model replicates quantitatively the empirical relation between firm profitability, Tobin’s Q , and investment. In Table 11 we report the median Tobin’s Q and investment rate for each profitability decile portfolio. The patterns in simulated model output (Panel B) are similar to those in the data (Panel A). Both in the model and in the data, Tobin’s Q has a U-shaped relation with profitability, and the median investment rate is mostly flat across profitability portfolios. Hence, even though our model features no direct link between the profitability of assets in place and firms’ investment opportunities, its implications for the relations between profitability and investment or Tobin’s Q are consistent with empirical evidence. Further, the model matches closely the empirical relation between profitability and risk exposures: both in the model and in the data IMC and market betas are declining across the profitability deciles.

5.2 Profitability and value premium

Next, we show that our model can replicate the empirical evidence in Novy-Marx (2012). We form 25 value-weighted portfolio sorted on profitability and Q . First, we sort firms into quintiles based on profitability; then, within each profitability quintile, we sort firms into deciles based on Tobin's Q . We repeat the same procedure in simulated data, and compare the results in Table 12.

Panels *A.i* and *B.i* show that the findings of Novy-Marx (2012) hold in our sample: sorting firms on profitability leads to an increasing pattern in risk premia. The difference in average returns across the top and bottom profitability quintiles ranges from 3.3% to 10.1%; the difference in CAPM alphas ranges from 2.1% to 13.6%. Further, controlling for past profitability, there are substantial differences in average returns and CAPM alphas across the Q -quintiles. Across the profitability quintiles, high- Q firms have returns that are -4.4% to -12.1% lower than those of low- Q firms, and the corresponding differences in CAPM alphas range from -3.5% to -15%. Last, the value premium is weakest among the firms in the top profitability quintile.

Our model reproduces these empirical patterns quantitatively, as we show in panels *A.ii* and *B.ii*. Sorting firms on profitability leads to a positive spread in risk premia (ranging from 0.6% to 6.6%) and CAPM alphas (ranging from 2.3% to 9.7% across the top and bottom quintiles). Further, controlling for profitability leads to large differences across the top and bottom Q -quintile portfolios; the difference in risk premia ranges from -2.1% to -8.1%, while the difference in alphas ranges from -2.5% to -9.9%. As in the data, the value premium in the model is roughly the same across the first four quintiles of profitability, and is much for the fifth quintile, containing the most profitable firms. This pattern arises because the very profitable firms derive most of their value from the value of existing assets; hence controlling for profitability, there is a smaller spread in $PVGO/V$.

5.3 Profitability and IST risk exposure

The empirical evidence on the risk premia of the profitability- Q double-sorted portfolios provide a powerful set of moments that differentiates our setup from existing structural models linking firm characteristics to average returns. Here, we explore the additional restriction of the model, that differences in IST risk exposure largely account for the differences in risk premia for these portfolios.

We repeat the empirical tests in Section 2.3 using the cross-section of 25 profitability- Q portfolios. We summarize the results in Tables 13. We see that including proxies for the IST shock in the stochastic discount factor (Δz^I or R^{ime}) leads to a substantial reduction in the pricing errors relative to the specification with only TFP shocks or the market portfolio; the sum of squared errors drops by about approximately two thirds. In comparison, the specification with the market portfolio and the HML portfolio leads to 50% higher pricing errors than either one of the specifications with IST shocks.

We compare the performance of the empirical two-factor model that includes the market and the common risk factor $PC1$ constructed in Section 2.2 to the pricing errors generated by the specifications for the SDF that includes proxies for the IST shock. Panels B, C and D in Figure 4 show that all three models perform similarly in pricing the cross-section of profitability- Q double-sorted portfolios. In most cases, all three models perform well, with the exception that they all fail to price the lowest-profitability/highest- Q portfolio (P1Q5). We conclude that the first principal component $PC1$ extracted from the cross-section of Q -, P/E-, IK-, MBETA- and IVOL-portfolios contains similar information about systematic risk to our two proxies for the IST shock.

6 Conclusion

We provide a unified explanation for several of the previously documented relations between firm characteristics and average returns, and the patterns of return comovement of firms with similar characteristics. The key idea is that past investment, profitability, valuation ratios, market betas and idiosyncratic volatility are related to the ratio of growth opportunities to firm value. Growth opportunities have a different exposure to technological changes embodied in new capital compared to asset in place. The endogenous relation between firms' characteristics and their IST risk exposures generates stock return comovement among firms with similar characteristics and a predictive relation between firm characteristics and future stock returns. Both of these patterns are quantitatively consistent with the data.

Our work has connections to several broad lines of research. In our setting, risk characteristics of

new investments are distinct from those of the firm. The risk of new investments is identical to that of assets in place, while the risk in firm value also includes the risk of future growth opportunities. This distinction has important implications for how historical asset returns should be used to estimate discount rates for discounted cash-flow analysis.

In our current setting there is no role for financing decisions; we assume that all firms are financed by equity. If we allow firms to issue debt and assume that firm's debt capacity is constrained by its stock of tangible assets, that is, the value of its existing projects, our model would give rise to a nontrivial relation between firms' leverage and their growth opportunities. Our model can therefore be useful in studying the interaction between financial frictions, asset tangibility and firm growth.

Our analysis is focused on the cross-sectional relations between stock returns and firm characteristics. However, the key mechanism – the relation between the share of growth opportunities to value and risk premia – also applies in the time-series dimension. Thus, in addition to its cross-sectional implications, our model also has implications for the joint time-series dynamics of aggregate stock returns and macroeconomic quantities, as well as the dynamic risk-return tradeoff in the aggregate stock returns.

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Appendix A: Data construction

We measure the investment rate as the ratio of capital expenditures (*capx*) to the lagged book value of capital (*ppeg*_t). We define Tobin's *Q* as the ratio of the market value of common equity (CRSP December market capitalization) plus the book value of debt (*dltt*) plus the book value of preferred stock (*pstkrv*) minus inventories (*invt*) and deferred taxes (*txdb*) divided by the book value of capital (*ppeg*_t). We construct the firm's price-to-earnings ratio (PE) as the ratio of the market value of the firm (*mkcap* + *dltt* + *pstkrv* - *txdb*) to operating income (*ib*) plus interest expenses (*xint*). We follow Novy-Marx (2012) and define profitability as the ratio of gross profitability (*gp*) to the book value of assets (*at*).

We estimate the firm's market beta (*MBETA*) and IMC beta (*BIMC*) using weekly returns

$$r_{ftw} = \alpha_{ft} + \beta_{ft}^F r_{tw}^F + \varepsilon_{ftw}, \quad w = 1 \dots 52, \quad (31)$$

where r_{ftw} refers to the log return of firm f in week w of year t , and $r_{ftw}^F \in \{r_{tw}^{mkt}, r_{tw}^{imc}\}$ refers to the log excess return of the market, or IMC portfolio, in week w of year t . Thus, $MBETA_{ft} \equiv \hat{\beta}_{ft}^{mkt}$ and $BIMC_{ft} \equiv \hat{\beta}_{ft}^{imc}$ are constructed using information only in year t . We also use weekly returns to estimate the firm's idiosyncratic volatility (*IVOL*)

$$r_{ftw} = \alpha_{ft} + \beta_{ft}^{mkt} r_{tw}^{mkt} + \beta_{ft}^{imc} r_{tw}^{imc} + \varepsilon_{ftw}, \quad w = 1 \dots 52, \quad (32)$$

where r_{ftw}^{imc} refers to the log return of the IMC portfolio in week w of year t . Our measure of idiosyncratic volatility $IVOL_{ft} \equiv \sqrt{var_t(\varepsilon_{ftw})}$ is also constructed using information only in year t . We estimate idiosyncratic volatility from the two-factor specification (32) rather than the market model (31) to ensure that our measure of idiosyncratic variance is not mechanically reflecting variation in IMC betas across firms.

Tables

Table 1: Characteristics, comovement and risk premia

| | 10 minus 1 decile portfolios | | | | |
|---------------|------------------------------|------------------|-------------------|------------------|-------------------|
| | Tobin's Q | I/K | P/E | MBETA | IVOL |
| $E(R_p)$ (%) | -8.79 (-3.26) | -4.94 (-1.42) | -8.91 (-3.00) | -1.97 (-0.52) | -5.86 (-0.95) |
| σ (%) | 20.75 | 24.86 | 19.72 | 26.47 | 37.05 |
| α (%) | -10.27 (-3.64) | -8.04 (-2.41) | -10.15 (-3.51) | -5.71 (-1.87) | -10.78 (-1.83) |
| β^{mkt} | 0.29 (1.66) | 0.62 (2.91) | 0.25 (1.62) | 0.75 (4.26) | 0.98 (3.97) |
| R^2 (%) | 6.55 | 20.13 | 5.11 | 25.93 | 22.85 |

Table 1 reports moments of the top minus decile portfolio of firms sorted on Tobin's Q , past investment (IK), price-to-earnings (PE), market beta (MBETA), and idiosyncratic volatility (IVOL). We report the mean and volatility of portfolio returns, and the results of CAPM regressions. See main text for variable definitions. All portfolios are value-weighted and rebalanced annually; portfolios based on accounting variables (Q , IK, and PE) are rebalanced every June; portfolios based on firm moments (MBETA and IVOL) are rebalanced at the beginning of every calendar year. Estimation is done at annual frequencies. We report t -statistics in parenthesis computed using Newey-West errors with 3 lags. The sample period is 1964-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 2: Return comovement

| | Cross-sections | | | | | Eigenvalues | |
|--------------|----------------|--------|--------|--------|--------|-------------|----------------------------|
| | I/K | P/E | IVOL | MBETA | Q | ALL | $\lambda_1/\sum \lambda_i$ |
| I/K | | | | | | | 35.5 |
| (p-value) | | | | | | | (0.00) |
| P/E | 72.2 | | | | | | 35.6 |
| (p-value) | | | | | | | (0.00) |
| IVOL | 40.7 | 22.4 | | | | | 51.5 |
| (p-value) | | | | | | | (0.00) |
| MBETA | 81.0 | 63.9 | 53.7 | | | | 41.0 |
| (p-value) | | | | | | | (0.00) |
| Q | 80.6 | 62.1 | 11.4 | 60.6 | | | 35.9 |
| (p-value) | | | | | | | (0.00) |
| ALL (PC1) | 92.0 | 77.9 | 46.8 | 89.7 | 74.2 | | 33.2 |
| (p-value) | (0.00) | (0.00) | (0.03) | (0.00) | (0.00) | | (0.00) |
| IMC | 61.3 | 60.8 | 54.0 | 67.5 | 69.6 | 68.6 | |
| -HML | 77.0 | 61.7 | 28.2 | 72.6 | 73.9 | 81.7 | |
| Δz^I | 31.9 | 35.2 | 14.2 | 49.3 | 19.0 | 38.4 | |

Table 2 shows return comovement across the five decile portfolio sorts, on I/K, P/E, IVOL, MBETA, and Q. We extract the first principal component in each of the five cross-sections from the return residuals from a market model regression, $\tilde{R}_p^e = R_p^e - \beta_{pm} R_m^e$. We normalize the sign of the first principal component so that it loads positively on portfolio 10. We repeat the exercise the pooled cross-section on the extreme decile portfolios (1, 2, 9 and 10) sorted on Q, IK, EP, MBETA and IVOL. We extract principal components using the correlation matrix. We report the correlation matrix of these principal components, along with their correlations with IMC, HML and the real proxy for the IST shock Δz^I . The real measure IST shocks Δz_t^I is equal to the change in the detrended quality-adjusted relative price of new capital goods; see Kogan and Papanikolaou (2012b) for more details. The IMC portfolio refers to a zero investment portfolio long investment firms and short consumption firms; see Papanikolaou (2011) for more details. The HML portfolio is constructed excluding firms producing investment goods. We compute p -values based on 10,000 permutations, where we randomly and independently permute the time-series order of each cross-section. See main text and notes to Table 1 for more details.

Table 3: Loadings of 1st PC on corner portfolios

| Portfolio | 1 | 2 | 9 | 10 |
|-----------|--------|--------|-------|-------|
| Q | -0.260 | -0.206 | 0.075 | 0.156 |
| I/K | -0.207 | -0.309 | 0.258 | 0.135 |
| P/E | -0.149 | -0.023 | 0.209 | 0.197 |
| MBETA | -0.181 | -0.292 | 0.222 | 0.146 |
| IVOL | -0.371 | -0.142 | 0.075 | 0.043 |

Table 3 reports the loadings of the first principal component of the pooled cross-section on the extreme decile portfolios (1,2, 9 and 10) sorted on Q, I/K, P/E, MBETA and IVOL, orthogonalized with respect to the market. We construct PC1 as follows: first, we orthogonalizing the excess returns of the 1,2, 9 and 10 portfolios with respect to the excess returns of the market portfolio, $\tilde{R}_p^e = R_p^e - \beta_{pm} R_m^e$; second, we perform principal component analysis on the pooled cross-section of 20 portfolio de-meaned returns – using the correlation rather than the covariance matrix – and extract the first principal component, scaled to 0.1 standard deviation; third, we recover the first principal component (PC1) as the linear combination – using the estimated loadings – of the extreme decile portfolio returns \tilde{R}_p^e . See notes to Table 1 for more details.

Table 4: The empirical factor model

| | 10 minus 1 decile portfolios | | | | |
|---------------|------------------------------|-----------------|------------------|-----------------|------------------|
| | Tobin's Q | I/K | P/E | MBETA | IVOL |
| α (%) | -2.18 (-1.00) | 1.32 (0.62) | -3.17 (-1.41) | 3.21 (1.27) | -4.44 (-0.94) |
| β^{mkt} | 0.29 (3.83) | 0.62 (5.16) | 0.25 (2.66) | 0.75 (6.94) | 0.98 (4.24) |
| β^{PC1} | 0.63 (7.32) | 0.73 (10.01) | 0.54 (5.83) | 0.69 (8.16) | 0.49 (3.75) |
| R^2 (%) | 66.13 | 75.75 | 54.24 | 70.47 | 34.32 |
| GRS | 0.647 (0.91) | 0.668 (0.89) | 1.303 (0.21) | 0.947 (0.57) | 0.688 (0.88) |

Table 4 summarizes the performance of the two-factor model that includes the market portfolio and the first principal component of the pooled cross-section (PC1, normalized to standard deviation of 10%) of Q , I/K, P/E, MBETA and IVOL portfolios. We report the alpha, market beta and PC1-beta of the top minus decile portfolio of firms sorted on each characteristic, as well as the result of the GRS test that the proposed factor model prices the decile portfolios in each cross-section. We report t -statistics in parenthesis computed using Newey-West errors with 3 lags. See main text and notes to Tables 1 to 3 for more details.

Table 5: Stochastic discount factor

| Factor price | (1) | (2) | (3) | (4) | (5) |
|--------------|----------------------|-------------------------|----------------------|-------------------------|-------------------------|
| Δx | 0.75 [0.04, 1.46] | -0.77 [-1.80, 0.25] | | | |
| R^{mkt} | | | 0.21 [0.02, 0.40] | 0.52 [0.31, 0.73] | 0.37 [0.17, 0.57] |
| Δz^I | | -1.35 [-2.24, -0.46] | | | |
| R^{imc} | | | | -0.71 [-1.06, -0.36] | |
| $-R^{hml}$ | | | | | -0.65 [-0.98, -0.32] |
| SSQE (%) | 4.23 | 0.92 | 3.61 | 0.75 | 1.40 |
| MAPE (%) | 3.85 | 1.87 | 3.69 | 1.53 | 2.00 |

Table 5 reports empirical estimates of γ_x and γ_z from the model SDF: $m = a - \gamma_x \Delta x - \gamma_z \Delta z$. We proxy for IST shocks Δz with the changes in the detrended relative price of new equipment, Δz^I ; for the disembodied shock Δx we use the change in the log total factor productivity in the consumption sector from Basu et al. (2006). We also consider two-factor specifications for the SDF with the market portfolio and either the IMC portfolio, or the HML portfolio. We sort firms on Q , IK, PE, MBETA and IVOL, and use portfolios 1, 2, 9, and 10 from each sort, resulting in a total of 20 portfolios. We report first-stage estimates; 90%-confidence intervals around the point estimates computed using the Newey-West procedure with 3 lags; the sum of squared errors (SSQE); and mean absolute pricing errors (MAPE). See main text and notes to Tables 1 and 2 for more details.

Table 6: IMC-beta and characteristics

| $BIMC_t$ | a. Market Beta | b. Investment | c. Earnings/Price | d. Idiosyncratic Volatility | e. Tobin's Q | f. All |
|---------------|-----------------|------------------|-------------------|-----------------------------|------------------|-------------------|
| $BIMC_{t-1}$ | 0.155 (9.21) | 0.226 (10.07) | 0.233 (10.26) | 0.211 (9.80) | 0.222 (10.05) | 0.130 (8.81) |
| $MBETA_{t-1}$ | 0.307 (7.93) | | | | | 0.264 (7.38) |
| IK_{t-1} | | 0.487 (9.62) | | | | 0.205 (5.93) |
| EP_{t-1} | | | -0.215 (-3.26) | | | 0.188 (5.72) |
| $IVOL_{t-1}$ | | | -0.152 (-2.89) | 5.066 (9.67) | | -0.006 (-0.14) |
| $\ln Q_{t-1}$ | | | | 3.774 (7.45) | | 3.273 (7.18) |
| R^2 | 0.257 | 0.221 | 0.213 | 0.233 | 0.225 | 0.277 |
| | | 0.261 | 0.257 | 0.268 | 0.264 | 0.287 |
| | | | | | 0.097 (8.78) | 0.044 (4.72) |
| | | | | | | |

Table 6 reports results of regressing a firm's IMC-beta (BIMC) on lagged estimates of IMC-beta and firm characteristics – equation (4) in main text. All specifications include year-fixed effects. We report t -statistics in parenthesis computed using clustered errors by firm and year. Estimation sample is 1964-2008. See main text and notes to Tables 1 for more details.

Table 7: Fama-MacBeth regressions

| R_{ft} | a. Market Beta | b. Investment | c. Earnings/Price | d. Idiosyncratic Volatility | e. Tobin's Q |
|-------------------|-------------------|-------------------|-------------------|-----------------------------|-------------------|
| $E_{t-1}[BIMC_t]$ | -0.299 (-6.56) | -0.292 (-5.69) | -0.264 (-6.73) | -0.256 (-4.04) | -0.276 (-6.28) |
| $MBETA_{t-1}$ | -0.013 (-1.10) | -0.006 (-0.57) | -0.013 (-1.24) | 0.095 (4.41) | -0.003 (-0.31) |
| IK_{t-1} | | -0.151 (-6.01) | | | 0.103 (5.85) |
| EP_{t-1} | | | 0.255 (4.60) | | |
| $IVOL_{t-1}$ | | | | | |
| $\ln Q_{t-1}$ | | | | -1.831 (-6.63) | -0.031 (-4.15) |
| | | | 0.132 (3.64) | -0.429 (-1.26) | -0.008 (-1.30) |

Table 7 reports results of Fama and MacBeth (1973) tests. As return predictions we use lagged firm characteristics (Q , I/K , E/P , $MBETA$ and $IVOL$) and the predicted values of IMC-beta $E_{t-1}[BIMC_t]$ using the specification corresponding to the last column of Table 6. To minimize look-ahead bias, we estimate equation (4) in main text using the 1965-1986 subsample. We then use the point estimates to construct predicted values of $E_{t-1}[BIMC_t]$ in the 1987-2008 subsample for the Fama and MacBeth (1973) regressions. Estimation is done at annual frequencies, with the stock return on fiscal year t defined as the stock return from June of calendar year t to May of calendar year $t + 1$. See main text and notes to Tables 1 for more details.

Table 8: Investment comovement – response to IST shock

| IK_{ft} | A. (Δz^I) | | | | | B. (IMC) | | | | |
|----------------------------------|---------------------|----------------|------------------|------------------|------------------|------------------|----------------|----------------|----------------|----------------|
| | MBETA | IK | P/E | IVOL | Q | MBETA | IK | P/E | IVOL | Q |
| Δz_{t-1} | 0.18 (1.14) | 0.20 (0.90) | 0.22 (1.20) | 0.32 (1.19) | 0.20 (0.91) | 0.80 (4.63) | 0.64 (3.56) | 0.42 (2.00) | 0.55 (2.49) | 0.57 (2.88) |
| $D(G_f)_2 \times \Delta z_{t-1}$ | 0.10 (0.72) | 0.14 (1.46) | -0.01 (-0.15) | 0.09 (0.57) | -0.01 (-0.01) | -0.29 (-2.30) | 0.10 (0.97) | 0.27 (2.76) | 0.23 (2.70) | 0.10 (1.17) |
| $D(G_f)_3 \times \Delta z_{t-1}$ | 0.04 (0.26) | 0.41 (3.67) | 0.15 (1.90) | 0.17 (0.95) | 0.15 (1.02) | 0.14 (1.17) | 0.09 (0.67) | 0.19 (1.73) | 0.23 (1.95) | 0.36 (3.74) |
| $D(G_f)_4 \times \Delta z_{t-1}$ | 0.26 (1.66) | 0.55 (3.27) | 0.37 (2.46) | 0.19 (1.04) | 0.40 (2.57) | 0.02 (0.11) | 0.31 (1.67) | 0.49 (2.01) | 0.33 (1.30) | 0.48 (3.84) |
| $D(G_f)_H \times \Delta z_{t-1}$ | 0.67 (2.18) | 0.55 (1.96) | 0.66 (1.97) | -0.10 (-0.53) | 0.58 (2.15) | 0.70 (2.74) | 0.96 (3.88) | 1.23 (3.62) | 0.75 (3.56) | 0.51 (1.73) |

Table 8 shows the differential response of investment of firms with different characteristics $G_f \in \{Q, IK, PE, MBETA, IVOL\}$ on measures of the IST shock, shocks to equipment price and returns to the IMC portfolio $\Delta z \in \{\Delta z^I, R^{ime}\}$, normalized to unit standard deviation. We show the estimated coefficients $b_1 \dots b_5$ from equation (6), along with t -statistics computed using standard errors clustered by firm and year. We control for industry fixed effects and lagged investment across specifications. The sample period is 1964-2008 and excludes firms producing investment goods, financial firms (SIC6000-6799) and utilities (SIC4900-4949).

Table 9: Calibration

| Moment | Data | Model | | |
|---|-------|--------|--------|-------|
| | | Median | 5% | 95% |
| <i>Aggregate moments, real variables</i> | | | | |
| Mean of aggregate dividend growth | 0.025 | 0.025 | -0.048 | 0.076 |
| Volatility of aggregate dividend growth | 0.118 | 0.088 | 0.055 | 0.221 |
| Volatility of aggregate investment growth | 0.157 | 0.155 | 0.100 | 0.228 |
| <i>Aggregate moments, asset prices</i> | | | | |
| Mean excess return of market portfolio | 0.059 | 0.061 | 0.034 | 0.092 |
| Volatility of market portfolio return | 0.161 | 0.145 | 0.100 | 0.199 |
| Mean return of HML portfolio | 0.035 | 0.038 | 0.024 | 0.047 |
| Volatility of HML portfolio | 0.141 | 0.066 | 0.041 | 0.090 |
| Relative market capitalization of I- and C-sector | 0.149 | 0.138 | 0.060 | 0.211 |
| <i>Firm characteristics, time-series moments</i> | | | | |
| Median firm investment rate | 0.116 | 0.100 | 0.041 | 0.177 |
| Correlation between investment and lagged stock returns | 0.177 | 0.180 | 0.068 | 0.249 |
| Correlation between investment and lagged Tobin's Q | 0.280 | 0.280 | 0.198 | 0.387 |
| Serial correlation of return on assets | 0.825 | 0.841 | 0.821 | 0.899 |
| Serial correlation of firm investment rate | 0.478 | 0.524 | 0.421 | 0.604 |
| <i>Firm characteristics, cross-sectional dispersion (IQR)</i> | | | | |
| Firm investment rate | 0.157 | 0.107 | 0.048 | 0.180 |
| Cashflows-to-Capital | 0.234 | 0.234 | 0.203 | 0.298 |
| Tobin's Q | 3.412 | 2.327 | 1.339 | 4.280 |
| β^{imc} | 0.990 | 0.691 | 0.477 | 0.937 |
| Firm size relative to average size | 0.830 | 0.778 | 0.725 | 0.931 |

Table 9 compares sample moments to moments in simulated data. The moments of investment growth are estimated using the series on real private nonresidential investment in equipment and software. Moments of firm-specific variables are estimated using Compustat data, where we report time series moments of the investment rate and cash flows over capital, Tobin's Q and IMC-beta. Moments of dividend growth are from the long sample in Campbell and Cochrane (1999). We construct the value factor (HML) in the consumption sector as $1/2(LV - LG) + 1/2(SV - SG)$ where LV , LG , SV , SG refer to the corner portfolios of a 2-by-3 sort on ME and BE/ME using consumption firms only and NYSE breakpoints. We report median and 5-th and 95-th percentiles of the point estimates across 1,000 simulations, each with length of 50 years.

Table 10: Characteristics, comovement and risk premia, simulated data

| | 10 minus 1 decile portfolios | | | | |
|---------------|------------------------------|-------------------|--------------------|-------------------|-------------------|
| | Tobin's Q | I/K | P/E | MBETA | IVOL |
| $E(R_p)$ (%) | -5.27 (-5.71) | -4.81 (-7.85) | -7.21 (-3.93) | -5.55 (-4.06) | -5.24 (-5.38) |
| σ (%) | 6.89 | 4.41 | 13.33 | 9.84 | 7.03 |
| β^{mkt} | 0.31 (7.71) | 0.22 (7.10) | 0.74 (9.53) | 0.54 (9.21) | 0.36 (6.99) |
| α (%) | -7.13 (-10.83) | -6.23 (-13.88) | -11.94 (-10.47) | -8.98 (-10.14) | -7.56 (-10.68) |
| R^2 (%) | 53.34 | 53.01 | 64.23 | 63.49 | 54.79 |

Table 10 replicates the analysis in Table 1 in simulated data. We report the mean and volatility of portfolio excess returns, and the results of CAPM regressions. We report median point estimates of the parameters and the t -statistics across 1,000 simulations, each with length of 50 years. See main text and notes to Tables 1 for more details.

Table 11: Firm profitability and firm characteristics: Data vs Model

| A. Data | | | | | | | | | | |
|----------------------|------|------|------|------|------|------|------|------|------|------|
| Profitability decile | Lo | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Hi |
| β^{imc} | 1.36 | 0.60 | 0.33 | 0.41 | 0.40 | 0.29 | 0.41 | 0.18 | 0.21 | 0.10 |
| β^{mkt} | 1.44 | 1.12 | 0.92 | 0.98 | 1.04 | 0.92 | 1.02 | 0.83 | 0.86 | 0.89 |
| I/K | 0.09 | 0.08 | 0.08 | 0.09 | 0.09 | 0.10 | 0.10 | 0.10 | 0.11 | 0.11 |
| Q | 6.53 | 1.10 | 0.90 | 1.06 | 1.23 | 1.45 | 1.73 | 2.01 | 2.47 | 2.48 |
| B. Model | | | | | | | | | | |
| Profitability decile | Lo | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | Hi |
| β^{imc} | 1.05 | 0.83 | 0.73 | 0.68 | 0.64 | 0.63 | 0.62 | 0.61 | 0.60 | 0.55 |
| β^{mkt} | 1.25 | 1.07 | 0.98 | 0.94 | 0.91 | 0.90 | 0.89 | 0.89 | 0.88 | 0.83 |
| I/K | 0.11 | 0.11 | 0.10 | 0.09 | 0.08 | 0.08 | 0.07 | 0.08 | 0.08 | 0.09 |
| Q | 3.48 | 3.06 | 2.86 | 2.88 | 2.99 | 3.16 | 3.39 | 3.69 | 4.11 | 5.12 |

Table 11 reports characteristics for decile portfolios sorted on profitability to assets, in the data (Panel A) and the model (Panel B). We follow Novy-Marx (2012) and define profitability in the data as gross profits (item `gp`) divided by book assets (item `at`); in simulated data profitability is earnings (E_{ft}) over replacement value of capital (B_{ft}). We report the portfolio IMC and market beta, and the time-series average of the median investment rate and Tobin's Q in each portfolio. In panel B, we report median and 5-th and 95-th percentiles of the point estimates across 1,000 simulations. See main text for more details.

Table 12: Double sorted portfolios: profitability and Tobin's Q

| A. Average excess returns | | | | | | | | | | | | | |
|---------------------------|-----------------|-----------------|----------------|----------------|-----------------|-------------------|-------------------|----------------|----------------|----------------|----------------|------------------|------------------|
| Profitability quintile | i. Data | | | | | | | ii. Model | | | | | |
| | Q quintile | | | | | | | Q quintile | | | | | |
| | L | 2 | 3 | 4 | H | H-L | L | 2 | 3 | 4 | H | H-L | |
| | L | 8.20 (4.67) | 4.41 (2.21) | 3.05 (0.80) | 0.76 (0.26) | -3.91 (-1.23) | -12.11 (-4.08) | 9.44 (5.13) | 5.80 (2.90) | 4.31 (1.98) | 3.07 (1.42) | 1.36 (0.59) | -8.08 (-7.57) |
| 2 | 8.40 (3.43) | 9.03 (4.41) | 3.44 (1.77) | 2.92 (1.13) | 0.22 (0.09) | -8.19 (-2.94) | 10.97 (7.25) | 8.05 (4.52) | 6.03 (3.17) | 4.44 (2.20) | 2.88 (1.35) | -8.09 (-7.19) | |
| 3 | 11.86 (4.23) | 9.64 (4.42) | 6.95 (3.12) | 4.12 (2.04) | 0.62 (0.24) | -11.24 (-4.28) | 11.34 (7.68) | 9.13 (5.34) | 7.46 (4.15) | 5.23 (2.69) | 3.38 (1.63) | -7.97 (-7.67) | |
| 4 | 13.19 (4.06) | 8.57 (2.85) | 7.79 (3.24) | 6.57 (2.47) | 2.77 (1.02) | -10.41 (-3.04) | 11.04 (7.21) | 8.97 (5.63) | 7.60 (4.45) | 5.79 (3.14) | 4.60 (2.14) | -6.44 (-6.06) | |
| H | 11.47 (3.27) | 10.28 (3.54) | 8.97 (3.52) | 8.10 (2.94) | 7.06 (2.83) | -4.40 (-1.21) | 10.00 (6.91) | 8.42 (5.34) | 6.95 (4.12) | 6.97 (4.55) | 7.93 (4.95) | -2.06 (-3.47) | |
| H-L | 3.27 (1.18) | 5.87 (2.70) | 5.92 (1.65) | 7.34 (2.82) | 10.98 (3.73) | | 0.56 (0.90) | 2.61 (3.90) | 2.64 (3.57) | 3.90 (4.17) | 6.58 (5.24) | | |

| B. CAPM alphas | | | | | | | | | | | | | |
|------------------------|----------------|----------------|------------------|------------------|------------------|-------------------|-------------------|-----------------|------------------|-------------------|-------------------|--------------------|-------------------|
| Profitability quintile | i. Data | | | | | | | ii. Model | | | | | |
| | Q quintile | | | | | | | Q quintile | | | | | |
| | L | 2 | 3 | 4 | H | H-L | L | 2 | 3 | 4 | H | H-L | |
| | L | 4.31 (2.37) | 0.38 (0.27) | -2.73 (-1.25) | -4.89 (-3.09) | -10.72 (-3.93) | -15.03 (-5.39) | 2.53 (9.51) | -1.78 (-4.90) | -3.94 (-7.74) | -5.34 (-8.08) | -7.31 (-8.78) | -9.84 (-20.18) |
| 2 | 4.15 (2.08) | 4.50 (2.82) | -0.67 (-0.49) | -1.89 (-1.20) | -4.83 (-3.53) | -8.98 (-4.04) | 5.47 (13.70) | 1.53 (6.12) | -0.94 (-4.66) | -3.37 (-10.57) | -5.22 (-8.81) | -10.69 (-21.19) | |
| 3 | 6.87 (3.35) | 4.98 (3.13) | 2.42 (2.19) | -0.83 (-0.58) | -5.46 (-4.15) | -12.33 (-3.48) | 6.16 (12.26) | 3.05 (7.33) | 0.73 (3.33) | -2.04 (-7.14) | -4.66 (-11.92) | -10.82 (-20.91) | |
| 4 | 7.60 (2.69) | 4.23 (2.01) | 3.31 (1.75) | 2.01 (1.44) | -2.40 (-1.58) | -10.00 (-3.80) | 5.68 (9.79) | 2.98 (7.38) | 1.17 (3.29) | -1.29 (-4.89) | -3.29 (-10.22) | -8.96 (-12.40) | |
| H | 6.37 (2.78) | 5.79 (2.75) | 4.87 (2.35) | 3.89 (2.31) | 2.89 (1.75) | -3.48 (-1.11) | 4.84 (10.72) | 2.48 (10.18) | 0.75 (2.28) | 0.83 (3.07) | 2.36 (5.50) | -2.48 (-3.36) | |
| H-L | 2.06 (0.67) | 5.41 (2.07) | 7.60 (3.33) | 8.78 (2.53) | 13.61 (4.06) | | 2.32 (2.70) | 4.26 (4.75) | 4.70 (5.92) | 6.17 (7.44) | 9.67 (13.03) | | |

Table 12 reports average excess returns (Panel A) and CAPM alphas (Panel B) for 25 value-weighted portfolios double sorted first on profitability and the Tobin's Q , in the data (i) and in the model (ii). We first sort all firms, excluding those producing investment goods, financials (SIC6000-6799) and utilities (SIC4900-4949), into five portfolios based on profitability; within each profitability quintile, we sort firms into five portfolios based on Tobin's Q . We report t -statistics in parenthesis computed using Newey-West errors with 3 lags. In Panels A.i and B.i, the sample period is 1964-2008; in Panels A.ii and B.ii we report median and 5-th and 95-th percentiles of the point estimates across 1,000 simulations, each with length of 50 years. See main text and notes to Table 11 for more details.

Table 13: Asset pricing: 25 portfolios sorted on profitability and Tobin's Q

| Factor price | (1) | (2) | (3) | (4) | (5) |
|--------------|----------------------|-------------------------|----------------------|-------------------------|-------------------------|
| Δx | 1.40 [0.66, 2.14] | 0.18 [-0.35, 0.71] | | | |
| R^{mkt} | | | 0.33 [0.14, 0.53] | 0.60 [0.39, 0.82] | 0.39 [0.19, 0.59] |
| Δz^I | | -0.85 [-1.20, -0.50] | | | |
| R^{ime} | | | | -1.08 [-1.42, -0.75] | |
| $-R^{hml}$ | | | | | -0.68 [-0.96, -0.39] |
| SSQE | 5.63 | 1.87 | 5.34 | 1.95 | 3.17 |
| MAPE | 3.79 | 2.15 | 3.97 | 2.28 | 2.88 |

Table 13 reports empirical estimates of γ_x and γ_z from the model SDF: $m = a - \gamma_x \Delta x - \gamma_z \Delta z$ using the cross-section of 25 portfolios double sorted on profitability and Q . We report first-stage estimates; 90%-confidence intervals around the point estimates computed using the Newey-West procedure with 3 lags; the sum of squared errors (SSQE); and mean absolute pricing errors (MAPE). See main text and notes to Tables 5 and 11 for more details.

Figure 1: Empirical versus model-implied risk premia

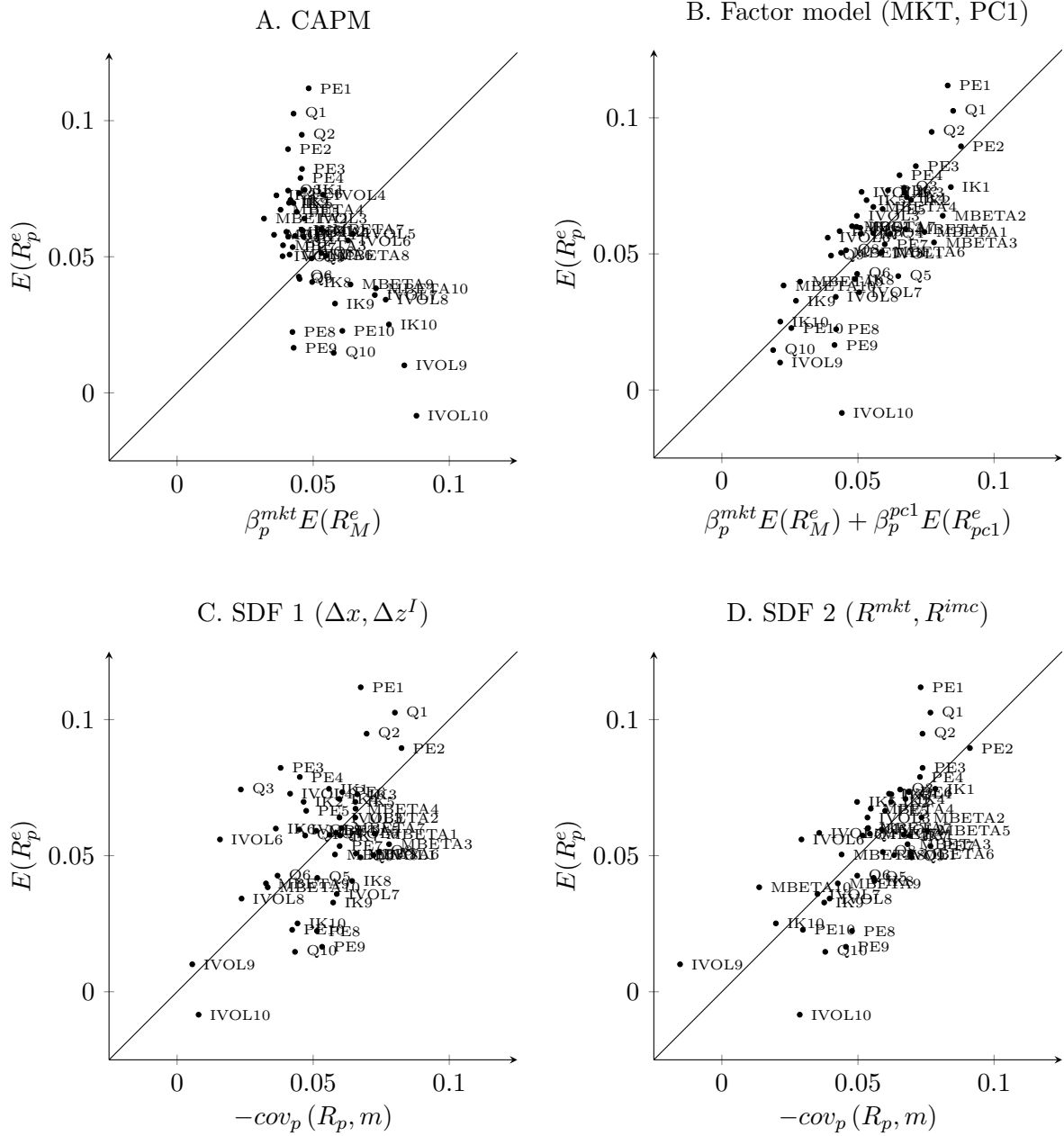


Figure 1 plots predicted expected returns versus realized average returns from four asset pricing models: A) the CAPM; B) the two factor model with the market portfolio and PC1; C) the SDF implied by the model using two real proxies for the IST shock (column 2 of Table 5); and D) the SDF using market portfolio and IMC returns (column 5 of Table 5). We consider the pooled cross-section of IK, PE, Q, MBETA, and IVOL portfolios. See main text and notes to Tables 1 and 5 for more details.

Figure 2: Risk premia and characteristics: data vs model

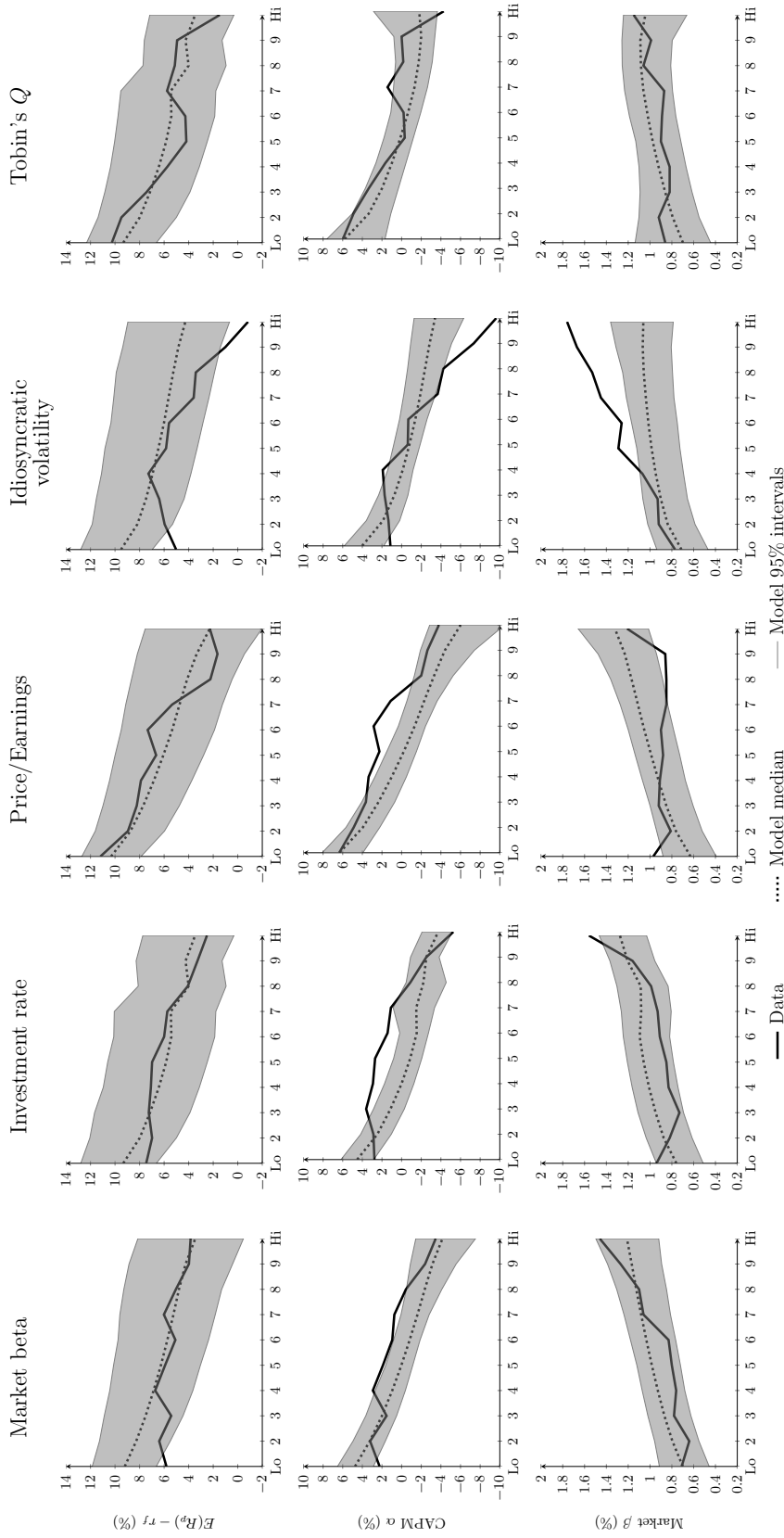


Figure 2 compares average excess returns, CAPM alphas and betas for decile portfolios sorted on I/K , P/E , Q , $MBETA$, and $IVOL$ in the data versus the model. The black solid line corresponds to the empirical estimates. The dotted line corresponds to the median point estimate across 1,000 simulations, each with length of 50 years; grey shaded area corresponds to the 5-th and 95-th percentiles across simulations. See main text and notes to Table 1 for more details.

Figure 3: Investment shock and firm output comovement

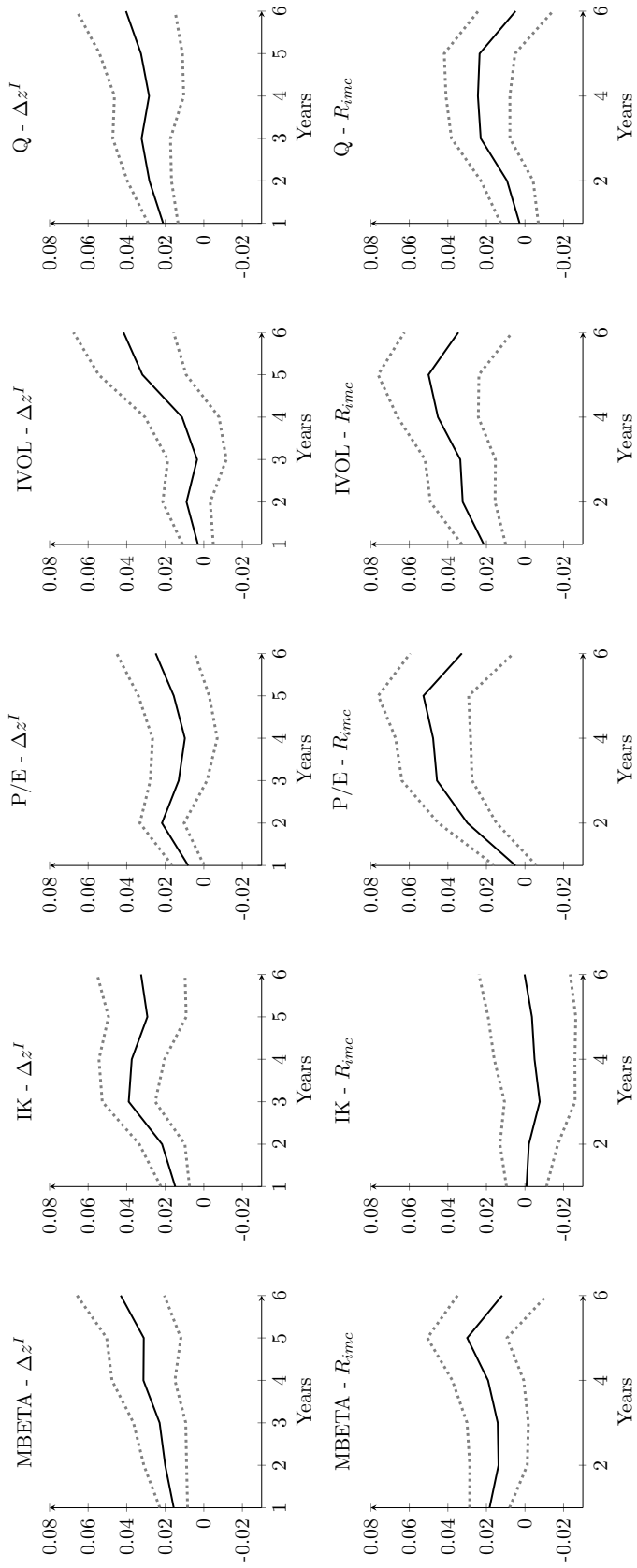


Figure 3 plots the differential response of output growth on the IST shock between firms in the top versus bottom quintile in terms of I/K, P/E, Q, IVOL or MBETA – the estimate of coefficient b_5 in equation (7) in the text. The top panel shows the response to the real proxy for the IST shock Δz^I ; the bottom panel shows the response to returns to the IMC portfolio. Dotted lines correspond to 95% confidence intervals, constructed using standard errors clustered by firm and year. See text for more details.

Figure 4: Profitability and growth: Empirical versus model-implied risk premia

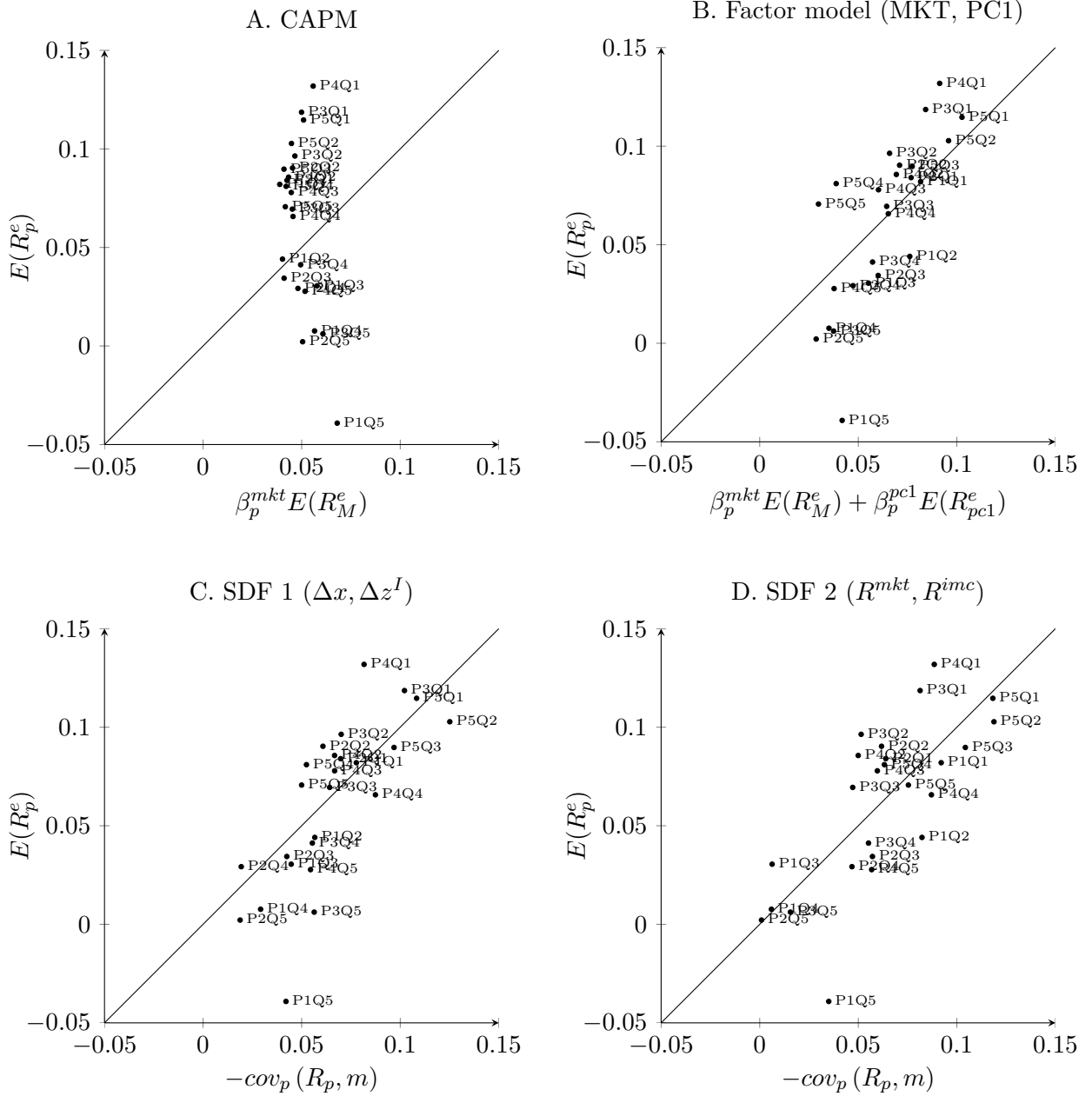


Figure 4 plots predicted expected returns versus realized average returns from four asset pricing models: A) the CAPM; B) the two factor model with the market portfolio and PC1; C) the SDF implied by the model using two real proxies for the IST shock (column 2 of Table 5); and D) the SDF using market portfolio and IMC returns (column 5 of Table 5). We consider the 25 value-weighted portfolios double sorted on profitability (P) and Tobin's Q (Q). See main text and notes to Tables 12 and 13 for more details.