CREEP, ELASTIC HYSTERESIS AND DAMPING IN BAKELITE

by

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for the Degree of

MASTER OF SCIENCE

from the

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Signature of Author ....

Department of Mechanical Engineering, October 3, 1938.

Signature of Professor in

Charge of Research ....

Signature of Chairman of Department Committee

on Graduate Students ....

Cambridge,

Massachusetts.

October 3, 1938.

Professor George W. Swett,

Secretary of the Faculty,

Massachusetts Institute of Technology.

Dear Professor Swett:

I submit herewith a thesis, in partial fulfillment of the requirements for the degree of Master of Science, entitled,

CREEP, ELASTIC HYSTERESIS AND DAMPING IN BAKELITE.

Your faithfully,

Herbert Leaderman.

CONTENTS

Preface.

This work deals with an experimental study of the inelastic properties of four types of bakelite material. It is shown that the damping of free torsional oscillations is accounted for by two phenomena, namely logarithmic creep and elastic hysteresis. Due to the presence of creep, a theory has to be developed to show how the existence of elastic hysteresis may be determined experimentally. This theory is developed in Part II. In Part III is given first an experimental proof of the assumptions of the theory, and also the study of the inelastic properties by both static and dynamic methods. Part IV deals with a few experiments suggested by the results of Part III.

I wish to express my appreciation of the encouragement and advice Professor A. V. de Forest gave me during the course of this work. I also wish to acknowledge the assistance of Professor J. T. Norton, Mr. R. Fanning, and Mr. W. Walsh. I am indebted to Messrs. Bakelite Limited of England and the Bakelite Corporation of America for the supply of materials.

In particular, I wish to express my gratitude to the London County Council for the award of a Robert Blair Fellowship, enabling me to undertake this work.

CREEP, HYSTERESIS, AND DAMPING IN BAKELITE.

Part I. Introduction.

I. Importance of the Problem.

An important property of materials which recently has received much attention is damping capacity, or the ability to dissipate mechanical energy as heat in the working range. This dissipation, though small for ordinary metals in the normal condition, is very sensitive to structure and to strain history, and the study of damping capacity is throwing much light on the differences in behaviour of materials under the same conditions.

The origin of damping in metals is understood to be due mainly to two causes, elastic hysteresis and primary creep. By elastic hysteresis is meant the appearance of a narrow stress-strain loop when the stress is varied cyclically either in steps or continuously; it is a phenomenon independent of time. It has been shown by Rowett[1] that for cold drawn steel, the energy dissipated under free torsional oscillations up to two-thirds of the elastic limit is exactly accounted for by the areas of the hysteresis loops determined by static experiments.

By primary creep is meant that part of creep which proceeds at a decreasing rate with time. It has been shown by Bennewitz[2] that the creep of glass is of this nature and that the damping of glass is entirely accounted for by primary creep.

Owing to the difficulties of experimentation, the phenomenon of creep in metals at room temperature is not well understood; similarly, the inelastic properties of metals at stresses where both primary creep and elastic hysteresis are present has not yet been explored. It is therefore of interest to examine in detail the <u>inelastic</u> properties of synthetic resins (in particular, bakelite) where these properties, being of a large magnitude, can be easily studied. In addition, apart from the contribution to the general understanding of the properties of materials a study of bakelite would give, such a study would be of importanee in the production of synthetic materials with better elastic properties, and in giving a clearer knowledge of allowable working stresses in such materials.

II. Nature of Problem.

If a hysteresis test be made on bakelite by taking a specimen (say) in torsion around a stress cycle, a stress-strain loop will be obtained. The following questions then arise:

1. Is the loop due to the action of primary creep, or is part

of it due to an elastic hysteresis, of a static nature as explained above?

2. If the loop is due to creep, we must be able to demonstrate that this is so; to do this, we must know the law of primary creep for load applied (or removed) in increments at different times. This law we will call the <u>superposition law</u> of creep.
3. If both elastic hysteresis and creep are present, then the area of the loop obtained from a stress-strain test made under definite conditions is greater than the area due to creep deduced from the superposition law. From such a test, we can calculate the damping due both to creep and to elastic hysteresis; this should agree with the damping measured by the decrement of free oscillations of the specimen.

III. <u>Scheme of Thesis</u>.

It is now agreed that the creep of materials can be divided into two components, one proceeding at a decreasing rate with time and proportional to the stress (primary creep) and the other proportional to time but not to stress (plastic flow). For the time being, we will neglect the second component; we will show later that with certain precautions we can eliminate the effect of plastic flow. The main problem is therefore to study the laws of superposition of primary creep, and the application of these laws to the separation of creep components from true elastic hysteresis in a cyclic stress-strain test. This work is divided into a theoretical part

(Part II) and an experimental part (Part III).

Part II is concerned firstly with devising suitable means of testing the superposition law due to Boltzmann,[3] in such a manner that plastic flow will not be an interfering factor in the experiment; and secondly with the calculation of the deflections that would be obtained if primary creep only were present, the load being varied cyclically and in steps. By this means we can determine if elastic hysteresis is present in addition to creep. Finally, we have to calculate the damping due to primary creep on the basis of the superposition law, and then to calculate the total damping due to primary creep and hysteresis.

On the experimental side, the problem was to devise an apparatus by which creep, hysteresis, and damping could be measured on the same specimen. The method chosen was that of torsion of a cylindrical specimen with squared ends, one end being fixed and the other fastened to a torsion bar pivoted about the axis of the specimen. The experimental work in Part III is concerned, first, with the testing of the superposition law according to the schemes developed in Part II for each of the materials investigated; and secondly, with stress-strain observations for loads applied cyclically, in increments. Damping measured directly is shown to agree well with that predicted from stress-strain step-by-step measurements.

Part IV of this thesis gives an account of experiments which shed some light on the nature of creep.

Previous Work.

De Bruyne[4] and also Parzich[5] have measured the damping of reinforced bakelite; they do not explain the origin of the damping. Jordan[6] has studied the creep of soft metals under pure bending with superposed loading. His results are plotted on the basis of the logarithm of time elapsed from the beginning of the experiment. When replotted on the basis of logarithm of equivalent time (see p. 7), his results suggest that Boltzmann's law is true.

IV. Conclusions.

Four types of material were tested in torsional shear.

1. Using a concept of equivalent time, it is shown that the creep follows Boltzmann's superposition law.
2. By means of observations of deflection obtained when the load is varied cyclically in equal increments in equal intervals of time, it is shown that the area of the stress-strain loop is greater than that due to creep calculated from the Boltzmann law; hence an elastic hysteresis exists in bakelite.
3. Measurement of the decrement of free oscillations shows that the measured specific damping capacity at a surface shear stress of 2520 lbs./in.^{2} agrees well with the value deduced from the magnitude of the primary creep and of the elastic hysteresis.

Part II. Theory.

I. Creep Superposition and Equivalent Time.

Boltzmann's superposition law of primary creep can be summarised as follows:

1. Creep under Single Load. The deflection at time t due to a single load W applied at time t_o is given by:

$$x = W\left[b + a \log(t - t_o)\right] \qquad \text{.........(1)}$$

i.e., it consists of a part proportional to the load and independent of the time, and a part proportional to the load and to the logarithm of the time elapsed since the load was applied. In considering creep under complex loading, we may omit for simplicity the quasi-elastic or time-independent part of the deflection.

2. Creep Recovery. If at any subsequent time the load W be removed there will be a creep recovery or negative creep. It is assumed that the effect of removing the load W is that of applying a negative load of magnitude W, at the same time the creep due to the original loading continuing indefinitely. The material is thus assumed to possess a memory for all past loading actions. If the load be removed at time t_1 the creep recovery at a subsequent time t will be given by:

$$W\left[b + a \log(t - t_o)\right] - W\left[b + a \log(t - t_1)\right], \text{ i.e., by}$$

$$Wa \log (t - t_o)/(t - t_1) \qquad \text{...........(2)}$$

3. Increase or Decrease of Load. If the load be increased or decreased suddenly, then the effect on the creep is that of adding the increment or removing the decrement of load, the creep due to previous loading actions being assumed as before to continue indefinitely. If for example loads W_0, W_1, W_2, be applied at times t_0, t_1, t_2, the creep at a subsequent time t, neglecting the time-independent deflection, is:

$$W_o a \log(t - t_o) + (W_1 - W_o) a \log(t - t_1) + (W_2 - W_1) a \log(t - t_2) \qquad \ldots\ldots\ldots\ldots(3)$$

If we apply a single load to a specimen of bakelite, say in torsion, the creep deflection is not found to be logarithmic with time. We can assume either that Boltzmann's law is untrue for bakelite, or that alternatively the logarithmic creep is masked by an additional plastic effect. This plastic effect might be analogous to fluid flow, and therefore in existence whenever the specimen is under load; alternatively, it might represent an initial plastic adjustment.

If now we remove this load that we have applied above, the creep recovery indeed gives a straight line when plotted against $\log (t - t_o)/(t - t_1)$ as indicated by equation (2); but this is not sufficient (as has been assumed many times in the past) for assuming the correctness of Boltzmann's superposition law. If however we consider the creep and creep recovery due to repeated application and removal of the load at different time instants, then by comparison with the creep and creep recovery as expected by the superposition law we can determine the validity of the law.

Equivalent Time.

In carrying out the experiments to confirm Boltzmann's theory it is expedient to add or remove loads which are of the same magnitude or at least simple multiples of each other; for example, if in equation (3) above, $W_1 = 2W_o$, $W_2 = 3W_o$, then the theoretical creep at time t is given by:

$$W_o a \log(t - t_o)(t - t_1)(t - t_2),$$

i.e., the creep is proportional to the logarithm of a simple function of time. A simple method of checking the superposition law for any complex loading therefore presents itself; let us plot the observed creep (or creep recovery) deflection against the logarithm of the appropriate time function. Then for any set of tests we should obtain a set of parallel straight lines, the distance between any two lines corresponding to the elastic together with any hysteretic deflection. We thus have a very powerful means of checking the superposition law.

Let us call this function of time the <u>equivalent time</u>. The procedure is thus to calculate the equivalent time corresponding to each observation, and to plot the scale deflections against equivalent times on semi-logarithmic paper. In this way, deviations from the superposition law can be immediately observed during the course of the experiment.

Confirmation of the Superposition Law, Using the Concept of Equivalent Time.

Three tests will be considered which together are sufficient to confirm the superposition law, and to indicate the nature of the departures therefrom.

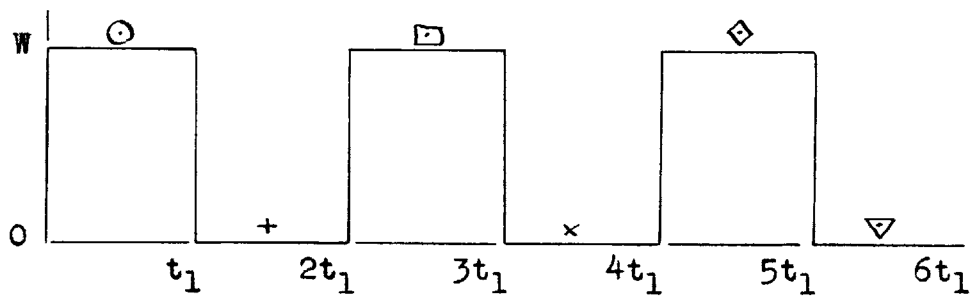


Fig. 1.

Test I. Repeated Loading.

Fig. 1 represents a load-time diagram for an experiment in which a load W is applied and removed repeatedly. The equivalent times are therefore as follow:

If t lies in time interval:	expression for equivalent time is:
$0 - t_1$	t
$t_1 - 2t_1$	$t/(t - t_1)$
$2t_1 - 3t_1$	$t(t - 2t_1)/(t - t_1)$
$3t_1 - 4t_1$	$t(t - 2t_1)/(t - t_1)(t - 3t_1)$
$4t_1 - 5t_1$	$t(t - 2t_1)(t - 4t_1)/(t - t_1)(t - 3t_1)$
$5t_1 - 6t_1$	$\dfrac{t(t - 2t_1)(t - 4t_1)}{(t - t_1)(t - 3t_1)(t - 5t_1)}$

If the theory is rigidly true, we expect that all the creep deflections when plotted against the logarithms of the corresponding equivalent times to fall on one straight line; and the same be true for observations of creep recovery. These two lines should be parallel and separated by a distance corresponding to the elastic deflection due to a load W.

Test II. Creep and Stress.

We can show that creep is proportional to load by repeating any experiment, with, say, half the load. We then expect the slopes of all the lines to be halved.

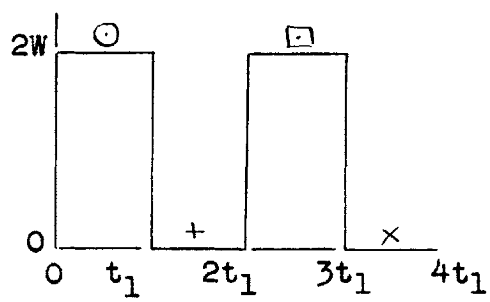


Fig. 2a.

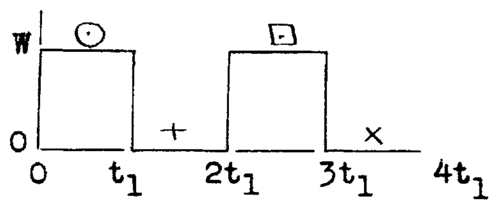


Fig. 2b.

Loading diagrams according to figs. 2a and 2b represent experiments which could be made to test the proportionality of logarithmic creep with applied load. This proportionality is important. If experiments carried out by torsion of cylindrical specimens show a proportionality of creep with load, we can say that the creep is proportional to stress; furthermore, the stress must be always linearly distributed across any cross-section, and therefore primary creep would not cause a redistribution of stress. The results of a torsion experiment would then be of significance.

Test III. Memory Action.

According to the superposition law, the creep due to each loading action persists indefinitely, i.e., the material remembers past loading actions. The purpose of the following experiment is to demonstrate this effect in such a manner that plastic flow will not be an interfering factor; the experiment is to show that if a specimen be loaded first in one sense and then in the other, and then the load removed, the material will tend to recover in opposite senses from the two loading actions. The times can be so arranged that in fact after a certain instant the memory of the first loading action outweighs that of the second, i.e., that creep recovery proceeds first in one direction and then in the other. Such an experiment demonstrates qualitatively the memory action, and is well known. We are now going to use the concept of equivalent time to demonstrate that in fact Boltzmann's law is obeyed quantitatively.

Mathematically, the condition that the creep recovery should stop and then reverse its direction is that the equivalent time for the recovery should possess a maximum (or minimum).

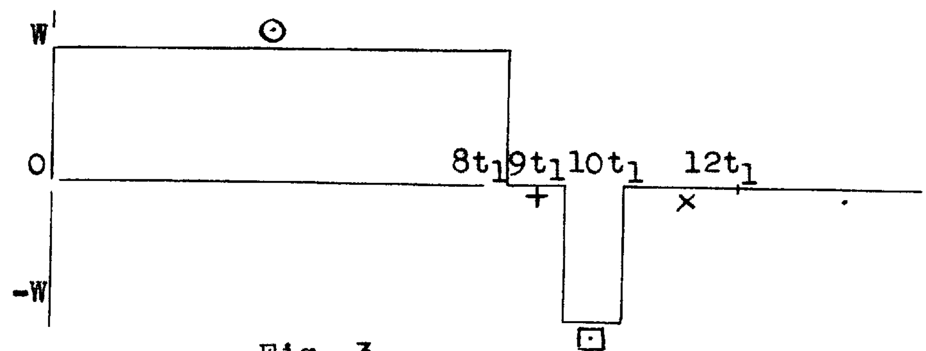


Fig. 3.

The loading diagram in fig. 3 represents the application of a load W from $t = 0$ to $t = 8t_1$, and the application of a load -W from $t = 9t_1$ to $t = 10t_1$. The equivalent time for the subsequent recovery is therefore:

$$\frac{t(t - 10t_1)}{(t - 8t_1)(t - 9t_1)}$$

This has a stationary value at $t = 12t_1$, hence at this instant we would expect the direction of the creep recovery to reverse. Furthermore, we would expect all the creep recovery observations from $t = 10t_1$ onwards when plotted against the logarithm of the equivalent time to fall on a straight line, retracing the line backwards from $t = 12t_1$ onwards. This in fact happened in all the materials tested. In an otherwise ideally elastic material, the creep recovery observations from $t = 8t_1$ to $t = 9t_1$ should fall on the same line. In actual fact, they fall on a parallel line, indicating the existence of an elastic hysteresis loop.

The above is an account of the ideal behaviour of a material under experiments to check the validity of the superposition law of primary creep. In Part III is given an account of the behaviour of various commercial types of bakelite under these tests. Results were such as to show that plastic flow was not, after the first application of load, a seriously interfering factor.

II. Separation of Creep and Elastic Hysteresis.

The next stage in the investigation is the demonstration of the existence of an elastic hysteresis loop, and the experimental determination of its magnitude. Let us assume first that elastic hysteresis is absent, and let us imagine that a material which possesses the property of primary creep obeying the superposition law is loaded cyclically by applying (or removing) equal increments of load in equal intervals of time. Then the deflections due to creep at the end of each of these intervals could be calculated, if the magnitude of the creep be known; this latter can be obtained from a repeated loading test. All the observed deflections obtained in the cyclic test could be corrected for creep by means of the calculated creep deflections. In the absence of elastic hysteresis, the experimentally observed loop widths at each load station should be the same as the corresponding theoretically calculated widths, that is, the calculated stress-strain loop due to primary creep should be the same as the experimentally determined loop. If the width of the measured loop at any load station is greater than the calculated width, the difference must be the width of the loop due to elastic hysteresis at that load.

Thus if we take a sufficient number of steps in the cycle, it is possible to find the shape and area of the elastic hysteresis loop. The area thus determined can be checked in two ways. First, we can repeat the experiment, using different values of time interval between the load increments. The observations corrected for creep should then be the same for all tests, and the hysteresis loop area should be the same; this has been found to be true within the order of accuracy of the experiment. Secondly, the internal energy absorption (or damping capacity) as measured by the decrement of free oscillations should agree with the area of the hysteresis loop and the magnitude of the primary creep as measured statically: again in the experiments reasonable agreement was found.

Stress-Strain Loop due to Creep.

We now have to calculate the deflections which would be obtained, due to creep, if a material be loaded cyclically. We assume:

1. Boltzmann's superposition law to be true.
2. The load to be applied (or removed) in equal amounts in equal intervals of time.
3. The deflections to be read at the end of each time interval.

In the experimental work, there were four loading steps, i.e., sixteen loading operations per cycle. However we will at first keep the investigation general by assuming that there are n loading steps, and that each load increment is of magnitude W. The creep constant is to be a as before, and the time interval between load changes t_1.

<u>Time-variable part of deflection</u>.

At the outset, we are confronted by a great difficulty: it appears impossible to separate the deflection into an instantaneous component and a subsequent creep component; the deflection follows a logarithmic law with time, and this seems to be true for the smallest measurable values of deflection and time. In other words, the curve of deflection against time is as shown in fig. 4(b) and not as shown in fig. 4(a).

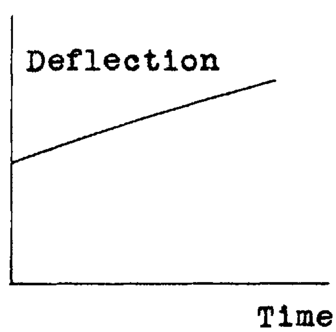


Fig. 4(a)

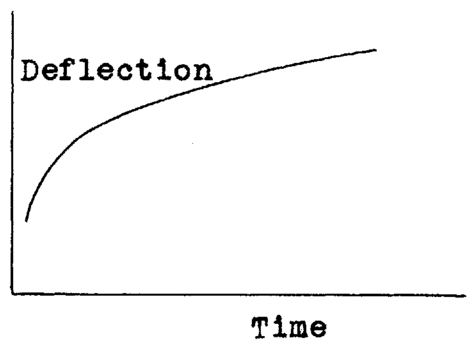


Fig. 4(b)

Of course the logarithmic law cannot hold down to zero time, since the deflection would then have an infinitely negative value. It is not possible to say where the logarithmic law ceases to hold, and therefore to find any value of instantaneous deflection.

Since the deflection due to a single load according to Boltzmann's law varies with time according to the equation:

$$x = B + A \log(t/T)$$

where t is the elapsed time and T is the value of the time unit, then we may conceive the deflection as consisting of a <u>time-independent</u> part B, and a <u>time-dependent</u> part A log(t/T).

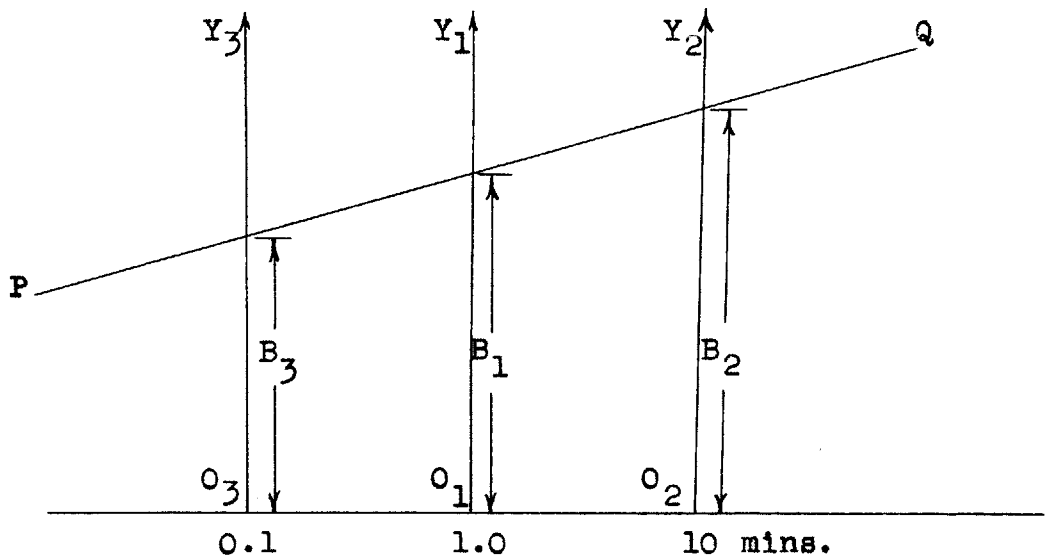


Fig. 5.

Let PQ in fig. 5 represent the relation between the deflection (plotted vertically) and the logarithm of the elapsed time in minutes (plotted horizontally). If the time unit T is chosen to be 1 minute, we draw the axis of ordinates O_1Y_1 through the point corresponding to t = 1 minute. The intersection of this line with the line PQ gives us the value of B_1, the time-independent part of the deflection. If however we choose our time unit T equal to 10 minutes or to 0.1 minute, the time-independent part of the deflection becomes B_2 or B_3. Thus the values of the time-independent and time-dependent parts of the deflection depend on the value we choose for our time-unit.

Only at one stage in this work is the choice of the time unit of weight. We will calculate the shape of the hysteresis loop corresponding to the time-dependent part of the deflection for cyclic step-by-step loading, and use this loop to correct the observed loop. The effect of variation of time unit is to shear over the calculated loop, and consequently also the corrected loop, without altering the actual areas. Furthermore, if loops be traced with different values of time interval between the load increments, the calculated and corrected loops will all be affected similarly. It therefore appears that our problem is a purely philosophical one. In order to assess the damping due to elastic hysteresis, however, we have to choose a value of total strain range; unlike loop area, this quantity is affected by the choice of the time unit.

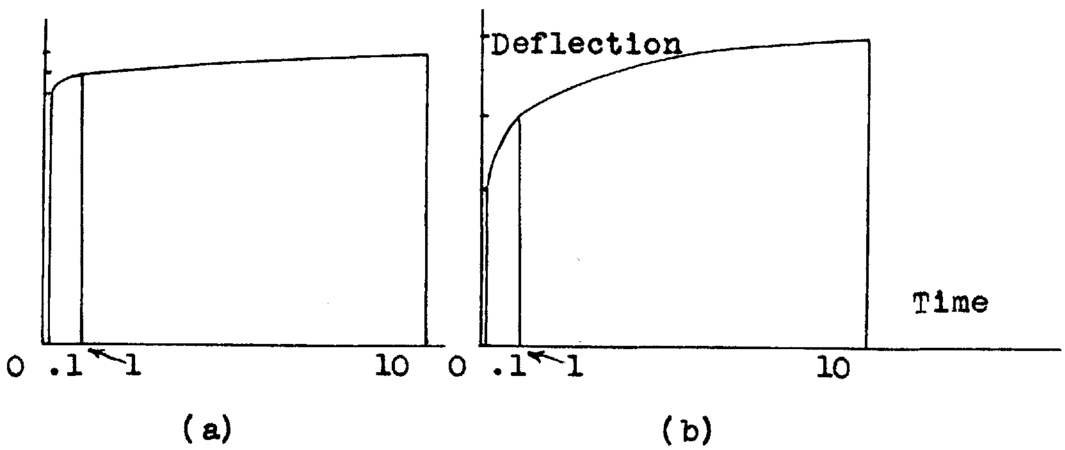


Fig. 6.

Fig. 6 represents diagrammatically the relative creep behaviour of glass and bakelite. In glass the change in deflection is relatively so slow that the value of the time-unit is not of significance; in the case of bakelite, however, where the deflection changes relatively more rapidly with time, the problem is a practical and not merely a philosophical one.

Two values of time unit suggest themselves. The first is that value corresponding to the stiffness as given by the natural frequency of oscillation; the second is the period of this oscillation. In general, a value of time unit T of 1 minute will be chosen except (as in the case above) where this unit is of importance; it will then be taken to be 10^{-3} minute, which is of the order of magnitude of the natural period of oscillation.

In the experiments, stress-strain loops were measured by applying four equal loads in equal intervals of time, which were 2, 1, or 0.5 minutes. The general investigation assumes that there are n loading steps, at intervals of time t_1. Each increment of load is of magnitude W.

Let deflection due to load W applied at zero time be

$$W(b + a \log t)$$

at current time t. Let an additional load W be applied at a subsequent time t_1. Then the deflection at time t, where $2t_1 > t > t_1$, is

$$W\left[2b + a \log t + a \log(t - t_1)\right]$$

Similarly, deflection at time t, where $3t_1 > t > 2t_1$, is

$$W\left[3b + a \log t + a \log(t - t_1) + a \log(t - 2t_1)\right]$$

After the n th load increment has been applied, deflection at time t where $(n - 1)t_1 < t < nt_1$, is

$$W\left[nb + a \log t + a \log(t - t_1) + \ldots + a \log(t - \overline{n - 1}\,t_1)\right]$$

From these expressions, we can find the deflections at the end of each time interval, by substituting the appropriate value of t. The first three deflections are given in the table below.

Time.	Load.	Deflection.
t_1	W	$W(b + a \log t_1)$
$2t_1$	2W	$2Wb + Wa(\log 2t_1 + \log t_1)$
$3t_1$	3W	$3Wb + Wa(\log 3t_1 + \log 2t_1 + \log t_1)$

This last expression can be written:

$$3W(b + a \log t_1) + W.a \log \underline{|3}$$

The above table can thus be rewritten as follow:

Time.	Deflection
t_1	$W\left[b + a(\log t_1 + \log \underline{|1})\right]$
$2t_1$	$W\left[2b + a(2\log t_1 + \log \underline{|2})\right]$
$3t_1$	$W\left[3b + a(3\log t_1 + \log \underline{|3})\right]$
nt_1	$W\left[nb + a(n \log t_1 + \log \underline{|n})\right]$

These are the deflections for a material which possesses the property of primary creep only, for the first quarter of the first loading cycle. It is observed that the deflection at a load k.W is givem by the time-independent deflection k.W.b together with a time-dependent deflection $W.a(k \log t_1 + \log F)$, where F is an expression containing factorial quantities, dependent on the loading history.

Unloading, n.W to 0.

Proceeding similarly, we can find the deflections from $t = nt_1$ to $t = 2nt_1$, during which period the load is being reduced in steps from n.W to 0.

Deflection at time t, where $(n + 1)t_1 > t > nt_1$, is

$$W.b(n - 1) + W.a\left[\log t + \ldots + \log(t - \overline{n - 1}\, t_1) - \log(t - nt_1)\right]$$

Deflection at time t, where $(n + 2)t_1 > t > (n + 1)t_1$, is

$$W.b(n - 2) + W.a\left[\log t + \ldots + \log(t - \overline{n - 1}\, t_1) - \log(t - nt_1) - \log(t - \overline{n + 1}\, t_1)\right]$$

Finally, deflection at time t, where $2nt_1 > t > (2n - 1)t_1$, is

$$W.a\left[\log t + \ldots + \log(t - \overline{n - 1}\, t_1) - \log(t - nt_1) - \ldots - \log(t - \overline{2n - 1}\, t_1)\right]$$

Substituting $t = (n + 1)t_1$, $(n + 2)t_1$, $\ldots 2nt_1$, in the above expressions (corresponding to the elapsed time at the end of each of the time intervals), we obtain the corresponding deflections if primary creep only is present.

Elapsed Time.	Load.	Deflection.
$(n + 1)t_1$	$(n - 1)W$	$W.b(n-1) + W.a\left[(n-1)\log t_1 + \log \underline{\lvert n+1}\right]$
$(n + 2)t_1$	$(n - 2)W$	$W.b(n-2) + W.a\left[(n-2)\log t_1 + \log \dfrac{\underline{\lvert n+2}}{\underline{\lvert 2}\,\underline{\lvert 2}}\right]$
$2nt_1$	0	$W.a \log(\underline{\lvert 2n} / \underline{\lvert n}\,\underline{\lvert n})$

Comparing these deflections with the deflections for the loading period, we see that the deflections during unloading are greater; in other words, a stress-strain loop due to creep has begun to appear. At any given stress level, the time-independent deflections are the same, and proportional to load. It is clear then, as stated previously, that the effect of change of time-unit is to alter the time-independent and time-dependent parts by amounts proportional to the load, i.e., to shear over the diagram of time-dependent deflections. For the time being, we are interested only in hysteresis loop widths, and hence we need not be interested in the value of the time unit.

For the first half-cycle, the time-dependent part of the deflection, dropping the factor W.a, can be tabulated as follow:

<table>
<tr><th rowspan="2">Load.</th><th colspan="2">Deflection.</th></tr>
<tr><th>Load Increasing.</th><th>Load Decreasing.</th></tr>
<tr><td>n.W</td><td colspan="2">$n \log t_1 + \log \underline{\lfloor n}$</td></tr>
<tr><td>(n-1)W</td><td>$(n-1)\log t_1 + \log \underline{\lfloor n-1}$</td><td>$(n-1)\log t_1 + \log \underline{\lfloor n+1}$</td></tr>
<tr><td>2W</td><td>$2 \log t_1 + \log \underline{\lfloor 2}$</td><td>$2\log t_1 + \log(\underline{\lfloor 2n-2} / \underline{\lfloor n-2}\,\underline{\lfloor n-2})$</td></tr>
<tr><td>W</td><td>$\log t_1 + \log \underline{\lfloor 1}$</td><td>$\log t_1 + \log(\underline{\lfloor 2n-1} / \underline{\lfloor n-1}\,\underline{\lfloor n-1})$</td></tr>
<tr><td>0</td><td>0</td><td>$\log(\underline{\lfloor 2n} / \underline{\lfloor n}\,\underline{\lfloor n})$</td></tr>
</table>

The difference between these pairs of quantities is the loop width due to creep, and this difference is seen to be merely the logarithm of a function of factorial quantities dependent on the load history, multiplied by the slope of the logarithmic creep plot W.a. The unit of time measurement T and the time interval t_1 are eliminated; we can thus say that the loop width due to creep is independent of time considerations, and must be the same (if plastic flow is not an interfering factor) irrespective of the value chosen for the time interval; furthermore, the time-dependent deflections at zero load appear to be independent of the time interval, but dependent on the load history. Thus we can conclude:

1. Due to primary creep, a stress-strain loop appears. The observations at zero load depend upon the loading history, but are independent of t_1.
2. The observations at other loads depend on the time interval also. The loop widths are always independent of time interval, the effect of change of t_1 being merely to shear over the observed loop by an amount proportional to the logarithm of the time interval. In this way loops traced with different values of t_1 can be compared.

The next section will show how these theoretical predictions are borne out by experimental results. By continuing now the theoretical development, we will trace the variation of the time-dependent deflections for the first two complete loading cycles. From these deflections the loop widths at each station and hence the loop area due to primary creep can be obtained. Attention in the experimental section will be concentrated on the second complete loop: it will be assumed that in the actual experiment all plastic adjustment takes place in the first loading cycle, and that the second loading cycle represents solely the action of elasticity, primary creep, and elastic hysteresis. If the observed loop width at any station is greater than the calculated value, the difference must be due to the static effect which we call elastic hysteresis.

Our object therefore is to find the values of time dependent deflections for the first two complete loading cycles, at first generally with n loading steps at intervals t_1 apart, and then with four loading steps at intervals 0.5, 1, and 2 minutes apart. This time dependent deflection, as has been remarked above, contains a part k.W.a log t_1 (when the load is k.W) and a part W.a log F where F is a factorial function. In the calculations, we may omit the first term, and reintroduce it later, for the cases where the time interval t_1 differs from one minute. The following are the factorial expressions for the first complete loading cycle.

Time.	Load.	Factorial.
	W	
t_1		$\lfloor\underline{1}$
	$2W$	
$2t_1$		$\lfloor\underline{2}$
	$3W$	
nt_1		$\lfloor\underline{n}$
Unloading, nW to 0.		
	$(n - 1)W$	
$(n + 1)t_1$		$\lfloor\underline{n + 1}$
	$(n - 2)W$	
$(n + 2)t_1$		$\lfloor\underline{n + 2}/\lfloor\underline{2}\lfloor\underline{2}$
	$(n - 3)W$	
$2nt_1$		$\lfloor\underline{2n}/\lfloor\underline{n}\lfloor\underline{n}$
Loading, 0 to $-nW$.		
	W	
$(2n + 1)t_1$		$\dfrac{\lfloor\underline{2n + 1}}{\lfloor\underline{n + 1}\lfloor\underline{n + 1}}$
	$2W$	
$(2n + 2)t_1$		$\dfrac{\lfloor\underline{2n + 2}}{\lfloor\underline{n + 2}\lfloor\underline{n + 2}}$
$3nt_1$		$\dfrac{\lfloor\underline{3n}}{\lfloor\underline{2n}\lfloor\underline{2n}}$
Unloading, $-nW$ to 0.		
$4nt_1$		$\dfrac{\lfloor\underline{4n}\lfloor\underline{n}\lfloor\underline{n}}{\lfloor\underline{3n}\lfloor\underline{3n}}$

The third quarter of the loading cycle is obtained as before by adding the appropriate terms to the memory function and substituting for the current value of time t as before. It will be noticed that the shape of the factorial expression changes on passing through the maximum load points, where a discontinuity in the loading occurs, but not when passing through the zero points.

From this observation, it is possible to write down the factorial expressions for the zero and maximum load points for the second cycle. The factorial expression for any intermediate loading can then be obtained from the nearest subsequent expression tabulated.

Second Loading Cycle.

Time.	Factorial.
$5nt_1$	$(\lfloor\underline{5n}\,\lfloor\underline{2n}\,\lfloor\underline{2n})/(\lfloor\underline{4n}\,\lfloor\underline{4n})$
$6nt_1$	$(\lfloor\underline{6n}\,\lfloor\underline{3n}\,\lfloor\underline{3n})/(\lfloor\underline{5n}\,\lfloor\underline{5n}\,\lfloor\underline{n}\,\lfloor\underline{n})$
$7nt_1$	$(\lfloor\underline{7n}\,\lfloor\underline{4n}\,\lfloor\underline{4n})/(\lfloor\underline{6n}\,\lfloor\underline{6n}\,\lfloor\underline{2n}\,\underset{2}{\lfloor\underline{2n}})$
$8nt_1$	$\lfloor\underline{8n}\,\left[(\lfloor\underline{5n}\,\lfloor\underline{n})/(\lfloor\underline{7n}\,\lfloor\underline{3n})\right]$

Numerical Evaluation.

The following table gives the value of the factorial function F evaluated for n = 4 for the first two loading cycles.

First Cycle.		Second Cycle.	
Time/t_1	F	Time/t_1	F
1	1	17	1/7.57
2	2	18	1/2.29
3	6	19	1.81
4	24	20	9.04
5	120	21	53.2
6	180	22	90.2
7	140	23	77.3
8	70	24	41.7
9	25.2	25	16.0
10	7.0	26	4.67
11	1.57	27	1.1
12	1/3.4	28	1/4.67
13	1/21.2	29	1/28.1
14	1/37.8	30	1/48.8
15	1/33.9	31	1/42.7
16	1/19.1	32	1/23.6

The values of $\log_{10}F$ are tabulated below. These figures, multiplied by the slope of the logarithmic creep plot for load W (that is, W.a), give the time-dependent deflections for step-by-step cyclic loading, when the time interval is equal to one minute.

Load.	First Cycle.					Second Cycle.				
4W		1.380					.956			
3W	.778		2.079			.257		1.725		
2W	.301		2.255			-.360		1.954		
W	0		2.146			-.879		1.888		
0			1.845		-1.280	-1.280		1.620		-1.376
-W			1.401		-1.530			1.204		-1.630
-2W			.845		-1.577			.668		-1.688
-3W			.196		-1.326			.041		-1.449
-4W				-.530					-.671	

When t_1 differs from 1 minute, we have to add the quantity k.Walog t_1 when the load is k.W to obtain the coefficients of time-dependent deflection. The values for t_1 = 2 minutes and for t_1 = 0.5 minute for the second loading cycle are given below.

Load.	t_1 = 0.5 min.					t_1 = 2 mins.				
4W		-.248					2.160			
3W	-.646		.822			1.160		2.628		
2W	-.962		1.352			.242		2.556		
W	-1.180		1.587			-.578		2.189		
0	-1.280		1.620		-1.376	-1.280		1.620		-1.376
-W			1.505		-1.329			.903		-1.931
-2W			1.270		-1.086			.066		-2.290
-3W			.944		-.546			-.862		-2.352
-4W				.533					-1.875	

FIG. 7

TIME - DEPENDENT DEFLECTIONS

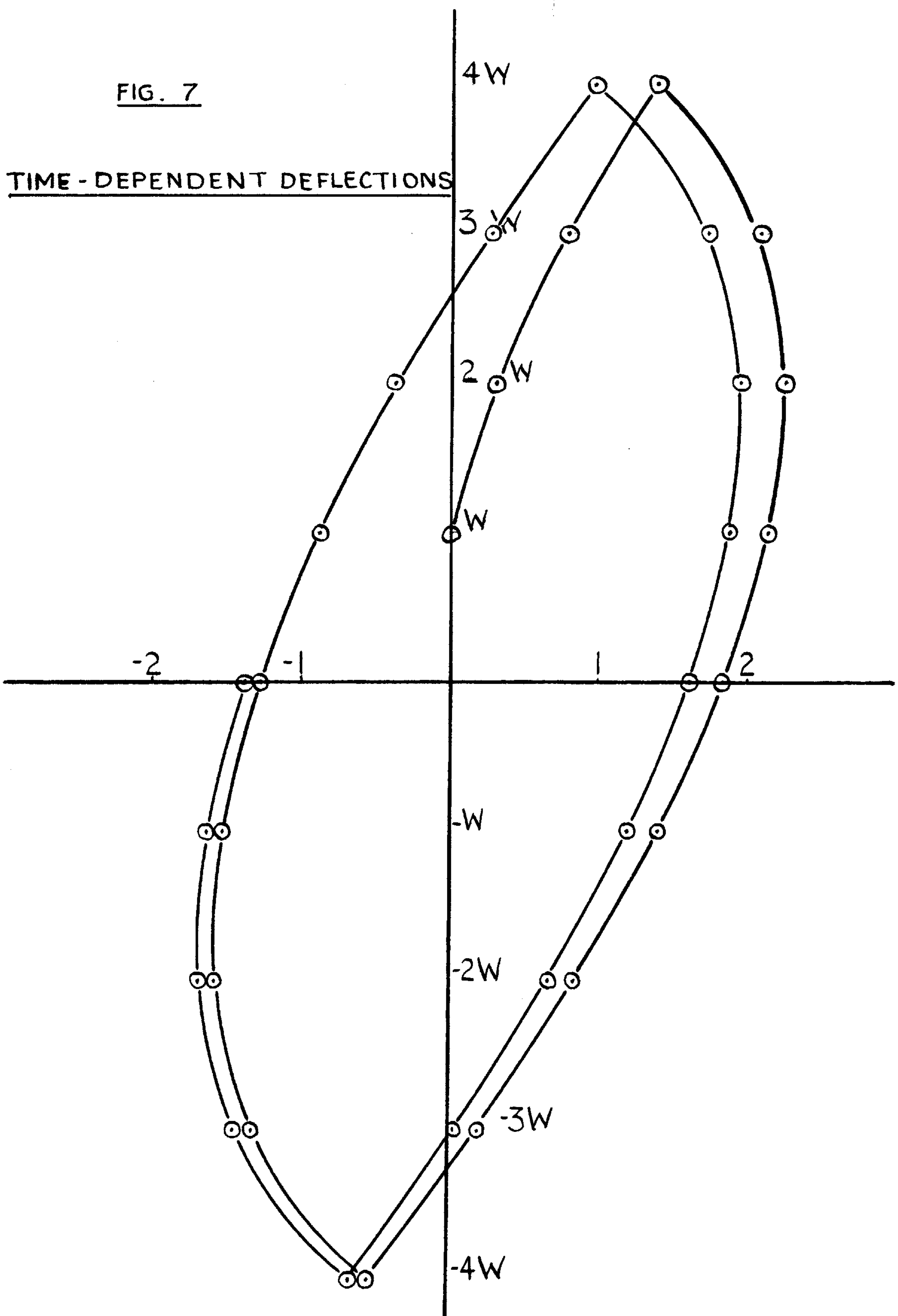


FIG. 8.

TIME - DEPENDENT

DEFLECTIONS.

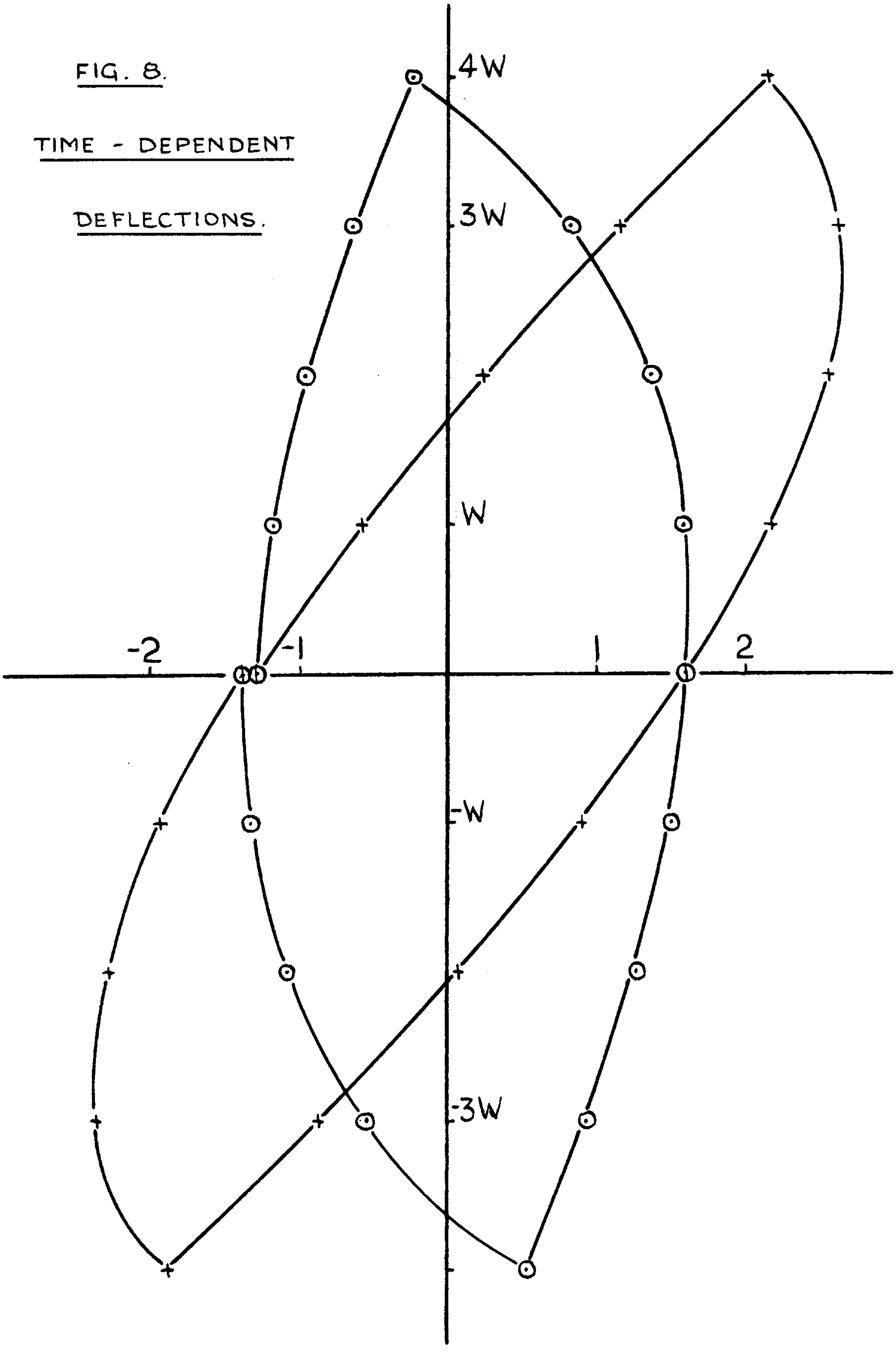


Fig. 7 is a plot of the first table on p. 26; fig. 8 is a plot of the second. Each point represents of course a time-dependent deflection, which is the difference between the deflection obtained at a given load after the loading history corresponding to the point in question, and that deflection which would be obtained one minute after an equal load were placed instantaneously in position (the time-independent deflection). This assumes primary creep according to Boltzmann's law to be the only inelastic effect. In Part III it will be shown that only part of the observed loop can be accounted for by creep, and the remainder must therefore be ascribed to elastic hysteresis.

III. Damping due to Hysteresis and Creep.

The decrement of free torsional oscillations of a system, of which the material under test in the form of a cylinder forms the elastic member, is due in the absence of mechanical friction and air damping (and these can be made very small) to the material damping of the specimen. In the case of the bakelite specimens under consideration, damping appears to be due to primary creep and to elastic hysteresis. To confirm this quantitatively, we wish to measure the damping directly, and compare this with the value computed from the magnitudes of the primary creep and of the elastic hysteresis obtained from the cyclic loading test.

Specific Damping Capacity.

A measure of damping is the specific damping capacity ψ, which is the ratio of the energy dissipated in a cycle at a given strain range to the maximum strain energy in the cycle. In the case of torsional shear of solid cylindrical specimens, each annulus is working through a strain cycle of different range. The ratio of the energy dissipated in one cycle to the maximum strain energy in the specimen at a given range of surface shear strain thus represents a mean value over the cross-section; this value we refer to as the mean specific damping capacity ψ_m. If we consider a thin tube of the same diameter working through the same range of twist per unit length, the damping capacity of the tube will be different from that of the solid cylinder, though they be working through the same strain range, since the damping capacity of the thin tube represents the value for the annulus of material on the surface only of the solid cylinder, and therefore we expect this to be different from the mean specific damping capacity ψ_m. The damping capacity of this surface annulus is called accordingly the surface specific damping capacity ψ_o. The ψ_o curve can be derived from the ψ_m curve.While in general the ψ_o curve is of greater scientific importance, we are here interested only in the ψ_m curve, since we are measuring the mean value of elastic hysteresis over the cross-section.

Logarithmic Decrement.

If A_1 and A_2 are successive amplitudes of free oscillation, then the logarithmic decrement is

$$\delta = \ln A_1/A_2.$$

This value is taken to refer to the strain corresponding to $1/2(A_1 + A_2)$, though this may not be strictly true when the decrement is high, as in bakelite. If we write δ according to the formula:

$$\delta = \frac{A_1 - A_2}{1/2(A_1 + A_2)} = \frac{\text{Difference in Amplitude}}{\text{Mean Amplitude}}$$

then δ represents the value of the logarithmic decrement within 1% up to $\delta = 0.35$; the expression

$$\delta(1 + \delta^2/12)$$

represents the decrement accurately over the whole working range.

Logarithmic Decrement and Mean Specific Damping Capacity.

If the damping is small, then Hooke's Law is very nearly obeyed, and the strain energy is therefore proportional to the square of the amplitude. The strain energies corresponding to amplitudes A_1 and A_2 are $k.A_1^2$ and $k.A_2^2$, say. The mean specific damping capacity is then

$$\psi_m = \frac{k.A_1^2 - k.A_2^2}{1/2.k(A_1^2 + A_2^2)}$$

referred to a strain corresponding to $1/2(A_1 + A_2)$.

$$\text{Then } \psi_m = \frac{(A_1 - A_2)(A_1 + A_2)}{1/4\left[(A_1 + A_2)^2 + (A_1 - A_2)^2\right]}$$

$$= \frac{(A_1 - A_2)}{1/4(A_1 + A_2)\left[1 + \left(\frac{A_1 - A_2}{A_1 + A_2}\right)^2\right]}$$

$$= \frac{2\delta}{1 + \delta^2/4} \simeq 2\delta$$

where δ is as before the ratio of the difference in amplitude to the mean amplitude. Thus the mean specific damping capacity is equal to twice the logarithmic decrement within 1% if δ is less than .20, provided that the strain energy is proportional to the square of the amplitude; the reasons we have for assuming that this is so are given below. We will find in our tests that δ will not exceed .20, and hence we will use the above formula for finding ψ_m.

Relation between Strain Energy and Strain.

We wish to show that in the case of bakelite, it is reasonable to assume that the strain energy for specific damping capacity purposes is proportional to the square of the strain. If we can do this, then we may use the formula obtained above for the mean specific damping capacity. Furthermore, we have a basis for assessing the strain energy corresponding to the observed hysteresis loop, so that the contribution of elastic hysteresis to the specific damping capacity may be computed.

<u>The Static Loading Curve and Free Oscillations</u>.

Let us assume at first that creep is absent. When the damping due to hysteresis is high, Hooke's Law is no longer even approximately obeyed, and the strain energy given to the specimen in loading up to a given value of surface stress is appreciably greater than the strain energy recovered on unloading, the difference of course corresponding to half the area of the hysteresis loop. Let us consider the known behaviour of metals which possess the property of elastic hysteresis to a large degree.

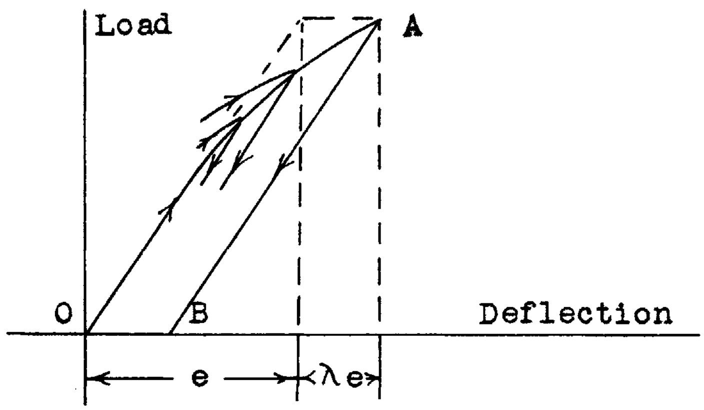


Fig. 9.

If the material originally free from any permanent set be loaded from O to A, then on unloading a permanent set OB is produced. If on the other hand the specimen be provided with a mass or inertia bar so that it can execute free oscillations with a starting amplitude corresponding to A, the oscillations will gradually die away, and the specimen will finally be left without any permanent set; this is analogous to the demagnetisation of a magnetised specimen by placing it in an alternating field of intensity decreasing to zero.

It is assumed that the successive amplitude maxima of the free oscillation, when plotted on the load-extension diagram (fig. 9), lie on the original static loading curve OA. We take the mean strain energy corresponding to any given deflection to be equal to the area under the original loading curve up to that deflection. In the case of metals, this curve is linear up to a certain point and then (under certain conditions) becomes distinctly curved. It is assumed that the total strain consists of an elastic part e and an inelastic part λe, the ratio λ increasing rapidly at a critical stress. In the case of bakelite, λ appears to be nearly constant, and therefore the tips of successive hysteresis loops lie on a straight line passing through the origin. The strain energy is then proportional to the square of the amplitude.

The reason for this statement is as follows. If c is the ratio of the mean to the maximum width of the hysteresis loop, then we have approximately

$$\psi_m = 8 . c . \lambda$$

and c is a function of λ. Damping tests show that ψ_m for bakelite is nearly constant over a wide range of stress; hence λ may be assumed nearly constant.

Specific Damping Capacity due to Creep.

The specific damping capacity due to logarithmic creep following the superposition law of Boltzmann has been calculated for sinusoidal oscillations by different methods, by Boltzmann,[3] by Bennewitz,[2] and by Becker.[7] Boltzmann calculated the decrement of free (torsional) oscillations. Bennewitz and Becker calculated the specific damping capacity under forced harmonic vibrations. The conclusions were the same: the specific damping capacity was independent of amplitude and of frequency. If the logarithmic creep be given by:

$$x = B + A \log_{10} t$$

where B is the deflection in unit time, then the specific damping capacity due to creep is given by:

$$\psi_c = \frac{\pi^2 A}{M.B}, \text{ where } M = \log_e 10.$$

Then $\psi_c = 4.28\ A/B.$

Here B will be taken to be the elastic part of the deflection, that is, half the difference between the total strain range and the hysteresis loop width at zero load. The total strain range will be the value observed in the hysteresis tests, corrected for time-dependent deflection and with the time unit adjusted to 10^{-3} minutes.

Specific Damping Capacity due to Elastic Hysteresis.

The difference between each pair of observations at the same stress level, corrected for time-dependent deflection, gives the elastic hysteresis loop width; the sum of these widths multiplied by the mean distance between the stations gives a measure of the area of the loop. If P is the total corrected strain range, adjusted for a time unit of 10^{-3} minute, and 4W the maximum load, the value of the strain energy corresponding to the maximum load can be represented by P.W. The specific damping capacity due to hysteresis is then given by

$$\Psi_h = \frac{\text{loop area.}}{\text{strain area}}.$$

The mean damping at a given stress range computed from the creep and from the elastic hysteresis tests thus becomes $\Psi_c + \Psi_h$. Owing to the theoretical and experimental difficulties involved, we would expect only a fair agreement of the computed value with the observed value of damping at the same stress range.

Part III. Experimental.

I. Description of Materials Used.

Four types of material with bakelite (phenol or cresol formaldehyde) base were used: transparent resin, wood-flour filled resin, cord-reinforced material and fabric-filled moulding material. To explain the differences in structure of these materials, it would be well to describe briefly the mode of manufacture.

Phenol formaldehyde and cresol formaldehyde resins are made by heating phenol (or cresol) with an aqueous solution of formaldehyde in the presence of a suitable catalyst. After some time an amber-coloured fluid separates out: this fluid is formed by a polymerisation process, and has the characteristics of a resin in that it has no definite melting point or molecular weight, and that when solid is amorphous in structure. On further heating, the degree of polymerisation increases, and this resin solidifies to a brown mass which is insoluble in most solvents and infusible, but which chars slowly at an elevated temperature. This is called the C stage resin; the first stage is referred to as the A stage. In practice, the process is arrested at the first stage, and the resin run off and allowed to solidify.

It can then be cast or moulded under temperature and pressure to the desired shape. This is the usual transparent bakelite resin as used for photoelasticity work. One of the specimens tested was of this type. It is doubtful whether the material as supplied is completely polymerised.

Alternatively, the A stage resin can be ground, and mixed with wood flour, accelerator, and dye, thus forming the commercial moulding powder. From a slab of material moulded from such a powder a second specimen was prepared. In order to obtain higher strength, it is possible to reinforce the resin with textile material. The A stage resin is dissolved in alcohol, and the textile reinforcement drawn through the solution until sufficiently impregnated; it is then baked in ovens to remove the solvent and to cause a partial polymerisation. The textile material is then moulded under pressure and temperature in the usual way; the resin changes to the infusible form, acting as a bond for the textile reinforcement. By using cotton cloth, the familiar laminated material is obtained. By using a material consisting almost entirely of parallel cotton cords, we obtain a moulded material of increased strength and stiffness; this is cord material, a specimen of which was used for the tests. Finally, if small pieces of fabric be used instead of cotton cloth, the resulting material possesses a certain amount of flow when under pressure and temperature. It is therefore marketed as shock-proof fabric moulding material, since its resistance to mechanical shock is much greater than that of the wood flour material. Of the four specimens used, three were isotropic, and the fourth (cord material) had an axis of symmetry along the axis of the specimen.

II. <u>Description of Apparatus</u>.

The method of reversed torsion was chosen as being the simplest method of obtaining reversed static and dynamic loading, in which creep, elastic hysteresis, and damping could all be measured without removing the specimen. Possible trouble from temperature fluctuations was greatly reduced by the use of specimens in shear; this method enabled the deflections to be projected optically on to a large graduated scale, so that corresponding values of time and deflection could be observed and recorded by a single observer. Solid cylindrical specimens, with squared ends, as shown in fig. 10, were used.

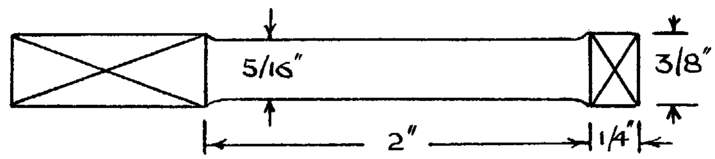


Fig. 10.

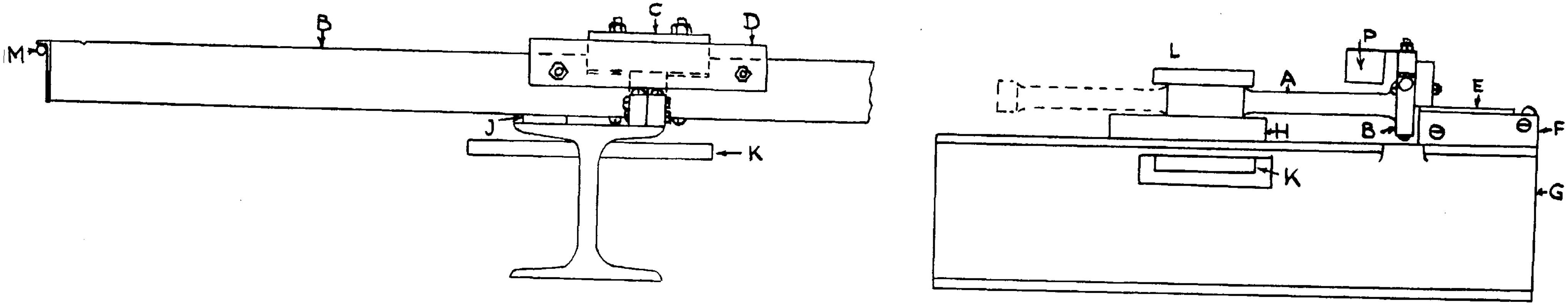


Fig. 11.

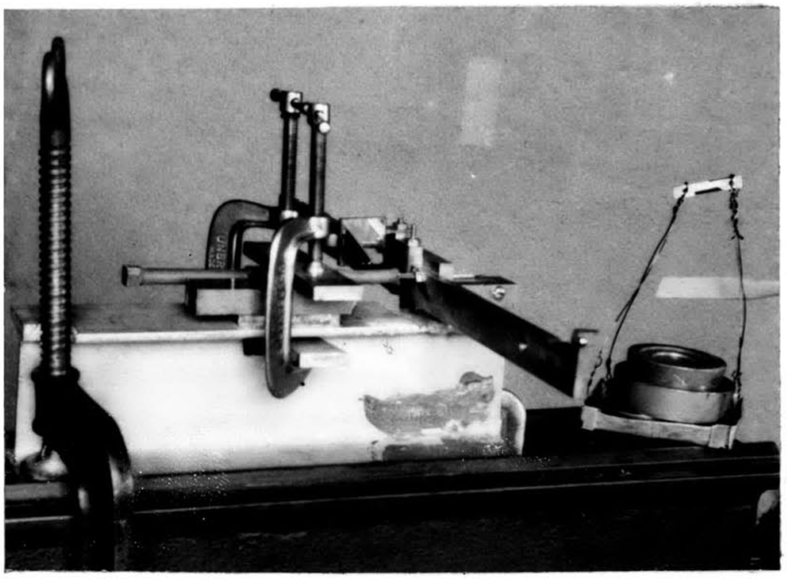


Fig. 12.

Torsion Apparatus.

A sketch of the apparatus, approximately half size, is shown in fig. 11. A photograph of the apparatus is shown in fig. 12. The specimen A was in many cases double-ended. The specimen was clamped to the bar B by means of the piece C. The lower surface of the piece D contained the axis of the specimen produced. This piece rested on a knife-edge E consisting of a razor-blade, the razor-blade being fixed in a holder F which in turn was fixed to a length of duralumin girder G. The fixed end of the specimen lay on a block H. A stiff brass strip K was passed through a hole in the web of the girder, and another strip L was laid on top of the squared end of the specimen and on a block J (not shown in the front elevation in fig. 11). This block was of the correct height so that the bar L was horizontal. By means of two clamps (not shown in fig. 11) over the block J and the specimen respectively, the fixed end of the specimen could be firmly clamped to the girder. Before clamping down, care was taken to align the specimen accurately so that the axis of the specimen produced lay along the edge of the blade. A small steel ball M was fixed to one end of the bar B. The front-silvered mirror P was attached to the bar in such a way that it could easily be removed without disturbing the specimen. The bar B was provided on its upper surface with two grooves exactly 7.5 inches from the vertical plane containing the axis of the specimen; a scale pan with knife edge support wighing exactly 2 ozs. was constructed. The girder G was clamped down to a lathe-bed, where it was free from disturbance and reasonably free from building vibration.

<u>Optical Arrangement</u>.

All observations were obtained using optical methods: the creep and hysteresis readings were made on a long graduated scale, the mirror P being used to reflect a light beam: the damping tests were recorded on 35 mm. cinematographic film, a high-light reflected from the steel ball tracing the record.

A metal plate having a sharp straight lower edge was used to cover the upper half of the lens of a projection lamp. The light from the lamp was passed through a focussing lens on to the mirror P, and then reflected back to a vertical scale about five feet long divided into gradations of .05 inch. By means of the focussing lens a sharp image of the horizontal edge of the metal plate could be focussed on the scale, and thus deflections corresponding to .01 inch on the scale could be read. The optical lever arm was 148.4 inches. Hence a deflection of .01 inch on the scale corresponded to a twist of 6.73×10^{-5} radian, and to a surface shear strain (bar diameter .312 inch) of 5.25×10^{-6}.

For the damping tests, the lamp was placed several feet away, in the left hand direction with respect to the front elevation in fig. 11. The light was adjusted to shine on the steel ball. A box placed over the apparatus was provided with two vertical slits, one at the side to allow the light from the lamp to fall on the steel ball, and one at the front facing the end of the bar B. An electrically driven cinematograph camera with the intermittent motion removed was placed in front of this slit, and focussed so that a sharp image of the high-light on the ball was formed on the film. In order to obtain a reference line, a pin was fixed to an iron rod clamped to the girder, so that the head of the pin lay as near as possible to the steel ball, and also reflected light from the lamp into the camera. This light from the pin traced a straight line on the film, which was used as a reference line.

III. Method of Conducting Tests.

In the hysteresis tests, the load increments were 8 ozs. each, the weights being applied and removed in the same order. In the case of high creep materials, difficulty was experienced when the specimen was carrying large torque. The theory requires that the load be changed immediately the scale be read. In practice, two or three seconds were required to effect this change, by which time the deflection must have altered appreciably. With the low damping materials each change of load set up a slight vibration which caused some difficulty in making the subsequent reading. The former difficulty could have been overcome by making the readings in the middle of the load interval, and modifying the theory accordingly. In all the creep tests a load of 2 lbs. was used (corresponding to a torque of 15 lbs. ins. and a nominal surface shear stress of 2520 lbs./sq.in.).

Before conducting a damping test, the laboratory was darkened as far as possible. In order to take a record a length of cotton was attached to the scale pan and to the torsion bar, the cotton being lightly secured by plasticine in the groove in the bar. The motor driving the camera was started, and the scale pan allowed to hang freely. The load was released by burning through the cotton with a match. The oscillations of the bar were then recorded on the moving film by means of the high light reflection from the steel ball. The runs were usually repeated several times, good agreement being obtained between successive runs.

Since the static hysteresis tests were carried out with a maximum load range of ± 2 lbs. on the torsion bar, it was required to find the damping at a deflection corresponding to this load range. It will be realised that with high damping materials, a certain amount of extrapolation is necessary to obtain the damping at the stress range corresponding to the deflecting load. The precaution was usually taken, therefore, of making damping runs with a deflecting load slightly in excess of 2 lbs. One or two dummy runs were made before the recorded runs in order to condition the specimen.

Reduction of Observations.

The developed film was placed in a photographic enlarger and the record focussed on to a sheet of paper. The successive deflections as well as the base line were then marked on the paper. From this record, the perpendicular distances of the marked points from the base line were measured, and hence the successive amplitudes obtained. By measuring also the distances from the base line of the original deflection and of the ultimate position of rest, a length corresponding to the applied load could be obtained. Since linear distribution of stress has been assumed in the theoretical section, this length gives a scale for surface stress, since the length corresponding to a deflecting load of 2 lbs. is equivalent to a surface shear stress of 2520 lbs./sq.in.

By comparison, an actual photographic enlargement was made of one record, and the damping calculated from measurements taken from this enlargement. No appreciable divergence was found between this and the other method. It was accordingly decided that the preparation of photographic enlargements was unnecessary.

Neither the speed of the film, frequency of vibration, nor the actual magnitude of the deflections have been measured, since these quantities were not essential.

Arrangement of Experimental Work.

The first part of the experimental work is concerned with the testing of the superposition laws of creep by means of the three creep tests of Part I (figs. 1, 2, and 3) for each of the four materials. The second part of the experimental work involved for each material three tests, the tests for each material being carried out on the same day, to reduce the effect of temperature and humidity variations. These tests comprised at least two step-by-step cyclic loading tests of two complete cycles. A creep test was carried out with the maximum load, following the scheme of fig. 2(a), to determine the creep constant for correcting the observations, and finally a damping test was made.

IV. Results of Creep Tests.

In figs. 13 to 24 are shown the results of creep tests I, II, and III (figs. 1, 2, and 3) on cord material, fabric moulding material, wood-flour filled material, and transparent (unfilled) resin. For Test I, each loading period was of 10 mins. duration, the load being 2 lbs. at 7.5 ins. radius, except in the case of bakelite resin, when the load was 1.5 lbs. In Test II, the experiment was carried out with loads of 2lbs. and of 1 lb., the load being applied and removed twice. The creep and recovery intervals were of 10 mins. each, with the exception of the case of the bakelite resin, when they were of 15 mins. duration. In this test, observations in the first creep period have not been plotted. In Test III, a load of 2 lbs. was applied from $t = 0$ mins. to $t = 16$ mins., and then removed; the load was then applied on the opposite arm of the torsion bar from $t = 18$ mins. to $t = 20$ mins., and then removed. Deflections were obtained for the original and reversed creep, and for the first and second recovery periods.

The creep results show the connection between scale reading in inches and equivalent time. The tables below give the equivalent times corresponding to the observed times for Test I, Test II (time interval 15 mins.), and Test III. The table for Test I refers also to Test II for the first three materials. In all cases, the scale or scales of deflection referring to creep are plotted on the left-hand side, and those referring to recovery on the right-hand side. Semi-logarithmic paper was used to facilitate plotting, the equivalent times of 0.1, 1, 10, and 100 mins. being marked in.

A consistent symbolism was used on all the graphs; the symbol referring to each creep or recovery period will be found on reference to figs. 1, 2, and 3. The arrows on each line indicate the direction in which the equivalent time is changing.

Observed and Equivalent Times.

Test I and Test II.

Loading and recovery intervals are of 10 mins. duration.

Time from beginning of loading interval, mins.	Equivalent Times in Minutes.				
	First Recovery	Second Creep	Second Recovery	Third Creep	Third Recovery
.33	31	.66	46.2	.87	57.6
.50	21	.98	31.2	1.3	39
.75	14.3	1.45	21.2	1.92	26.4
1.0	11	1.91	16.2	2.5	20.2
1.5	7.7	2.8	11.2	3.7	14.0
2	6	3.7	8.7	4.8	10.8
3	4.3	5.3	6.2	6.9	7.7
5	3	8.3	4.2	10.7	5.1
7	2.43	11.1	3.33	14.1	4.03
9.75	2	14.7	2.7	18.4	3.2

Test II.

Loading and recovery intervals of 15 mins. duration.

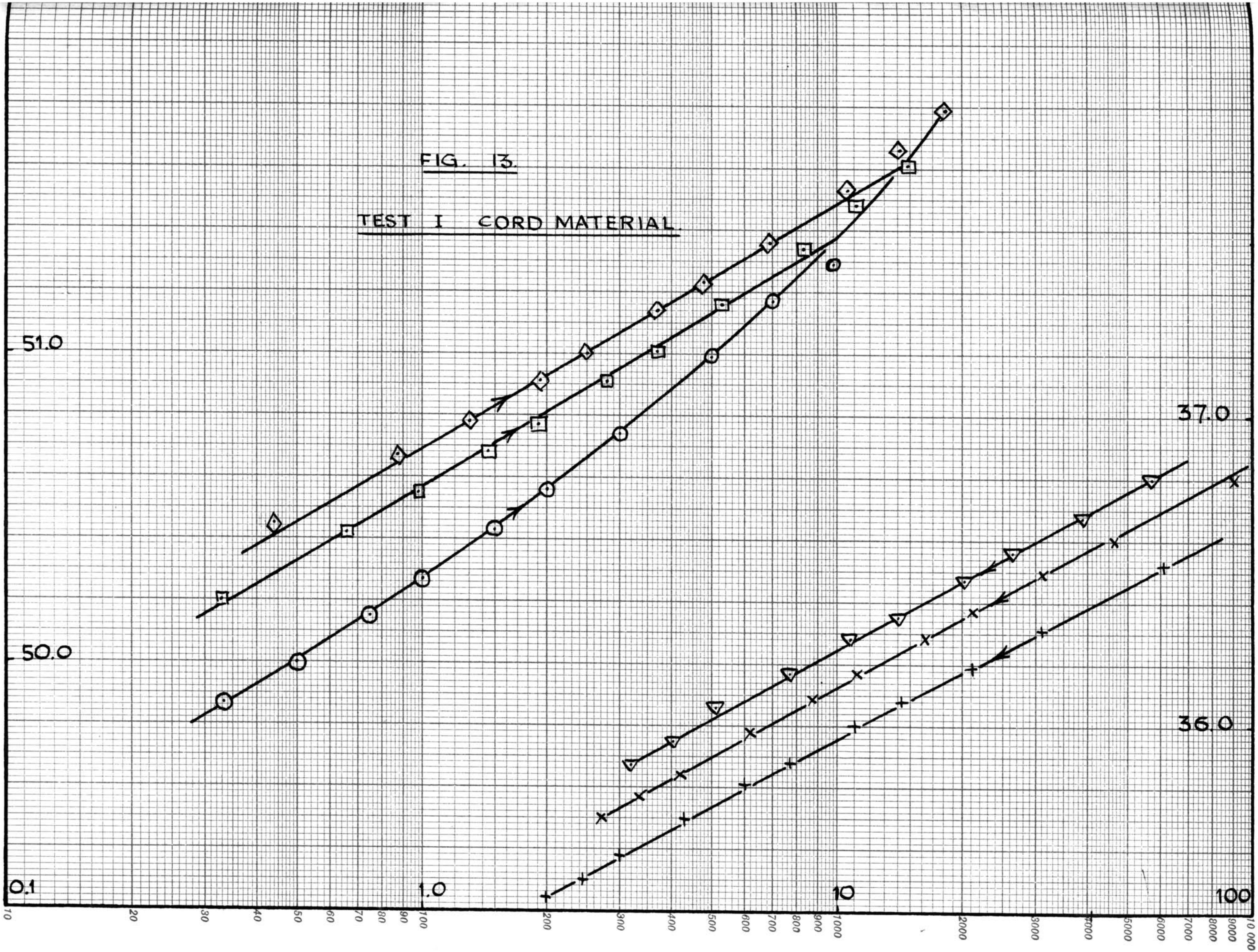
Time from beginning of loading interval, mins.	Equivalent Times.		
	First Recovery	Second Creep	Second Recovery
.33	46	.66	68.8
.50	31	.98	46.3
.75	21	1.46	31.2
1.0	16	1.94	23.8
1.5	11	2.86	16.2
2	8.5	3.76	12.5
3	6	5.5	8.72
5	4	8.75	5.71
7	3.14	11.8	4.42
10	2.5	16.0	3.44
14.75	2	22.5	2.67

Test III.

	Time from beginning of test, mins.	Equivalent time, mins.
First Recovery	16.17	97
	16.33	49
	16.50	33
	16.75	22.3
	17	17
	17.5	11.7
	17.75	10.1
Reversed Loading	18.17	50.3
	18.33	23.6
	18.50	14.8
	18.75	9.1
	19	6.3
	19.5	3.7
	19.75	3.0
Second Recovery	20.17	.37
	20.33	.67
	20.50	.91
	20.75	1.19
	21	1.4
	21.5	1.67
	22	1.83
	23	1.97
	24	2.00
	26	1.95
	28	1.87
	30	1.79
	32	1.68
	36	1.60
	40	1.51
	50	1.38
	60	1.30
	70	1.25

FIG. 13.

TEST I CORD MATERIAL.



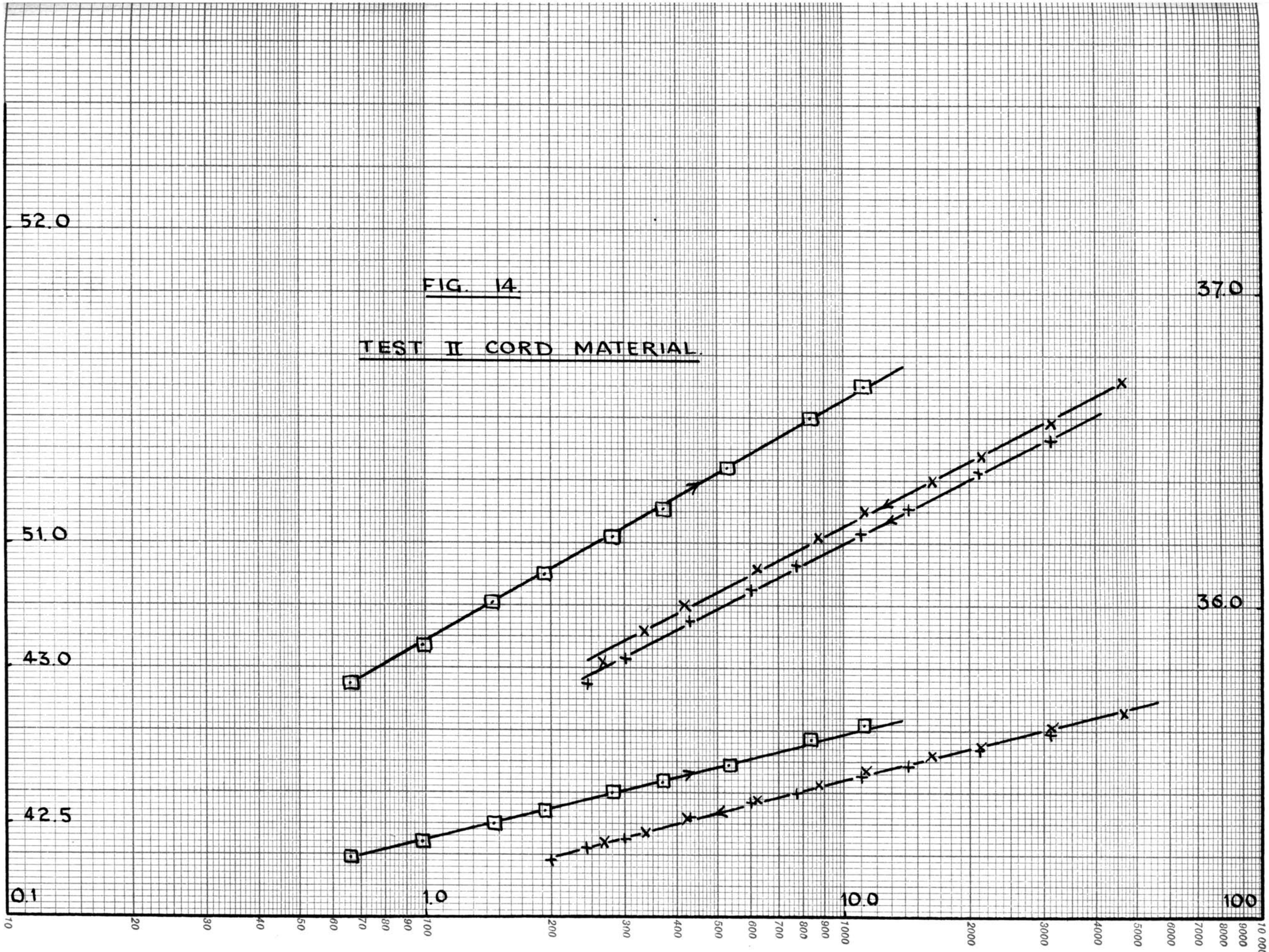


FIG. 14.

TEST II CORD MATERIAL.

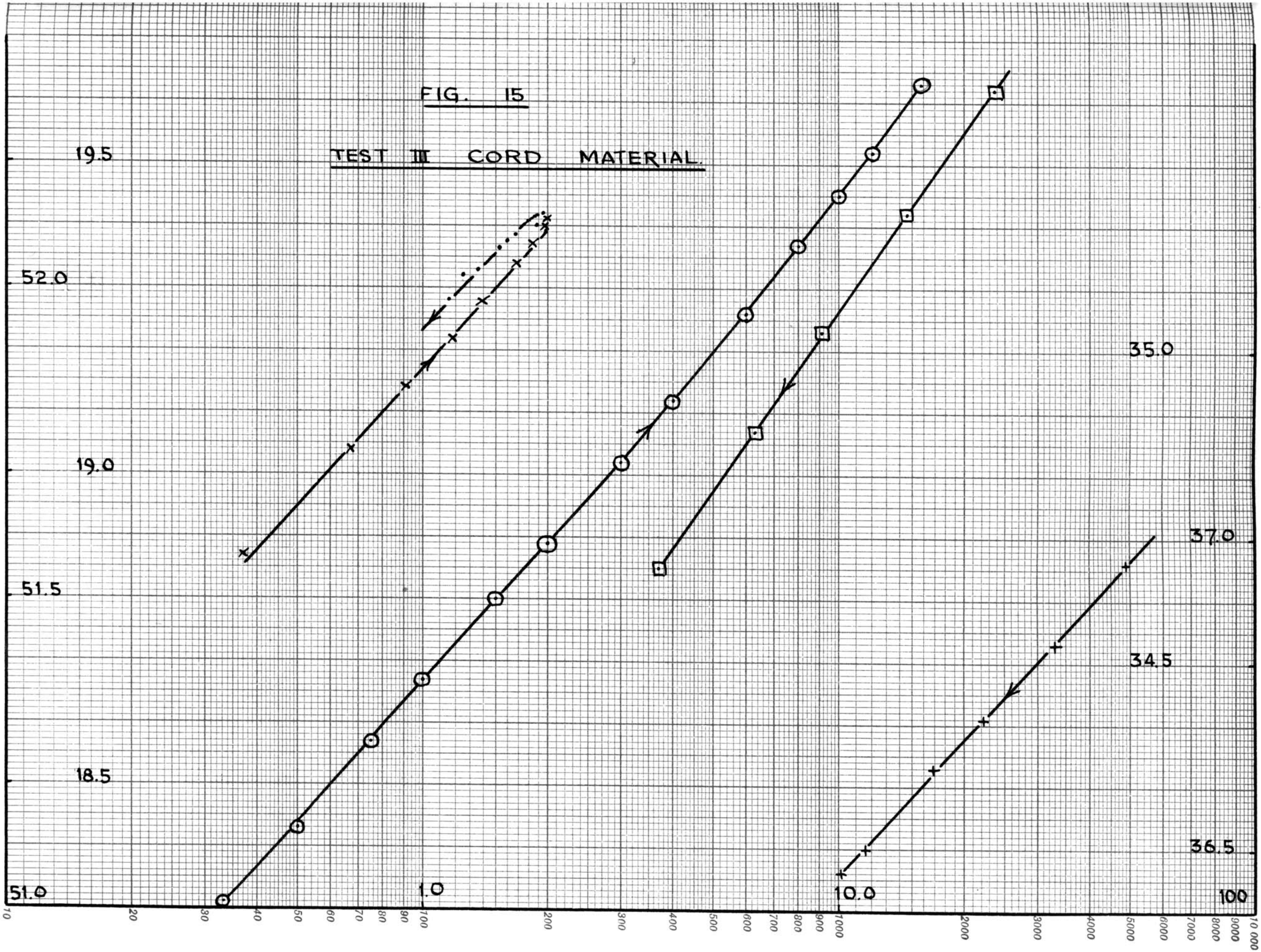
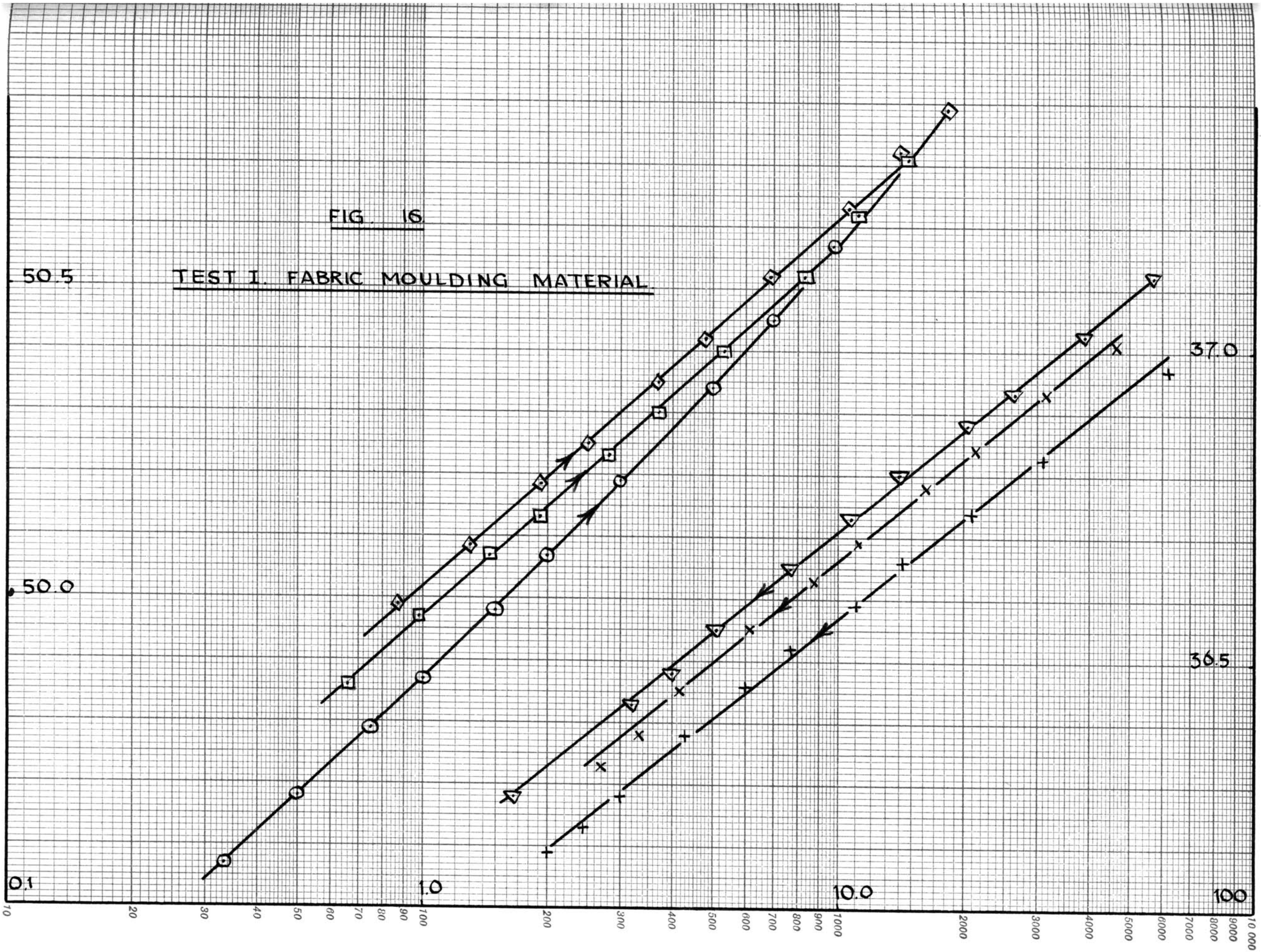


FIG. 15

TEST III CORD MATERIAL.

FIG. 16

TEST I. FABRIC MOULDING MATERIAL



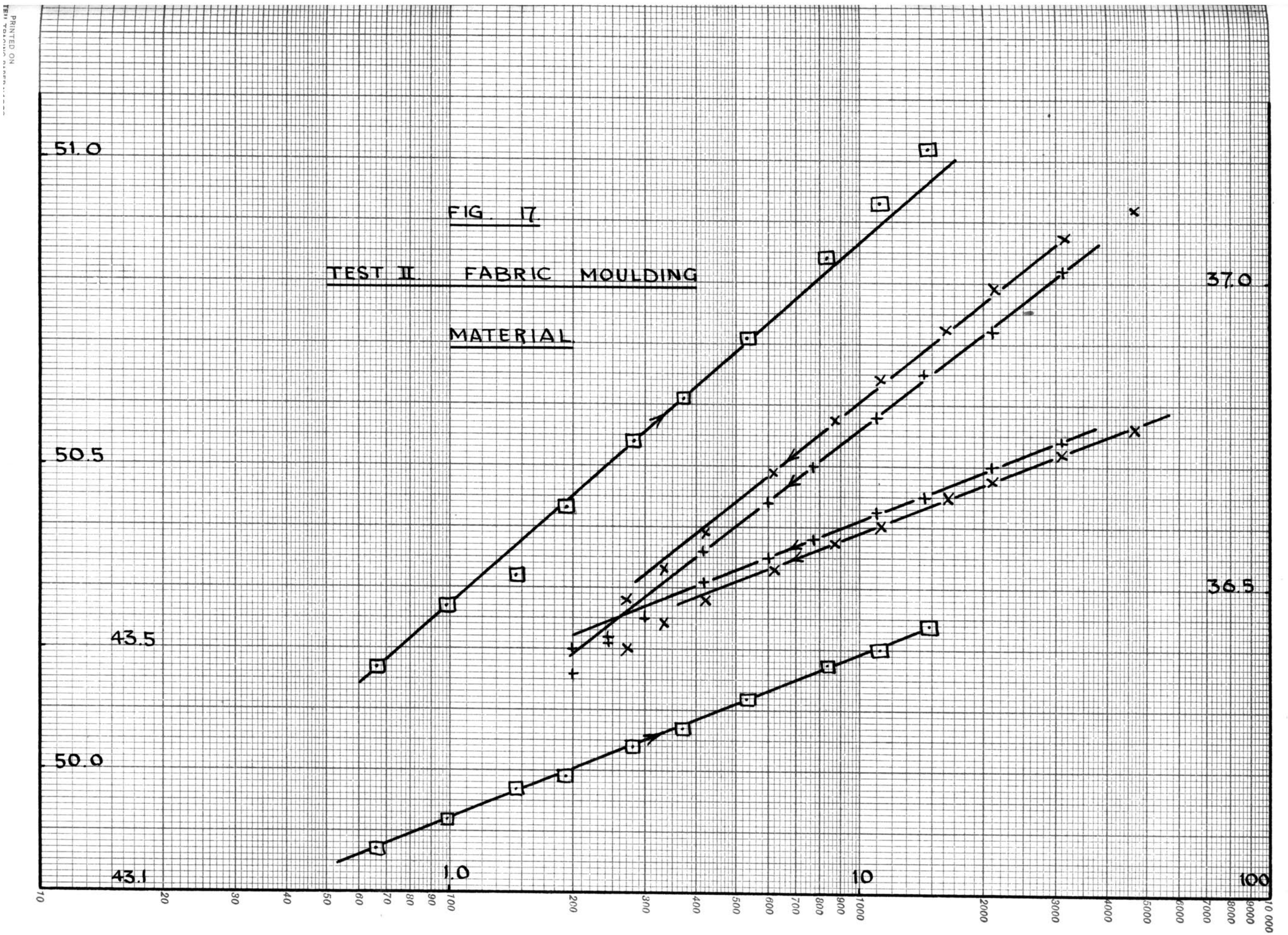
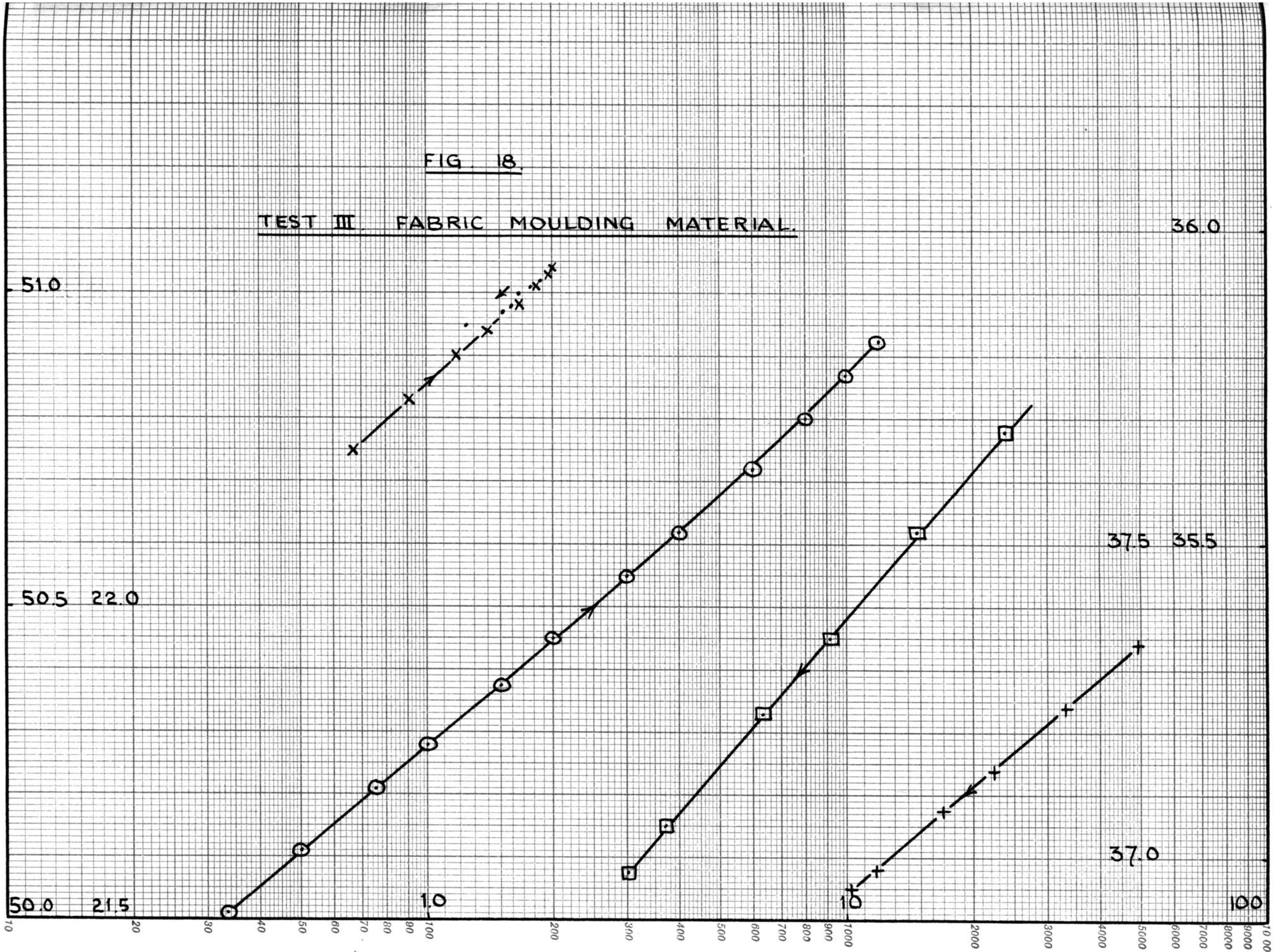


FIG. 17.
TEST II. FABRIC MOULDING
MATERIAL.
51.0
50.5
50.0
43.5
43.1
37.0
36.5
1.0
10
100

FIG. 18.

TEST III. FABRIC MOULDING MATERIAL.



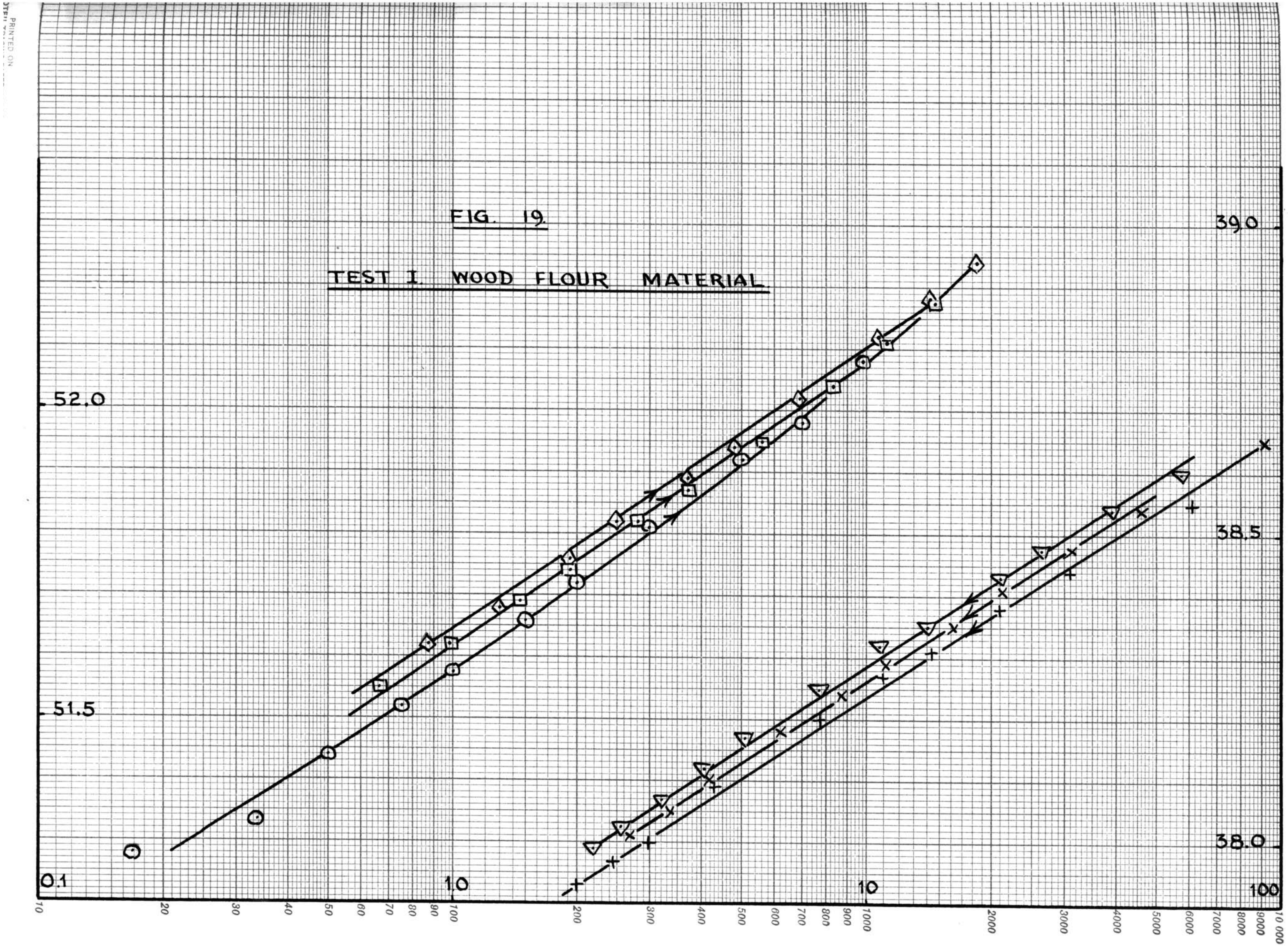


FIG. 19.
TEST I. WOOD FLOUR MATERIAL
52.0
51.5
39.0
38.5
38.0
0.1
1.0
10
100

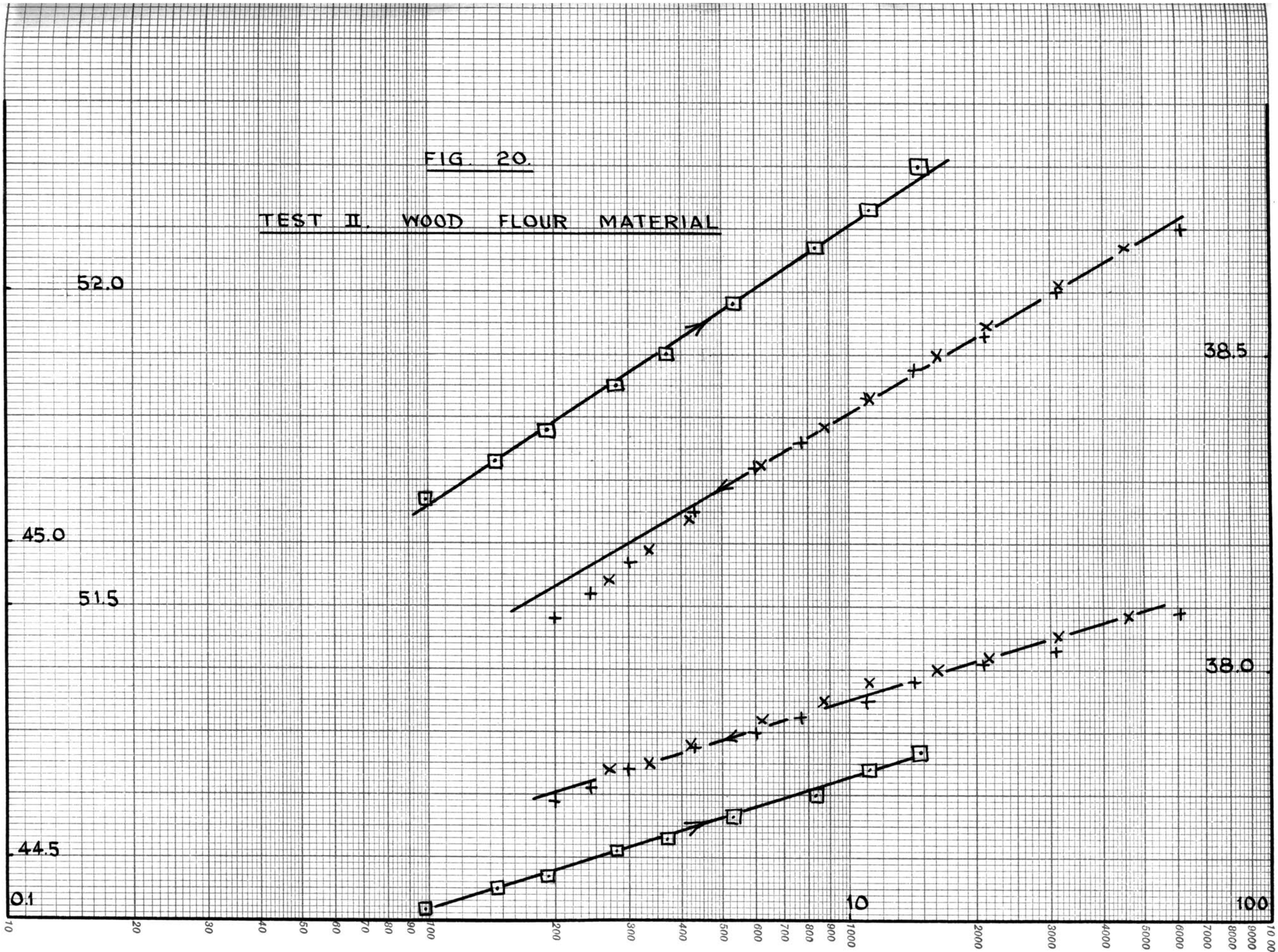


FIG. 20.
TEST II. WOOD FLOUR MATERIAL
52.0
45.0
51.5
44.5
38.5
38.0
0.1
10
100

FIG. 21.

TEST III. WOOD FLOUR MATERIAL.

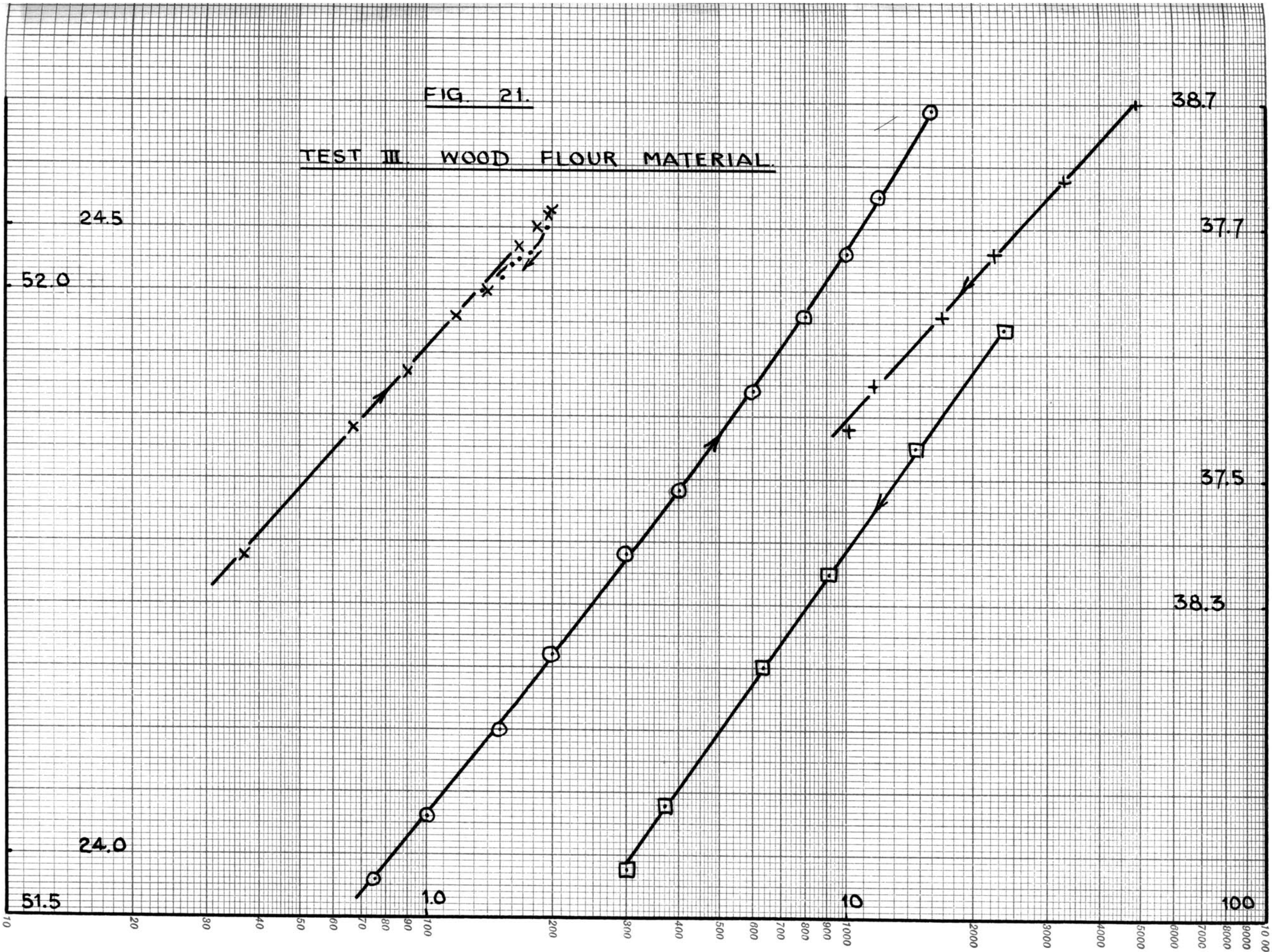
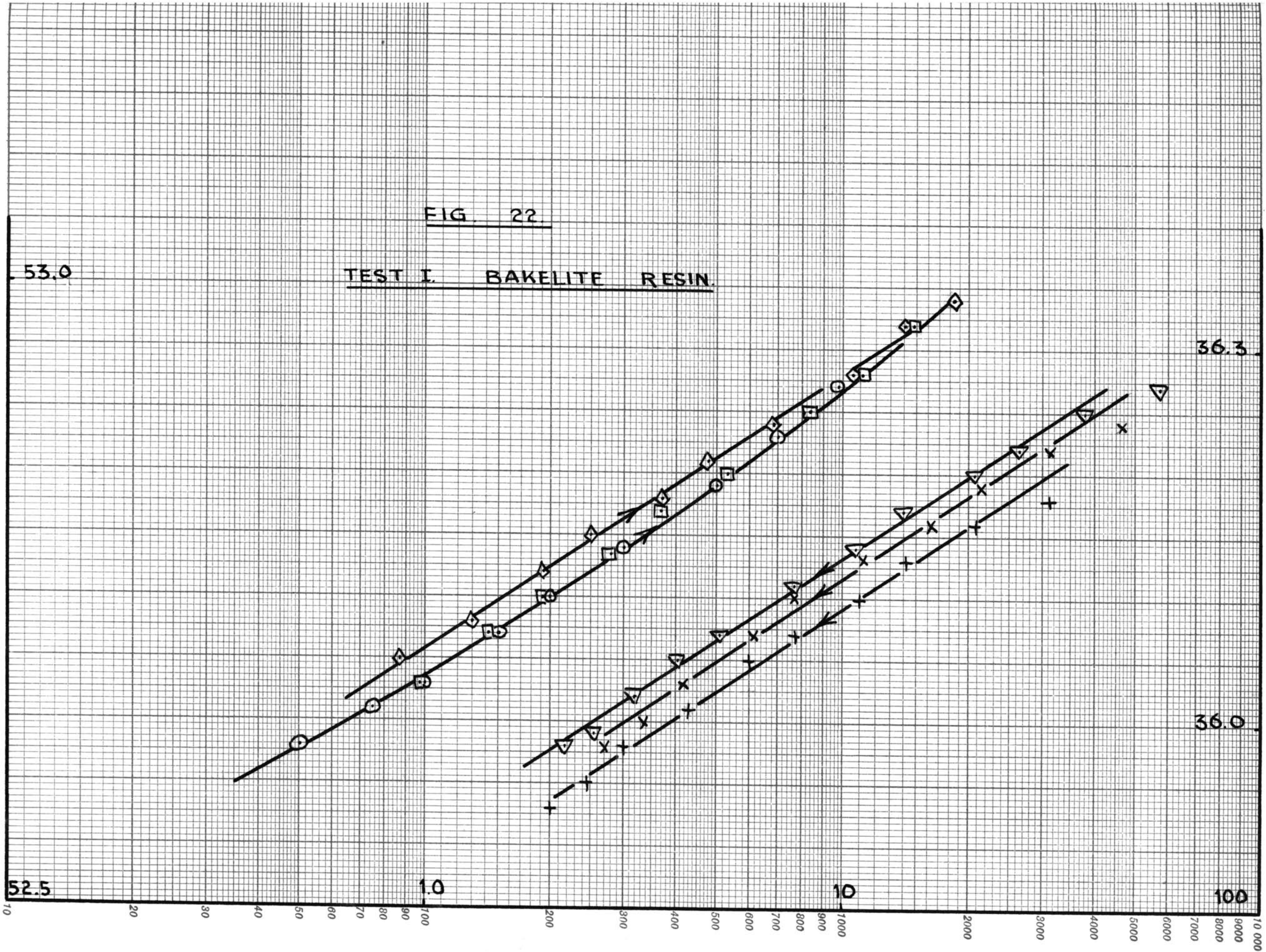


FIG. 22.

TEST I. BAKELITE RESIN.



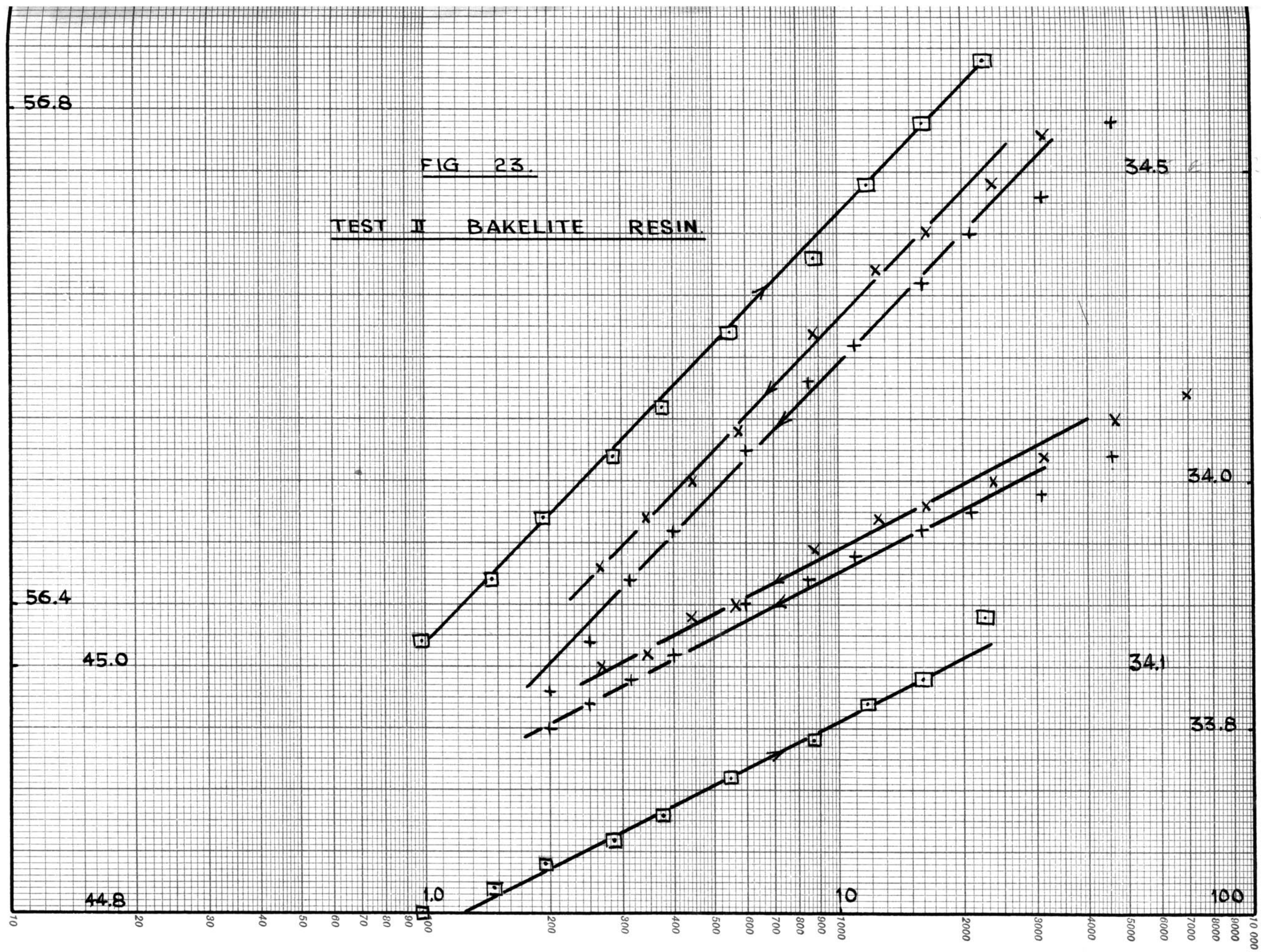
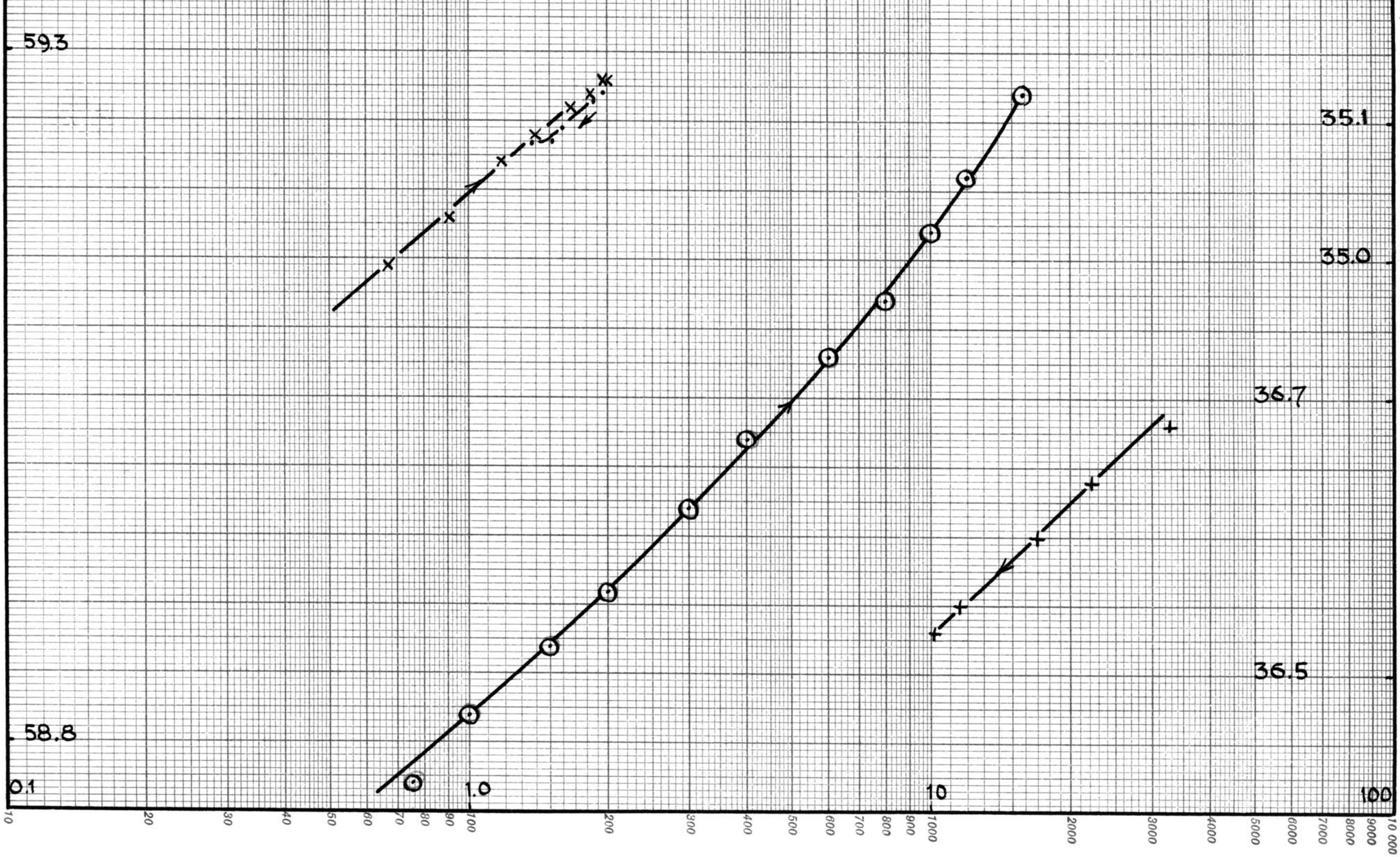


FIG. 23.
TEST II BAKELITE RESIN.
56.8
56.4
45.0
44.8
34.5
34.0
34.1
33.8
1.0
10
100

FIG 24.

TEST III. BAKELITE RESIN.



Discussion of Results.

Test I: In the case of cord material (fig. 13) it seems at first sight that the superposition law is not obeyed. Though the recovery curves do indeed give parallel straight lines when plotted against equivalent times on semi-logarithmic paper, these lines are not coincident. The curves for the second and third creep periods are straight over part of their range, and parallel to the recovery lines, while the first creep curve does not exhibit any linearity at all.

On closer inspection of the creep curves, however, it is seen that the second creep curve is linear up to the maximum strain reached in the first creep period. Above this strain an additional non-logarithmic creep takes place. The same applies to the creep curve for the third loading period. Examination of the recovery curves shows us that this additional non-logarithmic creep which takes place when the previous maximum strain is exceeded, is in fact non-recoverable. The recovery lines are straight, indicating that the memory action is following the laws previously enunciated. Furthermore, we see that the displacement in the direction of the strain axis between each pair of recovery lines is nearly equal to the plastic flow occurring in the intermediate creep stage.

We can thus enunciate the following law:
For loads applied in one sense, the creep recovery obeys Boltzmann's law; the creep obeys this law only if the previous maximum strain is not exceeded, otherwise an additional non-recoverable non-logarithmic creep takes place.

This additional creep appears to be of the nature of a plastic flow, i.e., proportional to time. It was not possible to investigate this further with long-time creep tests with the specimens used, since the secondary creep appeared not to be proportional to stress. After a long-time creep test a redistribution of stress across the cross-section was caused; the subsequent recovery was therefore non-logarithmic. To investigate this plastic flow, tubular specimens would be required.

Plastic flow proportional to time but not necessarily to stress is a phenomenon which in metals has been studied intensively in recent years. It is now considered to be a diffusion process, and therefore the secondary creep rate must be a function of temperature only and not of strain history. In these tests this interfering factor to the logarithmic creep appears to be of the nature of plastic flow and yet dependent on the strain history of the specimen. If this phenomenon is a real effect, and not a result of the experimental method, then it must be concluded that secondary creep in polycrystalline materials is different from that found in amorphous bodies.

Some justification is required for drawing the creep curves in fig. 13 in the manner shown. On the evidence of one or two points we have drawn straight creep lines up to the previous maximum strain, and then continued these along an extension of the original creep curve. This justification is found in an experiment in Part IV, in which the deflections at the upper end of the second creep curve were taken very frequently to determine the exact course of the upper part of the curve. In order to test that the displacements between the recovery curves are due to the plastic flow in the intervening creep test, a special repeated loading test was devised in which the maximum equivalent time in the creep test was nearly the same for all three loading periods, the same applying to the minimum equivalent time in the recovery periods. In this case the displacements between the three recovery tests, and the second and third creep tests were very small, and were probably due to experimental error (p. 94).

In all cases, the slopes of the recovery lines and of the straight parts of the creep lines are nearly the same. All the same conclusions apply to this test on the other three materials; in the case of the wood-flour material and the unfilled resin the plastic flow appears to be of a much smaller order. The general conclusion of this test is that with the above provisos, the material remembers past loading actions; this memory appears to persist indefinitely.

Test II. In this test for all materials, the mean slope of the lines for full load (2 x 7.5 lbs.ins.) was approximately double that for the half load (1 x 7.5 lbs.ins.), indicating that the logarithmic creep is proportional to stress in the experimental range.

<u>Test III</u>. In this test, there are two creep periods and two recovery periods. From the results of Test I, we understand why the creep under the first loading is not linear. The creep under reversed loading from 18 mins. to 20 mins. we must also ignore; first, owing to the rapid increase in deflection, these points in this part of the test are not as accurate as the other observations; though these points appear in some cases to fall on a straight line, this is probably due to the nature of the time function. The time function for the second recovery has a maximum at an elapsed time (from the beginning of the test) of 24 mins. i.e., 4 mins. after the reversed load has been removed. We find in all cases that the creep recovery does reverse its direction at this instant. Theoretically, it should retrace the line corresponding to the creep recovery from 20 mins. to 24 mins. Usually this is so. Cord material is an exception in that it retraces a line parallel and very close to the earlier part of the recovery curve (fig. 15), thus forming a very narrow loop. This displacement of the line corresponding to observations from t = 24 mins. onwards from the line corresponding to observations up to 24 mins. might be due to an internal friction effect. In the other three materials, the creep deflections from t = 24 mins. onwards does retrace as accurately as can be determined the creep from t = 20 mins. to 24 mins. Thus the correctness of the memory action law is strikingly shown.

We remember that the ideal theory predicted that the short recovery period from t = 16 mins. to t = 18 mins.

should give the same line as that yielded by the second recovery period (t greater than 20 mins.). We obtain two parallel lines in fact. The displacement between these two lines is due to elastic hysteresis, since these two lines represent the recovery from creep due to equal loads in opposite directions.

The Separation of Creep and Elastic Hysteresis.

The creep experiments on specimens of each of the four materials show that Boltzmann's superposition law is a sufficient working hypothesis. We are thus justified in using the theory developed in Part II of this thesis, to determine the area of the hysteresis loop. The method was as follows. Loads were applied or removed in four increments of 8 ozs. each, the loading being applied cyclically and in equal intervals of time; these intervals of time were in the various tests 0.5 min., 1 min., and 2 mins. In each test, two complete cycles, involving 32 steps, were traced. It was endeavoured to read the scale as near as possible to the instant when the load was changed. A certain amount of error was inevitable, especially when the creep was rapid. In each case the step-by-step cyclic loading test was repeated with at least one other value of time-interval.

Following the hysteresis test, a repeated loading creep test was carried out; this involved two creep periods of 10 mins. each, and two recovery periods also of 10 mins. The second creep and the two recovery deflections when plotted against equivalent time gave nearly parallel straight lines. Since a load of two pounds was used in the creep tests, the mean slope was divided by four to obtain the value of W.a.

To obtain quantitative confirmation of the values of creep and hysteresis obtained by this test, a decrement test was then made as described on p. 45. In the reduction of the hysteresis results, the corrections were evaluated by means of the tables on p. 26, and of the value of W.a obtained from the creep test. These corrections are of course not creep corrections, which we have shown are impossible to obtain, but represent what we have called time-dependent deflections, that is, referred to the behaviour of a specimen under a given load for unit time. As far as hysteresis loop width is concerned, the value of time unit is immaterial; this is also the case if we wish to compare loops obtained by using different time-intervals, since the effect of altering the time unit is to shear over all the loops by an equal amount.

In order to assess the damping capacity due to the hysteresis loop however, we have to know the total strain range, and this must be a value corresponding to a time unit of the order of the period of vibration. Except for the determination of this quantity, all values are expressed in terms of a time unit of one minute.

V. Results of Hysteresis Tests.

The observations (scale reading in inches) are recorded for the first two loading cycles for each of the four materials. For cord material and fabric moulding material tests were carried out with three values of time interval (0.5 min., 1 min., 2 mins.). For the wood flour material and the bakelite resin only two tests were made.

In all cases the corrections for the second loop have been calculated from the mean slope of the creep curves, and applied to the observations. The corrected loop width at each station and loop area (in units of inches of scale multiplied by lbs. load on beam) were calculated. In the case of the second test recorded (cord material, time interval 1 minute), the corrections have been applied to both first and second cycles. Good agreement appears to be obtained between corresponding observations for the first and second loops. The agreement between the areas of the first and second loops (2.305 and 2.34 units respectively) is also good.

<u>Cord Material</u>. Time Interval 0.5 minute.

<u>Observations.</u>

Load, lbs.

2		52.83				52.81			
1.5	48.49		49.12		48.44		49.11		
1	44.28		45.22		44.15		45.20		
0.5	40.22		41.19		39.91		41.16		
0	36.33		37.07		35.73		37.04		35.70
-0.5			32.89		31.57		32.82		31.53
-1			28.58		27.45		28.53		27.42
-1.5			24.14		23.43		24.07		23.38
-2				19.53				19.47	

<u>Corrections, Second Cycle.</u>

Creep Factor .189

Load, lbs.					
2		-.05			
1.5	-.12		.16		
1	-.18		.26		
0.5	-.22		.30		
0	-.23		.31		-.26
-0.5			.28		-.25
-1			.24		-.21
-1.5			.18		-.10
-2				.10	

<u>Second Cycle, Corrected Obsns.</u>

					<u>Loop Widths.</u>
	52.86				0
48.56		48.95			.39
44.33		44.94			.61
40.13		40.86			.73
35.96		36.73		35.96	.77
		32.54		31.78	.76
		28.29		27.63	.66
		23.89		23.48	.41
			19.37		0

Cord Material. Time Interval 1 Minute.

Observations.

Load, lbs.									
2		53.08				53.04			
1.5	48.66		49.32		48.59		49.27		
1	44.32		45.36		44.23		45.32		
0.5	40.07		41.28		39.93		41.24		
0	36.03		37.10		35.68		37.04		35.64
-0.5			32.86		31.47		32.79		31.41
-1			28.48		27.31		28.41		27.24
-1.5			23.97		23.23		23.89		23.15
-2				19.28				19.19	

Corrected Observations. Creep Factor .189

Load, lbs.										Second Cycle Loop Width.
2		52.82				52.86				0
1.5	48.51		48.93		48.54		48.94			.40
1	44.26		44.88		44.30		44.95			.65
0.5	40.07		40.87		40.10		40.88			.78
0			36.75		35.92		36.73		35.90	.83
-0.5			32.60		31.76		32.56		31.72	.84
-1			28.36		27.61		28.28		27.56	.72
-1.5			23.93		23.48		23.88		23.42	.46
-2				19.38				19.32		0

Cord Material. Time Interval 2 Minutes.

Observations.

Load, lbs.

2		53.33				53.28			
1.5	48.82		49.51		48.75		49.47		
1	44.40		45.51		44.31		45.48		
0.5	40.09		41.37		39.94		41.34		
0	35.99		37.12		35.63		37.10		35.62
-0.5			32.82		31.33		32.77		31.32
-1			28.37		27.12		28.32		27.10
-1.5			23.78		22.98		23.73		22.94
-2				18.98				18.93	

Second Cycle.

Corrected Observations. Creep Factor .189 Loop Width.

Load, lbs.

						Loop Width
2		52.87				0
1.5	48.53		48.97			.44
1	44.26		45.00			.74
0.5	40.05		40.93			.88
0	(35.67)		36.79		35.88	.91
-0.5			32.54		31.68	.86
-1			28.30		27.53	.77
-1.5			23.89		23.38	.51
-2				19.28		0

Cord Material.

Calculation of Specific Damping Capacity from Cyclic Loading Tests.

	t_1 = 0.5 min.	t_1 = 1 min.	t_1 = 2 mins.
Elastic hysteresis loop area	2.17	2.34	2.51
Strain range: obs.	33.34	33.85	34.35
corrected, T = 1 min.	33.49	33.54	33.59
corrected, T = 10^{-3} min.	28.96	29.01	29.06
A = .5(strain range - max. loop width)	14.10	14.09	14.08
Strain area	14.48	14.51	14.53
Ψ_c = 428 x .755/A %	22.9	22.9	22.9
Ψ_h, %	15.0	16.1	17.3
Computed Ψ, %	37.9	39.0	40.2

Fabric Moulding Material. Time Interval 0.5 Minute.

Observations.

Load, lbs.

Load									
2		50.79				50.78			
1.5	46.87		47.37		46.84		47.36		
1	42.99		43.82		42.91		43.81		
0.5	39.30		40.18		39.06		40.16		
0	35.75		36.44		35.27		36.41		35.26
-0.5			32.63		31.54		32.57		31.52
-1			28.69		27.83		28.64		27.80
-1.5			24.68		24.16		24.62		24.12
-2				20.55				20.49	

Second Cycle.

Corrected Observations. Creep Factor .145 Loop Width.

Load, lbs.

Load						Loop Width
2		50.82				0
1.5	46.93		47.24			.31
1	43.05		43.61			.56
0.5	39.23		39.93			.70
0	35.46		36.18		35.46	.72
-0.5			32.35		31.71	.64
-1			28.46		27.96	.50
-1.5			24.48		24.20	.28
-2				20.41		0

Fabric Moulding Material. Time Interval 1 Minute.

Observations.

Load, lbs.									
2		51.06				51.00			
1.5	47.06		47.58		47.00		47.53		
1	43.08		43.98		43.02		43.94		
0.5	39.20		40.30		39.11		40.25		
0	35.51		36.49		35.27		36.45		35.23
-0.5			32.63		31.49		32.58		31.44
-1			28.62		27.73		28.57		27.68
-1.5			24.55		24.01		24.50		23.95
-2				20.35				20.28	

Second Cycle.

Corrected Observations. Creep Factor .145 Loop Width.

Load, lbs.						Loop Width
2		50.86				0
1.5	46.96		47.28			.32
1	43.07		43.66			.59
0.5	39.24		39.98			.74
0	(35.46)		36.22		35.43	.79
-0.5			32.41		31.68	.73
-1			28.47		27.92	.55
-1.5			24.49		24.16	.33
-2				20.38		0

Fabric Moulding Material. Time Interval 2 Minutes.

Observations.

Load, lbs.

2		51.37				51.27			
1.5	47.58		47.86		47.21		47.75		
1	43.69		44.23		43.19		44.12		
0.5	40.00		40.51		39.23		40.40		
0	36.43		36.70		35.36		36.57		35.29
-0.5			32.82		31.54		32.66		31.46
-1			28.77		27.74		28.62		27.65
-1.5			24.62		23.98		24.47		23.89
-2				20.27				20.18	

Second Cycle.

Corrected Observations. Creep Factor .145 Loop Width.

Load, lbs.

2		50.96				0
1.5	47.04		47.37			.33
1	43.15		43.75			.60
0.5	39.31		40.08			.77
0	(35.55)		36.34		35.49	.85
-0.5			32.53		31.74	.79
-1			28.61		27.98	.63
-1.5			24.59		24.23	.36
-2				20.45		0

Fabric Moulding Material.

Calculation of Specific Damping Capacity from Cyclic Loading Tests.

	t_1 = 0.5 min.	t_1 = 1 min.	t_1 = 2 mins.
Elastic hysteresis loop area	1.81	2.03	2.17
Strain range: obs.	30.29	30.72	31.09
corrected, T = 1 min.	30.41	30.48	30.51
corrected, T = 10^{-3} min.	26.93	27.00	27.03
A = .5(strain range - max. loop width)	13.11	13.11	13.09
Strain Area	13.47	13.50	13.52
Ψ_c = 428 x .58/A, %	18.9	18.9	18.9
Ψ_h, %	13.4	15.0	16.0
Computed Ψ, %	32.3	33.9	34.9

Wood Flour Material. Time Interval 0.5 Minute.

Observations.

Load, lbs.

2		51.43				51.41			
1.5	47.72		48.06		47.60		48.03		
1	44.06		44.60		43.79		44.57		
0.5	40.49		41.08		39.88		41.04		
0	36.98		37.45		35.95		37.41		35.93
-0.5			33.72		32.27		33.67		32.25
-1			29.79		28.62		29.73		28.62
-1.5			25.65		25.00		25.59		24.99
-2				21.40				21.39	

Second Cycle.

Corrected Observations. Creep Factor .110 Loop Width.

Load, lbs.

						Loop Width
2		51.44				0
1.5	47.67		47.94			.27
1	43.90		44.42			.52
0.5	40.01		40.87			.86
0	(36.09)		37.23		36.08	1.15
-0.5			33.50		32.40	1.10
-1			29.59		28.74	.85
-1.5			25.49		25.05	.44
-2				21.33		0

Wood Flour Material. Time Interval 1 Minute.

Observations.

Load, lbs.

2		51.51				51.47			
1.5	47.71		48.10		47.66		48.07		
1	43.88		44.62		43.82		44.58		
0.5	39.95		41.06		39.87		41.02		
0	36.06		37.40		35.92		37.37		35.90
-0.5			33.65		32.22		33.61		32.19
-1			29.68		28.55		29.63		28.51
-1.5			25.52		24.90		25.46		24.85
-2				21.28				21.23	

Second Cycle.

Corrected Observations. Creep Factor .110 Loop Width.

Load, lbs.

						Loop Width
2		51.36				0
1.5	47.63		47.88			.25
1	43.86		44.36			.50
0.5	39.97		40.81			.84
0	(36.06)		37.19		36.05	1.14
-0.5			33.48		32.37	1.11
-1			29.56		28.69	.89
-1.5			25.46		25.01	.45
-2				21.30		0

Wood Flour Material.

Calculation of Specific Damping Capacity from Cyclic Loading Tests.

	t_1 = 0.5 min.	t_1 = 1 min.
Elastic hysteresis loop area	2.60	2.59
Strain range: observed	30.02	30.24
corrected, T = 1 min.	30.11	30.06
corrected, T = 10^{-3} min.	27.46	27.41
A = .5(strain range - max. loop width)	13.16	13.14
Strain area	13.73	13.71
Ψ_c = 428 x .441/A, %	14.4	14.4
Ψ_h, %	18.9	18.9
Computed Ψ, %	33.3	33.3

Bakelite Resin. Time Interval 1 Minute.

Observations.

Load, lbs.

2		57.22				57.18			
1.5	51.17		51.45		51.14		51.42		
1	45.29		45.71		45.23		45.68		
0.5	39.47		39.94		39.39		39.92		
0	33.72		34.13		33.58		34.10		33.57
-0.5			28.24		27.73		28.22		27.71
-1			22.20		21.76		22.15		21.72
-1.5			15.91		15.64		15.88		15.60
-2				9.30				9.26	

Second Cycle.

Corrected Observations. Creep Factor .086 Loop Width.

Load, lbs.

2		57.10				0
1.5	51.12		51.27			.15
1	45.26		45.51			.25
0.5	39.47		39.76			.29
0	(33.69)		33.96		33.69	.27
-0.5			28.12		27.85	.27
-1			22.09		21.87	.22
-1.5			15.88		15.72	.16
-2				9.32		0

Bakelite Resin. Time Interval 2 Minutes.

Observations.

Load, lbs.

2		57.47				57.30			
1.5	51.32		51.64		51.23		51.53		
1	45.37		45.78		45.30		45.77		
0.5	39.52		39.99		39.43		39.98		
0	33.73		34.18		33.60		34.16		33.59
-0.5			28.27		27.72		28.25		27.70
-1			22.14		21.72		22.16		21.69
-1.5			15.89		15.58		15.83		15.55
-2				9.25				9.18	

Second Cycle.

Corrected Observations. Creep Factor .086 — Loop Width.

Load, lbs.

2		57.11				0
1.5	51.13		51.30			.17
1	45.28		45.55			.27
0.5	39.48		39.79			.31
0	(33.71)		34.02		33.71	.31
-0.5			28.17		27.87	.30
-1			22.15		21.89	.26
-1.5			15.90		15.75	.15
-2				9.34		0

Bakelite Resin.

Calculation of Specific Damping Capacity from Cyclic Loading Tests.

	t_1 = 1 min.	t_1 = 2 mins.
Elastic hysteresis loop area	.80	.89
Total strain range: observed	47.92	48.12
corrected, T = 1 min.	47.78	47.77
corrected, T = 10^{-3} min.	45.72	45.71
A = .5(strain range - max. loop width)	22.73	22.70
Strain area	22.86	22.86
Ψ_c = 428 x .343/A, %	6.5	6.5
Ψ_h, %	3.5	3.9
Computed Ψ, %	10.0	10.4

Discussion of Hysteresis Results.

The following general conclusions are seen to hold for the hysteresis tests.

1. Steady State. Excepting the first quarter of the first cycle, there is good agreement between the observed deflections for the first and second cycles. The slight difference appears to be accounted for within the order of accuracy of the experiment by the slight difference between the corrections due to creep for the two cycles (fig. 7).
2. Uncorrected Observations. Considering the second cycle the zero load observations appear to be almost unaffected by the time interval of the cycle. This is not the case with the other observations. The greater the time interval, the greater is the total strain range.
3. Corrected Observations. After applying to the second cycle observations the corrections for time dependent deflection, it is seen that good agreement is obtained between the corresponding observations for different time intervals. In spite of the fact that the observations at the maximum load points are likely to be in error for reasons already mentioned, they are pulled nearly into agreement on applying the corrections. Whereas the observed total strain ranges for different time intervals differ greatly, these ranges when adjusted by the theory are found to be nearly equal. The results thus bear out the theoretical conclusions that the cycles for different time intervals are sheared over by amounts

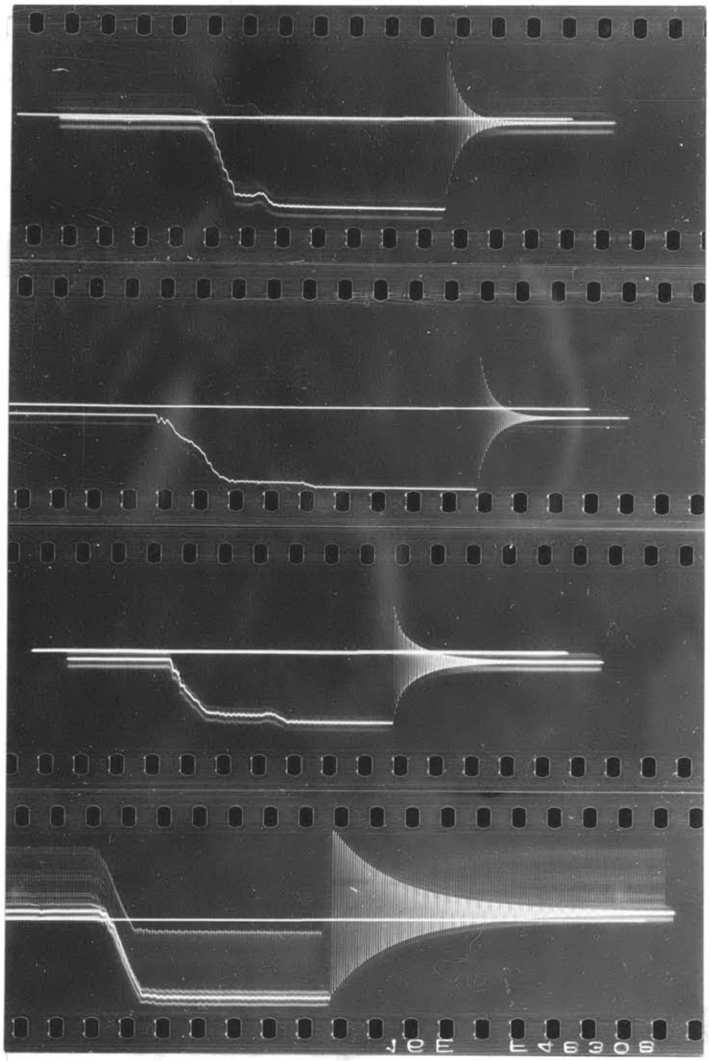
proportional to the logarithm of the time interval.

4. Elastic Hysteresis Loops. The difference between pairs of corrected observations gives the hysteresis loop width in terms of inches of scale. The sum of these widths for the nine stations (i.e., eight intervals) when divided by two gives the loop area in terms of lbs. wt. on beam x inches on scale. The loop widths represent the difference between two nearly equal magnitudes, the observations being taken rapidly and without any possibility of a check. Because of this and some other unknown causes, there is only a fair agreement between the sizes and areas of the hysteresis loops; the width at each station and the total area is slightly greater the greater the time interval. Agreement is good between the two values of loop area in the case of the wood flour material and the unfilled resin. Except in the case of the wood flour material, the loop widths are roughly symmetrical with respect to the load.

VI. Damping Capacity Results.

The logarithmic decrement δ was evaluated from an amplitude record by means of the simple formula, and the mean specific damping capacity obtained by multiplying by two. The stress scale was determined by assuming linear distribution of stress, the initial semi-amplitude thus giving a connection with surface stress.

Fig. 25 shows amplitude records as obtained for the various materials. Figs. 26 - 29 show the relation



Cord Material

Fabric Moulding Material

Wood Flour Material

Bakelite Resin

Fig. 25.

Amplitude of Free Oscillation

(Contact Print from Film Record)

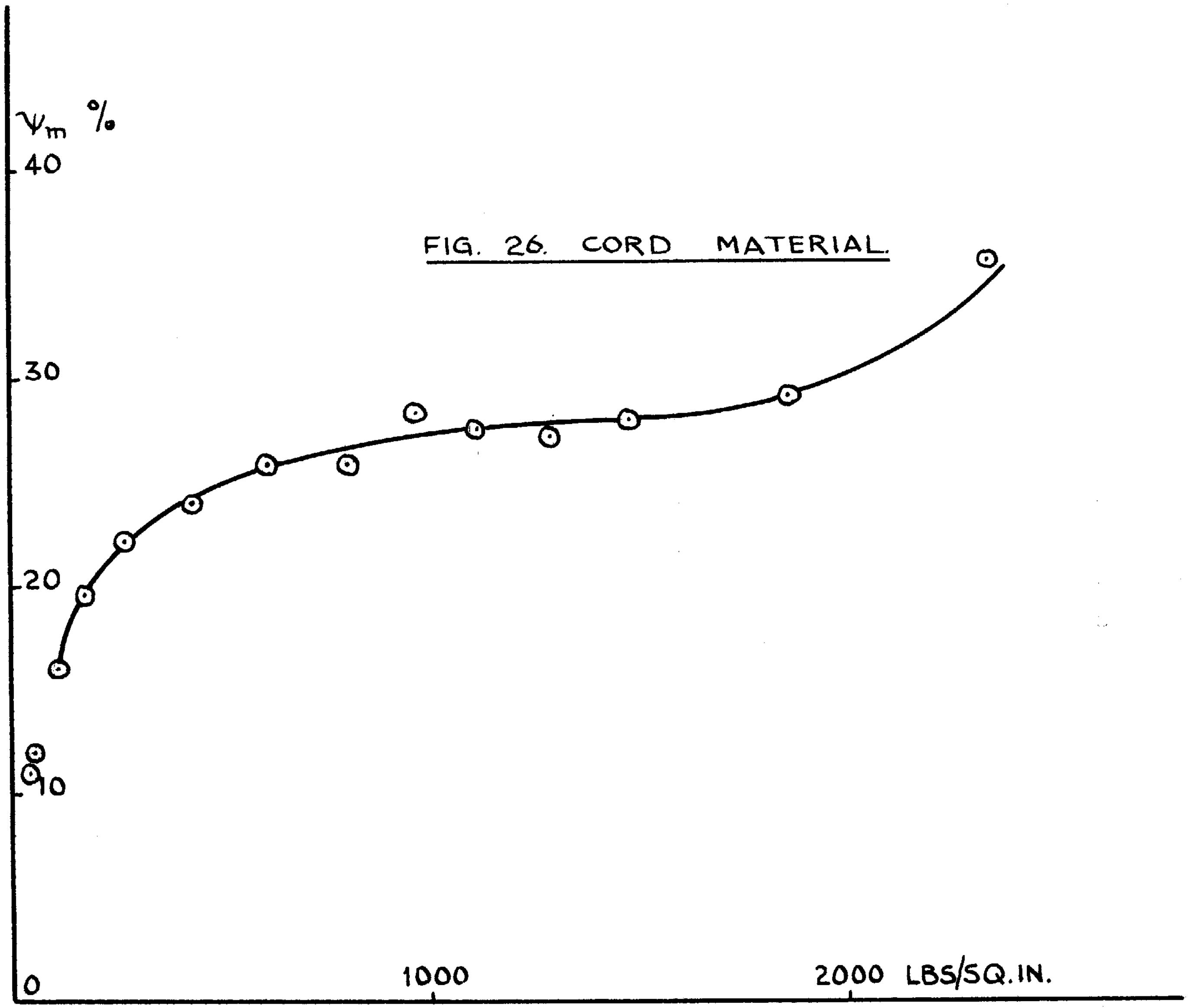
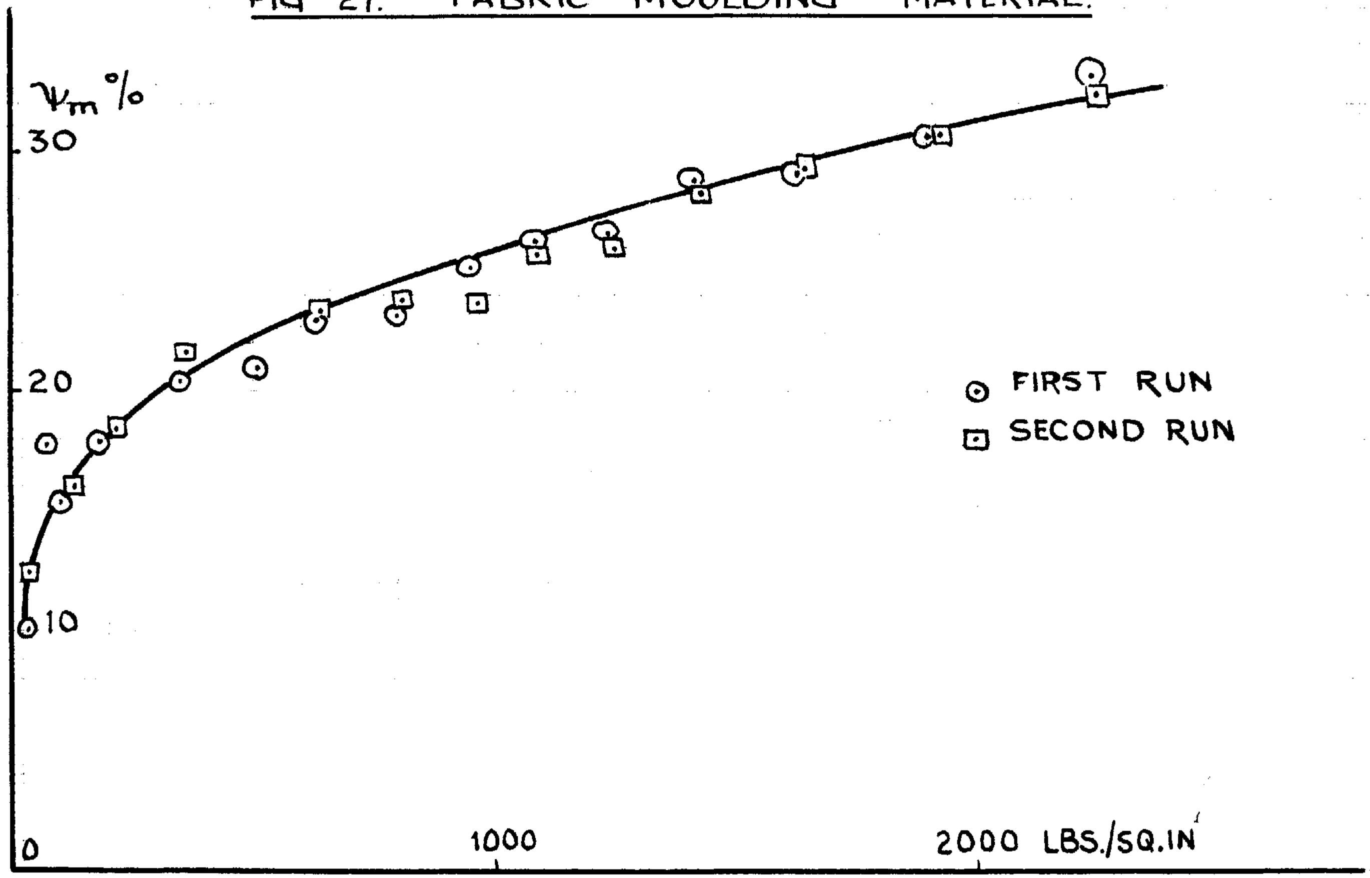
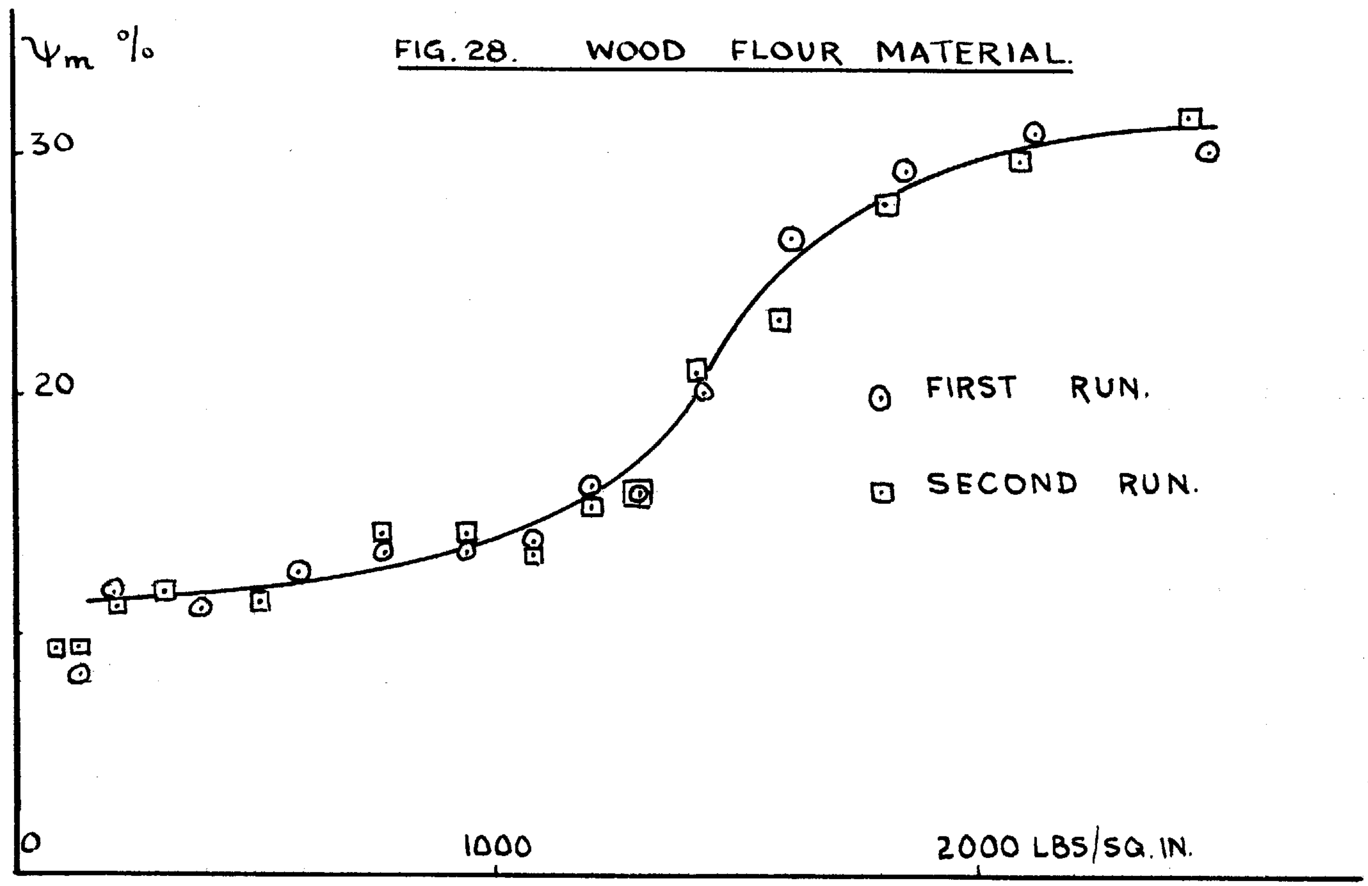


FIG. 26. CORD MATERIAL.

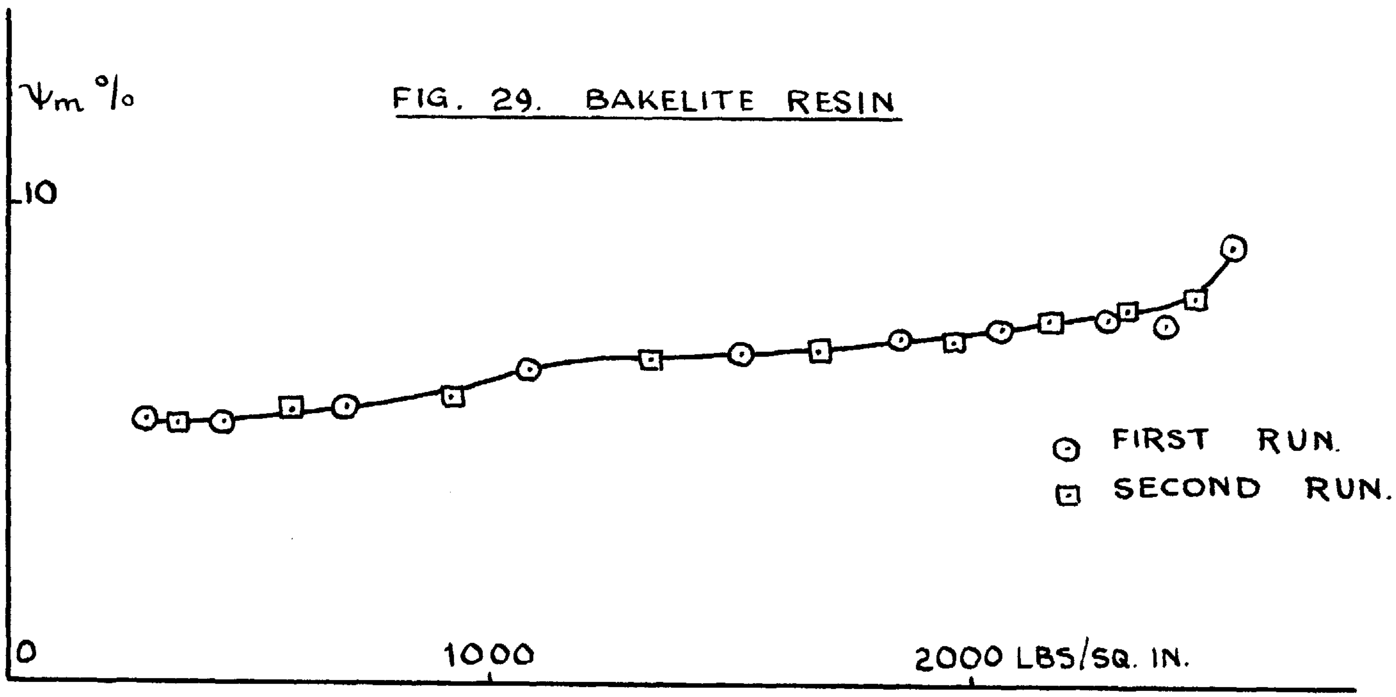
FIG 27. FABRIC MOULDING MATERIAL.

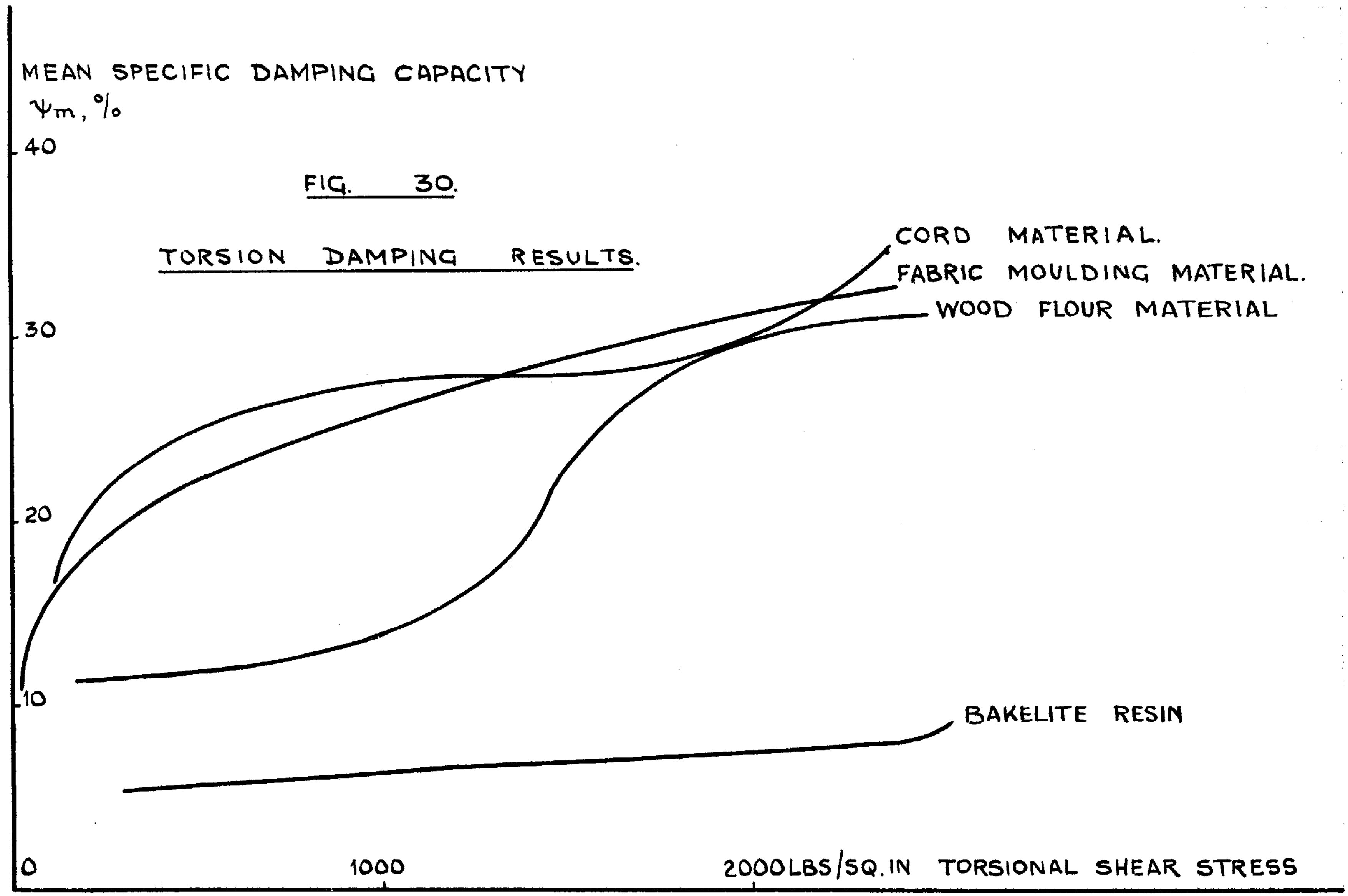




ψ_m %
FIG. 28. WOOD FLOUR MATERIAL.
30
20
0
1000
2000 LBS/SQ. IN.
FIRST RUN.
SECOND RUN.

FIG. 29. BAKELITE RESIN




MEAN SPECIFIC DAMPING CAPACITY
Ψm, %
40
FIG. 30.
TORSION DAMPING RESULTS.
CORD MATERIAL.
FABRIC MOULDING MATERIAL.
WOOD FLOUR MATERIAL
30
20
10
BAKELITE RESIN
0
1000
2000 LBS/SQ.IN TORSIONAL SHEAR STRESS

between mean specific damping capacity in torsion and nominal surface shear stress for the four materials in the as received condition. In many cases two runs are recorded; the agreement is usually close. Fig. 30 shows on one diagram how the faired records compare. An example of the calculation of decrement for cord material is given below.

<u>Stress Scale</u>. 2 lbs. load ≡ 2520 lbs./in^2 2.25 lbs. ≡ 2 x 3.47 ins. hence 1 in. on record ≡ 408 lbs./in^2

Cycle	Amplitude, ins.	Ampl. Difference	Mean Ampl.	Log. Dec., %	Stress, lbs./in^2.
1	6.23				
		1.02	5.72	17.85	2340
2	5.21				
		1.33	4.55	14.6	1860
4	3.88				
		.51	3.63	14.0	1480
5	3.37				
		.43	3.16	13.6	1290
6	2.94				
		.38	2.75	13.8	1120
7	2.56				
		.34	2.39	14.2	976
8	2.22				
		.51	1.97	12.95	805
10	1.71				
		.39	1.505	12.95	614
12	1.32				
		.51	1.065	12.00	435
16	.81				
		.29	.665	11.1	272
20	.52				
		.17	.435	9.78	178
24	.35				
		.17	.265	8.02	108
32	.18				
		.07	.145	6.03	59
40	.11				
		.04	.09	5.56	37
48	.07				

Comparison of Dynamic and Static Results.

The following table gives the value of ψ_m at a surface shear stress of 2520 lbs./in.^{2} (corresponding to a load on the torsion bar of 2 lbs. at 7.5 ins. radius) obtained from the damping curves, extrapolation being resorted to where necessary. This table gives also the values of ψ_m statically determined at this stress-range by the cyclic step-by-step loading tests.

Material	Cord	Fabric Mldg.	Wood Flour	Resin
Dynamic Test	36.6	33.2	31.1	8.9
Static Test $t_1 = 0.5$	37.9	32.3	33.3	---
$t_1 = 1$	39.0	33.9	33.3	10.0
$t_1 = 2$	40.2	34.9	----	10.4

The static and dynamic tests in the above table are in reasonable agreement, indicating that the theory is probably correct, and therefore that damping in bakelite in the as received condition is due partly to elastic hysteresis and partly to primary (logarithmic) creep. At the stress range considered, for three materials rather more than half the damping was due to creep; for the fourth material (wood flour filler) the reverse was true.

Part IV. Conclusion.

I. Experiments on Non-Recoverable Creep.

Three experiments will be dealt with, which represent a preliminary study of the nature of the non-recoverable creep. The cord material specimen was used in these tests.

Test I.

This is a repeated loading test similar to Test I of Part II, with the difference that the loading and unloading instants are so adjusted that the equivalent time of 15 minutes is not exceeded during the creep periods, and that the equivalent time never falls below 2.0 minutes during the recovery periods. The loading diagram is shown in fig. 31. The equivalent times are given below; the observations are plotted against equivalent time as before in fig. 32.

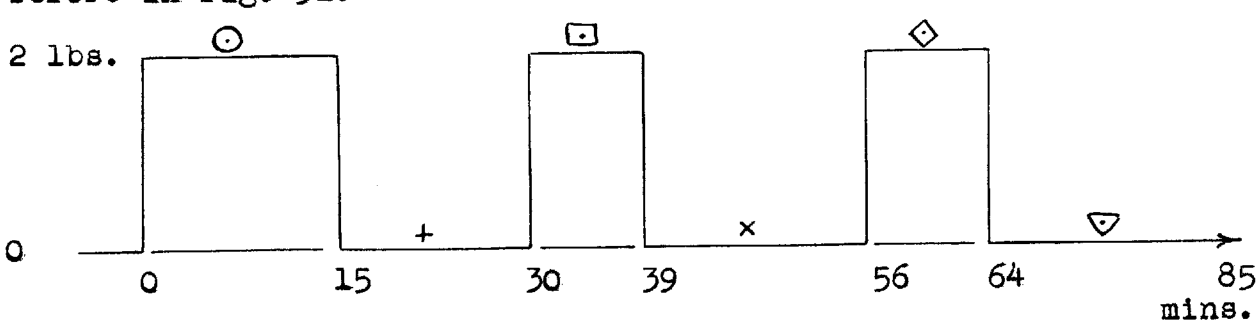


Fig. 31.

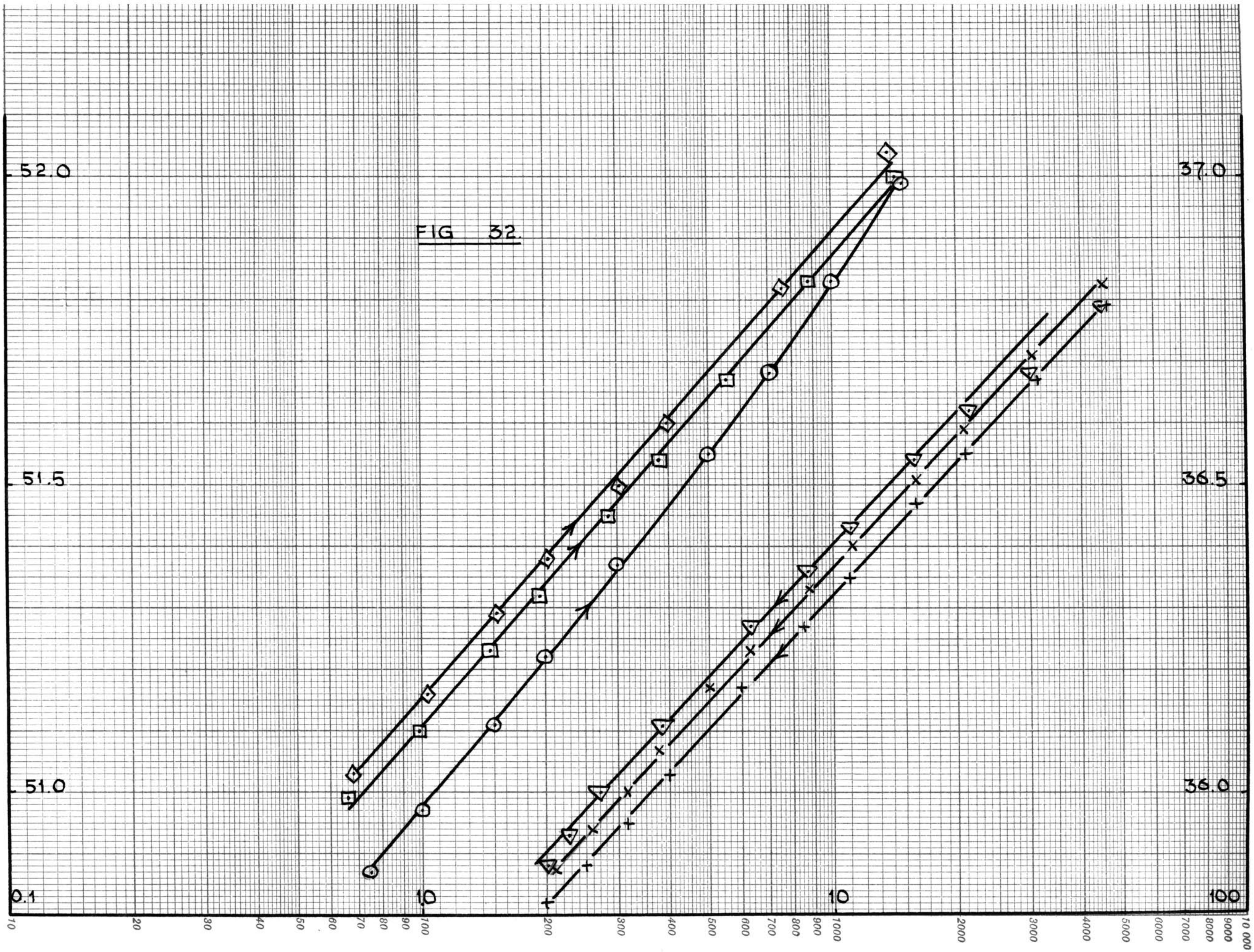


FIG 32.
52.0
51.5
51.0
37.0
36.5
36.0
0.1
10
10
100

The equivalent times up to t = 39 minutes are the same as in Test II (p. 49). The subsequent times are given below.

Recovery		Creep		Recovery	
Time from beginning of test.	Equiv. time.	Time from beginning of test.	Equiv. time.	Time from beginning of test.	Equiv. time.
39.33	45.3	56.33	.68	64.33	44.2
39.50	30.6	56.50	1.03	64.50	30
39.75	20.9	56.75	1.52	64.75	21.4
40	16.0	57	2.04	65	15.75
40.50	11.1	57.50	3.02	65.50	11.0
41	8.68	58	3.98	66	8.63
42	6.22	60	7.6	67	6.23
43	5.00	63.75	13.8	70	3.84
45	3.75			75	2.70
47	3.12			80	2.26
50	2.6			85	2.01
55.75	2.1				

It is seen that the displacements between the recovery lines, and between the second and third creep lines are very much reduced (cf. fig. 13). The slight displacement that remains might be due to defects in the apparatus.

Test II.

This experiment deals with the creep following an initial creep and intermediate recovery. Fig. 33 shows the loading diagram; the equivalent times are of course the same as for Test II of Part II up to 45 minutes from the beginning of the test. This test was continued for 70 minutes.

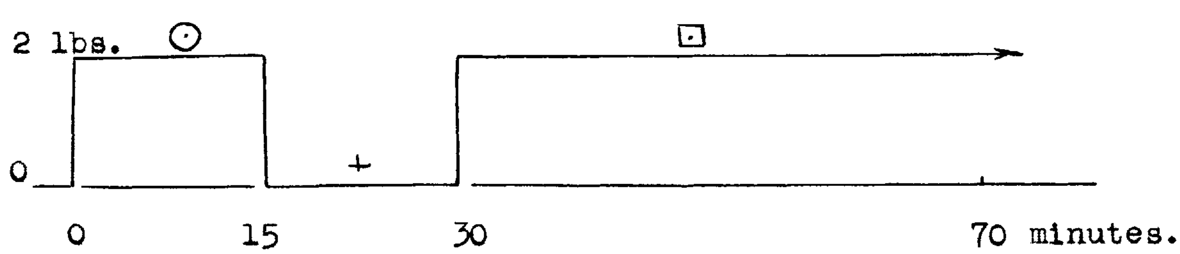


Fig. 33.

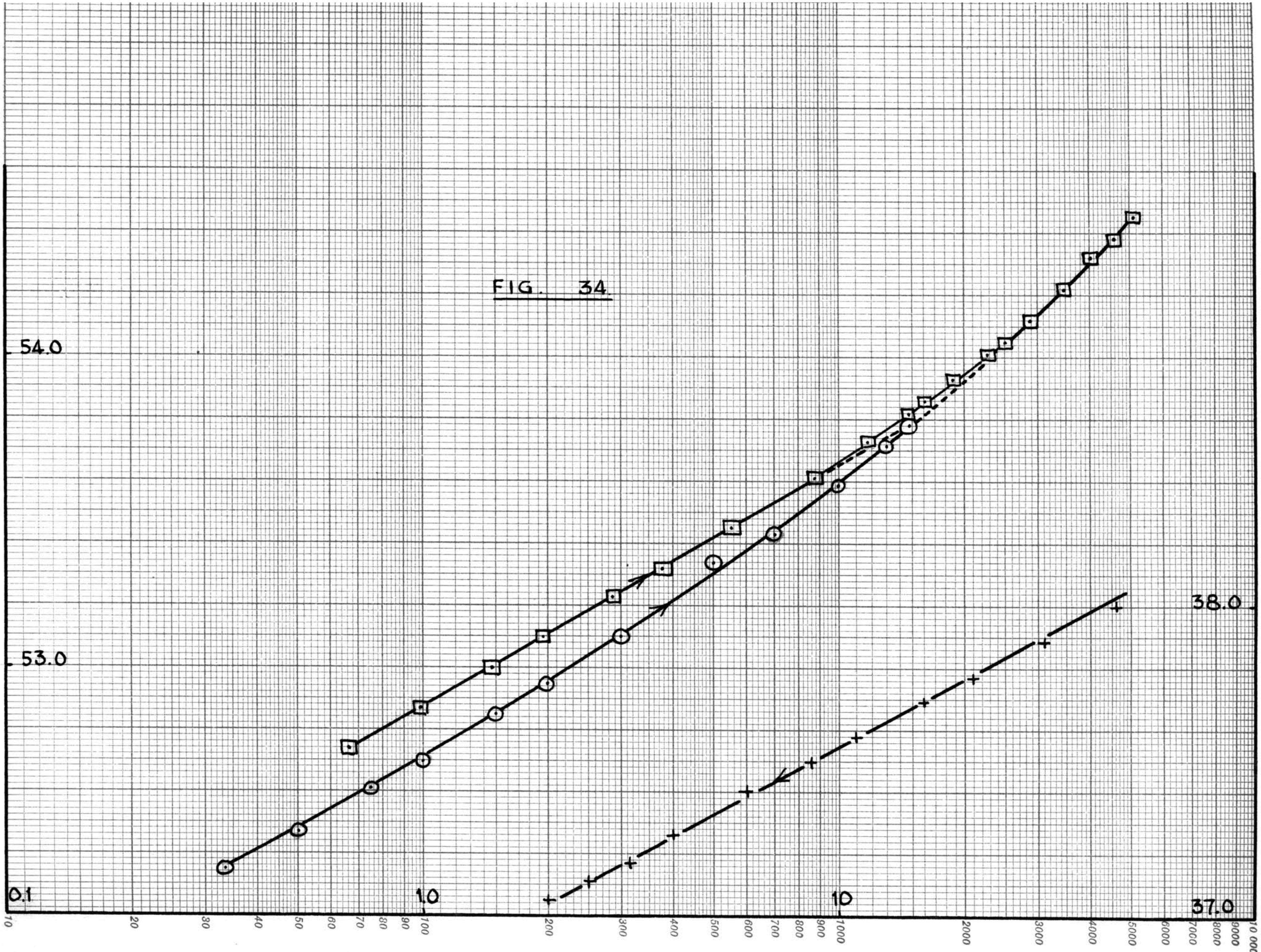

54.0
53.0
38.0
37.0
0.1
1.0
10
10
20
30
40
50
60
70
80
90
100
200
300
400
500
600
700
800
900
1000
2000
3000
4000
5000
6000
7000
8000
9000
10 000

FIG. 34.

The results in fig. 34 show that the second creep curve is represented by a straight line parallel to the recovery line up to an equivalent time of 8.6 mins. This line when extrapolated to 15 mins. intersects the original creep curve at the deflection corresponding to the termination of the first creep. From about 25 mins. onwards, the second creep curve lies on a continuation of the first. Between these points the second creep curve follows a transition curve between the straight line and the continuation of the first curve. In the curves in Part III the transition curve has been replaced by the broken line shown in fig. 34.

Test III.

This test represents an extension of Test II of Part II (with 10 minute loading periods) in which the second recovery is observed for a further 60 minutes. The loading diagram is shown in fig. 35; the creep and creep recovery observations are plotted against the corresponding equivalent times in fig. 36.

Fig. 35.

After about 35 minutes from the beginning of the second recovery period (corresponding to an equivalent time of 1.52 minutes) the creep appears to recover at a much faster rate than expected. The extrapolation of the second

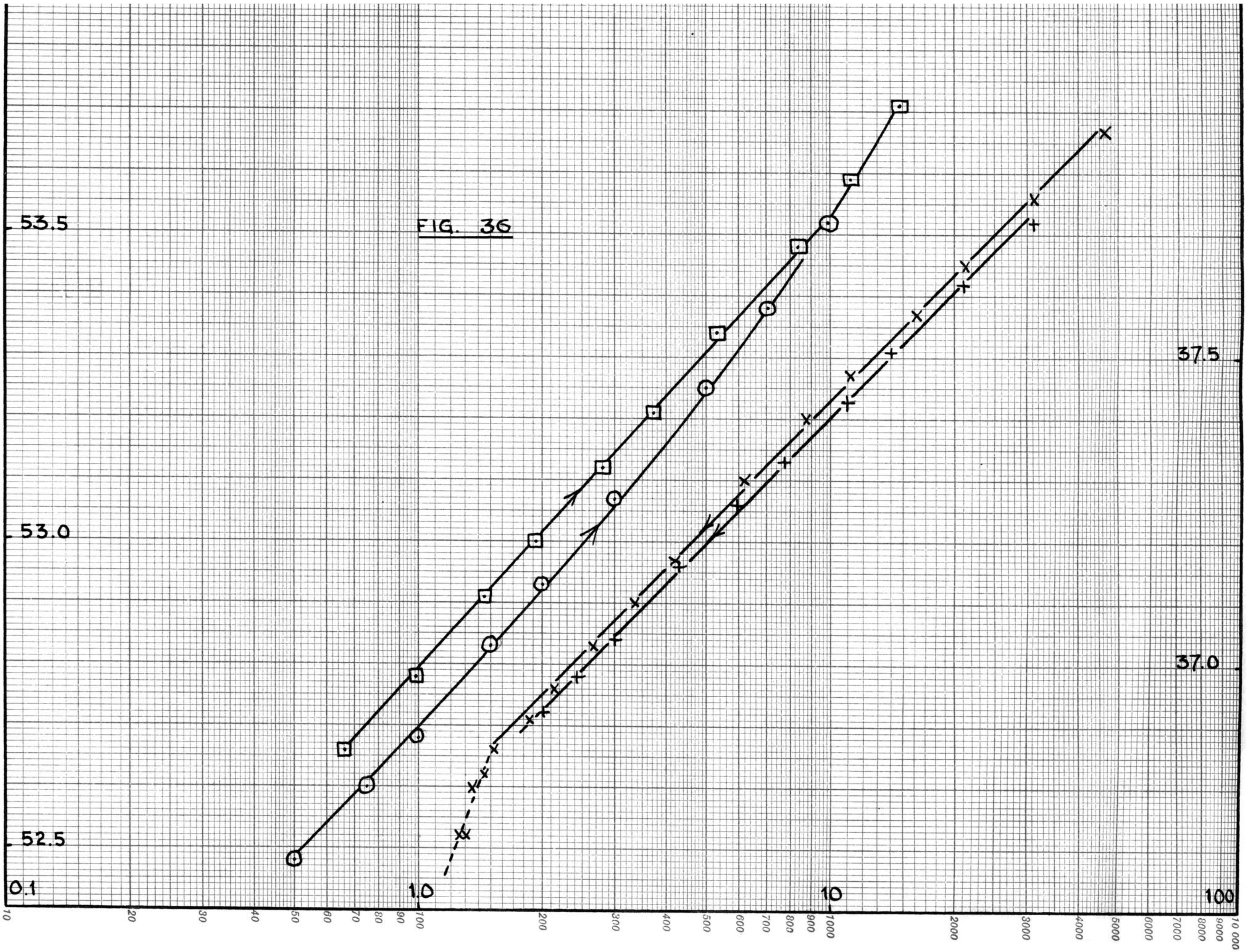


FIG. 36

recovery curve to unit equivalent time (i.e., infinite elapsed time) suggests a final additional recovery of the order of magnitude of the non-logarithmic creep in the first and the latter part of the second creep periods. The experiment suggests therefore that the non-recoverable creep deflection does probably recover after a long period of time.

II. Suggestions for Further Work.

During the course of the work several improvements to the apparatus suggested themselves. First, thin tubular specimens would have been preferable to solid cylindrical ones, since the stress-distribution would then be nearly uniform across the cross-section. The enlarged squared ends of the specimen are best clamped between V-blocks. A pivot which would be an improvement on a knife-edge would be one consisting of two flat strips at right angles in flexure.

The greatest experimental difficulty in the cyclic loading tests was the application of the loads at the exact time instants, without disturbing the loads already on the beam. A possible solution would be to apply the loads by some electromagnetic device, leaving the observer's attention free for making scale readings.

The static and dynamic tests have been compared on the basis of specific damping capacity. Difficulty and uncertainty arises both in the translation of loop area and of logarithmic decrement into this unit. If a machine be devised for measuring unit damping (i.e., the energy dissipated per unit volume per cycle) the results can immediately be compared with the hysteresis loop area, and thus the difficulties disappear.

The experimental work has been concerned only in showing that at a particular stress range the damping is due to hysteresis and primary creep. According to the theory, the specific damping capacity due to hysteresis may be a function stress, whereas that due to creep should be nearly constant. That the variation in damping capacity over the stress range is due to variation in the hysteresis component is yet to be shown. The effect of annealing at say, 100° C., the effect of overstrain, and the relations between moulding pressure, nature of filler, and type of damping curve afford further interesting problems in the damping of bakelite materials.

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