Perceptually Based Learning of Shape Descriptions for Sketch Understanding

by

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B.S. Computer Science
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ABSTRACT

We are interested in enabling a generic sketch recognition system that would allow more natural interaction with design tools in various domains. Instead of writing recognizer code for each new domain, new shapes should be added by describing them in a shape description language. While writing such descriptions is easier than writing code, it is still not a particularly easy or natural mode of interaction. The most natural way to teach new symbols to the system would be simply drawing them. This thesis presents a learning system that takes in a drawn symbol and produces a textual description of it. The main challenge is to decide which properties of the example are relevant. People cope with this task in part, we believe, through the use of perceptual biases. We use studies of human perception of geometry to understand these biases and use them to help select the relevant properties from a single example. The main generalization power is derived from two sources: 1) a qualitative description vocabulary that reflects properties that people pay attention to and 2) mechanisms, derived from the observations about perception, that adjust the relative importance of different properties based on the particular configuration of the geometric primitives in the example. The system is able to describe complex symbols and adequately filtering the irrelevant properties.

Thesis supervisor: Randall Davis
Title: Professor of Electrical Engineering and Computer Science

Chapter 1 Introduction

1.1 Research context: multi-domain sketch understanding

Informal sketches are often an important part of early stage design in many domains [Ullman, 1990]. Sketching helps people explore new ideas, brainstorm designs and reduces cognitive load of the design process. Many designers still use pen and paper for trying out ideas, since CAD tools available to date do not accept free-hand input. These tools require precise specification of all parameters and well-formed designs. Only when the design matures, can it be entered using a CAD tool for more detailed analysis and documentation. Often the valuable information about the design intent expressed in the paper sketches never gets documented. The designers also lose the benefit that computers could potentially provide even at early stages of design. Useful analysis, qualitative simulations, or exploration of alternatives can be done even on a rough sketch, if only the computer could recognize the objects sketched.

We feel that interaction with design tools could be made more natural if they not only provided powerful analysis of precise designs, but also recognized sketched input at the early design stages.

The work reported here is part of the effort by the Design Rationale Group (DRG) that has developed sketch understanding systems for several design domains including mechanical engineering and software [Alvarado and Davis, 2001], [Hammond and Davis, 2002]. Those systems used hand-coded recognizers for the domain shapes, which made creating a system for each new domain or adding more shapes very tedious and time-consuming.

The DRG is currently interested in enabling generic sketch recognition [Alvarado and Davis, 2002], and is building a system that would reuse the recognition engine for multiple domains. The intent is that new domain be added simply by providing descriptions of the domain symbols using a shape description language. Each symbol is described in terms of geometric primitives (lines, arcs, ovals, etc.) and constraints between them (connects, parallel, above, horizontal, shorter, etc.) [Hammond, 2003]. Symbolic, easily readable textual descriptions make shape representation explicit and allow any user to define new symbols.

While being able to type new shape descriptions is clearly easier than writing code, describing shapes textually is itself not a particularly natural mode of interaction. This thesis describes a system we have developed that is capable of learning a symbolic description of a shape from the user's drawing. The system provides a way to automatically produce the textual descriptions needed by the generic recognition engine from examples provided by the designer of the domain. These descriptions can be further checked or edited by the designer, if required. Figure 1.1 presents the overall view of the generic sketch understanding system and shows the role of our work.

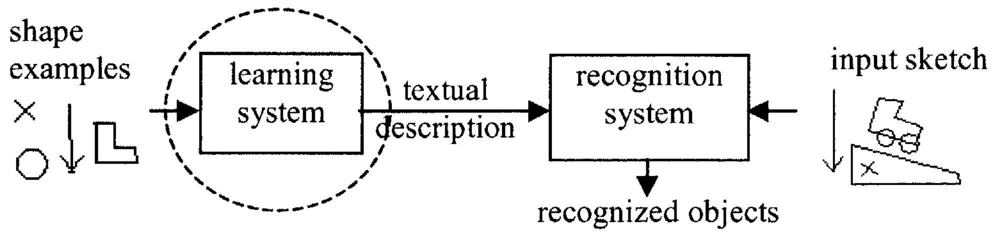


Figure 1.1 Generic sketching system

1.2 The learning problem

Like handwritten characters, symbols in commonly used graphical languages can be drawn with some variation. For instance, all of the drawings in Figure 1.2 are examples of an inverter symbol in electric circuits:

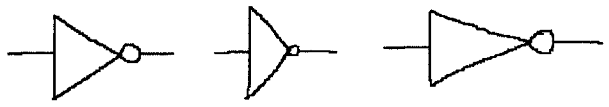


Figure 1.2 Variations of the inverter symbol

Despite the variations, there are important properties that are going to be present in all the examples, such as the lines forming the triangle or the relative size of the circle and the triangle, and unimportant properties that can be varied, such as the relative sizes of the sides of the triangle. We are faced with a classic problem in learning from examples: how can we generalize, i.e., how can we identify which subset of properties is relevant?

One common approach to this is to ask the user to draw the symbol numerous times (e.g., hundreds of times for neural nets), in the belief that the inessential elements will "average out." We find this undesirable for our task of teaching the system new symbols. The system would be more natural if one could interact with it as if communicating with another person. And typically, one example of each symbol is sufficient for people to learn a new domain. In this work, we have focused on the problem of learning as much as possible from a single example.

1.3 Motivating example

Consider how people learn new symbols such as the one in Figure 1.3.

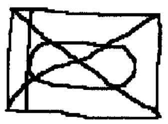


Figure 1.3 Symbol for mechanized infantry used in military planning diagrams

Most people would describe this symbol as a rectangle with diagonals, with an oval in the center and a vertical line adjacent to the oval. A single example is often enough to understand the structure of the symbol. People are likely to recognize it again, even if drawn with some variations (Figure 1.4). The goal of the learning system is to do the same, producing a description of the symbol that is adequate for later recognition.

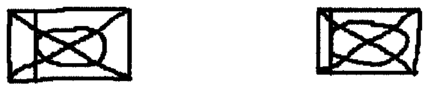


Figure 1.4 Perceptually similar symbols

Both instances of the mechanized infantry symbol in Figure 1.4 differ from the original example (eg. in the aspect ratio of the rectangle, the orientations of the slanted lines, and the relative size of the oval). Yet most people would recognize these instances. They do not pay attention to the exact values of the varied properties in the original example from Figure 3.

To understand what properties people attend to we have turned to studies of human perception and memory of geometric shapes. We looked at Goldmeier's studies of similarity [Goldmeier, 1972], [Goldmeier, 1982], Arnheim's work on art and visual perception [Arnheim, 1974], and the perceptual grouping principles identified by the gestalt psychologists. Inspired by the phenomena described in these bodies of work and following our own introspection, we have developed a number of heuristics for ranking different geometric properties on perceptual saliency. We show that they are an important step towards matching people's ability to learn from one example.

Our approach clearly depends on the assumption that the drawings in Figure 1.5 are in fact to be interpreted as the same symbol.

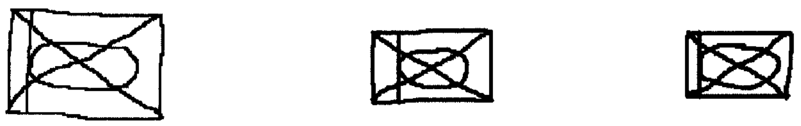


Figure 1.5 Perceptually similar symbols

We feel that it is reasonable to assume that the above figures should be the same symbol, because similarity and perceptual saliency play an important role in the design of graphical languages. If two symbols that are perceptually similar – i.e. differ on a property that people don't pay attention to – it would be unwise to use them to mean different things. They would be easily confused and the difference would be hard to remember. We thus suggest that a well-designed graphical language is unlikely to contain such ambiguous symbols.

1.4 *Domain-specific knowledge*

Geometric saliency is not the only source of people's capacity to learn symbols. In some cases we also use domain-specific information. Consider the symbol of an AND-gate in Figure 1.6.

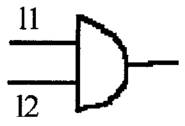


Figure 1.6 AND-gate symbol

When students first learn this symbol in a logic design class they know that the lines labeled l1 and l2 do not have to be the same length because they represent wires. Domain-specific knowledge and graphical conventions sometimes help identify which properties are not important, even if these properties are perceptually salient. Our system currently does not incorporate such knowledge. Yet we feel that relying only on geometric information is still a step in the right direction. There are domains, like military diagrams, where most of the symbols are quite abstract and rarely resemble the objects that they represent (like symbol in Figure 1.3). Most people would still be able to learn these symbols from one example, using only the geometric clues. In the future, the system could be extended to incorporate domain information or common conventions.

1.5 *Measure of success*

Ideally, the measure of success for the system is whether the produced descriptions are adequate for recognition. By adequate we mean that the description would cause the recognition engine to admit all and only the instances that the user intended to be recognized when teaching the system an example of the symbol. As the system uses only geometric information, it is bound to make domain-related errors in some cases. For example, it would conclude from Figure 1.6 that lines l1 and l2 have to be the same length. Hence, we prefer to evaluate the system's descriptions by comparing them to the geometric properties a person shown the same symbol would pay attention to, *without* taking into account the knowledge of the domain or of how the symbol is to be used.

A possible way to test this is to show people a symbol from an unfamiliar domain and to ask whether different variations of it should be recognized as the original symbol. The accepted variations should match the description and the rejected should not. We have conducted such a study with several military planning symbols. On examples with high agreement between the subjects the system achieved 83% accuracy (i.e. percentage of the times when it agreed with the majority answer). We describe the study in more detail in Chapter 6.

1.6 *Example and approach*

This section illustrates what the system does on a brief example. Suppose the user would like to teach the system the symbol in Figure 1.7.

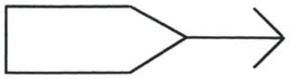
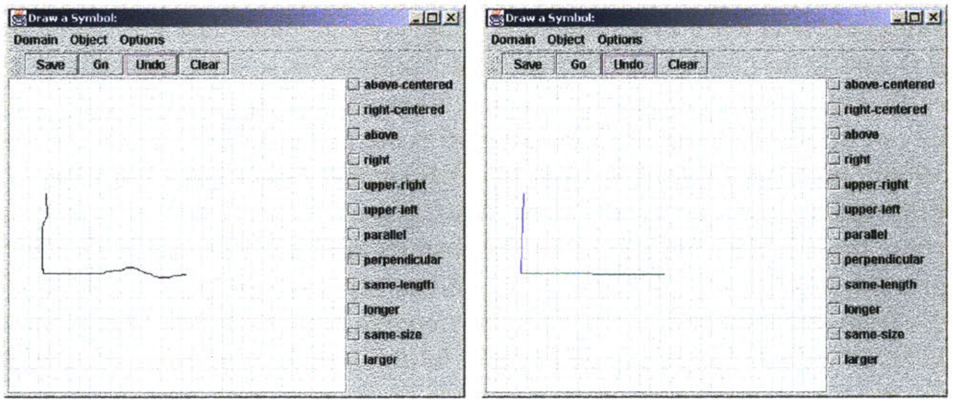
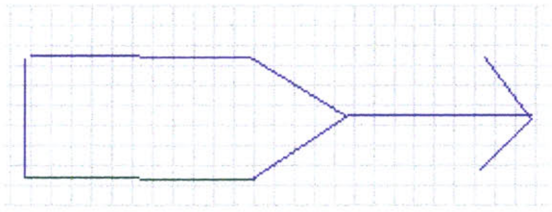


Figure 1.7 Military planning symbol

The system expects the user to draw carefully. We think that this is a reasonable requirement for the teaching phase, since the symbol only has to be drawn once. As the user draws the symbol in the drawing window (with a mouse or pen-based input), each individual stroke is segmented into simple geometric primitives (like lines and ovals) using pen-speed and stroke curvature data [Sezgin, 2001]. After the drawing is completed the user presses "Go" to start generating the description.



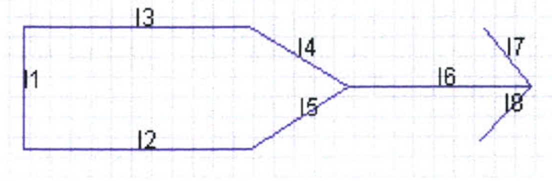
a) b)



c)

Figure 1.8 a) Single stroke. b) Segmentation into geometric primitives. c) Completed symbol

The system straightens out the primitives that are almost horizontal or vertical and properly connects the line endpoints if their separation is within a small threshold. The straightened and labeled primitives are shown in Figure 1.9 below:



Straightened and labeled primitives

Figure 1.9 a) Single stroke. b) Segmentation into geometric primitives. c) Completed symbol

Next, the system finds all pairwise constraints that hold in the drawing. The constraints do not have to hold exactly. The system includes small thresholds on distances and angles to account for noise. For example, even if there is a small horizontal offset between the centers of lines l2 and l3, the system will consider these centers to be on the same vertical line.

There were 96 pairwise constraints found for the symbol above (see Appendix A). The challenge is to pick only the relevant subset of these constraints for the description. For example, both of the constraints "same-length (l4 l1)" and "same-length (l4 l5)" hold in the drawing. However, people would typically include only the second of those in their description of the symbol.

The system uses several mechanisms to generalize the description (i.e. filter out irrelevant constraints), inspired by the results of psychological studies and our introspective analysis. Here we give a brief summary of these mechanisms and provide more details in chapters 3 and 5:

- **Qualitative vocabulary:** Initial generalization is achieved by using qualitative terms to describe constraints and properties. For example, the orientation of a line is described as "horizontal", "vertical", "positive-slope", or "negative-slope."
- **Different default relevance scores:** On average, different types of constraints have different perceptual importance. For example, the system assigns higher relevance scores to "connects" constraints than "longer" constraints.
- **Score adjustment based on global properties:** The system increases or decreases the relevance score of each constraint using three heuristics that analyze the global properties of the symbol:
    1. *Obstruction*: This heuristic relies on the assumption that it is harder to pay attention to constraints between two primitives if several other primitives separate them (create obstruction). For example, in Figure 1.9 there are several lines between lines l1 and l7. Hence, the relevance of constraints like "longer (l1 l7)" will be decreased.
    2. *Tension lines*: People pay attention to horizontal and vertical alignments of primitives. We call such alignments tension lines. We increase relevance of constraints that cause the primitives to be aligned. For example, in Figure 1.9 line l3 is centered above line l2 and the lengths of these lines are the same. Hence, their endpoints are aligned vertically. The relevance of "above-centered" and "same-length" constraints would be increased even if these primitives were separated by several others.
    3. *Grouping*: People tend to group primitives together and see them as a whole. The system currently supports grouping by connectedness and familiarity of shape (although perceptual grouping also results from proximity, similarity, continuity, and closure of primitives). When people

see primitives as one whole, they pay less attention to individual detail. The system decreases the relevance of a constraint on a pair of primitives if they belong to different groups. In Figure 1.9 lines l6, l7, and l8, are recognized as a previously learned arrow, so the relevance of constraints like "longer l6 l4" will be decreased.

The system uses these mechanisms to calculate a relevance score between 0 and 1 for each constraint. Constraints with relevance less than 0.5 are filtered out. The description shown below was produced for the symbol in Figure 7, after filtering removed half of the initial constraints:

GROUP HIERARCHY: Group g1 connected-component: l5 l4 l3 l1 l2 l6 l8 l7 Group g2 symbol - right arrow: l8 l7 l6 Group g3 other: l5 l4 l3 l1 l2 CONSTRAINTS: elongated: (g3) connects: (l5.p2 l6.p1) (l4.p2 l5.p2) (l4.p2 l6.p1) (l3.p2 l4.p1) (l2.p2 l5.p1) (l1.p1 l3.p1) (l1.p2 l2.p1) horizontal: (l3) (l2) vertical: (l1)	pos-slope: (l5) neg-slope: (l4) right: (l5 l1) (l4 l1) upper-right: (l5 l2) (l4 l1) (l4 l2) (l3 l1) upper-left: (l3 l4) (l3 l5) (l1 l2) above-centered: (l4 l5) (l3 l2) same-length: (l4 l5) (l2 l3) longer: (l3 l1) (l3 l4) (l2 l1) (l2 l5)

This description reasonably captures the salient properties of the symbol. It would cause the recognition to admit all the variations of the symbol in Figure 1.10 and reject the variations in Figure 1.11.

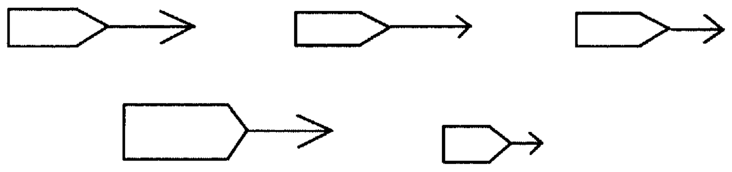


Figure 1.10 Variations that would fit the description

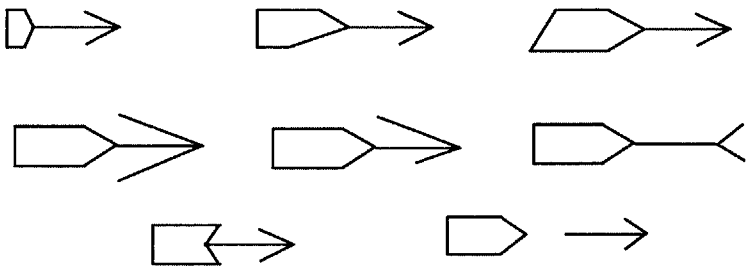


Figure 1.11 Variations that would contradict the description

We have started exploring ways of displaying the system's conclusions graphically, in order to enable the user to check the result without having to read the textual description. Figure 1.12 shows constraints displayed for line l3.

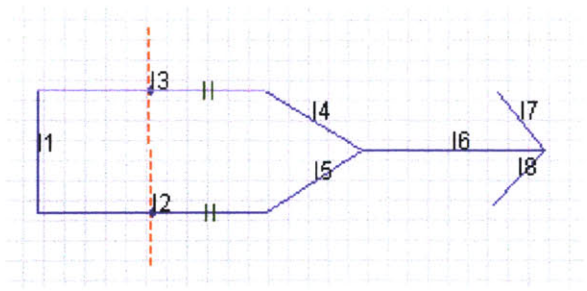


Figure 1.12 Graphical display of the constraints

The user selects line l3 and the system shows all constraints related to this line. Short double dashes indicate the "same-length (l3 l2)" constraint and dashed lines indicate relative position and center alignment – "above-centered (l3 l2)".

1.7 Scope and limitations

The system currently supports symbols composed of lines and ovals, and could be easily extended to support arcs. It can describe symbols that can be expressed in terms of qualitative constraints. So for example, it would not be able to learn a constraint like "three times longer," since the main assumption is that such constraints are unlikely to be important in a lot of cases.

Our qualitative vocabulary lumps several property values into one term and does not capture that some values may be "too much." The system would describe the relative size of the circles in Figure 13a as "larger o1 o2". The drawing in Figure 1.13b would fit the description, even though most people would probably say that the difference between the sizes is too large for Figure 1.13b to be recognized as Figure 1.13a.

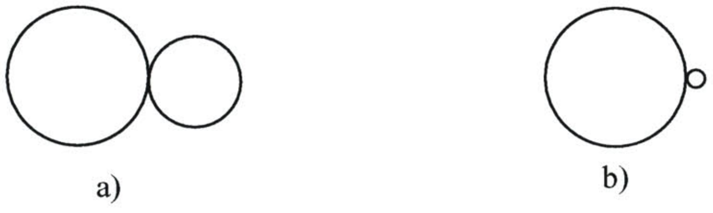


Figure 1.13 a) Original symbol. b) Variation that fits the description of the original symbol

A potential solution would be to add a "much larger" constraint, however it may not be easy to define a good boundary between "larger" and "much larger."

Another limitation comes from using only positive constraints, i.e. specifying only which constraints should hold in the symbol. The system does not include "must not" constraints. So, for example, the system would not be able to describe a closed shape (say, a quadrilateral) that should not have self-intersections.

Currently the system uses only pairwise constraints. So certain constraints like interval equality or alignment of multiple elements are not represented, which makes it impossible to properly describe configurations like the one in Figure 1.14:

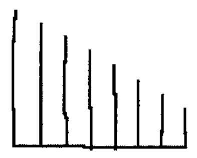


Figure 1.14 A symbol requiring alignment and interval equality constraints

All the limitations mentioned above refer to the system's inability to sufficiently constrain the description of certain symbols. This may create a problem if symbols in the domain are distinguished only based on such properties – for example, if a normal rectangle and a very long thin rectangle are intended to be two different symbols. The system would have the same description for both. We feel that even with this limitation the system is still applicable for describing a large variety of symbols, since in many domains symbols have more than one structural property that distinguishes them. A more serious problem arises for the symbols that the system is bound to overconstrain. A whole class of symbols that can have an arbitrary number of certain primitives (Figure 1.15) falls into this category, because the system always specifies exactly the number of primitives that a symbol should have. Hence, springs, resistors, inductors, dashed lines, etc. would be impossible to describe properly. We address potential approaches to this problem in the Future work chapter.

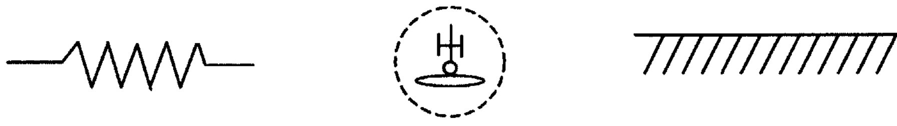


Figure 1.15 Symbols with an arbitrary number of primitives

1.8 Structure of the thesis

Chapter 2 discusses related work on sketching and learning shape descriptions. In Chapter 3 we describe the findings in the perceptual literature that served as inspiration for our approach. Chapter 4 illustrates the performance of the system on several examples and shows how each of the generalization mechanisms is applied. We discuss the details of the implementation in Chapter 5, followed by user study analysis in Chapter 6 and ideas for future work in Chapter 7.

Chapter 2 Related Work

There has been a substantial amount of work on making human-computer interaction more natural by adding interfaces that support free-hand sketches. Work on sketching systems to date falls into two categories: systems that use sketching interfaces without attempting to interpret what the input means and systems that actually perform recognition of the sketched objects.

The work in the first category includes systems that transform the user's free-hand input to beautify it [Arvo and Novins, 2000], [Igarashi et al., 1997], systems that support intelligent editing of sketches by allowing perceptually based selection of strokes [Saund et al, 2002], and systems that allow capturing sketches for documenting designs or knowledge but minimize recognition of shapes, so that the user is free to draw anything [Lin et al, 2002], [Forbus and Usher, 2002].

The systems that are more relevant for our work are the ones that perform recognition of the sketched input. Here we mainly discuss two aspects of these systems:

- **Representation:** We are interested in how the recognized symbols are represented, what features are recorded, and what the descriptive ability of the chosen representation is. For our system we have chosen a symbolic, qualitative representation that corresponds to properties that people typically find perceptually relevant. It is an important source of generalization, because it throws out information on properties that we expect to vary in different instances of the symbol we want to learn. We examine differences and similarities to this approach for the reviewed work.

- **Learning:** We look at how the recognizers for the symbols are acquired, i.e. whether they are specified by hand or can be obtained automatically through training, and if yes, how many training examples are required. In our system we choose to learn symbols from a single example, while most of the systems reviewed here need several examples. Yet, some systems are able to learn from much fewer examples than others. So it is important to look at the sources of power for the generalization mechanisms. We believe that in our system, apart from the qualitative vocabulary, such source is the prior knowledge about human perception of geometry. While other systems rely on looking at several examples to "average out" the properties in the symbol that are irrelevant, our system obtains that information from the relevance ranking of the properties based on studies of human perception.

One of the earlier sketching systems that several other approaches are based on is Rubine's GRANDMA [Rubine, 1991]. Rubine describes a trainable recognizer for single-stroke gestures. Gestures are represented by global features, like length and angle of the bounding box diagonal, the total angle traversed, the sum of the angles at each mouse point, the duration of the gesture, the initial angle of the gesture, etc. The gestures are classified according to a linear function of the features, where the weights are determined during training on multiple examples (typically around 50).

Apart from handling only single strokes, the limitation of this approach is that it only uses aggregate properties of the stroke. The representation does not explicitly capture the detail that may help disambiguate between two gestures with very similar aggregate properties. The representation used in our system makes explicit the properties and constraints on parts of the symbol (like individual lines or ovals).

[Caetano et al., 2002] presents JavaSketchIt – a system that can recognize sketched UI components (buttons, scroll-bars, check-boxes, etc.) and automatically create Java code for them. To recognize UI components, JavaSketchIt uses CALI, a shape recognizer described in [Fonseca et al., 2002]. CALI can recognize simple shapes: squares, rectangles, diamonds, triangles, arrows, crosses, and simple single stroke gestures.

CALI is similar to Rubine's recognizer in that it also uses aggregate properties to represent shapes. Shapes are specified in terms of features of special polygons: enclosing rectangle, convex hull, largest inscribed triangle and largest inscribed quadrilateral. Using these global features provides certain flexibility. For example, CALI recognizes multi-stoke shapes. Also, shapes can be drawn with overtraced and dashed lines. However, as mentioned above, the recognizer only handles simple shapes. For symbols with more internal detail, like military diagram symbols (see Figure 1.3), the given features would be clearly insufficient. Moreover, the number of training examples used to achieve a sufficiently high level of recognition was over 50 for each shape.

Landay and Meyers also describe a sketching tool for designing user interfaces – SILK [Landay and Meyers, 2001]. SILK recognizes sketched interface widgets composed of primitive components – rectangle, squiggly line, straight line, and ellipse. The recognizers for primitive components are based on Rubine's algorithm. A similarity to our system is that SILK uses symbolic spatial relationships (like containment, nearness, and vertical or horizontal sequence) between the primitive components to determine the interface widget that the designer is trying to draw. For example, a scroll-bar is a long skinny rectangle with a box contained in it. However, there is no mechanism for learning these relationships. SILK creators specified the relationships for each UI widget manually. Only the recognizers for the primitive components can be trained (using Rubine's algorithm). Our system, on the other hand, provides mechanisms to learn such relationships from an example. Notice also that the set of spatial relationships in SILK is limited by what is needed for the application at hand. It may be insufficient for describing more complicated symbols in general (for example, the set does not include parallelism or same-length properties).

The Electronic Cocktail Napkin (which is the recognition core of a later system for sketching in conceptual design [Gross and Do, 2000]) uses two level representation for symbol similar to that of SILK: low level glyphs and combinations of glyphs described by symbolic spatial relationships between them [Gross *et al.,* 1996]. A glyph is a single-stroke or multi-stroke symbol represented by a transition sequence of the pen through the cells of a three-by-three grid. For each glyph the aggregate properties like allowed number of strokes, number of corners, aspect ratio, and size, are recorded. More complicated symbols can be composed from several glyphs, by specifying spatial relationships between the glyphs. Spatial relationships include adjacent, containing, and overlapping shapes, and intersection, parallel, and tee conditions among line segments.

This representation allows describing a larger variety of symbols than SILK and CALI. The Electronic Cocktail Napkin also lets the user to specify new glyphs and glyph combinations. Yet, when learning these combinations, the system records all the spatial relationships that it finds for the combination and the user has to manually remove the ones that are unimportant. There is again no automatic generalization mechanism.

Shilman et al. treat symbol recognition as visual language parsing [Shilman et al., 2002]. The visual language consists of the declarative grammar that specifies ranges of allowed values for a set of constraints between elements (distance, angle, width and height ratios, overlap). Training on many examples is used to turn these ranges into distributions, so that the maximum likelihood parse tree can be calculated during recognition. Again, the visual grammar has to be written by hand, i.e. the designer has to determine relationships that are significant for the statistical model. Only the distributions are obtained through training.

This system, as well as SILK and the Electronic Cocktail Napkin, deal with the potential variability of symbol instances partially through using a small symbolic vocabulary of spatial constraints. However, none of the systems provides capabilities for learning which of these spatial constraints are important – the constraints have to be provided by the user or the designer of the system.

Ferguson and Forbus describe GeoRep – a spatial reasoning engine that generates qualitative spatial descriptions from perfect line drawings [Feguson and Forbus, 1999]. It has been applied for symmetry detection tasks, critiquing simple diagrams of physical phenomena, and spatial reasoning about military course of action diagrams. The paper mentions future applications of GeoRep to sketching, once it is modified to process free-hand input rather than exact line drawings. Apart from using a qualitative vocabulary of spatial constraints, GeoRep also includes generalization capabilities.

The part of GeoRep that is relevant to our work is the Low-Level Relational Describer (LLRD). LLRD produces qualitative spatial descriptions of the input in terms of geometric primitives and relations between them. It handles lines, circular arcs, circles, ellipses, splines, and text strings. It records position constraints like above, beside, etc.; orientation constraints, like horizontal, vertical; connection relations, parallel lines, interval relations, presence of polygons, and boundary description.

Similar to our system, GeoRep attempts to limit the number of recorded constraints between different primitive elements, because not all of them are visually important. The single mechanism it uses for this purpose is proximity. LLRD only looks at constraints between proximal elements. Proximity is calculated as a function of size, shape type, and distance between elements.

Our system also prefers local interactions. Though locality is defined not through distance but through the obstruction mechanism. The primitives are considered "close" if there are no other primitives between them, regardless of the actual distance. Chapter 3 explains how we chose such definition based on analyzing human perception.

In addition, our system adjusts the relevance of different properties of the symbol based on alignments (tension lines) and grouping. We show that even if two primitives are far away from each other, the constraints on them may still be relevant for the description of the symbol, and these mechanisms help detect this.

Connell's work on learning shape descriptions for images of physical objects (airplanes, tools, household items, etc.) contains several ideas that are also reflected in our work [Connell, 1985]. The goal of their system is to generalize a description for a class of objects (e.g., "airplane") from images for several objects in the class (e.g.

individual types of airplanes) in order to be able to recognize a new instance of the object. Objects are represented in terms of non-overlapping elongated blobs and qualitative properties (like straight, curved, tapered, etc.) and constraints on these blobs (like joins, bigger-than, etc.). The description is recorded as a semantic network.

The parallel to our work is in the idea that the representation vocabulary should correspond to perceptually salient properties of objects. The description should make the visually important parts explicit. Connell talks about the importance of reflecting people's notion of visual similarity: "syntactic difference should reflect semantic difference: similar things should give rise to similar descriptions, dissimilar things should yield manifestly different descriptions."

Connell's system generalizes descriptions from a very small set of examples by comparing their semantic networks and removing constraints and properties that are not common between the examples. We believe that the ability to generalize from only a few examples stems mostly from the qualitative description vocabulary that already gets rid of a lot of information about the detailed properties of an object. The system does not have to go through a lot of examples to "average out" the unimportant properties.

Our system differs from Connell's approach in that it defines a ranking of the constraints. It does not have to discover which constraints are unimportant by seeing their absence in additional examples. The ranking already provides this information. This, however, depends on how well ranking reflects the actual biases in people's perception.

Calhoun et al. presents a system that is most similar to our approach. It is a system that learns and recognizes symbols from relatively few examples. The recognizers are used for interpreting sketches of physical devices [Kurtoglu and Stahovich, 2002].

Like Connell, Calhoun uses semantic networks. The nodes are primitives in the symbol (lines and arcs) and the links are constraints between them. Constraints include: intersections, relative location of intersections, angle between intersecting primitives, existence of parallel lines. The lines and arcs also have properties: type, length, length relative to the sum of all lengths, slope, and radius. To train the recognizers the system uses several examples of each symbol, including only the relationships and properties that appear with high frequency in the examples. During recognition some degree of matching error is allowed. The important part is that different weights are assigned to different kinds of errors during matching, reflecting different perceptual importance. In other words, if the learned descriptions mandate some unimportant constraints to hold, the system can compensate for that during the recognition stage, because the weights on matching errors for such constraints will be low.

The error weights, in some sense, play the same role as the default relevance scores in our system. For example, the relative length constraint in Calhoun's system is always allowed a larger error than relative orientation constraint (the same is true for the default scores in our system – relative length is less relevant than relative orientation). Yet our system also adjusts relevance scores from the default scores, based on the configuration of primitives in the particular symbol. We believe that our system approximates more accurately the fact that the same type of constraint may have different importance for different primitives. (Give example)

So far we have talked about approaches that use mostly symbolic descriptions (except for Rubine's system). Commonly used statistical machine learning techniques are mostly not applicable for our system because we have chosen to learn from only a single example and these approaches typically require a very large number of examples. For example, classifiers for handwritten character recognition such as LeCun et al.'s convolutional networks, that achieve performance that is close to human subjects, use 6000 samples of each character [LeCun et al. 1995].

Handwritten character recognition is most applicable from the field of machine vision, since it deals with pen input, rather than physical images. The most relevant work for our system is by Miller et. al. on learning characters or digits from one example [Miller et al., 2000] The authors create a classifier that is based on only a single training example for each class. They achieve this by including "prior knowledge", which is the shared probability density on common transforms (deformations) of digits or characters. They create an artificial data set by sampling transforms from the distribution and applying them to the single example. Then a classifier, like nearest neighbor, for example, can be trained using this data set.

Our system would not be able to use this approach directly because their current work is limited to affine transformations (translate, rotate, scale, and sheer). We believe that affine transformations are not the only variations that produce an image perceptually similar to the original, so the distribution would not capture all the possible varaitions. Consider the example in Figure 2.1. The second arrow cannot be obtained by an affine transformation on the first arrow, because it would involve disproportionately scaling different parts of the symbol:

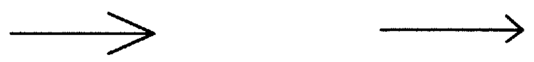


Figure 2.1 The second arrow cannot be obtained by an affine transformation of the first arrow.

Even though Calhoun's et al. approach is not directly applicable, the idea of including prior knowledge to be able to learn from one example is very similar to our approach. By providing perceptually based constraint ranking we allow the system to extract the important information from a single example of the symbol.

In summary, our approach is strongly determined by the fact that we are learning from one example. Partially the generalization power comes from the qualitative vocabulary of constraints that reflects the relevant properties. Several systems have used this approach to address the variability of the symbol instances. In addition, the generalization is guided by the relevance ranking, instead of using several examples, as have been done in several other systems.

Chapter 3 Knowledge About Human Perception

The challenge in learning symbols from a single example is to extract just the right subset of properties from it. We believe that the relevant properties are the ones that people pay attention to when looking at the symbol. A well-designed graphical language would not distinguish symbols by properties that people tend not to notice. If two symbols are very perceptually similar, but are intended to mean different things, they would be often confused, making the language ineffective. Thus, we suggest that it is the perceptually salient properties that constitute the essence of the symbol and should be learned by the system for each example.

We have turned to studies of human perception and memory to understand what people attend to and what they ignore in a geometric configuration. We rely mostly on Goldmeier's work on perceived similarity of geometric shapes and on memory traces [ref]. We also draw useful insights from Arnheim's book on art and visual perception [ref] and studies of the perceptual grouping by the gestalt psychologists [ref]. This chapter describes the sources of inspiration from the perceptual studies for the five main generalization mechanisms used by the system:

- Qualitative vocabulary
- Default relevance ranking
- Adjusting relevance scores based on global properties:
    - Tension lines
    - Obstruction
    - Grouping

3.1 Singularities as the basis for qualitative vocabulary

Goldmeier attempted to discover which properties of a geometric figure are *phenomenally realized*, i.e. which properties people notice when looking at a symbol. He uses people's perception of similarity to explore this: "Some features of a figure are more important for the over-all impression than others, so that changes of these features have a marked effect on similarity" [Goldmeier, 1972]. Figure 3.1 illustrates a typical experiment. Examine the symbol in Figure 3.1a and ask yourself which of 3.1b and 3.1c is more similar to 3.1a?

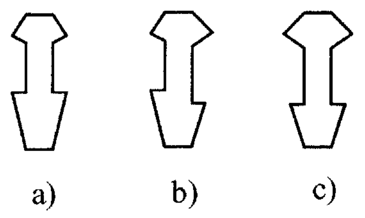


Figure 3.1 Which of b and c is more similar to a?

The majority of subjects chose c. Note that the left side of b is exactly the same as a. Yet even though in c all the lengths and angles are slightly changed, it is considered more similar because of preserved symmetry. It is the symmetry that was perceptually salient in the original figure.

Goldmeier's experiments showed that people frequently attend to properties that he called *singularities*, special cases in the space of geometric configurations (see examples below).

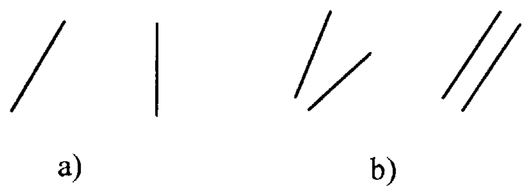


Figure 3.2 a) A vertical (or horizontal) line is a special case of possible line orientations. b) Parallel lines are a special case for possible angles between two lines

Features such as verticality, horizontality, parallelism, etc., are singular in the sense that a small variation in them makes a qualitative difference: Rotate a vertical line slightly and it is no longer vertical; do the same to a line described as "slanting upward" (i.e., positive slope) and its qualitative description stays the same.

Goldmeier's work showed that, while singularities significantly affect perception of the symbol, people are relatively insensitive to variations in nonsingular properties. Consider Figure 3.3a:

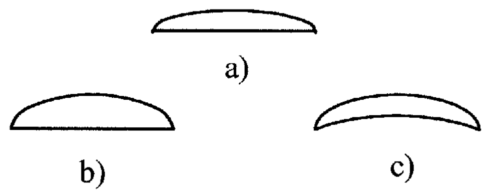


Figure 3.3 Which is of b and c is more similar to a?

Even though the thickness of the figure is preserved in c, the figure does not preserve the straight line, so the majority of subjects chose b. The subjects tolerated a large distortion in thickness and curvature, which are non-singular properties, because the more salient singular property (straightness) was preserved in b.

Goldmeier describes the way people generalize geometric properties that they see in a symbol. For each property "the value is coded either as singular, nearly singular, or nonsingular… This system combines coding accuracy in the narrow singular range with information reduction in the broad nonsingular range" [Goldmeier, 1982]. We use this observation to reduce the description vocabulary to a few qualitative states, lumping the range of nonsingular values into one term. For example, for a slanted line, it is not necessary to record the exact angle. It is enough to learn only that it has a positive or a negative slope.

Goldmeier notes that the nearly singular values are perceived as a distortion to the singularity. Taking into account the nature of sketching where it is natural to expect sloppy drawing, this distortion can typically be considered accidental. Our system interprets nearly singular values as intended singularities, so the vocabulary consists only of singular and nonsingular terms. Goldmeier explicitly mentions some of the singularities, like parallelism, horizontality, verticality, and straightness. We have picked the rest of the terms for the vocabulary based on our own introspection and using Goldmeier's description of singularities as the "more regular, better, more unique" [Goldmeier, 1982, p. 44] and as properties a change in which significantly alters the perception of the symbol.

The system records the properties of the symbol in the form of unary and binary constraints on the geometric primitives (lines and ovals) in the symbol. The table below shows the list of supported constraints (singular constraints are shown in bold)

"Touch" constraints:	**Connects, meets, intersects, touches, tangent, overlaps**
Orientation:	**Horizontal, vertical**, positive-slope, negative-slope
Aspect ratio:	Elongated, **non-elongated**
Relative position:	**Above-centered, right-centered, left-centered, below-centered**, above, below, right, left, upper-right, upper-left, lower-right, lower-left, **inside-centered**, inside
Relative orientation:	**Parallel, perpendicular**
Relative length:	**Same-length**, longer
Relative size:	**Same-size**, larger

3.2 Default ranking: relative importance of different singularities

In addition to showing that singular properties are perceptually more important than nonsingular ones, Goldmeier also compared singular properties among each other. Figures 3.4 and 3.5 illustrate how this is done for different axes of symmetry. The subjects were asked which of b and c is more similar to a.

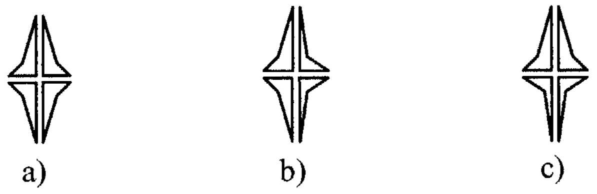


Figure 3.4 Which of b and c is more similar to a?

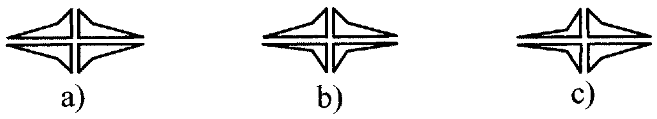


Figure 3.5 Which of b and c is more similar to a?

In Figure 3.4 the majority of the subjects chose c, while in Figure 3.5 the choice was b, even though the shapes in Figure 3.5 are simply rotated versions of Figure 3.4. In both cases the viewers preferred the vertical axis of symmetry.

This example is not directly applicable to our system, since it currently does not support symmetry detection, however, it illustrates the experimental framework in which the importance of different properties can be compared. Goldmeier presents several similar experiments, however they are not sufficient to construct a ranking of different constraints used by our system. We had to use our own introspective analysis of common symbols in several domains (electric circuits, military planning, mechanical engineering, etc.) to rank the average perceptual importance of different types of symbol properties. The list below shows the order of decreasing importance:

1. The parts that the symbol is composed of.
2. "Touch" constraints.
3. Orientation.
4. Relative orientation.
5. Relative position.
6. Relative length and relative size.

Note, however, that this is only the default ranking of constraints. Goldmeier argues that the saliency of a given property depends on the particular configuration of the primitives in a shape. The next section describes the observations that helped us develop heuristics for adjusting the relevance of different constraints based on global properties like alignment (tension lines), obstruction, and grouping of primitives.

3.3 Effect of global properties on constraint relevance

3.3.1 Tension Lines

In his book *Art and Visual Perception*, Arnheim argues that people pay attention to regular alignments of geometric primitives in a symbol, particularly horizontal and vertical alignments. In Figure 3.6a the circle is perceived to be "out of balance," while placing it on one of the dashed lines in 3.6b would create a more "stable" configuration [Arnheim, 1974]:

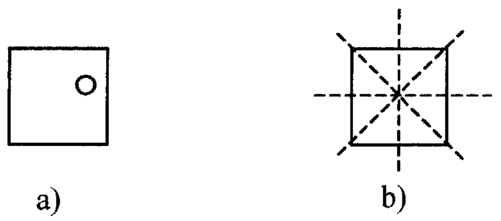


Figure 3.6 Regular alignments

Arnheim talks about "the hidden structure of a square" that can be explored by placing the circle in different places inside the square. The lines shown in Figure b emerge as axes of most stability, especially the horizontal and vertical lines. The alignment of corners and the centers of the sides of the square form a kind of perceptual grid that other elements are "pulled" toward.

In our system we call these alignments *tension lines*, which we define by looking at alignments of line endpoints and midpoints. The system identifies a tension line wherever at least two such line points align horizontally or vertically (currently, we do not support diagonal alignments). Although this definition of tension lines may not capture the full complexity of the perceptual mechanisms that create the hidden structure, we believe that it can serve as a useful approximation.

Since the hidden structure grid represented by the tension lines is a salient element of the symbol, we increase the relevance of relative length, position, and orientation constraints that contribute to the creation of these lines.

3.3.2 Obstruction

We looked a variety of symbols to try to understand the perceptual importance of different constraints. In the process we have noticed that in the symbols that contain a lot of primitives our attention seems to be limited to the local interactions between primitives. Consider the example in Figure 3.7a below:

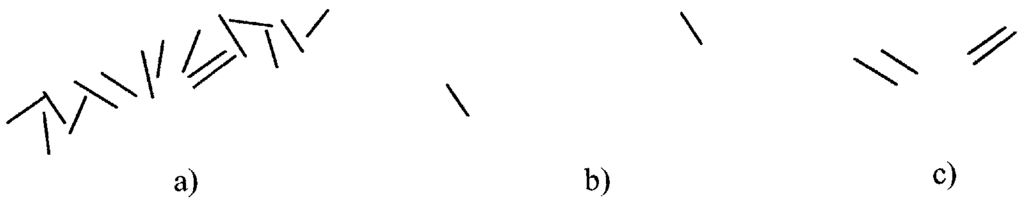


Figure 3.7 a) Pattern of lines. b) Two parallel lines that are part of the pattern. c) Other pairs of parallel lines that are part of the pattern

The lines in Figure 3.7b are part of the pattern in a. In b it is noticeable that the lines are parallel, while in a, that is not a constraint that we pay attention to. Part of the reason for this might be that we perceive the pattern as a whole – a slanted elongated blob of lines. Nevertheless, notice that the parallelism of the pairs of lines in c is more noticeable in the original pattern than the parallelism of pair b. It is easier to pay attention to the local interaction of these lines because there are no other lines separating them. We try to approximate this effect by the notion of *obstruction*, which is measured by the number of geometric primitives between a given pair. The relevance of constraints is decreased for higher obstruction values.

3.3.3 Grouping

Finally, we also use observations of perceptual bias from the Gestalt psychologists, who noted that people tend to combine individual primitives into a greater whole, grouping them by proximity, similarity, etc. For example, Figure 3.8a is perceived as two rows of circles, rather than six individual circles. Properties of a row as a whole are also perceptually more important than properties of its components. We don't tend to notice the vertical alignment of the circles in two columns the way we do in Figure 3.8b:

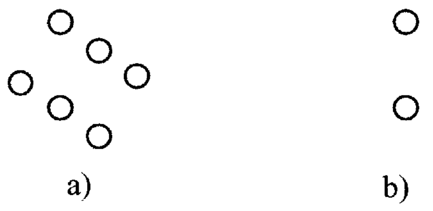


Figure 3.8 Perceptual grouping. It is generally not noticeable that parts of a are the same as b

Grouping allows describing symbols more concisely. In the figure below the group consisting of the circle and the arrow is centered inside the rectangle. Conveying the same relationship using individual constraints on each of the primitives would be much harder.

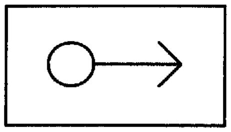


Figure 3.9 Military planning symbol for mortar

Our system currently supports two grouping principles: connectedness and familiarity of shape, i.e. previously learned shapes are recognized as separate groups within a new symbol. We decrease the relevance of constraints between pairs of primitives that belong to different groups.

3.4 Challenges

Studies show that people's view of geometric properties, such as sizes, angles, orientation, and curvature, do not easily map onto exact measurements from the drawing. Goldmeier notes: "Experiments demonstrate that similarity does not vary parallel with simple and obvious geometric parameters." Consider the example below, taken from Goldmeier:

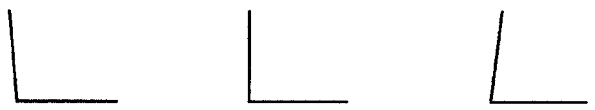


Figure 3.10 Which angle is 90°?

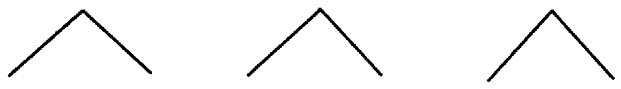


Figure 3.11 Which angle is 90°?

It is much harder to tell which angle is 90°, even though the angles in Figure 3.10 are simply rotated versions of the ones in Figure 3.11.

Another example of a common misjudgment is the famous perceptual illusion given below:

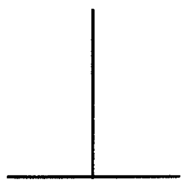


Figure 3.12 Do the lines seem the same length?

Even though the mechanisms causing such misjudgments are not understood well enough to exactly replicate such biases computationally, we include heuristics for the most common cases, decreasing the relevance of perpendicularity, for example, if it is found in other than a horizontal/vertical configuration.

Chapter 4 Examples of Performance

This chapter demonstrates examples of the system's descriptions for several symbols. We show how the mechanisms described in the previous chapter affect the system's decisions on the relevance of different constraints.

Currently the system can handle symbols composed of straight lines, ovals, and circles. Note that certain symbols, like symbols for electric circuits, may be valid in any orientation. The system creates the description for the given orientation and the user can indicate separately whether the symbol can be rotated.

The symbols can be drawn with a mouse or pen-based input. The segmentation of each stroke into geometric primitives is shown after the user lifts the pen. Primitives are shown in alternating colors to make the segmentation clear.

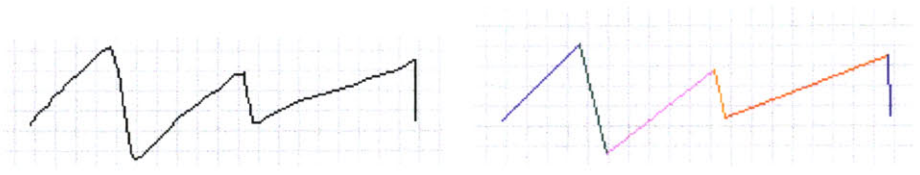


Figure 4.1 a) Original stroke. b) Geometric primitives

The description is based on the primitives, rather than the strokes, so the symbol can be drawn in any order and number of strokes, as long as the same primitives are identified in the drawing.

4.1 Example

Consider the symbol for in the Figure 4.2 below:

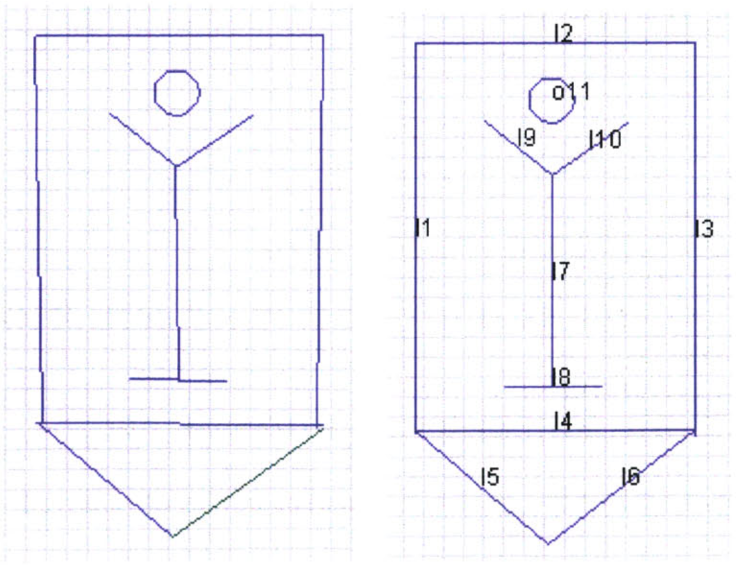


Figure 4.2 Military symbol: a) Strokes segmented into primitives. b) Straightened and labeled primitives

For this example, we assume that the system has not been previously taught the rectangle or the triangle shapes, so it will not be able to identify them in the symbol. Further we will illustrate the description the system produces after if it has learned common shapes beforehand.

The system initially identifies 166 pair-wise constraints in the symbol, if we count each pair of symmetrical constraints (like "same-length (l6 l5) (l5 l6)") only once. Each of the constraints is given a default relevance score between 0 and 1, yet all the default scores are above the 0.5 filtering threshold (see Appendix A for the list of constraints).

The initial number of constraints grows quadratically with the number of geometric primitives in the symbol. A lot of these constraints are not relevant. The core of our system deals with the reduction of the number of constraints by adjusting the default scores based on the global properties of the symbol:

- Obstruction
- Tension lines
- Grouping

After adjusting the scores, filtering removes 86 constraints, leaving 80 constraints in the description:

CONSTRAINTS:
connects: (l5.p2 l6.p1) (l4.p2 l6.p2) (l4.p1 l5.p1) (l3.p2 l6.p2) (l3.p2 l4.p2) (l2.p2 l3.p1) (l1.p2 l5.p1) (l1.p2 l4.p1) (l1.p1 l2.p1) (l9.p2 l10.p1) (l7.p1 l10.p1) (l7.
p1 l9.p2)
meets: (l7.p2 l8.c)
horizontal: (l4) (l2) (l8)
vertical:(l3) (l1) (l7)
pos-slope:(l10) (l6)
neg-slope:(l5) (l9)
above:(l10 l8) (l4 l5) (l4 l6) (o11 l10) (l9 l8)
right:(o11 l1)
below:(l5 l4) (l8 l10) (l6 l4)
left:(o11 l3)
upper-right:(l10 l7) (o11 l9) (l3 l4) (l3 l6) (l2 l1)
upper-left:(l2 l3) (l1 l4) (l1 l5) (l9 l7)
lower-right:(l5 l1) (l10 o11) (l4 l1) (l3 l2) (l3 o11) (l8 l9) (l7 l9)
lower-left:(l4 l3) (l1 l2) (l1 o11) (l9 o11)
above-centered:(o11 l4) (o11 l7) (o11 l8) (l2 l4) (l2 l7) (l2 l8) (l2 o11) (l8 l4) (l7 l4) (l7l8)
right-centered:(l10 l9) (l3 l1) (l6 l5)
same-length:(l5 l6) (l2 l4) (l1 l3) (l9 l10)
longer:(l4 l5) (l4 l6) (l3 l2) (l3 l4) (l3 l6) (l1 l2) (l1 l4) (l1 l5) (l7 l9) (l7 l10)

We illustrate the effect of each of the mechanisms and examples of removed constraints in detail.

4.1.1 Obstruction

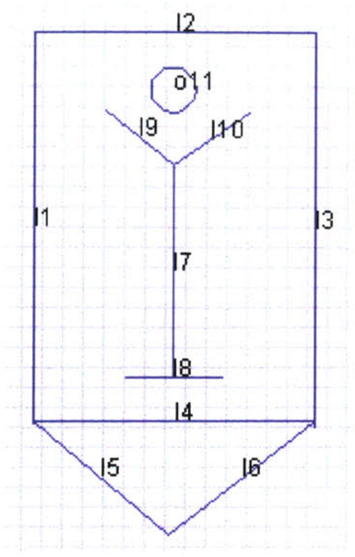


Figure 4.3 Military symbol

The first heuristic applied to adjust the relevance scores is the obstruction mechanism. For each pair of primitives it measures the approximate number of other primitives between this pair (chapter 5 gives the precise definition). For example, there are 5 primitives between lines l2 and l4. Larger obstruction values result in greater decrease of the relevance scores for the constraints on the pair. This mechanism affects relative orientation, position, length, and size constraints.

For the symbol in Figure 4.3, the relevance of 41 constraints was decreased, pushing some of the scores below the threshold (although, they may be brought back by the tension line mechanism). Examples include:

parallel: (l10 l5) (l9 l6)
longer: (l1 l5) (l3 l5)
same-length: (l10 l8)
above: (l10 l5)
upper-right: (o11 l6)

4.1.2 Tension Lines

Tension lines are horizontal and vertical alignments of two or more line endpoints or center points. Figure 4.4 shows the tension lines in the example symbol.

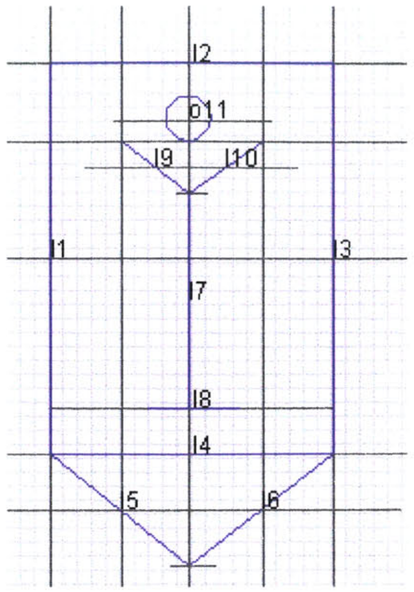


Figure 4.4 Tension lines formed by the end-points and center-points of lines and the circle

The formation of tension lines depends, in part, on the relative position and relative length constraints. For example, in Figure 4.4, lines l3 is centered to the right of line l1 and these lines have the same length. Hence, their endpoints are aligned and create two tension lines. Alignments are salient properties of the symbol. Hence, we increase the relevance of relative length and position constraints that support them. This may bring back above the filtering threshold the constraints previously pushed down by obstruction.

The system adjusts the relevance of 15 relative position and length constraints – for each pair of primitives the endpoints of which support two tension lines. Examples of such constraints are

same-length: (l4 l2) (l1 l3) (l6 l5)
right-centered: (l3 l1) (l10 l9)
above-centered: (l2 l4)

4.1.3 Grouping

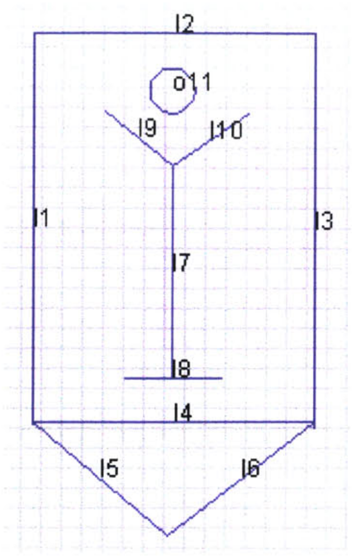


Figure 4.5 Military symbol

People tend to group together subsets of primitives in a symbol, especially when it contains a lot of primitives. The group is perceived as one whole and relationships between individual primitives belonging to different groups become less important. Our system currently supports two of the perceptual grouping principles: connectedness and familiarity of shape. In Figure 4.5, for example, there are 3 connected components:

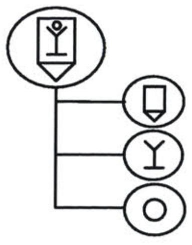


Figure 4.6 Grouping of the primitives in Figure 5 based on connected components

For this symbol, the system decreases the relevance of 35 constraints between primitives in different connected components. In most cases the obstruction factor alone was enough to push some of the same constraints below the filtering threshold. However, constraints like "longer (l4 l8) (l3 l7)" were only filtered out due to grouping factor.

4.2 More effects of tension lines

In the previous example the system increases the relevance of a constraint between a pair of primitives if the endpoints of these primitives contribute to the formation of two tension lines. The second example of this mechanism applies to one tension line formed by centers of more than two primitives. Alignment of several centers creates a "stronger" tension line. The system increases the relevance of each "above-centered" or "right-centered" constraint that contributes to such a line.

Consider the symbol for DC voltage in Figure 4.7:

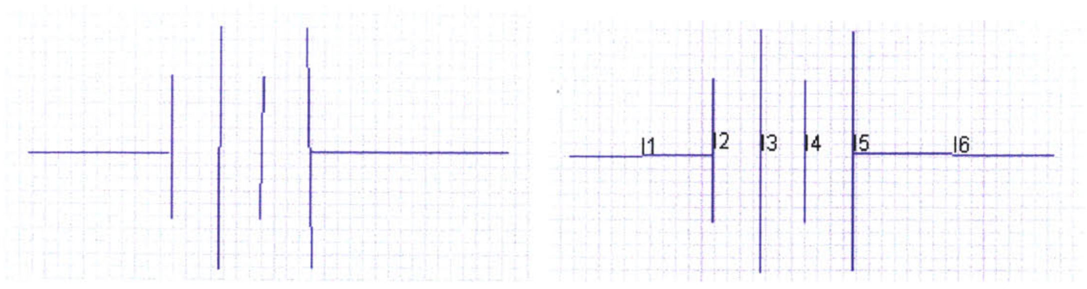


Figure 4.7 Symbol for DC voltage: a) Drawn strokes segmented into geometric primitives. b) Strokes straightened out and labeled

The description for this symbol is shown below:

CONSTRAINTS:
meets: (l1.p2 l2) (l6.p1 l5)
horizontal: (l1) (l6)
vertical: (l5) (l4) (l3) (l2)
right-centered: (l5 l1) (l5 l2) (l5 l3) (l5 l4) (l4 l1) (l4 l2) (l4 l3) (l3 l1) (l3 l2) (l2 l1) (l6 l1) (l6 l2) (l6 l3) (l6 l4) (l6 l5)
same-length: (l3 l5) (l2 l4)
longer: (l5 l4) (l3 l2) (l3 l4)

STATS:
No. of initial: 45
No.of affected by obstruction: 15
No.of affected by tension lines: 15
No. of final: 28

All the primitives form a strong horizontal tension line. The constraint "right-centered l6 l1" would have been filtered by the obstruction mechanism since there are several lines between lines l6 and l1. However, the presence of the tension line caused the system to increase of the relevance of this constraint.

Examples of the constraints the system filtered out for this description are "longer (l5 l2) (l5 l1) (l6 l1)" etc.

4.3 Familiar shapes

A second grouping factor supported by the system, apart from connectedness, is the familiarity of shape. This factor groups together primitives within the symbol that form an already known shape. In many domains (e.g. military planning) symbols are composed of common shapes like rectangles, triangles, diamonds, circles, etc. or a combination of other simpler symbols. A much more concise description of the symbol is often possible in terms of constraints on those shapes as a whole. Also constraints between individual primitives that belong to different shapes become less perceptually relevant.

To group primitives based on the familiarity of shape, the system checks whether any of the previously learned symbols are contained in the new symbol and identifies such subparts as separate groups. Consider the symbol in Figure 4.8. Before learning it, the system has been shown a rectangle, a square, a triangle, and a cross and produced descriptions for those symbols. It will search for them in the new symbol.

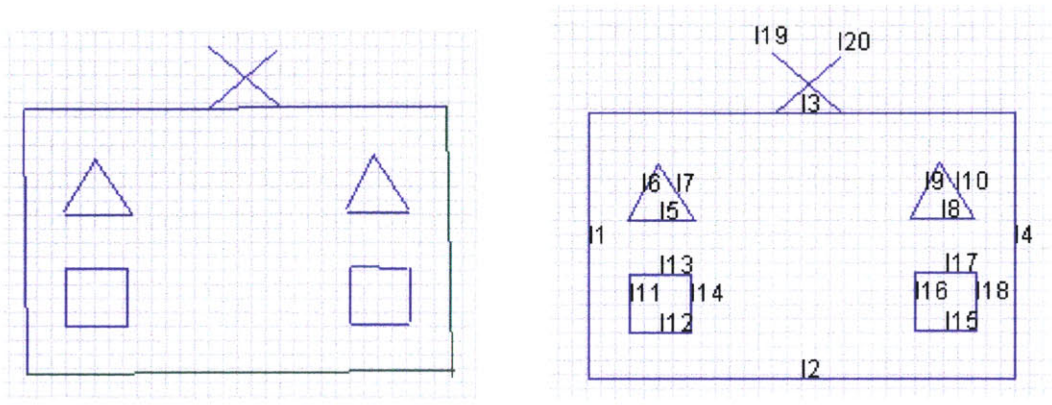


Figure 4.8

Below is the system's description for the symbol:

```
GROUP HIERARCHY:
Group g1: l1 l2 l3 l4 l5 l6 l7 l8 l9 l10 l11 l12 l13 l14 l15 l16 l17 l18 l19 l20
        Group g2 subobject - regular triangle: l5 l7 l6
        Group g3 subobject - regular triangle: l8 l10 l9
        Group g4 subobject - square: l14 l13 l12 l11
        Group g5 subobject - square: l18 l17 l15 l16
        Group g6 connected-component: l19 l4 l3 l1 l2 l20
                Group g7 subobject - cross: l20 l19
                Group g8 subobject - rectangle: l4 l3 l2 l1

CONSTRAINTS:
upper-right:(g3 g4)
upper-left:(g2 g5)
above-centered:(g3 g5) (g2 g4)
right-centered:(g5 g4) (g3 g2)
inside:(g5 g8) (g4 g8) (g3 g8) (g2 g8)
meets:(l19.p2 l3) (l20.p1 l3)
above-centered:(l20 l3) (l19 l3)

STATS
No. of initial: 500
No. of affected by obstruction: 300
No. of affected by to tension lines: 60
No. of affected by grouping: 340
```

No. of removed, because already listed in previously learned descriptions: 93
No. of final: 14

Multiple constraints have been filtered out due to the grouping factor. For example, the relative length and position constraints between lines l4 and l6 are not affected by obstruction and support horizontal tension lines. Yet they receive a low relevance score because these lines belong not only to different connected components but also to different previously learned symbols, which makes the relevance decrease even greater.

The symbol is described in terms of more general constraints on the shapes as a whole. There is also no need to include the constraints that are already listed in the descriptions for previously learned symbols. As a result the description is compact even though the symbol has a lot of primitives and the initial number of constraints is very large.

Chapter 5 Implementation

This chapter describes the processing steps the system goes through from the time the user starts drawing the symbol to the generated description. We provide the definition of all supported constraints and their default relevance scores and show how the system adjusts these scores to filter out irrelevant constraints, based on three factors: obstruction, tension lines, and grouping. We also present a graphical interface for displaying the resulting constraints.

Figure 5.1 below briefly outlines the processing steps to generate the textual description:

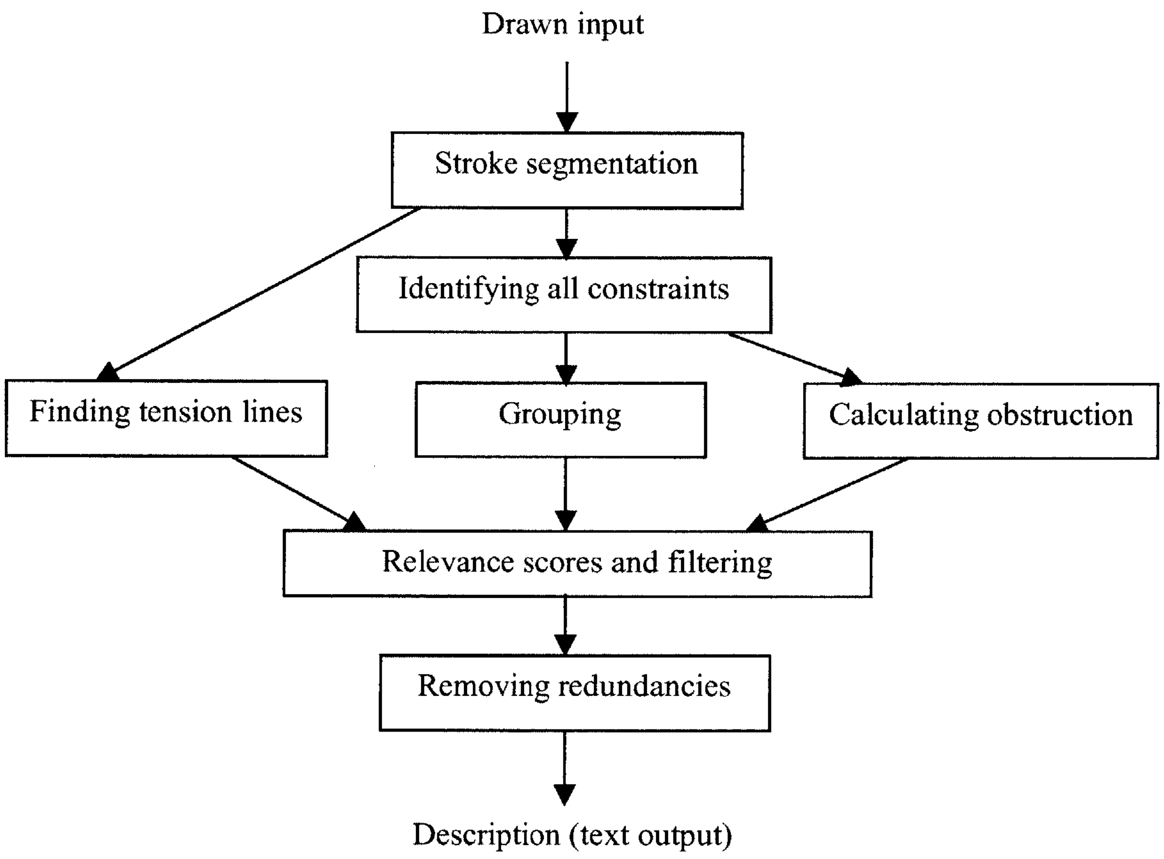


Figure 5.1 Processing steps to produce the description of a symbol

Each section describes one of these steps and illustrates the work of the system on the symbol in Figure 5.2:

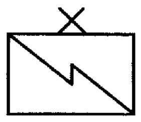


Figure 5.2 Military planning symbol

5.1 Stroke segmentation

The user draws the symbol in the system's drawing window (Figure 5.3), which provides a grid to make it easier for the user to draw carefully. The program accepts any mouse or pen-based input.

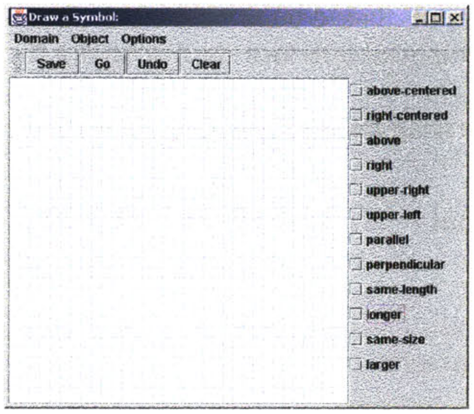
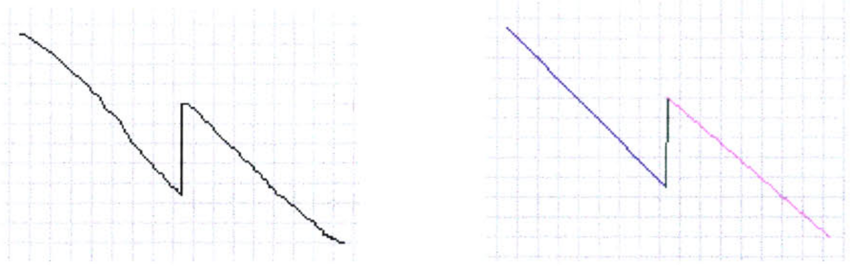


Figure 5.3 Drawing window

We use a toolkit developed by Sezgin [2001] to segment the strokes into simple geometric primitives. The toolkit takes into account both stroke curvature and pen speed data to find separate geometric primitives, based on the observation the people often slow down the pen at corners. Our system instructs the toolkit to classify each stroke as either a polyline or an oval.

If the user slows down accidentally (which often happens when using a mouse, rather then pen input) the segmentation may produce spurious corners. We use alternating segment colors for each primitive within a stroke to provide feedback on segmentation (Figure 5.4). The user can press "Undo" and redraw the stroke, if the segmentation is incorrect.



a) b)

Figure 5.4 a) Original single stroke. b) Segmentation of the stroke

Figure 5.5 below shows the symbol from Figure 5.2 in the system's drawing window:

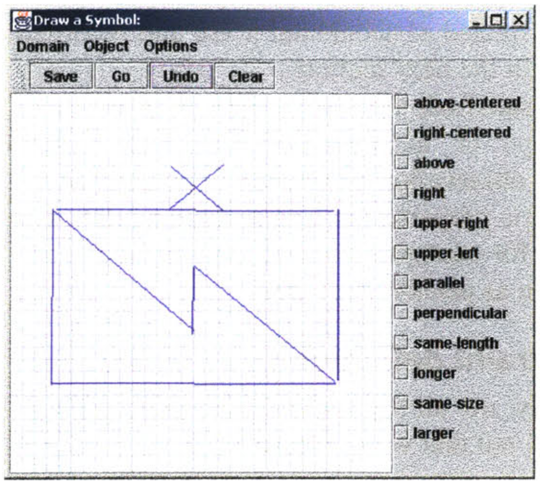


Figure 5.5 Military planning symbol: strokes segmented into geometric primitives

The user presses the "Go" button after completing the drawing to generate the description. All the strokes on the surface are considered to be one symbol.

As we have already mentioned, generalization is done on the primitives, not on the stroke data, so the order and the number of strokes does not effect the produced description. The advantage of this approach is that the user is not required to draw the symbol in exactly the same way during sketching as during the teaching phase. It is natural to expect the sketching system to recognize the symbol based on what it looks like, regardless of how it is drawn, and our approach supports this.

On the other hand, some ways of drawing are more likely to occur than others. For example, one would often draw a rectangle starting from the top-left corner and all in one stroke. The stroke order and number information may give additional clues for the recognition engine for distinguishing between symbols in cases of ambiguity. Our system does not explicitly record this information, although the order is implicitly contained in the primitive labeling.

5.2 Identifying all constraints

Once the drawing is completed, the system records all the constraints in the drawing. Each constraint type is represented by a graph – one graph for "connects", one graph for "above", etc. Geometric primitives are nodes in the graph and edges signify whether the constraint holds between a pair of nodes. Unary constraints, like "horizontal", reuse the same data structure for uniformity, with all the edges as self-loops. The edges in the graphs are directional, so for each symmetric constraint like "same-length" or "parallel" two edges will be found for a given pair of primitives. The final output description includes only one of each pair of symmetric constraints to minimize redundant information.

To identify constraints the system considers each primitive for unary constraints and each pair of primitives for binary constraints. As mentioned in Chapter 3, the vocabulary consists of singular and non-singular constraints. The system first tests whether a singular constraint holds for the primitive (or pair). For example, for line orientation, the system tests whether a it can be considered horizontal or vertical. Nearly singular values, like almost horizontal, are treated as accidental noise and recorded as singular. We describe the noise thresholds in the next section.

Only if the singular constraint is not satisfied, the system then records one of the non-singular constraints (like "positive-slope" or "negative-slope" for line orientation). This approach corresponds to Goldmeier's observation that people's perception is sensitive to singularities and codes geometric properties in terms of their relation to the singularity [Goldmeier, 82, p. 43]. That is, for example, if a line is perceived as horizontal, it cannot be simultaneously seen as positively sloped.

5.2.1 Noise thresholds and constraint definition

It is hard to draw the symbol perfectly; not all lines intended to be exactly horizontal, connected, or aligned will come out that way (Figure 5.6).

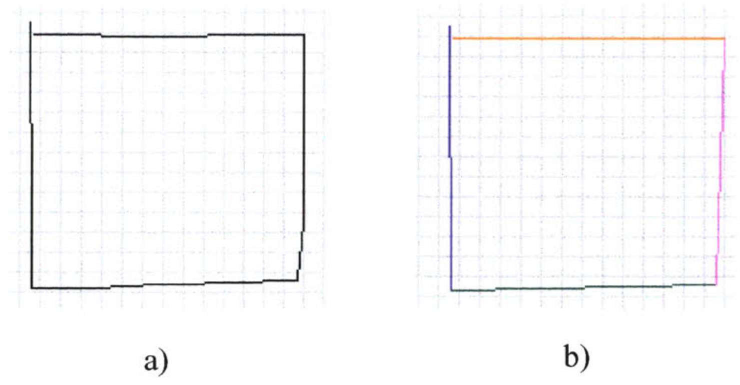
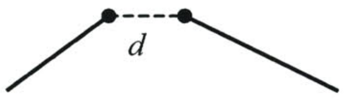
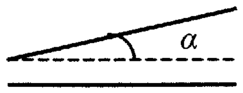
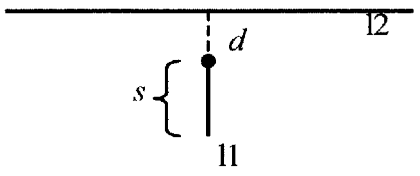


Figure 5.6 Noisy drawing of a square: a) Original stroke. b) Stroke segmented into lines

The system allows a certain amount of noise when testing for presence of constraints. Noise tolerances are governed by three constants:

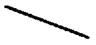
Constant	Value	Example
MAX_OFFSET: This constant is used for testing any constraints where the system needs to determine whether the distance between some two points can be considered negligible. The constant specifies that the distance should be less than 7 pixels.	7 pixels	Two lines will not be considered connected if the distance *d* between their endpoints is greater than 7 pixels. 

MAX_ANGULAR_OFFSET This constant specifies the maximum angular difference for which the angle can be considered negligibly small. It is used for constraints like line orientation or relative orientation.	10°	Two lines will not be considered parallel if their angle difference α is more than 10°. 
SIZE_TO_OFFSET_RATIO We do not want to consider the distance *d* between two points negligible if the size of the primitives in question is small (even when the MAX_OFFSET threshold is satisfied). The constant specifies the minimum ratio between the size of the smallest primitive and the distance *d*. The size should be at least 3 times larger than *d* for *d* to be considered negligible	3 times	Line l1 is not considered to meet line l2 because its length *s* is less than 3 times larger than the distance *d* to line l2. 

We use the noise tolerance constants to determine when the system can decide that a constraint holds in the drawing. Below we describe the definitions for each constraint in the vocabulary. In the definition tables singular constraints are shown in bold.

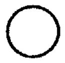
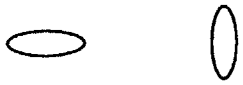
5.2.1.1 Orientation

"Horizontal" and "vetical" constraintsis hold if the angle difference between the ideal and the actual orientation of the line in the drawing is less than MAX_ ANGULAR_OFFSET and if the change in y (for horizontal) or x (for vertical) coordinates from the center to the endpoints is less than MAX_OFFSET. If the "horizontal" or "vertical" constraint is not satisfied, the orientation is recorded as either "positive slope" or "negative slope," depending on the slope of the line. The orientation of an oval is defined by the orientation of its longer axis and only applies to ovals satisfying the "elongated" constraint (see definition further).

Constraint	Applies to	Example
Horizontal	lines, elongated ovals	
Vertical	lines, elongated ovals	
Positive Slope	lines	
Negative Slope	lines	

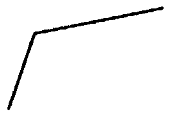
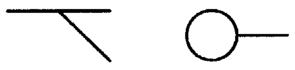
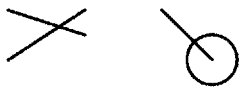
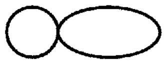
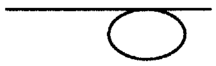
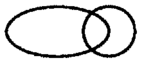
5.2.1.2 Aspect ratio

Ovals are considered "non-elongated" if the ratio of their length to their thickness is less than 1.5. Otherwise the oval is "elongated".

Constraint	Applies to	Example
Non-elongated	ovals	
Elongated	ovals	

5.2.1.3 "Touch" constraints

We refer to "connects," "meets," "tangent," etc. as the "touch" constraints. The distance *d* between the points of the primitives that are supposed to be coincident should be less than MAX_OFFSET. The table below shows how we define *d* for each constraint. Also, the ratio of the size of the smallest primitive and *d* should be greater than SIZE_TO_OFFSET_RATIO. The size of a line is its length and the size of an oval is the maximum of its width and height.

Constraint	Applies to	Definition of tested distance *d*	Example
Connects	lines	The distance between line endpoints.	
Meets	lines, line and oval	The perpendicular distance from the line endpoint to the line segment or oval boundary.	
Intersects	lines and ovals	Not applicable. The system tests for the presence of intersection.	
Touches	ovals	The smallest perpendicular distance between oval boundaries.	
Tangent	line and oval	The smallest perpendicular distance between the line and the oval.	
Overlaps	ovals	Not applicable.The system tests for the presence of intersection of oval boundaries.	

It may happen that several of the "touch" constraints are satisfied for a given pair of primitives at the same time. For example, both "meets" and "intersects" constraints would be satisfied in Figure 5.7.

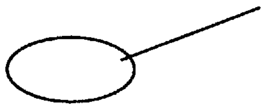


Figure 5.7 The symbol satisfies both "meets" and "intersects" constraints

We only want to choose one interpretation. We define an order in which "touch" constraints are tested and record only the first satisfied constraint:

For lines	For line and oval	For ovals
1. Connects 2. Meets 3. Intersects	1. Meets, tangent 2. Intersects	1. Touches 2. Overlaps

For some "touch" constraints the system also specifies where exactly the primitives touch. For example, Figure 5.8 shows the kinds of cases we would like to distinguish:

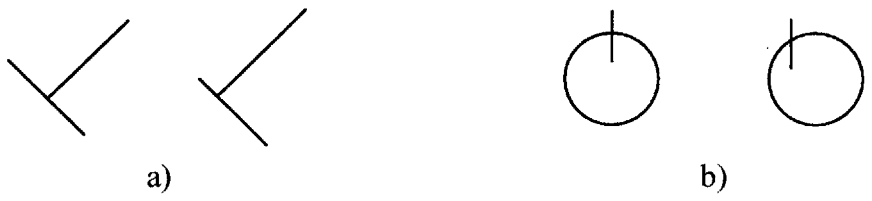
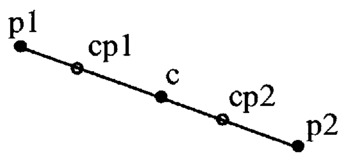


Figure 5.8 a) Different points where one line may meet the other. b) Different points of intersection of a line with the oval

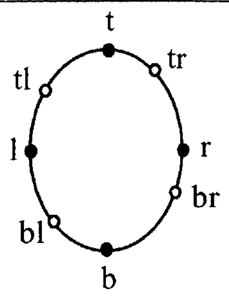
For each of the "connects," "meets," "intersects," "touches," and "tangent" constraints the system records the points of coincidence on both primitives. For example, "meets (l1.p1 o1.t)" means that point p1 of line l1 meets oval o1 at the top.

As with all constraints, we attempt to reflect perceptual singularities in the specification of coincidence points. The table below shows the definitions of possible coincidence points on a line, with singular points shown in bold. Endpoint labels p1 and p2 are assigned arbitrarily.

Point on a line	Notation	Example
Endpoint 1	p1	
Any point between center and endpoint1	cp1	
Center	c	
Any point center and endpoint2	cp2	
Endpoint 2	p2	

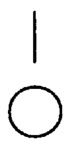
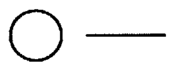
The end and center points are singular, so the system always starts by testing whether a constraint holds for one of these points, that is if the distance from the coincidence point on the other primitive to one of these points is less than MAX_OFFSET and satisfies the SIZE_TO_OFFSET_RATIO threshold. If not, one of the non-singular points is recorded.

Similarly, coincidence points are defined for ovals. Singular points that are tested first are shown in bold.

Point on an oval	Notation	Example
Top	t	
Top right	tr	
Right	r	
Bottom right	br	
Bottom	b	
Bottom left	bl	
Left	l	
Top left	tl	

5.2.1.4 Singular position constraints

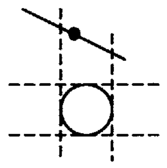
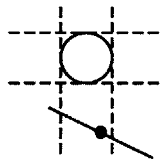
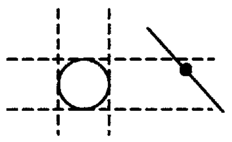
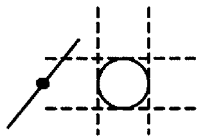
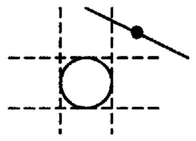
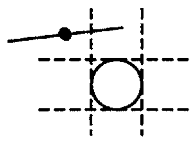
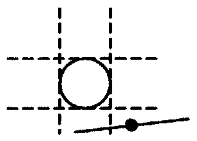
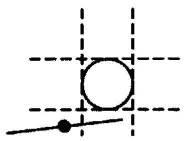
These constraints specify relative position of the primitives and the horizontal or vertical alignment of their geometric centers. The centers are considered horizontally or vertically aligned if the difference in their respective y or x coordinates is less than MAX_OFFSET and SIZE_TO_OFFSET_RATIO is satisfied.

Constraint	Applies to	Example (the position of line relative to oval)
Above-centered	lines and ovals	
Right-centered	lines and ovals	

We do not test for "below-centered" and "left-centered" constraints because their definition is symmetric to "above-centered" and "right-centered" respectively, so these constraints would only provide redundant information.

5.2.1.5 Non-singular position constraints

These constraints specify relative positions of the primitives and are recorded only in the absence of the corresponding singular position constraints described in the previous section. The recorded constraint depends on the position of the center of the first primitive relative to the bounding box of the second primitive. To test the constraint "above l1 l2," for example, the system would look at the center of line l1 and the bounding box of line l2.

Constraint	Applies to	Example (the position of line relative to oval)
Above	lines and ovals	
Below	lines and ovals	
Right	lines and ovals	
Left	lines and ovals	
Upper-right	lines and ovals	
Upper-left	lines and ovals	
Lower-right	lines and ovals	
Lower-left	lines and ovals	

The limitation of these definitions is that the boundaries between these terms do not correspond to clear qualitative perceptual boundaries. For example, the difference between the two drawings in Figure 5.9 is almost unnoticeable, while the produced descriptions would be different – one would be "above (o1 l2)" and the other "upper-right (o1 l2)." This means that the description produced for the first symbol would prevent the second symbol to be considered an instance of the first one.

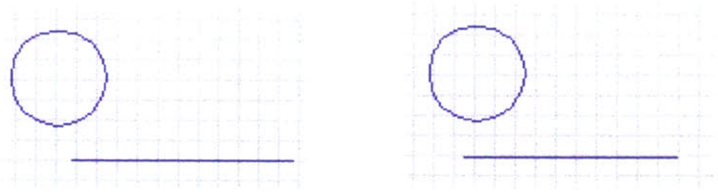


Figure 5.9 Two very similar drawings that produce dissimilar descriptions

5.2.1.6 "Inside" and "inside-centered" position constraints

These constraints apply to primitives inside ovals. The "Inside-centered" constraint holds if the primitive is inside the oval and the coordinate difference between its center and the center of the oval is less than MAX_OFFSET and satisfies the SIZE_TO_OBJECT_RATIO. Otherwise only the "inside" constraint holds.
"Inside" constraints do not have to hold exactly in the actual drawing, as long as they hold if noise were removed from the drawing. For example, in Figure 5.10, the line is considered to be inside the oval because the system decides that it satisfies the "meets" rather than "intersects" constraint with the oval:

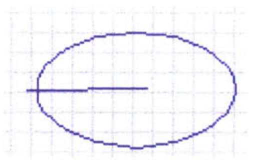
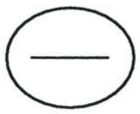
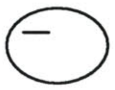


Figure 5.10 The line is considered to be inside the oval

Constraint	Applies to	Example (the position of line relative to oval)
Inside and centered	line and oval, oval and oval	
Inside	line and oval, oval and oval	

5.2.1.7 Relative orientation

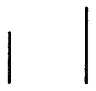
"Parallel" and "perpendicular" constraints hold if the actual angle between the lines in the drawing differs from the ideal angle by less than MAX_ANGULAR_OFFSET. The system records these constraints only for lines that it identified as positively or negatively sloped. This is done because, as we show further in Section 5.6, the system never filters out “horizontal” and “vertical” constraints. “Parallel” and “perpendicular” constraints only would only provide redundant information for horizontal and vertical lines, so the system does not record them.

Also, as shown in Chapter 3, it is hard for people to accurately tell the angle between the two connected slanted lines (see Figure 3.10 and Figure 3.11), so we do not record the “perpendicular” constraint for such lines.

Constraint	Applies to	Example (the position of line relative to oval)
Parallel	lines	
Perpendicular	lines	

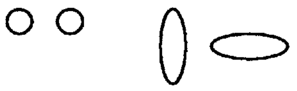
5.2.1.8 Relative length

Two lines are considered to have the same length if the ratio of the length difference over the length sum is less than 0.05. Otherwise a “longer” constraint is recorded.

Same length	lines	
Longer	lines	

5.2.1.9 Relative size

The size of the oval is defined as the maximum of its width and its height. Two ovals are considered to have the same size if the ratio of the size difference over the size sum is less than 0.08. Otherwise a “larger” constraint is recorded.

Constraint	Applies to	Example (the position of line relative to oval)
Same size	ovals	
Larger	ovals	

5.2.2 Possible contradictions.

Tolerances for noise make it possible to record contradicting constraints, because the system tests constraints for each pair of primitives separately. For example, in Figure 5.11a, the system will decide that both lines l1 and l2 connect to the endpoint of line l3 if the distance between lines l1 and l2 is smaller than MAX_OFFSET. Remember that a "meets" constraint is never recorded if "connects" is found first. The system examines pairs (l1 l3) and (l2 l3) separately (Figure 5.11b). For both of these pairs, the distance between line endpoints is small enough for the system to identify a "connects" constraint. Yet it also decides that both lines l1 and l2 are vertical, which contradicts the "connects" constraints.

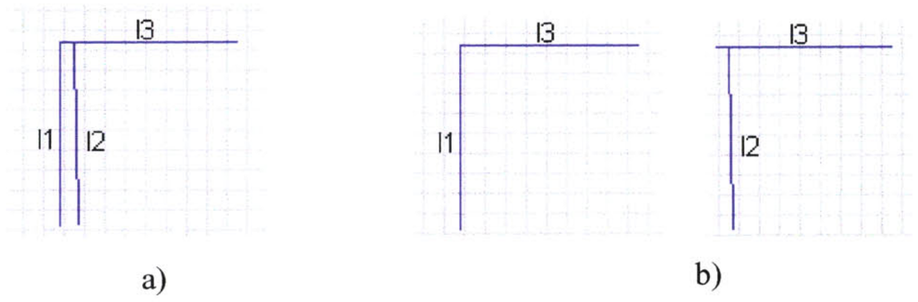


Figure 5.11 a) Drawing resulting in potential contradictory constraints. b) Pairs of primitives separately examined by the system

Figure 5.12 shows another example that may cause contradictions. If the noise tolerance is large enough compared to the length of line l3, the system will decide that both lines l2 and l3 are centered to the right of l1. Yet l3 is also above l2.

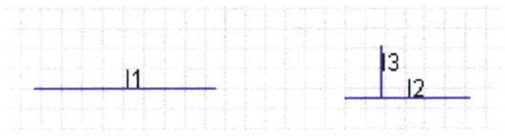


Figure 5.12 Drawing resulting in potential contradictions

We have not implemented a mechanism to detect and correct such contradictions. Currently, the only solution for the user is to draw carefully, keeping in mind the magnitude of the tolerance thresholds. The MAX_OFFSET threshold is indicated by the size of the grid cells. The smallest primitives and distances in the symbol should be larger than the grid size. And the level of noise, like accidental gaps and misalignments, should be smaller than the grid size.

If the physical size of the pixels on the device is too small, it may be hard to keep the noise under the MAX_OFFSET (which is specified in pixels) when drawing. We allow the user to change this constant, which will be reflected visually in the grid size.

Absolute noise thresholds may be somewhat unnatural. Consider the lines in Figure 5.13:

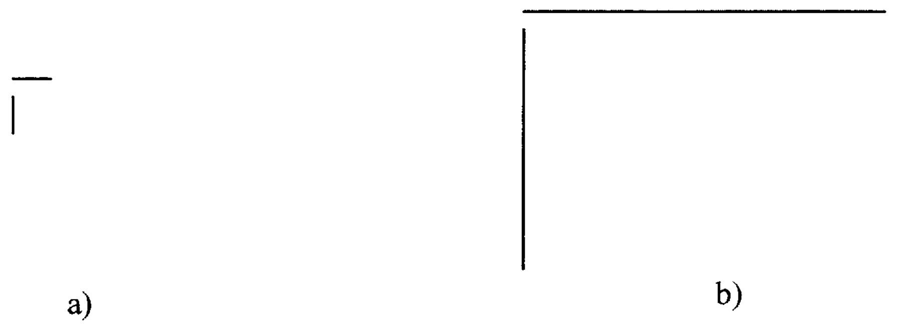


Figure 5.13 a) Short lines. b) Long lines

Although the distance between the endpoints of the two lines is the same in both cases, the lines in Figure 5.13b are much more likely to be perceived as connected than in the lines in Figure 5.13a. That means that the maximum tolerance for line connectivity could be larger for longer lines.

We do decrease the noise threshold if primitives are small, which is achieved by the mandatory minimum SIZE_TO_OFFSET_RATIO. This constant always limits the noise threshold to less than a third of the primitive size. Yet the system does not increase the noise threshold beyond MAX_OFFSET if primitives get larger.

We chose to have an absolute maximum threshold for all primitive sizes, so that it is clearer to the user what the system's maximum noise tolerance is. We believe that this would make it easier to determine how carefully one should draw, though we have not verified this assumption in user studies. In the future, if the system includes contradiction resolution, size-dependant noise thresholds will probably be more appropriate.

Although, there is no generic mechanism for contradiction detection, we have included several routines to correct one type of common mistakes with relative length and size constraints. These routines enforce the transitive closure in "same-length" and "same-size" constraints and remove the "longer" and "larger" constraints that contradict the closure.

Consider the triangle in Figure 5.14a. In Figure 5.14b the sides of the triangle are lined up to better show their relative length.

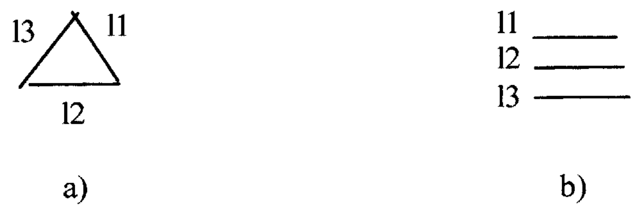


Figure 5.14 a) Triangle. b) Lengths of sides of the triangle

Suppose, for example, that the system considers the length difference for line pairs (l1 l2) and (l2 l3) negligible and records the constraints "same-length: (l1 l2) (l2 l3)" and "longer: (l3 l1)." Using transitive closure the system finds that for consistency with "same-length: (l1 l2) (l2 l3)" constraints lines l1 and l3 also need to have the same length.

Hence, it removes the "longer" constraint and replaces it with "same-length (l1 l3)". A similar mechanism is used for relative oval size.

Clearly, the limitation of this mechanism is that it may interpret a series of very gradually increasing lines to be the same length, even if the length of the first and the last line in the sequence are significantly different.

5.2.3 Example result of identifying all constraints

Figure 5.15 shows a drawing of a military symbol with strokes segmented into geometric primitives. We label the primitives for convenience.

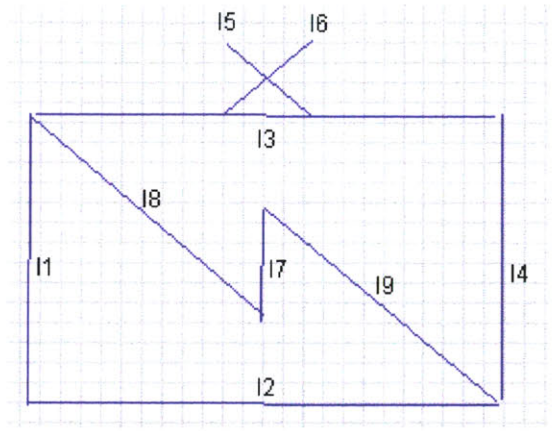


Figure 5.15 Military symbol

The table below shows 122 constraints that the system finds in the symbol, if we count symmetric constraints like "same-length (l8 l9) (l9 l8)" only once.

connects: (l4.p1 l3.p2) (l4.p2 l2.p2) (l4.p2 l9.p2) (l3.p1 l1.p1) (l3.p1 l8.p1) (l2.p1 l1.p2) (l2.p2 l9.p2) (l1.p1 l8.p1) (l9.p1 l7.p1) (l7.p2 l8.p2)
meets: (l6.p1 l3.cp1) (l5.p2 l3.cp2)
intersects: (l6 l5)
horizontal: (l3) (l2)
vertical: (l4) (l1) (l7)
pos-slope: (l6)
neg-slope: (l5) (l9) (l8)
above: (l5 l9) (l3 l8) (l9 l2) (l8 l2) (l6 l9)
right: (l4 l8) (l4 l9) (l9 l1) (l9 l7) (l9 l8) (l8 l1) (l7 l8)
below: (l2 l9) (l9 l3) (l8 l3)
left: (l1 l8) (l1 l9) (l9 l4) (l8 l4) (l8 l7) (l8 l9) (l7 l9)
upper-right: (l5 l1) (l5 l8) (l4 l2) (l3 l1) (l6 l1) (l6 l8)
upper-left: (l5 l4) (l3 l4) (l3 l9) (l1 l2) (l6 l4)
lower-right: (l4 l3) (l4 l5) (l4 l6) (l2 l1) (l2 l8) (l9 l5) (l9 l6)
lower-left: (l2 l4) (l1 l3) (l1 l5) (l1 l6) (l8 l5) (l8 l6)
above-centered: (l6 l2) (l6 l3) (l6 l7) (l5 l2) (l5 l3) (l5 l7) (l3 l2) (l3 l7) (l7 l2)
right-centered: (l4 l1) (l4 l7) (l7 l1)
parallel: (l5 l8) (l5 l9) (l9 l8)
perpendicular: (l5 l6) (l8 l6)
same-length: (l5 l6) (l2 l3) (l1 l4) (l9 l1) (l9 l4) (l8 l1) (l8 l4) (l8 l9) (l7 l5) (l7 l6)
longer: (l4 l5) (l4 l6) (l4 l7) (l3 l1) (l3 l4) (l3 l5) (l3 l6) (l3 l7) (l3 l8) (l3 l9) (l2 l1) (l2 l4) (l2 l5) (l2 l6) (l2 l7) (l2 l8) (l2 l9) (l1 l5) (l1 l6) (l1 l7) (l9 l5) (l9 l6) (l9 l7) (l8 l5) (l8 l6) (l8 l7)

5.3 Tension lines

The next processing step is to find tension lines – the horizontal and vertical alignments of primitives in the symbol. The system starts by creating a list of *tension points*. The list includes all line centers and endpoints and points on the top, bottom, left, right, and center of the ovals. The horizontal or vertical alignment of two or more tension points defines a tension line (Figure 5.16).

These alignments are found by a horizontal and vertical sweep through the list of tension points sorted by y and x coordinates respectively. Each group of consecutive tension points for which the maximum vertical (or horizontal) difference between point coordinates is less than MAX_ OFFSET corresponds to a different tension line. This means that the maximum misalignment of points on the a tension line is MAX_OFFSET, consistent with the overall noise threshold in the system.

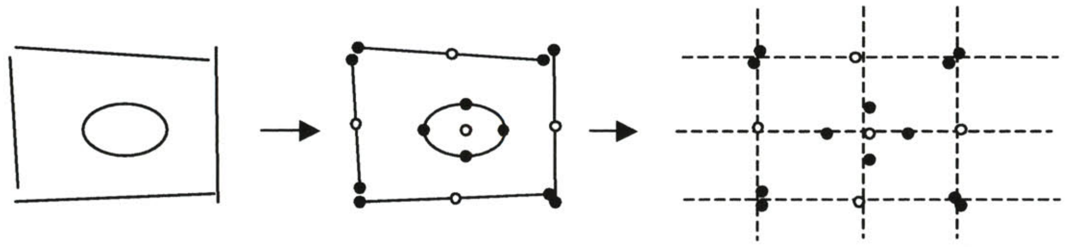


Figure 5.16 Tension lines defined by groups of tension points

Grey lines in Figure 5.17b show tension lines for the military symbol in Figure 5.17a.

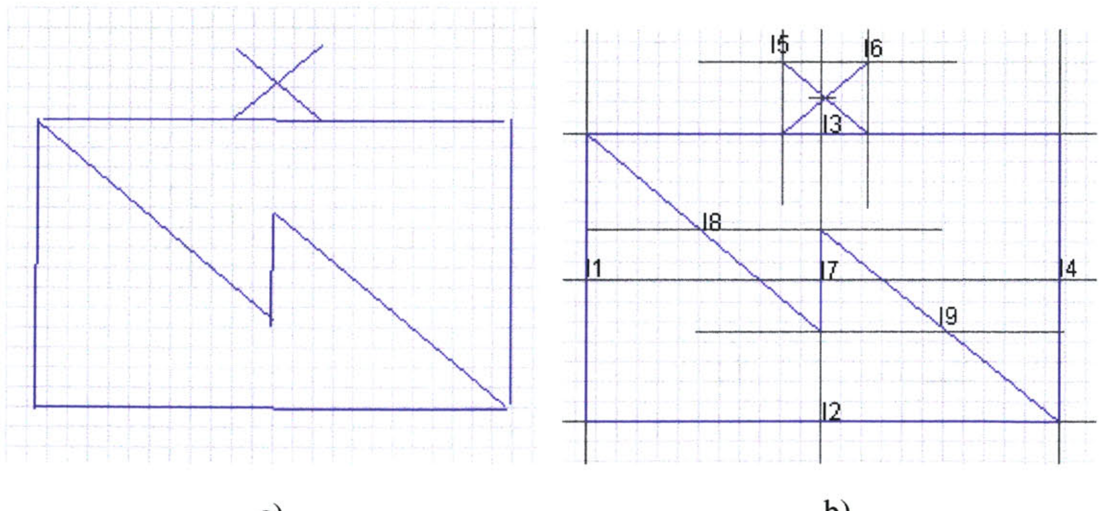


Figure 5.17 a) Symbol b) Tension lines for the symbol

5.4 Obstruction

After finding all constraints and tension lines the system proceeds to calculate obstruction. The obstruction value for each pair of primitives is roughly the number of other primitives between the pair. This section explains how obstruction values are calculated.

Consider Figure 5.18. There are 4 lines between the lines l1 and l6.

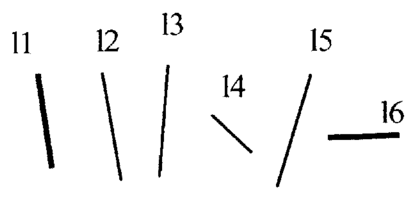


Figure 5.18 Four lines separate lines l1 and l6

Notice, however, that it is not always clear whether a primitive is "between" a given pair. If we look at lines l1 and l3 in Figure 5.19, it is hard to decide whether line l2 is between them.

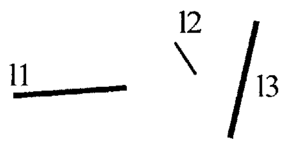


Figure 5.19 Is line l2 "between" lines l1 and l3?

In Figure 5.19 line l2 does not completely separate l1 and l3, but it creates some obstruction. In this case we would like to assign an obstruction value that is somewhere between 0 (as, for example, in Figure 5.20a) and 1 (as in Figure 5.20b), so we use non-integer obstruction values.

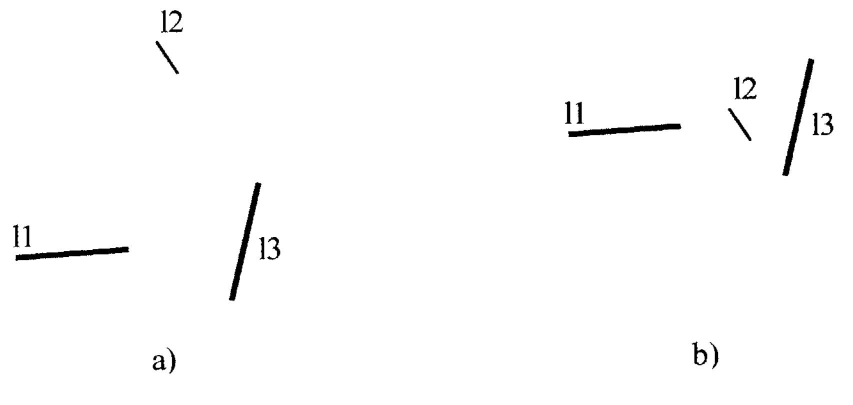
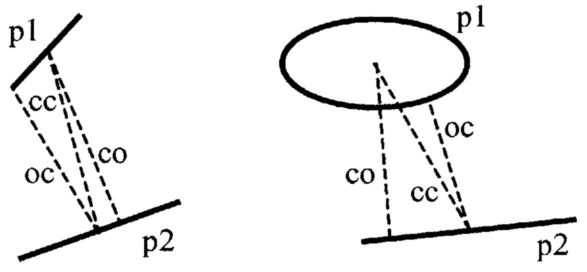


Figure 5.20 a) Line l2 creates no obstruction for the pair (l1 l3). b) Line l2 is clearly between l1 and l3

To calculate obstruction values for a pair of primitives, we define three special lines connecting them:

Connecting line	Examples
A line connecting the centers of two primitives (cc)	
A line connecting the center of the first primitive to the closest point on the second primitive (co)	
A line connecting the center of the second primitive to the closest point on the first primitive (oc)	

The contribution of every remaining primitive p_i to the obstruction value for the pair (p_1,p_2) is an exponentially decreasing function of the distance between p_i and each of the connecting lines. This distance is taken relative to the size s of the smaller primitive in the pair.

$$O(p_1,p_2)=\begin{cases}\sum\limits_{i\neq 2,3}(\alpha^{\text{distance}(p_i,cc)/s}+\alpha^{\text{distance}(p_i,co)/s}+\alpha^{\text{distance}(p_i,oc)/s})/3, \\ \\ 0,\text{ if } p_1 \text{ and } p_2 \text{ connect, meet, overlap, touch, are tangent, or one is inside} \\ \text{the other}\end{cases}$$

where s = min(size(p_1), size(p_2)) and α is set to 0.2.

We examine obstruction calculation for the pair of lines (p1, p2) in Figure 5.21.

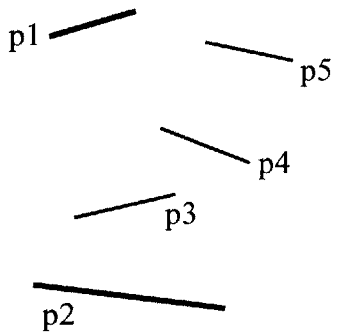


Figure 5.21 Example lines

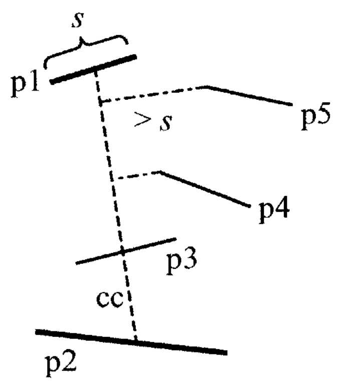


Figure 5.22 Positions of primitives relative to the connecting line cc

Figure 5.22 shows the special line cc connecting the centers of lines p1 and p2. In this case s is equal to the length of p_1 – the smaller of the two lines. Line p3 intersects cc, so distance(p_3, cc) / s = 0 and $\alpha^{\text{distance}(p_3, cc)/s} = \alpha^0 = 1$. Line p4 causes less obstruction: $\alpha^{\text{distance}(p_4, cc)/s} = \alpha^{0.5} = 0.45$. When distance($p_3$, cc) exceeds s the exponent becomes greater than 1, and the obstruction will become less than $\alpha = 0.2$, which is relatively small.

The analysis is analogous for the connecting lines oc and co. We divide the obstruction values obtained for each of the connecting lines by 3, so that if some primitive intersects all three of them the total value would come to 1 (Figure 5.23):

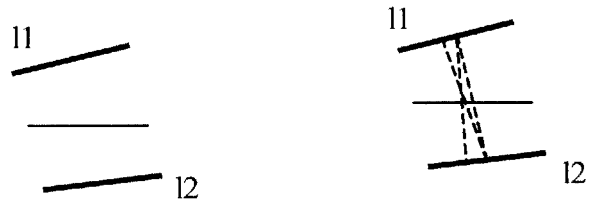


Figure 5.23 One line separates lines l1 and l2

Notice, however, that there is a problem with defining obstruction in terms of the distance to the connecting lines. Consider the example in Figure 5.24:

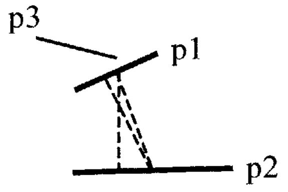


Figure 5.24 Line p3 should not obstruct p1 and p2

Line p3 should not obstruct the pair (p1, p2), but it is very close to the connecting lines so the obstruction formula would give a value close to 1. To deal with this problem we remove from consideration all the primitives that are behind what we call the *boundary infinite lines* for the primitives p1 and p2. These lines narrow down the region where a primitive can obstruct the pair (p1, p2) (Figure 5.25):

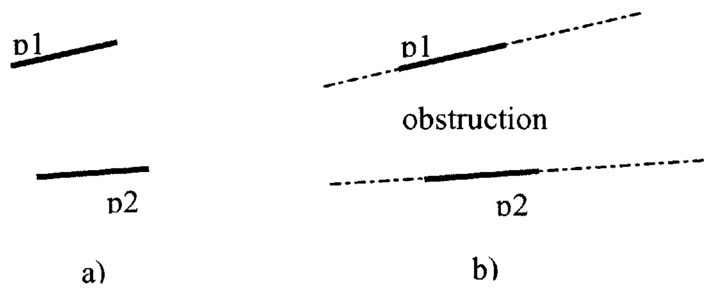


Figure 5.25 a) Pair of lines. b) Boundary lines for the pair (p1, p2) and the obstruction region

The obstruction values are only calculated for primitives that are fully or partially contained in the obstruction region between the boundaries. For each of the primitives in a pair (p1, p2), the boundary is defined differently depending on the relative orientation of the primitive and the line cc connecting the centers of p1 and p2. The goal is to always keep the boundaries close to parallel. We define two cases, depending on the angle α between the primitive and the line cc:

- $\alpha \geq 72°$: This means that the primitive is close to being perpendicular to the connecting line cc. The boundary in this case is simply the extension of the line:

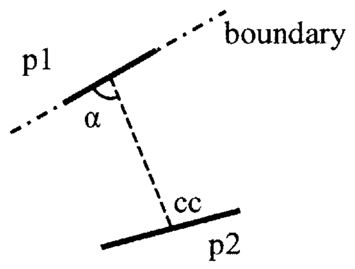


Figure 5.26 Boundary for the primitive p1

- $\alpha < 72°$: In this case the primitive is close to facing the other primitive in the pair with its endpoint. The boundary is perpendicular to the line cc and passes through the endpoint of the primitive p1, with a small offset (MAX_OFFSET). The offset is included so that lines connected to this endpoint would not be considered behind the boundary:

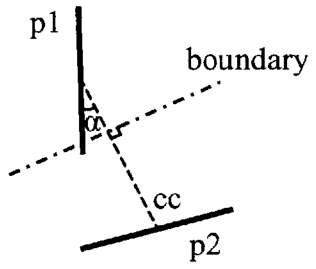


Figure 5.27 Boundary for the primitive p2

For ovals, the boundary is perpendicular to the line cc. As in the previous case, there is a small offset (MAX_OFFSET) that exposes part of the oval, so that a line tangent or meeting the oval at that part would not fall behind the boundary:

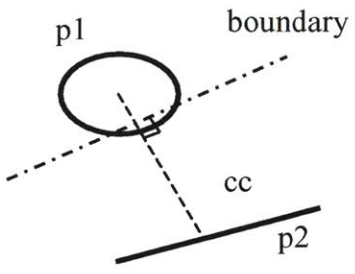


Figure 5.28 Boundary for the oval

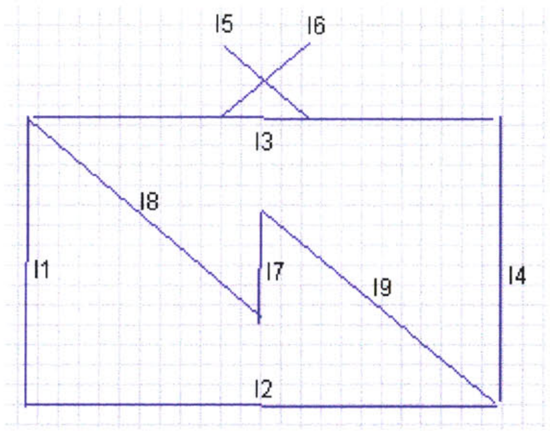


Figure 5.29 Military symbol

The table below shows obstruction values for the symbol in Figure 5.29. As we can see, the obstruction value for the pair (l2 l3), for example, is 3.9. It is caused by the lines l7, l8, and l9, and somewhat by the lines l4 and l1. As defined by the obstruction equation, when two primitives touch, the obstruction will be zero, as for the pair (l3 l1), for example.

	l1	l2	l3	l4	l5	l6	l7	l8	l9
l1	0								
l2	0	0							
l3	0	3.9	0						
l4	4.8	0	0	0					
l5	3.2	4.7	0	1.5	0				
l6	2.5	4.7	0	2.3	0	0			
l7	1.5	0.5	0.6	1.5	2.4	2.4	0		
l8	0	0.8	0	3.9	2.6	1.7	0	0	
l9	3	0	0.8	0	1.9	1.9	0	0	0

5.5 Grouping

This is the final processing step before relevance scores can be calculated for all the constraints. We support two grouping principles: connectedness and familiarity. The system produces candidate groups by segmenting the drawing into connected components and identifying previously learned symbols as drawing subparts. It then combines these groups into a hierarchy and merges any groups that share the same primitives. This section describes these steps in detail.

5.5.1 Connected components

Any two primitives that touch in some way are considered to be in the same connected component. To compute the components, the system constructs a graph in which nodes are primitives and an undirected edge exists for any pair of primitives constrained by "connects", "meets", "intersects", "touches", "overlaps", or "tangent." The system performs a depth-first search on this graph to find its connected components, which correspond to the connected components in the symbol.

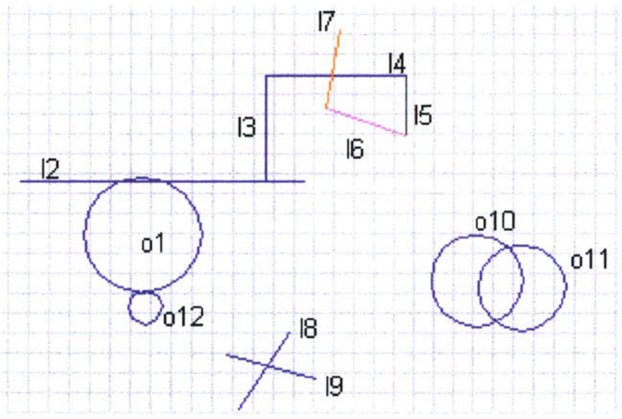


Figure 5.30 Examples of touching primitives

The system identifies three connected components in Figure 5.30:

Component 1: l6, l7, l4, l5, l3, l2, o1, o12
Component 2: o11, o10
Component 3: l9, l8

5.5.2 Previously learned symbols

To identify the second set of candidate groups the system looks for previously learned symbols as subparts of the new symbol. For each stored symbol it searches for a mapping of primitives that makes its constraints a subset of all the constraints in the new symbol.

For example, the primitives l7, l6, l8, and l5 in Figure 5.31b satisfy the constraints of the rectangle symbol in Figure 5.31a, given the mapping: (l1→l8) (l2→l5)(l3→l6) (l4→l7).

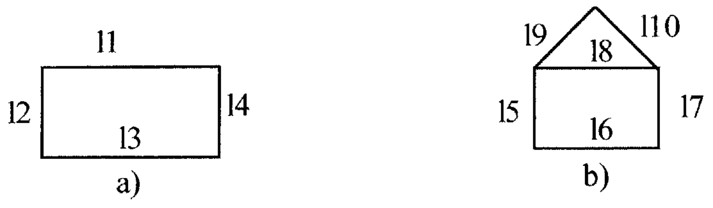


Figure 5.31 a) Rectangle. b) New symbol

Identifying previously learned symbols is a subgraph isomorphism problem on the symbol graphs, where the primitives are nodes and constraints are edges. We use Ullman's algorithm to compute the isomorphism [Ullman, 1976]. It proceeds by trying one mapping pair at a time and checking edges given the pairs so far, until it fails or finds the compete mapping. For example, if the algorithm is looking for the rectangle from Figure 5.31a in the symbol in Figure 5.31b, it can try setting (l1→l8). The "horizontal l1" constraint is satisfied for l8, so it proceeds to set the mapping for l2, now trying to ensure that the mapped primitives in the new symbol satisfy the same constraint as l1 and l2 in the rectangle symbol, and so on.

The running time of this algorithm is exponential in the number of primitives and linear in the number of previously learned symbols. We find that in practice it runs reasonably fast because most symbols have a small number of primitives and mappings are quickly pruned when constraints involve only a few primitives.

Previously learned symbols may be related. For example, an isosceles triangle is a subclass of a triangle in general. The isosceles triangle has more constraints. The system keeps track of the subclass relationships between the learned symbols in a multiple-inheritance domain graph. Figure 5.32 shows such a graph for different kinds of triangles.

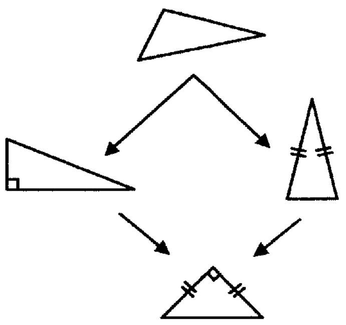


Figure 5.32 Domain graph for different types of triangles

The lines l8, l9, l10 in Figure 18b would match all of these triangles. In such cases the system chooses the most specific interpretation, i.e. the one with most constraints. To achieve this, the matching process starts from the bottom of the domain graph. Once a set of primitives is matched to a symbol in the domain graph, there is no need to match this set to the ancestors of the symbol. We know that they are all guaranteed to match because they contain fewer constraints.

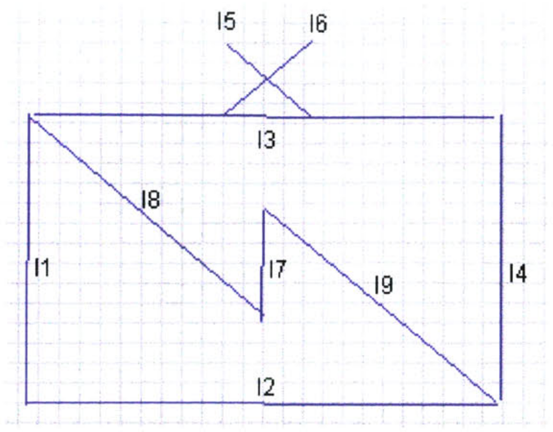


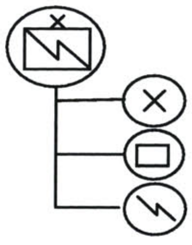
Figure 5.33 Military symbol

The system identifies previously learned cross and rectangle symbols as subparts of the symbol in Figure 5.33.

5.5.3 Combining grouping factors

The system combines candidate groups – connected components and previously learned symbols – into a group hierarchy.

Figure 5.34 shows the group hierarchy for the symbol in Figure 5.33.



Group g1 connected-component

Group g2 symbol – cross

Group g3 symbol – horizontal rectangle

Group g4 other (the remaining primitives in the connected component)

Figure 5.34 Group hierarchy of the symbol in Figure 5.33

If one group shares primitives with another, but cannot be the other's child or parent in the hierarchy, the two groups are merged into one. For example, in Figure 5.35 the system would find a triangle and a rectangle (the whole figure). They share the same primitive, so they will be merged into one group.

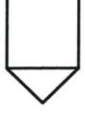


Figure 5.35 Symbol with competing groupings

This approach has does not always produce the most salient grouping hierarchy. For example, Figure 26b shows the grouping hierarchy for the symbol in Figure 26a:

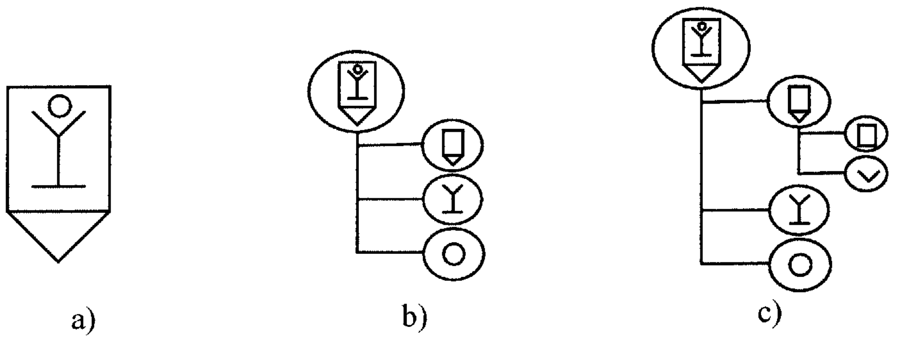


Figure 5.36 a) Symbol. b) Grouping produced by the system. c) Alternative grouping

As a result of merging, the grouping in Figure 5.36b does not recognize the rectangle as a salient part of the symbol. Consequently, the system will not record, for example, constraints like "inside (circle rectangle)", which would be more concise than specifying interactions of the circle with each of the primitives in the rectangle instead. The grouping in Figure 5.36c would be more appropriate.

A potential approach to this problem would be to resolve competitions between groups by picking a "winner" that gets to keep the shared primitives, rather than merging the groups. The winner could be defined, for example, as the group with the largest number of primitives. We that approach the system would produce a grouping shown in Figure 5.36c.

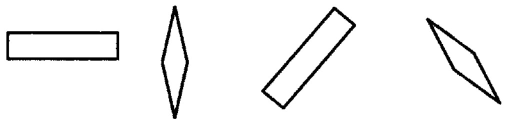
5.5.4 Group constraints

The system finds constraints between every two groups in the hierarchy that do not have an ancestor-descendant relationship. We currently support aspect ratio, orientation, relative position and relative size constraints, which are defined similarly to constraints on ovals and lines:

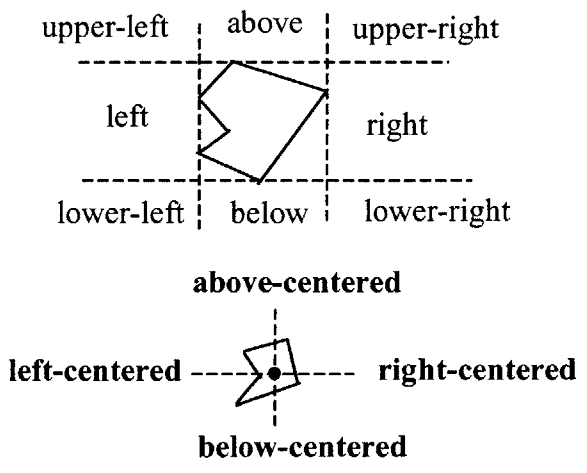
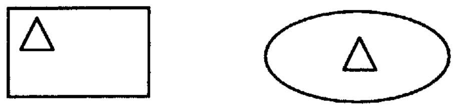
- **Aspect ratio:** The aspect ratio of a group is defined by the aspect ratio of the group's smallest-area bounding box (which does not have to be axis-parallel). This constraint is only identified for closed shapes. The group is "non-elongated" if the ratio of its length to its thickness is less than 1.5. Otherwise the group is elongated.

Group constraint	Example
Non-elongated, Elongated	

- **Orientation:** The constraint applies only to elongated groups. The orientation of the group is defined by the orientation of the longer axis of the smallest-area bounding box of the group. So it is computed as defined for lines in section 5.2.1.1:

Group constraint	Example
Horizontal, Vertical, Positive slope, Negative slope	

- **Relative position:** Position constraints are defined the same way as for lines and ovals in section 5.2.1 using the axis-parallel bounding box of the second group and the center of the first group (defined as the geometric center of the smallest-area bounding box). The position of the center relative to the bounding box determines the constraint (see the table below).
  "Inside" and "inside-centered" constraints are only identified if the outer group is a closed shape. "Inside-centered" holds if one group is inside another and the difference between the center coordinates of the groups is less than MAX_OFFSET and satisfies the SIZE_TO_OBJECT_RATIO. The size of the group is defined as the length of the smallest-area bounding box of the group. "Inside" constraints hold in the same loose sense as we mentioned for lines and ovals. The primitives of the inner group are allowed to touch the boundary of the outer group as long as the system does not identify "intersects" constraints.

Group constraint	Example
 upper-left above upper-right left right lower-left below lower-right above-centered left-centered right-centered below-centered	
Inside, **Inside-centered**	

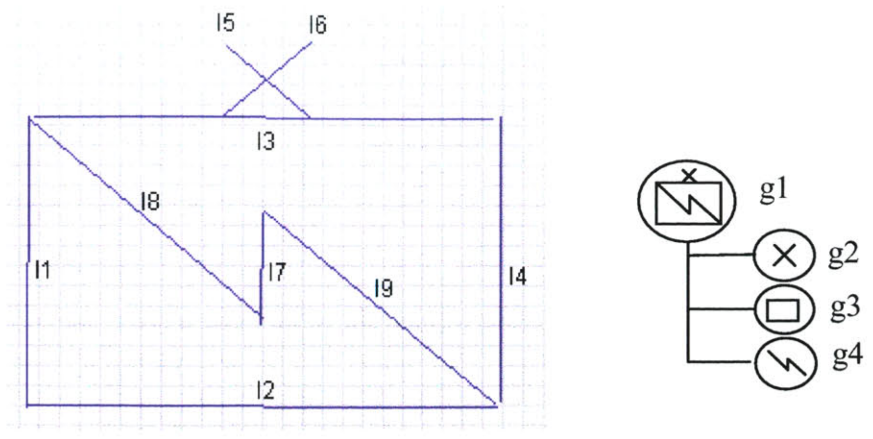


Figure 5.37 Military symbol

For the symbol in Figure 5.37 the system finds 4 group constraints:
above-centered: (g2 g4)
inside-centered: (g4 g3)
elongated: (g3)
horizontal: (g3)

5.6 *Assigning relevance scores*

A relevance score between 0 and 1 is computed for every constraint. This section explains how relevance scores are calculated based on:

- Default scores
- Obstruction
- Tension lines
- Grouping

5.6.1 Default scores

The default score for every constraint type is selected to approximate the relative perceptual relevance of the type:

Constraints	Default relevance score
Connects	1.0
Meets, Intersects, Tangent, Inside, Inside-centered	0.95
Touches, Overlaps	0.9
Horizontal (lines), Vertical (lines)	0.8
Positive slope, Negative slope, Position constraints (except inside), Parallel, Perpendicular	0.7
Horizontal (ovals), Vertical (ovals), Elongated, Non-elongated, Same-length, Same-size	0.6
Longer, Larger	0.55

We obtained these scores by ordering different types of constraints by their perceptually saliency, based on our introspection with various symbols, and assigning scores spread out in the interval between 0.5 and 1.0 according to this ordering.

More accurate relevance ordering could potentially be obtained through looking at a large variety of symbols, using a setup like Goldmeier's similarity experiments. In such an experiment the subjects would look at a symbol and two variations of it, produced by changing two constraints that we want to compare. The subjects would be asked which of the variations looks more similar to the original symbol and their choice would indicate which of the two compared constraints is less important. The constraint varied to produce the more similar symbol is the less perceptually relevant of the two, because changing it altered the perception of the symbol less. Section 3.1 provides such an experiment for comparing the importance of the degree of curvature to the importance of line straightness (Figure 3.3).

Three factors – obstruction, tension lines, and grouping – are used to increase and decrease the default scores of relative position, size, length, and orientation constraints. The score of all "touch" (i.e. connect, intersects, etc.), individual orientation, aspect ratio, and group constraints is not changed. As a result, these constraints will always remain in the description. In Chapter 7 on future work, we discuss, why it may still be useful to rank these constraints by relevance and what could be done to enable the system to learn that in certain cases even these constraints may be irrelevant.

Each of the three adjustment factors pushes the relevance of a constraint up or down depending on the strength and direction of influence δ of this factor. For the relevance score r, the new score after adjustment will be:

$$r' = \begin{cases} r + \delta(1-r), \text{if the factor increases the relevance.} \\ r + \delta r, \text{if the factor decreases the relevance.} \end{cases}$$

This formula achieves an asymptotic approach towards both 0 and 1.

The factors are applied in the order of:

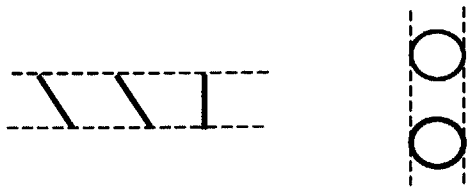
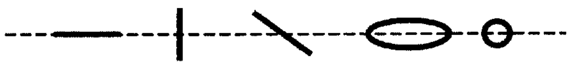
1. Obstruction
2. Tension lines
3. Grouping

5.6.2 Obstruction

An obstruction value is calculated for each pair of primitives, corresponding roughly to the number of primitives between the pair. The relevance of relative orientation, position, length, and size constraints for this pair will be decreased according to the amount of obstruction $O(p_1, p_2)$. This is intended to mimic the psychological observation that the more primitives are between a given pair the less we pay attention to the constraints for it. The influence constant for this factor is $\delta_{ob} = \downarrow 0.15\ O(p_1, p_2)$.

5.6.3 Tension lines

Tension lines represent salient alignments of the primitives in a symbol. Hence, this factor increases the relevance of the relative position, length, and size constraints that contribute to the formation of tension lines. We deal with cases where the pair of primitives supports either one or two tension lines:

Relevance increased	Example
Affected constraints: Above- , below-, right-, and left-centered; Same-length; Same-size. **Condition:** The constraint is between two primitives that have endpoints on two parallel tension lines (formed by these or other primitives).	
Affected constraints: Above- , below-, right-, and left-centered. **Condition:** The constraint is between two primitives the centers of which are on the tension line with at least one more center point of another primitive.	 The relevance of the "right-centered" constraint for all of these pairs will be increased.

The influence constant for tension lines is $\delta_{tl} = \uparrow 0.5$.

5.6.4 Grouping

Grouping affects relative orientation, position, length, and size constraints. The factor approximates people's tendency to pay attention only to aggregate properties of the grouped primitives and to ignore the individual interactions of primitives in different groups.

The system decreases the relevance of the constraints between a pair of primitives if they belong to two different groups. Examples of such primitives are shown in Figure 5.38 in bold:

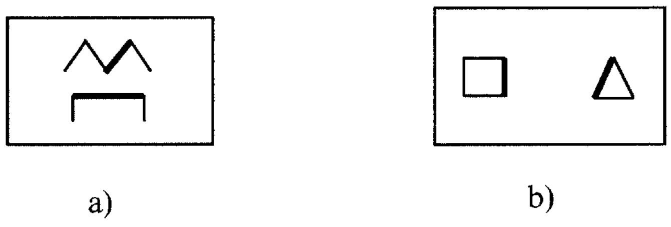


Figure 5.38 a) Two primitives in different connected components. b) Two primitives in previously learned shapes

The influence constant for a pair of primitives in different groups when neither of the groups is a previously learned symbol is $\delta_{dg} = \downarrow 0.2$. If one or both of the groups is a previously learned shape we expect the tendency to aggregate to be even stronger so the constant is $\delta_{ds} = \downarrow 0.4$.

5.6.5 Example

After calculating the relevance scores, the system removes constraints with scores below the 0.5 threshold.

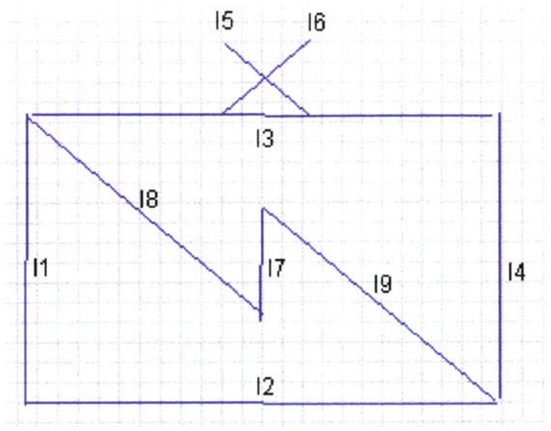


Figure 5.39 Military symbol

67 constraints were removed for the symbol in Figure 5.39. Examples include:

parallel: (l5 l8)
same-length: (l7 l6)
upper-right: (l6 l1)
upper-left: (l5 l4)
(List all constraints)

5.7 Removing redundancies

The descriptions for previously learned symbols are available in the domain graph, so there is no need to list the constraints for those symbols in the new description. To produce the final description the system filters out all such constraints if its group hierarchy contains previously learned symbols.

The final description for the symbol in Figure 5.39 contains 26 constraints, after removing 29 constraints related to the descriptions of the cross and the rectangle:

GROUP HIERARCHY: Group g1 connected-component: 16 13 15 14 12 11 18 17 19 Group g2 symbol - cross: 16 15 Group g3 symbol - horizontal rectangle: 13 14 12 11 Group g4 other: 18 17 19 CONSTRAINTS: elongated: (g3) above-centered: (g2 g4) inside-centered: (g4 g3) connects: (14.p2 19.p2) (13.p1 18.p1) (12.p2 19.p2) (11.p1 18.p1) (17.p1 19.p1) (17.p2 18.p2)	meets: (15.p2 13.cp2) (16.p1 13.cp1) vertical: (17) neg-slope: (19) (18) right: (19 17) (19 18) (17 18) upper-right: (16 18) upper-left: (13 19) (18 12) above-centered: (15 13) (15 17) (13 17) (17 12) (16 13) (16 17) right-centered: (14 17) (17 11) parallel: (18 19) same-length: (14 19) (11 18) (18 19) longer: (13 17) (13 18) (12 17) (12 19) (19 17) (18 17)

Applying the mechanisms inspired by the studies on human perception and removing redundant information has allowed the system to reduce the number of constraints for this symbol from the initial 122 to 26.

Figures below demonstrate the variations of the symbol in Figure 5.39 that would and would not fit the description:

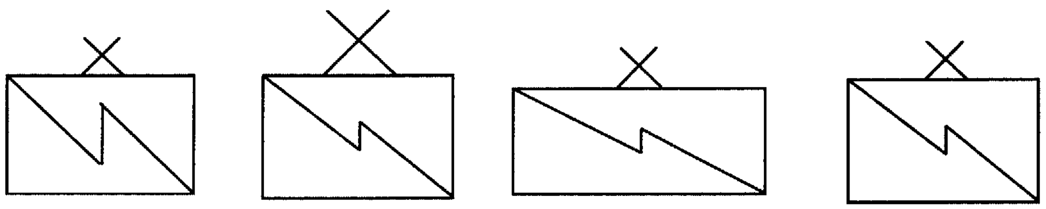


Figure 5.40 Examples of variations that would fit the description

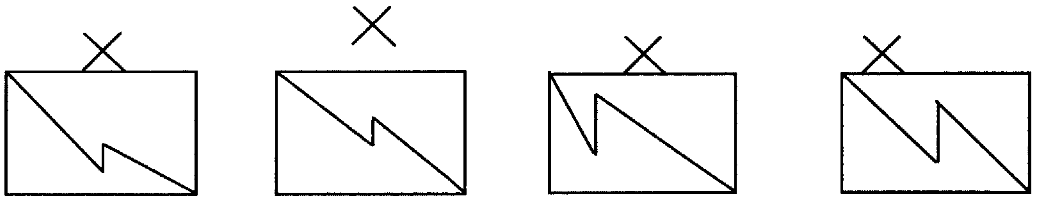


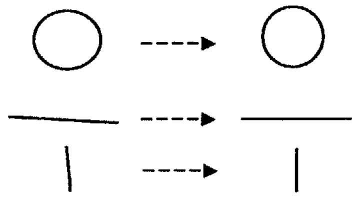
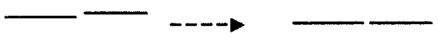
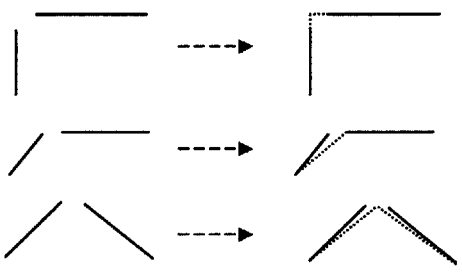
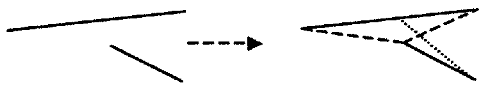
Figure 5.41 Variations that would not fit the description

5.8 User interface

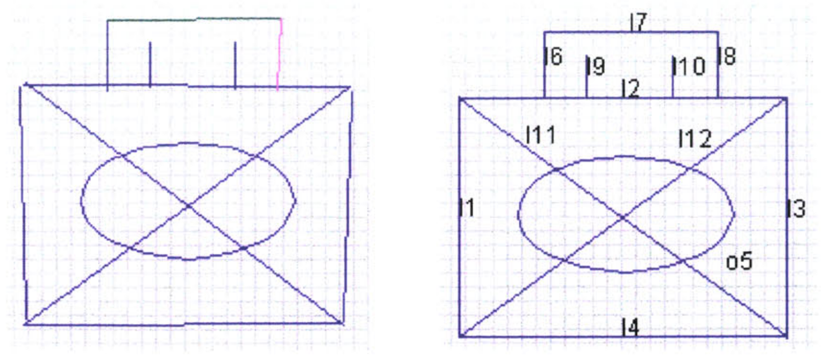
We would like the user to be able to check descriptions output by the system without having to read the text. We have taken initial steps towards creating a suitable interface for this purpose. It combines straightening the symbol to enforce some of the constraints in the description and displaying the rest of the constraints using simple graphical notation similar to the conventions in geometry textbooks. We discuss potential alternative approaches in the future work chapter.

5.8.1 Straightening the symbol

The system attempts to straighten the primitives in the symbol and enforce the constraints from the description. Currently, only orientation, aspect ratio, connects, and meets constraints are taken into account. The system proceeds through four steps:

Step	Example
1. Straighten individual primitives: Ovals satisfying the "non-elongated" constraint are turned into circles. Lines that the system identified as horizontal or vertical are rotated through the center to achieve perfect alignment with the axes.	
2. Align collinear primitives: Axis-parallel lines that have the same orientation and satisfy "connects" constraints are made collinear.	
3. Enforce connections: Endpoints of lines satisfying "connects" constraints are adjusted in three ways: • If both lines are not slanted, their endpoints are extended to the point of intersection. • If one of the lines is slanted, its endpoint is connected to the other. • If both lines are slanted, the connection point is set to be the midpoint.	
4. Enforce meets constraints: The endpoint of the line that should "meet" the other line is adjusted to be on that line in such a way that the ratio of distances from the endpoint of the first line to the endpoints of the second line is preserved.	

Steps three and four are performed for each constraint without consideration of whether the transformation may break other constraints, so it is possible that not all of these constraints will hold in the final drawing. In practice, however, this algorithm works reasonably well. Figure 5.42b shows the straightened version of the symbol in Figure 5.42a.



a) b)

Figure 5.42 a) Original primitives. b) Straightened symbol

We provide an alternative way to straighten the symbol using tension lines. For each tension line, an average x or y coordinate (depending on the orientation of the line) is calculated from the coordinates of the tension points on this line. Then this coordinate is set for the endpoints and center points of each primitive that contributes a tension point to the line.

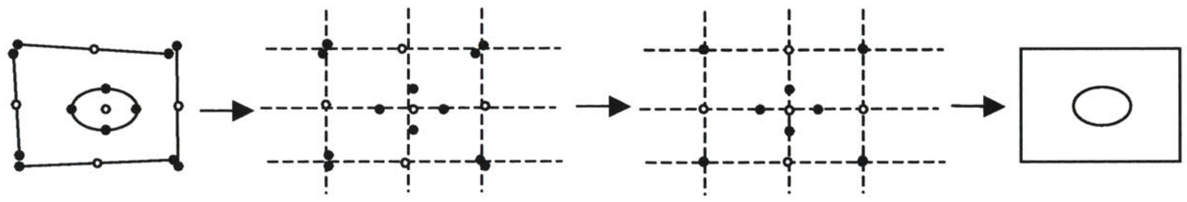
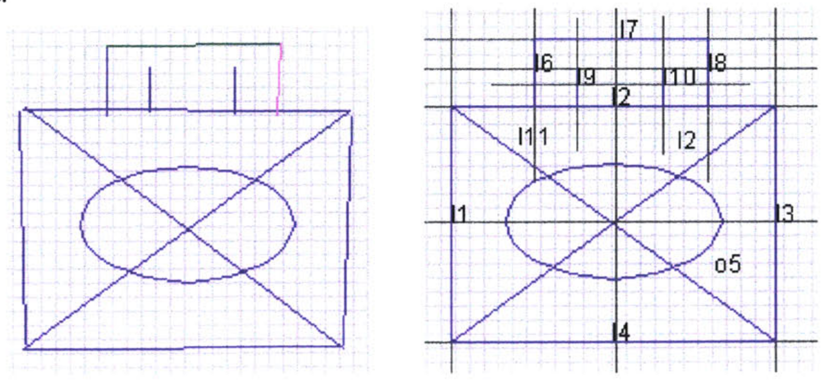


Figure 5.43

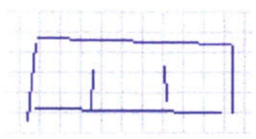
This mechanism usually produces more accurate results than the straightening algorithm described previously. Consider the example in Figure 5.44b for the symbol in 5.44a.



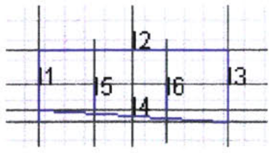
a) b)

Figure 5.44 a) Original primitives. b) Straightened symbol

Unfortunately, the identified tension lines are sometimes contradictory. For example, several points from a short vertical line may appear on the same horizontal tension line because of the tolerance for noise. This makes straightening out much less reliable. Consider the result in Figure 5.45 below:



a)



b)

```
TENSION LINES:
Horizontal: l2.p1 l2.center l1.p1 l2.p2 l3.p1
Horizontal: l6.p1 l5.p1 l3.center l1.center l6.center
Horizontal: l5.center l6.p2 l4.p1 l5.p2 l4.center
Horizontal: l3.p2 l4.p2 l1.p2
```

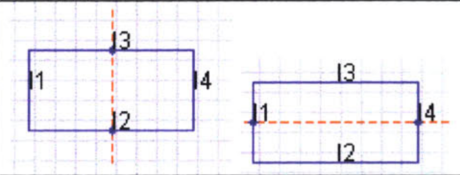
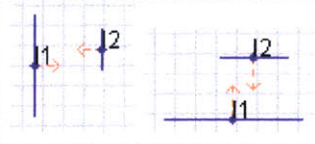
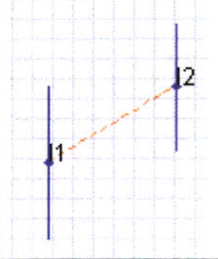
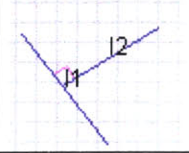
c)

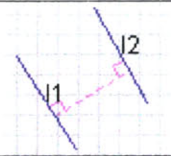
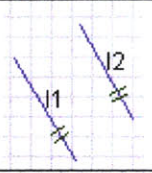
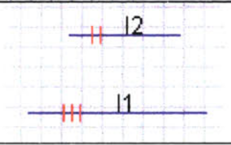
Figure 5.45 a) Original primitives. b) Drawing straightened according to tension lines. c) The list of horizontal lines the system identified in the drawing

We have not yet implemented a mechanism to remove such contradictions, so we mostly rely on the first method to straighten the symbol.

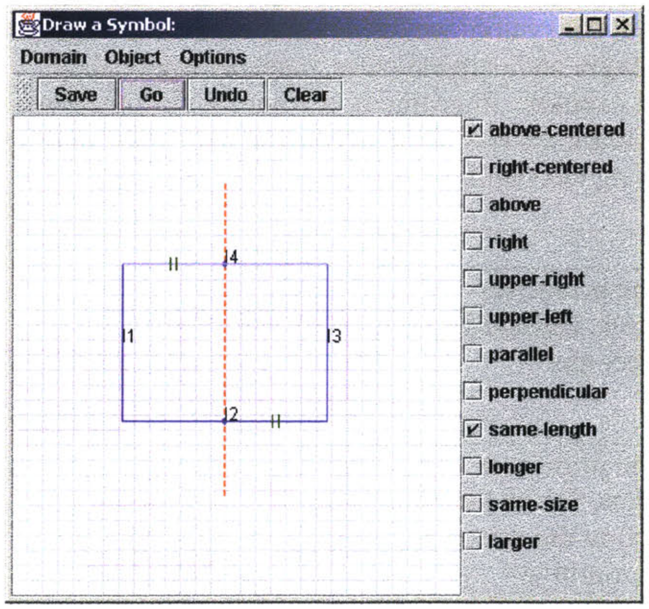
5.8.2 Graphical notation

In addition to straightening out the symbol we display some constraints graphically. For certain constraints, like same length and perpendicular, there are established conventions, like the ones used, for example, in geometry textbooks. For others we have created our own notation. We only mark the less obvious constraints, i.e. the ones that may not be evident from straightening the symbol:

Constraint	Notation	Example
Above-centered, below-centered	Centers of the primitives marked by dots. Dashed-line through the centers	
Above, below, left, right	Centers of the primitives marked by dots. Dashed arrow (axis-parallel) in the direction of the other primitive	
Upper-right, upper-left, lower-right, lower-left	Centers of the primitives marked by dots. Dashed line connecting the centers.	
Perpendicular	Square at the line intersection.	

Parallel	Squares at the corners of the lines and a line perpendicular line to them	
Same-length	Two short dashes through both lines	
Longer	Three dashes on the longer line and two dashes on the shorter line	

All the constraints related to a given primitive are displayed whenever the user clicks on it. The drawing may get cluttered if constraints of all types are displayed at the same time, so we provide a set of check boxes to specify which constraints should be shown:



[✓] above-centered
[ ] right-centered
[ ] above
[ ] right
[ ] upper-right
[ ] upper-left
[ ] parallel
[ ] perpendicular
[✓] same-length
[ ] longer
[ ] same-size
[ ] larger

Figure 5.46 Choosing constraint types to display

Chapter 6 Evaluation

The ideal evaluation of the system would be to use the produced descriptions in a sketch recognition engine and test the recognition accuracy. The user would teach a symbol to the system and then draw multiple variations of it. All and only the variations that the user intended to be recognized would have to fit the description. However, the recognition engine is only being developed, so the learning system had to be evaluated in isolation.

Our primary goal was to test whether the system accurately generalized the symbols using knowledge about human perception of geometry. We wanted to verify that the system captured the same properties that a person would pay attention to when learning a new symbol. To do this, we conducted a user study where subjects were shown an unknown symbol and several variations of it and asked whether they should be recognized as the original symbol. We tested whether the users accepted and rejected the same variations that would be accepted or rejected using the system's description.

In some domains, people may also use domain-specific information to decide what properties are important. For example, in electric circuit symbols we know that the lines representing wires can have arbitrary length. Since the system only uses geometric information, we picked symbols from military planning – a domain the subjects were not likely to be familiar with and where symbols have little resemblance to the actual objects they represent.

We describe the procedure and the results of the study in the next sections.

6.1 Data set and study procedure

We used 9 symbols from the military planning domain (Figure 6.1). The symbols were chosen to have a good balance in the use of different shapes and the level of detail.

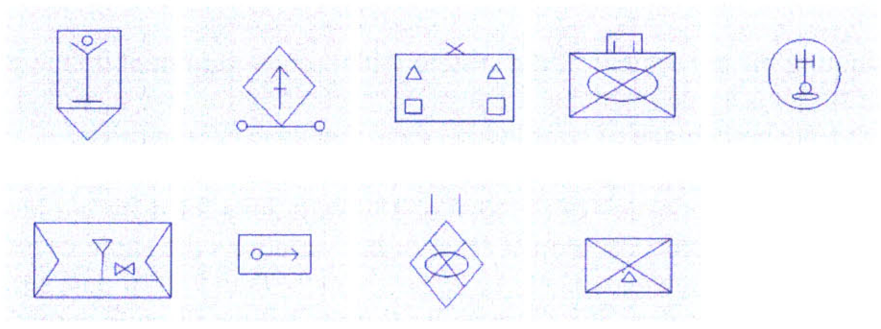


Figure 6.1 Test symbols

We examined the descriptions produced for these symbols and created 20 variations for each one: 10 variations that would be accepted and 10 variations that would be rejected, according to the description.

The goal in constructing the variations was to have a uniform distribution of the changes across different properties and across degrees of change. For each variation we randomly picked to vary one of the eight parts of the description:

- Touch constraints: connects, meets, intersects, etc.

- Orientation
- Relative position
- Relative length and size
- Relative orientation
- Group aspect ratio
- Group relative position
- Number of primitives

Then we randomly chose either a large or a small degree of variation. To produce each variation we tried, if possible, to change only one property of the original symbol without violating other constraints. The original symbols and the variations were drawn with very low levels of noise (i.e. satisfying all constraints almost perfectly) so that people would not attribute variations to sloppy drawing, but rather see them as non-accidental changes to the original symbol. The variations that the system would accept or reject were randomly mixed. Appendix B presents the complete data set.

The subjects completed the study online. The symbols and the variations were presented one at a time, occupying the whole browser window. The subject would first see the original symbol and then 20 variations of it, though the original symbol was also shown repeatedly before each variation. For each variation the subject was asked whether it should be recognized as the original symbol. Only "yes" and "no" options were provided. The subjects could take as much time as they needed to decide on the answer. We also provided the option to look at the original symbol by pressing the "Back" button on the browser. The order of the original symbols was randomized for each subject to average out potential order effects. We surveyed 33 subjects getting judgments for a total of 180 variations (20 for each symbol).

6.2 Results

Before evaluating the agreement of the system with human judgment it is important to see whether the subjects agreed with each other. For each variation, we recorded the majority answer and the percentage of people who gave that answer (majority percentage). The chart in Figure 6.2 gives an assessment of the agreement levels.

The y-axis shows the proportion of the total of 180 variations for different levels of majority percentage on the x-axis. For almost 40 % of the variations the subjects had high agreement – the majority percentage was above 90%. On more than half of data set the majority percentage was higher than 80%. Appendix B gives the detail on the votes and majority percentages for each variation in the data set.

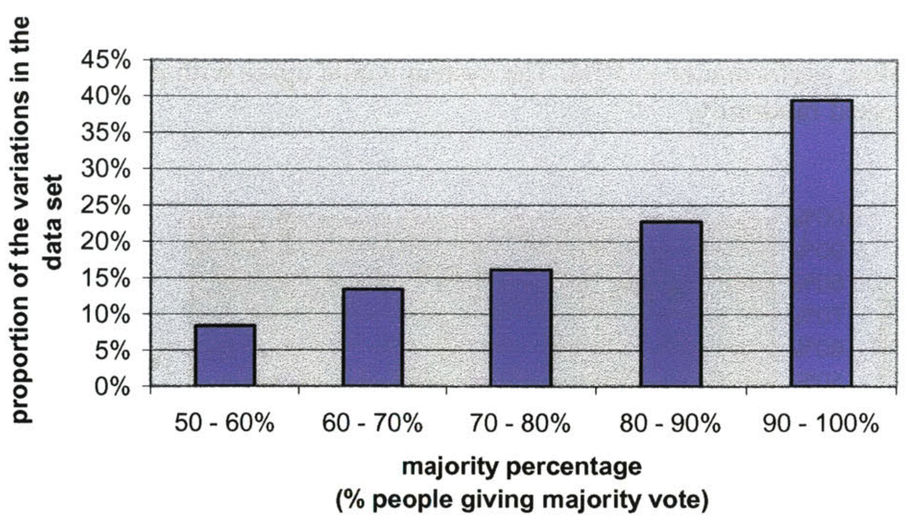


Figure 6.2 Levels of agreement for different variations

The chart shows that there were still a substantial number of cases (more than a third) where the subjects did not reach agreement, i.e. the opinions were strongly divided. Examples include:

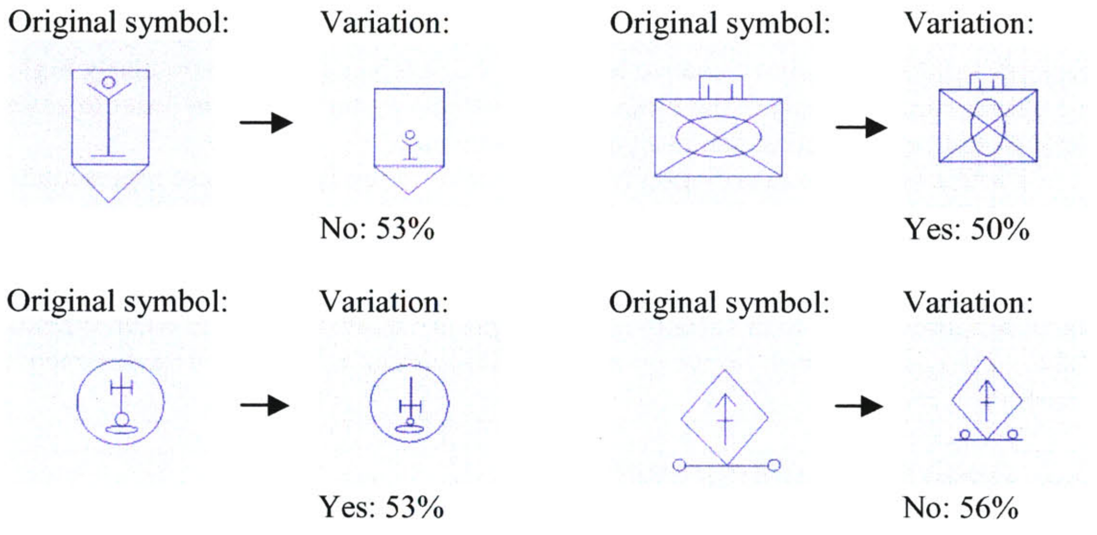


Figure 6.3 Variations that caused divided opinions

We think that it is reasonable to expect divided opinions in some cases. Perceptual similarity is a continuous property, yet we were forcing the subjects to make a binary decision. Subjects may differ on the exact threshold for whether a symbol should be recognized.

For such borderline cases, it makes less sense to evaluate the performance of the system (i.e. level of agreement with people) since people did not even agree with each other. Hence, we report the results not only for the complete data set, but also for just the subsets of variations with high agreement (with majority $\geq 80\%$ and majority $\geq 90\%$). We measured the proportion of times that the system agreed with the majority answer. Notice that the baseline performance is 50%. The system would agree with people half of the time if it guessed randomly.

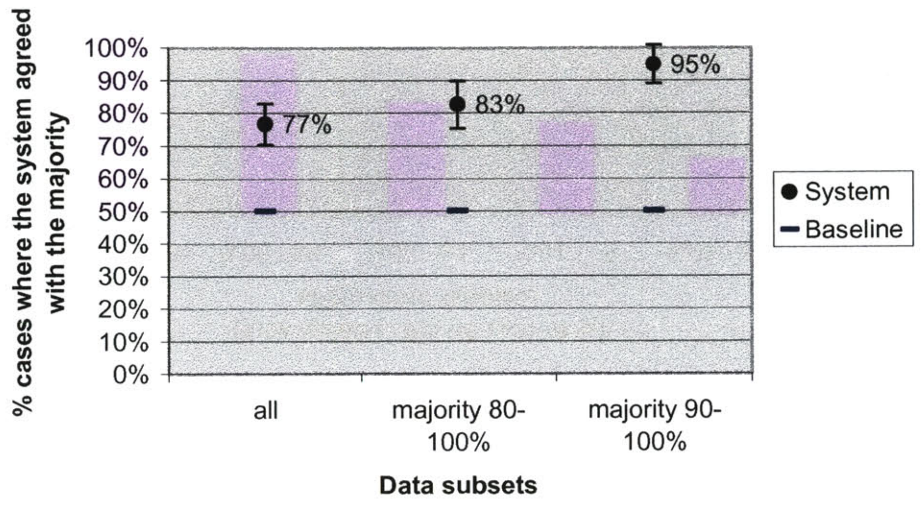


Figure 6.4 Percentage of cases where the system agreed with the majority answer

The system clearly captures enough relevant information about the symbol to perform significantly above chance level. Yet the numbers are not particularly high. In the next section we analyze the kinds of mistakes the system makes in order to assess what would be required to achieve better performance.

Notice that if the distribution of the examples in the data set were representative of the kinds of variations people make during sketching, these numbers would show potential recognition performance. However, the data set was not based on people drawing the symbol many times. It was created to reflect variations over all properties in the description, rather than variations that people are likely to produce when intending to draw the original symbol. So we do not think this would be an accurate assessment of recognition accuracy.

6.3 *Analysis of disagreements*

The system has disagreed with both "yes" and "no" majority answers, though there were significantly fewer mistakes on the "no" examples.

6.3.1 Disagreement on the "no" examples

These are cases where the variation fit the description, but the majority vote was not to recognize it as the original symbol. These mistakes fall into two categories.

In the first category the variation introduces connects, intersects, meets, or touches constraints that were not originally in the description. For example:

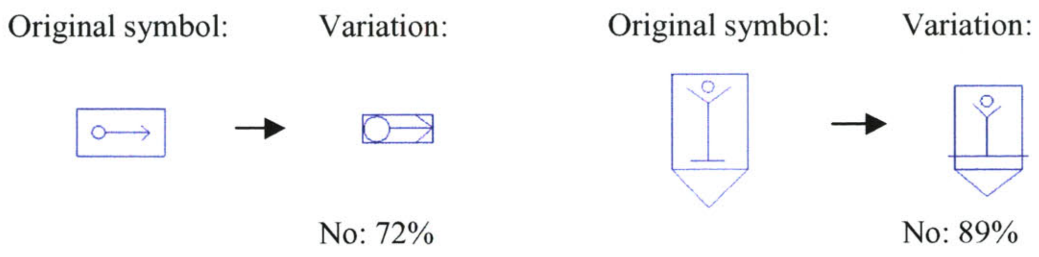


Figure 6.5 Variations accepted by the description but rejected by the majority of the subjects

These examples fit the description, because the symbols are represented in the system by specifying which constraints should hold, rather than which constraints should not hold. Yet the majority of the subjects reject the variation because, perceptually, the symbol is altered significantly. To correct this kind of errors the system would have to be extended to support "must-not" constraints. We think that these constraints would only be relevant for "touch" properties, like "connects", "meets", "intersects", "touches", "overlaps", etc., since these are most perceptually salient and can strongly alter the perception of the symbol.

The second type of disagreement is caused by the lack of explicit symmetry detection in the system. The variation below satisfies the description of the original symbol, even though it lacks symmetry. The majority, however, rejects the example (though there is only a slight majority).

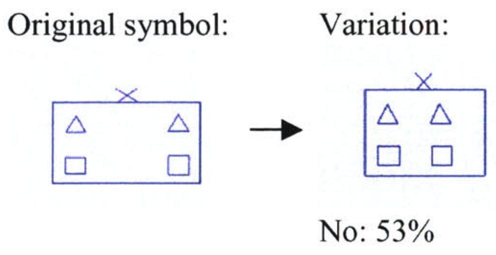


Figure 6.6 Variation that fits the description but is rejected by the majority

In summary, the disagreement on the "no" examples arises because the system does not capture properties of the symbol that have high perceptual relevance. The system does not look for these properties due to limited description vocabulary.

6.3.2 Disagreement on the "yes" examples

These examples represent cases where variations of the symbol violate some description constraints, but the majority of the subjects still consider them similar enough to the original symbol to be recognized. Four fifths of the errors the system makes belong to this category.

One type of these errors occurs when the aspect ratio of a subpart of the symbol is changed, but people do not consider this change of the symbol significant:

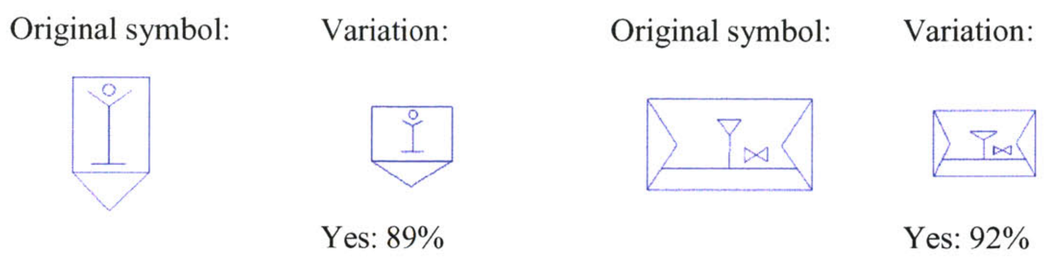


Figure 6.7 Changes in aspect ratio

The description of the "vertical rectangle" – the previously learned shape that the system finds in the first symbol – declares that the vertical sides should be longer than horizontal sides. So the rectangle in the variation of the symbol does not fit the description of the "vertical rectangle." Yet, for the subjects, it seems sufficient to just see a rectangle, regardless of the aspect ratio. We think that this effect may be related to the number of primitives in the symbol. When there is enough other detail to discriminate the symbol people tend to generalize more.

A potential fix would be to record more general versions of the previously learned shapes from the domain graph (Figure 6.8), when the shape is discovered in the new symbol that has many other details. Currently the system always prefers the most specific versions.

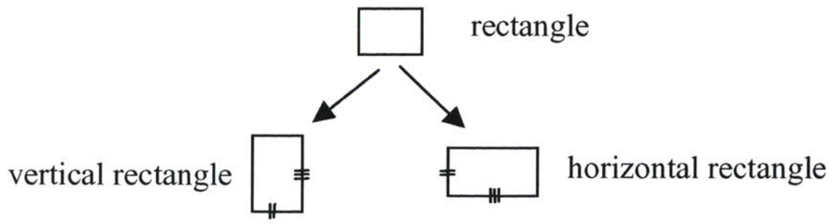


Figure 6.8 Domain graph that the system searches for previously learned shapes

There are a few cases where the system found "longer" constraints to be important and included them in the description, yet the majority of the subjects accepted the variation with these constraints violated, for example:

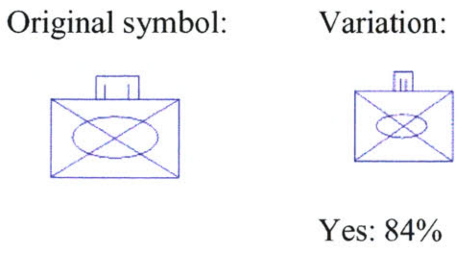
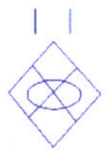


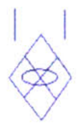
Figure 6.9 Changes in relative length constraints

We think that here the problem is similar to the one regarding the aspect ratio. The system may need to generalize more when the symbol has a lot of detail.

The system also made one error related to position constraints:

Original symbol: Variation:





Yes: 74%

Figure 6.10 Changes in position constraints

The system records, for example, that top-left side of the diamond in the original symbol is to the lower-left of the short vertical line above. When the two vertical lines are moved apart, the constraint no longer holds. The perceptual change is not very significant, however. It would have been enough to record that the vertical lines are above the top sides of the diamond.

System errors on “yes” examples are cases where the system does not generalize sufficiently. All the examples above are composed of several high-level shapes: diamond, oval, rectangle, etc. It seems that the most perceptually relevant feature is the combination of these high-level shapes, and people pay less attention to the individual detail. The system needs to include more mechanisms for decreasing relevance of constraints on the primitives that constitute detail, when multiple previously learned shapes are present.

Chapter 7 Future Work

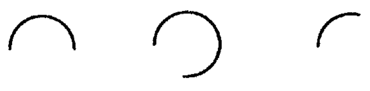
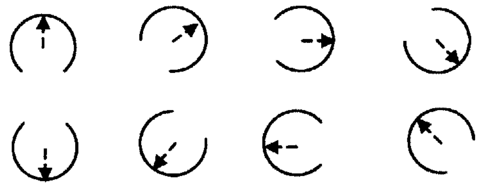
This section describes our ideas for improving the system descriptive ability, achieving better relevance ranking by using domain information, and ideas for alternative approaches to the user interface.

7.1 Extending the system's descriptive ability

To be able to represent a larger variety of symbols the system needs added support for arcs, curved elements, and symbols that contain an arbitrary number of certain elements (like a resistor, or a dashed line). In addition, many symbols could potentially be described more concisely, if the system used higher-level constraints that include more than two primitives. The next sections describe potential steps needed to make these improvements.

7.1.1 Arcs

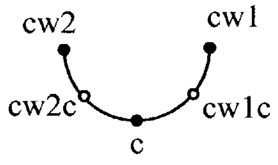
Incorporating arcs into the system would require defining a set of constraints that correspond to singular and non-singular arc properties. The table below shows a potential list of such properties:

Properties (singular ones shown in bold)	Example
Arc angle: **half-arc**, >half-arc, <half-arc	
Arc orientation: **top**, top-right, **right**, bottom-right, **bottom**, bottom-left, **left**, top-left	

Constraints defined similar to those for lines and ovals could also be used with arcs:

- Connects, meets, intersects, touches.
- Position constraints (referring to the center of the bounding box): above, right, left, below, upper-right, upper-left, lower-left, lower-right, above-centered, below-centered, left-centered, right-centered, inside, inside-centered
- Same-size, larger (referring to largest dimension of the bounding box)

Parameterized constraints like meets, connects, intersects, and touches would refer to the points on the arc in the table below:

Part	Notation	Example
First point on the arc in clockwise direction	**cw1**	
Any point between cw1 and the center of the arc curve	cw1c	
Center of the arc curve	**c**	
Any point between cw2 and the center of the arc curve	cw2c	
Second point on the arc in clockwise direction	**cw2**	

The descriptive power of these properties and constraints would have to be tested on a variety of symbols.

7.1.2 Curve representation

A large number of symbols contain spirals, waves, and other curved elements:

Figure 7.1 Symbols with curved elements

In many systems curves have been represented by parameters that do not easily capture the important perceptual characteristics. For example, two of the Bezier control points don't lie on the curve. It would also be hard for a person to judge the positions of these points when looking at a given curve:

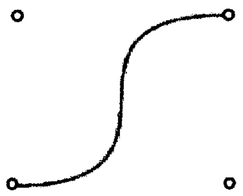


Figure 7.2 The circles show the control points of the Bezier curve

Bezier control points are not the perceptually salient elements of the curve. The positions of the endpoints, the existence of an inflexion point, and the "angular distance" traversed by the two segments separated by the inflexion point are probably more perceptually relevant. A description in these terms captures the perceptual similarity between different curves in Figure 7.3, even though some of them are composed from more than one Bezier curve segment or from two arcs:

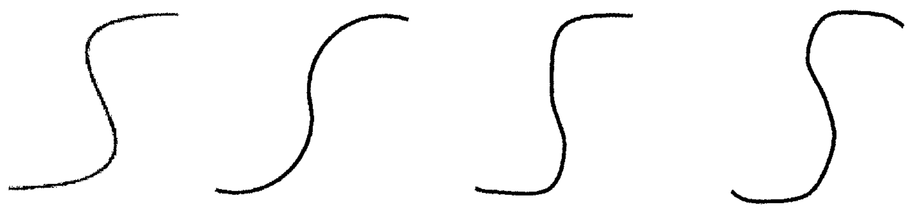


Figure 7.3 Perceptually similar curves

Future work should explore perceptually salient properties of curves to create a qualitative for describing curved symbols.

7.1.3 Arbitrary number of elements

Symbols often have components that can be repeated an arbitrary number of times:

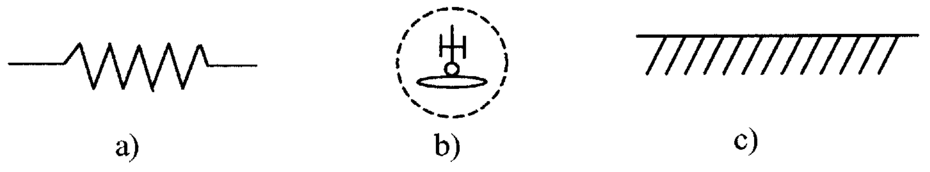


Figure 7.4 Symbols with varying number of primitives. a) Resistor symbol. b) Symbol from military planning. c) Symbol for ground or surface in mechanical engineering

Learning such configurations presents two challenges. The system first has to be able to identify a group of repeated components and, second, decide whether an arbitrary number of them were intended. Goldmeier's studies provide some hints on how this may be done. He distinguishes the geometric elements perceived by people as either *material* or *form*. Consider two experiments in Figure 7.5. Which of the b and c is more similar to a?

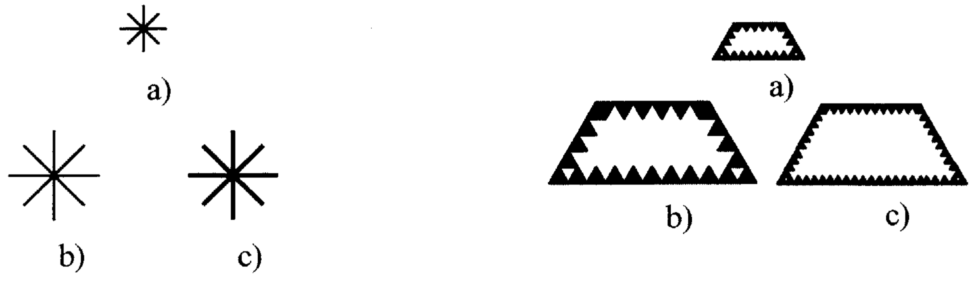


Figure 7.5 Which of b and c is more similar to a?

Even though uniform scaling of the symbol should not, supposedly, affect similarity, most of the subjects pick the example where the line width or the size of the small triangles remains the same, rather than increased. Goldmeier argues that the lines of a certain width or the small triangles are perceived as material that makes up a larger shape (form). For the symbol to remain perceptually more similar, he claims that "the form is best preserved by proportional enlargement; material properties are best preserved by keeping the measurements of the material elements constant." However, ask yourself the same question for Figure 7.6:

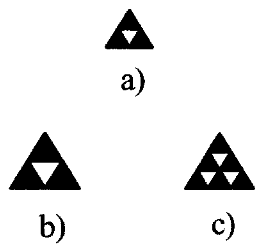


Figure 7.6 Which of b and c is more similar to a?

Most subjects choose b. In this case smaller triangles are not considered material. The difference between the cases when repeated elements can be viewed as material and when they should be viewed as form is best illustrated by Figure 7.7:

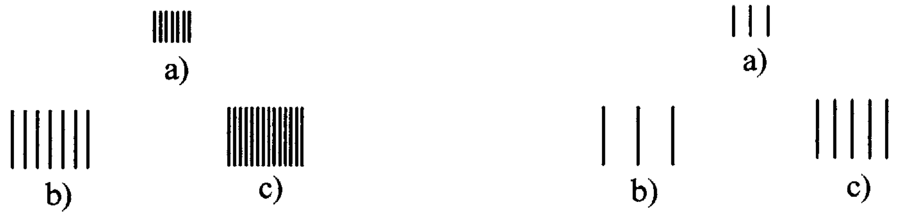


Figure 7.7 Which of b and c is more similar to a?

In the first experiment most subjects have picked c, treating the lines as material. However, in the second experiment they chose b. The presence of exactly three lines is perceived as a salient part of the form (structure) of the symbol.

According to Goldmeier, when the repeated elements are small compared to the size of the symbol and there is a large number of them, people start perceiving them as material rather than form and hence become insensitive to the variation in number of such components. The difficult task is defining quantitatively the terms "small relative to the symbol size" and "large number of elements."

7.1.4 Higher-level constraints

Due to the restriction of the vocabulary to binary constraints, the system cannot capture several constraints that are often perceptually noticeable.

For example, the system does not represent symmetry, although tension lines sometimes implicitly capture some of the constraints that contribute to horizontal or vertical symmetry.

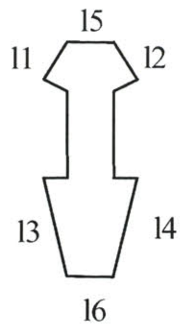


Figure 7.8 Symmetrical symbol

In Figure 7.8, for example, the relevance of constraints "same-length: (l1 l2) (l3 l4)" and "above-centered (l5 l6)" is increased due to the tension lines formed by these primitives. In general, any two primitives symmetrical across the vertical or horizontal axis will form one or more tension lines, helping increase the constraints on their relative position and sometimes length:

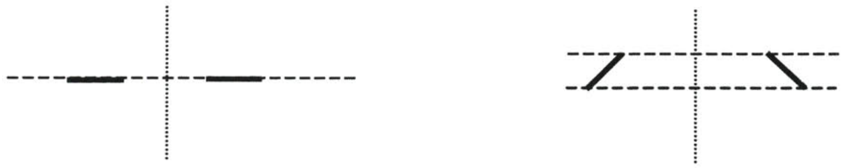


Figure 7.9 Symmetrical segments form tension lines

However, currently there is no mechanism to require that the two elements should be equidistant from the symmetry axis or that they should have the same absolute slope. For the system there is no difference in the constraints produced by the system for the symmetrical and non-symmetrical symbols in the pair of examples below:

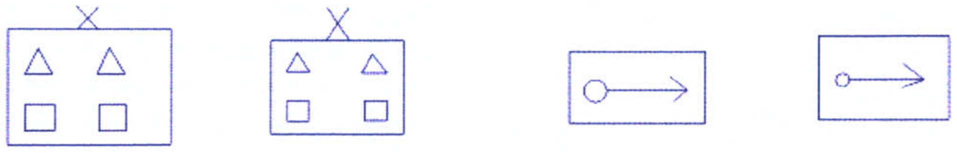


Figure 7.10 Symmetrical segments form tension lines

The system would also benefit from addition of constraints like interval equality between pairs of lines and alignment of several endpoints of different primitives. With these constraint symbols like the ones in Figure 7.11 could be described more concisely:

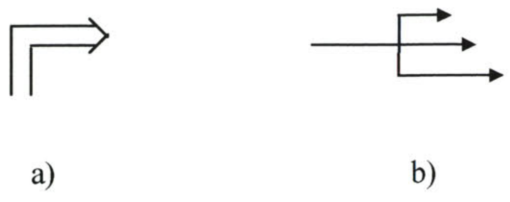


Figure 7.11 a) Symbol requiring interval equality constraints. b) Symbols requiring alignment constraints

The only way the system currently allows constraining more than two primitives at a time is through group constraints. Improving grouping would help identify more accurate global constraints. The system supports only two grouping principles: connectedness and familiarity of shape. Proximity, similarity, continuity, and closure factors need to be added to better approximate perceptually relevant grouping of the primitives within the symbol. Drawing order may possibly provide additional clues for grouping. We think that people will be more likely to draw perceptually salient components consecutively, without overlap.

Including more grouping factors, however, would pose the challenge of resolving the competition between different grouping principles. This problem is hard, since in some symbols even people are not always clear on the grouping choice.

7.2 Domain knowledge

Using domain knowledge may help in learning symbol descriptions. For example, the system could ensure that all of the symbols it learned would be distinguishable if it compared all the produced descriptions. With that safeguard, it is possible that some domains would permit the system to generalize more aggressively, i.e. throw away more constraints from a particular drawing. Consider the example from the user study set:

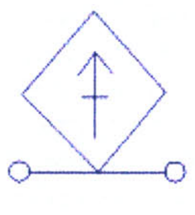


Figure 7.12 Military planning symbol

This is a symbol from military planning diagrams. We assume that most study subjects were not familiar with other symbols in the domain and based their judgment on purely geometric criteria. Most users replied that the drawings in Figure 7.13 should not be recognized as the symbol above.

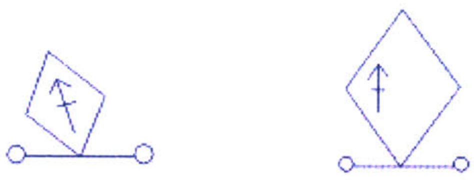


Figure 7.13 Variations of the symbol

The description produced by the system would also cause these variations to be rejected. However, if we consider other symbols in the domain (for example, other drawings in the test set), the variations in Figure 7.13 still look perceptually closest to the symbol in Figure 7.12. It may be reasonable to still recognize these distorted instances during sketching. The description below would suffice for this goal:

"Diamond above (and touching) a horizontal line with two ovals at the end-points. An arrow with a line across it inside the oval".

Notice that this description is very underconstrained, yet it captures the properties that make this symbol different form other symbols in the same domain. With this approach, however, it is important to be careful not to reduce the description only to the discriminating features. The system has to be able to reliably reject symbols that are not from the domain.

7.3 Improved user interface

It would be more natural to be able to draw more sloppily. The goal of the system is to match the ease of person-to-person interaction. And even when learning new symbols, people are typically able to tolerate higher levels of noise than currently allowed by the system.

The system would need to have looser noise thresholds and also make them dependent on the size of the primitives so that more noise is tolerated when people draw larger symbols. Higher noise thresholds, however, may cause the system to find erroneous or contradictory constraints. It would need a mechanism to detect and correct such cases. This would involve disambiguating between possible alternatives (eg. Figure 7.14b). We believe that here again the system would need to make use of the knowledge about human perception in order to decide which of the alternatives is more salient.

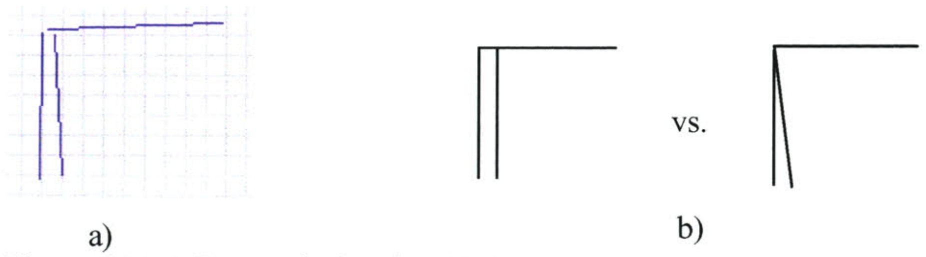


Figure 7.14 a) The user's drawing. b) Alternative interpretations of the drawing

Graphical display of the system's conclusions needs further improvement. This would include creating a better mechanism for straightening up the symbol, so that the configuration of the primitives fully satisfies the constraints in the description. Also it would be worth exploring a more intuitive set of graphical notation for each type of constraints.

An alternative approach to checking produced descriptions, based on variations of the symbol, is described in the next section.

7.3.1 Automatic generation of potential "near misses"

Instead of displaying constraints graphically, the system could show different variations of the symbol that fit and do not fit the description and ask the user to accept or reject them. Then it would modify the description based on the responses.

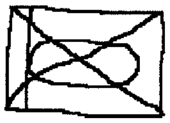


Figure 7.15 Military planning symbol

In Figure 1, the horizontal elongation of the rectangle and the oval may or may not be a required constraint. One way to verify that would be to ask the user whether the following examples should be recognized as the symbol:

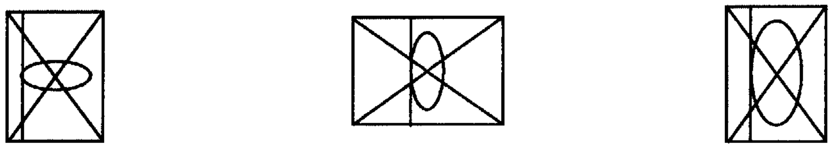


Figure 7.16 Examples with questionable constraints removed

The system would remove the constraints that are violated in the accepted examples and include missing constraints that differentiate the original symbol from the rejected examples.

The space of variations may be too large to explore exhaustively. For example, if a description contains 30 constraints and the option is to drop or keep each constraint, there may be up to $2^{30} \sim 1$ billion variations. Even if we assume that it is enough to check each constraint individually, the user would still have to look at 60 symbols. The main challenge is to generate only the few variations that the system could benefit from, i.e. the variations that explore the constraints that the system is "not sure" about.

The system could take advantage of relevance scores to identify such constraints, as they approximate the degree of perceptual salience. For example, there is no need to check the constraints that have a high score (like connects or meets). Removing those constraints would produce a symbol that is significantly different and that the user would be likely to reject. That would give no new information to the system. On the other hand, varying constraints with scores near the filtering threshold is more likely to provide "near misses" that the system can learn from, because its judgment may differ from that of the user.

7.4 Relevance ranking for recognition robustness

A generic recognition engine will use the system's descriptions to identify symbols in user's sketches. If relevance scores were included in the description, the engine could use them for error-tolerant matching, making the recognition potentially more robust in the cases when the description is overconstrained. Consider, for example, one of the constraints for the symbol in Figure 7.17.

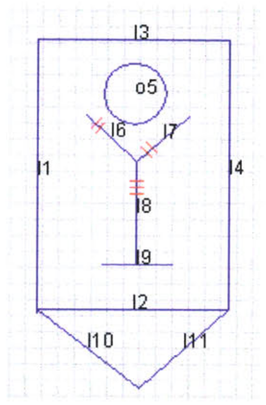


Figure 7.17 Military symbol

The system decides that line l8 should be longer than l6 and l7. Now assume that the user still wants the system to recognize the variations of this symbol where these constraints do not hold. The system gave these constraints relevance scores of 0.55, which are only slightly above the filtering threshold and lower than the scores of most other constraints (eg. connects has a score of 1.0 and meets has a score of 0.9). Error-tolerant recognition would proceed by computing the matching error by summing the number of discrepancies between the input sketch and the constraints in the description, weighted by their relevance scores. Any input with a total error below a certain threshold would be considered to fit the description. When the description is incorrectly overconstrained, the engine may still recognize the input symbol, as long as the constraints that are require by the description but are missing from the input have low relevance.

Chapter 8 Conclusion

We have presented a system for learning shape descriptions from a single example of a symbol. By explicitly putting in knowledge about human perception we attempt to guide the generalization process. The generalization power derives from two sources:

1. Qualitative vocabulary of constraints based on perceptual singularities:

The vocabulary contains singular and non-singular terms, reflecting the property values that people attend to (singularities) and aggregating values that they ignore (non-singularities). This aggregation is an important initial generalization step.

In spite of the qualitative nature, the vocabulary is adequate for describing a large variety of symbols because it captures perceptually salient properties that we expect to be the basis for creation of graphical languages.

2. Perceptually inspired mechanisms for ranking constraints by relevance:

Constraints are assigned default relevance scores, based on their average perceptual importance. In addition, obstruction, tension lines, and grouping mechanisms that take into account the particular configuration of the primitives in the symbol cause these scores to be increased or decreased. These mechanisms reflect the observation that people pay attention to global properties of the symbol and that perceptual relevance of constraints is context-dependent.

As shown on several examples the system is capable of adequately describing complicated symbols with a lot of detail. We measure the success of the system in learning a new symbol by how well it captures the properties that people would pay attention to. The user study has shown that the system performs reasonably on the examples where the subjects agreed among each other.

Future work on the system would include improving its descriptive ability by providing support for curves and symbols with an arbitrary number of elements and by extending the constraint vocabulary to support higher-level constraints like symmetry, interval equality, and multiple alignments. As we have shown, knowledge about perception may provide further clues on how to achieve these extensions.

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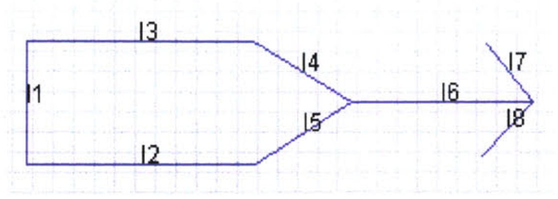
Appendix A



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vertical:(l1)
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neg-slope:(l4) (l7)
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right:(l5 l1) (l4 l1) (l8 l1) (l7 l1)
below:(l8 l6) (l6 l7)
left:(l1 l5)
upper-right:(l5 l2) (l4 l2) (l3 l1) (l8 l2) (l7 l2) (l7 l5) (l6 l2) (l6 l5)
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lower-left:(l5 l7) (l2 l4) (l2 l6) (l2 l7) (l2 l8) (l1 l3) (l1 l4) (l1 l7)
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right-centered:(l8 l5) (l7 l4) (l6 l1)
perpendicular:(l5 l7) (l8 l7) (l7 l5) (l7 l8)
same-length:(l5 l1) (l5 l4) (l4 l1) (l4 l5) (l3 l2) (l2 l3) (l1 l4) (l1 l5) (l8 l7) (l7 l8)
longer:(l5 l7) (l5 l8) (l4 l7) (l4 l8) (l3 l1) (l3 l4) (l3 l5) (l3 l6) (l3 l7) (l3 l8)
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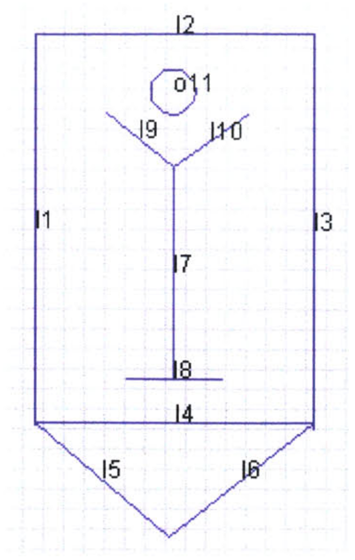

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(l7l8)
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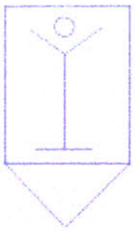

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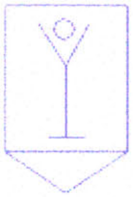	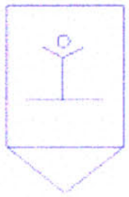	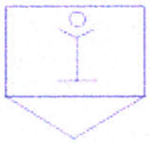	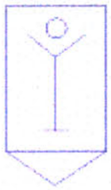
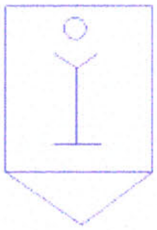	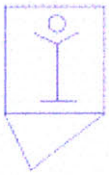	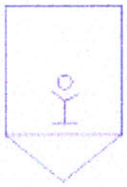	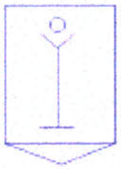
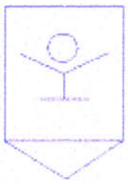	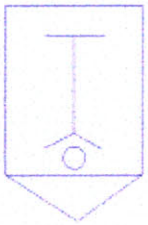	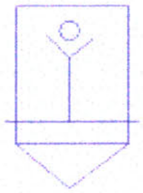	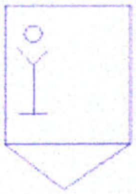
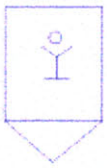	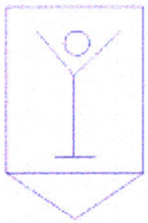	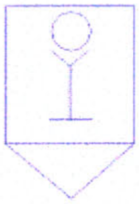	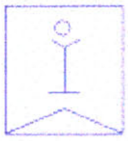
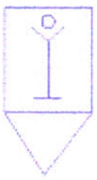	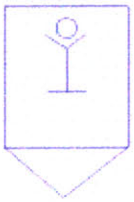	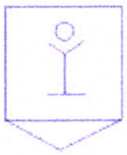	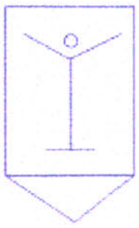

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System:	NO	NO	YES	NO
Majority:	NO	NO	NO	NO
Majority %:	50%	92%	89%	76%
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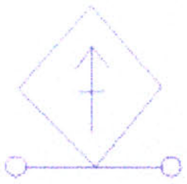

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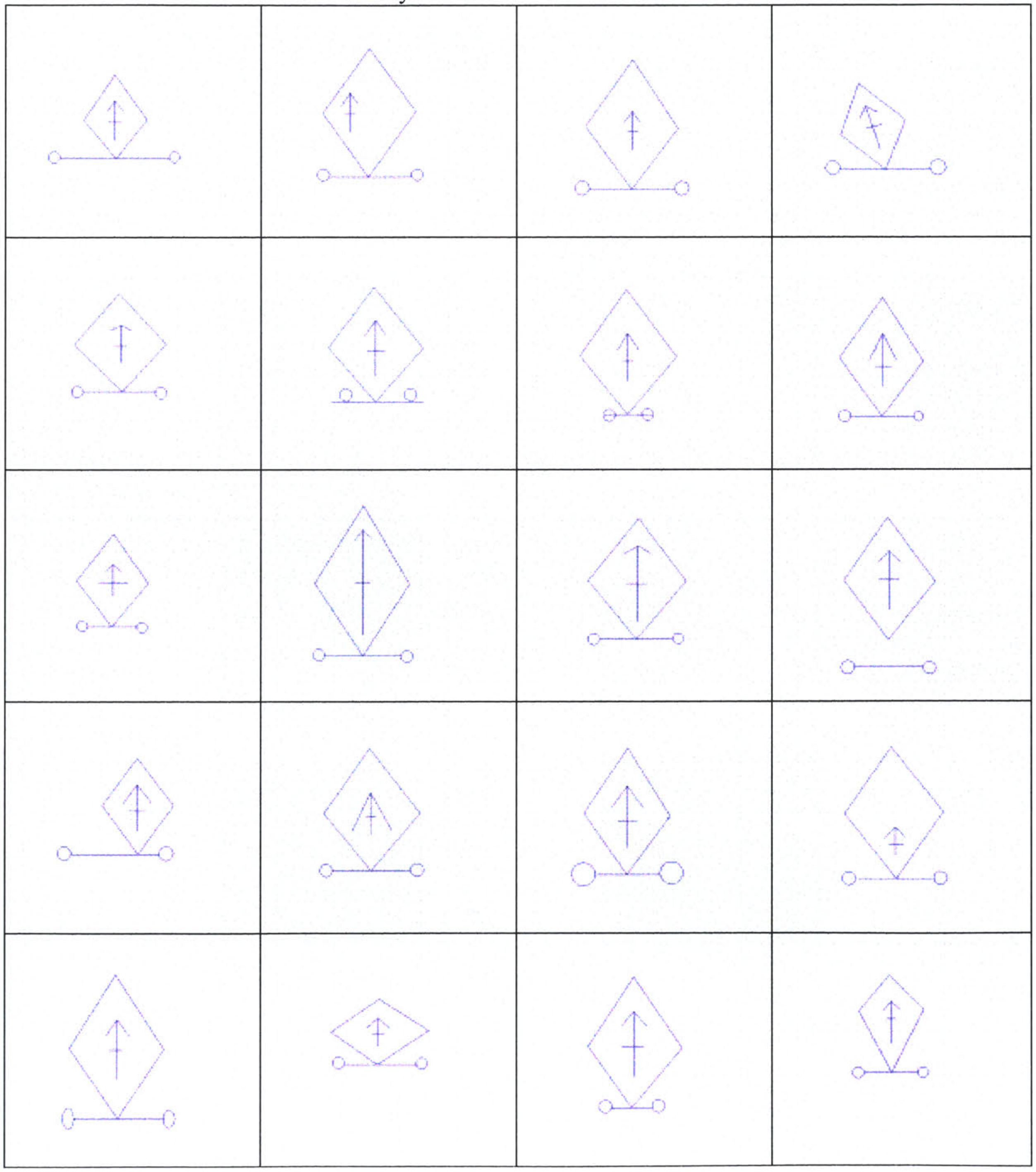

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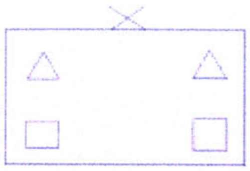

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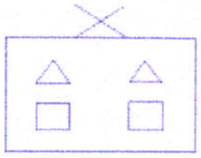	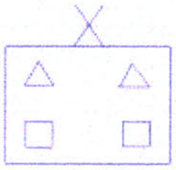	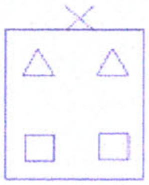	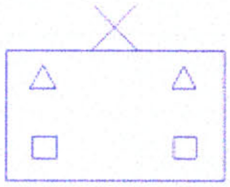
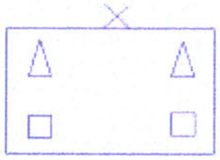	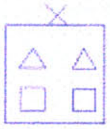	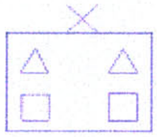	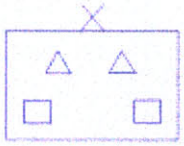
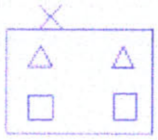	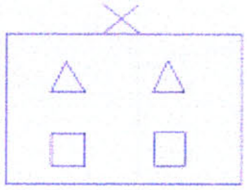	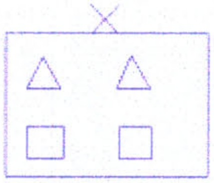	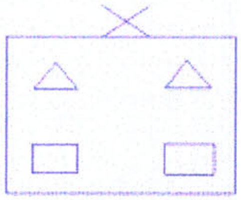
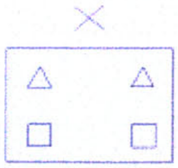	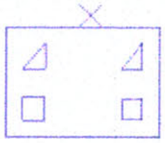	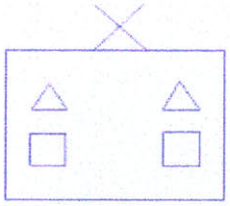	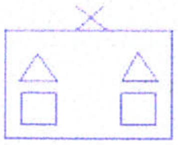
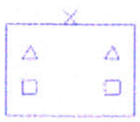	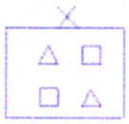	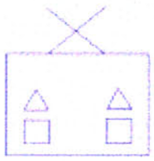	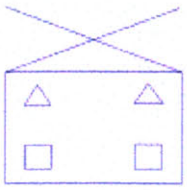

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Majority %:	83%	75%	100%	85%
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Majority %:	83%	85%	53%	90%
System:	NO	NO	YES	YES
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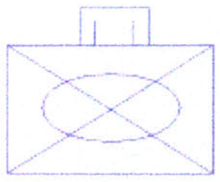

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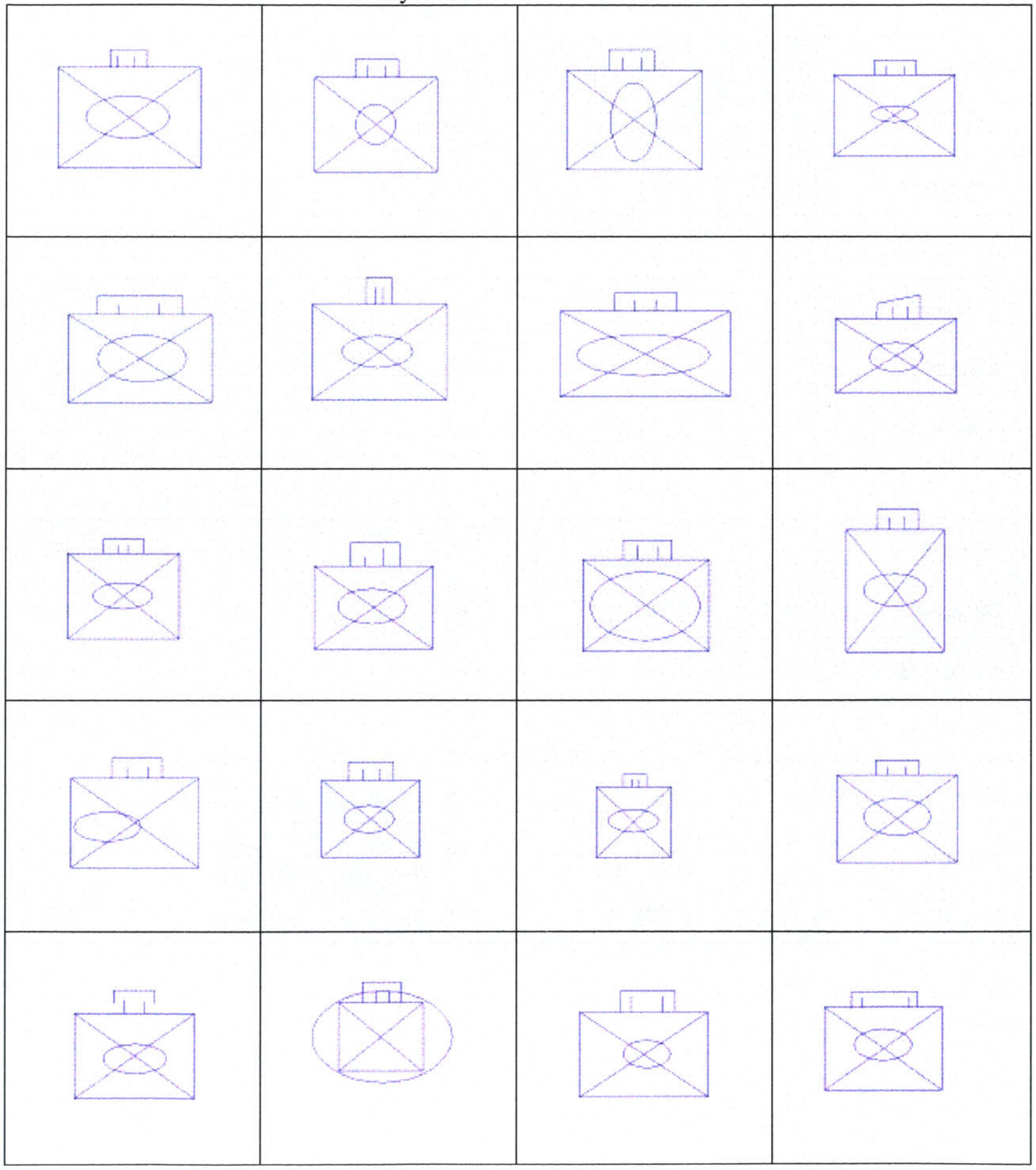

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System: Majority: Majority %:	YES YES 84%	NO YES 84%	YES YES 100%	NO NO 61%
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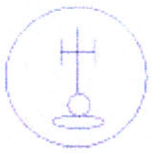

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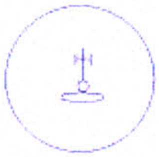	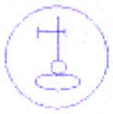	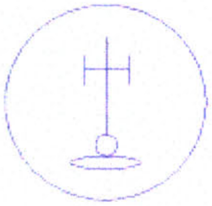	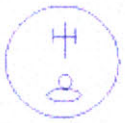
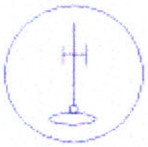	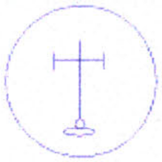	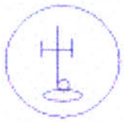	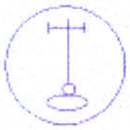
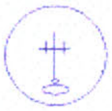	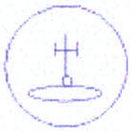	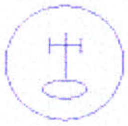	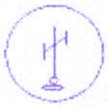
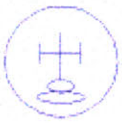	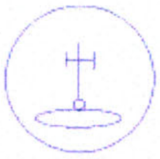	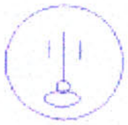	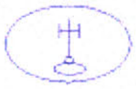
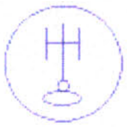	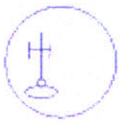	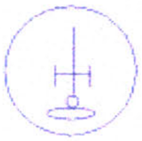	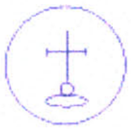

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Majority:	YES	YES	NO	NO
Majority %:	50%	73%	73%	90%
System:	NO	YES	NO	NO
Majority:	YES	YES	NO	YES
Majority %:	85%	80%	100%	78%
System:	YES	NO	YES	YES
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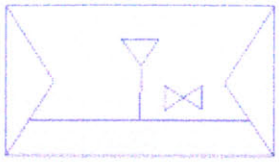
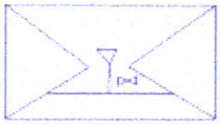
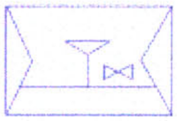
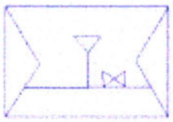
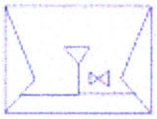
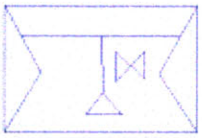
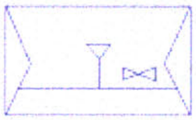
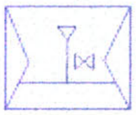
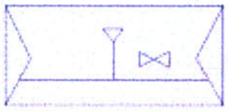
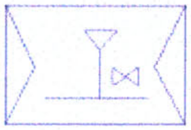
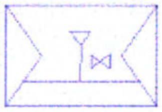
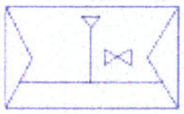
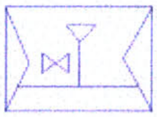
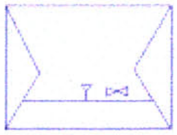
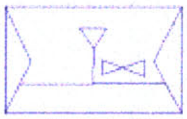
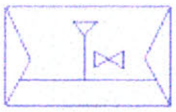
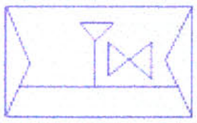
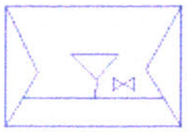
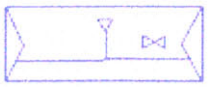
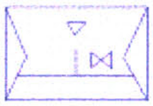


Symbol 6

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System:	NO	YES	YES	NO
Majority:	NO	YES	YES	NO
Majority %:	79%	97%	79%	95%
System:	NO	YES	YES	YES
Majority:	NO	YES	YES	YES
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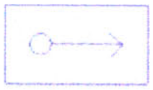
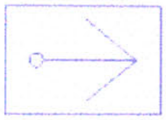
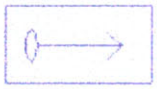
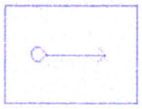
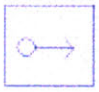
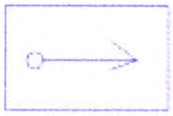
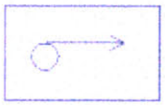
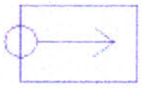
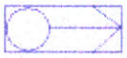
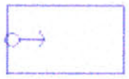
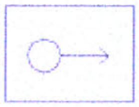
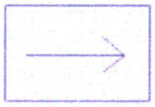
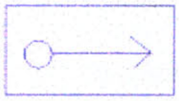
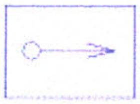
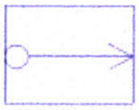
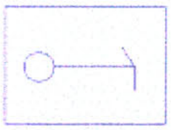
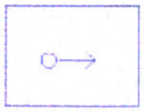
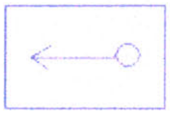
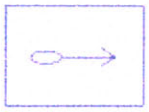
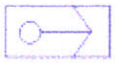
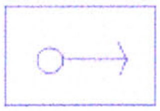


Symbol 7

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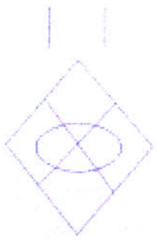
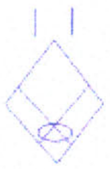
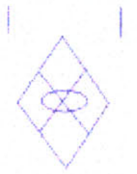
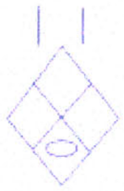
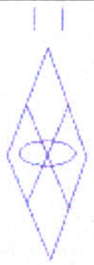
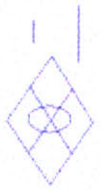
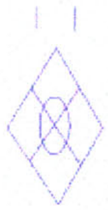
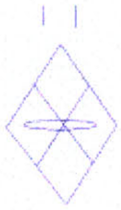
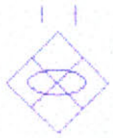
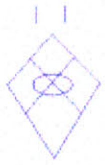
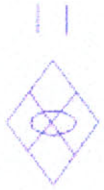
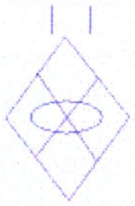
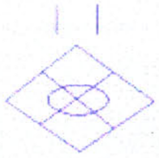
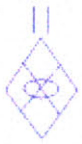
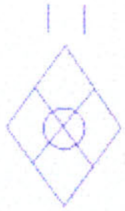
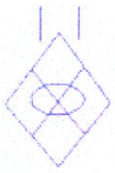
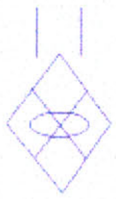
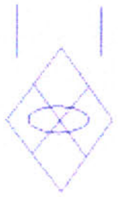
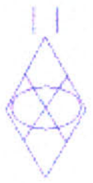
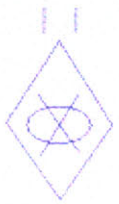
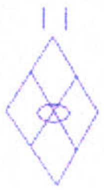


Symbol 8

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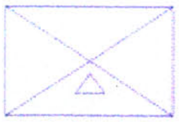
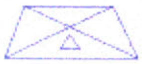
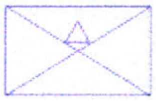
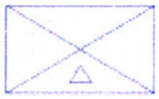
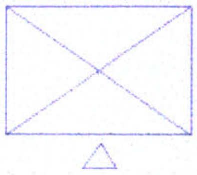
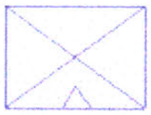
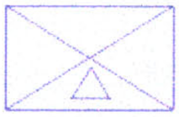
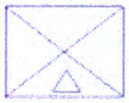
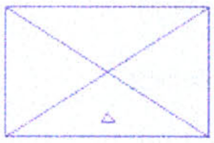
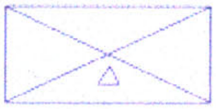
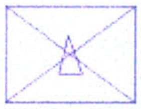
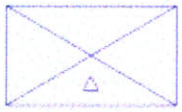
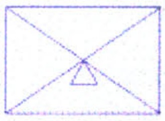
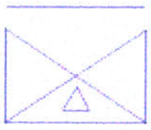
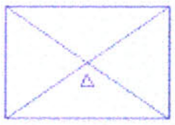
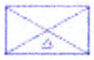
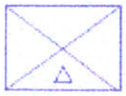
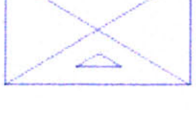
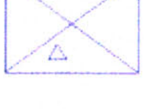
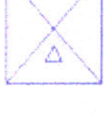
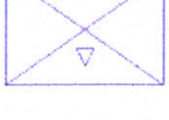


Symbol 9

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Majority %:	95%	97%	89%	81%
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