Channel Assignment Algorithms and Blocking Probability Analysis for Connection-Oriented Traffic in Wireless Networks

by

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Abstract

We address the problem of dynamic channel assignment and blocking probability for connection-oriented traffic in a multihop wireless network. Specifically, we present an exact blocking probability analysis for a single channel wireless line network and derive blocking probability formulas for both bi-directional and uni-directional calls. In the multiple channel case, we present a simplified analytical model and obtain approximate blocking probability formulas that predict the simulation results very accurately, especially, for low to moderate number of channels. We apply the formulas derived to consider the effect of transmission radius on blocking probability. We show that in a line network with equal length calls, it is preferable to use larger transmission radius; while for a more dense grid network we make simplified analytical arguments showing that it is more desirable to use smaller transmission radius. Finally, we present a novel channel assignment algorithm that aims at reducing blocking probability by cleverly reusing the channels while also satisfying the wireless transmission/reception constraints. We compare its performance with other channel assignment algorithms such as the rearrangement, random and the first fit algorithm.

Thesis Supervisor: Eytan Modiano
Title: Associate Professor

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Chapter 1

Introduction

Wireless communication witnessed a tremendous growth in the last decade and continues to expand rapidly with the development of new technologies. Cellular networks, for wireless voice telephony, have been successfully deployed around the world. However with a rapidly expanding set of wireless users there is a growing need for local wireless networks that can be quickly setup in any terrain. Unfortunately, cellular networks do not provide this flexibility as they require a massive infrastructure investment. Multihop wireless networks are a promising alternative towards achieving this goal.

A multi-hop wireless network (also called an ad-hoc network) is a cooperative network where data streams may be transmitted over multiple wireless links (multiple hops) to reach the destination. The nodes in such a network operate not only as hosts but also as routers forwarding data streams for other nodes that are not within direct transmission range of each other. Unlike its cellular counterpart, this network can be easily deployed in any terrain without the need for any fixed infrastructure. Though easily deployed, a wireless ad-hoc network introduces added complexity into problems of routing [1], scheduling and network connectivity [2]. In addition, such a network should also be able to provide quality of service guarantees to have widespread application as its cellular counterpart.

Recently, there has been increased focus on issues relating to Quality of Service (QoS) guarantees for wireless ad-hoc networks [3], [4], [5], [6]. The work in QoS guarantees has focussed mainly on finding source-destination routes that satisfy end to end QoS requirements, often given in terms of the required channels (bandwidth). The problem of which channels to allocate from amongst the available channels and the effect of transmission radius on the blocking probability of calls is the focus of this thesis. In this work, we study a line and a grid network as they are good representatives of a sparse and a dense network respectively.

The problem of dynamic channel assignment has been considered in the context of cellular networks [14], [16], [17], [18]. However, in a multi-hop wireless network the problem differs significantly from its counterpart in the cellular network. In a cellular network (potentially many) nodes communicate only with the nearest base-station. In contrast, in a multi-hop wireless network, transmission of data takes place over multiple wireless links to reach the destination. This imposes additional complexity as non-conflicting channels must be allocated on the wireless links along the source-destination path. Another difference between the two networks is that a cellular network has a regular cell structure which makes the set of interfering cells fixed. While, in a multi-hop wireless network, the set of interfering nodes can be varied by varying the strength of the transmitted signal. This also has the effect of altering the hop length of the calls. With a larger signal strength, a node can communicate with a node further away and reach the destination node in fewer hops albeit at the expense of interference with a larger set of nodes.

The performance metric considered in this thesis is the steady state blocking probability of a call. Blocking probability as a performance metric has been widely used by researchers in studying various networks. Some of the work on blocking probability includes [21], [22] in all-optical networks, [9], [10], [11], [12], in circuit-switched networks and [13], [16], [18], in cellular networks. In [13], Kelly considered a cellular network (where nodes communicate with the nearest base-station) with a linear topology and compared the fixed and the rearrangement channel assignment algorithm. The result was derived in the limit of the length of the line network tending to infinity. We consider a similar limiting argument for the blocking probability analysis of the wireless line network.

The primary contribution of this thesis includes the development of an exact blocking probability analysis for a single channel wireless line network with single hop calls and a good approximate model for the multiple channel case. We then apply the blocking probability formulas derived to consider the effect of transmission power on blocking probability in a line and a grid network. We show that in a line network it is preferable to use larger transmission radius and communicate directly rather than go multihop to reach the destination (assuming all calls are of the same length); while in a more dense grid network it is more desirable to use smaller transmission radius. Since the line and grid network represent a sparse and a dense network respectively, these results have significant implications in the design of such networks. The result also clearly brings forth the significance of the density of the network on blocking probability in a multi-hop environment. Finally, we develop a channel assignment algorithm for single and multi-hop calls in a wireless network. Our algorithm significantly outperform the naive random algorithm. More importantly, our results show that the effect of the channel assignment algorithm is more significant when a) the network has a dense topology (e.g., grid) and b) the call length is large. This observation is somewhat intuitive as when the network is dense, efficient packing of the channels becomes critical.

The rest of the thesis is organized as follows. In Chapter 2, we describe the wireless interference and the traffic model. Chapter 3 presents blocking probability analysis for a line network including an exact analysis for the single channel case and an approximation in the multiple channel case. Chapter 4 considers the effect of transmission radius on blocking probability in a line and a grid network. In Chapter 5, we present channel assignment algorithms for multihop calls in a general topology wireless network and simulation results that compare their performance. Finally, Chapter 6 concludes the work and provides future research directions.

Chapter 2

Network Model

2.1 Wireless Interference Model

A wireless network is modeled as a two-dimensional graph $G \equiv (N, A)$ where N is the set of nodes and A is the set of wireless links which are assumed to be bi-directional. We say that a wireless link exists between any two nodes if they are within direct transmission range of each other. All nodes in the network have an omnidirectional antenna and are assumed to transmit with the same constant power. We investigate a static wireless network whose topology does not change with time. We refer to the shared resource in the network to service a call as a channel. For example, in a time division multiaccess (TDMA) network, a channel is a time slot within the time-division multiaccess frame and for a frequency division multiaccess (FDMA) network a channel is a distinct frequency band. Any transmission or reception is assumed to completely utilize the channel.

To understand the call service mechanism, consider a time division multi-access (TDMA) system. Time is divided into frames which are further sub-divided into slots (called time slots). The total number of time slots in a frame is the total channel resources available in the network. The data transfer between any two adjacent nodes (nodes that are within direct transmission range), takes place in the time slots within a frame. Since we consider only connection-oriented traffic (explained in Section 2.2), a time slot must be reserved a priori to allow the data transfer without any interference from the neighboring nodes. Once a time slot is reserved, the call is serviced in that time slot over multiple frames for the entire duration of the call. In case of multi-hop calls, such a mechanism is repeated over all the hops along the entire length of the path. However, neighboring links cannot share the same time slot as interference imposes certain constraints on the simultaneous use of a channel. A frequency division multi-access (FDMA) system is identical to a TDMA system with the time slots being replaced by distinct frequency bands.

We assume a disk model of interference and define the transmission radius of a node (say T) as the radius of a circle (called the transmission circle) outside which the interference due to node T is negligible. Within the transmission radius of node T, we assume complete interference of the signal transmitted by node T with other ongoing transmissions. For any two nodes T and R, we say that node R is a neighbor of node T if R lies within the transmission radius of T. Since the links are assumed to be bi-directional, T is also a neighbor of R. Let the set consisting of neighbors of node T and node R be denoted as $\mathcal{N}_T$ and $\mathcal{N}_R$ respectively. To explain the interference model, we consider the following example of a data transfer from node T to node R ($T \rightarrow R$) in channel γ. We refer to node T, as the transmitting node and R, as the receiving node. For call $T \rightarrow R$ to be successful, the following criteria needs to be satisfied.

1. Nodes T and R must **not** be involved in any other call **transmission/reception** in channel γ. This criterion ensures that a node cannot simultaneously serve two calls in the same channel.

2. Neighbors of the transmitting node ($P \in \mathcal{N}_T$) must **not receive** any other data in channel γ. Otherwise the transmission from node T will result in interference and data loss at node P. However, note that a node $P \in \mathcal{N}_T$ can transmit in channel γ if this does not violate Condition 3.

3. Neighbors of the receiving node ($Q \in \mathcal{N}_R$) must **not transmit** any other data

in channel γ. Otherwise the transmission from node Q will result in the loss of data received at node R. However, note that a node $Q \in \mathcal{N}_R$ can receive in channel γ if this does not violate Condition 2 (This happens for nodes that are neighbors of both T and R).

The above "idealized" model approximates realistic interference assumptions and is commonly used in the study of wireless networks [3], [19], [20]. We consider two types of calls in this work; one involving a uni-directional data transfer and the other a bi-directional data transfer. The next two sections describe each type of call in detail and present examples that illustrate the simultaneous channel use constraints (referred to as the wireless constraints) in either case.

2.1.1 Uni-directional Calls

A uni-directional call from the source node S, to the destination node D, involves data transfer in one direction from node S to node D (S $\rightarrow$ D). Each call is assumed to require a single channel for service on each hop. For a multihop call, a channel must be reserved on every link along the entire length of the path. The wireless constraints on a single link are explained in the example below (Figure 2-1). Let the path of a multi-hop call be $\{S, N_1, .., N_k, D\}$ where, S is the source node, D is the destination node and $N_1, .., N_k$ are the intermediate nodes. The data transfer takes place as $S \rightarrow N_1 \rightarrow N_2 .. \rightarrow D$. Let $\gamma_1, \gamma_2, .., \gamma_{k+1}$ be the channels selected on the respective links along the path from S to D. The channel assignment is feasible, if for every link i the single hop wireless constraints (Figure 2-1) are satisfied in the chosen channel γ_i for that link.

Figure 2-1 example : Consider a single hop data transmission from node $Z \rightarrow Y$ in some channel γ. Nodes Z and Y **cannot** participate in any other **transmission/reception** in channel γ. Nodes A, B are neighbors of Z and they **cannot receive** any transmission from their neighbors in channel γ. Note that, node A can transmit to node P which is its neighbor but not a neighbor of Z. Neighbors of node

Y are C, D and they **cannot transmit** in channel γ. Note again that node C can receive from node Q which is its neighbor but not a neighbor of Y. In Figure 2-1, the set of interfering data transfers are marked 'X'.

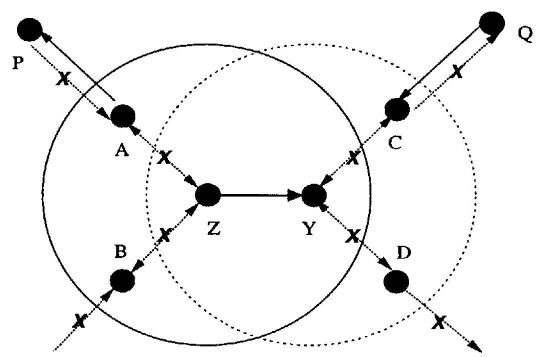

Figure 2-1: Interference model for uni-directional data transfer ($Z \rightarrow Y$ in the figure).

2.1.2 Bi-directional Calls

A call between any two nodes S, D is defined as bi-directional if there is data transfer in both directions ($S \rightarrow D$ and $D \rightarrow S$). We continue to make the assumption that each call requires a single channel for service on every link along the entire length of the path. The actual mechanism by which the data transfer takes place, in either direction on each link, using a single channel is immaterial. One possible way would be to further subdivide the channel reserved on each link. However, we ignore this detail by assuming a bi-directional data transfer on a single "super" channel. The wireless constraints for a single hop bi-directional call are explained in the example below (Figure 2-2). For a multihop call, these constraints must be satisfied at every link on the entire path. Let the path of the multi-hop call be $\{S, N_1, .., N_k, D\}$ along links $S \leftrightarrow N_1$, $N_1 \leftrightarrow N_2$,..., $N_k \leftrightarrow D$. Let $\gamma_1, \gamma_2, .., \gamma_{k+1}$ be the channels selected on the respective links along the path from S to D. The channel assignment is feasible, if for every link i the single hop wireless constraints (Figure 2-2), are satisfied in the chosen channel γ_i on that link.

Figure 2-2 example : Consider a single hop bi-directional data transfer between nodes A and B in channel γ. Since both nodes A and B transmit and receive data (in channel γ) during the duration of the call, all the three conditions as stated earlier apply to both the nodes. It follows from Conditions 2 and 3 that neighbors of node A cannot service any other bi-directional call. A similar condition holds for the neighbors of node B. Let a node be labelled inactive (in γ) if it is not involved in transmission on channel γ and active (in γ) otherwise. With this notation, we get a more simplified single hop simultaneous channel use constraint as follows. For the bi-directional call $A \leftrightarrow B$ to be successful (in channel γ), neighbors of node A (excluding B) and neighbors of node B (excluding A) must be inactive. As shown in Figure 2-2, nodes A_1, A_2 are neighbors of A and they cannot service any other call in channel γ (while call $A \leftrightarrow B$ is active). Neighbors of node B are B_1, B_2 and they cannot service any other call in channel γ. In the figure, the set of interfering data transfers are marked 'X'.

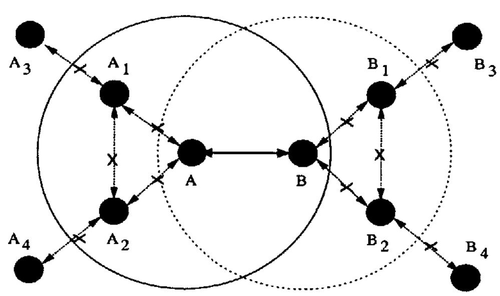

Figure 2-2: Interference model for bi-directional data transfer ($A \leftrightarrow B$ in the figure).

2.2 Traffic Model

All calls in the network are assumed to be connection-oriented. A connection-oriented call requires dedicated resources for service during the entire duration of the call. These resources are held up while the call is in progress and simultaneously released at the end of the call. We assume that all calls require a single channel for service on each link. The call arrival and departure process is described next. Let C_{ij} represent the call arriving at node i and destined for node j. Let $X_{ij}(t)$ denote the arrival process of this call and Z_{ij} denote the time this call remains in service. The arrival process $X_{ij}(t)$ is independent of all the other call arrival processes and is assumed to be Poisson with rate λ_{ij}. The service time Z_{ij} is independent of the arrival times and service periods of other calls and is distributed according to an Exponential distribution with mean $1/\mu_{ij}$. In this work, we assume a uniform traffic model and set all the arrival rates $\lambda_{ij} = \lambda$ and the service rates $\mu_{ij} = \mu$.

We consider bi-directional and uni-directional calls which differ in the set of wireless constraints on the successful service of a call. To keep the analysis tractable, we consider networks in which calls are either all bi-directional or all uni-directional.

Chapter 3

Line Network

3.1 Introduction

In wireless networks, dynamic channel assignment algorithms can be compared using various performance metrics. As noted earlier, the performance metric in this work is the probability that in steady state an arriving call is blocked. To compute this blocking probability, we construct a stochastic model of the system and analyze its steady state behavior. However, analyzing a general network using the stochastic model is very difficult. Therefore, we consider simpler networks (line network and grid network) with symmetrical loads. These networks help us understand the behavior of blocking probability under different network parameters and the conclusions drawn here can be applied to more general networks. In this chapter, we analyze a line network where the nodes are located unit distance apart from each other, the transmission radius of each node is unity and all the links have uniform load.

We first consider the simplest non-trivial case of single hop calls and a single channel available in the network. By considering the limiting behavior (length of the line network $\to \infty$), we obtain an elegant formula that computes the exact blocking probability in the single channel case. We then present a simplified approximate model for computing the blocking probability in the multiple channel case for the random channel allocation policy. This policy is explained in detail in the later sections. The next chapter builds upon this work and extends it to a more general line network. The insights gained and the formulas derived in this chapter are used in the subsequent chapter to study the effect of transmission radius on blocking probability in a line and a grid network.

The rest of the chapter is organized as follows. In Section 3.2 we consider a line network with a single channel. We first analyze the blocking probability in a line network with all single hop bi-directional calls (Section 3.2.1) followed by the analysis for single hop uni-directional calls (Section 3.2.2). Section 3.3 presents a simplified model for analyzing blocking probability in the multiple channel case for the random channel assignment policy. Using the simplified model, we derive blocking probability formulas that predict very well the values obtained from simulation results.

3.2 Single Channel Wireless Line Network

Consider a Wireless Line Network with nodes located at positions $x = -m$, $-m+1$,...., $m-1$, m. We label these nodes as X_{-m}, X_{-m+1},..., X_{m-1}, X_m with each node having a transmission radius of unity as shown in Figure 3-1. Since the transmission range of every node is unity, each node can communicate directly with a node on its left and a node on its right.

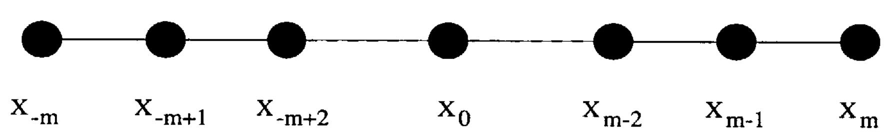

Figure 3-1: A Line Network.

The reason behind considering a line network with even number of links ($2m$) is to simplify the blocking probability analysis when we later consider the limiting behavior (with the length of the line tending to infinity). This simplification does not in any way affect the limiting results. The reason for considering an infinite line network is that the edge effects can be eliminated and each node then has an identical environment. This simplifies the blocking probability analysis and gives elegant and useful results that have applicability for finite length line networks. Before proceeding with the analysis, we first present an observation that plays a central role in the subsequent proofs.

***Observation**: If $\mathcal{X}$ and $\mathcal{Y}$ are disjoint discrete sets and $f(x)$ and $g(y)$ are any two functions defined on $\mathcal{X}$ and $\mathcal{Y}$, then*

$$\sum_{(x,y)\in\mathcal{X}X\mathcal{Y}} f(x)g(y) = (\sum_{x\in\mathcal{X}} f(x))(\sum_{y\in\mathcal{Y}} g(y)) \tag{3.1}$$

Equation 3.1 can be trivially proved as follows. Let the set $\mathcal{X}$ be $x_1, x_2, ..., x_k$ and the set $\mathcal{Y}$ be $y_1, y_2, ..., y_l$ then,

$$(\sum_{x\in\mathcal{X}} f(x))(\sum_{y\in\mathcal{Y}} g(y)) = (f(x_1) + ... + f(x_k))(g(y_1) + ... + g(y_l)) \tag{3.2}$$

Expanding the above expression, it can be shown to equal the left hand side of Equation 3.1.

We begin by considering single hop calls and a single channel available in the network. Single hop calls are between nodes that are within direct transmission range of each other. The line network either has all bi-directional calls or all uni-directional calls. In the subsections that follow, we treat each case separately.

3.2.1 Bi-directional Calls

A wireless link (or simply link) is said to exist between any two nodes if they are within direct transmission range of each other. For a node X_k in the line network, there is a link between nodes (X_{k-1}, X_k) and between nodes (X_k, X_{k+1}). We label the links in an increasing order with link (X_{-m}, X_{-m+1}) labelled L_{-m}, link (X_{-m+1}, X_{-m+2}) labelled L_{-m+1}, .., link (X_{m-1}, X_m) labelled L_{m-1}. Thus, there are total $2m$ links (L_{-m}, L_{-m+1}, .., L_{m-1}) in the network. A link is said to be active if there is a call in service on that link.

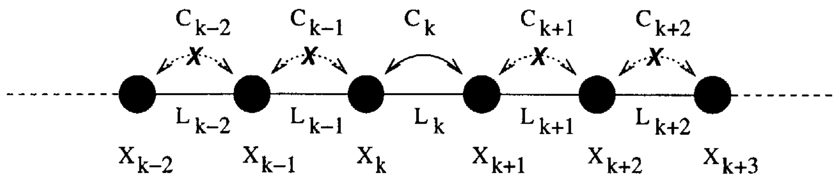

Figure 3-2: Constraints on the simultaneous service of adjacent bi-directional calls.

Let C_k denote a call between nodes X_k and X_{k+1}. $C_k = 0$, if the call is inactive and $C_k = 1$, if the call is active. Following the constraints as noted in Section 2.1.2, call C_k can be successfully serviced in the single available channel if node X_{k-1} (neighbor of X_k) and node X_{k+2} (neighbor of X_{k+1}) are inactive. This implies that calls C_{k-2}, C_{k-1}, C_{k+1}, C_{k+2} must be inactive (Figure 3-2). We refer to these constraints as the wireless constraints.

- $C_{k-1} = 0$, (Node X_k cannot service any other call).
- $C_{k+1} = 0$, (Node X_{k+1} cannot service any other call).
- $C_{k-2} = 0$, (Node X_{k-1} cannot service any other call as it is within the transmission range of node X_k).
- $C_{k+2} = 0$, (Node X_{k+2} cannot service any other call as it is within the transmission range of node X_{k+1}).

The traffic model follows from Section 2.2. Calls (C_k) arrive according to an independent Poisson process of rate λ. The call holding period of all calls is independent of earlier arrival times and holding periods of other calls and identically distributed according to an Exponential distribution with mean $1/\mu$. There is no admission control and no buffering of calls in the network. If a call cannot be accepted then it is dropped. Otherwise it holds the channel for the entire duration of the call. We refer to this network as WLN-1.

Theorem 1. *The blocking probability of a call in a WLN-1 line network with the length of the line network tending to infinity and finite $\nu = \lambda/\mu$ is,*

$$P_B = 1 - \frac{x^3}{1 + 2\nu x^3} \tag{3.3}$$

where x is the unique root in (0,1] of $\nu x^3 + x = 1$.

Proof : Let $n_k(t)$ be the number of calls C_k in progress at time t. Let $\nu = \lambda/\mu$ and define the vector $\mathbf{n}(t) = \{n_k(t)\}$, $k \in -m, ..., m-1$. State $\mathbf{n}$ is admissible if $\mathbf{n} \geq 0$ and satisfies the wireless constraints as described earlier. Let $\mathcal{G}(m)$ denote the set of all admissible states for a line network with m links (or $m+1$ nodes). Since we consider a line network with $2m$ links, the set of all admissible states is $\mathcal{G}(2m)$. We could express the set $\mathcal{G}(m)$ mathematically but the analysis that follows does not require such an explicit description of the state space. The local constraints on the simultaneous use of a channel at a node suffice for the analysis.

The stochastic process $(\mathbf{n}(t), t \geq 0)$ is an aperiodic, irreducible, finite state Markov process and hence has a unique stationary distribution $\pi(\mathbf{n}) = P(\mathbf{n} = (n_{-m}, .., n_{m-1}))$ given by the following product form solution [7]. The normalization constant in the product form solution is denoted as $S(2m)$, where $2m$ denotes a line network with $2m$ links.

$$\pi(\mathbf{n}) = \frac{1}{S(2m)} \prod_{r=-m}^{m-1} \frac{\nu^{n_r}}{n_r!} \quad , \qquad \mathbf{n} \in \mathcal{G}(2m) \tag{3.4}$$

The normalization constant $S(2m)$ makes $\pi(\mathbf{n})$ a probability distribution and it can be computed by summing $\pi(\mathbf{n})$ over all $\mathbf{n} \in \mathcal{G}(2m)$.

$$S(2m) = \sum_{\mathbf{n} \in \mathcal{G}(2m)} \prod_{r=-m}^{m-1} \frac{\nu^{n_r}}{n_r!} \tag{3.5}$$

Since we are dealing with a single channel network $n_r = 0$ or 1 and $n_r! = 1$. Hence we can drop the term $n_r!$ from Expression 3.4 and re-write as,

$$
\begin{aligned}
\pi(\mathbf{n}) &= \frac{1}{S(2m)} \prod_{r=-m}^{m-1} \nu^{n_r} \quad , \qquad \mathbf{n} \in \mathcal{G}(2m) & (3.6) \\
S(2m) &= \sum_{\mathbf{n}\in\mathcal{G}(2m)} \prod_{r=-m}^{m-1} \nu^{n_r} & (3.7) \\
&= \sum_{\mathbf{n}\in\mathcal{G}(2m)} \nu^{n_{-m}+..+n_{m-1}} & (3.8)
\end{aligned}
$$

Let $n_{total} = n_{-m} + .. + n_{m-1}$, then,

$$
\begin{aligned}
\pi(\mathbf{n}) &= \frac{1}{S(2m)} \nu^{n_{total}}, \qquad \mathbf{n} \in \mathcal{G}(2m) & (3.9) \\
S(2m) &= \sum_{\mathbf{n}\in\mathcal{G}(2m)} \nu^{n_{total}} & (3.10)
\end{aligned}
$$

Consider the call C_0 of the line network. The non-blocking states for call C_0 are $\{\mathbf{n} : \mathbf{n} \in \mathcal{G}(2m)$ and $n_{-2}, n_{-1}, n_0, n_1, n_2 = 0\}$, assuming $m \geq 2$. Denote this set as $\mathcal{N}B_0$ and let $P^0_{NB,2m}$ be the probability that in steady state call C_0 is not blocked. Then,

$$
\begin{aligned}
P^0_{NB,2m} &= \sum_{\mathbf{n}\in\mathcal{N}B_0} \pi(\mathbf{n}) & (3.11) \\
P^0_{NB,2m} &= \frac{\sum_{\mathbf{n}\in\mathcal{N}B_0} \nu^{n_{total}}}{\sum_{\mathbf{n}\in\mathcal{G}(2m)} \nu^{n_{total}}} & (3.12)
\end{aligned}
$$

The set of non-blocking states for call C_0 must have calls C_{-2}, C_{-1}, C_0, C_1, C_2 inactive. Therefore, to evaluate the numerator in Equation 3.12 we must characterize the feasible states of the remaining calls ($C_{-m}, .., C_{-3}$ and $C_3, .., C_{m-1}$). It turns out that we do not have to explicitly describe this set. Rather, we can exploit the symmetry in the line network to evaluate the numerator. Before proceeding forward we make the following definitions.

Let $\mathcal{G}_L$ be the state space of calls C_{-m},.., C_{-3} and $\mathcal{G}_R$ be the state space of calls C_3,.., C_{m-1}. Then,

$$
\begin{aligned}
\mathcal{G}_L &\equiv \text{state space of WLN-1 with } m-2 \text{ links} \\
&= \mathcal{G}(m-2); \\
\mathcal{G}_R &\equiv \text{state space of WLN-1 with } m-3 \text{ links} \\
&= \mathcal{G}(m-3)
\end{aligned}
$$

Calls $C_{-m},..,C_{-3}$ are not affected by the simultaneous service of calls $C_3,..,C_{m-1}$ which makes the state space $\mathcal{G}_L$ independent of $\mathcal{G}_R$. Therefore, the set $\mathcal{N}B_0$ is the cartesian product of the sets $\mathcal{G}_L$ and $\mathcal{G}_R$ (by independence) and can be written as, $\mathcal{N}B_0 \equiv \{\mathcal{G}_L X \mathcal{G}_R,\ n_{-2}, n_{-1}, n_0, n_1, n_2 = 0\}$. We can now evaluate Expression 3.12 using Equation 3.1. Let, $n_L = n_{-m} + .. + n_{-3}$ and $n_R = n_3 + .. + n_{m-1}$. With this notation and $n_{-2}, n_{-1}, n_0, n_1, n_2 = 0$ we get,

$$
\sum_{\mathcal{N}B_0} \nu^{n_{total}} = \sum_{\mathcal{N}B_0} (\nu^{n_{-m}+..+n_{-3}})(\nu^{n_3+..+n_{m-1}}) \tag{3.13}
$$

$$
= (\sum_{\mathcal{G}_L} \nu^{n_L})(\sum_{\mathcal{G}_R} \nu^{n_R}) \tag{3.14}
$$

$$
P^0_{NB,2m} = \frac{(\sum_{\mathcal{G}_L} \nu^{n_L})(\sum_{\mathcal{G}_R} \nu^{n_R})}{\sum_{\mathbf{n}\in\mathcal{G}(2m)} \nu^{n_{total}}} \tag{3.15}
$$

$$
P^0_{NB,2m} = \frac{(\sum_{\mathcal{G}(m-2)} \nu^{n_L})(\sum_{\mathcal{G}(m-3)} \nu^{n_R})}{\sum_{\mathbf{n}\in\mathcal{G}(2m)} \nu^{n_{total}}} \tag{3.16}
$$

Using our notation for the normalization constant we get,

$$
\begin{aligned}
\sum_{\mathcal{G}(m-2)} \nu^{n_L} &= S(m-2) \\
\sum_{\mathcal{G}(m-3)} \nu^{n_R} &= S(m-3) \\
\sum_{\mathcal{G}(2m)} \nu^{n_{total}} &= S(2m)
\end{aligned}
$$

The expression for the probability of non-blocking of call C_0 can now be expressed in a more simplified way as,

$$P^0_{NB,2m} = \frac{S(m-2)S(m-3)}{S(2m)} \tag{3.17}$$

The size of the state space, $\mathcal{G}(m)$, increases very rapidly with the length of the line. Hence, calculating $S(m)$ by summing over all the feasible states is not practical. However, the symmetry of the line network facilitates an iterative evaluation of $S(m)$. The set of feasible states can be partitioned into a set of states for which the leftmost call is inactive and the set of states for which the leftmost call is active. For the former state space $S(m)$ equals $S(m-1)$. In the latter state space, the wireless constraint forces the next two calls on the right (of the leftmost call) to be inactive and the state of the remaining calls on the line is independent of the leftmost active call. Thus, for the latter state space, $S(m)$ equals $\nu S(m-3)$. Let $S()|\{constraint\}$ represent the evaluation of the function $S()$ under the specified *constraint*.

$$S(m) = S(m)|\{\text{leftmost call inactive}\} + S(m)|\{\text{leftmost call active}\} \tag{3.18}$$

$$S(m) = S(m-1) + \nu S(m-3), \quad m \geq 3 \tag{3.19}$$

$S(k) = 1, k \leq 0$ (*defn*), $S(1) = 1 + \nu$, and $S(2) = 1 + 2\nu$.

Using the above equations we can evaluate $P^0_{NB,2m}$ for a line network with $2m$ links for any m. Following a similar methodology, we can easily generalize the approach and evaluate the probability of non-blocking $P^k_{NB,m}$ of a call C_k, $\forall k$ and for any finite m. It turns out that if we look at the limiting behavior ($\lim_{m \to \infty}$), an elegant formula for the blocking probability of any call in the network is obtained. Simulation results (Table 3.1) have shown that this formula very closely approximates the blocking probability for finite length line networks and is much easier to evaluate than the iterative formula presented earlier. The iterative formula becomes cumbersome to deal with as the length of the line increases. We now proceed to consider the limiting behavior. Re-writing Equation 3.17 and taking limits we get,

$$P^0_{NB,2m} = \frac{\left(\frac{S(m-2)S(m-3)}{S(m-1)S(m-1)}\right)}{\left(\frac{S(2m)}{S(m-1)S(m-1)}\right)} \quad (3.20)$$

$$\lim_{m\to\infty} P^0_{NB,2m} = \lim_{m\to\infty} \frac{\left(\frac{S(m-2)S(m-3)}{S(m-1)S(m-1)}\right)}{\left(\frac{S(2m)}{S(m-1)S(m-1)}\right)} \quad (3.21)$$

To evaluate the denominator in the above equation, we need to evaluate $S(2m)$. This is done by partitioning the state space of WLN-1 into a set of states conditioned on all the possible states of calls (C_{-1}, C_0). We then evaluate $S(2m)$ over each of the partitioned state space and sum them up. There are four cases that need to be considered.

1. C_{-1}, C_0 both inactive. In this case, calls $C_{-m}, .., C_{-2}$ do not interfere with calls $C_1, .., C_{m-1}$. Thus, the state of calls $C_{-m}, .., C_{-2}$ is independent of the state of calls $C_1, .., C_{m-1}$ and we get, (Note that using the earlier notation, feasible state space of $\{C_{-m}, .., C_{-2}\} = \mathcal{G}(m-1)$ and the feasible state space of $\{C_1, .., C_{m-1}\}$ $= \mathcal{G}(m-1)$)

$$\begin{aligned} S(2m) &= \sum_{\mathcal{G}(m-1)} \nu^{n_{-m}+..+n_{-2}} \sum_{\mathcal{G}(m-1)} \nu^{n_1+..+n_{m-1}} \\ S(2m) &= S(m-1)S(m-1) \end{aligned}$$

2. C_{-1} active, C_0 inactive. Since C_{-1} is active, calls C_{-3}, C_{-2}, C_1 must be inactive. This leaves the state of calls $C_{-m}, .., C_{-4}$ independent of the state of the calls $C_2, .., C_{m-1}$. The feasible state space of $\{C_{-m}, .., C_{-4}\} = \mathcal{G}(m-3)$ and the feasible state space of $\{C_2, .., C_{m-1}\} = \mathcal{G}(m-2)$.

$$\begin{aligned} S(2m) &= \left(\sum_{\mathcal{G}(m-3)} \nu^{n_{-m}+..+n_{-4}}\right) \nu \left(\sum_{\mathcal{G}(m-2)} \nu^{n_2+..+n_{m-1}}\right) \\ S(2m) &= \nu S(m-3)S(m-2) \end{aligned}$$

3. C_{-1} inactive, C_0 active. By symmetry (with case 2),

$$S(2m) = \nu S(m-2)S(m-3).$$

4. C_{-1}, C_0 both active. This state is infeasible.

Thus we have,

$$\begin{aligned} S(2m) &= S(2m)|\{C_{-1}, C_0 = 0\} + S(2m)|\{C_{-1} = 1, C_0 = 0\} \\ &\quad + S(2m)|\{C_{-1} = 0, C_0 = 1\} \end{aligned} \tag{3.22}$$

$$S(2m) = S(m-1)S(m-1) + 2\nu S(m-2)S(m-3) \tag{3.23}$$

Plugging Equation 3.23 in Equation 3.21 we get,

$$\lim_{m\to\infty} P^0_{NB,2m} = \lim_{m\to\infty} \frac{\frac{S(m-2)S(m-3)}{S(m-1)S(m-1)}}{1 + 2\nu\frac{S(m-2)S(m-3)}{S(m-1)S(m-1)}} \tag{3.24}$$

When $m \to \infty$ the probability of non-blocking P_{NB} of any call, by symmetry, is equal to the probability of non-blocking of call C_0. We can drop the super script 0 and re-write as (assuming the limit exists which we later show that it does exist),

$$P_{NB} = \lim_{m\to\infty} \frac{\frac{S(m-2)S(m-3)}{S(m-1)S(m-1)}}{1 + 2\nu\frac{S(m-2)S(m-3)}{S(m-1)S(m-1)}} \tag{3.25}$$

To prove the existence of the limit and evaluate its value, we go back and examine Equation 3.19 that evaluates $S(m)$ iteratively.

$$S(m) = S(m-1) + \nu S(m-3) \tag{3.26}$$

$$1 = \frac{S(m-1)}{S(m)} + \nu\frac{S(m-3)}{S(m)} \tag{3.27}$$

$$1 = \lim_{m\to\infty} \frac{S(m-1)}{S(m)} + \nu \lim_{m\to\infty} \frac{S(m-3)}{S(m)} \tag{3.28}$$

Since the left hand side of the above equation is 1, the limits on the right hand side must exist. Let,

$$\lim_{m\to\infty} \frac{S(m-1)}{S(m)} = x$$

Then,

$$\lim_{m\to\infty} \frac{S(m-3)}{S(m)} = \lim_{m\to\infty} \frac{S(m-3)}{S(m-2)} \lim_{m\to\infty} \frac{S(m-2)}{S(m-1)} \lim_{m\to\infty} \frac{S(m-1)}{S(m)}$$

Making change of variables, $(k = m-2, l = m-1)$

$$\begin{aligned} \lim_{m\to\infty} \frac{S(m-3)}{S(m)} &= \lim_{k\to\infty} \frac{S(k-1)}{S(k)} \lim_{l\to\infty} \frac{S(l-1)}{S(l)} \lim_{m\to\infty} \frac{S(m-1)}{S(m)} \\ &= x^3 \end{aligned}$$

In terms of x, Equation 3.28 can be written as,

$$1 = x + \nu x^3 \tag{3.29}$$

We go back and examine what x represents. The normalization constant $S(m)$ is a non-negative monotonically increasing function of m, $\forall\ \nu > 0$ and gives an indication of the size of the state space. With this interpretation, x is the state space expansion factor (in terms of the normalization constant $S(m)$) in the limit $(m \to \infty)$. For all finite $\nu \geq 0$, $S(k) \leq S(k+1)$, $k \geq 1$ which implies that $x \in (0, 1]$. The existence of a root of the cubic Equation 3.29 in $(0, 1]$ for any finite $\nu \geq 0$ can be proved as follows. Re-write Equation 3.29 as,

$$\nu x^2 + 1 = \frac{1}{x}$$

The function $1/x$ is a positive decreasing function and takes values between $[1, \infty)$ in the interval $x \in (0, 1]$. Function, $\nu x^2 + 1$ is a positive non-decreasing function taking values between $(1, 1+\nu]$ in $x \in (0, 1]$. Since we assumed that $\nu \geq 0$, the two curves must intersect at a unique point in $(0, 1]$. This is illustrated in Figure 3-3.

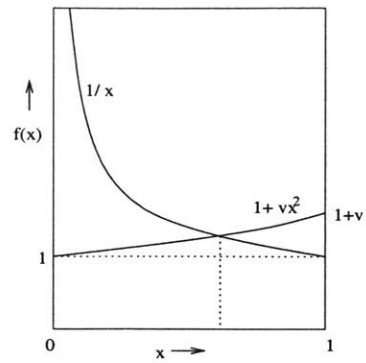

Figure 3-3: Plot indicating the intersection point between $1/x$ and $1 + \nu x^2$.

P_{NB} can now be evaluated in terms of x as,

$$P_{NB} = \lim_{m\to\infty} \frac{\frac{S(m-2)S(m-3)}{S(m-1)S(m-1)}}{1 + 2\nu \frac{S(m-2)S(m-3)}{S(m-1)S(m-1)}} = \frac{x^3}{1 + 2\nu x^3}$$

Finally, the probability of blocking $P_B = 1 - P_{NB}$, is,

$$P_B = 1 - \frac{x^3}{1 + 2\nu x^3}, \qquad \nu x^3 + x = 1 \tag{3.30}$$

This completes the proof of Theorem 1.

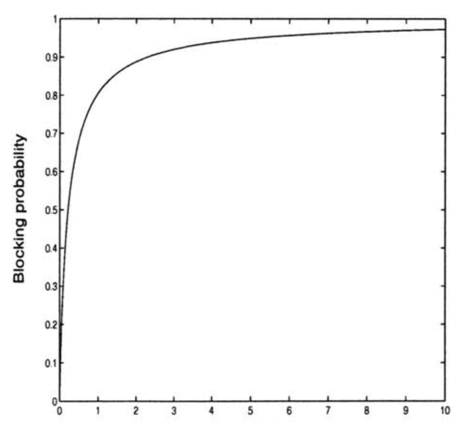

Figure 3-4: Blocking probability plot for bi-directional calls (plot of $P_B = 1 - \frac{x^3}{1+2\nu x^3}$).

Figure 3-4 is a plot of P_B (Equation 3.30) for various values of the load ν. The value of x is computed by finding a root in (0,1] of $\nu x^3 + x = 1$. Being a cubic polynomial, the root can be evaluated very easily either analytically or using numerical methods.

Table 3.1 compares the blocking probability values computed theoretically using Expression 3.30 with the values obtained from simulations of a line network with 10 nodes. In the simulations, blocking probability of the center call is computed as the edge effects are minimal for this call. As seen from the table, the blocking probability formula (Equation 3.30) accurately predicts the values even for small length line networks.

ν	Length = 10 nodes	
	Theoretical Blocking prob.	Simulated Blocking prob.
0.00005	0.000249	0.000244
0.00040	0.001995	0.001987
0.00160	0.007929	0.007933
0.01280	0.059734	0.059813
0.10240	0.328020	0.331024
0.81920	0.775250	0.780030

Table 3.1: Comparison of theoretically computed and simulated blocking probability values for finite length line network and bi-directional calls.

To understand Equation 3.30 in more detail, we compare it with the standard M/M/1/1 blocking probability expression. The steady state blocking probability in a M/M/1/1 system with load ν' is given by,

$$P_B = \frac{\nu'}{1 + \nu'} \tag{3.31}$$

We make the comparison by calculating an equivalent load ν' in the M/M/1/1 blocking probability expression (Eqn 3.31) that has the same blocking as that obtained from Equation 3.30 for load ν. The significance of the effective load is that, if we isolate a particular link of the line network then load ν' on this isolated link would have the same blocking probability as experienced by the link within the line network (with symmetrical load ν). To calculate ν', we equate Equation 3.30 and 3.31.

$$\begin{aligned}\frac{\nu'}{1+\nu'} &= 1 - \frac{x^3}{1+2\nu x^3} \\ \nu' &= \frac{1+(2\nu-1)x^3}{x^3}\end{aligned}$$

Define factor g as, $g = \nu'/\nu$, then g can be expressed as,

$$g = \frac{1+(2\nu-1)x^3}{\nu x^3} \tag{3.32}$$

A plot of g for different values of load ν is presented in Figure 3-5. Evaluating the limits in Equation 3.32, we get $\lim_{\nu\to 0} g = 5$ and $\lim_{\nu\to\infty} g = 3$. The understanding behind these values of g is that at light load ($\lim_{\nu\to 0}$) each arriving call contributes an equivalent load of 5 calls; while at high loads ($\lim_{\nu\to\infty}$) each arriving call contributes an equivalent load of 3 calls. These values can be explained in detail as follows.

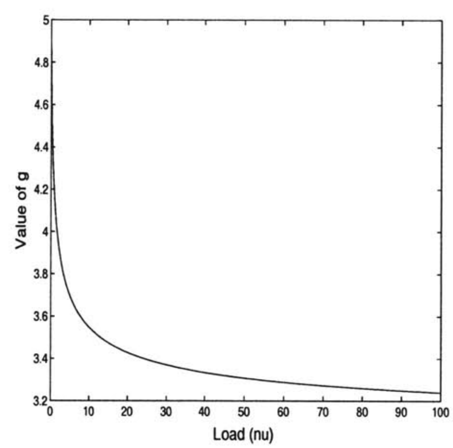

Figure 3-5: Plot of $g = \nu'/\nu$ for bi-directional calls.

$\boldsymbol{\nu \longrightarrow 0}$: Each single hop call, C_k, in WLN-1 has four other neighboring interfering calls (C_{k-2}, C_{k-1}, C_{k+1}, C_{k+2}). To serve any of these 5 calls, a channel on link L_k must be free. In the low load case almost all the calls get served and the chances of more than one call being active from among the 5 calls, $C_{k-2}, .., C_{k+2}$, is negligible. Thus, the total rate seen by link L_k is five times the arrival rate and the value of g

as $\nu \longrightarrow 0$ is 5.

$\nu \longrightarrow \infty$: At very high loads, the highly probable states of the network are the maximally packed states [15]. In WLN-1 network, the maximally packed states have an active call every two hops apart. The largest set of mutually interfering calls is three and at very high loads (due to maximal packing), these calls mutually block the channels. Thus, the total rate seen by link L_k is three times the arrival rate and the value of g as $\nu \longrightarrow \infty$ is 3.

3.2.2 Uni-directional Calls

We extend the blocking probability analysis of Section 3.2.1 to the case of unidirectional calls in a line network. A unidirectional call from node X_k to node X_{k+1} involves data transfer only in one direction ($X_k \longrightarrow X_{k+1}$). Therefore, in this case, calls $X_k \longrightarrow X_{k+1}$ and $X_{k+1} \longrightarrow X_k$ are distinct. This adds more complexity to the set of interfering calls and manifests itself in a more tedious blocking probability analysis. However, interestingly, an exact expression can be obtained even in this case in the limit of the length of the line network tending to infinity.

The line network is identical to that considered in Section 3.2.1. The wireless interference model follows from Section 2.1. The notation for the calls is as follows. We label the call from node $X_l \longrightarrow X_{l+1}$ as C_{2l} and the call from node $X_{l+1} \longrightarrow X_l$ as C_{2l+1}. Thus, the set of distinct calls in the network are $C_{-2m}, C_{-2m+1}, .., C_{2m-2}, C_{2m-1}$. Calls arrive according to independent Poisson processes each of rate λ. There is a single channel available in the network. If the arriving call cannot be accommodated for service then it is lost and does not reattempt service request. Otherwise the call is connected and holds the channel for the holding period of the call. The call holding period of all calls is independent of earlier arrival times and holding periods of other calls and Exponentially distributed with mean $1/\mu$.

Consider a particular call C_k (k, even). This call is from node $X_{k/2} \longrightarrow X_{k/2+1}$. The set of local constraints for servicing this call are as follows.

- Since node $X_{k/2}$ is only transmitting for the entire duration of call C_k, neighbors ($X_{k/2-1}$ and $X_{k/2+1}$) of this node including itself cannot receive any other call. This implies that calls $C_{k-4}, C_{k-2}, C_{k-1}, C_{k+1}, C_{k+3}$ must be inactive.

- Node $X_{k/2+1}$ is the receiver for call C_k. Therefore neighbors ($X_{k/2}$ and $X_{k/2+2}$) of this node including itself cannot transmit any other call while call C_k is in progress. This constrains the calls $C_{k-1}, C_{k+1}, C_{k+2}, C_{k+3}, C_{k+4}$ to be inactive.

Combining all the constraints, we find that for call C_k to be successfully serviced in the available channel, calls $C_{k-4}, C_{k-2}, C_{k-1}, C_{k+1}, C_{k+2}, C_{k+3}, C_{k+4}$ must be inactive. Figure 3-6 shows these constraints for call C_k, k being even. Calls marked 'X' must be inactive for call C_k to be successfully serviced . A similar set of conditions can be obtained for the case when k is odd. We refer to this network as WLN-1(uni).

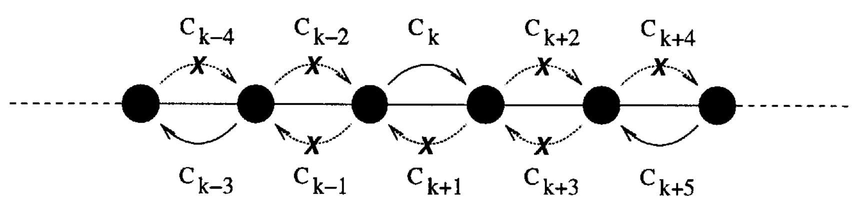

Figure 3-6: Constraints on the simultaneous service of adjacent uni-directional calls.

Theorem 2. *The blocking probability of a call in a WLN-1(uni) line network with the length of the line network tending to infinity and finite* $\nu = \lambda/\mu$ *is,*

$$P_B = 1 - \frac{xy}{\nu^2(1-x)^2 + 4\nu xy} \tag{3.33}$$

x and y satisfy the following relations,

$$\begin{aligned} x(1-x)^2 + 4x^2 &= \nu(1-x)^2 \\ y^2 &= \nu x \end{aligned}$$

and x is the unique root in [0,1) of $x(1-x)^2 + 4x^2 = \nu(1-x)^2$.

Proof: Let $n_k(t)$ be the number of calls, C_k, in progress at time t. Let $\nu = \lambda/\mu$ and define the vector $\mathbf{n}(t) = (n_k(t), k \in -2m, ..., 2m-1)$. State $\mathbf{n}$ is admissible if $\mathbf{n} \geq 0$ and satisfies the constraints as described earlier. Let $\mathcal{H}(m)$ denote the set of all admissible states for a line network with m links (or $m+1$ nodes). Since we consider a line network with $2m$ links (nodes $-m, .., m$), the set of all admissible states is $\mathcal{H}(2m)$. As in the bidirectional case, an explicit description of the admissible state space is not required. The local constraints on the simultaneous use of a channel suffice for the analysis. The stochastic process $(\mathbf{n}(t), t \geq 0)$ is an aperiodic, recurrent Markov process and hence has a unique product form stationary distribution $\pi(\mathbf{n}) = P(\mathbf{n} = (n_{-2m}, .., n_{2m-1}))$. The normalization constant in the product form expression is denoted as $N(2m)$.

$$\pi(\mathbf{n}) = \frac{1}{N(2m)} \prod_{r=-2m}^{2m-1} \frac{\nu^{n_r}}{n_r!}, \qquad \mathbf{n} \in \mathcal{H}(2m)$$

The normalization constant $N(2m)$ makes $\pi(\mathbf{n})$ a probability distribution and it can be computed by summing $\pi(\mathbf{n})$ over all $\mathbf{n} \in \mathcal{H}(2m)$. For a single channel network $n_r = 0$ or 1 and $n_r! = 1$ and we get,

$$\pi(\mathbf{n}) = \frac{1}{N(2m)} \prod_{r=-2m}^{2m-1} \nu^{n_r}, \qquad \mathbf{n} \in \mathcal{H}(2m) \tag{3.34}$$

$$N(2m) = \sum_{\mathbf{n}\in\mathcal{H}(2m)} \prod_{r=-2m}^{2m-1} \nu^{n_r} \tag{3.35}$$

$$= \sum_{\mathbf{n}\in\mathcal{H}(2m)} \nu^{n_{-2m}+..+n_{2m-1}} \tag{3.36}$$

Let $n_{total} = n_{-2m} + .. + n_{2m-1}$, then,

$$\pi(\mathbf{n}) = \frac{1}{N(2m)} \nu^{n_{total}}, \qquad \mathbf{n} \in \mathcal{H}(2m) \tag{3.37}$$

$$N(2m) = \sum_{\mathbf{n}\in\mathcal{H}(2m)} \nu^{n_{total}} \tag{3.38}$$

Following Section 3.2.1 we focus on call C_0 of the line network. The non-blocking states for call C_0 are $\{\mathbf{n} : \mathbf{n} \in \mathcal{H}(2m),\ n_{-4},\ n_{-2},\ n_{-1},\ n_0,\ n_1,\ n_2,\ n_3,\ n_4 = 0\}$. Denote this state space as $\mathcal{N}B_0$. The probability that call C_0 is not blocked is,

$$P^0_{NB} = \sum_{\mathbf{n}\in\mathcal{N}B_0} \pi(\mathbf{n}) \tag{3.39}$$

$$P^0_{NB} = \frac{\sum_{\mathbf{n}\in\mathcal{N}B_0} \nu^{n_{total}}}{\sum_{\mathbf{n}\in\mathcal{H}(2m)} \nu^{n_{total}}} \tag{3.40}$$

To evaluate the numerator in Equation 3.40, we must characterize the set of non-blocking states. It turns out that we do not have to explicitly describe the set $\mathcal{N}B_0$. Rather, as done in the bidirectional case, we can exploit the symmetry in the line network to evaluate the numerator.

Let C_L represent the set of calls, $C_{-2m}, C_{-2m+1}, .., C_{-5}, C_{-3}$ (call C_{-4} is excluded as it is inactive in the set $\mathcal{N}B_0$) and C_R represent the set of calls, $C_5, C_6, .., C_{2m-1}$. The set $\mathcal{N}B_0$ consists of all the feasible states of calls C_L, C_R with $\{C_{-4},\ C_{-2},\ C_{-1},\ C_0,\ C_1,\ C_2,\ C_3,\ C_4\} = 0$. Nodes for the calls C_L are not within direct transmission range of the nodes for C_R. Therefore, the calls in C_L are not affected by the simultaneous service of calls in C_R which makes the state $\mathbf{n_L} = \{n_{-2m}, .., n_{-5}, n_{-3}\}$ independent of the state $\mathbf{n_R} = \{n_5, .., n_{2m-1}\}$ and we can apply Equation 3.1. Let,

$$\begin{aligned}
n_L &= n_{-2m} + .. + n_{-5} + n_{-3} \\
\mathcal{H}_L &\equiv \text{feasible state space of calls } C_L \\
n_R &= n_5 + .. + n_{2m-1} \\
\mathcal{H}_R &\equiv \text{feasible state space of calls } C_R
\end{aligned}$$

We can now evaluate Expression 3.40 using the above notation and Equation 3.1.

$$\sum_{\mathcal{N}B_0} \nu^{n_{total}} = \sum_{\mathcal{H}_L X \mathcal{H}_R} (\nu^{n_{-2m}+..+n_{-5}+n_{-3}})(\nu^{n_5+..+n_{2m-1}}) \tag{3.41}$$

$$= (\sum_{\mathcal{H}_L} \nu^{n_L})(\sum_{\mathcal{H}_R} \nu^{n_R}) \tag{3.42}$$

$$P_{NB}^{0} = \frac{(\sum_{\mathcal{H}_L} \nu^{n_L})(\sum_{\mathcal{H}_R} \nu^{n_R})}{\sum_{\mathbf{n}\in\mathcal{H}(2m)} \nu^{n_{total}}} \tag{3.43}$$

To make the evaluation of the above expression easier, we take the limit, $\lim_{m\to\infty}$. The limiting condition eliminates the edge effects of finite length line networks and leads to an elegant formula for the blocking probability of a call. Simulation results (Table 3.2) have shown that this formula very closely approximates the actual blocking probability for finite length line networks. Thus, the result obtained here is not restrictive. When $m \to \infty$ the probability of non-blocking, P_{NB}, of any call is by symmetry equal to the probability of non-blocking of call C_0 and we can drop the super script 0. Re-writing Equation 3.43 and taking the limit, $\lim_{m\to\infty}$, we get,

$$\lim_{m\to\infty} P_{NB}^{0} = \lim_{m\to\infty} \frac{(\frac{\sum_{\mathcal{H}_L} \nu^{n_L}}{N(m-1)})(\frac{\sum_{\mathcal{H}_R} \nu^{n_R}}{N(m-1)})}{\frac{\sum_{\mathbf{n}\in\mathcal{H}(2m)} \nu^{n_{total}}}{N(m-1)N(m-1)}} \tag{3.44}$$

$$P_{NB} = \frac{\lim_{m\to\infty}(\frac{\sum_{\mathcal{H}_L} \nu^{n_L}}{N(m-1)}) \lim_{m\to\infty}(\frac{\sum_{\mathcal{H}_R} \nu^{n_R}}{N(m-1)})}{\lim_{m\to\infty} \frac{N(2m)}{N(m-1)N(m-1)}} \tag{3.45}$$

To evaluate the limits in the above expression, we use the conditioning argument and the symmetry in the line network as used in the case of bidirectional calls.

$$\sum_{\mathcal{H}_R} \nu^{n_R} = \sum_{\mathcal{H}_R, n_5=0} \nu^{n_R} \ \{n_5 \text{ inactive}\} + \sum_{\mathcal{H}_R, n_5=1} \nu^{n_R} \ \{n_5 \text{ active}\}$$

when $n_5 = 0$ (inactive), the feasible state space of calls $\{C_6, .., C_{2m-1}\} \equiv \mathcal{H}(m-3)$. Thus we get,

$$\sum_{\mathcal{H}_R, n_5=0} \nu^{n_R} = \sum_{\mathcal{H}(m-3)} \nu^{n_6+\ldots+n_{2m-1}} = N(m-3)$$

when $n_5 = 1$, we have $n_6, n_7, n_8 = 0$ and conditioning on n_9 we get,

$$\sum_{\mathcal{H}_R, n_5=1, n_9=0} \nu^{n_R} = \nu N(m-5) \ \{n_9 \text{ inactive}\} + \sum_{\mathcal{H}_R, n_5=1, n_9=1} \nu^{n_R} \ \{n_9 \text{ active}\}$$

Proceeding this way,

$$\sum_{\mathcal{H}_R} \nu^{n_R} = N(m-3) + \nu N(m-5) + \nu^2 N(m-7) + .. + \nu^{\frac{(m-3)}{2}+1}, \quad m \text{ odd}$$
$$\sum_{\mathcal{H}_R} \nu^{n_R} = N(m-3) + \nu N(m-5) + \nu^2 N(m-7) + .. + \nu^{\frac{(m-4)}{2}+1}, \quad m \text{ even}$$

Taking the limit ($\lim_{m\to\infty}$) we get,

$$\lim_{m\to\infty} \frac{\sum_{\mathcal{H}_R} \nu^{n_R}}{N(m-1)} = \lim_{m\to\infty} \frac{N(m-3)}{N(m-1)} + \nu \lim_{m\to\infty} \frac{N(m-5)}{N(m-1)} + \nu^2 \lim_{m\to\infty} \frac{N(m-7)}{N(m-3)} + ... \tag{3.46}$$

Define the following limits (the existence of these limits is shown later). We choose this definition of the limit as it simplifies the evaluation of the blocking probability expression.

$$\begin{aligned} \lim_{m\to\infty} \frac{\nu N(m)}{N(m+2)} &= x \\ \lim_{m\to\infty} \frac{\nu N(m)}{N(m+1)} &= y \\ y^2 &= \nu x \end{aligned}$$

In terms of x and y, we can rewrite expression 3.46 as,

$$\lim_{m\to\infty} \frac{\sum_{\mathcal{H}_R} \nu^{n_R}}{N(m-1)} = \frac{1}{\nu}(x + x^2 + x^3 + x^4...) \tag{3.47}$$
$$= \frac{x}{\nu(1-x)} \tag{3.48}$$

Following a similar reasoning, we can evaluate $\lim_{m\to\infty} (\sum_{H_L} \nu^{n_L}/N(m-1))$ and the denominator $\lim_{m\to\infty}(N(2m)/N(m-1)N(m-1))$ in Equation 3.45.

$$\lim_{m\to\infty} \frac{\sum_{H_L} \nu^{n_L}}{N(m-1)} = \lim_{m\to\infty} \frac{N(m-2)}{N(m-1)} + \lim_{m\to\infty} \nu \frac{N(m-4)}{N(m-1)} + ... \tag{3.49}$$
$$= \frac{y}{\nu}(1 + x + x^2 + x^3 + ...) \tag{3.50}$$
$$= \frac{y}{\nu(1-x)} \tag{3.51}$$

To evaluate $N(2m)$, we partition the state space $\mathcal{H}(2m)$ into a set for which $\{C_{-2}, C_{-1}, C_0, C_1\} = 0$ (inactive) and a set for which atleast one of the call amongst $\{C_{-2}, C_{-1}, C_0, C_1\}$ is active. For the former state space $N(2m) = N(m-1)N(m-1)$ and for the latter state space we make a conditioning argument identical to that made earlier, to obtain the second term in the following equation.

$$
\begin{aligned}
N(2m) &= N(m-1)^2 + \\
&\quad 4\nu(N(m-3) + \nu N(m-5) + ..)(N(m-2) + \\
&\quad \nu N(m-4) + ..) \qquad (3.52)\\
\lim_{m\to\infty} \frac{N(2m)}{N(m-1)^2} &= 1 + \frac{4}{\nu}(x + x^2 + ..)(y + yx + yx^2 + ..) \qquad (3.53)\\
&= 1 + \frac{4xy}{\nu(1-x)^2} \qquad (3.54)
\end{aligned}
$$

Using the above results, we can evaluate P_B in terms of x and y. However to complete the proof, we need to prove the existence of the limit x and evaluate its value (Note that y can be calculated from x using $y^2 = \nu x$). Using the conditioning argument and symmetry of the line network, we evaluate the function $N(m+1)$ as,

$$
\begin{aligned}
N(m+1) &= N(m) + 2\nu N(m-2) + \qquad (3.55)\\
&\quad 2\nu^2 N(m-4) + .. + 2\nu^{\frac{m}{2}+1},\ m \text{ even}\\
N(m+1) &= N(m) + 2\nu N(m-2) + \qquad (3.56)\\
&\quad 2\nu^2 N(m-4) + .. + 2\nu^{\frac{m+1}{2}},\ m \text{ odd}
\end{aligned}
$$

Dividing by $N(m+1)$ and taking limit($\lim_{m\to\infty}$) we get,

$$
1 = \frac{1}{\nu} \lim_{m\to\infty} \frac{\nu N(m)}{N(m+1)} + \frac{1}{\nu} \lim_{m\to\infty} \frac{\nu^2 N(m-2)}{N(m+1)} + ... \qquad (3.57)
$$

Since the left hand side (LHS) of the above equation is 1, the limits on the right hand side (RHS) must exist. This proves the existence of the limits and also provides a way to compute its value. Rewriting Equation 3.57 in terms of x and y we get,

$$1 = \frac{y}{\nu} + 2x\frac{y}{\nu} + 2x^2\frac{y}{\nu} + \ldots \quad (3.58)$$

$$= \frac{y}{\nu}(1 + 2x + 2x^2 + \ldots) \quad (3.59)$$

The normalization constant $N(m)$ is a non-negative monotonically non-decreasing function of m, $\forall\ \nu \geq 0$. This implies that $y \geq x$. Since Equation 3.59 is satisfied for all values of ν, the infinite series in x must converge and $x \in [0, 1)$ for all finite values of ν. Using $y = \sqrt{\nu x}$ and excluding the cases $x = 0$ ($\nu = 0$) and $x \to 1$ ($\nu \to \infty$), we get the following cubic equation in x.

$$\sqrt{\frac{x}{\nu}}(1 + \frac{2x}{1-x}) = 1 \quad (3.60)$$

$$x(1-x)^2 + 4x^2 = \nu(1-x)^2 \quad (3.61)$$

To prove that there exists a unique solution in $(0, 1)$ of the cubic equation, for any finite value of $\nu > 0$, we re-write Equation 3.61 as, (Note that by neglecting $\nu = 0$ case, we have $x > 0$)

$$1 + \frac{4x}{(1-x)^2} = \frac{\nu}{x}$$

The function $\frac{\nu}{x}$ is a positive decreasing function and takes values between $[\nu, \infty)$ in the interval $x \in (0, 1)$. Function, $1 + \frac{4x}{(1-x)^2}$ is a positive increasing function taking values between $(1, \infty)$ in $x \in (0, 1)$. Since ν is assumed to be positive, the two curves must intersect at a unique point in $(0, 1)$. Finally, the blocking probability of a call, P_B, is given by,

$$P_B = 1 - \frac{xy}{\nu^2(1-x)^2 + 4\nu xy} \quad (3.62)$$

where x and y satisfy the following relations

$$x(1-x)^2 + 4x^2 = \nu(1-x)^2$$

$$y^2 = \nu x$$

This completes the proof of Theorem 2.

Table 3.2 compares the blocking probability values computed theoretically using Expression 3.62 with the values obtained from simulations of a line network with 10 nodes. In the simulations, blocking probability of the center call is computed as the edge effects are minimal for this call. As seen from the table, the blocking probability formula (Equation 3.62) accurately predicts values even for small length line networks.

ν	Length = 10 nodes Theoretical Blocking prob.	Simulated Blocking prob.
0.00001	0.0000799	0.000082
0.00004	0.0003198	0.000313
0.00016	0.0012781	0.001285
0.00256	0.0200130	0.019928
0.04096	0.2396400	0.239738
0.65536	0.8061500	0.807249

Table 3.2: Comparison of theoretically computed and simulated blocking probability values for finite length line network and uni-directional calls.

Following the arguments of the bidirectional case, we compare the blocking probability expression (Equation 3.62) with the standard M/M/1/1 formula. We compute an effective load ν' in the M/M/1/1 expression (Equation 3.31) that gives the same blocking as that obtained from Equation 3.62 for load ν. The significance of the effective load is that if we isolate a particular link of the line network then load ν' on this isolated link would have the same blocking probability as experienced by the link within the line network (with symmetrical load ν). The effective load ν' can be computed by equating Equation 3.62 and 3.31.

$$\begin{aligned}\frac{\nu'}{1+\nu'} &= 1-\frac{xy}{\nu^2(1-x)^2+4\nu xy}\\ \nu' &= \frac{\nu^2(1-x)^2+(4\nu-1)xy}{xy}\end{aligned}$$

For unidirectional calls the factor g defined as, $g = \nu'/\nu$, is given by,

$$g = \frac{\nu^2(1-x)^2 + (4\nu - 1)xy}{\nu xy} \quad (3.63)$$

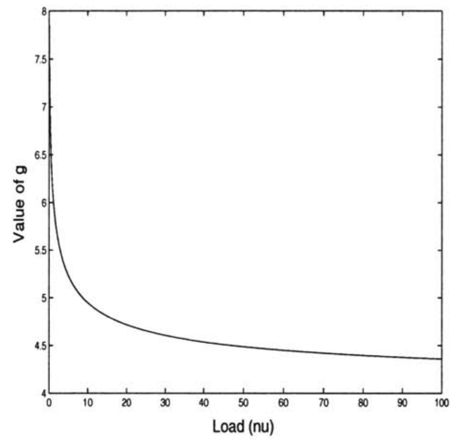

Figure 3-7: Plot of $g = \nu'/\nu$ for uni-directional calls.

A plot of g for different values of the load ν is presented in Figure 3-7. When $\nu \rightarrow 0$, the value of g equals 8 which is the number of interfering calls plus one (the particular call). When $\nu \rightarrow \infty$, g equals 4, which is the size of the largest set of mutually interfering calls. These values conform to the reasoning presented for bidirectional calls and it is applicable here as well.

3.3 Multiple Channels

The earlier sections considered a line network with a single channel. In the single channel case, if the channel is available (allocating the channel to the incoming call satisfies the wireless constraints) then it is assigned otherwise the call is dropped. However, in the multiple channel case, we may have situations when there are many free channels available and a channel allocation decision must be made. In this section, we analyze a line network with multiple channels and single hop calls. We consider the random channel allocation policy and attempt to compute the blocking probability of a call for this policy. Extending the analysis of the single channel case to multiple channels is not straight forward. The difficulty arises in evaluating the normalization constant in the steady state probability distribution expression. Therefore, we consider a simplified analytical model and derive approximate blocking probability formulas that predict very well the values obtained from simulations.

3.3.1 Random Channel Allocation Policy

A random channel assignment policy, assigns a channel on a link randomly from among the free channels on that link. Free channels refer to those channels that do not have any calls in progress and the acceptance of a call in those channels does not violate the wireless transmission/reception constraints. The random policy is very easy to implement practically and its performance study helps us evaluate other dynamic resource allocation algorithms. We then look for policies which perform better than the random policy in terms of steady state blocking probability. Our region of comparison will always be the region of low blocking probability. It has been shown that in the high blocking regime, the random policy outperforms the rearrangement policy in a linear cellular network, [13]. However such a regime is not very interesting as in practice, networks operate in the low blocking regime.

The analysis of the random policy cannot be carried out along the same lines as the single channel analysis. This is because the state vector $\mathbf{n}$ defined earlier as, ($\mathbf{n} = \{n_k\}$,where n_k = number of calls in progress on link L_k) does not completely describe the system behavior. To make a channel allocation decision for a new arriving call, we must have knowledge of the channels already occupied by the ongoing calls. The vector $\mathbf{n} = \{n_k\}$ does not provide this information and is not sufficient to describe the system. We could expand the state space to include information about the state of the channels on each link but this would make the state space very large. Even if we could characterize such a state space, it would be hard to find a steady state probability distribution over this expanded state space. Thus, we devise an approximate model of the system from which we compute the steady state blocking probability. We then present plots that compare the theoretically predicted blocking probability values and the simulation results. What follows next is a description of this approximate model.

The traffic model is similar to that considered in Sections 3.2.1 and 3.2.2. Calls arrive according to independent Poisson processes each of rate λ and the holding period of each call is Exponentially distributed with mean μ. The load ν is defined as $\nu = \lambda/\mu$. There is no buffering of calls in the network. If a call cannot be accepted then it is dropped. Otherwise it holds the channel for the entire duration of the call. Instead of looking at the entire network, we focus on the behavior of a single link. We first construct a model of the behavior of a single link in the single channel case and then extend it to the multiple channel case. Though we present the model for a line network, it can also be applied to a general network with minor modifications.

Let us consider a link L_k of the line network. For the present, assume that there is only a single channel γ in the network and denote its state on link L_k as S_k. We model S_k as a three state process as shown in Figure 3-8. The three states being the free state (denoted $\mathcal{F}$), the busy state (denoted $\mathcal{B}u$) and the blocked state (denoted $\mathcal{B}l$). The link L_k is said to be in the blocked state if the channel is occupied by a call on a neighboring interfering link thereby making the channel unavailable on link L_k. The amount of time that the state is in the free state before making a transition to the busy state is Exponentially distributed with rate λ. Let $Y_{\mathcal{F}\to\mathcal{B}l}$ be the random variable that denotes the amount of time the state S_k is in the free state before going to the blocked state.

Suppose that the present state $S_k = \mathcal{F}$ (free state). If we knew the state of channel γ on other links then $Y_{\mathcal{F}\to\mathcal{B}l}$ is an Exponential random variable with rate equal to the sum of the rates of the competing arrival processes on interfering links. The number of such competing processes will vary depending on the present state of other links. Thus, conditioned on the state of the network, $Y_{\mathcal{F}\to\mathcal{B}l}$ is Exponentially distributed. However, unconditionally $Y_{\mathcal{F}\to\mathcal{B}l}$ has a general distribution. We approximate $Y_{\mathcal{F}\to\mathcal{B}l}$ as an Exponential random variable with an average rate λ'. The amount of time that the state S_k is in the blocked state before making a transition to the free state is taken as a general distribution with mean $1/\mu'$. With these assumptions, S_k is a three state random process having a steady state distribution identical to a Markov process [7] with transition rates as shown in Figure 3-8.

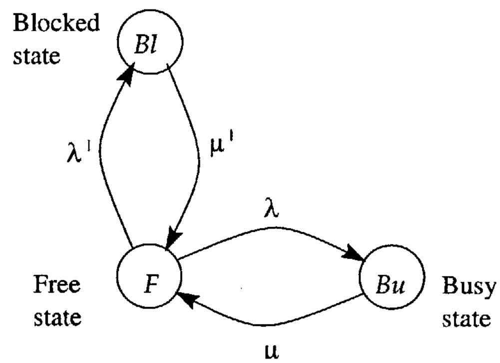

Figure 3-8: Three state Markov process model of the channel on a link.

Consider the following notation for the steady state probability distribution of the process S_k.

$$\begin{aligned} \pi_{\mathcal{F}} &\equiv \text{probability of being in the free state.} \\ \pi_{\mathcal{B}u} &\equiv \text{probability of being in the busy state.} \\ \pi_{\mathcal{B}l} &\equiv \text{probability of being in the blocked state.} \end{aligned}$$

Let P_B denote the steady state probability that an incoming call is blocked. Then as a function of the transition rates, we can compute P_B by solving the detailed balance equation of the three state Markov process. Let, $\nu' = \lambda'/\mu'$, and $\nu = \lambda/\mu$, then,

$$\begin{aligned} \pi_{\mathcal{B}u} &= (1/\nu)\pi_{\mathcal{F}} \\ \pi_{\mathcal{B}l} &= (1/\nu')\pi_{\mathcal{F}} \\ \pi_{\mathcal{F}} + \pi_{\mathcal{B}u} + \pi_{\mathcal{B}l} &= 1 \\ P_B &= \pi_{\mathcal{B}u} + \pi_{\mathcal{B}l} \end{aligned}$$

Solving these equations we get,

$$\tilde{\nu} = \nu' + \nu = \frac{P_B}{1 - P_B} \tag{3.64}$$

In Section 3.2.1 and Section 3.2.2, we computed the exact blocking probability, P_B, of a call in a single channel line network for both bi-directional and uni-directional calls. Combining the results of those sections with Equation 3.64, we can calculate the equivalent load ν' in the model (note that load ν is a known parameter). Given an arrival rate λ and the departure rate μ of the calls, we can interpret λ' as an independent rate of call arrivals that block the channel on a link. The offered load of such calls is ν'.

In the multiple channel case, define the state of a link as $X(t) \equiv (X_{bu}(t), X_{bl}(t))$ where X_{bu} is the number of busy channels and X_{bl} the number of blocked channels on that link, at time t. Let p be the total number of channels in the network. At any time t, the state $X(t) \equiv (X_{bu}(t), X_{bl}(t))$ must satisfy $X_{bu}(t) + X_{bl}(t) \leq p$. We assume that the transition rates among the states of the process $X(t)$ do not depend on the state of the system and model these transitions as shown in Figure 3-9. The transition time from $(X_{bu}(t), X_{bl}(t)) \rightarrow (X_{bu}(t)+1, X_{bl}(t))$ is Exponentially distributed with rate λ and the transition time from $(X_{bu}(t), X_{bl}(t)) \rightarrow (X_{bu}(t), X_{bl}(t)+1)$ is modeled as Exponentially distributed with rate λ'. The steady state distribution of $X(t)$ is identical to that of a Markov process with state transition diagram as shown in Figure 3-9.

Let $\pi(i,j)$ denote the steady state probability that X takes value (i,j). Then the steady state probability of blocking P_B^{rand} is equal to $\sum_{i+j=p} \pi(i,j)$. Solving the detailed balance equations [7] we get,

$$\begin{aligned} P_B^{rand} &= \sum_{i+j=p} \pi(i,j) \\ &= \frac{\frac{\tilde{\nu}^p}{p!}}{1 + \tilde{\nu} + \frac{\tilde{\nu}^2}{2!} + \ldots + \frac{\tilde{\nu}^p}{p!}} \end{aligned}$$

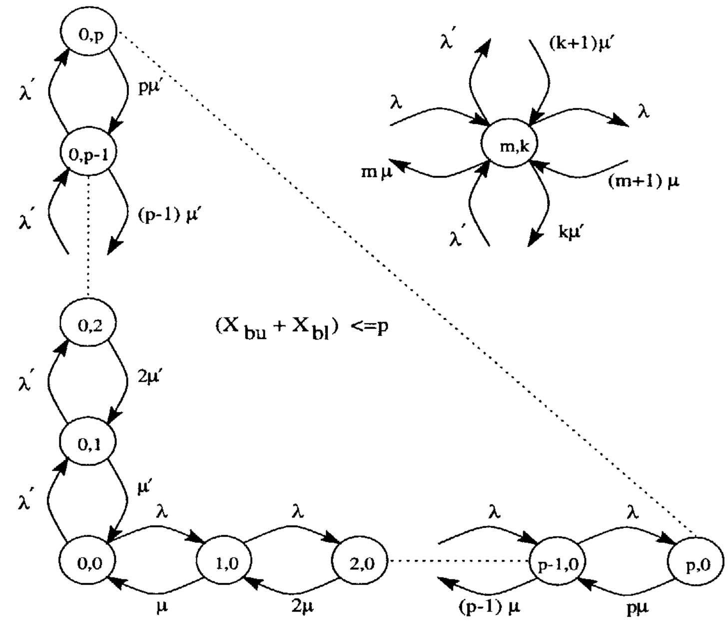

Figure 3-9: State transition diagram for the random channel allocation policy.

Define $E(\nu, p)$ as the Erlang B formula [8] for a load ν and p servers.

$$E(\nu, p) = \frac{\frac{(\nu^p)}{p!}}{1 + \nu + \frac{(\nu^2)}{2!} + \ldots + \frac{(\nu^p)}{p!}} \tag{3.65}$$

Then in terms of the Erlang B formula we have,

$$P_B^{rand} = E(\tilde{\nu}, p) \tag{3.66}$$

We next consider bi-directional and uni-directional calls and present formulas for $\tilde{\nu}$ and P_B^{rand} in terms of the offered load ν. Plots comparing the theoretically computed P_B^{rand} values and the simulation results are also presented.

Bi-directional calls

In Section 3.2.1, we derived the exact single channel blocking probability, P_B, expression for an infinite WLN-1 network. We use that result (Equation 3.30) and

Equation 3.64, to evaluate $\tilde{\nu}$.

$$\begin{aligned}\tilde{\nu} &= \frac{P_B}{1-P_B} && (3.67)\\ &= \frac{1+(2\nu-1)x^3}{x^3} && (3.68)\end{aligned}$$

where $x \in (0,1]$ satisfies $\nu x^3 + x = 1$.

Using Equation 3.66, we compute the steady state blocking probability of a call for p channels and the random channel allocation policy.

$$\begin{aligned}P_B^{bi\_rand} &= E(\tilde{\nu}, p) && (3.69)\\ \tilde{\nu} &= \frac{1+(2\nu-1)x^3}{x^3}\,, \quad \nu x^3 + x = 1 && (3.70)\end{aligned}$$

Plots comparing the predicted blocking probability values and the simulation results for 20 and 50 channels are shown in Figure 3-10. The length of the line network is 30 nodes and blocking probability is computed for the center call (the edge effects are minimal for this call). As seen in Figure 3-10, the predicted values are very accurate for low to moderate values of p. In the next chapter, we build upon this work and use the formulas derived here to study the effect of transmission radius on blocking probability.

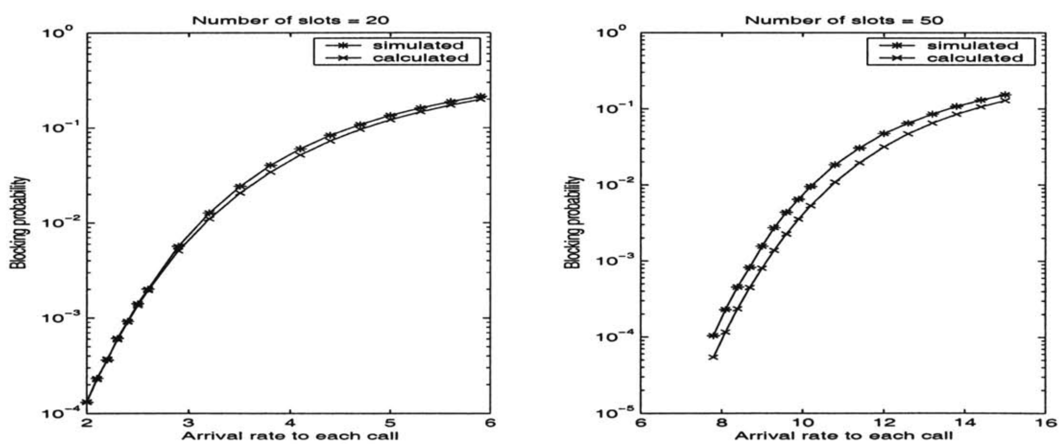

Figure 3-10: Comparison of theoretical and simulated values for bi-directional calls and random channel allocation policy.

Uni-directional calls

The exact expression for the blocking probability of calls in an infinite WLN-1(uni) network was derived in Section 3.2.2. Using Equation 3.62 and Equation 3.64 we evaluate $\tilde{\nu}$,

$$\tilde{\nu} = \frac{P_B}{1 - P_B} \tag{3.71}$$

$$= \frac{\nu^2(1-x)^2 + 4\nu xy - xy}{xy} \tag{3.72}$$

where $x \in [0,1)$ satisfies $x(1-x)^2 + 4x^2 = \nu(1-x)^2$ and $y^2 = \nu x$.
Using Equation 3.66, we compute the steady state blocking probability of a call for p channels and the random channel allocation policy.

$$P_B^{uni\_rand} = E(\tilde{\nu}, p) \tag{3.73}$$

$$\tilde{\nu} = \frac{\nu^2(1-x)^2 + 4\nu xy - xy}{xy} \tag{3.74}$$

Figure 3-11 presents plots comparing the predicted blocking probability values and the simulation results for 20 and 50 channels. The length of the line network is 30 nodes and blocking probability is computed for the center call. As seen in Figure 3-11, the predicted values are very accurate for low to moderate values of p.

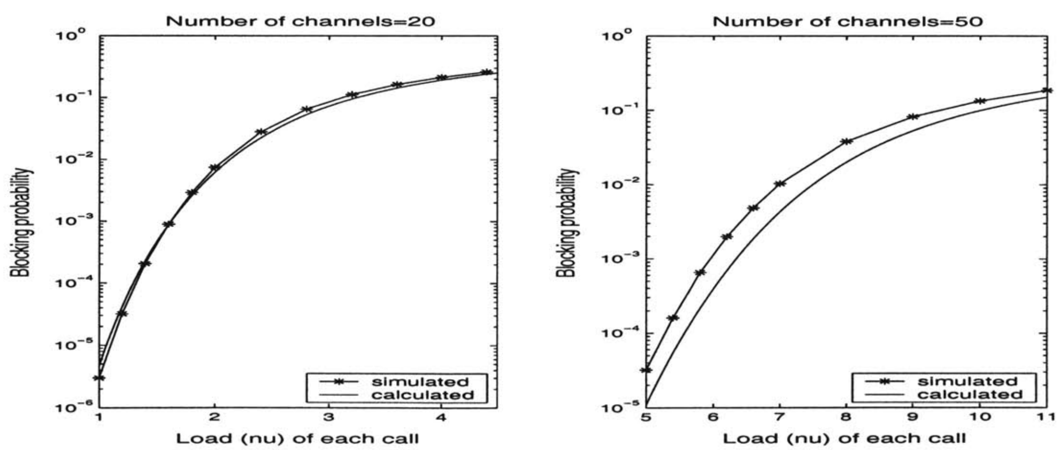

Figure 3-11: Comparison of theoretical and simulated values for uni-directional calls and random channel allocation policy.

3.4 Summary

In this chapter, we derived exact blocking probability formulas for bi-directional (Equation 3.3) and uni-directional calls (Equation 3.33) in the single channel line network case. We obtained the result in the limit of the length of the line network tending to infinity, however, the result is very accurate for finite length line networks as well (Table 3.1 and 3.2) . We compared these formulas to the standard M/M/1/1 blocking probability expression and obtained useful insights on the effective load (ν') that helped us construct the model for the multiple channel case. Our methodology of analysis is not restrictive to the cases considered in this chapter but can be applied to other symmetrical systems as well. One extension of the methodology is the blocking probability analysis for a generalized line network presented in the next chapter. Finally in Section 3.3, we presented a simplified analytical model in the multiple channel case for the random channel allocation policy. We derived approximate blocking probability formulas for both bi-directional and uni-directional calls that accurately predict the values obtained from simulation results, especially, for low to moderate number of channels (Figures 3-10 and 3-11).

Chapter 4

Effect of Transmission Radius on Blocking Probability

4.1 Introduction

We have modeled an ad-hoc wireless network as a two-dimensional graph $G \equiv (N, A)$ where N denotes the set of nodes and A is the set of wireless links. Assuming that all nodes transmit with constant power, an edge exists between any two nodes if they are both within direct transmission range of each other. Thus, given the transmission radius of each node, the topology of the network as a graph is well defined. This topology can be varied by changing the transmission radius of the nodes (which can be achieved by changing the transmission power). For dynamic channel allocation policies, the blocking probability of calls in the network is influenced by the topology of the network. In a sparsely connected network, a node has fewer neighbors and less constraints on the spatial re-use of a channel while in a densely connected network, a node has more neighbors and hence more constraints on the spatial re-use of a channel.

Consider a multi-hop call in the network between nodes S and D, where the route of the call involves multiple hops. In this case, a different channel must be assigned over adjoining hops and the set of channels chosen must satisfy the wireless constraints as noted in Section 2.1. We could however increase the transmission radius of the two nodes S,D and have a direct transmission of the call. In this case the multi-hop call becomes a single hop call and a single channel is required to service the call. However, the nodes now have more neighbors and more constraints on the use of a channel to service this call. Thus, we have the following tradeoff in the above example. In the first case (smaller transmission radius), we require multiple channels to service the call but fewer constraints on the use of a channel at each hop along the length of the path. In the second case (larger transmission radius), a single channel is required to service the call but there are more constraints on the use of a channel. In this chapter, we explore this tradeoff in more detail and study the influence of transmission radius of the nodes on blocking probability of calls. We first analyze the tradeoff in a line network with nodes located in a straight line unit distance apart and then consider a network with nodes located as a grid.

The chapter is organized as follows. In Section 4.2, we generalize the blocking probability analysis to a line network with transmission radius r. The section is further subdivided into an exact blocking probability analysis for the single channel case and an approximation in the multiple channel case. Section 4.3 considers the effect of transmission radius on blocking probability in a line network. The conclusion drawn in this section is that it is preferable in terms of blocking probability to use a larger transmission radius in a line network . Finally, Section 4.4 considers a grid network and concludes that it is more preferable to use a smaller transmission radius. Thus, we see that varying the transmission radius has different effects depending on the density of the nodes in the wireless network.

4.2 Blocking Probability Analysis in a Generalized Wireless Line Network

A line network, as define earlier, consists of nodes located in a straight line at unit distance apart from each other. In order to have connectivity in the network, each node must have a minimum transmission radius of unity. In the analysis that follows, we assume that all nodes transmit with the same constant power. Thus, the transmission radius of each node is $r, r \geq 1$ and r is an integer. We consider only bi-directional calls as it is more amenable to a simple analysis. The work can be easily extended to unidirectional calls albeit at a more complicated mathematical calculation.

We consider the following traffic model. All calls in the network are bi-directional and of length r which means that the calls are between two nodes that are r units apart. Since the transmission radius of the nodes is r, all calls in the network are single hop. Calls arrive according to independent Poisson processes each of rate λ. The call holding period of all calls is independent of earlier arrival times and holding periods of other calls and identically distributed according to an Exponential distribution with mean $1/\mu$. There is no buffering in the network. If a call cannot be accepted then it is dropped. Otherwise it holds the channel for the entire duration of the call.

First, we analyze the single channel network, followed by an extension to the multiple channel case and the random channel allocation policy.

4.2.1 Single Channel

Consider a line network with $2m+1$ nodes located at positions $x = -m$, $-m+1$,...., m (m is assumed much larger than r). The transmission radius of each node is r and all calls are of length r. We label the nodes as X_{-m}, X_{-m+1},..., X_m. Let C_k denote the call between nodes X_k and X_{k+r}. $C_k = 0$ if the call is inactive and $C_k = 1$ if the call is active. Note that there are $2m+1-r$ distinct calls, $C_{-m}, C_{-m+1}, .., C_{m-r}$.

We say that a node X_l is active if either call C_{l-r} or call C_l is active and node X_l is inactive otherwise. For call C_k (C_k is between nodes X_k and X_{k+r}) to be successfully serviced, neighbors of node X_k and neighbors of node X_{k+r} must be inactive. Since the transmission radius of each node is r, neighbors of node X_k are nodes X_{k-r},..,

X_{k+r} and neighbors of node X_{k+r} are nodes $X_k,..,\ X_{k+2r}$. This implies that nodes $X_{k-r},..,\ X_{k+2r}$ must be inactive or equivalently calls $C_{k-2r},..,\ C_{k+2r}$ must be inactive. In summary, we have the following local constraint,

$$\text{Call } C_k \text{ is successfully serviced if, calls } C_{k-2r},..,C_{k+2r} \text{ are inactive.} \tag{4.1}$$

We refer to this network as WLN-r. It is a generalization of the WLN-1 network considered in Section 3.2.1. The key difference from WLN-1 is that, now, a node interferes with many more nodes.

Theorem 3. *The blocking probability of a call in a WLN-r ($r \in Z^+$) line network with the length of the line network tending to infinity and finite $\nu = \lambda/\mu$ is,*

$$P_B = 1 - \frac{x^{2r+1}}{1 + 2r\nu x^{2r+1}} \tag{4.2}$$

x is the unique root in (0,1] of $\nu x^{2r+1} + x = 1$

Proof: Let $n_k(t)$ denote the number of calls C_k in progress. Let $\nu = \lambda/\mu$ and define the vector $\mathbf{n}(t) = (n_k(t), k \in -m, ..., m-r)$. State $\mathbf{n}$ is admissible if $\mathbf{n} \geq 0$ satisfies the wireless constraints as described in Section 2.1. Let $\mathcal{G}^r(m)$ denote the set of all admissible states for a network with m nodes. Since we consider a line network with $2m+1$ nodes, the set of all admissible states is $\mathcal{G}^r(2m+1)$. The analysis that follows does not require an explicit description of the complete state space. The local constraints on the spatial re-use of a channel suffice for the analysis.

The stochastic process $(\mathbf{n}(t), t \geq 0)$ is an aperiodic, irreducible, finite state Markov process and hence has a unique stationary distribution $\pi(\mathbf{n}) = P(\mathbf{n} = (n_{-m}, .., n_{m-r}))$ given by a product form solution. The normalization constant in the product form solution is denoted as $S^r(2m+1)$. The product form distribution is same as in the unit transmission radius case but over a different admissible state space and is expressed as,

$$\pi(\mathbf{n}) = \frac{1}{S^r(2m+1)} \prod_{i=-m}^{m-r} \frac{\nu^{n_i}}{n_i!}, \quad \mathbf{n} \in \mathcal{G}^r(2m+1) \tag{4.3}$$

$$S^r(2m+1) = \sum_{\mathbf{n}\in\mathcal{G}^r(2m+1)} \prod_{i=-m}^{m-r} \frac{\nu^{n_i}}{n_i!}, \quad \mathbf{n} \in \mathcal{G}^r(2m+1) \tag{4.4}$$

For a single channel network $n_i = 0$ or 1 and $n_i! = 1$. Let $n_{total} = n_{-m} + .. + n_{m-r}$, then we can re-write $\pi(\mathbf{n})$ as,

$$\pi(\mathbf{n}) = \frac{1}{S^r(2m+1)} \nu^{n_{total}}, \quad \mathbf{n} \in \mathcal{G}^r(2m+1) \tag{4.5}$$

$$S^r(2m+1) = \sum_{\mathbf{n}\in\mathcal{G}^r(2m+1)} \nu^{n_{total}} \tag{4.6}$$

Consider the call C_0 of the line network. Let $\mathcal{G}_N^r(2m+1)$ represent the set of non-blocking states, $\mathbf{n}$, for the call C_0. Let P_{NB}^0 denote the probability that in steady state call C_0 is not blocked. Then P_{NB}^0 can be expressed as,

$$P_{NB}^0 = \sum_{\mathbf{n}\in\mathcal{G}_N^r(2m+1)} \pi(\mathbf{n}) \tag{4.7}$$

$$P_{NB}^0 = \frac{\sum_{\mathbf{n}\in\mathcal{G}_N^r(2m+1)} \nu^{n_{total}}}{S^r(2m+1)} \tag{4.8}$$

Using condition 4.1, call C_0 can be successfully serviced, if calls C_{-2r},.., C_{2r} are inactive. The state space $\mathcal{G}_N^r(2m+1)$ can now be defined as the set of states $\mathbf{n}$ where n_{-2r},.., $n_{2r} = 0$.

To characterize $\mathcal{G}_N^r(2m+1)$, we need to determine the feasible state space of the remaining calls (C_{-m},.., C_{-2r-1}) and (C_{2r+1},.., C_{m-r}). Given (C_{-2r},.., C_{2r}) $= 0$, the state of calls (C_{-m},.., C_{-2r-1}) is not constrained by the state of calls (C_{2r+1},.., C_{m-r}). Therefore, the state of calls C_{-m},.., C_{-2r-1} is independent of the state of calls C_{2r+1},.., C_{m-r}. Let,

$$
\begin{aligned}
\mathcal{H} &\equiv \text{feasible state space of calls } C_{-m},..,\ C_{-2r-1} \\
&\equiv \text{state space of WLN-r with } m-r \text{ nodes} \\
&= \mathcal{G}^r(m-r); \\
\mathcal{M} &\equiv \text{feasible state space of calls } C_{2r+1},..,\ C_{m-r} \\
&\equiv \text{state space of WLN-r with } m-2r \text{ nodes} \\
&= \mathcal{G}^r(m-2r)
\end{aligned}
$$

Since the set of calls $(C_{-m},..,\ C_{-2r-1})$ and $(C_{2r+1},..,\ C_{m-r})$ are independent given $C_{-2r},..,\ C_{2r} = 0$, the set $\mathcal{G}_N^r(2m+1)$ is the cartesian product of $\mathcal{G}^r(m-r)$ and $\mathcal{G}^r(m-2r)$.

$$\mathcal{G}_N^r(2m+1) = \mathcal{G}^r(m-r)X\mathcal{G}^r(m-2r)$$

Using Equation 3.1, it follows that,

$$
\begin{aligned}
\sum_{\mathbf{n}\in\mathcal{G}_N^r(2m+1)} \nu^{n_{total}} &= \sum_{\mathbf{n}\in\mathcal{G}^r(m-r)X\mathcal{G}^r(m-2r)} (\nu^{n_{-m}+..+n_{-2r-1}})(\nu^{n_{2r+1}+..+n_{m-r}}) && (4.9) \\
&= \sum_{\mathcal{G}^r(m-r)} \nu^{n_{-m}+..+n_{-2r-1}} \sum_{\mathcal{G}^r(m-2r)} \nu^{n_{2r+1}+..+n_{m-r}} && (4.10) \\
&= S^r(m-r)S^r(m-2r) && (4.11)
\end{aligned}
$$

We can re-write Equation 4.8 as,

$$P_{NB}^0 = \frac{S^r(m-r)S^r(m-2r)}{S^r(2m+1)} \tag{4.12}$$

Following a similar analysis as in the unit transmission radius case, we consider the limiting behavior ($m \to \infty$) with the length of the line network tending to infinity. In an infinite line network the edge effects vanish and each node then has an identical environment. Since we consider a uniform traffic model, the blocking probability of each call becomes identical and an elegant formula is obtained in this limiting case. Before proceeding to take the limits, we first evaluate the denominator in

Equation 4.12 in terms of the $S^r(.)$ function of lower arguments. This is done by conditioning on the state of calls (C_{-r},.., C_0,.., C_{r-1}). For these conditioned calls, other than the all zero state, there are $2r$ distinct cases corresponding to each call being active and the rest inactive, i.e. $C_p = 1, C_{l \neq p} = 0 \ \forall p, l \in \{-r, .., r-1\}$. Note that a state with more than one call being active among C_{-r},.., C_0,.., C_{r-1} is infeasible. Let $S^r()|\{constraint\}$ represent the evaluation of the function $S^r()$ under the specified *constraint* and $p, l \in \{-r, .., r-1\}$ in the equations that follow.

$$\begin{aligned} S^r(2m+1) &= S^r(2m+1)|\{C_{-r}, .., C_{r-1} = 0\} + \\ &\quad \sum_{p=-r}^{r-1} S^r(2m+1)|\{C_p = 1, C_{l \neq p} = 0\} \qquad (4.13) \\ &= S^r(m)S^r(m-r+1) + \\ &\quad \sum_{p=-r}^{r-1} S^r(2m+1)|\{C_p = 1, C_{l \neq p} = 0\} \qquad (4.14) \end{aligned}$$

To evaluate the above equation, we reverse the order of summation and let $p = r - j$. As the reversed summation of p runs from $r-1$ to $-r$, j runs from 1 to $2r$. For a particular term in the summation, the condition $C_p = 1$ can be written as $C_{r-j} = 1$. Using condition 4.1, the set of calls that must be inactive (for $C_{r-j} = 1$) are C_{-r-j},.., C_{3r-j}. This leaves the state of calls ($C_{-m}, .., C_{-r-j-1}$) independent of the state of calls ($C_{3r-j+1}, .., C_{m-r}$). Thus for this particular term, the normalization constant $S^r(2m+1)$ (under the constraint $C_p = 1, C_{l \neq p} = 0, l \in \{-r, .., r-1\}$) equals $S^r(m-j)S^r(m-3r+j)$.

$$\begin{aligned} S^r(2m+1) &= S^r(m)S^r(m-r+1) + \\ &\quad \sum S^r(2m+1)|\{C_p = 1, C_{l \neq p} = 0\} \qquad (4.15) \\ &= S^r(m)S^r(m-r+1) + \\ &\quad \nu \sum_{j=1}^{2r} S^r(m-j)S^r(m-3r+j) \qquad (4.16) \end{aligned}$$

$$\frac{S^r(2m+1)}{S^r(m)S^r(m-r+1)} = 1 + \nu \sum_{j=1}^{2r} \frac{S^r(m-j)S^r(m-3r+j)}{S^r(m)S^r(m-r+1)} \qquad (4.17)$$

Define the following limit (the existence of the limit is proved later).

$$\lim_{m\to\infty} \frac{S^r(m-1)}{S^r(m)} = x \tag{4.18}$$

When the length of the line network tends to infinity, we have $\lim_{m\to\infty} P^0_{NB} = P_{NB}$, the probability of non-blocking of any call. Taking the limit on both sides of Equation 4.12 and re-writing it we get,

$$\begin{aligned} P_{NB} &= \lim_{m\to\infty} \frac{\frac{S^r(m-r)S^r(m-2r)}{S^r(m)S^r(m-r+1)}}{\frac{S^r(2m+1)}{S^r(m)S^r(m-r+1)}} && (4.19)\\ &= \frac{\lim_{m\to\infty} \frac{S^r(m-r)S^r(m-2r)}{S^r(m)S^r(m-r+1)}}{1+\nu\sum_{j=1}^{2r}\lim_{m\to\infty}\frac{S^r(m-j)S^r(m-3r+j)}{S^r(m)S^r(m-r+1)}} && (4.20)\\ &= \frac{x^{2r+1}}{1+2r\nu x^{2r+1}} && (4.21) \end{aligned}$$

To prove the existence of the limit and evaluate its value, we express $S^r(m)$ in terms of the function $S^r(.)$ of lower arguments. This is achieved by conditioning on the state of the call associated with the leftmost node of the line network. The two cases include the call being inactive and the call being active.

$$\begin{aligned} S^r(m) &= S^r(m-1) + \nu S^r(m-2r-1) && (4.22)\\ 1 &= \frac{S^r(m-1)}{S^r(m)} + \frac{\nu S^r(m-2r-1)}{S^r(m)} && (4.23)\\ 1 &= \lim_{m\to\infty}\frac{S^r(m-1)}{S^r(m)} + \lim_{m\to\infty}\frac{\nu S^r(m-2r-1)}{S^r(m)} && (4.24) \end{aligned}$$

Since the left hand side of the above equation is 1, the limits on the right hand side exist. The value of the limit x can be computed by solving the following polynomial obtained by re-writing Equation 4.24 in terms of x.

$$1 = x + \nu x^{2r+1} \tag{4.25}$$

Combining Equations 4.25 and 4.21, the blocking probability of a call is given by,

$$P_B = 1 - \frac{x^{2r+1}}{1 + 2r\nu x^{2r+1}} \quad , \quad x + \nu x^{2r+1} = 1 \tag{4.26}$$

We next show the existence of a unique value of $x \in (0, 1]$. We exclude the case of $\nu \to \infty$. The normalization constant $S^r(m)$ is a non-negative monotonically non-decreasing function of $m, \forall \nu \geq 0$. Therefore, $S^r(m) \leq S^r(m+1)$ and x must lie in $(0, 1]$. To show that Equation 4.25 has a unique root in $(0, 1]$ for finite ν, we re-write it as,

$$\nu x^{2r} + 1 = \frac{1}{x} \tag{4.27}$$

The function $1/x$ is a positive decreasing function and takes values between $[1, \infty)$ in the interval $x \in (0, 1]$. Function, $\nu x^{2r} + 1, (\forall r \geq 1)$ is a positive increasing function taking values between $(1, 1 + \nu]$ in $x \in (0, 1]$. Since we assumed that $\nu \geq 0$, the two curves must intersect at a unique point in $(0, 1]$. This completes the proof of Theorem 3.

4.2.2 Multiple Channels.

This section extends the single channel analysis to the case of multiple channels. The transmission radius of each node of the line network is r units, all calls are of length r and the random channel allocation policy is followed to assign the channels. As argued in the unit transmission radius case (Section 3.3), the exact blocking probability analysis of the random assignment policy is not easy. The difficulty lies in computing the steady state probability distribution over the admissible state space. However, we can construct an approximate model identical to that of the unit transmission radius case (Section 3.3.1). Following Section 3.3.1, we evaluate the equivalent load ν' from the single channel model as,

$$\begin{aligned} \nu' + \nu &= \frac{P_B}{1 - P_B} && (4.28) \\ &= \frac{1 + (2r\nu - 1)x^{2r+1}}{x^{2r+1}} \quad , \quad (\nu x^{2r} + 1 = \frac{1}{x}) && (4.29) \end{aligned}$$

If the total number of channels is p then the blocking probability for the random policy is computed from the Erlang B formula with load $\tilde{\nu} = \nu' + \nu$. Thus,

$$P_B^{rand} = E(\tilde{\nu}, p) \ , \quad (\tilde{\nu} = \nu' + \nu) \tag{4.30}$$

$$E(\tilde{\nu}, p) = \frac{\frac{\tilde{\nu}^p}{p!}}{1 + \tilde{\nu} + \frac{\tilde{\nu}^2}{2!} + .. + \frac{\tilde{\nu}^p}{p!}} \tag{4.31}$$

where, $E(\nu, p)$ is the Erlang B formula for a load ν and p servers. We next present plots comparing the blocking probability values calculated from Equation 4.31 and the values obtained from simulations. Figure 4-1 shows comparison plots for $r = 2$ and $r = 10$ with 20 channels. As seen from the figure, in the case of moderate number of channels the theoretically calculated values are very accurate, even for longer length calls ($r = 10$).

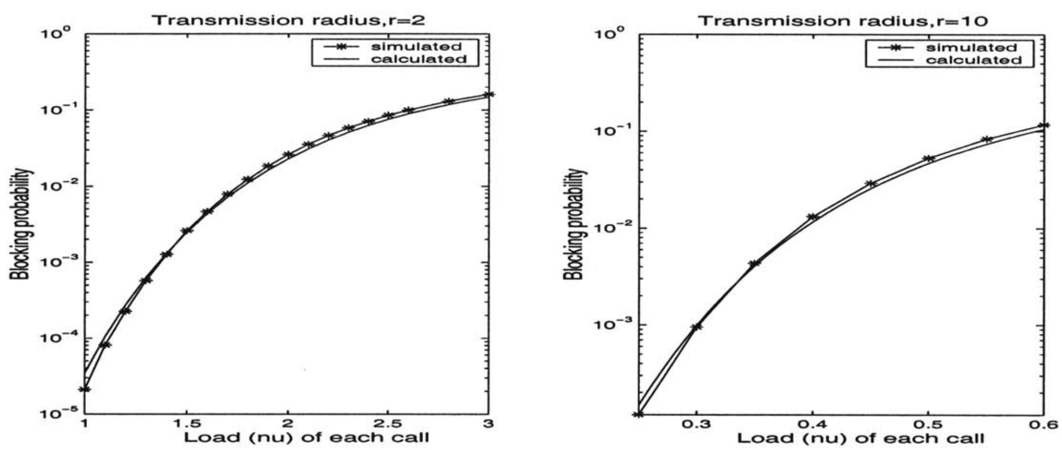

Figure 4-1: Comparison of theoretical and simulated values for $r = 2$ and $r = 10$ with 20 channels.

Figure 4-2 presents comparison plots for $r = 2$ and $r = 10$ with 50 channels. We observe that as the number of channels becomes large (larger value of p), the theoretically computed values are less accurate. However, even in this case the theoretical curve follows closely the simulation results.

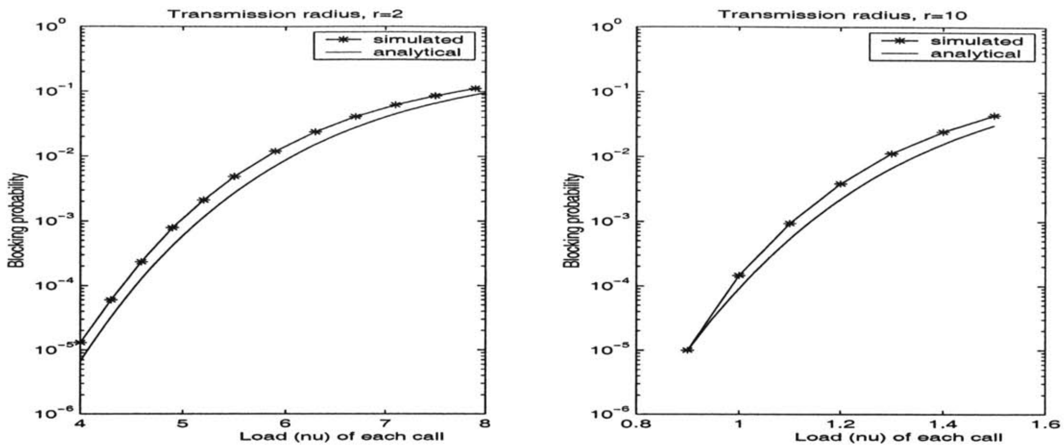

Figure 4-2: Comparison of theoretical and simulated values for $r = 2$ and $r = 10$ with 50 channels.

4.3 Effect of Transmission Radius in a Line Network

The topology of a wireless network, as a two dimensional graph, can be varied by changing the transmission radius of the nodes. This in turn affects the blocking probability of the calls. In case of multi-hop calls, if we use a smaller transmission radius then multiple channels are required to service the call but there are fewer constraints on the use of a channel at each hop along the length of the path. If we use a larger transmission radius (such that the destination is within the transmission range), a single channel is required to service the call but there are more constraints on the use of a channel. To understand this tradeoff and its effect on blocking probability, we first consider the simplest non-trivial example in a line network and then generalize the conclusions drawn from this example. In this section, we conclude that in a line network it is preferable in terms of blocking probability to use a larger transmission radius.

Consider first, the following simplest non-trivial example. Consider a line network with **two channels** and all calls of length two (all calls are between nodes that are two hops apart). The traffic model is identical to that considered earlier. Consider two scenarios, the first in which all nodes have a transmission radius of unity ($r = 1$) and thus all calls are two hop long. Each call requires a distinct channel on each hop as adjacent links are interfering. In this case, we use the rearrangement channel assignment policy (Section 5.2) and compute the exact blocking probability of a call.

The second scenario is one in which all nodes have a transmission radius of two units ($r = 2$) and hence all calls are single hop. Here, analyzing the exact blocking probability of a call for the rearrangement policy is difficult. Therefore, we consider a sub-optimal policy that selects a channel randomly from the two channels for each new arriving call. If the channel is free (non-blocked and non-busy) then it is allocated otherwise the incoming call is dropped. The policy clearly under utilizes the channels, as it rejects a call (if the randomly selected channel is not free) without considering the state of the other channel. It performs a simple random splitting of the incoming arrival stream into two independent Poisson processes of rate $\lambda/2$ with the split load applied to each channel. We compute the blocking probability for this case and compare it with the unit transmission radius case. The following result shows that, even with this very inefficient random policy, it is better to transmit with a larger radius.

Theorem 4. *The blocking probability for case $r = 2$ is lower than the blocking probability for case $r = 1$, for $p = 2$ (2 channels) and all finite load $\nu > 0$.*

$$P_B \{r = 2\} < P_B \{r = 1\}, \ \nu \in (0, \infty). \tag{4.32}$$

Proof: We analyze each case separately and then compare the blocking probability expression obtained for each case.

Case a : Transmission radius of all nodes is unity

The analysis of blocking probability follows the methodology used in Section 4.2.1

with an identical line network model (but with each node having a transmission radius of unity). Since all calls are of length two, C_k denotes the call between nodes X_k and X_{k+2}. Let $n_k(t)$ be the number of calls C_k in progress and define the state vector as, $\mathbf{n}(t) = (n_k(t), k \in -m, ..., m-2)$. Since we use the rearrangement policy for assigning channels, the vector $\mathbf{n} = (n_{-m}, ..., n_{m-2})$ completely describes the system behavior. The stochastic process $\mathbf{n}(t)$ is an aperiodic, irreducible, finite state Markov process with a product form steady state distribution. $S(2m+1)$ is the normalization constant and $\mathcal{G}(2m+1)$ denotes the admissible state space (where $2m+1$ denotes a line network with $2m+1$ nodes).

$$\pi(\mathbf{n}) = \frac{1}{S(2m+1)} \prod_{j=-m}^{m-2} \frac{\nu^{n_j}}{n_j!}, \quad \mathbf{n} \in \mathcal{G}(2m+1) \tag{4.33}$$

Since adjacent hops cannot share the same channel, each call requires two channels to get served. As there are only two channels in the network, we get the constraint $0 \leq n_j \leq 1$, $\forall j$ and $n_j! = 1$. With this constraint, Equation 4.33 reduces to,

$$\pi(\mathbf{n}) = \frac{1}{S(2m+1)} \prod_{j=-m}^{m-2} \nu^{n_j}, \quad \mathbf{n} \in \mathcal{G}(2m+1) \tag{4.34}$$

$$S(2m+1) = \sum_{\mathbf{n} \in \mathcal{G}(2m+1)} \prod_{j=-m}^{m-2} \nu^{n_j}, \quad \mathbf{n} \in \mathcal{G}(2m+1) \tag{4.35}$$

Let $n_{total} = n_{-m} + .. + n_{m-2}$, then,

$$\pi(\mathbf{n}) = \frac{1}{S(2m+1)} \nu^{n_{total}}, \quad \mathbf{n} \in \mathcal{G}(2m+1) \tag{4.36}$$

$$S(2m+1) = \sum_{\mathbf{n} \in \mathcal{G}(2m+1)} \nu^{n_{total}}, \quad \mathbf{n} \in \mathcal{G}(2m+1) \tag{4.37}$$

Following a similar reasoning as done in Section 4.2.1, the wireless constraints translated in terms of the constraints on the state of calls reduce to,

Call C_k is successfully serviced if, calls $C_{k-2}, C_{k-1}, C_{k+1}, C_{k+2}$ are inactive. (4.38)

We consider the probability of non-blocking, P^0_{NB}, of call C_0 and then take the limit ($\lim_{m\to\infty}$). The set of non-blocking states of call C_0 are $\{\mathbf{n} : \mathbf{n} \in \mathcal{G}(2m+1)$ and $n_{-2}, .., n_2 = 0\}$. Let this state be denoted as $\mathcal{G}^0_N(2m+1)$. To characterize $\mathcal{G}^0_N(2m+1)$, we need to determine the feasible state space of the remaining calls (C_{-m},.., C_{-3}) and (C_3,.., C_{m-2}). Given (C_{-2},.., C_2) = 0, the state of calls (C_{-m},.., C_{-3}) is not constrained by the state of calls (C_3,.., C_{m-2}). Therefore, the state of calls (C_{-m},.., C_{-3}) is independent of the state of calls (C_3,.., C_{m-2}). Since the state space of calls (C_{-m},.., C_{-3}) is $\mathcal{G}(m)$ and the state space of calls (C_3,.., C_{m-2}) is $\mathcal{G}(m-2)$, we can express $\mathcal{G}^0_N(2m+1)$ as $\mathcal{G}(m)X\mathcal{G}(m-2)$. We can now apply Equation 3.1 and $n_{-2}, .., n_2 = 0$ to evaluate P^0_{NB}.

$$P^0_{NB} = \sum_{\mathbf{n}\in\mathcal{G}^0_N(2m+1)} \pi(\mathbf{n}) \tag{4.39}$$

$$= \frac{\sum_{\mathbf{n}\in\mathcal{G}^0_N(2m+1)} \nu^{n_{total}}}{S(2m+1)} \tag{4.40}$$

$$= \frac{\sum_{\mathbf{n}\in\mathcal{G}(m)X\mathcal{G}(m-2)} \nu^{n_{-m}+..+n_{-3}}\nu^{n_3+..+n_{m-2}}}{S(2m+1)} \tag{4.41}$$

$$= \frac{(\sum_{\mathcal{G}(m)} \nu^{n_{-m}+..+n_{-3}})(\sum_{\mathcal{G}(m-2)} \nu^{n_3+..+n_{m-2}})}{S(2m+1)} \tag{4.42}$$

$$= \frac{S(m)S(m-2)}{S(2m+1)} \tag{4.43}$$

To evaluate $S(2m+1)$, we partitioning the state space $\mathcal{G}(2m+1)$ into a set of states conditioned on all the possible states of calls (C_{-1}, C_0). We then evaluate $S(2m+1)$ over each of the partitioned state space and sum them up. The conditioning argument uses the constraint 4.38 and is identical to the argument made in Section 4.2.1. The possible states of calls (C_{-1}, C_0) are,

1. C_{-1}, C_0 both inactive. In this case, calls $C_{-m}, .., C_{-2}$ do not interfere with calls $C_1, .., C_{m-2}$. Thus, the state of calls $C_{-m}, .., C_{-2}$ is independent of the state of calls $C_1, .., C_{m-2}$ and we get, (Note that using the earlier notation, feasible state space of $(C_{-m}, .., C_{-2}) = \mathcal{G}(m+1)$ and the feasible state space of $(C_1, .., C_{m-2})$ $= \mathcal{G}(m)$)

$$\begin{aligned} S(2m+1) &= \sum_{\mathcal{G}(m+1)} \nu^{n_{-m}+..+n_{-2}} \sum_{\mathcal{G}(m)} \nu^{n_1+..+n_{m-2}} \\ S(2m+1) &= S(m+1)S(m) \end{aligned}$$

2. C_{-1} active, C_0 inactive. Since C_{-1} is active, calls C_{-3}, C_{-2}, C_1 must be inactive. This leaves the state of calls $C_{-m}, .., C_{-4}$ independent of the state of the calls $C_2, .., C_{m-2}$. The feasible state space of $(C_{-m}, .., C_{-4}) = \mathcal{G}(m-1)$ and the feasible state space of $(C_2, .., C_{m-2}) = \mathcal{G}(m-1)$.

$$\begin{aligned} S(2m+1) &= (\sum_{\mathcal{G}(m-1)} \nu^{n_{-m}+..+n_{-4}})\ \nu\ (\sum_{\mathcal{G}(m-1)} \nu^{n_2+..+n_{m-2}}) \\ S(2m+1) &= \nu S(m-1)S(m-1) \end{aligned}$$

3. C_{-1} inactive, C_0 active. Since C_0 is active, calls C_{-2}, C_{-1}, C_1, C_2 must be inactive. This leaves the state of calls $C_{-m}, .., C_{-3}$ independent of the state of the calls $C_3, .., C_{m-2}$. The feasible state space of $(C_{-m}, .., C_{-3}) = \mathcal{G}(m)$ and the feasible state space of $(C_3, .., C_{m-2}) = \mathcal{G}(m-2)$.

$$\begin{aligned} S(2m+1) &= (\sum_{\mathcal{G}(m)} \nu^{n_{-m}+..+n_{-3}})\ \nu\ (\sum_{\mathcal{G}(m-2)} \nu^{n_3+..+n_{m-2}}) \\ S(2m+1) &= \nu S(m)S(m-2) \end{aligned}$$

4. C_{-1}, C_0 both active. This state is infeasible.

Summing up all the above cases we get,

$$S(2m+1) = S(m+1)S(m) + \nu S(m)S(m-2) + \nu S(m-1)S(m-1) \tag{4.44}$$

Taking the limit ($\lim_{m\to\infty}$), the probability of non-blocking of a call is given by,

$$P_{NB} = \frac{\lim_{m\to\infty} \frac{S(m)S(m-2)}{S(m+1)S(m)}}{1 + \nu \lim_{m\to\infty} \frac{S(m)S(m-2)}{S(m+1)S(m)} + \nu \lim_{m\to\infty} \frac{S(m-1)S(m-1)}{S(m+1)S(m)}} \tag{4.45}$$

Let,

$$\lim_{m\to\infty} \frac{S(m-1)}{S(m)} = y \tag{4.46}$$

Then P_{NB} can be expressed in terms of y as,

$$P_{NB} = \frac{y^3}{1+2\nu y^3} \tag{4.47}$$

To evaluate the limit and prove its existence, we evaluate $S(m)$ by conditioning on the state of the leftmost call. This gives,

$$S(m) = S(m-1) + \nu S(m-3) \tag{4.48}$$

$$1 = \lim_{m\to\infty} \frac{S(m-1)}{S(m)} + \nu \lim_{m\to\infty} \frac{S(m-3)}{S(m)} \tag{4.49}$$

Since the left hand side is 1, the limits on the right hand side must exist and we get the following cubic equation.

$$1 = y + \nu y^3 \tag{4.50}$$

From the definition (Equation 4.46) it is clear that y lies in (0,1] for all finite $\nu \geq 0$. To show that Equation 4.50 has a unique root in (0,1] we re-write it as,

$$\nu y^2 + 1 = \frac{1}{y}$$

The function $1/y$ is a positive decreasing function and takes values between $[1,\infty)$ in the interval $y \in (0,1]$. Function, $\nu y^2 + 1$ is a positive non-decreasing function taking values between $(1, 1+\nu]$ in $y \in (0,1]$. Since we assumed that $\nu \geq 0$, the two curves must intersect at a unique point in $(0,1]$. Finally, the expression for the blocking probability of a call is,

$$P_B = 1 - \frac{y^3}{1+2\nu y^3}, \quad \nu y^3 + y = 1 \tag{4.51}$$

Case b : Transmission radius of all nodes is two units

In this case, as noted earlier, for each arriving call a channel is randomly selected with equal probability from the two channels. If the chosen channel is free (non-blocked and non-busy) then it is allocated otherwise the incoming call is dropped. Clearly, this policy is sub-optimal. It amounts to simply splitting the incoming Poisson arrival stream and assigning the split streams to each channel respectively. Since the splitting is done with equal probability, the split streams are Poisson with rate $\lambda/2$. Thus, the blocking probability of a call is equal to the blocking probability with load $(\lambda/2\mu)$ in a single channel WLN-2 network (as each node has a transmission radius of two units and all calls are of length two). This system has been solved exactly in Section 4.2.1 and Equation 4.26 gives the exact blocking probability when the length of the line network tends to infinity. Plugging $\nu/2$ $(\nu = \lambda/\mu)$ and $r = 2$ in Equation 4.26 we get,

$$P_B = 1 - \frac{x^5}{1 + 4\frac{\nu}{2}x^5} \quad , \quad x + \frac{\nu}{2}x^5 = 1 \tag{4.52}$$

To compare Cases (a) and (b), we need to compare Equations 4.51 and 4.52 for the same load ν. For load $\nu = 0$, P_B is zero for both cases (a) and (b). Excluding the $\nu \to \infty$ case, we will show that for all finite ν, there does not exist a load $\hat{\nu}$ for which the two blocking probabilities are equal. Since $P_B \geq 0$ is a bounded continuous function of ν, we conclude that the blocking probability in one case is always higher than in the other case. To complete the proof, we finally show that $P_B(Case\ b) < P_B(Case\ a)$ for a specific ν (here we take $\nu = 1$).

Let $\hat{\nu} > 0$ be such that the blocking probability in Cases (a) and (b) are equal. Let $\tilde{x}$ and $\tilde{y}$ be the unique root in (0,1) of the polynomials $\frac{\hat{\nu}}{2}x^5 + x = 1$ and $\hat{\nu}y^3 + y = 1$ respectively (Note that since $\hat{\nu}$ is strictly greater than 0, $\tilde{x}$ and $\tilde{y}$ are strictly less than 1). Equating 4.51 and 4.52, we get,

$$\frac{\tilde{x}^5}{1 + 4\frac{\hat{\nu}}{2}\tilde{x}^5} = \frac{\tilde{y}^3}{1 + 2\hat{\nu}\tilde{y}^3} \tag{4.53}$$

$$\tilde{x}^5 = \tilde{y}^3 \tag{4.54}$$

$\tilde{x}$ and $\tilde{y}$ also satisfy the following equalities,

$$\frac{\hat{\nu}}{2}\tilde{x}^5 + \tilde{x} = 1 \quad (4.55)$$

$$\hat{\nu}\tilde{y}^3 + \tilde{y} = 1 \quad (4.56)$$

Substituting, $\hat{\nu}\tilde{x}^5 = \hat{\nu}\tilde{y}^3 = 1 - \tilde{y} = 1 - \tilde{x}^{5/3}$, in Equation 4.55 we get,

$$2\tilde{x} - \tilde{x}^{5/3} = 1 \quad (4.57)$$

However, no value of $\tilde{x}$ lying in (0,1) satisfies the above equation. This can be proved by noting that the function $2x - x^{5/3}$ is a monotonically increasing function in (0,1) (its derivative, $2 - (5/3)x^{2/3}$ is positive in (0,1)). It takes value 0 at $x = 0$ and value 1 at $x = 1$. Thus, for $x \in (0,1)$, $2x - x^{5/3} \neq 1$ and we arrive at a contradiction. The conclusion drawn is that for finite $\nu > 0$, blocking probability for Cases (a) and (b) cannot be equal. Since $P_B \geq 0$ is a continuous function of ν, the blocking probability in one case is always higher than in the other case. By substituting $\nu = 1$, it can be easily shown that $P_B(Case\ b) < P_B(Case\ a)$ (Note that if there exists a ν for which $P_B(Case\ b) > P_B(Case\ a)$, then the two P_B curves must cross each other at some $\hat{\nu}$ but we have shown that no such $\hat{\nu}$ exists). Thus, we conclude that for all finite load, ν, the blocking probability for $r = 2$ case is smaller than $r = 1$ case and it is preferable to use a larger transmission radius. Figure 4-3 is a plot of blocking probability for Cases (a) and (b) (Equations 4.51 and 4.52).

Line network with p channels and all calls of length k units

To generalize the conclusion drawn above, we consider a line network with p channels and all calls of length $k > 1$ ($k = 1$ is the single hop case for which the transmission radius must be 1). Here again, we consider two scenarios one in which all nodes have a transmission radius of unity and thus all calls are k hop long. The other in which all nodes have a transmission radius of k units and hence all calls are single hop.

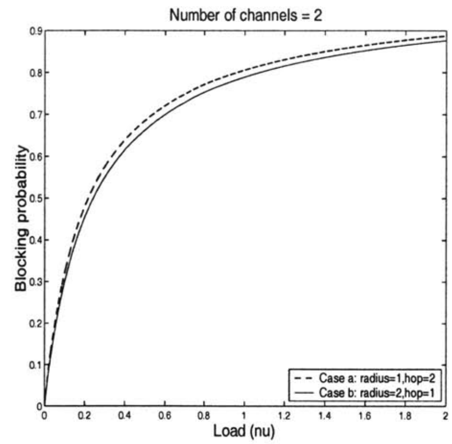

Figure 4-3: Blocking probability for calls of length 2 with radius 1 (Eqn. 4.51) and radius 2 (Eqn. 4.52).

The arrival process of each call is an independent Poisson process of rate λ and the call holding period is Exponentially distributed with mean $1/\mu$ (define $\nu = \lambda/\mu$). In both cases, we assume a random channel assignment policy and use the approximate formulas derived in Section 4.2.2 to make the comparison.

Case A : Transmission radius of all nodes is one unit

An exact analysis of blocking probability for all values of the traffic load is difficult. Therefore, we consider a simplified model and focus on the low blocking probability regime. In practice, networks operate in this regime and so the conclusions drawn here have practical significance. In wireline circuit switched networks, blocking probability analysis of multihop calls is based on the reduced load approximation [12]. We make a similar approximation but also consider a further low blocking simplification.

In the low blocking probability regime, almost all calls get serviced and the average load on each link is $\approx k\nu$ (sum of the loads of all calls hopping through a link). Assuming this load to be Poisson, the probability that none of the channels on a link are free (either busy or blocked) can be computed by considering this as an equivalent system with load $k\nu$ on each link and $r = 1$ (Equation 4.30). Let this probability be denoted as P_L and $E()$ be the Erlang B function.

$$\tilde{\nu} = \frac{1 + (2k\nu - 1)x^3}{x^3}, \quad (k\nu x^2 + 1 = \frac{1}{x}) \tag{4.58}$$

$$P_L = E(\tilde{\nu}, p) \tag{4.59}$$

For a k hop call to be served, it must not be blocked on all the hops along the entire length of the path. Therefore, the probability of blocking of the call is greater than the probability of blocking on the first hop. But, the probability of blocking on the first hop is equal to P_L. Denoting the blocking probability of a call as P_B^{caseA} we have,

$$P_B^{caseA} > P_L = E(\tilde{\nu}, p) \tag{4.60}$$

This is a very weak lower bound on blocking probability, yet it suffices to show that using a larger transmission radius yields an even lower blocking probability. To see this consider Case B.

Case B : Transmission radius of all nodes is k units

For this system, the blocking probability of a call has been computed in Section 4.2.2. Let P_B^{case2} denote this blocking probability. Substituting, $r = k$ and load $= \nu$, in Equation 4.30, we get,

$$\tilde{\nu} = \frac{1 + (2k\nu - 1)x^{2k+1}}{x^{2k+1}}, \quad (\nu x^{2k} + 1 = \frac{1}{x}) \tag{4.61}$$

$$P_B^{caseB} = E(\tilde{\nu}, p) \tag{4.62}$$

First, we show that $P_B^{caseB} < P_L$ from which it follows (using inequality 4.60) that $P_B^{caseB} < P_B^{caseA}$. To prove $P_B^{caseB} < P_L$, we make an argument identical to that made in the simpler example considered earlier. For load $\nu = 0$, P_B^{caseB} and P_L are both zero. Excluding the $\nu \to \infty$ case, we will show that for all finite loads and all $k > 1$, there does not exist a load ν for which the two blocking probabilities are equal. Since

P_B^{caseB} and P_L are continuous functions of ν, we conclude that one function is always higher than the other. To complete the proof, we finally show that $P_B^{caseB} < P_L$ for a specific ν and k (here we take $\nu = 1$ and $k = 2$).

Let us assume that $\nu > 0$ is such that P_B^{caseB} and P_L are equal. Let x_1 be the root in (0,1) of the function $k\nu x^3 + x = 1$ and x_2 be the root in (0,1) of the function $\nu x^{2k+1} + x = 1$. Since by assumption $P_B^{caseB} = P_L$, we can equate the effective load $\tilde{\nu}$ in the Erlang B formula for P_B^{caseB} and P_L.

$$\frac{1 + (2k\nu - 1)x_2^{2k+1}}{x_2^{2k+1}} = \frac{1 + (2k\nu - 1)x_1^3}{x_1^3} \tag{4.63}$$

$$x_2^{2k+1} = x_1^3 \tag{4.64}$$

x_1 and x_2 also satisfy the following equalities,

$$k\nu x_1^3 + x_1 = 1 \tag{4.65}$$

$$\nu x_2^{2k+1} + x_2 = 1 \tag{4.66}$$

Substituting, $\nu x_1^3 = \nu x_2^{2k+1} = 1 - x_2 = 1 - x_1^{3/(2k+1)}$, in Equation 4.65 we get,

$$k - kx_1^{3/(2k+1)} + x_1 = 1 \tag{4.67}$$

However, no value of x_1 lying in (0,1) satisfies the above equation. This can be proved by noting that the function $k - kx^{3/(2k+1)} + x$ is a monotonically decreasing function in (0,1) (its derivative, $1 - \frac{3k}{2k+1}x^{(2-2k)/(2k+1)}$ is negative in (0,1), for $k > 1$). It takes value $k(> 1)$ at $x = 0$ and value 1 at $x = 1$. Thus, for $x \in (0, 1)$, $k - kx^{3/(2k+1)} + x \neq 1$ and we arrive at a contradiction. The conclusion drawn is that for finite $\nu > 0$, P_B^{caseB} and P_L cannot be equal. Similarly, we can show that for a fixed ν, there does not exist $k > 1$ (taking k real) such that $P_B^{caseB} = P_L$ (they are equal at $k = 1$). Since P_B^{caseB} and P_L are continuous functions of ν and k (taking k real), we conclude that one function is always higher than the other. For $\nu = 1$ and $k = 2$, it can be easily shown that $P_B^{caseB} < P_L$ (Note that if there exists a (ν, k) for which $P_B^{caseB} > P_L$, then the two curves must cross each other at some $(\hat{\nu}, \hat{k})$ but we have shown that no such $(\hat{\nu}, \hat{k})$ exists). This completes the proof that for all finite ν and real $k > 1$, $P_B^{caseB} < P_B^{caseA}$. The result also holds true for k being integer (since it holds for real k). Thus, we conclude that it is preferable, in terms of blocking probability, to use a larger transmission radius.

Figure 4-4 presents simulation results verifying this claim. The blocking probability of the center call is computed in each simulation (as the edge effects are minimal for this call). The first plot has all calls of length 3 and two scenarios of radius 1 and 3. The second plot has all calls of length 6 with radius 1 and 6. Note that the reduction in blocking probability by using a larger transmission radius is a few orders of magnitude and this difference increases with the length of the calls.

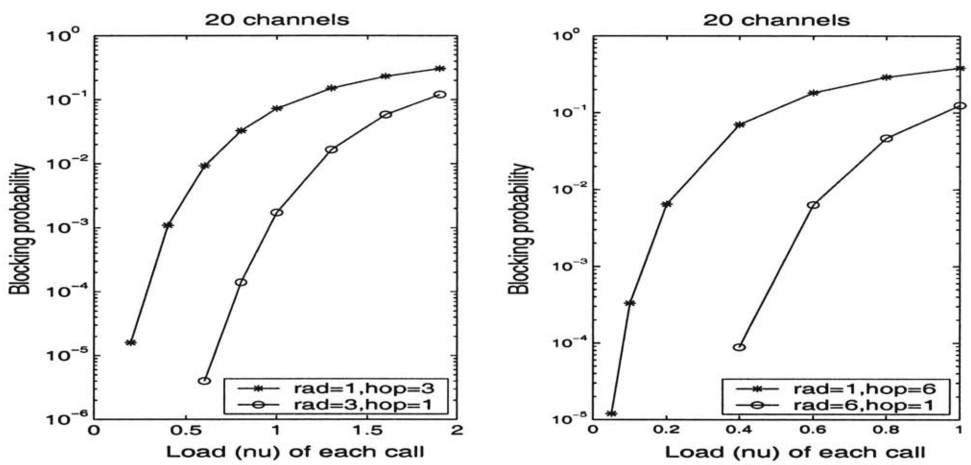

Figure 4-4: Comparison of blocking probability in a line network for calls of length 3 and 6 and different transmission radius.

4.4 Effect of Transmission Radius in a Grid Network

In the single channel line network with all calls single hop, we observed that as $\nu \to 0$, $\tilde{\nu} = 5\nu$ (total load of interfering calls plus the load on the concerned link). This observation can be shown to hold for a single channel general network as well by making the low load approximation in the steady state probability distribution i.e. as $\nu \rightarrow 0$, $\tilde{\nu} = \alpha\nu$ where α = total number of interfering calls + 1 (this includes the concerned call). This is proved as follows.

Consider a single channel network with a general topology. Since there is only a single channel, such a network can have only single hop calls. Let calls arrive according to independent Poisson processes of rate λ and the service times be independent identically distributed according to an Exponential distribution with mean $1/\mu$. As before, define the state of the network as $\mathbf{n}(t) = \{n_k(t)\}, k \in$ set of calls, where n_k = number of calls k in progress ($0 \le n_k \le 1$). The stochastic process $(\mathbf{n}(t), t \ge 0)$ is an aperiodic, irreducible, finite state Markov process and hence has a unique stationary distribution $\pi(\mathbf{n})$ given by the following product form solution ($\mathcal{G}$ is the feasible state space, N is the normalization constant and $\nu = \lambda/\mu$).

$$\pi(\mathbf{n}) = \frac{\prod_k \nu^{n_k}}{N}, \quad \mathbf{n} \in \mathcal{G} \tag{4.68}$$

$$N = \sum_{\mathcal{G}} \prod_k \nu^{n_k} \tag{4.69}$$

The blocking probability, P_B, of a particular call can be computed by summing $\pi(\mathbf{n})$ over all those states that are blocking for that call. Let this set be denoted as $\mathcal{B}$.

$$P_B = \frac{\sum_{\mathcal{B}} \prod_k \nu^{n_k}}{\sum_{\mathcal{G}} \prod_k \nu^{n_k}} \tag{4.70}$$

In the low load regime ($\nu \rightarrow 0$), we can re-write Equation 4.70 by partitioning $\mathcal{B}$ into a set of states (denote as $\mathcal{B}_1$) for which there is only one active call and the set of states (denote as $\mathcal{B}_2$) in which more than one call is active. The set of blocking states for which only one call is active must consist of states in which either the concerned call is active or one of the neighboring interfering calls is active. Let α = number of interfering calls + 1. Before proceeding forward we make the following definition.

Let $f(x)$ be a function of x. We say that $f(x)$ has order $o(x)$ if,

$$\lim_{x \to 0} \frac{f(x)}{x} = 0 \tag{4.71}$$

With this definition we have,

$$\begin{aligned}
\sum_{\mathcal{B}} \prod_k \nu^{n_k} &= \sum_{\mathcal{B}_1} \prod_k \nu^{n_k} + \sum_{\mathcal{B}_2} \prod_k \nu^{n_k} && (4.72)\\
&= \alpha\nu + o(\nu) && (4.73)\\
N &= \sum_{\mathcal{G}} \prod_k \nu^{n_k} = 1 + \beta\nu + o(\nu) && (4.74)
\end{aligned}$$

where β is some finite constant. We can now compute the effective load $\tilde{\nu}$ for small ν by equating the M/M/1/1 expression and Equation 4.70.

$$\begin{aligned}
\frac{\tilde{\nu}}{1+\tilde{\nu}} &= \frac{\sum_{\mathcal{B}} \prod_k \nu^{n_k}}{\sum_{\mathcal{G}} \prod_k \nu^{n_k}} && (4.75)\\
&= \frac{\alpha\nu + o(\nu)}{1 + \beta\nu + o(\nu)} && (4.76)\\
\tilde{\nu} &\approx \alpha\nu, \qquad \text{small } \nu && (4.77)
\end{aligned}$$

To extend the approximation to the multiple channel case, we follow the model of Section 3.3.1. Thus, at low loads, the blocking probability of a single hop call is $P_B = E(\tilde{\nu}, p)$, where $\tilde{\nu}$ = the total load of interfering calls plus the load of the concerned call. However, to consider the effect of transmission radius, we need to compute the blocking probability in the multihop case as well. In this case, we make the reduced load approximation [12] with the low blocking simplification.

We consider the example of a grid network where nodes are located as a two dimensional mesh. To be concise, we consider a grid network with all calls of length 3 and load ν (calls are between nodes $\{x, y\} \to \{x+3, y\}$ and $\{x, y\} \to \{x, y+3\}$). The approach can be easily generalized to calls of any arbitrary length. In the first scenario, the transmission radius of each node is 1. Here, each link has 23 interfering links (including itself). Figure 4-5 shows the set of interfering links for link $A \leftrightarrow B$.

In the figure the interfering links are marked 'X' and the transmission radius of all the nodes is unity. Note that there are 22 links marked 'X' such that a bi-directional data transfer on these links interferes with the data transfer $A \leftrightarrow B$ (thus, $\alpha = 22{+}1 = 23$). We first make the reduced load approximation to compute the average load on each link. In the low blocking regime, almost all calls get served and the average load on each link is $\approx 3\nu$. Treating the system as an equivalent network with load 3ν on each link, the probability that no channel is free at a link is $E(23 * 3\nu, p)$. Making a further approximation that the links block independently, the probability that a 3-hop call is blocked is $\approx 1 - (1 - E(69\nu, p))^3 \approx 3E(69\nu, p)$.

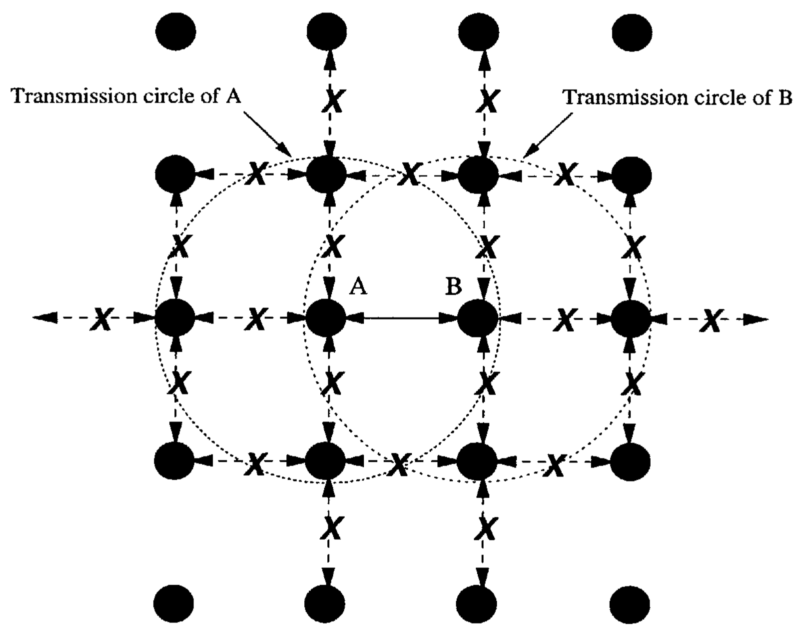

Figure 4-5: Constraints on the service of a bi-directional call in a grid network with nodes having unit transmission radius.

In the second scenario, transmission radius of each node is 3 and hence all calls are single hop. As stated earlier, for small ν, the effective load on a single hop link is equal to the sum of the load on neighboring interfering links and its own load. For a grid network with radius 3 and single hop calls, each link has 135 interfering links (including the concerned link) all of which carry load ν. Thus, the value of $\tilde{\nu} = 135\nu$

and the blocking probability of a call is $P_B = E(135\nu, p)$. Clearly, for low ν and moderate number of channels, $E(135\nu, p) > 3E(69\nu, p)$ (to see this take $\nu = 1/135$ then, $E(1, p) > 3E(\frac{69}{135}, p)$ for $p \geq 3$), which suggests that it is preferable in terms of blocking probability to use a smaller transmission radius.

Figure 4-6 presents simulation results that justify this conclusion. The plot shows the blocking probability of the center call in a 20X20 grid with 30 channels. All calls are of length of 3 and two cases of radius 1 and 3 are considered.

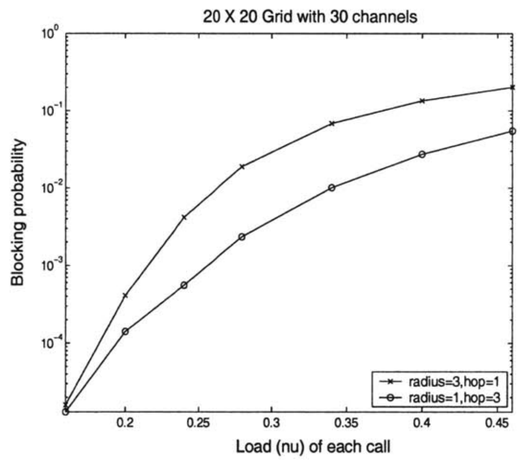

Figure 4-6: Comparison of blocking probability in a grid network for calls of length 3 and different transmission radius.

4.5 Summary

In this chapter, we considered the effect of transmission radius on blocking probability in a line and a grid network. We studied the following tradeoff. If we use a smaller transmission radius then a call would require more hops and multiple channels would be needed to service the call (as neighboring hops cannot share the same channel). If we use a larger transmission radius (such that the destination is within the transmission range), the call can be serviced in a single hop and a single channel is required to serve the call. However, the increased transmission power will cause interference with many more nodes.

We first presented the blocking probability analysis in a generalized wireless line network with bi-directional calls (Section 4.2). The result obtained in the single channel case is (Equation 4.2),

$$P_B = 1 - \frac{x^{2r+1}}{1 + 2r\nu x^{2r+1}}\ , \qquad \nu x^{2r+1} + x = 1$$

In the multiple channel case (Section 4.2.2), we considered a simplified model (that uses the single channel result) and obtained approximate blocking probability formulas that very accurately predict the values obtained from simulations, especially for low to moderate number of channels (Figure 4-1).

To study the effect of transmission radius, we first considered a simple non-trivial example in a line network that clearly highlights this tradeoff (Theorem 4). Then using the approximate blocking probability formulas derived for the multiple channel case we draw the following conclusion. Assuming that all calls are of the same length, in the sparse line topology it is preferable to use larger transmission radius and communicate directly rather than go multihop to reach the destination; while in the dense grid topology it is more desirable to use smaller transmission radius.

Chapter 5

Dynamic Channel Assignment Algorithms

5.1 Introduction

In the last chapter, we have seen that in a wireless network, the blocking probability of calls is affected by the transmission radius of the nodes (which in turn depends on the transmission power used by the nodes). However, once we fix the transmission radius of the nodes, blocking probability of calls also depends on how we assign the channels to the incoming calls. The channel assignment algorithms have a profound impact on the blocking of calls. In this chapter, we study the performance of some dynamic channel allocation policies. We also propose an algorithm called the Local Channel Re-use Algorithm and show that it performs better than other policies.

We will consider two network topologies for comparing the different policies. The first one is the line network where the nodes are located in a line at unit distance apart from each other. The second one is the grid network where the nodes are located as a two dimensional mesh. The reason for choosing these two topologies is that they are good representatives of a sparse network and a dense network respectively and the conclusions drawn here have significant implications in the design of such networks. To independently study the effect of channel assignment on blocking probability (eliminate the effect of transmission radius), we fix the transmission radius of each node at unity by appropriately choosing the transmission power.

We will consider only dynamic channel allocation policies where the available channels are shared by all the nodes in the network and the channels are allocated to the calls in real-time. We could also do a fixed spatial partitioning of the channels such that the spatial channel reuse constraints are satisfied. In this case the set of interfering links will be allocated different sets of channels. Each link then acts independently and a channel is allocated to a requesting call whenever the set of available channels for that link is non-empty. Such a channel partitioning scheme would be highly network topology dependent. Since a wireless ad-hoc network does not have a fixed topology, a general scheme of channel partitioning might be highly inefficient.

Corson and Zhu [3] considered the problem of calculating the maximum available bandwidth (number of channels) on a multihop path. They showed the problem to be NP-complete and proposed a heuristic algorithm to calculate the bandwidth. However, they did not consider the problem of channel assignment to minimize the call blocking probability when the incoming call does not require the entire bandwidth. Our goal in this chapter is to first present channel assignment algorithms for one-hop calls where the source and the destination nodes are neighbors. We then extend it to the case of multihop calls. We regard a multihop call as a sequence of single hop calls and implement the channel assignment algorithm repeatedly along the entire length of the multihop path.

The dynamic channel allocation algorithms that we consider in this work include the rearrangement algorithm, the random algorithm, the first fit algorithm and the local channel re-use algorithm. These algorithms will be explained in more detail in the sections that follow. Whereas the rearrangement algorithm requires rearrangement of the channels allocated to the calls in progress, the other policies do not require any rearrangement of the channels. Hence we classify the other policies as non-rearranging policies. Non-rearranging policies are practically more appealing in large wireless networks as they can be implemented much more easily than the rearrangement policy.

The rest of the chapter is organized as follows. Section 5.2 describes the rearrangement algorithm and gives mathematical conditions for the feasibility of rearrangement in a line network. Section 5.3 explains the non-rearranging channel assignment policies which include the random, first fit and the LCRA algorithms. Finally, Section 5.4 presents simulation results for the line and the grid network. In this section, we also draw conclusions about the effect of other parameters such as the hop length of the calls and the degree of the nodes (the number of neighbors a node has) on call blocking. We consider only bi-directional calls for the simulations. However, similar results can be obtained for the uni-directional case as well. In the simulations, blocking probability of a call is computed as the ratio of the number of calls rejected to the number of call arrivals over a very large simulation time.

5.2 Rearrangement Algorithm

The rearrangement algorithm was first presented in cellular networks by Everitt and Macfadyen [14]. Since then it has been widely used as a benchmark to compare the performance of other policies. The rearrangement policy has a unique feature that it does not do any admission control. Under this policy, in any state of the network, if there are resources available to admit the incoming call then the call will be accepted even if this requires rearranging the channels allocated to the calls in progress. Thus to describe this policy, we need to make some definitions about the admissibility of a state. We will assume a fixed route system where the route of each call is fixed and does not change dynamically with time. If a call cannot be accepted on a particular route then it is dropped and does not attempt service through another route.

We first describe the algorithm for a network with a general topology and then show that in the case of a line network it has a very simple description. For a general network, let $\mathcal{Q}$ be the set of all source-destination pairs and $q \in \mathcal{Q}$ be a particular call. Note that the set $\mathcal{Q}$ can consist of both single and multi-hop calls. Let α_q denote the number of calls of type q in progress. Let the state of the network be $\boldsymbol{\alpha} = \{\alpha_q\}, q \in \mathcal{Q}$.

Definition: State $\boldsymbol{\alpha}$ is admissible if there exists an allocation of channels to the calls $(\alpha_q, \in \boldsymbol{\alpha})$ such that the wireless transmission/reception constraints as described in Section 2.1, are satisfied.

The rearrangement policy accepts a call whenever this leads to a state $\boldsymbol{\alpha}$ which is admissible. The policy thus accepts a call whenever possible, even if this involves a rearrangement of the channels allocated to calls already in progress. Implementing this algorithm in large general networks is practically impossible. Even simulating this policy involves a search over a large set of feasible channel assignments which in most networks is practically infeasible. However, in case of a line network there is a simple characterization of the state space in terms of necessary and sufficient conditions for a state to be admissible. The following is a description of these conditions for a line network.

Let us consider a line network with $m + 1$ nodes located unit distance apart and transmission radius of each node equal to unity. This implies that each node can communicate directly with a node on its left and a node on its right. For a node $X_k, k \in \{1, .., m+1\}$ in the line network (other than the edge nodes that have only one neighbor), there is a link between nodes (X_{k-1}, X_k), labeled L_{k-1}, and between nodes (X_k, X_{k+1}), labeled L_k. Thus, there are in total m links $L_1, L_2, ..., L_m$, in the line network. Let n_k be the number of calls in progress on link L_k and define the vector $\mathbf{n} = (n_1, n_2, .., n_m)$. Let p be the number of channels available in the network. Since we consider a fixed route system, given a particular state $\boldsymbol{\alpha}$ (state of the calls, as described earlier) of the line network, we can determine the number of calls (n_k) on each link. Therefore, given $\boldsymbol{\alpha}$, we can determine the state $(\mathbf{n})$ of the links. The necessary and sufficient conditions for a state $\mathbf{n}$ to be admissible are,

$$n_k \geq 0 \quad \forall k \in \{1, ..., m\} \tag{5.1}$$

$$n_1 \leq p \tag{5.2}$$

$$n_1 + n_2 \leq p \tag{5.3}$$

$$n_k + n_{k-1} + n_{k-2} \leq p \quad , \forall k \in \{3, ..., m\} \tag{5.4}$$

Necessity: The necessity of the above conditions is proved by noting that calls on links L_k, L_{k-1}, L_{k-2} form a set of mutually interfering calls and hence any channel assignment must satisfy $n_k + n_{k-1} + n_{k-2} \leq p$.

Sufficiency: The sufficiency of conditions 5.2-5.4 is proved by presenting a channel assignment scheme that assigns channels to the calls whenever the state $\mathbf{n}$ satisfies those conditions. Thus, we begin by assuming that $\mathbf{n}$ satisfies the above conditions. Consider an assignment scheme that assigns channels starting from the leftmost link, L_1, of the line network. The first three links, L_1, L_2, L_3, are assigned a set of n_1, n_2, n_3 non-overlapping channels respectively. Condition $n_1 + n_2 + n_3 \leq p$ ensures that such a set of non-overlapping channels exists. We next assign channels to calls on link L_4. There are two cases to be considered as follows.

- If $n_4 \leq n_1$, then we assign n_4 channels from among the set of n_1 channels that were assigned to calls on link L_1. Since calls on link L_1 do not interfere with calls on link L_4, the assignment of channels to calls on links L_1, L_2, L_3, L_4 is admissible.

- If $n_4 > n_1$, then we assign n_1 channels that were assigned to calls on link L_1 plus the additional channels $n_4 - n_1$. The additional channels are chosen such that they are non-overlapping with the channels assigned to calls on links L_2, L_3. Condition $n_2 + n_3 + n_4 \leq p$ ensures that such a set exists. Thus, we have an admissible assignment of channels to calls on links L_1, L_2, L_3 and L_4.

Proceeding this way, let us assume that we have an admissible assignment on links L_1,

.., L_{k-1}. We next assign channels to calls on link L_k by comparing n_k and n_{k-3} and following the above two steps of assignment replacing n_1 with n_{k-3} and n_4 with n_k. Thus by induction, if the above set of conditions on $n_k, k \in \{1, 2, .., m\}$ are satisfied, we have a feasible assignment of channels to the calls.

The above set of conditions and the channel assignment approach can be easily generalized to a line network where the nodes have transmission radius $r, r \in \mathbf{Z}^+$.

5.3 Non-rearranging Algorithms

This section considers algorithms that do not require rearrangement of the channels allocated to the calls already in progress. The algorithms that we study are the random algorithm, the first fit algorithm and the local channel re-use algorithm. These algorithms base their decision on the set of free channels available at a node. Free channels refer to those channels such that the acceptance of a call in those channels does not violate the wireless transmission/reception constraints as explained earlier in Section 2.1. If a call cannot be assigned a channel then it is dropped.

Let $\mathcal{F}_N$ denote the set of free channels at node N. $\mathcal{F}_N$ contains all those channels in which node N and its neighbors are inactive (not transmitting/receiving). Similarly we can define the set of free channels for a link $N \leftrightarrow M$ (N, M are two nodes) as the set of all those channels that are free at both nodes N and M. This set can be obtained from the sets $\mathcal{F}_N$ and $\mathcal{F}_M$ as, $\mathcal{F}_{N \leftrightarrow M} = \mathcal{F}_N \cap \mathcal{F}_M$.

Single Hop Calls

Let there be a single hop call between nodes S and D. $\mathcal{F}_S$ and $\mathcal{F}_D$ are the set of free channels associated with nodes S and D respectively. Since the call is single hop, nodes S and D are neighbors. The set of available channels for this call using link $S \leftrightarrow D$ is the intersection between sets $\mathcal{F}_S$ and $\mathcal{F}_D$ ($\mathcal{F}_S \cap \mathcal{F}_D$). Let $\tilde{g}()$ be the decision function which selects one channel from the set $\mathcal{F}_S \cap \mathcal{F}_D$. The difference between the random, first fit and the local channel re-use algorithms is in the decision function $\tilde{g}()$. These differences are explained in the later sections. Let γ_c be the chosen channel then γ_c is given by $\gamma_c = \tilde{g}(\mathcal{F}_S \cap \mathcal{F}_D)$.

Multihop Calls

A multihop call is regarded as a sequence of single hop calls where the first call arrives on the first link followed by an arrival on the second link and so on until the last link of the multihop path. With this interpretation, we assign channels for the multihop call by repeating the single hop assignment procedure in a sequence over the multihop path. After choosing a channel on a particular hop, the information about the chosen channel is communicated to the next hop node before the channel decision is made on the next hop.

To be more precise, consider a multihop call between nodes S and D along the path S, N_1, N_2,.., D. To allocate channels to this call start at the source node S. Let the channel chosen on link $S \leftrightarrow N_1$ be denoted as γ_1. Then, $\gamma_1 = \tilde{g}(\mathcal{F}_S \cap \mathcal{F}_{N_1})$. The information about the selected channel on link $S \leftrightarrow N_1$ is communicated to node N_2. Neighbors of nodes S and N_1 along the path including themselves (nodes S, N_1, N_2) cannot choose channel γ_1. Thus, nodes S, N_1 and N_2 update their set of free channels. Let the updated sets of nodes N_1 and N_2 be denoted as $\tilde{\mathcal{F}}_{N_1}$, $\tilde{\mathcal{F}}_{N_2}$. Channel γ_2 is then chosen from the set $\tilde{\mathcal{F}}_{N_1} \cap \tilde{\mathcal{F}}_{N_2}$ as $\gamma_2 = \tilde{g}(\tilde{\mathcal{F}}_{N_1} \cap \tilde{\mathcal{F}}_{N_2})$. This information about the selected channel (γ_2) is communicated to node N_3. Channel γ_3 is chosen from the updated sets of free channels of nodes N_2 and N_3. The process repeats until a channel is allocated on the last link $N_k \leftrightarrow D$. If the assignment is successful, all other neighboring nodes (neighbors of $S, N_1, .., D$) other than those along the path update their set of free channels. If at any step there are no free channels available on that link then the call is dropped and the nodes along the path once again update their set of free channels to the set present before the call request was made.

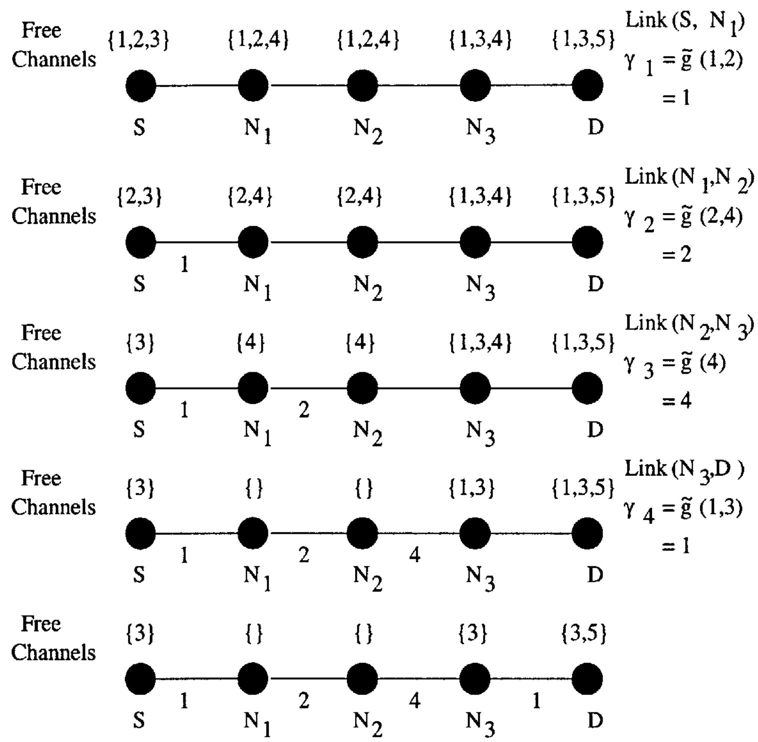

Figure 5-1: Multihop channel assignment (S, N_1, N_2, N_3, D is the path of the multihop call).

The procedure is illustrated in the example shown in Figure 5-1. The figure shows only the source-destination path while all the other nodes of the network are not shown. Since the call is 4-hop long, there are 4 steps involved and an arbitrary channel assignment policy is followed for the purpose of illustration. The last step shows the final allocated set of channels. In the first step $\mathcal{F}_S \cap \mathcal{F}_{N_1} \equiv \{1,2\}$ and channel 1 is chosen. Neighbors of nodes S and N_1 along the path including themselves (nodes S, N_1, N_2) cannot choose channel 1. Thus, Nodes S, N_1 and N_2 update their set of free channels as shown in the second step. In the second step, the set of free channels on link $N_1 \leftrightarrow N_2$ is $\tilde{\mathcal{F}}_{N_1} \cap \tilde{\mathcal{F}}_{N_2} \equiv \{2,4\}$ and channel 2 is chosen. Neighbors of nodes N_1 and N_2 along the path including themselves (nodes S, N_1, N_2, N_3) cannot choose channel 2. They update their set of free channels shown in the third step. $\tilde{\mathcal{F}}_{N_2} \cap \tilde{\mathcal{F}}_{N_3} \equiv \{4\}$ and channel 4 is chosen. Neighbors of nodes N_2 and N_3 along the path including themselves (nodes N_1, N_2, N_3, D) cannot choose channel 4. They update their sets of free channels and in the fourth step, $\tilde{\mathcal{F}}_{N_3} \cap \tilde{\mathcal{F}}_D \equiv \{1, 3\}$ of which channel 1 is chosen. The final assignment is shown in the last step. All other neighbors of S, N_1, N_2, N_3, D (not shown in the figure) update their sets of free channels based on this assignment.

As noted earlier, the random, first fit and the local channel re-use algorithms differ in the decision function $\tilde{g}()$ given a particular set Γ of free channels. These differences are explained next.

5.3.1 Random Algorithm

The random channel assignment algorithm allocates a channel randomly from the set of free channels available on the link on which the call request was made. Thus, the channel decision function $\tilde{g}(\Gamma)$ chooses a channel γ_c randomly from among the channels in the set Γ.

5.3.2 First Fit Algorithm

The first fit algorithm orders the channels $(\gamma_1, \gamma_2, .., \gamma_p)$ by assigning them an index number. Assuming that all the channels are equivalent, the index numbers are assigned to the channels arbitrarily. The channel decision function $\tilde{g}(\Gamma)$ chooses a channel γ_c that has the lowest index among the channels in the set Γ. This policy has been studied earlier by researchers for wavelength assignment in WDM optical networks [23].

Intuitively, the first fit algorithm uses the channel resources more efficiently than the random algorithm. In the first fit algorithm, channels with lower index numbers are used more often for servicing the calls than the higher index numbered channels. This causes a packing of the calls onto the lower index numbered channels. Thus two calls which are non-interfering are more likely to share a channel in the first fit policy than in the random assignment policy. This effectively entails re-use of the channels already in use in the network leaving more resources free for the future calls.

5.3.3 Local Channel Re-use Algorithm (LCRA)

The local channel re-use algorithm assigns an index number to the channels (γ_1, γ_2,..,γ_p) in a way identical to the first fit algorithm. The channel decision function, $\tilde{g}()$, minimizes a certain criterion Ω (explained later) over the set of available free channels. We first present a mathematical description of the decision function followed by an intuitive explanation of why the algorithm uses the channels more efficiently than the random policy.

To explain the channel decision function $\tilde{g}()$, consider a link $S \leftrightarrow D$ on which the channel needs to be allocated. $\mathcal{F}_S$ and $\mathcal{F}_D$ are the set of free channels in the present state associated with nodes S and D respectively. Let Γ represent the set of free channels available on the link $S \leftrightarrow D$. The set Γ is given by $\Gamma = \mathcal{F}_S \cap \mathcal{F}_D$. The elements of the set Γ (the free channels) are denoted as $\gamma_1, \gamma_2, .., \gamma_{|\Gamma|}$ where, $|\Gamma|$ = number of elements in the set Γ. To make a channel decision, the algorithm also takes into account the free channels available at the neighbors of nodes S and D (the neighboring nodes are represented by the sets $\mathcal{N}_S$ and $\mathcal{N}_D$ respectively). Let the nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ be denoted as $N_1, N_2, .., N_{|\mathcal{N}_S \cup \mathcal{N}_D|}$ and let $\mathcal{F}_{N_1}, \mathcal{F}_{N_2}, ..$ be their set of free channels in the present state when the service request is made on link $S \leftrightarrow D$. We want to choose a channel, γ_c, such that γ_c minimizes the number of nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ that have channel γ_c as a free channel in the present state. Choosing such a channel will make that channel blocked for the least number of neighboring nodes. Define an indicator variable I_N and a function $\Omega()$ as follows,

$$
\begin{aligned}
I_{N_i}(\gamma_k) &= 1, \text{ if } \gamma_k \text{ is free at node } N_i \text{ in the present state} \\
&= 0, \text{ otherwise.}
\end{aligned}
$$

$$\begin{aligned}\Omega(\gamma_k) &= \text{Number of neighbors of S and D that have } \gamma_k \text{ as a free channel} \\ &= \sum_{N \in \mathcal{N}_S \cup \mathcal{N}_D} I_N(\gamma_k)\end{aligned}$$

Choose the channel γ_c that minimizes the function $\Omega()$ over the set, Γ, of free channels. Thus we get,

$$\gamma_c = \tilde{g}(\Gamma) = \arg\ \min_{\gamma_k \in \Gamma}\ \Omega(\gamma_k) \tag{5.5}$$

If there are more than one γ_k that minimize $\Omega()$ then the smallest indexed γ_k is selected.

To understand how this algorithm uses the channels in an efficient manner, consider that we choose channel γ_c from the set Γ. Then, the neighbors of node S (set $\mathcal{N}_S$) and node D (set $\mathcal{N}_D$) cannot use channel γ_c as long as the allocated call is active. Therefore, given that we choose γ_c, all those nodes in $\mathcal{N}_S \cup \mathcal{N}_D$ that had γ_c as a free channel before the call request was made, remove γ_c from their set of free channels. It might be beneficial to have this set of nodes that get blocked in channel γ_c, to be as small as possible. The fact that some nodes (in $\mathcal{N}_S \cup \mathcal{N}_D$) do not have γ_c in their set of free channels also implies that there is presently an active call in the neighborhood of those nodes but that call does not interfere with the new incoming call on $S \leftrightarrow D$. Choosing such a channel will then lead to a local re-use of the channels. Thus we see that by having an optimization criterion as discussed earlier, the algorithm indirectly tries to locally re-use the channels as much as possible.

In the above formulation, we assumed that all the nodes in the network experience the same call arrival rates. However in a general network with multihop calls, there might be more channel requests at some nodes than at other nodes. In such a situation, we can generalize the LCRA algorithm by assigning weights to the nodes. These weights represent a priority ordering of the nodes. The generalized form of the

LCRA algorithm is as follows.

Let w_N be the weight assigned to node N. A node with a higher priority is assigned a higher weight. These weights can be chosen based on the objective that needs to be achieved. In the case of minimizing blocking probability, the weight of a node can be chosen in proportion to the rate at which channel requests are made at that node.

Let function $\Omega_W()$ be defined as,

$$\Omega_W(\gamma_k) = \sum_{N \in \mathcal{N}_S \cup \mathcal{N}_D} I_N(\gamma_k) w_N$$

Choose channel γ_c that minimizes the function $\Omega_W()$ over the set, Γ, of free channels. Thus we get,

$$\gamma_c = \tilde{g}(\Gamma) = \arg\ \min_{\gamma_k \in \Gamma}\ \ \Omega_W(\gamma_k) \tag{5.6}$$

If there are more than one γ_k that minimize $\Omega_W()$ then the smallest indexed γ_k is selected.

5.4 Simulation Results

In this section, we present simulation results that compare the performance of the above stated algorithms in a line and a grid network. We compute the blocking probability of the center call as the edge effects for this call are minimal. The simulation results also show trends about the effects of other network parameters such as density of the nodes and hop length of the calls. Since a line and a grid network are good representatives of a sparse and a dense network respectively, these conclusions have significant implications in the design of such networks. In both networks, the transmission radius of each node is fixed at unity. The arrival process of all the calls is Poisson and of the same rate while the departure time is Exponentially distributed with mean 1. The load in the plots is in Erlangs and all calls are assumed to be bi-directional. All gains are calculated with respect to the random channel assignment policy.

Figure 5-2 compares the blocking probability in a line network with 30 nodes, unit length calls and 50 channels. In the low blocking regime (10^{-3}-10^{-1}), substantial gain in blocking probability as compared to the random policy is achieved. LCRA performs better than both the random and the first fit algorithms. The significance of having lower blocking probability is that if we fix a particular value of blocking probability, then, LCRA can support a higher load for each call as compared to random and first fit algorithms. As expected the rearrangement algorithm has the lowest blocking probability.

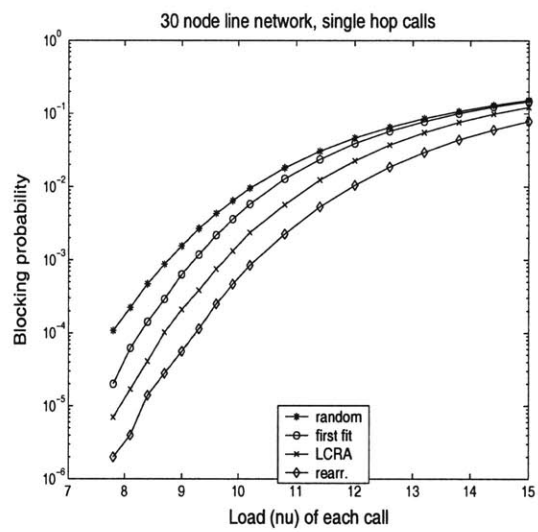

Figure 5-2: Comparison of blocking probability in a line network for unit length calls and different channel assignment algorithms.

Simulating the rearrangement policy in a grid network is practically difficult. Therefore, in a grid network we compare the blocking probability for the random, first fit and the LCRA algorithms. Figure 5-3 shows the comparison plot for a 20X20 grid with 50 channels and unit length calls. Blocking probability gains as compared to the random policy are higher than in the line network. This observation is somewhat intuitive as in a grid network a node has more interfering neighbors as compared to a line network. Therefore, spatially re-using the channels will pack the calls onto significantly lesser number of channels, thereby having a greater impact on reducing the blocking probability. This shows that an efficient channel assignment algorithm becomes more critical in dense networks.

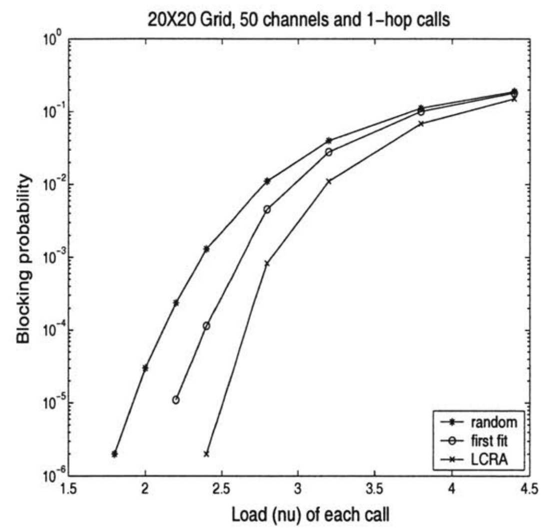

Figure 5-3: Comparison of blocking probability in a grid network for unit length calls and different channel assignment algorithms.

We next consider the effect of hop length of the calls in a line network. We consider a line network with 50 channels and all calls 6-hop long (between nodes 6 units apart). Figure 5-4 compares the blocking probability for random, first fit, LCRA and the rearrangement algorithms. As in the case of unit length calls LCRA outperforms random and first fit algorithms while the rearrangement algorithm has the lowest blocking probability curve.

Next, we compare the performance of LCRA for the case of 1-hop and 6-hop calls by computing the percentage gain in the load (compared to the random algorithm) for a fixed blocking probability. As shown in Figure 5-5, the percentage gain in the load is higher in the 6-hop call case than in the 1-hop case. This can be easily understood by noting that as we maximize the channel reuse over each hop, the gains become

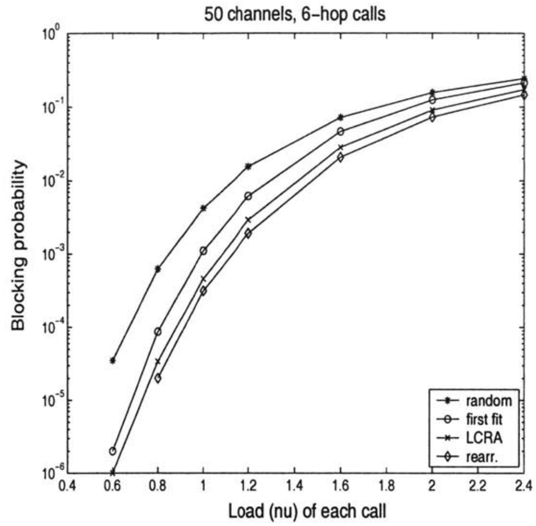

Figure 5-4: Comparison of blocking probability in a line network for 6-hop calls (length 6 units) and different channel assignment algorithms.

higher with increasing path length. The efficient use of the channels has a greater impact for calls with longer length. Thus, we conclude that efficient channel reuse becomes important as the hop length of the calls increase.

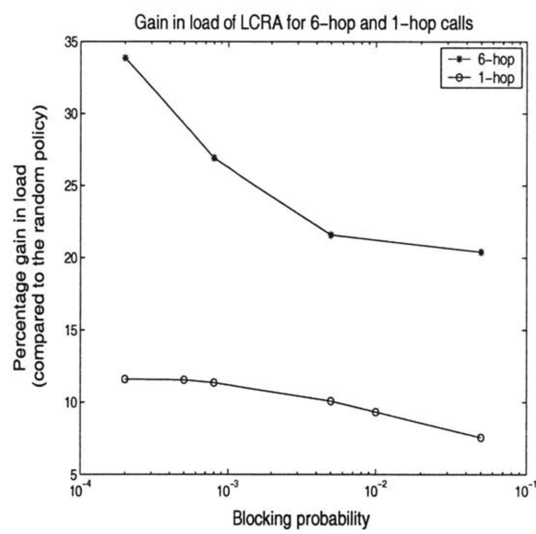

Figure 5-5: Comparison of gain in the load for LCRA for 6-hop and 1-hop calls in a line network.

Chapter 6

Conclusion

We considered the problem of dynamic channel assignment in multihop wireless networks. We followed an analytical approach and derived exact blocking probability formulas for bi-directional and uni-directional calls in a single channel line network.

- **Bi-directional Calls** (Section 3.2.1)

$$\begin{aligned} P_B &= 1 - \frac{x^3}{1 + 2\nu x^3} \\ \nu x^3 + x &= 1 \end{aligned} \tag{6.1}$$

- **Uni-directional Calls** (Section 3.2.2)

$$\begin{aligned} P_B &= 1 - \frac{xy}{\nu^2(1-x)^2 + 4\nu xy} \\ x(1-x)^2 + 4x^2 &= \nu(1-x)^2 \ , \ y^2 = \nu x \end{aligned} \tag{6.2}$$

We compared these formulas to the standard M/M/1/1 blocking probability expression and obtained useful insights on equivalent load (ν') that helped us construct the model for the multiple channel case. Our methodology of analysis is not restrictive to the cases considered in this work but can be applied to other symmetrical systems as well. For example in Section 4.2.1, we used a similar analytical technique to consider a generalization in the single channel line network. We obtained the following blocking probability result for bi-directional calls (The case of $r = 1$ reduces to the earlier result Equation 6.1).

$$\begin{aligned} P_B &= 1 - \frac{x^{2r+1}}{1 + 2r\nu x^{2r+1}} \\ \nu x^{2r+1} + x &= 1 \end{aligned} \tag{6.3}$$

Extending the work to the multiple channel case, we presented a simplified analytical model (Section 3.3) based on the single channel results for the random channel allocation policy. We derived approximate blocking probability formulas that accurately predict the values obtained from simulation results, especially, for low to moderate number of channels (Figures 3-10, 3-11 and 4-1).

We, then, applied the formulas derived to consider the effect of transmission radius on blocking probability. Specifically (assuming that all calls are of the same length), we showed that in the sparse line topology it is preferable to use larger transmission radius and communicate directly rather than go multihop to reach the destination (Section 4.3); while in the dense grid topology it is more desirable to use smaller transmission radius (Section 4.4). This result clearly highlights the significance of the density of the network on blocking probability in a multihop environment.

Finally, we developed a novel channel assignment algorithm (Local Channel Reuse Algorithm, LCRA) that aims at reducing blocking probability by cleverly reusing the channels while also satisfying the wireless transmission/reception constraints (Section 5.3). We compared our algorithm to other channel assignment algorithms such as the rearrangement, random and the first fit algorithms. We showed through simulations that an efficient channel assignment algorithm can significantly reduce blocking probability; especially for densely connected networks and multihop calls (Section 5.4).

Important extensions to this work include a detailed study of the effects of network parameters such as the density of the nodes and the hop length of the calls. It would also be interesting to investigate channel assignment schemes when the nodes are mobile and the network topology changes with time.

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