Surface Wave Distributed Actuators for Planar Manipulation with Application to Human Transport

by

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ABSTRACT

A surface wave distributed actuation method and its proper design for safely transporting bedridden patients is explored in this thesis. Natural surface waves, created by periodic elliptical motion of material particles, can transfer a long object placed upon the crests of the waves in an arbitrary direction within the horizontal plane. Inspired by this natural behavior, a surface wave distributed actuation method and its potential for transporting bedridden patients is explored. First, the basic principle of surface wave distributed actuation is presented, followed by kinematic modeling and analysis. Based on the analysis, modifications to natural wave transport are made to enhance transport efficiency and human comfort. Specifically it is revealed that a new feature called an 'extender' augments natural surface wave motion for enhanced transport efficiency. Analysis reveals that by using the extender feature an object can be transferred by a simplified actuator architecture that makes the concept amenable to hardware realization with a unique 'hidden trough' property that is particularly suited for human transport.

Human tissue physiology is studied to establish worst-case criteria for safe and healthy interactions between the human and the support surface that depends on the pressure and time duration of the interaction. Static models are developed and solved using finite element methods to calculate interaction stresses for realistic, worst-case human-surface wave interaction scenarios. Based on these results a new two-mode surface is designed to secure safe interactions for both long-term support and short term transport tasks.

Several prototypes are designed, built, and tested. Experiments demonstrate the surface wave actuation concept and verify the analytical results. Design trade-offs and guidelines for developing a feasible and practical surface wave bed are discussed based on the prototyping and experiments.

Thesis Supervisor: H. Harry Asada
Title: Ford Professor of Mechanical Engineering
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Abstract

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Chapter 1
Introduction

In this thesis a new type of distributed actuator is introduced to provide flexible, tangential manipulation of objects about their support surface. Specifically this new technology is applied to the tangential transport of bedridden human patients. The kinematic analysis of this new mechanism and the design guidelines for application of surface waves to transport humans are the major topics of this thesis. In this chapter, the motivation for the thesis work is provided along with a description of prior work, a list of specific objectives, and an outline of the thesis.

1.1 Thesis Motivation

There is an increasing need for distributed actuators to provide tangential traction forces in multiple dimensions over a wide contact area. Applications exist on a variety of physical scales. Potential applications on the scale of human dexterous handling ability include parts sorting and reordering, assembly of planar objects, flexible routing of goods and packages, grasp and handling of objects with varying geometries and elastic properties, and general transportation tasks.

On a scale much smaller than human dexterous ability, applications include new designs for micro-manipulators that include active surfaces with a crawling motion for mobility or for manipulation of microscopic objects such as surface mount electronic components. On a much larger scale, a distributed actuator could be utilized to safely handle loads that are beyond the human strength capability or in situations where human safety is compromised.

Emerging requirements in the elder care community for mobility aids to assist the transportation of bedridden patients have fostered a pressing need for technology that
provides hands-off, powered movement of sedentary patients in multiple directions across the surface. This is useful because it allows the caregivers to change sheets, alter patient position of support, and provide care without excessive, forceful physical interaction. To realize this goal requires the development of a novel distributed actuator system with the appropriate kinematic properties and a design of sufficient simplicity to be realistically implemented.

Figure 1.1 - Typical patient-caregiver interaction (Boston Globe-1/99)

Bedridden patients require frequent reconfiguration and transport on and off of their support surface for a variety of tasks to be performed. Traditionally these activities have been handled using the physical exertion and intervention by caregivers in both the home and institutional setting illustrated in Figure 1.1. These exertions are the cause of physical injuries and pain to legions of caregivers. This is a recognized and recently well documented problem. [Owen and Garg, 1989] reported that 89% of the back injury reports filed by hospital nursing personnel claimed a patient handling task as a causal
factor. Nursing personnel in general show a high level of lower-back pain, [Klein et.al., 1984] reported that based on worker compensation claims nursing ranked fifth among all occupations, trailing only heavy labor groups such as construction. Nursing assistants are recognized as being the most vulnerable of all nursing personnel. [Fuortes et.al., 1994] reported that nursing assistants have back injury rates 3.3 times higher than registered nurses and licensed practical nurses. In particular [Garg et.al., 1992] reported that 51% of nursing assistants visited a health care provider in the last three years for work related lower back pain.

1.2 Assessment of Current Technology

In an attempt to mitigate the difficulties associated with physically manipulating patients, numerous mechanical devices have emerged. The Hoyer lift, Trans-Aid, Med-Lifter, Molift, Ambulift, etc. are common commercial lifting assistive devices on the market and purchased by hospitals to lessen the physical burden on nursing aids. [Garg, Owen, et.al., 1991] presented an ergonomic study of three of these devices and five common manual techniques for patient transfer from bed to wheelchair and vice-versa. The results showed that two out of three hoists were perceived to be as physically stressful as manual methods by the nurses. Patients found that these two hoists were more uncomfortable and felt less secure than three of the five manual methods. Ambulift was found to be the least stressful and most comfortable of all methods. This is encouraging in the sense that at least mechanical aids can be beneficial in the care environment. However, [Garg et.al., 1992] through video taping and observations of daily nursing activities, showed that assistive devices such as lifts and belts were used less than 2% of the time. Time to use device, lack of knowledge, patient discomfort, and accessibility
issues contributed heavily to their lack of use. While the problem of patient manipulation is well recognized, and the use of mechanical aids has been considered, no clear, widely accepted and implemented solution has emerged.

In the field of robotics, research on distributed actuation originated in the seminal work by [Hirose and Umetani, 1976]. They developed a series of distributed actuation systems inspired by biological systems. Initially they introduced snake robots consisting of many servoed joints connected in series that can walk in an unconventional manner. Later, they developed an abalone robot, a two-dimensional active surface that can reconfigure and move. [Chirikjian and Burdick, 1991] extended Hirose's snake robot concept by introducing partial differential equations to describe the distributed nature of snake behavior. [Luntz, Rollins, and Messner, 1995] developed a two-dimensional array of manipulator cells that can transfer and orient an object. Each cell consisted of a Swedish wheel for omni-directional vehicles placed upside down to manipulate an object placed on top. Work in the cellular robotics area is concerned with the coordination of individual robot units over a cellular space. [Beni, 1988] details the concept of cellular robotics and treats the problem from a theoretical perspective. Numerous researchers have worked in the area of parts manipulation. [Bohringer, Bhatt, and Goldberg, 1995] detail a manipulator based on the transverse vibrations of a plate.

In this paper, a new approach to distributed actuation inspired by natural surface waves will be presented. Natural waves propagate through continuous media; therefore they are truly distributed. In the past, surface wave motion was utilized in the field of traveling wave type ultrasonic motors actuated by piezoelectric ceramics developed by several groups including [Kenjo and Sashida, 1991], [Hirata and Ueha, 1995], and
[Fleischer, Stein, and Meixner, 1989]. Other novel active materials have been used to generate traveling waves for micro-manipulation such as [Tadokoro, et.al., 1997]. [Suzumori and Asaad, 1996] show the use of coordinated pneumatic actuators for small-scale manipulation. However there appears to be no work seeking to exploit surface wave behavior for the manipulation of large-scale, elastic bodies such as humans.

1.3 Objectives of Thesis

The goal of this paper is two-fold: to demonstrate the utility of surface waves for transport of both rigid and elastic bodies and to present appropriate designs that satisfy the conditions for safely supporting and transporting human bodies. Surface wave distributed actuation may be a solution to the transport of the bedridden, because it does not lift the patient, but transfers horizontally while the patient is lying on the support surface. The development of such a surface wave distributed actuator requires a novel design and sound understanding of the transport mechanism. In particular, to transport a human body, special requirements and conditions must be satisfied with regard to elasticity of the body, comfort, and safety. In the following sections, basic properties of natural surface waves and the principle of surface wave actuation will be presented, followed by kinematic modeling and analysis. Most theoretical development will be performed in the context of transporting bedridden patients, although the utility of the kinematic analysis and design is broad.

Additional consideration must be given to the use of surface waves to transport bedridden patients. In addition to transport functionality, the body must be gently supported, and concentration of tissue pressure must be avoided. This requires a sound understanding of human tissue physiology and its interaction with surface wave actuators.
In this thesis the basic requirements for safely supporting humans are obtained based on human physiological data. Worst-case support interaction models are developed to characterize the human-surface interaction and the results are benchmarked with physiologically based acceptable requirements. Based on the analysis results, a surface design is proposed that ensures safe support at all times, under worst-case conditions.

1.4 Overview of Thesis

The thesis begins with Chapter 2, which introduces the actuation concept of surface waves for transporting human patients. This includes a discussion of why surface waves are suitable for transport of humans. Finally, a new feature called the extender is introduced which leads to a large improvement of efficiency in the transport of humans by surface waves by radically changing the manipulation properties of surface waves in ways that prove to be quite advantageous.

Chapter 3 presents the kinematic analysis of surface waves both with and without extenders. Constraints on design and interesting non-linear properties are discovered and discussed in detail. In addition design synthesis of surface waves is discussed as surface waves are parameterized and optimal design choices can be made.

Chapter 4 introduces several prototype surface wave actuators. The first prototype is a water-based, proof of concept prototype which demonstrates many of the interesting features of surface wave actuators in a very natural, simple setting. With the limitations of this water-based prototype in mind, a new prototype is constructed that is entirely mechanical and it is used to transport humans and verify the kinematic results of Chapter 3 for both cases, with and without extenders. Finally, a third prototype with an actuated rubber membrane is shown which further displays some interesting properties of surface
waves discovered in the kinematic analysis of Chapter 3 and makes a more realistic support surface for human, bedridden patients.

Chapter 5 introduces the human considerations necessary to ensure a safe, efficient, and comfortable support surface. Models are built to predict the human interaction with the support surface under different, realistic, worst-case scenarios, and conclusions are made regarding the suitability of surface waves for supporting patients.

Chapter 6 introduces a new, integrated surface design based on the results of the static interaction analysis of Chapter 5. It emerges that surface wave actuators are indeed a suitable support and transport mechanism for bedridden patients if proper design considerations are made. A simple design is implemented to demonstrate the functionality of surface waves for both gentle, safe support and transport ability.
In this chapter the principle of surface wave actuation for the transport of bedridden patients is presented. In addition the inspiration provided by natural surface waves is explained and the benefits and limitations of these naturally occurring waves are enumerated. A new concept, termed the extender, is introduced to mitigate the limitations of natural surface waves and a workable manipulation concept is born.

2.1 Actuation Principle

Figure 2.1 shows the principle of surface wave actuation for transporting elastic bodies. A body supported at the crests of the surface waves is moved horizontally, along the surface, as the waves propagate. As shown in Figure 2.2, the waves are created by periodic movements of individual points, steps (a) to (d). First a particular surface particle moves upwards (a), contacts the body, supports the body weight, moves in a horizontal direction (b), moves downwards, detaches from the body (c), moves horizontally back to the original point (d), and repeats the same process. The individual points along the surface repeat the same trajectory, but with different phase angles. As a result, some segments of the surface are in contact with the body and the others are detached from the body, creating a wave-like contour as shown in Figure 2.1-(b). When the segment is in contact with the body, it moves together with the body in the horizontal direction and thereby transports the body.
2.2 Natural Water Waves

This surface wave actuation was inspired by natural surface waves. Figure 2.3 shows traces of water particles in natural wave motion by [Van Dyke, 1982]. Note that each particle motion clearly follows an elliptic trajectory. At the upper portion of each trajectory, a horizontal velocity is generated in one direction, as shown by the arrow in Figure 2.3. Note also that the two gentle curves in the figure indicate the contours of the waves; one shows the crest and the other shows the trough of the waves. The left-bound
horizontal velocity is created along the crest of the waves, which tend to move an object placed on the wave surface.

![Figure 2.3 - Traces of water particles in wave motion [Van Dyke, 1982]](image_url)

This surface wave actuation would be used for transporting an object with a membrane placed on the wave surface to support the object. Such a system would exhibit these salient features:

- It would allow a body to move horizontally while lying on the membrane,
- It would allow a body to move in an arbitrary direction within a plane; two-dimensional motion,
- It can be applied to a water bed, providing flexible, comfortable support to diverse body shapes, and
- Exploiting the natural resonance of fluidic surfaces would allow a small number of actuators to create continuous, distributed wave motion with low power consumption.

Natural surface waves, however, are limited in performance and are not optimal for the purpose of transporting humans and objects. Moreover, natural waves are not necessarily convenient or feasible to generate by artificial means. In this paper, several modifications and improvements to the natural surface wave actuation will be made. The
wave pattern will be modified so that it provides faster motion, higher efficiency, and more gentle and comfortable contours. In addition conditions are derived which yield design simplifications that make the proposed system concept feasible to realize in hardware. Specifically, it is shown that by proper design the elliptical base particle motion trajectories can be replaced by vertical trajectories. This greatly reduces the actuation design complexity because the required base particle motion trajectory is reduced from a three-dimensional ellipsoidal shape (for arbitrary planar manipulation) to a one-dimensional linear trajectory.

Figure 2.4 shows one of the improvements that would significantly increase transportation speed and efficiency. Instead of placing the body directly on a smooth surface membrane, the body is placed on a membrane with a bed of protrusions, called extenders. In addition to the horizontal motion of each base particle along the membrane, there is a rotation of the surface membrane due to the wave propagation. This magnifies the horizontal motion at the tip of the extenders placed normal to the surface membrane. This additional forward velocity can be utilized to increase the transport speed and efficiency of the surface wave actuator. The kinematic behavior of surface wave actuators, both with and without extenders, will be analyzed in Chapter 3 and useful design guidelines obtained. Chapter 4 introduces several physical prototypes that demonstrate the properties of surface waves for the transport of rigid and elastic objects.

Figure 2.4 - Detail of surface waves with extenders
Chapter 3

**Kinematic Analysis of Surface Waves**

In this chapter the kinematics of surface wave actuation is introduced. First, the case of surface waves without extenders is analyzed and kinematic constraints on realizable systems and design of systems for the transport of humans are introduced. Next, surface waves with extender features are analyzed and the non-linear properties of this mechanism are illuminated in light of application to human transport. Design simplifications, special waveforms, and design guidelines are introduced.

### 3.1 Kinematics of Artificial Surface Waves

#### 3.1.1 Kinematic Modeling

In this section we analyze the particle motion of the surface membrane with no extenders.

![Base particle kinematics](figure)

Let \( P \) be a particle at distance \( u \) from origin \( O \) along the surface when the wave is not present. As waves are created, the particle follows the path of an ellipse centered at \( x=u \) and \( y=0 \):

\[
\frac{(x-u)^2}{r^2} + \frac{y^2}{R^2} = 1
\]

(3.1)
where parameters, \( r \) and \( R \), are the lengths of the principal axes in the \( x \) and \( y \) directions, respectively. This trajectory is called a base particle trajectory in the following analysis. Using the angle \( \theta_u \) measured from the vertical axis, as shown in Figure 5, the solution to equation (3.1) is expressed as

\[
\begin{align*}
    x_p(u,t) &= u - r \sin \theta_u \\
y_p(u,t) &= R \cos \theta_u
\end{align*}
\]

Equations (3.2) and (3.3) provide the \( x \)-\( y \) coordinates of the particle \( P(x_p,y_p) \), as shown in the figure. When \( \theta_u \) takes on a value of \( 2n\pi, n=0,1,2,3,\ldots \), the particle is at the highest point, called a crest, while the particle is at the lowest point, called a trough, when \( \theta_u=(2n+1)\pi \).

Angle \( \theta_u \) varies with respect to time, and adjacent base particles have a phase lag that depends on the distance \( u \). We represent \( \theta_u \) as

\[
\theta_u = \omega t - 2\pi \frac{u}{\lambda}
\]

where \( \omega \) is the angular velocity of the cyclic motion, and the second term accounts for position dependent phase lag that varies continually along the surface. Parameter \( \lambda \) shown in Figure 3.1, is the wavelength, the distance between two successive crests or troughs. From equations (3.2) and (3.4), the velocity of the particle at each crest is obtained:

\[
v_{crest} = \left. \frac{\partial x_p}{\partial \theta_u} \right|_{\theta_u=0} = -r\omega
\]

Similarly, the velocity of the particle at each trough is obtained as

\[
v_{trough} = \left. \frac{\partial x_p}{\partial \theta_u} \right|_{\theta_u=\pi} = r\omega
\]
When an object lies on the surface as shown in Figure 2.1-(b) it moves at the same speed as the crest speed $v_{\text{crest}}$. This transport speed, denoted $v_{\text{trans}}$, is the same as $v_{\text{crest}}$ under ideal conditions where the top speed of the wave surface at the crests is completely transmitted to the object.

$$v_{\text{trans}} = v_{\text{crest}} = -r \omega$$  \hspace{1cm} (3.7)

Due to slip or slack of the object, the actual transport speed may be lower.

The contour of the waves shifts in time. This wave propagation speed, $v_{\text{wave}}$, is determined by observing the $x$ coordinate of a particular part of the wave contour. The crest at $\theta_b=0$, for example, is used for determining $v_{\text{wave}}$. Substituting $\theta_b=0$ into equation (3.4) yields

$$u = \frac{\lambda \omega}{2 \pi} t$$  \hspace{1cm} (3.8)

Substituting the above equation and $\theta_b=0$ into equation (3.2), the $x$ coordinate of the crest is given by

$$x_{\text{crest}} = \frac{\lambda \omega}{2 \pi} t$$  \hspace{1cm} (3.9)

Therefore the wave velocity is given by

$$v_{\text{wave}} = \frac{dx_{\text{crest}}}{dt} = \frac{\lambda \omega}{2 \pi}$$  \hspace{1cm} (3.10)

Note that $dx_{\text{crest}}/dt$ represents the time rate of change in the crest position, whereas equation (3.5) is the speed of the particle when it reaches the crest, that is, the speed of the object placed on the waves. Note also that the wave propagation is in the direction opposite to the object motion in the above formulation.

3.1.2 Shaping the Waveform
There are several issues to overcome to transport a human by surface waves. Since a human body is a multi-d.o.f. system consisting of many flexible bodies connected by articulated joints, it may conform to the wave surface as shown in Figure 2.1-(a). As it slacks, the body may contact the trough side of the wave surface, which moves in the direction opposite the crests as shown in equation (3.6). This results in low efficiency and, more importantly, leaves the human in an uncomfortable situation.

To avoid slack and uncomfortable situations,

The wavelength $\lambda$ must be shortened,
The waves must be deep, and
The crest must be gentle and broad.

The shorter the wavelength, the less the body slacks as more crests support the body. The deeper the troughs, the less likely the body will contact the troughs. The third design guideline addresses the shape of the crests to provide an effective solution to the slack and comfort problem. The design parameters, $\lambda$, $R$, and $r$, must be chosen to meet these requirements.

![Surface waveforms](image)

(a) $r/\lambda = 1/12$ (Negative phase difference)
(b) $r/\lambda = 1/12$ (Positive phase difference)
(c) $r/\lambda = 1/2\pi$ (Critical waveform)
(d) $r/\lambda = 1/4$ (Infeasible waveform)

Figure 3.2 - Surface waveforms
Figure 3.2 shows various waveforms for different parameter values. Figure 3.2-(a) shows the profile for \( r/\lambda = 1/12 \) and \( R = r \), while Figure 3.2-(b), the mirror image of Figure 3.2-(a), is created by reversing the sign of the phase angle definition of equation (3.4):

\[
\theta_u = \omega t + 2\pi \frac{u}{\lambda} \quad (3.11)
\]

Natural waves with no membrane have this phase-lead waveform. Note that this natural waveform entails sharp crests that are undesirable for human transfer. Phase-lag waveforms, as in Figure 3.2-(a), have gentle and wide crests. Although phase-lag waveforms do not exist in natural systems, they can be created by artificial means. As the wavelength, \( \lambda \), becomes smaller, the proportion of the gradual side to the sharp side becomes larger. As shown in Figure 3.2-(c), the crest side becomes broad and more gentle as \( \lambda \) gets shorter. However, at a certain point the trough becomes a sharp edge and the waves collapse beyond this point. Namely, as shown in Figure 3.2-(d), the contour of the wave surface crosses over. This waveform, although very gradual on one side, is not physically realizable because no continuous surface can be manufactured that continually wraps over itself. Therefore the waveform with the sharp edge, termed the critical waveform in Figure 3.2-(c), provides the lower limit of the wavelength \( \lambda \). The sharp edge of the critical waveform is a stagnation point, where the contour of the wave surface has a zero gradient in the \( u \) direction. Namely,

\[
\frac{dx}{du} = 0 \quad (3.12)
\]

Evaluating the gradient at \( \theta_u = \pi \) where the sharp edge exists,

\[
\frac{dx}{du} \bigg|_{\theta_u=\pi} = 1 - r \cos \theta_u \left( -\frac{2\pi}{\lambda} \right) = 1 - \frac{2\pi}{\lambda} r = 0 \quad (3.13)
\]
Therefore, the stagnant condition is given by

\[ \lambda = 2\pi r \]  

(3.14)

Namely, the lower bound for the absolute value of the wavelength is given by

\[ a = \frac{2\pi r}{\lambda} < 1 \]  

(3.15)

This means that the wavelength must be longer than the circumference of the circle whose radius is equivalent to the horizontal principal length, the horizontal amplitude of the particle trajectory. In this thesis, the parameter \( a = 2\pi r / \lambda \) is called a 'shape factor', which determines the waveform contour.

From this waveform analysis, the parameter values of desired waveforms that avoid stress concentration and body slack can be determined. The sharp crests in Figure 3.2-(b) cause a concentration of stress, which provides an uncomfortable and even dangerous situation for patients with fragile skin conditions. The gradual contours in Figure 3.2-(a) would significantly reduce the stress concentration and gently support the body. By choosing a small wavelength \( \lambda \) and a relatively large radius \( r \) that satisfy the kinematic limit of equation (3.15) and the correct sign for the phase angle, one can obtain a broad area of contact surface supporting the human body and thereby reduce the stress concentration. Note that the length of the vertical principal axis, \( R \), involved in the elliptic trajectory, determines the depth of the waves, but it does not influence the critical wave condition. However, the parameter \( R \) can be utilized to deepen the waves which is beneficial for transporting humans.

3.2 Surface Wave Actuators with Extenders

3.2.1 Extender Position and Velocity Analysis
As discussed earlier the addition of an extender normal to the membrane can be used to amplify the motion created by the surface waves. This added feature is modeled in this section.

Figure 3.3 - Kinematic model of surface waves with extenders

The position of point $C$ shown in Figure 3.3 is needed to model what is occurring at the tip of the extender in contact with the body undergoing tangential transport by surface waves. First we evaluate the slope of the contour of surface waves at point $P$.

\[
b = \frac{dy_p(u,t)}{dx_p(u,t)} = \frac{\frac{dy_p(u,t)}{du}}{\frac{dx_p(u,t)}{du}} = \frac{R\mu \sin \theta_u}{1 + \mu \cos \theta_u}
\]

where $\mu = 2\pi/\lambda$. Note that this slope is not the tangent of the elliptic particle trajectory, but tangent to the wave contour. From this slope, the tip position of the extender can be obtained. Let $l_o$ be the length of the extender, $x_A$ and $y_A$ be the $x$-$y$ components of the extender shown in Figure 3.3. Then,

\[
x_A = \frac{-b l_o}{\sqrt{1 + b^2}}
\]

\[
y_A = \frac{l_o}{\sqrt{1 + b^2}}
\]

The coordinates of point $C$ are then given by
\[ x_c(u,t) = x_p(u,t) + x_s(u,t) = u - r \sin \theta_s - \frac{bl_0}{\sqrt{1 + b^2}} \]  
\[ y_c(u,t) = y_p(u,t) + y_s(u,t) = R \cos \theta_s + \frac{l_0}{\sqrt{1 + b^2}} \]  

(3.18)

With the position analysis completed the tip velocity can be derived. By differentiation of the position equations with respect to time one obtains

\[ \frac{\partial x_c(u,t)}{\partial t} = -r_0 \omega_0 \cos \theta_s - \frac{l_0 \omega_0 R}{(1 + b^2)^2} \left[ \frac{\mu_r + \cos \theta_s}{(1 + \mu_r \cos \theta_s)^2} \right] \]  
\[ \frac{\partial y_c(u,t)}{\partial t} = -R \omega \sin \theta_s - \frac{l_0 \omega_0 b}{(1 + b^2)^2} \left[ \frac{\mu_r + \cos \theta_s}{(1 + \mu_r \cos \theta_s)^2} \right] \]  

(3.19)

The transport speed, that is, the speed of the extender tip at a crest, is given by

\[ v_{trans} = -r_0 \omega - \frac{l_0 \omega_0 R}{1 + \mu_r} \]  

(3.20)

Note that the absolute value of the transport speed has increased substantially due to the addition of the second term in equation (3.20). By adding extenders, the x-velocity at the point of contact has increased, \( l_0 \omega_0 R / (1 + \mu_r) \). This not only increases efficiency, but also allows us to simplify the mechanism of surface wave actuation, as discussed in the following section. In addition, one can shape the waveform further beyond the critical waveform by exploiting the extenders so that the wave contour may be more gentle and comfortable in transporting a human body. This feature will be addressed in Section 3.3.

### 3.2.2 Simplifying the Actuator Mechanism

To generate the surface waves described above, actuators are needed to move the surface in both the vertical and horizontal directions, requiring two axes of motion. This leads to complexity in physical design. Use of extenders allows us to eliminate the horizontal actuators and significantly simplify the mechanism. Specifically, vertical
movements alone at individual base particles on the wave surface can create horizontal movements at the extender tip.

Figure 3.4 - Extender kinematics with vertical base point motion

When the base particle motion trajectory reduces to a vertical line, as shown in Figure 3.4, the horizontal principal axis length, \( r \), becomes zero. Replacing \( r \) in equation (3.16) by zero, one obtains the slope of the line tangent to the overall surface wave contour at point \( P \).

\[
b = \frac{dy_c(u,t)}{dx_c(u,t)} = R \mu \sin \theta_u \quad (3.21)
\]

Also substituting \( r=0 \) in equation (3.18) yields the position equations

\[
x_c(u,t) = u - \frac{b l_0}{\sqrt{1+b^2}} \quad (3.22)
\]

\[
y_c(u,t) = R \cos \theta_u + \frac{l_0}{\sqrt{1+b^2}}
\]

and the velocity equations follow

\[
\frac{dx_c(u,t)}{dt} = -\frac{l_0 \omega u R \cos \theta_u}{(1+b^2)^{3/2}}
\]

\[
\frac{dy_c(u,t)}{dt} = -R \omega \sin \theta_u - \frac{l_0 \omega u R^2 \cos \theta_u \sin \theta_u}{(1+b^2)^{3/2}}
\]

Note that although the trajectory of base particle, \( P \), does not include horizontal movements, the tip of the extender moves horizontally at a velocity of -
As shown by equation (3.23). As a result, the transport speed is given by

\[
v_{\text{trans}} = \frac{\partial x_{(u,t)}}{\partial t} \bigg|_{\theta_t=0} = \frac{2\pi \omega R}{\lambda}
\]  

(3.24)

and the object can move horizontally without horizontal actuators.

3.3 Kinematic Synthesis: Trade-offs and the hidden trough condition

In this section, the non-linear kinematic behavior of surface wave actuation will be further investigated in simulation to obtain gentle surfaces that transport an elastic human body despite slack. A drawback of surface wave actuation is that surface particles must move backwards for some duration in each periodic motion cycle. These particles may drag the body backwards when the body slacks on the surface. Therefore it is desirable to maximize the fraction of surface particles available to support the body at any time and minimize the fraction of backward moving surface particles in contact with the body. For the case of surface waves without extenders the theory of Section 3.1 shows that all surface particles are available for support, by the no-looping condition, and half are moving backwards at any time, by equations (3.2) and (3.4), regardless of parameter choice. However, due to the non-linear kinematics of the extender feature a unique waveform termed 'hidden troughs' can be designed that allows better performance than possible without extenders. In this section hidden trough conditions will be illustrated and kinematic design guidelines for generating desired waveforms will be obtained.

Figure 3.5 illustrates the trajectory of an individual extender tip for one complete cycle of motion. For each trajectory plotted a constant increment of input angular displacement, \( \theta_t \), is specified so that adjacent points that appear closer together on the
plot indicates that the extender tip spends more time in that particular region relative to regions of the trajectory where the points are more spread out. A shape factor of $a = 2\pi r/\lambda = 0.75$ and $r = R$ is chosen for these simulations. The three separate trajectories represent the extender tip trajectories for normalized extender lengths of $l_o/\lambda = 0.0, 0.15,$ and $0.30$. Note that for the case of $l_o/\lambda = 0.0$ the trajectory is circular as expected for a surface with no extenders. For the two trajectories with extenders, the shape varies substantially from that of a circle and some of the special properties of the extender feature are revealed.

![Figure 3.5 - Trajectory of extender tips for $2\pi r/\lambda = 0.75$, $R=r$ and $l_o/\lambda = 0.0$, 0.15, and 0.30. Backward moving regions of the trajectory are indicated by a heavy, dark line.](image-url)
Assume that we desire to move an object from left to right across the figure. For this to occur we will specify the input sense such that the extender tip moves in a clockwise manner about the trajectory shown. Note that as the $l_o/\lambda$ ratio is increased the extender tip spends less time in the retraction phase of the cycle where the tip moves from the extreme right hand side of the trajectory pictured to the extreme left side and more time in the forward moving phase. For the case of $l_o/\lambda = 0.0$, half of the nodes are moving backward, half are moving forward. For the case of $l_o/\lambda = 0.15$, 36 points are moving forward and 27 points are retracting. For the case of $l_o/\lambda = 0.15$, 42 points are moving forward and 21 points are moving backward. In all of the plots a heavy black line indicates the regions of backward motion. Note that as the $l_o/\lambda$ ratio is increased the percentage of backward moving nodes is decreased.

Figure 3.6 shows the distribution of the normalized horizontal velocity at the extender tip, $dx/dt$, from equation (3.19) plotted along the normalized $u$ axis. Horizontal velocity below zero represents backward motion, which may cause dragging of the human body. Note that the percentage of nodes moving backwards is reduced and the magnitude of the peak backward velocity increases as the extender length gets longer. This is desirable because a larger fraction of extender tips are moving forward and may be available to support the body as the body slacks on the extender surface.
Using the same parameters, Figure 3.7 shows the surface wave contour generated by plotting a series of coordinates of the extender tips for one wavelength based on equation (3.18). This is essentially the locus of extender tips or 'snapshot' of one cycle of waveform where \( u \) varies from 0 to \( \lambda \). Depending on the extender length used, the contours shown in Figure 3.7 can intersect at \( x_r/\lambda = 0.5 \), creating a loop of extender tips in the cycle. Note that the body is in contact with the region of the contour above the
intersection at \(x/\lambda = 0.5\), hence the looping segment is detached from the body. Extender tip looping is a unique feature that can be used to solve the backward motion problem by 'hiding' all extender tips undergoing backward velocity inside the loop. In the figure, thicker lines highlight the backward velocity extender tips. Note that the contour segment of backward velocity can be completely contained in the looping region in the case of extender length \(l/\lambda = 0.15\) and \(0.30\). Consequently, all the extender tips in the backward velocity region are detached from the body. This phenomenon is termed the 'hidden trough' condition and has powerful implications for human transport. Although the body is very soft, no backward dragging force is transmitted to the body. By proper parameter selection all extender tips on the exposed support surface are moving in the same direction at all times, a remarkable feature considering the local nature of the actuator.

In Section 3.1 we discussed that looping of base particles was impossible to physically realize, but extender tip looping interference can be avoided by proper selection of extender diameter, density, and pattern. Figure 3.8 is an example of the extender tip loop created by the rubber mat used for the experiments in the previous section. The extenders of the rubber mat are interlaced in a diagonal pattern detailed in the figure, leaving sufficient room for a loop without interference. Note that extenders marked white appear to interfere with extenders marked black when looking from the end view. However, if the white and black extenders are separated in the third dimension as shown in the picture, no interference occurs and for this case the two black extenders nearest to the trough cover the white extender at the trough and a small extender tip loop is formed. The parameters of the waveform in this case are \(\lambda = 76.4\, mm\), \(r = 6.0\, mm\), \(R\)
= 6.0 \text{ mm}, l_0 = 9.5 \text{ mm}, while the diameter of the extender is 3.50 \text{ mm} and the interval between adjacent extenders is 6.36 \text{ mm}.

Figure 3.8 - Detail of looping condition

It is important to interpret the results of the kinematic analysis for the purpose of transporting humans. The two most important factors for effective transport of the human body are transport efficiency and reduction of interaction stresses between the human and the surface wave. These two factors are contradictory and a trade-off is needed. Since the human body is compliant and will slack into the trough regions of the waveform it is most important to reduce contact with backward moving nodes for the purpose of high transport efficiency. It is also important to reduce stress concentration as much as possible by allowing the human to be supported by as many nodes as possible.

For human transport it is optimal to hide all of the backwards moving nodes in the hidden trough so that only forward moving nodes are available to transport the body forward no matter how much the body slacks from the crest of the wave. In addition, it is important to maximize the number of extender tips available for support. Specifically, it is not useful to have forward moving nodes tucked away in the hidden trough. These nodes should be exposed to the human for transport. Looking at Figure 3.7, the case of \( l_0/\lambda = 0.0 \) is not optimal because none of the backward moving nodes are hidden. The
case of $l/\lambda = 0.30$ is also not optimal because, although all of the backwards moving nodes are hidden, many forward moving nodes which could be used to support the body load are hidden from the human body. The case of $l/\lambda = 0.15$ is optimal because all of the backward moving nodes are hidden and all of the forward moving nodes are available for support and forward transport. To quantify the desire to have all of the forward moving nodes in contact with the body for support and transport and all of the backward moving nodes hidden from the body we define two metrics as follows:

$$
\alpha = \frac{\# \text{ hidden extenders}}{\# \text{ backward velocity nodes}} \tag{3.25}
$$

$$
\beta = \frac{\# \text{ exposed extenders}}{\# \text{ extenders}} \tag{3.26}
$$

For an optimal choice of parameters, $\alpha$ should equal 100% and $\beta$ should be as large as possible under the condition that $\alpha = 100\%$. The condition for a hidden trough is provided by a complex combination of waveform parameters as well as extender dimensions. The results of simulation with many trial design combinations are shown in Figure 3.9 for the case of circular trajectories. Solid lines of constant $\alpha$ and dashed lines of constant $\beta$ are plotted. Note that $\alpha$ and $\beta$ are roughly parallel and only diverge for small shape factors. A trade-off between $\alpha$ and $\beta$ exists, as expected. By assuming extender interference is eliminated by density reduction techniques, the design space of hidden troughs is hatched above the $\alpha = 100\%$ line with this line being the lower limit of the hidden trough condition. The space with no extender tip looping is hatched below the $\beta = 100\%$ line. In general, as the shape factor $a=2\pi r/\lambda$ becomes larger, the hidden trough condition can be met with shorter extenders.
As mentioned in Section 3.1, the waveform becomes more gentle, as the shape factor, \( a \), becomes larger, but it cannot exceed 1 for all nonzero, \( r \). In practice, interference due to looping of extenders about the base particles occurs as \( a \) gets closer to 1. Again this is impossible to physically realize regardless of extender density. In the figure, the practical bound on design parameters due to interference is shown. The shaded region in the design parameter space is recommended, since it provides a gentle waveform satisfying the hidden trough conditions, and is practically feasible to build. The parameter choice illustrated in Figure 3.8 is marked on this plot with a star.

Figure 3.9 - Results of parameter study (Case \( r=R \)). Solid lines of constant \( \alpha \) and dashed lines of constant \( \beta \) are plotted. The design space of hidden troughs is hatched above the \( \alpha = 100\% \) line. The space with no extender tip looping is hatched below the \( \beta = 100\% \) line.

A simplified design results when \( r = 0 \), as studied in Section 3.3.2. Figure 3.10 illustrates the results for this case. We note that for the case of \( r = 0 \), \( \alpha \) and \( \beta \) are no longer independent because the number of extender tips undergoing backwards velocity is constant for the \( r = 0 \) case, regardless of parameter choice. Equation (3.23) evaluated
at \( \frac{dx}{dt} = 0 \) yields a zero velocity at \( \theta_a = \pi/2 \) which corresponds to \( u = l/4 \) by equation (3.4). This means that with vertical base particle trajectories exactly half the surface particles are moving backward at any time. Note that the theoretical limit for base point motion looping is valid only for non-zero \( r \), so for the \( r = 0 \) case this limit does not apply.

The shaded region illustrates the desirable region of the hidden trough condition with the minimum lengths of \( l_o \) and \( R \).

![Diagram showing parameter study results](image)

**Figure 3.10 - Results of parameter study (Case \( r=0 \))**

In Section 4.3 a prototype is designed to transport a human. This prototype takes advantage of the simplified actuator architecture, which results from vertical base point trajectories. The design parameters represented by the star in Figure 3.10 were used in that prototype. With this choice we should expect some extender tip looping, but not all extender tips will be hidden \( (\alpha<100\%) \).
Chapter 4

Experiments with Surface Wave Actuation

In this chapter, three physical prototypes are introduced that are used to explore the different physical properties of surface waves for tangential transport. First, a water-based prototype demonstrates the proof-of-concept, yet also illustrates some of the shortcomings of using water waves for human transport. A mechanical prototype is designed and constructed to transport human subjects by surface waves. Finally, a second mechanical prototype with a special molded rubber surface is constructed to more fully explore surface waves with extenders for manipulation.

4.1 Water Waves

The benefits of using water waves as the wave propagation medium were discussed in Chapter 2. Seeking to realize these benefits, an experiment was designed to investigate how a modified water-based system with extenders could be used to transfer objects in accordance with the theory of Chapter 3. Figure 4.1 shows an apparatus for experiments with natural water waves. A vat, 39cm by 72cm, was filled with water to a depth of approximately 20cm. A thin plastic sheet was placed over the water to separate the water from the support mat. As shown in the detailed picture of Figure 4.1, a rubber mat with numerous protuberances of 9.5mm in height was placed on top of the plastic sheet.
A Lorentz force actuator provided excitation specified by a sinusoidal signal generator through a vertical board. The board was fashioned across the width of the vat to force waves to travel in one-dimension in the direction perpendicular to the board. A small object was placed on the rubber mat, and the distance traveled was measured by a rule and timed by the frame rate of a digital video camera. Knowing the excitation frequency of 6.0Hz and evaluating the resulting video frame by frame it can be determined that the waves travel at a rate of 114cm/sec with a wavelength $\lambda = 19\, \text{cm}$. Figure 4.2 shows that by monitoring a marked point at the surface-rubber mat interaction one could observe the vertical and horizontal amplitudes of the base particle trajectory beneath the object to be $R = 1.9\, \text{mm}$ and $r = 0.0\, \text{mm}$. The tip of the extenders traveled approximately $1.0\, \text{mm}$ horizontally in their periodic motion. The object itself moved at a speed of $1.9\, \text{cm/sec}$ towards the actuator. Evaluating equation (3.22) for a range of $0<\theta<2\pi$, one can calculate the amplitude of the $x$-component displacement to be $1.2\, \text{mm}$. Substituting the above values for $\lambda, \omega, r, R$, and $l_o$ into equation (3.24) yields the theoretical transport speed of $2.24\, \text{cm/sec}$, which is close to the experimental value. Note
that multiplying the horizontal stroke (1.2 mm) by the excitation frequency (6Hz) grossly underestimates the transport velocity. The wave propagation action with extenders greatly amplifies the transport speed and the small, local extender tip displacement limits the strain on an elastic body being transported by the surface. The experiments using water waves have shown that surface waves can transport objects with the use of extenders.

The propagation of the waves was in the direction opposite the object motion. This is consistent with the gentle waveform design described in Chapter 3 with phase defined as equation (3.4). This implies that the waves created with the rubber mat are suitable for patient transport. This is contrary to the case for natural waves exposed to air with phase described by equation (3.11). With the plastic membrane and rubber mat placed on the water surface, the properties of the waves have changed for improved patient transport characteristics.

![Figure 4.2 - Base particle motion](image)

4.2 Mechanical Waves
Figure 4.3 shows an experimental testbed of artificial surface waves produced by purely mechanical elements. The body lying on a mat is supported by a number of bars across the bed as illustrated in the figure. The testbed is composed of a series of mechanical linkages that are coordinated to generate an overall surface wave behavior. These waves can be tuned and shaped as desired. Each mechanism traces a particular base particle trajectory.

A matching pair of mechanisms are driven in tandem and the base particle motion produced by this pair is traced by each long bar. Adjacent mechanism pairs are set out of phase to create the desired discrete wave shape. Twenty-eight of these pairs are driven in a coordinated manner by a DC motor of 1160 W output power to generate the wave motion that travels across the discretized surface of bars. Speed control of this motor sets the input angular velocity of the coordinated mechanisms. The bed dimensions were specified to be $213\, \text{cm}$ by $198\, \text{cm}$, approximately the size of a queen-size bed.
Figure 4.4 - Detail of rocker-slider mechanisms for base particle motion

Figure 4.4 shows the rocker-slider mechanism chosen as the fundamental mechanism for each base particle of the surface wave actuator. Using this mechanism one can achieve a trajectory that is very close to an elliptical shape with tunable parameters $r_{crank}$ and $R_{crank}$ that approximately correspond to $r$ and $R$ defined in Chapter 3.

Figure 4.5 - Schematic of slider-crank mechanism

Figure 4.5 details the mechanism kinematics. By adjusting the length of the radius of each crank one can adjust the vertical displacement of the base particle, $R_{crank}$ and consequently the amplitude of the waveform. By adjusting the position of the cross-
bar on each connecting rod, \( l_2 \), one can adjust the horizontal displacement of the base particle, \( r_{\text{crank}} \). Note, however, that the angular velocity used in the kinematic analysis, \( \omega \), differs from the crank velocity \( \omega_{\text{crank}} \).

The theory of Chapter 3 was developed based on the assumption of an elliptical base particle trajectory, an angular displacement defined as in Figure 3.1, and a constant time rate of change of angular displacement was assumed. The experimental prototype utilizing the rocker-slider mechanism approximates the elliptical trajectory closely, but the base particle angular velocity changes with position. To apply our general analysis to the particular rocker-slider implementation we will assume that the trajectory of the rocker-slider mechanism is elliptical, but we will more carefully calculate the angular velocity. In particular we will evaluate the angular velocity at the point where it is needed, at the wave peak, hence we can evaluate the transport velocity using the theory developed.

We assume that the rocker-slider trajectory is an ellipse with identifiable major and minor axis lengths, \( R \) and \( r \). We define these lengths in a way consistent with the definitions of Figure 3.1. The principle axis length in the y-direction \( R \) equals the crank radius \( R_{\text{crank}} \) by inspection. The principle axis length in the x-direction can be derived with simple geometric arguments.

\[
r_{\text{crank}} = R_{\text{crank}} \left( 1 - \frac{l_2}{\sqrt{h^2 + R_{\text{crank}}^2}} \right)
\]  \hspace{1cm} (4.1)

The derivations of Chapter 3 yield the transport speed as a function of several design parameters including the constant angular velocity, \( \omega \), the time derivative of the
displacement $\theta_n$ defined in Figure 3.1. The angular velocity of the rocker-slider mechanism is not constant and we must identify the angular velocity of the operating point of interest in a way consistent with our derivations and in terms of the geometric variables and input crank angular velocity of the rocker-slider mechanism. The operating point of interest is the velocity of the wave particle, $P$, at the wave peak, where the object transport is occurring. Figure 4.6 illustrates the rocker-slider mechanism at this particular configuration.

![Diagram of rocker-slider mechanism at peak position]

Figure 4.6 - Rocker-slider at peak position

We define $\omega$ as the angular velocity of the pivot joint at the top of the rocker-slider mechanism. From simple kinematics it is established that

$$
\omega = \frac{R_{\text{crank}}\alpha_{\text{crank}}}{h - R_{\text{crank}}}
$$

(4.2)
Next we define $\omega^-$, the rate of change of the angle particle $P$ makes with the $y$-direction principle axis of the ellipse. This can be expressed as

$$\omega^- = \frac{\omega_{\text{crank}} (h - R_{\text{crank}} - l_2)}{h - R_{\text{crank}}} \tag{4.3}$$

Now we must express $\omega^-$ in terms of the $\omega$ defined in Chapter 3. In other words we must reconcile the definition of displacement, $\theta_u$ in Chapter 3 and the displacement, $\theta$, whose time rate of change is $\omega^-$.

![Figure 4.7 - Detail of Particle Displacement](image)

Figure 4.7 details the two definitions of displacement. Using the definitions in equations 3.2 and 3.3 that describe the position of particle $P$, we note that

$$\tan \theta = \frac{r \sin \theta_u}{R \cos \theta_u} \tag{4.4}$$
This expression relates the two definitions of angular displacement. Taking the time derivative of both sides yields

\[
\frac{\dot{\theta}}{\cos^2 \theta} = \frac{\dot{\theta}_u (r \cos \theta_u R \cos \theta_u + R \sin \theta_u r \sin \theta_u)}{R^2 \cos^2 \theta_u}
\]  

(4.5)

Now evaluating this expression about the operating point of interest \(\theta = \theta_u = 0\) yields:

\[
\dot{\theta} = \frac{\dot{L}}{R} \dot{\theta}_u
\]  

(4.6)

Finally we have the desired expression which connects our general theoretical derivations to the specific rocker-slider mechanism. Putting it all together yields

\[
\omega = \dot{\theta}_u = \frac{R \omega_{\text{crank}} (h - R_{\text{crank}} - l_2)}{r (h - R_{\text{crank}})}
\]  

(4.7)

With the results of equations 4.1 and 4.7 we were able to compare experimental results with the theoretical predictions for the surface waves without extenders. An experiment was devised where a rigid object was transported across the discrete surface wave manipulator. A magnetic tracker was attached to a board placed on top of the surface to measure the displacement of the board as a function of time during manipulation. Trials were run to illustrate the transport velocity for surfaces without extenders as the parameter, \(r_{\text{crank}}\) was changed. A wavelength of 228.6 mm, an input crank angular velocity of 1.15 rad/sec, and a crank radius of 25.4 mm were utilized. The rod length, \(l_2\), was adjusted to different values and the resulting displacement-time profiles are plotted in Figure 4.8. Theoretical predictions based on equation (3.7) with the appropriate corrections provided by equations (4.1) and (4.7) are plotted as dashed lines. The results are close to theory with errors due to slip and the discretized nature of
the mechanical prototype. Note that despite the discretization, the experimental curves are quite smooth and linear indicating steady transportation at constant velocity.

![Transport Displacement Profile](image)

Figure 4.8 - Experiment of transport velocity for different ellipse shapes (No extender used)

To examine the effects of an extender surface, a rubber mat surface with extenders of 9.5 \text{mm} in length was attached to a portion of the prototype at each cross-bar as shown in Figure 4.9, thus interpolating the discrete nodes to make an overall continuous surface. Using $r_{crank} = 6.3 \text{ mm}$, a comparison of transport velocity, both with and without the extender surface, was made. The experimental and theoretical results from equation (3.20) are plotted in Figure 4.10. The difference between theory and experiment can be attributed to slip, the discretized nature of the manipulator and the non-zero thickness of the rubber mat. Despite a mere 9.5 \text{mm} extender length on the surface shown in Figure 4.9 the transport velocity has been nearly doubled. Thus, extenders significantly increase transport efficiency.
Finally various trials were run while varying the input crank angular velocity, $\omega_{crank}$, the wavelength $\lambda$, and the $x$-direction principal axis length, $r_{crank}$. The $y$-direction principal axis length, $R_{crank}$, was held constant. Figure 4.11 shows the results of these experiments as normalized transport velocity vs. normalized principal length. Using the theory of equation (3.20) also with the corrections of equations (4.1) and (4.7) yields two theoretical lines, one with extenders of length 9.5 mm, the other without. Note that, although the ellipse trajectory collapses to a vertical line, i.e. $r = 0$, the transport velocity is still non-zero with the use of extenders as was illustrated in the water bed experiments.
Transport velocity results in non-dimensional form

Figure 4.11 - Aggregate data

The experiments using the two prototypes have verified the analytical results and demonstrated the feasibility of surface wave actuation. Furthermore, the second prototype has demonstrated that a human body can be transported by the surface wave actuator. However, body elasticity and slack as well as stress concentration and comfort must be further investigated. The mechanical prototype used for human transport was designed for large load bearing capacity and for experimental convenience. It has limited design flexibility and is inappropriate for exploring design possibilities to better deal with the specific issues of human transport.

4.3 Rubber-based Prototype

A prototype surface wave distributed actuator has been developed in the laboratory to examine the features of surface wave actuation discussed in Chapter 3. This
prototype generates vertical trajectories for each of the base membrane nodal points and the overall surface waves propagate in one-dimension as shown in Figure 4.12.

![Direction of Wave Propagation](image)

Figure 4.12 - Rubber extender prototype

The waves propagate through a rubber base membrane surface with molded extender features. The molded rubber is of shore hardness 30A with a base membrane thickness of 1.5mm and with conical shaped extender features 25mm in length. The mold for this surface was created by drilling conical holes, using a specially designed tool, into a large piece of Delrin engineering plastic shown in Figure 4.13.
Wood boards are screwed into the Delrin block along the edges to establish a recess where the rubber compound can pool while it is curing. A two-part pourable urethane rubber was mixed and poured into the mold, filling the voids left by the machining and leaving a layer of material on top of the mold. This is illustrated in Figure 4.14.
When the urethane cures, the mold can be released and the result is a thin, rubber base membrane with the conical extender features fully integrated as illustrated in Figure 4.15.

The basic structural frame of the prototype is illustrated in Figure 4.16. The frame consists of two base bars that are driven vertically by a group of four cam-wheels, one placed at each end of each base bar. The structural frame is constrained to move vertically by a set of pins, which also serve as connection points for the set of cross bars.

Figure 4.16 - Basic structural frame

The rubber membrane is attached to this set of cross-bars as illustrated in Figure 4.17. This is accomplished by attaching the circular base of each conical extender tip to the cross-bar by coarse threaded wood screws.
Figure 4.17 - Attachment of cross-bars to rubber membrane

Figure 4.18 shows that each cross-bar is designed to pivot along its longitudinal axis. This pivoting action allows the base membrane to rotate freely with the changing contour of the surface wave. This is essential for surface wave actuation with extenders to be successful, as qualitatively discussed in Chapter 2 and quantitatively analyzed in Chapter 3.
In Figure 4.19, three identical frames, offset by 12.7 mm in the direction of wave propagation and 25.4 mm in the direction perpendicular to wave propagation have been assembled. The rubber membrane is attached to this bed of bars.
Figure 4.20 illustrates a bottom view of the prototype. The cam-wheels in contact with a particular frame at each corner are exactly in phase with one another at their respective locations. The cam-wheels are distributed on two drive shafts, which are rotated in tandem by a drive motor and a chain connecting the two shafts. Therefore the three structural frames are each driven by a group of four cam-wheels and the three distinct groups of four cam-wheels are set out of phase with one another by 120 degrees.

Figure 4.20 - Bottom view of prototype

Figure 4.21 focuses on one corner of the surface wave prototype. Each set of camwheels is set out of phase with one another by 120 degrees to create the overall wave surface. By using a 120 degree rotational phase separation among each structural frame three distinct, vertical actuation points per wavelength are formed. Figure 4.21 also illustrates the bias springs placed on each structural frame. These springs maintain contact between the camwheel and the base bar at all times. The natural unstretched position of the rubber extender surface is completely flat with no wave shape. During the upward motion stroke the stretching of the rubber surface pulls down the structural frame onto the camwheel. However, during the downward stroke of the wave motion the stretching of the rubber extender surface tends to lift the structural frame off of the camwheel. The
bias springs resist this lifting tendency and force the structural frame down onto the camwheel for full wave motion at all times.

Figure 4.21 - View of camwheel and bias spring arrangement

The prototype was designed with a wavelength of 38\text{mm} and a vertical peak to peak amplitude of 7.6\text{mm}. This design takes advantage of the design simplification discussed in Chapter 3, providing horizontal motion from vertical actuation only. This design is marked on the design plot of Figure 3.10 and is close to the optimal design choice discussed in Section 3.3. This choice of parameters shows a small amount of extender tip looping. Figure 4.22 shows the trajectory of an individual extender tip for this particular choice of design parameters.
Figure 4.22 - Trajectory of extender tip

Figure 4.23 shows the extender tips of the prototype highlighted with white dots which match with the theoretical locus of extender tips calculated in simulation which is overlaid in a heavy, black line also in Figure 4.23. Note that the backward motion of the extender tip takes place at a lower position so that the top surface of the bed is moving forward at all times. The slight differences between the simulation trace and the real extender tip position can be explained by the discrete nature of the extender and actuator placement in the real prototype.
Several experiments were run to examine the transport properties of this system. The experimental scenario is illustrated in Figure 4.24. A rigid board of area 850 cm\(^2\) was placed on the bed of extenders for transport under conditions of no load and loads of 40N, 100N, and 200N. A magnetic tracker was utilized to measure the object displacement as a function of time. Figure 4.25 illustrates the results. It is noted that the displacement vs. time curves are straight and smooth indicating smooth transport with the discrete extender surface. However, transport velocity decreases as the load is increased. This can be explained by the differences in experimental conditions as the load is increased. Maximum transport speed is obtained when point contact between the wave crest and the object being transported is made. For the case of our prototype with three extenders per wavelength, point contact with the exact wave crest is not possible at all times. In addition any deformation of the extender tips under load results in contact with a group of extenders rather than those at the peak so real transport velocity is reduced. Note that the transport velocity is reduced as more load is added until with a 200N load the transport velocity is almost zero. At this point the extender tips had collapsed onto
one another and smooth transfer of load from one group of extenders to the others is no longer possible. In addition, some of the heavier loads show a slight decrease in speed with time. This is due to

Figure 4.24 - Rigid board transport with 200N load

![Rigid board transport with 200N load](image)

**Figure 4.24 - Rigid board transport with 200N load**

Figure 4.25 illustrates the results for the exact set of experiments above, but with the extender length reduced to 12.7mm. This was achieved by trimming the extenders

![Performance under load with rigid body](image)

**Figure 4.25 - Performance under load with rigid body**

Figure 4.26 illustrates the results for the exact set of experiments above, but with the extender length reduced to 12.7mm. This was achieved by trimming the extenders

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shown in Figure 4.23 in half. The extender length is reduced and the lateral stiffness of the halved extender is much higher than the full length extender due to the conical shape of the molded extender. Note that the sensitivity of performance on load is much less with the shorter extenders. Even under the large 200N load very little reduction in transport velocity is observed. Another observation is that system performance is actually improved by the presence of some load over the unloaded condition. Watching the experiment take place shows that by adding some load performance is improved because the object "tracks" better on the bed of extenders when the rigid object is pressed down in contact with more extenders. For the unloaded case the rigid board contacts only some extenders due to small length deviations among the bed of extenders and this reduces performance.

![Figure 4.26 - Performance of short extender](image)

Realistically, contact between the flexible, rubber extender mat and an elastic body will always cause a patch contact that will reduce the transport efficiency, but also allow for a larger region of support and less stress concentration. A piece of gelatin was
transported by the surface wave actuator to illustrate the transport of soft, elastic objects with properties very close to human tissue. This experiment is shown in Figure 4.27. A small board connected to the magnetic tracker was placed on top of the gelatin.

![Figure 4.27 - Gelatin transport](image)

The result for the case of 25.4mm extenders is shown in Figure 4.28 and compared with the case of the unloaded rigid board. As expected the transport efficiency was compromised by deformations of the gelatin.

![Figure 4.28 - Jello with long extender](image)
Figure 4.29 illustrates the same experiment for the 12.7mm extender. Again, elasticity reduces the performance, but the difference between rigid object and elastic object transport is not as large for the case of short extenders.

![Graph showing elastic body transport with 12.7mm extender](image)

**Figure 4.29 - Jello with short extender**

An infant of 10kg and 66cm in height was transported using the surface wave actuator prototype with the half length extenders shown in Figure 4.30. This yields a design choice of $l_o/\lambda=.33$ and $R/\lambda=.10$ and is indicated with a star on the design plot of Figure 3.10. The infant was transported at a rate of 1.0mm/sec. This result is much lower than the unloaded board, but is consistent with the reduction in transport efficiency for the case of high load and body elasticity which are both present in the case of infant transport.
The objective of these tests is to illustrate that large scale, elastic bodies can be successfully transported by this new type of actuator. The positive result of the human transport experiment was promising, but more careful analysis of the deformation of the object to be transported and the surface wave actuator is needed to more accurately estimate the transport efficiency of the system. Specifically, models must be developed to accurately predict performance under particular loading conditions. Before this is undertaken, however, issues of human surface-wave interaction safety must be considered.
Chapter 5

Human Considerations in Surface Wave Actuator Design

The prototype surface wave actuator described in the previous section supports the human body by the tips of the extenders, hence stress concentration occurs at the contact surface. This chapter considers the important physiological implications of transporting humans by surface wave actuators. This is accomplished by checking the worst-case support scenario to compare the resulting stresses with acceptable limits. Our models of this scenario under different support surface design conditions are presented in this chapter. The results indicate that the stresses induced are not within acceptable levels for long-term support without modification. An appropriate design modification is presented in Chapter 6 to ensure safe, long-term support under all realistic conditions.

5.1 Human Constraints on Interface Pressure

While Chapter 4 verifies that surface waves can indeed transport the human, the vital question concerning whether or not surface waves can safely support the human body remains. This chapter is dedicated to ensuring that the proposed system is clinically viable by offering a physiologically acceptable (healthy) support surface. To begin, criteria for healthy interactions between the human and the support surface are quantitatively defined. Next, worst-case interaction scenarios are proposed and modeled, and the resulting stresses are calculated. These stresses are compared with the established criteria to check the acceptability of our particular design concept.

The physiology of soft tissue/support surface interactions has been studied within the fields of rehabilitation, biomechanics, and physical medicine. When an individual lies upon a support surface, stresses are induced which deform the soft tissue. The changes in
pressure result in occlusion of blood vessels and lymphatics, and stimulates nerve endings which could signal discomfort to the central nervous system.

[Ferguson-Pell, 1990] has written a primer on the clinical criteria of seat cushion selection. In his report the most important factor that determines functionality of a support surface is the proper distribution of stress in soft tissues, followed by control of moisture accumulation and heat. Clearly from a clinical and physiological perspective these are the criteria that all support surface designs must meet to be acceptable. This report is a general overview and more detailed quantified data is needed for a rigorous mechanical analysis and design. Work has been done to study the human interaction with the support surface. [Brienza, 1996] experimentally studied the interaction pressures of the seated person and designed an adjustable contour surface to minimize these pressures. However, no work has been done to study the human interaction with surface waves.

Of primary importance is the data showing the relationship between allowable pressure and time duration for support that will not result in permanent tissue damage. The data presented by [Reswick and Rogers, 1975] is shown in Figure 5.1 and is a clinically established pressure-time relationship for normal pressures based on clinical observations of nearly 1,000 patients. This data illustrates clearly the maximum allowable pressure for a given period of time. This data demonstrates that below some pressure threshold, tissue can sustain itself without damage for very long periods of time. However, this data also indicates that higher pressures, above the threshold can be maintained for shorter periods of time.
Based on the clinical results of Reswick and Rogers, a normal, resting interaction pressure of 4,000Pa is chosen as the limit of acceptable interaction pressure for long-term support. This ensures a safe interaction of approximately 10 hours duration. Pressure 10 times this value is permissible for short-term support during the time required for reconfiguration. Those values are considered the bounds of acceptable pressure for a candidate design.

### 5.2 Modeling of Human Interaction with Surface Wave Actuator

With the surface wave actuation described in Chapter 2 it is possible to transport the human body with or without extenders. Section 5.2.1 models the situation without extenders and Section 5.2.2 models the case with extenders as shown in Figure 5.2. It is important to capture the worst-case support scenario to obtain specifications for designing the transport surface. Numerous clinical data support the fact that bedsores develop due to the peak interaction pressures at the bony protuberances of the human
body. Therefore our models include the influence of bone at the interaction to capture the stress concentration that occurs in the real worst-case human/surface interaction as shown in Figure 5.2. In addition we will also model the long-term support scenario because of the much more restrictive allowance for safe interaction pressure established in Section 5.1. In this scenario, the surface waves have been relaxed to yield a flat surface.

![Diagram showing physical interactions](image)

Figure 5.2 - Physical interactions to be investigated

5.2.1 Contact with Support Surface Without Extenders

A finite element model of tissue in contact with a support surface without extenders is developed in this section to investigate the contact pressures induced from support by an ideal frictionless surface that would be used to transport the human body by surface waves without extenders. The mathematical model employed to explore the problem includes:

- Large displacement, large strain analysis
- Hyperelastic, nonlinear material law
- Contact analysis
- Nearly incompressible material analysis
Each of these components adds to the complexity of the problem and contributes to a more accurate picture of the real physical situation. The physical scenario considered in this chapter is a critical section of human tissue in contact with the support surface under gravity load as illustrated in Figure 5.3.

![Diagram of human section in contact with support surface under gravity load](image)

**Figure 5.3 - Human section in contact with support surface under gravity load**

To begin let us assume that the human section in some portion of the body is uniform along its length and statically isolated from the other portions of the body. We will analyze a section of unit length and perform a 2-D plane strain analysis. We assume that the contact between the human section and the support surface is frictionless. Let us assume that the bulk soft tissue is homogeneous and the material density and the parameters describing the material properties are kept constant throughout the body. In all of the models a realistic, non-linear, hyperelastic material model for bulk soft human tissue due to [Vannah, 1996] is used. The model is a non-linear Mooney-Rivlin material model. Fully non-linear strain-displacement relations are used and equilibrium is taken in the deformed geometry. Incremental analysis is used to handle the resulting large displacements and strains. The constitutive behavior of the bulk soft tissue is captured using the a strain energy function of the form

\[ W = c_{10}(I_3) + c_{01}(I_2) + c_{11}(I_1) \]  

(5.1)
where $I_1$ and $I_2$ are the first and second strain invariants and $c_{10}$, $c_{01}$, and $c_{11}$ are material parameters given by Vannah as $c_{10}=0.0026\text{Mpa}$, $c_{01}=0.00064\text{Mpa}$, and $c_{11}=0.0057\text{Mpa}$. The non-linearity comes in the third term of the strain energy function. The density of bulk soft tissue is given by [Fung, 1993] to be $910\text{kg/m}^3$. Using the approximation of the initial elastic modulus, $E$, given in [Bathe, 1996],

$$E=6(c_{01}+c_{10})$$

and specifying a Poisson’s ratio of .4997, one can calculate the bulk modulus to be

$$K=10.8\times10^6$$

Figure 5.4 - Physical interaction schematic of support without extender tips

Figure 5.4 shows a detail of the model that we wish to solve. To capture the effect of stress concentration due to a thin tissue layer separating bone from its support surface we introduce a circular bone inclusion close to the contact surface. The bone inclusion will be modeled as perfectly rigid and homogeneous with constant density. The bone inclusion will also be modeled with a no slip boundary condition between it and the surrounding bulk soft tissue as is the case in real human tissue. The material properties for bone are given by [Fung, 1993]: Young’s modulus, $E_{\text{bone}}=1.8\times10^9\text{Pa}$, Poisson’s ratio, $\nu_{\text{bone}}=.3$, and density, $\rho_{\text{bone}}=1950\text{kg/m}^3$. The geometry used in this model is bone radius, $R_{\text{bone}}=.015\text{m}$, section radius, $R_{\text{section}}=.050\text{m}$ and the distance between the center of the
bone and the initial point of contact between the undeformed section and the support surface, \( h = 0.025 \text{m} \).

Figure 5.5 illustrates the results of the finite element solution to the problem posed in Figure 5.4. Note that with the inclusion of the bone, peak pressure is 4,000 Pa and the strains are in the nonlinear regime (>>.01) justifying the non-linear analysis used.

![ADINA Time 1.000 Pressure](image)

**Figure 5.5 - Pressure solution for model without extenders**

From the modeling results one could conclude that this design would be marginally acceptable for long and short term support. However, there is no factor of safety for the long-term support condition, which is our main concern with this modeling effort. For this reason further action will need to be taken to ensure a safe interaction between the human and the surface wave actuator for long-term, rest conditions.

### 5.2.2 Contact with support surface with extenders
We replace the model scenario discussed in Section 5.2.1 with a similar scenario, but with the smooth surface replaced by a bed of spherically tipped extenders to capture the impact of extenders on the interaction with the human shown in Figure 5.6.

Figure 5.6 - Physical scenario for support by extender tips

The assumptions for this model follow from the model of Section 5.2.1 with the following modifications. Because we want to capture the effect of spherically tipped extenders we will perform a 2-D axisymmetric analysis by examining a 'core' sample of the section subject to the assumptions listed above.

Figure 5.7 - Physical interaction schematic for extender tip support
With this analysis we will assume that the body load is shared by each extender and that each extender tip is responsible for carrying the load of the axisymmetric core projected above itself as shown in Figure 5.7. In addition we assume that the radius of the spherically tipped extenders is much less than the bone, $R_{sphere} < R_{bone}$.

![Mathematical model schematic](image)

Figure 5.8 - Mathematical model schematic

Figure 5.8 illustrates a schematic of the mathematical model that we use to characterize the physical interaction scenario of Figure 5.6. We are interested in the highest stress concentration, the worst-case scenario, which happens when the bone inclusion is closest to the skin surface rather than when the bone curves away from the skin surface. This also corresponds to the location of highest loading. The extender tip is modeled as a semi-spherical rigid body of radius, $R_{sphere}=5.0mm$, which contacts the soft tissue of thickness, $h_{bone}=10.0mm$. The extender tip is assumed to be completely rigid, which is a conservative assumption considering that the rubber extenders deflect during operation. The tissue and the bone are layered as shown in Figure 5.8. The thickness of the bone layer is $d_{bone}=30mm$, whereas the total thickness of the limb is $d_{section}=150mm$. These dimensions are the same as the model of Section 5.2.1 so the results can be compared. To capture the loading on each support sphere it is assumed that each sphere
bears the load of a cylindrical portion of tissue equivalent to the mean area of support of each extender tip. This area of support is related to the spacing of the extender tips, $L$. 

Figure 5.9 - Spherical foundation matrix layout

Assuming a diagonal matrix of support spheres illustrated in Figure 5.9, by simple geometric arguments it can be shown that the equivalent mean area of support for each support sphere is given by

$$A_s = \frac{\sqrt{3}}{2} L^2$$  

(5.4)

In this way the gravity load on each sphere can be calculated to be the density of tissue and bone multiplied by the area of support, $A_s$, multiplied by the depth of tissue to be supported. Not that as the spacing between extenders, $L$, increases, the load increases. A particular surface design can be characterized by two parameters, the radius of the spheres, $R_{\text{sphere}}$, and the distance between adjacent spheres, $L$. The separation distance among adjacent spheres is bounded by the radius of the spheres as follows:

$$L > 2R_{\text{sphere}}$$  

(5.5)

This constraint is introduced to recognize that it is physically impossible to have the same space occupied by two extenders.
This model was solved using finite element analysis and the peak pressure was calculated for several parameter choices. It was found that the major impact on peak pressure is due to the ratio of the sphere radius, $R_{sphere}$, and the spacing, $L$. In Figure 5.10, the results are plotted at the dots and the regions of constraint are also shown. The infeasible region represents the machine design constraint that $L > 2R_{sphere}$. The undesirable region for short-term support is the peak pressure level that exceeds the short-term acceptable pressure levels established in Section 5.1. The undesirable region for long-term support is the peak pressure level that exceeds the long-term acceptable pressure levels established in Section 5.1. The result is that the body cannot be supported by a bed of rigid extenders during long-term, rest conditions. In addition, if the extender tips are spaced too far apart, the body cannot be safely supported during short-term, transport conditions.

![Figure 5.10 - Model results for bed of extenders](image)

The results of this analysis indicate that consideration of extender tip spacing is important when transporting the body. These results also agree with the results of the modeling of
Section 5.2.1 that indicate that long-term support requires a soft, compliant support regardless of whether or not extenders are used in the design of surface wave actuators.

5.2.3 Contact with an Elastic Surface

A long period of resting time requires that the support surface be made more compliant. This is particularly important for bodies transported using surface waves with extenders. The following model is used to investigate the results of introducing a smooth, elastic foundation to relieve the stresses that are incurred due to the rigid support surface needed for transport.

An elastic foundation supporting the limb has been introduced to illustrate how stress is reduced due to the introduction of a low impedance support surface shown in Figure 5.11. Material parameters for the support surface were chosen to be the same as soft tissue. The depth of the surface is .0125m. Otherwise, all assumptions are the same as Section 5.2.1.
Figure 5.12 illustrates the solution for pressure of the model illustrated in Figure 5.11. By introducing an elastic foundation the peak pressure has been reduced to 3200Pa. The results indicate that with the introduction of the compliant foundation the contact pressure has been reduced below the 4,000Pa limit and a safe, healthy support surface can be realized, although the factor of safety is not as large as one would like.

5.3 Hydrostatic Support

Another support condition is that of hydrostatic support such as that can be approximately provided by an air cushion. In this ideal situation the entire gravity load is evenly spread across the body area projected onto the surface as shown in Figure 5.13.
In this ideal support configuration the pressure required to support the human limb is given by

$$P = \frac{W}{2R_{\text{limb}}}$$

For the geometry and material properties used in this Section the required pressure is 2,200Pa. This represents the best possible support configuration and also provides a factor of safety of 1.8 for safe long-term support. Chapter 6 is focused on the design of an integrated surface that can come close to achieving the ideal hydrostatic support condition.
Chapter 6

Design of Surface Wave Actuators for Human Support and Transport

In this chapter a new, modified surface is designed that possesses two modes of support. In transport mode, the extenders are revealed to provide transport the human patient. In rest mode the extenders are hidden by an inflatable pad that is suitable for long-term support specified in Chapter 5.

6.1 Two-mode surface design concept

Chapter 5 explains that the peak interaction pressures incurred with the extender tip surface can only safely be allowed for short periods of time. For long periods of time, when the patient is resting, a smooth, elastic foundation is required for acceptable interaction stresses to be achieved. However, to realize this elastic foundation in hardware requires a modification of the surface wave active surface. Particularly, the surface of extender tips attached to the base membrane and required for transport must be transformed into a smooth, compliant foundation surface for rest conditions. To reconcile this problem an additional design feature has been added to the system that allows the surface to be used as both a long-term support surface and a short-term active transport surface. The proposed design consists of two distinct modes of support illustrated in Figure 6.1.
The idea is to attach a thin, continuous elastic membrane on top of the bed of extender tips. This thin, elastic membrane is sealed to the base membrane at the perimeter and is attached to each individual extender tip. When long-term support is needed the inflatable chamber between the base membrane and the thin elastic layer is pressurized. Figure 6.1-(a) illustrates that the thin, elastic membrane bulges upward above the extenders as the chamber is pressurized. This lifts the body weight off of the extender tips and supports the body by a thin air mattress, which meets the conditions for safe, long-term support. When the body needs to be transported, a vacuum is pulled on the inflatable chamber. By deflating and then evacuating the chamber the thin, elastic layer is pulled down over the extender tips illustrated in Figure 6.1-(b). In this way the extender tips are revealed for support of the human body and the thin, elastic layer is pulled down toward
the base membrane where it will not interfere with the operation of the surface wave transport mechanism.

6.2 Implementation of Two-mode Surface

Figure 6.2 illustrates a real implementation with a) the untreated surface including the base membrane and extenders before the thin, elastic membrane has been attached, b) the pressurized condition where the thin, elastic membrane bulges upward to support the body, and c) the evacuated condition where the extender tips are revealed for transport. With the two-mode support design, the high impedance, rigid support surface needed for surface wave transport is transformed into a smooth, compliant surface for long-term support.
To implement the two-mode surface design on the one-dimensional prototype discussed in Section 4.3 a modification of the molding process and some small changes in the support structure are needed to achieve the desired results. In the laboratory it was not possible to mold the inflatable extender surface in one piece. Instead a two-step approach was implemented where the extender surface was molded as before and the thin, elastic layer was then glued onto the extender tips. One caveat is that it is difficult to glue directly to urethane rubber that has already cured, which is a necessary condition for the mold to release. To resolve this issue a wooden substrate layer was molded directly into the extender surface to ensure an excellent bond with the urethane rubber and then the thin, elastic latex layer was then glued to the wooden substrate. To mold the wooden substrate into the extender surface the mold was modified from the form discussed in Section 4.3. Figure 6.3-(a) illustrates the cross-section of the mold discussed in Section 4.3. This mold has been machined down so that the cone-shaped cavities have been replaced by conically shaped through holes as illustrated by Figure 6.3-(b). This revised Delrin block is then placed on a flat, smooth sheet of Lexan to seal the bottom of
each through hole and ensures that the pourable rubber stays in the conical voids while curing.

![Diagram](image)

(a) - Original cross-section

(b) - Revised cross-section for substrate insertion

Figure 6.3 - Mold cross-sections

The wooden substrate is simply a thin wooden disk that is placed into the bottom of each conical cavity. The rubber compound is then poured into the mold, over the top of the wooden disks, and allowed to cure. When released, the wooden disks are firmly attached to the tips of the extenders as shown in Figure 6.4 and the latex sheet is then bonded to the wooden disks.
To complete the implementation on the existing prototype, the perimeter of the extender surface must be sealed so that the latex sheet can be inflated and evacuated about the extenders. To achieve this goal the edges of the base membrane and the latex sheet are pinched between two metal straps attached to the prototype frame as shown in Figure 6.5. At one end of the prototype the base membrane and the latex sheet are separated by a header block where the ports for compressed air and vacuum are located, allowing actuation of the surface.
Figure 6.6 illustrates a block being transported while the inflatable chamber is evacuated, thus revealing the extenders for transport.

Figure 6.6 - Inflatable chamber is evacuated for transport
Chapter 7

Conclusions

This work presents the first consideration of large-scale mechanically generated surface waves as a flexible, tangential transport mechanism for bedridden patients. In this thesis, the non-linear kinematic properties of surface waves are formulated and examined. Position and velocity analyses have been presented that characterize one-dimensional surface waves in terms of the design parameters \( \{r, R, I, \lambda, \omega, \text{phase angle sense} \} \). The analysis provides a base particle looping condition that represents a kinematic constraint on base point motion. In addition the analysis reveals the need for a judicious choice of phase angle sense for most effective human transport. The extender design feature has been introduced to enhance the performance of surface waves and simplify the design of waves synthesized to transport humans or elastic bodies. Analysis and simulation indicate conditions for smooth, gentle support, enhanced load sharing capacity, actuator dimension reduction, and hidden trough conditions. Each of these properties is either enhanced or is a direct result of the extender feature. These properties are the critical features needed to physically realize the potential for patient transport by surface waves. Experimentally, a low impedance actuator based on water support is explored without significant loading effect to utilize resonance conditions of a fluidic base as a dramatic simplification of the design, and a mechanical prototype verifies analytical results.

A fully functional prototype has been constructed that explicitly takes advantage of the features discovered in this thesis. A mechanical analysis of the interaction between the surface wave actuator and elastic bodies has been performed to ensure the
viability of surface waves in service of bedridden patients. Meaningful human-based performance criteria have been established to characterize the effectiveness of a candidate design in terms of support and transport quality. Combining the kinematic analysis results of this paper with static analysis provides the suite of tools necessary to safely tune the surface and transport properties for specific applications.

The performance of surface wave actuation has been illustrated with a prototype that shows the efficiency of transport of both rigid and elastic bodies. Special consideration has been given to the transport of bedridden patients by surface waves. Design constraints based on clinical data have been developed to quantitatively describe the human-surface interaction. Finite element models have been developed for realistic worst-case interaction scenarios to calculate the interaction pressure between the human and the support surface. Based on these results a revised surface has been designed to provide a clinically viable support surface for both short-term transport and long-term static support of bedridden patients.
References


