Robust Control Design and Simulation of the Maneuvering Dynamics of an Arleigh Burke Class Destroyer

by

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Submitted to the Department of Ocean Engineering in partial fulfillment of the requirements for the degrees of

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and

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Abstract

This thesis presents the design of a Robust Multivariable Control System for
an Arleigh Burke Class destroyer. The system is designed to control the ship
motions in surge, sway and yaw, and the engine-propeller dynamics. The controller
designed in this thesis is intended to be used on a scaled model of the destroyer in
the Charles River.

A nonlinear model of the ship dynamics was developed based on the
available ship data and models for ship motions. Most of the ship's geometry was
available, but the hydrodynamic coefficients required approximation using
regression formulae. The nonlinear model was used to obtain Linear-Time-
Invariant models, required for controller design. The designs were completed using
the Linear-Quadratic-Gaussian optimal control with Loop-Transfer-Recovery
(LQG/LTR) technique and then tested with the nonlinear model in different
conditions; including sensor noise, currents, actuator saturation and rate limits.
The final controller is a non-linear controller based on the LTI designs and gain
scheduling.

The impact of ship maneuvering automatic control in surface combatants is
analyzed, based on current U.S. Navy programs and practices. The control system
designed in this thesis is going to be used in a scaled model of the ship to simulate
maneuvers performed manually at present.

Thesis Supervisor: M. S. Triantafyllou
Title: Professor of Hydrodynamics and Ocean Engineering
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This thesis would not have been possible without the encouraging and tender support of my wife Giovanna, and the expert assistance of my children Rosario and Tomas.

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To Giovanna, Rosario and Tomas

Cambridge, Massachusetts, August 1999.
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Chapter 1
Introduction and Summary

1.1 Background

Ship maneuvering by means of automatic controls has been attempted since the introduction of the gyrocompass in the early years of this century. The first autopilots introduced by Sperry in the 1930's consisted of Single-Input-Single-Output (SISO) proportional controllers for ship heading. Further refinement of these devices included Proportional, Integral and Derivative (PID) control and adaptive algorithms. All of these approaches somehow failed in the more complex cases, or the ones requiring better performance, because ship motions and engine dynamics are multivariable systems requiring multiple inputs and outputs, in order to accurately model and control their behavior.

With the development of modern control theory, new tools are available to represent multiple degree of freedom systems, and design the appropriate controllers. Nowadays state space methods, optimal controllers and Kalman Filters are used with robust methodologies allowing for better controller designs. None of these Multivariable techniques were viable before the explosive development of digital computing, which eases the computation of matrix equations inherent to the Modern Control Theory.

Modern applications of Robust Control to ship maneuvering include roll control, ship-propeller speed control, heading control, ship tracking, station keeping and fuel consumption optimization. Most of these control systems have
been developed to specific ship applications, using dedicated computers and with limited published information concerning details of the methods used and design tradeoffs. Moreover, despite the awesome progress in navigation aids and control systems, few ships, naval or merchant, have implemented modern control systems for maneuvering. Most applications include only course keeping by means of PID controllers. Nevertheless, nowadays-maneuvering control systems find application in:

- Open seas: roll, course keeping, and fuel economy
- Restricted water: Track keeping, anchoring and docking
- Underwater: depth, speed, course, pitch, etc...

The naval applications have been concentrated in station keeping for minehunters, roll control (with rudders and lateral active fins) for surface combatants, depth control for submarines and some full automated maneuvering for small unmanned Autonomous Underwater Vehicles (AUV's). This is expected to change, as ship crews will shrink significantly in the years to come as a consequence of cost reductions and functionality. The surface combatant of the XXI century is expected to reduce crew significantly (if not completely unmanned), and be capable of executing multiple preplanned actions, including maneuvering. In this scenario, knowledge of the ship’s maneuvering dynamics and the use of true Multiple Input-Multiple Output (MIMO) control design techniques are a must. There is also necessary a deeper understanding of the limitations and capabilities of actual simulation and design techniques, to include maneuvering from the first stages in the design spiral. This represents a major change from the way ship maneuvering is commonly approached today: as a sub-product of the design process. Therefore, the available information to model the dynamics of ships, propellers, and power plants is incomplete from the maneuvering control perspective.

* An excellent paper on the state of the art and the necessary research for ship maneuvering simulation is the one by I.W. Dand [Da.1].
With some of the above challenges in mind, an autonomous scale model of the U.S. Navy's surface fleet backbone, the *Arleigh Burke* Class destroyer (DDG51), is being built by a team of graduate students at the MIT Towing Tank. The model is being constructed to investigate ship dynamics and control, and to assist the design curriculum of the Ocean Engineering Department students, specifically course 13.49, *Maneuvering and Control of Surface and Underwater Vehicles*. This thesis is part of this project and intends to develop a mathematical model for DDG51 and design a controller of the ship motions in surge, sway, yaw and the engine-propeller dynamics.

### 1.2 Objectives of the Thesis

The main objective of this thesis is the design of a controller for the maneuvering motions of an Arleigh Burke destroyer, including engine-propeller dynamics. In order to design and test this controller it is necessary to develop a mathematical model of the ship that includes some nonlinear dynamics that are inherent to ship's motions. To achieve the thesis main goal is necessary to consider some implicit specific objectives and limitations that bound this work.

Among the specific objectives of this thesis is the development of models and control systems that may grow in the future and are based in physical principles, were the data may be filled by parametric or experimental results. Another objective is the use of a true MIMO control system, with guaranteed robustness and stability. It is also considered that the system description and modeling have to be based as much as possible on the full scale DDG51 rather than the model, but dimensionless quantities should be used throughout the study to keep generality and allow the use of standard hydrodynamics notation and results. The use of dimensionless quantities also proved a good approach for controller design. Finally, it is considered of importance to show how this control system and maneuvering simulation fit in the development of the unmanned surface combatant of the next century.

The global objective of this thesis is extremely ambitious, and is limited to focus the effort on the essential performance expected for the first maneuvers with the scaled model. The controller design will be focused on the forward motions
including changes in heading and speed at rates that reflect normal operating conditions. Nonlinear effects will be considered, as lift and drag forces, which are both very important, are quadratic functions of speed. Nevertheless, the magnitude of the nonlinearities should be kept within reasonable margins, keeping in mind that inaccuracies in the estimation of many linear parameters may have a much larger effect that the inclusion of these terms.

1.3 Contributions of the Thesis

This thesis presents a complete design process of a Robust MIMO controller for ships motions, based in general parameters that may be known in the early design stages. This thesis allows an engineer or graduate student to learn the basics of the LQG/LTR technique and apply it to a multivariable system.

The thesis also provides a simulation model of DDG51 maneuvering dynamics, based in a proven mathematical model enhanced with actuators saturation, rate limits and improved robustness. The simulations are presented in a graphic interface that allows for ease of simulation and results visualization. All of the work was developed in the Matlab® computing environment, which allows for connections with C or Fortran codes if required in the future.

Maybe one of the most important contributions of this thesis is that it will help to provide the Ocean Engineering graduate students with practical experience in the maneuvering dynamics and control of surface ships. The availability of a scaled model of DDG51 with automated maneuvering will help them significantly to understand the dynamics of surface vehicles.

This thesis also presents the driving factors for naval ships maneuvering automation and includes the main requirements to fully automate ship motions. Some "typical" preplanned maneuvering of an automated surface combatant are simulated using the controller design for DDG51 and the nonlinear simulation.

1.4 Thesis Outline

Chapter two provides a general description of DDG51 and develops a nonlinear model of the ship. The non-linear model is tested against the expected dynamic performance of the actual destroyer based in existing rules of thumb. The
LTI models are then obtained linearizing the nonlinear equations about selected operating points. These are the models that will be used for controller design. Finally the linear models are validated comparing their behavior with the nonlinear model.

Chapter three investigates the LTI models dynamics in order to visualize system characteristics important for controller design. The models first undergo modal analysis and then their pole/zero structure and singular values are also analyzed. Finally, the models are balanced and their order reduced using balanced truncation.

Chapter four explains and develops the linear MIMO controller design. First of all, the controller expected performance is defined, and then the reader is introduced to the LQG/LTR controller design methodology. Following is a complete example of the design for one of the LTI models with the results of the simulation carried on with the nonlinear model.

Chapter five describes the implementation of the nonlinear MIMO controller based in the use of Gain Scheduling. Further simulation results with the use of the nonlinear controller are presented, along with some standard maneuvers on the earth coordinate system.

Chapter six briefly introduces the reader to a vision of the 21st century surface combatant, based in current U.S. navy programs and fundaments the need for automated maneuvering. Further analysis of the functions expected from such a ship and the difficulties to achieve this goal are presented using a Risk Rating Matrix.

Chapter seven summarizes the work done in this thesis and gives recommendations on further improvements to the model and simulation.
Chapter 2
System Description and Models Development

2.1 Introduction
The global objective of this chapter is to obtain two types of ship models: First a nonlinear model to simulate, as accurately as possible, DDG51 motions to test the controller before onboard setup and second, LTI models that will be used for linear controller design.

In order to obtain these models the ship will be discussed in physical terms, describing its main features and dynamical characteristics. Following this description, the nonlinear equations of motion used for the ship's maneuvering simulation are presented alongside with some simulation results. These equations of motion are then non-dimensionalized and linearized to obtain Linear Time Invariant (LTI) models to be used for controller design. The LTI models performance in the description of the ship motions is verified comparing the results with the nonlinear model dynamics.

2.2 System Description
The Arleigh Burke Class or DDG51 is a 142-meter destroyer displacing 8630 metric tons at design waterline (DWL). DDG51 principal characteristics are presented in table 2.1. For the purposes of this study, the ship is considered to be floating at the design waterline with zero trim and list.
<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Length (LBP)</td>
<td>142.04 m</td>
</tr>
<tr>
<td>Beam (B)</td>
<td>17.98 m</td>
</tr>
<tr>
<td>Draft (DWL)</td>
<td>6.31 m</td>
</tr>
<tr>
<td>Displacement (DWL)</td>
<td>8630 MT</td>
</tr>
<tr>
<td>Waterplane Area (A_w)</td>
<td>2777.2 m²</td>
</tr>
<tr>
<td>Longitudinal Center of Gravity (LCG)</td>
<td>0.85 mts aft amidships</td>
</tr>
<tr>
<td>Block Coefficient (C_B)</td>
<td>0.522</td>
</tr>
<tr>
<td>Prismatic Coefficient (C_P)</td>
<td>0.615</td>
</tr>
</tbody>
</table>

Table 2.1 Arleigh Burke Class Main Characteristics at DWL

DDG51 hull form is bulkier than prior US destroyer designs, but with C_P 0.615 and C_B 0.52 it still is a very streamlined vessel when compared with most ships of its size. Figure 2.1 shows DDG51 Body Plan, where the transom stern and part of the sonar dome are the most remarkable features in the streamlined hullform.

Figure 2.1 DDG51 Body Plan
The ship propulsion plant consists of two shafts, each powered by two General Electric LM2500 gas turbines coupled through a gearing box and shafting to a controllable pitch propeller which allows for astern motion. This power plant is able to drive the ship to speeds up to 32 knots at full power.

Course control is achieved by means of two mechanically linked rudders that are located downstream of each propeller. There are not other fins in the hull, neither for course control or roll stabilization.

The ship is assumed to be following a track with forward speed during normal maneuvers. This scheme leads to model the propellers as fixed pitch propellers operating at pitch corresponding to maximum power as they are normally operated in this class of ships for any speed from about 5 knots and above. Both propellers are assumed to be running at the same speed, also reflecting actual operational procedures. This study does not consider emergency maneuvers such as crash astern or collision avoidance.

The ship has port-starboard symmetry, as is shown in figure 2.2, with two coordinate systems: One is a ship-based coordinate system with the x-axis along the ship and the origin amidships. The ship moves with general forward speed U, which is not necessarily over the x-axis, but at a drift angle β. The ship mass is supposed to be concentrated in the CG, and positive speed and forces are towards the bow for the x-axis and to port side in the case of the y-axis. The second system is an earth reference and defines a heading angle ψ with respect to the speed vector U.

Figure 2.3 shows the relative position of rudders, propellers and ship’s LCG with respect to the ship’s coordinate system. The vertical center of gravity is not taken into consideration because the model assumes motions only in the x-y plane, while yG is assumed to be over the ship’s centerline (x axis) as is normal in a symmetric ship without list. It was chosen not to set the origin of the ship coordinate system at the LCG to preserve model generality and flexibility.
Figure 2.2 Reference Coordinate System

Figure 2.3 Relative Position of Ship's Rudders and Propellers
2.3 Nonlinear Model

The mathematical nonlinear model of the ship maneuvering motions in this thesis consists of the coupled equations of surge, sway, yaw and propeller revolution and is based in the mathematical model developed by Inoue [In.1]. These equations make possible to calculate the x-y plane motion of the ship due to changes in engine fuel rate and rudder angle. A detailed explanation of the nonlinear model, including numeric data, is enclosed in Appendix 1. With reference to the coordinate system shown in figure 2.1, the Equations of Motion for the ship can be written in the classic form first presented by Abkowitz [Ab.1] with the addition of a fourth equation for the propeller revolution dynamics (2.4). For a complete derivation of these equations the reader is referred to references [PNA.1] and [Tr.1]. This ODE includes the polar moment of inertia of the whole propulsion plant, $I_{pp}$, that comprise added polar moment of inertia of the gas turbines, gearbox, shafting, propeller and water entrapped in the propeller [PDI.1]. In the RHS are the engine and propeller dynamics, matched at the propeller shaft by the gear ratio, $\lambda$, and respective efficiencies.

\begin{align}
\mathbf{m} \cdot (\mathbf{u} - \mathbf{v}_{r} - \mathbf{x}_{r} \mathbf{r}^2) &= \mathbf{X} \quad (2.1) \\
\mathbf{m} \cdot (\mathbf{v} + \mathbf{u}_{r} + \mathbf{x}_{t} \mathbf{r}) &= \mathbf{Y} \quad (2.2) \\
I_{zz} \dot{\mathbf{r}} + \mathbf{m} \mathbf{x}_{O} \cdot (\dot{\mathbf{v}} + \mathbf{u}_{r}) &= \mathbf{N} \quad (2.3) \\
\frac{2\pi I_{pp}}{\eta_{P}} n_{P} &= \lambda \eta_{E} Q_{E} - \frac{Q_{P}}{\eta_{R}} \quad (2.4)
\end{align}

The RHS terms in the equations of surge, sway and yaw can be decomposed in forces and moments due to the Hull, Propeller and Rudder hydrodynamics.

\begin{align}
\mathbf{X} &= \mathbf{X}_{H} + \mathbf{X}_{R} + \mathbf{X}_{P} \quad (2.5) \\
\mathbf{Y} &= \mathbf{Y}_{H} + \mathbf{Y}_{R} \mathbf{m} \quad (2.6) \\
\mathbf{N} &= \mathbf{N}_{H} + \mathbf{N}_{R} \quad (2.7)
\end{align}
2.3.1 Hull Forces

The hydrodynamic forces acting on the hull due to the ship motion are expressed as functions of polynomials and hydrodynamic derivatives. The hydrodynamic derivatives and coefficients can be derived from model tests - rotating arm and/or planar movement generator - or estimated based on regression formulae derived for this purpose.

Despite the fact that the later method is not always accurate, it was used in this design because the actual coefficients were not available. Nevertheless, this method was in part chosen to prove if a satisfactory control system can be designed with the ship data available in early design stages with the modern Robust Control techniques that include model and parametric uncertainty. Taking this into consideration, the model includes up to third order hydrodynamic derivatives in surge, sway and yaw.

\[ X_H = X_0 \dot{u} + X(u) + (X_{vr} - Y_v) vr \]  
\[ Y_H = Y_v \dot{v} + X_0 ur + Y_p \beta + Y_r \gamma + Y_{pp} \beta \dot{\beta} + Y_{ppr} \beta \gamma r + Y_{pp\beta} \beta r \]  
\[ N_H = N_f \dot{\gamma} + N_p \beta + N_r \gamma + N_{p\beta} \beta^2 \gamma + N_{p\beta r} \beta^2 \gamma r \]

Equation (2.8) describes the forces acting in the X direction due to the hull, which are approximated to be the added mass, Drag and a term that accounts for increased draft due to drift angle. This term is approximated using Inoue model and Hasegawa [Ha.1] formulae that depend on the added mass in the Y direction and \( C_B \). All other hull hydrodynamic derivatives are calculated using Inoue formulae [In.2].

Equation (2.9) includes the forces in the Y direction, which correspond to the added mass in surge and sway and the standard derivatives due to the drift angle and yaw rate up to second order.

Finally, equation (2.10) includes the added moment of inertia about the z-axis, and linear and nonlinear derivatives functions of the drift angle, \( \beta \), and yaw rate, \( r \).
2.3.2 Propeller Forces

Propeller forces are modeled using the classic Propeller Thrust and Torque equations (2.11) and (2.12) with $K_T$ and $K_Q$ being second order polynomials in $J$ fitted by least squares method to match the actual DDG51 propellers data at pitch ratio 1.72. $X_p$ has a factor 2 in front to account for the two propellers installed onboard. $Q_p$ does not have this factor because the Propeller Dynamics equation (2.4) is set up for one shaft. There is not total propeller-induced force acting in the Y-axis direction ($Y_p$), nor moment ($N_p$) about the z-axis, because of the arrangement with opposite rotation. The wake deduction fraction includes a correction for drift angle, with the effective thrust deduction factor, $w_p$, being a function of axial and lateral velocity, $u$ and $v$, yaw rate, $r$, and $x_p$.

$$X_p = 2 \rho n_p^2 D^4 K_T(J) \tag{2.11}$$
$$Q_p = \rho n_p^2 D^5 K_Q(J) \tag{2.12}$$

$$J = \frac{u(1 - w_p)}{n_p D} \tag{2.13}$$

This model fails in some conditions such as zero propeller revolutions and astern thrust. As these conditions are not considered in this design, the model was improved in order to provide only qualitative solution to these problems and simulation robustness. As a result, astern propulsion and zero propeller revolution are feasible, but no accuracy is granted.

2.3.3 Rudder Forces

Rudder forces are assumed to be related only with induced drag and lift. All viscous drag is supposed to be included with the ship's resistance. The model assumes a force normal to the rudder, $F_n$, that is a function of the rudder inflow velocity, $V_R$, and angle, $a_R$. There is a factor $(1+a_H)$ that accounts for ship geometry based in the chart presented by Kijima [Ki.1]. $F_n$ has a factor of two to account for both DDG51 rudders.
\[ X_R = -F_N \sin \delta \]  
\[ Y_R = -(1 + a_h)F_N \cos \delta \]  
\[ N_R = -(1 + a_h)X_R F_N \cos \delta \]  
\[ F_N = \rho \frac{6.13 \Lambda}{\Lambda + 2.22} A_R V_R^2 \sin \alpha_R \]  

Details of the relations to compute the different factors and variables are described in Appendix 1.

2.3.4 Engine Torque

Engine torque is modeled as a function of fuel rate and shaft revolutions. In fact it is a look-up table, where given the fuel rate and shaft revolutions per second it is possible to find the engine torque output. The coefficients were selected in order to match the LM2500 performance*.

\[ Q_E = Q_M \left\{ \left( a \frac{f}{f_M} + b \right) \frac{n_N}{n_M} + \left( c \frac{f}{f_M} \right) \right\} \]  

2.3.5 Simulation Results

The equations presented above were implemented in a Runge-Kutta 5th order ODE solver from the Matlab\textsuperscript* package, and several simulations were carried on to verify the model's ship-like behavior and robustness. A user-friendly display was assembled with Matlab\textsuperscript* to ease visualizing of the maneuvering performance of the model. The interface allows simultaneous presentation of all the time variables and maps the ship position on an earth-based grid. The simulation data is also saved into files for post-simulation analysis.

* Gas turbine performance is strongly affected by several variables difficult to simulate in this model. Therefore, this simulation oversights many of the important factors in the LM2500 dynamics from the engine standpoint. Nevertheless, it offers a model feasible to simulate and accurate enough to represent the engine dynamics effect on DDG51 maneuvering. For a detailed model of the DDG51 engine dynamics the reader is deferred to reference [PDI.1].
Most of the existing data about the actual maneuvering performance of DDG51 is classified as is a normal practice when referring to Navy's surface combatants. Nevertheless, it can be said that the model adequately represented DDG51 turning dynamics when compared with ship sea trials. Figure 2.4 illustrates about some standard parameters in ship maneuvering with respect to turning ability that were used to validate the model. Figure 2.5 shows an x-y earth based plot of the model motions in a turn at 20 knots and 15, 25 and 35 degrees rudder angle, where the results show how the turning circle shrinks as rudder angle increases. At 10 degrees rudder angle, the motions almost resemble steady...
turning, but at 30 degrees deflection it takes about a complete turn to achieve steady motions.

Figure 2.5 Nonlinear Model Turning Circles at 20 knots

Figure 2.6 shows plots of the axial and lateral velocities and yaw rate for 25 degrees rudder deflection angle at initial speeds of 10, 20, 25 and 35 knots. This plot shows how the lateral component of the ship's speed, \( v \), considerably increases with respect to the axial velocity as the turn progresses, and how it also increase at larger rudder deflections. Similarly, yaw rate also grows with rudder angle and time. It must be remarked that this "well-behaved" nonlinear dynamics for the ship are no longer valid for larger rudder deflections. In effect, the lift coefficient, \( C_L \) may be assumed linear for small rudder deflections but even for this low aspect ratio
wing, stall is supposed to start at about 30 degrees and lift will have a significant degradation at about 35 degrees. For this reason, the action of the rudders for control design is limited to 30 degrees.

![DDG51 Nonlinear model kinematics at 20 knots](image)

Figure 2.6 DDG51 Nonlinear Model Dynamic Responses at 20 knots

**Unforced Response**

Once examined the model behavior, the first case to be considered was the unforced response of the model to non-zero initial conditions. For this purpose, both inputs were set to zero (fuel rate and rudder angle) and unitary dimensionless initial conditions set for $u$, $v$ and $r$ with respect to a 20 knots ship speed. Remarks are made that the lateral velocity has a negative value in order to set initial conditions that are feasible for the ship without external forces and that propeller revolutions initial condition was set to zero in order to have insight in the ship hull dynamics. The resulting time domain response is shown in figure 2.7.
Several conclusions can be reached from this time response plot. The ship relative velocities and yaw rate seem to have a stable first-order response, with damping depending on hydrodynamic drag. It is easy to visualize that these drag forces are smaller in the X direction (due to streamlined body) than in the case of sway and yaw rates.

In addition, an unforced response simulation taking into account the ship straightforward motion was considered. With both inputs set to zero, initial conditions in r and v were also set to zero and axial speed and propeller rpm were set to a pre-computed equilibrium point corresponding to 20 knots. Again, both unforced responses, shown in figure 2.8, seemed stable first-order systems, with the ship's longitudinal motion having larger inertial forces than the propeller and engines. The later ones start slowing fast, but have to follow the shape of the axial
speed due to the propeller torque conditions (propeller trailing with positive rotation but negative thrust).

Figure 2.8 Nonlinear Model Unforced Response, Surge-Propeller

**Forced Response — Rudder Step Simulation**

Next, the model response to step inputs was considered. First the ship was considered in steady motions at 20 knots with no rudder deflection and the appropriate fuel rate and propeller rpm. Lateral velocity and yaw rate initial conditions were zero. A rudder deflection of 15 degrees (midrange) was considered as step input at time zero, obtaining the results plotted in figure 2.9.
This time, both lateral velocity and yaw rate appear as highly damped second order systems, and surge appear as a very slow first-order system. This plot also shows that there is some coupling between surge and the sway-yaw couple that can be attributed to the increased drag due to drift angle.

** Forced Response – Fuel Step Simulation  

Finally the system was tested with a fuel rate step corresponding to the 20 knots steady value. All initial conditions and inputs were set to zero, and the fuel step imposed. The resulting response is shown in figure 2.10 and ratifies some of the characteristics already outlined in the unforced response simulation. The larger forces involved in the coupled dynamics correspond to the ship's forward motions, which are a first order, very slow system. The propeller dynamics appear also as a first order stable system, but its dynamics are somehow governed by the ships...
motion. This effect can be seen in figure 2.10, where the propeller revolutions per second increase rapidly, because for that small-scale ship speed it is fairly unloaded. As ship speed rises, the propeller gets loaded and its dynamics follow that of the ship axial velocity, just a little faster.

![Nonlinear Model Fuel Step Response](image)

**Figure 2.10 Nonlinear Model Unforced Response, Fuel Rate Step**

### 2.4 Full Order LTI Model

For the linear control systems design techniques, a non-linear model as presented in Section 2.3 can not be used. The LQG/LTR design methodology requires that the dynamics of DDG51 have to be converted into a Linear Time Invariant (LTI) model of the form:

\[ \dot{x} = A \cdot x + B \cdot u \quad (2.19) \]
\[ y = C \cdot x + D \cdot u \quad (2.20) \]
In this system of linear equations, expressed in matrix form, \( \mathbf{x} \) is know as the state vector and contains the fundamental system variables that embed its dynamics, \( \mathbf{u} \) is the inputs vector, which represents the external inputs to the system, and \( \mathbf{y} \) is the outputs vector. \( \mathbf{D} \) is a direct feed-through matrix which is zero in the case of our system. As it is easy to visualize from the previous sections, the state vector and input vectors are:

\[
\mathbf{x} = \begin{bmatrix} \mathbf{u} \\ \mathbf{v} \\ \mathbf{r} \\ \mathbf{n}_f \end{bmatrix} \quad \mathbf{u} = \begin{bmatrix} f_R \\ \delta \end{bmatrix}
\]

The output vector will be selected during control design, but for now it will be assumed that the output vector is the same as the full state vector.

Nevertheless, these LTI models represent the system behavior accurately only around a small domain and accurate modeling will require several LTI systems to be used for different operating conditions.

2.4.1 Linearization

Linearization consists in finding a linear model that describes the dynamics about an equilibrium point. In order to find a linear model, it is first necessary to figure out an equilibrium condition and then obtain an incremental model about that point. Consider the nonlinear time-invariant system defined as follow:

\[
\begin{align*}
\dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}, \mathbf{u}) \\
\mathbf{y} &= \mathbf{h}(\mathbf{x}, \mathbf{u})
\end{align*}
\]

(2.21) \quad (2.22)

Given a constant value of the inputs, \( \mathbf{u}^* \), and initial conditions, it is possible to compute an equilibrium state, \( \mathbf{x}^* \), that satisfy:

\[
\begin{align*}
\mathbf{f}(\mathbf{x}^*, \mathbf{u}^*) &= 0 \\
\mathbf{h}(\mathbf{x}^*, \mathbf{u}^*) &= \mathbf{y}^*
\end{align*}
\]

(2.23) \quad (2.24)
The solution of this problem requires solving the ODE's of the previous sections with the desired constant inputs until the derivatives become zero.

Once the Equilibrium State is determined the variables should be written in incremental form around the equilibrium condition.

\[ x(t) = x^* + \Delta x(t) \quad (2.25) \]
\[ u(t) = u^* + \Delta u(t) \quad (2.26) \]
\[ y(t) = y^* + \Delta y(t) \quad (2.27) \]

As \( x^* = 0 \), and substituting equations 2.25, 2.26 and 2.27 in 2.23 and 2.24, the incremental model of the nonlinear equations become:

\[ \Delta x = f(x^* + \Delta x, u^* + \Delta u) \quad (2.28) \]
\[ \Delta y = h(x^* + \Delta x, u^* + \Delta u) - y^* \quad (2.29) \]

This incremental model is then expanded in Taylor series about the operating point. For example \( f \) is expanded about the operating point:

\[
\begin{align*}
    f_i(x^* + \Delta x, u^* + \Delta u) &= f_i(x^*, u^*) + \frac{\partial f_i}{\partial x_1} \Delta x_1 + \frac{\partial f_i}{\partial x_2} \Delta x_2 + \cdots \\
    &\quad + \frac{\partial f_i}{\partial x_n} \Delta x_n + \frac{\partial f_i}{\partial u_1} \Delta u_1 + \cdots + \frac{\partial f_i}{\partial u_r} \Delta u_r + \text{higher order terms in } \Delta x, \Delta u \quad (2.30)
\end{align*}
\]

Assuming that the higher order terms in \( \Delta x \)'s and \( \Delta u \)'s are small enough, the system can be written in matrix form as:

\[ \Delta \dot{x} = \left[ \frac{\partial f}{\partial x} \right] \Delta x + \left[ \frac{\partial f}{\partial u} \right] \Delta u \quad (2.31) \]

In similar manner, the outputs equation become:

\[ \Delta y = \left[ \frac{\partial h}{\partial x} \right] \Delta x + \left[ \frac{\partial h}{\partial u} \right] \Delta u \quad (2.32) \]
In this system, the partial differential of vectors is known as the Jacobian. For example, in the ship's system of 4 differential equations, the A matrix will be the Jacobian of \( \mathbf{f} \) with respect to the state vector:

\[
\frac{\partial \mathbf{f}}{\partial \mathbf{x}} = \begin{bmatrix}
\frac{\partial f_1}{\partial u} & \frac{\partial f_1}{\partial \nu} & \frac{\partial f_1}{\partial r} & \frac{\partial f_1}{\partial \eta_p} \\
\frac{\partial f_2}{\partial u} & \frac{\partial f_2}{\partial \nu} & \frac{\partial f_2}{\partial r} & \frac{\partial f_2}{\partial \eta_p} \\
\frac{\partial f_3}{\partial u} & \frac{\partial f_3}{\partial \nu} & \frac{\partial f_3}{\partial r} & \frac{\partial f_3}{\partial \eta_p} \\
\frac{\partial f_4}{\partial u} & \frac{\partial f_4}{\partial \nu} & \frac{\partial f_4}{\partial r} & \frac{\partial f_4}{\partial \eta_p}
\end{bmatrix}
\]

(2.33)

As these Jacobians are evaluated at the equilibrium point, they are constant matrices, and because the equations above are linear, the incremental system is linear and time-invariant (LTI).

Linearization was carried out as outlined above, using existing pre-coded Matlab\textsuperscript{*} routines\textsuperscript{*} about several operating points (total of 18: 6 speeds and 0, 10 and 20 degrees rudder angle). The LTI models performance was then evaluated to decide how many models were required for adequate nonlinear control design (gain scheduling), selecting a total of 10 LTI models, whose matrices A, B, C and D, besides equilibrium points are included in Appendix 2. The most important consideration in this selection was to include two types of models, speed changing models (fuel rate in this case) and turning rate (rudder deflection angle). Despite the disadvantage that it will be computationally expensive, neither of the models captured by itself the dynamics of the ship. If a model about a turning trajectory were chosen, the controller would predict a change in surge and yaw forces and would act in consequence even in straightforward motions in the event of any speed changes. By the other hand, a model linearized about straightforward motions would completely fail to predict the reduction in speed during turns.

\* For the reader interested in using the Matlab functions their names are: trim (get equilibrium points) and linmod (get linear model).
2.4.2 LTI models performance

In order to establish the validity of the LTI models generated and to determine the number of them required for controller design, several LTI models were generated and tested. These models were computed selecting appropriate fuel and rudder angle inputs to cover a wide range of speeds and turnings to ascertain the region of the space in which each model was accurate enough. Equally perturbing the non-linear and the LTI models performs the validation process that finished with the selection of the 10 LTI models shown in Table 2.2.

<table>
<thead>
<tr>
<th>LTI Mod</th>
<th>Fuel ratio (%)</th>
<th>Rudder deflection (degrees)</th>
<th>$u^*/V$</th>
<th>$v^*/V$</th>
<th>$r^*L/V$</th>
<th>$n_{p^*L/V}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F5R0</td>
<td>5</td>
<td>0</td>
<td>0.1653</td>
<td>0.0000</td>
<td>0.0000</td>
<td>4.6769</td>
</tr>
<tr>
<td>F10R0</td>
<td>10</td>
<td>0</td>
<td>0.3476</td>
<td>0.0000</td>
<td>0.0000</td>
<td>7.8472</td>
</tr>
<tr>
<td>F25R0</td>
<td>25</td>
<td>0</td>
<td>0.6089</td>
<td>0.0000</td>
<td>0.0000</td>
<td>13.3025</td>
</tr>
<tr>
<td>F50R0</td>
<td>50</td>
<td>0</td>
<td>0.8025</td>
<td>0.0000</td>
<td>0.0000</td>
<td>18.2458</td>
</tr>
<tr>
<td>F100R0</td>
<td>100</td>
<td>0</td>
<td>1.0003</td>
<td>0.0000</td>
<td>0.0000</td>
<td>24.0010</td>
</tr>
<tr>
<td>F5R10</td>
<td>5</td>
<td>10</td>
<td>0.1213</td>
<td>-0.0242</td>
<td>0.0537</td>
<td>4.4059</td>
</tr>
<tr>
<td>F10R10</td>
<td>10</td>
<td>10</td>
<td>0.2397</td>
<td>-0.0424</td>
<td>0.0962</td>
<td>7.0526</td>
</tr>
<tr>
<td>F25R10</td>
<td>25</td>
<td>10</td>
<td>0.4532</td>
<td>-0.0759</td>
<td>0.1738</td>
<td>12.0659</td>
</tr>
<tr>
<td>F50R10</td>
<td>50</td>
<td>10</td>
<td>0.6332</td>
<td>-0.1063</td>
<td>0.2433</td>
<td>16.9282</td>
</tr>
<tr>
<td>F100R10</td>
<td>100</td>
<td>10</td>
<td>0.8231</td>
<td>-0.1406</td>
<td>0.3208</td>
<td>22.7063</td>
</tr>
</tbody>
</table>

Table 2.2 LTI Models Selected and Corresponding Operating Points

To illustrate the selection process, figure 2.11 shows the performance of a model linearized about 20 knots speed and 10 degrees rudder angle, with initial conditions corresponding to 20 knots speed and zero degrees rudder deflection. The model at time zero is forced with a 15 degrees rudder deflection. The plots show a very accurate response in all variables, especially when rate is considered. As the final, and initial, condition are different that the equilibrium point, some steady differences show up, especially in propeller rpm and speed, due to the high
rate of change of the ship resistance. Nevertheless, the differences are small and the curves initial slopes are very accurate.

![Graphs showing LTI and Nonlinear model responses](image)

Figure 2.11 20 knots 10 degrees LTI Model and Nonlinear Model Response

The same rudder step was applied to the LTI model linearized about 20 knots and zero rudder deflection. Figure 2.12 shows the performance of that model, which is characterized by no response in axial speed and propeller revolutions and narrow response in lateral velocity and yaw rate for a short period.
Figure 2.11 20 knots 0 degrees LTI Model and Nonlinear Model Response

The approach explained above was repeated with several models, using fuel and rudder deflection steps. Although for this design a gain scheduling with 2D interpolation (speed-rudder angle) will be used, it is possible to develop a simpler, and computationally cheaper, non-linear controller with gain scheduling based only on speed, but with degraded performance. Therefore, the selected models are the ones summarized in table 2.2.
2.5 Summary

This chapter has introduced DDG51 maneuvering characteristics and a nonlinear model to simulate the ship's motions. Some first conclusions about coupling and system response were foreseen from the forced and unforced response of the nonlinear model. Based on this model, the LTI systems required for controller design were obtained.

The next chapter will further investigate the LTI models, including modal analysis, pole/zero structure and singular values.
3.1 Introduction

In this chapter, a detailed analysis of the eigenstructure of two of the LTI models is performed in order to support the design of the LQG/LTR controller. The models to be analyzed are the F50R10 (50% fuel rate, 10°-rudder deflection) and the F50R0 (50% fuel rate, no rudder deflection), both corresponding to ship's speeds of about 20 knots. All the other models were investigated in similar extension, but are not presented here for document conciseness. All models were explored in detail, including modal analysis (eigenvalues and eigenvectors), MIMO pole/zero structure, Controllability and Observability, and frequency response (singular values). Furthermore, given the results of these analyses, some alternatives are explored in order to simplify the models.

The computed data from this chapter, as well as singular value plots of all LTI models can be found in Appendix 3.

3.2 Setting up the model for controller design

The LQG/LTR technique ensures, under certain conditions, that the control design will have some guarantees with respect to gain and phase margins*. The linear system must meet certain requirements for the validity of these margins to be certain. Among these conditions the system should be square, detectable and reachable. The first of these conditions impose a limit in the number of outputs that can be controlled, in this case 2. The second condition is translated into

* These guarantees are not absolute and exist in the sense that the LTR process recovers a loop shape that is similar to the one of the Kalman filter design, which in fact have the robustness qualities [Li.1].
determine if the system is controllable and observable, system characteristics that
are stronger than the original requirement but easier to check.

3.2.1 Outputs selection

The LQG/LTR design methodology requires the system to be square, i.e. an
equal number of control inputs, \( u \), and outputs, \( y \). As the inputs are already
defined to be fuel rate, \( f \), and rudder deflection, \( \delta \), there is a selection to be made
with respect to the outputs. The compensator will be designed as a rate controller
to control changes in axial velocity, \( u \), and yaw rate, \( r \). This alternative was
selected, because it allows to control ship speed and heading.

The LTI Models F50R0 and F50R10 can be expressed in state space form:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]  

And the system input and outputs will be defined as:

\[
\begin{align*}
u &= \begin{bmatrix} f_r \\ \delta \end{bmatrix} \\
y &= \begin{bmatrix} u \\ r \end{bmatrix} = Cx = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} u \\ v \\ r \\ n_p \end{bmatrix}
\end{align*}
\]

Therefore, the LTI models in Appendix 2 require only a change in their \( C \)
matrix to the one of equation 3.4.

3.2.2 Controllability and Observability

As mentioned in the introduction of this section, the LQG with LTR
technique requires the LTI system to be reachable and detectable. These properties
of the LTI model are weaker conditions than controllability and observability, but
enclose the same concept. If the system is controllable and observable, reachabilty and detectability are ensured. As Controllability and Observabiliuty are easier to check, all systems will be studied for unobservable or uncontrollable states.

A LTI system is said to be controllable if an input function exist, \( u(t) \) \( 0 < t < T \), which can bring the state from rest \( (x=0) \) to any given state at any \( T > 0 \) [Be.1]. Controllability is ensured if the Controllability matrix \( Co \) has full rank. The controllability matrix (3.5) is a function of the matrices \( A \) and \( B \), and reflects the capacity of the LTI system to reach any state with the existing input structure. For a space state system:

\[
Co = [B \ AB \ A^2B \ A^3B \ 
\ldots \ldots A^{n-1}B]
\] (3.5)

A LTI system is said to be observable if the initial state \( x(0)=x_0 \) can be uniquely deduced from knowledge of the input \( u(t) \) and output \( y(t) \) for all \( t \) between 0 and any \( T > 0 \) [Be.1]. Observability is ensured if the Observability matrix \( O \) has full rank. The Observability matrix (3.6) is a function of the matrices \( A \) and \( C \), and reflects the ability of the sensors to capture the dynamical behavior of the system.

\[
O = \begin{bmatrix}
C \\
CA \\
CA^2 \\
CA^3 \\
\vdots \\
CA^{n-1}
\end{bmatrix}
\] (3.6)

These matrices (3.5 and 3.6) were computed for all the LTI models and none of them has unobservable or uncontrollable states.

3.3 Modal Analysis

The analysis of the natural modes of the system is accomplished analyzing the modal matrix and the system eigenvalues. To obtain these entities it is necessary to first define a new state vector:
\[ x(t) = Tz(t) \] (3.7)

Where \( T \), by now, is an \( n \times n \) invertible matrix, and the state space equations become:

\[ T\dot{z} = ATz + Bu \] (3.8)
\[ y = CTz \] (3.9)

Then equation (3.8) is multiplied by \( T^{-1} \) on the left:

\[ \dot{z} = T^{-1}ATz + T^{-1}Bu \] (3.10)
\[ y = CTz \] (3.9)

If \( T \) is such that \( T^{-1}AT \) is diagonal, then the new state vector \( z \) defines a new state space with the modes decoupled and the entries in the diagonal in \( T^{-1}AT \) are the \textit{eigenvalues} of \( A \). The following relations hold:

\[ \Lambda = T^{-1}AT \] (3.11)
\[ AT = T\Lambda \] (3.12)

If the \( T \) matrix is represented by its column vectors:

\[ T = [v_1 \ v_2 \ \cdots \ v_n] \] (3.13)

Then equation (3.12) can be written as:

\[ Av_i = v_i\lambda_i \quad \text{for } i = 1 \text{ to } n \quad (\Lambda = \begin{bmatrix} \lambda_1 & 0 & 0 & 0 \\ 0 & \lambda_2 & 0 & 0 \\ 0 & 0 & \ddots & 0 \\ 0 & 0 & 0 & \lambda_n \end{bmatrix}) \] (3.14)
The columns of $T$ are the eigenvectors of $A$ and this diagonalization is possible only if $A$ has distinct eigenvalues.

Therefore, the $T$ matrix is the modal matrix, where each eigenvector defines the importance of each state ($u,v,r$ and $n_p$) for a particular mode, which is defined by the eigenvalues. As any analysis of the modal matrix is based on comparison of the importance of the different modes and states, it is important that all units are the same, and if possible non-dimensional. Fortunately the models are already in non-dimensional form, and there are no complex eigenvalues or eigenvectors, so there is no loss of information through loss of phase information. Figures 3.1 and 3.2 show the magnitude of the eigenvectors for each mode in the direction of the different states. The values shown are the actual eigenvector magnitudes. The benefits of using non-dimensional units in the modeling process are evident.

![Figure 3.1 LTI Modal Analysis for F50R0](image-url)
The 4 modes indicated in figures 3.1 and 3.2 correspond to the eigenvalues presented in Table 3.1. It is important to notice that the modes shown represent the natural modes of the ship at 50% fuel rate and with 0 and 10 degrees rudder deflection angle. Therefore, although the bar plots conveniently show the modes of the LTI model; in this case is not straightforward to visualize physical
interpretations out of the chart. Nevertheless, the following conclusions are obtained from the analysis of this data:

1. All eigenvalues are in the LHP, and therefore the system is stable.
2. Mode 1 represents is the surge mode, with its eigenvalue far apart from the imaginary axis due to the large inertia of the ship moving at 50% fuel rate.
3. The propeller revolutions state ($n_p$) has little importance in all modes in both models, and further study is necessary to evaluate the feasibility of eliminating this state.
4. Mode 3 appears to be a sway mode, with dampening effect of the forward speed. This is actually the only mode that effectively dampens the ship forward motion in the F50R10 Model.
5. Mode 4 appears to be a yaw mode.
6. The F50R0 Model shows no coupling between the pairs surge-propeller and sway-yaw.

The eigenvalues and eigenvectors for all models are contained in Appendix 3.

### 3.4 Pole/Zero Structure

The poles of the system (open loop) correspond to the eigenvalues already mentioned in the above section. All poles are in the LHP, therefore the system is stable; furthermore, all poles have no imaginary part, which confirms that the system has a structure consisting of several first order subsystems. The system poles are function of the A matrix and are not modified by input or output selections (B and C matrices).

The zeros of the system can become very important if there is one or more in the RHP. These zeros, known as *non-minimal phase (NMP)* zeros can be very troublesome because they limit the system bandwidth and there is not known solution to this problem. Unfortunately, some of five LTI models corresponding to zero rudder deflection have one zero in the RHP. The F50R0 model does not show a zero (due to poor scaling), but once the realization is balanced, the NMP will show up. The transmission zeros for the F50R0 and F50R10 models are listed in table
3.2. It is necessary to remark that the zeroes are function of the system transfer function, i.e. the matrices A, B and C, and any change in input or output selection will require a re-analysis of transmission zeros.

<table>
<thead>
<tr>
<th>F50R0</th>
<th>F50R10</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0039</td>
<td>-4.1773</td>
</tr>
<tr>
<td>-1.8211</td>
<td>-1.5540</td>
</tr>
<tr>
<td>-1.4704</td>
<td>-7.0630</td>
</tr>
<tr>
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<tr>
<td>-29.8496</td>
<td>-29.1040</td>
</tr>
<tr>
<td>-0.5414</td>
<td>-1.4011+0.1750i</td>
</tr>
<tr>
<td>-1.0996</td>
<td>-1.4011-0.1750i</td>
</tr>
<tr>
<td></td>
<td>-29.3217</td>
</tr>
<tr>
<td></td>
<td>-0.2677</td>
</tr>
<tr>
<td></td>
<td>-1.4324</td>
</tr>
</tbody>
</table>

Table 3.2 LTI Models Transmission Zeros

3.5 Singular Values

Singular Values of a MIMO system are the extension of the Bode Plot concept in Single-Input-Single-Output (SISO) systems. Singular values are a function of frequency and for a transfer matrix $G$, the $i^{th}$ singular value is defined as a function of $G$ and the $i^{th}$ eigenvalue $\lambda$:

$$\sigma_i(G) = \sqrt{\lambda_i(G^*G)}$$  \hspace{1cm} (3.15)

In the case of the models under investigation in this thesis, we want to first investigate the open loop singular values from input to output, i.e. the singular values of the LTI model transfer matrix. Then $G(s)$ is defined in the standard form to obtain the transfer matrix from a state space realization:

$$G(s) = C(sI - A)^{-1}B$$  \hspace{1cm} (3.16)

Then the singular values are obtained using a computer program for different frequency values and plotted in a logarithmic grid as the Bode plots.
Figure 3.3 F50R10 LTI Model Open Loop Singular Values

Figure 3.4 F50R0 LTI Model Open Loop Singular Values
Figure 3.3 shows the open loop singular values for the F50R10 LTI model computed using a Matlab routine. The maximum singular value is dominated by the yaw rate and the minimum singular value by the speed, reflecting a quicker response in the former. It is also important to notice the need of integrators in both channels for the F50R10 model to achieve satisfactory command following performance in the loop transfer matrix. Figure 3.4 shows the model linearized in the straightforward trajectory.

Further information about the effect of the inputs in the outputs can be obtained from the dc gains of the open loop transfer functions. Tables 3.3 and 3.4 show the dc gains of either system, where it is easy to visualize in the first rows the positive contribution of fuel rate increases in axial speed $u$, and the negative effect of rudder angle deflections (further from 10 degrees). In the second row it is possible to appreciate the difference between both models, with the F50R0 model having decoupled gains.

<table>
<thead>
<tr>
<th></th>
<th>$f$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.5349</td>
<td>-1.0482</td>
</tr>
<tr>
<td>$r$</td>
<td>0.2124</td>
<td>0.3336</td>
</tr>
</tbody>
</table>

Table 3.3 F50R10 LTI Model DC Gains

<table>
<thead>
<tr>
<th></th>
<th>$f$</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$u$</td>
<td>0.5624</td>
<td>-0.4478</td>
</tr>
<tr>
<td>$r$</td>
<td>0</td>
<td>643.2358</td>
</tr>
</tbody>
</table>

Table 3.4 F50R0 LTI Model DC Gains
3.6 Model Order Reduction

It is apparent from the modal analysis section (3.2) that the propeller revolution equation is weakly coupled with the sway and yaw equations and that its response is much faster than all the other ship dynamics, and that the ship is a very slow system. These characteristics of the LTI model makes necessary to investigate the feasibility of reducing the system order. The Model Order reduction will be executed using Balanced Truncation, which is one of the simplest but most effective existing techniques. The basics of this method will be addressed in this section, and the complete proof and algorithms for the solution of this problem can be found in references [Zh.1] and [Mo.1].

3.6.1 Balanced Realization

In order to have a properly balanced system to evaluate model order reduction it is first necessary to obtain a balanced realization of the LTI system from the input-output standpoint. This realization is found in the one that gives balanced gramians for controllability and observability.

The gramians are computed by means of the Lyapunov equations:

\[ A^*Q + QA + CC^* = 0 \]  
\[ AP + PA^* + BB^* = 0 \]

Q and P in the above Lyapunov equations are the observability and controllability gramians respectively. These gramians are positive definite if A is stable and the pairs (A,C) and (B,A) are observable and controllable, which is the case in the 10 LTI models considered.

The balanced realization of the system is then defined as the minimal realization that have equal controllability and observability gramians. This realization is found by applying a nonsingular state transformation T to the system described in 3.4 and 3.5, that yields a realization with:
\[ \hat{x} = Tx \]  
\[ \hat{A} = TAT^{-1} \]  
\[ \hat{B} = TB \]  
\[ \hat{C} = CT^{-1} \]

The gramians are also transformed to:

\[ \hat{P} = TPT^{*} \]  
\[ \hat{Q} = (T^{-1})^{*}QT^{-1} \]

The state transformation \( T \) is chosen to obtain equal gramians, which also will have the property of being diagonal matrices with the diagonal composed of the singular values of the balanced realization.

\[ \hat{P} = \hat{Q} = \Sigma = \text{diag}(\sigma_{1}I_{s1}, \sigma_{2}I_{s2}, \ldots, \sigma_{N}I_{sN}) \]

This singular values, called Hankel Singular Values, are ordered such that \( \sigma_{1} > \sigma_{2} > \ldots > \sigma_{N} \). Then the balanced realization is a state realization ordered by its degree of controllability and observability, with the last states being the least observable and controllable. Therefore, the truncation of these states will not result in loss of much information about the system.

Balanced realizations and Hankel singular values of all models were obtained using the Matlab Control Toolbox and enclosed in Appendix 3. The F50R0 and F50R10 LTI model have the Hankel singular values shown in table 3.4. Inspecting the results, the last state show singular values two orders of magnitude smaller than the first two states, and therefore it may be eliminated and the order of the system reduced. It should be noticed that the Hankel singular values of the model linearized in the straightforward motions show that there is only on important mode to describe the dynamics. This can be confusing, because despite
the forward motions are important, they are not critical from a control standpoint, because even an open loop strategy may work with the F50R0 model description. (Further analysis of the consequences in the model and the validity of this approach are explored in section 3.6.3.)

<table>
<thead>
<tr>
<th></th>
<th>F50R0</th>
<th>F50R10</th>
</tr>
</thead>
<tbody>
<tr>
<td>321.1665</td>
<td>0.6379</td>
<td></td>
</tr>
<tr>
<td>0.4515</td>
<td>0.3103</td>
<td></td>
</tr>
<tr>
<td>0.2860</td>
<td>0.0177</td>
<td></td>
</tr>
<tr>
<td>0.0048</td>
<td>0.0065</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.5 Hankel Singular Values of LTI Model F50R10

3.6.2 Model Order Reduction by Balanced Truncation

Model order reduction is executed eliminating the desired states from the balanced realization obtained as explained in the previous section. This method ensures a good dynamic response and excellent approximation in the frequency domain. In this case the order of all systems considered in this thesis was reduced from 4 to 3, obtaining simpler and computationally cheaper models that adequately represent the ship's dynamics. Nevertheless, the following analysis showed the inconvenience of this methodology in this case.

3.6.3 System Characteristics after Model Order reduction

In order to verify the system characteristics of the reduced order model, eigenvalues, transmission zeros and singular values were computed and analyzed. The following is a summary of the results for the F50R10 and F50R0 LTI models.

**Pole/zero structure**

The poles of the reduced order LTI model are all in the LHP and are real valued; therefore, the system kept its stability conditions and is a damped system (first order). Table 3.6 shows the eigenvalues for the F50R10 and F50R0 Reduced Order LTI models. There are NMP zeros in the F50R0 Model (Table 3.7), and some of them are in the expected system bandwidth (\(\omega' = 0.52\) radians). As the NMP of the original balanced F50R0 system was at frequencies far above the system's
bandwidth, the models with zero rudder deflection will not be reduced. This
decision forces to keep the models with 10 degrees rudder angle at order 4, in order
to be able to develop a gain scheduling algorithm with same dimensions gain
matrices. Other factor taken in account to keep the propeller revolutions equation
was that it may be required to control this state in the future for specific
applications such as noise and cavitation control (This will require a new set of
gains).

<table>
<thead>
<tr>
<th></th>
<th>F50R0</th>
<th>F50R10</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.0039</td>
<td>-0.5171</td>
<td></td>
</tr>
<tr>
<td>-1.8211</td>
<td>-2.3975</td>
<td></td>
</tr>
<tr>
<td>-0.5053</td>
<td>-4.0478</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.6 Eigenvalues of Reduced Order Models

<table>
<thead>
<tr>
<th></th>
<th>F50R0</th>
<th>F50R10</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.8211</td>
<td>-4.0422</td>
<td></td>
</tr>
<tr>
<td>-0.0039</td>
<td>-2.7145</td>
<td></td>
</tr>
<tr>
<td>-1.2467</td>
<td>-4.8823</td>
<td></td>
</tr>
<tr>
<td>45.4984</td>
<td>-3.7097</td>
<td></td>
</tr>
<tr>
<td>-0.0293</td>
<td>-3.3927</td>
<td></td>
</tr>
<tr>
<td>0.5227</td>
<td>-1.1647</td>
<td></td>
</tr>
<tr>
<td>-1.0996</td>
<td>-2.0102</td>
<td></td>
</tr>
<tr>
<td>-0.5053</td>
<td>-0.2769</td>
<td></td>
</tr>
</tbody>
</table>

Table 3.7 Transmission zeros of the Reduced Order Model

The open loop singular values of the reduced order system are an excellent
match of the Full Order system. Figure 3.5 is included to show the excellent
performance of this method in the frequency domain. The only differences are in
the high frequency range of the minimum singular value, were the minimum
singular value drop off a little faster.
3.7 Summary

In this chapter the dynamics of the LTI models generated in chapter 2 were investigated using several techniques suitable for linear systems. As a result the stability of the system about the operating point was confirmed, as well as the damped characteristic of the model dynamics. The modal analysis gave some insight in the possibility of reducing the system order, but that alternative was dismissed based in practical considerations regarding the overall system requirements. Finally, all LTI models are expressed in the form of balanced (input-output) realizations for ease of controller design. Some of the systems have NMP zeros, but located well above the expected system bandwidth.
Chapter 4
MIMO Controller Design

4.1 Introduction

This chapter presents the design of the LQG/LTR controller for DDG51. The controller, also known as Model Based Compensator, MBC, is designed to meet singular values loop shape requirements. These performance and robustness requirements are first introduced to the reader in sections 4.2 and 4.3. Next, the LQG/LTR methodology is explained with some theoretic background, mainly focused in the Kalman Filter design to for loopshaping. Afterwards, a complete example is shown for the LTI Model F50R10, including some remarks that ease the loopshaping process. Finally, simulations with the nonlinear model are carried on to visualize the controller performance.

4.2 Controller Performance Requirements

There is no clear statement given beforehand as to the requirements for controller performance; therefore, it will be necessary to impose conditions to the design. Two sets of conditions will be imposed: time domain and frequency domain requirements.

First of all, with respect to time domain conditions, the system should be able to respond favorably to step inputs, have zero steady state error and, as the system is not designed for critical maneuvers, the speed of response is not a primary goal. An overdamped response is preferred to avoid overshooting and oscillations.

In the frequency domain, the requirements are defined with respect to the plant singular values. It will be shown that the requirements specifically apply to
Figure 4.1 show a sketch of the desired singular values of that product, where the minimum singular value should overcome a low frequency performance barrier, and the maximum singular value at high frequencies should be small and roll off rapidly. The objective of this shape is to ensure good command following and input-output disturbance rejection at low frequencies and attenuation of filter sensor noise and unmodeled dynamics at high frequencies.
4.2.1 Crossover frequency limits

It is necessary to explore the frequency limits in order to choose and appropriate crossover frequency for design. The desired crossover frequency is limited to the ability of the ship to respond to disturbances. This crossover frequency is obtained perturbing the nonlinear model and obtaining the system time constant. The system step response to 50% fuel rate increase with zero rudder deflection angles was used for this purpose. This test was selected because surge is the slowest mode at any operating point, and a 50% fuel increase is considered a maximum foreseeable step input in normal operating conditions.

Inspection of the step responses at different speeds allowed to obtain settling times to within 90% of the steady state condition for all models up to 50% fuel rate. In the case of the ship at 50% fuel rate, the settling time is about 25 seconds, corresponding to a crossover frequency of 0.25 radians/sec (\(\omega' = 1.1\)).

The results for tests at 5 different fuel rates confirmed that settling time increases as the ship slows requiring different crossover frequencies for each model. The following crossover frequency limits were finally selected for the LTI models to be used for controller design:

<table>
<thead>
<tr>
<th>LTI Models</th>
<th>Crossover Frequencies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(\omega) (radians/s)</td>
</tr>
<tr>
<td>F5R0-10</td>
<td>0.12</td>
</tr>
<tr>
<td>F10R0-10</td>
<td>0.15</td>
</tr>
<tr>
<td>F25R0-10</td>
<td>0.17</td>
</tr>
<tr>
<td>F50R0-10</td>
<td>0.25</td>
</tr>
<tr>
<td>F100R0-10*</td>
<td>0.31</td>
</tr>
</tbody>
</table>

Table 4.1 Crossover Frequencies for Linear Models

* Correspond to interpolated value using least squares fit on ship's fuel rate
4.2.2 Augmented dynamics

The zero steady error requirement will be met by placing integrators in each input channel, in order to correct the error signal. This method consists in adding integral control to the compensator that will be before the plant in the feedback loop.

In order to include these integrators, the submarine model is augmented. An augmentation plant $G_A(s)$ is defined as:

$$G_A(s) = C_A (sI - A_A)^{-1} B_A$$

(4.1)

with $A_A = 0_{2x2}$, $B_A = I_{2x2}$ and $C_A = I_{2x2}$

(4.2)

The nominal model $G_p(s)$ and the augmentation dynamics are combined simply multiplying the transfer functions state space realizations, forming $G(s)$ as described in figure 4.2.

Figure 4.2 Nominal Plant and Augmentation Transfer Function Connection
The input vector to the plant is named now $u_p$, and $u$ corresponds to the input to the integral controller. Despite the fact that the augmentation dynamics will be lumped into the compensator, during the LQG/LTR design it will be rather included to the plant dynamics until the design is complete.

In order to visualize the effect of the integral action included before the nominal plant, a plot of the open loop singular values for the nominal and augmented plants of the F50R10 LTI Model is shown in figure 4.3. The effect is most noticeable at low frequencies, where the gain was increased 40 decibels at $\omega'=0.1$ radians and the slope is clearly "one integrator-20db per decade".

![Augmented and Unaugmented Models Singular Values](image)

**Figure 4.3 Effect of Plant Augmentation in Open Loop Singular Values**
4.3 Robustness

The LQG/LTR methodology was selected in part, because it matches the requirements and environment for the ship maneuvering controller design. The LQG/LTR methodology, properly applied, provide the following guaranteed properties for the closed loop system:

1. Closed loop stability
2. Infinite upward gain margin
3. 6 dB downward gain margin
4. Phase margins of ±60°
5. Robustness: $\sigma[I+G_{KF}(j\omega)] \geq 1$
   $$\sigma[I+G_{KF}^{-1}(j\omega)] \geq 1/2$$

$G_{KF}(s)$ is the transfer matrix of the Kalman Filter, and as it will be seen later, the closed loop transfer matrix is expected to have improved robustness compared with $G_{KF}(s)$, if the recovery phase is properly applied.

4.4 The LQG/LTR Design Methodology

This section introduces the MIMO LQG/LTR design methodology based in the paper by Athans [Ath.1] that shows a practical example for a submarine compensator. The process can be executed in the four steps described below and consists of obtaining a low frequency plant linear model, defining the performance requirements, and the design of a Model Based Compensator, MBC. This MBC has first designed its optimal estimator, such that the obtained Kalman Filter loop shape has properties similar to those specified in the performance requirements. Then the optimal controller gains are obtained such as to recover the loop shape achieved with the Kalman Filter design.

4.4.1 Low Frequency Nominal Plant and Crossover frequency

The first step consists in the development of a low frequency model of the nominal plant with the corresponding model uncertainties. The methodology assumes command following and disturbance rejection is required in the range of
low frequencies; similarly, the process considers that uncertainty in the model, including sensor noise and uncertain dynamics are present at high frequencies. The break-off point of what constitutes high and low frequencies is set by the crossover frequency fixed for nominal design.

The crossover frequencies were defined in section 4.2, while the low frequency nominal plant will consist of the augmented dynamics LTI model. No attempts are made to establish modeling uncertainties in this thesis.

4.4.2 Definition of Performance Requirements

This step sets the low frequency specific performance requirements for the compensator. Consider the block diagram of the controlled plant in figure 4.4.

![Figure 4.4 MIMO Feedback System Block Diagram](image)

Where the different signals are:

- \( r(s) \) = reference signal
- \( e(s) \) = error signal
- \( u(s) \) = control input vector to plant
- \( y_p(s) \) = plant output vector
- \( d(s) \) = disturbance vector reflected at the plant output
\[ y(s) = \text{actual output vector} \]
\[ K(s) = \text{Controller Transfer Function Matrix} \]
\[ G(s) = \text{Plant Transfer Function Matrix (augmented)} \]

G(s) is assumed to be the augmented plant, as described in section 4.2.2. In order to find the requirements for the controller, K(s), it is necessary to investigate the transfer function of the closed loop system, which is given in equation 4.3 for a SISO system.

\[ y(s) = \frac{d(s) + G(s)K(s)r(s)}{1 + G(s)K(s)} \quad (4.3) \]

Inspection of equation 4.3 tells that good command following requires \( y(s) \approx r(s) \) and good disturbance rejection is achieved if the contribution of \( d(s) \) is relatively small. Considering that this is at low frequencies, \( s = j\omega \), both requirements will be met if \( G(j\omega)K(j\omega) \) is very large. Specifically, the minimum singular values of \( G(s)K(s) \) below crossover frequencies are required to be large with respect to one.

Similarly, smaller values of the maximum singular values of \( G(s)K(s) \) at high frequencies (over crossover) will attenuate or minimize the system response to sensor noise and other high frequency effects.

The overall purpose of these two initial steps was to set the performance requirements for \( G(j\omega)K(j\omega) \) at the different frequency ranges as shown in figure 4.1.

### 4.4.3 MBC and Loopshaping with Kalman Filter Design

As \( G(s) \) is determined by the augmented plant, the work to be done is finding the compensator transfer matrix \( K(j\omega) \) to match the desired \( G(j\omega)K(j\omega) \) open loop transfer matrix singular values (figure 4.1). This process is know as Loop Shaping and is the principal step of the LQG/LTR design methodology.

First of all, the MBC is constructed using the results from optimal control theory (Linear Quadratic Regulator) and optimal estimation theory (Kalman Filter).
This controller has the property of being stabilizing if the proper gains are chosen. Thus the system parameters of the MBC, if properly designed, can be chosen to achieve the loop shape imposed by frequency performance requirements. It is remarked that despite the use of Linear-Quadratic-Gaussian stochastic optimal control theory to determine the gains of the MBC, the overall system is not designed under an optimal control perspective. The designer will vary some free parameters in a systematic way in order to obtain the desired loop shape.

A state space representation of the MBC and plant are shown in figure 4.5, where it can be seen that the compensator includes the plant matrices $A$, $B$ and $C$, therefore its name "Model - Based".

![Figure 4.5 State Space Description of the Feedback Loop with MBC](image)

The open loop dynamics of the plant can be written as in equations 4.4 and 4.5, with $L$ a design parameter that may be varied to allow the designer to construct an open loop system that accomplishes the required specifications.

$$\dot{x} = Ax + Bu + Ld \quad (4.4)$$

$$y = Cx \quad (4.5)$$

The plant transfer matrix corresponds to the augmented model, and can be obtained as in equation 4.6.
\[ G(s) = C(sI - A)^{-1}B \]  

(4.6)

The MBC dynamics can be expressed as in equations 4.7 and 4.8 [Tr.2].

\[ \dot{x} = (A - BG - HC)x - He \]  

(4.7)

\[ u = -G\dot{x} \]  

(4.8)

The MBC transfer matrix, \( K_{LQG}(s) \), can be found to be:

\[ K_{LQG}(s) = G(sI - A + BG + HC)^{-1}H \]  

(4.9)

And the compensator's input and output are related by the above transfer matrix, such that:

\[ u(s) = K_{LQG}(s)e(s) \]  

(4.10)

The new variable \( \hat{x}(t) \) is the estimated state vector of the same dimension as the state vector \( x(t) \). The control gain matrix, \( G \), and filter gain matrix, \( H \), are design products of the LQG methodology that together determine the pole/zero structure of the MBC. A closed loop of the plant and compensator can be found* by defining a new state vector \( z \) that includes both the actual state vector and the estimated state vector. Equations 4.12 and 4.13 completely describe the closed loop dynamics, which will ultimately depend on the selection of \( H \) and \( G \).

* A derivation can be found in [Tr.2]
The design for closed loop stability is separated, in accordance with the *separation principle*, into two problems:

First, given a stabilizable pair, \( A \) and \( B \), find \( G \) such that the real part of the eigenvalues of \( A-BG \) are negative. Second, given a detectable pair, \( A \) and \( C \), find \( H \) such that the real part of eigenvalues of \( A-HC \) are negative. As it was already stated, all the ship’s LTI models satisfy the stronger conditions of controllability and observability, thus they are stabilizable.

\[
\begin{align*}
  \mathbf{z} &= \begin{bmatrix} \mathbf{x} \\ \dot{\mathbf{x}} \end{bmatrix} \\
  \dot{\mathbf{z}} &= \begin{bmatrix} A & -BG \\ HC & A-BG-HC \end{bmatrix} \mathbf{z} + \begin{bmatrix} 0 \\ -HC \end{bmatrix} \mathbf{r} \\
  \mathbf{y} &= \{ C \ 0 \} \mathbf{z}
\end{align*}
\] (4.11)

To solve the Kalman Filter problem, the nominal state space description is formulated to produce the modified dynamics:

\[
\begin{align*}
  \dot{\mathbf{x}} &= A\mathbf{x} + L\xi \\
  \mathbf{y} &= C\mathbf{x} + \vartheta
\end{align*}
\] (4.14, 4.15)

\( \xi \) and \( \vartheta \) in equations 4.14 and 4.15 represent process white noise and measurement white noise respectively. The covariance of the signals is:

\[
\begin{align*}
  \text{cov}[\xi(t);\xi(\tau)] &= I\delta(t-\tau) \\
  \text{cov}[\vartheta(t);\vartheta(\tau)] &= \mu I\delta(t-\tau) \quad (\mu > 0)
\end{align*}
\] (4.16, 4.17)

Therefore, \( \mu \) and \( L \) can be used to produce the desired loop shapes of the Kalman Filter transfer matrix \( G_{KF}(s) \).
\[ G_{KF}(s) = C(sI - A)^{-1}H \quad (4.18) \]
\[ H = \Sigma C^T \quad (4.19) \]

The matrix \( \Sigma \) is the solution to the Filter Algebraic Riccati Equation (FARE) of the Kalman Filter:

\[ A\Sigma + \Sigma A^T + LL^T - \frac{1}{\sqrt{\mu}} \Sigma C^T \Sigma = 0 \quad (4.20) \]

The iteration process is skipped using the Kalman Filter identity; \( G_{POL}(s) \) can approximate \( G_{KF}(s) \) when \( \mu << 1 \).

\[ G_{POL}(j\omega) = C(sI - A)^{-1}L \quad (4.21) \]

\[ \sigma_i\{G_{KF}(j\omega)\} = \frac{1}{\sqrt{\mu}} \sigma_i\{G_{POL}(j\omega)\} \quad (4.22) \]

In practical terms, \( L \) can be chosen to produce the desired loop shape, and \( \mu \) can be used to adjust the singular values up and down to the desired crossover frequencies. With \( L \) and \( \mu \) obtained to match the desired loop shape, it is only left to solve the Kalman filter CARE one time to obtain the gain matrix \( H \).

### 4.4.4 Loop Transfer Recovery

The final step in the design process involves the recovery of the loop shape of \( G_{KF}(s) \) by the compensated plant transfer matrix \( G(s)K(s) \). To solve this is necessary to compute another problem, analogous to the FARE, the Control Algebraic Riccati Equation (CARE).

\[ KA + A^TK + \frac{1}{\rho} C^TC - KBB^TK = 0 \quad \rho > 0 \quad (4.23) \]
This equation is used with the design parameter $\rho$ as a very small number to obtain the optimal gain matrix $G$ by.

$$G = B^T K$$  \hspace{1cm} (4.24)

By choosing $\rho$ as a very small number it can be shown that the singular values of $G_{KF}(s)$ and $K_{LQG}(s)G(s)$ converge for frequencies below crossover. This is ensured if the plant is minimum phase. If not, recovery will depend upon the location of the NMP zeros. For frequencies above crossover, an extra pole of roll-off is produced by the recovery phase, further enhancing robustness.

4.5 Model Based Compensator Design

In this section the detailed design process is carried on and explained for the F50R10 augmented model. Numeric data for all the models is included in Appendix 4.

4.5.1 Kalman Filter Design

As it was stated in the previous section, the singular values of the Kalman Filter transfer matrix $G_{KF}(s)$ can be well approximated by $1/\sqrt{\mu}$ times the singular values of $G_{POL}(s)$. Moreover, it is only needed to vary $L$ to adjust to the required singular value specifications. Nevertheless, $L$ is still a matrix of dimension 6x2 in the case of the models considered in this case, giving place for a large variety of combinations that makes the loop shaping process difficult.

Fortunately, if one considers meeting the low and high frequency requirements with the maximum and minimum singular values of $G_{POL}(s)$ matched, the choice of $L$ is simplified. As it was shown when the ship model was augmented, $G(s)=G_A(s)G_F(s)$. A state space realization of the transfer matrix is as in equation 4.6, with the matrices $A$ and $C$ partitioned as shown below:

$$A = \begin{bmatrix} A_p & B_p \\ 0 & 0 \end{bmatrix} \hspace{1cm} C = \begin{bmatrix} C_p & 0 \end{bmatrix}$$  \hspace{1cm} (4.24)
Then \((sI-A)\) and its inverse are:

\[
sI - A = \begin{bmatrix} sI - Ap & -Bp \\ 0 & sI \end{bmatrix}
\]

(4.25)

\[
(sI - A)^{-1} = \begin{bmatrix} (sI - Ap)^{-1} & Bp(sI - A)^{-1} / s \\ 0 & I / s \end{bmatrix}
\]

(4.26)

At low frequencies,

\[
\omega \to 0, \quad sI - Ap \approx -Ap, \quad (sI - Ap)^{-1} \approx -Ap^{-1}
\]

The inverse of \(Ap\) exists, as \(Ap\) has distinct and non-zero eigenvalues.

Conveniently, \(L\) is partitioned into \(L_1\) and \(L_2\), obtaining the following description for \(G_{FOL}(s)\) at low frequencies:

\[
G_{FOL}(s)_{\omega \to 0} = C(sI - A)^{-1}L \approx \begin{bmatrix} C \end{bmatrix} \begin{bmatrix} -Ap^{-1} & -Ap^{-1}Bp / s \\ 0 & I / s \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}
\]

(4.27)

\[
G_{FOL}(s)_{\omega \to 0} \approx -CpAp^{-1}L_1 - CpAp^{-1}BpL_2 / s
\]

(4.28)

If \(L_2\) is appropriately chosen as:

\[
L_2 = -(CpAp^{-1}Bp)^{-1}
\]

(4.29)

Then with \(L_1\) unspecified, for low frequencies, the approximation for \(G_{FOL}(s)\) become:

\[
G_{FOL}(s)_{\omega \to 0} \approx \frac{I}{j\omega} + M
\]

(4.30)
Where \( M \) is a 2x2 matrix that is dominated by the first term for frequencies low enough.

At high frequencies,

\[
\omega \to \infty, \quad sI - Ap \approx sI, \quad (sI - Ap)^{-1} \approx I/s
\]

and

\[
G_{POL}(s)_{\omega \to \infty} = C(sI - A)^{-1}L \approx \begin{bmatrix} C_p & 0 \\ 0 & I/s \end{bmatrix} \begin{bmatrix} I/s & Bp/s^2 \\ 0 & I/s \end{bmatrix} \begin{bmatrix} L_1 \\ L_2 \end{bmatrix}
\]  

(4.31)

\[
G_{POL}(s)_{\omega \to \infty} \approx CpL_1/s + CpBpL_2/s^2
\]  

(4.32)

As \( 1/s \) is increasingly larger than \( 1/s^2 \) as \( \omega \to \infty \), the second term will be dominant at high frequencies and one can choose \( L_1 \) in similar manner as \( L_2 \) for low frequencies.

\[
L_1 = C_p^T(CpC_p^T)^{-1}
\]  

(4.33)

To obtain the desired match at high frequencies:

\[
G_{POL}(s)_{\omega \to \infty} \approx \frac{-CpBpL_2}{\omega^2} + \frac{I}{j\omega}
\]  

(4.34)

Therefore, \( L \) is easily obtained from equations 4.29 and 4.33, and the singular values of \( G_{POL}(s) \) are plotted for an initial value of \( \mu \). The maximum and minimum singular values should match at low and high frequencies, and \( m \) can be used to move the singular values up and down. A plot of the singular values of
$G_{ \text{FOL}(s)}$ for the LTI Model (augmented) F50R10 is shown in figure 4.6. The result can lead to confusion, because in this case there is convergence of the singular values at all frequencies. This result is not a consequence of this technique, because usually the mid-range singular values will be far from each other, requiring frequency scaling of the outputs to close them together. In this thesis the results are significantly better because of the early use of non-dimensional units in the system model, and due to the balanced realization of the system.

![Singular Values of $G_{\text{FOL}}$](image)

**Figure 4.6** Singular Values of $G_{\text{FOL}(j\omega)}$ for F50R10 LTI Model

The crossover frequency was adjusted with $\mu$, and then the Kalman Filter gain matrix $H$ was found solving the corresponding CARE (equation 4.20).
Table 4.2 shows the values of $m$ for the 5 models considered in this design phase.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\mu$</th>
</tr>
</thead>
<tbody>
<tr>
<td>F5R10</td>
<td>0.7</td>
</tr>
<tr>
<td>F10R10</td>
<td>0.5</td>
</tr>
<tr>
<td>F25R10</td>
<td>0.45</td>
</tr>
<tr>
<td>F50R10</td>
<td>0.19</td>
</tr>
<tr>
<td>F100R10</td>
<td>0.14</td>
</tr>
<tr>
<td>F5R0</td>
<td>0.5</td>
</tr>
<tr>
<td>F10R0</td>
<td>0.4</td>
</tr>
<tr>
<td>F25R0</td>
<td>0.25</td>
</tr>
<tr>
<td>F50R0</td>
<td>0.15</td>
</tr>
<tr>
<td>F100R0</td>
<td>0.08</td>
</tr>
</tbody>
</table>

Table 4.2 $\mu$ Values for LTI Models

4.5.2 Loop Transfer Recovery Design

Once the Kalman filter design is complete, the remaining tasks are straightforward. The constant $p$ is selected to be between $1e^{-4}$ to $1e^{-6}$ for all models, resulting in the main matrix $G$ obtained solving the CARE of equation 4.23. The complete design sequence is summarized in Figures 4.7 to 4.9, where the singular value plots of $G_{PO}(s)$, $G_{KF}(s)$ and $G(s)K(s)$ for the F50R10 LTI model design are presented. In this plots is notable the recovery of the Kalman Filter singular values for frequencies below crossover, and the expected improved robustness for frequencies above crossover. The values of $p$ correspond to revised numbers after simulation, so that crossover frequencies of $G(s)K(s)$ are slightly slower than $\omega_c$, to have a response slightly overdamped.
Figure 4.7 Singular Values of $G_{\text{POL}}(j\omega)$

Figure 4.8 Singular Values of $G_{\text{KP}}(j\omega)$
Figure 4.9 Singular Values of $G(j\omega)K(j\omega)$

The maximum and minimum crossover frequencies for the transfer matrix $G(s)K(s)$ are shown in table 4.3.

<table>
<thead>
<tr>
<th>Model</th>
<th>$\omega'_{\text{cmax}}$ (rad)</th>
<th>$\omega'_{\text{cmin}}$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>F5R10</td>
<td>1.0</td>
<td>0.7</td>
</tr>
<tr>
<td>F10R10</td>
<td>1.3</td>
<td>0.9</td>
</tr>
<tr>
<td>F25R10</td>
<td>1.4</td>
<td>0.9</td>
</tr>
<tr>
<td>F50R10</td>
<td>2.0</td>
<td>1.5</td>
</tr>
<tr>
<td>F100R10</td>
<td>2.7</td>
<td>2.0</td>
</tr>
<tr>
<td>F5R0</td>
<td>0.9</td>
<td>0.6</td>
</tr>
<tr>
<td>F10R0</td>
<td>1.0</td>
<td>0.85</td>
</tr>
<tr>
<td>F25R0</td>
<td>1.2</td>
<td>1.1</td>
</tr>
<tr>
<td>F50R0</td>
<td>1.6</td>
<td>1.6</td>
</tr>
<tr>
<td>F100R0</td>
<td>2.0</td>
<td>2.0</td>
</tr>
</tbody>
</table>

Table 4.3 $G(s)K(s)$ Crossover Frequencies
4.6 Analysis of the Closed Loop System

The closed loop system can be represented as in equations 4.10-4.12, with the poles of the closed loop system including the poles of A-BG and A-HC from the optimal controller and Kalman Filter design. Figure 4.10 shows all closed-loop system poles and zeros in the LHP, as expected from the stabilizing solution of the LQG technique.

If the disturbance input vector is set to zero, one may obtain a singular value plot of the closed loop system, where the dc-gain is unity (0 dB) for low frequencies and up to crossover, and then roll off for frequencies above cross over.
4.7 Simulation of the compensated model

In order to evaluate and verify the designed controllers, simulations with the nonlinear model were run. It should be remarked that the MBC was designed based on a linear system subject to small perturbations about an operating point. Therefore, the controller is expected to perform as designed under those conditions. In this section, all MBC are named under the code name of the linear model they came from. So, for example, MBC F50R10 was designed based in the LTI model.
linearized about the equilibrium state corresponding to 50% fuel rate and 10 degrees rudder deflection. This model is the one presented as an example in this chapter, along with some characteristics of F50R0 to illustrate the need for inclusion of rudder angle and speed in gain scheduling.

All ten MBC designed in this chapter were tested for 10% and 20% steps in axial speed reference and yaw rate reference, symmetric response and actuator saturation. The models presented in this chapter were the first tested, and they were also investigated out of their operation point and their response to perturbations such as current and high frequency sensor noise was also explored. The results were extremely satisfactory as explained below.

4.7.1 Step Response

The F50R10 MBC was tried with the system at equilibrium state, with the corresponding reference signals in axial speed, $u'$, and yaw rate, $r'$. The system was at equilibrium with the expected 10 degrees rudder angle and 50% fuel rate. The first test was 10% step in yaw rate reference, with the responses shown in figures 4.12 to 4.14.

The response was very stable for the four state variables ($u$, $v$, $r$ and $n_p$) and the actuators. The controlled variables smoothly reach the steady value with zero steady error and without overshoot. The perturbation in axial speed (Fig.4.12) is very small, with a max perturbation of 0.2% of the steady value. As mentioned above, the response is not very fast, but considering the inherent dynamic characteristics of the ship, it is about as fast as possible without overshoot. In fact for the 10% yaw rate step, it takes it about 9 seconds to reach 90% of the steady value. The response of up to 40% yaw rate steps showed the same performance. Speed and combined (speed and yaw rate) steps were also tried with similar responses.

The F50R0 MBC, i.e. one linearized about an operating point with zero rudder angle showed similar response, with the exception of yaw rate. For yaw rate, the compensator had good performance only up to very small yaw rate steps, say the corresponding to about 2 degrees steady rudder angle. Over that values the response is excessively damped.
Figure 4.12 MBC F50R10, System Response to 10% Step in Yaw Rate, $u'$ (top) and $v'$ (bottom)
Figure 4.13 MBC F50R10, System Response to 10% Step in Yaw Rate, 
$r'$ (above) and $n_p/n_m$ (bottom)
Figure 4.14 MBC F50R10, Actuators Response to 10% Step in Yaw Rate,
Fuel Rate (top) and Rudder Deflection in degrees (bottom)
4.7.2 Symmetry and Actuator Saturation Tests

The Symmetry of the response and commands was checked by means of an ad-hoc method. The symmetry test consisted in applying positive and negative 10% steps in yaw rate. Despite the system inherent nonlinearities, and the antisymmetry of the equilibrium point used for controller design, the response proved to be very satisfactory, with similar overdamped responses. This can be seen in figure 4.15 that corresponds to the yaw rate and rudder deflection angle during the maneuver.

Actuator saturation was also explored, because it is a highly non-linear condition that may lead to system instability. The systems were tested with large steps in yaw rate and speed. As an example, plots are presented for the F50R10 MBC with a 75% of the nominal yaw rate step. This produced a high perturbation in both speed and yaw rate. Figure 4.16 shows on top the speed response, where it slows down due to the maneuver, but the control signal (figure 4.18 top) starts working fast and saturates. This in turn produces an increment in the control signal that makes it take a long time to come back. By the time the control signal get lower than the saturation point, speed has risen and overshot, and then comes back reasonable well after the perturbation is removed. The lateral velocity (figure 4.16 bottom) and yaw rate (4.17 top) show similar patterns, with the yaw rate unable to meet the reference signal, because it was slightly out of its operating point. It is remarked that it was necessary to test the system with saturation in this way rather than large perturbation inside the operating condition range, because the controller fell out of their design range for such large steps*. Finally, figure 4.18 presents the controllers response and signal. The most visible, and important, effect is the overshoot in the desired position signal which produces that the system is not only slow in response, but also add an oscillatory component due to the difficult to come back from such high values.

Overall, the system response to actuator saturation is acceptable, with some oscillations in the response, but stable.

---

* Section 4.7.3 further explains this concept.
Figure 4.15 Symmetry Tests, Yaw Rate and Rudder Response
Figure 4.16 Actuator Saturation Conditions, Axial Velocity $u'$ (top) and Lateral Velocity $v'$ (bottom)
Figure 4.17 Actuator Saturation Conditions, Yaw Rate \( r \) (top) and Propeller Revolutions \( n_p \) (bottom)
Figure 4.18 Actuator Saturation Conditions,
Fuel Rate (top) and Rudder Angle (bottom) Actual Position and Desired Signal
4.7.3 MBC and Gain Scheduling

As mentioned earlier, the objective is to design a non-linear controller based on gain scheduling, because of the inherent non-linear characteristics of the ship. In general, it is known that gain scheduling will be needed for speed, but in first instance it was not clear that a requirement of gain scheduling for yaw was needed.

In fact, based on the works from references [Li.1] and [Mi.1] involving submarine maneuvers, one may infer that linearization about a turning trajectory, and gain scheduling with speed may be sufficient to achieve good control. That was the first attempt made in this thesis and worked until the controller tests were done. During the testing phase it became evident that it was necessary to have both, gain scheduling with respect to speed and yaw rate. Figure 4.19 shows F50R10 MBC used at straightforward motions (10-degree rudder angle out of its design trajectory) with a step input in speed required. The response is of very poor quality, highly oscillatory in axial velocity (top), but the worst is in the bottom, where there is significant rudder action (~1 degree) and small yaw rate oscillations. The oscillation is mainly a gain problem, and the rudder oscillations are related with the multivariable nature of the controller and linearization point. At ten degree- rudder angle the controller "knows" that an increase in speed will affect the required rudder angle to maintain yaw rate, because the sway and yaw equations had components depending on u at that point that are zero at straight forward motions.

This effect is not considered in most maneuvering control designs, because usually surge is considered decoupled form sway and yaw, or there is no design provision for propeller rotational speed control, and they are assumed constant, allowing the ship speed to vary freely.
Figure 4.17 Response of the F50R10 Model to Step in Axial Velocity with Zero Initial Rudder Deflection. $u$ and $n_p$ (top), $r$ and $\delta$ (bottom)
4.8 Summary

This chapter presented the LQR/LTR MIMO control design methodology as it was applied to 10 LTI models of DDG51. A complete example with the model F50R10 was used to explain in detail the methods and considerations that were used along this chapter. The reader is referred to Appendix 4 for the complete numeric data and singular value plots of the other model designs. Finally, simulations of the MBC with the plant, including step responses and symmetry and saturation tests were successfully carried on, validating the design. As a final product, at this stage there are 10 sets of gains (H and C) and LTI plants (A,B,C,D) that enables us to have as many MBC, valid for small perturbations close to the respective design point. The following chapter will treat the subject of how to obtain an overall controller for the complete range of operating power and rudder angles.
Chapter 5

Nonlinear MIMO Controller Design

5.1 Introduction

This chapter introduces the final Compensator design for DDG51. As already discussed in the previous chapter, the nonlinearities of the original system require the use of a non-linear controller, made out of several linear controllers, each valid for small perturbations about the respective linearization point. This approach is known as gain scheduling and consists of changing the controller gains according with a schedule based on selected variables.

The need of modeling about a trajectory in order to capture the ship dynamics accurately and the problems presented by these LTI models when in straight motion, mandated gain scheduling both with respect to speed and yaw rate. The considerations taken into account and the procedure to obtain the nonlinear MIMO controller will be discussed and presented in this chapter and simulations of the nonlinear controller (with the nonlinear model) are evaluated to further validate the design.

5.2 Gain Scheduling

Ten LTI models were developed in the previous chapters, alongside with their respective MBCs. Unfortunately, at this stage of the study, some of the procedures and selections made during the linear controller design proved to be troublesome for gain scheduling.
First, during the linearization process, the equilibrium points were chosen with reference to the inputs, i.e. fuel rate and rudder deflection angle. Although it is possible to use these input variables for gain scheduling, the dynamics will be better captured if gain scheduling is referred to ship states, in this specific case axial velocity, \( u \), and yaw rate, \( r \). Therefore, a problem exists to relate the models at zero rudder angles with the ones at 10 degrees rudder angle, because they are linearized at different equilibrium axial velocities (\( u^* \)), as shown in table 5.1.

<table>
<thead>
<tr>
<th></th>
<th>Fuel 5%</th>
<th>Fuel 10%</th>
<th>Fuel 25%</th>
<th>Fuel 50%</th>
<th>Fuel 100%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rudder 0</td>
<td>0.1653</td>
<td>0.3476</td>
<td>0.6089</td>
<td>0.8025</td>
<td>1.003</td>
</tr>
<tr>
<td>Rudder 10</td>
<td>0.1213</td>
<td>0.2397</td>
<td>0.4532</td>
<td>0.6332</td>
<td>0.8231</td>
</tr>
</tbody>
</table>

Table 5.1 Equilibrium Axial Velocities \( u^* \) for the LTI models

The second problem, and a more difficult one, is due to the use of a balanced state space realization. This method facilitated the LQG/LTR design, but presented a serious problem in this step of the process. When revising the MBC Kalman Filter gains for the different models in table 5.2, many coefficients lack correlation, and thus gain scheduling seemed impossible.

<table>
<thead>
<tr>
<th>F5RO</th>
<th>F10RO</th>
<th>F25RO</th>
<th>F50RO</th>
<th>F100RO</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0000</td>
<td>-2.3133</td>
<td>0.0000</td>
<td>-1.3958</td>
<td>0.0000</td>
</tr>
<tr>
<td>-1.3341</td>
<td>0.0000</td>
<td>-1.5441</td>
<td>-0.0009</td>
<td>-2.2925</td>
</tr>
<tr>
<td>0.0000</td>
<td>-1.3361</td>
<td>0.0001</td>
<td>0.6017</td>
<td>0.0001</td>
</tr>
<tr>
<td>0.0936</td>
<td>0.0000</td>
<td>0.0754</td>
<td>0.0000</td>
<td>0.0743</td>
</tr>
<tr>
<td>0.3373</td>
<td>0.0000</td>
<td>0.5439</td>
<td>0.0004</td>
<td>1.7923</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.0056</td>
<td>0.0000</td>
<td>0.0053</td>
<td>0.0000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>F5R10</th>
<th>F10R10</th>
<th>F25R10</th>
<th>F50R10</th>
<th>F100R10</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.4745</td>
<td>-0.2774</td>
<td>1.7925</td>
<td>-0.3454</td>
<td>1.8866</td>
</tr>
<tr>
<td>-1.0512</td>
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<td>-0.7324</td>
<td>1.9494</td>
<td>0.1037</td>
</tr>
<tr>
<td>-0.1626</td>
<td>0.6617</td>
<td>-0.0082</td>
<td>0.3184</td>
<td>0.0820</td>
</tr>
<tr>
<td>-0.0945</td>
<td>0.3633</td>
<td>-0.0447</td>
<td>0.2105</td>
<td>-0.0289</td>
</tr>
<tr>
<td>0.2394</td>
<td>0.6530</td>
<td>0.3081</td>
<td>1.2564</td>
<td>0.6802</td>
</tr>
<tr>
<td>-2.5097</td>
<td>8.0446</td>
<td>-1.4624</td>
<td>5.0101</td>
<td>-0.8950</td>
</tr>
</tbody>
</table>

Table 5.2 Kalman Filter Gains
This problem was due to the mentioned model balancing, which produced input-output equivalent models of the LTI systems, but obscured the explicit dynamics from the coefficients of the State Space, A, B and C matrices*. This in turn affected the gains, finally obtaining efficient controllers, but with coefficients not correlated. Nevertheless, the compensators are correlated, as can be easily seen by looking at the MBC eigenvalues in tables 5.3 and 5.4, and there are ways to uncover this correlation.

<table>
<thead>
<tr>
<th>F5RO</th>
<th>F10RO</th>
<th>F25RO</th>
<th>F50RO</th>
<th>F100RO</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.4015</td>
<td>-1.6118</td>
<td>-2.1049</td>
<td>-2.6325</td>
<td>-3.3797</td>
</tr>
<tr>
<td>-0.0099</td>
<td>-0.4674</td>
<td>-0.0031</td>
<td>-0.0065</td>
<td>-0.0050</td>
</tr>
<tr>
<td>-0.2285</td>
<td>-0.0026</td>
<td>-0.7930</td>
<td>-1.0812</td>
<td>-1.4543</td>
</tr>
<tr>
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<td>-0.7965</td>
<td>-1.0326</td>
<td>-1.4276</td>
<td>-1.3729</td>
</tr>
<tr>
<td>-0.1728</td>
<td>-0.2491</td>
<td>-0.5933</td>
<td>-0.9780</td>
<td>-2.2520</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>FSRO</th>
<th>FI0R10</th>
<th>F25R10</th>
<th>F50R10</th>
<th>F100R10</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.1163+0.3635i</td>
<td>-0.2523</td>
<td>-3.0070</td>
<td>-6.438</td>
<td>-9.254</td>
</tr>
<tr>
<td>-1.1163-0.3635i</td>
<td>-1.6103+0.3465i</td>
<td>-1.7665</td>
<td>-6.631</td>
<td>-9.046</td>
</tr>
<tr>
<td>-0.8209</td>
<td>-1.6103-0.3465i</td>
<td>-0.4535</td>
<td>-2.633</td>
<td>-3.064</td>
</tr>
<tr>
<td>-0.2183</td>
<td>-0.9713</td>
<td>-1.0652</td>
<td>-1.8202</td>
<td>-2.2980</td>
</tr>
<tr>
<td>-0.2453</td>
<td>-0.5228</td>
<td>-0.9351</td>
<td>-1.4459</td>
<td>-1.9318</td>
</tr>
</tbody>
</table>

Table 5.3 Kalman Filter Eigenvalues

<table>
<thead>
<tr>
<th>FSRO</th>
<th>F10RO</th>
<th>F25RO</th>
<th>F50RO</th>
<th>F100RO</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.2264</td>
<td>-11.0951+11.0951i</td>
<td>-10.7529+10.7529i</td>
<td>-25.7115+25.7115i</td>
<td>-32.9875+32.9875i</td>
</tr>
<tr>
<td>-3.3667+3.3667i</td>
<td>-11.0951-11.0951i</td>
<td>-10.7529-10.7529i</td>
<td>-25.7115-25.7115i</td>
<td>-32.9875-32.9875i</td>
</tr>
<tr>
<td>-3.3667-3.3667i</td>
<td>-0.4762</td>
<td>-0.8338</td>
<td>-1.0996</td>
<td>-1.3698</td>
</tr>
<tr>
<td>-3.7337+4.4503i</td>
<td>-6.2923-7.9511i</td>
<td>-4.0567-4.0567i</td>
<td>-6.7377-6.7377i</td>
<td>-6.2639-6.3787i</td>
</tr>
</tbody>
</table>

Table 5.4 Optimal Controller Eigenvalues

* State Space Matrices of the LTI models before and after balancing can be found in Appendix 3.
The sequence problem was solved converting the MBC representation from state space model to transfer function or zero-pole-gain model and backward to an appropriate transfer function realization. This process was applied and correlation clearly showed up for all models, with their zeros, poles and gains increasing or decreasing sequentially.

The data at zero rudder deflection was used for least squares approximation to obtain second order polynomial coefficients, \( C_{0ij}, C_{1ij} \) and \( C_{2ij} \). Thus, each matrix coefficient, \( X_{ij} \), can be computed as polynomial functions:

\[
X_{ij}(u) = C_{0ij} + C_{1ij}u + C_{2ij}u^2
\]  

Using equation 5.1, the Controller Coefficients were obtained for 5 models corresponding to zero rudder angle, but at the axial velocities corresponding to the models with 10 degrees rudder angle equilibrium point (second row of table 5.1). Then the difference of the coefficients, \( X_{ij} \), with 10 degrees and zero was computed, as well as the difference in equilibrium yaw rate, \( r \), with the latter set equal to zero for the straightforward motion model.

\[
\Delta X_{ij} = X_{ij}\big|_{\delta=10} - X_{ij}\big|_{\delta=0}
\]
\[
\Delta r = r\big|_{\delta=10} - r\big|_{\delta=0} = r
\]

This allowed to obtain a slope for linear interpolation at each equilibrium point \((u^*,r^*)\) as shown in equation 5.4. This slope for the five equilibrium points was obtained as a quadratic function of speed for each coefficient (equation 5.5), which allows for computing any controller coefficient as function of \( r \) for any speed \( u \). The computation of a coefficient \( X_{ij} \) starts by solving for straightforward trajectory \( X_{ij}(r=0) \) with equation 5.1, then the slope for the specific coefficient, \( m_{ij} \), at that speed is computed using equation 5.5. Finally the coefficient value is calculated using equation 5.6.
These equations were used continuously to compute the controller gains during simulations and proved faster than 2D interpolation.

The numeric data and different representations of the controller are included in Appendix 5.

5.3 Simulation with the Nonlinear Compensator

The simulations carried with the nonlinear compensator were focused in testing system stability and controller smoothness over the operating range.

The system showed immediately good response, for the whole range of speeds and yaw rates, with the exemption of yaw rate, when approaching straightforward motion. The problem is due to the low gain of the controller for small values of $r'$. The controller matrix coefficients have a linear fit with respect to yaw rate, which proved satisfactory for a wide range of $r'$ values but failed close to zero $r'$ values. One way to solve this, but costly (in computer time) is to develop more LTI models for smaller rudder angles to obtain a better fit. An alternative, which is the one used in this design, consisted of compensating externally for $r$ before its input to the compensator, to obtain a controller with larger gains. This was achieved introducing a switch in the code for values of $r$ less than 0.02 in which the values of $r$ (only for controller scheduling) are given by a sinusoidal function:

\[ r = r + r' \frac{k}{0.2} \sin(\frac{r\pi}{0.2}) \] (5.7)
5.3.1 Step Response

The controller was tested at different speeds and yaw rates for step perturbations in yaw rate and speed. Figures 5.1 to 5.6 show the system response for a yaw rate step of $r'=0.1$ at different speeds. The controller performance is very close to the linear controller, but a little slower in the axial velocity response for yaw perturbations.

Figure 5.1 Gain Scheduled Response at $u'=0.8$; Yaw Rate Step
Figure 5.2 Gain Scheduled Response at $u' = 0.8$; Yaw Rate Step
Figure 5.3 Gain Scheduled Response at $u'=0.4$; Yaw Rate Step
Figure 5.4 Gain Scheduled Response at u'=0.4; Yaw Rate Step
Figure 5.5 Gain Scheduled Response at \( u' = 0.25 \); Yaw Rate Step
Figure 5.6 Gain Scheduled Response at $u'=0.25$; Yaw Rate Step
Figure 5.7 Gain Scheduled Response to Axial Velocity Steps

Figure 5.7 shows the excellent system response to speed disturbances, with smooth control and overdamped response. Yaw rate and lateral velocity did not experience any perturbation.
5.3.2 Ramp Response

Finally, the system was tested with ramps in speed and with a combination of speed ramp and yaw rate step. The system showed good stability and command following. The response in yaw rate is slow, but acceptable in consideration of the high nonlinearities involved in the maneuver and the large deviations in both variables.

Figure 5.8 Gain Scheduled Response to Speed Ramp
Figure 5.9 Gain Scheduled Response to Speed Ramp and Yaw Rate Step
Figure 5.10 Gain Scheduled Response to Speed Ramp and Yaw Rate Step
5.4 Summary

The chapter presented the procedure used to design the nonlinear controller. Balancing the realizations for controller design hide the physical correlation of the state space matrices, requiring a reformulation and a new realization of the state space models to perform gain scheduling. A simple algorithm was developed to perform gain scheduling and was implemented using ten linear models. The system employs linear interpolation for yaw rate and a quadratic function for speed, resulting in a stable and smooth nonlinear controller. The Gain Scheduled system was tested with steps and ramps, with no apparent problems. Next step is to implement and install onboard.
Chapter 6
System Analysis of the Automated Maneuvering Surface Combatant

6.1 Introduction

This chapter is intended to give insight into the risks and benefits of including automatic control in executing maneuvering to be used for the next generation of surface combatants. In order to introduce the reader into the topic, the benefits (economical and functional) of reduced manning are presented along with important considerations with respect to automation. After the introductory sections, the technical considerations deemed important for the system, as well as some expected performance requirements are presented, and will constitute the base of the risk assessment analysis. As will be shown in the first part of this chapter, the economical and functional benefits of automation for the ship as a whole are well known. Nevertheless, the system risk assessment section explores the different functions and technologies necessary to any development project for such a system. Important conclusions arise, showing the feasibility of automated maneuvering and further reinforcing the need of more experimental evaluation of some ship dynamic characteristics.

6.2 Costs and Manning Considerations

"The bridge of the Autonomic Ship should be located with the Combat Information Center: the Command Center. In the command center of the Autonomic Ship, the functions of the thirteen people standing on the bridges of today's ships will be performed by two watchstanders."

This definition corresponds to part of the vision of the US Navy's 21st Century Surface Combatant as developed by the Naval Surface Warfare Center,
Carderock Division [Di.1]. Nowadays large manning reductions are the core requirement for all programs involving surface ship designs. These reductions have strong impact on ship design and life cycle cost. Despite the fact that LCC is difficult to compute, estimates of savings in the new generation of surfaces combatants, if expected manning reduction is achieved, run up to 11 billion dollars for the 30 years lifecycle period [An.1]. Manning reduction does not only affect costs, it also substantially influences ship characteristics and design. The reduction of onboard personnel will significantly reduce space requirement, allowing for larger payloads, or smaller ships, more maneuverable, and with reduced signature. Reduced manning also allows fewer people in harm during a conflict, which is becoming a very important factor in the type of conflicts the US Military is confronted with these days. After the cold war, a survival war changed to interests wars that are limited offensive wars. These conflicts happen overseas and usually without direct impact to US citizens; therefore, lives of US sailors are difficult to risk, and unmanned or low risk operations are preferred.

It is also true that due to the high complexity of systems and size, US surface combatants trailed other navies when focused in manning, as can be seen in figure 6.1.

**Figure 6.1 Manning for Frigates (3500~4000 tons)**

* Data from Jane Fighting Ships [Ja.1]
The prime mover in manning reduction was, is and will be cost, but it is not the only one, there are several important motivations to reduce crews, including:

- Manpower cost*  
  - 60% of US Navy budget  
  - 30% of DDG51 direct O&S cost  
  - 15% of DDG51 direct Life Cycle Cost  
- Risk of crew lives  
- Reduced reaction/response times  
- Job enrichment  

But all of this seems feasible due to the technological advances of the last years, mainly the awesome computational power of digital computers that can be installed onboard. The most important technologies that allow for the concept of an automated ship are:

- Digital Computers/CD-ROMs/Software  
- GUI's  
- Large Flat Displays  
- Expert Systems  
- Reliable Sensors  
- Fiber Optics  
- GPS  
- Electronic Charting and Navigation (ECDIS)  
- Corrosion and wear resistant coatings  
- Robotics  

Despite the advantages and technology availability, manning reduction in a surface combatant is not an easy task. Warship personnel work with a multidimensional workload resulting from:

* Cost data from "Notes from Methods in Total Ship System Design"[Ma.1], other factors based on reference [Di.1]
- Functional requirements and tasks
- Maintenance requirements
- Training requirements
- Safety concerns

Figure 6.2, obtained from reference [Ma.1], shows the Total Workload Distribution for DDG51. This chart is very illustrative with respect to the many tasks a ship’s crew has to execute.

**DDG51 Workload Distribution**

![Graph showing workload distribution for DDG51](image)

Figure 6.2 Total Workload Distribution for DDG51

This workload is usually uneven, with manning driven by the highest workload periods such as Replenishment at Sea, firefighting, etc...

There is no doubt that automation is a desirable objective with many advantages for a surface combatant, cost reduction being probably the most important among all. Nevertheless, manning reduction cannot be resolved only
with technology, it is a multidimensional problem involving human resources that requires major changes in doctrines, traditions, culture, training and procedures.

6.3 Automation Considerations

6.3.1 Warship automation

An autonomic ship can be described as a ship where the people decide *what* to do and the ship *makes it happen* [Di.1]. To achieve this objective, it is necessary a complete a re-design of the surface combatants, or more accurately a complete a re-thinking of the design process.

First of all, consider that such a ship will have some intelligence level. For example it may be designed based in a Rational Behavior scheme that has been used for control of experimental Autonomous Underwater Vehicles (AUV's)[Byr.1]. This approach considers three intelligence levels: strategic, tactical and execution levels. The strategic level involves all decisions involved with the specific mission objectives. The tactical level is the link between the former and the execution level and consists of preplanned actions based in the system state. The execution level is responsible for controlling the actual machinery, sensors, weapons, etc... For the case of ship automation, the crew will take the strategic level decisions, while the ship will automatically execute all tactic and execution level tasks.

Having in mind the objective and intelligence scheme shown above, a highly automated and low manned ship should consider meeting the following general requirements during the design process*:

- Automated ship motions control and navigation (tactical)
- Automated Command and Control (tactical level-strategic with veto)
- Combat Systems (tactical-strategic with veto)
- Engineering and Damage Control (strategic with veto)
- Design for redundancy, reliability and survivability
- Highly trained crew for flexible cross-utilization
- Extensive off-ship support

* The use of veto allows the intelligent ship to propose alternatives which may be accepted, rejected or changed by the operator.
6.3.2 Maneuvering automation

One of the mentioned areas of automation is ship motions and navigation control. Nowadays, ship piloting in surface combatants is sometimes carried on simultaneously by two teams, the Pilothouse and Combat Information Center, including manual control of the rudder, even if autopilots are available. This is mostly due to the complexity of some maneuvers (formations, replenishment at sea, evasion maneuvers, etc...) and culture. There are many examples of highly complex operations that human operators are not able to perform adequately and control systems have been designed to work automatically in a most efficient manner. The execution of these tasks by computers has not been a concern, mainly because people have never executed them or just can't perform them. It seems that ship maneuvering somehow suffers from this effect, where despite the advances in technology, automatic maneuvering has not being a goal, because in people's mind it is unsafe, despite the fact that human errors account for more than 60% of accidents at sea [Bo.1] and requires human cognitive skills.

Ship maneuvering control is a tactic level ability that the automated ship is supposed to perform itself, given certain strategic inputs. It is considered a tactic level action because it would consist mainly of preplanned maneuvers executed according to some external command from the operator or a superior level computer. The control system developed in this thesis is the link between the tactical and execution levels. It is able to receive the preprocessed desired motions and compute outputs to feed the corresponding machinery control systems.

6.4 Automated Maneuvering System

In order to analyze the risks involved in developing this project, it is necessary to first specify what the automated maneuvering system is and what it is expected to do. It is also explained what is technically necessary to accomplish the goals.

It is desirable that the automated maneuvering system be able to perform the following functions, without the help of land based navigation aids:
1. Track keeping
   The ship is required to accurately follow a track in open seas, at the desired speed and heading. This is the basic requirement and does not contemplate rapid changes in course and speed.

2. Evasion maneuvers
   The ship is required to follow a preplanned track ordered from the ship command computer due to an emergency such as a missile, torpedo or other harm. This programmed track may very often, if not always, consider rapid changes in speed and turning with large rudder angles.

3. Collision avoidance
   Collision avoidance is similar to the former case, but the command computer should order a collision avoidance maneuver that may include astern motions.

4. Station Keeping – RAS
   Consists of highly accurate maneuvering to maintain relative position with other ship, usually in the case of surface combatants during RAS. The main requirements are that smaller deviations from the required heading and speed are allowed, and that the hydrodynamic effect of the proximity of both hulls should be taken into account.

5. Restricted Waters navigation
   Very similar to the station keeping requirement, but the use of electronic charts may be beneficial. Shallow water effect on ship motions need to be considered.

6. Preplanned Evolutions
   There will be preplanned evolutions that are feasible. The ship's geometric dimensions limit the maneuvering performance, thus the use of optimized performance maneuvers avoids uncertainties.
7. Different Operational Profiles
Different preplanned maneuvering or ship controllers can be designed for different operations. It is possible to control some propeller parameters to avoid excessive noise, slow down engines for similar purposes, adjust speed for fuel consumption reduction.

8. Roll and Pitch attenuation
Roll can be attenuated with stabilizing fins or the rudder, while pitch with ship's speed and heading. Attenuation of one or both of them may be required for helicopter launching or capture, missile launching, etc..

These functions are based in different physic and engineering models. The following issues are considered of importance in the development of each function:

1. Track keeping
Mainly ship hydrodynamic coefficients and propulsion plant data. As no fast changes are expected, dynamics will be relatively slow, models are not complicated. Determination of ship coefficients is very important and can be accomplished with scale model tests.

2. Evasion maneuvers
Similar to Track keeping, but fast changes in speed and turning with large rudder deflections expected. Need better non-linear models of rudder at large angles and include engine and propeller dynamics in detail. It is complex to model and requires further development of transient models of propeller-engine dynamics.

3. Collision avoidance
This is one of the most complex cases. It may be possible to have qualitative results, but the crash astern maneuver and any model of the ship moving astern is highly complex and beyond actual practice.
4. Station Keeping – RAS
   It has similar requirements with track keeping, just a better controller is required and modeling of the ship nearby to be considered. There are solutions to this problem using potential flow theory.

5. Restricted Waters navigation
   Very similar to the station-keeping requirement, shallow water effect on ship motions can be evaluated with potential flow theory.

6. Preplanned Evolutions
   This is absolutely feasible, requires simulation of ship maneuvering

7. Different Operational Profiles
   This can be achieved in the measure that the variables desired to control can be modeled with respect to ship motions variables.

8. Roll and Pitch attenuation
   This is feasible, but still roll models are difficult due to the viscous nature of the motions dampening. Pitch is very difficult to attenuate without changing heading and speed, which usually will have priority to be kept.

The functions that the system is expected to perform were defined and very briefly presented the difficulties and capabilities in modeling and designing for this objectives. The following section will make use of a system analysis tool for Risk Assessment called Risk Rating Matrix to visualize the relative risks and importance of the different functions.
6.5 Risk Assessment

Risk Assessment is used to describe the risk in a project, program or any set of events, in a way that helps to isolate the cause and predict its impact. A Risk Event is something that could go wrong. In this case, Risk Events are all 8 functions expected from the automatic maneuvering system, described in the above section. Therefore, Risk Rating is an indication of the probability of failure of a specific Risk Event.

Table 6.1 shows point assignment basis for the different Risk Events and table 6.2 shows the basis for Risk Rating of the Risk Events.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
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<tbody>
<tr>
<td>1</td>
<td>Low</td>
</tr>
<tr>
<td>2</td>
<td>Medium</td>
</tr>
<tr>
<td>3</td>
<td>High</td>
</tr>
</tbody>
</table>

Table 6.1 Risk Event Importance in Project Outcome

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<tbody>
<tr>
<td>1</td>
<td>High</td>
</tr>
<tr>
<td>2</td>
<td>Medium</td>
</tr>
<tr>
<td>3</td>
<td>Low</td>
</tr>
</tbody>
</table>

Table 6.2 Risk Rating

According with these value scales, the following point assignment (table 6.3) was used for the different functions based on the relative importance of them for the project and the modeling difficulties (Risk Rating)

110
<table>
<thead>
<tr>
<th>Risk Event</th>
<th>Importance</th>
<th>Risk Rating</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Track keeping</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>2. Evasion maneuvers</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>3. Collision avoidance</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>4. Station keeping – RAS</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>5. Restricted Waters navigation</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>6. Preplanned Evolutions</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td>7. Different Operational profiles</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>8. Roll and Pitch Attenuation</td>
<td>1</td>
<td>2</td>
</tr>
</tbody>
</table>

Table 6.3 Points Assignment for Risk Events

The table 6.3 was filled based on the author’s discussion in section 6.4, but it may be completed in different ways. Examples of methods for completing this table are experts’ opinion, mathematical models, and random numbers, depending on the final objectives. In this case a mathematical model is not necessary to obtain the Risk Rating Matrix of table 6.4.

<table>
<thead>
<tr>
<th>High Importance</th>
<th>1</th>
<th>2</th>
<th>4</th>
<th>5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Medium</td>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low</td>
<td></td>
<td></td>
<td>7</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>Low Likelihood</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>High</td>
</tr>
</tbody>
</table>

Table 6.4 Risk Rating Matrix

According with the results of the Risk Rating Matrix, the functions to be most concerned with are evasion maneuvers, station keeping, restricted waters navigation and Collision Avoidance, all with a High importance to be achieved for
the system to be successful, but with medium to high difficulty to be achieved. It is clear that the main problems are the transient dynamics and astern motions. To have better answers to these questions, further experimental data need to be gathered. This reinforces the importance of this thesis, where an automatic controller has been designed for a scale model of DDG51 to explore the mentioned dynamics.

6.6 Summary

This chapter presented the big picture of ship automation and ship maneuvering, including cost and manning reduction issues. Current trends and automation goals were presented, alongside with proposed functions of the automatic maneuvering system. The functions of the automated maneuvering system were analyzed using a Risk Rating Matrix, identifying the most difficult aspects of developing such a system.
Chapter 7

Conclusion

7.1 Summary

This thesis presented a Robust Multivariable control design for an Arleigh Burke Class Destroyer intended to be used in a scaled autonomous model. The design comprised the following steps:

1. Selection, development and testing of a mathematical nonlinear model to describe the surge, sway, yaw and propeller-engine ship dynamics, based on geometric data of DDG51 and regression models to compute the hydrodynamic coefficients.

2. Linearization of the model about selected operating points to obtain LTI models for controller design. These models were exhaustively investigated prior to controller design, including modal analysis, singular values, pole/zero structure and Hankel singular values.

3. General system specifications were developed prior to controller design, selecting to have a rate controller for axial velocity and yaw rate with overdamped response and robust performance for low frequency disturbances and high frequency noise.

4. The LQG/LTR MIMO design methodology was introduced and applied to the LTI models obtaining 10 Model Based Compensators, which were tested to satisfaction around their design points. Nevertheless, the controllers showed poor performance far from the operating points, due to the inherent system nonlinearities.
Finally, a nonlinear MIMO controller, using gain scheduling, for the whole operating range was designed and successfully tested.

After completion of the controller design, the thesis dedicates a chapter to emphasize the importance of this study. Moreover, given the actual demand for manning reduction and automated performance, it was shown that automated maneuvering is a must for the upcoming surface combatant designs. The key functions of an automated maneuvering system, were also considered concluding that this study is concurrent with the research needs to have a fully operational system in the future.

7.2 Conclusions

Several conclusions and experiences appeared important to the author during this thesis; among them, the following are the most remarkable:

- The need of both linearized about straightforward and turning motions LTI models to adequately capture ship dynamics.
- The LQG/LTR is a very systematic methodology that allowed for computer programming of most tasks, allowing for almost effortless design repetition.
- The maximum bandwidth of the compensated system is limited by the plant’s open loop bandwidth, ensuring a realistic controller design.
- The hydrodynamic coefficients are the heart of the system model, and therefore the lack of actual values from model tests limits the confidence in the model.
- The development of this control system and simulation, including propulsion plant dynamics, helped visualize important differences with the more common maneuvering control designs without it.
- Automated ship maneuvering is a core subject in manning reduction and a must of the automated warship of the 21st century. The control design of this thesis, and autonomous scaled model construction are in the right track to investigate the transient dynamics characteristics of ships and propulsion plants required for high-performance automated maneuvering.
7.3 Recommendations for further Study

This topic is open for further design, with the following being the logical next steps in controller and simulation improvement:

- Include pitch and roll motions
- Include variable pitch propeller. This will allow the use of a third output variable
- Include trim effects
- Obtain hydrodynamic coefficients from scaled model and introduce to mathematical model
- Include transient effects in engine-propeller dynamics
- Improve the engine model using complete simulation from [PDI.1]
- Include discrete operation of engines
- Compare results of simulation with model testing, correct model and introduce model uncertainty.
- Explore the use of other MIMO techniques such as H-infinity and μ syntheses
References


[Tr.1] M.S. Triantafyllou, "Class Notes for Maneuvering and Control of Surfaces and Underwater Vehicles", MIT, 1996.


Appendix 1

Full Order Nonlinear Model

A1.1 Nonlinear Model development

The Inoue nonlinear model is based in the classic equations of motion derived by Abkowitz. The coordinate system is moving with the ship, as presented in chapter 2. A derivation of these equations can be found in many references, including SNAME’s Principles of Naval Architecture. A fourth equation has been included to allow for propeller shaft dynamics.

\[ X_H + X_P + X_R = m \left\{ \frac{du}{dt} - rv - x_G r^2 \right\} \]
\[ Y_H + Y_P + Y_R = m \left\{ \frac{dv}{dt} + ru + x_G \frac{dr}{dt} \right\} \]
\[ N_H + N_P + N_R = I_{zz} \frac{dr}{dt} + mx_G \left\{ \frac{dv}{dt} + ru \right\} \]
\[ \frac{dn_p}{dt} = \frac{Q_E + Q_P}{2\pi I_{pp}} \]

In these equations the subscripts H, P, R represent the hull forces, the propeller forces and the rudder forces respectively, and \( Q_E \) stands for the main engines torque.
A1.1.1 Hull forces

Hull hydrodynamic forces are expressed as in Inoue's model.

\[ X_H = X_0 \dot{u} + X(u) + (X_{\nu r} - Y_\nu) \dot{\nu} \]
\[ Y_H = Y_\nu \dot{v} + X_\nu u \dot{r} + \frac{1}{2} \rho L T U^2 \left\{ Y_{\beta \beta} \dot{\beta} + Y_{\beta r} \dot{r} \beta' + Y_{r \beta} \dot{\beta} + Y_{r r} \dot{r} \beta' \right\} + Y_{r r} \dot{r} \beta' \]
\[ N_H = N_r \dot{r} + \frac{1}{2} \rho L^2 T U^2 \left\{ N_{\beta \beta} \dot{\beta} + N_{\beta r} \dot{r} \beta' + N_{r \beta} \dot{\beta} + N_{r r} \dot{r} \beta' \right\} \]

The added inertia terms are calculated using slender body theory for the case of \( m_{22} \) and \( m_{66} \), and the actual value for \( m_{11} \) is obtained from the PDI Report about the DDG51 propulsion plant dynamics.

\[ X_\dot{u} = -m_{11} \]
\[ Y_\dot{v} = -m_{22} \]
\[ N_\dot{r} = -m_{66} \]

The term \( X(u) \) denotes the hull resistance, which was estimated with a 3rd order polynomial fitted by least squares method. The curve fitted was the towed resistance curve of DDG51, and when computing the forces it was accounted for the thrust deduction coefficient:

\[ X(u) = -\frac{R(u)}{(1-t)} = -\frac{(R_0 + R_1 u + R_2 u^2 + R_3 u^3)}{(1-t)} \]

The term \( X_{\nu r} + m_{22} \) accounts for increased resistance in the axial direction due to drift angle, and is estimated using Hasewaga formula, which gives the total coefficient a value of 0.66. This value falls in the range expected by Inoue Model as mentioned in his original report (range 0.50-0.75). Inoue computed the linear hydrodynamic derivatives using regression formulae. These formulas were obtained after the analysis of several full-scale trials of different ships and a wide range of hull geometries. Inoue also proposed a correction for trim that is not used in this study.
The nonlinear derivatives $-Y_{\beta\beta}$, $Y_{r_1r_1}$, $Y_{\beta r_1}$, $N_{\beta\beta}$, $N_{\beta r}$ and $N_{r_1r_1}$ were obtained by Inoue in similar way as the linear derivatives, but the data is presented in charts.

A1.1.2 Propeller Forces

Propeller forces are evaluated using the classic thrust and torque equations. As both propeller are assumed to be operating in the same conditions, the total lateral forces, $Y_P$ and propeller induced moments, $N_P$, are assumed to be zero. This model has limited value outside the first quadrant (positive $J$ and positive $K_T$) and some approximations were made with the software to avoid problems with the codes in operations out of that quadrant. It was explored to use other models, but there is not data available for the DDG51 propeller in other formats, and as the main purpose of this thesis is forward motions, it was preferred to keep the better quality of this model in the first quadrant.

$$X_P = T_P = \rho n_P^2 D^4 K_T(J)$$
$$Q_P = \rho n_P^2 D^5 K_Q(J)$$
$$J = \frac{u(1 - w_P)}{n_P D}$$
In these equations the wake fraction is corrected for turning motions, and the curves for $K_T$ and $K_Q$ were fitted from the actual propeller data by 2$^{nd}$ order polynomials. The propeller was assumed at a fixed pitch ($P/D=1.72$).

$$w_p = w_{PG}e^{-4(\beta-x_p')^2}$$

$$K_T = K_0 + K_1J + K_2J^2$$

$$K_Q = Q_0 + Q_1J + Q_2J^2$$

A1.1.3 Rudder Forces

$$X_R = -F_N \sin \delta$$

$$Y_R = -(1+a_H)F_N \cos \delta$$

$$N_R = -(1+a_H)X_R F_N \cos \delta$$

There are assumed axial forces, lateral forces and moments. All these forces are based in a force component normal to the rudder, $F_N$, that is a function of the rudder inflow velocity, $V_R$, and angle, $\alpha_R$. There is a factor $(1+a_H)$ that accounts for ship geometry based in the chart presented by Kijima. $F_N$ has a factor of two in this study to account for both DDG51 rudders.

$F_N$ includes a factor related with the rudder lift coefficient, called by some authors the normal coefficient, $C_N$, which is function of the rudder aspect ratio, $\Lambda$. The rudder inflow velocity and inflow angle are functions of lateral velocity, yaw rate and rudder angle.

$$F_N = \rho \frac{6.13\Lambda}{\Lambda + 2.22} A_R V_R^2 \sin \alpha_R$$
These equations account for the relative velocity corrections between rudder and ship coordinates. The rudder wake fraction was assumed to be the same as the propeller wake fraction. The flow rectifying coefficient, $\gamma$, is considered to be made two factors, a flow rectifying effect due to ship hull, $C_s$, and one due to the propeller, $C_p$. These factors are given in the following form, based on model experimental results.

$$C_p = \frac{1}{\sqrt{1 + 0.6\eta(2 - 1.4s)s}}$$

$$C_s = 0.45\beta_R \quad \beta_R \leq \frac{C_{SO}}{0.45}$$

$$C_s = C_{SO} \quad \beta_R > \frac{C_{SO}}{0.45}$$

$$C_{SO} = 0.5$$
A1.1.4 Engine Torque

Engine torque is modeled as a function of fuel rate and shaft revolutions. In fact it is a look-up table, where given the fuel rate and shaft revolutions per second it is possible to find the engine torque output. The coefficients were selected in order to match the LM2500 performance.

\[ Q_E = Q_M \left( (af_R + b) \frac{n_p}{n_m} + (cf_R) \right) \]

Gas turbine performance is strongly affected by several variables difficult to simulate in this model. Therefore, this simulation neglects many of the important factors in the LM2500 dynamics from the engine standpoint. Nevertheless, it offers a model feasible to simulate and accurate enough to represent the engine dynamics effect on DDG51 maneuvering, and as it can be seen in chapter three, its influence in the control system design is minimal. For a detailed model of the DDG51 engine dynamics the author recommends the PDI Report of reference [PDI.1].

A1.2 Conversion to dimensionless units

In order to ease the model analysis and allow the use of standard hydrodynamics coefficients, the equations described in section A1.1 were non-dimensionalized. One of the most important results, and one that should be kept in mind during control design is that frequencies and times are also expressed in dimensionless units, not rad/sec or seconds.

The above equations comprise forces, X and Y, moments, N, and torque, Q. This forces will be converted to dimensionless units using the following parameters:

Length: \( L \), ship's LBP (142.04 m)
Mass: \( \rho L^3 \), where \( \rho \) is saltwater density (1024.7 kg/m³)
Time: \( L/V \), where \( V \) is the ship's full power maximum speed (15.8171 m/s)
Using these basic units, the forces are non-dimensionalized:

\[
X', Y' = \frac{(X,Y)}{\rho L^2 V^2} \\
N' = \frac{N}{\rho L^3 V^2} \\
Q' = \frac{Q}{\rho L^3 V^2}
\]

The following is a list of all non-dimensional units used for the nonlinear simulation equations.

a. Inputs

Both inputs, rudder deflection \( \delta \) and fuel ratio \( f_R \) are non-dimensional, \( \delta \) is in radians and the later is the actual fuel rate to maximum fuel rate ratio.

b. Variables

\[
u' = \frac{u}{V} \quad \dot{u}' = \frac{\dot{u}L}{V^2} \quad v' = \frac{v}{V} \quad \dot{v}' = \frac{\dot{v}L}{V^2} \\
r' = \frac{rL}{V} \quad \dot{r}' = \frac{\dot{r}L^2}{V^2} \quad n'_p = \frac{n_pL}{V} \quad \dot{n}'_p = \frac{\dot{n}_pL^2}{V^2}
\]

c. Mass and inertia terms

\[
m' = \frac{m}{\rho L^3} \quad m'_{11} = \frac{m_{11}}{\rho L^3} \quad m'_{22} = \frac{m_{22}}{\rho L^3} \\
I'_{zz} = \frac{I_{zz}}{\rho L^5} \quad m'_{66} = \frac{m_{66}}{\rho L^5} \quad I'_{PP} = \frac{I_{PP}}{\rho L^5}
\]

d. Surge equation

Drag

\[
X'(u) = \frac{X(u)}{\rho L^2 V^2} = -\frac{1}{(1-t)} \left\{ \frac{R_0}{\rho L^2 V^2} + \frac{R_1 u'}{\rho L^2 V} + \frac{R_2 u'^2}{\rho L^2} + \frac{R_3 u'^3 V}{\rho L^2} \right\}
\]
Propeller thrust

\[ X'_p = \frac{X_p}{\rho L^2 V^2} = n'_p^2 D'^4 K_T(J) \]

In these equations one must be careful to compute \( J \) with dimensionless numbers:

\[ J = \frac{u'(1 - w_p)}{n'_p D'} \]

And for the corrected wake fraction it must be noted that \( r' \) in Inoue's formulae is related to \( U \), which is a varying velocity, so when using dimensionless units, \( w_p \) should be computed with \( r' \) as:

\[ r' = \frac{rL}{U} = \frac{L}{U'} \frac{r'V}{L} = \frac{r'V}{U'} = \frac{r'}{U'} \]

Rudder forces

\[ X'_R = -\frac{F_n \sin \delta}{\rho L^2 V^2} = -F'_n \sin \delta \]

\[ F'_n = \frac{\rho C_N A R V'^2 \sin \alpha_R}{\rho L^2 V^2} = C_N A'_R V'^2 \sin \alpha_R \]

With the following considerations:

\[ V'_R = U'(1 - w_R)\sqrt{1 + g(s)} \]

\[ s = 1 - \frac{u'(1 - w_p)}{n'_p P'} \]
e. Sway equation

Hull forces (other than inertial)

\[
Y'_H = \frac{Y_H}{\rho L^2 V^2} = \frac{1}{2} \rho L T U^2 \left\{ Y_{\beta}' \beta + Y_r' r' + Y_{\beta r}' \beta r' \right\} + \frac{Y_{\beta r} \beta r'}{\rho L^2 V^2}
\]

\[
Y'_H = \frac{1}{2} \frac{T U'^2}{L} \left\{ Y_{\beta}' \beta + Y_r' \frac{r'}{U'} + Y_{\beta r}' \beta \frac{r'}{U'} + Y_{r r}' \frac{r'}{U'} + Y_{\beta r} \beta \frac{r'}{U'} \right\}
\]

In this equation it is important to visualize the correction of the non-dimensional values for yaw rate in Inoue's formula to fit the definition in use in the model.

Rudder Forces

Non-dimensionalization of rudder forces is as in the surge equation

f. Yaw equation

Hull forces (other than inertial)

\[
N'_H = \frac{N_H}{\rho L^3 V^2} = \frac{1}{2} \rho L^2 T U^2 \left\{ N_{\beta}' \beta + N_r' r' + N_{\beta r}' \beta^2 r' \right\} + \frac{N_{r r} \beta^2 r'^2}{\rho L^2 V^2}
\]

\[
N'_H = \frac{1}{2} \frac{T U'^2}{L} \left\{ N_{\beta}' \beta + N_r' \frac{r'}{U'} + N_{\beta r}' \beta^2 \frac{r'}{U'} + N_{r r}' \frac{r'^2}{U'^2} \right\}
\]

Rudder Forces

\[
N'_R = -(1 + a_H) x'_R F'_n \cos \delta
\]

g. Propeller revolutions equation

Propeller torque

\[
Q'_p = \frac{Q_p}{\rho L^3 V^2} = n_p^2 D^{15} K_T (J)
\]
Engine Torque

\[ Q'_E = \frac{1}{\rho L^3 V^2} Q_M \left\{ \left( a f_R + b \right) \frac{n_p \lambda}{n_M} + (c f_R) \right\} \]

\[ Q'_E = \frac{1}{\rho L^3 V^2} Q_M a f_R \left( \frac{\lambda n_p V}{n_M L} \right) + \frac{1}{\rho L^3 V^2} Q_M b \left( \frac{\lambda n_p V}{n_M L} \right) + \frac{1}{\rho L^3 V^2} Q_M c f_R \]

\[ Q'_E = \frac{Q_M a \lambda}{n_M \rho L^4 V} f_R n_p' + \frac{Q_M b \lambda}{n_M \rho L^4 V} n_p' + \frac{Q_M c}{\rho L^3 V^2} f_R \]
Appendix 2

LTI Models

The following models were derived from the nonlinear model presented in chapter 2 and Appendix 1. The linear time-invariant systems correspond to incremental models about an operating point. This operating point corresponds to the system response to a constant input vector $u^*$ that forces the state vector to an equilibrium state $x^*$. The equations are in state space form, where the $C$ matrix is assumed to be arranged to provide the full state vector as system output ($I_{4x4}$), and $D$ is a 4x2 matrix of zeros for all models.

A2.1 Straightforward Motions

<table>
<thead>
<tr>
<th></th>
<th>A</th>
<th>B</th>
<th>$u^*$</th>
<th>$x^*$</th>
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<td>F5R0</td>
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<td>0 0.0000</td>
<td>0.0500</td>
<td>0.1653</td>
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<td>-0.0001</td>
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### A2.2 Turning Motions

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<tr>
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<td>-0.9266</td>
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<tr>
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<tr>
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</table>

**Values:**

- **x**: -0.1449, 0.1042, -0.0351, 0.0074...
- **y**: 0.0538, -0.0970, 0.0075...
- **z**: -0.0148, 0.0352, 0.1673...
- **r**: 426.4669, 406.3216, 331.1554...
- **t**: 0.6500, 0.1745, 0.1745...
- **u**: 0.1213, -0.0242, 0.0537...

**Unit:**

- **m**: 1.0257
- **cm**: 0.0443
- **mm**: 0.0003
- **°**: 154.4853
- **°**: 100.0000
- **°**: 180.0000
- **°**: 216.5897
- **°**: 1.3773
- **°**: 1.3773

**Additional Values:**

- **F25R10**: 0.3692, 0.3388, -0.1148, 0.0185
- **F50R10**: -0.5994, 0.4743, -0.1606, 0.0260
- **F100R10**: -0.9266, 0.6252, -0.2117, 0.0353
Appendix 3

LTI Models Analysis

The following is the most important data concerning modal analysis and balancing of the state space realization of the LTI as described in Chapter 3. Results were obtained with functions of the Matlab® Control Toolbox. For further detail in the codes used see Appendix 6.

F5RO

Model Eigenvalues
-0.0858  0  0  0
0  -9.2354  0  0
0  0  -0.0005  0
0  0  0  -0.3750

Transmission Zeros
-0.0005
-0.3750
-0.3750
-0.0005
-9.1844
-9.2354
-0.0858
-0.2264

Balanced Realization
ab =
-0.0005  0.0000  -0.0009  0.0000
0.0000  -0.0828  0.0000  0.1671
-0.0009  0.0000  -0.3750  0.0000
0.0000  -0.1671  0.0000  -9.2385
bb =
0.0000 -0.3701
-0.5918 0.0000
0.0000 -0.2987
-0.5918 0.0000

cb =
0.0000 -0.5918 0.0000 0.5918
-0.3701 0.0000 -0.2987 0.0000

F10R0

Model Eigenvalues

-13.2803 0 0 0
0 -0.1259 0 0
0 0 -0.0016 0
0 0 0 -0.7887

Transmission Zeros

-0.0016
-0.7887
-26.1125
-0.6457
-13.1840
-23.3603
-0.2837
-13.2803
-0.1259
-0.4762

Balanced Realization

ab =

-0.0016 0.0000 0.0026 0.0000
0.0000 -0.1214 0.0000 0.2450
0.0026 0.0002 -0.7887 0.0000
0.0000 -0.2450 -0.0005 -13.2848

bb =

0.0000 -0.6867
-0.5967 -0.0007
-0.0002 0.5535
-0.5967 0.0000

cb =

0.0006 -0.5967 0.0000 0.5967
-0.6867 -0.0002 0.5535 0.0000
F25RO

Model Eigenvalues
-21.2813 0 0 0
0 -0.3069 0 0
0 0 -1.3809 0
0 0 0 -0.0019

Transmission Zeros
-0.0019
-1.3809
-1.3173
0.3886
-21.1087
1.0e+004 *
-21.2813
-0.3069
-0.8338

Balanced Realization
ab =
-0.0019 0.0000 -0.0030 0.0000
0.0000 -0.2904 0.0000 0.5887
-0.0030 0.0000 -1.3809 0.0000
0.0000 -0.5887 0.0000 -21.2978

bb =
0.0000 1.1817
-0.5731 0.0000
0.0000 0.9540
-0.5731 0.0000

cb =
0.0000 -0.5731 0.0000 0.5731
1.1817 0.0000 0.9540 0.0000

F50RO

Model Eigenvalues
-0.5414 0 0 0
0 -29.8496 0 0
0 0 -1.8211 0
0 0 0 0 -0.0039

Transmission Zeros
-0.0039
-1.8211
-1.4704
-29.6156
-29.8496
-0.5414
-1.0996
Balanced Realization

\[ a_\beta = \begin{bmatrix} -0.0039 & -0.0063 & 0.0000 & 0.0000 \\ -0.0063 & -1.8210 & 0.0049 & -0.0039 \\ 0.0000 & 0.0074 & -0.5054 & -1.0280 \\ 0.0000 & 0.0026 & 1.0280 & -29.8857 \end{bmatrix} \]

\[ b_\beta = \begin{bmatrix} 0.0000 & -1.5916 \\ -0.0020 & -1.2823 \\ -0.5377 & 0.0045 \\ 0.5377 & 0.0001 \end{bmatrix} \]

\[ c_\beta = \begin{bmatrix} 0.0011 & -0.0033 & -0.5377 & -0.5377 \\ -1.5916 & -1.2823 & 0.0048 & 0.0000 \end{bmatrix} \]

**F100R0**

Model Eigenvalues

\[ \begin{bmatrix} -0.8752 & 0 & 0 & 0 \\ 0 & -41.6968 & 0 & 0 \\ 0 & 0 & -2.2687 & 0 \\ 0 & 0 & 0 & -0.0030 \end{bmatrix} \]

Transmission Zeros

\[ \begin{bmatrix} -0.0030 \\ -2.2687 \\ -2.2687 \\ -0.0030 \\ -41.4029 \\ -41.6968 \\ -0.8752 \\ -1.3698 \end{bmatrix} \]

Balanced Realization

\[ a_\beta = \begin{bmatrix} -0.0031 & 0.0049 & 0.0000 & 0.0000 \\ 0.0049 & -2.2687 & 0.0000 & 0.0000 \\ 0.0000 & -0.0001 & -0.8087 & 1.6493 \\ 0.0000 & 0.0000 & -1.6493 & -41.7633 \end{bmatrix} \]

\[ b_\beta = \begin{bmatrix} 0.0000 & -2.0404 \\ 0.0000 & 1.6474 \\ 0.4883 & 0.0000 \\ 0.4883 & 0.0000 \end{bmatrix} \]

\[ c_\beta = \begin{bmatrix} 0.0000 & 0.0000 & 0.4883 & -0.4883 \\ -2.0404 & 1.6474 & 0.0000 & 0.0000 \end{bmatrix} \]
**F5R10**

Model Eigenvalues
-9.2210 0 0 0
0 -0.1256 0 0
0 0 -1.0242 0
0 0 0 -0.3314

Transmission Zeros
-0.3305
-0.9929
-1.6309
-0.3965
-9.1765
-0.3452 + 0.0290i
-0.3452 - 0.0290i
-9.2245
-0.0623
-0.2827

Balanced Realization

\[ \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} -0.1301 & -0.0245 & -0.2527 & 0.0862 \\ -0.1236 & -0.7967 & -1.3859 & 0.3669 \\ 0.2666 & 1.5444 & -7.7757 & 3.6623 \\ -0.0889 & -0.5601 & 3.5395 & -1.9997 \end{bmatrix} \]

\[ \begin{bmatrix} b \\ a \end{bmatrix} = \begin{bmatrix} 0.6054 & -0.0250 \\ 0.3065 & 0.3276 \\ -0.6079 & 0.0754 \\ 0.2060 & 0.0113 \end{bmatrix} \]

\[ \begin{bmatrix} c \\ b \end{bmatrix} = \begin{bmatrix} 0.5554 & -0.1003 & 0.4480 & -0.1608 \\ 0.2422 & 0.4373 & 0.4179 & -0.1292 \end{bmatrix} \]

**F10R10**

Model Eigenvalues
-13.0154 0 0 0
0 -0.1608 0 0
0 0 -1.7547 0
0 0 0 -0.6176

Transmission Zeros
-1.6842
-0.6086
-2.8262
-0.7473
-12.9370
-0.5474 + 0.0886i
-0.5474 - 0.0886i
-13.0225  -0.0631  -0.5458

Balanced Realization
ab =
-0.1625  -0.1145  -0.3078  0.1494
-0.0768  -1.6573  -2.0569  1.0261
0.3341   1.2459  -8.9677  6.2138
-0.1672  -0.2754  5.8705  -4.7610

bb =
0.5950  -0.0967
0.2844   0.6719
-0.6128   0.0258
0.2993  -0.0761
cb =
0.5644  -0.0247   0.4355  -0.2070
0.2116   0.7292   0.4319  -0.2292

F25R10

Model Eigenvalues
-20.7372  0 0 0
  0  -0.3056  0 0
  0  0  -3.1224  0
  0  0  0  -1.1328

Transmission Zeros
-2.9818
-1.1105
-5.0431
-1.3831
-20.5951
-0.9705+ 0.1177i
-0.9705- 0.1177i
-20.7509
-0.1334
-1.0237

Balanced Realization
ab =
-0.2552   0.2208  -0.2478   0.3185
-0.4976  -3.2882   1.4701  -2.3768
0.1855  -1.1961  -4.2257   9.3065
-0.4391   0.2666   5.9388  -17.5288

bb =
0.4925  -0.3497
-0.2690  -1.2411
-0.3584  -0.1380
0.5683  -0.0886
cb =
  0.6041  -0.0627    0.2885  -0.3713
-0.0017  -1.2684    0.2535  -0.4392

F5OR10

Model Eigenvalues
-29.3025    0    0    0
  0  -0.5154    0    0
  0    0  -4.3720    0
  0    0    0  -1.5814

Transmission Zeros
-4.1773
-1.5540
-7.0630
-1.9344
-29.1040
-1.4011 + 0.1750i
-1.4011 - 0.1750i
-29.3217
-0.2677
-1.4324

Balanced Realization
ab =
-0.5361  -0.1249  -0.1166    0.1228
-1.7666  -4.5542  1.0048  -2.7073
-0.1636  -1.3000  -1.8721    7.9851
-0.3622    0.4008  1.9052  -28.8089
bb =
  0.3333    -0.7569
-0.2705  -1.6592
-0.1521    -0.2076
  0.6107    -0.0352
cb =
  0.6966  -0.1693    0.2079  -0.4034
-0.4459  -1.6726    0.1516  -0.4598

F10OR10

Model Eigenvalues
-41.2714    0    0    0
  0    0.8299    0    0
  0    0  -5.7874    0
  0    0    0  -2.0683
Transmission Zeros
-5.5409
-2.0429
-9.3466
-2.5310
-41.0159
-1.9292+ 0.2341i
-1.9292- 0.2341i
-41.2956
-0.4890
-1.8702

Balanced Realization
ab =
\[
\begin{bmatrix}
-1.3881 & -0.8338 & 0.0075 & -0.2064 \\
-3.5472 & -5.4856 & 1.0759 & -2.5838 \\
-0.4904 & -1.4354 & -1.8062 & 7.7981 \\
-0.1752 & 0.3249 & 0.0961 & -41.2771 \\
\end{bmatrix}
\]

bb =
\[
\begin{bmatrix}
0.1872 & -1.3428 \\
-0.2246 & -2.0125 \\
-0.0602 & -0.2501 \\
0.5670 & -0.0079 \\
\end{bmatrix}
\]

cb =
\[
\begin{bmatrix}
0.8229 & -0.3335 & 0.2062 & -0.3819 \\
-1.0776 & -1.9973 & 0.1537 & -0.4191 \\
\end{bmatrix}
\]
Open Loop Singular Values

LTI Model F25R0

Open Loop Singular Values

LTI Model F50R0
Open Loop Singular Values

LTI Model FI00R0
LTI Model F5R10

LTI Model F10R10
Open Loop Singular Values

LTI Model F25R10

Open Loop Singular Values

LTI Model F50R10
Open Loop Singular Values

Singular Values (dB)

Frequency (rads*L/V)

LTI Model F100R10
### Appendix 4
### LQR/LTR Design Data

#### A4.1 L matrices

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#### A4.2 Kalman Filter Gains H

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### A4.3 Optimal Controller Gains G

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### A4.4 Kalman Filter Poles (A-HC)

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### A4.5 Optimal Controller Poles (A-BG)

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<td>-10.7529+10.7247i</td>
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<td>-32.9875+32.9627i</td>
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<td>-3.3667+3.3600i</td>
<td>-11.0951-11.0862i</td>
<td>-10.7529-10.7247i</td>
<td>-25.7115-25.6910i</td>
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A4.7 Singular Values of $G(s)K(s)$

LTI Model F5R0

LTI Model F10R0
Singular Values of $G(s)K(s)$ after LTR

LTI Model F25R0

Singular Values of $G(s)K(s)$ after LTR

LTI Model F50R0
Singular Values of $G(s)K(s)$ after LTR

LTI Model F100R0

Singular Values of $G(s)K(s)$ after LTR

LTI Model F5R10
Singular Values of $G(s)K(s)$ after LTR

LTI Model F10R10

Singular Values of $G(s)K(s)$ after LTR

LTI Model F25R10
Singular Values of $G(s)K(s)$ after LTR

LTI Model F50R10

Singular Values of $G(s)K(s)$ after LTR

LTI Model F100R10
Appendix 5
Gain Scheduling Data

A5.1 Transfer Matrices

The system in state space form

\[ \dot{x} = Ax + Bu \]
\[ y = Cx \]

was converted to the transfer matrix representation

\[ H(s) = \begin{bmatrix} H_{11}(s) & H_{12}(s) \\ H_{21}(s) & H_{22}(s) \end{bmatrix} \]

using the Matlab command \texttt{zpk} that executes the transformation:

\[ H(s) = C(sI - A)^{-1}B \]

Each transfer function \( H_{ij}(s) \) can be factorized into the zero, pole, gain form:
with \( n > n \) for the transfer function to be strictly proper. \( z_i \) are the zeros, \( p_i \) are the poles and \( k \) is the gain of \( H_{ij}(s) \).

**F5R0 LTI Model**

Zero/pole/gain from input 1 to output...

\[
\begin{align*}
H_{ij}(s) &= \frac{k(s - z_1)(s - z_2)\cdots(s - z_m)}{(s - p_1)(s - p_2)\cdots(s - p_n)}
\end{align*}
\]

81.2123 \((s+9.177)\ (s+0.1504)\ (s+0.2264)\ (s^2 + 7.989s + 30.93)\)

#1:  
\((s+0.2264)\ (s+10.34)\ (s^2 + 7.989s + 30.93)\ (s^2 + 7.82s + 40.6)\)

-0.00014792 \((s+7.644)\ (s+1.779)\ (s+0.001684)\ (s+0.1363)\)

#2:  
\((s+0.2264)\ (s+10.34)\ (s^2 + 7.989s + 30.93)\ (s^2 + 7.82s + 40.6)\)

**F10R0 LTI Model**

Zero/pole/gain from input 1 to output...

94.9633 \((s+0.2102)\ (s+13.25)\ (s+0.4762)\ (s^2 + 13.78s + 93.66)\)

#1:  
\((s+0.4762)\ (s+13.89)\ (s^2 + 8.67s + 44.87)\ (s^2 + 13.78s + 93.66)\)

-0.028585 \((s+13.52)\ (s+1.348)\ (s-0.001332)\ (s^2 + 7.872s + 34.37)\)

#2:  
\((s+0.4762)\ (s+13.89)\ (s^2 + 8.67s + 44.87)\ (s^2 + 13.78s + 93.66)\)

Zero/pole/gain from input 2 to output...

-0.023278 \((s+13.47)\ (s-0.8942)\ (s+0.5021)\ (s^2 + 12.59s + 78.07)\)

#1:  
\((s+0.4762)\ (s+13.89)\ (s^2 + 8.67s + 44.87)\ (s^2 + 13.78s + 93.66)\)
125.1959 \( (s+13.89) \) \( (s+0.002611) \) \( (s+0.6025) \) \( (s^2 + 8.67s + 44.87) \)

\#2:  
\[ (s+0.4762) (s+13.89) (s^2 + 8.67s + 44.87) (s^2 + 13.78s + 93.66) \]

F25RO

Zero/pole/gain from input 1 to output...

137.8937 \( (s+0.4454) \) \( (s+21.27) \) \( (s+0.8338) \) \( (s^2 + 23.02s + 262.9) \)

\#1:  
\[ (s+0.8338) (s+21.53) (s^2 + 9.28s + 46.9) (s^2 + 23.02s + 262.9) \]

\[ 0.00065181 (s+21.7) (s+2.601) (s-0.002634) (s^2 + 8.395s + 51.71) \]

\#2:  
\[ (s+0.8338) (s+21.53) (s^2 + 9.28s + 46.9) (s^2 + 23.02s + 262.9) \]

F50RO

Zero/pole/gain from input 1 to output...

196.9931 \( (s+0.7097) \) \( (s+29.84) \) \( (s+1.1) \) \( (s^2 + 30.84s + 472) \)

\#1:  
\[ (s+1.1) (s+29.99) (s^2 + 9.527s + 47.12) (s^2 + 30.84s + 472) \]

\[ -0.31303 (s+3.059) (s-0.004323) (s+29.87) (s^2 + 7.622s + 29.41) \]

\#2:  
\[ (s+1.1) (s+29.99) (s^2 + 9.527s + 47.12) (s^2 + 30.84s + 472) \]

Zero/pole/gain from input 2 to output...

-0.28281 \( (s+29.88) \) \( (s+0.3848) \) \( (s+1.175) \) \( (s^2 + 28.97s + 418.1) \)

\#1:  
\[ (s+1.1) (s+29.99) (s^2 + 9.527s + 47.12) (s^2 + 30.84s + 472) \]

\[ 185.6636 (s+29.99) (s+0.006466) (s+1.536) (s^2 + 9.527s + 47.12) \]

\#2:  
\[ (s+1.1) (s+29.99) (s^2 + 9.527s + 47.12) (s^2 + 30.84s + 472) \]
F100R0
Zero/pole/gain from input 1 to output...

\[ 293.4906 \left( s+1.054 \right) \left( s+41.69 \right) \left( s+1.37 \right) \left( s^2 + 39.66s + 780.6 \right) \]

\#1: 
\[ \left( s+1.37 \right) \left( s+41.79 \right) \left( s^2 + 9.837s + 47.33 \right) \left( s^2 + 39.66s + 780.6 \right) \]

\[ 0.0040172 \left( s+41.75 \right) \left( s+3.515 \right) \left( s-0.003999 \right) \left( s^2 + 6.942s + 28.68 \right) \]

\#2: 
\[ \left( s+1.37 \right) \left( s+41.79 \right) \left( s^2 + 9.837s + 47.33 \right) \left( s^2 + 39.66s + 780.6 \right) \]

Zero/pole/gain from input 2 to output...

\[ 0.0057356 \left( s+41.72 \right) \left( s+0.8209 \right) \left( s+1.656 \right) \left( s^2 + 37.63s + 704.3 \right) \]

\#1: 
\[ \left( s+1.37 \right) \left( s+41.79 \right) \left( s^2 + 9.837s + 47.33 \right) \left( s^2 + 39.66s + 780.6 \right) \]

\[ 248.1621 \left( s+41.79 \right) \left( s+0.005018 \right) \left( s+1.958 \right) \left( s^2 + 9.837s + 47.33 \right) \]

\#2: 
\[ \left( s+1.37 \right) \left( s+41.79 \right) \left( s^2 + 9.837s + 47.33 \right) \left( s^2 + 39.66s + 780.6 \right) \]

F5R10
Zero/pole/gain from input 1 to output...

\[ 67.6698 \left( s+9.124 \right) \left( s+0.137 \right) \left( s+0.2586 \right) \left( s^2 + 7.831s + 29.25 \right) \]

\#1: 
\[ \left( s+10.74 \right) \left( s+0.2723 \right) \left( s^2 + 6.217s + 18.6 \right) \left( s^2 + 8.892s + 53.43 \right) \]

\[ -53.079 \left( s+11.47 \right) \left( s+0.438 \right) \left( s+0.2427 \right) \left( s^2 + 9.292s + 64.6 \right) \]

\#2: 
\[ \left( s+10.74 \right) \left( s+0.2723 \right) \left( s^2 + 6.217s + 18.6 \right) \left( s^2 + 8.892s + 53.43 \right) \]

Zero/pole/gain from input 2 to output...

\[ 43.2742 \left( s+9.208 \right) \left( s+3.761 \right) \left( s+0.2902 \right) \left( s^2 + 0.7842s + 3.255 \right) \]

\#1: 
\[ \left( s+10.74 \right) \left( s+0.2723 \right) \left( s^2 + 6.217s + 18.6 \right) \left( s^2 + 8.892s + 53.43 \right) \]

\[ 117.5772 \left( s+10.45 \right) \left( s+1.022 \right) \left( s+0.2839 \right) \left( s^2 + 8.214s + 45.56 \right) \]

\#2: 
\[ \left( s+10.74 \right) \left( s+0.2723 \right) \left( s^2 + 6.217s + 18.6 \right) \left( s^2 + 8.892s + 53.43 \right) \]
F10R10
Zero/pole/gain from input 1 to output...
90.2977 (s+12.95) (s+0.133) (s^2 + 12.39s + 72.14)
#1: -----------------------------------------------
(s+14.01) (s+0.5356) (s^2 + 8.453s + 37.94) (s^2 + 11.99s + 79.61)

-50.5591 (s+15.38) (s+0.602) (s^2 + 11.19s + 97.54)
#2: ---------------------------------------------------
(s+14.01) (s+0.5356) (s^2 + 8.453s + 37.94) (s^2 + 11.99s + 79.61)

Zero/pole/gain from input 2 to output...
40.4179 (s+13.02) (s+4.955) (s+0.6025) (s^2 + 1.929s + 14.28)
#1: -----------------------------------------------
(s+14.01) (s+0.5356) (s^2 + 8.453s + 37.94) (s^2 + 11.99s + 79.61)

139.8461 (s+13.68) (s+0.5678) (s+1.625) (s^2 + 9.162s + 49.51)
#2: ---------------------------------------------------
(s+14.01) (s+0.5356) (s^2 + 8.453s + 37.94) (s^2 + 11.99s + 79.61)

F25R10
Zero/pole/gain from input 1 to output...
111.1942 (s+20.71) (s+0.2071) (s+1.017) (s^2 + 20.18s + 193.5)
#1: -----------------------------------------------
(s+21.09) (s+1.009) (s^2 + 9.255s + 46.29) (s^2 + 19.7s + 190.9)

-40.8689 (s+22.76) (s^2 + 1.974s + 0.9811) (s^2 + 12.71s + 143)
#2: ---------------------------------------------------
(s+21.09) (s+1.009) (s^2 + 9.255s + 46.29) (s^2 + 19.7s + 190.9)

Zero/pole/gain from input 2 to output...
33.5522 (s+20.75) (s+5.282) (s+1.182) (s^2 + 9.539s + 90.22)
#1: -----------------------------------------------
(s+21.09) (s+1.009) (s^2 + 9.255s + 46.29) (s^2 + 19.7s + 190.9)

149.2916 (s+20.91) (s+1.07) (s+2.916) (s^2 + 9.643s + 46.44)
#2: ---------------------------------------------------
(s+21.09) (s+1.009) (s^2 + 9.255s + 46.29) (s^2 + 19.7s + 190.9)

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F50R10
Zero/pole/gain from input 1 to output...
   190.5599 (s+1.436) (s+29.29) (s+0.3105) (s^2 + 28.12s + 378.2)
#1: ---------------------------------------------------------------
   (s+29.5) (s+1.41) [s^2 + 9.845s + 50.06] (s^2 + 27.68s + 372.2)

-61.0106 (s+30.79) [s^2 + 2.863s + 2.079] [s^2 + 13.23s + 159.4]
#2: ---------------------------------------------------------------
   (s+29.5) (s+1.41) [s^2 + 9.845s + 50.06] (s^2 + 27.68s + 372.2)

Zero/pole/gain from input 2 to output...
   55.6008 (s+29.31) (s+1.701) (s+5.288) (s^2 + 20.18s + 258.2)
#1: ---------------------------------------------------------------
   (s+29.5) (s+1.41) (s^2 + 9.845s + 50.06) (s^2 + 27.68s + 372.2)

   228.02 (s+29.38) (s+1.507) (s+4.281) (s^2 + 10.05s + 46.75)
#2: ---------------------------------------------------------------
   (s+29.5) (s+1.41) [s^2 + 9.845s + 50.06] (s^2 + 27.68s + 372.2)

F100R10
Zero/pole/gain from input 1 to output...
   238.0425 (s+0.4674) (s+41.27) (s+1.883) (s^2 + 36.66s + 646.4)
#1: ---------------------------------------------------------------
   (s+41.36) (s+1.834) [s^2 + 9.853s + 47.29] [s^2 + 36.22s + 636.3]

   -78.9197 (s+42.01) [s^2 + 3.919s + 3.901] [s^2 + 13.21s + 137.8]
#2: ---------------------------------------------------------------
   (s+41.36) (s+1.834) [s^2 + 9.853s + 47.29] [s^2 + 36.22s + 636.3]

Zero/pole/gain from input 2 to output...
   79.7796 (s+41.27) (s+2.283) (s+5.278) (s^2 + 31.02s + 518.8)
#1: ---------------------------------------------------------------
   (s+41.36) (s+1.834) [s^2 + 9.853s + 47.29] [s^2 + 36.22s + 636.3]

   261.6077 (s+41.29) (s+1.985) (s+6.038) (s^2 + 9.732s + 41.75)
#2: ---------------------------------------------------------------
   (s+41.36) (s+1.834) [s^2 + 9.853s + 47.29] [s^2 + 36.22s + 636.3]